THE IMPLICATIONS OF A WEALTH PREFERENCE IN A MONETARY EQUILIBRIUM MODEL OF THE BUSINESS CYCLE

BY

PAMELA DUKE MORRIS

A DISSERTATION SUBMITTED TO
THE GRADUATE SCHOOL AT
MIDDLE TENNESSEE STATE UNIVERSITY
IN PARTIAL FULFILLMENT
OF THE REQUIREMENT FOR THE DEGREE

DOCTOR OF PHILOSOPHY/ECONOMICS

MURFREESBORO, TENNESSEE

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DEDICATION PAGE

To my family

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Preface

The "Implications of a Wealth Preference in a Monetary Equilibrium Model of the Business Cycle" is composed of two papers: "The Economic Impact of a Preference for Wealth" and "The Welfare Costs of Inflation When Agents Have a Relative Preference for Wealth." In both of the papers a relative preference for wealth is added to the households' utility function.[1] Wealth is defined as the sum of the cash and capital holdings of the households and is the means in which money is directly incorporated into the households' utility function. Adjusting the household's preference for wealth allows for a sensitivity analysis showing not only differences attributable to incorporating a preference for wealth but also the degree to which this preference affects households' allocation decisions in response to a money growth rate innovation.

The first paper incorporates a demand for money through the use of a cash-in-advance constraint. The cash-in-advance constraint is designed to look at an economy that demands cash for the purchase of consumption goods only as well as an economy that demands cash for the purchase of consumption and investment goods. Households' response to a money growth rate innovation is qualitatively different under these two money demand rules. The addition of a preference for wealth causes a persistent response even when the money is demanded only for consumption goods. Increasing the relative preference for wealth dampens the quantitative response of output but does not affect its qualitative features.

[1] The weights placed on consumption and wealth sum to unity.

The dynamic response of wages and rental rates to a money growth rate innovation inspired an investigation into the long-run properties of the model. When money is demanded for consumption goods, inflation has no effect on the long-run real marginal cost of labor or rental rates. However, when a relative preference for wealth exists, an inflationary effect on these variables presents itself. In the case where money is demanded for consumption and investment, superneutrality does not hold regardless of the wealth preferences. However, the capital-to-hours ratio determining the real wage and real returns to capital declines as inflation increases and increases with the preference for wealth. This translates to a reduction in the real wage and an increase in the real return to capital as the level of inflation increases regardless of the preference for wealth.

The first paper points to differences in the economic impacts that are due to wealth preferences and to the important role that money demand plays in the economy's response to inflation. When money is used for consumption purposes only, the velocity of money is set to unity. However, when money is used for both consumption and investment, the velocity of money is no longer equal to one. The vast differences in the responses associated with differences in money demand naturally led to the welfare cost investigation of Paper 2, in which the velocity of money plays a larger role.

The second paper incorporates a transaction cost mechanism that is dependent on the velocity of money. At the Pareto optimal velocity, no transaction costs are present. However, as the velocity of money moves further away from the optimal level, households realize a cost in terms of lost consumption. The households' demand for money adjusts as interest rates affect the velocity of money. Thus, the welfare costs of inflation are determined under two different policy approaches.

Policymakers are first assumed to target only money growth rates. The second policy approach analyzed assumes that policymakers target interest rates in order to maintain price stability and a sustainable output growth. This second assumption is more in line with the stated goals of monetary policymakers in the U.S. The welfare costs are determined for various levels of inflation and various wealth preferences under a utility function in which households' have separable preferences over consumption and wealth and two non-separable cases in which consumption and wealth are complements and substitutes.[2] Benabou (1991) suggests that a welfare investigation of this nature using a variety of model specifications would be a valuable addition of the literature.

When policymakers target money growth rates only, welfare costs of inflation are small at moderate rates of inflation between 0 and 4.25 percent. In the separable case the welfare costs of inflation decrease with the preference for wealth. The welfare costs of inflation are not as affected by wealth preferences under the non-separable cases, though slightly larger differences are present when consumption and wealth are assumed to be substitutes.

When the policymakers target interest rates, setting the velocity of money, interesting results are found. Welfare costs in terms of goods are associated with money growth rates greater than 2 to 3 percent, and welfare gains in terms of goods are present

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[2] The model incorporates the indivisible labor assumption of Hansen (1985), providing a channel through which inflationary impacts on employment enter the model.

at money growth rates 2 percent or less. The money growth rate at which the welfare costs are zero rises as the preference for wealth increases. This result is similar under the separable and non-separable cases, although the separable case is much more sensitive to wealth preference changes. There are many different types of investors in the U.S. economy, and this investigation sheds light on how the welfare costs of inflation can be extremely different for each type.

The sensitivity of the results obtained in Papers 1 and 2 to the preference for wealth suggest that the next logical step in this stream of literature would be an attempt to quantify the relative preference for wealth. If a positive preference for wealth is found to exist based on the U.S. economic data, the model suggests that this level would have important policy implications.

Paper 1. The Economic Impact of a Preference for Wealth

Abstract

This paper evaluates the short- and long-run impacts of incorporating a preference for wealth in a monetary equilibrium model of the business cycle. The household's utility function assumes relative preferences for consumption and wealth along with an indivisible labor assumption. A demand for money emerges through the inclusion of a cash-in-advance (CIA) constraint. The properties of the model are affected by the specification of the cash constraint as well as the relative preference for wealth assumed in the utility function. Two specifications of the CIA constraint are considered. Under the first, agents require cash for the purchase of consumption goods only, and under the second, agents require cash for both consumption and investment goods.

The preference for wealth under either CIA specification has increasing effects on the short-run volatility of output and hours worked while having decreasing effects on the volatility of other economic variables such as consumption, investment, capital, wages, and wealth. The introduction of a preference for wealth does not materially alter the relationship between inflation and the business cycle. However, the business cycle effects are more pronounced under the second CIA constraint and increase with the preference for wealth.

Inflation acts as a tax on all goods for which money is required. The impact of an inflation tax on the economy's equilibrium is sensitive to the form of the CIA constraint. Investment goods provide agents with a source of future consumption and hence are subject to the inflation tax of the future. When investment is subject to a cash constraint, the consumption goods are essentially taxed twice. Thus, greater substitution effects take place under the second CIA constraint.

Additionally, the effects of an inflation tax on the economy's capital-to-labor ratio differ with the specification of the CIA constraint directly impacting real wages and returns to capital. Real wages increase with inflation under the first CIA specification and decrease under the second CIA specification. Returns to capital move in the opposite manner. The capital-labor tradeoff is the channel through which inflation impacts the steady-state ratios of an economy with a preference for wealth.

Keywords: preference for wealth, cash-in-advance model, indivisible labor

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Section 1. Introduction

Household preferences are the basis on which all economic models rely. This paper explores the properties of a general equilibrium model of the business cycle in which the average household is assumed to have relative preferences over consumption and wealth along with an indivisible-labor assumption. Money is the economic distortion in this model economy and is introduced through the use of a cash-in-advance (CIA) constraint. The CIA constraint is designed in such a manner as to allow for a comparison between two cases: (a) the case in which money is used for consumption only and (b) the case in which money is used for both consumption and investment.

Extending the monetary business cycle literature, this paper addresses three main questions:

- "How reliant are the business cycle effects of inflation on the specification of the CIA constraint and the economy's preference for wealth?"
- "How does the dynamic response of a money growth rate innovation in the model economy compare to those seen in the U.S. economy?"
- "What are the differences in the long-run properties of the economy under different money growth rate levels as the economy's preference for wealth changes?"

Traditionally, the role of wealth is to smooth consumption profiles when shocks occur. In this paper, households are assumed to have a direct preference for wealth although the consumption-smoothing benefits of wealth are not ignored. This analysis embeds Veblen's ideas into a standard CIA monetary model to examine the differences in the long- and short-run properties of the economy as households' relative preference for wealth is adjusted. Inflation acts as a tax distorting the economic choices of households. Thus, in the examination of the long- and short-run properties, various money growth rates are considered.

This analysis compares the business-cycle properties of a deterministic economy in which the money growth rate is constant to those in a stochastic economy in which households are subject to money growth rate innovations.[3] The economy's dynamic response to innovations is graphically presented. The paper examines the steady-state properties including a steady-ratio analysis in order to clearly show that superneutrality does not exist when households have a preference for wealth. This analysis of the business-cycle and steady-state properties of an economy in which households have preferences for consumption, status, and leisure provides an important addition to the monetary real business cycle literature.

The remainder of Section 1 consists of a background on the development of the household preference structure, a background on the supporting literature, and an overview of the model implications. Section 2 analyzes why wealth matters and lays a foundation for its inclusion in the household's utility function. Section 3 presents the stochastic CIA model. The problem faced by utility-maximizing households in this three-sector economy is detailed in subsections 3.1 through 3.3. Subsection 3.4 fully

[3] As in Cooley and Hansen (1989), the households in both deterministic and stochastic economies are subject to technological innovations.

characterizes the solution of the household's problem by deriving the full set of general equilibrium conditions. Section 4 explains solution techniques along with calibrated parameters used in the estimation process. Section 5 includes an examination of the impacts of inflation on the business cycle as well as steady-state properties and ratios determined by the model and how these respond to changes in the economy's preference for wealth. Section 6 concludes.

1.1 Background for Household Preference Development

This paper provides a bridge connecting two branches of research: (a) the work of Cooley and Hansen (1989), who analyze the effects of inflation on the business cycle and the steady-state properties of a monetary real business cycle model in which agents have preferences for consumption and leisure, and (b) the wealth-is-status long-run analyses of Gong and Zou (2001) and Chang and Tsai (2003), in which agents have a preference for consumption and status (or wealth). In both of these branches of research, money is an economic distortion introduced through a cash-in-advance (CIA) constraint.

The theories of Thorstein Veblen (1899), who argues that the acquisition of wealth increases the physical comforts of life that consumption goods provide, provide a rationale for the inclusion of both wealth and leisure in the household preference structure. Wealth is an "invidious distinction," the basis of one's reputation or social status. Whether wealth is earned or inherited, wealth brings with it merit or status. Veblen states, *"The desire for wealth can scarcely be satiated in any individual instance, and evidently a satiation of the average or general desire for wealth is out of the question"* (Veblen 1899).

If wealth is meant only to increase the physical comforts provided by consumption goods, then as industrial efficiency increases one would think that the desire for wealth would be satiated, but if the struggle for wealth is a race for "invidious comparison," no attainment is possible (Veblen 1899).

Societies generally find labor distasteful and leisure favorable (Veblen, 1899). Veblen states,

"This direct, subjective value of leisure and of other evidences of wealth is no doubt in great part secondary and derivative. It is in part a reflex of the utility of leisure as a means of gaining the respect of others, and in part it is the result of a mental substitution ... insistence on the meritoriousness of wealth leads to a more strenuous insistence on leisure" (Veblen 1899).

Supporting the theories of Veblen, this study's addition of leisure to household preferences as well as consumption and wealth extends the current literature.

1.2 Background Literature Supporting Analysis

The spirit-of-capitalism literature introduced by Weber (1958) assumes man focuses on earning money not only to meet material needs but also to accumulate wealth for wealth's sake (Weber 1958; Keynes 1971). Wealth is defined as capital accumulation. Spirit-of-capitalism research spawning from Weber (1958) includes Zou (1992, 1994, 1995) and Bakshi and Chen (1996), among others.

The analysis incorporates a preference for wealth and, like Cooley and Hansen (1989), a labor-leisure choice that enters the monetary economy through the introduction of an employment lottery using Hansen's (1985) indivisible labor assumption. The effects of an "inflation tax" on the steady-state values and business-cycle properties of a monetary real business cycle model form the focus of their analysis. They find increases in the growth rate of money or inflation to cause substitution effects. Agents decrease their demand for goods requiring money (consumption goods) and increase demand for goods that do not require money (investment and leisure). Cooley and Hansen also find that anticipated changes in the growth rate of money have rather small business-cycle implications.

The spirit-of-capitalism literature is extended by the wealth-is-status approach of Gong and Zou (2001) and the "wealth-induced social status" approach of Chang and Tsai (2003). These lines of research extend wealth to include both accumulated capital and real money balances. Unlike Cooley and Hansen (1989), they do not allow for labor-leisure adjustments in response to inflationary adjustments.

Gong and Zou (2001) examine the effects of inflation on capital accumulation in an economy in which agents have a preference for consumption and wealth. They find that when agents have a preference for status (wealth) and the CIA constraint applies to consumption and investment (Stockman 1981), superneutrality of money no longer holds. Through comparative static analysis, Gong and Zou show that inflation increases capital accumulation when both consumption and investment are subject to the CIA constraint.

Gong and Zou's (2001) article, while definitely an important expansion of the literature, provoked a commentary by Chang and Tsai (2003).

Chang and Tsai (2003) contradict one of Gong and Zou's findings, stating that under the Stockman (1981) CIA constraint inflation decreases capital accumulation. Under a Lucas and Stokey (1987) CIA specification in which consumption goods only are liquidity-constrained, Chang and Tsai (2003) find that inflation increases steady-state capital accumulation. Chang and Tsai (2003) also add to the wealth-induced social status literature by performing comparative static analysis on the effects of an increasing preference for status on steady-state capital accumulation. As the agent's desire for status increases, steady-state capital accumulation increases (Chang and Tsai 2003).

1.3 Overview of Model Implications

This study assumes that an increase in the preference for wealth increases the marginal utility of wealth. A similar assumption was made by Friedman and Savage (1948) and Becker, Murphy, and Werning (2005), who assume higher status increases the marginal utility of income. This paper goes on to introduce wealth by making it an explicit argument in the agent's utility function. An increase in the agent's preference for wealth is captured by the model as an increase in the weight placed on wealth relative to consumption. The relative weights placed on consumption and wealth sum to unity.

Agents are assumed to receive disutility from labor hours worked. The agent's labor-leisure decision enters the model economy through the same indivisible labor assumption made by Hansen (1985) and Cooley and Hansen (1989). Thus, agents are assumed to have preferences for consumption, wealth, and leisure.

A positive demand for money emerges through a standard CIA constraint (Lucas 1980, 1982; Stockman 1981; Svensson 1985; Lucas and Stokey 1987; Cooley and Hansen 1989; Gong and Zou 2001; Chang and Tsai 2003). Two CIA constraint specifications are considered. Under the first, agents require cash for the purchase of consumption goods only, and under the second, agents require cash for both consumption and investment goods.

The economic impacts are sensitive to both the household's assumed preferences and the CIA constraint specification. The addition of a relative preference for wealth and consumption along with a labor-leisure choice examined under two distinctly different CIA constraint specifications lead to differences in both the business-cycle properties found in Cooley and Hansen (1989) and the long-run properties of the wealth-induced social status research of Gong and Zou (2001) and Chang and Tsai (2003).

When the CIA constraint takes the first form, a rise in the growth rate of money causes households to substitute consumption (cash goods) for leisure. Under the second CIA constraint specification, greater substitution effects emerge. Households faced with an inflation tax rationally substitute non-taxed goods (leisure) for taxed goods (consumption and investment). Investment financing future consumption is subject to current and future inflation taxation. The consumption investment provides is in essence subject to double taxation. The incorporation of a labor-leisure decision shows a substitution toward leisure is linked to increased inflation levels and corresponds to reductions in consumption, investment, capital, and output.

The long-run implications of an increase in the economy's relative preference for wealth are increases in households' demand for leisure. Even more interesting, there exists a preference level for which steady-state consumption is maximized. The short-run volatility of output and hours worked increases with the preference for wealth under either CIA specification, while the volatility of consumption, investment, capital, wages, and wealth decrease with the preference for wealth.

The business-cycle effects differ with the CIA constraint specification and the preference for wealth. More pronounced responses are observed under the second CIA constraint and are dampened as the preference for wealth increases. The differences become obvious when the dynamic responses of the economy to both productivity and money growth rate innovations under each CIA specification are compared.

Section 2. Evolution of the Role of Wealth

Veblen (1899) and Duesenberry (1949) argue that individuals may seek wealth just for the sake of status and may get satisfaction from feeling they are relatively "better off" than others. The struggle for wealth is not only a struggle for subsistence (Veblen 1899). Veblen (1899) focuses on wealth and its accompanying status, noting the link between conspicuous consumption and leisure and the association of both with wealth.

As the status associated to wealth increases, the demand for leisure increases (Veblen 1899).

The desire for status is thought to increase with wealth (Weiss 1976). Individuals concerned with their own wealth may also be concerned with the relative standing in society that wealth brings (Robson 1992). Agents may have higher wealth preferences not necessarily due to their desire for status but because of the consumption opportunities wealth affords (Postlewaite 1998; Hopkins & Kornienko 2004).

The direct preference for wealth has entered the utility functions of other economic researchers. Stockman (1981) showed that money exhibited superneutrality when the CIA constraint includes only consumption; however, when the CIA constraint includes investment, inflation is inversely related to capital accumulation in the long run. Gong and Zou (2001) extended the Stockman model, adding wealth-induced social status. They found that inflation leads to a higher level of consumption as well as more capital accumulation in the long run when the CIA constraint applies to consumption only.

Gong and Zou (2001) also found that when the CIA constraint applies to both consumption and investment so that cash is required for both, the strength of the agent's desire for wealth-induced social status is the factor that determines the effect of inflation on long-run capital accumulation. Gong and Zou show that under the perfect-foresight assumption the steady state of consumption and capital accumulation increase as the strength of the wealth-induced social status increases.

Gong and Zou (2002) have examined a real economy in which individuals derive utility from both consumption and wealth. Gong and Zou suggest that a model that contains real money balances would be the appropriate direction for future research. In this analysis real money balances not only are included as the liquid portion of wealth but also, through the CIA constraint, are required for consumption or investment.

Becker, Murphy, and Werning (2005) argue that all individuals can be assumed to have some interest in status. They assume the utility of economic agents depends on both consumption and status and introduce a market for status in which agents indirectly purchase status. Increases in the agents' willingness to participate in fair lotteries are linked with increasing status preferences, represented in their model as the interaction between status and consumption (Becker et al. 2005). The authors allude to Veblen's (1899) work but omit leisure for the sake of simplicity.

Other studies incorporating wealth in the individual's or household's utility include St. Amour (2005) and Corneo and Jeanne (2001), among others. However, most fail to include a preference for leisure in the utility function.[4] In an effort to capture the dynamics of U.S. real economic data, this analysis assumes that households have a preference for consumption, wealth, and leisure.

Since the U.S. economy has moved from an agrarian to an industrial and then a service orientation and per capita wealth continues to increase, the inclusion of the wealth-induced social status in the agent's utility function is appropriate in the cash-in-

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[4] In the only working paper found so far, Young and Lou (2006) examine wealth distributions in a heterogeneous agent model in which money is not a distortion.

advance economy of Cooley and Hansen (1989), making utility separable into consumption, wealth, and leisure.

The introduction of a preference for wealth into a CIA economy allows for an examination of how an increase in wealth-induced social status affects the substitution from consumption to leisure in response to an inflation tax, a comparison of the economy's dynamic response to a money supply shock when the preference for wealth is adjusted, and a simulated economy for testing the business-cycle and steady-state differences resulting from an increase in relative preference for wealth.

Wealth is assumed to matter to the economic agents in this model economy whether they derive utility from wealth itself or from the social status derived from it. The preference for wealth or its associated social status drives households' behavior and choices regarding capital accumulation, consumption, and potentially labor versus leisure.

Section 3. The Stochastic CIA Model

The household's decision problem takes account of utility-optimizing consumers faced with budget and CIA constraints choosing optimal bundles of consumption, wealth, and leisure; a profit-maximizing firm's demands for capital and labor; and the money growth rate policies set in place by the monetary authority. The problem includes the derivation of the first-order and envelope conditions of the model. These conditions combined with the constraints affect the household's decision to form a set of general equilibrium conditions.

3.1 *The Consumer's Optimum*

In this analysis, households have a direct preference for consumption, leisure, and wealth. The direct preference for wealth is assumed in order to capture the effects of an increasing desire for wealth-induced social status on the utility-maximizing household's behavior in terms of consumption, output, capital accumulation, and leisure choices.

Households faced with uncertainty are assumed to make choices on consumption, wealth, and leisure in an attempt to maximize of their expected lifetime utility:

$$E_0 \sum_{i=0}^{\infty} \beta^i \, U(c_{t+i}, a_{t+i}, h_{t+i}) = E_0 \sum_{i=0}^{\infty} \beta^i \, \frac{1}{1-\sigma}\left[\gamma\, c_{t+i}^{\;1-\eta} + (1-\gamma) a_{t+i}^{\;1-\eta}\right]^{\frac{1-\sigma}{1-\eta}} - Bh_{t+i} \qquad (3.10)$$

where $0 < \beta < 1$.[5] The household's utility is a function of real consumption, c_t, the fraction of total time that is supplied to market activities or work, h_t, and real wealth, a_t. The parameters γ, σ, and η are restricted to only positive values.[6]

The household is subject to the same indivisible labor restriction seen in Hansen (1985). The indivisible labor restriction assumes that the households are allotted one unit of time from which they allocate a portion to labor market activities, $h_0 < 1$, and the rest to leisure activities. Households having full unemployment insurance are subject to an

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[5] The stochastic version of this model was chosen so that dynamic properties of the model can be examined. The dynamic response of the real economic variables is included in Section 5.1.3.

[6] In the case where $\gamma = \sigma = \eta = 1$, the household's utility is specified in the same manner as in Cooley and Hansen (1989).

employment lottery. The probability of being employed, ψ_t, determines the expected number of hours worked by each household in any given period t to be defined as $h_t = \psi_t h_0$. The parameter, B, measures the marginal disutility the household derives from market activities.[7]

The households are subject to a cash-in-advance constraint modified from that of Cooley and Hansen (1989) in order to examine the case in which cash is required for the purchase of consumption goods and that in which cash is required for both consumption and investment:

$$\mu_1 c_t + \mu_2 x_t \leq \frac{m_{t-1}}{\Pi_t} + \tau_t \tag{3.11}$$

The gross rate of inflation, denoted by Π_t, τ_t represents real government transfers, and m_{t-1} represents the real money balances carried over from the previous period for consumption and investment in period t. When $\mu_1 = 1$ and $\mu_2 = 0$, cash is used to aid in consumption purchases; however, when $\mu_1 = \mu_2 = 1$, cash is required for consumption as well as investment.

Wealth, a_t, is defined as in Gong and Zou (2001) to be composed of accumulated capital, k_{t-1}, real money holdings, $\frac{m_{t-1}}{\Pi_t}$, and government transfers, τ_t, in the following manner:

$$a_t = k_{t-1} + \frac{m_{t-1}}{\Pi_t} + \tau_t \ . \tag{3.12}$$

[7] For a full description of indivisible labor, see Cooley and Hansen (1989) or Hansen (1985).

The parameter $(1-\gamma)$ is a measure of the household's preference for wealth relative to consumption.

The households supply labor to the firm and receive compensation in the form of wages, w_t. The households also supply capital to the firm for which they charge a rental rate, r_t. The households receive real transfers from the government, τ_t, and may have real money holdings from the previous period. Thus, the household's budgets are constrained in the following manner:

$$c_t + x_t + m_t \leq w_t h_t + r_t k_{t-1} + \tau_t + \frac{m_{t-1}}{\Pi_t}. \tag{3.13}$$

Investment, x_t, is constrained as:

$$x_t = k_t - (1-\delta)k_{t-1} \tag{3.14}$$

where $\delta \in [0,1]$ is the capital depreciation rate.

The household's decision problem is represented by the following value function:

$$V(k_{t-1}, m_{t-1}, s_t) = \underset{c_t, h_t, m_t, x_t, k_t}{Max} \{ U(c_t, a_t, h_t) + \beta E_t [V(k_t, m_t, s_{t+1})] \}$$

where s_t is a vector of state variables. The decision problem is subject to the CIA constraint (3.11), the budget constraint, (3.13), and the investment constraint (3.14).

3.2 *The Firm's Optimum*

Firms use the labor and capital supplied by households to produce output. The firm's production function is Cobb-Douglas with constant returns to scale such that per capita output, Y_t, is a function of capital accumulated at the end of the previous period, K_{t-1}, and the percentage of total time allocated to market activities, H_t, so that:

$$Y_t = f(K_{t-1}, H_t, z_t) = e^{z_t} K_{t-1}{}^{\alpha} H_t{}^{1-\alpha} \tag{3.15}$$

and $\alpha \in [0,1]$. The production function is subject to exogenous productivity shocks, z_t. This linear shock takes the following form:

$$z_{t+1} = \rho_z z_t + \xi_{z_{t+1}}.$$

The persistence of the shock, $\rho_z \in [0,1]$, and the innovation term, $\xi_{z_{t+1}}$, is such that $E_t\{\xi_{z_{t+1}}\} = 0$ and variance of the innovation is $\sigma_z{}^2$.

The firm is a price-taker in wages and rental rates and acts optimally, choosing the amount of capital and labor necessary and sufficient for profit maximization. The firm's profit maximization problem takes the following form:

$$\underset{\{K_{t-1}, H_t\}}{Max} \; Y_t - w_t H_t - r_t K_{t-1} \quad \textit{such that} \;\; Y_t = e^{z_t} K_{t-1}{}^{\alpha} H_t{}^{1-\alpha}$$

where $\alpha \in [0, 1]$ and both K and H are restricted to positive values. Thus, for all $t \in [0, \infty)$, the firm's necessary and sufficient optimality conditions must hold,

$$K_{t-1} : \alpha e^{z_t} K_{t-1}{}^{\alpha-1} H_t{}^{1-\alpha} = r_t$$

$$H_t : (1-\alpha) e^{z_t} K_{t-1}{}^{\alpha} H_t{}^{-\alpha} = w_t.$$

3.3 Monetary Authority Policy

The nominal money supply is assumed to grow at the rate of θ_t.[8] The government regulates the growth rate of the nominal money supply such that:

$$P_t M_t = \Theta_t P_{t-1} M_{t-1},$$

otherwise written as:

$$M_t = \frac{\Theta_t}{\Pi_t} M_{t-1},$$

where M_t represents the per capita real money supply. The government also provides per capita real transfers, τ_t, to the households at the start of each period, so that the following equality holds:

$$\tau_t = M_t - \frac{M_{t-1}}{\Pi_t}.$$

Otherwise stated, the nominal transfer, $P_t \tau_t$, equals the product of the growth rate of money, Θ, and the nominal money supply from the previous period, $P_{t-1} M_{t-1}$.

The growth rate of money follows the following process,

$$\log \Theta_{t+1} = (1 - \rho_\theta)\Theta + \rho_\theta \log \Theta_t + \xi_{\theta_{t+1}}.$$

The persistence associated with the money growth rate process is measured by ρ_θ such that $\rho_\theta \in [0,1]$, and the innovation to the money growth rate process, $\xi_{\theta_{t+1}}$, has mean zero and variance $\sigma_\theta{}^2$. Shocks to the growth rate of the money supply feed through to the

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[8] The real money supply grows in the following manner: $m_t = \frac{(1+\theta_t)}{\Pi_t} m_{t-1} = \frac{\Theta_t}{\Pi_t} m_{t-1}$.

household through lump-sum transfers that affect the household's budget and CIA constraints.

When formulating its plan, the household takes as parametric (not individual-specific) the following: $w_t = w(z_t, K_{t-1}, H_t)$, $r_t = r(z_t, K_{t-1}, H_t)$, $\tau_t = M_t - \frac{M_{t-1}}{\Pi_t}$, and Π_t, where $K_{t-1}, H_t, \text{and}\, M_t$ represent the per capita capital stock, hours worked, and real balances.[9] Households also take as given the productivity and money supply growth rates, $\{z_t,\ \theta_t\}$, and the autoregressive processes, which take the following form:

$$z_{t+1} = \rho_z z_t + \xi_{z_{t+1}}$$

$$\log\theta_{t+1} = (1-\rho_\theta)\theta + \rho_\theta \log\theta_t + \xi_{\theta_{t+1}}.$$

In the competitive equilibrium, the per capita capital demanded by the firm is the capital supplied by the representative household, $K_t = k_t$, the per capita hours demanded by the firm equal the hours supplied by the household, $H_t = h_t$, and the per capita real money balances supplied by the government equal the real balances demanded by the representative household.

3.4 *The Household's Problem*

The solution of the household's decision problem relies on the assumption made by Cooley and Hansen (1989) that both the budget constraint and the CIA constraint hold

[9] Capital letters are used to distinguish per capita variables that a competitive household takes as parametric from individual-specific variables that the household chooses. In equilibrium these will be the same.

with equality. As long as nominal interest rates are positive and households purchase goods after the observation of current shocks, the CIA constraint always holds with equality (Lucas 1980; 1982).

Thus, the household's decision problem is to maximize the following value function:

$$V(k_{t-1}, m_{t-1}, s_t) = \underset{c_t, h_t, m_t, x_t, k_t}{Max} \left\{ U(c_t, a_t, h_t) + \beta E_t \left[V(k_t, m_t, s_{t+1}) \right] \right\}$$

such that

$$c_t + x_t + m_t \leq w_t h_t + r_t k_{t-1} + \tau_t + \frac{m_{t-1}}{\Pi_t}$$

$$x_t = k_t - (1-\delta) k_{t-1}$$

$$\mu_1 c_t + \mu_2 x_t \leq \frac{m_{t-1}}{\Pi_t} + \tau_t$$

for all $t \in [0, \infty)$.

The household's problem can be written in Lagrange form, where the Lagrangian multipliers associated with the budget, investment, and CIA constraints are λ_t, q_t, and φ_t, respectively. The household's problem then takes the following form:

$$\underset{c_t, h_t, m_t, x_t, k_t, \lambda_t, q_t, \varphi_t}{Max \; \mathfrak{L}} = \left\{ \begin{array}{l} U(c_t, a_t, h_t) + \beta E_t \left[V(k_t, m_t) \right] \\ \quad + \lambda_t \left[w_t h_t + r_t k_{t-1} + \tau_t + \dfrac{m_{t-1}}{\Pi_t} - c_t - x_t - m_t \right] \\ \quad + q_t \left[x_t + (1-\delta) k_{t-1} - k_t \right] \\ \quad + \varphi_t \left[\dfrac{m_{t-1}}{\Pi_t} + \tau_t - \mu_1 c_t - \mu_2 x_t \right] \end{array} \right\}$$

The household's problem yields the following first-order conditions:

$$c_t : U_c(c_t, a_t, h_t) - \lambda_t - \varphi_t \mu_1 = 0$$

$$h_t : U_h(c_t, a_t, h_t) + \lambda_t w_t = 0$$

$$m_t : \beta E_t \left[V_m(k_t, m_t) \right] - \lambda_t = 0$$

$$x_t : -\lambda_t + q_t - \varphi_t \mu_2 = 0$$

$$k_t : \beta E_t \left[V_k(k_t, m_t) \right] - q_t = 0$$

$$\lambda_t : c_t + x_t + m_t = w_t h_t + r_t k_{t-1} + \tau_t + \frac{m_{t-1}}{\Pi_t}$$

$$q_t : k_t = x_t + (1-\delta) k_{t-1}$$

$$\varphi_t : \mu_1 c_t + \mu_2 x_t = \frac{m_{t-1}}{\Pi_t} + \tau_t$$

and envelope conditions:

$$V_m(k_{t-1}, m_{t-1}, s_t) = \left(U_a(c_t, a_t, h_t) + \lambda_t + \varphi_t \right) \left(\frac{1}{\Pi_t} \right), \text{ and}$$

$$V_k(k_{t-1}, m_{t-1}, s_t) = U_a(c_t, a_t, h_t) + \lambda_t r_t + q_t (1-\delta).$$

The updated envelope conditions needed to combine the first order conditions with the envelope conditions are listed below:

$$V_m(k_t, m_t, s_{t+1}) = \left(U_a(c_{t+1}, a_{t+1}, h_{t+1}) + \lambda_{t+1} + \varphi_{t+1} \right) \left(\frac{1}{\Pi_{t+1}} \right), \text{ and}$$

$$V_k(k_t, m_t, s_{t+1}) = U_a(c_{t+1}, a_{t+1}, h_{t+1}) + \lambda_{t+1} r_{t+1} + q_{t+1} (1-\delta).$$

The combination of the first-order conditions and the time-consistent envelope conditions gives the following set of equilibrium conditions for the household:

$$c_t : U_c(c_t, a_t, h_t) - \lambda_t - \varphi_t \mu_1 = 0$$
$$h_t : U_h(c_t, a_t, h_t) + \lambda_t w_t = 0$$
$$m_t : \beta E_t \left[\left(U_a(c_{t+1}, a_{t+1}, h_{t+1}) + \lambda_{t+1} + \varphi_{t+1}\right)\left(\frac{1}{\Pi_{t+1}}\right)\right] - \lambda_t = 0$$
$$x_t : -\lambda_t + q_t - \varphi_t \mu_2 = 0$$
$$k_t : \beta E_t \left[U_a(c_{t+1}, a_{t+1}, h_{t+1}) + \lambda_{t+1} r_{t+1} + q_{t+1}(1-\delta)\right] - q_t = 0$$
$$\lambda_t : c_t + x_t + m_t = w_t h_t + r_t k_{t-1} + \tau_t + \frac{m_{t-1}}{\Pi_t}$$
$$q_t : k_t = x_t + (1-\delta)k_{t-1}$$
$$\varphi_t : \mu_1 c_t + \mu_2 x_t = \frac{m_{t-1}}{\Pi_t} + \tau_t$$
$$\text{where } a_t = k_{t-1} + \frac{m_{t-1}}{\Pi_t} + \tau_t \ \forall\, t \geq 0$$

These nine equations together with initial conditions k_{-1}, m_{-1}, and the appropriate transversality conditions fully characterize the representative household's optimal plan for $\{c_t, a_t, m_t, k_t, h_t, \lambda_t, q_t, \varphi_t\}_{t=0}^{\infty}$.

To close the model, the following conditions must hold in the case where all agents are identical:

$$m_t = \frac{\Theta_t}{\Pi_t} m_{t-1}$$

$$\tau_t = m_t - \frac{m_{t-1}}{\Pi_t}$$

The homogeneous household conditions together with the first-order conditions with respect to the Lagrange multipliers λ_t, q_t, and φ_t, after some algebraic manipulation, yield the following simplified constraints:

$c_t + x_t = w_t h_t + r_t k_{t-1}$ or $c_t + x_t = y_t$,

$k_t = x_t + (1-\delta)k_{t-1}$, and

$\mu_1 c_t + \mu_2 x_t = m_t$.

The optimality conditions of the household and the firm, together with the policy rules of the government, form the following system of equations, which fully define the economy's competitive equilibrium:

$$\lambda_t + \varphi_t \mu_1 = U_c(c_t, a_t, h_t)$$
$$\lambda_t w_t = -\ U_h(c_t, a_t, h_t)$$
$$\lambda_t = \beta E_t\left[\ (U_a(c_{t+1}, a_{t+1}, h_{t+1}) + \lambda_{t+1} + \varphi_{t+1})\left(\frac{1}{\Pi_{t+1}}\right)\right]$$
$$\lambda_t + \varphi_t \mu_2 = q_t$$
$$q_t = \beta E_t\left[U_a(c_{t+1}, a_{t+1}, h_{t+1}) + \lambda_{t+1} r_{t+1} + q_{t+1}(1-\delta)\right]$$
$$c_t + x_t = y_t$$
$$y_t = e^{z_t} k_{t-1}{}^{\alpha} h_t{}^{1-\alpha}$$
$$k_t = x_t + (1-\delta)k_{t-1}$$
$$\mu_1 c_t + \mu_2 x_t = m_t$$
$$a_t = k_{t-1} + m_t$$
$$w_t = (1-\alpha)e^{z_t}\left(\frac{k_{t-1}}{h_t}\right)^{\alpha}$$
$$r_t = \alpha e^{z_t}\left(\frac{h_t}{k_{t-1}}\right)^{1-\alpha}$$
$$m_t = \Theta_t \frac{m_{t-1}}{\Pi_t}$$

These 13 equations form a nonlinear system of difference equation that characterizes the dynamics of $\{c_t, h_t, m_t, a_t, x_t, k_t, y_t, w_t, r_t, \pi_t, \lambda_t, q_t, \varphi_t\}$.

The system is subject to the following random shocks $\{\xi_{z_{t+1}}, \xi_{\theta_{t+1}}\}$ affecting both productivity and the money growth rate in the following manner:

$$z_{t+1} = \rho_z z_t + \xi_{z_{t+1}}$$

$$\log\theta_{t+1} = (1-\rho_\theta)\theta + \rho_\theta \log\theta_t + \xi_{\theta_{t+1}}.$$

The solution of the system of equations is a time-invariant function of $\{z_t, \theta_t, k_{t-1}, m_{t-1}\}$.

Section 4. Solution Techniques and Calibration

In this section, the methodology used to solve the household's decision problem is shown in detail. The technique used by Klein (2000) relies on a log-linearized system of equations derived from the system of competitive equilibrium equations obtained from the solution of the household's decision problem. The system of log-linearized equations is solved for their unique real general equilibrium values. The parameters of the model are calibrated to emulate the characteristics of the U.S. data over the sample period ranging from 1964:Q1 to 2004:Q4. The calibration is done in order to create a set of simulated data from the model that attempts to capture the business cycle properties of the U.S. economy.

4.1 Solution Technique

The generalized Schur decomposition method for solving a linear system of dynamic equations is used to solve the household's decision problem. Klein (2000) explains this technique in full detail. The technique requires the derivation of a system of log-linearized equations unique to the specification of the household's utility function. The set of log-linearized competitive equilibrium conditions for households under the separable utility assumption ($\sigma = \eta = 1$) fully characterizes the household's decision.

The household, however, takes as given the productivity and money supply growth rates $\{z_t,\ \theta_t\}$, and autoregressive processes, which have the following log-linearized form:

$$\hat{z}_{t+1} = \rho_z \hat{z}_t + \xi_{z_{t+1}} \tag{4.10}$$

$$\log \hat{\theta}_{t+1} = \rho_\theta \log \hat{\theta}_t + \xi_{\theta_{t+1}}. \tag{4.11}$$

The technological and monetary growth processes along with the identities used to tie past capital and money into the future,

$$E_t\{k_t \mid k_{t-1}\} = k_t$$
$$E_t\{m_t \mid m_{t-1}\} = m_t$$

are mapped into the system of equations along with the following system of log-linearized competitive general equilibrium conditions[10]:

$$0 = c\hat{c}_t + x\hat{x}_t - y\hat{y}_t$$
$$0 = \mu_1 c\hat{c}_t + \mu_2 x\hat{x}_t - m\hat{m}_t$$

[10] A detailed derivation of each of the log-linearized equilibrium conditions is contained in the attached technical appendix.

$$0 = k\hat{k}_t - x\hat{x}_t - (1-\delta)k\hat{k}_{t-1}$$

$$0 = a\hat{a}_t - k\hat{k}_{t-1} - m\hat{m}_t$$

$$0 = z_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{h}_t - \hat{y}_t$$

$$0 = z_t + \alpha\hat{k}_{t-1} - \alpha\hat{h}_t - \hat{w}_t$$

$$0 = z_t + (1-\alpha)\hat{h}_t + (\alpha-1)\hat{k}_{t-1} - \hat{r}_t$$

$$0 = \lambda\hat{\lambda}_t + \varphi\mu_2\hat{\varphi}_t - q\hat{q}_t$$

$$0 = \hat{\Theta}_t + \hat{m}_{t-1} - \hat{\Pi}_t - \hat{m}_t$$

$$0 = \hat{\lambda}_t + \hat{w}_t$$

$$0 = \frac{\lambda}{\lambda + \varphi\mu_1}\hat{\lambda}_t + \frac{\varphi\mu_1}{\lambda + \varphi\mu_1}\hat{\varphi}_t + \hat{c}_t$$

$$E_t\left\{\lambda\hat{\Pi}_{t+1} - \frac{\beta}{\Pi}\left(-(1-\gamma)\frac{1}{a}\hat{a}_{t+1} + \lambda\hat{\lambda}_{t+1} + \varphi\hat{\varphi}_{t+1}\right)\right\} = -\lambda\hat{\lambda}_t$$

$$E_t\left\{\beta\left((1-\gamma)\frac{1}{a}\hat{a}_{t+1} - \lambda r\left(\hat{\lambda}_{t+1} + \hat{r}_{t+1}\right) - (1-\delta)q\hat{q}_{t+1}\right)\right\} = -q\hat{q}_t$$

yielding a uniquely solvable system of 17 equations and 17 variables.

The system of equations can be written in matrix format as:

$$A_1 * E_t\{X_{t+1}\} = A_2 * X_t + \xi_{t+1} \tag{4.12}$$

where A_1 and A_2 are 17 x 17 matrices used to map the log-linearized autoregressive productivity and money growth rate processes, the time invariant conditions, and the log-linearized equations into the matrix format given by equation (4.12). Specifically, A_1 and A_2 contain the coefficients of the variables contained in X_{t+1} and X_t, respectively. The matrix X_{t+1} is the updated version of X_t, a vector composed of two specific vectors,

$Z_{1,t}$ and $Z_{2,t}$. $Z_{1,t}$ contains the technological and monetary growth rates along with the predetermined variables. $Z_{2,t}$ contains the jump variables. The transposes of each of these vectors are identified below:

$$X_t^T = [Z_{1,t} \ Z_{2,t}]$$

where $Z_{1,t}^T = [\hat{z}_t, \hat{\theta}_t, \hat{k}_{t-1}, \hat{m}_{t-1}]$ and $Z_{2,t}^T = [\hat{c}_t, \hat{h}_t, \hat{m}_t, \hat{a}_t, \hat{x}_t, \hat{k}_t, \hat{y}_t, \hat{w}_t, \hat{r}_t, \hat{\pi}_t, \hat{\lambda}_t, \hat{q}_t, \hat{\varphi}_t]$.

The Klein (2000) solution technique allows for A_1 to be singular, which allows for the inclusion of the predetermined variables contained in $Z_{1,t}$ when solving for the household's general equilibrium. The generalized Schur decomposition decomposes the (4.1.2) into stable and unstable blocks. The unstable blocks are solved forward and stable blocks backward to derive the household's equilibrium (Klein, 2000).

The size of the unstable block is determined by the number of unstable eigenvalues of the system. Blanchard and Kahn (1980) argue that a system has a determinate solution if and only if the number of unstable eigenvalues is equal to the number of forward-looking variables. In solving (4.1.2), the number of unstable eigenvalues was indeed calculated to equal the number of forward-looking variables contained in $Z_{2,t}$: 13. This result indicates the determinacy condition is satisfied.

The solution of the system yields two matrices, M and C, which are used to determine the dynamic behavior of the economic variables represented in $Z_{2,t}$ in response to a technology and money growth rate shock. The percent deviation from steady-state j periods after the shocks occur is calculated using the following equation:

$$Z_{2,t+j} = C * M^j * N * S_{t+j} * 100 \; \forall \; j = 0 \; to \; T.$$

The dimensions of C and M are 13 x 4 and 4 x 4, respectively. The matrix N is a 4 x 2 matrix of the form:

$$N = \begin{bmatrix} I_2 \\ 0_{2x2} \end{bmatrix}.$$

The matrix N is multiplied onto the matrix S_{t+1}, which contains the standard deviation of the productivity and money growth rate shocks, σ_z and σ_θ, respectively, and takes the following form:

$$S_{t+1} = \begin{bmatrix} \sigma_z \\ \sigma_\theta \end{bmatrix}.$$

Monetary and technological disturbances cause households to deviate from the economy's real equilibrium values. The real effects of disturbances or shocks are attributable to the changes they cause in the household's inflationary expectations (Walsh 2003). Since the size of the shock plays an important role, the determination of how the real equilibrium is changed in the event of a shock is dependent on the parameterization of the model.

4.2 Calibration

The parameterization of the model was based on a calibration exercise using U.S. data collected over the sample period from 1964:Q1 to 2004:Q4. The U.S. data was collected from the Bureau of Labor Statistics (BLS) and the Bureau of Economic Analysis (BEA). Table 4.2 contains the calibrated parameters used throughout the remainder of this analysis.

Table 4.2.1: Calibrated Parameters

α	δ	σ_θ	ρ_θ
0.375	0.020	0.009	0.638
B	β	σ_z	ρ_z
2.426	0.986	0.006	0.950

The parameter α is calibrated to equal the average capital share of output over the sample period. The capital share measure is a ratio. The numerator is the sum of the measured value of capital income and the imputed flow of services from both the stock of consumer durables and the stock of government capital. The denominator is the sum of GDP and the flow of services from both consumer durables and government capital. This ratio is calculated for each quarter of the sample period, and the average is then taken.

The parameter δ is calibrated to equal the quarterly average investment-to-capital ratio over the sample period. Investment is total real investment expenditures, and capital is the real value of the total stock of capital. The investment-to-capital ratio is calculated and then divided by 4 to obtain a quarterly measure.

The discount factor, β, is calibrated to equal the reciprocal of the gross real interest rate in the following manner:

$$\beta = \frac{1}{R}.$$

The gross real interest rate, R, is derived using the monthly measures of the three- month Treasury rate. These monthly measures are converted to effective annual rates, which are then converted to nominal quarterly measures. A gross quarterly interest rate series is then derived by adding 1 to the adjusted periodic interest rate, and R is set equal to the average of this series. The parameter β is calibrated to equal approximately 0.986.

The M1 money stock and the U.S. civilian non-institutional population (16 and over) monthly time series were converted from monthly to quarterly measures. These time series were used to calculate per capita money balance measures. A quarterly money growth rate time series is formed by taking the logged difference of the quarterly per capita money balance measure. The average gross quarterly money growth rate over the sample period, θ, is approximately 0.00977, which corresponds to an annual money growth rate of approximately 3.908 percent.

The persistence of the money growth rate is denoted by ρ_θ. The value of ρ_θ is calibrated by estimating the following OLS regression:

$$\theta_t = \alpha_0 + \rho_\theta \ \theta_{t-1} + \xi_{\theta_t}$$

where θ_t represents the money growth rate at time *t*. The parameter σ_θ is the standard deviation of the error term, ξ_{θ_t}, and is approximately 0.00886.

The scaling coefficient in the household's utility function, B, is calibrated using α, β, the average of the output-to-consumption ratio $\left(\frac{y}{c} = 1.403942\right)$, and the fraction of time U.S. households spend working over the sample period, h_0 = 0.337559.[11] The calibration of B also takes into account Θ = 1.03908, the average gross annual growth

[11] The parameter h used is calibrated by first calculating the fraction of time spent working, h_0= 0.337559, which is the average weekly hours (35.44) worked over the sample period divided by 105 (the number of hours available for labor-leisure choices). Finally, h_0 is multiplied by likelihood of employment, taken to be the average employment rate over the sample period (94.08%), to obtain h = 0.317634.

rate of money over the sample period. Finally, the calibrated parameter B is derived in the following manner:

$$\mathrm{B} = \frac{\left[(1-\alpha)\beta\left(\frac{y}{c}\right)\right]}{h_0\Theta} \, .$$

The quarterly population series used to calculate per capita real money balances is again used to calculate per capita real output. Taking the logged difference of this series, a technology growth series is determined. The persistence of the technology growth rate, $\rho_z = 0.95$, is calibrated using the technology growth rate series by estimating the following OLS regression: $z_t = \alpha_0 + \rho_z \; z_{t-1} + \xi_{z_t}$

where z_t represents the technology growth rate at time t. The parameter σ_z is the standard deviation of the residual, ξ_{z_t} and is approximately 0.0056.

Table 4.2.2: U.S. Time Series Data:
Standard Deviations and Correlations with Output

U.S.Time Series Data 1964:01 to 2004:04	s.d.	corr. w/ output
Output	1.378	1.000
Consumption	0.743	0.654
Investment	4.334	0.945
Capital	1.019	0.432
Hours	1.064	0.892
Wages	0.892	0.682
Inflation	0.320	0.161

The calibration is done in order to create a set of simulated data from the model that attempts to capture the business-cycle properties of the U.S. economy. To allow for comparison, Table 4.2.2 displays the standard deviations and correlations with output for the U.S. Time Series data over the full sample period, 1964:01 to 2004:04.

Section 5. Business-Cycle Effects and Steady-State Properties

In this analysis, the households are assumed to have log-separable preferences for consumption and wealth. I define the household's utility function in the following manner:

$$U(c_t, a_t, h_t) = \gamma\, ln(c_t) + (1-\gamma) ln(a_t) - Bh_t. \tag{4.30}$$

This is a unique specification of equation (3.10) for which $\sigma = \eta = 1$. In this economy, agents' preferences are log-separable in consumption, c_t, and wealth, a_t, while incorporating the indivisible labor assumption as described fully in Hansen (1985).

The short-run impacts are derived in order to show how the preference for wealth affects the business-cycle properties of the model and to evaluate the effects of inflation on the business cycle. The analysis includes the business-cycle results under a deterministic and a stochastic money growth rate. The steady-state values and ratios associated with the model are derived in order to determine if the model economy's long-run equilibrium is dependent on the nominal growth rate of money. The steady-state values and ratios will also aid in the determination of the economy's sensitivity to changes in status preferences.

5.1 Short-Run Impacts

The business-cycle properties of economies that differ in their preference for wealth are examined for various inflation rates. The deterministic results represent the short-run effects of an economy that is subject to a technology shock only. In the deterministic case, money growth rates remain constant. The economy is not subject to a shock on the growth rate of money that could lead to changes in the agent's inflation expectations. The stochastic results allow for the examination of the short-run effects of an economy that is subject to both a technology and a money growth rate shock.

5.1.1: CIA for Consumption Goods Only

The business-cycle statistics presented in Table 5.1.1 are the short-run results from the analysis of an economy for which the CIA constraint takes the first specification. Money is used for consumption goods only. Various money growth rates ranging from -5.6 percent to 100 percent are used to show the impact of an inflation tax on the business cycle. The ranges of the standard deviation estimates and their correlation with output are presented for economies having no, low, and moderate preference for wealth. The size of the inflationary impact is measured through the width of the window represented by the range.[12]

[12] If only one value is listed for the standard deviation or correlation with output measures, then inflation did not have a deviating effect on the respective measures.

Table 5.1.1: Business Cycle Statistics for Log-Separable Utility — CIA on Consumption Goods

Relative Preference for Wealth	$(1-\gamma) = 0$		$(1-\gamma) = 0.05$		$(1-\gamma) = 0.25$	
Deterministic Case	Standard Deviation	Corr. w/ Output	Standard Deviation	Corr. w/ Output	Standard Deviation	Corr. w/ Output
Output	1.36	1.00	1.38 - 1.39	1.00	1.45 - 1.46	1.00
Consumption	0.38	0.87	0.36 - 0.37	0.87	0.30 - 0.31	0.87
Investment	5.05	0.99	4.66 - 4.73	0.99	3.52 - 3.72	1.00
Capital	0.35	0.35	0.32 - 0.33	0.33 - 0.34	0.25 - 0.26	0.29 - 0.30
Hours	1.05	0.98	1.08 - 1.09	0.99	1.19 - 1.21	0.99
Wages	0.38	0.87	0.36 - 0.37	0.87	0.30 - 0.31	0.87
Wealth	0.34	0.12	0.32	0.10	0.24 - 0.26	0.03 - 0.04
Stochastic Case	Standard Deviation	Corr. w/ Output	Standard Deviation	Corr. w/ Output	Standard Deviation	Corr. w/ Output
Output	1.37	1.00	1.38 - 1.39	1.00	1.45 - 1.47	1.00
Consumption	0.63 - 0.77	0.52 - 0.61	0.63 - 0.77	0.49 - 0.59	0.61 - 0.76	0.39 - 0.52
Investment	5.27 - 5.4	0.90 - 0.93	4.92 - 4.97	0.91 - 0.94	3.68 - 3.83	0.94 - 0.96
Capital	0.36 - 0.37	0.32 - 0.34	0.33 - 0.34	0.31 - 0.32	0.25 - 0.26	0.28 - 0.29
Hours	1.06 - 1.07	0.98	1.09 - 1.10	0.98	1.19 - 1.22	0.99
Wages	0.39	0.83 - 0.85	0.37	0.83 - 0.85	0.30 - 0.31	0.85 - 0.86
Wealth	0.35 - 0.36	0.12	0.33	0.10 - 0.11	0.25 - 0.26	0.04 - 0.05

Note: The use of various money growth rates ranging from -5.6% to 100% are used to show the impact of an inflation tax on the business cycle under different specifications. The impact of inflation on the business cycle is measured by the width of the window represented by the ranges of the estimated percent standard deviation and correlation with output measures that are presented above. If only one value is listed for the standard deviation or correlation with output measures, then inflation did not have a deviating effect on the respective measures.

The volatility of output, capital, hours worked, wages, and wealth show only small differences between the case of deterministic and the case of stochastic money growth. However, in the stochastic case, consumption and investment become more volatile, and consumption, investment, capital, and wages become less correlated with output.

In both the deterministic and the stochastic case, the volatility of the economic variables differs with the preference for wealth. As the preference for wealth increases, the volatility of output and hours worked increases; the volatility of the other economic variables—consumption, investment, capital, wages, and wealth—decreases; and inflation has a greater impact on the volatility of investment. The size of the inflationary impact is measured by the width of the window of standard deviation measures obtained by increasing the level of inflation from -5.6 percent to 100 percent.

In the deterministic case, the effects of an increasing preference for wealth on the economic variables' correlation with output are minimal except for capital and wealth. As the preference for wealth moves from low to moderate, output becomes less correlated with both capital and wealth. The effects of changes in output on wealth and capital accumulation are reduced. On the other hand, the reduced correlation means that changes in per capita wealth or capital accumulation have a greater impact on output as the preference for wealth increases.

In the stochastic case, the volatilities of consumption and wages are found to differ, unlike those in the deterministic case. Households subject to money growth rate shocks change their inflationary expectations, causing the volatility of consumption to be much higher than that of wages. The correlation with output of consumption, capital, and wealth decreases as the preference for wealth increases. Changes in consumption, capital, or wealth have a larger impact on output; however, changes in output have a lesser impact on consumption, capital, and wealth when the preference for wealth is higher.

5.1.2: CIA for Consumption and Investment Goods

The business-cycle statistics presented in Table 5.1.2 are the short-run results from the analysis of an economy for which the CIA constraint takes the second specification. Money is used for both consumption and investment goods. The effects of inflation on the business cycle are greater when both consumption and investment require the use of money. In both the deterministic and the stochastic case, output and hours worked become more volatile as the preference for wealth increases. The volatility of the other variables is reduced by an increase in the preference for wealth. Significant differences in the volatilities of the variables exist between the deterministic and stochastic cases under this CIA specification.

Table 5.1.2: Business Cycle Statistics for Log-Separable Utility – CIA on Consumption and Investment

Relative Preference for Wealth	$(1-\gamma)=0$		$(1-\gamma)=0.05$		$(1-\gamma)=0.25$	
Deterministic Case	Standard Deviation	Corr. w/ Output	Standard Deviation	Corr. w/ Output	Standard Deviation	Corr. w/ Output
Output	1.16 - 1.23	1.00	1.18 - 1.24	1.00	1.26 - 1.31	1.00
Consumption	0.37 - 0.42	0.90 - 0.93	0.35 - 0.40	0.90 - 0.93	0.30 - 0.33	0.89 - 0.92
Investment	4.44 - 4.86	0.99	4.18 - 4.52	0.99	3.31 - 3.46	1.00
Capital	0.31 - 0.34	0.35 - 0.37	0.29 - 0.31	0.33 - 0.35	0.23 - 0.24	0.29 - 0.30
Hours	0.71 - 0.83	0.98	0.75 - 0.86	0.98	0.89 - 0.97	0.99
Wages	0.44 - 0.48	0.94 - 0.95	0.43 - 0.47	0.94 - 0.95	0.38 - 0.40	0.94 - 0.95
Wealth	0.31 - 0.33	0.39 - 0.43	0.29 - 0.31	0.37 - 0.41	0.23 - 0.24	0.32 - 0.34
Stochastic Case	Standard Deviation	Corr. w/ Output	Standard Deviation	Corr. w/ Output	Standard Deviation	Corr. w/ Output
Output	1.34 - 1.36	1.00	1.36 - 1.38	1.00	1.42 - 1.46	1.00
Consumption	0.37 - 0.44	0.82 - 0.85	0.36 - 0.41	0.82 - 0.85	0.30 - 0.34	0.82 - 0.85
Investment	5.10 - 6.22	0.98 - 0.99	4.76 - 5.70	0.99	3.69 - 4.17	0.99 - 1.00
Capital	0.33 - 0.41	0.35 - 0.37	0.31 - 0.37	0.34 - 0.36	0.25 - 0.27	0.30 - 0.31
Hours	1.19 - 1.33	0.88 - 0.91	1.21 - 1.36	0.89 - 0.91	1.30 - 1.46	0.92 - 0.93
Wages	0.56 - 0.66	0.36 - 0.40	0.55 - 0.64	0.35 - 0.39	0.51 - 0.60	0.31 - 0.34
Wealth	0.33 - 0.40	0.39 - 0.43	0.31 - 0.36	0.37 - 0.41	0.24 - 0.27	0.32 - 0.34

Note: The use of various money growth rates ranging from -5.6% to 100% are used to show the impact of an inflation tax on the business cycle under different specifications. The impact of inflation on the business cycle is measured by the width of the window represented by the ranges of the estimated percent standard deviation and correlation with output measures that are presented above. If only one value is listed for the standard deviation or correlation with output measures, then inflation did not have a deviating effect on the respective measures.

When subject to a money growth rate shock, the volatility of output, investment, and hours worked is much higher than in the deterministic case regardless of the preference for wealth. The volatility of consumption, capital, and wealth is not as affected by the money growth rate shock. Consumption volatility is the least affected by the money supply shock; however, the correlation of consumption, hours worked, and wages with output is significantly reduced. Wages become much less correlated with output. The correlation of wages with output falls from approximately 0.94 in the deterministic case to around 0.34 in the stochastic case.

The impact of the money supply shock on the correlation of wages with output causes inflationary changes in the volatility of wages to have a greater impact on the volatility of output. Since the correlation of consumption and hours worked with output is rather high, the impact of inflation on the volatility of wages will also have a greater impact on the volatility of hours worked and consumption.

While consumption is smoother than the U.S. data suggests, the results displayed in Tables 5.1.1 and 5.1.2 for the stochastic case capture many of the aspects of the real U.S. data displayed in Table 4.2.2. Wealth used to facilitate future consumption is composed of capital and, in this case, consumption and investment. When there is no preference for wealth, households are concerned with consumption smoothing. When a wealth preference is present, households still concerned with consumption smoothing are also concerned with smoothing their wealth. Thus, wealth and all of its individual components become less volatile as the preference for wealth increases.

5.1.3: Dynamic Response to Money Growth Rate or Technology Shock

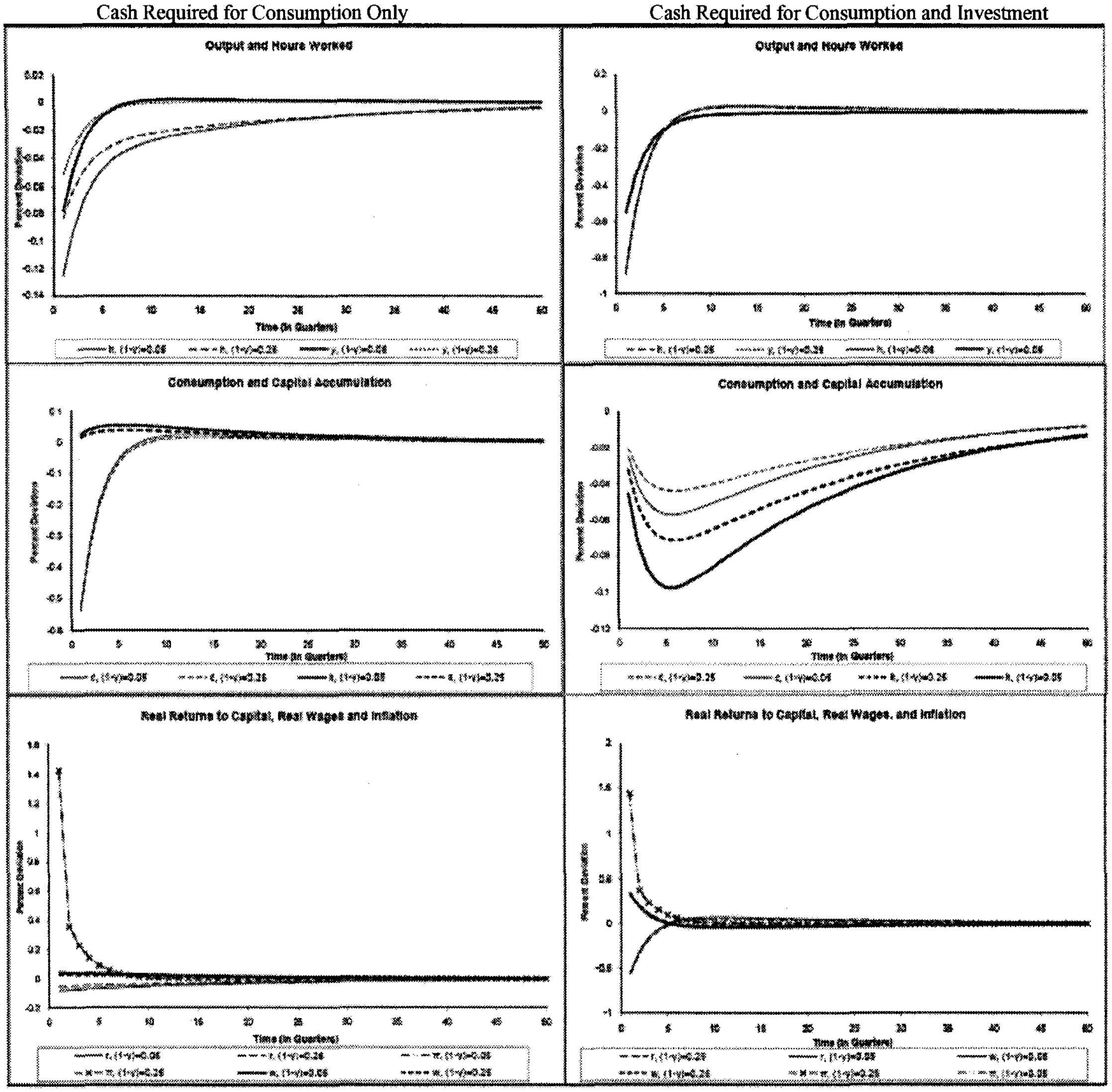
The impact of the monetary shock on the correlation with output is sensitive to the specification of the CIA constraint. The business cycle statistics from Sections 5.1.1 and 5.1.2 have shown that larger business-cycle effects take place in the stochastic case where households are subject to both technology and money growth rate shocks. This result logically leads to a need for further investigation into the economy's dynamic response to each of these innovations individually. Impulse response functions are commonly used in monetary economics to measure the dynamic response of the economic variables when a monetary innovation occurs.

The impulse response functions displayed in Figure 5.1.3.a and 5.1.3.b show how the dynamic responses of the economic variables to a money rate innovation differ from the responses attributable to a technology innovation. The response functions are shown for the two specifications of the CIA constraint and for low and moderate wealth preferences. The impulse response functions for output, hours worked, consumption, capital, wages, rental rates, and inflation graphically represent the percent deviation from the general equilibrium values caused by the innovation.

As shown in Figure 5.1.3.a, a positive shock to the money growth rate causes inflation to deviate from its steady-state value. The tax effects of inflation cause households to substitute away from any goods that require the use of money. The impulse-response functions on the left side of Figure 5.1.3.a show the effects of a monetary innovation when households hold money for the purchase of consumption goods only. A positive deviation in the inflation rate initially has a negative effect on the rental rates. Households decrease their demand for investment goods. The real wage rises initially in response to the shock. Firms substitute away from the relatively more expensive input leading to a reduction in hours worked. Increased wages along with the decrease in capital allow households to increase current consumption and to smooth their consumption profile.

Figure 5.1.3.a: Sensitivity of Impulse Response Functions to the Specification of the CIA Constraint and the Relative Preference for Wealth (Money Shock Only)

Cash Required for Consumption Only — Cash Required for Consumption and Investment



As the preference for wealth increases, wages and rental rates become less volatile. Thus, the initial decline in hours worked is smaller. As a result, the magnitude of the response of output to the money growth rate innovation decreases as the preference for wealth increases.

On the right side of Table 5.1.3.a, money is demanded to facilitate consumption and investment. In this case the responses of output, hours, rental rates, and wages are similar to those seen when money is demanded for consumption only. However, these responses are not sensitive to the relative preference for wealth. Interesting substitution effects are seen in the response of consumption and capital accumulation to the money growth rate innovation.

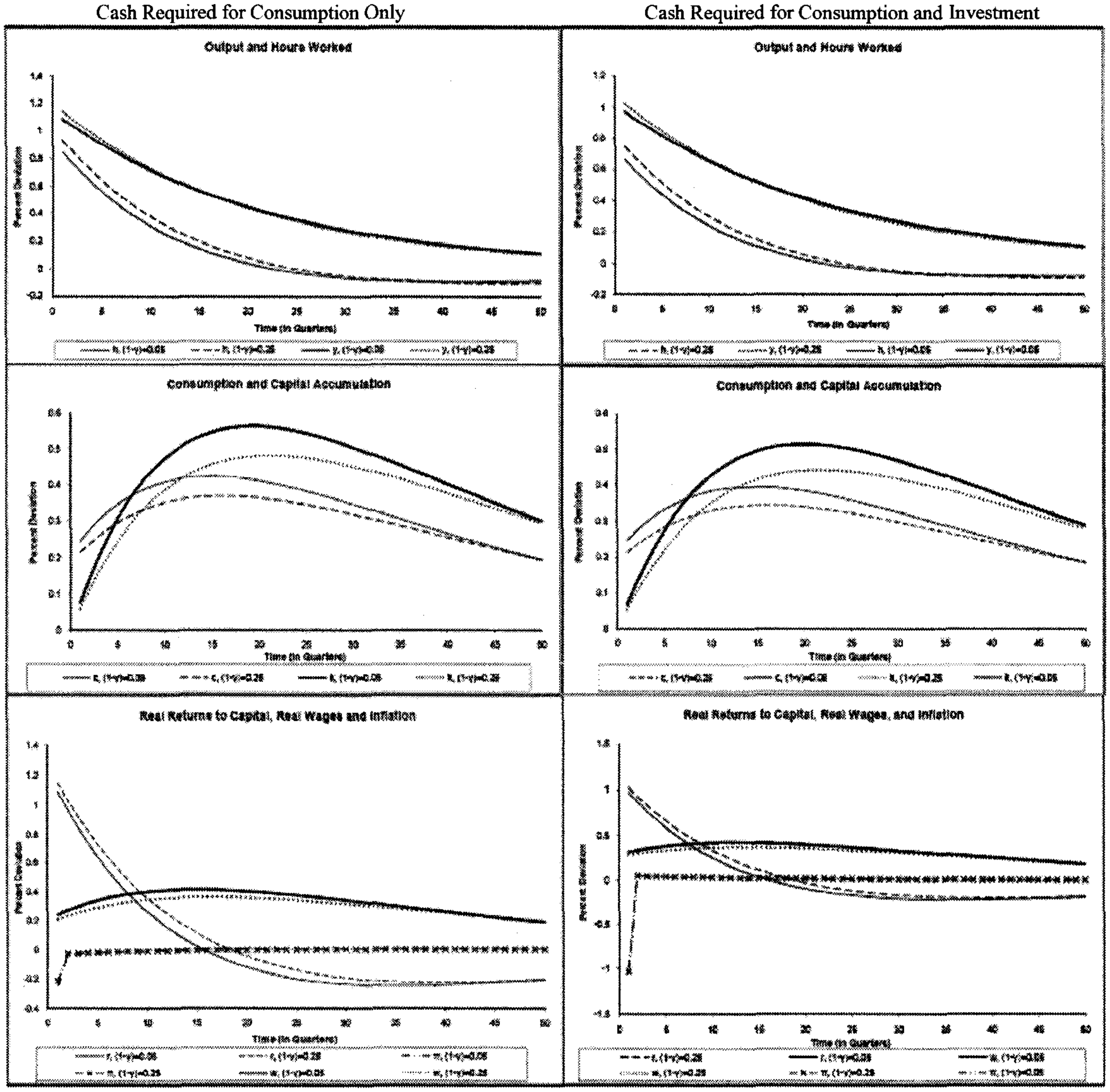
Inflation taxes both consumption and investment. The effects of this tax result in a persistent negative deviation of both consumption and capital accumulation from their steady-state level. For consumption and capital accumulation, the effects of the monetary shock are sensitive to the relative preference for wealth. As the preference for wealth increases, consumption and capital accumulation are less volatile; this is consistent with the idea of consumption as well as wealth smoothing.

Figure 5.1.3.b displays the dynamic responses of the economic variables to a positive technology innovation. The responses to a technology innovation differ not only in size but nature from the responses associated with a money growth rate innovation. For both CIA specifications, output and hours worked rise in response to the productivity shock, though the magnitude of these responses is larger when cash is used for consumption goods only. Wages and rental rates both increase. Firms, substituting toward the relatively cheaper input, become more labor intensive. When cash is used for both consumption and investment, wages are slightly more sensitive and rental rates are

slightly less sensitive to the innovation. Thus, the initial rise in hours worked is smaller, as is the rise in output.

Figure 5.1.3.b: Sensitivity of Impulse Response Functions to the Specification of the CIA Constraint and the Relative Preference for Wealth (Technology Shock Only)

Cash Required for Consumption Only — Cash Required for Consumption and Investment



Under both CIA specifications, the supply of goods increases, placing a downward pressure on prices in the initial period, making both consumption and investment relatively cheaper. Households increase their consumption and capital. Wages based on productivity are persistent and higher, allowing households to attain higher consumption profiles. Again, when the preference for wealth is higher, the capital and consumption profiles become smoother as agents attempt to smooth wealth.

To graph the impulse-response functions in Figures 5.1.3.a–5.1.3.b, a steady-state money growth rate of 3.908 percent is used.[13] The steady-state values of the economic variables for which the percent deviations of the impulse response are derived are affected by the steady-state value of inflation used. Thus, further analysis of the long-run impact of inflation on the steady-state values of the economic variables is undertaken.

5.2 Long-Run Impacts

From the analysis of the competitive equilibrium conditions, the effects of changes in the preference for wealth and the steady-state level of inflation are revealed. The differences in the steady-state levels of the economic variables are dependent on the definition of the CIA constraint (or the specification of the economy's goods whose purchase requires the use of real money balances). The steady-state solutions that result from the competitive equilibrium are derived in the following manner:

$$m = \Theta \frac{m}{\Pi}$$

$$\Pi = \Theta.$$

[13] The steady-state money growth rate is the average inflation rate derived from the U.S. data over the 1964:Q1 to 2004:Q4 sample period.

In the long run, the gross growth rate of inflation is equal to the gross growth rate of the money supply, and if capital from one period to the next is not allowed to vary, then steady-state investment must equal the capital lost from depreciation.

$$k = x + (1-\delta)k$$
$$x = \delta k$$

The remainder of the steady-state general equilibrium conditions can be manipulated to obtain steady-state identities for $x, y, w, r, c, m,$ and a, which are all functions of k and h. These identities are listed below:

$$x = \delta k$$
$$y = k^{\alpha} h^{1-\alpha}$$
$$w = (1-\alpha)\left(\frac{k}{h}\right)^{\alpha}$$
$$r = \alpha\left(\frac{h}{k}\right)^{1-\alpha}$$

$$c = k^{\alpha} h^{1-\alpha} - \delta k$$
$$m = \mu_1 \left(k^{\alpha} h^{1-\alpha} - \delta k\right) + \mu_2 \delta k$$
$$a = k + \mu_1 \left(k^{\alpha} h^{1-\alpha} - \delta k\right) + \mu_2 \delta .$$

After solving for λ and q,

$$\lambda = U^{ss}{}_{c}(c,a,h) - \varphi\mu_1$$

$$q = U^{ss}{}_{c}(c,a,h) - \varphi\mu_1 + \varphi\mu_2$$

the three remaining conditions can be written as functions of three variables, $\{k,h,\varphi\}$.

$$\left[U^{ss}{}_c(c,a,h)-\varphi\mu_1\right]w=-U^{ss}{}_h(c,a,h)$$

$$U^{ss}{}_c(c,a,h)-\varphi\mu_1=\beta\left[\left(U^{ss}{}_a(c,a,h)+U^{ss}{}_c(c,a,h)-\varphi\mu_1+\varphi\right)\left(\frac{1}{\Pi}\right)\right]$$

$$U^{ss}{}_c(c,a,h)-\varphi\mu_1+\varphi\mu_2=\beta\left[\begin{array}{l}U^{ss}{}_a(c,a,h)+\left(U^{ss}{}_c(c,a,h)-\varphi\mu_1\right)r\\ \qquad+\left(U^{ss}{}_c(c,a,h)-\varphi\mu_1+\varphi\mu_2\right)(1-\delta)\end{array}\right]$$

Thus, a unique solution of the system can be determined.

The CIA constraint is designed in such a way that multiple cases can be examined by the model. The first case considered is that in which households hold real money balances to facilitate the purchase of consumption goods only ($\mu_1=1$ and $\mu_2=0$). The second is one for which cash is required for the purchase of both consumption and investment goods ($\mu_1=1$ and $\mu_2=1$).

5.2.1: CIA for Consumption Goods Only

The steady-state values of the real economic variables are found to be affected by the preference for wealth as well as the growth rate of money. Table 5.2.1 displays the impact of an increase in the preference for wealth as well as the impact of inflation on the steady-state values under the first CIA specification.

As the preference for wealth increases, the steady-state values of output, investment, capital, real wages, leisure, and welfare increase. The steady-state real return to capital is inversely related to the preference for wealth when cash is required for consumption only. The effect of a preference for wealth on the steady-state level of consumption is indeterminate. The steady-state level of consumption increases with the preference for wealth up to a given preference level, $(1-\gamma)^*$, where $0.1<(1-\gamma)^*<0.25$. For preferences greater than $(1-\gamma)^*$, the steady-state level of consumption declines as the preference for wealth increases.

Table 5.2.1: Steady-State Values Separable Utility — CIA on Consumption Only

Wealth Preference	**(1 − γ) = 0**	**(1 − γ) = 0.1**	**(1 − γ) = 0.25**
Output	1.392 - 1.098 (D)	1.496 - 1.229 (D)	1.634 - 1.406 (D)
Consumption	1.085 - 0.856 (D)	1.098 - 0.888 (D)	1.074 - 0.886 (D)
Investment	0.307 - 0.242 (D)	0.397 - 0.340 (D)	0.560 - 0.520 (D)
Capital	15.205 - 11.994 (D)	19.684 - 16.861 (D)	27.759 - 25.772 (D)
Hours	0.330 - 0.260 (D)	0.317 - 0.254 (D)	0.297 - 0.244 (D)
Wages	2.632 (No change)	3.019 - 2.943 (I)	3.593 - 3.432 (I)
Welfare	(-0.720) - (-0.788) (D)	(-0.382) - (-0.435) (D)	(0.173) - (0.137) (D)
Rental Rates	0.034 (No change)	0.029 - 0.027 (D)	0.022 - 0.020 (D)

Note: The ranges of the steady-state values presented on the table above represent the impacts of inflation on the steady-state values. The inflation rates that correspond to the money growth rates used in approximating the state-values range from -5.60% to 100%. All ranges are expressed from the maximum to minimum. To see the direction of the impact of inflation, the symbol (D), (I), or (No change) is used to denote the decreasing, increasing, or non-distortionary effect of inflation on the steady-state values of the given economic variables, respectively.

Steady-state values of the real economic variables other than wages are found to decline as the growth rate of money increases when households have a preference for wealth. When there is no preference for wealth and money is required only for the purchase of consumption goods, real wages and rental rates are not affected by inflation. Real wages increase and rental rates decrease with inflation when there is a preference for wealth under the first CIA constraint.[14]

[14] This is because superneutrality no longer holds when there is a preference for wealth. Inflation causes changes in the steady-state capital-hours ratio of the monetary economy. Movements of the economy's steady-state ratios are discussed more thoroughly in Section 5.3.

5.2.2: CIA for Consumption and Investment Goods

The steady-state values of the real economic variables are found to be affected by the preference for wealth as well as the growth rate of money. Table 5.2.2 displays the impact of an increase in the preference for wealth as well as the impact of inflation on the steady-state values under the second CIA specification. When both consumption and investment goods require the use of money, the effects of an increasing preference for wealth are similar to those seen when money is required for consumption only; however, the impacts of inflation on the steady-state values of the economic variables differ.

Table 5.2.2: Steady-State Values Separable Utility — CIA on Consumption and Investment

Wealth Preference	$(1-\gamma)=0$	$(1-\gamma)=0.1$	$(1-\gamma)=0.25$
Output	1.392 - 0.898 (D)	1.499 - 1.000 (D)	1.642 - 1.135 (D)
Consumption	1.085 - 0.742 (D)	1.100 - 0.778 (D)	1.077 - 0.790 (D)
Investment	0.307 - 0.156 (D)	0.399 - 0.222 (D)	0.566 - 0.345 (D)
Capital	15.205 - 7.740 (D)	19.776 - 10.985 (D)	28.021 - 17.107 (D)
Hours	0.330 - 0.246 (D)	0.318 - 0.236 (D)	0.298 - 0.222 (D)
Wages	2.632 - 2.282 (D)	2.948 - 2.640 (D)	3.441 - 3.193 (D)
Welfare	(-0.720) - (-0.895) (D)	(-0.379) - (-0.551) (D)	(0.180) - (0.011) (D)
Rental Rates	0.044 - 0.034 (I)	0.034 - 0.028 (I)	0.025 - 0.022 (I)

Note: The ranges of the steady-state values presented on the table above represent the impacts of inflation on the steady-state values. The inflation rates that correspond to the money growth rates used in approximating the state-values range from -5.60% to 100%. All ranges are expressed from the maximum to minimum. To see the direction of the impact of inflation, the symbol (D), (I), or (No change) is used to denote the decreasing, increasing, or non-distortionary effect of inflation on the steady-state values of the given economic variables, respectively.

The deviating effects of inflation on the steady-state values of the economic variables are greater in magnitude. Some variables, such as real wages and rental rates, show key differences. Real wages decline as inflation rises, and real rental rates on capital increase with inflation. These results are opposite those reported in Table 5.2.1 for which money is required for consumption only. These differences are due to the effects of inflation on the capital-to-hours ratio under the second CIA specification on which wages and rental rates are based.

The differences in the impacts of inflation on the steady-state values of wages and rental rates lead to a need to examine the economy's steady-state ratios. Thus, the next section, Section 5.3, compares the differences in these steady-state ratios under the two CIA specifications and the different ways in which inflation affects these ratios.

5.3 *Steady-State Ratio Analysis*

When consumption goods are the only goods classified as "cash goods," the CIA constraint equates consumption with real money balances, $c_t = m_t$, when the CIA and budget constraints are binding. Thus, if money is needed only for consumption (not for investment), then consumption is equal to money in the steady state, $c^{ss} = m^{ss}$, and the steady-state values for the consumption capital and money-capital ratios are equal. More interesting, under this specification wealth would be a function of accumulated capital and consumption. This specification supports the theory of conspicuous consumption and on the surface is thought to emulate the behavior exhibited in the United States. [15]

When cash is required for consumption and investment, the CIA constraint together with the budget constraint, if both are binding, equate real money balances with

[15] Hopkins and Kornienko (2004) mention that in societies with relatively higher per capita income measures most society members have higher levels of conspicuous consumption. Hopkins and Kornienko form a model in which individuals care about status, allowing individuals' utility to be dependent on the utility of others.

output, $y_t = m_t$. If cash is needed for consumption and investment, then output equals money in the steady state, $y^{ss} = m^{ss}$. Thus, the output-capital ratio and the money-capital ratio have equal long-run steady-state values.

The steady-state ratios for the economy have been derived from the steady-state equations for the real variables, $\{k, c, x, y, a\}$, and are presented in Table 5.3.1. All of the ratios are in terms of the steady-state rental rate of capital, r, and the calibrated parameters of the model, presented in Table 4.2.1.[16] When the CIA constraint applies to consumption only, the money-to-capital ratio is equal to the consumption-to-capital ratio. However, when the CIA constraint applies to both consumption and investment, the money-to-capital ratio is equal to the output-to-capital ratio.

Table 5.3.1: Steady-State Ratios

$\frac{k}{h}$	$\frac{c}{k}$	$\frac{m}{k}$	$\frac{y}{k}$	$\frac{a}{k}$
$\left(\frac{r}{\alpha}\right)^{\frac{1}{\alpha-1}}$	$\frac{r}{\alpha}-\delta$	when $\mu_1 = 1$ and $\mu_2 = 0$ $\frac{r}{\alpha}-\delta$ or when $\mu_1 = 1$ and $\mu_2 = 1$ $\frac{r}{\alpha}$	$\frac{r}{\alpha}$	$1+\frac{m}{k}$

Note: The CIA constraint is defined by equation 3.11. When $\mu_1 = 1$ and $\mu_2 = 0$, cash is used for consumption goods only; however, when $\mu_1 = \mu_2 = 1$, cash is required for both consumption and investment goods.

[16] The steady-state ratios presented in Table 5.3.1 could have been expressed in terms of real wages instead of rental rates, since real wages are a multiple of rental rates, $w = \frac{(1-\alpha)}{\alpha}\left(\frac{k}{h}\right) r$.

Under either specification of the CIA constraint, the wealth-to-capital ratio exceeds the money-to-capital ratio by one, and the consumption-to-capital ratio is less than the output-to-capital ratio by the amount of the depreciation rate of capital. This is understandable because by the definition of steady state the capital from one time period must equal the capital from the next time period. In order for this definition to hold, the steady-state investment for each period must exactly replace the depreciated capital from the previous period.

In Tables 5.2.1 and 5.2.2, the impacts of inflation on real wages and rental rates of capital differ with the specification of the CIA constraint. The rental rates decrease with inflation under the first CIA specification and increase under the second. The effects of inflation on real wages are opposite in direction when compared to the inflationary effects on rental rates. Both rental rates and real wages in equilibrium are dependent on the steady-state capital-to-labor ratio. Thus, the inflationary effects on the capital-to-labor ratio as well as the other steady-state ratios initially addressed in Table 5.3.1 are presented in Table 5.3.2.

In the case for which the preference or status is absent (i.e., $(1-\gamma)=0$) and the CIA constraint is on consumption goods only, inflation does not significantly affect the steady-state ratios of the economy; superneutrality holds.[17] However, when there is no preference for wealth or the CIA constraint applies to both consumption and investment,

[17] This specification of the CIA constraint and the household's utility function are the same as those used by Cooley and Hansen (1989), and the results support the superneutrality reported in their analysis.

superneutrality no longer holds.[18] This analysis hinges on the differences in impact of a preference for wealth on the economy. Thus, the steady-state ratios obtained in the absence of a preference for wealth are presented to allow for ease of comparison.

Table 5.3.2: Sensitivity of Steady-State Ratios to Inflation and Wealth Preference

Steady State Ratios	CIA - Consumption Only					CIA - Consumption & Investment				
	$\frac{k}{h}$	$\frac{c}{k}$	$\frac{m}{k}$	$\frac{y}{k}$	$\frac{a}{k}$	$\frac{k}{h}$	$\frac{c}{k}$	$\frac{m}{k}$	$\frac{y}{k}$	$\frac{a}{k}$
$(1-\gamma) = 0$										
$\pi = -5.6$	46.05	0.07	0.07	0.09	1.07	46.05	0.07	0.09	0.09	1.09
$\pi = 0.0$	46.05	0.07	0.07	0.09	1.07	45.02	0.07	0.09	0.09	1.09
$\pi = 3.908$	46.05	0.07	0.07	0.09	1.07	44.33	0.07	0.09	0.09	1.09
$\pi = 10$	46.05	0.07	0.07	0.09	1.07	43.28	0.08	0.10	0.10	1.10
$\pi = 100$	46.05	0.07	0.07	0.09	1.07	31.49	0.10	0.12	0.12	1.12
$\pi = 400$	46.05	0.07	0.07	0.09	1.07	14.84	0.17	0.19	0.19	1.19
$(1-\gamma) = 0.05$										
$\pi = -5.6$	53.61	0.06	0.06	0.08	1.06	53.72	0.06	0.08	0.08	1.08
$\pi = 0.0$	53.72	0.06	0.06	0.08	1.06	52.66	0.06	0.08	0.08	1.08
$\pi = 3.908$	53.80	0.06	0.06	0.08	1.06	51.94	0.07	0.09	0.09	1.09
$\pi = 10$	53.91	0.06	0.06	0.08	1.06	50.85	0.07	0.09	0.09	1.09
$\pi = 100$	55.65	0.06	0.06	0.08	1.06	38.48	0.08	0.10	0.10	1.10
$\pi = 400$	61.47	0.06	0.06	0.08	1.06	20.40	0.13	0.15	0.15	1.15
$(1-\gamma) = 0.25$										
$\pi = -5.6$	93.36	0.04	0.04	0.06	1.04	94.04	0.04	0.06	0.06	1.06
$\pi = 0.0$	94.01	0.04	0.04	0.06	1.04	92.88	0.04	0.06	0.06	1.06
$\pi = 3.908$	94.46	0.04	0.04	0.06	1.04	92.10	0.04	0.06	0.06	1.06
$\pi = 10$	95.17	0.04	0.04	0.06	1.04	90.90	0.04	0.06	0.06	1.06
$\pi = 100$	105.48	0.03	0.03	0.06	1.03	77.07	0.05	0.07	0.07	1.07
$\pi = 400$	137.53	0.03	0.03	0.05	1.03	55.22	0.06	0.08	0.08	1.08

Note: The steady-state ratio values are derived by adjusting the preference for wealth, $(1-\gamma)$, and the steady-state growth rate of money. The symbol π represents the annual percentage of inflation corresponding to the steady-state growth rate of money used in the estimation of the model.

When a preference for wealth is assumed in the household's utility function, superneutrality does not hold regardless of the CIA constraint imposed on the household. However, the direction and magnitude of the inflationary impact on the steady-state

[18] This result is robust to the findings of Stockman (1981).

values are sensitive to the CIA specification. The steady-state ratio most affected by changes in the growth rate of money, even low inflationary adjustments, is the capital-to-labor ratio.

The capital-to-labor ratio rises with inflation under the first CIA constraint and falls with inflation under the second.[19] The magnitude of the inflationary impact on the capital-to-labor ratio is greater under the second CIA constraint. The taxing effects of inflation cause the capital-to-labor ratio to fall more rapidly when money is required for the purchase of both consumption and investment goods. Thus, real returns to capital increase and real wages decrease.

As the preference for wealth increases, the steady-state output-to-capital ratio is reduced and capital's share of wealth increases regardless of the CIA specification. The impact of inflation on capital's share of wealth is dependent on the CIA specification. Capital's share of wealth increases with inflation under the first CIA specification, and the changes become more pronounced as the preference for wealth increases. Under the second CIA specification, capital's share of wealth decreases with inflation, and as the preference for wealth increases the share of wealth allocated to capital becomes less sensitive to the money growth rate.

[19] The differences in the inflationary movements are directly responsible for the differences in the dynamic responses resulting from a shock to the growth rate of money previously discussed in section 5.1.3.

Section 6. Conclusion

This paper evaluates the short- and long-run impacts of a preference for wealth in a monetary equilibrium model of the business cycle. Status is introduced by making wealth an explicit argument in the agent's utility function, and a demand for money emerges through the inclusion of a cash-in-advance (CIA) constraint. I consider two different specifications of the CIA constraint. Under the first, agents require cash for the purchase of consumption goods only, and under the second, agents require cash for both consumption and investment goods. I find that the properties of the model are affected by the specification of the cash constraint as well as the role of status in the utility function.

The results of Chang and Tsai (2003) on the effect of inflation on capital accumulation are supported by this model under the second CIA specification; however, households having a preference for leisure as well as consumption and wealth decrease their capital accumulation in response to inflation under either CIA specification. Agents attempt to smooth not only consumption but wealth. As the relative wealth preferences increase, the weight placed on consumption decreases. However, money that facilitates consumption is a component of wealth, and consumption remains smooth. This result is in contrast with the results of Gong and Zou (2001) and Chang and Tsai (2003), who find, under the first CIA specification, that household having preferences for consumption and wealth increase their capital accumulation as inflation increases.

In fact, this study finds that an increase in the level of inflation not only decreases long-run capital accumulation but also output, consumption, investment, hours worked, and welfare under either of the two CIA constraints. The effects of inflation on the real wages and rental rates of capital differ due to the specification of the CIA constraint. When there is a preference for wealth, real wages increase with inflation under the first CIA specification and decrease with inflation under the second. Rental rates of capital are affected in the opposite manner. The inflationary effect on the marginal return to labor is the channel through which the dynamic properties of the model differ with the CIA specification.

The effects of an increasing preference for wealth are also one of the major contributions of this analysis. Households with wealth preferences attempt to smooth not only consumption but also wealth and its components. Under either CIA specification, an increase in households' preference for wealth increases the volatility of output and hours worked while decreasing the volatility of consumption, investment, capital accumulation, wages, and wealth. Thus, an increase in the preference for wealth magnifies the volatility of output and labor hours in response to the business cycle. The business-cycle dynamics found when the CIA constraint applies to both consumption and investment yield output volatility measures much smaller than those seen in the U.S. data when there is a shock to technology only. However, the results from the stochastic case where households are subject to both technology and money growth rate shocks are more in line with the business-cycle statistics derived from the U.S. data.

The long-run effects of an increasing preference for wealth on consumption are indeterminate, increasing with low preference levels and decreasing at higher preference levels. Steady-state rental rates decrease in response to a rising preference for wealth. The steady-state values of output, investment, capital accumulation, wages, and welfare increase with the preference for wealth. This is in contrast to the finding of Fershtman and Weiss (1993) that steady-state output decreases with the preference for wealth. The results support the theory of Veblen (1899) that steady-state hours worked decrease, which translates to an increase in leisure, as the preference for wealth rises.

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Technical Appendix

Log-Linearization Process for the Log Separable Utility Case

The log-linear approximation is needed in order to implement the technique used in Klein (2000). The log-linearization process for each of the competitive equilibrium conditions derived from the households' problem and listed in subsection 3.4 is detailed below:

$$c_t + x_t = y_t$$
$$ce^{\hat{c}_t} + xe^{\hat{x}_t} = ye^{\hat{y}_t}$$
$$c(1+\hat{c}_t) + x(1+\hat{x}_t) = y(1+\hat{y}_t)$$
$$c\hat{c}_t + x\hat{x}_t - y\hat{y}_t = 0$$

$$\mu_1 c_t + \mu_2 x_t = m_t$$
$$\mu_1 ce^{\hat{c}_t} + \mu_2 xe^{\hat{x}_t} = me^{\hat{m}_t}$$
$$\mu_1 c(1+\hat{c}_t) + \mu_2 x(1+\hat{x}_t) = m(1+\hat{m}_t)$$
$$\text{In the steady state, } \mu_1 c + \mu_2 x = m.$$
$$\mu_1 c\hat{c}_t + \mu_2 x\hat{x}_t - m\hat{m}_t = 0$$

$$k_t = x_t + (1-\delta)k_{t-1}$$
$$ke^{\hat{k}_t} = xe^{\hat{x}_t} + (1-\delta)ke^{\hat{k}_{t-1}}$$
$$k(1+\hat{k}_t) = x(1+\hat{x}_t) + (1-\delta)k(1+\hat{k}_{t-1})$$
$$\text{In the steady state, } k = x + (1-\delta)k.$$
$$k\hat{k}_t - x\hat{x}_t - (1-\delta)k\hat{k}_{t-1} = 0$$

$a_t = k_{t-1} + m_t$

$ae^{\hat{a}_t} = ke^{\hat{k}_{t-1}} + me^{\hat{m}_t}$

$a(1 + \hat{a}_t) = k(1 + \hat{k}_{t-1}) + m(1 + \hat{m}_t)$

In the steady state, $a = k + m$.

$a\hat{a}_t - k\hat{k}_{t-1} - m\hat{m}_t = 0$

$y_t = e^{z_t} k_{t-1}{}^{\alpha} h_t{}^{1-\alpha}$

$ye^{\hat{y}_t} = k^{\alpha} h^{1-\alpha} e^{(z_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{h}_t)}$

In the steady state, $y = k^{\alpha} h^{1-\alpha}$ and $z = 0$.

$e^{\hat{y}_t} = e^{(z_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{h}_t)}$

$1 + \hat{y}_t = 1 + z_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{h}_t$

$z_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{h}_t - \hat{y}_t = 0$

$w_t = (1-\alpha)e^{z_t}\left(\frac{k_{t-1}}{h_t}\right)^{\alpha}$

$we^{\hat{w}_t} = (1-\alpha)\left(\frac{k}{h}\right)^{\alpha} e^{(z_t + \alpha\hat{k}_{t-1} - \alpha\hat{h}_t)}$

In the steady state, $w = (1-\alpha)\left(\frac{k}{h}\right)^{\alpha}$ and $z = 0$.

$e^{\hat{w}_t} = e^{(z_t + \alpha\hat{k}_{t-1} - \alpha\hat{h}_t)}$

$1 + \hat{w}_t = 1 + z_t + \alpha\hat{k}_{t-1} - \alpha\hat{h}_t$

$z_t + \alpha\hat{k}_{t-1} - \alpha\hat{h}_t - \hat{w}_t = 0$

$$r_t = \alpha e^{z_t}\left(\frac{h_t}{k_{t-1}}\right)^{1-\alpha}$$

$$re^{\hat{r}_t} = \alpha\left(\frac{h}{k}\right)^{1-\alpha} e^{\left(z_t+(1-\alpha)\hat{h}_t+(\alpha-1)\hat{k}_{t-1}\right)}$$

$$e^{\hat{r}_t} = e^{\left(z_t+(1-\alpha)\hat{h}_t+(\alpha-1)\hat{k}_{t-1}\right)}$$

In the steady state, $r = \alpha\left(\frac{h}{k}\right)^{1-\alpha}$ and $z = 0$.

$$1+\hat{r}_t = 1+z_t+(1-\alpha)\hat{h}_t+(\alpha-1)\hat{k}_{t-1}$$

$$z_t+(1-\alpha)\hat{h}_t+(\alpha-1)\hat{k}_{t-1}-\hat{r}_t = 0$$

$$\lambda_t+\varphi_t\mu_2 = q_t$$

$$\lambda e^{\hat{\lambda}_t}+\varphi e^{\hat{\varphi}_t}\mu_2 = qe^{\hat{q}_t}$$

$$\lambda\left(1+\hat{\lambda}_t\right)+\varphi\mu_2\left(1+\hat{\varphi}_t\right) = q\left(1+\hat{q}_t\right)$$

In the steady state, $\lambda+\varphi\mu_2 = q$.

$$\lambda\hat{\lambda}_t+\varphi\mu_2\hat{\varphi}_t-q\hat{q}_t = 0$$

$$m_t = \Theta_t\frac{m_{t-1}}{\Pi_t}$$

$$me^{\hat{m}_t} = \Theta\frac{m}{\Pi}e^{\left(\hat{\Theta}_t+\hat{m}_{t-1}-\hat{\Pi}_t\right)}$$

In the steady state, $m = \Theta\frac{m}{\Pi}$.

$$e^{\hat{m}_t} = e^{\left(\hat{\Theta}_t+\hat{m}_{t-1}-\hat{\Pi}_t\right)}$$

$$1+\hat{m}_t = 1+\hat{\Theta}_t+\hat{m}_{t-1}-\hat{\Pi}_t$$

$$\hat{\Theta}_t+\hat{m}_{t-1}-\hat{\Pi}_t-\hat{m}_t = 0$$

$$\lambda_t w_t = -U_h(c_t, a_t, h_t)$$

$$\lambda_t w_t = \mathrm{B}$$

$$\lambda w e^{(\hat{\lambda}_t + \hat{w}_t)} = \mathrm{B}$$

In the steady state, $\lambda w = \mathrm{B}$.

$$e^{(\hat{\lambda}_t + \hat{w}_t)} = 1$$

$$1 + \hat{\lambda}_t + \hat{w}_t = 1$$

$$\hat{\lambda}_t + \hat{w}_t = 0$$

$$\lambda_t + \varphi_t \mu_1 = U_c(c_t, a_t, h_t)$$

$$\lambda_t + \varphi_t \mu_1 = \gamma \frac{1}{c_t}$$

$$\lambda e^{\hat{\lambda}_t} + \varphi \mu_1 e^{\hat{\varphi}_t} = \gamma \frac{1}{c} e^{-\hat{c}_t}$$

In the steady state, $\lambda + \varphi \mu_1 = \gamma \frac{1}{c}$ or $\frac{\lambda}{\lambda + \varphi \mu_1} + \frac{\varphi \mu_1}{\lambda + \varphi \mu_1} = 1$.

$$\frac{\lambda}{\lambda + \varphi \mu_1} e^{\hat{\lambda}_t} + \frac{\varphi \mu_1}{\lambda + \varphi \mu_1} e^{\hat{\varphi}_t} = e^{-\hat{c}_t}$$

$$\frac{\lambda}{\lambda + \varphi \mu_1} \left(1 + \hat{\lambda}_t\right) + \frac{\varphi \mu_1}{\lambda + \varphi \mu_1} \left(1 + \hat{\varphi}_t\right) = 1 - \hat{c}_t$$

$$\frac{\lambda}{\lambda + \varphi \mu_1} \hat{\lambda}_t + \frac{\varphi \mu_1}{\lambda + \varphi \mu_1} \hat{\varphi}_t + \hat{c}_t = 0$$

$$\lambda_t = \beta E_t \left[\left(U_a(c_{t+1}, a_{t+1}, h_{t+1}) + \lambda_{t+1} + \varphi_{t+1} \right) \left(\frac{1}{\Pi_{t+1}} \right) \right]$$

$$\lambda_t = \beta E_t \left\{ \left((1-\gamma) \frac{1}{a_{t+1}} + \lambda_{t+1} + \varphi_{t+1} \right) \left(\frac{1}{\Pi_{t+1}} \right) \right\}$$

$$\lambda e^{\hat{\lambda}_t} = \beta E_t \left\{ (1-\gamma) \frac{1}{a\Pi} e^{\left(-\hat{a}_{t+1} - \hat{\Pi}_{t+1}\right)} + \frac{\lambda}{\Pi} e^{\left(\hat{\lambda}_{t+1} - \hat{\Pi}_{t+1}\right)} + \frac{\varphi}{\Pi} e^{\left(\hat{\varphi}_{t+1} - \hat{\Pi}_{t+1}\right)} \right\}$$

In the steady state, $\lambda = \frac{\beta}{\Pi} \left((1-\gamma) \frac{1}{a} + \lambda + \varphi \right)$.

$$\lambda \left(1 + \hat{\lambda}_t\right) = \beta E_t \left\{ \begin{array}{l} (1-\gamma) \frac{1}{a\Pi} \left(1 - \hat{a}_{t+1} - \hat{\Pi}_{t+1}\right) + \frac{\lambda}{\Pi} \left(1 + \hat{\lambda}_{t+1} - \hat{\Pi}_{t+1}\right) \\ + \frac{\varphi}{\Pi} \left(1 + \hat{\varphi}_{t+1} - \hat{\Pi}_{t+1}\right) \end{array} \right\}$$

$$\lambda \hat{\lambda}_t = \frac{\beta}{\Pi} E_t \left\{ -(1-\gamma) \frac{1}{a} \left(\hat{a}_{t+1} + \hat{\Pi}_{t+1}\right) + \lambda \left(\hat{\lambda}_{t+1} - \hat{\Pi}_{t+1}\right) + \varphi \left(\hat{\varphi}_{t+1} - \hat{\Pi}_{t+1}\right) \right\}$$

$$q_t = \beta E_t \left\{ (1-\gamma) \frac{1}{a_{t+1}} + \lambda_{t+1} r_{t+1} + q_{t+1} (1-\delta) \right\}$$

$$q e^{\hat{q}_t} = \beta E_t \left\{ (1-\gamma) \frac{1}{a} e^{-\hat{a}_{t+1}} + \lambda r e^{\hat{\lambda}_{t+1} + \hat{r}_{t+1}} + (1-\delta) q e^{\hat{q}_{t+1}} \right\}$$

In the steady state, $q = \beta \left[(1-\gamma) \frac{1}{a} + \lambda r + q(1-\delta) \right]$.

$$q(1 + \hat{q}_t) = \beta E_t \left\{ (1-\gamma) \frac{1}{a} \left(1 - \hat{a}_{t+1}\right) + \lambda r \left(1 + \hat{\lambda}_{t+1} + \hat{r}_{t+1}\right) + (1-\delta) q \left(1 + \hat{q}_{t+1}\right) \right\}$$

$$q \hat{q}_t = \beta E_t \left\{ -(1-\gamma) \frac{1}{a} \hat{a}_{t+1} + \lambda r \left(\hat{\lambda}_{t+1} + \hat{r}_{t+1}\right) + (1-\delta) q \hat{q}_{t+1} \right\}$$

PAPER 2. WELFARE COSTS OF INFLATION AND THE BUSINESS CYCLE WHEN AGENTS HAVE A RELATIVE PREFERENCE FOR WEALTH

Abstract

The current analysis extends the literature by investigating the welfare costs of inflation when households have relative preferences over consumption and wealth along with a preference for leisure. Defining a household's preference structure in this manner and incorporating a transaction cost mechanism that varies with the velocity of money yields welfare costs that differ based on the way monetary policy is assumed to be conducted. Welfare costs increase with inflation and decrease with the relative preference for wealth under a money growth rate rule. In fact, the size of the welfare cost is tied to the velocity of money itself: as the equilibrium velocity deviates from the optimal rate, the welfare costs rise. The analysis alters the time-separable characteristics of the household's utility function, determining the welfare cost of inflation to be highest when there is no wealth preference and time-separable preferences are assumed.

When policymakers are assumed to set the velocity of money by targeting a short-term interest rate, welfare costs in terms of output present themselves at moderate levels of inflation. Setting positive short-term rate targets high enough helps to ensure positive real interest rates at moderate levels of inflation. Also, transaction costs associated with the velocity of money are held constant. An increase in the level of inflation does not increase the transaction costs associated with each unit of consumption as is the case with money growth rate targets. The results of the analysis support a monetary policy targeting a short-term interest rate consistent with a level of price stability. The results suggest that the selection of the short-term rate is dependent on the economy's relative preference for wealth.

Keywords: Welfare Costs, Inflation, Business Cycle, and Preference for Wealth

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Section 1: Introduction

The real business cycle literature of Cooley and Hansen (1989) examines the effects of inflation over the business cycle and the associated welfare costs. The wealth-induced social status literature of Gong and Zou (2001) addresses the effects of the inflation level on steady-state capital accumulation when a concern for status (wealth) is incorporated into the preference structure of economic agents. This paper combines these two branches of the literature, investigating the welfare costs of inflation when agents have relative preferences for consumption and wealth along with a preference for leisure. A welfare cost investigation with these underlying preference assumptions has not been undertaken in the welfare cost literature.

Unlike the literature of Cooley and Hansen (1989) and Gong and Zou (2001), who incorporate money into the economy through the use of a cash-in-advance (CIA) constraint, this study uses a transaction cost mechanism borrowed from Collard and Dellas (2005) and Schmitt-Grohe and Uribe (2004) that is dependent on the velocity of money as the channel through which money enters the economy, detracting from consumption and/or investments share of output. Money also enters the households' utility function directly as a component of wealth when a relative preference for wealth is assumed. In this economy, as in that of Cooley and Hansen (1989), households are subject to an employment lottery, and an indivisible labor assumption holds. Unemployment is the means through which hours worked fluctuates. The addition of a relative preference for wealth, in a model that allows not only for labor market adjustments but also for adjustments in the velocity of money, makes this study a unique addition to the literature.

Through the determination of the welfare costs of inflation, this study addresses three important questions:

- Do the welfare costs of inflation differ widely based on the channel through which money enters the economy?
- When agents having relative preferences for consumption and wealth make allocation decisions that impact the market for labor, what is the relationship between the relative preference for wealth and the welfare costs of inflation?
- Do the welfare costs of inflation differ significantly based on the time-separable nature of the household's utility function and the means in which monetary policy is conducted?

1.1 Theoretical Background

The idea that economic agents have preferences for wealth is not a new one. The underlying assumption of Thorstein Veblen's 1899 work *Theory of the Leisure Class* is that all agents have some desire for status and that desire for status increases with wealth. Many other researchers have acknowledged preferences for wealth in the form of a preference for status (Weiss 1976; Corneo and Jeanne 2001; Gong and Zou 2001, 2002; Chang and Tsai 2003; St. Amour 2005; among others), a market for status (Becker, Murphy, and Werning 2005), or even a "Spirit of Capitalism" (Weber 1958; Zou 1992, 1994, 1995; Bakshi and Chen 1996, Lou and Young 2006; among others). However, a relative preference for wealth along with a labor-leisure decision has not yet been addressed in the literature. [1]

Friedman and Savage (1948) and Becker, Murphy, and Werning (2005) assume that the marginal utility of wealth increases with the preference for status. Becker et al. (2005) mention that the incorporation of a labor-leisure decision would enhance their model but was left out for simplicity's sake. This analysis adds to the literature by incorporating labor-leisure decisions and a demand for money into a model in which the marginal utility of wealth is assumed to increase with the preference for wealth.

The economy is modeled in order to analyze differences in the welfare costs of the business cycle as changes in the economy's preference for wealth occur under various money growth rate policies. In this economy, consumption goods are in essence subject to a tax, which presents itself through a transaction cost function that applies to consumption purchases. This transaction cost is a function of the velocity of money. At the optimal velocity, v^{opt}, the transaction costs equal zero. Any deviation from v^{opt} will result in a positive transaction cost on consumption purchases. The specification of the transaction cost function is the same as that used by Collard and Dellas (2005) and Schmitt-Grohe and Uribe (2004).

[1] A working paper written by Lou and Young (2006) looks at wealth distributions in a heterogeneous agent model. Money does not enter their model, since wealth is defined to be capital only.

In this monetary economy, an increase in the level of inflation increases the consumption velocity, which in turn increases the transaction costs associated with consumption purchases. The increase in the velocity of money is associated with increases in nominal rates when policymakers target inflation rates and reductions in real interest rates when short-term rates are targeted. The increase in the effective tax on consumption goods causes substitution effects to occur. The welfare costs are determined by comparing allocations associated with the monetary economy and the cashless economy using the techniques employed by Lucas (1987) and Cooley and Hansen (1989). As in Cooley and Hansen (1989), steady-state welfare costs are determined for various money growth rate rules.

Welfare costs are hypothesized to differ with respect to the economy's money growth rate rule as well as the economy's specified preference for wealth. A sensitivity analysis will be performed in order to determine how sensitive the results are to changes in these specifications.[2]

1.2 Overview of Results

When dealing with computable general equilibrium models, Leeper (1995) gives some words of advice. This model incorporates the three key components Leeper

[2] The solution of the stochastic model can be determined through the use of second-order approximations of the competitive equilibrium conditions of the economy. This solution technique is borrowed from Sutherland (2002). These results are included in the appendix for future estimation of the business cycle dynamics corresponding with a shock to the growth rate of money. These estimates will allow for the estimation of the welfare costs associated with the business cycle.

suggests must be present in a general equilibrium model that is to be used to evaluate policy:

- In calibration of this model, the characteristics of technology growth innovations, money growth innovations, and velocity measures that minimize the transaction costs associated with consumption are approximated.
- The model is also calibrated to fit the U.S. data.
- Monetary policy conducted by setting a short-term rate consistent with a level of price stability in order to maintain a sustainable level of output growth is addressed and gives outcomes that resemble policy choices made by policymakers in the U.S.

Another important feature of this model is the inclusion of both money and interest rates, which work together to transmit monetary policy (Leeper and Roush 2003). Money enters the model through a direct preference for wealth assumed by the households. Wealth is composed of capital accumulation and real money balances. Also, the transaction cost mechanism and money demand rule used by Collard and Dellas (2005) and Schmitt-Grohe and Uribe (2004) are incorporated into the model. The money demand rule is used to allow the short-term rate targeted by policymakers to set an equilibrium velocity of money that is not dependent on the money growth level.[3]

The results of the analysis show that when the target interest rate is set to 6.56 percent compounded quarterly, the average of the federal funds rate over the sample

[3] This analysis addresses the steady-state characteristics; however, in the stochastic economy a shock to the growth rate of money causes households to make allocation adjustments. Interest rates change, causing deviations in the velocity of money from the set rate.

period, 1964:01 – 2004:04, the money growth rate necessary to eliminate the cost in terms of total goods ranges from 2 to 4 percent and is dependent on the preference for wealth as well as the time-separable nature assumed in the households' utility function. In the separable case, the results confirm the results of Cooley and Hansen (1989). Welfare costs of inflation are small when the velocity of money is held constant by targeting short-term interest rates. However, the results also show substantial welfare costs in terms of output associated with moderate money growth rates in the non-separable cases.

The analysis of the different time-separable cases can be looked at as exemplifying the outcomes associated with different types of investors. Though the welfare costs associated with these investors differ, the investors share a common expectation: they expect policymakers to act in a manner that takes investors' welfare into consideration.

The remainder of this paper is broken into multiple sections. The model is presented in Section 2. Section 3 discusses the calibration of the model. Section 4 presents the results of the welfare cost analysis. A time-separable analysis in which multiple utility function specifications takes place in subsections 4.1–4.3. The case that is time-separable and two that are not are examined. The first non-separable case assumes consumption and wealth are complements, and the second assumes they are substitutes. Section 5 concludes.

Section 2: The Model

2.1 The Consumer's Optimum

Households faced with uncertainty are assumed to make choices on consumption, wealth, and leisure in an attempt to maximize their expected lifetime utility:

$$E_0 \sum_{i=0}^{\infty} \beta^i \; U(c_{t+i}, a_{t+i}, h_{t+i}) = E_0 \sum_{i=0}^{\infty} \beta^i \; \frac{1}{1-\sigma} \left[\gamma \, c_{t+i}^{\;\;1-\eta} + (1-\gamma) a_{t+i}^{\;\;1-\eta} \right]^{\frac{1-\sigma}{1-\eta}} - Bh_{t+i} \quad (2.1.0)$$

where $0 < \beta < 1$. Thus, the household's utility is a function of real consumption, c_t, the fraction of total time supplied to labor market activities, h_t, and real wealth, a_t. The parameters γ, σ, and η are restricted to only positive values.

The household is subject to the same indivisible labor restriction seen in Hansen (1985), which assumes that the households are allotted one unit of time and allocate a portion of it to labor market activities, $h_0 < 1$, and the rest to leisure. Households having full unemployment insurance are subject to an employment lottery. The probability of being employed, ψ_t, determines the expected number of hours worked by each household in any given period *t*, to be defined as $h_t = \psi_t h_0$. The parameter, *B*, measures the marginal disutility the household derives from labor market activities.[4]

[4] For a full description of indivisible labor, see Cooley and Hansen (1989).

The introduction of money into this model is through a transaction cost function. The transaction cost function is a function of the consumption velocity of money, v_t. The velocity of money is defined as:

$$v_t = \frac{c_t}{m_t}, \tag{2.1.1}$$

where c_t represents household consumption of non-durables and services at a given time t, and m_t represents household's real money balances at time t. The households are subject to the same transaction cost function employed by Collard and Dellas (2005) and Schmitt-Grohe and Uribe (2004), which takes the following form:

$$\mathrm{T}_t(v_t, \zeta) = \zeta\left(A^{\mathrm{T}} v_t + \frac{B^{\mathrm{T}}}{v_t} - 2\sqrt{A^{\mathrm{T}} B^{\mathrm{T}}} \right). \tag{2.1.2}$$

ζ is set equal to unity in a monetary economy and converges to zero in the cashless society.[5] Money is the distorting factor in this model economy. The money supply is taken as given by the households and grows at a rate of θ_t, determined by the government, and the growth rate is subject to shocks. However, households choose the quantity of money demanded in the economy. The transaction cost mechanism described by equation (2.1.2) is designed so that transaction costs equal zero at the optimal rate of velocity. When the velocity of money rises above or falls below this optimal rate, the households are subject to transaction costs.

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[5] For model estimation purposes, ζ is set equal to $1 * e^{-12}$.

The household's demand for money affects the velocity of money. Thus, the functional form of the household's money demand is written with respect to the velocity of money and therefore will deviate as the money supply changes over time. The money demand function assumed by the model is borrowed from Schmitt-Grohe and Uribe (2004) and is defined in the following manner:

$$v_t^2 = \frac{B^{\mathrm{T}}}{A^{\mathrm{T}}} + \frac{1}{A^{\mathrm{T}}}\left(\frac{I_t - 1}{I_t}\right). \tag{2.1.3}$$

The optimal rate of velocity on which the transaction costs are based is the Pareto optimal rate as defined by the Friedman Rule, suggesting no transaction costs when $v_t = \sqrt{\frac{B^T}{A^T}}$.

Wealth, a_t, is composed of accumulated capital, k_{t-1}, real money holdings, $\frac{m_{t-1}}{\Pi_t}$, real bond holdings, $\frac{I_{t-1}b_{t-1}}{\Pi_t}$, and government transfers, τ_t, where Π_t is the gross rate of inflation. Therefore, the functional form of household wealth

$$a_t = k_{t-1} + \frac{m_{t-1}}{\Pi_t} + \frac{I_{t-1}b_{t-1}}{\Pi_t} + \tau_t \tag{2.1.4}$$

is similar to that of Gong and Zou (2001).[6] The parameters γ and $(1-\gamma)$ are relative weights households place on consumption and wealth, respectively.

[6] Gong and Zou (2001) did not include bond holdings because in their steady-state analysis of a homogeneous agent economy bond holdings at any given time will equal zero. This analysis also assumes homogeneous households in order to analyze average aggregate behavior of households and how allocations differ with respect to the aggregate economy's preference for wealth. However, bond holdings were included in solving the model in order to determine the economy's nominal interest rate.

The households supply labor to the firm and receive compensation in the form of wages, w_t. The households also supply capital to the firm, for which they charge a rental rate, r_t. The households receive real transfers from the government, τ_t, and may have real money holdings from the previous period. Thus, the household's budgets are constrained in the following manner:

$$(1+\mathrm{T}_t)c_t + x_t + m_t + b_t \leq w_t h_t + r_t k_{t-1} + \tau_t + \frac{m_{t-1}}{\Pi_t} + \frac{b_{t-1}}{\Pi_t} . \tag{2.1.5}$$

Investment, x_t, is constrained as:

$$x_t = k_t - (1-\delta)k_{t-1} \tag{2.1.6}$$

where $\delta \in [0,1]$ is the capital depreciation rate.

The household's decision problem is represented by the following value function:

$$V(k_{t-1}, m_{t-1}, b_{t-1}) = \underset{c_t, h_t, m_t, b_t, x_t, k_t}{Max} \left\{ U(c_t, a_t, h_t) + \beta E_t \left[V(k_t, m_t, b_t) \right] \right\}. \tag{2.1.7}$$

The decision problem is subject to the transaction cost function (2.1.2), the budget constraint, (2.1.5), and the investment constraint (2.1.6).

2.2 *The Firm's Optimum*

The firms use labor and capital to produce output. The firm's production function is Cobb-Douglas with constant returns to scale such that per-capita output, Y_t, is a function of capital accumulated at the end of the previous period, K_{t-1}, and the percentage of total time that is allocated to market activities, H_t, so that:

$$Y_t = f(K_{t-1}, H_t, z_t) = e^{z_t} K_{t-1}{}^{\alpha} H_t^{1-\alpha} \tag{2.2.1}$$

and $\alpha \in [0,1]$. The production function is subject to exogenous productivity shocks, z_t, which are linear taking the following form:

$$z_{t+1} = \rho_z z_t + \xi_{z_{t+1}}. \tag{2.2.2}$$

The persistence of the shock $\rho_z \in [0,1]$ and the innovation term, $\xi_{z_{t+1}}$, is such that $E_t\{\xi_{z_{t+1}}\} = 0$ and variance of the innovation is $\sigma_z{}^2$.

The firm is a price-taker. Wages and rental rates are taken as given and firms act optimally, choosing the amount of capital and labor necessary and sufficient for profit maximization. The firm's profit maximization problem takes the following form:

$$\underset{\{K_{t-1}, H_t\}}{Max}\ Y_t - w_t H_t - r_t K_{t-1} \quad such\ that\ \ Y_t = e^{z_t} K_{t-1}{}^{\alpha} H_t^{1-\alpha} \tag{2.2.3}$$

where $\alpha \in [0,\ 1]$ and both K and H are restricted to positive values. Thus, the firm's necessary and sufficient optimality conditions are such that

$$K_{t-1} : \alpha e^{z_t} K_{t-1}{}^{\alpha-1} H_t^{1-\alpha} = r_t \quad \text{and} \tag{2.2.4}$$

$$H_t : (1-\alpha) e^{z_t} K_{t-1}{}^{\alpha} H_t^{-\alpha} = w_t \tag{2.2.5}$$

must hold for all $t \in [0, \infty)$.

2.3 *The Monetary Policy Authority*

The nominal money supply is assumed to grow at the rate of θ_t.[7] The government regulates the growth rate of the nominal money supply so that:

$$P_t M_t = \Theta_t P_{t-1} M_{t-1}, \tag{2.3.1}$$

otherwise written as:

$$M_t = \frac{\Theta_t}{\Pi_t} M_{t-1}, \tag{2.3.2}$$

where M_t represents the per capita real money supply. The government also provides per capita real transfers, τ_t, to the households at the start of each period, so that the following equality holds:

$$\tau_t = M_t - \frac{M_{t-1}}{\Pi_t}. \tag{2.3.3}$$

Otherwise stated, the nominal transfer, $P_t \tau_t$, is equal to product of the gross growth rate of money, Θ_t, and the nominal money supply from the previous period , $P_{t-1} M_{t-1}$.

The growth rate of money follows the following process:

$$log\,\theta_{t+1} = (1-\rho_\theta)\theta + \rho_\theta\, log\,\theta_t + \xi_{\theta_{t+1}}. \tag{2.3.4}$$

The persistence associated with the money growth rate process is measured by ρ_θ so that $\rho_\theta \in [0,1]$, and the innovation to the money growth rate process, $\xi_{\theta_{t+1}}$, has mean zero and variance ${\sigma_\theta}^2$. Shocks to the growth rate of the money supply feed through to the

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[7] The real money supply grows in the following manner: $m_t = \frac{(1+\theta_t)}{\Pi_t} m_{t-1} = \frac{\Theta_t}{\Pi_t} m_{t-1}$.

household through lump-sum transfers that affect the household's budget constraint and the velocity of money. Thus, money supply shocks affect households' transaction costs and money demand.

Monetary policy in the U.S. not only targets a short-term interest rate. These targets are chosen in order to maintain price stability as well as promote a sustainable level of output growth. Thus, in the latter part of the analysis a further restriction is placed on the model, one for which the monetary policy targets a short-term interest rate by restricting the money demand function in the following way:

$$v^2 = \frac{B^T}{A^T} + \frac{1}{A^T}\left(\frac{TI-1}{TI}\right), \tag{2.3.5}$$

where *TI* is the targeted short-term interest rate.

Nominal interest rates are still determined by households' money demand. The monetary policymaker's short-term interest rate target just restricts the economy's velocity of money, holding real rates constant. Thus, an increase in inflation causes nominal rates to rise. Removing this restriction will allow for the examination of an economy that has only money growth rate targets, whereas the implementation of this constraint will allow for the examination of the welfare costs of inflation in a model economy in which policymakers target short-term interest rates. These welfare costs are stated in terms of output for differing levels of price stability.

2.4 *The Household's Problem*

When households make their allocation decisions, they take (2.2.4), (2.2.5), (2.3.3), and the gross rate of inflation as parametric.[8] The households also take as given the productivity and money supply growth rates $\{z_t,\ \theta_t\}$, and the autoregressive processes which take the form previously defined by equations (2.2.2) and (2.3.4).

In the competitive equilibrium, the per capita capital demanded by the firm is the capital supplied by the representative household, $K_t = k_t$, the per capita hours demanded by the firm equal the hours supplied by the household, $H_t = h_t$, and the per capita real money balances supplied by the government equals the real balances demanded by the representative household.

The household's decision problem takes account of utility optimizing consumers constrained by their budget choosing optimal bundles of consumption, wealth, and leisure; profit maximizing firm's demands for capital and labor; and the money growth rate policies set in place by the monetary authority. The solution of the model is included in the appendix, which is broken into 3 parts: solution of the households' decision problem, competitive equilibrium conditions, steady state analysis, and first- and second-order logarithmic approximations of the competitive equilibrium conditions.

[8] Capital letters are used to distinguish per capita variables that a competitive household takes as parametric from individual-specific variables that are chosen by the household. In equilibrium these will be the same.

Section 3: Calibration

The first- and second-order logarithmic approximations of the competitive equilibrium conditions are used to determine the dynamic response of the model to productivity and money growth rate innovations. The solution technique described in Sutherland (2002) is used in solving the stochastic general equilibrium model of the business cycle. The dynamics of the model are affected by the calibrated parameters of the model.

The parameterization of the model was based on a calibration exercise using U.S. data collected over the sample period from 1964:Q1 to 2004:Q4. The U.S. data was collected from the Bureau of Labor Statistics (BLS) and the Bureau of Economic Analysis (BEA). Table 3.1 contains the calibrated parameters used throughout the remainder of this analysis.

Table 3.1: Calibrated Parameters

α	δ	A^T	σ_θ	ρ_θ
0.375	0.020	0.011	0.009	0.638
B	β	B^T	σ_z	ρ_z
2.426	0.986	0.075	0.006	0.950

Note: The average capital share of output, α, is the sum of the measured value of capital income and the imputed flow of services from both the stock of consumer durables and the stock of government capital over the sum of GDP and the flow of services from both consumer durables and government capital.

The quarterly average investment-to-capital ratio, δ, is calibrated to equal total real investment expenditures over the real value of the total stock of capital.

The discount factor, β, is calibrated to equal the reciprocal of the gross real interest rate. The nominal real interest rate is calibrated to equal the average three-month T-bill rate adjusted to quarterly measures.

The scaling coefficient in the household's utility function, B, is calibrated using α; β; the average of the output-to-consumption ratio, 1.404, and the fraction of time U.S. households spend working over the sample period, $h_0 = 0.338$; the average gross annual growth rate of money, $\Theta = 1.039$; and the calibrated parameters α and β. Thus, the calibrated parameter B is derived in the following manner: $B = \dfrac{\left[(1-\alpha)\beta\left(\frac{y}{c}\right)\right]}{h_0\,\Theta}$.

The parameters for the transaction cost function, A^T and B^T, are the same as those used by Schmitt-Grohe and Uribe (2004) and Collard and Dellas (2005).

Taking the logged difference of the per capita output series yields a technology growth series from which the technology growth rate persistence is determined. The persistence of the technology growth rate, ρ_z, is obtained by estimating the following OLS regression: $z_t = \alpha_0 + \rho_z\, z_{t-1} + \xi_{z_t}$ where z_t represents the technology growth rate at time *t*. The parameter σ_z is the standard deviation of the residual, ξ_{z_t}.

A quarterly money growth rate time series is formed by taking the logged differences of the quarterly per capita money balance measures. The persistence of the money growth rate is denoted by ρ_θ. The value of ρ_θ is calibrated by estimating the following OLS regression: $\theta_t = \alpha_0 + \rho_\theta\, \theta_{t-1} + \xi_{\theta_t}$ where θ_t represents the money growth rate at time *t*. The parameter σ_θ is the standard deviation of the error term, ξ_{θ_t}.

The parameters of the model are calibrated using the U.S. data over the sample period ranging from 1964:Q1 to 2004:Q4. The calibration is done in order to create a set of simulated data from the model that attempts to capture the business cycle properties of the U.S. economy. The attempt to replicate the U.S. economy makes the welfare cost results obtained from this analysis more meaningful.

Section 4: Welfare Costs

This study addresses both the welfare costs of inflation and the welfare costs of the business cycle when households in the model economy have relative preferences over consumption and wealth along with the indivisible labor assumption. Money is the distorting factor in the model. An increase in the growth rate of money increases the velocity of money and the associated transaction cost measured as a share of output. Changes in the growth rate of money impose welfare costs on households. As in Lucas (1987), when there is no preference for wealth, welfare costs are measured in terms of lost consumption. However, when there is a preference for wealth, the welfare costs estimates are measured in terms of a composite good, G, which is defined in the following manner:

$$G_t = \gamma c_t^{1-\eta} + (1-\gamma) a_t^{1-\eta} \tag{4.1}$$

and accounts for households' preferences for both consumption and wealth. The analysis addresses differences due to the time-separable characteristic assumed by the household's utility function, and the welfare costs are then evaluated under the alternate preference structure assumptions to allow for comparison.

First, the separable case assumes a modified version of the utility function used by Cooley and Hansen (1989), in which the utility function of the household is separable in consumption and wealth and the indivisible labor assumption holds. Second, the preference structure is extended, assuming households' utility function to be non-separable over consumption and wealth. This extension allows for the examination of the cases for which households view consumption and wealth as complements or substitutes. Sections 4.1 and 4.2 expound upon the welfare costs of inflation and the business cycle determined under the separable and non-separable assumptions, respectively.

As mentioned, the welfare costs of inflation are determined using the methods of Lucas (1987) as did Cooley and Hansen (1989). Using this method, the welfare cost of inflation equals the additional amount of the composite good, G, required to equate the utility from the "cash economy", U, to that of the "cashless economy", $\overline{U}$, in the following manner:

$$\overline{U} = U[G(1+\Delta G), h]. \tag{4.0.1}$$

The estimated ΔG is translated into a percentage of GDP to create a more meaningful economic measure for comparison.

Households may view fluctuations in consumption caused by money growth rate innovations differently. The differences manifest themselves in the magnitude of the welfare costs determined by the model and may be tied to the time-separable characteristics of the utility function chosen (Otrok 2001). In the following sections, I attempt to account for the differences in the welfare costs through the analysis of the separable case and unique specifications of the non-separable case. The welfare costs approximations are determined using different policy rules. The first rule considered, the policy rule used in Cooley and Hansen (1989), assumes policymakers target a money growth rate, and the welfare costs are presented for moderate money growth rates ranging from 0 percent to 4.25 percent. The second policy rule considered is similar to those used by U.S. policymakers. Under this rule, policymakers target a short-term interest rate consistent with a level of price stability and a sustainable level of output growth.[9]

4.1 Separable Preferences

When households are assumed to have time-separable preferences over consumption and wealth, households' marginal utility of consumption today does not directly affect their marginal utility of consumption tomorrow. Households are not as concerned with fluctuations in consumption as they are with the persistence associated with the change in consumption (Otrok 2001). These are the characteristics associated with risk-neutral investors. The case where monetary policymakers target only money growth rates and the case where short-term interest rates are targeted by policymakers interested in price stability and output growth are examined. The two cases yield vastly different welfare cost results.

[9] This policy rule is consistent with the goals stated in the minutes of the Open Market Committee Meetings, available at the Federal Reserve website.

The results are given for different wealth preference levels including no preference for wealth. When there is no preference for wealth, the welfare costs of inflation increase with the level of inflation targeted by the monetary policymakers. As a sign of money demand, the velocity of money increases with the money growth rate target as do nominal interest rates. The channel through which dynamic effects of monetary shocks are seen is that of the opportunity cost of holding money, nominal interest rates.

The importance of the money growth rate increase in response to a positive money growth rate innovation is the effect that this increase has on the velocity of money. The transaction cost mechanism, incorporated into the model, ties transaction costs to the velocity of money. Given that the budget constraint (2.1.5) is binding, the steady-state resource constraint takes the following form:

$$(1+T)c + x = y. \tag{4.1.1}$$

Thus, the share of output not associated with consumption or investment, *Tc*, is viewed as a cost. The attempt to recoup or mitigate this cost could in itself suggest policy implications of an appropriate money growth rate target. The welfare cost estimates for various money growth rates are presented in Table 4.1.1.[10]

[10] In Table 4.1.1 as well as all other tables in this paper, the ** denotes that the measures are presented as percentages. The transaction costs as well as some of the welfare costs are very small and stated in hundredths of one percent.

Table 4.1.1: Welfare Costs by Inflation Rate and Relative Preference for Consumption and Wealth

	Separable Preferences over Consumption and Wealth (Money Growth Rate Target)											
	Annual Inflation Rate											
γ=1	0.00%	1.00%	2.00%	2.25%	2.50%	2.75%	3.00%	3.25%	3.50%	3.75%	4.00%	4.25%
Wealth	14.73	14.73	14.72	14.72	14.72	14.72	14.72	14.72	14.71	14.71	14.71	14.71
Labor	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31
Capital	14.37	14.37	14.37	14.37	14.37	14.37	14.37	14.37	14.37	14.37	14.37	14.37
Wages	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63
Capital Returns**	3.44	3.44	3.44	3.44	3.44	3.44	3.44	3.44	3.44	3.44	3.44	3.44
Velocity	2.84	2.87	2.91	2.92	2.93	2.94	2.95	2.96	2.97	2.98	2.98	2.99
C/Y**	77.93	77.93	77.92	77.92	77.92	77.92	77.91	77.91	77.91	77.91	77.91	77.91
TC/Y*100**	1.64	2.20	2.82	2.98	3.15	3.32	3.49	3.66	3.84	4.02	4.20	4.39
Nom Interest **	5.68	6.68	7.68	7.96	8.20	8.44	8.68	8.92	9.20	9.44	9.68	9.92
Real Interest **	5.68	5.68	5.68	5.68	5.68	5.68	5.68	5.68	5.68	5.68	5.68	5.68
WCI:ΔG**	1.27	1.28	1.29	1.29	1.30	1.30	1.30	1.30	1.31	1.31	1.31	1.31
WCI:ΔG*G/Y**	0.97	0.97	0.98	0.98	0.99	0.99	0.99	0.99	0.99	1.00	1.00	1.00
OPT INF RATE	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60
γ=0.9												
Wealth	19.47	19.47	19.46	19.46	19.46	19.46	19.46	19.46	19.46	19.46	19.45	19.45
Labor	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
Capital	19.09	19.09	19.09	19.09	19.09	19.09	19.09	19.09	19.09	19.09	19.09	19.09
Wages	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97	2.97
Capital Returns**	2.81	2.81	2.81	2.81	2.81	2.81	2.81	2.81	2.81	2.81	2.81	2.81
Velocity	2.74	2.78	2.81	2.82	2.83	2.84	2.85	2.86	2.87	2.88	2.89	2.90
C/Y**	73.01	73.00	73.00	72.99	72.99	72.99	72.99	72.99	72.99	72.98	72.98	72.98
TC/Y*100**	0.51	0.86	1.28	1.40	1.52	1.64	1.76	1.89	2.03	2.16	2.30	2.44
Nom Interest **	3.16	4.16	5.16	5.40	5.64	5.88	6.12	6.36	6.64	6.88	7.12	7.36
Real Interest **	3.16	3.16	3.16	3.16	3.16	3.16	3.16	3.16	3.16	3.16	3.16	3.16
WCI:ΔG**	0.95	0.95	0.96	0.96	0.96	0.96	0.96	0.97	0.97	0.97	0.97	0.97
WCI:ΔG*G/Y**	0.66	0.67	0.67	0.67	0.67	0.67	0.68	0.68	0.68	0.68	0.68	0.68
OPT INF RATE	-3.12	-3.12	-3.12	-3.12	-3.12	-3.12	-3.12	-3.12	-3.12	-3.12	-3.12	-3.12
γ=0.8												
Wealth	24.92	24.92	24.92	24.91	24.91	24.91	24.91	24.91	24.91	24.91	24.91	24.91
Labor	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29
Capital	24.53	24.53	24.54	24.54	24.54	24.54	24.54	24.54	24.54	24.54	24.54	24.54
Wages	3.32	3.32	3.32	3.32	3.32	3.32	3.32	3.32	3.32	3.32	3.32	3.32
Capital Returns**	2.34	2.34	2.34	2.34	2.34	2.34	2.34	2.34	2.34	2.34	2.34	2.34
Velocity	2.66	2.70	2.74	2.75	2.76	2.77	2.78	2.79	2.80	2.81	2.82	2.83
C/Y**	67.62	67.61	67.61	67.61	67.61	67.61	67.60	67.60	67.60	67.60	67.60	67.60
TC/Y*100**	0.08	0.26	0.51	0.59	0.67	0.75	0.84	0.93	1.02	1.12	1.22	1.33
Nom Interest **	1.28	2.28	3.28	3.52	3.76	4.00	4.24	4.48	4.76	5.00	5.24	5.48
Real Interest **	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28
WCI:ΔG**	0.66	0.67	0.67	0.67	0.67	0.68	0.68	0.68	0.68	0.68	0.68	0.68
WCI:ΔG*G/Y**	0.43	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.45	0.45	0.45
OPT INF RATE	-1.28	-1.27	-1.27	-1.27	-1.27	-1.27	-1.27	-1.27	-1.27	-1.27	-1.27	-1.27
γ=0.7												
Wealth	31.12	31.11	31.11	31.11	31.11	31.11	31.11	31.11	31.11	31.11	31.11	31.11
Labor	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28
Capital	30.73	30.73	30.74	30.74	30.74	30.74	30.74	30.74	30.74	30.75	30.75	30.75
Wages	3.67	3.67	3.67	3.67	3.67	3.67	3.67	3.67	3.67	3.67	3.67	3.67
Capital Returns**	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Velocity	2.60	2.64	2.68	2.69	2.70	2.71	2.72	2.73	2.74	2.75	2.76	2.77
C/Y**	61.73	61.73	61.72	61.72	61.72	61.72	61.72	61.72	61.72	61.71	61.71	61.71
TC/Y*100**	0.00	0.03	0.16	0.20	0.24	0.30	0.35	0.41	0.48	0.54	0.62	0.69
Nom Interest **	-0.15	0.84	1.84	2.08	2.32	2.56	2.80	3.08	3.32	3.56	3.80	4.04
Real Interest **	-0.15	-0.15	-0.15	-0.15	-0.15	-0.15	-0.15	-0.15	-0.15	-0.15	-0.15	-0.15
WCI:ΔG**	0.52	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.43	0.44	0.44
WCI:ΔG*G/Y**	0.32	0.26	0.26	0.26	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
OPT INF RATE	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16

The results from the separable case when there is no preference for wealth, $\gamma = 1$, show that the welfare costs of inflation are higher using the transaction cost mechanism than the welfare costs determined by Cooley and Hansen (1989) using the cash-in-advance constraint that sets a unitary velocity of money. However, both the current study and that of Cooley and Hansen infer that the money growth rate target has no impact on real wages or capital returns, which are both based on the capital-to-hours ratio of the economy. Since real interest rates do not adjust with money growth rate changes when there is no preference for wealth, an increase in the money growth rate target would affect nominal interest rates on a one-to-one basis. Moderate declines in real wealth result as inflation levels increases. This result is likely linked to the increase in transaction costs resulting from inflationary adjustments moving the velocity of money further away from its Pareto optimal value.

When there is no preference for wealth, the welfare costs of inflation estimates for a 2 percent money growth rate target measure approximately a 1.29 percent loss in terms of composite goods, *G,* and 0.98 percent loss in terms of GDP. The welfare costs show only slight differences with respect to moderate inflation levels ranging from 0 to 4.25 percent. The welfare costs of inflation are very different when a preference for wealth is assumed.

When there is a preference for wealth, $\gamma < 1$, the welfare costs of the inflation are smaller than in the case of no wealth preference. These costs become gradually smaller as the preference for wealth grows (or the relative preference for consumption, γ, falls). The preference for wealth affects the optimal rate of inflation. When a preference for wealth is present, the Freidman rule no longer holds. The money growth rates examined are not as far from the optimal rate, and thus the welfare costs are reduced as the economy's preference for wealth increases. The loss falls to approximately 0.43 percent in terms of goods and 0.26 percent in terms of GDP at a 2 percent inflation rate when the relative preference for wealth increases to 0.20, $\gamma = 0.80$.

Policymakers do not choose households' preference levels and in this case choose only the money growth rate for the economy. However, an increase in the preference for wealth is tied to decreases in the velocity of money, moving the velocity closer to the Pareto optimal level. At the Pareto optimal level, the transaction costs converge to zero as do the welfare costs of inflation. The wealth preference level necessary to eliminate these costs may be considered unrealistically high. The policymakers in an attempt to alleviate the welfare costs set the velocity of money closer to the Pareto optimal rate. Using the money demand equation borrowed from Schmitt-Grohe and Uribe (2004), this attempt would involve a short-term interest rate target.

Policymakers in the U.S. target short-term rates, the federal funds rate, in an attempt to maintain a level of price stability. Short-term rate targets are announced in the minutes of the Federal Open Market Committee regularly. The average federal funds rate over the sample period, 1964:01–2004:04, is approximately 6.56 percent compounded quarterly.[11] The welfare costs derived after incorporating this average short-term rate target are displayed in Table 4.1.2 for various money growth rates and wealth preference levels.[12]

Accounting for short-term rate targets makes the welfare costs of inflation very different and in many ways much more meaningful in comparison with U.S. monetary policy. If high enough, the short-term rate target helps to ensure positive real interest rates at moderate rates of inflation.

The welfare costs in terms of goods increase as the money growth rate target increases, positing a theory for implementing positive money growth rate targets. The welfare cost estimates over these moderate rates of inflation increase with inflation, but these increases are very small. Keeping this in mind, policymakers would attempt to target money growth rates at which the welfare costs presented in Table 4.1.2 converge to zero. The results suggest a positive money growth rate target lying between 2 and 4 percent. The results obtained in this analysis suggest the money growth rate target is dependent on the relative preference for wealth present in the economy. As the preference for wealth rises, the money growth rate target eliminating the welfare costs of inflation increases as well. This behavior seems to stem from increasing wages and decreasing rental rates, changing labor force demands as wealth preferences increase.

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[11] The federal funds rate data is published monthly by the Federal Reserve. The average quarterly rate of 6.56 percent is the nominal quarterly rate that has the same effective rate as the average nominal monthly rate over the sample period.

[12] Tables 4.1.2, 4.2.1.b, and 4.2.2.b display the welfare costs derived by comparing the cashless economy to the Pareto optimal cash economy.

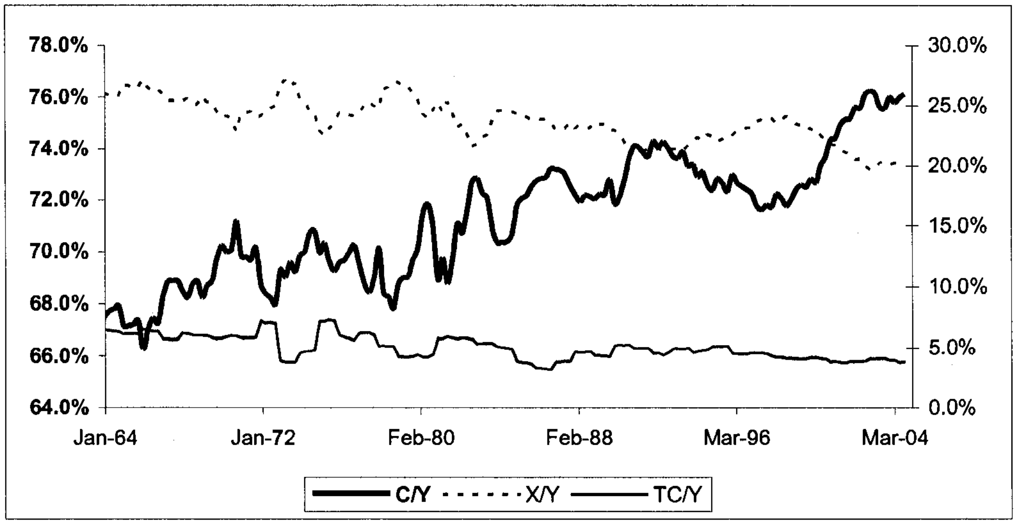
Table 4.1.2: Welfare Costs by Inflation Rate and Relative Preference for Consumption and Wealth

	Separable Preferences over Consumption and Wealth (Target Interest Rate = 6.56%)											
	Annual Inflation Rate											
γ=1	0.00%	1.00%	2.00%	2.25%	2.50%	2.75%	3.00%	3.25%	3.50%	3.75%	4.00%	4.25%
Wealth	13.17	15.00	17.26	17.91	18.59	19.32	20.10	20.92	21.80	22.74	23.75	24.83
Labor	0.31	0.31	0.32	0.32	0.32	0.33	0.33	0.33	0.33	0.34	0.34	0.34
Capital	12.83	14.64	16.88	17.53	18.21	18.93	19.70	20.52	21.40	22.33	23.33	24.41
Wages	2.54	2.65	2.77	2.80	2.84	2.87	2.91	2.94	2.98	3.02	3.06	3.10
Capital Returns**	3.65	3.40	3.16	3.10	3.04	2.98	2.92	2.86	2.80	2.74	2.68	2.62
Velocity	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87
C/Y**	79.22	77.71	75.98	75.51	75.02	74.51	73.98	73.44	72.87	72.28	71.66	71.02
TC/Y*100**	2.16	2.12	2.07	2.06	2.04	2.03	2.01	2.00	1.98	1.97	1.95	1.93
Nom Interest **	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56
Real Interest **	6.56	5.56	4.56	4.28	4.04	3.80	3.56	3.32	3.08	2.84	2.60	2.36
WCI:ΔG*100**	(0.16)	(0.12)	(0.03)	0.01	0.05	0.11	0.18	0.27	0.40	0.58	0.84	1.25
WCI:ΔG*G/Y*100**	(0.13)	(0.09)	(0.02)	0.00	0.04	0.08	0.13	0.20	0.29	0.42	0.60	0.89
γ=0.9												
Wealth	11.86	13.52	15.57	16.16	16.78	17.44	18.15	18.90	19.70	20.55	21.47	22.45
Labor	0.28	0.28	0.29	0.29	0.29	0.29	0.30	0.30	0.30	0.30	0.31	0.31
Capital	11.55	13.19	15.23	15.81	16.43	17.09	17.79	18.54	19.33	20.18	21.09	22.07
Wages	2.54	2.65	2.77	2.80	2.84	2.87	2.91	2.95	2.98	3.02	3.06	3.11
Capital Returns**	3.65	3.40	3.16	3.09	3.03	2.97	2.91	2.85	2.79	2.73	2.67	2.61
Velocity	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87
C/Y**	79.22	77.69	75.95	75.48	74.98	74.47	73.94	73.39	72.82	72.22	71.60	70.95
TC/Y*100**	2.16	2.12	2.07	2.06	2.04	2.03	2.01	2.00	1.98	1.97	1.95	1.93
Nom Interest **	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56
Real Interest **	6.56	5.56	4.56	4.28	4.04	3.80	3.56	3.32	3.08	2.84	2.60	2.36
WCI:ΔG*100**	(0.27)	(0.24)	(0.19)	(0.16)	(0.13)	(0.09)	(0.03)	0.04	0.15	0.29	0.51	0.86
WCI:ΔG*G/Y*100**	(0.47)	(0.45)	(0.36)	(0.31)	(0.25)	(0.17)	(0.06)	0.09	0.30	0.60	1.06	1.80
γ=0.8												
Wealth	10.54	12.03	13.88	14.40	14.97	15.56	16.20	16.87	17.59	18.36	19.18	20.07
Labor	0.25	0.25	0.26	0.26	0.26	0.26	0.26	0.27	0.27	0.27	0.27	0.27
Capital	10.27	11.74	13.57	14.10	14.66	15.25	15.88	16.55	17.27	18.03	18.85	19.73
Wages	2.54	2.65	2.77	2.81	2.84	2.88	2.91	2.95	2.99	3.03	3.07	3.11
Capital Returns**	3.65	3.40	3.15	3.09	3.03	2.97	2.91	2.84	2.78	2.72	2.66	2.60
Velocity	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87
C/Y**	79.22	77.68	75.92	75.44	74.94	74.42	73.89	73.33	72.75	72.15	71.52	70.87
TC/Y*100**	2.16	2.12	2.07	2.05	2.04	2.03	2.01	2.00	1.98	1.96	1.95	1.93
Nom Interest **	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56
Real Interest **	6.56	5.56	4.56	4.28	4.04	3.80	3.56	3.32	3.08	2.84	2.60	2.36
WCI:ΔG*100**	(0.37)	(0.37)	(0.35)	(0.33)	(0.31)	(0.28)	(0.24)	(0.19)	(0.11)	0.00	0.18	0.47
WCI:ΔG*G/Y*100**	(1.02)	(1.07)	(1.05)	(1.02)	(0.97)	(0.90)	(0.78)	(0.62)	(0.37)	0.01	0.62	1.63
γ=0.7												
Wealth	9.22	10.55	12.18	12.65	13.15	13.68	14.25	14.84	15.48	16.17	16.90	17.68
Labor	0.21	0.22	0.22	0.23	0.23	0.23	0.23	0.23	0.23	0.24	0.24	0.24
Capital	8.98	10.30	11.92	12.39	12.88	13.41	13.97	14.56	15.20	15.88	16.61	17.39
Wages	2.54	2.65	2.78	2.81	2.84	2.88	2.92	2.95	2.99	3.03	3.07	3.12
Capital Returns**	3.65	3.40	3.14	3.08	3.02	2.96	2.90	2.84	2.78	2.72	2.66	2.60
Velocity	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87
C/Y**	79.22	77.66	75.87	75.39	74.88	74.36	73.82	73.25	72.67	72.06	71.43	70.76
TC/Y*100**	2.16	2.11	2.07	2.05	2.04	2.03	2.01	1.99	1.98	1.96	1.95	1.93
Nom Interest **	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56
Real Interest **	6.56	5.56	4.56	4.28	4.04	3.80	3.56	3.32	3.08	2.84	2.60	2.36
WCI:ΔG*100**	(0.48)	(0.50)	(0.50)	(0.50)	(0.49)	(0.48)	(0.45)	(0.42)	(0.37)	(0.28)	(0.15)	0.07
WCI:ΔG*G/Y*100**	(1.79)	(1.97)	(2.12)	(2.13)	(2.13)	(2.10)	(2.04)	(1.91)	(1.70)	(1.34)	(0.73)	0.36

When a relative preference for wealth exists, results suggest adjustments in the money growth rate lead to changes in the real wage as well as real and nominal interest rates. Moreover, if policymakers target money growth rate levels around 2 to 3 percent like current monetary policy in the U.S., changes in the real wage that could be explained in part by the business cycle could also be explained by changes in the economy's relative preference for wealth. An increase in the relative preference for wealth is associated with not only an increase in real wages but also a decrease in both real and nominal interest rates and declines in consumption's share of output. Conversely, a decrease in wealth preferences has the opposite effect on these economic variables.

Additionally, households deriving utility from consumption would prefer a smooth consumption profile rather than one that is more volatile. When a preference for wealth is present, households are not only interested in smoothing their consumption profile but also smoothing their wealth profile. Wealth provides for future consumption. Thus, increasing the relative weight households place on wealth has a smoothing effect on both consumption and wealth. Households derive disutility from work and as the preference for wealth increases the percentage of hours supplied to the labor market has declined. This substitution effect may explain the rise in real wages determined by the model when the relative preference for wealth increases.

Figure 4.1: Adjustments in the U.S. Share of Output from 1964:01 to 2004:04



Note: C/Y denotes consumption's share of output and X/Y denotes investment's share of output. The ratio, TC/Y, denotes the share of output that is neither consumed nor invested: transaction cost share of output.

U.S. quarterly data from 1964:01 to 2004:04 support this economic phenomenon. Figure 4.1 shows changes in consumption's share of output, investment's share of output, and the statistical difference that would reflect output not consumed or invested (i.e., transaction costs). When relating the U.S. data to the results from the separable case, it is clear that the transaction cost estimates determined by the model are much lower than those reflected in the U.S., implying other distortions that are thought to affect the velocity of money might be a valuable extension of the model. However, when addressing the average consumption-to-output ratio, C/Y, the figure shows this measure has increased over this time period. At the same time, the investment-to-output ratio and unemployment rates have declined, and money growth rate targets have been set at moderate rates similar to those analyzed here.

4.2 *Non-Separable Preferences*

The time non-separable assumption implies that households are concerned with the fluctuation in the level of consumption as well as the volatility of consumption. These characteristics are associated with risk-averse investors. The results based on the non-separable assumption are reported for the case in which consumption and wealth are complements and the case in which they are substitutes. The results are dependent on the specific of the way monetary policy is conducted and are reported for various money growth rate rules when no short-term rate is targeted and again when a short-term rate target by policymakers is assumed.

4.2.1 Consumption and Wealth Complements ($\sigma = 1.5$ and $\eta = 2.5$)

When consumption and wealth are complements an inflation increase has a decreasing effect on both consumption and wealth. Under this specification of the household's utility function, the employment of a money growth rate target policy yields welfare costs associated with moderate inflation. Like the findings of Cooley and Hansen (1989), inflation has no effect on wages and rental rates, and the relatively small positive welfare costs do not significantly differ at low levels of inflation. The welfare cost estimates are presented in Table 4.2.1a.

Table 4.2.1a: Welfare Costs by Inflation Rate and Relative Preference for Consumption and Wealth

	Non-Separable Preferences: Consumption and Wealth are Compliments (Money Growth Rate Target Only)											
	Annual Inflation Rate											
γ=1	0.00%	1.00%	2.00%	2.25%	2.50%	2.75%	3.00%	3.25%	3.50%	3.75%	4.00%	4.25%
Wealth	14.61	14.61	14.60	14.60	14.60	14.60	14.60	14.60	14.60	14.59	14.59	14.59
Labor	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31
Capital	14.25	14.25	14.25	14.25	14.25	14.25	14.25	14.25	14.25	14.25	14.25	14.25
Wages	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63
Capital Returns**	3.44	3.44	3.44	3.44	3.44	3.44	3.44	3.44	3.44	3.44	3.44	3.44
Velocity	2.84	2.87	2.91	2.92	2.93	2.94	2.95	2.96	2.97	2.98	2.98	2.99
C/Y**	77.93	77.93	77.92	77.92	77.92	77.92	77.91	77.91	77.91	77.91	77.91	77.91
TC/Y*100**	1.64	2.20	2.82	2.98	3.15	3.32	3.49	3.66	3.84	4.02	4.20	4.39
Nom Interest **	5.68	6.68	7.68	7.96	8.20	8.44	8.68	8.92	9.20	9.44	9.68	9.92
Real Interest **	5.68	5.68	5.68	5.68	5.68	5.68	5.68	5.68	5.68	5.68	5.68	5.68
WCI:ΔG**	0.85	0.85	0.86	0.86	0.87	0.87	0.87	0.87	0.87	0.88	0.88	0.88
WCI:ΔG*G/Y**	0.63	0.64	0.64	0.65	0.65	0.65	0.65	0.65	0.65	0.66	0.66	0.66
OPT INF RATE	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60
γ=0.9												
Wealth	14.36	14.36	14.35	14.35	14.35	14.35	14.35	14.35	14.35	14.35	14.35	14.34
Labor	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
Capital	14.01	14.01	14.01	14.01	14.01	14.01	14.01	14.01	14.01	14.01	14.01	14.01
Wages	2.64	2.64	2.64	2.64	2.64	2.64	2.64	2.64	2.64	2.64	2.64	2.64
Capital Returns**	3.42	3.42	3.42	3.42	3.42	3.42	3.42	3.42	3.42	3.42	3.42	3.42
Velocity	2.83	2.87	2.91	2.92	2.93	2.94	2.95	2.96	2.96	2.97	2.98	2.99
C/Y**	77.84	77.83	77.83	77.82	77.82	77.82	77.82	77.82	77.82	77.81	77.81	77.81
TC/Y*100**	1.61	2.16	2.78	2.94	3.11	3.27	3.44	3.62	3.79	3.97	4.15	4.34
Nom Interest **	5.60	6.64	7.64	7.88	8.12	8.36	8.64	8.88	9.12	9.36	9.60	9.88
Real Interest **	5.60	5.60	5.60	5.60	5.60	5.60	5.60	5.60	5.60	5.60	5.60	5.60
WCI:ΔG**	0.84	0.85	0.86	0.86	0.86	0.86	0.87	0.87	0.87	0.87	0.88	0.88
WCI:ΔG*G/Y**	0.60	0.61	0.61	0.61	0.61	0.62	0.62	0.62	0.62	0.62	0.62	0.63
OPT INF RATE	-5.54	-5.54	-5.54	-5.54	-5.54	-5.54	-5.54	-5.54	-5.54	-5.54	-5.54	-5.54
γ=0.8												
Wealth	14.10	14.10	14.09	14.09	14.09	14.09	14.09	14.09	14.08	14.08	14.08	14.08
Labor	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29
Capital	13.76	13.76	13.76	13.76	13.76	13.76	13.76	13.76	13.76	13.76	13.76	13.76
Wages	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65	2.65
Capital Returns**	3.41	3.41	3.41	3.41	3.41	3.41	3.41	3.41	3.41	3.41	3.41	3.41
Velocity	2.83	2.87	2.91	2.92	2.93	2.93	2.94	2.95	2.96	2.97	2.98	2.99
C/Y**	77.72	77.72	77.71	77.71	77.71	77.70	77.70	77.70	77.70	77.70	77.70	77.69
TC/Y*100**	1.57	2.12	2.73	2.89	3.05	3.22	3.39	3.56	3.73	3.91	4.09	4.27
Nom Interest **	5.56	6.56	7.56	7.80	8.04	8.32	8.56	8.80	9.04	9.28	9.56	9.80
Real Interest **	5.56	5.56	5.56	5.56	5.56	5.56	5.56	5.56	5.56	5.56	5.56	5.56
WCI:ΔG**	0.84	0.85	0.85	0.86	0.86	0.86	0.86	0.86	0.87	0.87	0.87	0.87
WCI:ΔG*G/Y**	0.57	0.57	0.58	0.58	0.58	0.58	0.58	0.58	0.59	0.59	0.59	0.59
OPT INF RATE	-5.47	-5.47	-5.47	-5.47	-5.47	-5.47	-5.47	-5.47	-5.47	-5.47	-5.47	-5.47
γ=0.7												
Wealth	13.82	13.81	13.81	13.81	13.81	13.81	13.81	13.80	13.80	13.80	13.80	13.80
Labor	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29
Capital	13.49	13.49	13.49	13.49	13.49	13.49	13.49	13.49	13.49	13.49	13.49	13.49
Wages	2.66	2.66	2.66	2.66	2.66	2.66	2.66	2.66	2.66	2.66	2.66	2.66
Capital Returns**	3.38	3.38	3.38	3.38	3.38	3.38	3.38	3.38	3.38	3.38	3.38	3.38
Velocity	2.83	2.87	2.90	2.91	2.92	2.93	2.94	2.95	2.96	2.97	2.98	2.99
C/Y**	77.58	77.57	77.56	77.56	77.56	77.56	77.56	77.55	77.55	77.55	77.55	77.55
TC/Y*100**	1.52	2.06	2.67	2.82	2.99	3.15	3.32	3.49	3.66	3.84	4.02	4.20
Nom Interest **	5.44	6.48	7.48	7.72	7.96	8.20	8.48	8.72	8.96	9.20	9.44	9.68
Real Interest **	5.44	5.44	5.44	5.44	5.44	5.44	5.44	5.44	5.44	5.44	5.44	5.44
WCI:ΔG**	0.83	0.84	0.85	0.85	0.85	0.85	0.86	0.86	0.86	0.86	0.87	0.87
WCI:ΔG*G/Y**	0.53	0.53	0.54	0.54	0.54	0.54	0.54	0.55	0.55	0.55	0.55	0.55
OPT INF RATE	-5.28	-5.28	-5.28	-5.28	-5.28	-5.28	-5.28	-5.28	-5.28	-5.28	-5.28	-5.28

When policymakers target money growth rates only, the welfare costs of inflation estimates are approximately 0.86 percent in terms of goods and 0.64 percent of real GDP. Just as the welfare cost estimates did not significantly differ due to changes in the level of inflation, the welfare costs when consumption and wealth are complements do not significantly differ with changes in the economy's relative preference for wealth. The loss falls to approximately 0.85 percent in terms of goods and 0.54 percent in terms of GDP at a 2 percent inflation rate when the relative preference for wealth increases to 0.20, $\gamma = 0.80$.

When policymakers target short-term interest rates, welfare costs of inflation between the cashless and cash economies with the same rate target show significantly high costs in terms of goods associated with increasing the money growth rate target; however, remembering that the cost in terms of goods from the Pareto optimal economy increase moderately with inflation, the policymakers would set money growth rates so that the welfare costs associated with setting the interest rate above zero are eliminated. This result is displayed in Table 4.2.1b. These costs could reflect the increase in work hours required as upward inflation rate adjustments reduce the real returns to capital. A money growth rate target between 2 and 2.5 percent results over various wealth preference levels within the range $0 \leq \gamma \leq 0.70$. Again, under a short-term interest rate target policy, the costs increase with inflation and decrease with the preference for wealth.

Table 4.2.1b: Welfare Costs by Inflation Rate and Relative Preference for Consumption and Wealth

	Non-Separable Preferences: Consumption and Wealth are Compliments (Target Interest Rate = 6.56%)											
	Annual Inflation Rate											
γ=1	0.00%	1.00%	2.00%	2.25%	2.50%	2.75%	3.00%	3.25%	3.50%	3.75%	4.00%	4.25%
Wealth	13.23	14.85	16.83	17.39	17.99	18.62	19.29	19.99	20.75	21.55	22.40	23.31
Labor	0.31	0.31	0.31	0.31	0.31	0.31	0.32	0.32	0.32	0.32	0.32	0.32
Capital	12.88	14.49	16.46	17.02	17.62	18.24	18.91	19.61	20.36	21.16	22.01	22.92
Wages	2.54	2.65	2.77	2.80	2.84	2.87	2.91	2.94	2.98	3.02	3.06	3.10
Capital Returns**	3.65	3.40	3.16	3.10	3.04	2.98	2.92	2.86	2.80	2.74	2.68	2.62
Velocity	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87
C/Y**	79.22	77.71	75.98	75.51	75.02	74.51	73.98	73.44	72.87	72.28	71.66	71.02
TC/Y*100**	2.16	2.12	2.07	2.06	2.04	2.03	2.01	2.00	1.98	1.97	1.95	1.93
Nom Interest **	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56
Real Interest **	6.56	5.56	4.56	4.28	4.04	3.80	3.56	3.32	3.08	2.84	2.60	2.36
WCI:ΔG**	(18.40)	(13.98)	(4.08)	0.13	5.55	12.65	22.23	35.55	54.95	84.90	135.26	231.37
WCI:ΔG/Y**	(14.57)	(10.86)	(3.10)	0.10	4.16	9.43	16.44	26.11	40.04	61.36	96.93	164.31
γ=0.9												
Wealth	12.91	14.49	16.43	16.98	17.56	18.17	18.83	19.52	20.25	21.04	21.87	22.76
Labor	0.30	0.30	0.30	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31
Capital	12.57	14.15	16.07	16.62	17.20	17.81	18.46	19.15	19.88	20.66	21.49	22.38
Wages	2.54	2.65	2.77	2.80	2.84	2.87	2.91	2.94	2.98	3.02	3.06	3.10
Capital Returns**	3.65	3.40	3.16	3.10	3.04	2.98	2.92	2.86	2.80	2.74	2.68	2.62
Velocity	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87
C/Y**	79.22	77.71	75.98	75.51	75.02	74.51	73.98	73.44	72.87	72.28	71.66	71.02
TC/Y*100**	2.16	2.12	2.07	2.06	2.04	2.03	2.01	2.00	1.98	1.97	1.95	1.93
Nom Interest **	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56
Real Interest **	6.56	5.56	4.56	4.28	4.04	3.80	3.56	3.32	3.08	2.84	2.60	2.36
WCI:ΔG**	(18.56)	(14.14)	(4.24)	(0.04)	5.37	12.47	22.03	35.34	54.70	84.61	134.90	230.86
WCI:ΔG/Y**	(32.82)	(25.87)	(8.06)	(0.07)	10.41	24.42	43.61	70.73	110.75	173.32	279.67	484.59
γ=0.8												
Wealth	12.56	14.10	15.99	16.52	17.09	17.69	18.33	19.00	19.72	20.48	21.29	22.16
Labor	0.29	0.29	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.31
Capital	12.23	13.77	15.64	16.17	16.74	17.33	17.97	18.64	19.35	20.11	20.92	21.79
Wages	2.54	2.65	2.77	2.80	2.84	2.87	2.91	2.94	2.98	3.02	3.06	3.10
Capital Returns**	3.65	3.40	3.16	3.10	3.04	2.98	2.92	2.86	2.80	2.74	2.68	2.62
Velocity	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87
C/Y**	79.22	77.71	75.98	75.51	75.02	74.51	73.98	73.44	72.87	72.27	71.66	71.01
TC/Y*100**	2.16	2.12	2.07	2.06	2.04	2.03	2.01	2.00	1.98	1.97	1.95	1.93
Nom Interest **	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56
Real Interest **	6.56	5.56	4.56	4.28	4.04	3.80	3.56	3.32	3.08	2.84	2.60	2.36
WCI:ΔG**	(18.75)	(14.33)	(4.45)	(0.25)	5.15	12.24	21.78	35.07	54.40	84.25	134.44	230.23
WCI:ΔG/Y**	(51.50)	(41.32)	(13.53)	(0.78)	16.10	38.81	70.12	114.63	180.63	284.28	461.13	803.04
γ=0.7												
Wealth	12.17	13.67	15.50	16.02	16.57	17.16	17.77	18.43	19.12	19.86	20.65	21.50
Labor	0.28	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.30	0.30
Capital	11.85	13.34	15.16	15.68	16.23	16.81	17.42	18.08	18.77	19.51	20.29	21.13
Wages	2.54	2.65	2.77	2.80	2.84	2.87	2.91	2.94	2.98	3.02	3.06	3.10
Capital Returns**	3.65	3.40	3.16	3.10	3.04	2.98	2.92	2.86	2.80	2.74	2.68	2.62
Velocity	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87
C/Y**	79.22	77.71	75.98	75.51	75.02	74.51	73.98	73.43	72.87	72.27	71.66	71.01
TC/Y*100**	2.16	2.12	2.07	2.06	2.04	2.03	2.01	2.00	1.98	1.97	1.95	1.93
Nom Interest **	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56
Real Interest **	6.56	5.56	4.56	4.28	4.04	3.80	3.56	3.32	3.08	2.84	2.60	2.36
WCI:ΔG**	(19.01)	(14.59)	(4.72)	(0.53)	4.87	11.94	21.46	34.72	54.01	83.79	133.86	229.42
WCI:ΔG/Y**	(70.77)	(57.41)	(19.72)	(2.23)	20.99	52.35	95.71	157.50	249.32	393.82	640.77	1118.90

4.2.2 Consumption and Wealth Substitutes ($\sigma = 2.5$ and $\eta = 1.5$)

Under the time non-separable assumption for which consumption and wealth are substitutes, households respond to expected increases in inflation by substituting away from capital markets and toward consumption purchases in an attempt to sustain their wealth. Households potentially increase their demand for consumer durables whose values increase in an inflationary low interest rate environment (e.g., demands for housing). Targeting money growth rates leads to positive welfare costs of inflation that increase but only slightly with the inflation level targeted.

When consumption and wealth are substitutes and there is no preference for wealth, the welfare cost estimates are smaller than those seen in the other two cases. When there is no preference for wealth, the welfare cost of inflation is approximately 0.52 percent in terms of goods and 0.40 percent of output for moderate levels of inflation. The results are presented in Table 4.2.2a. Yet, unlike the results from section 4.2.1 in which consumption and wealth were complements, as the preference for wealth increases, the wealth-reducing impact of inflation seems somewhat mitigated, possibly due to the portfolio adjustments made by the households. Changes in the preference for wealth have an effect on welfare costs of inflation similar to that seen in the separable case. The welfare costs of inflation decrease as the preference for wealth increases. In fact, the welfare costs at a wealth preference level of 0.30 fall to 0.28 percent of goods and 0.24 percent of output.

Table 4.2.2a: Welfare Costs by Inflation Rate and Relative Preference for Consumption and Wealth

Non-Separable Preferences: Consumption and Wealth are Substitutes (Money Growth Rate Target Only)												
	Annual Inflation Rate											
γ=1	0.00%	1.00%	2.00%	2.25%	2.50%	2.75%	3.00%	3.25%	3.50%	3.75%	4.00%	4.25%
Wealth	14.52	14.51	14.51	14.51	14.51	14.50	14.50	14.50	14.50	14.50	14.50	14.50
Labor	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31
Capital	14.16	14.16	14.16	14.16	14.16	14.16	14.16	14.16	14.16	14.16	14.16	14.16
Wages	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63
Capital Returns**	3.44	3.44	3.44	3.44	3.44	3.44	3.44	3.44	3.44	3.44	3.44	3.44
Velocity	2.84	2.87	2.91	2.92	2.93	2.94	2.95	2.96	2.97	2.98	2.98	2.99
C/Y**	77.93	77.93	77.92	77.92	77.92	77.92	77.91	77.91	77.91	77.91	77.91	77.91
TC/Y*100**	1.64	2.20	2.82	2.98	3.15	3.32	3.49	3.66	3.84	4.02	4.20	4.39
Nom Interest **	5.68	6.68	7.68	7.96	8.20	8.44	8.68	8.92	9.20	9.44	9.68	9.92
Real Interest **	5.68	5.68	5.68	5.68	5.68	5.68	5.68	5.68	5.68	5.68	5.68	5.68
WCI:ΔG**	0.51	0.52	0.52	0.52	0.53	0.53	0.53	0.53	0.53	0.54	0.54	0.54
WCI:ΔG*G/Y**	0.39	0.40	0.40	0.40	0.40	0.41	0.41	0.41	0.41	0.41	0.41	0.42
OPT INF RATE	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60	-5.60
γ=0.9												
Wealth	14.27	14.27	14.27	14.26	14.26	14.26	14.26	14.26	14.26	14.26	14.26	14.26
Labor	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28
Capital	13.94	13.94	13.95	13.95	13.95	13.95	13.95	13.95	13.95	13.95	13.95	13.95
Wages	2.72	2.72	2.73	2.73	2.73	2.73	2.73	2.73	2.73	2.73	2.73	2.73
Capital Returns**	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.24
Velocity	2.81	2.84	2.88	2.89	2.90	2.91	2.92	2.93	2.94	2.95	2.96	2.96
C/Y**	76.62	76.62	76.61	76.61	76.61	76.61	76.60	76.60	76.60	76.60	76.60	76.60
TC/Y*100**	1.23	1.73	2.29	2.44	2.59	2.75	2.91	3.07	3.23	3.40	3.57	3.74
Nom Interest **	4.88	5.92	6.92	7.16	7.40	7.64	7.88	8.16	8.40	8.64	8.88	9.12
Real Interest **	4.88	4.88	4.88	4.88	4.88	4.88	4.88	4.88	4.88	4.88	4.88	4.88
WCI:ΔG**	0.43	0.44	0.44	0.44	0.45	0.45	0.45	0.45	0.45	0.46	0.46	0.46
WCI:ΔG*G/Y**	0.34	0.35	0.35	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.37	0.37
OPT INF RATE	-4.84	-4.84	-4.84	-4.84	-4.84	-4.84	-4.84	-4.84	-4.84	-4.84	-4.84	-4.84
γ=0.8												
Wealth	13.96	13.96	13.95	13.95	13.95	13.95	13.95	13.95	13.95	13.95	13.95	13.95
Labor	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
Capital	13.66	13.66	13.66	13.66	13.66	13.66	13.66	13.66	13.66	13.66	13.66	13.66
Wages	2.83	2.83	2.83	2.83	2.83	2.83	2.83	2.83	2.83	2.83	2.83	2.83
Capital Returns**	3.05	3.05	3.05	3.05	3.05	3.05	3.05	3.05	3.05	3.05	3.05	3.05
Velocity	2.77	2.81	2.85	2.86	2.87	2.88	2.89	2.90	2.91	2.92	2.93	2.94
C/Y**	75.15	75.14	75.14	75.14	75.13	75.13	75.13	75.13	75.13	75.13	75.12	75.12
TC/Y*100**	0.88	1.31	1.81	1.95	2.08	2.23	2.37	2.52	2.67	2.82	2.98	3.14
Nom Interest **	4.12	5.12	6.12	6.36	6.60	6.88	7.12	7.36	7.60	7.84	8.08	8.36
Real Interest **	4.12	4.12	4.12	4.12	4.12	4.12	4.12	4.12	4.12	4.12	4.12	4.12
WCI:ΔG**	0.35	0.35	0.36	0.36	0.36	0.36	0.37	0.37	0.37	0.37	0.37	0.37
WCI:ΔG*G/Y**	0.29	0.30	0.30	0.30	0.30	0.31	0.31	0.31	0.31	0.31	0.31	0.31
OPT INF RATE	-4.08	-4.08	-4.08	-4.08	-4.08	-4.08	-4.08	-4.08	-4.08	-4.08	-4.08	-4.08
γ=0.7												
Wealth	13.57	13.57	13.57	13.57	13.56	13.56	13.56	13.56	13.56	13.56	13.56	13.56
Labor	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21
Capital	13.30	13.30	13.30	13.30	13.30	13.30	13.30	13.30	13.30	13.31	13.31	13.31
Wages	2.94	2.94	2.94	2.94	2.94	2.94	2.94	2.94	2.94	2.94	2.94	2.94
Capital Returns**	2.86	2.86	2.86	2.86	2.86	2.86	2.86	2.86	2.86	2.86	2.86	2.86
Velocity	2.74	2.78	2.82	2.83	2.84	2.85	2.86	2.87	2.88	2.89	2.90	2.91
C/Y**	73.46	73.45	73.45	73.45	73.44	73.44	73.44	73.44	73.44	73.44	73.43	73.43
TC/Y*100**	0.58	0.94	1.38	1.50	1.62	1.75	1.88	2.01	2.15	2.29	2.43	2.57
Nom Interest **	3.32	4.36	5.32	5.60	5.84	6.08	6.32	6.56	6.80	7.08	7.32	7.56
Real Interest **	3.32	3.32	3.32	3.32	3.32	3.32	3.32	3.32	3.32	3.32	3.32	3.32
WCI:ΔG**	0.27	0.27	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.29	0.29	0.29
WCI:ΔG*G/Y**	0.24	0.24	0.24	0.24	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
OPT INF RATE	-3.27	-3.27	-3.27	-3.27	-3.27	-3.27	-3.27	-3.27	-3.27	-3.27	-3.26	-3.26

Table 4.2.2b: Welfare Costs by Inflation Rate and Relative Preference for Consumption and Wealth

	Non-Separable Preferences: Consumption and Wealth are Substitutes (Target Interest Rate = 6.56%)											
	Annual Inflation Rate											
γ=1	0.00%	1.00%	2.00%	2.25%	2.50%	2.75%	3.00%	3.25%	3.50%	3.75%	4.00%	4.25%
Wealth	13.27	14.73	16.49	16.99	17.52	18.07	18.66	19.28	19.94	20.64	21.38	22.17
Labor	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31
Capital	12.92	14.38	16.13	16.63	17.15	17.71	18.29	18.91	19.57	20.26	21.00	21.79
Wages	2.54	2.65	2.77	2.80	2.84	2.87	2.91	2.94	2.98	3.02	3.06	3.10
Capital Returns**	3.65	3.40	3.16	3.10	3.04	2.98	2.92	2.86	2.80	2.74	2.68	2.62
Velocity	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87
C/Y**	79.22	77.71	75.98	75.51	75.02	74.51	73.98	73.44	72.87	72.28	71.66	71.02
TC/Y*100**	2.16	2.12	2.07	2.06	2.04	2.03	2.01	2.00	1.98	1.97	1.95	1.93
Nom Interest **	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56
Real Interest **	6.56	5.56	4.56	4.28	4.04	3.80	3.56	3.32	3.08	2.84	2.60	2.36
WCI:ΔG**	(5.89)	(4.35)	(1.26)	(0.06)	1.39	3.15	5.34	8.06	11.51	15.99	21.95	30.26
WCI:ΔG/Y**	(4.66)	(3.38)	(0.96)	(0.05)	1.04	2.35	3.95	5.92	8.39	11.55	15.73	21.49
γ=0.9												
Wealth	11.98	13.28	14.86	15.31	15.78	16.28	16.80	17.35	17.94	18.57	19.23	19.93
Labor	0.28	0.28	0.28	0.28	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
Capital	11.66	12.97	14.54	14.98	15.45	15.95	16.47	17.02	17.61	18.23	18.89	19.60
Wages	2.54	2.65	2.77	2.80	2.84	2.87	2.91	2.94	2.98	3.02	3.06	3.10
Capital Returns**	3.65	3.40	3.16	3.10	3.04	2.98	2.92	2.86	2.80	2.74	2.68	2.62
Velocity	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87
C/Y**	79.22	77.70	75.97	75.50	75.01	74.50	73.97	73.43	72.86	72.26	71.65	71.00
TC/Y*100**	2.16	2.12	2.07	2.06	2.04	2.03	2.01	2.00	1.98	1.97	1.95	1.93
Nom Interest **	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56
Real Interest **	6.56	5.56	4.56	4.28	4.04	3.80	3.56	3.32	3.08	2.84	2.60	2.36
WCI:ΔG**	(6.82)	(5.39)	(2.43)	(1.27)	0.13	1.85	3.98	6.65	10.04	14.43	20.31	28.50
WCI:ΔG/Y**	(12.06)	(9.86)	(4.62)	(2.45)	0.25	3.63	7.89	13.31	20.32	29.57	42.11	59.84
γ=0.8												
Wealth	10.70	11.86	13.26	13.65	14.07	14.51	14.97	15.46	15.98	16.53	17.11	17.74
Labor	0.25	0.25	0.25	0.25	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24
Capital	10.42	11.58	12.97	13.36	13.78	14.21	14.68	15.17	15.68	16.23	16.82	17.44
Wages	2.54	2.65	2.77	2.80	2.84	2.87	2.91	2.94	2.98	3.02	3.06	3.10
Capital Returns**	3.65	3.40	3.16	3.10	3.03	2.97	2.91	2.85	2.79	2.73	2.67	2.61
Velocity	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87
C/Y**	79.22	77.70	75.96	75.49	75.00	74.49	73.96	73.41	72.84	72.25	71.63	70.98
TC/Y*100**	2.16	2.12	2.07	2.06	2.04	2.03	2.01	2.00	1.98	1.97	1.95	1.93
Nom Interest **	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56
Real Interest **	6.56	5.56	4.56	4.28	4.04	3.80	3.56	3.32	3.08	2.84	2.60	2.36
WCI:ΔG**	(7.89)	(6.59)	(3.80)	(2.70)	(1.35)	0.32	2.38	4.97	8.28	12.58	18.35	26.40
WCI:ΔG/Y**	(21.68)	(19.02)	(11.57)	(8.32)	(4.22)	1.00	7.67	16.27	27.51	42.49	62.97	92.16
γ=0.7												
Wealth	9.45	10.46	11.68	12.03	12.39	12.77	13.17	13.60	14.05	14.53	15.04	15.58
Labor	0.22	0.22	0.22	0.22	0.22	0.22	0.21	0.21	0.21	0.21	0.21	0.21
Capital	9.20	10.21	11.43	11.77	12.13	12.51	12.91	13.34	13.79	14.27	14.78	15.32
Wages	2.54	2.65	2.77	2.80	2.84	2.87	2.91	2.95	2.98	3.02	3.06	3.10
Capital Returns**	3.65	3.40	3.16	3.09	3.03	2.97	2.91	2.85	2.79	2.73	2.67	2.61
Velocity	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87	2.87
C/Y**	79.22	77.69	75.95	75.48	74.98	74.47	73.94	73.39	72.82	72.23	71.61	70.96
TC/Y*100**	2.16	2.12	2.07	2.06	2.04	2.03	2.01	2.00	1.98	1.97	1.95	1.93
Nom Interest **	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56	6.56
Real Interest **	6.56	5.56	4.56	4.28	4.04	3.80	3.56	3.32	3.08	2.84	2.60	2.36
WCI:ΔG**	(9.16)	(8.02)	(5.44)	(4.40)	(3.12)	(1.53)	0.45	2.96	6.16	10.35	15.97	23.85
WCI:ΔG/Y**	(34.10)	(31.59)	(22.77)	(18.70)	(13.46)	(6.71)	2.03	13.43	28.49	48.70	76.56	116.49

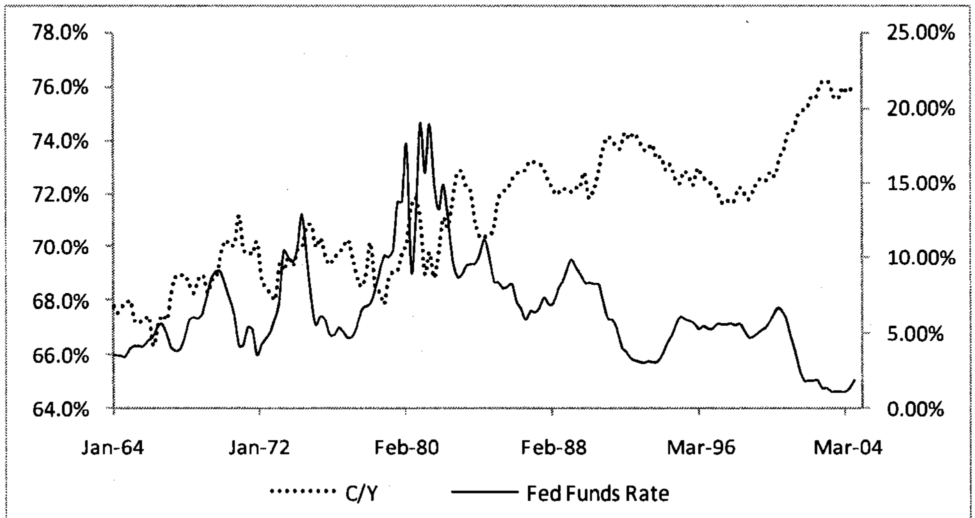
Table 4.2.2b displays the results obtained when policymakers are assumed to target short-term rates. Resultant welfare costs of inflation continue to suggest money growth rate targets between 2.25 and 3 percent when households view consumption and wealth as substitutes and wealth preferences are within the range examined. Further examination of the results in Table 4.2.2b shows the effect of interest rate targeting is directly seen in the real interest rate. Faced with a choice to increase or decrease the inflation target, the results suggest that an increase in the inflation target above 3 percent could lead to welfare costs in terms of goods for most wealth preferences and all assumptions of policy implementation. However, a money growth rate of 2 percent would result in welfare gains, though the gains are higher for some wealth preferences than others.

Section 5: Conclusions

The welfare cost estimates determined in the time-separable analysis suggest that the costs of inflation and the business cycle differ based on the time-separable nature assumed in the household preference structure. The separable assumption of household preferences seems to portray the outcome in which the average investor is risk neutral, taking only the expected consumption level and not the volatility of consumption into consideration when making allocation choices. The non-separable assumptions seem to portray the outcome in which the average investor is risk averse, taking long-run consumption expectations as well as short-term consumption volatility into consideration when making investment decisions. Thus, the policymakers must act in a manner that considers the concerns of various types of investors, who have different risk characteristics and wealth preferences, when making policy decisions.

The results obtained when assuming policymakers target short-term interest rates show that the impact of targeting short-term interest rates is a rise in the real interest rate as the preference for wealth increases. The results suggest that the appropriate target has a great impact on the viability of U.S. financial markets and should be chosen carefully, taking the economy's preferences into consideration. The estimates from the separable and non-separable case in which consumption and wealth are considered substitutes are consistent with the long-run consumption-to-output and investment-to-output ratios derived from the U.S. economic data over the sample period, 1964:01 to 2004:04, and also consistent with a moderate inflation target slightly above 2 percent.

Figure 5.1: Comparison of the Consumption-to-Output Ratio and the Federal Funds Rate from 1964:01 to 2004:04



Note: C/Y denotes consumption's share of output, and the Fed Funds Rate is the Federal Funds Rate as reported in the Federal Reserve Historical Database.

Upon analysis of Figure 5.1, which graphs the federal funds rate along with the consumption-to-output ratio, it is seen that policy determination cannot be attributed to short-term rate targeting alone. The age makeup of the workforce could account for the rise in the consumption-to-output ratio. In this analysis, the higher consumption-to-output ratios were associated with the non-separable case of the risk-averse investor, though as the federal funds rate decline, the real interest rate would also decline at any given money growth rate. A reduction in the real interest rate plays a critical role in explaining the increase in consumption and the decrease in investment's share of output.

An important caveat pertaining to representative agent models noted by Otrok (2001) points out that the welfare costs may not be proportionately distributed when incomplete markets such as the labor market exist in the model economy. In this model economy the caveat can be extended to the capital market, the market for savings or investment. Disproportionate welfare costs are determined based on the differences in the time preferences assumed for the households in this model.

Risk neutrality is consistent with the suggested rational asset allocation strategies for young investors with relatively long investment horizons, and risk aversion is consistent with rational asset allocation strategies of older investors who focus on maintaining their accumulated wealth. Given that expected welfare costs of the business cycle are higher for the older investors and are reduced as the targeted rate of inflation increases under all specifications, the results suggest the age distributions of the U.S. workforce might play an important role when monetary policy decisions are considered. An increase in the average age of the U.S. workforce might suggest increases in the average preference for retirement wealth and decreases in the average share of output invested. The results of this welfare cost analysis suggest that policymakers, in an attempt to reduce the welfare costs of inflation, would support moderately low inflation targets slightly higher than 2 percent.

Thoughts for Future Research

An empirical investigation using the maximum likelihood approach to determine whether the data supports a positive relative preference for wealth seems to be the logical next step in this stream of literature. If this test suggests a positive relative wealth preference level associated with the U.S. data, further segmentation of the data can help to determine the direction that the relative preference for wealth has taken over time in the U.S.

Additionally, the model has been designed for the future examination of the welfare costs of the business cycle. The incorporation of a relative preference for consumption and wealth in a monetary model of the business cycle has shown that both the effects of a money growth rate innovation and the response to a technology innovation differ with wealth preferences. Thus, the welfare costs of the business cycle are thought to differ based on the time-separable assumptions of the household's utility function as well as the wealth preference levels of the household. After identifying the relative wealth preference suggested by the U.S. data, the estimation of the welfare costs of the business cycle seems to be the next logical step in this stream of literature.

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APPENDICES

APPENDIX A

A.1 Solution of the Household's Decision Problem

The household's problem can be written in Lagrange form, where the Lagrangians associated with the budget, investment, and transaction cost function constraints are λ_t, q_t, and φ_t, respectively. The Lagrangian of the household's decision problem then takes the following form:

$$\underset{\{c_t,h_t,m_t,b_t,x_t,k_t,\lambda_t,q_t,\varphi_t,T_t\}_{t=0}^{\infty}}{Max\ \mathfrak{L}} = \left\{ \begin{array}{l} U(c_t,a_t,h_t)+\beta E_t\left[V(k_t,m_t,b_t)\right] \\ +\lambda_t\left[w_t h_t + r_t k_{t-1} + \tau_t + \dfrac{m_{t-1}}{\Pi_t} + \dfrac{I_{t-1}b_{t-1}}{\Pi_t} - (1+T_t)c_t - x_t - m_t - b_t\right] \\ +q_t\left[x_t + (1-\delta)k_{t-1} - k_t\right] \\ +\varphi_t\left[\zeta\left(A^T v_t + \dfrac{B^T}{v_t} - 2\sqrt{A^T B^T}\right) - T_t\right] \end{array} \right\}. \quad \text{(A.1.1)}$$

The household's decision problem includes the derivation of the first-order and time-consistent envelope conditions of the model. These conditions are then used along with the constraints that affect the household's decision to form a set of competitive equilibrium conditions. The household's problem yields the following first-order conditions:

$$c_t : U_c(c_t,a_t,h_t) - (1+T_t)\lambda_t - \varphi_t\zeta\left(\frac{A^T}{m_t} - \frac{\boldsymbol{B}^T m_t}{c_t^2}\right) = 0 \qquad \text{(A.1.2)}$$

$$h_t : U_h\left(c_t, a_t, h_t\right) + \lambda_t w_t = 0 \tag{A.1.3}$$

$$m_t : \beta E_t \left\{V_m\left(k_t, m_t, b_t\right)\right\} - \lambda_t - \varphi_t \zeta \left(\frac{A^T c_t}{m_t^2} + \frac{B^T}{c_t}\right) = 0 \tag{A.1.4}$$

$$b_t : \beta E_t \left\{V_b\left(k_t, m_t, b_t\right)\right\} - \lambda_t = 0 \tag{A.1.5}$$

$$x_t : -\lambda_t + q_t = 0 \tag{A.1.6}$$

$$k_t : \beta E_t \left[V_k\left(k_t, m_t, b_t\right)\right] - q_t = 0 \tag{A.1.7}$$

$$\lambda_t : w_t h_t + r_t k_{t-1} + \tau_t + \frac{m_{t-1}}{\Pi_t} + \frac{I_{t-1} b_{t-1}}{\Pi_t} - \left(1 + \mathrm{T}_t\right) c_t - x_t - m_t - b_t = 0 \tag{A.1.8}$$

$$q_t : x_t + \left(1 - \delta\right) k_{t-1} - k_t = 0 \tag{A.1.9}$$

$$\varphi_t : \zeta \left(A^T v_t + \frac{B^T}{v_t} - 2\sqrt{A^T B^T}\right) - T_t = 0 \tag{A.1.10}$$

$$\mathrm{T}_t : -c_t \lambda_t - \varphi_t = 0 \tag{A.1.11}$$

and envelope conditions:

$$V_m\left(k_{t-1}, m_{t-1}, b_{t-1}\right) = \left(U_a\left(c_t, a_t, h_t\right) + \lambda_t\right)\left(\frac{1}{\Pi_t}\right) \tag{A.1.12}$$

$$V_b\left(k_{t-1}, m_{t-1}, b_{t-1}\right) = \left(\frac{I_{t-1}}{\Pi_t}\right)\left[\lambda_t + U_a\left(c_t, a_t, h_t\right)\right] \tag{A.1.13}$$

$$V_k\left(k_{t-1}, m_{t-1}, b_{t-1}\right) = U_a\left(c_t, a_t, h_t\right) + \lambda_t r_t + q_t\left(1 - \delta\right). \tag{A.1.14}$$

The time-consistent envelope conditions needed to combine the first-order conditions with the envelope conditions are listed below:

$$V_m(k_t,m_t,b_t)=[U_a(c_{t+1},a_{t+1},h_{t+1})+\lambda_{t+1}]\left(\frac{1}{\Pi_{t+1}}\right) \tag{A.1.15}$$

$$V_b(k_t,m_t,b_t)=\left(\frac{I_t}{\Pi_{t+1}}\right)[\lambda_{t+1}+U_a(c_{t+1},a_{t+1},h_{t+1})] \tag{A.1.16}$$

$$V_k(k_t,m_t,b_t)=U_a(c_{t+1},a_{t+1},h_{t+1})+\lambda_{t+1}r_{t+1}+q_{t+1}(1-\delta). \tag{A.1.17}$$

The combination of the first-order conditions and the time-consistent envelope conditions gives the following set of equilibrium conditions for the household.

$$c_t : U_c(c_t,a_t,h_t)-(1+\mathrm{T}_t)\lambda_t-\varphi_t\zeta\left(\frac{A^{\mathrm{T}}}{m_t}-\frac{\mathbf{B}^{\mathrm{T}}m_t}{c_t^{\,2}}\right)=0 \tag{A.1.18}$$

$$h_t : U_h(c_t,a_t,h_t)+\lambda_t w_t=0 \tag{A.1.19}$$

$$m_t : \beta E_t\left\{(U_a(c_{t+1},a_{t+1},h_{t+1})+\lambda_{t+1})\left(\frac{1}{\Pi_{t+1}}\right)\right\}-\lambda_t-\varphi\zeta\left(\frac{A^T c_t}{m_t^{\,2}}+\frac{B^T}{c_t}\right)=0 \tag{A.1.20}$$

$$b_t : \beta E_t\left\{\left(\frac{I_t}{\Pi_{t+1}}\right)[\lambda_{t+1}+U_a(c_{t+1},a_{t+1},h_{t+1})]\right\}-\lambda_t=0 \tag{A.1.21}$$

$$x_t : -\lambda_t+q_t=0 \tag{A.1.22}$$

$$k_t : \beta E_t\{U_a(c_{t+1},a_{t+1},h_{t+1})+\lambda_{t+1}r_{t+1}+q_{t+1}(1-\delta)\}-q_t=0 \tag{A.1.23}$$

$$\lambda_t : w_t h_t+r_t k_{t-1}+\tau_t+\frac{m_{t-1}}{\Pi_t}+\frac{I_{t-1}b_{t-1}}{\Pi_t}-(1+T_t)c_t-x_t-m_t-b_t=0 \tag{A.1.24}$$

$$q_t : x_t+(1-\delta)k_{t-1}-k_t=0 \tag{A.1.25}$$

$$\varphi_t : \zeta\left(A^T v_t+\frac{B^T}{v_t}-2\sqrt{A^T B^T}\right)-T_t=0 \tag{A.1.26}$$

$$T_t : \lambda_t c_t - \varphi_t = 0 \tag{A.1.27}$$

These 10 equations together with initial conditions k_{-1}, m_{-1}, b_{-1}, and the appropriate transversality conditions fully characterize the representative household's optimal plan for $\{c_t, a_t, m_t, x_t, k_t, h_t, \lambda_t, q_t, \varphi_t\}_{t=0}^{\infty}$.

To close the model, the following conditions must hold in the case where all agents are identical:

$$m_t = \frac{\Theta_t}{\Pi_t} m_{t-1} \tag{A.1.28}$$

$$\tau_t = m_t - \frac{m_{t-1}}{\Pi_t} \tag{A.1.29}$$

$$b_t = 0 \;\forall\; t \tag{A.1.30}$$

A.2 Competitive Equilibrium Conditions

The optimality conditions of the homogeneous households and the firm, together with the policy rules of the government, form the following system of equations, which fully define the economy's competitive equilibrium:

$$(1 + \mathrm{T}_t)c_t + x_t = y_t \tag{A.2.1}$$

$$k_t = x_t + (1 - \delta)k_{t-1} \tag{A.2.2}$$

$$T_t = \zeta\left(A^T v_t + \frac{B^T}{v_t} - 2\sqrt{A^T B^T}\right) \tag{A.2.3}$$

$$v_t = \frac{c_t}{m_t} \tag{A.2.4}$$

$$y_t = e^{z_t} k_{t-1}{}^{\alpha} h_t{}^{1-\alpha} \tag{A.2.5}$$

$$a_t = k_{t-1} + m_t \tag{A.2.6}$$

$$r_t = \alpha e^{z_t} k_{t-1}{}^{\alpha-1} h_t{}^{1-\alpha} \tag{A.2.7}$$

$$w_t = (1-\alpha) e^{z_t} k_{t-1}{}^{\alpha} h_t{}^{-\alpha} \tag{A.2.8}$$

$$m_t = \frac{\Theta_t}{\Pi_t} m_{t-1} \tag{A.2.9}$$

$$U_c(c_t, a_t, h_t) = (1+T_t)\lambda_t + \varphi_t \zeta \left(\frac{A^T}{m_t} - \frac{\boldsymbol{B}^T m_t}{c_t^2} \right) \tag{A.2.10}$$

$$U_h(c_t, a_t, h_t) = -\lambda_t w_t \tag{A.2.11}$$

$$\lambda_t = \beta E_t \left\{ \left(U_a(c_{t+1}, a_{t+1}, h_{t+1}) + \lambda_{t+1} \right) \left(\frac{1}{\Pi_{t+1}} \right) \right\} - \varphi_t \zeta \left(\frac{A^T c_t}{m_t^2} + \frac{B^T}{c_t} \right) \tag{A.2.12}$$

$$\lambda_t = \beta E_t \left\{ \left(\frac{I_t}{\Pi_{t+1}} \right) \left[\lambda_{t+1} + U_a(c_{t+1}, a_{t+1}, h_{t+1}) \right] \right\} \tag{A.2.13}$$

$$\lambda_t = q_t \tag{A.2.14}$$

$$q_t = \beta E_t \left\{ U_a(c_{t+1}, a_{t+1}, h_{t+1}) + \lambda_{t+1} r_{t+1} + q_{t+1}(1-\delta) \right\} \tag{A.2.15}$$

$$\lambda_t c_t = \varphi_t \tag{A.2.16}$$

These 16 equations form a nonlinear system of difference equation that characterizes the dynamics of $\{c_t, h_t, m_t, a_t, x_t, k_t, v_t, y_t, w_t, r_t, \Pi_t, I_t, \lambda_t, q_t, \varphi_t, T_t\}$.

The system is subject to the following random shocks $\{\xi_{z_{t+1}}, \xi_{\theta_{t+1}}\}$ affecting both productivity and the money growth rate in the following manner:

$$z_{t+1} = \rho_z z_t + \xi_{z_{t+1}} \tag{A.2.17}$$

$$\log \theta_{t+1} = (1-\rho_\theta)\theta + \rho_\theta \log \theta_t + \xi_{\theta_{t+1}}. \tag{A.2.18}$$

The solution of the system of equations is a time-invariant function of $\{z_t, \theta_t, k_{t-1}, m_{t-1}\}$.

A.3 Steady-State Analysis

In this section the steady-state values of the economic variables are derived from the competitive equilibrium conditions from Section A.2. The dynamic properties of the model are removed by assuming that all variables remain constant for all time. The steady-state properties give the long-run properties of the model and are used as a basis from which deviations occur in the stochastic model. The simplified steady-state properties derived from the competitive equilibrium conditions are given by the following equations:

$$(1+T)c + x = y \tag{A.3.1}$$

$$x = \delta k \tag{A.3.2}$$

$$T = \zeta\left(A^T v + \frac{B^T}{v} - 2\sqrt{A^T B^T}\right) \tag{A.3.3}$$

$$v = \frac{c}{m} \tag{A.3.4}$$

$$\frac{y}{h}=\left(\frac{k}{h}\right)^{\alpha} \tag{A.3.5}$$

$$a=k+m \tag{A.3.6}$$

$$r=\alpha\left(\frac{k}{h}\right)^{\alpha-1} \tag{A.3.7}$$

$$w=(1-\alpha)\left(\frac{k}{h}\right)^{\alpha} \tag{A.3.8}$$

$$\Theta=\Pi \tag{A.3.9}$$

$$U_c(c,a,h)=(1+T)\lambda+\lambda\zeta\left(A^T v-\frac{\boldsymbol{B}^T}{v}\right) \tag{A.3.10}$$

$$U_h(c,a,h)=-\lambda w \tag{A.3.11}$$

$$\lambda=\beta(U_a(c,a,h)+\lambda)\left(\frac{1}{\Pi}\right)-\lambda\zeta\left(A^T v^2+B^T\right) \tag{A.3.12}$$

$$\lambda=\beta\left(\frac{I}{\Pi}\right)[\lambda+U_a(c,a,h)] \tag{A.3.13}$$

$$\lambda=\beta(U_a(c,a,h)+\lambda r+\lambda(1-\delta)) \tag{A.3.14}$$

Substitution of (A.2.14) and (A.2.16) simplifies (A.3.10), (A.3.12), and (A.3.14) and reduces the number of steady-state equations.

Welfare Costs of Inflation

The welfare costs of inflation are measured as the percent change in goods that would equate the steady-state utility in the "cashless economy" with the utility in the "cash economy." The welfare costs are calculated using the following formula:

$$\overline{U} = U[G(1+\Delta G),h]$$

where

$$G = \gamma c^{1-\eta} + (1-\gamma)a^{1-\eta}$$

$$U[G(1+\Delta G),h] = \frac{1}{1-\sigma}\left[\left(\gamma c^{1-\eta} + (1-\gamma)a^{1-\eta}\right)(1+\Delta G)\right]^{\frac{1-\sigma}{1-\eta}} - Bh$$

$$\overline{U} = \frac{1}{1-\sigma}\left[\gamma c^{1-\eta} + (1-\gamma)a^{1-\eta}\right]^{\frac{1-\sigma}{1-\eta}}(1+\Delta G)^{\frac{1-\sigma}{1-\eta}} - Bh$$

$$(\overline{U} + Bh)(1-\sigma) = \left[\gamma c^{1-\eta} + (1-\gamma)a^{1-\eta}\right]^{\frac{1-\sigma}{1-\eta}}(1+\Delta G)^{\frac{1-\sigma}{1-\eta}}$$

$$(1+\Delta G)^{\frac{1-\sigma}{1-\eta}} = \frac{(\overline{U} + Bh)(1-\sigma)}{\left[\gamma c^{1-\eta} + (1-\gamma)a^{1-\eta}\right]^{\frac{1-\sigma}{1-\eta}}}$$

$$(1+\Delta G) = \frac{[(\overline{U} + Bh)(1-\sigma)]^{\frac{1-\eta}{1-\sigma}}}{\left[\gamma c^{1-\eta} + (1-\gamma)a^{1-\eta}\right]}$$

$$\Delta G = \frac{[(\overline{U} + Bh)(1-\sigma)]^{\frac{1-\eta}{1-\sigma}}}{\left[\gamma c^{1-\eta} + (1-\gamma)a^{1-\eta}\right]} - 1$$

In the separable case, $\sigma = \eta = 1$. To allow for the solution to converge in the limit, the value of σ used was 0.9999, and the value of η used was 1.0001.

A.4 Second-Order Logarithmic Approximation

The second-order logarithmic approximations of the competitive equilibrium conditions have been determined in order to use Sutherland's (2002) quadratic solving technique. The list of the second-order log-approximations is composed of the following[13]:

$$c(1+T)\hat{c}_t + cT\hat{T}_t + x\hat{x}_t - y\hat{y}_t + \frac{1}{2}\left(c(1+T)\hat{c}_t^{\,2} + 2cT\hat{c}_t\hat{T}_t + cT\hat{T}_t^2 + x\hat{x}_t^{\,2} - y\hat{y}_t^{\,2}\right) = 0 \tag{A.4.1}$$

$$k\hat{k}_t - x\hat{x}_t - (1-\delta)k\hat{k}_{t-1} + \frac{1}{2}\left(k\hat{k}_t^{\,2} - x\hat{x}_t^{\,2} - (1-\delta)k\hat{k}_{t-1}^{\,2}\right) = 0 \tag{A.4.2}$$

$$T\hat{T}_t - \zeta\left(A^T v\hat{v}_t - \frac{B^T}{v}\hat{v}_t\right) + \frac{1}{2}\left(T\hat{T}_t^2 - \zeta\left(A^T v\hat{v}_t^{\,2} + \frac{B^T}{v}\hat{v}_t^{\,2}\right)\right) = 0 \tag{A.4.3}$$

$$v\hat{v}_t - \frac{c}{m}\left(\hat{c}_t - \hat{m}_t\right) + \frac{1}{2}\left(v\hat{v}_t^{\,2} - \frac{c}{m}\left(\hat{c}_t^{\,2} + \hat{m}_t^{\,2} - 2\hat{c}_t\hat{m}_t\right)\right) = 0 \tag{A.4.4}$$

$$\hat{y}_t - z_t - \alpha\hat{k}_{t-1} - (1-\alpha)\hat{h}_t + \frac{1}{2}\left(\begin{array}{l}\hat{y}_t^{\,2} - z_t^{\,2} - \alpha^2\hat{k}_{t-1}^{\,2} - (1-\alpha)^2\hat{h}_t^{\,2} \\ -2\alpha z_t\hat{k}_{t-1} - 2(1-\alpha)z_t\hat{h}_t - 2\alpha(1-\alpha)\hat{k}_{t-1}\hat{h}_t\end{array}\right) = 0 \tag{A.4.5}$$

$$a\hat{a}_t - k\hat{k}_{t-1} - m\hat{m}_t + \frac{1}{2}\left(a\hat{a}_t^{\,2} - k\hat{k}_{t-1}^{\,2} - m\hat{m}_t^{\,2}\right) = 0 \tag{A.4.6}$$

$$\begin{aligned} &z_t + (\alpha-1)\hat{k}_{t-1} - (\alpha-1)\hat{h}_t - \hat{r}_t \\ &\quad + \frac{1}{2}\left(\begin{array}{l} z_t^{\,2} + (\alpha-1)^2\hat{k}_{t-1}^{\,2} + (\alpha-1)^2\hat{h}_t^{\,2} - \hat{r}_t^{\,2} \\ +2(\alpha-1)z_t\hat{k}_{t-1} - 2(\alpha-1)z_t\hat{h}_t - 2(\alpha-1)^2\hat{k}_{t-1}\hat{h}_t \end{array}\right) = 0 \end{aligned} \tag{A.4.7}$$

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[13] A detailed derivation of each of the second-order log-approximations of the competitive general equilibrium conditions can be found in the technical appendix.

$$z_t + \alpha\hat{k}_{t-1} - \alpha\hat{h}_t - \hat{w}_t + \frac{1}{2}\begin{pmatrix} z_t{}^2 + \alpha^2\hat{k}_{t-1}{}^2 + \alpha^2\hat{h}_t{}^2 - \hat{w}_t{}^2 \\ + 2\alpha z_t\hat{k}_{t-1} - 2\alpha z_t\hat{h}_t - 2\alpha^2\hat{k}_{t-1}\hat{h}_t \end{pmatrix} = 0 \tag{A.4.8}$$

$$\hat{\Theta}_t + \hat{m}_{t-1} - \hat{\Pi}_t - \hat{m}_t + \frac{1}{2}\begin{pmatrix} \hat{\Theta}_t{}^2 + \hat{m}_{t-1}{}^2 + \hat{\Pi}_t{}^2 - \hat{m}_t{}^2 \\ + 2\hat{\Theta}_t\hat{m}_{t-1} - 2\hat{\Theta}_t\hat{\Pi}_t - 2\hat{m}_{t-1}\hat{\Pi}_t \end{pmatrix} = 0 \tag{A.4.9}$$

$$\begin{gathered} \lambda\left(1 + T + \zeta\left(A^T v - \frac{\boldsymbol{B}^T}{v^2}\right)\right)\hat{\lambda}_t + \lambda T\hat{T}_t + \zeta\lambda\left(A^T v\hat{v}_t + 2\frac{\boldsymbol{B}^T}{v^2}\right)\hat{v}_t + \left(\frac{\gamma}{c}\right)\hat{c}_t \\ + \frac{1}{2}\begin{pmatrix} \lambda\left(1 + T + \zeta A^T v - \zeta\frac{\boldsymbol{B}^T}{v^2}\right)\hat{\lambda}_t{}^2 + \zeta\lambda\left(A^T v - 4\frac{\boldsymbol{B}^T}{v^2}\right)\hat{v}_t{}^2 + \lambda T\hat{T}_t{}^2 \\ + 2\lambda\zeta\left(A^T v + 2\frac{\boldsymbol{B}^T}{v^2}\right)\hat{\lambda}_t\hat{v}_t + 2\lambda T\hat{\lambda}_t\hat{T}_t - \left(\frac{\gamma}{c}\right)\hat{c}_t{}^2 \end{pmatrix} = 0 \end{gathered} \tag{A.4.10}$$

$$\hat{\lambda}_t + \hat{w}_t + \frac{1}{2}\left(\hat{\lambda}_t{}^2 + \hat{w}_t{}^2 + 2\hat{\lambda}_t\hat{w}_t\right) = 0 \tag{A.4.11}$$

$$-\beta E_t\left\{\begin{matrix} \begin{pmatrix} (1-\gamma)\left(\frac{1}{a\Pi}\right)\hat{a}_{t+1} + \frac{\lambda}{\Pi}\hat{\lambda}_{t+1} \\ + \left((1-\gamma)\left(\frac{1}{a\Pi}\right) + \frac{\lambda}{\Pi}\right)\hat{\Pi}_{t+1} \end{pmatrix} \\ -\frac{1}{2}\begin{pmatrix} (1-\gamma)\left(\frac{1}{a\Pi}\right)\left(\hat{a}_{t+1}{}^2 + \hat{\Pi}_{t+1}{}^2 + 2\hat{a}_{t+1}\hat{\Pi}_{t+1}\right) \\ + \frac{\lambda}{\Pi}\left(\hat{\lambda}_{t+1}{}^2 + \hat{\Pi}_{t+1}{}^2 - 2\hat{\lambda}_{t+1}\hat{\Pi}_{t+1}\right) \end{pmatrix} \end{matrix}\right\} = \begin{bmatrix} 2\zeta A^T\lambda v^2\hat{v}_t \\ + \left(\zeta B^T + \zeta A^T v^2 + 1\right)\lambda\hat{\lambda}_t \\ + \frac{1}{2}\begin{pmatrix} 4\zeta A^T\lambda v^2\hat{v}_t{}^2 \\ + 4\zeta A^T\lambda v^2\hat{\lambda}_t\hat{v}_t \\ + \left(\zeta B^T + \zeta A^T v^2 + 1\right)\lambda\hat{\lambda}_t{}^2 \end{pmatrix} \end{bmatrix} \tag{A.4.12}$$

$$\begin{bmatrix} -\frac{\beta I}{\Pi}\left(\lambda+\frac{(1-\gamma)}{a}\right)\hat{I}_t+\lambda\hat{\lambda}_t \\ +\frac{1}{2}\begin{pmatrix} \lambda\hat{\lambda}_t^{\,2} \\ -\beta\frac{I}{\Pi}\left(\lambda+(1-\gamma)\left(\frac{1}{a}\right)\right)\hat{I}_t^{\,2} \end{pmatrix} \end{bmatrix}=\frac{\beta I}{\Pi}E_t\left\{\begin{matrix} \lambda\hat{\lambda}_{t+1}-(1-\gamma)\left(\frac{1}{a}\right)\hat{a}_{t+1}-\left(\lambda+\frac{(1-\gamma)}{a}\right)\left(\hat{\Pi}_{t+1}\right) \\ +\frac{1}{2}\lambda\begin{pmatrix} \hat{\lambda}_{t+1}^{\;\;2}+\hat{\Pi}_{t+1}^{\;\;2}+2\hat{I}_t\hat{\lambda}_{t+1} \\ -2\hat{I}_t\hat{\Pi}_{t+1}-2\hat{\lambda}_{t+1}\hat{\Pi}_{t+1} \end{pmatrix} \\ +\frac{1}{2}(1-\gamma)\left(\frac{1}{a}\right)\begin{pmatrix} \hat{\Pi}_{t+1}^{\;\;2}+\hat{a}_{t+1}^{\;\;2}-2\hat{I}_t\hat{\Pi}_{t+1} \\ -2\hat{I}_t\hat{a}_{t+1}+2\hat{\Pi}_{t+1}\hat{a}_{t+1} \end{pmatrix} \end{matrix}\right\} \qquad \text{(A.4.13)}$$

$$2v^2\hat{v}_t-\frac{1}{A^{\mathrm{T}}}\frac{1}{I_t}\hat{I}_t+\frac{1}{2}\left(4v^2\hat{v}_t^{\,2}+\frac{1}{A^{\mathrm{T}}}\frac{1}{I_t}\hat{I}_t^{\,2}\right)=0 \qquad \text{(A.4.13)'}$$

$$\lambda\hat{\lambda}_t+\frac{1}{2}\lambda\hat{\lambda}_t^{\,2}=\beta E_t\left\{\begin{matrix} -(1-\gamma)\left(\frac{1}{a}\right)\hat{a}_{t+1}+\lambda r\hat{\lambda}_{t+1}+\lambda r\hat{r}_{t+1}+(1-\delta)\lambda\hat{\lambda}_{t+1} \\ +\frac{1}{2}\begin{pmatrix} (1-\gamma)\left(\frac{1}{a}\right)\hat{a}_{t+1}^{\;\;2}+(1-\delta)\lambda\hat{\lambda}_{t+1}^{\;\;2} \\ +\lambda r\left(\hat{\lambda}_{t+1}^{\;\;2}+\hat{r}_{t+1}^{\;\;2}+2\hat{\lambda}_{t+1}\hat{r}_{t+1}\right)^2 \end{pmatrix} \end{matrix}\right\} \qquad \text{(A.4.14)}$$

$$E_t\{z_{t+1}\}=\rho_z z_t+E_t\{\xi_{z_{t+1}}\} \qquad \text{(A.4.17)}$$

$$\hat{\theta}_{t+1}=\rho_\theta\hat{\theta}_t+\xi_{\theta_{t+1}}. \qquad \text{(A.4.18)}$$

$$A_t=\left[\hat{z}_t,\hat{\theta}_t,\hat{k}_{t-1},\hat{m}_{t-1},\hat{I}_{t-1},\hat{c}_t,\hat{h}_t,\hat{m}_t,\hat{a}_t,\hat{x}_t,\hat{k}_t,\hat{v}_t,\hat{y}_t,\hat{w}_t,\hat{r}_t,\hat{\Pi}_t,\hat{I}_t,\hat{T}_t\right]$$
$$A_{t+1}=\left[\hat{z}_{t+1},\hat{\theta}_{t+1},\hat{k}_t,\hat{m}_t,\hat{I}_t,\hat{c}_{t+1},\hat{h}_{t+1},\hat{m}_{t+1},\hat{a}_{t+1},\hat{x}_{t+1},\hat{k}_{t+1},\hat{v}_{t+1},\hat{y}_{t+1},\hat{w}_{t+1},\hat{r}_{t+1},\hat{\Pi}_{t+1},\hat{I}_{t+1},\hat{T}_{t+1}\right]$$

For the non-separable case, (A.4.10), (A.4.12), and (A.4.15) differ due to differences in $U_c(c_t, a_t, h_t)$ and $U_a(c_t, a_t, h_t)$, and an additional equation for the variable q_t is added to simplify these new conditions.

$$\lambda\left(1+T+\zeta\left(A^{\mathrm{T}}v-\frac{\mathbf{B}^{\mathrm{T}}}{v^2}\right)\right)\hat{\lambda}_t+\lambda T\hat{T}_t+\zeta\lambda\left(A^{\mathrm{T}}v+2\frac{\mathbf{B}^{\mathrm{T}}}{v^2}\right)\hat{v}_t+\gamma\left(\frac{q^{\frac{\eta-\sigma}{1-\eta}}}{c^{\eta}}\right)\left(\eta\hat{c}_t-\frac{\eta-\sigma}{1-\eta}\hat{q}_t\right)$$

$$+\frac{1}{2}\left(\begin{array}{l}\lambda\left(1+T+\zeta A^{\mathrm{T}}v-\zeta\frac{\mathbf{B}^{\mathrm{T}}}{v^2}\right)\hat{\lambda}_t^{\,2}+\zeta\lambda\left(A^{\mathrm{T}}v-4\frac{\mathbf{B}^{\mathrm{T}}}{v^2}\right)\hat{v}_t^{\,2}+\lambda T\hat{T}_t^2+2\lambda T\hat{\lambda}_t\hat{T}_t\\-\gamma\left(\frac{q^{\frac{\eta-\sigma}{1-\eta}}}{c^{\eta}}\right)\left(\eta^2\hat{c}_t^{\,2}+\left(\frac{\eta-\sigma}{1-\eta}\right)^2\hat{q}_t^{\,2}-2\eta\left(\frac{\eta-\sigma}{1-\eta}\right)\hat{c}_t\hat{q}_t\right)+2\lambda\zeta\left(A^{\mathrm{T}}v+2\frac{\mathbf{B}^{\mathrm{T}}}{v^2}\right)\hat{\lambda}_t\hat{v}_t\end{array}\right)=0$$

(A.4.10)′

$$\frac{\beta}{\Pi}E_t\left\{\begin{array}{l}(1-\gamma)a^{-\eta}q^{\frac{\eta-\sigma}{1-\eta}}\left(-\eta\hat{a}_{t+1}+\frac{\eta-\sigma}{1-\eta}\hat{q}_{t+1}-\hat{\Pi}_{t+1}\right)+\lambda\left(\hat{\lambda}_{t+1}-\hat{\Pi}_{t+1}\right)\\+\frac{1}{2}\left(\begin{array}{l}(1-\gamma)a^{-\eta}q^{\frac{\eta-\sigma}{1-\eta}}\left(\begin{array}{l}\eta^2\hat{a}_{t+1}^{\,2}+\left(\frac{\eta-\sigma}{1-\eta}\right)^2\hat{q}_{t+1}^{\,2}+\hat{\Pi}_{t+1}^{\,2}\\-2\eta\left(\frac{\eta-\sigma}{1-\eta}\right)\hat{a}_{t+1}\hat{q}_{t+1}+2\eta\hat{a}_{t+1}\hat{\Pi}_{t+1}-2\left(\frac{\eta-\sigma}{1-\eta}\right)\hat{q}_{t+1}\hat{\Pi}_{t+1}\end{array}\right)\\+\lambda\left(\hat{\lambda}_{t+1}^{\,2}+\hat{\Pi}_{t+1}^{\,2}-2\hat{\lambda}_{t+1}\hat{\Pi}_{t+1}\right)\end{array}\right)\end{array}\right\}$$

$$=\left[\begin{array}{l}2\zeta A^{\mathrm{T}}\lambda v^2\hat{v}_t\\+\left(\zeta B^{\mathrm{T}}+\zeta A^{\mathrm{T}}v^2+1\right)\lambda\hat{\lambda}_t\\+\frac{1}{2}\left(\begin{array}{l}4\zeta A^{\mathrm{T}}\lambda v^2\hat{v}_t^{\,2}+4\zeta A^{\mathrm{T}}\lambda v^2\hat{\lambda}_t\hat{v}_t\\+\left(\zeta B^{\mathrm{T}}+\zeta A^{\mathrm{T}}v^2+1\right)\lambda\hat{\lambda}_t^{\,2}\end{array}\right)\end{array}\right]$$

(A.4.12)′

$$\lambda\hat{\lambda}_t + \frac{1}{2}\lambda\hat{\lambda}_t^{\,2} = \beta E_t \left\{ \begin{array}{l} (1-\gamma)a^{-\eta} q^{\frac{\eta-\sigma}{1-\eta}} \left(-\eta \hat{a}_{t+1} + \frac{\eta-\sigma}{1-\eta}\hat{q}_{t+1} \right) + \lambda r\left(\hat{\lambda}_{t+1} + \hat{r}_{t+1}\right) + (1-\delta)\lambda\hat{\lambda}_{t+1} \\ + \frac{1}{2}\left(\begin{array}{l} (1-\gamma)a^{-\eta} q^{\frac{\eta-\sigma}{1-\eta}} \left(\eta^2 \hat{a}_{t+1}^{\,2} + \left(\frac{\eta-\sigma}{1-\eta}\right)^2 \hat{q}_{t+1}^{\,2} - 2\eta\left(\frac{\eta-\sigma}{1-\eta}\right)\hat{a}_{t+1}\hat{q}_{t+1} \right) \\ + \lambda r\left(\hat{\lambda}_{t+1}^{\,2} + \hat{r}_{t+1}^{\,2} + 2\hat{\lambda}_{t+1}\hat{r}_{t+1}\right)^2 + (1-\delta)\lambda\hat{\lambda}_{t+1}^{\,2} \end{array} \right) \end{array} \right\}$$

(A.4.15)′

$$0 = \gamma c^{1-\eta}(1-\eta)\hat{c}_t + (1-\gamma)a^{1-\eta}(1-\eta)\hat{a}_t - q\hat{q}_t + \frac{1}{2}\left(\gamma c^{1-\eta}(1-\eta)^2\hat{c}_t^{\,2} + (1-\gamma)a^{1-\eta}(1-\eta)^2\hat{a}_t^{\,2} - q\hat{q}_t^{\,2}\right)$$

(A.4.19)′

$$A_t = \left[\hat{z}_t, \hat{\theta}_t, \hat{k}_{t-1}, \hat{m}_{t-1}, \hat{I}_{t-1}, \hat{c}_t, \hat{h}_t, \hat{m}_t, \hat{a}_t, \hat{x}_t, \hat{k}_t, \hat{v}_t, \hat{y}_t, \hat{w}_t, \hat{r}_t, \hat{\Pi}_t, \hat{I}_t, \hat{T}_t, \hat{q}_t\right]$$

$$A_{t+1} = \left[\hat{z}_{t+1}, \hat{\theta}_{t+1}, \hat{k}_t, \hat{m}_t, \hat{I}_t, \hat{c}_{t+1}, \hat{h}_{t+1}, \hat{m}_{t+1}, \hat{a}_{t+1}, \hat{x}_{t+1}, \hat{k}_{t+1}, \hat{v}_{t+1}, \hat{y}_{t+1}, \hat{w}_{t+1}, \hat{r}_{t+1}, \hat{\Pi}_{t+1}, \hat{I}_{t+1}, \hat{T}_{t+1}, \hat{q}_{t+1}\right]$$

A.5 Technical Appendix

Second-Order Log-Approximation Process for the Log Separable Utility Case

Many of the competitive equilibrium conditions are the same for the separable and the non-separable cases; however, the few conditions that contain either U_c or U_a will differ. Thus, after showing the detailed derivation of all of the second-order log-approximations for the log-separable case, the derivation of those few additional equations that differ will be given for the non-separable case.

Second-order log-approximation for equation (A.2.1):

$$(1+T_t)c_t + x_t = y_t$$

$$c_t + c_tT_t + x_t = y_t$$

$$ce^{\hat{c}_t} + cTe^{(\hat{c}_t+\hat{T}_t)} + xe^{\hat{x}_t} = ye^{\hat{y}_t}$$

$$\text{In the steady state, } c(1+T)+x = y$$

$$c\left(1+\hat{c}_t+\frac{1}{2}\hat{c}_t^2\right) + cT\left(1+\hat{c}_t+\hat{T}_t+\frac{1}{2}\left(\hat{c}_t+\hat{T}_t\right)^2\right) + x\left(1+\hat{x}_t+\frac{1}{2}\hat{x}_t^2\right) = y\left(1+\hat{y}_t+\frac{1}{2}\hat{y}_t^2\right)$$

$$c\left(1+\hat{c}_t+\frac{1}{2}\hat{c}_t^2\right) + cT\left(1+\hat{c}_t+\hat{T}_t+\frac{1}{2}\left(\hat{c}_t^2+2\hat{c}_t\hat{T}_t+\hat{T}_t^2\right)\right) + x\left(1+\hat{x}_t+\frac{1}{2}\hat{x}_t^2\right) = y\left(1+\hat{y}_t+\frac{1}{2}\hat{y}_t^2\right)$$

$$c(1+T)\hat{c}_t + cT\hat{T}_t + x\hat{x}_t - y\hat{y}_t + \frac{1}{2}\left(c(1+T)\hat{c}_t^2 + 2cT\hat{c}_t\hat{T}_t + cT\hat{T}_t^2 + x\hat{x}_t^2 - y\hat{y}_t^2\right) = 0$$

Second-order log-approximation for equation (A.2.2):

$$k_t = x_t + (1-\delta)k_{t-1}$$

$$ke^{\hat{k}_t} = xe^{\hat{x}_t} + (1-\delta)ke^{\hat{k}_{t-1}}$$

$$k\left(1+\hat{k}_t+\frac{1}{2}\hat{k}_t^{\,2}\right) = x\left(1+\hat{x}_t+\frac{1}{2}\hat{x}_t^{\,2}\right)+(1-\delta)k\left(1+\hat{k}_{t-1}+\frac{1}{2}\hat{k}_{t-1}^{\,2}\right)$$

In the steady state, $k = x + (1-\delta)k$.

$$k\left(\hat{k}_t+\frac{1}{2}\hat{k}_t^{\,2}\right) - x\left(\hat{x}_t+\frac{1}{2}\hat{x}_t^{\,2}\right)-(1-\delta)k\left(\hat{k}_{t-1}+\frac{1}{2}\hat{k}_{t-1}^{\,2}\right)=0$$

$$k\hat{k}_t - x\hat{x}_t-(1-\delta)k\hat{k}_{t-1}+\frac{1}{2}\left(k\hat{k}_t^{\,2}-x\hat{x}_t^{\,2}-(1-\delta)k\hat{k}_{t-1}^{\,2}\right)=0$$

Second-order log-approximation for equation (A.2.3):

$$T_t = \zeta\left(A^T v_t + \frac{B^T}{v_t} - 2\sqrt{A^T B^T}\right)$$

$$\frac{T_t}{\zeta} = A^T v_t + \frac{B^T}{v_t} - 2\sqrt{A^T B^T}$$

$$\frac{T}{\zeta}e^{(\hat{T}_t)} = A^T v e^{\hat{v}_t} + \frac{B^T}{v}e^{-\hat{v}_t} - 2\sqrt{A^T B^T}e^0$$

$$\frac{T}{\zeta}\left(1+\hat{T}_t+\frac{1}{2}\hat{T}_t^{\,2}\right) = A^T v\left(1+\hat{v}_t+\frac{1}{2}\hat{v}_t^{\,2}\right)+\frac{B^T}{v}\left(1-\hat{v}_t+\frac{1}{2}\hat{v}_t^{\,2}\right)-2\sqrt{A^T B^T}$$

$$\frac{T}{\zeta}\left(1+\hat{T}_t+\frac{1}{2}\hat{T}_t^{\,2}\right) = A^T v\left(1+\hat{v}_t+\frac{1}{2}\hat{v}_t^{\,2}\right)+\frac{B^T}{v}\left(1-\hat{v}_t+\frac{1}{2}\hat{v}_t^{\,2}\right)-2\sqrt{A^T B^T}$$

In the steady state, $\frac{T}{\zeta} = A^T v + \frac{B^T}{v} - 2\sqrt{A^T B^T}$

$$\frac{T}{\zeta}\hat{T}_t - A^T v\hat{v}_t + \frac{B^T}{v}\hat{v}_t + \frac{1}{2}\left(\frac{T}{\zeta}\hat{T}_t^{\,2} - A^T v\hat{v}_t^{\,2} - \frac{B^T}{v}\hat{v}_t^{\,2}\right)=0$$

$$T\hat{T}_t - \zeta\left(A^T v\hat{v}_t - \frac{B^T}{v}\hat{v}_t\right)+\frac{1}{2}\left(T\hat{T}_t^{\,2} - \zeta\left(A^T v\hat{v}_t^{\,2}+\frac{B^T}{v}\hat{v}_t^{\,2}\right)\right)=0$$

Second-order log-approximation for equation (A.2.4):

$$v_t = \frac{c_t}{m_t}$$

$$ve^{\hat{v}_t} = \frac{c}{m}e^{\hat{c}_t - \hat{m}_t}$$

$$v\left(1 + \hat{v}_t + \frac{1}{2}\hat{v}_t^2\right) = \frac{c}{m}\left(1 + \hat{c}_t - \hat{m}_t + \frac{1}{2}(\hat{c}_t - \hat{m}_t)^2\right)$$

$$\text{In the steady state, } v = \frac{c}{m}$$

$$v\hat{v}_t + \frac{1}{2}v\hat{v}_t^2 = \frac{c}{m}(\hat{c}_t - \hat{m}_t) + \frac{1}{2}\frac{c}{m}(\hat{c}_t - \hat{m}_t)^2$$

$$v\hat{v}_t - \frac{c}{m}(\hat{c}_t - \hat{m}_t) + \frac{1}{2}\left(v\hat{v}_t^2 - \frac{c}{m}(\hat{c}_t - \hat{m}_t)^2\right) = 0$$

$$v\hat{v}_t - \frac{c}{m}(\hat{c}_t - \hat{m}_t) + \frac{1}{2}\left(v\hat{v}_t^2 - \frac{c}{m}\left(\hat{c}_t^2 + \hat{m}_t^2 - 2\hat{c}_t\hat{m}_t\right)\right) = 0$$

Second-order log-approximation for equation (A.2.5):

$$y_t = e^{z_t}k_{t-1}^{\alpha}h_t^{1-\alpha}$$

$$ye^{\hat{y}_t} = k^{\alpha}h^{1-\alpha}e^{\left(z_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{h}_t\right)}$$

$$\text{In the steady state, } y = k^{\alpha}h^{1-\alpha}$$

$$e^{\hat{y}_t} = e^{\left(z_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{h}_t\right)}$$

$$1 + \hat{y}_t + \frac{1}{2}\hat{y}_t^2 - \left(1 + z_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{h}_t + \frac{1}{2}\left(z_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{h}_t\right)^2\right) = 0$$

$$\hat{y}_t - z_t - \alpha\hat{k}_{t-1} - (1-\alpha)\hat{h}_t + \frac{1}{2}\left(\hat{y}_t^2 - \left(z_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{h}_t\right)^2\right) = 0$$

$$\hat{y}_t - z_t - \alpha\hat{k}_{t-1} - (1-\alpha)\hat{h}_t + \frac{1}{2}\begin{pmatrix}\hat{y}_t^2 - z_t^2 - \alpha^2\hat{k}_{t-1}^2 - (1-\alpha)^2\hat{h}_t^2 \\ -2\alpha z_t\hat{k}_{t-1} - 2(1-\alpha)z_t\hat{h}_t - 2\alpha(1-\alpha)\hat{k}_{t-1}\hat{h}_t\end{pmatrix} = 0$$

or

$$z_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{h}_t - \hat{y}_t +$$

$$\frac{1}{2}\left[z_t^2 + \alpha^2\hat{k}_{t-1}^2 + (1-\alpha)^2\hat{h}_t^2 + 2\alpha z_t\hat{k}_{t-1} + 2(1-\alpha)z_t\hat{h}_t + 2\alpha(1-\alpha)\hat{k}_{t-1}\hat{h}_t - \hat{y}_t^2\right] = 0$$

Second-order log-approximation for equation (A.2.6):

$$a_t = k_{t-1} + m_t$$

$$ae^{\hat{a}_t} = ke^{\hat{k}_{t-1}} + me^{\hat{m}_t}$$

$$a\left(1 + \hat{a}_t + \frac{1}{2}\hat{a}_t^{\,2}\right) = k\left(1 + \hat{k}_{t-1} + \frac{1}{2}\hat{k}_{t-1}^{\,2}\right) + m\left(1 + \hat{m}_t + \frac{1}{2}\hat{m}_t^{\,2}\right)$$

In the steady state, $a = k + m$.

$$a\hat{a}_t - k\hat{k}_{t-1} - m\hat{m}_t + \frac{1}{2}\left(a\hat{a}_t^{\,2} - k\hat{k}_{t-1}^{\,2} - m\hat{m}_t^{\,2}\right) = 0$$

Second-order log-approximation for equation (A.2.7):

$$r_t = \alpha e^{z_t} k_{t-1}^{\alpha-1} h_t^{1-\alpha}$$

$$re^{\hat{r}_t} = \alpha k^{\alpha-1} h^{1-\alpha} e^{z_t + (\alpha-1)\hat{k}_{t-1} - (\alpha-1)\hat{h}_t}$$

In the steady state, $r = \alpha k^{\alpha-1} h^{1-\alpha}$

$$e^{\hat{r}_t} = e^{z_t + (\alpha-1)\hat{k}_{t-1} - (\alpha-1)\hat{h}_t}$$

$$1 + \hat{r}_t + \frac{1}{2}\hat{r}_t^{2} = 1 + z_t + (\alpha-1)\hat{k}_{t-1} - (\alpha-1)\hat{h}_t + \frac{1}{2}\left(z_t + (\alpha-1)\hat{k}_{t-1} - (\alpha-1)\hat{h}_t\right)^2$$

$$z_t + (\alpha-1)\hat{k}_{t-1} - (\alpha-1)\hat{h}_t + \frac{1}{2}\left(z_t + (\alpha-1)\hat{k}_{t-1} - (\alpha-1)\hat{h}_t\right)^2 - \hat{r}_t - \frac{1}{2}\hat{r}_t^{2} = 0$$

$$z_t + (\alpha-1)\hat{k}_{t-1} - (\alpha-1)\hat{h}_t - \hat{r}_t + \frac{1}{2}\begin{pmatrix} z_t^{2} + (\alpha-1)^2\hat{k}_{t-1}^{\,2} + (\alpha-1)^2\hat{h}_t^{\,2} - \hat{r}_t^{2} \\ + 2(\alpha-1)z_t\hat{k}_{t-1} - 2(\alpha-1)z_t\hat{h}_t - 2(\alpha-1)^2\hat{k}_{t-1}\hat{h}_t \end{pmatrix} = 0$$

Second-order log-approximation for equation (A.2.8):

$$w_t = (1-\alpha)e^{z_t}\left(\frac{k_{t-1}}{h_t}\right)^{\alpha}$$

$$we^{\hat{w}_t} = (1-\alpha)\left(\frac{k}{h}\right)^{\alpha} e^{\left(z_t+\alpha\hat{k}_{t-1}-\alpha\hat{h}_t\right)}$$

In the steady state, $w = (1-\alpha)\left(\frac{k}{h}\right)^{\alpha}$

$$e^{\hat{w}_t} = e^{\left(z_t+\alpha\hat{k}_{t-1}-\alpha\hat{h}_t\right)}$$

$$1+\hat{w}_t+\frac{1}{2}\hat{w}_t^{\,2} = 1+z_t+\alpha\hat{k}_{t-1}-\alpha\hat{h}_t+\frac{1}{2}\left(z_t+\alpha\hat{k}_{t-1}-\alpha\hat{h}_t\right)^2$$

$$z_t+\alpha\hat{k}_{t-1}-\alpha\hat{h}_t-\hat{w}_t+\frac{1}{2}\begin{bmatrix} z_t^{\,2}+\alpha^2\hat{k}_{t-1}^{\,2}+\alpha^2\hat{h}_t^{\,2}-\hat{w}_t^{\,2} \\ +2\alpha z_t\hat{k}_{t-1}-2\alpha z_t\hat{h}_t-2\alpha^2\hat{k}_{t-1}\hat{h}_t \end{bmatrix}=0$$

Second-order log-approximation for equation (A.2.9):

$$m_t = \frac{\Theta_t}{\Pi_t}m_{t-1}$$

$$me^{\hat{m}_t} = \frac{\Theta}{\Pi}me^{\Theta_t+m_{t-1}-\Pi_t}$$

In the steady state, $\Theta = \Pi$

$$e^{\hat{m}_t} = e^{\hat{\Theta}_t+\hat{m}_{t-1}-\hat{\Pi}_t}$$

$$1+\hat{m}_t+\frac{1}{2}\hat{m}_t^{\,2} = 1+\hat{\Theta}_t+\hat{m}_{t-1}-\hat{\Pi}_t+\frac{1}{2}\left(\hat{\Theta}_t+\hat{m}_{t-1}-\hat{\Pi}_t\right)^2$$

$$\hat{\Theta}_t+\hat{m}_{t-1}-\hat{\Pi}_t-\hat{m}_t+\frac{1}{2}\left(\hat{\Theta}_t^{\,2}+\hat{m}_{t-1}^{\,2}+\hat{\Pi}_t^{\,2}-\hat{m}_t^{\,2}+2\hat{\Theta}_t\hat{m}_{t-1}-2\hat{\Theta}_t\hat{\Pi}_t-2\hat{m}_{t-1}\hat{\Pi}_t\right)=0$$

Second-order log-approximation for equation (A.2.10):

$$U_c(c_t,a_t,h_t)=(1+T_t)\lambda_t+\lambda_t\zeta\left(A^T v_t-\frac{\boldsymbol{B}^T}{v_t^{\,2}}\right)$$

when $\sigma=\phi=1$, $U_c(c_t,a_t,h_t)=\gamma\left(\frac{1}{c_t}\right)$

$$\gamma\left(\frac{1}{c_t}\right)=\lambda_t+\lambda_t T_t+\zeta A^T\lambda_t v_t-\zeta\boldsymbol{B}^T\frac{\lambda_t}{v_t^{\,2}}$$

$$\gamma\left(\frac{1}{c}\right)e^{-\hat{c}_t}=\lambda e^{\hat{\lambda}_t}+\lambda T e^{(\hat{\lambda}_t+\hat{T}_t)}+\zeta A^T\lambda v e^{(\hat{\lambda}_t+\hat{v}_t)}-\zeta\boldsymbol{B}^T\frac{\lambda}{v^2}e^{(\hat{\lambda}_t-2\hat{v}_t)}$$

$$\gamma\left(\frac{1}{c}\right)\left(1-\hat{c}_t+\frac{1}{2}\hat{c}_t^{\,2}\right)=\lambda\left(1+\hat{\lambda}_t+\frac{1}{2}\hat{\lambda}_t^{\,2}\right)+\lambda T\left(1+(\hat{\lambda}_t+\hat{T}_t)+\frac{1}{2}(\hat{\lambda}_t+\hat{T}_t)^2\right)$$
$$+\zeta A^T\lambda v\left(1+(\hat{\lambda}_t+\hat{v}_t)+\frac{1}{2}(\hat{\lambda}_t+\hat{v}_t)^2\right)-\zeta\boldsymbol{B}^T\frac{\lambda}{v^2}\left(1+(\hat{\lambda}_t-2\hat{v}_t)+\frac{1}{2}(\hat{\lambda}_t-2\hat{v}_t)^2\right)$$

In the steady state, $\gamma\left(\frac{1}{c}\right)=(1+T)\lambda+\lambda\zeta\left(A^T v-\frac{\boldsymbol{B}^T}{v^2}\right)$

$$\gamma\left(\frac{1}{c}\right)\left(-\hat{c}_t+\frac{1}{2}\hat{c}_t^{\,2}\right)=\lambda\left(\hat{\lambda}_t+\frac{1}{2}\hat{\lambda}_t^{\,2}\right)+\lambda T\left(\hat{\lambda}_t+\hat{T}_t+\frac{1}{2}(\hat{\lambda}_t+\hat{T}_t)^2\right)$$
$$+\zeta A^T\lambda v\left(\hat{\lambda}_t+\hat{v}_t+\frac{1}{2}(\hat{\lambda}_t+\hat{v}_t)^2\right)-\zeta\boldsymbol{B}^T\frac{\lambda}{v^2}\left(\hat{\lambda}_t-2\hat{v}_t+\frac{1}{2}(\hat{\lambda}_t-2\hat{v}_t)^2\right)$$

$$\lambda\left(1+T+\zeta\left(A^T v-\frac{\boldsymbol{B}^T}{v^2}\right)\right)\hat{\lambda}_t+\lambda T\hat{T}_t+\zeta\lambda\left(A^T v+2\frac{\boldsymbol{B}^T}{v^2}\right)\hat{v}_t+\gamma\left(\frac{1}{c}\right)\hat{c}_t$$
$$+\frac{1}{2}\left(\lambda\hat{\lambda}_t^{\,2}+\lambda T(\hat{\lambda}_t+\hat{T}_t)^2+\zeta A^T\lambda v(\hat{\lambda}_t+\hat{v}_t)^2-\zeta\boldsymbol{B}^T\frac{\lambda}{v^2}(\hat{\lambda}_t-2\hat{v}_t)^2-\gamma\left(\frac{1}{c}\right)\hat{c}_t^{\,2}\right)=0$$

$$\lambda\left(1+T+\zeta\left(A^T v-\frac{\boldsymbol{B}^T}{v^2}\right)\right)\hat{\lambda}_t+\lambda T\hat{T}_t+\zeta\lambda\left(A^T v+2\frac{\boldsymbol{B}^T}{v^2}\right)\hat{v}_t+\left(\frac{\gamma}{c}\right)\hat{c}_t$$
$$+\frac{1}{2}\left(\begin{array}{l}\lambda\left(1+T+\zeta A^T v-\zeta\frac{\boldsymbol{B}^T}{v^2}\right)\hat{\lambda}_t^{\,2}+\zeta\lambda\left(A^T v-4\frac{\boldsymbol{B}^T}{v^2}\right)\hat{v}_t^{\,2}+\lambda T\hat{T}_t^{\,2}-\left(\frac{\gamma}{c}\right)\hat{c}_t^{\,2}\\+2\lambda\zeta\left(A^T v+2\frac{\boldsymbol{B}^T}{v^2}\right)\hat{\lambda}_t\hat{v}_t+2\lambda T\hat{\lambda}_t\hat{T}_t\end{array}\right)=0$$

Second-order log-approximation for equation (A.2.11):

$U_h\left(c_t, a_t, h_t\right) = -\lambda_t w_t$

where $U_h\left(c_t, a_t, h_t\right) = -B$

$\lambda_t w_t = B$

$\lambda w e^{\left(\hat{\lambda}_t + \hat{w}_t\right)} = B e^0$

In the steady state, $\lambda w = B$

$e^{\left(\hat{\lambda}_t + \hat{w}_t\right)} = 1$

$$1 + \hat{\lambda}_t + \hat{w}_t + \frac{1}{2}\left(\hat{\lambda}_t + \hat{w}_t\right)^2 = 1$$

$$\hat{\lambda}_t + \hat{w}_t + \frac{1}{2}\left(\hat{\lambda}_t{}^2 + \hat{w}_t{}^2 + 2\hat{\lambda}_t \hat{w}_t\right) = 0$$

Second-order log-approximation for equation (A.2.12):

$$\lambda_t = \beta E_t \left\{ \left(U_a(c_{t+1}, a_{t+1}, h_{t+1}) + \lambda_{t+1} \right) \left(\frac{1}{\Pi_{t+1}} \right) \right\} - \lambda_t \zeta \left(A^T v_t^2 + B^T \right)$$

when $\sigma = \phi = 1$, $U_a(c_{t+1}, a_{t+1}, h_{t+1}) = (1-\gamma)\left(\frac{1}{a_{t+1}} \right)$

$$\lambda_t = \beta E_t \left\{ \left((1-\gamma)\left(\frac{1}{a_{t+1}} \right) + \lambda_{t+1} \right) \left(\frac{1}{\Pi_{t+1}} \right) \right\} - \lambda_t \zeta \left(A^T v_t^2 + B^T \right)$$

$$E_t \left\{ \beta (1-\gamma) \left(\frac{1}{a_{t+1} \Pi_{t+1}} \right) + \beta \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} - \zeta A^T \lambda_t v_t^2 - \zeta B^T \lambda_t - \lambda_t = 0$$

$$E_t \left\{ \beta (1-\gamma) \left(\frac{1}{a\Pi} \right) e^{-\hat{a}_{t+1} - \hat{\Pi}_{t+1}} + \beta \frac{\lambda}{\Pi} e^{\hat{\lambda}_{t+1} - \hat{\Pi}_{t+1}} \right\} - \zeta A^T \lambda v^2 e^{\hat{\lambda}_t + 2\hat{v}_t} - \zeta B^T \lambda e^{\hat{\lambda}_t} - \lambda e^{\hat{\lambda}_t} = 0$$

$$E_t \left\{ \begin{array}{l} \beta (1-\gamma) \left(\frac{1}{a\Pi} \right) \left(1 - \hat{a}_{t+1} - \hat{\Pi}_{t+1} + \frac{1}{2} \left(-\hat{a}_{t+1} - \hat{\Pi}_{t+1} \right)^2 \right) \\ + \beta \frac{\lambda}{\Pi} \left(1 + \hat{\lambda}_{t+1} - \hat{\Pi}_{t+1} + \frac{1}{2} \left(\hat{\lambda}_{t+1} - \hat{\Pi}_{t+1} \right)^2 \right) \end{array} \right\}$$

$$- \zeta A^T \lambda v^2 \left(1 + \hat{\lambda}_t + 2\hat{v}_t + \frac{1}{2} \left(\hat{\lambda}_t + 2\hat{v}_t \right)^2 \right) - \zeta B^T \lambda \left(1 + \hat{\lambda}_t + \frac{1}{2} \hat{\lambda}_t^2 \right) - \lambda \left(1 + \hat{\lambda}_t + \frac{1}{2} \hat{\lambda}_t^2 \right) = 0$$

In the steady state, $\beta \left((1-\gamma)\left(\frac{1}{a} \right) + \lambda \right) \left(\frac{1}{\Pi} \right) - \lambda \zeta \left(A^T v^2 + B^T \right) = \lambda$

$$\beta E_t \left\{ \begin{array}{l} \left(\begin{array}{l} \frac{\lambda}{\Pi} \hat{\lambda}_{t+1} - (1-\gamma)\left(\frac{1}{a\Pi} \right) \hat{a}_{t+1} \\ - \left((1-\gamma)\left(\frac{1}{a\Pi} \right) + \frac{\lambda}{\Pi} \right) \hat{\Pi}_{t+1} \end{array} \right) \\ + \frac{1}{2} \left(\begin{array}{l} (1-\gamma)\left(\frac{1}{a\Pi} \right) \left(\begin{array}{l} \hat{a}_{t+1}^2 + \hat{\Pi}_{t+1}^2 \\ + 2\hat{a}_{t+1} \hat{\Pi}_{t+1} \end{array} \right) \\ + \frac{\lambda}{\Pi} \left(\hat{\lambda}_{t+1}^2 + \hat{\Pi}_{t+1}^2 - 2\hat{\lambda}_{t+1} \hat{\Pi}_{t+1} \right) \end{array} \right) \end{array} \right\} = \left[\begin{array}{l} 2\zeta A^T \lambda v^2 \hat{v}_t \\ + \left(\zeta B^T + \zeta A^T v^2 + 1 \right) \lambda \hat{\lambda}_t \\ + \frac{1}{2} \left(\begin{array}{l} 4\zeta A^T \lambda v^2 \hat{v}_t^2 + 4\zeta A^T \lambda v^2 \hat{\lambda}_t \hat{v}_t \\ + \left(\zeta B^T + \zeta A^T v^2 + 1 \right) \lambda \hat{\lambda}_t^2 \end{array} \right) \end{array} \right]$$

Second-order log-approximation for equation (A.2.13):

$$\lambda_t = \beta E_t \left\{ \left(\frac{I_t}{\Pi_{t+1}} \right) \left[\lambda_{t+1} + U_a(c_{t+1}, a_{t+1}, h_{t+1}) \right] \right\}$$

when $\sigma = \phi = 1$, $U_a(c_{t+1}, a_{t+1}, h_{t+1}) = (1-\gamma)\left(\frac{1}{a_{t+1}} \right)$

$$\lambda_t = \beta E_t \left\{ \left(\frac{I_t}{\Pi_{t+1}} \right) \left[\lambda_{t+1} + (1-\gamma)\left(\frac{1}{a_{t+1}} \right) \right] \right\}$$

$$\lambda_t = E_t \left\{ \beta \frac{I_t \lambda_{t+1}}{\Pi_{t+1}} + \beta(1-\gamma)\left(\frac{I_t}{\Pi_{t+1} a_{t+1}} \right) \right\}$$

$$\lambda e^{\hat{\lambda}_t} = E_t \left\{ \beta \frac{I\lambda}{\Pi} e^{\hat{I}_t + \hat{\lambda}_{t+1} - \hat{\Pi}_{t+1}} + \beta(1-\gamma)\left(\frac{I}{\Pi a} \right) e^{\hat{I}_t - \hat{\Pi}_{t+1} - \hat{a}_{t+1}} \right\}$$

$$\lambda\left(1 + \hat{\lambda}_t + \frac{1}{2}\hat{\lambda}_t^2\right) = E_t \left\{ \begin{array}{l} \beta \frac{I\lambda}{\Pi}\left(1 + \hat{I}_t + \hat{\lambda}_{t+1} - \hat{\Pi}_{t+1} + \frac{1}{2}\left(\hat{I}_t + \hat{\lambda}_{t+1} - \hat{\Pi}_{t+1}\right)^2\right) \\ + \beta(1-\gamma)\left(\frac{I}{\Pi a}\right)\left(1 + \hat{I}_t - \hat{\Pi}_{t+1} - \hat{a}_{t+1} + \frac{1}{2}\left(\hat{I}_t - \hat{\Pi}_{t+1} - \hat{a}_{t+1}\right)^2\right) \end{array} \right\}$$

In the steady state, $\lambda = \beta \left(\frac{I}{\Pi} \right) \left[\lambda + (1-\gamma)\left(\frac{1}{a} \right) \right]$

$$\lambda\left(\hat{\lambda}_t + \frac{1}{2}\hat{\lambda}_t^2\right) = \beta \frac{I}{\Pi} E_t \left\{ \begin{array}{l} \lambda\left(\hat{I}_t + \hat{\lambda}_{t+1} - \hat{\Pi}_{t+1} + \frac{1}{2}\left(\hat{I}_t + \hat{\lambda}_{t+1} - \hat{\Pi}_{t+1}\right)^2\right) \\ + (1-\gamma)\left(\frac{1}{a}\right)\left(\hat{I}_t - \hat{\Pi}_{t+1} - \hat{a}_{t+1} + \frac{1}{2}\left(\hat{I}_t - \hat{\Pi}_{t+1} - \hat{a}_{t+1}\right)^2\right) \end{array} \right\}$$

$$\lambda\hat{\lambda}_t + \frac{1}{2}\lambda\hat{\lambda}_t^2 = \frac{\beta I}{\Pi} E_t \left\{ \begin{array}{l} \lambda\left(\hat{I}_t + \hat{\lambda}_{t+1} - \hat{\Pi}_{t+1}\right) + \frac{(1-\gamma)}{a}\left(\hat{I}_t - \hat{\Pi}_{t+1} - \hat{a}_{t+1}\right) \\ + \frac{1}{2}\left(\begin{array}{l} \lambda\left(\hat{I}_t^2 + \hat{\lambda}_{t+1}^2 + \hat{\Pi}_{t+1}^2 + 2\hat{I}_t\hat{\lambda}_{t+1} - 2\hat{I}_t\hat{\Pi}_{t+1} - 2\hat{\lambda}_{t+1}\hat{\Pi}_{t+1}\right) \\ + \frac{(1-\gamma)}{a}\left(\hat{I}_t^2 + \hat{\Pi}_{t+1}^2 + \hat{a}_{t+1}^2 - 2\hat{I}_t\hat{\Pi}_{t+1} - 2\hat{I}_t\hat{a}_{t+1} + 2\hat{\Pi}_{t+1}\hat{a}_{t+1}\right) \end{array} \right) \end{array} \right\}$$

$$\lambda\hat{\lambda}_t + \frac{1}{2}\lambda\hat{\lambda}_t^2 = \frac{\beta I}{\Pi} E_t \left\{ \begin{array}{l} \lambda\left(\hat{I}_t + \hat{\lambda}_{t+1} - \hat{\Pi}_{t+1}\right) + \frac{(1-\gamma)}{a}\left(\hat{I}_t - \hat{\Pi}_{t+1} - \hat{a}_{t+1}\right) \\ + \frac{1}{2}\left(\begin{array}{l} \lambda\left(\hat{I}_t^2 + \hat{\lambda}_{t+1}^2 + \hat{\Pi}_{t+1}^2 + 2\hat{I}_t\hat{\lambda}_{t+1} - 2\hat{I}_t\hat{\Pi}_{t+1} - 2\hat{\lambda}_{t+1}\hat{\Pi}_{t+1}\right) \\ + \frac{(1-\gamma)}{a}\left(\hat{I}_t^2 + \hat{\Pi}_{t+1}^2 + \hat{a}_{t+1}^2 - 2\hat{I}_t\hat{\Pi}_{t+1} - 2\hat{I}_t\hat{a}_{t+1} + 2\hat{\Pi}_{t+1}\hat{a}_{t+1}\right) \end{array} \right) \end{array} \right\}$$

Second-order log-approximation for equation (A.2.15):

$$\lambda_t = \beta E_t \left\{U_a\left(c_{t+1}, a_{t+1}, h_{t+1}\right) + \lambda_{t+1} r_{t+1} + \lambda_{t+1}(1-\delta)\right\}$$

$$\text{when } \sigma = \phi = 1, U_a\left(c_{t+1}, a_{t+1}, h_{t+1}\right) = (1-\gamma)\left(\frac{1}{a_{t+1}}\right)$$

$$\lambda_t = E_t \left\{\beta(1-\gamma)\left(\frac{1}{a_{t+1}}\right) + \beta\lambda_{t+1} r_{t+1} + \beta\lambda_{t+1}(1-\delta)\right\}$$

$$\lambda e^{\hat{\lambda}_t} = E_t \left\{\beta(1-\gamma)\left(\frac{1}{a}\right)e^{-\hat{a}_{t+1}} + \beta\lambda r e^{\hat{\lambda}_{t+1}+\hat{r}_{t+1}} + \beta(1-\delta)\lambda e^{\hat{\lambda}_{t+1}}\right\}$$

$$\lambda\left(1+\hat{\lambda}_t+\frac{1}{2}\hat{\lambda}_t^{\,2}\right) = E_t \left\{\begin{array}{l} \beta(1-\gamma)\left(\frac{1}{a}\right)\left(1-\hat{a}_{t+1}+\frac{1}{2}\hat{a}_{t+1}^{\,2}\right) \\ +\beta\lambda r\left(1+\hat{\lambda}_{t+1}+\hat{r}_{t+1}+\frac{1}{2}\left(\hat{\lambda}_{t+1}+\hat{r}_{t+1}\right)^2\right) \\ +\beta(1-\delta)\lambda\left(1+\hat{\lambda}_{t+1}+\frac{1}{2}\hat{\lambda}_{t+1}^{\,2}\right) \end{array}\right\}$$

$$\text{In the steady state, } \lambda = \beta(1-\gamma)\left(\frac{1}{a}\right) + \beta\lambda r + \beta\lambda(1-\delta)$$

$$\lambda\left(1+\hat{\lambda}_t+\frac{1}{2}\hat{\lambda}_t^{\,2}\right) = E_t \left\{\begin{array}{l} \beta(1-\gamma)\left(\frac{1}{a}\right)\left(1-\hat{a}_{t+1}+\frac{1}{2}\hat{a}_{t+1}^{\,2}\right) \\ +\beta\lambda r\left(1+\hat{\lambda}_{t+1}+\hat{r}_{t+1}+\frac{1}{2}\left(\hat{\lambda}_{t+1}+\hat{r}_{t+1}\right)^2\right) \\ +\beta(1-\delta)\lambda\left(1+\hat{\lambda}_{t+1}+\frac{1}{2}\hat{\lambda}_{t+1}^{\,2}\right) \end{array}\right\}$$

$$\lambda\hat{\lambda}_t + \frac{1}{2}\lambda\hat{\lambda}_t^2 = \beta E_t \begin{Bmatrix} -(1-\gamma)\left(\frac{1}{a}\right)\hat{a}_{t+1} + \frac{1}{2}(1-\gamma)\left(\frac{1}{a}\right)\hat{a}_{t+1}^2 \\ +\lambda r\hat{\lambda}_{t+1} + \lambda r\hat{r}_{t+1} + \frac{1}{2}\lambda r\left(\hat{\lambda}_{t+1} + \hat{r}_{t+1}\right)^2 \\ +(1-\delta)\lambda\hat{\lambda}_{t+1} + \frac{1}{2}(1-\delta)\lambda\hat{\lambda}_{t+1}^2 \end{Bmatrix}$$

$$\lambda\hat{\lambda}_t + \frac{1}{2}\lambda\hat{\lambda}_t^2 = \beta E_t \begin{Bmatrix} -(1-\gamma)\left(\frac{1}{a}\right)\hat{a}_{t+1} + \lambda r\hat{\lambda}_{t+1} + \lambda r\hat{r}_{t+1} + (1-\delta)\lambda\hat{\lambda}_{t+1} \\ +\frac{1}{2}\left((1-\gamma)\left(\frac{1}{a}\right)\hat{a}_{t+1}^2 + \lambda r\left(\hat{\lambda}_{t+1} + \hat{r}_{t+1}\right)^2 + (1-\delta)\lambda\hat{\lambda}_{t+1}^2\right) \end{Bmatrix}$$

$$\lambda\hat{\lambda}_t + \frac{1}{2}\lambda\hat{\lambda}_t^2 = \beta E_t \begin{Bmatrix} -(1-\gamma)\left(\frac{1}{a}\right)\hat{a}_{t+1} + \lambda r\hat{\lambda}_{t+1} + \lambda r\hat{r}_{t+1} + (1-\delta)\lambda\hat{\lambda}_{t+1} \\ +\frac{1}{2}\begin{pmatrix} (1-\gamma)\left(\frac{1}{a}\right)\hat{a}_{t+1}^2 + (1-\delta)\lambda\hat{\lambda}_{t+1}^2 \\ +\lambda r\left(\hat{\lambda}_{t+1}^2 + \hat{r}_{t+1}^2 + 2\hat{\lambda}_{t+1}\hat{r}_{t+1}\right)^2 \end{pmatrix} \end{Bmatrix}$$

Equations A.2.14 and A.2.16 were substituted out to reduce the number of equations used in the solution process. In the future, when calculating the welfare costs of the business cycle, the households that have bond holdings such that $\boldsymbol{B}_t = 0\ \forall\ t$ are assumed to have a demand for money that ties interest rates to the velocity of money. This money demand specification is borrowed from Schmitt-Grohe and Uribe (2004).

Second-order log-approximation of the money demand equation used by Schmitt-Grohe and Uribe (2004): $v_t^{\,2} = \frac{B^{\mathrm{T}}}{A^{\mathrm{T}}} + \frac{1}{A^{\mathrm{T}}}\left(\frac{I_t - 1}{I_t}\right)$

$$v_t^{\,2} = \frac{B^{\mathrm{T}}}{A^{\mathrm{T}}} + \frac{1}{A^{\mathrm{T}}}\left(\frac{I_t - 1}{I_t}\right)$$

$$v^2 e^{2\hat{v}_t} = \frac{1 + B^{\mathrm{T}}}{A^{\mathrm{T}}} e^0 - \frac{1}{A^{\mathrm{T}}}\frac{1}{I_t} e^{-\hat{I}_t}$$

$$v^2 e^{2\hat{v}_t} = \frac{1 + B^{\mathrm{T}}}{A^{\mathrm{T}}} e^0 - \frac{1}{A^{\mathrm{T}}}\frac{1}{I_t} e^{-\hat{I}_t}$$

$$v^2\left(1 + 2\hat{v}_t + \frac{1}{2}(2\hat{v}_t)^2\right) = \frac{1 + B^{\mathrm{T}}}{A^{\mathrm{T}}} - \frac{1}{A^{\mathrm{T}}}\frac{1}{I_t}\left(1 - \hat{I}_t + \frac{1}{2}\hat{I}_t^{\,2}\right)$$

In the steady state, $v^2 = \frac{1 + B^{\mathrm{T}}}{A^{\mathrm{T}}} - \frac{1}{A^{\mathrm{T}}}\frac{1}{I}$

$$2v^2\hat{v}_t + \frac{1}{2}\left(4v^2\hat{v}_t^{\,2}\right) = \frac{1}{A^{\mathrm{T}}}\frac{1}{I_t}\hat{I}_t - \frac{1}{2}\frac{1}{A^{\mathrm{T}}}\frac{1}{I_t}\hat{I}_t^{\,2}$$

$$2v^2\hat{v}_t - \frac{1}{A^{\mathrm{T}}}\frac{1}{I_t}\hat{I}_t + \frac{1}{2}\left(4v^2\hat{v}_t^{\,2} + \frac{1}{A^{\mathrm{T}}}\frac{1}{I_t}\hat{I}_t^{\,2}\right) = 0$$

Second-Order Log-Approximation Process for the Non-Separable Utility Case

The remaining second-order log-approximations are unique for the case in which non-separable preferences are assumed.

Second-order log-approximation for equation $q_t = \gamma c_t^{1-\eta} + (1-\gamma)a_t^{1-\eta}$

This equation was used in the non-separable case to simplify the second-order log-approximations of equations A.2.10, A.2.12, and A.2.15.

$$q_t = \gamma c_t^{1-\eta} + (1-\gamma)a_t^{1-\eta}$$

$$qe^{\hat{q}_t} = \gamma c^{1-\eta} e^{(1-\eta)\hat{c}_t} + (1-\gamma)a^{1-\eta} e^{(1-\eta)\hat{a}_t}$$

In the steady state, $q = \gamma c^{1-\eta} + (1-\gamma)a^{1-\eta}$

$$q\left(1 + \hat{q}_t + \frac{1}{2}\hat{q}_t^{\,2}\right) = \gamma c^{1-\eta}\left(1 + (1-\eta)\hat{c}_t + \frac{1}{2}(1-\eta)^2 \hat{c}_t^{\,2}\right)$$
$$+ (1-\gamma)a^{1-\eta}\left(1 + (1-\eta)\hat{a}_t + \frac{1}{2}(1-\eta)^2 \hat{a}_t^{\,2}\right)$$

$$q\left(\hat{q}_t + \frac{1}{2}\hat{q}_t^{\,2}\right) = \gamma c^{1-\eta}\left((1-\eta)\hat{c}_t + \frac{1}{2}(1-\eta)^2 \hat{c}_t^{\,2}\right)$$
$$+ (1-\gamma)a^{1-\eta}\left((1-\eta)\hat{a}_t + \frac{1}{2}(1-\eta)^2 \hat{a}_t^{\,2}\right)$$

$$0 = \gamma c^{1-\eta}(1-\eta)\hat{c}_t + (1-\gamma)a^{1-\eta}(1-\eta)\hat{a}_t - q\hat{q}_t$$
$$+ \frac{1}{2}\left(\gamma c^{1-\eta}(1-\eta)^2 \hat{c}_t^{\,2} + (1-\gamma)a^{1-\eta}(1-\eta)^2 \hat{a}_t^{\,2} - q\hat{q}_t^{\,2}\right)$$

Second-order log-approximation for equation (A.2.10):

$$U_c(c_t,a_t,h_t)=(1+\mathrm{T}_t)\lambda_t+\lambda_t\zeta\left(A^{\mathrm{T}}v_t-\frac{\mathbf{B}^{\mathrm{T}}}{v_t^2}\right)\text{and let } q_t=\gamma c_t^{1-\eta}+(1-\gamma)a_t^{1-\eta}$$

when $\sigma>1$ and $\phi>1$, $U_c(c_t,a_t,h_t)=\gamma\, c_t^{-\eta}q_t^{\frac{\eta-\sigma}{1-\eta}}$

$$\gamma c_t^{-\eta}q_t^{\frac{\eta-\sigma}{1-\eta}}=(1+\mathrm{T}_t)\lambda_t+\lambda_t\zeta\left(A^{\mathrm{T}}v_t-\frac{\mathbf{B}^{\mathrm{T}}}{v_t^2}\right)$$

$$\gamma c_t^{-\eta}q_t^{\frac{\eta-\sigma}{1-\eta}}=\lambda_t+\lambda_t\mathrm{T}_t+\zeta A^{\mathrm{T}}\lambda_t v_t-\zeta\mathbf{B}^{\mathrm{T}}\frac{\lambda_t}{v_t^2}$$

$$\gamma\left(\frac{q^{\frac{\eta-\sigma}{1-\eta}}}{c^{\eta}}\right)e^{-\eta\hat{c}_t+\frac{\eta-\sigma}{1-\eta}\hat{q}_t}=\lambda e^{\hat{\lambda}_t}+\lambda Te^{(\hat{\lambda}_t+\hat{T}_t)}+\zeta A^{\mathrm{T}}\lambda ve^{(\hat{\lambda}_t+\hat{v}_t)}-\zeta\mathbf{B}^{\mathrm{T}}\frac{\lambda}{v^2}e^{(\hat{\lambda}_t-2\hat{v}_t)}$$

In the steady state, $\gamma\left(\frac{q^{\frac{\eta-\sigma}{1-\eta}}}{c^{\eta}}\right)=(1+\mathrm{T})\lambda+\lambda\zeta\left(A^{\mathrm{T}}v-\frac{\mathbf{B}^{\mathrm{T}}}{v^2}\right)$

$$\begin{aligned}\gamma\left(\frac{q^{\frac{\eta-\sigma}{1-\eta}}}{c^{\eta}}\right)\left(-\eta\hat{c}_t+\frac{\eta-\sigma}{1-\eta}\hat{q}_t+\frac{1}{2}\left(-\eta\hat{c}_t+\frac{\eta-\sigma}{1-\eta}\hat{q}_t\right)^2\right)&=\lambda\left(\hat{\lambda}_t+\frac{1}{2}\hat{\lambda}_t^2\right)\\&+\lambda T\left(\hat{\lambda}_t+\hat{T}_t+\frac{1}{2}(\hat{\lambda}_t+\hat{T}_t)^2\right)+\zeta A^{\mathrm{T}}\lambda v\left(\hat{\lambda}_t+\hat{v}_t+\frac{1}{2}(\hat{\lambda}_t+\hat{v}_t)^2\right)\\&-\zeta\mathbf{B}^{\mathrm{T}}\frac{\lambda}{v^2}\left(\hat{\lambda}_t-2\hat{v}_t+\frac{1}{2}(\hat{\lambda}_t-2\hat{v}_t)^2\right)\end{aligned}$$

$$\begin{aligned}&\lambda\left(1+T+\zeta\left(A^{\mathrm{T}}v-\frac{\mathbf{B}^{\mathrm{T}}}{v^2}\right)\right)\hat{\lambda}_t+\lambda T\hat{T}_t+\zeta\lambda\left(A^{\mathrm{T}}v+2\frac{\mathbf{B}^{\mathrm{T}}}{v^2}\right)\hat{v}_t+\gamma\left(\frac{q^{\frac{\eta-\sigma}{1-\eta}}}{c^{\eta}}\right)\left(\eta\hat{c}_t-\frac{\eta-\sigma}{1-\eta}\hat{q}_t\right)\\&+\frac{1}{2}\left(\begin{array}{l}\lambda\left(1+T+\zeta A^{\mathrm{T}}v-\zeta\frac{\mathbf{B}^{\mathrm{T}}}{v^2}\right)\hat{\lambda}_t^2+\zeta\lambda\left(A^{\mathrm{T}}v-4\frac{\mathbf{B}^{\mathrm{T}}}{v^2}\right)\hat{v}_t^2+\lambda T\hat{T}_t^2+2\lambda T\hat{\lambda}_t\hat{T}_t\\-\gamma\left(\frac{q^{\frac{\eta-\sigma}{1-\eta}}}{c^{\eta}}\right)\left(\eta^2\hat{c}_t^2+\left(\frac{\eta-\sigma}{1-\eta}\right)^2\hat{q}_t^2-2\eta\left(\frac{\eta-\sigma}{1-\eta}\right)\hat{c}_t\hat{q}_t\right)+2\lambda\zeta\left(A^{\mathrm{T}}v+2\frac{\mathbf{B}^{\mathrm{T}}}{v^2}\right)\hat{\lambda}_t\hat{v}_t\end{array}\right)=0\end{aligned}$$

Second-order log-approximation for equation (A.2.12):

$$\lambda_t = \beta E_t\left\{\left(U_a(c_{t+1},a_{t+1},h_{t+1})+\lambda_{t+1}\right)\left(\frac{1}{\Pi_{t+1}}\right)\right\}-\lambda_t\zeta\left(A^T v_t^{\,2}+B^T\right)$$

when $\sigma>1$ and $\phi>1$, $U_a(c_{t+1},a_{t+1},h_{t+1})=(1-\gamma)a_{t+1}^{\ -\eta}q_{t+1}^{\frac{\eta-\sigma}{1-\eta}}$

$$\lambda_t = \beta E_t\left\{\left((1-\gamma)a_{t+1}^{\ -\eta}q_{t+1}^{\frac{\eta-\sigma}{1-\eta}}+\lambda_{t+1}\right)\left(\frac{1}{\Pi_{t+1}}\right)\right\}-\lambda_t\zeta\left(A^T v_t^{\,2}+B^T\right)$$

$$E_t\left\{\beta(1-\gamma)a_{t+1}^{\ -\eta}q_{t+1}^{\frac{\eta-\sigma}{1-\eta}}\left(\frac{1}{\Pi_{t+1}}\right)+\beta\frac{\lambda_{t+1}}{\Pi_{t+1}}\right\}-\zeta A^T\lambda_t v_t^{\,2}-\zeta B^T\lambda_t-\lambda_t=0$$

$$\frac{\beta}{\Pi}E_t\left\{\begin{array}{l}(1-\gamma)a^{-\eta}q^{\frac{\eta-\sigma}{1-\eta}}e^{-\eta\hat{a}_{t+1}+\frac{\eta-\sigma}{1-\eta}\hat{q}_{t+1}-\hat{\Pi}_{t+1}}\\+\lambda e^{\hat{\lambda}_{t+1}-\hat{\Pi}_{t+1}}\end{array}\right\}-\zeta A^T\lambda v^2e^{\hat{\lambda}_t+2\hat{v}_t}-\zeta B^T\lambda e^{\hat{\lambda}_t}-\lambda e^{\hat{\lambda}_t}=0$$

$$\frac{\beta}{\Pi}E_t\left\{\begin{array}{l}(1-\gamma)a^{-\eta}q^{\frac{\eta-\sigma}{1-\eta}}\left(1-\eta\hat{a}_{t+1}+\frac{\eta-\sigma}{1-\eta}\hat{q}_{t+1}-\hat{\Pi}_{t+1}+\frac{1}{2}\left(-\eta\hat{a}_{t+1}+\frac{\eta-\sigma}{1-\eta}\hat{q}_{t+1}-\hat{\Pi}_{t+1}\right)^2\right)\\+\lambda\left(1+\hat{\lambda}_{t+1}-\hat{\Pi}_{t+1}+\frac{1}{2}\left(\hat{\lambda}_{t+1}-\hat{\Pi}_{t+1}\right)^2\right)\end{array}\right\}$$

$$-\zeta A^T\lambda v^2\left(1+\hat{\lambda}_t+2\hat{v}_t+\frac{1}{2}\left(\hat{\lambda}_t+2\hat{v}_t\right)^2\right)-\zeta B^T\lambda\left(1+\hat{\lambda}_t+\frac{1}{2}\hat{\lambda}_t^{\,2}\right)-\lambda\left(1+\hat{\lambda}_t+\frac{1}{2}\hat{\lambda}_t^{\,2}\right)=0$$

In the steady state, $\beta\left((1-\gamma)a^{-\eta}q^{\frac{\eta-\sigma}{1-\eta}}+\lambda\right)\left(\frac{1}{\Pi}\right)-\lambda\zeta\left(A^T v^2+B^T\right)=\lambda$

$$\frac{\beta}{\Pi}E_t\left\{\begin{array}{l}(1-\gamma)a^{-\eta}q^{\frac{\eta-\sigma}{1-\eta}}\left(-\eta\hat{a}_{t+1}+\frac{\eta-\sigma}{1-\eta}\hat{q}_{t+1}-\hat{\Pi}_{t+1}\right)+\lambda\left(\hat{\lambda}_{t+1}-\hat{\Pi}_{t+1}\right)\\+\frac{1}{2}\left((1-\gamma)a^{-\eta}q^{\frac{\eta-\sigma}{1-\eta}}\left(-\eta\hat{a}_{t+1}+\frac{\eta-\sigma}{1-\eta}\hat{q}_{t+1}-\hat{\Pi}_{t+1}\right)^2+\lambda\left(\hat{\lambda}_{t+1}-\hat{\Pi}_{t+1}\right)^2\right)\end{array}\right\}$$

$$=\left[\begin{array}{l}2\zeta A^T\lambda v^2\hat{v}_t\\+\left(\zeta B^T+\zeta A^T v^2+1\right)\lambda\hat{\lambda}_t\\+\frac{1}{2}\left(\begin{array}{l}4\zeta A^T\lambda v^2\hat{v}_t^{\,2}+4\zeta A^T\lambda v^2\hat{\lambda}_t\hat{v}_t\\+\left(\zeta B^T+\zeta A^T v^2+1\right)\lambda\hat{\lambda}_t^{\,2}\end{array}\right)\end{array}\right]$$

Second-order log-approximation for equation (A.2.12): (cont.)

$$\frac{\beta}{\Pi}E_t\left\{\begin{array}{l}(1-\gamma)a^{-\eta}q^{\frac{\eta-\sigma}{1-\eta}}\left(-\eta\hat{a}_{t+1}+\frac{\eta-\sigma}{1-\eta}\hat{q}_{t+1}-\hat{\Pi}_{t+1}\right)+\lambda\left(\hat{\lambda}_{t+1}-\hat{\Pi}_{t+1}\right)\\ +\frac{1}{2}\left(\begin{array}{l}(1-\gamma)a^{-\eta}q^{\frac{\eta-\sigma}{1-\eta}}\left(\begin{array}{l}\eta^2\hat{a}_{t+1}{}^2+\left(\frac{\eta-\sigma}{1-\eta}\right)^2\hat{q}_{t+1}{}^2+\hat{\Pi}_{t+1}{}^2\\ -2\eta\left(\frac{\eta-\sigma}{1-\eta}\right)\hat{a}_{t+1}\hat{q}_{t+1}+2\eta\hat{a}_{t+1}\hat{\Pi}_{t+1}-2\left(\frac{\eta-\sigma}{1-\eta}\right)\hat{q}_{t+1}\hat{\Pi}_{t+1}\end{array}\right)\\ +\lambda\left(\hat{\lambda}_{t+1}{}^2+\hat{\Pi}_{t+1}{}^2-2\hat{\lambda}_{t+1}\hat{\Pi}_{t+1}\right)\end{array}\right)\end{array}\right\}$$

$$=\left[\begin{array}{l}2\zeta A^{\mathrm{T}}\lambda v^2\hat{v}_t\\ +\left(\zeta B^{\mathrm{T}}+\zeta A^{\mathrm{T}}v^2+1\right)\lambda\hat{\lambda}_t\\ +\frac{1}{2}\left(\begin{array}{l}4\zeta A^{\mathrm{T}}\lambda v^2\hat{v}_t{}^2+4\zeta A^{\mathrm{T}}\lambda v^2\hat{\lambda}_t\hat{v}_t\\ +\left(\zeta B^{\mathrm{T}}+\zeta A^{\mathrm{T}}v^2+1\right)\lambda\hat{\lambda}_t{}^2\end{array}\right)\end{array}\right]$$

Second-order log-approximation for equation (A.2.15):

$$\lambda_t = \beta E_t \{U_a(c_{t+1}, a_{t+1}, h_{t+1}) + \lambda_{t+1} r_{t+1} + \lambda_{t+1}(1-\delta)\}$$

when $\sigma > 1$ and $\phi > 1$, $U_a(c_{t+1}, a_{t+1}, h_{t+1}) = (1-\gamma) a_{t+1}{}^{-\eta} q_{t+1}{}^{\frac{\eta-\sigma}{1-\eta}}$

$$\lambda_t = E_t \left\{ \beta(1-\gamma) a_{t+1}{}^{-\eta} q_{t+1}{}^{\frac{\eta-\sigma}{1-\eta}} + \beta\lambda_{t+1} r_{t+1} + \beta\lambda_{t+1}(1-\delta) \right\}$$

$$\lambda e^{\hat{\lambda}_t} = E_t \left\{ \beta(1-\gamma) a^{-\eta} q^{\frac{\eta-\sigma}{1-\eta}} e^{-\eta\hat{a}_{t+1} + \frac{\eta-\sigma}{1-\eta}\hat{q}_{t+1}} + \beta\lambda r e^{\hat{\lambda}_{t+1} + \hat{r}_{t+1}} + \beta(1-\delta)\lambda e^{\hat{\lambda}_{t+1}} \right\}$$

$$\lambda\left(1 + \hat{\lambda}_t + \frac{1}{2}\hat{\lambda}_t^2\right) = E_t \left\{ \begin{array}{l} \beta(1-\gamma) a^{-\eta} q^{\frac{\eta-\sigma}{1-\eta}} \left(1 - \eta\hat{a}_{t+1} + \frac{\eta-\sigma}{1-\eta}\hat{q}_{t+1} + \frac{1}{2}\left(-\eta\hat{a}_{t+1} + \frac{\eta-\sigma}{1-\eta}\hat{q}_{t+1}\right)^2\right) \\ + \beta\lambda r\left(1 + \hat{\lambda}_{t+1} + \hat{r}_{t+1} + \frac{1}{2}(\hat{\lambda}_{t+1} + \hat{r}_{t+1})^2\right) \\ + \beta(1-\delta)\lambda\left(1 + \hat{\lambda}_{t+1} + \frac{1}{2}\hat{\lambda}_{t+1}{}^2\right) \end{array} \right\}$$

In the steady state, $\lambda_t = E_t \left\{ \beta(1-\gamma) a^{-\eta} q^{\frac{\eta-\sigma}{1-\eta}} + \beta\lambda r + \beta\lambda(1-\delta) \right\}$

$$\lambda\hat{\lambda}_t + \frac{1}{2}\lambda\hat{\lambda}_t^2 = \beta E_t \left\{ \begin{array}{l} (1-\gamma) a^{-\eta} q^{\frac{\eta-\sigma}{1-\eta}} \left(-\eta\hat{a}_{t+1} + \frac{\eta-\sigma}{1-\eta}\hat{q}_{t+1}\right) + \lambda r(\hat{\lambda}_{t+1} + \hat{r}_{t+1}) + (1-\delta)\lambda\hat{\lambda}_{t+1} \\ + \frac{1}{2}\left(\begin{array}{l} (1-\gamma) a^{-\eta} q^{\frac{\eta-\sigma}{1-\eta}} \left(-\eta\hat{a}_{t+1} + \frac{\eta-\sigma}{1-\eta}\hat{q}_{t+1}\right)^2 \\ + \lambda r(\hat{\lambda}_{t+1} + \hat{r}_{t+1})^2 + (1-\delta)\lambda\hat{\lambda}_{t+1}{}^2 \end{array} \right) \end{array} \right\}$$

$$\lambda\hat{\lambda}_t + \frac{1}{2}\lambda\hat{\lambda}_t^2 = \beta E_t \left\{ \begin{array}{l} (1-\gamma) a^{-\eta} q^{\frac{\eta-\sigma}{1-\eta}} \left(-\eta\hat{a}_{t+1} + \frac{\eta-\sigma}{1-\eta}\hat{q}_{t+1}\right) + \lambda r(\hat{\lambda}_{t+1} + \hat{r}_{t+1}) + (1-\delta)\lambda\hat{\lambda}_{t+1} \\ + \frac{1}{2}\left(\begin{array}{l} (1-\gamma) a^{-\eta} q^{\frac{\eta-\sigma}{1-\eta}} \left(\eta^2\hat{a}_{t+1}{}^2 + \left(\frac{\eta-\sigma}{1-\eta}\right)^2 \hat{q}_{t+1}{}^2 - 2\eta\left(\frac{\eta-\sigma}{1-\eta}\right)\hat{a}_{t+1}\hat{q}_{t+1}\right) \\ + \lambda r(\hat{\lambda}_{t+1}{}^2 + \hat{r}_{t+1}{}^2 + 2\hat{\lambda}_{t+1}\hat{r}_{t+1})^2 + (1-\delta)\lambda\hat{\lambda}_{t+1}{}^2 \end{array} \right) \end{array} \right\}$$

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