

**MACRODYNAMIC EFFECTS OF EFFICIENCY  
WAGES AND WAGE DISCRIMINATION POLICIES  
MODELED WITH HABIT FORMATION IN  
CONSUMPTION**

by

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## ABSTRACT

This dissertation consists of two independent chapters, and an introductory first chapter with background and summary. While they have some theoretical and analytical tools in common, the models are independent of each other and each chapter can be read and understood by itself and makes its own contribution to the subject studied.

Chapter one reviews a segment of the literature on real business cycle research, focusing first on the modeling of efficiency wages to create equilibrium unemployment, then on the use habit formation in consumption to capture some dynamic features of macroeconomic variables. My innovation in the chapters that follow is adding habit formation in consumption to a shirking efficiency wage model, and applying DSGE techniques to create a laboratory where questions of the dynamic effects of altering model parameters can be addressed by Impulse response and simulations that include stochastic shocks to the economy. This chapter includes information about the differences between two models that are studied in chapters two and three, which are theoretical differences in how the habit formation is modeled, and applied to different questions.

Chapter two constructs an equilibrium model that combines external habit formation in consumption and efficiency wages arising from imperfectly observable effort to evaluate wage, employment, and output dynamics under fiscal and technology shocks. At certain levels of insurance and habit formation employment-output correlations and output volatilities match US data better than a model without habit formation. However, increased employment volatility and counter-

factual negative wage-employment correlations emerge. I use impulse response functions to explain the mechanisms that give rise to the observed changes in second moments.

Chapter three builds on the result by using a similar analytical framework to pose a policy question. Comparing results from two models, one where a wage gap arises from heterogeneous worker history, and another where an equal wage is enforced, impulse response experiments compare the respective welfare costs of a negative shocks to technology in either case. The welfare cost of a recession caused by a negative technology shock of one standard deviation in this simulation, when wage equality is enforced, as compared to when employers are allowed to negotiate wages with individuals based on past employment status, is about 1.0% per year for 45 years, if expressed as a compensating variation in consumption, due to a deeper and more persistent drop in employment before a return to steady state growth.

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## CHAPTER ONE: INTRODUCTION

This dissertation consists of two independent chapters, and an introductory first chapter with background and summary. Similar tools are used but the models are different, making different theoretical claims about habit formation, and applied differently to gain insight into the dynamics of the artificial economy created. Each chapter stands alone. For instance, chapter three represents a complete model and research lab, used to perform experiments to shed light on real policy questions. Measures of the welfare cost of wage discrimination prohibitions are developed and significant results thus expressed. Skipping chapters one or two would not make three any less understandable.

The purpose of this chapter is to provide some background in the literature that I build on in chapters two and three. Also, here is some discussion of theoretical claims I make about habit formation, comments about my experimental method and the value of using DSGE techniques to analyze this type of model. I will give a brief comparison and contrast of chapters two and three to provide understanding of how they differ in their theoretical claims and questions asked, and what tools and techniques they have in common.

The basic RBC Model of Kydland and Prescott (1982) predicts a steady-state involuntary employment rate of 0%. Variation in employment is on the intensive margin, the representative agent's leisure-labor decision. Hansen (1985) makes a model where employment can vary on the extensive margin, which is closer to the observed reality in labor markets. Shapiro and Stiglitz (1984) derive equilibrium involuntary unemployment from a shirking efficiency wage. Employers pay a wage high enough to make the risk of the cost unemployment for observed shirking high enough to get full effort from workers.

Alexopoulos (2004) innovates upon Shapiro and Stiglitz (1984) by adding a partial wage insurance so that the worker's decision accounts for the expected consumption to be received if shirking is detected. Again the employer chooses some unemployment and an efficiency wage. Givens (2008) makes the insurance

level a parameter that can be varied in the model. The Givens model is where I start in chapter two, with the case that is identical to that in Alexopoulos (2004).

My innovation is to add habit formation in consumption. Habit formation in consumption has been used to better explain some feature of macro data. For instance in Constantinides (1990) and Fuhrer (2000) and Gruber (1997). Adding it to an environment with steady-state unemployment in a model that gives me the capability of varying habit formation and insurance levels makes for the laboratory I use in chapter two. In chapter three I model habit formation in a different way and apply my artificial economy to wage regulation policy questions.

I use a set of standard macroeconomic research tools including calibration, difference equations, detrending by H-P filter, log-linear approximation into a linear vector autoregression, impulse response experiments, and stochastic simulations. I add habit formation to an existing literature on efficiency wage models and use similar tools to those of previous researchers. While I isolate my innovation to just the theoretical areas of interest of my work, and use standard tools, there is not a pre-packaged tool-kit available to work with exactly the models I have invented, so new technology had to be written to do these novel experiments. In chapter two the tools are developed and it is shown that habit formation makes a difference in predictions, improvement in some dimensions over not having it in the model. The use continues of the DSGE, log-liner approximation, calibration, impulse response and simulation tools, on a different model designed to address a certain type of policy question, in Chapter three. In other words, I use established tools to ask new questions. The results are striking, some expected and others not.

Calibration is an established practice in DSGE modeling, and it allows me to make direct comparisons across models and therefore design valid experiments that isolate a variable of interest. Some of the values I use in calibration and as constants in the model come from previous empirical work. For instance the

ratio of consumption between employed and unemployed consumers as .78, a value Alexopoulos (2004) uses and which I use in chapter three. One of the innovations in chapter two is to allow this value to vary, following Givens (2008). See chapters two and three for a list of values used in calibration and as constants in the model. These are used in both chapter two and three except where noted in the respective chapters. The calibration to a fixed steady state employment rate is a feature of both models, and of the control and treatment scenarios in chapter three.

Using values from Alexopoulos's (2004) GMM estimation facilitates making a comparison of covariance structure in Chapter two, and continuing the practice in chapter three makes comparisons possible that give support to the theoretical stance that habit formation in consumption is internal. Specifically, the positive correlation of output and wage is shown to be generated by internal habit formation, in contrast to external. The log-linear approximation of the artificial economy allows comparison to previous work in the RBC literature, and makes it possible to build a variation-in-consumption measure of the welfare cost of wage discrimination policies that is central to the main result of chapter three. Two models calibrated to the same steady-state unemployment retain identical consumer utility functions because of the way the firm's decision is modeled. The linear system allows this independence, and thus makes for a good experimental design and reliable inference about causes.

Chapter two analyzes the dynamics of a shirking efficiency wage model along the lines of Alexopoulos (2004). The key addition to the model is habit formation in consumption. I report simulation results that are directly comparable to Alexopoulos's partial insurance and full insurance setups. Indeed, with certain parameter values the model can be made equivalent to either case. In my model I parameterize the insurance level in a manner similar to that of Givens (2008). For certain values of the habit formation and insurance parameters, output volatility and output-employment correlations match the U.S. data better than those

produced by the Alexopoulos (2004) partial insurance model, although increased employment and counterfactual negative employment-wage correlations emerge.

A model incorporating both an efficiency wage and habit formation allows for a discussion of the dynamics arising from habit formation in an environment with structural unemployment. I use the model to focus on output-employment correlations and output volatility, observing the effects of habit formation under different levels of insurance. The insurance level models the extent to which workers contribute to a fund for the purpose of augmenting the income of the unemployed. It affects the penalty for shirking, and thus wage dynamics. With the addition of habit formation, dynamics are also affected through the utility of consumption, altering shirking penalty effects. While it is known that habit formation creates delayed and smoothed responses to shocks, the dynamics that result from including both habit formation and shirking efficiency wages with partial insurance are not well understood.<sup>1</sup>

The effects of habit formation, when combined with a partial-insurance shirking efficiency wage, are qualitatively more complex than a simple smoothing and delaying of responses. Alexopoulos (2004) finds that the shirking penalty effects of technology and fiscal shocks allow for improved wage and employment volatilities and correlations. However, her partial insurance model, which can be taken as a special case of mine, also produces excessive output volatility and output-employment correlations. In terms of a comparison to Alexopoulos's results, my model generates an improvement in output and employment dynamics that comes at the cost of a counterfactual negative wage-employment correlation and increased volatility of both wages and employment. Due to opposed effects of partial insurance and habit formation, the improvements can be achieved at different parameter combinations that imply different ratios of employed to unemployed consumption. Rather than calibrate to such a consumption ratio as Alexopoulos (2004) does, I report its implied steady-state value for different levels of habit and insurance.

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<sup>1</sup>For example Fuhrer (2000), Constantinides (1990), Dennis (2009), Christiano et al (2005).

Thus, it is shown that while habit and insurance values that generate the improved results are not unique, each pair of values implies a distinct consumption ratio. First, I present the model with habit formation and an exogenous insurance level.<sup>2</sup> I use external habit formation for this formulation, so workers are homogeneous with respect to the effects of habit formation.<sup>3</sup> The worker's decision whether to shirk leads to an efficiency wage and steady-state unemployment, exactly as described by Alexopoulos (2004). Insurance and habit formation levels affect shirking decisions, and thus the wage, in addition to affecting investment and consumption. Next, I focus on the short-run dynamics of employment, output, and wages, in simulations with technology and fiscal shocks. I solve a linearized system in log deviations from the steady states, and use the solution to simulate an economy. I describe the results of simulations, discussing the effects of habit formation and insurance levels on output volatility and output-employment correlation. Effects that depend on both the insurance and habit formation levels interact to bring about the improvements. I use impulse response diagrams to aid the discussion. The tables and figures I provide can be compared directly to Alexopoulos's (2004) findings. The main finding in chapter two is that a model where we have parameterized both habit formation and the partial wage insurance level can make better predictions of data moments in some dimensions. See chapter two for detailed results.

The key theoretical change between chapter two and chapter three is the matter of whether habit formation in consumption is internal or external. This shows up in the models in differing definitions of reference levels. These are different theoretical claims, and the models in chapter two and chapter three are different; see the respective chapters for the mathematical notation used. Either chapter

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<sup>2</sup>Insurance level is parameterized in a way similar to that of Givens (2008) but the insurance parameter does not have a direct interpretation as a consumption ratio unless habit formation is zero.

<sup>3</sup>Because workers have different previous individual consumption levels due to previous employment status, allowing different reference levels (internal habit formation) would result in different IC constraints and wages.

completely develops the model and notation it uses. Chapter two is not necessary for understanding chapter three. In comparing results of the simulations between chapter two and chapter three, we see compelling evidence that the internal habit formation hypothesis is true in that simulations yield second moments in the artificial economy with internal habit formation significantly closer to measured U.S. macroeconomic time series data. The value of the practice of modeling habit formation internally is demonstrated by this improvement from chapter two to chapter three. Additional benefit comes from the ability to model the results of changing rules about wage determination, which is the main inquiry and final result of the work in chapter three.

Chapter three explores the possibility that one plausible cause of wage discrimination is an employer's tendency to offer less to a job candidate who is currently unemployed. One cause for this decision by a hiring firm is that such an employee will accept a lower offer. While this is not the only cause of wage discrimination, such a practice would be restricted by any effective regulation meant to curtail wage discrimination. Such is the stated goal of Obama's executive order 13665<sup>4</sup> of 2014, which prohibits retaliation against employees for sharing wage information with each other. Such rules are a common practice where firms desire the option of offering different wages to different candidates for the same work. Henceforth in this paper 'pay transparency' or 'PT' will refer to this executive order. If the firm is unable to restrict this information sharing among employees, there is potentially an additional cost for the practice of wage discrimination. For instance, workers who are aware that they are paid less for the same work might find the original negotiated wage no longer satisfactory, and 'work to the rule' in protest, whereas if they never discover they are paid less they work with normal intensity and morale.

Setting aside questions of the effectiveness of an executive order of this type

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<sup>4</sup>Exec. Order. No. 13665, 72 Fed. Reg. 20749 (April 11, 2014), <https://www.gpo.gov/fdsys/pkg/FR-2014-04-11/pdf/2014-08426.pdf>. Hereafter to be referred to as 'pay transparency' or PT

with such an aim, we can still inquire as to the macroeconomic consequences of such regulation under an assumption that it is effective. I create a model along the lines of Alexopoulos (2004), and Shapiro, Stiglitz (1977), with the addition of habit formation in consumption. Adding habit formation in consumption to a model which has unemployment motivates the firm to offer a different wage to different workers, as a consequence of differing consumption history. In my model, the lower of the two wages goes to workers unemployed in the previous period because habit formation in consumption alters their incentive compatibility constraint (to be abbreviated hereafter as 'IC') which determines the least (and thus profit-maximizing best) wage that the employer can offer. The previously employed majority receives a higher wage. Since the firm decides employment levels, and prefers to pay two wages to two categories of workers, this model is amenable to creating a laboratory which can isolate any differences in dynamics that would arise from restricting the employer's wage decision; thus we can see the effects of prohibiting wage discrimination without placing any restriction on the decisions of other agents in the model. Allowing the labor market to determine wages is here compared to an economy where the firm is required to pay one wage in a scientifically valid experiment which isolates the variable of interest, which is the presence of an exogenous restriction on the firm's wage decision.

PT itself states it is undesirable to '... diminish market efficiency and decrease the likelihood that the most qualified and productive workers are hired at the market efficient price.' My experiments show how such a well-intentioned policy effort may have unintended and costly consequences by creating a fault-line later to be revealed after a negative shock to the U.S economy. In short, the wage-bill effect that wage discrimination allows makes for less costly recessions. I find a large cost when this effect is eliminated by disallowing wage discrimination.

Chapter three first describes the model with endogenous steady-state unemployment due to a shirking efficiency wage, including habit formation in consump-



tion which causes a lagged unemployment influence on present wage costs for employers. Then, an experimental laboratory is built which allows isolating a wage discrimination restriction for a comparison which holds all other factors equal, calibrated to make a valid comparison where one difference is allowed: the firm's choice to wage-discriminate. This can be thought of as an experiment designed to predict an effect that might come from a policy or rule imposed on firms against wage-discrimination, such as pay transparency which, if successful in its stated goals, would hamper any such discrimination. Finally there follows a discussion of the dynamic results in terms of impulse responses and general volatility, including a compensating variation measure to characterize the cost of such policies in terms of consumption preferences. The results are shown to be robust in terms of simultaneous improved predictions of multiple key macroeconomic variables.

When habit formation in consumption is accounted for in the utility function that determines a worker's incentive compatibility constraint, in a model where there is steady-state unemployment and employment varies on the extensive margin, the previous period's employment level becomes a determinant of the aggregate wage level chosen by the employer, because a portion of the workers requires a lower wage due to differing consumption standards: the previously unemployed. In my model, habit formation in the utility function brings about two wages, when an employer is allowed to make the decision to pay less to the fraction of employees previously consuming less; last period's unemployment rate is that fraction. Without habit formation, the wage difference goes to zero, but when it is present, and the employer can make the distinction, a relation between the present period wage bill and previous employment causes a change in dynamics.

The worker's decision whether to shirk leads to an efficiency wage and steady-state unemployment, exactly as described by Alexopoulos (2004). Insurance and habit formation levels affect shirking decisions, and thus the wage, in addition to affecting investment and consumption. Also, the employer can offer a lower wage

to a proportion of the workforce equal to the previous period's unemployment rate, due to those workers' lower reference consumption standard, which alters the IC constraint for those individuals.

The model in chapter three describes the interactions of three agents who each face a constrained optimization problem. First, the family decides between consumption allocated among the workers (both employed and unemployed) and future capital subject to a budget determined by the total output of the economy. Also, a profit-maximizing firm chooses an employment level, effort level for work, and wage levels for each category of worker. This wage decision is informed by the utility function of the worker in the manner of the shirking efficiency wage model of Shapiro Stiglitz (1977), with partial income insurance as in Alexopoulos (2004). In the present model, the utility function includes past consumption for the individual worker, and via the proportion of low to high wages in the employer's decision so that the past employment influences the present wage bill taken by the employer. The first order conditions from the solutions to these agent's problems, along with general equilibrium conditions and constraints of the model, give a system of difference equations which represents the laboratory economy. This makes it possible to build simulations which allow comparison to a scenario where wage discrimination is prohibited.

This laboratory is uniquely suited to scientific inquiry into the macrodynamic results of wage discrimination prohibition. Because of the way the wage decision falls to the employer, independent of the family's utility function, a welfare cost measure can be developed that compares the models directly because the utility functions of the family and workers are not altered by the regulation. Only the firm must modify its decisions, and this is naturally an accurate simulation of regulation placed on the firms hiring and wage decisions.

The main finding in chapter three is the nature and size of the macrodynamic welfare cost incurred if wage discrimination is successfully prohibited. See chap-

ter three for the detailed results. The laboratory allows us to ask questions and run experiments around the firm's wage and employment decisions, to detect dynamic consequences of exogenous restrictions such as pay transparency rules. We can also derive an intuitive and rigorous quantitative welfare measure using the representative family's utility function and impulse-response time series. When I compare artificial economies that are identical except for being with and without wage discrimination allowed by the firm, a one-standard-deviation negative technology shock causes welfare loss that would be offset by a .27% consumption increase for 180 quarters after the shock. Finally, I run simulations with stochastic technology and fiscal elements to get correlations and second moments for key macroeconomic measures for comparison to previous models in the literature and my model with and without the wage discrimination prohibition, showing some improvement in predictions in some areas, such as the output correlation. The predictive and explanatory value of my lab is demonstrated by the meaningful and robust results here described. My findings are significant and large, with real policy implications for the regulation of wage decisions by employers. This type of model is new, and its structure uniquely allows the for types of questions I ask to be studied in a scientifically valid manner, with measures of welfare costs developed that are both intuitive and experimentally valid in the context of the laboratory I built.

This dissertation extends macroeconomic knowledge into a fruitful new direction, adding a factor to models with involuntary unemployment that is already known to improve predictions in other models. Using these tools to evaluate fiscal and regulatory policy questions, beyond the monetary literature that has accumulated in previous decades, is an exciting area for up-and-coming research. The value of these models for experiments informing public policy is demonstrated with the large and experimentally valid results. In short, it is found that well-meaning policy changes can have effects that are not immediately observable because they

come into play in subsequent recessions. The warning that this study gives of that possibility could prevent unexpected suffering under wage policies meant to make the job market more fair and equitable for employees. While the economists job is not to make policy, results such as those shown here give the policy maker warning of the dangers of well-meaning interventions by authority into wage negotiations. The cost must be weighed against whatever positive benefit in terms of fairness or equality might be intended. The cost is here expressed in a measure friendly to its role in aiding such decisions. A dollar figure is understandable by anyone. \$680 a year can be compared, for perspective, with the \$1182 predicted benefit for a similar average family to come from the largest tax cut in U.S. history.<sup>5</sup>

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<sup>5</sup>Speaker Ryan's Floor Speech on Tax Reform Legislation, November 16, 2017

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**CHAPTER TWO: EMPLOYMENT, WAGE, AND  
OUTPUT DYNAMICS UNDER EXTERNAL HABIT  
FORMATION AND EFFICIENCY WAGES**

## Introduction

This study analyzes the dynamics of a shirking efficiency wage model along the lines of Alexopoulos (2004). The key addition to the model is habit formation in consumption. I report simulation results that are directly comparable to Alexopoulos's partial insurance and full insurance setups. Indeed, with certain parameter values the model can be made equivalent to either case. In my model I parameterize the insurance level in a manner similar to that of Givens (2008). For certain values of the habit formation and insurance parameters, output volatility and output-employment correlations match the U.S. data better than those produced by the Alexopoulos (2004) partial insurance model, although increased employment and counterfactual negative employment-wage correlations emerge.

A model incorporating both an efficiency wage and habit formation allows for a discussion of the dynamics arising from habit formation in an environment with structural unemployment. I use the model to focus on output-employment correlations and output volatility, observing the effects of habit formation under different levels of insurance. The insurance level models the extent to which workers contribute to a fund for the purpose of augmenting the income of the unemployed. It affects the penalty for shirking, and thus wage dynamics. With the addition of habit formation, dynamics are also affected through the utility of consumption, altering shirking penalty effects. While it is known that habit formation creates delayed and smoothed responses to shocks, the dynamics that result from including both habit formation and shirking efficiency wages with partial insurance are not well understood.<sup>1</sup>

The effects of habit formation, when combined with a partial-insurance shirking efficiency wage, are qualitatively more complex than a simple smoothing and delaying of responses. Alexopoulos (2004) finds that the shirking penalty effects of

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<sup>1</sup>For example Fuhrer (2000), Constantinides (1990), Dennis (2009), Christiano et al (2005).



technology and fiscal shocks allow for improved wage and employment volatilities and correlations. However, her partial insurance model, which can be taken as a special case of mine, also produces excessive output volatility and output-employment correlations. In terms of a comparison to Alexopoulos's results, my model generates an improvement in output and employment dynamics that comes at the cost of a counterfactual negative wage-employment correlation and increased volatility of both wages and employment. Due to opposed effects of partial insurance and habit formation, the improvements can be achieved at different parameter combinations that imply different ratios of employed to unemployed consumption. Rather than calibrate to such a consumption ratio as Alexopoulos (2004) does, I report its implied steady-state value for different levels of habit and insurance. Thus, it is shown that while habit and insurance values that generate the improved results are not unique, each pair of values implies a distinct consumption ratio. First, I present the model with habit formation and an exogenous insurance level.<sup>2</sup> I use external habit formation for this formulation, so workers are homogeneous with respect to the effects of habit formation.<sup>3</sup> The worker's decision whether to shirk leads to an efficiency wage and steady-state unemployment, exactly as described by Alexopoulos (2004). Insurance and habit formation levels affect shirking decisions, and thus the wage, in addition to affecting investment and consumption. Next, I focus on the short-run dynamics of employment, output, and wages, in simulations with technology and fiscal shocks. I solve a linearized system in log deviations from the steady states, and use the solution to simulate an economy. I describe the results of simulations, discussing the effects of habit formation and insurance levels on output volatility and output-employment correlation. Effects that depend on both the insurance and habit formation levels interact to bring

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<sup>2</sup>Insurance level is parameterized in a way similar to that of Givens (2008) but the insurance parameter does not have a direct interpretation as a consumption ratio unless habit formation is zero.

<sup>3</sup>Because workers have different previous individual consumption levels due to previous employment status, allowing different reference levels (internal habit formation) would result in different IC constraints and wages.

about the improvements. I use impulse response diagrams to aid the discussion. The tables and figures I provide can be compared directly to Alexopoulos's (2004) findings.

## The Model

Firms have imperfect capability to monitor worker effort, and exerting effort causes disutility for workers. Workers decide whether to shirk. Firms choose the smallest wage such that the expected utility of shirking detection equals the utility from exerting effort. Several things can vary in this model, including the probability of detection and the nature and extent of the penalty for shirking.<sup>4</sup> The extent to which workers share unemployment risk by redistributing consumption alters the incentive compatibility (IC hereafter) constraint that governs the wage decision of the firm.<sup>5</sup> My model adds habit formation to the utility function. Where Alexopoulos (2004) considers full and partial insurance setups, with partial insurance defined as the level where consumption is the same for detected shirkers and the unemployed, I consider different insurance arrangements by adding an exogenous parameter for insurance analogous to that used by Givens (2008).

A representative family rents capital to a representative firm. The family make an investment decision and allocates consumption among the members, attempting to maximize the expected utility of its members. The firm chooses employment and wage levels with the objective of maximum profit, knowing a shirking worker produces no output. Employed members decide whether to shirk. Since shirkers do no work, the firm chooses a wage level to just ensure that shirking does not occur.<sup>6</sup>

In other words, where a worker is indifferent between shirking and working.<sup>7</sup>

Government expenditures  $G_t$  appears as a cost (tax) to the family, and is added to simulate fiscal shocks. The technology coefficient in the production function  $A_t$

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<sup>4</sup>Alexopoulos (2004) considers a scenario where a wage penalty short of dismissal is possible, whereas Shapiro and Stiglitz (1984) penalize shirking with dismissal.

<sup>5</sup>Insurance options ranging from zero to perfect have been considered. Alexopoulos shows full insurance is equivalent to the model of Hansen (1984) and Givens (2008) allows the insurance level to vary in the full range.

<sup>6</sup>This is a result of the worker utility maximization problem described in detail by Alexopoulos (2004).

<sup>7</sup>I derive the function for effort in terms of wage from the indifference condition in Appendix A.

is used to simulate technology shocks. These are modeled with the AR processes

$$\tilde{G}_t = \rho_A \tilde{G}_{t-1} + \epsilon_{G_t}, \text{ and} \quad (1)$$

$$\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \epsilon_{A_t}, \quad (2)$$

where  $\epsilon_{A_t}$  and  $\epsilon_{G_t}$  are serially uncorrelated innovations with mean zero and standard deviations  $\sigma_a$  and  $\sigma_g$ .<sup>8</sup> I use parameter values from Alexopoulos's (2004) GMM estimation.<sup>9</sup>

## Family

I add habit formation to utility using a reference consumption level determined by the consumption levels of unemployed and employed members last period. The reference level is

$$\bar{c}_t = N_t c_t^e + (1 - N_t) c_t^u, \quad (3)$$

where  $N_t \in [0, 1]$  is the employment level, and  $c_t^e$  and  $c_t^u$  are the consumption levels of employed and unemployed family members, respectively. Utility for an employed worker, accounting for habit formation, is

$$U_t(c_t^e, \bar{c}_{t-1}) = \ln(c_t^e - b\bar{c}_{t-1}) + \theta \ln(T - he_t - \zeta), \quad (4)$$

where  $b$  is the habit formation parameter,  $T$  is the time endowment of an individual,  $e_t$  is the effort level expended,  $h$  is work hours, and  $\zeta$  is the fixed cost of exerting any effort greater than zero. An unemployed worker experiences utility

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<sup>8</sup>The equations presented here are expressed as log deviations from mean to equate to the linearized system presented in the solution section, while Alexopoulos (2004) writes them in levels. The two are exactly equivalent ( $\rho$  values are the same.)

<sup>9</sup>My simulation differs from hers in that I do not include technology and government spending growth. My technology and government spending shocks are independent of one-another. The results I am interested in are not affected.

based on unemployed consumption, and no disutility from working,

$$U_t(c_t^u, \bar{c}_{t-1}) = \ln(c_t^u - b\bar{c}_{t-1}) + \theta \ln(T). \quad (5)$$

A detected shirking employed worker consumes at the shirking consumption level,  $c_t^s$ , and experiences utility <sup>10</sup>

$$U_t(c_t^s, \bar{c}_{t-1}) = \ln(c_t^s - b\bar{c}_{t-1}) + \theta \ln(T). \quad (6)$$

The family decides a consumption contribution  $c_t^f$  to be given to each member. The shirking penalty  $s \in [0, 1]$  is the fraction of the full wage that a detected shirker will receive. The penalty for *detected* shirking, then, is  $w_t(1 - s)$ . Each employed family member contributes an amount  $F_t$  as insurance, to be paid to the unemployed. Thus, consumption levels are

$$c_t^u = c_t^f + \frac{N_t}{1 - N_t} F_t, \quad (7)$$

for the unemployed,

$$c_t^s = c_t^f + sw_t h - F_t, \quad (8)$$

for a detected shirker, and

$$c_t^e = c_t^f + w_t h - F_t, \quad (9)$$

for the employed. The insurance contribution from each member is exogenous to the family, determined by the wage chosen by the representative firm, and  $\sigma$ , the insurance parameter, by the rule

$$F_t = \sigma(1 - N_t)hw_t. \quad (10)$$

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<sup>10</sup>Since the IC constraint on wage assures employers will choose a wage which prevents all shirking, this utility expression applies to no one.

The family also chooses an investment level, and thus a capital level,  $K_{t+1}$ , for next period, subject to a budget constraint including investment in new capital and taxes.<sup>11</sup> The family makes its choices to solve

$$\max_{\{c_t^f, K_{t+1}\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ \begin{aligned} &(N_t) \ln(c_t^e - b\bar{c}_{t-1}) + (N_t) \theta \ln(T - he_t - \zeta) \\ &+ (1 - N_t) \ln(c_t^u - b\bar{c}_{t-1}^e) + (1 - N_t) \theta \ln(T) \end{aligned} \right\} \right\}, \quad (11)$$

$$\text{subject to } \bar{c}_t \leq [r_t K_t - G_t - [K_{t+1} - (1 - \delta)K_t]], \quad (12)$$

where  $\delta \in (0, 1]$  is the depreciation rate and  $r_t$  is the rental rate of capital. Maximizing the Lagrangian, the first order conditions are

$$\frac{(N_t)}{(c_t^e - b\bar{c}_{t-1})} + \beta \frac{(N_{t+1})(-b)}{(c_{t+1}^e - b\bar{c}_t)} + \frac{(1 - N_t)}{(c_t^u - b\bar{c}_{t-1}^e)} + \beta \frac{(1 - N_{t+1})(-b)}{(c_{t+1}^u - b\bar{c}_t)} = \lambda_t, \text{ and} \quad (13)$$

$$\lambda_t = \beta E_t \lambda_{t+1} \left( \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right), \quad (14)$$

where  $\lambda_t$  is the Lagrangian multiplier associated with Equation (12).

### Member Incentive Compatibility Constraint

The IC (incentive compatibility) constraint will apply to members who are employed. The IC constraint comes from calculating the wage level which will make a worker indifferent between shirking and working. The effort level is a function of the wage. Defining  $\chi_t$  as

$$\chi_t = \frac{c_t^e - b\bar{c}_{t-1}}{c_t^s - b\bar{c}_{t-1}}, \quad (15)$$

we can show  $\chi_t$  is constant,<sup>12</sup> determined by the relationship to time endowment, fixed effort cost, utility weight of leisure, detection probability, and shirking

<sup>11</sup>Because tax paid is equal to  $G_t$ ,  $G_t$  is substituted for tax cost in the budget constraint presented here.

<sup>12</sup>Appendix D Uses the wage FOC from the firm's problem (Solow condition) and the consumption equations to demonstrate this.

penalty defined by

$$\left( (T - \zeta) \chi^{1+\frac{d}{\theta}} - T\chi \right) (1 - s) = T \left( \frac{d}{\theta} \right) (1 - s\chi)(\chi - 1). \quad (16)$$

The IC constraint arising from the worker's shirking decision is obtained by finding the effort level which makes the worker indifferent between shirking and working at a given wage. The effort level is shown to be constant, because  $\chi_t$  is constant.<sup>13</sup> The effort function is used as a constraint in the firm's problem,

$$E_t = \Delta(w_t) = -\frac{T}{h} (\chi_t)^{-d/\theta} + \frac{T}{h} - \frac{\zeta}{h}. \quad (17)$$

It follows directly from consumption definitions that<sup>14</sup>

$$(1 - s) \frac{\chi}{\chi - 1} h w_t = c_t^e - b\bar{c}_{t-1} \quad (18)$$

and  $\mu(\sigma) = 1/\chi$ , a function of the insurance parameter  $\sigma$ <sup>15</sup>, following

$$\frac{c_t^u - b\bar{c}_{t-1}}{c_t^e - b\bar{c}_{t-1}} = \mu(\sigma) = 1 - \frac{1 - \sigma}{1 - s} \left( \frac{\chi_t - 1}{\chi_t} \right). \quad (19)$$

My definition of  $\chi$  differs from that of Alexopoulos (2004) when habit formation is not zero. While  $\chi$  is constant,  $c_t^u/c_t^e$  is not, but I report its steady-state value in my results, for a measure of insurance level which can be compared to other results, or potentially calibrated to observed data, such as the income ratio estimated by Gruber (1997).<sup>16</sup> When  $b=0$ , of course, these variables reduce to the values seen elsewhere and become constant.

## Firm

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<sup>13</sup>Appendix A.

<sup>14</sup>Appendix B.

<sup>15</sup>Appendix B. It is still true that the partial insurance case, as defined by Alexopoulos (2004), comes about when  $s=\sigma$ , as Givens (2008) shows when he introduces the insurance level parameter  $\sigma$ .

<sup>16</sup>I do not calibrate to this. I recover it from the steady state which yields a certain employment level.

The firm chooses a wage, an employment level, an effort level, and capital, to maximize profit. Costs are the wage bill and capital rental. The production function,

$$Y_t = A_t K_t^\alpha ((N_t) h E_t)^{1-\alpha}, \quad (20)$$

combines effective labor and capital.<sup>17</sup> The incentive compatibility constraint,

$$E_t = \Delta(w_t) = -\frac{T}{h} (\chi_t)^{-d/\theta} + \frac{T}{h} - \frac{\zeta}{h}, \quad (21)$$

enters the firm's decision as a binding constraint on the wage. The smallest wage that will prevent shirking is chosen. The firm maximizes profit by solving

$$\max_{\{w_t, N_t, E_t, K_t\}} (A_t K_t^\alpha (h E_t N_t)^{1-\alpha} - w_t h N_t - r_t K_t). \quad (22)$$

Maximizing profit yields the first order conditions<sup>18</sup>,

$$r_{t+1} = \alpha \frac{Y_{t+1}}{K_{t+1}}, \quad (23)$$

$$(1 - \alpha) A_t K_t^\alpha (h E_t N_t)^{-\alpha} h E_t = w_t h, \text{ and} \quad (24)$$

$$\frac{(w_t) \Delta'(w_t(e))}{\Delta(w_t(e))} = 1. \quad (25)$$

## General equilibrium

Expenditure equals income in equilibrium, and demand and supply for capital are equal. Aggregating the family budget constraint yields

$$C_t + G_t + I_t = Y_t. \quad (26)$$

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<sup>17</sup>The production function incorporates the finding that no shirking happens in equilibrium, so there is only one employment level and it fully contributes to production.

<sup>18</sup>Appendix C shows derivation of the Solow condition.



## Calibration

In each simulation I solve for parameters affected by changes in the utility function due to adding habit formation in order to yield a steady state employment rate of 0.941.<sup>19</sup> Other calibrations are borrowed from Alexopoulos's (2004) GMM estimates. Yet others are taken from other literature following Alexopoulos (2004). I depart from her estimates where necessary, since the utility function is altered under habit formation ( $b > 0$ ).  $d/\theta$  is affected, taking a different value under different values of  $b$ , for instance. Since  $(c_t^u - b\bar{c}_{t-1}) / (c_t^e - b\bar{c}_{t-1})$  is constant,  $c_t^u/c_t^e$  is not constant. I solve for  $\chi_t = (c_t^e - b\bar{c}_{t-1}) / (c_t^u - b\bar{c}_{t-1})$  at each parameter set considered, because it is a constant value. If desired, one could calibrate to a  $c_t^u/c_t^e$  as a steady state,<sup>20</sup> but in this study the dynamics of consumption are not of direct interest, so I calibrate to an employment steady state. I report  $c_t^u/c_t^e$  in all tables, and the trends in  $c_t^u/c_t^e$  with changes in  $b$  and  $\sigma$  are not surprising. My constant  $\chi$  (constant for given  $\sigma$  and  $b$ ) does not indicate a simple consumption ratio under any degree of habit formation besides zero. I use this relationship,

$$\left( (T - \zeta) \chi^{1+\frac{d}{\theta}} - T\chi \right) (1 - s) = T \left( \frac{d}{\theta} \right) (1 - s\chi)(\chi - 1), \quad (27)$$

to solve for  $\chi$ ,  $d/\theta$ , and  $s$  in order to yield steady state  $N = 0.941$ , at each set of  $\sigma$  and  $b$  parameters considered. The parameters that do not change with changes in  $b$  and  $\sigma$ , and their values, are  $\{\beta = 0.9796, \delta = 0.0203, \sigma_g = 0.0133, \sigma_a = 0.0074, \rho_g = 0.9797, \rho_a = 0.9699, \alpha = 0.4574, \zeta = 16, T = 1369, \log(g/y) = -1.6870\}$ .

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<sup>19</sup>This is the average from U.S. historical data.

<sup>20</sup>Alexopoulos (2004) does so, using a ratio derived from estimates by Gruber (1995). Givens (2008) generalizes to account for different values of this ratio, which characterizes the insurance level, since such a ratio is not directly measurable.

## Solution

I am interested in the dynamics of  $W$  (wage),  $N$  (employment), and  $Y$  (output), in a simulation using the estimated variances and persistence of the government spending and technology disturbances.<sup>21</sup> The linearized system is solved, and this solution is used to generate simulations. The results are expressed in log deviations from the steady state. I generate impulse response experiments to provide insight into mechanisms for changes in second moments at different insurance and habit formation levels. For reference, I present the linear system here. Steady states are recalculated at every set of  $\sigma$  (insurance) and  $b$  (habit formation) values, so the constants in these equations differ for different values, where the steady states appear in the equations. Klein's (2000) method yields a state-space solution in the predetermined and non-predetermined variable vectors.  $G_t$  and  $A_t$  are the exogenous forcing factors.<sup>22</sup>

Defining  $\tilde{x}_t = \ln x_t - \ln x$ , and using no subscript to indicate a steady-state value, the linearized system contains the equations

$$\tilde{A}_t + \alpha \tilde{K}_t + (1 - \alpha) e \tilde{N}_t = \tilde{Y}_t, \quad (28)$$

$$\bar{c} \tilde{c}_t + G \tilde{G}_t + K \tilde{K}_{t+1} - (1 - \delta) K \tilde{K}_t = Y \tilde{Y}_t, \quad (29)$$

$$\left( \frac{N}{c^e - b\bar{c}} - \frac{N}{c^u - b\bar{c}} \right) \tilde{N}_t - \frac{N}{(c^e - b\bar{c})} \tilde{c}_t^e - \frac{(1 - N)}{(c^u - b\bar{c})} \tilde{c}_t^u + \left( b \frac{N}{(c^e - b\bar{c})} + b \frac{(1 - N)}{(c^u - b\bar{c})} \right) \tilde{c}_{t-1} = \lambda \tilde{\lambda}_t, \quad (30)$$

$$\tilde{\lambda}_t = \beta \left( \alpha \frac{Y}{K} + 1 - \delta \right) \tilde{\lambda}_{t+1} + \beta \alpha \frac{Y}{K} \tilde{Y}_{t+1} - \beta \alpha \frac{Y}{K} \tilde{K}_{t+1}, \quad (31)$$

$$\tilde{Y}_t - \tilde{N}_t = \tilde{w}_t, \quad (32)$$

$$(1 - s) \frac{\chi}{\chi - 1} h w \tilde{w}_t = c^e \tilde{c}_t^e - b \bar{c} \tilde{c}_{t-1}, \quad (33)$$

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<sup>21</sup>Here I depart somewhat from Alexopoulos's model. She incorporates technology growth and a related government spending growth, where I have made the two series stationary and uncorrelated. In terms of implementing Klein's solution method: the autocorrelation matrix of the forcing factors vector is diagonal.

<sup>22</sup>The  $\sigma$  and  $\rho$  values are taken from Alexopoulos (2004). See the calibration section.

$$c^u \tilde{c}_t^u - bc \tilde{c}_{t-1} = \left(1 - \frac{1-\sigma}{1-s} \left(\frac{\chi-1}{\chi}\right)\right) (c^e \tilde{c}_t^e - b\bar{c} \tilde{c}_{t-1}), \text{ and} \quad (34)$$

$$(Nc^e - Nc^u) \tilde{N}_t + Nc^e \tilde{c}_t^e + (1-N)c^u \tilde{c}_t^u = \bar{c} \tilde{c}_t. \quad (35)$$

## Results

Second moments are taken from repeated simulations, with the series HP-filtered ( $\lambda = 1600$ ). I also supply impulse responses to aid the discussion of mechanisms for the second moment variation observed with different levels of habit formation and insurance. I report  $c_t^u/c_t^e$  values recovered from the steady state consumption values, to see what the implied insurance level is in those terms.<sup>23</sup>

My model is equivalent to Alexopoulos (2004) when my habit formation level is zero and insurance levels are set to certain levels.<sup>24</sup> At certain  $\sigma$  and  $b$  values, the initial technology-shock effect on  $N$  and the smoothed and delayed responses of all variables to shocks interact to yield lower  $N - Y$  correlations and lower  $Y$  volatility. This better matches U.S. Data than the model without habit formation. However, a much greater negative correlation between the wage and employment and increased wage and employment volatility are present, as compared to a model without habit formation. The improved second moments match the data most closely at sets of values ( $\sigma$  and  $b$ ) which are not unique. That is to say: for a given  $\sigma$  a  $b$  can be found to create dynamics which show the improvements. I report steady state  $c_t^u/c_t^e$  because it does differ within this parameter space. I focus on the dynamics which lead to these observations, using impulse responses to illustrate and discuss the effects.

### The Effect of Adding Habit Formation at Different Insurance Levels

Habit formation produces a 'hump-shaped' response to shocks. Since agents derive consumption utility relative to a reference level from the previous period,

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<sup>23</sup> $C_t^u/C_t^e$  becomes dynamic under habit formation, as discussed in the model section, but I do not make any analysis of that in this report, and consumption does not itself appear in tables or diagrams in this paper.

<sup>24</sup>The two cases are highlighted in Table 1 and Figures 1 and 2.

the response to shocks is delayed and smoothed. This effect of habit formation is known. In the absence of habit formation, the insurance effect appears as a change in the magnitude of fiscal-shock movements, by affecting the punishment associated with shirking, as described in Alexopoulos (2004). Under technology shocks, lower insurance levels dampen and delay the initial positive wage effect by a shirking penalty effect as well. As Alexopoulos (2004) explains,  $c_t^f$  increases more or less based on the relative magnitudes of investment increases and wage increases due to increased marginal product of labor.

Adding habit formation has the expected effect on responses to fiscal shocks, which can be seen in Figure 1. The response of employment to a technology shock is affected in a more surprising way, which is a consequence of the dynamics arising from imperfect effort monitoring. Adding habit formation in the presence of partial insurance has two effects which increase the initial wage response, and dampen the initial employment response, to the point of making it negative at high enough habit levels. First, the delayed investment response makes the shirking penalty effect on wages more pronounced earlier through its relative effect on initial  $c_t^f$ . Second, the shirking penalty effect itself is larger. The employer adjusts wages to keep effort constant, as before. Now, though, the constant  $\chi$  includes previous consumption.  $\chi_t = (c_t^e - b\bar{c}_{t-1}) / (c_t^s - b\bar{c}_{t-1})$ , so with greater  $b$  the initial wage increase is greater, to the point that the corresponding decrease in employment initially overwhelms the increase due to the marginal product of labor.

The qualitative difference in the initial movement of  $N$  under a technology shock drives the improvement (reduction) in  $N - Y$  correlation at higher habit formation values. Illustrating this effect, Figure 3 and Table 2 depict the results under increasing habit formation. It is clear in the table that increasing habit formation reduces the  $N - Y$  correlation. Viewing the figure makes it apparent that the source of this improvement is the initial drop in  $N$ . Figure 3 shows the transition to the negative initial response of  $N$ , and the trends in technology shock responses

of  $W$ ,  $N$ , and  $Y$ , with increasing habit formation. Table 2 corresponds to Figure 3 and should be viewed with it. The increasing negative  $W - N$  correlation seems to arise from the same effect that decreases the  $N - Y$  correlation and decreases  $Y$  volatility. I do not show the fiscal shock responses since they follow the pattern already demonstrated in Figure 1.

### Targeting Certain Improvements

Table 3 and Figure 4 should be viewed together. For different  $b$ , different values of  $\sigma$  produce the  $N - Y$  correlation and  $Y$  volatility improvements. Increased volatilities of  $N$  and  $W$  come about in these ranges, as well as a large negative  $W - N$  correlation, as can be predicted from viewing the impulse responses to the technology shock. The  $Y$  volatility and the  $N - Y$  correlation more closely match U.S. data than those the partial insurance model without habit formation produce. In Table 3 and the Figure 4 I have highlighted sets of values that bring us closer to the U.S. data in these respects. The dynamics of interest here seem to come from the technology shock effect on employment, so I do not show impulse responses for fiscal shocks.

As has been mentioned, under increased habit formation a technology shock causes a sharper initial increase in wage. The wage increase corresponds to a lesser employment increase, eventually a decrease, under increasing habit formation values. For the range of values shown in Figure 4 and Table 3, an increased insurance level counteracts this effect. Thus, imperfect monitoring and habit formation interact to determine the initial response of wages and employment. Increasing insurance and increasing habit formation have opposing effects. Examination of Figure 4 makes this clear. There are multiple combinations of  $\sigma$  and  $b$  which yield second moments to match U.S. data, and they correspond to impulse responses that look very similar. These results appear at different  $b$  levels given different  $\sigma$  levels, and

carry with them very similar volatility increases for  $W$  and  $N$ , and high negative  $W - N$  correlation. The best match to U.S. data is seen at parameter values highlighted in Table 3 and Figure 4, and the large negative  $W - N$  correlation can easily be predicted from the impulse responses. However, the  $c_t^e/c_t^s$  steady state does vary among parameterizations (see Table 3) so if we had some evidence for a certain consumption ratio then we would have some empirical support for a habit formation value.

## Conclusion

By examining dynamic effects of habit formation and variable insurance in a shirking efficiency wage model, we can see effects of habit formation in a model with structural unemployment. Adding habit formation implies that the ratio of consumption of the unemployed to that of the employed is no longer constant. Examination of second moments, along with impulse responses for technology and fiscal shocks, shows the interaction of habit formation and imperfect monitoring. Adding habit formation reduces output volatility and relaxes the high correlation of output and employment to a degree more consistent with observed data. However, a counterfactual negative correlation of wage and employment emerges, and the volatilities of both employment and wage increase, as compared to models such as the ones used by Alexopoulos (2004) and Givens (2008). Habit and insurance level pairs that cause the better match to U.S. data can imply different consumption ratios.

Perhaps one source of the inflated volatility of employment in my model comes from its failure to account for heterogeneity of workers' income histories that would affect the IC constraints and create a wage dispersion effect, if employers have access to information about individual employment history. Although the model simulated in this paper ignores the possibility by assuming a single reference consumption level, we could expect a smoothing effect on employment variation, since lagged employment would then affect the IC constraints determining wages and employment. High (low) unemployment in the recent past drives down (up) optimum wages, and thus raises (lowers) present employment, dampening movements away from the steady-state level in either direction. A model allowing for this heterogeneity in wages<sup>25</sup> may also yield a correlation of wage and employment closer to zero.

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<sup>25</sup>The model which allows differing IC constraints among workers, generating the wage dispersion, is work in progress.



Empirical inquiries which might support the usefulness of this type of model would include formal testing of the fit of this model against other proposed sources of labor market friction. In addition, a study evaluating whether there is a wage premium associated with jobs that are less closely monitored for performance would lend some support to this type of model. Finally, although not directly related to the model presented in this paper, because I use external habit formation, any empirical demonstration of a wage differential resulting from differences in workers' recent employment status could be taken to support a wage dispersion dynamic arising from heterogeneous work histories.

## APPENDICES CHAPTER TWO

## Appendix A: Deriving the incentive compatibility constraint

This function, contracted effort as a function of wage, constrains the contracted effort choice of the firm, appearing in section 2.2, Equation (16). Setting utility from working equal to utility expected from shirking, we arrive at a function for effort.

$$\begin{aligned}
 U(c_t^e - b\bar{c}_{t-1}, E_t) &= dU(c_t^s - b\bar{c}_{t-1}, 0) + (1 - d)U(c_t^e - b\bar{c}_{t-1}, 0) \\
 \ln(c_t^e - b\bar{c}_{t-1}) + \theta \ln(T - (hE_t + \zeta)) &= \\
 d(\ln(c_t^s - b\bar{c}_{t-1}) + \theta \ln(T)) + (1 - d)(\ln(c_t^e - b\bar{c}_{t-1}) + \theta \ln(T)) \\
 \ln(c_t^e - b\bar{c}_{t-1}) - (1 - d)(\ln(c_t^e - b\bar{c}_{t-1}) - d(\ln(c_t^s - b\bar{c}_{t-1})) &= \theta \ln(T) - \theta \ln(T - \\
 (hE_t + \zeta)) \\
 \left( \frac{c_t^e - b\bar{c}_{t-1}}{c_t^s - b\bar{c}_{t-1}} \right)^d &= \left( \frac{T}{T - (hE_t + \zeta)} \right)^\theta \\
 E_t = \Delta(w_t) &= -\frac{T}{h} \left( \frac{c_t^e - b\bar{c}_{t-1}}{c_t^s - b\bar{c}_{t-1}} \right)^{-d/\theta} + \frac{T}{h} - \frac{\zeta}{h}.
 \end{aligned}$$

I define  $\chi_t = (c_t^e - b\bar{c}_{t-1}) / (c_t^s - b\bar{c}_{t-1})$ , equivalent to the ratio used by Alexopoulos (2004) only when habit formation is zero.  $c_t^e/c_t^s$  is not constant, but  $\chi_t$  is. So  $E_t = \Delta(w_t) = -\frac{T}{h} (\chi_t)^{-d/\theta} + \frac{T}{h} - \frac{\zeta}{h}$  represents effort as a function of the wage and exogenous constants. It can be used as a constraint on the firms problem.

## Appendix B: Deriving Equations (18) and (19)

An expression for wage is used in the demonstration that  $\chi_t$  is constant. It comes directly from the consumption and insurance definitions. Section 2.2, Equation (18)

$$\begin{aligned} c_t^s &= c_t^f + sw_t h - F_t \\ c_t^e &= c_t^f + w_t h - F_t \\ c_t^e - c_t^s &= (1-s)hw_t \\ hw_t &= \frac{(c_t^e - b\bar{c}_{t-1}) - (c_t^s - b\bar{c}_{t-1})}{(1-s)} \\ (1-s)\frac{\chi_t}{\chi_t-1}hw_t &= (c_t^e - b\bar{c}_{t-1}). \end{aligned}$$

To derive the modified consumption ratio which is constant in a model with habit formation, we use consumption and insurance definitions exactly like those of Givens (2008), with the addition of habit formation to the definition of  $\mu$ .  $\mu = 1/\chi$  and is a function of  $\sigma$  and  $b$ . Section 2.2, Equation (19)

$$\begin{aligned} (1-s)\frac{\chi_t}{\chi_t-1}hw_t &= (c_t^e - b\bar{c}_{t-1}) \\ hw_t &= \frac{\chi_t-1}{\chi_t} \frac{1}{(1-s)} (c_t^e - b\bar{c}_{t-1}) \\ f_t &= \sigma(1-N_t)hw_t \\ c_t^u &= c_t^f + N_t\sigma hw_t \\ c_t^e &= c_t^f + hw_t - f_t \\ c_t^s &= c_t^f + shw_t - f_t \\ c_t^e &= c_t^f + hw_t - \sigma(1-N_t)hw_t \\ c_t^f &= c_t^e - hw_t + \sigma(1-N_t)hw_t \\ c_t^u &= c_t^e - hw_t + \sigma(1-N_t)hw_t + N_t\sigma hw_t \\ c_t^u &= c_t^e - \left( \frac{\chi_t-1}{\chi_t} \frac{1}{(1-s)} (c_t^e - b\bar{c}_{t-1}) \right) + \sigma(1-N_t) \left( \frac{\chi_t-1}{\chi_t} \frac{1}{(1-s)} (c_t^e - b\bar{c}_{t-1}) \right) + N_t\sigma \left( \frac{\chi_t-1}{\chi_t} \frac{1}{(1-s)} (c_t^e - b\bar{c}_{t-1}) \right) \\ c_t^u &= c_t^e - \left( \frac{\chi_t-1}{\chi_t} \frac{1}{(1-s)} (c_t^e - b\bar{c}_{t-1}) \right) + \sigma \left( \frac{\chi_t-1}{\chi_t} \frac{1}{(1-s)} (c_t^e - b\bar{c}_{t-1}) \right) \\ c_t^u &= c_t^e - \frac{1-\sigma}{1-s} \left( \frac{\chi_t-1}{\chi_t} \right) (c_t^e - b\bar{c}_{t-1}) \\ c_t^u - b\bar{c}_{t-1} &= c_t^e - b\bar{c}_{t-1} - \frac{1-\sigma}{1-s} \left( \frac{\chi_t-1}{\chi_t} \right) (c_t^e - b\bar{c}_{t-1}) \\ \frac{c_t^u - b\bar{c}_{t-1}}{c_t^e - b\bar{c}_{t-1}} &= \mu(\sigma) = 1 - \frac{1-\sigma}{1-s} \left( \frac{\chi_t-1}{\chi_t} \right) \end{aligned}$$

Notice this expression appears the same as that in Givens (2008), but in the

case where  $b > 0$ ,  $\chi_t$  is not the same. Thus, we have the expression relating the insurance level, the shirking detection probability, and the habit-modified consumption ratio  $\chi_t$ , which is constant.

## Appendix C: Deriving the Solow condition from first order conditions

Using the FOCs for  $N$  and  $W$ , we can arrive at the Solow condition. This is the source of Equation (25), in section 2.2.

$$\text{FOC } N_t: (1 - \alpha)A_t K_t^\alpha (hE_t N_t)^{-\alpha} hE_t = w_t h$$

$$\text{FOC } W_t: (1 - \alpha)A_t K_t^\alpha (h\Delta(w_t)N_t)^{-\alpha} \Delta'(w_t) = 1$$

$$\Delta'(w_t) = \frac{1}{(1-\alpha)A_t K_t^\alpha (h\Delta(w_t)N_t)^{-\alpha}}$$

$$\frac{(w_t)\Delta'(w_t)}{\Delta(w_t)} = 1$$

## Appendix D: Demonstrating that $\chi_t$ is constant

When  $\chi_t$  is defined as  $(c_t^e - b\bar{c}_{t-1}) / (c_t^s - b\bar{c}_{t-1})$ , it is constant. This finding is used in section 2.2. Under zero habit formation, this is exactly equivalent to the consumption ratio mentioned in Alexopoulos (2004) and calibrated to the value found by Gruber(1997). I recover steady-state  $c_t^e/c_t^s$  and report it with the other results, after calibrating for  $N = 0.941$ .

We know  $(1 - s)\frac{\chi_t}{\chi_t - 1}hw_t = (c_t^e - b\bar{c}_{t-1})$  and  $\Delta(w_t) = -\frac{T}{h}(\chi_t)^{-d/\theta} + \frac{T}{h} - \frac{\zeta}{h}$ .

Using the Solow condition,  $\frac{(w_t)\Delta'(w_t)}{\Delta(w_t)} = 1$ , simply substitute, yielding

$$\left((T - \zeta)\chi^{1+\frac{d}{\theta}} - T\chi\right)(1 - s) = T\left(\frac{d}{\theta}\right)(1 - s\chi)(\chi - 1).$$

## Appendix E: Tables

Table 1: Simulated second moments and consumption ratios at different habit formation ( $b$ ) and insurance ( $\sigma$ ) values. Corresponds to Figures 1 and 2.

| Parameters |                | $c_t^u/c_t^e$ | $\sigma_n$ | $\sigma_y$ | $\sigma_w$ | $\rho(n, y)$ | $\rho(n, w)$ | $\rho(y, w)$ |
|------------|----------------|---------------|------------|------------|------------|--------------|--------------|--------------|
| $b=0.00$   | $\sigma = .78$ | 0.7874        | 0.0168     | 0.0187     | 0.0063     | 0.9418       | 0.1318       | 0.4566       |
|            | $\sigma = .89$ | 0.8930        | 0.0117     | 0.0159     | 0.0070     | 0.9147       | 0.4070       | 0.7409       |
|            | $\sigma = 1$   | 1.0000        | 0.0106     | 0.0152     | 0.0073     | 0.9028       | 0.4329       | 0.7783       |
| $b=0.20$   | $\sigma = .78$ | 0.7744        | 0.0232     | 0.0218     | 0.0069     | 0.9547       | -0.3439      | -0.0492      |
|            | $\sigma = .89$ | 0.8231        | 0.0163     | 0.0180     | 0.0059     | 0.9461       | 0.1290       | 0.4425       |
|            | $\sigma = 1$   | 1.0000        | 0.0168     | 0.0186     | 0.0062     | 0.9437       | 0.1238       | 0.4448       |
| $b=0.40$   | $\sigma = .78$ | 0.7912        | 0.0262     | 0.0179     | 0.0134     | 0.8795       | -0.7731      | -0.3785      |
|            | $\sigma = .89$ | 0.8231        | 0.0230     | 0.0193     | 0.0093     | 0.9179       | -0.5622      | -0.1883      |
|            | $\sigma = 1$   | 1.0000        | 0.0233     | 0.0203     | 0.0089     | 0.9257       | -0.5097      | -0.1470      |
| U.S. Data  |                | N/A           | 0.0087     | 0.0154     | 0.0111     | 0.8656       | 0.0739       | 0.2414       |

*Notes:* Simulated series are HP-filtered ( $\lambda = 1600$ ).  $c_t^u/c_t^e$ , the ratio of unemployed to employed consumption, is recovered from the steady state if it cannot be calculated analytically ( $\sigma \neq 1$ ). Second moments reported for U.S. data are calculated directly from time series from FRED and BLS, Q1, 1964 through Q2, 2010, HP-filtered ( $\lambda = 1600$ ).



Table 2: Simulated second moments and consumption ratios at different habit formation ( $b$ ) values while holding insurance ( $\sigma$ ) level constant. Corresponds to Figure 3.

| Parameters            | $c_t^u/c_t^e$ | $\sigma_n$ | $\sigma_y$ | $\sigma_w$ | $\rho(n, y)$ | $\rho(n, w)$ | $\rho(y, w)$ |
|-----------------------|---------------|------------|------------|------------|--------------|--------------|--------------|
| $b=0.26$ $\sigma=.78$ | 0.7784        | 0.0265     | 0.0227     | 0.0087     | 0.9482       | -0.5702      | -0.2800      |
| $b=0.29$              | 0.7762        | 0.0286     | 0.0233     | 0.0101     | 0.9449       | -0.6591      | -0.3770      |
| $b=0.32$              | 0.7743        | 0.0319     | 0.0238     | 0.0127     | 0.9375       | -0.7571      | -0.4826      |
| $b=0.35$              | 0.7816        | 0.0288     | 0.0213     | 0.0126     | 0.9155       | -0.7329      | -0.3974      |
| $b=0.38$              | 0.7795        | 0.0275     | 0.0181     | 0.0145     | 0.8791       | -0.8045      | -0.4245      |
| $b=0.41$              | 0.7986        | 0.0254     | 0.0177     | 0.0128     | 0.8838       | -0.7625      | -0.3717      |
| $b=0.44$              | 0.8013        | 0.0235     | 0.0153     | 0.0136     | 0.8350       | -0.7861      | -0.3165      |
| $b=0.47$              | 0.8027        | 0.0218     | 0.0118     | 0.0148     | 0.7657       | -0.8594      | -0.3296      |
| $b=0.50$              | 0.8046        | 0.0201     | 0.0091     | 0.0152     | 0.7034       | -0.9057      | -0.3361      |
| U.S. Data             | N/A           | 0.0087     | 0.0154     | 0.0111     | 0.8656       | 0.0739       | 0.2414       |

*Notes:* Simulated series are HP-filtered ( $\lambda = 1600$ ).  $c_t^u/c_t^e$ , the ratio of unemployed to employed consumption, is recovered from steady state if it cannot be calculated analytically ( $\sigma \neq 1$ ). Second moments reported for U.S. data are calculated directly from time series from FRED and BLS, Q1 1964 through Q2 2010, HP-filtered ( $\lambda = 1600$ ). At a  $\sigma$  (insurance) value corresponding to Alexopoulos's (2004) partial insurance model, trends associated with increasing  $b$  (habit formation) values can be seen. Figure 3 shows the emergence of the initial  $N$  response which creates the  $N - Y$  correlation reduction seen here with increasing habit formation.

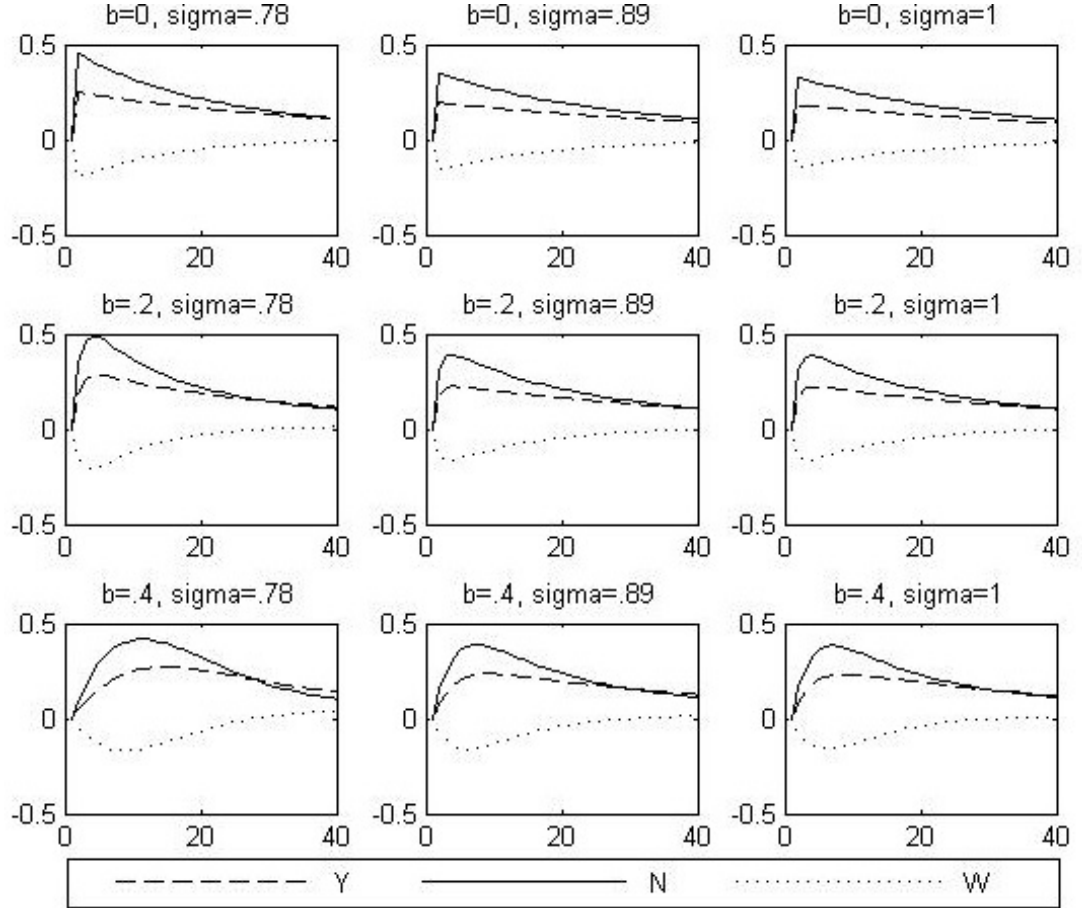
Table 3: Simulated second moments and consumption ratios at different habit formation ( $b$ ) and insurance ( $\sigma$ ) values. Corresponds to Figure 4.

| Parameters |              | $c_t^u/c_t^e$ | $\sigma_n$ | $\sigma_y$    | $\sigma_w$ | $\rho(n, y)$  | $\rho(n, w)$ | $\rho(y, w)$ |
|------------|--------------|---------------|------------|---------------|------------|---------------|--------------|--------------|
| $b=0.43$   | $\sigma=.77$ | 0.7911        | 0.0237     | <b>0.0142</b> | 0.0146     | <b>0.8156</b> | -0.8273      | -0.3502      |
|            | $\sigma=.86$ | 0.8117        | 0.0261     | 0.0191        | 0.0122     | 0.8995        | -0.7297      | -0.3582      |
|            | $\sigma=.95$ | 0.8311        | 0.0242     | 0.0197        | 0.0103     | 0.9079        | -0.6116      | -0.2248      |
| $b=0.48$   | $\sigma=.77$ | 0.7992        | 0.0207     | 0.0100        | 0.0151     | 0.7279        | -0.8906      | -0.3366      |
|            | $\sigma=.86$ | 0.8161        | 0.0227     | <b>0.0142</b> | 0.0138     | <b>0.8168</b> | -0.8036      | -0.3131      |
|            | $\sigma=.95$ | 0.8403        | 0.0224     | 0.0168        | 0.0113     | 0.8715        | -0.6846      | -0.2397      |
| $b=0.53$   | $\sigma=.77$ | 0.8107        | 0.0190     | 0.0075        | 0.0151     | 0.6590        | -0.9273      | -0.3298      |
|            | $\sigma=.86$ | 0.8095        | 0.0200     | 0.0080        | 0.0157     | 0.6755        | -0.9283      | -0.3537      |
|            | $\sigma=.95$ | 0.8412        | 0.0236     | <b>0.0148</b> | 0.0140     | <b>0.8270</b> | -0.8061      | -0.3341      |
| U.S. Data  |              | N/A           | 0.0087     | <b>0.0154</b> | 0.0111     | <b>0.8656</b> | 0.0739       | 0.2414       |

*Notes:* Bold numbers indicate  $b$  (habit formation) and  $\sigma$  (insurance) values that correspond to the bold-bordered diagrams in Figure 4. Simulated series are HP-filtered ( $\lambda = 1600$ ).  $c_t^u/c_t^e$ , the ratio of unemployed to employed consumption, is recovered from the steady state if it cannot be calculated analytically ( $\sigma \neq 1$ ). Second moments reported for U.S. data are calculated directly from time series from FRED and BLS, Q1 1964 through Q2 2010, HP-filtered ( $\lambda = 1600$ ). The opposed effects of the habit and insurance levels are apparent here and in Figure 4. Note how  $c_t^u/c_t^e$  varies even where the simulation is very similar with respect to deviations and correlations.

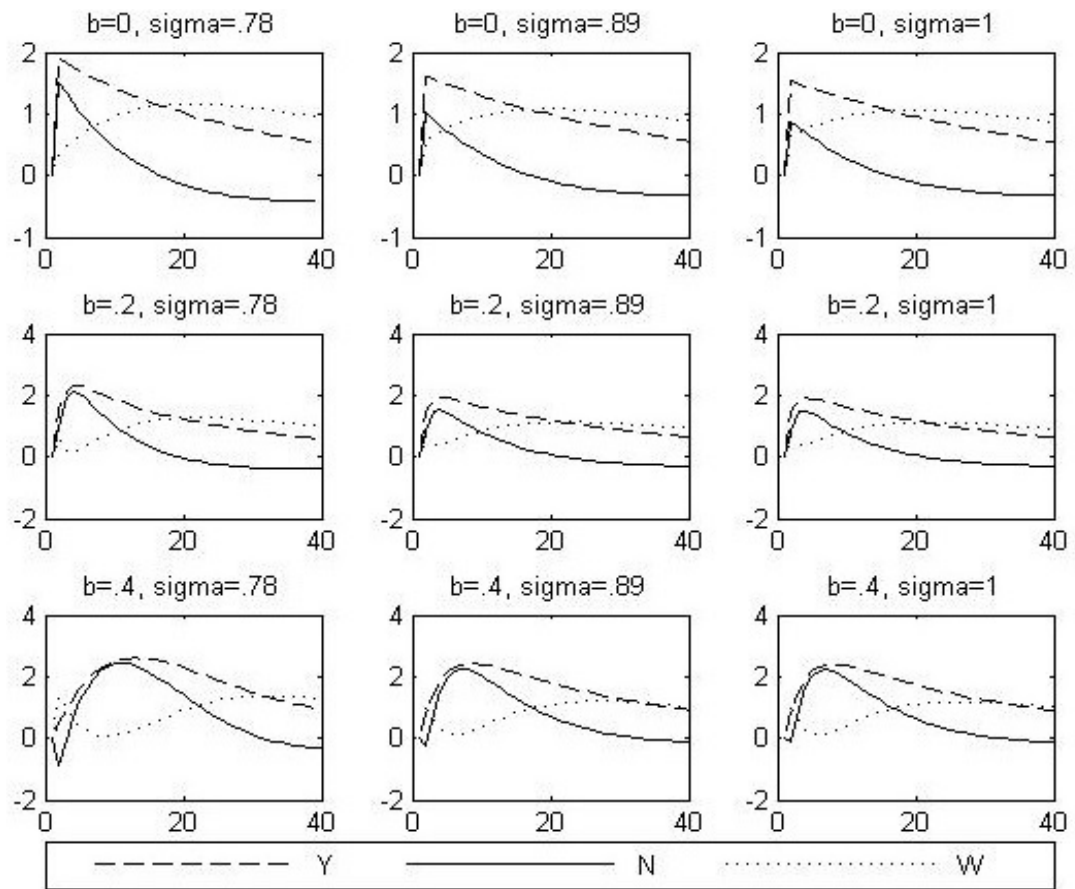
## Appendix F: Figures

Figure 1: Impulse response from fiscal shock.



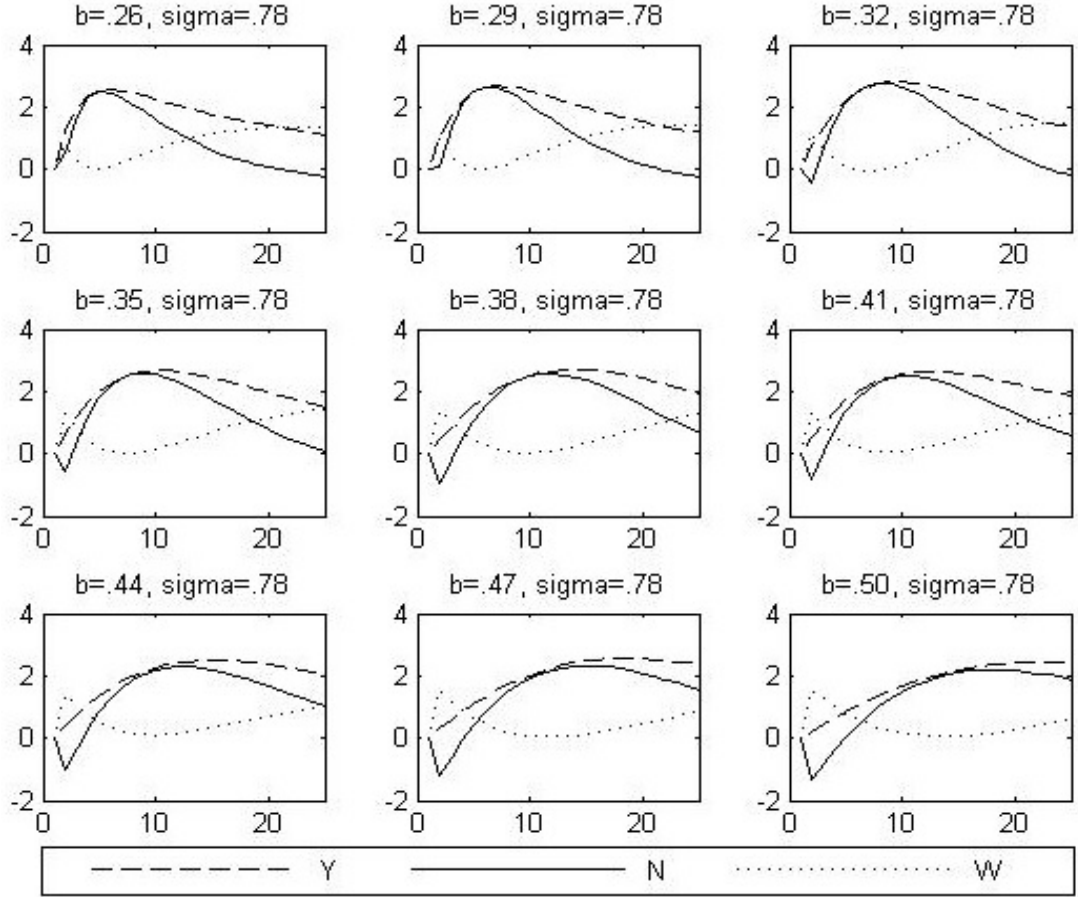
*Notes:* Varying  $b$  (habit formation) and  $\sigma$  (insurance) values yield qualitatively different responses. These diagrams correspond to Table 1. Compare the top, first and third diagrams to Alexopoulos's (2004) partial and full insurance setups. Fiscal shocks are not pictured in figures after this one since the interesting dynamics arise from the technology shocks.

Figure 2: Impulse response from technology shock.



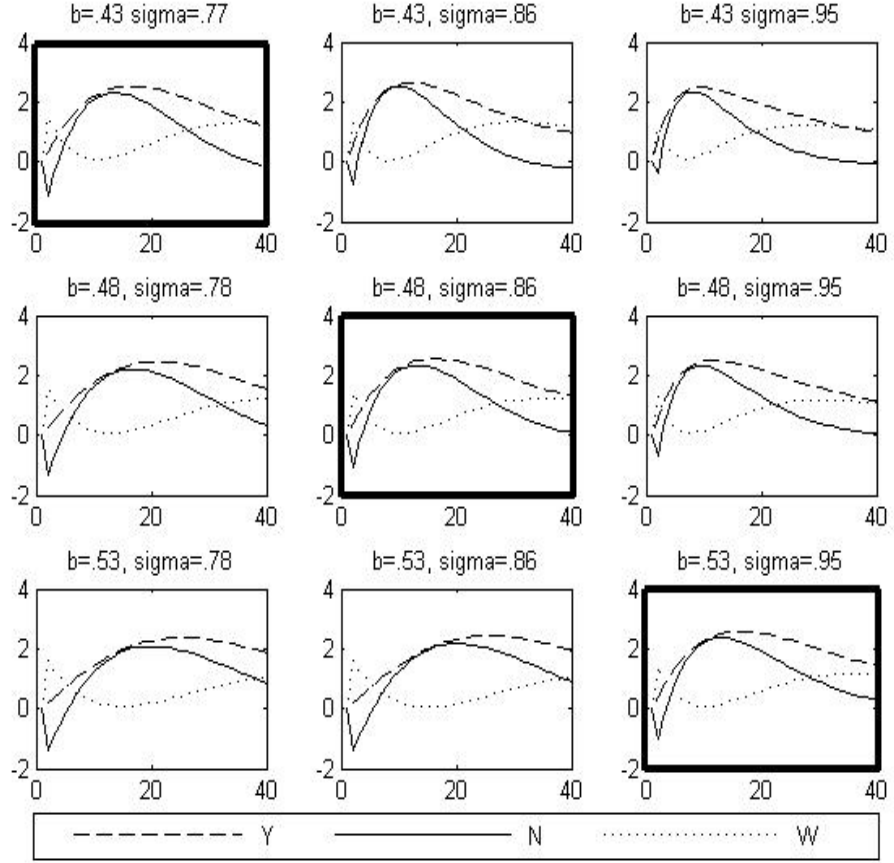
*Notes:* Varying  $b$  (habit formation) and  $\sigma$  (insurance) values yield qualitatively different responses. These diagrams correspond to Table 1. Compare the top, first and third diagrams to Alexopoulos's (2004) partial and full insurance setups.

Figure 3: Impulse response from a technology shock at increasing habit formation ( $b$ ) levels, at an insurance ( $\sigma$ ) level corresponding to Alexopoulos's (2000) partial insurance setup.



*Notes:* A progressively deeper initial drop in  $N$  arises with increased habit formation. Also visible is the lengthening period until the peak response. These effects contribute to the pattern of reduced  $\rho(n, y)$  with increasing habit formation seen in Table 2. A pattern of increasing (counterfactual) negative  $\rho(n, w)$  arises from the wage and employment responses.

Figure 4: Impulse response from a technology shock. Similar dynamics appear at different habit formation ( $b$ ) values for different insurance ( $\sigma$ ) values.



*Notes:* Bold boxes correspond with bold numbers in Table 3. While many dynamics are nearly identical, the implied consumption ratio, unemployed to employed,  $c_t^u/c_t^e$ , differs.

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**CHAPTER THREE: EXPRESSING THE WELFARE  
COST OF WAGE DISCRIMINATION REGULATIONS  
USING A DSGE MODEL WITH HABIT FORMATION  
IN CONSUMPTION AND EFFICIENCY WAGES**



## Introduction

One plausible cause of wage discrimination is an employer's tendency to offer less to a job candidate who is currently unemployed. One cause for this decision by a hiring firm is that such an employee will accept a lower offer. While this is not the only cause of wage discrimination, such a practice would be restricted by any effective regulation meant to curtail wage discrimination. Such is the stated goal of Obama's executive order 13665<sup>1</sup> of 2014, which prohibits retaliation against employees for sharing wage information with each other. Such rules are a common practice where firms desire the option of offering different wages to different candidates for the same work. Henceforth in this paper 'pay transparency' or 'PT' will refer to this executive order. If the firm is unable to restrict this information sharing among employees, there is potentially an additional cost for the practice of wage discrimination. For instance, workers who are aware that they are paid less for the same work might find the original negotiated wage no longer satisfactory, and 'work to the rule' in protest, whereas if they never discover they are paid less they work with normal intensity and morale.

Setting aside questions of the effectiveness of an executive order of this type with such an aim, we can still inquire as to the macroeconomic consequences of such regulation under an assumption that it is effective. I create a model along the lines of Alexopoulos (2004), and Shapiro, Stiglitz (1977), with the addition of habit formation in consumption. Adding habit formation in consumption to a model which has unemployment motivates the firm to offer a different wage to different workers, as a consequence of differing consumption history. In my model, the lower of the two wages goes to workers unemployed in the previous period because habit formation in consumption alters their incentive compatibility constraint (to be abbreviated hereafter as 'IC') which determines the least (and

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<sup>1</sup>Exec. Order. No. 13665, 72 Fed. Reg. 20749 (April 11, 2014), <https://www.gpo.gov/fdsys/pkg/FR-2014-04-11/pdf/2014-08426.pdf>. Hereafter to be referred to as 'pay transparency' or PT

thus profit-maximizing best) wage that the employer can offer. The previously employed majority receives a higher wage. Since the firm decides employment levels, and prefers to pay two wages to two categories of workers, this model is amenable to creating a laboratory which can isolate any differences in dynamics that would arise from restricting the employer's wage decision; thus we can see the effects of prohibiting wage discrimination without placing any restriction on the decisions of other agents in the model. Allowing the labor market to determine wages is here compared to an economy where the firm is required to pay one wage in a scientifically valid experiment which isolates the variable of interest, which is the presence of an exogenous restriction on the firm's wage decision.

PT itself states it is undesirable to '... diminish market efficiency and decrease the likelihood that the most qualified and productive workers are hired at the market efficient price.' My experiments show how such a well-intentioned policy effort may have unintended and costly consequences by creating a fault-line later to be revealed after a negative shock to the U.S economy. In short, the wage-bill effect that wage discrimination allows makes for less costly recessions. I find a large cost when this effect is eliminated by disallowing wage discrimination.

The remainder of this paper first describes the model with endogenous steady-state unemployment due to a shirking efficiency wage, including habit formation in consumption which causes a lagged unemployment influence on present wage costs for employers. Then, an experimental laboratory is built which allows isolating a wage discrimination restriction for a comparison which holds all other factors equal, calibrated to make a valid comparison where one difference is allowed: the firm's choice to wage-discriminate. This can be thought of as an experiment designed to predict an effect that might come from a policy or rule imposed on firms against wage-discrimination, such as pay transparency which, if successful in its stated goals, would hamper any such discrimination. Finally there follows a discussion of the dynamic results in terms of impulse responses and general volatility,

including a compensating variation measure to characterize the cost of such policies in terms of consumption preferences. The results are shown to be robust in terms of simultaneous improved predictions of multiple key macroeconomic variables.

## Model Description

When habit formation in consumption is accounted for in the utility function that determines a worker's incentive compatibility constraint, in a model where there is steady-state unemployment and employment varies on the extensive margin, the previous period's employment level becomes a determinant of the aggregate wage level chosen by the employer, because a portion of the workers requires a lower wage due to differing consumption standards: the previously unemployed. In my model, habit formation in the utility function brings about two wages, when an employer is allowed to make the decision to pay less to the fraction of employees previously consuming less; last period's unemployment rate is that fraction. Without habit formation, the wage difference goes to zero, but when it is present, and the employer can make the distinction, a relation between the present period wage bill and previous employment causes a change in dynamics.

The worker's decision whether to shirk leads to an efficiency wage and steady-state unemployment, exactly as described by Alexopoulos (2004). Insurance and habit formation levels affect shirking decisions, and thus the wage, in addition to affecting investment and consumption. Also, the employer can offer a lower wage to a proportion of the workforce equal to the previous period's unemployment rate, due to those workers' lower reference consumption standard, which alters the IC constraint for those individuals.

This section describes the interactions of three agents who each face a constrained optimization problem. First, the family decides between consumption allocated among the workers (both employed and unemployed) and future capital subject to a budget determined by the total output of the economy. Also, a profit-maximizing firm chooses an employment level, effort level for work, and wage levels for each category of worker. This wage decision is informed by the utility function of the worker in the manner of the shirking efficiency wage model of Shapiro Stiglitz (1977), with partial income insurance as in Alexopoulos (2004).

In the present model, the utility function includes past consumption for the individual worker, and via the proportion of low to high wages in the employer's decision so that the past employment influences the present wage bill taken by the employer. The first order conditions from the solutions to these agent's problems, along with general equilibrium conditions and constraints of the model, give a system of difference equations which represents the laboratory economy. This makes it possible to build simulations which allow comparison to a scenario where wage discrimination is prohibited.

## Family

The family decides between consumption allocated among the workers (both employed and unemployed) and future capital subject to a budget determined by the total output of the economy. In addition, the family extracts income from wage earners to fund insurance to partially replace the wage income lost by the unemployed. Expenditure equals income in equilibrium, and demand and supply for capital are equal. Aggregating the family budget constraint yields  $C_t + G_t + I_t = Y_t$  where  $I_t = K_{t+1} - (1 - \delta)K_t$  and  $Y_t = A_t K_t^\alpha ((N_t) h E_t)^{1-\alpha}$ .

## Utility and consumption reference level definitions

In this model, the family is the agent who makes a trade-off decision between future capital and current consumption, with the goal of maximizing total present and future discounted utility weighted by the proportion of each category of family member or worker. The utility functions of each category of worker will be noted as follows:

$U_t^u(u)$  for unemployed worker this period, who was unemployed last period,

$U_t^u(e)$  for unemployed worker this period, who was employed last period,  
 $U_t^e(u)$  for employed worker this period, who was unemployed last period,  
 $U_t^e(e)$  for employed worker this period, who was unemployed last period

These categories of worker occur in proportions determined by the employment rates last period and this period, so we can express the total utility the family hopes to maximize in terms of these proportions

$$U_t = \left\{ \begin{array}{l} (N_{t-1})(N_t)U_t^e(e) + (1 - N_{t-1})(N_t)U_t^e(u) \\ + (N_{t-1})(1 - N_t)U_t^u(e) + (1 - N_{t-1})(1 - N_t)U_t^u(u) \end{array} \right\} \quad (1)$$

where  $N_t \in [0, 1]$  is the employment level. The utilities for each category depend upon consumption this period, and upon a reference level from last period which is sensitive to previous employment status. Note the reference levels as follows:

$$\bar{c}_t^e = N_{t-1}c_t^e(e) + (1 - N_{t-1})c_t^e(u) \text{ for employed worker,}$$

$$\bar{c}_t^u = N_{t-1}c_t^u(e) + (1 - N_{t-1})c_t^u(u) \text{ for unemployed worker.}$$

These utility portions include habit formation in consumption which is informed by these reference levels depending upon the worker's category, so that we can write the utilities as functions of present and past consumption levels in the following notation:

$$U_t^u(u) = U_t(c_t^u(u), \bar{c}_{t-1}^u)$$

$$U_t^u(e) = U_t(c_t^u(e), \bar{c}_{t-1}^e)$$

$$U_t^e(u) = U_t(c_t^e(u), \bar{c}_{t-1}^u)$$

$$U_t^e(e) = U_t(c_t^e(e), \bar{c}_{t-1}^e)$$

which shows utility as a function of consumption in four categories in the present, and two in the past. The specific form for utility for the employed worker portion is in two forms in proportion to the past employment rate, which differ from each other in the consumption level and consumption reference level inside the consumption part of utility, giving these two:

$$U_t^e(e) = U_t(c_t^e(e), \bar{c}_{t-1}^e) = \ln(c_t^e(e) - b\bar{c}_{t-1}^e) + \theta \ln(T - he_t - \zeta) \quad (2)$$

$$U_t^e(u) = U_t(c_t^e(u), \bar{c}_{t-1}^u) = \ln(c_t^e(u) - b\bar{c}_{t-1}^u) + \theta \ln(T - he_t - \zeta) \quad (3)$$

where  $b$  is the habit formation parameter,  $T$  is the time endowment of an individual,  $e_t$  is the effort level expended,  $h$  is work hours, and  $\zeta$  is the fixed cost of exerting any effort greater than zero. Unemployed utility is based on unemployed consumption, and no disutility from working,

$$U_t^u(e) = U_t(c_t^u(e), \bar{c}_{t-1}^e) = \ln(c_t^u(e) - b\bar{c}_{t-1}^e) + \theta \ln(T) \quad (4)$$

$$U_t^u(u) = U_t(c_t^u(u), \bar{c}_{t-1}^u) = \ln(c_t^u(u) - b\bar{c}_{t-1}^u) + \theta \ln(T) \quad (5)$$

### Partial insurance

The insurance contribution from each member is exogenous to the family, determined by the wage chosen by the representative firm, and  $\sigma$ , the insurance parameter, by the rule

$$F_t = \sigma(1 - N_t)h\bar{w}_t.$$

where using similar notation to that used for consumption,  $\bar{w}_t$  is a representative wage level which is summed from the two current wage levels.

$$\bar{w}_t = N_{t-1}w_t(e) + (1 - N_{t-1})w_t(u)$$

In other words, the Family takes a sufficient contribution from wage-earning workers to replace some set fraction  $\sigma$  of the average wage level. The family decides a consumption contribution  $c_t^f$  to be given to each member. Employed family members contribute an amount  $F_t$  as insurance, to be paid to the unemployed. Thus, consumption levels are

$$c_t^u(e) = c_t^f + \frac{N_t}{1 - N_t} F_t,$$

$$c_t^u(u) = c_t^f + \frac{N_t}{1 - N_t} F_t,$$

for the unemployed,

$$c_t^e(e) = c_t^f + w_t(e)h - F_t$$

$$c_t^e(u) = c_t^f + w_t(u)h - F_t$$

for the employed.

### Family's constrained optimization problem

The family chooses  $c_t^f$  and  $K_{t+1}$  to maximize utility for members. Combining equations (1) through (5),

$$\max_{\{c_t^f, K_{t+1}\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} (N_{t-1}) (N_t) U_t^e(e) + (1 - N_{t-1}) (N_t) U_t^e(u) \\ + (N_{t-1}) (1 - N_t) U_t^u(e) + (1 - N_{t-1}) (1 - N_t) U_t^u(u) \end{array} \right\} \right\},$$

subject to

$$c_t^f \leq [r_t K_t - G_t - [K_{t+1} - (1 - \delta)K_t]]$$

$$\bar{c}_t^e = N_{t-1} c_t^e(e) + (1 - N_{t-1}) c_t^e(u)$$

$$\bar{c}_t^u = N_{t-1} c_t^u(e) + (1 - N_{t-1}) c_t^u(u)$$

$$c_t^e(e) = c_t^f + w_t(e)h - F_t(e)$$

$$c_t^e(u) = c_t^f + w_t(u)h - F_t(u)$$



$$c_t^u(e) = c_t^f + \frac{N_t}{1 - N_t} F_t(e)$$

where  $\delta \in (0, 1]$  is the depreciation rate and  $r_t$  is the rental rate of capital. Maximizing the Lagrangian, the first order conditions are

Family FOC  $C_t^f$

$$\begin{aligned} & \frac{(N_{t-1})(N_t)}{(c_t^e(e) - b\bar{c}_{t-1}^e)} + \beta \frac{(N_t)(N_{t+1})(-b)}{(c_{t+1}^e(e) - b\bar{c}_t^e)} + \frac{(1 - N_{t-1})(N_t)}{(c_t^e(u) - b\bar{c}_{t-1}^u)} \\ & + \frac{(1 - N_t)(N_{t-1})}{(c_t^u(e) - b\bar{c}_{t-1}^e)} + \beta \frac{(1 - N_{t+1})(N_t)(-b)}{(c_{t+1}^u(e) - b\bar{c}_t^e)} + \frac{(1 - N_t)(1 - N_{t-1})}{(c_t^u(u) - b\bar{c}_{t-1}^u)} \\ & + \beta \frac{(1 - N_{t+1})(1 - N_t)(-b)}{(c_{t+1}^u(u) - b\bar{c}_t^u)} + \beta \frac{(1 - N_t)(N_{t+1})(-b)}{(c_{t+1}^e(u) - b\bar{c}_t^u)} = \lambda_t \end{aligned}$$

Family FOC  $K_{t+1}$

$$\lambda_t = \beta E_t \lambda_{t+1} (r_{t+1} + 1 - \delta)$$

substitute  $r_{t+1} = \alpha \frac{Y_{t+1}}{K_{t+1}}$

$$\lambda_t = \beta E_t \lambda_{t+1} \left( \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right)$$

Note how the marginal utility of present consumption is composed of quantities for previously employed or unemployed workers that differ depending on reference levels. If there is only one reference level, or no habit formation ( $b = 0$ ) then the  $(1 - N_{t-1})$  and  $(N_{t-1})$  terms simply add to 1 and we get marginal utilities of present consumption for all employed or all unemployed in proportions determined by  $N_t$ . The discounted future consumption modified by the habit parameter  $b$  is subtracted, accounting for the family anticipating the effect of present consumption on raising the standard for future utility from consumption. This causes consumption smoothing. As consumption-capital bundles are considered by the family, the optimum is reached where the falling marginal utility of consumption meets the rising expected future marginal return on capital.

## Family Members Incentive Compatibility Constraint

Firms have imperfect capability to monitor worker effort, and exerting effort causes disutility for workers. Workers decide whether to shirk. Firms choose the smallest wage such that the expected utility of shirking detection equals the utility from exerting effort. The shirking penalty  $s \in [0, 1]$  is the fraction of the full wage that a detected shirker will receive. The penalty for *detected* shirking, then, is  $w_t(1 - s)$ . Thus the consumption for a detected shirker is

$$c_t^s = c_t^f + sw_th - F_t,$$

which is less than

$$c_t^e = c_t^f + w_th - F_t,$$

for the employed.

The IC (incentive compatibility) constraint will apply to members who are employed. The IC constraint comes from calculating the wage level which will make a worker indifferent between shirking and working. The effort level is a function of the wage. Defining  $\chi_t$  as

$$\chi_t = \frac{c_t^e - b\bar{c}_{t-1}}{c_t^u - b\bar{c}_{t-1}},$$

we can show  $\chi_t$  is constant,<sup>2</sup> determined by the relationship to time endowment, fixed effort cost, utility weight of leisure, detection probability, and shirking penalty defined by

$$\left( (T - \zeta) \chi^{1 + \frac{d}{\theta}} - T\chi \right) (1 - s) = T \left( \frac{d}{\theta} \right) (1 - s\chi)(\chi - 1).$$

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<sup>2</sup>Appendix D Uses the wage FOC from the firm's problem (Solow condition) and the consumption equations to demonstrate this.

The IC constraint arising from the worker's shirking decision is obtained by finding the effort level which makes the worker indifferent between shirking and working at a given wage. The effort level is shown to be constant, because  $\chi_t$  is constant.<sup>3</sup> The effort function is used as a constraint in the firm's problem,

$$E_t = \Phi(w_t) = -\frac{T}{h} (\chi_t)^{-d/\theta} + \frac{T}{h} - \frac{\zeta}{h}.$$

It follows directly from consumption definitions that <sup>4</sup>

$$(1-s)\frac{\chi}{\chi-1}hw_t = c_t^e - b\bar{c}_{t-1}$$

and  $\mu(\sigma) = 1/\chi$ , a function of the insurance parameter  $\sigma$  <sup>5</sup>, following

$$\frac{c_t^u - b\bar{c}_{t-1}}{c_t^e - b\bar{c}_{t-1}} = \mu(\sigma) = 1 - \frac{1-\sigma}{1-s} \left( \frac{\chi_t - 1}{\chi_t} \right).$$

My definition of  $\chi$  differs from that of Alexopoulos (2004) when habit formation is not zero. While  $\chi$  is constant,  $c_t^u/c_t^e$  is not, but I report its steady-state value in my results, for a measure of insurance level which can be compared to other results, or potentially calibrated to observed data, such as the income ratio estimated by Gruber (1997).<sup>6</sup> When  $b=0$ , of course, these variables reduce to the values seen elsewhere and become constant.

Following a similar notation to that used for consumption and wage, we can define  $\chi(u)$  and  $\chi(e)$  and show them to be equal when effort level is held constant.  $w(u)/w(e)$  can be approximated as  $1 - b/1 - b\sigma$ . This ratio of wage levels will be used inside the firm's profit maximizing decision to characterize the wage bill in a way that lends itself to our experimentation with wage discrimination decisions.

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<sup>3</sup>Appendix A.

<sup>4</sup>Appendix B.

<sup>5</sup>Appendix B. It is still true that the partial insurance case, as defined by Alexopoulos (2004), comes about when  $s=\sigma$ , as Givens (2008) shows when he introduces the insurance level parameter  $\sigma$ .

<sup>6</sup>I do not calibrate to this. I recover it from the steady state which yields a certain employment level.

## Firm

A representative family rents capital to a representative firm. The firm chooses employment and wage levels with the objective of maximum profit, knowing a shirking worker produces no output. Employed members decide whether to shirk. Since shirkers do no work, the firm chooses a wage level to just ensure that shirking does not occur.<sup>7</sup> In other words, where a worker is indifferent between shirking and working.<sup>8</sup> The employer chooses the minimum wage to prevent shirking for each category of employed worker, of which there are two, so that two wages are paid,  $w(u)$  and  $w(e)$  in proportions determined by last period's unemployment. Total employment is the sum of employment of workers who were employed last period and workers who were unemployed last period. The employer is forced to hire from these categories in a proportion determined by last period's employment rate, because this is the distribution of types of workers available and they are selected for employment from this distribution. The employer can offer one effort level (the 'same job') but can offer different wages based on knowledge of the history of the worker. In other words, selection for employment and the expected effort level of the job do not consider reference consumption level, but the wage determination does. Defining  $\bar{w}_t$  in a similar to way to that used previously for  $U_t$  and  $c_t$  as  $\bar{w}_t = w_t(e)N_{t-1} + w_t(u)(1 - N_{t-1})$  we can state the employer's constrained optimization problem in either of two equivalent ways, using  $\bar{w}_t$  or its expression in the two wage levels, since their proportion can be used for substitution.

A representative firm chooses two wages, total employment level, effort level, and a capital level, with the objective of maximum profit. Costs are the wage bill and capital rental. Note that the total employment level choice is informed

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<sup>7</sup>This is a result of the worker utility maximization problem described in detail by Alexopoulos (2004).

<sup>8</sup>I derive the function for effort in terms of wage from the indifference condition in Appendix A.

by employment levels of previously employed and unemployed workers due to the hiring proportion constraint. The total employment level and contracted effort level are decided with knowledge of the proportion of worker types that will be selected due to last period's employment levels. the representative firm cannot alter this proportion, but can offer different wages to workers once they are selected for hire. The firm chooses a wage, an employment level, an effort level, and capital, to maximize profit. Costs are the wage bill and capital rental. The production function,

$$Y_t = A_t K_t^\alpha ((N_t) h E_t)^{1-\alpha},$$

combines effective labor and capital.<sup>9</sup> The incentive compatibility constraint,

$$E_t = \Phi(w_t(e)) = -\frac{T}{h} (\chi_t)^{-d/\theta} + \frac{T}{h} - \frac{\zeta}{h} = \Phi(w_t(u)) = -\frac{T}{h} (\chi_t)^{-d/\theta} + \frac{T}{h} - \frac{\zeta}{h}$$

enters the firm's decision as a binding constraint on the wages. The smallest wage that will prevent shirking is chosen. The firm maximizes profit by solving

$$\max_{\{\bar{w}_t, N_t, E_t, K_t\}} (A_t K_t^\alpha (h E_t N_t)^{1-\alpha} - \bar{w}_t h N_t - r_t K_t).$$

Where

$$\bar{w}_t = w_t(e) N_{t-1} + w_t(u) (1 - N_{t-1})$$

Maximizing profit yields the first order conditions<sup>10</sup>,

$$r_{t+1} = \alpha \frac{Y_{t+1}}{K_{t+1}},$$

$$(1 - \alpha) A_t K_t^\alpha (h E_t N_t)^{-\alpha} h E_t = \bar{w}_t h, \text{ and}$$

$$\frac{(\bar{w}_t) \Phi'(\bar{w}_t)}{\Phi(\bar{w}_t)} = 1.$$

We can also express the problem in terms of  $w_t(e)$  and  $w_t(u)$

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<sup>9</sup>The production function incorporates the finding that no shirking happens in equilibrium, so there is only one employment level and it fully contributes to production.

<sup>10</sup>Appendix C shows derivation of the Solow condition.

$$\left( \begin{array}{c} \max \\ \{w_t(e), N_t, E_t, \\ w_t(u), \\ K_t\} \end{array} \right) \left( \begin{array}{c} A_t K_t^\alpha (h E_t N_t)^{1-\alpha} \\ -w_t(e) h N_t N_{t-1} - w_t(u) h N_t (1 - N_{t-1}) - r_t K_t \end{array} \right)$$

yielding

$$\alpha A_t K_t^{\alpha-1} (h E_t N_t)^{1-\alpha} = r_t$$

$$r_{t+1} = \alpha \frac{Y_{t+1}}{K_{t+1}}$$

$$(1 - \alpha) A_t K_t^\alpha (h E_t N_t)^{-\alpha} h E_t = w_t(e) h N_{t-1} + w_t(u) h (1 - N_{t-1})$$

$$\frac{(w_t(e) h N_{t-1} + w_t(u) h (1 - N_{t-1})) \Phi'(w_t(e))}{E_t} = N_{t-1}$$

$$\frac{(w_t(e) h N_{t-1} + w_t(u) h (1 - N_{t-1})) \Phi'(w_t(u))}{E_t} = (1 - N_{t-1})$$

Since we know that  $w_t(u)/w_t(e) = 1 - b/1 - \sigma b$  we can express  $\bar{w}_t$  in terms only of  $w(e)$ . Choosing either  $\bar{w}_t$  or  $w(e)$  chooses both wages, since the employer takes  $N_{t-1}$  as exogenous. To create an economy where wage discrimination is exogenously disallowed, we need only to change the definition of  $\bar{w}$  to include only  $w(e)$ , the higher wage, which guarantees non-shirking for all employment candidates, and complies with the rule that every employee must get the same wage for the same work. This is a higher wage bill for any  $N_t$  level chosen by the firm. The dynamic consequence of removing  $N_{t-1}$  from firm's decision in this way is what we will observe in our lab and discuss in the results section. Note that modeling the wage discrimination rule in this way allows an experiment where no other agent's decisions are exogenously altered, which makes for a good model of the effects of a wage discrimination regulation such as PT.

The model is a system of difference equations in our variables which allows defining an alternative model with a wage-discrimination prohibition by restricting the wage paid by the firm.  $\bar{w}_t$ , is composed of  $w_t(u)$  and  $w_t(e)$  in a proportion determined by  $N_{t-1}$  so the wage bill looks backwards in proportions that vary

with past  $N$ :  $-w_t(e)hN_tN_{t-1} - w_t(u)hN_t(1 - N_{t-1})$  Since we know the proportion of  $w_t(e)/w_t(u) = (1 - b\sigma) / (1 - b)$ ,  $-w_t(e)hN_tN_{t-1} - \frac{w_t(e)(1-b)}{(1-b\sigma)}hN_t(1 - N_{t-1})$  is the wage bill  $\bar{w}_t$  If we require the firm to pay one wage, and that wage must be high enough to prevent anyone shirking, then  $\bar{w}_t$  becomes simply  $w_t(e)$  and  $N_{t-1}$  is no longer part of the firm's decision, yielding the dynamic changes here reported.

## Laboratory and Experiment

This section describes making a stationary linear approximation of my economy that lends itself to experimental testing of scenarios with and without a wage discrimination rule such as PT. Isolating  $\bar{w}_t$ , the wage bill rate, as an experimental variable allows insight into the dynamic effects of such a change. This experiment places no regulation on any other agents besides the firm, so it is a good scientific test.

To focus on the short-run dynamics of employment, output, and wages, in simulations with technology and fiscal shocks, I solve a linearized system in log deviations from the steady states and use the solution to simulate an economy. I describe the results of simulations, discussing the effects of habit formation and insurance levels on output volatility and output-employment correlation. The tables and figures I provide can be compared directly to Alexopoulos's (2004) findings. Indeed, the  $b = 0$  case is computationally equivalent. The restriction is added that the employer must offer the 'same wage for the same work' which is the stated purpose of regulations meant to enforce wage equality or discourage 'wage discrimination.' Impulse response diagrams that accompany experiments that add negative fiscal and technology shocks to models that are equivalent with respect to everything but whether the employer can offer the second, lower wage reveal a large difference in the economy's ability to recover from such shocks, and this is clearly the result of the wage restriction. The divergence of the two cases only emerges in the time after such a shock, being not apparent in the steady state, so can be thought of as a kind of invisible fault line, a weakness that is revealed only when equilibrium is disrupted, causing a deeper and more persistent cost of the shock in one case than in the other.

It is relatively simple to construct a controlled experiment using this model, by simply isolating the dynamic effect of the wage differential and then comparing to a model where wage differential is exogenously disallowed. Wage discrimination



prohibition is the desired effect of some public policies, including Obama's 2014 PT, so this model and laboratory economy create an opportunity to see long-term macrodynamic effects arising from such restrictions.

## Calibration

In each simulation I solve for parameters affected by changes in the utility function due to adding habit formation in order to yield a steady state employment rate of 0.941.<sup>11</sup> Other calibrations are borrowed from Alexopoulos's (2004) GMM estimates. Yet others are taken from other literature following Alexopoulos (2004). I depart from her estimates where necessary, since the utility function is altered under habit formation ( $b > 0$ ).  $d/\theta$  is affected, taking a different value under different values of  $b$ , for instance. Since  $(c_t^u - b\bar{c}_{t-1}) / (c_t^e - b\bar{c}_{t-1})$  is constant,  $c_t^u/c_t^e$  is not constant. I solve for  $\chi_t = (c_t^e - b\bar{c}_{t-1}) / (c_t^u - b\bar{c}_{t-1})$  at each parameter set considered, because it is a constant value. If desired, one could calibrate to a  $c_t^u/c_t^e$  as a steady state,<sup>12</sup> but in this study the dynamics of consumption are not of direct interest, so I calibrate to an employment steady state. I report  $c_t^u/c_t^e$  in all tables, and the trends in  $c_t^u/c_t^e$  with changes in  $b$  and  $\sigma$  are not surprising. My constant  $\chi$  (constant for given  $\sigma$  and  $b$ ) does not indicate a simple consumption ratio under any degree of habit formation besides zero. I use this relationship,

$$\left( (T - \zeta) \chi^{1+\frac{d}{\theta}} - T\chi \right) (1 - s) = T \left( \frac{d}{\theta} \right) (1 - s\chi)(\chi - 1),$$

to solve for  $\chi$ ,  $d/\theta$ , and  $s$  in order to yield steady state  $N = 0.941$ , at each set of  $\sigma$  and  $b$  parameters considered. The parameters that do not change with changes

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<sup>11</sup>This is the average from U.S. historical data.

<sup>12</sup>Alexopoulos (2004) does so, using a ratio derived from estimates by Gruber (1995). Givens (2008) generalizes to account for different values of this ratio, which characterizes the insurance level, since such a ratio is not directly measurable.

in  $b$  and  $\sigma$ , and their values, are  $\{\beta = 0.9796, \delta = 0.0203, \sigma_g = 0.0133, \sigma_a = 0.0074, \rho_g = 0.9797, \rho_a = 0.9699, \alpha = 0.4574, \zeta = 16, T = 1369, \log(g/y) = -1.6870\}$ .

## Solution

I am interested in the dynamics of  $W$  (wage),  $N$  (employment), and  $Y$  (output), in a simulation using the estimated variances and persistence of the government spending and technology disturbances.<sup>13</sup> The linearized system is solved, and this solution is used to generate simulations. The results are expressed in log deviations from the steady state. I generate impulse response experiments to provide insight into mechanisms for changes in second moments at different insurance and habit formation levels. For reference, I present the linear system here. Steady states are recalculated at every set of  $\sigma$  (insurance) and  $b$  (habit formation) values, so the constants in these equations differ for different values, where the steady states appear in the equations. Klein's (2000) method yields a state-space solution in the predetermined and non-predetermined variable vectors.  $G_t$  and  $A_t$  are the exogenous forcing factors.<sup>14</sup> Defining  $\tilde{x}_t = \ln x_t - \ln x$ , and using no subscript to indicate a steady-state value, the linearized system contains the equations

$$\tilde{A}_t + \alpha \tilde{K}_t + (1 - \alpha) e \tilde{N}_t = \tilde{Y}_t, \quad (6)$$

$$\bar{c} \tilde{c}_t + G \tilde{G}_t + K \tilde{K}_{t+1} - (1 - \delta) K \tilde{K}_t = Y \tilde{Y}_t, \quad (7)$$

$$\left( \frac{N}{c^e - b\bar{c}} - \frac{N}{c^u - b\bar{c}} \right) \tilde{N}_t - \frac{N}{(c^e - b\bar{c})} \tilde{c}_t^e - \frac{(1 - N)}{(c^u - b\bar{c})} \tilde{c}_t^u + \left( b \frac{N}{(c^e - b\bar{c})} + b \frac{(1 - N)}{(c^u - b\bar{c})} \right) \tilde{c}_{t-1} = \lambda \tilde{c}_t, \quad (8)$$

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<sup>13</sup>Here I depart somewhat from Alexopoulos's model. She incorporates technology growth and a related government spending growth, where I have made the two series stationary and uncorrelated. In terms of implementing Klein's solution method: the auto-correlation matrix of the forcing factors vector is diagonal.

<sup>14</sup>The  $\sigma$  and  $\rho$  values are taken from Alexopoulos (2004). See the calibration section.

$$\tilde{\lambda}_t = \beta \left( \alpha \frac{Y}{K} + 1 - \delta \right) \tilde{\lambda}_{t+1} + \beta \alpha \frac{Y}{K} \tilde{Y}_{t+1} - \beta \alpha \frac{Y}{K} \tilde{K}_{t+1}, \quad (9)$$

$$\tilde{Y}_t - \tilde{N}_t = \tilde{w}_t, \quad (10)$$

$$\tilde{Y}_t - \tilde{N}_t = \tilde{w}_t - b\sigma \tilde{N}_{t-1}, \quad (11)$$

$$(1-s) \frac{\chi}{\chi-1} h w \tilde{w}_t = c^e \tilde{c}_t^e - b \bar{c} \tilde{c}_{t-1}, \quad (12)$$

$$c^u \tilde{c}_t^u - b \bar{c} \tilde{c}_{t-1} = \left( 1 - \frac{1-\sigma}{1-s} \left( \frac{\chi-1}{\chi} \right) \right) (c^e \tilde{c}_t^e - b \bar{c} \tilde{c}_{t-1}), \text{ and} \quad (13)$$

$$(N c^e - N c^u) \tilde{N}_t + N c^e \tilde{c}_t^e + (1-N) c^u \tilde{c}_t^u = \bar{c} \tilde{c}_t. \quad (14)$$

(10) or (11) is in effect when wage discrimination is prohibited or not prohibited, respectively.

## Simulation

Government expenditures  $G_t$  appears as a cost (tax) to the family, and is added to simulate fiscal shocks. The technology coefficient in the production function  $A_t$  is used to simulate technology shocks. These are modeled with the AR processes

$$\tilde{G}_t = \rho_A \tilde{G}_{t-1} + \epsilon_{G_t}, \text{ and}$$

$$\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \epsilon_{A_t},$$

where  $\epsilon_{A_t}$  and  $\epsilon_{G_t}$  are serially uncorrelated innovations with mean zero and standard deviations  $\sigma_a$  and  $\sigma_g$ .<sup>15</sup> I use parameter values from Alexopoulos's (2004) GMM estimation.<sup>16</sup> Second moments are taken from repeated simulations, with

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<sup>15</sup>The equations presented here are expressed as log deviations from mean to equate to the linearized system presented in the solution section, while Alexopoulos (2004) writes them in levels. The two are exactly equivalent ( $\rho$  values are the same.)

<sup>16</sup>My simulation differs from hers in that I do not include technology and government spending growth. My technology and government spending shocks are independent of one-another. The results I am interested in are not affected.

the series HP-filtered ( $\lambda = 1600$ ). I also supply impulse responses to aid the discussion of mechanisms for the second moment variation observed with different levels of habit formation and insurance. I report  $c_t^u/c_t^e$  values recovered from the steady state consumption values, to see what the implied insurance level is in those terms.

## Results

The lab described in section 3 allows us to ask questions and run experiments around the firm's wage and employment decisions, to detect dynamic consequences of exogenous restrictions such as pay transparency rules. We can also derive an intuitive and rigorous quantitative welfare measure using the representative family's utility function and impulse-response time series. When I compare artificial economies that are identical except for being with and without wage discrimination allowed by the firm, a one-standard-deviation negative technology shock causes welfare loss that would be offset by a .27% consumption increase for 180 quarters after the shock. Finally, I run simulations with stochastic technology and fiscal elements to get correlations and second moments for key macroeconomic measures for comparison to previous models in the literature and my model with and without the wage discrimination prohibition, showing some improvement in predictions in some areas, such as the wage-employment correlation. The predictive and explanatory value of my lab is demonstrated by the meaningful and robust results here described.

### The Effect of Adding Habit Formation at Different Insurance Levels

Habit formation produces a 'hump-shaped' response to shocks. Since agents derive consumption utility relative to a reference level from the previous period, the response to shocks is delayed and smoothed. This effect of habit formation is known. In the absence of habit formation, the insurance effect appears as a change in the magnitude of fiscal-shock movements, by affecting the punishment associated with shirking, as described in Alexopoulos (2004). Under technology shocks, lower insurance levels dampen and delay the initial positive wage effect by a shirking penalty effect as well. As Alexopoulos (2004) explains,  $c_t^f$  increases

more or less based on the relative magnitudes of investment increases and wage increases due to increased marginal product of labor.

The response of employment to a technology shock is affected in a way which is a consequence of the dynamics arising from imperfect effort monitoring. Adding habit formation in the presence of partial insurance has two effects which increase the initial wage response, and dampen the initial employment response, to the point of making it negative at high enough habit levels. First, the delayed investment response makes the shirking penalty effect on wages more pronounced earlier through its relative effect on initial  $c_t^f$ . Second, the shirking penalty effect itself is larger. The employer adjusts wages to keep effort constant, as before. Now, though, the constant  $\chi$  includes previous consumption.  $\chi_t = (c_t^e - b\bar{c}_{t-1}) / (c_t^s - b\bar{c}_{t-1})$ , so with greater  $b$  the initial wage increase is greater, to the point that the corresponding decrease in employment initially overwhelms the increase due to the marginal product of labor. Table 1 and Figure 1 show the pattern that emerges with increasing habit formation.

### **The Dynamic Effect of Restricting Wage Discrimination**

Because restriction of wage discrimination can be placed entirely in the definition of  $\bar{w}$  we can construct a controlled test of restricting the employer's wage decision. Figure 2 shows four variables in the the aftermath of a negative technology shock, in each model, calibrated to the same steady-state unemployment. My results show that a wage restriction makes for deeper and more persistent departures from the steady state after a technology shock. The difference is not all that surprising, because it is due to a positive relationship between the lagged employment rate and the total wage bill incurred by the firm at any given current employment level. Prohibiting wage discrimination removes this effect by forcing

the employer to pay the same wage regardless of past employment status. Something like PT intends to reduce wage discrimination for various stated reasons. One source of wage discrimination is generated by my theory and model. The employer will choose  $N_t$  based on the knowledge that  $(1 - N_{t-1})N_t$  of employment can be paid at a lower wage. So a proportion of workers determined by last period's unemployment will receive a lower wage. If regulations prohibit wage discrimination, then the firm is forced to pay the higher wage to everyone. I construct an experiment by modifying the model to force the firm to pay one wage to all workers.

The policy motivation for restricting wage discrimination is a general idea of 'fairness,' the 'same pay for the same work.' Like any intervention by authority into a market of free agents making their own negotiations, success of the policy goals will assist some individuals at the relative expense of others. In this model it is easy to characterize the winners and losers by looking at the wage bill chosen by the firm and how the wage is distributed between two categories of workers, the previously employed and the previously unemployed. Take one parameterization (the one I use for my experiment) where in the unregulated wage model we have two categories. The ratio of the low wage to the high wage, recall, is  $1 - b/1 - \sigma b$ , which is .89 when  $b = 0.4$  and  $\sigma = 0.79$ . In the steady state, the high to the low is paid in rate of roughly 95:5. It is easy to calculate that a successful flattening of the wage rate results in everyone making about .98 of what only the previously employed majority made before.

This trade-off may not appear so bad in the steady state, but a different cost to the whole economy emerges and deepens and worsens in two ways in the period after a negative shock. First, unemployment is higher in the period after a negative shock, so the lower wage is paid to a bigger fraction of the workforce during the recovery period. Second, and the main finding in my experiment, is the severe slowing of the process of returning to the steady state. In a recession,

unemployment and the suffering that goes with it persist much longer and are more severe, after a given shock, in a restricted (equal) wage scenario. Output follows a similar path, so the same negative shock causes a greater loss of growth while the economy recovers.

## Second Moments Comparison

Alexopoulos's (2004) partial insurance model can be taken as a special case of mine. In terms of a comparison to those results, my model generates an improvement in output and employment dynamics that comes at the cost of a counterfactual negative wage-employment correlation and increased volatility of both wages and employment. Table 2 includes a row with the  $b=0$  case that can be compared to results including habit formation. Once habit formation is added, two versions of the model can be compared, depending on whether the wage differential resulting from habit formation is allowed. Focusing on employment volatility and wage-output correlation, it is clear that the predictions of the two-wage model are an improvement. In every key macroeconomic variable in table 2, the two-wage habit formation model improves in terms of simulations matching the U.S. data. The data in this table are from before 2014, a world where wage discrimination is not reduced by PT. The two wage model is what my theory predicts in the world before PT is added. The treatment in the experiment is to add a wage-discrimination restriction such as PT. The general higher volatility and counter-cyclical wages are a predicted result of adding a prohibition on wage discrimination.



## Welfare Cost Measure

What would the representative family pay to live in the less regulated economy? I find a welfare cost expressed in terms of compensating variation in consumption. This is the extra consumption required to make the family equally willing to live in the economy with a wage discrimination prohibition. We can take the IR time series discussed in 4.2 and depicted in Figure 2, and supply the values back into the utility function to find the consumption change needed to make them equal. Simplifying our ability to make comparisons, I have calibrated each instance of the model to the same steady-state employment level, and because the only difference between the two is a simple restriction on the firm's hiring decision, again, we have a valid experimental method. The utility functions for family decisions are the same, so a compensating variation method of measuring the welfare cost of the wage restriction in the recovery from a negative shock is an intuitive and valid way to express the effect.

Recall our utility functions

$$U_t^e(e) = U_t(c_t^e(e), \bar{c}_{t-1}^e) = \ln(c_t^e(e) - b\bar{c}_{t-1}^e) + \theta \ln(T - he_t - \zeta)$$

$$U_t^e(u) = U_t(c_t^e(u), \bar{c}_{t-1}^u) = \ln(c_t^e(u) - b\bar{c}_{t-1}^u) + \theta \ln(T - he_t - \zeta)$$

$$U_t^u(e) = U_t(c_t^u(e), \bar{c}_{t-1}^e) = \ln(c_t^u(e) - b\bar{c}_{t-1}^e) + \theta \ln(T)$$

$$U_t^u(u) = U_t(c_t^u(u), \bar{c}_{t-1}^u) = \ln(c_t^u(u) - b\bar{c}_{t-1}^u) + \theta \ln(T)$$

where  $b$  is the habit formation parameter,  $T$  is the time endowment of an individual,  $e_t$  is the effort level expended,  $h$  is work hours, and  $\zeta$  is the fixed cost of exerting any effort greater than zero, combined thus

$$U_t = \left\{ \begin{array}{l} (N_{t-1}) (N_t) U_t^e(e) + (1 - N_{t-1}) (N_t) U_t^e(u) \\ + (N_{t-1}) (1 - N_t) U_t^u(e) + (1 - N_{t-1}) (1 - N_t) U_t^u(u) \end{array} \right\}$$

Define delta as a percent increase multiplier on the present consumption of the family. We can take our IR time series and solve for delta to get equal present and discounted future total utility equal to that of the other model. Solving, that is, for  $\Delta$  such that

$$\begin{aligned} & \sum_{tr=0}^{180} \beta^t \left\{ \begin{array}{l} (N_{tr-1}) (N_{tr}) U_{tr}^e(e, \Delta) + (1 - N_{tr-1}) (N_{tr}) U_{tr}^e(u, \Delta) \\ + (N_{tr-1}) (1 - N_{tr}) U_{tr}^u(e, \Delta) + (1 - N_{tr-1}) (1 - N_{tr}) U_{tr}^u(u, \Delta) \end{array} \right\} \\ &= \sum_{t0=0}^{180} \beta^t \left\{ \begin{array}{l} (N_{t0-1}) (N_{t0}) U_{t0}^e(e) + (1 - N_{t0-1}) (N_{t0}) U_{t0}^e(u) \\ + (N_{t0-1}) (1 - N_{t0}) U_{t0}^u(e) + (1 - N_{t0-1}) (1 - N_{t0}) U_{t0}^u(u) \end{array} \right\} \end{aligned}$$

where

$$U_t^e(e, \Delta) = U_t(c_t^e(e), \bar{c}_{t-1}^e) = \ln(c_t^e(e)(1 + \Delta) - b\bar{c}_{t-1}^e) + \theta \ln(T - he_t - \zeta)$$

$$U_t^e(u, \Delta) = U_t(c_t^e(u), \bar{c}_{t-1}^u) = \ln(c_t^e(u)(1 + \Delta) - b\bar{c}_{t-1}^u) + \theta \ln(T - he_t - \zeta)$$

$$U_t^u(e, \Delta) = U_t(c_t^u(e), \bar{c}_{t-1}^e) = \ln(c_t^u(e)(1 + \Delta) - b\bar{c}_{t-1}^e) + \theta \ln(T)$$

$$U_t^u(u, \Delta) = U_t(c_t^u(u), \bar{c}_{t-1}^u) = \ln(c_t^u(u)(1 + \Delta) - b\bar{c}_{t-1}^u) + \theta \ln(T)$$

and the  $t0$  subscript indexes the time series from the non-regulated economy, and  $tr$  subscript indicates the economy with the exogenous wage discrimination prohibition. Upon doing this, we find the welfare cost, so measured, is  $\Delta = .27\%$  consumption increase per quarter to compensate the wage-regulated economy dweller to the same satisfaction as the non-wage-regulated economy dweller, taken over a period of 180 quarters after a one standard deviation technology shock. This is a significant finding, since if average US consumption is \$58,000, then this amounts to about 1.0% or \$580 per year. With 126 million U.S. households, that

is \$73 billion annually. This annual compensating variation in consumption is a good measure because it is experimentally valid, as well as intuitively meaningful and expressed in a dollar cost, and therefore useful for informing policy decisions and discussions. For perspective, consider that the expected benefit to an average family of the largest tax cut in history has been stated as \$1158 per year<sup>17</sup>

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<sup>17</sup>Speaker Ryan's Floor Speech on Tax Reform Legislation, November 16, 2017

## Conclusion

In this paper a DSGE model that includes a shirking efficiency wage with partial insurance and habit formation leading to a two-wage economy allows the testing by simulation of the effect of an exogenous restriction on wage discrimination by the representative firm. Examining dynamic effects of habit formation and variable insurance in a shirking efficiency wage model shows the effects of habit formation in a model with structural unemployment. Examination of second moments, along with impulse responses for technology and fiscal shocks, shows the interaction of habit formation and imperfect monitoring. Adding habit formation while not restricting the two-wage decision yields an artificial economy that lends itself to a good experimental design because it is possible to restrict the firm's wage decision without making any change to the other agents in the model. A policy like PT also has the property of being a restriction only on employers. The main finding of this paper is that there is predictive and explanatory value to modeling the theory that wages are affected by past consumption via the firm's hiring decision in this way. Improvements in multiple macroeconomic measures lend some credibility to the predictions made here regarding unintended consequences of wage-equality policies. The magnitude of the cost is large, and in a future with longer recessions due to policy changes of this type, this study will become even more valuable to help explain them.

Two features of this laboratory demonstration constitute its value as to inform public policy. First, it demonstrates features of a recovery from recession, dynamics not visible in the steady state but which we are interested in having some warning about before a crisis - fault-lines, so to speak. Second, if we hope to evaluate possible effects of a policy change such as PT which hopes to curtail wage discrimination before it is implemented, then this type of study is important. It gives some persuasive warning in terms understandable by policymakers, of a fault-line potentially created by wage regulations imposed on firms. It shows

hidden long-run consequences of well-intentioned policies such as that of wage fairness behind PT. Because this cost appears in the short-run after a negative shock, and is not detectable until a recession happens, we have here evidence of a fault line that can be created by well-intentioned policies aimed at discouraging wage discrimination, a hidden dynamic cost beyond a static analysis of winners and losers under any new rule. This study can inform policy decisions as a warning of the possibility of deep long-term costs from such policies.

Empirical inquiries which might further support the usefulness of this type of model would include formal testing of the fit of this model against other proposed sources of labor market friction. In addition, a study evaluating whether there is a wage premium associated with jobs that are less closely monitored for performance would lend some support to this type of model. Finally, any empirical demonstration of a wage differential resulting from differences in workers' recent employment status could be taken to support a wage dispersion dynamic arising from heterogeneous work histories.

This model and these predictions of the results of policy changes will inform and enlighten future study of the effects of PT and other well-intentioned policies that aim for fairness and equality but may create costly fault-lines that emerge when the economy is stressed. The compensating variation measure developed here shows a cost of this policy change of \$580 a year for 45 years, which could be compared, for perspective, with the \$1158 annual benefit from the 2018 tax cut for a similar average family. In this paper a cost measure is developed which will be useful to policy makers for weighing against less quantifiable political or ethical goals such as fairness or equality. This cost may not outweigh such considerations, but the decision and debate can now be informed and warned of this potential unintended consequence of well-meaning reforms.

## **APPENDICES CHAPTER THREE**

## Appendix A: Deriving the incentive compatibility constraint

This function, contracted effort as a function of wage, constrains the contracted effort choice of the firm. Setting utility from working equal to utility expected from shirking, we arrive at a function for effort.

$$\begin{aligned}
 U(c_t^e - b\bar{c}_{t-1}, E_t) &= dU(c_t^s - b\bar{c}_{t-1}, 0) + (1 - d)U(c_t^e - b\bar{c}_{t-1}, 0) \\
 \ln(c_t^e - b\bar{c}_{t-1}) + \theta \ln(T - (hE_t + \zeta)) &= \\
 d(\ln(c_t^s - b\bar{c}_{t-1}) + \theta \ln(T)) + (1 - d)(\ln(c_t^e - b\bar{c}_{t-1}) + \theta \ln(T)) \\
 \ln(c_t^e - b\bar{c}_{t-1}) - (1 - d)(\ln(c_t^e - b\bar{c}_{t-1}) - d(\ln(c_t^s - b\bar{c}_{t-1})) &= \theta \ln(T) - \theta \ln(T - \\
 (hE_t + \zeta)) \\
 \left( \frac{c_t^e - b\bar{c}_{t-1}}{c_t^s - b\bar{c}_{t-1}} \right)^d &= \left( \frac{T}{T - (hE_t + \zeta)} \right)^\theta \\
 E_t = \Phi(w_t) &= -\frac{T}{h} \left( \frac{c_t^e - b\bar{c}_{t-1}}{c_t^s - b\bar{c}_{t-1}} \right)^{-d/\theta} + \frac{T}{h} - \frac{\zeta}{h}.
 \end{aligned}$$

I define  $\chi_t = (c_t^e - b\bar{c}_{t-1}) / (c_t^s - b\bar{c}_{t-1})$ , equivalent to the ratio used by Alexopoulos (2004) only when habit formation is zero.  $c_t^e/c_t^s$  is not constant, but  $\chi_t$  is. So  $E_t = \Phi(w_t) = -\frac{T}{h} (\chi_t)^{-d/\theta} + \frac{T}{h} - \frac{\zeta}{h}$  represents effort as a function of the wage and exogenous constants. It can be used as a constraint on the firms problem.

## Appendix B: Deriving modified consumption ratios

An expression for wage is used in the demonstration that  $\chi_t$  is constant. It comes directly from the consumption and insurance definitions.

$$\begin{aligned} c_t^s &= c_t^f + sw_t h - F_t \\ c_t^e &= c_t^f + w_t h - F_t \\ c_t^e - c_t^s &= (1-s)hw_t \\ hw_t &= \frac{(c_t^e - b\bar{c}_{t-1}) - (c_t^s - b\bar{c}_{t-1})}{(1-s)} \\ (1-s)\frac{\chi_t}{\chi_t-1}hw_t &= (c_t^e - b\bar{c}_{t-1}). \end{aligned}$$

To derive the modified consumption ratio which is constant in a model with habit formation, we use consumption and insurance definitions exactly like those of Givens (2008), with the addition of habit formation to the definition of  $\mu$ .  $\mu = 1/\chi$  and is a function of  $\sigma$  and  $b$ .

$$\begin{aligned} (1-s)\frac{\chi_t}{\chi_t-1}hw_t &= (c_t^e - b\bar{c}_{t-1}) \\ hw_t &= \frac{\chi_t-1}{\chi_t} \frac{1}{(1-s)} (c_t^e - b\bar{c}_{t-1}) \\ f_t &= \sigma(1-N_t)hw_t \\ c_t^u &= c_t^f + N_t\sigma hw_t \\ c_t^e &= c_t^f + hw_t - f_t \\ c_t^s &= c_t^f + shw_t - f_t \\ c_t^e &= c_t^f + hw_t - \sigma(1-N_t)hw_t \\ c_t^f &= c_t^e - hw_t + \sigma(1-N_t)hw_t \\ c_t^u &= c_t^e - hw_t + \sigma(1-N_t)hw_t + N_t\sigma hw_t \\ c_t^u &= c_t^e - \left( \frac{\chi_t-1}{\chi_t} \frac{1}{(1-s)} (c_t^e - b\bar{c}_{t-1}) \right) \\ &\quad + \sigma(1-N_t) \left( \frac{\chi_t-1}{\chi_t} \frac{1}{(1-s)} (c_t^e - b\bar{c}_{t-1}) \right) + N_t\sigma \left( \frac{\chi_t-1}{\chi_t} \frac{1}{(1-s)} (c_t^e - b\bar{c}_{t-1}) \right) \\ c_t^u &= c_t^e - \left( \frac{\chi_t-1}{\chi_t} \frac{1}{(1-s)} (c_t^e - b\bar{c}_{t-1}) \right) + \sigma \left( \frac{\chi_t-1}{\chi_t} \frac{1}{(1-s)} (c_t^e - b\bar{c}_{t-1}) \right) \\ c_t^u &= c_t^e - \frac{1-\sigma}{1-s} \left( \frac{\chi_t-1}{\chi_t} \right) (c_t^e - b\bar{c}_{t-1}) \\ c_t^u - b\bar{c}_{t-1} &= c_t^e - b\bar{c}_{t-1} - \frac{1-\sigma}{1-s} \left( \frac{\chi_t-1}{\chi_t} \right) (c_t^e - b\bar{c}_{t-1}) \\ \frac{c_t^u - b\bar{c}_{t-1}}{c_t^e - b\bar{c}_{t-1}} &= \mu(\sigma) = 1 - \frac{1-\sigma}{1-s} \left( \frac{\chi_t-1}{\chi_t} \right) \end{aligned}$$

Notice this expression appears the same as that in Givens (2008), but in the



case where  $b > 0$ ,  $\chi_t$  is not the same. Thus, we have the expression relating the insurance level, the shirking detection probability, and the habit-modified consumption ratio  $\chi_t$ , which is constant.

## Appendix C: Deriving the Solow condition from first order conditions

Using the FOCs for  $N$  and  $W$ , we can arrive at the Solow condition.

$$\text{FOC } N_t: (1 - \alpha)A_t K_t^\alpha (hE_t N_t)^{-\alpha} hE_t = w_t h$$

$$\text{FOC } W_t: (1 - \alpha)A_t K_t^\alpha (h\Phi(w_t)N_t)^{-\alpha} \Phi'(w_t) = 1$$

$$\Phi'(w_t) = \frac{1}{(1-\alpha)A_t K_t^\alpha (h\Delta(w_t)N_t)^{-\alpha}}$$

## Appendix D: Demonstrating that $\chi_t$ is constant.

When  $\chi_t$  is defined as  $(c_t^e - b\bar{c}_{t-1}) / (c_t^s - b\bar{c}_{t-1})$ , it is constant. Under zero habit formation, this is exactly equivalent to the consumption ratio mentioned in Alexopoulos (2004) and calibrated to the value found by Gruber(1997). I recover steady-state  $c_t^e/c_t^s$  and report it with the other results, after calibrating for  $N = 0.941$ .

We know  $(1 - s)\frac{\chi_t}{\chi_t - 1}hw_t = (c_t^e - b\bar{c}_{t-1})$  and  $\Phi(w_t) = -\frac{T}{h}(\chi_t)^{-d/\theta} + \frac{T}{h} - \frac{\zeta}{h}$ . Using the Solow condition,  $\frac{(w_t)\Phi'(w_t)}{\Phi(w_t)} = 1$ , simply substitute, yielding

$$\left((T - \zeta)\chi^{1+\frac{d}{\theta}} - T\chi\right)(1 - s) = T\left(\frac{d}{\theta}\right)(1 - s\chi)(\chi - 1).$$

## Appendix E: Tables

Table 4: Simulated second moments and consumption ratios at different habit formation ( $b$ ) values while holding insurance ( $\sigma$ ) level constant. Corresponds to Figure 3.

| Parameters            | $c_t^u/c_t^e$ | $\sigma_n$ | $\sigma_y$ | $\sigma_w$ | $\rho(n, y)$ | $\rho(n, w)$ | $\rho(y, w)$ |
|-----------------------|---------------|------------|------------|------------|--------------|--------------|--------------|
| $b=0.26$ $\sigma=.78$ | 0.7784        | 0.0265     | 0.0227     | 0.0087     | 0.9482       | -0.5702      | -0.2800      |
| $b=0.29$              | 0.7762        | 0.0286     | 0.0233     | 0.0101     | 0.9449       | -0.6591      | -0.3770      |
| $b=0.32$              | 0.7743        | 0.0319     | 0.0238     | 0.0127     | 0.9375       | -0.7571      | -0.4826      |
| $b=0.35$              | 0.7816        | 0.0288     | 0.0213     | 0.0126     | 0.9155       | -0.7329      | -0.3974      |
| $b=0.38$              | 0.7795        | 0.0275     | 0.0181     | 0.0145     | 0.8791       | -0.8045      | -0.4245      |
| $b=0.41$              | 0.7986        | 0.0254     | 0.0177     | 0.0128     | 0.8838       | -0.7625      | -0.3717      |
| $b=0.44$              | 0.8013        | 0.0235     | 0.0153     | 0.0136     | 0.8350       | -0.7861      | -0.3165      |
| $b=0.47$              | 0.8027        | 0.0218     | 0.0118     | 0.0148     | 0.7657       | -0.8594      | -0.3296      |
| $b=0.50$              | 0.8046        | 0.0201     | 0.0091     | 0.0152     | 0.7034       | -0.9057      | -0.3361      |
| U.S. Data             | N/A           | 0.0087     | 0.0154     | 0.0111     | 0.8656       | 0.0739       | 0.2414       |

*Notes:* Simulated series are HP-filtered ( $\lambda = 1600$ ).  $c_t^u/c_t^e$ , the ratio of unemployed to employed consumption, is recovered from steady state if it cannot be calculated analytically ( $\sigma \neq 1$ ). Second moments reported for U.S. data are calculated directly from time series from FRED and BLS, Q1 1964 through Q2 2010, HP-filtered ( $\lambda = 1600$ ). At a  $\sigma$  (insurance) value corresponding to Alexopoulos's (2004) partial insurance model, trends associated with increasing  $b$  (habit formation) values can be seen. Figure 3 shows the emergence of the initial  $N$  response which creates the  $N - Y$  correlation reduction seen here with increasing habit formation.

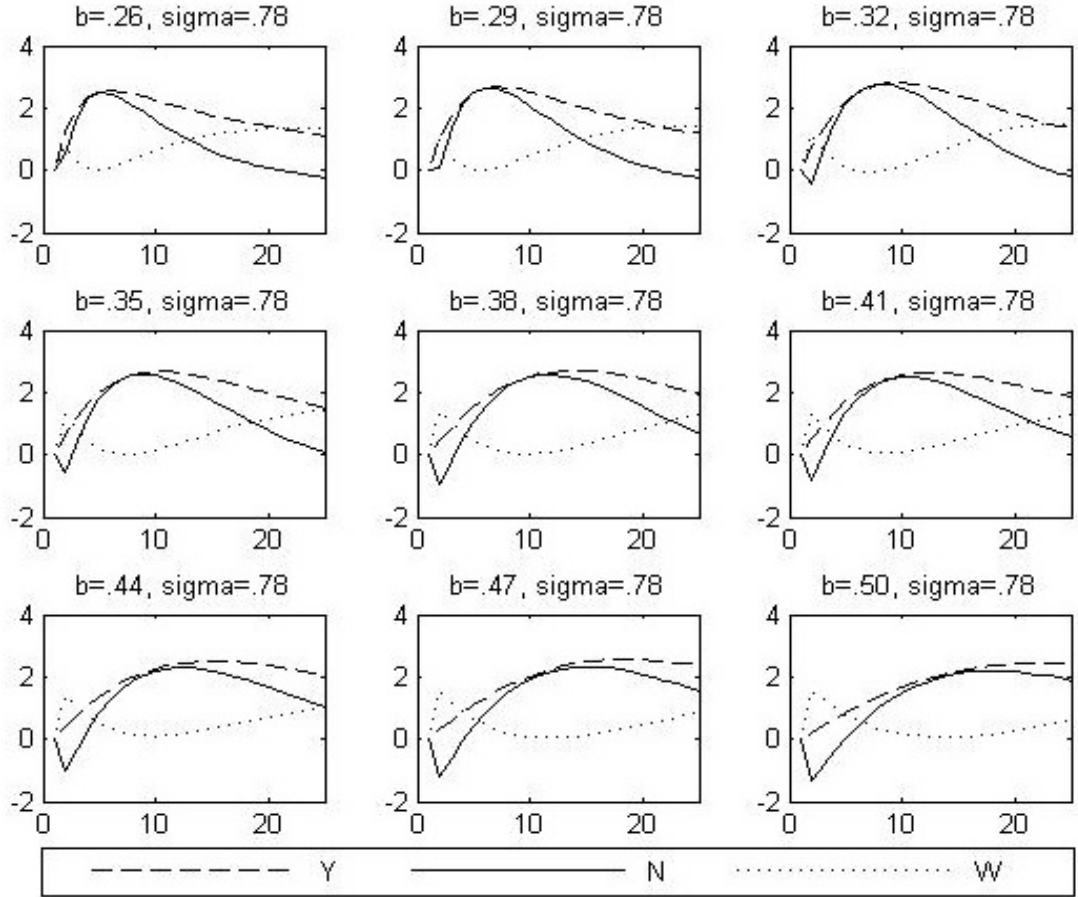
Table 5: Simulated second moments and consumption ratios for model with and without wage discrimination based on past consumption. Figure 2 shows how some of the differences arise.

| Parameters |                | $c_t^u/c_t^e$ | $\sigma_n$    | $\sigma_y$ | $\sigma_w$ | $\rho(n, y)$ | $\rho(n, w)$ | $\rho(y, w)$  |
|------------|----------------|---------------|---------------|------------|------------|--------------|--------------|---------------|
| 2w=yes     |                |               |               |            |            |              |              |               |
| $b=0.43$   | $\sigma = .77$ | 0.7911        | 0.0123        | 0.0132     | 0.0102     | 0.7438       | -0.7298      | -0.0972       |
| $b=0.33$   | $\sigma = .77$ | 0.7816        | <b>0.0095</b> | 0.0133     | 0.0082     | 0.7808       | -0.4817      | <b>0.1418</b> |
| 2w=no      |                |               |               |            |            |              |              |               |
| $b=0.00$   | $\sigma = .77$ | 0.7874        | 0.0168        | 0.0187     | 0.0063     | 0.9418       | 0.1318       | 0.4566        |
| $b=0.43$   | $\sigma = .77$ | 0.7911        | 0.0237        | 0.0142     | 0.0146     | 0.8156       | -0.8273      | -0.3502       |
| $b=0.33$   | $\sigma = .77$ | 0.7816        | 0.0308        | 0.0236     | 0.0118     | 0.9394       | -0.7272      | -0.4482       |
| U.S. Data  |                | N/A           | <b>0.0087</b> | 0.0154     | 0.0111     | 0.8656       | 0.0739       | <b>0.2414</b> |

*Notes:* Simulated series are HP-filtered ( $\lambda = 1600$ ).  $c_t^u/c_t^e$ , the ratio of unemployed to employed consumption, is recovered from the steady state if it cannot be calculated analytically ( $\sigma \neq 1$ ). Second moments reported for U.S. data are calculated directly from time series from FRED and BLS, Q1 1964 through Q2 2010, HP-filtered ( $\lambda = 1600$ ).

## Appendix F: Figures

Figure 5: Impulse response from a technology shock at increasing habit formation ( $b$ ) levels, at an insurance ( $\sigma$ ) level corresponding to Alexopoulos's (2000) partial insurance setup.



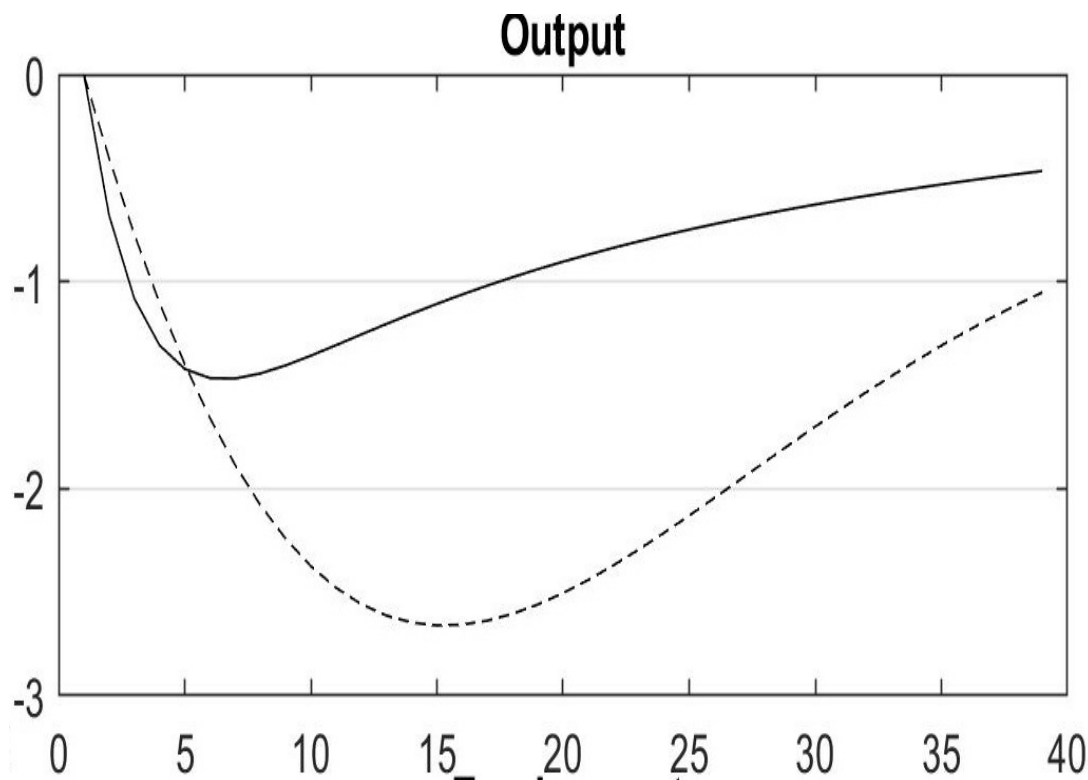
*Notes:* A progressively deeper initial drop in  $N$  arises with increased habit formation. Also visible is the lengthening period until the peak response. These effects contribute to the pattern of reduced  $\rho(n, y)$  with increasing habit formation seen in Table 1. A pattern of increasing (counterfactual) negative  $\rho(n, w)$  arises from the wage and employment responses.

Figure 6: Impulse response for employment with and without wage discrimination based on past consumption.



*Notes:* The wage-bill effect causes a return to steady-state employment sooner and after a shallower drop, when wage discrimination is allowed. The shallower loss and shorter time back to equilibrium results from the backward-looking wage bill which is less when employment in the last period is down. The consequence of restricting wage discrimination is a deeper and longer recession, with other variables recovering more slowly as a result of the employment effect.

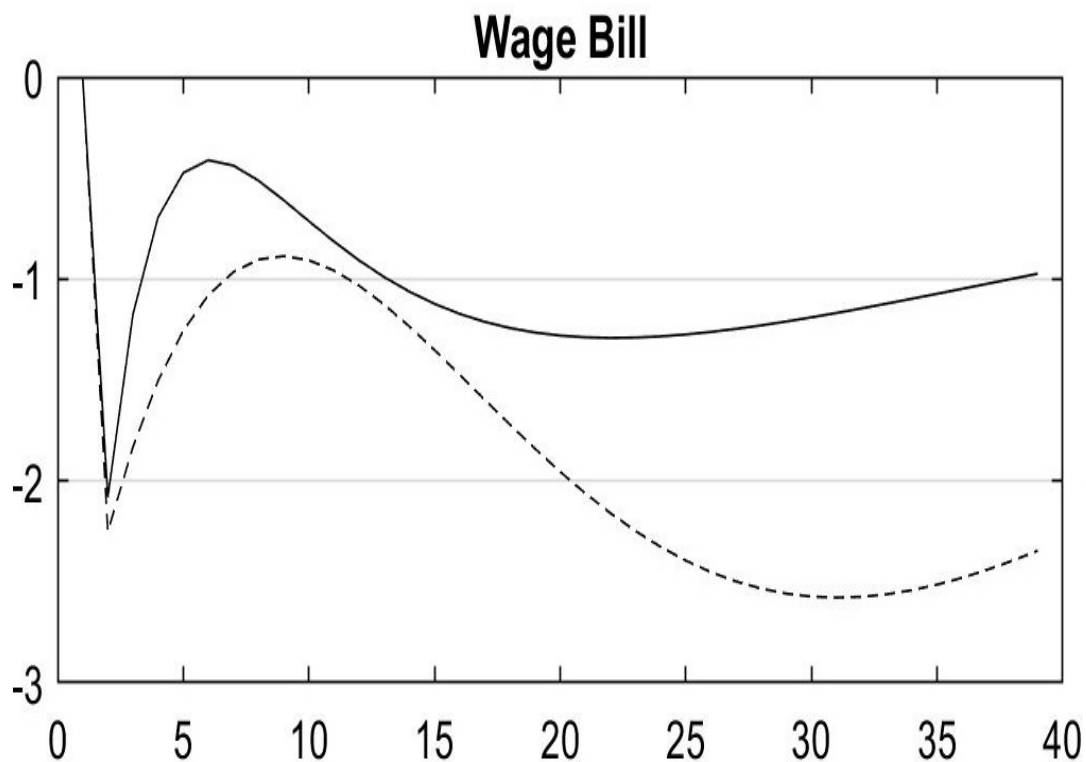
Figure 7: Impulse response for output with and without wage discrimination based on past consumption.



*Notes:* The wage-bill effect causes a return to steady-state employment sooner and after a shallower drop, when wage discrimination is allowed. The shallower loss and shorter time back to equilibrium results from the backward-looking wage bill which is less when employment in the last period is down. The consequence of restricting wage discrimination is a deeper and longer recession, with other variables recovering more slowly as a result of the employment effect.

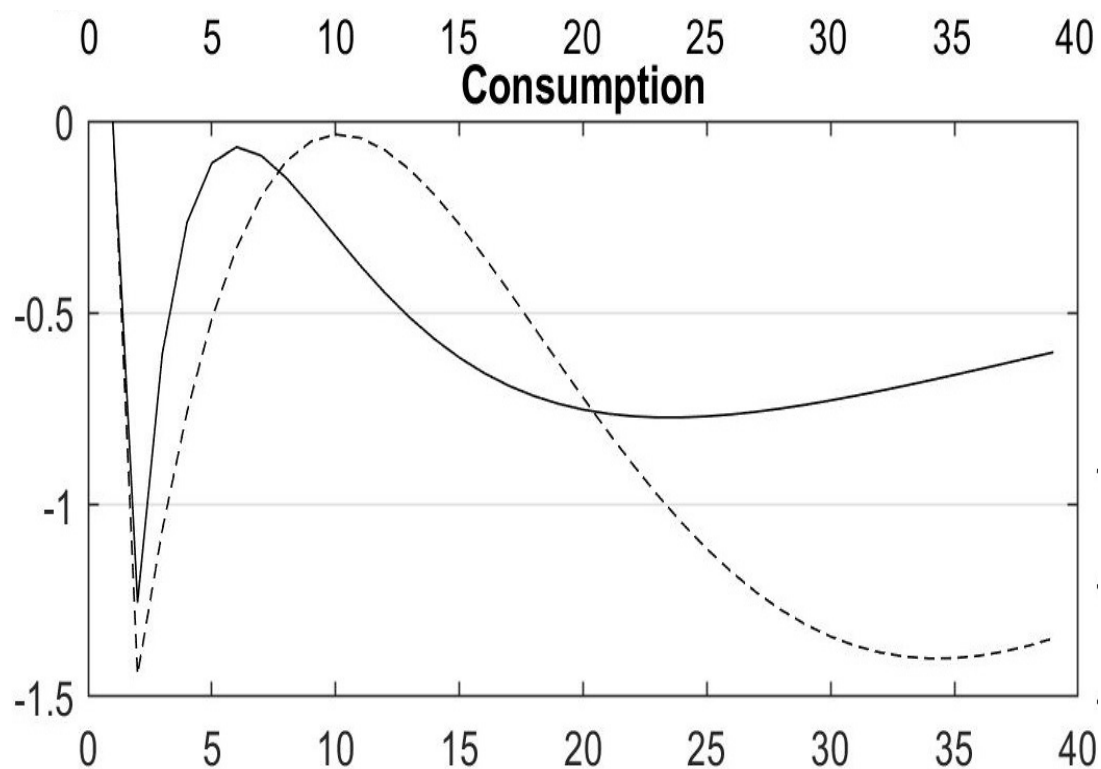


Figure 8: Impulse response for wage with and without wage discrimination based on past consumption.



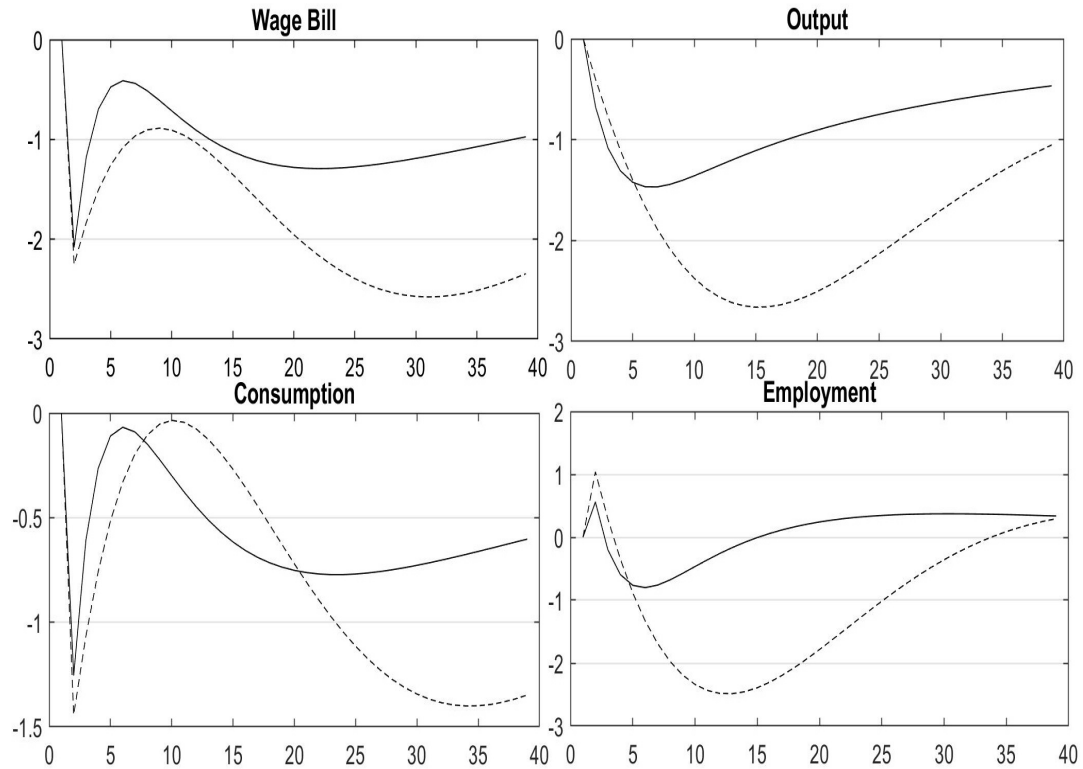
*Notes:* The wage-bill effect causes a return to steady-state employment sooner and after a shallower drop, when wage discrimination is allowed. The shallower loss and shorter time back to equilibrium results from the backward-looking wage bill which is less when employment in the last period is down. The consequence of restricting wage discrimination is a deeper and longer recession, with other variables recovering more slowly as a result of the employment effect.

Figure 9: Impulse response for consumption with and without wage discrimination based on past consumption.



*Notes:* The wage-bill effect causes a return to steady-state employment sooner and after a shallower drop, when wage discrimination is allowed. The shallower loss and shorter time back to equilibrium results from the backward-looking wage bill which is less when employment in the last period is down. The consequence of restricting wage discrimination is a deeper and longer recession, with other variables recovering more slowly as a result of the employment effect.

Figure 10: Impulse response for model with and without wage discrimination based on past consumption.



*Notes:* The wage-bill effect causes a return to steady-state employment sooner and after a shallower drop, when wage discrimination is allowed. The shallower loss and shorter time back to equilibrium results from the backward-looking wage bill which is less when employment in the last period is down. The consequence of restricting wage discrimination is a deeper and longer recession, with other variables recovering more slowly as a result of the employment effect.

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