LEARNING WITHIN CONTEXT: EXPLORING LESSON STUDY AS AN AID IN ENHANCING TEACHERS' IMPLEMENTATIONS, CONCEPTIONS, AND PERCEPTIONS OF THE MATHEMATICS TEACHING PRACTICES

by

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ABSTRACT

With traditional teaching methods pervasive in the U.S., it is crucial that mathematics teacher educators and professional development leaders understand what methods result in authentic changes in classroom instruction. Lesson study presents a promising approach to developing reform-oriented instruction, as it is situated within the classroom, draws upon rich discussions about lesson development, and creates opportunities for reflection upon practice. Although the literature has shown the usefulness of lesson study, evidence of how lesson study can support teachers' implementations, conceptions, and perceptions of reform-oriented instruction could be vital to the success of mathematics education locally and across the country. This study used an embedded case study design to explore how lesson study can be used to aid teachers in conceptualizing and implementing the Mathematics Teaching Practices and investigate how teachers' perceptions towards reform-oriented teaching change while participating in a Chinese form of lesson study, called *Keli* lesson study.

The researcher approached analysis from both individual and holistic perspectives. From an individual perspective, each participant made meaningful changes with respect to his or her implementation, conception, and perception of the Mathematics Teaching Practices. From a holistic perspective, the group made enhancements to the research lesson with respect to setting goals to focus learning, using goals to guide instructional decisions, designing the task, connecting representations, and teaching through problem solving. As a result, the research lesson became more focused and created rich opportunities for students to learn through problem solving. A cross-case analysis revealed that the most prominent changes across the embedded cases occurred with respect to four specific Mathematics Teaching Practices. The results of this study also revealed certain aspects of *Keli* lesson study that influenced change to participants' implementations, conceptions, and perceptions related to the Mathematics Teaching Practices. However, a disconnect was found, as there were 11 changes across the participants in this study related to conception and perception that were not associated with changes in implementation. The results of this study revealed the effectiveness of *Keli* lesson study in supporting teachers in transitioning to reform-oriented practices, as each participant made meaningful changes related to his or her implementation, conception, or perception. Moreover, this study revealed the aspects of professional development that influenced teacher change. Ongoing efforts are required, however, to influence sustained changes in implementation.

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CHAPTER I: INTRODUCTION

Introduction

Mathematics education in the U.S. has been under great scrutiny over the last ten years due to the results of many international comparison tests. Although the U.S. saw improvements in 2007 and 2011, only 68% of U.S. eighth-grade students performed intermediate or above in 2011, compared to 93% of Korean students (Third International Mathematics and Science Study [TIMSS], 2011). Moreover, just 7% of students reached the advanced level in eighth-grade mathematics in 2011, while 48% of eighth graders in Singapore and 47% of eighth graders in South Korea accomplished this goal. Students at the intermediate level are able to apply basic mathematical knowledge in straightforward situations, while the students at the advanced level can reason with information, draw conclusions, make generalizations, and solve linear equations. These results indicated that the U.S. is only moderately preparing students to understand procedures without connections, as defined by Smith and Stein (1998), while preparing very few to make connections and reason about mathematics.

The 2012 Program for International Student Assessment (PISA) further supported this notion. When compared to 64 other countries, U.S. teenagers scored slightly above average in reading and average in science, but below average in mathematics literacy and problem solving (Organisation for Economic Co-operation and Development [OECD], 2012). According to the OECD (2012), PISA results showed that “students in the United States have particular weaknesses in performing mathematics tasks with higher cognitive demands, such as taking real-world situations, translating them into mathematical terms, and interpreting mathematical aspects in real-world problems” (p. 1). Taken collectively, these results indicated poor mathematics understanding and proficiency in K-12 students and signified the need for change.

Traditional Teaching Methods

The effect of teachers on student achievement has been well documented (Garnett, 2013; Mathers & Olivia, 2008; Wright, Horn, & Sanders, 1997). Perhaps this is because “one of the most reliable findings from research on teaching and learning is that students learn what they are given opportunities to learn” (Hiebert, 2003, p. 10). Although reform-oriented teaching has been found to positively affect student learning and achievement (Firmender, Gavin, & McCoach, 2014; Gimbert, Bol, & Wallace, 2007), the majority of U.S. lessons are not aligned with such practices (Jacobs et al., 2006; Silver, Mesa, Morris, Star, & Benken, 2009; Wood, Shin, & Doan, 2006). In fact, TIMSS curricular reports suggested that one possible reason for the U.S. falling behind on international tests is because U.S. mathematics teaching is often times more skills oriented, more repetitive, and less conceptually deep than that of other nations (Stigler & Hiebert, 1999).

A number of other studies on mathematics teaching in the U.S. have revealed similar outcomes. When comparing video of classroom teaching in the U.S. to other countries, Hiebert et al. (2005) found the U.S. lessons contained lower levels of mathematical challenge, placed a greater emphasis on procedures and review, and were often unnecessarily fragmented. These are characteristics of the traditional teaching method in the U.S., which has remained largely unchanged for the previous 100 years (Ellis & Berry, 2005). Stigler and Hiebert (1999) labeled traditional mathematics education in the U.S. as “learning terms and practicing procedures” (p. 41), processes which do not provide students with valuable opportunities to learn. Therefore, when taken collectively, these studies demonstrate that many U.S. classrooms reflect traditional teaching methods, rather than reform-oriented practices.

Traditional Professional Development

Although U.S. teachers are aware of reformed-oriented practices (Stigler & Hiebert, 1999), they usually learn about new reform efforts through documents written by experts, outside of the context of the classroom (Stigler & Hiebert, 1999). Currently, the dissemination of the Common Core State Standards in Mathematics (CCSSM; Common Core State Standards Initiative [CCSSI], 2010) along with reform-oriented teaching practices appears to be no different. In many cases, teachers attempt to either translate written documents on their own or attend professional development programs instead of participating in collaborative communities at the school level (Wei, Darling-Hammond, & Adamson, 2010). Although many of the CCSSM teacher professional development projects have focused on helping teachers understand what the CCSSM entails, they often have neglected how to develop and implement lessons that meet the expectations of the CCSSM (Editorial Projects in Education Researcher Center, 2013).

Moreover, some professional development programs in the U.S. have focused on "quick fixes" (Ellis & Berry, 2005, p. 7) that supplement teachers' existing modes of teaching. As a result, most of the information learned through these experiences rarely transforms teaching in the classroom, what is known as the "paradox of change without difference" (Woodbury & Gess-Newsome, 2002, p. 1). In fact, researchers have argued that without the supporting context of the classroom, many teachers misinterpret reform teaching and only change surface features (Fullan, 2001; McLaughlin & Mitra, 2001), which could prove to be worse than what they were previously doing (Stigler & Hiebert, 1999).

Conclusion

When compared to its international counterparts, the U.S. has struggled to educate students to reason and think mathematically, draw conclusions, make generalizations, and perform mathematics tasks with higher cognitive demands (OECD, 2012). Although reform-oriented teaching practices have been found to positively affect student learning (Firmender et al., 2014), traditional teaching methods are still pervasive across the country (Banilower, Boyd, Pasley, & Weiss, 2006). Reform documents and professional development programs attempt to provoke change, but many of them fail to address teachers' underlying assumptions about the teaching and learning of mathematics (Stigler & Hiebert, 1999). With this in mind, mathematics teacher educators may be able to contribute to transforming teaching practices by investigating how teachers can transcend current norms in order to better support all students in developing deep understandings.

Background of Study

To improve the teaching and learning of mathematics in the U.S., it is important to understand the context in which teachers work and the documents that affect their practices. A number of recent documents have described not only what (i.e., CCSSM) and how (e.g., National Council of Teachers of Mathematics [NCTM], 2000) students should learn, but also how teachers should teach (NCTM, 2014). Moreover, specific objectives of student learning describe the expertise that teachers should support their students in developing (CCSSI, 2010). The combination of these documents describes goals for the teaching and learning of mathematics that can be leveraged to promote reform-oriented lessons. In the sections that follow, these documents will be described along with details about how lesson study, a specific type of professional development, can support the development of teachers. Then, a theoretical framework will be proposed as a lens to view teacher learning through the lesson study process.

Common Core State Standards for Mathematics

In 2010, state education chiefs and governors in 48 states collaborated to sponsor the creation of clear college- and career-ready standards for kindergarten through 12th grade in mathematics, what is known as the Common Core State Standards for Mathematics (CCSSI, 2010). A large majority of states adopted CCSSM or a modified version of CCSSM and were in the process of implementing the standards, which are designed to prepare high school graduates for introductory courses in two- or four-year college programs or enter a career (CCSSI, 2010). The CCSSM contains content standards that are intended to bring greater focus and coherence to mathematics education and counter the current practice in the U.S., labeled as a “mile wide and an inch deep” (CCSSI, 2010, p. 3). CCSSM content standards are separated into the thirteen conceptual categories found in Table 1. While the Standards for Mathematical Content describe the mathematics concepts and skills the student should learn, the Standards for Mathematical Practice (SMP) outline student “expertise” (CCSSI, 2010, p. 6) that teachers should develop in their students (see Table 1). These standards reaffirm the significance of habits of mind, mathematical processes, and proficiency as critical aspects of learning mathematics. The eight standards draw on both the process standards (NCTM, 2000) and the strands of mathematical proficiency (National Research Council [NRC], 2001).

Table 1

CCSSM Content and Mathematical Practice Standards

CCSSM Content Standard Domains	CCSSM Mathematical Practices
1. Counting and Cardinality	1. Make sense of problems and persevere in solving them. (MP1)
2. Number and Operations in Base Ten	2. Reason abstractly and quantitatively. (MP2)
3. Operations and Algebraic Thinking	3. Construct viable arguments and critique the reasoning of others. (MP3)
4. Geometry	4. Model with mathematics. (MP4)
5. Measurement and Data	5. Use appropriate tools strategically. (MP5)
6. Number and Operations in Fractions	6. Attend to precision. (MP6)
7. Ratios and Proportional Relationships	7. Look for and make use of structure. (MP7)
8. The Number System	8. Look for and express regularity in repeated reasoning. (MP8)
9. Expressions and Equations	
10. Functions	
11. Statistics and Probability	
12. Number and Quantity	
13. Algebra	

Note. Adapted from *Common Core State Standards for Mathematics*, by Common Core State Standards Initiative, 2010, National Governors Association Center for Best Practices and Council of Chief State School Officers, pp. 6-83.

National Council of Teachers of Mathematics

CCSSM has provided guidance and direction that help teachers focus and clarify common goals, but it does not address teaching practices at the classroom level. As a result, the primary purpose of a recently published book titled *Principles to Actions:*

Ensuring Mathematics Success for All (NCTM, 2014) was to “fill the gap between the development and adoption of CCSSM and other standards and the enactment of practices . . . required for their widespread and successful implementation” (p. 4). In *Principles to Actions*, NCTM provided a set of strongly recommended, research-based practices for all teachers. Many of the recommendations described how teachers can support the Process Standards and Principles outlined by NCTM in *Principles and Standards for School Mathematics* (2000). Together, both sets of standards aim to provide the vehicle by which teachers can support students in developing mathematical expertise, as defined by the CCSSM’s Standards for Mathematical Practice (CCSSI, 2010). In the following sections, an overview of each set of standards is provided.

Process standards. The Process Standards (NCTM, 2000) described the means by which students come to make sense of mathematics. These standards included communication, representation, reasoning and proof, connections, and problem solving (NCTM, 2000). The communication standard encouraged communication that allows students to “organize and consolidate . . . [and] analyze and evaluate the mathematical thinking” (p. 348). Moreover, this standard called for students to build understanding and permanence for ideas and make them visible. The representation standard encouraged the creation of representations to organize, record, and communicate mathematical ideas and model real-world phenomena. In addition, this standard called for students to select, apply, and translate among mathematical representations to solve problems.

The next standard, reasoning and proof, has been described as a mathematical process by which verification, explanation, systematization, discovery, and communication take place in a mathematics classroom (de Villiers, 1990). NCTM (2000) called for students to make and investigate mathematical conjectures as well as develop and evaluate mathematical arguments. The connections standard encouraged students to develop connections among mathematical ideas and application of mathematical ideas to contextual situations outside of mathematics. Moreover, it called for students to understand how mathematical ideas interconnect to produce a coherent mathematical knowledge structure. Problem solving, according to NCTM (2000), is not the application of previously learned concepts, but the “building [of] new mathematical knowledge” (p. 334). This standard also called for students to employ and modify a variety of suitable strategies to solve problems as well as monitor and reflect on the progression of mathematical problem solving. Collectively, the NCTM (2000) Process Standards described the processes in which instructional programs from kindergarten through grade 12 should engage students to learn mathematics meaningfully.

Principles. Along with the Process Standards, NCTM (2000) outlined six Principles that described characteristics of high-quality mathematics education. The six Principles included equity, curriculum, teaching, learning, assessment, and technology. The equity principle called for high expectations and support for all students, and the curriculum principle advocated for mathematics to be coherent and well-articulated across grade levels. In addition, the teaching principle suggested that teachers should understand what students currently know and need to learn and that they should support students in learning it well. The learning principle invited students to learn mathematics with understanding, and the assessment principle called for ongoing assessments that enhance student learning and serve as valuable tools for instructional decisions. Finally, the technology principle petitioned for students to use technology that enhances learning and impacts what mathematics is taught. Together, these principles created a vision to direct teachers, school administrators, and other professionals as they seek to improve mathematics education in classrooms, schools, and districts.

Mathematics teaching practices. *Principles to Actions* (NCTM, 2014) supported many of these ideas, reflecting current research from both cognitive science and mathematics education that have identified characteristics of effective mathematics teaching. Each practice described in *Principles to Actions* (NCTM, 2014) was supported by a collection of seminal studies relating to that particular topic. In addition, NCTM (2014) ended each description of the Mathematics Teaching Practices (MTP) with suggestions about what actions teachers and students should be doing to support the practice. The eight Mathematics Teaching Practices represented “high-leverage” (Ball & Forzani, 2010, p. 45) teaching practices necessary to promote deep learning of mathematics. Table 2 outlines these practices along with their connections to the Process Standards (NCTM, 2000), the Standards of Mathematical Practice (CCSSI, 2010), and the Principles (NCTM, 2000). A description of each practice follows, along with how these practices relate to earlier recommendations from NCTM (2000) and CCSSM (CCSSI, 2010).

Table 2

Connections Among the Mathematics Teaching Practices and Previous Documents

Mathematics Teaching Practices (NCTM, 2014)	Process Standards (NCTM, 2000)	Standards of Mathematical Practice (CCSSI, 2010)	Principles (NCTM, 2000)
1. Establish mathematics goals to focus learning	All	All	Equity Curriculum Technology
2. Implement tasks that promote reasoning and problem solving	Problem Solving, Reasoning and Proof	SMP1, SMP4, SMP7	Equity Learning
3. Use and connect mathematical representations	Representation	SMP4, SMP5	Curriculum Technology
4. Facilitate meaningful mathematical discourse	Connection, Communication	SMP3, SMP6	Curriculum Assessment
5. Pose purposeful questions	Connection, Reasoning and Proof	SMP3, SMP6, SMP2	Curriculum Assessment
6. Build procedural fluency from conceptual understanding	Reasoning and Proof	SMP2, SMP8	Learning
7. Support productive struggle in learning mathematics	Problem Solving, Reasoning and Proof	SMP1, SMP2	Learning
8. Elicit and use evidence of student thinking	Communication	SMP3	Assessment

Note. The Teaching Principle is supported by all of the Mathematics Teaching Practices (NCTM, 2014).

MTP 1 - Establish mathematics goals to focus learning. As teachers attend to students' learning progressions, they must identify clear and explicit learning goals and determine how they connect to prior knowledge, current curriculum standards, and students' future learning trajectories. The objects of learning (Marton, Runesson, & Tsui, 2004) that are selected should guide teachers' decision making throughout the planning, implementation, and reflection phases of the lesson (Hiebert & Grouws, 2007). For example, mathematical goals should serve as a guide for teachers as they facilitate meaningful discussions and help students make important connections (NCTM, 2014). These goals should be made clear to students throughout the lesson (NCTM, 2014) and evidence of students obtaining these goals should be collected (Wiliam, 2007).

This practice in particular is of chief importance because each of the other practices is informed by the mathematical goals that are selected (Hiebert & Grouws, 2007). The idea of formulating clear and concise mathematical goals can directly and indirectly support all of the Process Standards (NCTM, 2000) and Standards of Mathematical Practice (CCSSI, 2010). However, goal-related recommendations are specifically discussed within the equity, curriculum, and technology principles (NCTM, 2000). First, the equity principle called for high expectations for all students, which dictates the rigor of goals that are set. Second, goals are guided by the curriculum principle, which called for curricula that are "coherent and focused on important mathematics" (p. 11). Third, the technology principle emphasized technology's influence on the mathematics that is taught, and thus the goals of mathematics learning.

***MTP 2 - Implement tasks that promote reasoning and problem solving*.** Although selecting rich tasks can be difficult, implementing a task without diminishing its cognitive demand is extremely challenging (Stigler & Hiebert, 2004). Given that not all tasks provide the same opportunity for learning (Stein, Smith, Henningsen, & Silver, 2009), it is important to understand the various levels of tasks. Smith and Stein (1998) outlined four levels of increasing cognitive demand: memorization, procedures without connections, procedures with connections, and doing mathematics. The level of a task primarily depends on its multiple entry points and solution strategies, connections to mathematical concepts, and cognitive effort (Smith & Stein, 1998).

NCTM (2014) suggested teaching through carefully selected tasks that relate directly to the mathematical goals of the lesson. In doing so, teachers give students the opportunity to learn, which can lead to long-lasting and meaningful connections. Moreover, students experience mathematics as they explore and solve problems that "build on and extend their current mathematical understanding" (p. 24). This idea supports the problem-solving Process Standard (NCTM, 2000), which also focused on students learning through problem solving. In addition, this practice can support students in learning to make sense of problems and persevering in solving them (SMP 1), modeling with mathematics (SMP 4), and looking for and making use of structure (SMP 7) (CCSSI, 2010). Effectively implementing tasks with numerous entry points further supports the equity principle (NCTM, 2000) as well as the learning principle (NCTM, 2000), which called for students to "build knowledge from experiences and prior knowledge" (p. 11).

MTP 3 - Use and connect mathematical representations. Students exhibit deeper mathematical understanding and greater problem-solving abilities as they learn to represent, communicate, and make connections among mathematical ideas in multiple forms (Fuson, Kalchman, & Bransford, 2005). NCTM (2014) recommended that students make connections among physical, contextual, verbal, symbolic, and visual mathematical representations. By doing so, students experience the concept through a variety of lenses, each with its unique perspective that helps build richer and deeper understandings (Tripathi, 2008). Students can only draw upon these various representations if tasks are selected that allow students the freedom to decide which representation to use (NCTM, 2014). Revealing new representations offers students new tools that they can use to model and interpret mathematical ideas in the future (NCTM, 2014).

This mathematical practice supports the representation Process Standard (NCTM, 2000), which called for using appropriate representations of mathematics flexibly. In addition, as students represent “problems arising in everyday life” (CCSSI, 2010, p. 7), they learn to model with mathematics (SMP 4) and use appropriate tools (SMP 5) when necessary (CCSSI, 2010). Appropriate tools may include calculators and computer algebra systems, whose importance is described in the technology principle (NCTM, 2000). Furthermore, this particular teaching practice is related to the curriculum principle (NCTM, 2000), which called for students to make connections among mathematical ideas.

MTP 4 - Facilitate meaningful mathematical discourse. The purposeful exchange of verbal, visual, and written communication gives students opportunities to share ideas, refine understandings, build viable arguments, develop a common language, and experience mathematics from others’ perspectives (NCTM, 2000). Teachers can develop meaningful mathematical discourse by attending to Stein and Smith’s (2011) five practices, which include anticipating, monitoring, selecting, sequencing, and connecting students’ responses before and during whole-class discussion. This leads to engaged students sharing mathematical thoughts, reasoning, and approaches as the authors of ideas (NCTM, 2014).

These practices support recommendations described by the communication Process Standard (2000). Moreover, students participating in meaningful discourse have opportunities to strengthen their ability to construct viable arguments and critique the reasoning of others (SMP 3), as well as attend to precision (SMP 6) (CCSSI, 2010). The assessment principle (NCTM, 2000) relates to this practice, as students sharing their ideas create chances for teachers to assess student understanding, which guides subsequent instructional decisions. As connections are made between student responses and important mathematical ideas, the curriculum becomes more coherent, which supports the curriculum principle (NCTM, 2000).

MTP 5 - Pose purposeful questions. NCTM (2014) recommended asking questions to establish what students know, make connections among mathematical ideas, and reveal student reasoning. In other words, teachers should ask questions to both assess and advance student thinking during small-group and whole-class discussions. Although both focusing and funneling patterns of discourse (Wood, 1998) can be appropriate, a focusing pattern allows student thinking to lead the discussion. Regardless, the teacher should not hinder student thinking, but rather press students to explain and justify their reasoning (NCTM, 2014). In this environment, students are responsible for generating new ideas while the teacher focuses students' attention towards critical features of the intended object of learning (Marton et al., 2004). This also allows students to experience mathematics as a connected whole, instead of a set of disjoint parts (Skemp, 1976).

The reasoning and proof Process Standard (NCTM, 2000) is supported by this practice, in that purposeful questions require students to make conjectures and provide justification to support their ideas. The connection Process Standard (NCTM, 2000), which called for making connections among mathematical ideas and applying them to contexts outside of mathematics, is supported as well. In addition to SMP 3 and SMP 6, purposeful questions can also engage students in learning to reason abstractly and quantitatively (SMP 2) (CCSSI, 2010). Similar to facilitating meaningful discourse, this practice aligns with the assessment and curriculum principles (NCTM, 2000).

MTP 6 - Build procedural fluency from conceptual understanding. Teachers should develop in students a relational understanding (Skemp, 1976) of mathematics that allows them the flexibility to solve a variety of real-world problems. Instead of memorizing steps to complete a problem, NCTM (2014) challenged teachers to support students in understanding the procedures they choose to implement when solving a particular problem. Practicing procedures is not strictly prohibited, but NCTM (2014) recommended that students should have a conceptual understanding of the concepts first so that they will be able to use them appropriately and efficiently in the future. Discussing strategies, making connections with more efficient procedures, and reflecting on their work can support students in determining which procedures work best for certain types of problems (NCTM, 2014).

This practice relates to the reasoning and proof Process Standard (NCTM, 2000), as students learn to reason abstractly (SMP 2) and see regularity in repeating reasoning (SMP 8) (CCSSI, 2010). Further, these ideas relate well with the learning principle

(NCTM, 2000), which posits that "conceptual understanding is an important component to proficiency" (p. 20).

MTP 7 - Support productive struggle in learning mathematics. Students need time to grapple with mathematics and experience productive struggle in order to understand at a deeper level (Kapur, 2010). Teachers should constantly balance adversity and success, while rewarding students for effort and creativity instead of praising them for the right answer (Smith, 2000). Through tasks that promote reasoning and problem solving, teachers should encourage students to persevere and find ways to support students without removing all of the challenges (NCTM, 2014). One objective should be to instill a growth mindset in students that encourages them to embrace challenges, persist in the face of setbacks, view effort as a path to mastery, learn from criticism, and find inspiration in the success of others (Dweck, 2006). One way to support these thoughts is to "help students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles" (NCTM, 2014, p. 52).

The idea of productive struggle is described in detail in the learning principle (2000), and it supports many of the ideas found in the problem-solving and reasoning and proof Process Standards (NCTM, 2000). Moreover, it supports students in learning to reason abstractly (SMP 2) while making sense of problems and persevering in solving them (SMP 1) (CCSSI, 2010).

MTP 8 - Elicit and use evidence of student thinking. Eliciting and using evidence of students achieving mathematical goals not only helps teachers assess understanding, but also provides them with information that can alter future decision-making (NCTM, 2014). As a result, teachers need to incorporate opportunities for assessment during lessons and plan how they will respond to the results of these assessments. Students should understand that they are also responsible for “assessing and monitoring their own progress toward mathematics learning goals and identifying areas in which they need to improve” (NCTM, 2014, p. 56).

Although eliciting and using student thinking is not a specific Process Standard (NCTM, 2000), it heavily supports the communication standard as well as the assessment principle (NCTM, 2000). Moreover, students have opportunities to construct viable arguments and critique the reasoning of others (SMP 3) (CCSSI, 2010) as their thoughts are displayed.

Summary. Collectively, these eight practices align with previous recommendations in mathematics education (CCSSI, 2010; NCTM, 2000). *Principles to Actions* (NCTM, 2014) not only supported the ideas of previous documents, but it built upon them to form a vision of what effective teaching entails – under any standards or in any educational setting. As a result, teachers need to conceptualize the Mathematics Teaching Practices (NCTM, 2014) and be able to implement them effectively. However, many professional development programs continue to struggle to change classroom practices to be more aligned with NCTM (2000) recommendations for mathematics teaching and learning (Ellis & Berry, 2005). This could be one reason why there are many misconceptions related to the Process Standards (Sanchez, Lischka, Edenfield, & Gammill, 2015). To ensure teachers implement the newly released Mathematics Teaching Practices (NCTM, 2014) successfully, a non-traditional model for improving teaching is necessary.

Lesson Study

To alter the cultural activity of teaching, the U.S. is in need of a proven, sustainable system by which teachers can learn to implement new reform within the context of the classroom (Takahashi, Lewis, & Perry, 2013). Although there is not a perfect solution, a carefully designed lesson study approach could prove to be a viable option, as a number of studies have shown its usefulness in improving teaching in various situations and at many different levels (e.g., Huang & Li, 2009; Lewis, Fischman, Riggs, & Wasserman, 2013; Lewis, Perry, & Murata, 2006; Ricks, 2011; Takahashi et al., 2013; Yoshida, 2013).

Lesson study was first introduced to the U.S. in the late 1990's through Stigler and Hiebert's *The Teaching Gap* (1999), which was informed by Makoto Yoshida's (1999) dissertation. However, lesson study did not begin to gain acceptance with teachers and administrators in the U.S. until around 2005 (Yoshida, 2013). To describe it briefly, lesson study consists of a group of teachers working collaboratively and meticulously to carefully craft a lesson (Yoshida, 1999). A typical lesson study cycle includes teachers collaboratively planning, observing, and discussing classroom research lessons in order to improve their shared understanding of mathematics, teaching, learning, and students (Lewis, Friedin, Baker, & Perry, 2011). In some variations of lesson study, the cyclical process of teaching and revision culminates with an exemplary lesson and new ideas about the teaching and learning of mathematics that can be shared with others.

The lesson study approach is quite different from some traditional professional development models, which have been characterized as discontinuous and not focused on practice-based improvement of teaching and learning (Fernandez & Yoshida, 2004). Specifically, there are two main aspects of lesson study that separate it from traditional professional development, with regards to new reform. First, lesson study takes place in the context of the classroom, and thus clearly communicates what it looks like to implement reform-oriented practices (Takahashi et al., 2013). Second, lesson study involves the collaboration of teachers and knowledgeable others to better conceive reform-oriented practices and how they can be effectively implemented (Lewis, Perry, & Hurd, 2009). Therefore, lesson study represents a viable option for supporting teachers in conceptualizing and implementing the Mathematics Teaching Practices (NCTM, 2014).

Theoretical Framework

Due to the contextual and collaborative nature of lesson study, the theoretical framework chosen for this study drew upon both situated learning theories (Lave & Wenger, 1991) and cognitive theories of teacher learning (Remillard & Bryans, 2004). Although situated learning theorists focus on the context in which learning occurs, cognitive theorists focus on learning that arises from cognitive conflict and meaningful discussions. This combination informed the connection between lesson study and the learning and implementation of reform-oriented practices. Previous studies have used this theoretical perspective to help understand the unique features of lesson study (e.g., Lewis et al., 2009). The fusion of these two theories provided a powerful lens by which the process of lesson study was viewed in this study.

Statement of Purpose

Although the literature has displayed the advantages and the effectiveness of lesson study in general, few studies have discussed its value in supporting teachers' transition to reform-oriented teaching practices. Therefore, the purpose of this study was to explore how lesson study can be used to aid teachers in implementing the Mathematics Teaching Practices (NCTM, 2014) and investigate how teachers' conceptions and perceptions of reform-oriented teaching change while participating in lesson study. More specifically, the following research questions related to teachers transitioning to reform-oriented practices were posed.

1. How does lesson study support teachers in implementing the Mathematics Teaching Practices, if at all?
2. How do teachers' conceptions of the Mathematics Teaching Practices change while participating in lesson study, if at all?
3. How do teachers' perceptions of reform-oriented teaching practices change while participating in lesson study, if at all?

Significance of the Study

Effectively implementing reform-oriented teaching practices that support CCSSM is of chief importance due to the number of states enacting its content and practice standards (NCTM, 2014). Research has shown that making changes to how mathematics is taught is problematic (Franke, Kazemi, & Battey, 2007). The results from this study further informed current professional development and professional learning community practices, which is important given the need to improve mathematics teaching in the U.S. (Hiebert et al. 2005; NCTM, 2014; Stigler & Hiebert, 1999). Although the literature has shown the usefulness of lesson study, evidence of how lesson study can support teachers in conceptualizing and implementing reform-oriented instruction could be vital to the success of mathematics education locally and across the country. The results of this study provided much needed insight into how teachers transition from traditional teaching methods to reform-oriented instruction. Moreover, the results of this study provided information about how participating in lesson study can provide teachers with opportunities to learn and implement reform-oriented teaching practices.

Definitions

Throughout this study, the term *reform* in mathematics pedagogy is defined as a shift of instructional focus toward engaging students in mathematical reasoning and problem solving, encouraging students' conceptual understanding, and developing classrooms as mathematical learning communities (NCTM, 1991; CCSSI, 2010). In addition, *conception* in this study refers to a person's ideas about, knowledge of, and ability to recognize the Mathematics Teaching Practices (NCTM, 2014). *Perception* is defined as a person's views about how to best teach and learn mathematics. With respect to classroom activities, *bell work* is defined as an immediate assignment upon entering a classroom, so named because students are expected to be working on it as the bell rings. An *exit ticket* is an assignment students complete at the end of class. Typically, students must turn in the assignment to the teacher as their ticket to exit the classroom.

Chapter Summary

The context of the classroom (Stigler & Hiebert, 1999) and developing lessons that support reform practices (Editorial Projects in Education Researcher Center, 2013) are key components of learning new reform that professional development programs often neglect. As a result, some previous efforts to implement new curricula have resulted in teachers only changing surface features of their teaching without addressing certain underlying assumptions about the teaching and learning of mathematics (Fullan, 2001;

McLaughlin & Mitra, 2001). With traditional teaching methods pervasive in the U.S. (Hiebert et al., 2005), it is imperative that mathematics teacher educators and professional development leaders understand what professional development models result in authentic changes in classroom instruction. The lesson study model presents a promising approach to learning how to implement reform-oriented instruction (Yoshida, 2013), as it is situated within the classroom (Takahashi et al., 2013), draws upon rich discussions about lesson development (Lewis et al., 2009), and creates opportunities for reflection upon practice (Ricks, 2011). Guided by situated (Lave & Wenger, 1991) and cognitive (Kurt, Kugler, Coleman, & Liebovitch, 2014) theoretical perspectives on teacher learning, the purpose of this study was to investigate the effectiveness of lesson study in developing teachers' implementations, conceptions, and perceptions of reform-oriented teaching practices.

CHAPTER II: REVIEW OF LITERATURE

Introduction

Although reform-oriented teaching practices have been found to positively affect student learning (Firmender et al., 2014), transitioning to reform-oriented practices can be difficult to achieve and sustain (Richardson, 1990). The purpose of this study was to explore how lesson study can be used to aid teachers in conceptualizing and implementing the Mathematics Teaching Practices (NCTM, 2014) and investigate how teachers' perceptions of reform-oriented teaching change while participating in lesson study. This chapter draws upon previous research on teacher change and lesson study research in order to develop a coherent review of literature that joins the two together. The following section begins with an examination of the theory behind both student learning and teacher learning in order to develop a theoretical framework for this study. Various frameworks used to describe teachers in transition will then be outlined in the next section. The best practices of professional development will be described, including embedded professional development models such as professional learning communities. The chapter ends with a review of literature related to lesson study, which leads to the selection of a specific lesson study model that was implemented in this study.

Theoretical Framework

Theories from psychology have influenced the teaching and learning of mathematics (Lambdin & Walcott, 2007) as well as the professional development of teachers (Bowers, Cobb, & Mcclain, 1999). The following sections describe the theoretical perspectives on mathematics education and mathematics teacher education that were used to view effective teaching practices and the learning process of teachers.

Student Learning

The history of mathematics reform, as described by Schoenfeld (2007), has been set on a pendulum due to the paradigm shifts in mathematics education. The result has been approximately decade-long shifts (Lambdin & Walcott, 2007) between teaching for mastery (i.e., traditional) and teaching for understanding (i.e., reform) (Schoenfeld, 2007). Traditional teaching methods in the U.S., which emphasize procedures and review (Hiebert et al., 2005), are heavily influenced by connectionism (Lambdin & Walcott, 2007). In contrast, the reform-oriented practices suggested by NCTM (2014) align with teaching for understanding and were formed on the theoretical underpinnings of constructivism (Lambdin & Walcott, 2007). Therefore, constructivism served as the lens through which student learning was viewed in this study.

Constructivism was heavily influenced by the work of Jerome Bruner (1915-) and Jean Piaget (1896-1980) (Lefrancois, 2005). The main goal of constructivism is to transition students from assuming the role of listener to that of an active learner (Gabler & Schroeder, 2002). As independent learners, students should be provided opportunities to discover and build knowledge for themselves (Lefrancois, 2005) through problem solving (Lambdin & Walcott, 2007). Constructivists believe that teachers should act as mentors and facilitators that encourage students to think critically in order to make meaning and build understanding (Gabler & Schroeder, 2002). The Mathematics Teaching Practices (NCTM, 2014) were developed on similar theoretical underpinnings that describe mathematics learning as “an active process, in which each student builds his or her own mathematical knowledge from personal experiences, coupled with feedback from peers, teachers and other adults, and themselves” (p. 9).

Teacher Learning

When studying teacher learning through lesson study, it is important to consider the theoretical perspectives through which their participation was viewed. Two theoretical perspectives helped guide the decisions of the researcher and provided a lens through which the process of teacher learning was observed. The theoretical perspectives are situated learning theory (Lave & Wenger, 1991) and cognitive learning theory (Remillard & Bryans, 2004). Situated learning theory was selected because it helps capture the context in which lesson study takes place and the affordances brought about by authentic experiences. In contrast, cognitive learning theory was chosen to aid in understanding the collaborative nature of lesson study and how discussions between the participants may or may not lead to teacher change. Details of the two perspectives and their connection to lesson study follow.

Situated learning theory. Situated learning theorists support the notion that learning takes place in authentic contexts in which shared experiences can be then transferred to new situations (Lave & Wenger, 1991). Knowledge is viewed as arising conceptually through dynamic construction and reinterpretation within a particular social context (Clancey, 2009). In other words, learning needs to take place within authentic contexts, settings, and situations (Lave & Wenger, 1991). In a lesson study, the context can be described as a community of teachers working together to build norms (i.e., inquiry and accountability) and tools needed for instructional improvement (Lewis et al., 2009). Through such an experience, situated learning theorists suggest that teachers have the opportunity to develop tools that shape their identity in such a way that members are able to transfer forms of participation to new settings (Lave & Wenger, 1991), such as applying new practices in their classrooms.

Cognitive learning theory. Cognitive learning theorists suggest that learning is the result of cognitive conflict that occurs while trying to reach a consensus with others (Kurt et al., 2014). Alternatively, learning can be viewed as changes in individual's mental schema, which can take place when trying to make an idea visible or experiencing cognitive conflict with others (Linn, Eylon, & Davis, 2004; Remillard & Bryans, 2004). The lesson study process can provide teachers with opportunities to share their ideas and revise their own thinking based on discussions that arise during pre- or post-lesson discussions. Moreover, lesson study presses teachers to come to a consensus regarding the research lesson, which highlights new ideas about the teaching and learning of mathematics.

Lesson study. Forman and Cazden (1994) suggested that collaboration enhances the development of logical reasoning by reorganizing knowledge brought on by cognitive conflict. Lerman (2000) suggested that people in a group setting bring their various backgrounds and values together to create knowledge. In a lesson study, each member, with his or her own background, brings different perspectives to each conversation. It is this varying expertise and knowledge that can develop a sense of collective efficacy within a group (Suh & Parker, 2010). Situated learning theorists suggest that after teachers experience this, they are then able to transfer the forms of participation to their teaching and other conversations (Lave & Wenger, 1991).

Together, situated learning theory (Lave & Wenger, 1991) and cognitive learning theory (Lerman, 2000) combine to inform the collaborative process of lesson study and provide insight into teacher learning. With this lens, lesson study can be viewed as the means by which teachers dynamically construct or reinterpret new practices as well as create norms and tools for implementation (Lewis et al., 2009), which can then be transferred (Lave & Wenger, 1991) to their own teaching. It is with this lens that the researcher interpreted the advantages and challenges to conducting lesson study, which were used to support the rationale for this study as well as lay the foundation for the specific model of lesson study to be implemented.

Teachers in Transition

When studying teacher change, it is important to understand how and why changes are made. Moreover, examining previous research on teachers transitioning to reform-oriented practices helped develop a framework for this study. In the sections that follow, the process that teachers go through when transitioning to reform-oriented practices is described along with varying perspectives on how and why changes may be made.

Motivation

Prior to beginning the process of transitioning to reform-oriented practices, teachers must first feel the motivation to examine or change their practices. Goldsmith and Shifter (1997) suggested that these thoughts are often initiated when teachers feel the desire to implement reform pedagogy, recognize that their existing methods are not adequately serving some students, or realize that their students have a great amount of intuitive understanding that is not being invoked by their current practices. As a result, a conflict is created between their beliefs and observations in the classroom, which provokes teachers to question their role as an orchestrator instead of a facilitator (Goldsmith & Shifter, 1997).

Unproductive and Productive Beliefs

Cultural beliefs about the teaching and learning of mathematics can be an obstacle to reform-oriented practices because many parents and educators believe that students should learn as they were taught (Philipp, 2007). This could be due to the fact that they have no useful image of reform-oriented teaching (Goldsmith & Shifter, 1997). Therefore, as teachers experience the ongoing process of reconceiving ideas, they need time to reflect and observe (Goldsmith & Shifter, 1997). This progression presses teachers to resolve conflicts between their beliefs and observations in the classroom (Wood, Cobb, & Yackel, 1991). To outline beliefs that could possibly hinder or promote the implementation of effective instructional practices, NCTM (2014) described a set of unproductive and productive beliefs about the teaching and learning of mathematics (see Table 3). In general, teachers with productive beliefs believe that lessons should engage students in solving and discussing tasks that promote reasoning and problem solving (NCTM, 2009). In contrast, teachers with an unproductive belief system believe that students should learn to apply mathematics only after they have mastered the basic skills (NCTM, 2014).

Table 3

Beliefs About Teaching and Learning Mathematics

Unproductive beliefs	Productive beliefs
Mathematics learning should focus on practicing procedures and memorizing basic number combinations.	Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.
Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.	All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.
Students can learn to apply mathematics only after they have mastered the basic skills.	Students can learn mathematics through exploring and solving contextual and mathematical problems.
The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.	The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.
The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.	The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.
An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.	An effective teacher provides students with appropriate challenges, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

Note. Adapted from *Principles to actions: Ensuring mathematical success for all*, by National Council of Teachers of Mathematics, 2014, Reston, VA: Author, p. 11.

Frameworks of Teacher Change

There are numerous research studies that have aimed at understanding how teachers' changes in practice relate to their beliefs. However, there is widespread disagreement as to how this process takes place. In the following sections, four frameworks of teacher change will be presented. Then a paradigm will be selected to view teacher change in this study.

Classroom observation as catalyst. Guskey (1986) conducted research on 117 teachers from two school districts and their use of mastery learning. Within the 52 teachers who were trained in mastery learning, 34 of them implemented the teaching method during the first school year. Guskey (1986) found that the teachers who implemented the procedures and saw positive results expressed greater attitudes towards teaching and a greater responsibility towards student learning. These results were not found in the control group or the group that did not see positive outcomes. As a result, Guskey (1986) suggested teachers' beliefs are formed from seeing practices result in increased student learning (see Figure 1). In support of this framework is the idea that teachers tend to repeat practices that are successful and end practices that are unsuccessful. Guskey (1986) argued that this is because attitudes and beliefs about teaching are largely derived from classroom experiences.

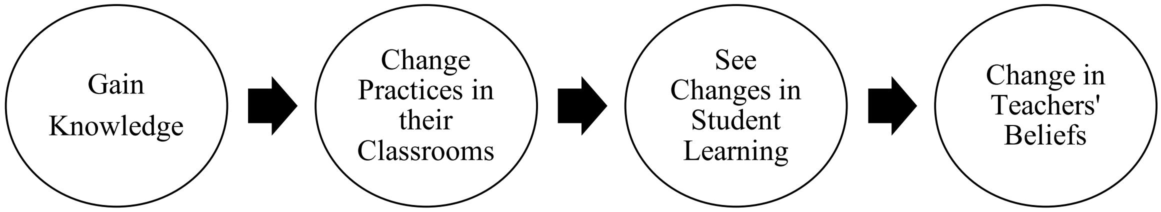

Figure 1. A model of the process of teacher change. Adapted from "Staff development and the process of teacher change," by T. R. Guskey, 1986, *Educational Researcher*, *5*, p. 7.

Change through extended professional development. Although Guskey's (1986) framework suggested that changes in student learning are necessary for teacher change, others have argued that beliefs must change prior to change in practices. For instance, Andreasen, Swan, and Dixon (2007) presented a framework of how teachers change their practice through extended professional development that results in a four-stage process (see Figure 2). First, teachers initially resist change and insist on continuing to do things as they have always been done. Second, Andreasen et al. (2007) posited that teachers then begin to talk about change and at least express some willingness to alter practices. Third, the framework suggests that teachers then duplicate activities presented at the professional development, but do not produce anything on their own. Finally, teachers change their practices. In this stage, teachers take what they have learned and apply it in their classrooms.

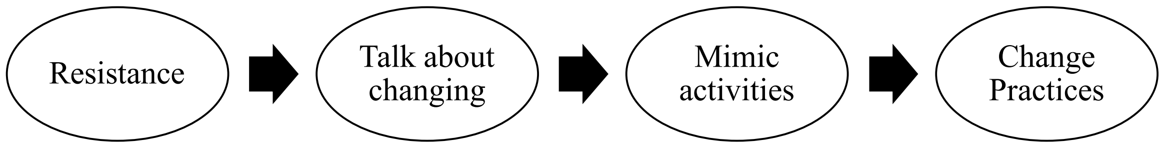

Figure 2. Stages of teacher change (Andreasen et al., 2007).

Continuous interplay between beliefs and practices. Cobb, Wood, and Yackel (1990) argued that Guskey's (1986) framework did not account for teachers' own reflection and rationalization of practices. Guskey (1986) assumed that teachers are merely receivers of knowledge from experts and only rely on experts' feedback and test scores for confirmation. In response, Cobb et al. (1990) recommended that there is not a linear causality between practices and beliefs, but rather a continuous interplay between practices and beliefs (see Figure 3). More specifically, Cobb et al. suggested that "beliefs are expressed in practice, and problems or surprises encountered in practice give rise to opportunities to reorganize beliefs" (p. 145).

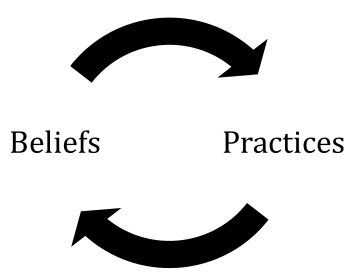

Figure 3. Beliefs and practices model (Cobb et al., 1990).

An interconnected model. Clarke and Hollingsworth (2002) posited that the linear models of teacher change aforementioned oversimplified the process of teacher growth and failed to capture the dynamic and interactive aspects of teacher change.

Moreover, unidirectional models did not recognize the individuality of every teacher's learning, but constrained teacher learning by describing it in a prescriptive linear manner (Clarke & Hollingsworth, 2002). In response, Clarke and Hollingsworth (2002) described an Interconnected Model that accounts for the possibility of multiple avenues of change through four domains (see Figure 4). In the Interconnected Model, the processes of enactment and reflection serve as the mediators by which change in one domain translates into change in another.

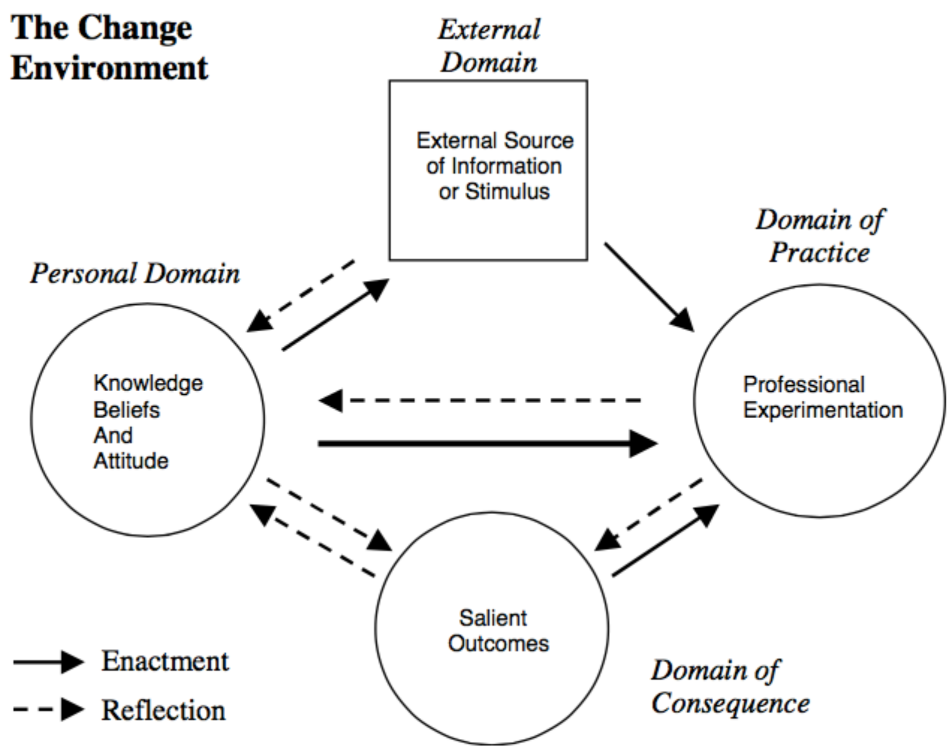

Figure 4. The Interconnected Model of professional growth. Reprinted from "Elaborating a model of teacher professional growth," by D. Clarke and H. Hollingsworth, *Teacher and Teacher Education*, *18*, p. 951, Copyright (2002), with permission from Elsevier.

The external domain represents the systems, information, and policies that shape teachers' learning (Clarke & Hollingsworth, 2002). For example, a teacher experiences a new teaching strategy related to group work during a professional development setting. According to Goldsmith, Doerr, and Lewis (2014), the personal domain represents teachers' characteristics such as attitudes, beliefs, dispositions, and knowledge. For example, a teacher's beliefs about the effectiveness of group work would exist in the personal domain. Next, the domain of practice signifies teachers' instructional practices (Clarke & Hollingsworth, 2002). For instance, a teacher may experiment with group work within his or her classroom. The domain of consequence represents the students' learning and other outcomes interpreted by teachers as consequences of their actions (Goldsmith et al., 2014). For example, a teacher might interpret an increase in student communication as a positive result of group work.

As teachers reflect or enact on changes in one domain, change in another domain occurs (see Figure 5). For example, after reading about strategies to maintain the demand of high cognitively demanding tasks (External Domain), a teacher enacts one of the strategies in his or her classroom (Domain of Practice) (Arrow 1). The teacher reflects upon the students' level of thinking through their discussions and interprets it as a result of the strategy (Domain of Consequence) (Arrow 2). Reflecting upon what happened, the teacher alters his or her beliefs related to this strategy (Personal Domain) (Arrow 3). The teacher then enacts these beliefs by continuing to incorporate this strategy in his or her classroom (Domain of Practice) (Arrow 4). Although this particular growth model is shown in Figure 5, there are many other possible paths within the interconnected model that can account for teacher change.

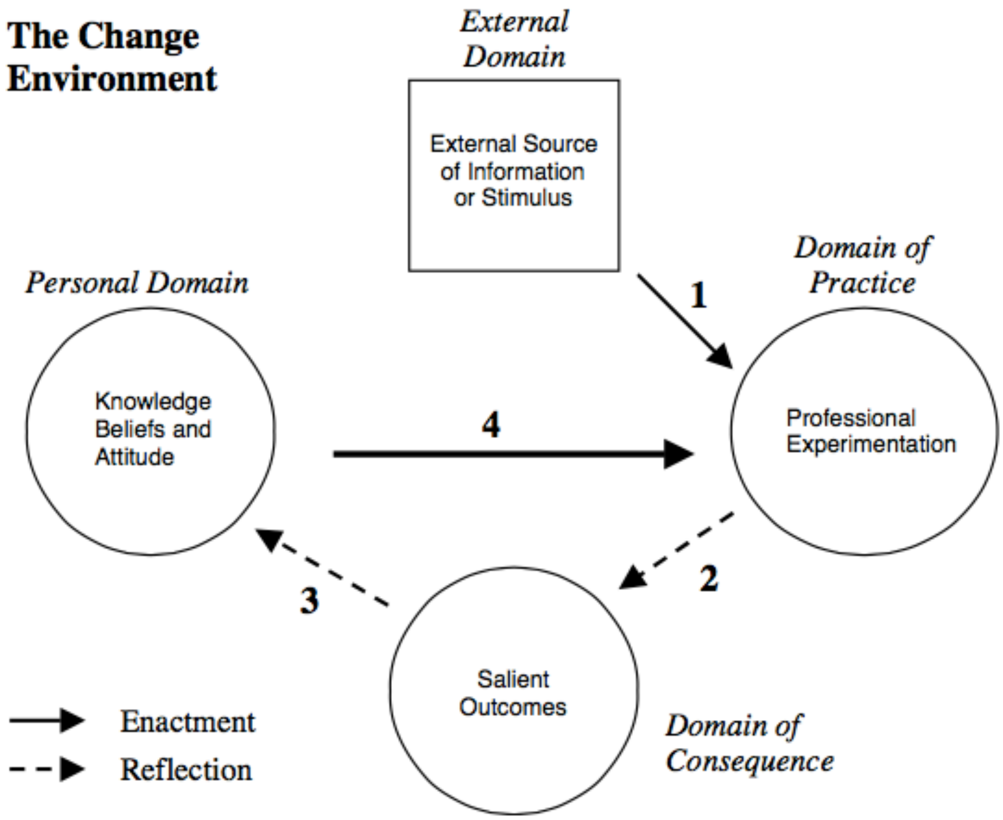

Figure 5. Sample growth model. Reprinted from "Elaborating a model of teacher professional growth," by D. Clarke and H. Hollingsworth, *Teacher and Teacher Education*, *18*, p. 961, Copyright (2002), with permission from Elsevier.

Change Environment

As teachers transition, their growth is situated within a change environment, which may influence teachers' transitions by hindering or promoting professional growth. Clarke and Hollingsworth (2002) identified four aspects of the school environment that can have a substantial impact on professional growth: access to opportunities or professional development, restriction or support for certain types of participation, support or opposition to experimentation with new teaching techniques, and administrative decisions related to long-term application of new ideas. The four aspects can promote or constrain any change that might occur in any one of the domains.

Summary

When teachers feel the motivation to transition to reform-oriented teaching, they need the support of their professional learning community, principal, and others (Lave & Wenger, 1991). Although cultural beliefs of parents and educators may hinder the implementation of these practices (Philipp, 2007), new images of mathematics learning can be developed through reflection and observation. Even with new images, transitioning from traditional methods of teaching to reform-oriented practices can be a very difficult task to achieve and sustain (Richardson, 1990). However, Wood et al. (1991) suggested that teachers can change their practices when given opportunities to learn within the classroom setting and observe new practices with their students. To view changes in beliefs and practices made by teachers, the Interconnected Model by Clarke and Hollingsworth (2002) was used as a framework, as it is a useful tool for describing reflection and enactment by teachers. Moreover, this model allowed the researcher to recognize the individuality of teachers' learning and account for the complexities of professional growth.

Professional Development

There are many professional development models that aim at initiating the forms of change described by the frameworks in the previous section. While the various types of models can differ in many aspects, researchers have identified sets of best practices in order to guide the development of professional learning opportunities. In this section, characteristics of effective professional development programs will be described. Next, a description of professional learning communities will follow.

Characteristics of Effective Professional Development

When assessing models of professional development, it is important to make comparisons to effective characteristics that have been identified. According to the literature, an effective professional development program should include: alignment with shared goals (school, district, and state) and assessment; a focus on core content and modeling of teaching strategies for the content; inclusion of opportunities for active learning of new teaching strategies; opportunities for collaboration among teachers; and inclusion of follow-up and continuous feedback (Archibald, Coggshall, Croft, & Goe, 2011; Desimone, 2009; Goldschmidt & Phelps, 2010). Similarly, Hill (2004) suggested that high-quality professional development should include: active or inquiry learning; examples for classroom practice; collaboration on the part of participants; modeling of effective and relevant pedagogy; opportunities for reflection, practice, and feedback; a focus on important content; a focus on student learning; and teacher involvement in planning. When comparing these two sets of recommendations, it is clear that both place emphasis on content, modeling, collaboration, and feedback.

Sztajn, Marrongelle, Smith, and Melton (2012) suggested that professional development programs aimed at supporting the implementation of CCSSM should engage teachers in both the CCSSM content and the CCSSM practices over an extended period of time, offer vivid images of mathematics teaching and learning that are consistent with CCSSM, foster strong working relationships among teachers, and use knowledgeable facilitators. In addition to these recommendations for professional development, NCTM (2000) suggested that the primary focus of professional development should be to help teachers teach their students using reform-oriented strategies. Moreover, NCTM (2000) advised programs to build teacher content knowledge, pedagogical knowledge, and knowledge of student thinking. Collectively, the research-based professional development practices identified in this section helped solidify the rationale for using lesson study in this study.

Professional Learning Communities

Professional learning communities (PLCs) have been acknowledged as a mechanism for school-embedded teacher professional development that contains many of the above effective characteristics (DuFour, DuFour, Eaker, & Many, 2010; Moss, Messina, Morley, & Tepylo, 2012). Currently, many public schools utilize PLCs to engage teachers in collaborative efforts to improve teaching (Vescio, Ross, & Adams, 2008). DuFour (2004) described a PLC as a "community of educators committed to working collaboratively in an ongoing process of collective inquiry and action research to achieve better results for the students they serve" (p. 10).

Hord (1997) identified five dimensions of successful PLCs: supportive and shared leadership; shared values and vision; collective learning and application of learning; shared personal practice; and results orientation. Within the context of CCSSM implementation, mathematics PLCs should work toward a shared vision of mathematics curriculum, instruction, and assessment tied to the school and district vision as aligned with the CCSSM expectations; engage in collective inquiry around rigorous mathematical practices and content, high-quality instruction, and formative assessment practices that provide meaningful feedback on student progress; remain focused on a collaborative action orientation, experimentation, and reflection by all team members; and use assessment data to guide continuous and formative improvement of student learning and teacher instruction (Zimmermann, Carter, Kanold, & Toncheff, 2012). Although PLCs have become increasingly popular, Murray (2013) argued that lesson study is one of the most effective strategies to build and sustain an effective PLC.

Lesson Study

There are a number of alternatives to traditional professional development that states and districts could use to aid teachers in understanding and implementing reform-oriented practices (Joyce & Calhoun, 2010; Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010). Lesson study is one alternative that has been shown to be useful for improving teaching (e.g., Huang & Li, 2009; Lewis et al., 2013; Lewis et al., 2006; Ricks, 2011; Takahashi et al., 2013; Yoshida, 2013). In general, lesson study consists of a group of teachers working collaboratively to plan, observe, and discuss lessons in order to improve their shared understanding of teaching, learning, and students (Lewis et al., 2011). However, there are a number of variations within the realm of lesson study. For example, some lesson study models suggest that the research lesson should be taught multiple times by one of the participating teachers (Huang & Han, 2015), while others recommend that re-teaching the research lesson is optional (Yoshida, 2013). Traditional lesson studies engage participants in live observations of the research lesson, while others settle for watching video of the lesson (Yoshida, 2013). Further, some designs involve a great deal of collaboration before the first teaching (Yoshida, 2013), while others suggest members wait until after the first research lesson to discuss making improvements (Huang & Bao, 2006). Regardless of the variations, one common theme remains:

developing lessons through a cyclical process of teaching and reflection that focuses on improving student learning. With this in mind, the purpose of this section is to further examine the advantages and obstacles to conducting lesson study in a general sense and suggest a model of lesson study that best fits transitioning to reform-oriented teaching practices. The two theoretical perspectives, situated learning theories (Lave & Wenger, 1991) and cognitive theories of learning (Remillard & Bryans, 2004) provided a lens by which to view the nature of lesson study and, thus, aid in making connections between lesson study and teacher learning.

To survey the literature available on the effectiveness of lesson study, articles were selected that focused on teacher learning as the result of lesson study. Beginning in March 2014, the researcher conducted a literature search within the EBSCO database. Articles were selected that appeared in the period between 1999 and 2014 in which the words *lesson study* or *exemplary lesson development* and *mathematics* were used in the title or the abstract of the paper. This search strategy resulted in 373 papers. Only relevant studies that clearly focused on teacher learning through lesson study were included (13). In order to synthesize the information provided, the papers were categorized into two topics: the advantages of conducting lesson study and the obstacles to effective lesson study. Once these are discussed, a Chinese lesson study model called *Keli* (Huang & Bao, 2006) is revealed as a promising solution to implementing reform-oriented practices.

Advantages

With respect to the advantages of lesson study, the following three characteristics emerged from the literature: context, collaboration, and opportunities to reflect. These three advantages to lesson study will be further explored in this section.

Context. Webster-Wright (2009) argued that professional learning should be related to practice, and that it should take place within the context of the classroom. Lesson studies operate based on what takes place within the classroom, with real students and in real-time. To examine this notion, Takahashi et al. (2013) observed six lesson study groups within Chicago Public Schools who were attempting to integrate teaching through problem solving (TTP) as a means to incorporate many of the mathematical practices described by CCSSM (CCSSI, 2010). In this study, the research lessons "brought to life TTP strategies in order to provide a more vivid and multi-dimensional experience" (p. 244). Teachers were able to observe students, collect data on student thinking, and discuss features of TTP and CCSSM with others who had viewed the same lesson. Takahashi et al. (2013) argued that lesson study allowed the teachers to form a common vision of what reform ideas actually look like in practice. This notion was in agreement with Yoshida (2013), who found that observing student learning during a live lesson within the classroom enriched the discussion and experience of professional development. In short, lesson study provides the necessary context recommended by situated learning theorists, in which teachers learn about reform ideas in a context that retains key complexities such as student characteristics, time constraints, materials, and the physical environment.

Collaboration. Teaching in the U.S. has been described as a private practice (Stigler & Hiebert, 1999). However, lesson study provides the opportunity for teachers to make it a public practice by working collaboratively to improve teaching while making ideas about new reform visible. In a case study with six teachers, Lewis et al. (2009) found that "lesson study enables teachers to strengthen professional community, and to build the norms and tools needed for instructional improvement" (p. 286). The structure of the lesson study, in which the lesson is a combined effort, supported participants in teaching outside of their traditional practices. As a result, these teachers developed their knowledge of mathematics and teaching as well as their ability to work with others.

These findings were further supported by Suh and Parker (2010), who found that varying expertise and abilities within a group of preservice and in-service teachers led to a sense of collective efficacy within the lesson study group, exemplifying an idea known as reciprocal learning (Patrick, Elliot, Hulme, & McPhee, 2010). The shared experience of these teachers helped them build collective knowledge as they worked collaboratively to comprehend and make sense of challenges. This shared experience provided an example of the social environment that cognitive learning theorists might suggest lesson study can provide. In fact, it is the discussions within the lesson study context that can create moments for teachers to reflect upon and alter their own practices to align with reform-oriented teaching (Jacobs, Franke, Carpenter, Levi, & Battey, 2007).

Reflection. Cognitive conflict can be an important factor within collaborative efforts of lesson study as teachers try to reach a group consensus on a particular matter. For example, cognitive conflict may occur as teachers try to decide what numbers to use in a problem or how to launch a particular task. Although conflict may seem like a disadvantage to lesson study, Perret-Clermont (1980) suggested that peer collaboration enhances the development of logical reasoning by recognizing knowledge caused by cognitive conflict. Identifying this knowledge is an important part of the reflection process during lesson study as teachers begin to reflect on their own practice and address areas that can be improved (Stigler & Hiebert, 1999). After all, teachers need to work together to understand the mathematics so that they can translate it from its original site and use it in another (Lerman, 2000).

As teachers begin to transfer what they have learned to their classroom, it is important to understand the reflection process. In a study by Ricks (2011), preservice teachers participated in a lesson study in which they taught one lesson to peers and a second lesson to high school students. The author used the process reflection framework (Dewey, 1933) to view how the preservice teachers effectively engaged in reflection during the lesson study. Process reflection contains four stages: experiential event, idea suspension and problem creation, idea formation, and idea testing. As described by the process reflection framework (Dewey, 1933), the author viewed reflection as a process of these stages by which participants generated and tested hypotheses. Each phase of the lesson study was connected to a stage of process reflection and revealed how their ideas were refined through moments of reflection and action. Although this study was not aimed directly at implementing reform-oriented practices, it revealed how substantial reflective experiences can provide a deeper level of engagement. Lesson study, as seen through this study, can be a viable option in enacting such opportunities to reflect. In particular, teachers can think about elements of instruction that are similar or different from their own current teaching, and thus connect reform ideas to their own practice (Takahashi et al., 2013).

Obstacles

A review of the literature revealed the following obstacles to lesson study: supporting community, rich curriculum resources, and expertise. These three obstacles to lesson study will be further examined in this section.

Supporting community. Once reform-oriented ideas are refined through the process of reflection, it is vital that the learned experiences of lesson study are shared with others. As Takahashi et al. (2013) explained:

> Public research lessons are an important way to spread ideas in the future, offering the possibility of rapid scale-up of curriculum materials, instructional strategies, and a learning structure (lesson study) that allows educators to experience first-hand not just materials and instruction, but also the culture and routines of a learning organization. (p. 250)

However, lesson study practitioners in the U.S. need to build a bigger community in which ideas and experiences can be shared. One of the major obstacles to creating such a community is time (Lewis, 2006; Parks, 2009; Yoshida, 2013). While Japanese teachers are given great support to continue lesson study (Yoshida, 1999), many U.S. teachers are not given the necessary support from their school districts or administrators. U.S. teachers are often overwhelmed with administrative tasks and grading papers, and not provided time to explore the benefits of lesson study (Lewis, 2006). Further, standardized testing and high administration turnover can make it difficult to conduct lesson study

(Yoshida, 2013). In order for the lesson study community to grow, teachers must be given time to conduct lesson study and observe research lessons.

Curriculum resources. With limited time, the U.S. lesson study groups that are formed often neglect the meticulous, yet critical process of *kyzaikenkyu* (Yoshida, 2013). In *kyzaikenkyu* (i.e., the first task of a lesson study), teachers investigate instructional materials to fully understand the mathematics and research-related teaching strategies. To help teachers successfully conduct *kyzaikenkyu,* Yoshida (2013) argued that teachers should be provided with the best curriculum materials available, which focus on strong content and pedagogical knowledge. Although this is ideal, many for-profit companies and non-profit organizations own resources for lesson study and only make them available to paid users (Lewis, 2006). As a result, Yoshida (2013) suggested that lesson study groups should examine the materials from Japan and Singapore that were used in the designing of CCSSM. These texts, which were written based on the results of *kyzaikenkyu* and lesson studies, would make it easier for lesson study members to see how mathematical concepts progress throughout grade levels (Yoshida, 2013). Regardless of the source, however, lesson study groups need to be provided with instructional materials that have the ability to reduce the time needed for *kyzaikenkyu* and, thus, maximize the effectiveness of the study.

Expertise. In order to conduct lesson study successfully, knowledgeable experts of the curriculum and lesson study experts must be involved (Yoshida, 2013). This creates a problem, as the U.S. lacks lesson study expertise and knowledge about research on teacher learning (Lewis, 2006). Without an experienced lesson study member, lesson studies might not be implemented as intended. One example might include teachers’ attitudes towards the collaborative lesson study process. Parks (2009) found that when teachers were not pressed to explain their thinking in a social setting, their underlying beliefs about the teaching and learning of mathematics were possibly strengthened instead of challenged. Further, Yoshida (2013) posited that a lack of pedagogical, curriculum, and content knowledge by teachers can lead to inappropriate lessons that do not reflect how the curriculum sequence builds student understanding, how student understanding progresses, and how different instructional methods and resources aid in the development of different types of mathematical proficiency. Although Japanese educators temporarily based in the U.S. have served as lesson study instructors (Lewis, 2006), more knowledgeable teachers need to be trained on how to conduct an effective lesson study in order to build expertise in the U.S.

Keli

In the midst of reform movements, lesson study can aid teachers in implementing new reform as teachers experience effective *kyzaikenkyu* and gain a deeper understanding of the content and instructional materials (Takahashi et al., 2013). To select a specific model for integrating new reform, it might be advantageous to specifically examine practices in China, as Huang and Li (2009) suggested that Chinese lesson study models focus more on teaching and learning related to new reform. Similar to CCSSM, a new mathematics curriculum in China has brought new standards as well as new perspectives towards teaching (Huang & Li, 2009). With the release of the National Mathematics Curriculum Standards (NMCS) (Education Department of P.R. China, 2001) in China, a lesson study model called *Keli* was developed to aid in its implementation (Huang & Bao, 2006). The model, originally described by Gu and Wong (2003), was first introduced to the U.S. by Huang and Bao (2006). There are two aspects that make *Keli* unique from other lesson study models. First, a participating teacher conducts the initial lesson independently, without any suggestions from the group. The purpose of this is to reveal differences between existing beliefs and new reform ideas. Second, the goal of each group meeting is to identify gaps between the research lesson and ideas presented by the new reform. Within this system, teachers can learn about the new reform, create and implement lessons informed by reform documents, and reflect upon and alter their own practice (Gu & Wong, 2003). *Keli*'s unique focus on integrating new reform into practice makes it a promising option for reform movements in the U.S.

Summary

Lesson study is one of the most effective strategies in building an effective PLC (Murray, 2013). The advantages of context, collaboration, and reflection within a lesson study can work cohesively to form a PLC environment that is ongoing (Moss et al., 2012) and not a discontinuous approach. Context allows teachers to experience new reform in a familiar environment that retains key complexities such as student characteristics, time constraints, materials, and the physical environment (Takahashi et al., 2013). The context provided by lesson study sets the stage for collaboration amongst its members and individual reflection upon practice (Takahashi et al., 2013). Collaboration makes new ideas visible and brings about change through cognitive conflict and meaningful discussion (Jacobs et al., 2007). Meanwhile, time to reflect and think about the lesson allows teachers to envision reform-oriented teaching within their own practices and alter underlying assumptions about the teaching and learning of mathematics (Lave & Wenger, 1991). Many of the features of lesson study discussed in this section are supported by the characteristics of effective professional development discussed previously, which further strengthened the credibility of lesson study.

Chapter Summary

Founded on the theoretical frameworks of situated and cognitive theories of learning, lesson study can be viewed as the means by which teachers dynamically construct or reinterpret new practices as well as create norms and tools for implementation (Lewis et al., 2009), which can then be enacted (Lave & Wenger, 1991) in their own classrooms. As teachers are motivated to reconsider their beliefs about the teaching and learning of mathematics, they need learning opportunities that align with shared goals, include opportunities for active learning of new teaching strategies, provide opportunities for collaboration among teachers, and allow adequate time for reflection (Hill, 2004). Most importantly, the primary focus of professional development should be to help teachers teach their students using reform-oriented strategies (NCTM, 2000). Although there are many frameworks on teachers' change, the Interconnected Model by Clarke and Hollingsworth (2002) was used in this study because it offers individualized pathways for describing reflection and enactment by teachers.

Together, the context, collaboration, and reflection provided by lesson study can aid teachers in avoiding the paradox of change without difference, and thus alter future practices to better reflect new reform ideas (Stigler & Hiebert, 1999). *Keli*, a Chinese approach to learning about new reform, provides an ideal system by which constant comparisons between current practice and reform-oriented practices can bring about these changes (Huang & Bao, 2006). Although there are a number of obstacles to conducting lesson study in the U.S., a supporting community is very important (Yoshida, 2013).

Given adequate time and support from administration, the community of lesson study practitioners can grow and with it will come the resources and expertise teachers need to conduct lesson study effectively in the U.S. (Lewis, 2006).

CHAPTER III: METHODOLOGY

Introduction

International comparison tests have revealed that the U.S. is not adequately preparing students to complete cognitively demanding tasks (OECD, 2012). As a result, a cultural change in teaching is necessary (Hiebert & Stigler, 1999). Although many traditional professional development programs have been characterized as being discontinuous and not focused on practice-based teaching and learning (Yoshida, 2013), lesson study offers some unique advantages that make it a promising alternative for conceptualizing and implementing reform-oriented practices (Lee & Ling, 2013). The three advantages described by the literature (i.e., context, collaboration, and reflection) provide a glimpse into the rationale for lesson study, but they lack specificity related to the development of reform-oriented practices. Therefore, the purpose of this qualitative study was to explore how lesson study can be used to aid teachers in conceptualizing and implementing the Mathematics Teaching Practices (NCTM, 2014) and investigate how teachers' perceptions of reform-oriented teaching change while participating in lesson study, if at all.

This chapter will begin with an overview of the research design and rationale for its selection. Then a description of the context of the study will be outlined, including detailed information about the state, district, school, grade-level, and lesson study group in which this study took place. This is followed by a depiction of the participants of this study as well as the instruments and procedures that were used to collect the data. Finally, a detailed account of how the data was analyzed and how this information was used to answer the research question will follow.

Research Overview

This study examined the following three research questions related to teachers transitioning to reform-oriented practices.

1. How does lesson study support teachers in implementing the Mathematics Teaching Practices, if at all?
2. How do teachers' conceptions of the Mathematics Teaching Practices change while participating in lesson study, if at all?
3. How do teachers' perceptions of reform-oriented teaching practices change while participating in lesson study, if at all?

To address the first research question, a *Keli* lesson study was conducted to qualitatively explore how lesson study can aid in the implementation of reform-oriented teaching practices. The second and third research questions were explored through interviews with the participating teachers and reflective journaling. The methodological design of this study is best characterized as an explanatory, embedded case study (Yin, 2014). A case study design was selected because it allowed for an in-depth description of how each teacher changed, if at all. Moreover, case study is a commonly used methodological approach to examining teacher change (Farmer, Gerretson, & Lassak, 2003; Goldsmith et al., 2014). An explanatory case study was used because the researcher attempted to explain presumed causal links between lesson study and teacher change that are too complex for survey or experimental methods (Yin, 2014). An embedded case study design was appropriate for this study because it allowed the researcher to examine the participants both as a group and as individuals. In particular, the embedded cases within the case study provided the researcher opportunities to examine specific phenomena within the larger group, what is known as subunit analysis (Yin, 2014). Given that the lesson study was a collaborative effort of the participants, the researcher examined the participants' lived experiences holistically as well.

Research Context

Describing the context in which this study was conducted is crucial because elements within the environment or culture may influence behavior (Lewin, 1935). Moreover, situated learning theory views knowledge as arising conceptually through dynamic construction and reinterpretation within a particular social context (Clancey, 2009). The participants in this study taught and collaborated within a certain context comprised of their state, district, school, and grade level. Each description is of particular interest because it may provide insight into the decisions that were made and behaviors that were observed in this study. A detailed account of the context follows.

State

This study took place within a southeastern state that has traditionally underperformed on national mathematics tests. This state ranked near the bottom on the mathematics subsection of the ACT in 2014 and only 30% of the students from this state met the mathematics benchmark score, compared to 43% nationally (ACT, 2014). With students scoring lower in mathematics than any other ACT subsection for five consecutive years, state officials identified drastic measures in an effort to seek change. First, state officials adopted the CCSSM, which they renamed as the state standards. Second, officials conducted the largest professional development program in the state's history to aid with the implementation of the state standards. Through this training, each teacher teaching a CCSSM course participated in at least one four-day workshop to learn about the CCSSM and its implications for teaching mathematics. These workshops were not held within the context of the classroom and primarily focused on the CCSSM standards and suggested teaching methods. It is within this state that this study was conducted.

District

This study was conducted in a suburban school district located outside one of the largest cities in this state. In 2015, this school district was one of the largest in the state, with more than 41,000 students enrolled. The enrollment demographics of the school district included 64.9% white students, 18.6% black or African American, 11.6% Hispanic, 4.5% Asian, and .2% Native American. Meanwhile, 39.7% of students in this district were economically disadvantaged and 11% required special education. The graduation rate within this district was approximately 94%. With respect to performance, students in this district scored almost a whole point above the state average on the ACT in 2014. Moreover, 32% of students in this district met the ACT college readiness benchmark in mathematics, compared to 30% statewide and 43% nationally. Approximately 66% of students in grades 3-8 mathematics scored proficient or advanced on the end-of-year assessments in 2015, earning the district an A in value-added or growth for this category.

School

The previously mentioned reports for grades 3-8 are important because this study took place at Tamarind Middle School (Pseudonym). In 2015, this school enrolled approximately 1,200 students in grades six through eight, of whom 73.2% were white, 15.8% were black or African American, 5.7% were Hispanic, and 5.1% were Asian. In addition, 28.7% of students at Tamarind Middle School were economically disadvantaged, while 12.4% had disabilities and 1.8% were English language learners. In 2015, 62% of students at Tamarind Middle School scored proficient or advanced on the end-of-year mathematics assessments, compared to 65.8% district-wide. However, Tamarind Middle School earned an A in mathematics achievement each of the previous three years. Meanwhile, the school saw a decline with respect to the Explore test, by meeting the Standard for Academic Growth in 2013 and 2014, but failing to meet it in 2015 (0.03, 0.3, -0.14).

Eighth Grade

This study explored the lived experiences of three eighth-grade teachers from Tamarind Middle School. Over the previous three years, the eighth-grade mathematics value-added for Tamarind Middle School had been 2.1 (2013), 3.2 (2014), and 2.5 (2015). Each of these scores indicated that students made substantially more progress during that year than the Standard for Academic Growth predicted by the state (score of zero). According to the state standards, teachers in the eighth grade should focus student learning on the following content domains: the number system, expressions and equations, fractions, geometry, and statistics and probability (CCSSI, 2010).

Eighth grade was selected for two reasons. First, eighth-grade mathematics teachers in this state were not required to have a degree in mathematics. Instead, they were required to have a degree in middle school education (grades 4-8). As a result, eighth-grade teachers typically had a background in general pedagogy, but one that was most likely not specific to mathematics. This created a unique opportunity to study teachers who do not typically have a strong background in mathematics pedagogy.

Second, the new state standards (formerly referred to as CCSSM) described mathematically rich content. Therefore, eighth-grade teachers needed opportunities to learn pedagogy related to these content areas. Specifically, the standards indicated that instructional time should primarily focus on three areas.

> Formulating and reasoning about expressions, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; grasping the concept of function and using functions to describe quantitative relationships; analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem. (CCSSI, 2010)

Providing opportunities to develop pedagogical and content knowledge related to these areas was crucial to the development of teachers.

PLC Group

A PLC is a community of educators who work collaboratively in an ongoing process of collective inquiry and action research to attain better results for students (DuFour, 2004). Teachers in this district were required to participate in a PLC. The three participants in this study worked together in a PLC group. They met weekly and collaborated on common formative assessments, sequencing and pacing, and sometimes specific resources such as worksheets or tasks. However, the PLC group had not participated in lesson study or regularly observed each other teach prior to this study. The PLC norms included: work collaboratively and keep student learning at the center of discussions; be engaged in all meetings by bringing all necessary materials and being prepared to collaborate; share your voice in a respectful manner; operate in a friendly atmosphere; stay solution-oriented; and avoid discussing confidential information. This PLC group as a whole served as the holistic case in this study as they participated in lesson study.

Knowledgeable Other

Studies have suggested the importance of including knowledgeable experts in lesson study groups (Groth, 2012; Huang, Li, & Zhang, 2011). As a result, Dr. Ross (Pseudonym), a mathematics education professor, guided the lesson study group in this study. Dr. Ross was selected because of his familiarity with the district and his availability. Dr. Ross was a former high school mathematics teacher who focused his research on mathematics classroom teaching cross-culturally and teacher learning through lesson study. He had extensive experience with conducting lesson studies and drawing teachers' attention towards student learning.

Pilot Study

In a separate study by Huang, Prince, Barlow, and Schmidt (in press), a lesson study group in this district used a form of *Keli* to address the implementation of CCSSM. The practicing teacher in this study developed competencies in supporting the Standards for Mathematical Practice (CCSSI, 2010) through appropriately launching and effectively implementing mathematically worthwhile tasks and strategically orchestrating student discussion. This lesson, created through a *Keli* lesson study, displayed characteristics of high quality instruction and was aligned with the *Principles and Standards for School Mathematics* (NCTM, 2000), the Mathematics Teaching Practices (NCTM, 2014), and the Standards for Mathematical Practice (CCSSI, 2010). The final exemplary lesson (Huang, Prince, & Schmidt, 2014) demonstrated important features, which met the expectations of the CCSSM (CCSSI, 2010) and thus, the goal of the lesson study group. Moreover, the teacher's personal view of what constitutes an effective lesson changed throughout the process. The teacher in their study stated in an interview:

> I realized that every change we made aligned with the process standards (NCTM, 2000): representation, reasoning and proof, presentation, problem solving, and communication. By making a connection between the changes in the lesson and the process standards, I am able to better identify strengths and weakness of a lesson. (Huang et al., in press, p. 21)

The results of this pilot study were important because time, support from administration, high quality instructional materials, and a knowledgeable lesson study instructor were all provided and led to the success of the lesson study. Therefore, even though obstacles to lesson study likely existed, this study provided evidence to show that it was indeed feasible in this particular district.

Participants

This case study was bound to a group of three Tamarind Middle School eighth-grade mathematics teachers. Each of the three participants served as embedded cases (Yin, 2014) within the single-case study. The three participants were the only eighth-grade mathematics teachers at Tamarind Middle School. This group of teachers was selected due to their availability and connection to a university faculty member. The faculty member led a professional development program called FormUp (Pseudonym), in which two of the teachers were participants. FormUp focused on developing teachers' abilities to implement reform-oriented teaching strategies, including those described in *Principles to Actions* (NCTM, 2014). Together, these three participants formed a lesson study group, which was considered the holistic case in this study. A description of each participant and their PLC follows.

Mark Gibson (Pseudonym)

Mark had been teaching mathematics for six years, four of which had been at Tamarind Middle School. He had been teaching eighth-grade mathematics for four years. Mark held a bachelor's degree in Middle School Education (4-8) with a mathematics endorsement. He had been participating in FormUp for the last year. In addition to his teaching responsibilities, Mark coached both football and track and field.

Sally Mills (Pseudonym)

Sally had been teaching mathematics for three years. This was her first year at Tamarind Middle School, but her third year teaching eighth-grade mathematics. Sally held a bachelor's degree in sports management with a minor in business administration. She also had a master's degree in curriculum and instruction. Sally had not been a participant in FormUp.

Britney Smyth (Pseudonym)

Britney had been teaching mathematics for five years, all of which had been at Tamarind Middle School. Britney held a bachelor's degree in Middle School Education (4-8) and had also been participating in FormUp for the last year. Britney was the leader of the PLC and their meetings typically occurred in her classroom.

Instruments and Data Sources

Multiple sources of data were collected before, during, and after the lesson study cycle to corroborate evidence and achieve data triangulation (Creswell, 2007). To form a detailed account of the lived experiences of the three participants, classroom observations and semi-structured interviews were conducted. In addition, the researcher collected research lesson plans, relevant student work, lesson video, audio of post-lesson debriefing sessions, written comments from the teachers, reflective journals, and field notes. Each of the preceding data sources will be described in the following sections. In addition, Table 4 summarizes the data sources in relation to the research questions. This is followed by the role of the researcher in this study.

Observation Protocol

The researcher developed the Observation Protocol (see Appendix A) to examine the alignment of each lesson with the Mathematics Teaching Practices (NCTM, 2014). A summary of each practice and its supporting actions (see Appendix B) was attached to the Observation Protocol to guide the thoughts of the researcher. To create the Mathematics Teaching Practices Summary, the researcher used specific actions described by NCTM (2014) to determine what constitutes high quality lessons that align with the Mathematics Teaching Practices. The protocol was pilot tested with three middle school lessons. Reflecting on the appropriateness of the instrument during pilot tests further aided the development of the instrument.

Data Sources

Research lesson plans. Participants' lesson plans for the research lesson were collected throughout the *Keli* lesson study process to analyze what changes they made and how observing and teaching the lesson influenced these decisions. Lesson plans also provided an initial basis by which to view the cognitive demand of the research lesson as well as the Mathematics Teaching Practices (NCTM, 2014). Participants submitted his or her research lesson plan each time significant changes were made.

Student materials. During the lesson study, any necessary student materials such as worksheets and individual work were collected after each lesson. This evidence was used to form a richer account of student understanding and, thus, the opportunities to learn that each lesson provided students.

Videos. Video of each lesson was recorded so that the researcher could confirm and make additions to field notes after the lesson. Also, video recordings were used to document specific statements that were made. The pre- and post-lesson study observations were also video-recorded to provide better accounts of each lesson.

Audio-recordings. Audio-recordings of post-lesson debriefing sessions provided additional accounts of participant statements and how ideas discussed in these sessions transformed the research lesson. These recordings were conducted using a handheld digital recorder and were transcribed for analysis purposes.

Written comments. During each lesson, written notes from the observing teachers were collected using the Observation Protocol. The Observation Protocol notes allowed the researcher to see how the observing teachers thought the lesson addressed the Mathematics Teaching Practices (NCTM, 2014). In addition, written comments from participants including the strengths and weaknesses of the lesson and suggestions for improvement were gathered (see Appendix C). The individual comments from the participants allowed the researcher to understand the thoughts of each individual participant that were not made apparent in the post-lesson debriefing sessions.

Self-reflection journals. Throughout the various stages of the lesson study, the participants completed an online self-reflection journal (see Appendix D). In general, the self-reflections focused on how lessons supported the Mathematics Teaching Practices

(NCTM, 2014) and how the participants' views toward the teaching and learning of mathematics changed, if at all. Therefore, the participants' answers to the self-reflection journals were vital to answering both research questions.

Interviews. A semi-structured interview with the participants was conducted before (see Appendix E) and after (see Appendix F) the lesson study. The interviews focused on soliciting participants' views of mathematics teaching and learning, their views towards changes of the instructional design, their conceptions of the Mathematics Teaching Practices (NCTM, 2014), and the usefulness of lesson study. A semi-structured interview protocol allowed the researcher to probe and ask clarifying or follow-up questions as necessary to reveal the participants' views about the teaching and learning of mathematics. Each interview was transcribed to aid analysis.

Table 4

Research Questions in Relation to Data Sources

Research Question	Primary Data Sources
1. How does lesson study support teachers in implementing the Mathematics Teaching Practices, if at all?	Classroom Observations Research Lesson Plans Interviews Self-Reflections
2. How do teachers' conceptions of the Mathematics Teaching Practices change while participating in lesson study, if at all?	Interviews Self-Reflections Observation Protocol Audio of Debriefing Sessions
3. How do teachers' perceptions of reform-oriented teaching practices change while participating in lesson study, if at all?	Interviews Self-Reflections Observation Protocol Audio of Debriefing Sessions

Note. Additional data sources were used when appropriate.

Role of the Researcher

Since the field notes and semi-structured interviews involved interpretation and guidance, the researcher was an instrument in this study (Creswell, 2007). The qualifications of the researcher included multiple experiences with lesson study, as the participating teacher and as a collaborating group member. In addition, the researcher had worked on numerous studies related to lesson study and collaborated on a state-funded grant to introduce teachers to the Mathematics Teaching Practices (NCTM, 2014) and lesson study. Six years of high school teaching experience and four years of coursework toward a Doctor of Philosophy degree in mathematics education provided the researcher with a familiarity of teaching mathematics as well as qualitative research methodology. Moreover, the researcher had extensive experience conducting qualitative research, including multiple publications as well as national and international presentations. Taken collectively, these experiences prepared the researcher for serving as an instrument in this study.

Procedures

The procedures of this study will be described in the paragraphs that follow. This section was organized according to activity: before the lesson study, reading assignment, lesson study, and after the lesson study. A research timeline is included to provide clarity as to when each event took place (see Table 5). It is important to note that prior to data collection, the researcher received approval from the Institutional Review Board to conduct this study (see Appendix G). Permission to conduct this study was also obtained from the school district.

Table 5

Research Timeline

Dates	Phase	Activities
September 3 – September 23	Before the Lesson Study	Pre-Interview Pre-Observation Self-Reflection 1
September 23 – October 14	Reading Assignment	Read *Principles to Actions* (NCTM, 2014) p. 5-57 Self-Reflection 2
October 14 and October 27	Planning Meeting 1 and 2	Self-Reflection 3 Discussed the reading and selected norms, goals, and content Discussed and planned the lesson
November 2	Research Lesson 1	Research Lesson 1 and Debrief Session Self-Reflection 4
November 6	Research Lesson 2	Research Lesson 2 and Debrief Session Self-Reflection 5
November 9	Research Lesson 3	Research Lesson 3 and Debrief Session Self-Reflection 6
November 18 – November 23	After the Lesson Study	Post-Observation Self-Reflection 7 Post-Interview

Before the Lesson Study

During the month of September, the researcher collected data to better understand the participants' thoughts and practices prior to the lesson study. To begin, each participant completed Self-Reflection 1 (see Appendix D) by reflecting upon their current views towards the teaching and learning of mathematics. The researcher then observed each of the three participants within their individual classrooms, utilizing the Observation Protocol (see Appendix A) and Mathematics Teaching Practices Summary (see Appendix B). The participants chose when they were observed and were asked to implement what they believed to be an exemplary lesson. Each of the three observation lessons focused on the same content: reflecting figures across lines of reflection.

Following the observed lessons, each participant completed Self-Reflection 2 (see Appendix D), which required his or her reflection upon the strengths, weaknesses, and suggestions for improvement of their lesson. Finally, a semi-structured interview (see Appendix E) was conducted with each participant to examine his or her initial perception of the teaching and learning of mathematics as well as his or her conception of the Mathematics Teaching Practices (NCTM, 2014) prior to the lesson study.

Reading Assignment

After the initial observation and interview, each of the participants read a detailed account of the Mathematics Teaching Practices in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014). They were assigned to read pages 5 – 57, which describe each of the practices, provide examples to illustrate the ideas, and outline supporting teacher and student actions. The teachers then individually completed Self-Reflection 3 (see Appendix D) to reveal which practices their beliefs and actions agreed or disagreed with most and how their observation lesson aligned with the practices. After self-reflection, the participants discussed the reading as a group during the first lesson study meeting, which took place in mid-October. A detailed description of the lesson study, including the first meeting, follows.

Lesson Study

During the months of October and November, the three participants participated in one round of *Keli* lesson study, consisting of three cycles. This lesson study included five key stages: Planning Meeting 1, Planning Meeting 2, teaching and observing the research lesson, revising and re-teaching the lesson, and summarizing the learning. The lesson study group consisted of Britney, Mark, Sally, and Dr. Ross. Each of the phases will be described in the following sections.

Planning Meeting 1. At the beginning of the lesson study, the lesson study group met to establish group norms, discuss the reading, and select goals. The meeting began with discussing group norms by considering the following questions: What are our expectations for how we will work together?; What conditions will contribute to our learning?; What conditions will create and sustain a sense of belonging and support?; and How will we resolve our differences and disagreements? The group discussed these questions and decided to use their current PLC norms. However, Dr. Ross emphasized the importance of providing feedback that is directed towards the lesson, not the teacher.

Once the group norms were set, the group discussed the reading of *Principles to Actions* (NCTM, 2014) by responding to the following questions: Which of the practices do you agree with the most?; Which of the practices do you agree with the least?; What are the obstacles to implementing the practices?; Which of the practices is most challenging to implement? The group agreed that implementing tasks that promote reasoning and problem solving as well as creating opportunities for productive struggle are the most difficult to achieve.

The participants then decided on a research theme by examining the following questions: What qualities would you like your students to have by the end of the year?; What are your students' qualities now?; and What would you like to learn about student learning? Lewis and Hurd (2011) suggested that these types of questions help participants locate the most meaningful focus for lesson study. Moreover, the research theme is "usually a broad goal that is compelling to teachers from all grade levels and many points of view, such as building students' desire to learn" (Lewis & Hurd, 2011, p. 43). Following the discussion about implementing tasks that promote reasoning and problem solving and productive struggle, the group decided to focus the research theme on developing students as problem solvers. Sally commented that she wanted students to be able "to think and work through things on their own. Because I feel like they are kind of lazy. They don't like to think through things and figure things out for themselves" (Planning Meeting 1, 10/14/15).

The participants then decided on a topic by examining the following questions: What topics or concepts are often difficult for students to learn?; What topics or concepts do teachers find most difficult to teach?; Are there any standards that you want to understand better?; What topics are being taught during the duration of the lesson study?; and What topics are essential to the curriculum? (Lewis & Hurd, 2011). The group chose systems of linear equations because, as Mark said, "It is a hard thing for them to understand at this level. Plus, it is around this time, so it flows with us" (Planning Meeting 1, 10/14/15). Systems of equations is an important topic in eighth grade as it is included in the first of three focus areas according to the state standards. The participants had already selected a few tasks that could be used in the lesson. The remaining discussion focused on narrowing the number of possible tasks down to two: the Umbrellas and Hats Task (see Appendix H) and the Fruits and Vegetables Task (see Appendix I).

Although Mark volunteered to develop and teach the first research lesson, each teacher was asked to develop a lesson plan in isolation prior to Planning Meeting 2. This allowed the researcher to track changes that each participant made to his or her lesson plan during the lesson study. Also, it allowed the researcher to make comparisons between each participant's lesson plan and the Mathematics Teaching Practices (NCTM, 2014).

Planning Meeting 2. The lesson study group met a second time to allow the participants to discuss the lesson. Dr. Ross directed the discussion by allowing each teacher to share his or her lesson plan with three questions in mind: What are your content goals for the lesson?; What does the implementation of the task look like?; and Is there anything you need help with? Once each participant discussed his or her thoughts on the lesson, the discussion primarily focused on setting content goals for the lesson. The broad content goal of systems of linear equations was already determined, but there was confusion about whether to focus on the idea of substitution or simply introduce students to the idea of systems of linear equations. At this point, Mark was given permission to take charge of the lesson and change it as he saw fit.

Teaching and observing the research lesson. As Mark taught the first research lesson, the researcher, Britney, Sally, and Dr. Ross observed the lesson using the Observation Protocol (see Appendix A) and Mathematics Teaching Practices Summary (see Appendix B) and provided written comments on the strengths and weaknesses of the lesson as well as suggestions for further revision. The three participants then met to debrief the lesson and share their ideas. Dr. Ross facilitated the debriefing sessions and provided additional feedback when necessary. Afterwards, each participant completed Self-Reflection 4 (see Appendix D) by reflecting upon how Research Lesson 1 supported or opposed the Mathematics Teaching Practices and what they learned by observing or teaching the lesson.

Revising and re-teaching the lesson. Sally took the suggestions from the lesson study group and made revisions to her lesson plan with Research Lesson 2 in mind. She taught the lesson a second time, but with a different set of students. The same process of debriefing, writing a self-reflection (see Appendix D), making revisions, and re-teaching was then repeated to include a third and final lesson that Britney taught. After the third lesson, the participants debriefed and completed Self-Reflection 6 (see Appendix D).

Summarizing learning. At the end of the Lesson 3 Debrief, the participants discussed and summarized the results of the lesson study. The discussion was guided by these questions: “What did we learn about the subject matter and about the curriculum?; What did we learn about student thinking and about teaching?; What insights did we gain from this lesson study about productive habits in our learning practices as teachers?” (Lewis & Hurd, 2011, p. 63). Teachers were then given a chance to share any other thoughts with the group.

After the Lesson Study

Once the lesson study was completed, the researcher observed each of the three participants using the Observation Protocol (see Appendix A) and Mathematics Teaching Practices Summary (see Appendix B). Each of the three post-observation lessons focused on the same content: slope and converting from standard form to slope-intercept form of linear functions. These observations allowed the researcher to examine each participant to see how the lesson study altered their implementation of the Mathematics Teaching Practices (NCTM 2014), if at all. The researcher then completed a final semi-structured interview (see Appendix F) with each participant. This interview focused on examining participants' changes in conception and perception of the Mathematics Teaching Practices (NCTM, 2014). Finally, each participant completed Self-Reflection 7 (see Appendix D), which asked participants to reflect upon their observation lesson.

Data Analysis

Detailed case descriptions were written for each participant, in which changes that the participants made throughout the lesson study process were described. Changes were defined to be any differences that were found between teachers' implementations, conceptions, and perceptions prior to the lesson study and their implementations, conceptions, and perceptions after the lesson study. Taken collectively, the participants' experiences were analyzed holistically as a lesson study group through common themes that emerged from the individual cases. Qualitative software was used to aid the coding and analysis of the embedded and holistic cases. Throughout the study, the researcher tried to appropriately deal with ethical considerations and issues of trustworthiness. Even still, there are always limitations that arise and therefore were anticipated and made apparent. These issues will be delineated in the following sections.

Embedded-Case Analysis

A detailed account of each participant's lived experiences was outlined through case descriptions (Yin, 2014). For each case description, a chronological narrative was written and major themes were extracted. The main data sources that were used to analyze the embedded cases were pre- and post-lesson study observations, interview responses, research lesson plans, and self-reflection responses. The three research questions served as a foundation on which the interview and self-reflection questions were designed (see Table 6).

Table 6

Research Questions in Relation to Interview and Self-Reflection Questions

Research Question	Interview or Self-Reflection Question
1. How does lesson study support teachers in implementing the Mathematics Teaching Practices, if at all?	R13, R14, R17-R18, R21-R22, R25-R26, R32, R23, R24, R32, B3, A4, A14-A16
2. How do teachers' conceptions of the Mathematics Teaching Practices change while participating in lesson study, if at all?	R8-R9, R14, R15-R16, R18, R19-R20, R22, R23-R24, R26, R30-R31, B2, B4-B12, A6-A16
3. How do teachers' perceptions of reform-oriented teaching practices change while participating in lesson study, if at all?	R1-R7, R10-R12, R14, R18, R22, R26-R29, B1, A1-A3, A5, A14-A16

Note. R = Self-Reflection, B = Before Lesson Study Interview, A = After Lesson Study Interview.

Classroom observations provided evidence of implemented practices that supported or opposed participants' statements. Pre- and post-observations and participant responses to questions before and after the lesson study were used to identify changes that were made, while responses during the lesson study were used to further support those claims. The Mathematics Teaching Practices Summary (see Appendix B) was used to aid in categorizing the data by Mathematics Teaching Practice (NCTM, 2014). To code data during the lesson study, open coding analysis (Yin, 2014) was used to discern relevant concepts within each Mathematics Teaching Practice (NCTM, 2014) (Iteration 2) (see Table 7). This allowed the researcher to identify evidence of changes that were made within each Mathematics Teaching Practice (NCTM, 2014) during the lesson study. Quotes and actions that occurred during the lesson study were coded according to these codes, and the researcher then narrowed the codes based on frequency (Iteration 3). The resulting data provided an auditable trail to support judgments made during data analysis concerning changes related to these specific teaching practices.

Table 7

Embedded-Case Codes

First Iteration: Mathematics Teaching Practices		
1. Establishing mathematics goals to focus learning 2. Implement tasks that promote reasoning and problem solving	3. Use and connect mathematical representation 4. Facilitate meaningful mathematical discourse 5. Pose purposeful questions	6. Build procedural fluency from conceptual understanding 7. Support productive struggle in learning mathematics 8. Elicit and use evidence of student thinking
Second Iteration: Initial Codes		
1a. Connect to prior knowledge 1b. Detail in goal setting 1c. Goals guide instructional decisions 1d. Referencing goals throughout lesson 1e. Situate goals within learning progression 1f. Students reflect upon goals 2a. Cognitive demand 2b. Tasks that promote high-order thinking	3a. Connections between representations 3b. Contextual representations 4a. Explanation and justification 4b. Facilitate mathematical discourse between students 5a. Questioning strategies 5b. Planning questions ahead of time	6a. Conceptual then procedural 6b. Students use their own reasoning strategies 7a. Learning from mistakes 7b. Persevere through problem solving 7c. Reveal common misconceptions 8a. Make in-the-moment decisions based on student thinking 8b. Share student work with other students
Third Iteration: Final Codes		
1a. Referencing and reflecting upon goals throughout lesson 1b. Situate goals within learning progression 1c. Detail in goal setting 1d. Goals guide instructional decisions	3a. Contextual representations 3b. Connections between representations 4a. Facilitate mathematical discourse 5a. Questioning strategies	6a. Conceptual then procedural 7a. Learning from mistakes 7b. Persevere through problem solving 8a. Share student work with other students

Implementations. The researcher began analyzing the embedded cases by comparing the implementation of the Mathematics Teaching Practices (NCTM, 2014) between pre- and post-lesson study observations. The researcher analyzed video of both pre- and post-observation lessons, listed occurrences of the Mathematics Teaching Practices (NCTM, 2014), and wrote summaries of each lesson. The corresponding Observation Protocols written during the lessons were used to corroborate evidence of the practices. After comparing and contrasting teaching practices within the two lessons, main differences between the lessons were identified for each participant.

Conceptions. With respect to how participants conceive the Mathematics Teaching Practices (NCTM, 2014), the researcher began by comparing and contrasting pre-interview questions B5-B12 with post-interview questions A6-A13. These questions elicited participants' thoughts about each of the Mathematics Teaching Practices (NCTM, 2014). Specifically, the researcher asked participants what each practice meant to them at that point in time. To detect differences, the researcher read the responses and labeled ideas found in the response that corresponded to main ideas or actions found on the Mathematics Teaching Practices Summary (see Appendix B). Once the main ideas were identified for each Mathematics Teaching Practice (NCTM, 2014), the researcher searched for ideas expressed in the post-interview response that were not present in the pre-interview response. Statements and actions made during the lesson study were then used to support these findings. Corroborating evidence included the participant recognizing the practice, ideas about the practice, and comments about learning related to the practice.

Perceptions. The researcher initially identified changes in participants' perceptions of reform-oriented instruction by comparing and contrasting responses to questions R1, R2, and B1 with responses to questions A1-A3, which asked participants what their views were about how to best teach and learn mathematics. Participants' views about the teaching and learning of mathematics were used to identify changes in their perceptions. Each idea presented in the pre- and post-lesson study responses were coded by Mathematics Teaching Practice (NCTM, 2014), then grouped accordingly so that the researcher could identify differences between the two responses. To detect differences, the researcher read the responses and labeled ideas found in the response that corresponded to main ideas or actions found on the Mathematics Teaching Practices Summary (see Appendix B). Once the ideas were identified for each Mathematics Teaching Practice (NCTM, 2014), the researcher searched for ideas expressed in the post-interview response that were not present in the pre-interview response. Statements and actions made during the lesson study were then used to support these findings. Corroborating evidence included the participant beliefs about the practice, statements about effective teaching, and comments about the success of a practice.

Holistic Analysis

The holistic analysis took place in two forms. First, the researcher examined the lesson study group as a whole to see how the collaboratively designed lessons aligned with the Mathematics Teaching Practices (NCTM, 2014) as well as changes to the lesson that were made during the process. To identify changes made by the lesson study group as a whole, the researcher examined each of the research lessons and subsequent debriefing sessions. The researcher identified changes that were made to the research lesson by examining both data sources. However, the researcher only used video of the researcher lessons to determine how the collaboratively designed lessons aligned with the Mathematics Teaching Practices (NCTM, 2014). Second, the researcher sought to develop patterns of change among the three participants. An inductive coding analysis (Yin, 2014) was used to discern relevant concepts. This helped “bring order, structure, and interpretation to the mass of data collected” (Marshall & Rossman, 1999, p. 150). Once the major changes were identified for each embedded case, the researcher grouped the changes according to Mathematics Teaching Practice (NCTM, 2014). The researcher then identified cross-case themes by removing Mathematics Teaching Practices (NCTM, 2014) that were only associated with one participant in that domain (implementation, conception, and perception) (see Appendix J). The remaining themes were used to describe the holistic case.

Together the themes extracted across the embedded cases also aided in determining the progression of the participants throughout the study, which was then represented in terms of the Interconnected Growth Model (Clarke & Hollingsworth, 2002). The Interconnected Growth Model (Clarke & Hollingsworth, 2002) allowed the researcher to describe growth patterns that were individualized for each type of change. The adapted model used to describe changes made in this study can be seen in Figure 6. The external domain in this study consisted of information from *Principles to Actions* (NCTM, 2014) and lesson study group discussions. The domain of practice included both implementation and observation of research lessons. The domain of consequence contained associations made by the participants between implementation and salient outcomes related to the group’s long-term goals (i.e., problem solving or student engagement). Finally, the personal domain consisted of participants' conceptions and perceptions of the Mathematics Teaching Practices (NCTM, 2014).

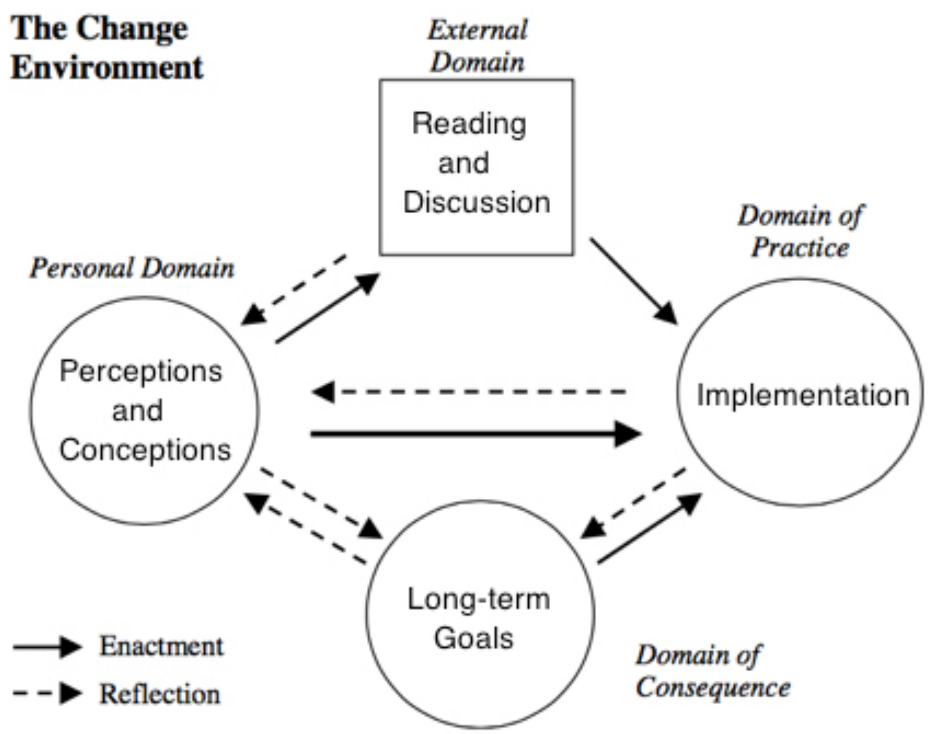

Figure 6. The adapted interconnected model for professional growth in this study. Adapted from "Elaborating a model of teacher professional growth," by D. Clarke and H. Hollingsworth, 2002, *Teacher and Teacher Education*, *18*, p. 951.

Qualitative Software

To aid with coding, the researcher used Atlas.ti to code transcriptions of debriefing sessions, pre- and post-interviews, and self-reflections. In addition, video of pre- and post-observations and three research lessons were coded according to the time at which certain events occurred. Atlas.ti made it possible to easily gather data related to each participant as well as each theme that emerged. Once coded, Atlas.ti allowed the researcher to easily see each piece of evidence within its context. This was important because it allowed the researcher to understand the context from which each quotation or event was situated and avoid mischaracterizing statements that were made.

Ethical Considerations

A carefully designed qualitative study should establish safeguards to protect the rights of participants and include informed consent, protect participants from harm, and ensure confidentiality (Bloomberg & Volpe, 2012). This study was presented to the Instructional Review Board (IRB), all involved risks were considered, and appropriate actions were taken to avoid potential issues. Furthermore, all audio-recordings and videotaped lessons were stored in a secure file and deleted from the video or audio recording instrument. In addition, physical materials such as research lesson plans and student work were securely stored in a lockable filing cabinet.

Issues of Trustworthiness

When assessing a study, it is important to consider its credibility, dependability, and transferability. Credibility is the degree to which qualitative data accurately gauge what we are trying to measure (Gay, Mills, & Airasian, 2012). To establish credibility in this study, the researcher employed a number of different strategies (see Table 8), which included member checks. To conduct member checks, the researcher emailed the individual case analysis to each participant once the final draft was complete. The researcher planned to alter each case analysis based on feedback, but no constructive feedback was received from the participants. With respect to bias, the researcher must clarify up front any bias that the researcher brings to the study (Bloomberg & Volpe, 2012). In this particular study, the researcher was a teacher in the same district as the participants. However, the researcher had never taught at Tamarind Middle School and had never worked with the participants prior to the study.

Dependability refers to whether one can track the processes and procedures used to collect and interpret the data (Bloomberg & Volpe, 2012). To achieve this, the researcher provided an auditable trail by being transparent and detailed about how the data was collected and analyzed. Finally, the fit or match between the research context and other contexts is known as transferability (Bloomberg & Volpe, 2012). The researcher developed transferability by providing rich descriptions of the context and of each particular case. Moreover, careful selecting of the participants created an authentic environment in which to study teacher change.

Table 8

Evidence of Research Quality and Rigor

Research Quality	Strategy Employed
Credibility	Bias explained and minimized
	Prolonged engagement in the field
	Triangulation of methods
	Triangulation of sources
	Provided alternative or disconfirming results
	Member Checks
Dependability	Auditable trail
Transferability	Provided thick descriptions of each case
	Purposeful selection of participants

Limitations of the Study

This study included certain limiting factors, some of which are common critiques of qualitative research methodology in general. Because analysis in qualitative research is ultimately determined by the researcher's thoughts and choices, researcher subjectivity is always a limiting condition. Moreover, internal validity is a key concern for explanatory case studies when a researcher tries to explain how and why a particular event led to some other event (Yin, 2014). Careful attention was given to explaining the cause of participant behaviors and perceptions, and the researcher used a number of data sources to support any claim that was made. In addition, knowledgeable experts are a recommended luxury in lesson study groups. Therefore, many lesson studies do not have access to an expert like Dr. Ross. Although the influence the knowledgeable other had on changes that were made was of interest, data aimed at revealing Dr. Ross' impact was not gathered.

Delimitations of the Study

Limitations related to time, location, grade-level, sample, and problem were under the control of the researcher and therefore considered delimitations of the study. The middle school used as the setting for this study may not be generalizable to other schools participating in lesson study. Moreover, a *Keli* lesson study model was implemented in this study. A *Keli* lesson study required multiple teachings, a knowledgeable other, and individual planning, which are not required in a general lesson study. Therefore, although the results of this study provided insight into the effectiveness and influence of *Keli* lesson study, this decision may have affected the transferability to other settings.

Chapter Summary

To explore lesson study as an aid in enhancing teachers' implementation, conception, and perception of the NCTM (2014) Mathematics Teaching Practices, an embedded case-study design was adopted. Each participant served as an embedded case within the lesson study group as a whole. The participants completed three cycles of *Keli* lesson study, in which all participants got a chance to take the lead role in teaching the lesson. Throughout the process, the researcher collected information from a variety of data sources to examine how and why the participants changed their implementations, conceptions, and perceptions, if at all. In addition, pre- and post- lesson study classroom observations were used to analyze specific changes that the participants made with respect to their teaching practices. An auditable trail, member checks, thick case descriptions, and other employed strategies support the credibility, dependability, and transferability of this study and its results. Moreover, careful attention was given to the analysis of data to limit concerns related to researcher subjectivity and explanatory statements.

CHAPTER IV: RESULTS

Introduction

With traditional teaching methods pervasive in the U.S. (Hiebert et al., 2005), it is crucial that mathematics teacher educators and professional development leaders understand what methods result in changes in classroom instruction. Lesson study presents a promising approach to developing reform-oriented instruction (Yoshida, 2013), as it is situated within the classroom (Takahashi et al., 2013), draws upon rich discussions about lesson development (Lewis et al., 2009), and creates opportunities for reflection upon practice (Ricks, 2011). Founded on the theoretical frameworks of situated and cognitive theories of learning, lesson study can be viewed as the means by which teachers dynamically construct or reinterpret new practices as well as create norms and tools for implementation (Lewis et al., 2009), which can then be enacted (Lave & Wenger, 1991) in their own classrooms. Although the literature has shown the usefulness of lesson study, evidence of how lesson study can support teachers in conceptualizing, perceiving, and implementing reform-oriented instruction could be vital to the success of mathematics education locally and across the country.

This study used an embedded case study design (Yin, 2014) to explore how lesson study can be used to aid teachers in conceptualizing and implementing the Mathematics Teaching Practices (NCTM, 2014) and investigate how teachers' perceptions of reform-oriented teaching change while participating in lesson study. Analysis of the embedded cases allowed the researcher to identify specific changes that were made by each participant. Conversely, the holistic case analysis revealed patterns within the embedded cases and allowed the researcher to investigate how the collaboratively designed lessons aligned with the Mathematics Teaching Practices (NCTM, 2014) throughout the lesson study. This section begins with a chronological narrative of the lesson study that provides a description of each event of the lesson study. An embedded case analysis, which describes rich accounts of each participant, will follow. Finally, a holistic case analysis of the lesson study group will be provided.

Lesson Study Narrative

In this section, a chronological narrative of the lesson study will be provided to give background information on the planning meetings, research lessons, and debriefing sessions. A synopsis of each planning meeting and debriefing session is provided. In addition, each research lesson is analyzed with respect to the Mathematics Teaching Practices (NCTM, 2014).

Planning Meeting 1

Prior to the first research lesson, the participants and Dr. Ross met twice for discussion. In Planning Meeting 1, the lesson study group discussed the reading assignment, group norms, broad goals for the lesson study, and specific content goals for the research lesson. The group, as a whole, agreed with what they had read in *Principles to Actions* (NCTM, 2014). However, the group commented on the difficulties of providing opportunities for productive struggle through meaningful tasks due to time constraints. Britney mentioned that the difficulty is “the whole, having a task and giving them time to struggle through it rather that guiding them through it . . . It's the time” (Planning Meeting 1, 10/14/15). As a result, the group decided to focus the lesson study on developing students as problem solvers through a lesson on systems of linear equations, a topic for which the group had brought with them two tasks to examine: the

Umbrellas and Hats Task (see Appendix H) and the Fruits and Vegetables Task (see Appendix I). At the conclusion of the Planning Meeting 1, the group considered selecting the Fruits and Vegetables Task (see Appendix I) and decided that the goal of the lesson would be to move students towards algebraic representation within systems of equations. Each participant was then tasked with writing an individual lesson plan for the research lesson.

Planning Meeting 2

In Planning Meeting 2, the group discussed their individual research lesson plans, possible tasks, and the goal for the lesson. Britney's lesson plan included two tasks (Fruits and Vegetables Task (see Appendix I) and a second task involving carrots) with a large portion of time devoted to individual and group work along with selecting and sequencing students to present their work. Sally's lesson plan was not as detailed, but it also used two tasks (solving equations from a real-life example and the Fruits and Vegetables Task (see Appendix I)) with students working in pairs. She explained that her lesson would conclude with the sharing of answers and different methods that were used. Similarly, Mark's lesson plan included two tasks (a task involving muffins and the Fruits and Vegetables Task (see Appendix I)) and both individual and group work. In Mark's lesson plan, the original bell work task involving muffins was as follows.

> I went to the store to buy a muffin. Muffins cost 25 cents each. I had a lot of change in my coin purse. I have quarters, dimes, nickels, and pennies. How many ways could I pay for the muffin? List the ways. (Mark Lesson Plan 1, 10/27/15)

However, Dr. Ross recommended narrowing the coins to just nickels and pennies to reflect only two variables. Mark also decided to have the groups present to reveal various strategies and representations.

The discussion in Planning Meeting 2 then returned to the specific goal for the lesson. Although the goal from Planning Meeting 1 was to move students towards using variables, Mark questioned it, saying, “What are we going to do to get them to think that way” (Planning Meeting 2, 10/27/15)? Dr. Ross then commented.

> You need to develop two linear equations, associated with each other . . . this is also a very important concept. And what does a solution of a system of linear equations mean? So that is our goal. So, we want students to develop those concepts. (Planning Meeting 2, 10/27/15)

The group refocused the goal, however, towards the substitution method for solving a system of equations, but they had difficulties matching the new objective with the selected tasks. In fact, there was still some confusion at the conclusion of the meeting as to how the tasks and the goal aligned. At that point, the group considered including the Umbrellas and Hats Task (see Appendix H). Without having reached consensus, since Mark volunteered to teach the first research lesson, he was given the freedom to finalize his lesson plan by adopting the ideas that were discussed in the planning meetings.

Research Lesson 1

The first research lesson, taught by Mark, began with the following bell work problem projected on the screen.

> I went to the store to buy a muffin. Muffins cost 25 cents each. I had a lot of change in my coin purse. I only have nickels and pennies. How many ways could I pay for the muffin? List the ways. (Research Lesson 1, 11/2/15)

Mark gave students approximately four minutes to investigate the problem prior to randomly calling on students to share their solutions. Mark listed students' responses on the board and then provided an additional constraint to the problem: "How could I pay for my muffin if the clerk said she could only take nine coins per purchase" (Research Lesson 1, 11/2/15)? Once students found the solution, Mark summarized the main points of solving the problem and stated the objective for the lesson, which was for students to solve real-world and mathematical problems leading to two linear equations in two variables. Mark shared his expectation that "all students will use strategies to solve problems and communicate with their partners" (Research Lesson 1, 11/2/15). In addition, Mark posed four questions for students to think about throughout the lesson: "What mathematics is being learned? Where are these mathematical ideas going? Why is it important? How does it relate to what has already been learned" (Research Lesson 1, 11/2/15)?

Mark gave students approximately 1.5 minutes to think individually about the picture in the Fruits and Vegetables Task (see Appendix I) with the question covered. Mark then stopped the class to discuss problem-solving strategies in general and asked students to share problem-solving strategies that they knew. Students proposed the following strategies: look at similar problems, break it down into steps, eliminate possibilities, and use pictures, symbols, and graphs. Mark wrote these strategies on the board and encouraged students to try to incorporate the strategies with their groups as they attempted to solve the Fruits and Vegetables Task (see Appendix I).

After five minutes, Mark asked for additional strategies. One student added logic to the list, then Mark presented a student's work, which used equations with words (see Figure 7). Mark presented yet another student's work in which the fruits were given specific weights to figure it out (see Figure 8). Mark asked the second student to "talk about what you were thinking and how to do that" (Research Lesson 1, 11/2/15). The student responded, "Since you don't know what they weigh, there is different numbers that you could pick, but they can't change" (Research Lesson 1, 11/2/15). To conclude the task, Mark revealed to students three example solution strategies on the projector screen that he had created himself: equations with words, equations with variables, and equations with pictures. With the various representations on the screen, the following dialogue occurred comparing the efficiency of using words and variables compared to drawing pictures.

> Mark: Could you use pictures as a strategy?
>
> Student: You could draw pictures, but, like, it wasn't really necessary for this because that would take longer and we used a simpler way.
>
> Mark: What do you mean by necessary?
>
> Student: It was already drawn out for us, so . . . we didn't really need to modify it. It is easier, like writing stuff down [symbols] instead of drawing a picture.
>
> Mark: So you're saying that maybe sometimes using these symbols would be easier than drawing a picture. Is that what you are saying?
>
> Student: Yeah. (Research Lesson 1, 11/2/15)

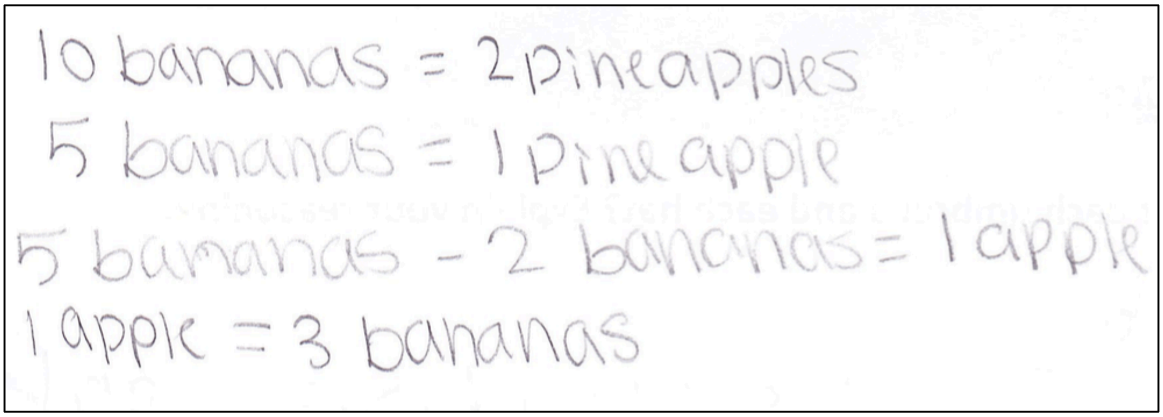
10 bananas = 2 pineapples
5 bananas = 1 pineapple
5 bananas - 2 bananas = 1 apple
1 apple = 3 bananas

Figure 7. Student solution using equations to solve the Fruits and Vegetables Task in Research Lesson 1.

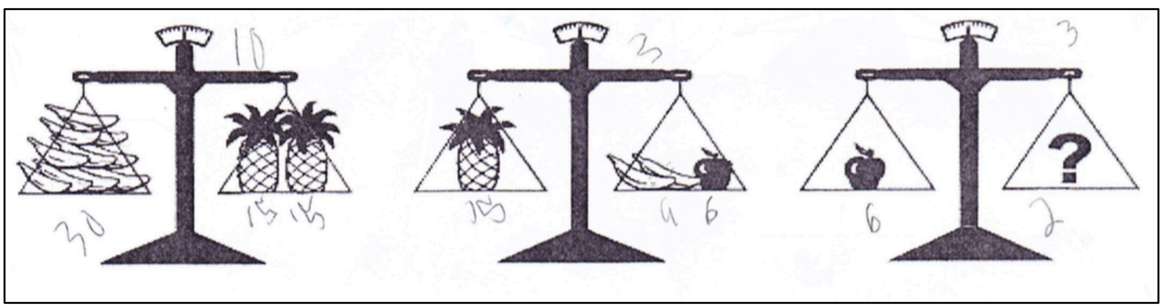

Figure 8. Student solution using specific values to solve the Fruits and Vegetables Task in Research Lesson 1.

The lesson transitioned to the Umbrellas and Hats Task (see Appendix H) next, as Mark allowed students two minutes to investigate it on their own. After individual work, Mark gave students 20 minutes to discuss the problem in their groups. As students worked, Mark circulated the room to assess and advance student thinking. For example, Mark made the statement "show proof" (Research Lesson 1, 11/2/15). Most students used a guess-and-check strategy to find the solution (see Figure 9). The class ended without time to discuss students' solutions to the task, so Mark asked the students to continue looking at the problem that night so that they could go over it more the next day.

With respect to the practices (see Table 9), Mark promoted problem solving and reasoning (MTP 2) as students attempted to complete the muffin task and the Umbrellas and Hats Task (see Appendix H). Moreover, he facilitated mathematical discourse (MTP 4) by allowing students to share their ideas and work together during the tasks. As students worked, Mark provided students with opportunities to think critically and struggle through the tasks (MTP 7). Although Mark made connections among representations during the Fruits and Vegetables Task (see Appendix I), he did not use various representations to help students understand the concept of a system of equations during the Umbrellas and Hats Task (MTP 3). Moreover, Mark did not pose purposeful questions (MTP 5) to press students to explain their reasoning, begin to build procedural fluency upon the conceptual understanding (MTP 6), or include a summary or conclusion.

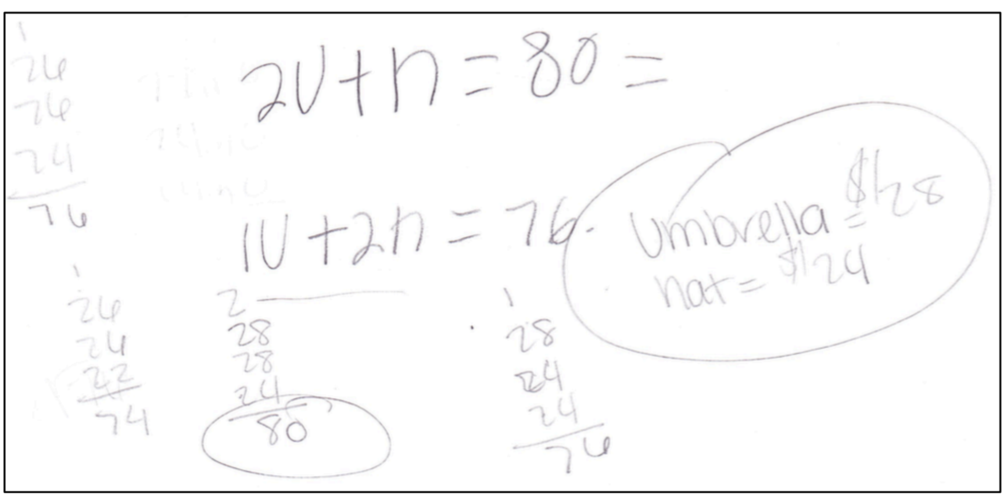

Figure 9. Student solution to the Umbrellas and Hats Task in Research Lesson 1.

Table 9

Research Lesson 1 Teaching Practices

MTP	Evidence
1	Mark posed four questions for students to think about throughout the lesson: What mathematics is being learned? Where are these mathematical ideas going? Why is it important? How does it relate to what has already been learned?
	Mark shared the objective with students prior to beginning the Fruits and Vegetables Task (see Appendix I).
2	The muffin problem pressed students to reason and problem solve in order to find the solution.
	Mark shared his expectation that all students will use problem solving.
	The Fruits and Vegetables Task (see Appendix I) pressed students to reason and problem solve in order to find the solution.
	Mark led a discussion about strategies and said, "Think about using these strategies that we just talked about" (Research Lesson 1, 11/2/15) as students began the task.
	The Umbrellas and Hats Task (see Appendix H) forced students to reason abstractly and required a high level of cognitive demand.
	Mark supported students in exploring the Umbrellas and Hats Task (see Appendix H) without taking over student thinking.
3	Mark presented three teacher-generated representations of the Fruits and Vegetables Task (see Appendix I) simultaneously: equations with words, equations with variables, and equations with pictures.

Table 9 continued

MTP	Evidence
4	Students shared their solutions to the muffin problem until they thought all had been shared. Mark shared his expectation that all students will communicate with their partners. Students worked together in groups on Fruits and Vegetables Task (see Appendix I). Mark gave students time to work with their group on the Umbrellas and Hats Task (see Appendix H).
5	Mark used assessing and advancing questions to facilitate group discussions during the Umbrellas and Hats Task (see Appendix H). For example, "Show proof" (Research Lesson 1, 11/2/15).
6	There was a class discussion about which solution method was the most efficient.
7	Mark gave students ample time to struggle through the Umbrellas and Hats Task (see Appendix H). Students persevered on the Umbrellas and Hats Task (see Appendix H) and never gave up.
8	Mark presented two students' work so that he could bring attention to two new strategies. "Talk about what you were thinking and how to do that" (Research Lesson 1, 11/2/15).

Lesson 1 Debrief

During each debriefing session, participants used notes from their observation protocols to aid the discussion. Mark began Lesson 1 Debrief by discussing his thoughts about the lesson with respect to the Mathematics Teaching Practices (NCTM, 2014) and provided suggestions for improvement of the lesson.

> It didn't go the way I was hoping . . . because we kind of stalled when we got together, when we started looking for strategies . . . Even though we had talked about strategies before [listed on the board], they kind of just went out the window. And then when we got started, everybody went straight to guess-and-check. And I guess that is their default. If you don't figure it out, just kind of plug-and-chug. That's what they have been taught in the past and so we got caught in a loop of just guess-and-check, guess-and-check, guess-and-check, and so I was just going around just trying to get them to do anything other than guess-and-check. (Lesson 1 Debrief, 11/2/15)

Britney and Sally followed Mark with their own strengths, weaknesses, and suggestions for improvement of the lesson. Britney provided a suggestion.

> I like the idea, when you are asking “What method did you use?”, having another student explain what another student said. So and so said this, “Can you explain what they were doing?”, “Can you put that in your words?”, “What are you understanding about what you see here?” Stuff like that. I know a couple of groups got it, so go back to them and just be like, “Hey why don't you try one of these other methods we have listed on the board.” (Lesson 1 Debrief, 11/2/15)

Then Sally commented on strengths of the lesson.

> I thought it went really well, too. I even made a comment to Britney about how organized that you are. It just seemed so . . . everything flowed . . . and I think I struggle with that. But you are very organized. I also really liked the bell work question, and how you had to narrow it down the second time [to the restriction of nine coins]. (Lesson 1 Debrief, 11/2/15)

Each of the three participants commented on students solely relying on a guess-and-check strategy despite Mark's attempts to reveal numerous strategies. Mark commented that discussing the general problem-solving strategies took too much time. Mark also suggested removing the Fruits and Vegetables Task (see Appendix I), saying that he "didn't feel like [the Fruits and Vegetables Task (see Appendix I)] led to systems as much as it did to just substituting and exchanging" (Lesson 1 Debrief, 11/2/15). The group agreed and focused on narrowing the goal for the lesson.

Dr. Ross directed the group's attention towards the idea of a solution of a system of equations by saying, "Why did you come up with only one solution? Why not others? So if you just look at one condition, there are many solutions. So why did you only select that one? They will notice that it satisfies both [conditions]" (Lesson 1 Debrief, 11/2/15). The group accepted this idea and then discussed ways to present the idea of a table if a student did not share one. Britney stated, "I would maybe make a table in advance and pretend it is someone else's [work]" (Lesson 1 Debrief, 11/2/15). As the meeting concluded, the group agreed that a tabular representation would serve two purposes: provide students with a systematic way of guessing and checking and serve as a transition to writing the equations. Sally had previously volunteered to teach Research Lesson 2, so she was then given the authority to alter her lesson plan as she thought best.

Research Lesson 2

Research Lesson 2 began with a similar bell work problem.

> I went to the store to buy a muffin. Muffins cost 25 cents each. I had a lot of change in my coin purse. I only have nickels and pennies. How many ways could

> I pay for the muffin? Write an equation to solve the problem. (Research Lesson 2, 11/6/15)

Sally added the last sentence to the original bell work question to press students to write an equation to solve the problem. Sally gave students six minutes to work on the muffin task individually, then two minutes to discuss their solutions. Sally then asked the class for various solutions and equations. Students quickly found the solutions, but struggled to create the appropriate equation. Sally recorded student-generated equations similar to $2n + 15p = 25$ (see Figure 10) on the board and then redirected student thinking by calling on a student who had written the equation correctly to explain her thinking. Once the student had shared her equation of $5n + 1c = 25$ (see Figure 10), Sally attempted to focus student thinking on the structure of the equation by saying, "What does *c* represent" (Research Lesson 2, 11/6/15)? Sally then revealed to students how creating a table (see Figure 11) would allow them to "organize [their solutions] in a way that would be easier for us to read" (Research Lesson 2, 11/6/15). Sally then introduced the second stipulation, which restricted the amount of coins the clerk could accept to exactly nine. The students discussed this situation in their groups and found the solution by inspecting the table on the board (see Figure 11). Next, Sally discussed the objective for the day: solve real-world and mathematical problems that develop the concept of systems of equations.

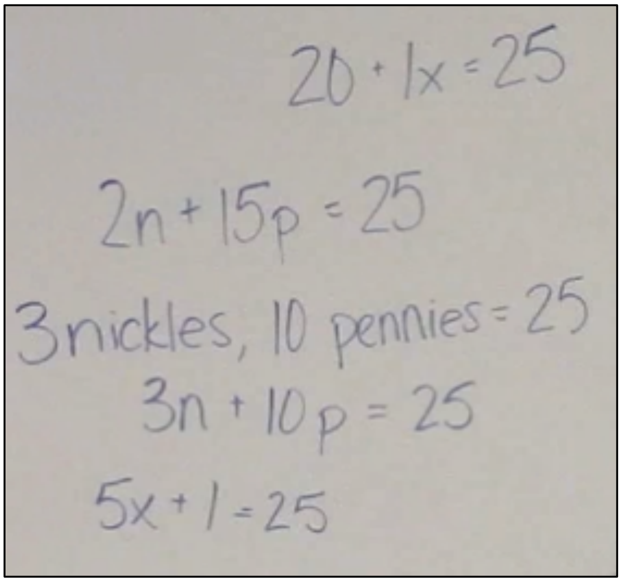

Figure 10. Sally’s writing of incorrect equations on the board in Research Lesson 2.

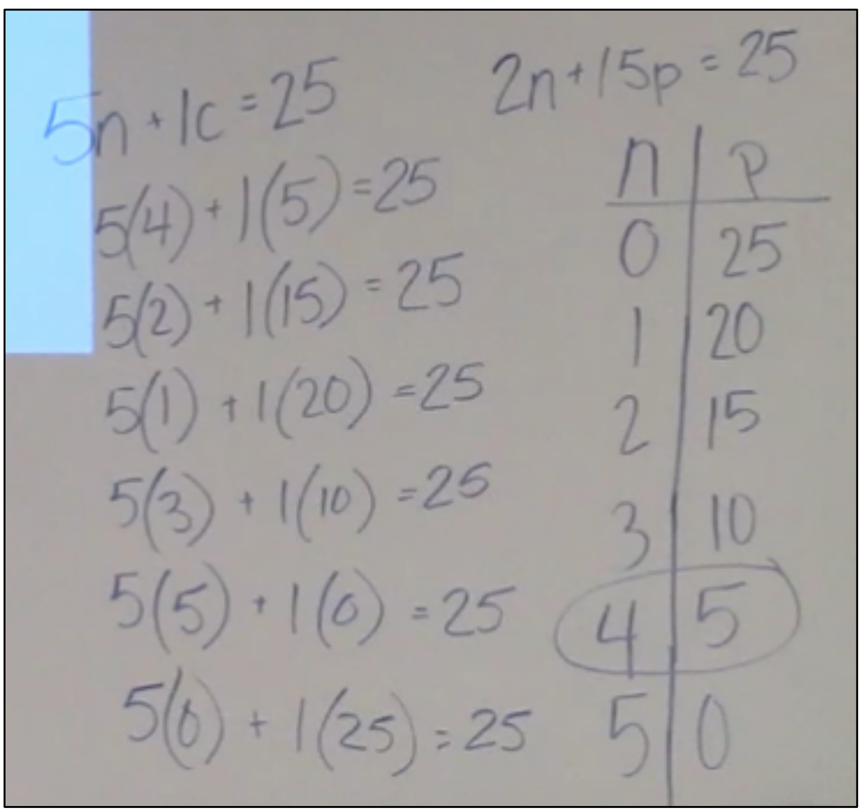

Figure 11. Sally’s writing of the solutions on the board in Research Lesson 2.

Sally displayed the four questions, originally used by Mark in Research Lesson 1, in order to get students to reflect upon the mathematics as they engaged in solving the task. Sally kept these questions on the screen as students worked individually for three minutes on the Umbrellas and Hats Task (see Appendix H). Students then worked with their groups for 20 minutes before Sally called on various students to share their strategies. Evidence of Sally selecting and sequencing the discussion was evident as she started with simple solution strategies such as estimating the solutions, then called on students who had created equations to represent the problem in order to guess-and-check more effectively. Sally recorded students' solutions on the board and asked students to explain how they reached those values. Finally, Sally asked another student, who had used a more complex method (see Figure 12), to explain his reasoning by asking, "How did you come up with 84 and 72?" (Research Lesson 2, 11/6/15) After Sally wrote the student's table on the board, the student explained, "I assumed there was a pattern going on that if you subtract an umbrella and add a hat, then it would be four dollars less" (Research Lesson 2, 11/6/15). Sally rephrased the student's thinking to help the rest of the class understand his method, then ended class by having students answer one of the four reflection questions posed earlier.

With respect to the Mathematics Teaching Practices (NCTM, 2014) (see Table 10), Sally implemented tasks that promoted reasoning and problem solving (MTP 2) as students worked on the muffin task and the Umbrellas and Hats Task (see Appendix H). One of the major strengths of Research Lesson 2 was when Sally selected and sequenced student work (MTP 4) in order to discuss various strategies for solving the Umbrellas and Hats Task (see Appendix H). However, Sally did not begin to build procedural fluency upon conceptual understanding (MTP 6) or elicit student thinking beyond having students explain their thinking as she wrote their solution methods on the board (MTP 8).

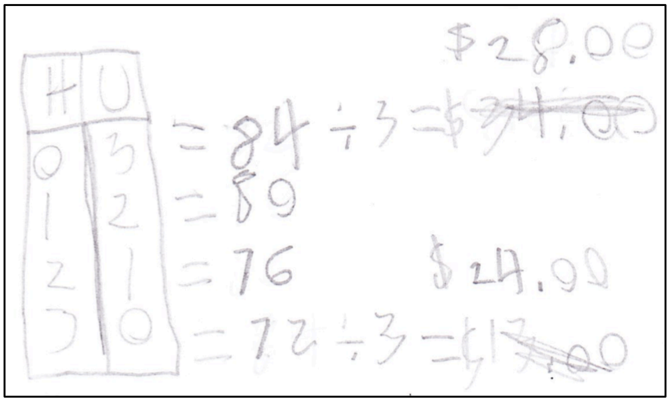

Figure 12. Student solution to the Umbrellas and Hats Task in Research Lesson 2.

Table 10

Research Lesson 2 Teaching Practices

MTP	Evidence
1	Sally posed four questions for students to think about throughout the lesson: What mathematics is being learned? Where are these mathematical ideas going? Why is it important? How does it relate to what has already been learned? Sally shared the objective with students prior to beginning the Umbrellas and Hats Task (see Appendix H). Sally referred back to the four questions listed above and asked students to pick one to respond to as an exit ticket.
2	The muffin problem pressed students to reason and problem solve in order to find the solution. The Umbrellas and Hats Task (see Appendix H) forced students to reason abstractly and required a high level of cognitive demand. Sally supported students in exploring the Umbrellas and Hats Task (see Appendix H) without taking over student thinking.

Table 10 continued

MTP	Evidence
3	Sally pressed students to create equations during the muffin task by adding the last sentence in the task.
	Sally introduced students to a tabular representation of their solutions and made connections to the equation.
4	Students shared their solutions to the muffin problem until they thought all had been shared.
	Sally gave students time to work with their group on the Umbrellas and Hats Task (see Appendix H).
	Sally called on students to share their solution strategies, and sequenced the discussion based on the complexity of the methods used.
5	Sally used questioning strategies to focus student thinking on the structure of equations for the muffin problem. For example, after a student shared the equation $5n + 1c = 25$, Sally said, "What does *c* represent?" (Research Lesson 2, 11/6/15)
	Sally asked purposeful questions to make the concept of systems of equations more visible in the discussion that followed the Umbrellas and Hats Task (see Appendix H). For example, "How did you come up with 84 and 72?" (Research Lesson 2, 11/6/15)
6	Sally connected student-generated strategies following the Umbrellas and Hats Task (see Appendix H).
7	Sally gave students ample time to struggle through the Umbrellas and Hats Task (see Appendix H).
	Students persevered on the Umbrellas and Hats Task (see Appendix H) and never gave up.
8	Sally asked students to explain their thinking as she wrote their solution methods on the board.

Lesson 2 Debrief

Lesson 2 Debrief began with Sally sharing her thoughts about the strengths and weaknesses of the lesson. Sally stated, "I tried to narrow their thinking more with the bell work. I said write an equation because I kind of wanted them to take the hint and of what I was trying to get them to do" (Lesson 2 Debrief, 11/6/15). In terms of a weakness, Sally discussed how she was disappointed with "the guessing and checking" (Lesson 2 Debrief, 11/6/15) in the Umbrellas and Hats Task (see Appendix H), but she was glad that "they weren't just choosing a random number. Some of them were saying, 'I know it is between 20 and 30'" (Lesson 2 Debrief, 11/6/15).

Each of the three participants further discussed the importance of removing the Fruits and Vegetables Task (see Appendix I) from the lesson. For example, Britney commented, "I know [during Research Lesson 1] we didn't get through it all, and I think removing [the Fruits and Vegetables Task (see Appendix I)] definitely got rid of that problem" (Lesson 2 Debrief, 11/6/15). These statements made by the group reaffirmed the decision Sally had made to remove the task after Research Lesson 1, due to time constraints.

The group also agreed that one of the strengths of Research Lesson 2 was Sally's questioning. Britney said that she "liked the questions, making them explain, what does it mean?" (Lesson 2 Debrief, 11/6/15), and that "even the one girl asked, with the $5n + 15p = 25$. She said, 'Why couldn't n be 2 and p be 1?' and [Sally was] like, 'Well, could it be that every time?'" (Lesson 2 Debrief, 11/6/15). Dr. Ross was also positive about Sally's discussion of the Umbrellas and Hats Task (see Appendix H). He said, "So you intentionally picked two or three different ways and asked them to explain how they got their solution. I think that is very important. So, overall, you organized it very well" (Lesson 2 Debrief, 11/6/15).

The group then discussed various ways to press students to build the correct equation and avoid the common misconception associated with equations similar to $2n + 15p = 25$. Dr. Ross suggested using multiple representations to make the idea easily visible to students. “So what I am thinking is, if you have three representations: verbal, table, and symbolic on the board . . . this will really help them develop the concept of systems of linear equations” (Lesson 2 Debrief, 11/6/15). Dr. Ross also recommended that the muffin task be split into three separate questions: one question on finding the solutions and equation for the amount of nickels and pennies yielding 25 cents; another question on finding the solutions and equation for the amount of nickels and pennies yielding nine coins; and one question connecting them together to find a solution that fits both situations. Britney agreed with this approach and added, “Our [goal] isn't specific, it’s just about understanding of systems. I wonder if I might change it to develop an understanding of the solution to a system of equations” (Lesson 2 Debrief, 11/6/15). Britney was scheduled to teach Research Lesson 3, so she now had the opportunity to alter her lesson plan to reflect this idea.

Research Lesson 3

During Research Lesson 3, Britney’s objective was to support students in developing the concept of the solution of a system of equations. The lesson began with a similar bell work problem that only contained the first question of the muffin task:

> I have a change purse with only nickels and pennies. I would love to buy a muffin that costs \$.25. Write an equation to represent the situation. List as many possible ways to pay as possible. (Research Lesson 3, 11/9/15)

Students worked for five minutes before Britney presented various students' work to make connections among visual, verbal, and tabular representations of the solutions (see Figure 13).

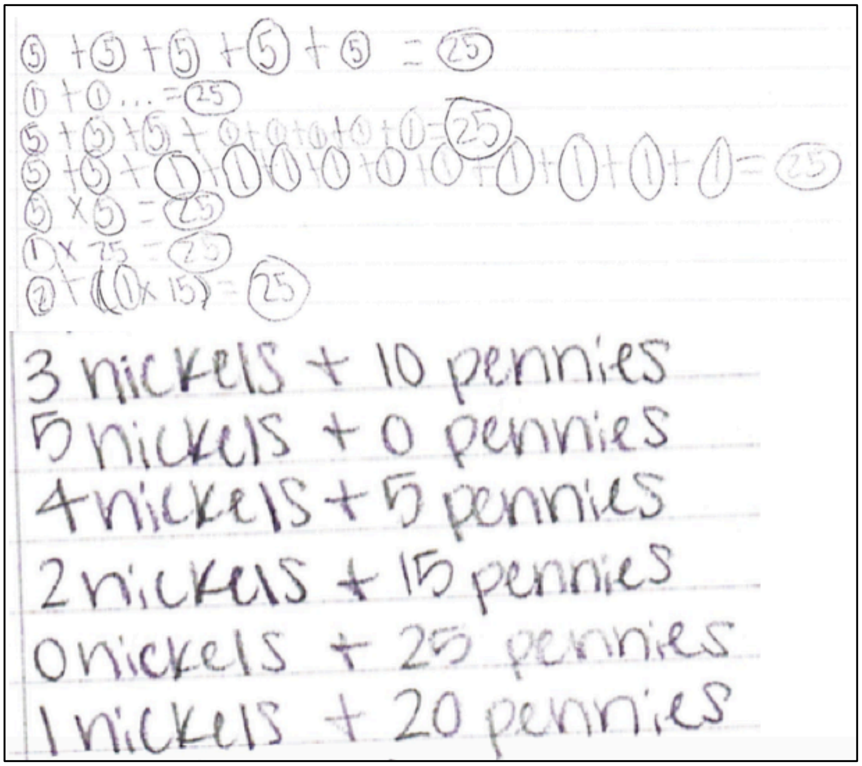

Figure 13. Student solutions to part 1 of the muffin task in Research Lesson 3.

Then Britney wrote various student-generated equations on the board and led a discussion about which one represented the situation best. Responding to one of the proposed equations, Britney asked, "What does this mean in terms of the problem we are talking about" (Research Lesson 3, 11/9/15)? Britney then addressed a common misconception by helping students understand why the equation $n + p = 25$ could not represent the situation.

Britney: So what you're saying is that the number of nickels plus the number of pennies is 25. Does that fit the solutions we have in our table? [pointing to the table] Does 2 plus 15 equal 25?

Students: No.

Britney: So if we are saying *n* is 2 and *p* is 15, does that work here?

Students: No.

Britney: So, class, what are your thoughts [on the equation] now that we have talked about this?

Students: No.

Britney: Not if we are using *n* as the number of nickels and *p* as the number of pennies.

Student: Can I revise?

Britney: You want to revise it. Ahh, revise. Fancy! Go ahead.

Student: You could just do *n* plus *p* equals total.

Britney: *n* plus *p* equals total what? What is a nickel? What is a penny?

Students: Coins.

Britney: So *n* plus *p*, if you add the nickels and pennies, you get the total number of coins [wrote $n + p$ = total coins on the board]. (Research Lesson 3, 11/9/15)

Soon after this discussion, Britney asked, "Do you see an equation on the board that incorporates the value of the coins as well as the number of coins" (Research Lesson 3, 11/9/15)? A student selected the equation $5n + 1p = 25$ and Britney asked the class, "Does it work for all of these values in my table [pointing to table of the left] (see Figure 14)" (Research Lesson 3, 11/9/15)? She then chose various students to substitute the values into the equation to test to see if it worked. These students confirmed that the equation worked for all of the values in the table. Britney tried to reveal the structure of the equation to students by asking, “What is the importance of this 5 beside the *n*” (Research Lesson 3, 11/9/15)?

Following this discussion, Britney gave students the second question of the muffin task:

> I pull 9 coins from my change purse. How much money could I be holding? Write an equation to represent the situation. List the possible amounts of money I could have. (Research Lesson 3, 11/9/15)

After five minutes of individual work, Britney presented two student strategies and used questions to scaffold student thinking towards the correct equation to represent this situation ($n + p = 9$, which was written on the board from the previous problem).

> Britney: Lexi, what are you thinking?
>
> Lexi: That *n* plus *p* = total coins.
>
> Britney: Check that out, see if it fits all of these solutions [pointing to the table on the right (see Figure 14)].
>
> Students: Yes.
>
> Britney: Does this [wrote $n + p = 9$ on the board] fit all of these solutions?
>
> Students: Yes.
>
> Britney: So, 4 plus 5 equals 9, that works. (Research Lesson 3, 11/9/15)

To bring the results together, Britney presented the last question of the muffin task. “I am purchasing that muffin. I still only have nickels and pennies, but the clerk tells me he can only accept 9 coins. How can I buy the muffin” (Research Lesson 3, 11/9/15)?

Britney gathered ideas from various students and summarized what a system of equations is and how the solution to this problem fit both equations by comparing both situations on the board simultaneously (see Figure 14).

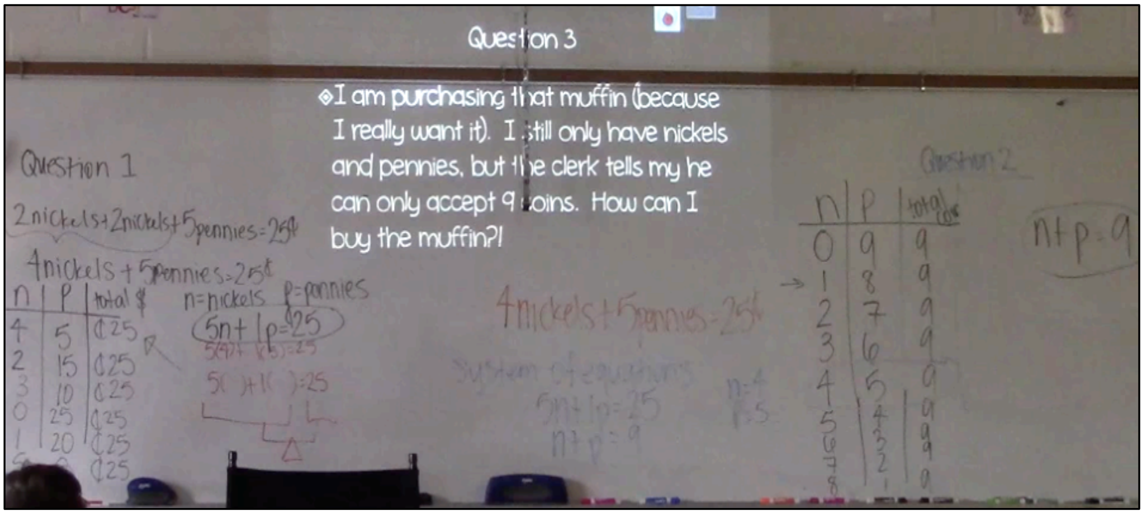

Figure 14. Britney’s writing on the board in Research Lesson 3.

Britney then stated the objective, which was for students to develop an understanding of what a system of equations is and what the solution means. Britney kept the objective on the screen while students worked independently on the Umbrellas and Hats Task (see Appendix H). After five minutes of individual work, students worked in groups for 10 minutes as Britney circulated the room to assess and advance student thinking by making statements like “prove to me” (Research Lesson 3, 11/9/15). Britney then shared two students’ work, one that wrote the system of equations and a second that found the correct solution. As Britney presented student work, she called on a group of students to explain their reasoning to the class: “Tell me about this, how did you get this?” (Research Lesson 3, 11/9/15) and “Can somebody explain this equation that was written" (Research Lesson 3, 11/9/15)? However, the discussion that followed did not reveal their solution methods or make connections between the students' work.

As the class concluded, Britney told students that they would finish the discussion the next day and asked them to answer one of the four questions, initially used by Mark in Research Lesson 1, in order to get students to reflect upon the mathematics. With respect to the Mathematics Teaching Practices (NCTM, 2014) (see Table 11), Britney presented tasks that promoted reasoning and problem solving (MTP 2), as she gave students time to work on the muffin task and the Umbrellas and Hats Task (see Appendix H). In addition, she provided opportunities for students to discuss the mathematics in groups and as a class (MTP 4) and posed numerous questions that required students to explain their thinking (MTP 5). Britney also displayed student work (MTP 8) during the Umbrellas and Hats Task (see Appendix H) and had students explain their solution strategies. However, Research Lesson 3 did not present many opportunities for students to build procedural fluency upon conceptual understanding (MTP 6).

Table 11

Research Lesson 3 Teaching Practices

MTP	Evidence
1	Britney shared the objective with students prior to beginning the Umbrellas and Hats Task (see Appendix H).
	Britney referred to last week's lesson to remind students how to check to see if a solution works or not.
	Britney referred back to the following four questions and asked students to pick one to respond to as an exit ticket: What mathematics is being learned? Where are these mathematical ideas going? Why is it important? How does it relate to what has already been learned?
	Britney made a connection to the lesson's goal by letting students know that what they had just created in the muffin problem was called a system of equations.
2	The muffin problem pressed students to reason and problem solve in order to find the solution.
	Britney encouraged and praised multiple solution strategies.
	The Umbrellas and Hats Task (see Appendix H) forced students to reason abstractly and required a high-level of cognitive demand.
	Britney supported students in exploring the Umbrellas and Hats Task (see Appendix H) without taking over student thinking.
3	Britney pressed students to create equations during the muffin task.
	Britney introduced students to a tabular representation of their solutions and made connections to the equation.

Table 11 continued

MTP	Evidence
4	Students shared their solutions to the muffin problem until they thought all had been shared. Britney gave students time to work individually and with their group on the Umbrellas and Hats Task (see Appendix H). Students shared their thoughts about the work of others in response to Britney's question, "Can somebody explain this equation that was written" (Research Lesson 3, 11/9/15)? As Britney presented student work, she called on a group of students to explain their reasoning on the Umbrellas and Hats Task (see Appendix H) to the class. "Tell me about this, how did you get this" (Research Lesson 3, 11/9/15)?
5	Britney separated the questions within the muffin task to make the mathematics more visible and accessible for students. Britney used questioning strategies to focus student thinking on creating equations for the muffin problem. As a student shared her equation, Britney asked, "What does this mean in terms of the problem we are talking about" (Research Lesson 3, 11/9/15)? This pressed the student to explain her reasoning. Britney focused student thinking on what each variable represented during the muffin task. Britney drew students' attention to the correct equation by asking them, "Do you see an equation on the board that incorporates the value of the coins as well as the number of coins" (Research Lesson 3, 11/9/15)? Britney tried to reveal the structure of the equation to students by asking, "What is the importance of this 5 beside the *n*" (Research Lesson 3, 11/9/15)? As Britney displayed student work during the muffin task, she asked students to explain and critique the reasoning of others.
6	Britney led students to writing equations by asking them to think of a more efficient method to represent 4nickels + 5pennies = 25cents.

Table 11 continued

MTP	Evidence
7	Britney addressed the misconception involved with the equation $n + p = 25$ by asking students, "Does that fit the solutions we have in our table" (Research Lesson 3, 11/9/15)? When a student realized she had shared an incorrect equation, Britney encouraged her to revise the equation. Britney showed students that errors are a natural part of the learning process. Britney gave students ample time to struggle through the Umbrellas and Hats Task (see Appendix H), and asked questions to scaffold students' thinking without stepping in to do the work for them. Students persevered on the Umbrellas and Hats Task (see Appendix H) and never gave up.
8	Britney presented two students' work during the first question of the muffin task so that she could bring attention to two new strategies: pictorial and equation. Britney presented two more students' work during the muffin task to connect the tabular representation with substituting into the equation. Britney presented two students' work at the conclusion of the Umbrellas and Hats Task (see Appendix H) to reveal the equations.

Lesson 3 Debrief

After Research Lesson 3, the participants and Dr. Ross met to discuss the lesson as well as debrief the entire lesson study. The conversation began with Britney's thoughts on the strengths, weaknesses, and suggestions for improvement.

> I was disappointed that they didn't come up with $n + p = 9$ [right away] . . . But I know that the bell work went pretty well and they were kind of catching on . . . It was good for them to have, because when we got the equation [in the Umbrellas and Hats Task (see Appendix H)] it was easy for me to say, "Okay, let's see if the equation matches the solutions." Because, they all agreed on the solutions. So, it

> was good for them to see, oh wait that equation didn't work with the numbers that I know are correct. I wish it had taken a little less time for the bell work. On the task, a lot of them went to guess-and-check. (Lesson 3 Debrief, 11/9/15)

All three of the participants commented that they finally felt like they had specified the goal in enough detail. For instance, Sally said,

> I agree with you about the goal. I think finally that we have narrowed it down enough to really get what they are capable of at this moment. Just working with two variables and being able to identify a solution. (Lesson 3 Debrief, 11/9/15)

Dr. Ross then added his comments, which primarily addressed student misconceptions and task design. With respect to misconceptions, Dr. Ross posed the question, “So how can we convince the student that no, you cannot [add the prices up and] divide by 3 [during the Umbrellas and Hats Task (see Appendix H)]” (Lesson 3 Debrief, 11/9/15)? He further suggested using incorrect student work to address common errors during the lesson as well as find ways for students to realize that hats and umbrellas are not necessarily the same price. When addressing the task design, Dr. Ross suggested that using smaller numbers in the Umbrellas and Hats Task (see Appendix H) could save time. “If we achieve the same goal, why not just use simple numbers and save time” (Lesson 3 Debrief, 11/9/15)?

The second half of Lesson 3 Debrief focused on debriefing the entire lesson study in order to summarize participants’ learning. When asked about what they learned about the subject matter (i.e., systems of linear equations), the participants made comments related to using multiple representations including real-life contexts (MTP 3). Mark said, “I'd agree. Multiple representations. Not just looking at systems as just Algebra” (Lesson

3 Debrief, 11/9/15). Britney added that “talking about different ways to re-do the tasks has helped me see more real-world situations” (Lesson 3 Debrief, 11/9/15). In response to a question regarding what they learned about student thinking, Sally discussed how the group focused their learning goal based on student thinking.

> Every time we taught it, we got a little bit more specific . . . which I think helped [the students], because in Mark's lesson we saw that they were all going to go to guess-and-check. So, that kind of helped us trying to guide them. (Lesson 3 Debrief, 11/9/15)

A final question was asked regarding what participants learned about productive learning habits as teachers. The group mentioned the importance of detail in developing goals and questions. Mark commented on both ideas. “I find myself looking at details a little bit more about each little individual thing . . . and being intentional as far as what the goal is, what questions I want to ask” (Lesson 3 Debrief, 11/9/15). Britney added that she learned the importance of “being more specific about our expectations of students [as a PLC]” (Lesson 3 Debrief, 11/9/15). Finally, Sally mentioned the importance of asking guiding questions to advance student thinking. Then, the debriefing session ended, the lesson study came to an end, and the participants thanked Dr. Ross for his assistance during the lesson study.

Summary of Changes

There were many changes made between Research Lesson 1 and Research Lesson 3. The goal became more detailed throughout the research lessons and the participants attempted to specify their expectations to better align with students’ learning progressions and the selected task (Umbrellas and Hats Task (see Appendix H)). In addition, the bell work task involving the purchase of muffins was refined to make the idea of systems of equations visible and direct students towards writing equations to represent the constraints provided. Lesson 1 Debrief and Lesson 2 Debrief were instrumental in these changes, as it gave the group time to discuss the goal of the lesson and how to alter the bell work task to better lead into the Umbrellas and Hats Task (see Appendix H). Moreover, Mark's reflections and Dr. Ross' suggestions related to Research Lesson 1 were crucial in making changes relating to teaching through problem solving instead of teaching problem solving. As a result, the discussion of problem-solving strategies and the Fruits and Vegetables Task (see Appendix I) were removed so that students could have more time to reason and solve problems. Overall, the research lesson became more focused, and thus, allowed more opportunities for students to learn what a solution to a system of equations is and how it can be found.

Embedded Case Analysis

Each participant was considered an embedded case within the lesson study group. The researcher conducted pre- and post-interviews and gathered data on each participant through a number of other sources including self-reflection journals, video of research lessons, audio of debriefing sessions, and research lesson plans. To begin analysis, the researcher compared participants' pre- and post-observation lessons to detect subtle differences in their teaching practices. Responses to pre- and post-interview questions as well as journal prompts were then analyzed to identify changes in conceptions and perceptions. Differences found in the observations, interviews, and journals were extracted and coded to develop common themes to describe the change of participants. These themes were used to code evidence as the researcher analyzed occurrences during the lesson study. As a result, the researcher was able to triangulate evidence and further develop each participant's case analysis. The following sections provide in-depth case descriptions of each embedded case and are organized according to participant.

Mark Gibson

The results of changes made by Mark will be described in the sections to follow. First, a description of Mark's typical lesson structure will be described. Second, major changes in implementation, conception, and perception of the Mathematics Teaching Practices (NCTM, 2014) will be described along with the aspects of the lesson study that influenced these changes.

Description of teaching. Mark, who had been teaching for six years, focused on setting the right atmosphere in his class to ensure that students felt comfortable talking and sharing ideas in class. He also mentioned that he stopped periodically during lessons to make sure students understood what the lesson was about that day and where they were in achieving the goal for the day. In terms of structure, Mark usually followed the schedule that appeared on his board. Mark described it as follows.

> We look at the objective, we intro the lesson, we talk about the lesson, we'll pause, we'll review, we'll look at the objective . . . we do some practice and share again and usually then we do some sort of conclusion whether it be an exit ticket or partner share or something like that. (Mark Pre-Interview, 9/9/15)

Mark's view of his teaching prior to the lesson study emphasized creating an atmosphere that allowed students to feel comfortable discussing ideas with one another. Moreover, it focused on students monitoring their learning during a lesson. In the section that follows, Mark's implementation will be examined to see how these ideas were enacted before and after the lesson study.

Changes in implementation. Analysis of Mark's changes in implementation follows. To begin, a description of his pre- and post-observation will be provided. Then, a synthesis comparing the two lessons will be provided along with evidence as to what changes were made.

Pre-observation. Mark's pre-observation lesson began with bell work that asked students three questions pertaining to translations. During the bell work, Mark connected the representations of graphs, mapping notation, and verbal descriptions (see Figure 15). During dialogue regarding the bell work, Mark led a discussion between students to decide which figure was the pre-image.

> Mark: Tim, which one is our pre-image? The blue one or the red one?
>
> Tim: Blue?
>
> Mark: He says blue, but he's not real sure of that. Lauren, can you help him out, yes, no, maybe?
>
> Lauren: That's right.
>
> Mark: She said that's right. Can someone explain to me why that is right?
>
> Student: Because the red ones have the prime notation. (Mark Pre-Observation, 9/23/15)

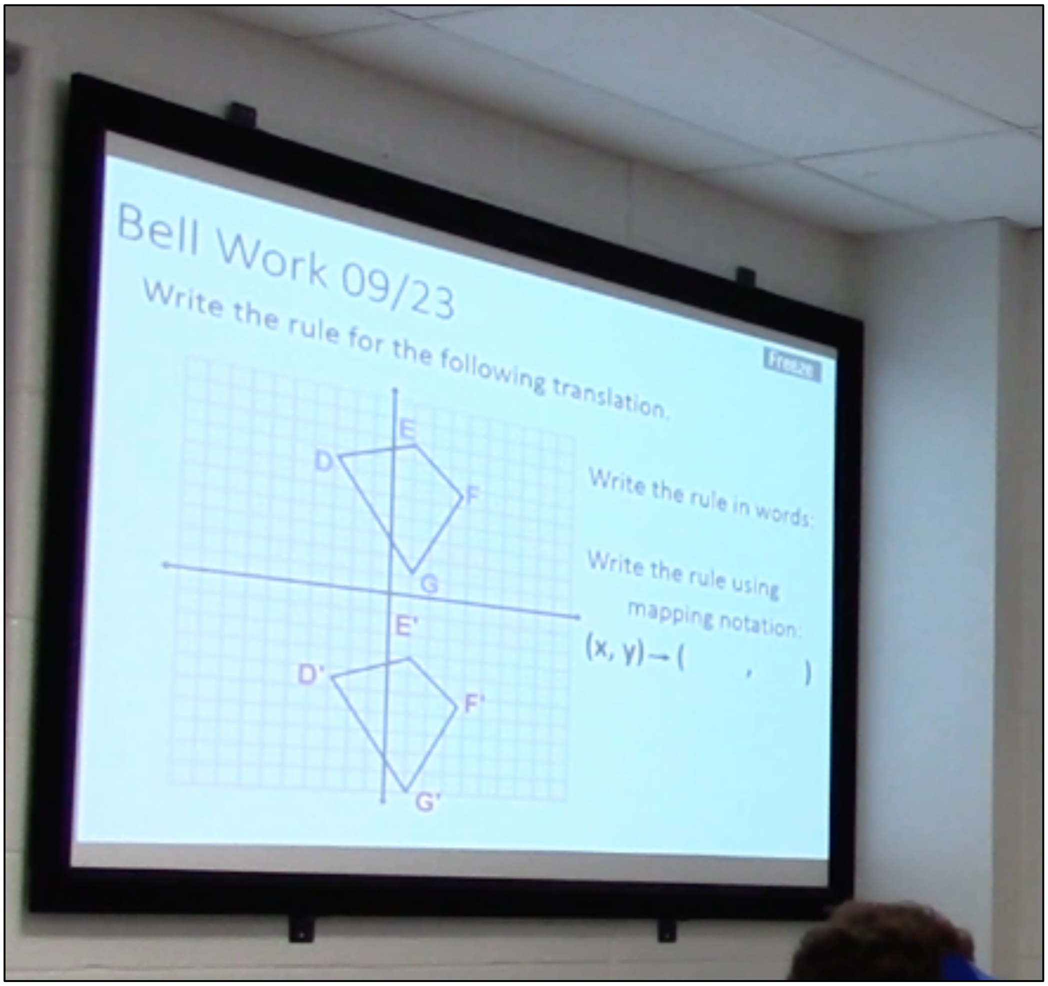

Figure 15. Mark's bell work connecting multiple representations in pre-observation.

The goal of the lesson was for students to be able to reflect figures across various horizontal, vertical, and diagonal lines. After Mark stated the objective for the day, he told students, "That is what you need to make sure you're able to do today" (Mark Pre-Observation, 9/23/15). Mark then described the three transformations in the unit (i.e., translations, reflections, and rotations) and said that during this day's lesson they would be learning about reflections. "What we said was a flip, right" (Mark Pre-Observation, 9/23/15)? To introduce reflections, Mark began by reminding students of what a reflection is and how it can be seen in a picture of a crane with its reflection in the water. Then, he discussed with students specific lines; vertical and horizontal lines such as $x = 0$, $y = 0$, and $x = 2$ and diagonal lines $y = x$ and $y = -x$. Next, Mark emphasized that corresponding points are equidistant from the line of reflection, "A and A prime are the same distance from the line of reflection" (Mark Pre-Observation, 9/23/15). To provide opportunities for students to understand how to graph $x = 0$ and $y = 0$, Mark asked students to give him the *x*-value at certain points. After students saw that all the points in which x is 0 formed the *y*-axis, Mark said, "So when we see $x = 0$, we really know that that is the *y*-axis." A similar discussion occurred for the $y = 0$, except this time Mark called a student to the board to identify some points for which $y = 0$.

Similarly, discussions about the line $x = 2$, $y = 2$, $y = x$, and $y = -x$ occurred. While discussing the last two lines, Mark generalized $y = x$ as "when y equals x" (Mark Pre-Observation, 9/23/15) and generalized $y = -x$ as "when y is the opposite of x" (Mark Pre-Observation, 9/23/15). Mark then displayed some examples of reflecting across the *x*-axis and *y*-axis to help students understand that corresponding points are equidistant from the line of reflection. Mark checked distances in an example of reflecting across $y = x$ to reinforce the idea that corresponding points are the same distance away from the line of reflection.

When presenting another example, Mark said, "Look at the coordinates of the pre-image and the coordinates of the image. Do you see anything going on there" (Mark Pre-Observation, 9/23/15)? Mark's intent was to support students in discovering the pattern when reflecting across the *x*-axis. A student responded, "We are changing all of the *y*-values to their opposite" (Mark Pre-Observation, 9/23/15). Similarly, students located the points when reflecting a figure across the *y*-axis, and Mark said, "Do the same thing as we did in the first example. Compare those real quick" (Mark Pre-Observation, 9/23/15). As before, Mark aimed to get students to discover the pattern when reflecting across the *y*-axis. A student revealed that "the *x* [coordinate] changes" (Mark Pre-Observation, 9/23/15). However, Mark pressed the student to explain. "What do you mean by changes" (Mark Pre-Observation, 9/23/15)? The student responded, "The negative [x-coordinate] becomes positive" (Mark Pre-Observation, 9/23/15).

Mark then gave students general rules to follow (see Figure 16), sometimes referring back to the cases they had already examined. Students then attempted to apply the general patterns to reflections across certain lines. At this point, Mark discussed Gary's work on the projector (see Figure 17) and said, "Notice how it just reflected, just like a mirror image, across [$y = x$]" (Mark Pre-Observation, 9/23/15). Mark used this example to confirm that the general rule actually worked. The class then attempted to reflect a figure across the line $y = -x$. Mark presented another student's work to further explain to students how "switching the coordinates and change it to its opposite, it works, right" (Mark Pre-Observation, 9/23/15)? Finally, Mark revisited the objective and concluded the lesson with three problems on an exit ticket. "In one word describe a reflection; a reflected figure has _______ size and shape as the original figure; a figure reflected over the line of reflection is _______ distance from the line as the original figure" (Mark Pre-Observation, 9/23/15).

<u>Reflecting with graph paper</u>: Simply flip the figure over the x-axis or y-axis (whichever is directed).

<u>Reflecting without graph paper:</u>

- To reflect over the **x-axis,** keep the x ordinate the same and make the **y coordinate its opposite.**
  (x, y)→(x, -y). **Example:** (3, 5)→(3, -5)
- To reflect over the **y-axis,** keep the y ordinate the same and make the **x coordinate to its opposite**
  (x, y)→(-x, y). **Example:** (4, 7)→(-4, 7)
- To reflect over **y = x**, switch the **x coordinate** and **y coordinate** in the ordered pair.
  (x, y) →(y, x). **Example**: (5, 8) →(8, 5)
- To reflect over **y = -x**, switch the **x coordinate** and **y coordinate** in the ordered pair and make **both** the **x coordinate** and **y coordinate** its opposite.
  (x, y) →(-y, -x). **Example**: (2, 4) →(-4, -2)

Figure 16. Mark's notes on how to reflect across certain lines in pre-observation.

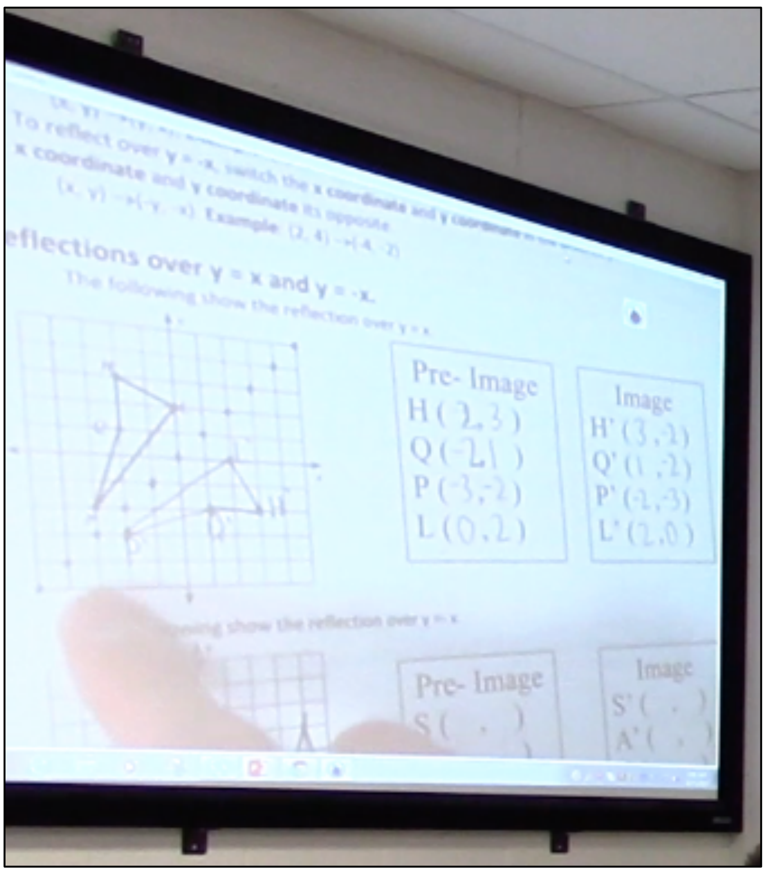

Figure 17. Mark presenting student work in pre-observation.

In terms of the Mathematics Teaching Practices (NCTM, 2014) (see Table 12), Mark used questioning strategies that mainly assessed student learning with procedural answers (MTP 5). Higher-order thinking questions were limited to the two instances in which he asked students to look for patterns when reflecting across the *x*-axis and *y*-axis (MTP 6). He referenced the objective of the lesson and how it fit into the larger unit (MTP 1). Also, Mark tried to make sure that students understood what a reflection was (using distances) prior to looking at examples and developing the general pattern (MTP 6). He also attempted to develop students' understanding of certain lines by explaining why they look the way they do (by examining coordinates) (MTP 6). However, student discourse was minimal (MTP 4) and meaningful tasks were not provided (MTP 2).

Table 12

Mark's Pre-Observation Teaching Practices

MTP	Evidence
1	Mark described the three transformations in the unit [i.e., translations, reflections, and rotations] and said that they would be learning about reflections. "What we said was a flip, right" (Mark Pre-Observation, 9/23/15)? Mark stated the objective for the day and told students "that is what you need to make sure you're able to do today" (Mark Pre-Observation, 9/23/15). Revisited the objective at the end of the lesson.
2	N/A
3	Made connections between words (description), graphical, and symbolic representation of translations during the bell work. Connected coordinates and graphical representations while teaching students about reflections.

Table 12 continued

MTP	Evidence
4	Students had a discussion about which figure is the pre-image.
5	Mark said, "Can someone explain to me why that is right" (Mark Pre-Observation, 9/23/15)? Questioned students in order to help them understand why $x = 0$ is the y-axis and $y = 0$ is the x-axis. Mr. Gibson used purposeful questioning strategies to help students understand $y = x$ and $y = -x$. "Look at the coordinates of the pre-image and the coordinates of the image. Do you see anything going on there" (Mark Pre-Observation, 9/23/15)? Mark was trying to get students to discover the pattern when reflecting across the x-axis. "What do you mean by changes" (Mark Pre-Observation, 9/23/15)?
6	Mark showed students a picture of a swan and its reflection in water. Mark emphasized that corresponding points are the same distance away from the line of reflection. Generalized $y = x$ as "when y equals x" (Mark Pre-Observation, 9/23/15) and generalized $y = -x$ as "when y is the opposite of x" (Mark Pre-Observation, 9/23/15). Mark checked distances in an example of reflecting across $y = x$ to reinforce the idea that corresponding points are the same distance away from the line of reflection. "Do the same thing as we did in the first example" (Mark Pre-Observation, 9/23/15). Mr. Gibson was trying to get students to discover the pattern when reflecting across the y-axis.
7	Mark gave students time to grapple with reflections and try to figure out the general pattern for reflecting across certain lines.

Table 12 continued

MTP	Evidence
8	Mark displayed Gary's work on the projector so that everyone could see it and so that he could use it to discuss ideas with the class. Mark looked at student work as he circulated the room and noticed that some students had already found the rule for reflecting across the line $y = -x$. Mark displayed Cindy's work on the projector so that everyone could see it and so that he could use it to discuss ideas with the class. Mark assessed student learning with an exit ticket with three questions.

Post-observation. Mark began the post-observation lesson by sharing the goal for the lesson, which was for students to write and graph linear equations. The lesson began with two questions for bell work. Both questions asked students to convert linear functions in standard form to slope-intercept form: $7x + 3y = 12$ and $x - 4y = -12$. It appeared as if students had already been taught how to convert from one form to the other in a previous lesson, as he expected them to know how to do so in the bell work. To review, Mark asked, "What was slope-intercept form again" (Mark Post-Observation, 11/18/15)? A student responded with, "*y* equals *m x* plus *b*" (Mark Post-Observation, 11/18/15). As Mark discussed the bell work first problem, he asked, "Why did I put the 7*x* before the 3. What was the point of that" (Mark Post-Observation, 11/18/15)? After a student said that the variable must come first, Mark asked, "Why does the variable go first" (Mark Post-Observation, 11/18/15)? A different student responded, "Because we want to put it in slope-intercept form" (Mark Post-Observation, 11/18/15). When discussing the second bell work problem, Mark stated, "I heard some of you say $\frac{x}{4} + 3$

[instead of $\frac{1}{4}x + 3$]. Could we write it this way? Okay, that means the same thing" (Mark Post-Observation, 11/18/15).

After the bell work, Mark restated the objective for the lesson before introducing new concepts. Mark distributed a worksheet that included problems that involved finding the equation of a line given its graph. In addition, he gave them detailed instructions on how to find the slope and *y*-intercept of the first graph. Then, Mark gave students six minutes to practice before discussing the worksheet as a class. Mark called on students to share each slope and *y*-intercept and drew the equation on the screen (see Figure 18). Then Mark distributed another worksheet that required students to graph equations from slope-intercept form. When transitioning to the next worksheet, Mark asked, "What do we notice about these equations" (Mark Post-Observation, 11/18/15)? A student said, "They are already in slope-intercept form" (Mark Post-Observation, 11/18/15). Again, Mark explained how to complete the first problem with help from students. Then he gave students two minutes to finish the worksheet.

As Mark explained the problems, he emphasized the idea that there are multiple ways to locate points of a linear function. "Instead of going down and to the right, I can go where" (Mark Post-Observation, 11/18/15)? Students responded with "up and to the left" (Mark Post-Observation, 11/18/15). During this discussion a student noticed a mistake in the line Mark graphed in the example and brought it to his attention. "Isn't it negative seven over one" (Mark Post-Observation, 11/18/15)? Mark corrected the line and gave students a final worksheet that combined the two procedures by asking them to convert from standard form to slope-intercept form and then graph the function. Mark said, "Last step in our progression here" (Mark Post-Observation, 11/23/15). Then Mark told the students that in order to reach the lesson's objective they needed to rewrite equations from standard form to slope-intercept form. After explaining the first example (graph $4x + y = 0$), students worked on the second problem on their own. Mark then modeled how to work the second problem and restated the objective of the lesson. Finally, Mark discussed the four reflection questions he originally used in Research Lesson 1 and distributed the homework for the night.

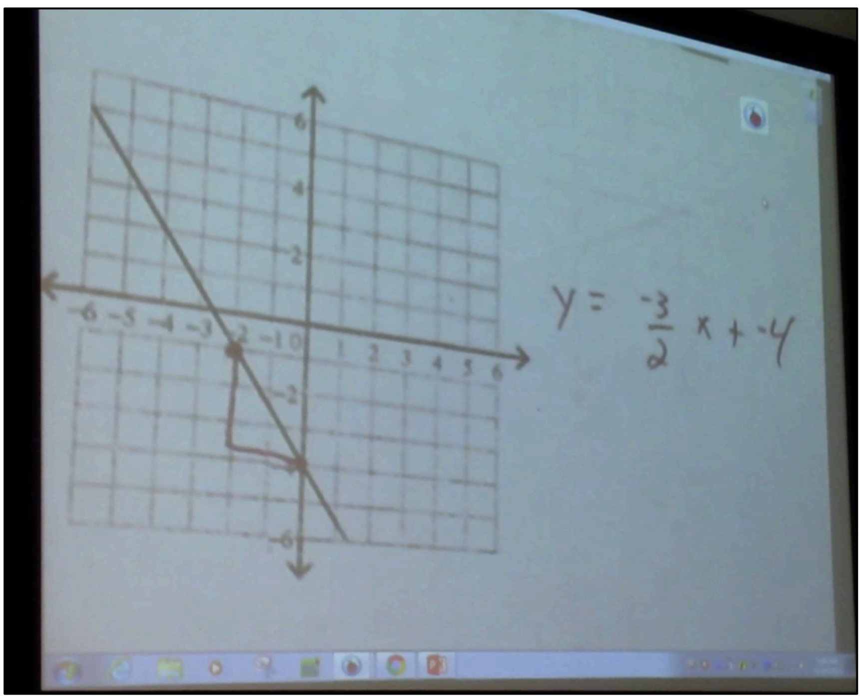

Figure 18. Mark's writing during a discussion of a worksheet in post-observation.

In regards to the Mathematics Teaching Practices (NCTM, 2014), Mark's lesson mainly contained practice problems and did not require students to reason or think abstractly. There were no real-life examples in the lesson (MTP 3). However, Mark made connections among representations of functions (MTP 3) and continually referred back to the objective (MTP 1). In fact, at the end of class Mark asked students to reflect upon the objective.

> Mark: Remember the things we want to be asking ourselves. Where is this going? What is the point of this? Why are we doing this? What are we going to do with this slope stuff? So ask yourself that. What mathematics is being learned? What is being learned here? What are we doing?
>
> Students: Slope.
>
> Mark: I hear people saying slope, solving for *y*. What else?
>
> Students: Graphing.
>
> Mark: Graphing. So we are doing a lot of different things. How does it relate to what we have already done? What did we do today that relates to what we have already done before?
>
> Students: Plotted points.
>
> Mark: Okay, we have plotted points in the past, so this relates to plotting points.
>
> Students: Pythagorean theorem.
>
> Mark: Okay, we talked about Pythagorean theorem with the triangles. So we looked at the little small triangles and how they worked with the line. Then that final question I want you to ask yourself, why is this important? So these are things we can learn from and use on a day-to-day basis depending on our job. We get older we are going to have jobs. We may need to find the slope of a hill. We need to understand those things. (Mark Post-Observation, 11/18/15)

Mark helped students reflect upon what they had learned and revealed to students a possible real-life application. Table 13 summarizes evidence of the Mathematics Teaching Practices (NCTM, 2014) in this lesson.

Table 13

Mark's Post-Observation Teaching Practices

MTP	Evidence
1	Mark told students the objective for today: convert to slope-intercept form and graph.
	Mark connected the bell work to yesterday's lesson and reviewed slope-intercept form.
	Mark revisited the objective for today before introducing new content.
	Mark revisited the objective to let students know where they were in the learning process and tell them where they are about to go.
	Mark stated, "Last step in our progression here" (Mark Post-Observation, 11/18/15). Then Mark said that in order to reach our objective we need to rewrite equations from standard form to slope-intercept form.
	Discussed the objective one last time during the final discussion of the class (transcription above).
2	N/A
3	Reviewed the connections between graph, table, and equation.
	Made connections between the graph and the equation throughout.
4	Student noticed a mistake by Mark and brought it to his attention.
5	Mark said, "Why did I put the 7*x* before the 3. What was the point of that" (Mark Post-Observation, 11/18/15)? After a student answered, Mark asked, "Why does the variable go first" (Mark Post-Observation, 11/18/15)?
	When transitioning to the next worksheet, Mark asked, "What do we notice about these equations" (Mark Post-Observation, 11/18/15)?

Table 13 continued

MTP	Evidence
6	N/A
7	Gave students time to try finding the equation from the graph. Brought up misconception of $6x - 4y = 3$ becoming $4y = -6x + 3$ because students think the negative went with the $6x$.
8	Mark stated, "I heard some of you say $x/4 + 3$. Could we write it this way" (Mark Post-Observation, 11/18/15)?

Comparison. There were two subtle differences between Mark's pre- and post-observation lessons. The first difference was related to MTP 1. Mark placed goal-related questions on his board and used those as talking points at the end of the lesson. During the pre-observation, Mark only referenced the objective twice (beginning and end). However, in the post-observation, Mark referenced the objective four times throughout the lesson, concluding with the new goal-related questions on his board. Mark used the questions he found in *Principles to Actions* (NCTM, 2014) and used during Research Lesson 1. The second difference was in terms of MTP 7. Mark identified a common misconception regarding negative signs, which he did not do in the pre-observation. This helped reveal to students that mistakes are a natural part of the learning process.

There were also two practices that were better supported in the pre-observation lesson: MTP 6 and MTP 8. Mark provided more opportunities during the pre-observation lesson for students to build procedural fluency upon conceptual understanding. In addition, Mark presented student work during the pre-observation, but did not do so in the post-observation. This could be because students had already learned how to convert to slope-intercept form prior to the post-observation lesson. A comparison of the two lessons with respect to the Mathematics Teaching Practices (NCTM, 2014) is provided in Table 14.

Table 14

Mark's Pre- and Post-Observation Comparison

MTP	Comparison
1	Although Mark referenced the goals twice during the pre-observation lesson, he did so four times in the post-observation lesson. Moreover, he placed four goal-related reflection questions above his board and led a discussion of those questions at the end of class.
2	Neither lesson included a true problem-solving task that required a high level of cognitive demand.
3	Mark made connections between multiple representations in both lessons. He did not include contextual representations in either lesson.
4	Students were not given much opportunity in either lesson to share and discuss their ideas.
5	Mark used purposeful questions in both lessons to assess and advance student reasoning.
6	Mark's pre-observation lesson included more opportunities for students to reason through why procedures they were using worked when compared to the post-observation lesson.
7	Mark gave students time to work individually during both lessons. However, Mark also addressed a common misconception in his post-observation lesson.
8	During the pre-observation lesson, Mark displayed two students' work on the projector so that other students could see it and so that he could use it to discuss ideas with the class. This was not enacted in the post-observation lesson.

Changes in conception and perception. Occurrences during the lesson study, pre- and post-observations, and pre- and post-interview responses were used to identify changes that were made with respect to Mark's conception and perception of the

Mathematics Teaching Practices (NCTM, 2014). Mark's major changes were related to using goals to guide instructional decisions, referencing and reflecting upon goals throughout a lesson, questioning strategies, productive struggle, and sharing student work with other students. Each of these major changes will be described in detail in the paragraphs that follow along with an auditable trail that provides support for these claims.

Goals guide instructional decisions. Mark made improvements in his conception of MTP 1 related to using goals to guide instructional decisions. Mark made multiple comments during the lesson study that supported Mark's change in conception of MTP 1. For example, during Research Lesson 1, Mark decided to wait until class the following day to conclude the discussion of the Umbrellas and Hats Task (see Appendix H). His rationale was that he "didn't want to cut it short and stop it and sum it up when they were nowhere close to being at the goal we were trying to get to" (Lesson 1 Debrief, 11/2/15).

Similarly, Mark stated that he did not "feel like [the Fruits and Vegetables Task (see Appendix I)] led as much to systems as it did just substituting and exchanging" (Lesson 1 Debrief, 11/2/15). Mark later added, "It took time out, and I feel like it didn't focus on what we were trying to do" (Lesson 1 Debrief, 11/2/15). When asked about what changes he had made to his research lesson plan, Mark revealed how important it was to relate back to the goal of the lesson. "I made changes to my bell work to better reflect systems of equations from the discussion we had last week" (Lesson 1 Debrief, 11/2/15). "We decided that it was not as clear as I was wanting it to be. All to better reflect our goals of the lesson" (Mark Self-Reflection 4, 11/3/15). Each of these statements signified Mark's desire to align instructional decisions with the research lesson's goal.

Although Mark did not mention how goals guide instructional decisions in his pre-interview, he mentioned this idea multiple times in his post-interview. "Without all of those [distractions] . . . minimizing all the extra stuff. That would help reduce the amount of time we spend on things – just pinpointing exactly the stuff that we are supposed to be teaching" (Mark Post-Interview, 11/18/15). Mark understood how important goals are when making instructional decisions to focus learning. He added, "Just being able to tweak our task that we wanted to do to get where we wanted to be. I think we went through two or three tasks before we started" (Mark Post-Interview, 11/18/15). In this statement, Mark was referring to the multiple changes that were made to tasks between Research Lesson 1 and Research Lesson 3. Finally, Mark emphasized the importance of clarifying goals in helping make instructional decisions based on his experience in the lesson study. "To be able to focus the goals as teachers, I feel like it helps with the lesson planning for sure" (Mark Post-Interview, 11/18/15). Mark realized how the specificity of the goal of a lesson impacts decisions that are made during planning. Overall, these statements signified the changes Mark made to his conception of using goals to guide instructional decisions.

Referencing and reflecting upon goals throughout a lesson. Perhaps the most substantial change that Mark made was on MTP 1 related to referencing and reflecting upon goals during a lesson. Evidence supporting claims about Mark's change in conception and perception of MTP 1 will be delineated in the paragraphs that follow.

Conception. In his first research lesson plan, Mark stated that he would "introduce the objective" (Mark Lesson Plan 1, 10/27/15). However, Mark expanded on this idea in his second research lesson plan by posing four goal-oriented questions that he found in

Principles to Action (NCTM, 2014). After Mark posed the questions to his students during Research Lesson 1, Sally commented, “Oh, and I really liked the four questions that you had at the beginning when you were making your goals. I thought that was good” (Lesson 1 Debrief, 11/2/15). Mark responded by discussing how he planned to have students reflect upon goals.

> That was going to be my exit ticket [as well] . . . to answer those four questions. That was the conclusion. To try to figure out where we were going and how did it relate to what we were doing and why is it important . . . I have been starting to ask [students] those questions over everything that we are doing. It really helps them just kind of think about how it connects and where it is going. I think that is useful. (Lesson 1 Debrief, 11/2/15)

This statement signified Mark’s long-term change in this area, as he desired to continue this practice.

As he observed other research lessons, Mark recognized how the goals were being used in the lessons. In fact, the only weakness Mark noted of Research Lesson 2 was that “[Sally] introduced the objective as developing [the] concept of systems but did not explain what that meant after that” (Mark Lesson 2 Observation Protocol, 11/6/15). Expanding on this idea in the Lesson 2 Debrief, Mark said:

> We just said, systems of equations . . . they have never heard of those three words and we never talked about it again. So, I felt that was the weakness of the [lesson], because we just gave them a goal, but we never hit that goal or expressed what that goal actually was. (Lesson 2 Debrief, 11/6/15)

Similarly, Mark wrote that a weakness of Research Lesson 3 was the "timing and conclusion of objective" (Mark Lesson 3 Observation Protocol, 11/9/15).

After the lesson study, Mark stated that he had changed the way he approached goals as a result of the lesson study. "My goals and questions are more defined than before [the lesson study]" (Mark Self-Reflection 7, 11/20/15). Moreover, Mark was able to clarify the purpose of a goal as a result of his experience. "I think just being able to talk about the goals of my lesson . . . I think [the lesson study] really clarified a lot of it for me and just what the purpose of the goal was" (Mark Post-Interview, 11/18/15). His conception of MTP 1 now included referencing and reflecting upon goals. "They need to be thinking about [goals] the entire time, I feel like" (Mark Post-Interview, 11/18/15).

Perception. Although Mark briefly mentioned "keeping the goals of what is being taught in mind" during his pre-interview, his perception of this idea was altered during the lesson study. Near the beginning of the lesson study, Mark provided evidence that he valued referencing goals. "During a lesson, teachers should make sure students understand the goal they are trying to get to each day" (Mark Self-Reflection 1, 9/23/15). In addition, Mark commented that "students should stay engaged and have an active role in the lesson, keeping the goals of what is being taught in mind" (Mark Self-Reflection 1, 9/23/15). In fact, when asked which of the Mathematics Teaching Practices (NCTM, 2014) he agreed with most, Mark stated:

> Establish goals to focus learning. I really think that having the kids understand what they are expected to do and what they can focus in on . . . it helps them. Because I have been in classrooms where we did a lesson that day…and it's like what in the world did we just talk about? I have no idea. What was the point? So

> having those goals I think helps a lot. So that's what attracted me the most. Just having a focused goal and everyone knows where they are going. (Planning Lesson 1, 10/14/15)

After the lesson study Mark stated:

> I feel like [students] learn best when there's clear goals. They know what they're going to be learning . . . I think it helps because they can ask themselves, “How is this working? Am I doing what I am supposed to be doing? Is the idea coming across that is supposed to be coming across?” (Mark Post-Interview, 11/18/15)

Mark’s emphasis on referencing and reflecting upon goals during a lesson in these statements signified the meaningful changes he made to his perception of this aspect of MTP 1.

Posing purposeful questions. Mark made improvements in his perception of MTP 5 related to questioning strategies. At the beginning of the lesson study, Mark believed that his use of questions was one of his major weaknesses. The lesson study group discussed Mark’s questioning strategies after Research Lesson 1.

> Mark: My questioning can always be better. That is my weakness. Purposeful questions. I think I have always fell short there. I try to work on that.
>
> Sally: Yeah, you said questioning is your weakness, but I thought your questioning was good.
>
> Mark: I feel like the higher-order thinking. You know, those things, I think I just struggle on.
>
> Sally: But you were good at leading them to the next thing. Like asking leading questions.

> Britney: Like when the conversation stalled, you got it going again.
>
> Mark: I just have a bad habit of asking a question that is a short answer, instead of having them ask and talk to each other and ask and clarify like you said. I feel like that is one thing I need to work on with my questioning. Having them talk instead of me asking all the time. Just have more elaborate questions I guess . . . higher-order thinking. (Lesson 1 Debrief, 11/2/15)

Contrary to other group members' thoughts, Mark believed that questioning was one of his weaknesses and an area in which he wanted to make improvements.

However, after Research Lesson 1, Mark wrote in his self-reflection that he thought his "questions [had] included more higher-order thinking" (Mark Self-Reflection 4, 11/3/15) since the beginning of the lesson study. Subsequently, Mark appreciated Britney's use of questioning strategies in Research Lesson 3. "I heard you say several times 'Tell me why you did that', and so I liked that question to try to get them to explain" (Lesson 3 Debrief, 11/9/15). During Lesson 3 Debrief, Britney was uncertain about whether she should have guided students more towards the specific goal. In response, Mark said, "I guess the difference there is the focusing versus the funneling. Trying to focus what we want them to do without funneling them to it" (Lesson 3 Debrief, 11/9/15).

When asked how his research lesson plan had changed after Research Lesson 3, Mark commented, "Asking questions that would lead to our objective of using equations" (Mark Self-Reflection 5, 11/11/15). In fact, Mark developed a greater confidence in developing questions throughout the lesson study. When he was asked about what productive habits he had developed by participating in the lesson study, he remarked, "Being intentional as far as what the goal was, what questions do I want to ask" (Lesson 3 Debrief, 11/9/15). Even though he began the lesson study by commenting that questioning was his weakness, he said that posing "purposeful questions to focus the lesson" (Mark Self-Reflection 7, 11/20/15) was a major strength of his post-observation. After the lesson study, Mark stated, "My goals and questions are more defined than before [the lesson study]" (Mark Self-Reflection 7, 11/20/15). Mark's statements revealed the alterations he made with respect to his conception of MTP 5 with regards to planning purposeful questions.

Productive struggle. Mark made enhancements in his conception of MTP 7 related to persevering through problem solving. Mark's primary change related to MTP 7 occurred in terms of his conception of productive struggle. Specifically, his conception changed regarding giving students time to struggle and not taking over the thinking. In Planning Meeting 1, the group discussed productive struggle, and Mark voiced a concern.

> Just the curriculum that we have to teach and trying to get all that in and being able to do the way I would love to be able to do it. I would love to be able to do that. I just feel like I would need two to three days to get a good lesson in, but I only have one day to do the whole entire concept and then I quiz the next day. So, unfortunately that is the world we live in. (Planning Meeting 1, 10/14/15)

As the group discussed the long-term goal of developing problem solvers, Mark revealed what he desired for his students.

> I think productive struggle is a big part of it. I want my kids to know the importance of why they are here. Are they going to use dilations in real life? Probably not, but it's not about that, it's about problem solving and using critical

> thinking . . . That's what I would like to see. My students putting in the effort and realizing it takes effort sometimes. (Planning Meeting 1, 10/14/15)

Mark’s statement focused on students struggling through a difficult task and realizing that struggling is a natural part of the learning process.

With this idea in mind, Mark recognized that students made progress towards the long-term goal during Research Lesson 1. “I definitely think we supported the productive struggle. I think we worked towards things and kept moving and chugging along . . . so I think that was good” (Lesson 1 Debrief, 11/2/15). Mark also added:

> I was happy that they were just trying to persevere through the struggle, you know. So I was happy with that, just because they were working. No one shut down and quit on me. I like that. They stalled a little bit, but none of them quit. (Lesson 1 Debrief, 11/2/15)

This statement still primarily concentrated on students struggling. However, changes to Mark’s conception of MTP 7 were evident at the end of the lesson study, as he began to focus on how teachers can support students in productive struggle. Mark stated in his post-interview, “To me what it means is, just to let them work. Let them try to persevere through that” (Mark Post-Interview, 11/18/15).

After the lesson study, Mark also reflected on changes that he had made as a result of the lesson study. “That’s one thing I’ve tried to do [since] . . . is [to] build productive struggle . . . but don't guide them too much to where they are just doing what you told them to do. Let them try to work it out on their own” (Mark Post-Interview, 11/18/15). Overall, Mark made meaningful changes to his conception related to giving students time to struggle and not guiding them too much.

Share student work with other students. Mark made changes in his conception of MTP 8 related to sharing student work. Although Mark discussed student thinking under MTP 8 in both the pre- and post-interview, there was no evidence that his conception included ideas about sharing student work with other students so that they can further develop their understanding. However, Mark enhanced his conception to include this idea throughout the lesson study. First, Mark described in his research lesson plans how students would share their work. “Allow students to discuss the task within their groups. While the groups are sharing their ideas and thoughts, be sure to walk around and look at which groups you would like to share” (Mark Lesson Plan 2, 11/2/15). He also stated that he would “allow students to present their thoughts and conclusions” (Mark Lesson Plan 2, 11/2/15). Mark’s rationale for students sharing their work was “to assess the students’ thoughts and thought processes” (Mark Lesson Plan 2, 11/2/15).

In Research Lesson 1, Mark presented two students’ work so that he could bring attention to two new strategies. He asked students to “talk about what you were thinking and how to do that” (Research Lesson 1, 11/2/15). While discussing Research Lesson 1, Mark commented, “We tried for sure to use evidence of student thinking and trying to write their work” (Lesson 1 Debrief, 11/2/15). Mark also discussed the reasoning behind presenting one student’s work. “She ended up getting the wrong answer because she added wrong, but I wanted her to show her work because I thought it was a neat way to go about doing it” (Lesson 1 Debrief, 11/2/15). Mark’s rationale was further evidence of his enhanced conception of MTP 8.

Mark further displayed evidence of this while commenting on Research Lesson 2. Mark wrote that a strength of the lesson was the “gathering information . . . copied down student work that was presented on the board" (Mark Lesson 2 Observation Protocol, 11/6/15). Similarly, during Research Lesson 3, Mark recognized that Britney "showed students' evidence of thinking by showing off their work, so I thought that was good - both with the bell work and at the end of the task" (Lesson 3 Debrief, 11/9/15). After the lesson study, Mark revealed his conception of MTP 8.

> Being able to get kids to tell you what they are thinking along with the work that they are putting down on paper and being able to use that to share and let others look at and understand. But just talking about how each person is thinking and being able to share that with the kids and . . . how Johnny did it, showing that to Susie . . . and letting them understand how others could think about the situation. Just being able to use their work and being able to see each other's is a good thing. (Mark Post-Interview, 11/18/15)

Although Mark focused on teachers understanding student thinking in his pre-interview, this statement signified Mark's change to his conception of MTP 8 related to using student work to aid the learning of other students.

Major influences. It is important to consider the aspects of lesson study that caused Mark to change. In the sections that follow, Mark's rationale for the different types of change will be provided along with evidence that confirms or disconfirms each claim.

Implementation. In terms of his changes in implementation of the Mathematics Teaching Practices (NCTM, 2014), Mark stated very confidently that it was observing the research lessons that was most valuable. When he was asked what was most influential to his implementation, Mark commented:

> That was definitely the observing. Just being able to see them do it . . . then how did I do it and then being able to see someone else do it. I think being able to watch someone else implement those things was the best part of that and helped me the most. (Mark Post-Interview, 11/18/15)

Due to the fact that Mark taught Research Lesson 1 and only made changes in implementation regarding goals, this claim could not be confirmed or disconfirmed.

Conception. In terms of his conception of the Mathematics Teaching Practices (NCTM, 2014), Mark indicated that the changes were partly due to reading *Principles to Actions* (NCTM, 2014), but primarily a result of his discussions with the lesson study group. When he was asked what was most helpful, he commented:

> Just reading was part of it, but . . . I think the reflection was, for me, the best part of that. Just being able to, after me reading it, being able to discuss it with the others and clarify some things that were in the book, but didn't quite click until I actually was talking. I would rather talk than read off a page. I feel like the practices became more [clear] after I heard opinions and how others read it off the page . . . than I read it off the page. So I think that was the biggest part for me. (Mark Post-Interview, 11/18/15)

This statement by Mark was confirmed by analyzing changes Mark made in his conception early in the lesson study. He enhanced his conception related to reflecting and referencing goals during a lesson and productive struggle prior to the first research lesson. This supported the idea that reading and discussing the Mathematics Teaching Practices (NCTM, 2014) was the most beneficial to altering Mark's conception of the Mathematics Teaching Practices (NCTM, 2014).

Perception. In terms of his perception of reform-oriented practices, Mark stated that the changes were a result of reading *Principles to Actions* (NCTM, 2014) and discussions with the lesson study group. When he was asked what was most valuable, he commented:

> I would probably again say the talking with the group . . . just being able to talk it out. Obviously, I felt a certain way after I read it and was able to clarify what I was thinking after we talked about it. So I would probably say again, just being able to talk with the group was nice. (Mark Post-Interview, 11/18/15)

This statement by Mark was supported by the notion that Mark altered his perception about reflecting and referencing goals during a lesson early in the lesson study process. However, it appeared as if Mark's transition related to questioning strategies took place over the entire lesson study process.

Summary. In Mark's case, the most significant changes were made in relation to MTP 1, MTP 5, MTP 7, and MTP 8. In fact, Mark altered his implementation, conception, and perception of MTP 1. Although Mark's most substantial change was in terms of using goals to guide instructional decisions (MTP 1) and referencing and reflecting on the goals of a lesson (MTP 1), he also made improvements in questioning strategies (MTP 5), productive struggle (MTP 7), and sharing student work with other students (MTP 8). A summary of all major and minor changes is provided in Table 15. Mark indicated that changes to his conceptions were initially influenced by reading *Principles to Actions* (NCTM, 2014), but were primarily a result of his discussions with the lesson study group. Moreover, Mark stated that observing the research lessons was influential in altering his implementation. Finally, Mark viewed reading *Principles to*

Actions (NCTM, 2014) and participating in discussions with the lesson study group as the most impactful to changes in his perception.

Table 15

Summary of Changes: Mark

MTP	Implementation	Conception	Perception
1	Mark placed goal-related questions above the board. Mark referenced the objective 2 times (beginning and end) during the pre-observation. In the post-observation, Mark referenced the objective 4 times throughout the lesson. Mark concluded by discussing the goal related problems above his board.	Have students think about the goals of the lesson. Have students use the goals of the lesson to reflect on how they are doing. Goals should guide instructional decisions (provide focus).	Getting students thinking about the goals (questions on board).
2	N/A	N/A	More tasks that promote higher learning.
3	N/A	N/A	Real-world context
4	N/A	N/A	N/A
5	N/A	N/A	Planning before the lesson what questions to ask.
6	N/A		"I wish I could do more conceptual learning" (Mark Post-Observation, 11/23/15).

Table 15 continued

MTP	Implementation	Conception	Perception
7	Mark brought up a common misconception.	Give students time to struggle. Do not take over the thinking.	N/A
8	N/A	Share student work with other students to show how others think about the situation.	N/A

Sally Mills

The results of changes made by Sally will be described in the sections to follow. A description of Sally's typical lesson structure will be outlined. Then major changes in implementation, conception, and perception of the Mathematics Teaching Practices (NCTM, 2014) will be described along with the aspects of the lesson study that influenced these changes.

Description of teaching. Sally, who was in her third year teaching, described her lessons as very structured. When a lesson starts, she said, "We go over the bell work and . . . we either do notes one day and activity the next day" (Sally Pre-Interview, 9/8/15). For example, on a typical day Sally might have "a few practice problems, and then [the next day] we will have some sort of activity that brings it all together. Like [the] 'I have who has' [game] or matching" (Sally Pre-Interview, 9/8/15). Sally explained how she liked to keep students engaged, and she tried to make it fun. Sally described one means for engaging students in this way. "Instead of having a worksheet, cut them up and make it task cards . . . that to them is more fun than a worksheet because they can just pass a card" (Sally Pre-Interview, 9/8/15).

Sally's views of her teaching prior to the lesson study emphasized students being engaged in the lesson and enjoying the lesson. Moreover, it focused on alternating between activities and taking notes. In the section that follows, Sally's implementation will be examined to explore how these ideas were enacted in her classroom before and after the lesson study.

Changes in implementation. Analysis of Sally's changes in implementation follows. To begin, a description of her pre-observation will be provided. Then, an account of her post-observation will be described. Finally, a synthesis comparing the two lessons will be provided along with evidence as to what changes were made.

Pre-observation. The goal of Sally's pre-observation lesson was for students to be able to reflect figures across lines. The lesson began with a bell work problem that asked students to translate a figure on the graph given a description (e.g., left and down) and to locate one of the new points. It also asked students to translate the figure given the symbolic notation (e.g., $x + 4$, $y - 5$). Conversation between Sally and the students involving words, symbolic notation, and graphical representation continued as Sally discussed the bell work problems. During this portion of the lesson, she at times randomly called on students and at other times called on volunteers. As Sally told students about the upcoming common formative assessment, she mentioned the main topic from the previous day's lesson (translations), the present lesson (reflections), and the following day's lesson (rotations). Then, she reviewed the previous day's homework and responded to questions regarding translations. While reviewing the homework, Sally again connected the same three representations (i.e., words, symbolic notation, and graphical representation) to describe translations (see Figure 19). After she demonstrated how to solve a few problems, she gave students two minutes to finish the remaining problems. All together, the bell work and homework comprised approximately half of the class period.

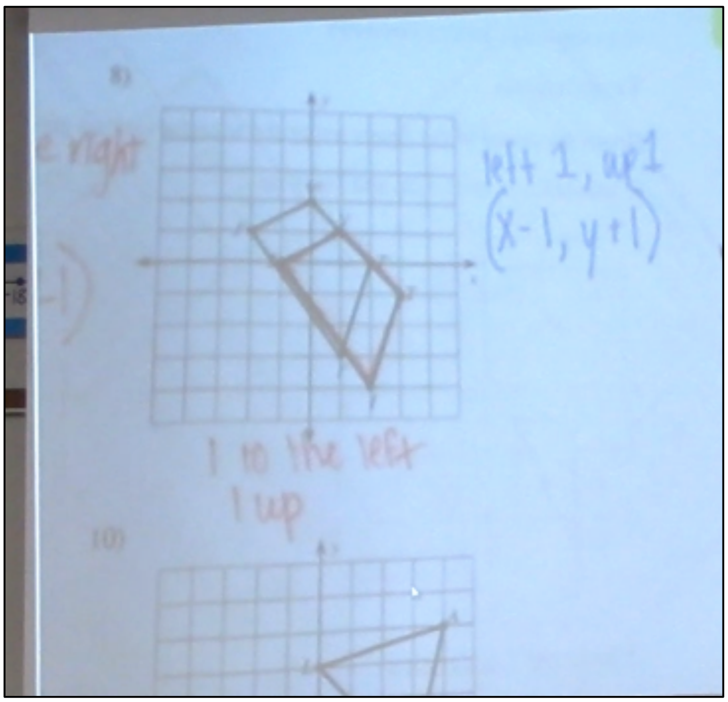

Figure 19. Sally's use of multiple representations in pre-observation.

Sally then shared the purpose of the lesson and began introducing new content by relating the concept of a reflection to the term flip.

> Sally: When you think of reflection, which is what we are doing today, what do you think of?
>
> Student: A mirror.
>
> Sally: Okay, a mirror. What is different about a mirror image of yourself?

Students: Opposite.

Sally: Yes, it is opposite. It is switched. It is?

Students: Flipped.

Sally: Flipped. Flipped is the word we are going to use. (Sally Pre-Observation, 9/23/15)

Sally stated that they would use the term flip and compared this term to the previous day's term, which was a slide. She then distributed a worksheet that students were to complete. Right away, she described for students how the coordinates change when reflecting across the *y*-axis and *x*-axis (see Figure 20). Then, Sally demonstrated for the class how to solve a few examples. During this time, Sally asked a student about the point (0, 5). "Why is it the same [reflecting across the y-axis]" (Sally Pre-Observation, 9/23/15)? The student then explained why by saying, "Zero can't be negative" (Sally Pre-Observation, 9/23/15). The student seemed to rely on the previously stated rule and was not pressed to think about what it meant to reflect in terms of distance.

Next, Sally distributed the homework sheet. However, there was a problem on the homework sheet that asked students to reflect across the line $y = 3$, a type of problem that had not been previously presented. As students received their homework sheet, Sally demonstrated how to complete that problem. To introduce students to the line $y = 3$, Sally said, "Put a point on three on the *y*-axis and draw a line through it" (Sally Pre-Observation, 9/23/15). Then, Sally allowed students to see that points of reflection were equidistant from the line of reflection. "We are not going to use a rule on this one. If this point is two below the line [of reflection] . . . if I flipped it, where would it be" (Sally Pre-Observation, 9/23/15)? Students responded by saying, "Two above" (Sally Pre-

Observation, 9/23/15). This provided students with the opportunity to learn what it means to reflect a figure in a more general sense.

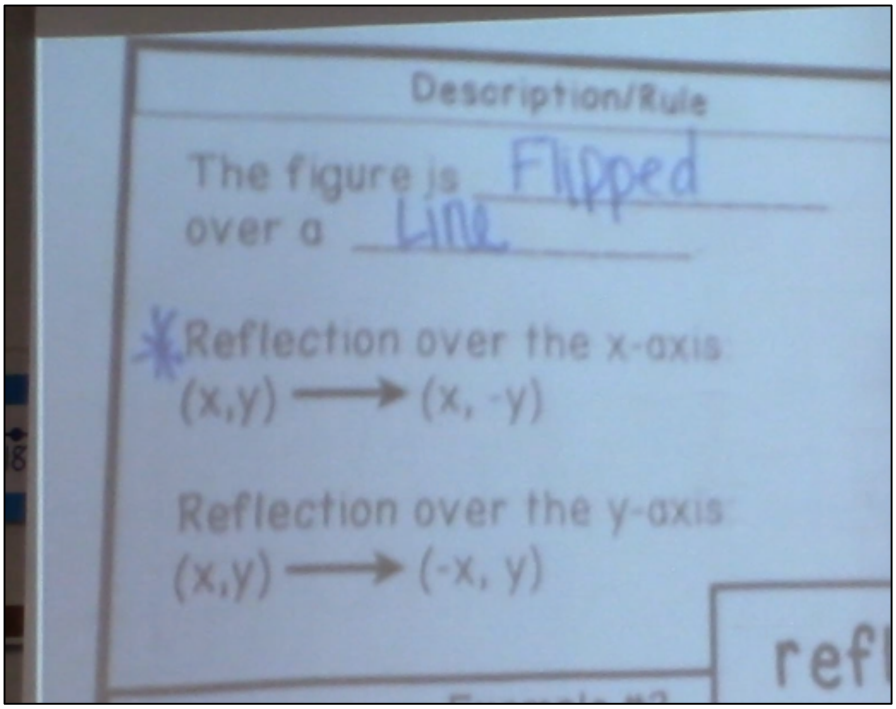

Figure 20. Sally's worksheet in pre-observation.

With respect to the Mathematics Teaching Practices (NCTM, 2014) (see Table 16), Sally made connections between the various representations during the bell work activity (MTP 3). However, Sally began teaching students about reflections by telling students exactly what to do when reflecting across the *x*-axis and *y*-axis (MTP 6). The concept of equal distances was not mentioned until the homework, when students were faced with a new situation for which they did not have a rule. This resulted in a procedural lesson (MTP 6) and did not engage students in doing mathematics. Moreover, students were not given many opportunities to discuss the mathematics or share their ideas (MTP 4). Overall, Sally's strongest connection to the Mathematics Teaching

Practices (NCTM, 2014) occurred near the beginning of class, as she made connections between multiple representations (MTP 3) and told students the goal for the day (MTP 1).

Table 16

Sally's Pre-Observation Teaching Practices

MTP	Evidence
1	Sally briefly described the purpose of the lesson. Sally connected the lesson's objective to the previous day's objective. "Yesterday, we wrote 'translation is a slide.' Today, you are going to write 'reflection is a flip'" (Sally Pre-Observation, 9/23/15). Made a connection back to the prior knowledge of the word congruency.
2	N/A
3	Used multiple representations (graph, words, and symbols) during the bell work (all representing how to translate a figure).
4	After a student responded with (0,5), Sally asked the student, "Why is it the same [reflecting across the *y*-axis]" (Sally Pre-Observation, 9/23/15)? The student then explained, "0 can't be negative" (Sally Pre-Observation, 9/23/15). Students responded throughout the lesson with factual answers (no explanation, outside of the example above.)
5	N/A
6	"We are not going to use a rule on this one. If this point is two below the line [of reflection] . . . if I flipped it, where would it be" (Sally Pre-Observation, 9/23/15)? Students responded by saying two above. This allowed students the opportunity to learn what it means to reflect a figure in a more general sense.
7	Sally gave students time to practice a few problems on their own.
8	N/A

Post-observation. Sally began the lesson by sharing the lesson's goal, "Today we are writing equations from standard form to slope-intercept form" (Sally Post-

Observation, 11/18/15). The class began with a series of problems for bell work regarding slope (see Figure 21). One of the problems involved a graph of gallons remaining given miles driven. The students had to make sense of the graph and describe the situation. Throughout the three problems, Sally pressed the students to analyze slope with various representations: graph, context, and equation. Sally then discussed the bell work and asked students to explain their reasoning throughout. For example, after a student correctly labeled the slope in problem one of the bell work, Sally asked the student, "How do you know that" (Sally Post-Observation, 11/18/15)? The student said, "M" (Sally Post-Observation, 11/18/15). Sally followed up with this student and said, "What equation are we going off of" (Sally Post-Observation, 11/18/15)? The student said, "*y* equals *m x* plus *b*" (Sally Post-Observation, 11/18/15). After another student located the *y*-intercept, Sally asked, "How do you know that it is negative" (Sally Post-Observation, 11/18/15)?

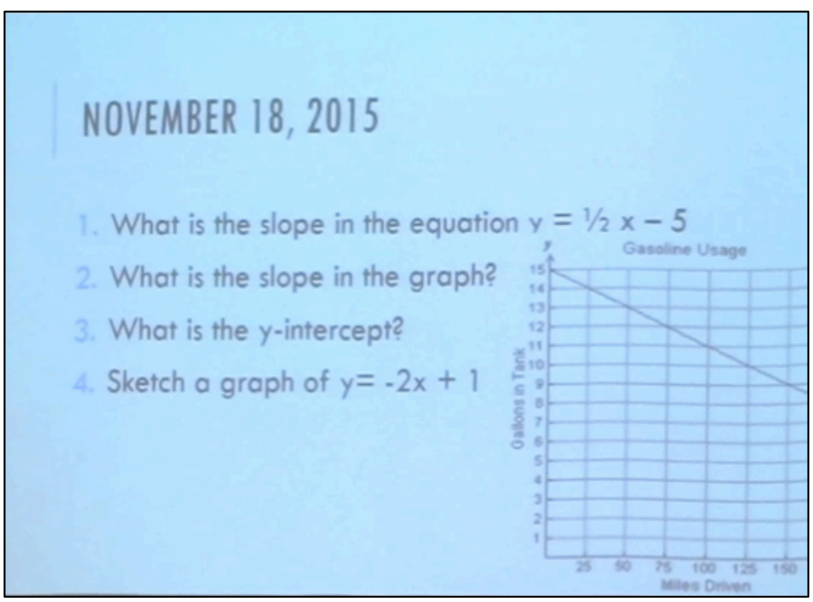

Figure 21. Sally's bell work questions in post-observation.

In terms of the gasoline graph (see Figure 21), Sally continued to ask questions that required students to provide explanation. After a student shared the wrong value of the slope, Sally responded, “Why did you say that it was -1” (Sally Post-Observation, 11/18/15)? The student’s explanation was incorrect because she did not consider the scale of the *x*-axis. Without indicating that the explanation was incorrect, Sally asked for other ideas. After another student shared the correct slope, Sally said, “Why do you think that” (Sally Post-Observation, 11/18/15)? The student referenced the scale of the *x*-axis then Sally asked the class with whom they agreed. “Do you agree with Gale or Macy” (Sally Post-Observation, 11/18/15)? The class agreed with the second explanation, and Sally said, “Yes . . . this graph looks a little bit different than what we have been doing, but I wanted you to see a real-world situation . . . What is this graph telling us . . . what is happening as they drive” (Sally Post-Observation, 11/18/15)? One student said, “Their gas is going down . . . 1 gallon per 25 miles” (Sally Post-Observation, 11/18/15).

Next, Sally gave students a worksheet that contained a four-step process for converting from standard form to slope-intercept form (see Figure 22). Sally demonstrated this four-step process on a few examples of one-step problems (e.g., $4 + y = 7x$) and then asked students to work individually on several practice problems. During this time, Sally continued asking questions that required explanation on behalf of the students. For example, she asked, “Why am I subtracting it?” (Sally Post-Observation, 11/18/15) and “Why are you dividing by -1” (Sally Post-Observation, 11/18/15)? When addressing the problem $12 + y = 4x$, Sally connected student-generated strategies to generate a discussion about which method would be the most efficient.

Sally: Ali said that we should move the 4*x* over [to the left side of the equation] and the *y* over [to the right side of the equation]. How many steps would that take?

Students: Two.

Sally: Two. I could do it that way, but is there a simpler way?

Student: Yes.

Sally: Taylor, what would you move?

Taylor: The 12.

Sally: The 12. Yes. If I move the 12, how many steps do I have to do?

Students: One.

Sally: One. Is that better than two?

Students: Yes. (Sally Post-Observation, 11/18/15)

y = mx + b

m = slope **b** = y-intercept

1- Underline the side of the equation that contains the y variable.
2- **ON THE SIDE WITH *Y***, circle the term that is farthest from y.
3- Use inverse operations to "peel" away the terms/coefficients that are with y. You can keep fractions –even improper ones- but the fraction must be in its simplest form. (Make certain that y is not negative. The negative sign has to go, too! You can eliminate it by dividing the entire equation by -1.)
4- Once one side has y completely by itself, the coefficient with x is the slope, and the constant is the y-intercept.

Figure 22. Sally's notes on how to convert to slope-intercept form in post-observation.

Sally continued by addressing two misconceptions. First, she addressed adding non-like terms by saying, "Can I put my 12 with my 4" (Sally Post-Observation, 11/18/15)? Second, while discussing two-step equations, Sally addressed the negative in the equation $5x - 3y = 15$. “There is a misconception that because I am subtracting we have to add $5x$ to both sides . . . a lot of people would add that because I am subtracting” (Sally Post-Observation, 11/18/15). Sally continued by solving more examples of two-step problems with the class, which was followed by a group activity. The activity required students to work together to connect cards that paired the correct equation in standard form with its corresponding slope-intercept form. At the end of class, Sally foreshadowed the next day’s lesson. “Can we graph them now” (Sally Post-Observation, 11/18/15)?

In terms of the Mathematics Teaching Practices (NCTM, 2014) (see Table 17), the bell work provided discussion regarding slope that involved multiple representations, including a contextual representation (MTP 3). In addition, Sally posed questions throughout the lesson that required students to explain themselves (MTP 5). Although the introduction to converting to slope-intercept form was focused on a procedure (MTP 6), Sally pressed students to explain their reasoning (MTP 5). In addition, students worked together during the group activity (MTP 4).

Table 17

Sally's Post-Observation Teaching Practices

MTP	Evidence
1	"Today we are writing equations from standard form to slope-intercept form" (Sally Post-Observation, 11/18/15). Sally asked students to think about what can be done with these equations now. "Can we graph them now" (Sally Post-Observation, 11/18/15)?
2	N/A
3	Made connections between the graph and the context of the car's gasoline level by miles driven. Analyzed slope with various representations: graph, context, and equation.
4	Students worked together to connect the correct cards.
5	After a student correctly labeled the slope in problem 1 of the bell work, Sally asked the student, "How do you know that" (Sally Post-Observation, 11/18/15)? The student said, "M" (Sally Post-Observation, 11/18/15). Sally followed up with this student and said, "What equation are we going off of" (Sally Post-Observation, 11/18/15)? The student said, "*y* equals *m x* plus *b*" (Sally Post-Observation, 11/18/15). After another student located the *y*-intercept, Sally said, "How do you know that it is negative" (Sally Post-Observation, 11/18/15)? "How did you know that it was -1" (Sally Post-Observation, 11/18/15)? "Why do you know that" (Sally Post-Observation, 11/18/15)? "Do you agree with Gale or Macy" (Sally Post-Observation, 11/18/15)? "What is this graph telling us" (Sally Post-Observation, 11/18/15)? "Why I am subtracting it" (Sally Post-Observation, 11/18/15)? "Why are you dividing by -1" (Sally Post-Observation, 11/18/15)?

Table 17 continued

MTP	Evidence
6	Very procedural list of steps to convert to slope-intercept form. Connected student-generated strategies to form a discussion about which method would be the most efficient.
7	Sally addressed a common misconception by saying, “There is a misconception that because I am subtracting that we have to add 5*x* to both sides” (Sally Post-Observation, 11/18/15). Gave students time to work together to try and figure it out.
8	The questions used by Sally elicited student thinking.

Comparison. During the bell work of the post-observation lesson, Sally presented students with a problem about gas consumption and mileage. Not only did this problem connect slope to its graphical representation, but it also connected slope to a contextual representation. This was significant because there were no real-life examples present in Sally’s pre-observation. Sally also addressed a common misconception related to solving for *y*, which did not occur in the pre-observation. In addition, Sally asked more questions that pressed students to provide explanation. During the pre-observation, Sally asked one question that asked students to explain their reasoning. In contrast, during the post-observation, Sally asked seven questions that called for students to explain their reasoning. Finally, Sally gave students time to work together, which was not the case in the pre-observation lesson. Overall, the introduction of new content during the post-observation was procedural, as was the case in the pre-observation. Sally made changes, however, in both questioning and mathematical discourse. A comparison of the two lessons with respect to the Mathematics Teaching Practices (NCTM, 2014) is provided in Table 18.

Table 18

Sally's Pre- and Post-Observation Comparison

MTP	Comparison
1	Sally briefly described the purpose of both lessons. She did not refer back to the goals in either lesson.
2	Neither lesson included a true problem-solving task that required a high level of cognitive demand.
3	Although Sally connected multiple representations in both lessons, she also included contextual representation of slope in her post-observation lesson.
4	Sally's pre-observation lesson was primarily teacher-directed. However, in her post-observation, Sally allowed students to work together in groups on an activity.
5	Only one of Sally's questions in the pre-observation pressed students to explain their reasoning, compared to seven in the post-observation.
6	Even though the post-observation bell work included the gasoline problem and some purposeful questions, the introduction of new content in both lessons was very procedural and did not allow for students to use their own reasoning strategies.
7	Sally gave students time to work during both lessons. However, Sally also addressed a common misconception in her post-observation lesson.
8	Sally did not have students share their work in either of the two lessons. Some student thinking was revealed as a result of her questions in the post-observation lesson.

Changes in conception and perception. Occurrences during the lesson study, pre- and post-observations, and pre- and post-interview responses were used to identify changes that were made with respect to Sally's conception and perception of the

Mathematics Teaching Practices (NCTM, 2014). Sally's major changes were related to situating goals within students' learning progression, using contextual representations, facilitating mathematical discourse, using purposeful questioning, and supporting productive struggle. Each of these major changes will be described in detail in the paragraphs that follow along with an auditable trail that will provide support for these claims.

Situate goals within learning progression. With respect to MTP 1, Sally developed her conception of how goals should be situated within the learning progression. Evidence of this change emerged early in the lesson study when Sally said, "Teachers should guide their learners through rigorous instruction and activity that ties in content with the learners' individual needs" (Sally Self-Reflection 1, 9/23/15). Sally had students' prior knowledge in mind as she reflected upon her pre-observation. "Up to this point in their mathematical lives, students really only know how to plot points on a graph" (Sally Self-Reflection 2, 10/11/15). Sally's conception of MTP 1 was supported throughout the lesson study as the group worked to refine their goal. Sally described a change she made to the research lesson to fit within students' learning progressions.

> Since [Research Lesson 1], we kind of refined our objective a little bit more . . . [In my lesson] everything was the same, except "write the equation" or "write an equation that we could use" and just getting them to see that first before giving them this task. (Lesson 2 Debrief, 11/6/15)

In this statement, Sally was referring to how she altered the bell work in the research lesson to prepare students for the Umbrellas and Hats Task (see Appendix H).

After Research Lesson 3, Sally commented on how the goal for the lesson addressed students' capabilities.

> I agree with you about the goal. I think finally that we have narrowed it down enough to really get what they are capable of at this moment. Just working with two variables and being able to identify a solution. (Lesson 3 Debrief, 11/9/15)

Sally later reflected upon her experience in the lesson study, thus affirming her enhanced conception of MTP 1 after the lesson study. "I think it is just seeing where the students are and then where we need to get them and then just keep pushing towards the big goals with smaller goals . . . [so that] students can connect what they have learned in the past and see how it all comes together" (Sally Post-Interview, 11/23/15). Sally began to consider how the goal of a lesson is situated within students' learning progression.

Contextual representations. Sally's primary change related to MTP 3 related to her conception of contextual representations. Before the lesson study, Sally had no conception of MTP 3, as she was confused about what mathematical representations meant. Sally stated, "I don't know that one. So what's a mathematical representation" (Sally Pre-Interview, 9/8/15)? However, she made substantial changes throughout the lesson study with respect to contextual representations. Near the beginning of the lesson study, Sally stated, "I believe learning is enhanced when a more memorable task is involved, and mathematics is connected to students' lives. Mathematics becomes more meaningful if the content is put into context of real-world experiences" (Sally Self-Reflection 1, 9/23/15). Sally instituted this idea in her second research lesson plan that read, "The class will discuss how these tasks were helpful and how we might use these strategies and problem solving in the real world" (Sally Lesson Plan 2, 11/2/15). Sally did not mention contextual representations during her pre-interview. Yet, she articulated its importance in her post-interview by saying that a teacher's main role is to "facilitate their [students'] discussion about the problem-solving and real-world questions" (Sally Post-Interview, 11/23/15).

Facilitate mathematical discourse. During the lesson study, Sally enhanced her conception and perception of MTP 4. These changes are described in the paragraphs that follow.

Conception. A comparison of Sally's responses regarding MTP 4 revealed changes related to questioning strategies and facilitating the sharing of students' reasoning within real-life situations. Sally began including these ideas in her research lesson plans, which incorporated opportunities for students to discuss the mathematics. "[The teacher will] lead the class in discussion over different strategies to use when problem solving" (Sally Lesson Plan 3, 11/6/15). Then in her fourth research lesson plan, she incorporated purposefully selecting groups to present. "[The teacher will] select groups to explain their thinking and their answers. The groups will then present their answers to the whole class" (Sally Lesson Plan 4, 11/9/15).

Prior to writing her fourth research lesson plan, Sally recognized effective mathematical discourse during Research Lesson 1, stating that, "Mark did a great job of facilitating meaningful mathematical discourse. The whole class was engaged, they were putting their ideas together, and were having great conversations with one another" (Self-Reflection 4, 11/6/15). Sally applied what she had learned in order to facilitate mathematical discourse in Research Lesson 2. Afterwards, other group members made positive comments about how she facilitated the discourse in her lesson.

Building upon what she had gained from the lesson study, Sally argued that teachers should ask “purposeful questions that kind of lead into discussion within the classroom. So that also keeps the students focused because they are having to have conversations with their peers and also the teacher . . . so facilitating that mathematical discourse” (Sally Post-Interview, 11/23/15). This statement signified Sally’s changes to her conception of MTP 4, as she did not mention facilitating mathematical discourse in her pre-interview.

Perception. When asked about her views about the teaching and learning of mathematics, Sally did not mention mathematical discourse in her pre-interview. However, after reading *Principles to Actions* (NCTM, 2014), Sally stated that MTP 4 was the practice she agreed with most.

> I agree most with facilitating meaningful discourse in the mathematics classroom. I believe students can learn a lot from each other, and exchanging ideas through discussion within the mathematics classroom is a great way to do that. This allows for students to be actively engaged in their own learning process. This type of environment also encourages students to listen to each other's ideas and hear other views or approaches to a particular problem. (Sally Self-Reflection 3, 10/14/15)

Sally continued to display changes related to MTP 4 in Planning Meeting 1.

> I put facilitate mathematical discourse just because I think in an open environment classroom where the kids are having a discussion about mathematics. I think that shows a deeper level of understanding if they are able to talk about it and go off of each others’ conversations . . . back up their own point and justify their

> thinking. I think that deepens the conceptual understanding, which leads to the other principles. (Planning Meeting 1, 10/14/15)

When the time came for her post-observation lesson, she stated that she “paid more attention to the questions I was asking, and the mathematical discourse that was happening. I also tried to choose an activity that may cause some productive struggle, but let them work with groups to problem solve together” (Sally Self-Reflection 7, 11/23/15).

After the lesson study, she made a connection between MTP 4 and MTP 5, stating, “I think [Purposeful Questioning and Facilitating Mathematical Discourse] kind of go hand-in-hand. I think when you get really good at this, they all kind of flow . . . because if you are asking questions, that leads to the mathematical discourse” (Sally Post-Interview, 11/23/15). When asked again about her views about the teaching and learning of mathematics, Sally commented about students having “conversations with their peers and also the teacher” (Sally Post-Interview, 11/23/15) and how teachers can facilitate mathematical discourse.

Purposeful questioning. Sally made changes related to her perception of posing purposeful questions that probe and extend student thinking. When asked about her views about the teaching and learning of mathematics before the lesson study, Sally did not mention questions or questioning. However, Sally experienced change in this area during the lesson study. Evidence of Sally beginning to value questions that press students to explain and justify occurred after Research Lesson 1.

> I heard [Mark] one time say that you need to prove . . . because they guess-and-checked . . . and you were like, “Is there a way you can prove that that is right?”

> And I thought that was good because, just guessing is not going to prove it. (Lesson 1 Debrief, 11/2/15)

Sally later stated that Mark was “good at leading them to the next thing . . . like asking leading questions” (Lesson 1 Debrief, 11/2/15).

In Research Lesson 2, Sally used probing questions like, “How did you know it was more?” (Research Lesson 2, 11/6/15) and “How could we write that like this . . . if we are following this pattern?” (Research Lesson 2, 11/6/15) to get students to explain themselves. She explained, “I was trying to get them to at least explain why they were guessing random numbers or where they were getting the random numbers” (Lesson 2 Debrief, 11/6/15). However, Sally realized that she should have used different questions to focus students’ attention on writing equations. “So I think I just need to refine that a little bit and maybe come up with different questions that I could ask to get them in that direction” (Lesson 2 Debrief, 11/6/15). These statements revealed the value that Sally placed on questioning strategies.

Sally responded to a comment made by Mark about intentionality by describing changes she had made. “You're right because I never really think about the questions I'm going to ask beforehand . . . [Now] I find myself planning a lesson thinking ‘okay, if they think this way, what can I ask’” (Lesson 3 Debrief, 11/9/15)? Further, Sally stated, “I paid more attention to the questions I was asking, and the mathematical discourse that was happening [during the post-observation lesson]” (Sally Self-Reflection 7, 11/23/15) because of her experience in the lesson study. Moreover, Sally made changes to the process by which she chooses questions.

> I am more purposeful, I guess, in planning lessons. Like trying to ask more in-depth questions. I am more intentional when I am teaching, because I am thinking about these eight practices . . . and are my questions good enough? Is the task hard enough? Or is this promoting problem solving? I really do think about these things now when I plan and as I am teaching . . . when I am asking questions and using student thinking. (Sally Post-Interview, 11/23/15)

This statement signified Sally's change in perception of MTP 5, as she valued planning questions prior to a lesson and critiquing the rationale of the questions she asked.

Productive struggle. Sally made alterations to her conception and perception of MTP 7 related to persevering through problem solving. These changes will be described in the paragraphs that follow.

Conception. Evidence of Sally's changes related to her conception of MTP 7 can begin to be seen in her comments on how Mark supported productive struggle in Research Lesson 1. Sally stated, "While walking around checking on groups, [Mark] never gave students the answer or told them if they were correct or not" (Sally Research Lesson 1 Observation Protocol, 11/2/15). She later wrote that "the students' thinking was extended, but they continued to persevere . . . [Mark] allowed them to sort of figure out their own mistakes and how to fix them. No student just gave up either" (Self-Reflection 4, 11/6/15). This statement signified Sally's ability to identify MTP 7 in practice.

After observing Research Lesson 1, Sally successfully provided opportunities for students to experience productive struggle in Research Lesson 2, especially during the Umbrellas and Hats Task (see Appendix H). Based on her experience in the lesson study, Sally described MTP 7 as when students were "struggling . . . but they were trying and they weren't giving up . . . don't just give them an answer because they want it" (Sally Post-Interview, 11/23/15). These statements related to persevering through problem solving signified changes to her conception of MTP 7 during the lesson study process.

Perception. Evidence of Sally's changes related to her perception of MTP 7 appeared throughout the lesson study. After reading *Principles to Actions* (NCTM, 2014), Sally stated:

> The mathematics teaching practice that I agree with least is supporting productive struggle. Although I do agree that teachers should not just give students the answer when struggling, I find that a lot of students just give up if the task at hand is too challenging. (Sally Self-Reflection 3, 10/14/15)

Sally supported this statement when asked which Mathematics Teaching Practice (NCTM, 2014) was the most challenging. "The struggle, I think that one is challenging . . . I want them to try, but it is hard if they are just going to give up, to not be like, here is how you do it" (Planning Meeting 1, 10/14/15). With this challenge in mind, the group chose their long-term goal for the lesson study, which Sally explained was for students "to be problems solvers. To let them think and work through things on their own. Because I feel like they are kind of lazy. They don't like to think through things and figure things out for themselves" (Planning Meeting 1, 10/14/15).

Sally saw first-hand during the lesson study how students can persevere given the right task. She explained:

> I thought that was one of the best parts about [the Umbrellas and Hats Task (see Appendix H)], is that they really were struggling because they didn't know how to

> do it, but they were trying and they weren't giving up . . . and we weren't giving them the answer. (Sally Post-Interview, 11/23/15)

This statement signified Sally's shift in her perception with respect to students' capabilities to persevere through problem solving.

This shift was important given that MTP 7 was the Mathematics Teaching Practice (NCTM, 2014) she initially agreed with least. However, Sally eventually argued after the lesson study that teachers should "provide problem-solving tasks . . . and that struggle that I was talking about for the students. Promoting those tasks that make it harder for the students" (Sally Post-Interview, 11/23/15). In addition, she stated:

> I think [students] need to have productive struggle. That is something that I have really tried to incorporate more in my classroom, just because I feel like if it is more rigorous then they are more focused and they are trying to figure it out. (Sally Post-Interview, 11/23/15)

These statements made after the lesson study revealed the meaningful changes Sally made to her perception of productive struggle in learning mathematics.

Major influences. It is important to consider the aspects of lesson study that caused Sally to change. In the sections that follow, Sally's rationale for the different types of change will be provided along with evidence that supports or disconfirms each claim.

Implementation. In terms of her implementation of the Mathematics Teaching Practices (NCTM, 2014), Sally indicated that the changes were primarily due to the lesson study and not necessarily because of the reading. When she was asked what was most influential in terms of her implementation, she stated:

> That was the lesson study. Seeing other teachers do it and then afterwards, discussing [the practices] used and what could be better, I think, as far as implementation goes. Also watching them do it and having to write it down on [the Observation Protocol] because you were looking for it. I thought that was helpful. So definitely just the lesson study itself as far as implementing those eight practices. (Sally Post-Interview, 11/23/15)

This claim could not be confirmed or disconfirmed by examining the evidence because data related to her implementation after the reading, but before the lesson study, were not gathered.

Conception. In terms of her conception of the Mathematics Teaching Practices (NCTM, 2014), Sally stated that the changes were primarily due to reading *Principles to Actions* (NCTM, 2014) and completing her self-reflections. When she was asked what was most valuable, she commented:

> I think that was probably reading the book. And the journals also, because you had to look back at the practices every time you wrote a journal. So I think that is probably the most helpful in understanding the eight [practices] . . . was reading the book and then having to review them every time to do a journal. (Sally Post-Interview, 11/23/15)

This claim was supported by the fact that Sally was the most detailed and descriptive of the participants in her self-reflection responses. For example, Sally described her suggestions for the research lesson after Research Lesson 3.

> The only thing that I would do differently is change the bell work question a little bit to be the quantity of something instead of the value. That idea was brought up

> by Mark. I think it was a little confusing for the students because the bell work incorporated the value of nickels and dimes in the equations, whereas the task was only concerned with the number of items. I would change the bell work problem so that it was more along the same lines of the task. (Sally Self-Reflection 6, 11/23/15)

Moreover, Sally made some changes early in the lesson study process that supported the claim that reading was influential. For example, it appeared as if Sally made changes in her conception of contextual representations and situating goals within learning progressions early in the lesson study. However, it seemed as if changes in her conception of facilitating mathematical discourse and productive struggle did not occur until later in the lesson study, thus, possibly occurring as a result of completing her self-reflections.

Perception. With respect to her perception of reform-oriented practices, Sally stated that the changes were primarily due to observing the practices in action. When she was asked what was most valuable, she commented, "Probably the lesson study again. Because when you see them actually in action, I think you believe in [the Mathematics Teaching Practices (NCTM, 2014)] more . . . because they were working. So the lesson study for that one for sure" (Sally Post-Interview, 11/23/15). This statement was supported by the two changes she made in her perception of the Mathematics Teaching Practices (NCTM, 2014). In terms of change in her perception of MTP 5, Sally commented on Mark's use of the phrase "need to prove" (Lesson 1 Debrief, 11/2/15) during Research Lesson 1 and received a positive response to her questioning strategies used in Research Lesson 2. With respect to change in her perception of MTP 7, Sally saw in the research lessons how students can persevere given the right task.

Summary. In Sally's case, the most significant changes were made in relation to MTP 1, MTP 3, MTP 4, MTP 5, and MTP 7. Sally altered her implementation, conception, and perception of MTP 3, MTP 4, and MTP 7. However, Sally's strongest single change was her implementation of MTP 5, as there was a substantial difference between her questioning strategies in the pre-observation and those found in the post-observation. Sally also made interesting connections between the practices in her response. Sally connected MTP 3 and MTP 6 as well as MTP 4 and MTP 5. She stated, "I think when you get really good at this, they all kind of flow" (Sally Post-Interview, 11/23/15) and support each other. A summary of all major and minor changes is provided in Table 19. Sally indicated that changes to her conception were provoked by reading Principles to Actions (NCTM, 2014) and completing her self-reflections. Moreover, she viewed the lesson study and not necessarily the reading as influential in her alterations to her implementation. Finally, Sally stated that observing the practices being successfully implemented in the research lessons was the most impactful to changes in her perception.

Table 19

Summary of Changes: Sally

MTP	Implementation	Conception	Perception
1	N/A	Situates goals within learning progressions. Reference goals throughout the lesson. Connect to prior learning so that students can use their own reasoning strategies.	N/A
2	N/A	N/A	N/A
3	Sally presented students with a problem about gas and mileage. Not only did this problem connect slope to its graphical representation, but it connected it to the context as well.	Provided examples of multiple representations. Contextual representation.	Sally connected MTP 3 to MTP 6. “[They] go hand in hand” (Sally Post-Interview, 11/23/15).
4	Sally gave students time to work together in groups.	Questioning strategies. Facilitate students sharing their reasoning within varied representations.	Facilitate mathematical discourse. Sally connected MTP 4 to MTP 5. “If you are asking questions, that leads to the mathematical discourse” (Sally Post-Interview, 11/23/15).

Table 19 continued

MTP	Implementation	Conception	Perception
5	Sally's questioning was much improved from the first lesson. Her questions required explanations.	Questioning that probes and extends.	Purposeful questions that led into discussion. Ask more in-depth questions. Sally connected MTP 5 to MTP 4. "If you are asking questions, that leads to the mathematical discourse" (Sally Post-Interview, 11/23/15).
6	N/A	Various solution strategies.	Building procedural fluency. Sally connected MTP 6 and MTP 3. "[They] go hand in hand" (Sally Post-Interview, 11/23/15).
7	Sally brought up a common misconception.	Persevere through problem solving. Require a high level of cognitive demand.	Productive struggle. More rigorous.
8	N/A	Make in-the-moment decisions based on student thinking.	N/A

Britney Smyth

The results of changes made by Britney will be described in the sections to follow. A description of Britney's typical lesson structure will be outlined. Major changes in implementation, conception, and perception of the Mathematics Teaching Practices (NCTM, 2014) will then be described along with the aspects of lesson study that influenced these changes.

Description of teaching. Britney, in her fifth year teaching, said that her typical lesson began with bell work. Normally, "bell work is a review of a previous skill that relates to the current skill . . . so that it links" (Britney Pre-Interview, 9/9/15). This usually utilized the first five to 10 minutes of her lessons. Then, using their composition books, students took notes and worked through examples together as a class before practicing on their own. If discovery was involved, it typically came between bell work and notes or "it could come the day before. So it would be a separate lesson" (Britney Pre-Interview, 9/9/15). Britney further described how her beliefs were enacted in her lessons.

> As far as students learning. The whole like multiple pathways. Allowing them to kind of find the way that works best. I kind of . . . when we are going over stuff, they have to explain what they did. Show me their steps. Kind of talk through well why, "Why did you do that?" I ask a lot of whys. I answer a lot of questions with questions, which drives them nuts sometimes. I do allow for group work. A lot of times when they are working on stuff in class, I let them talk with a neighbor if they get confused. Check with a neighbor before you check with me.

> Learn from each other. Not necessarily only from me. (Britney Pre-Interview, 9/9/15)

Britney's views of her teaching prior to the lesson study emphasized students explaining their thinking and discussing with one another. Moreover, it focused on discovery and taking notes in composition notebooks. In the section that follows, Britney's implementation will be examined to see how these ideas were enacted in her classroom.

Changes in implementation. An analysis of Britney's changes in implementation follows. To begin, a description of her pre-observation will be given. Then, an account of her post-observation will be described. Finally, a synthesis comparing the two lessons will be provided.

Pre-observation. The goal of Britney's pre-observation lesson was for students to be able to reflect figures across horizontal and vertical lines. The lesson followed Britney's description of a typical lesson in her classroom. The lesson began with a bell work problem that reviewed the previous day's concept of translating geometric figures. When discussing the bell work, Britney randomly selected students to share and called on some to rephrase responses of other students. For example, Britney said, "In your own words, what does it mean for two figures to be congruent?" (Britney Pre-Observation, 9/23/15) and "Tell me how you know which way it goes" (Britney Pre-Observation, 9/23/15). Britney also asked, "I like what he said. Can someone rephrase what he just said" (Britney Pre-Observation, 9/23/15)? To transition to that day's lesson, Britney said, "So let's reflect fondly on yesterday's class, shall we. We talked about translations. And what was another word for translations" (Britney Pre-Observation, 9/23/15)? The class responded with "slide" (Britney Pre-Observation, 9/23/15), to which Britney said, "Slide

. . . all we did was slide the figure over and down. See that? [referring to the previous page in students' composition notebook]. Things are changing today" (Britney Pre-Observation, 9/23/15).

This led into the lesson on reflections. However, a connection between the bell work and the lesson's topic was not offered. Britney then showed students what a reflection is using an example with wax paper (see Figure 23). Britney asked students to make comparisons with the visual of the wax paper method and the graphical representation of the coordinates. "See if you can figure out what happens to the ordered pairs. Something magical happens" (Britney Pre-Observation, 9/23/15). Then Britney encouraged them to figure out the general pattern for reflecting. "You know how yesterday the trick with the ordered pairs was that you add the x change with the x and the y change with the y? Now, I am not telling you the trick" (Britney Pre-Observation, 9/23/15).

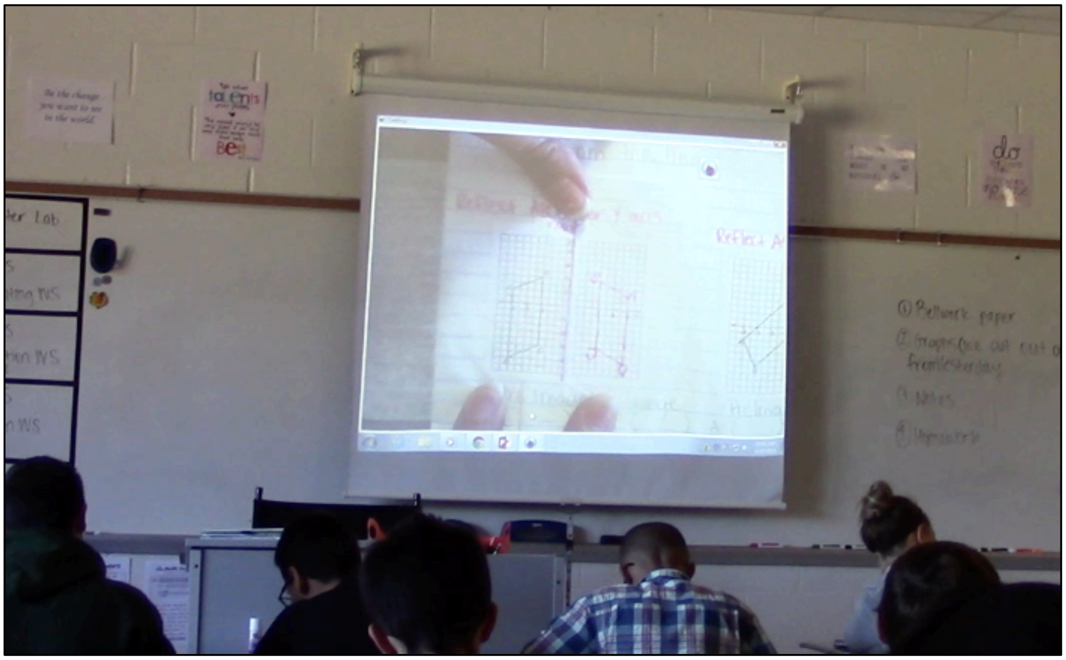

Figure 23. Britney showing students how to reflect a figure using wax paper in pre-observation.

After, Britney gave students two examples to practice, one reflecting across the *y*-axis and one reflecting across the *x*-axis. Britney used higher-order questions during this time to press students to think and discuss what it means to reflect.

> What do you think a reflection is going to do? Take a moment in your head, not with your mouth, just in your head. And think about what is going to happen when you reflect a shape . . . discuss that with your neighbor. (Britney Pre-Observation, 9/23/15)

In the second example that included points of the pre-image on both sides of the line of reflection, Britney asked, “Last time they were all on one side . . . we moved them all to the other. What is different here” (Britney Pre-Observation, 9/23/15)? Britney then called on two students to come up to the board and share how to reflect certain points. After, Britney attempted to summarize what happened to the ordered pairs. "Did anybody see what happened to the ordered pairs" (Britney Pre-Observation, 9/23/15)? A student responded by describing what happens when reflecting across the y-axis, "The y-values all stayed the same, but then the little x-coordinates went from negative to positive or positive to negative" (Britney Pre-Observation, 9/23/15).

Students practiced two additional problems, in which they had to reflect across the lines $x = -1$ and $y = 2$. A discussion about how to create equations for horizontal and vertical lines followed. Britney gave students time to think about where the line $x = -1$ would be located. She did not tell them right away. Instead, she said, "In your heads, think about . . . where would I put that line? What makes sense" (Britney Pre-Observation, 9/23/15)? Britney then stated, "Discuss with your groups. Where do you think [the line $x = -1$] would go" (Britney Pre-Observation, 9/23/15)? Britney called on various students to share their thoughts. After a student answered with horizontal, Britney encouraged the student to provide an explanation. "Okay, why do you say that" (Britney Pre-Observation, 9/23/15)? Britney then revealed to students why the $x = -1$ is a vertical line by asking students for the x-value of various points on the horizontal line she drew on the board. By doing so, Britney revealed to students that every point on the line $x = -1$ has an x-value of -1.

In order to help students understand these two problems, Britney explained how the corresponding points are equidistant from the line of reflection. "See how my counting started at the line of reflection? I started there. [Point A] was five to the right, so I flipped [point A] and put [point A'] five to the left" (Britney Pre-Observation, 9/23/15). Students were able to graph the reflection with Britney's help. Then, a similar discussion occurred with the last problem (reflecting across $y = 2$), but was shortened because of time.

With respect to the practices (see Table 20), Britney facilitated mathematical discourse (MTP 4) by allowing students to share their ideas and urging students to try to discover the “trick” (Britney Pre-Observation, 9/23/15) with the coordinates. Moreover, she used various representations (wax paper and coordinates) (MTP 3) to help students understand the concept of reflection. Britney aided students in conceptually understanding why the graph of $x = -1$ is a vertical line (MTP 6). However, the goals for the lesson were not made explicit (MTP 1) and the lesson lacked a task that promoted problem solving and reasoning (MTP 2). As a result, students were not provided with opportunities to think critically or struggle through a task (MTP 7). Moreover, the lesson did not make any connections to real life (MTP 3), nor did it include a summary or conclusion.

Table 20

Britney’s Pre-Observation Teaching Practices

MTP	Evidence
1	“So let’s reflect fondly on yesterday’s class, shall we. We talked about translations. And what was another word for translations” (Britney Pre-Observation, 9/23/15)? The class responded with “slide” (Britney Pre-Observation, 9/23/15). Britney said, “Slide . . . all we did was slide the figure over and down” (Britney Pre-Observation, 9/23/15).
2	N/A
3	“See if you can figure out what happens to the ordered pairs. Something magical happens” (Britney Pre-Observation, 9/23/15). Britney asked students to make comparisons with the visual of the wax paper method and the graphical representation of the coordinates.

Table 20 continued

MTP	Evidence
4	Britney asked, “I like what he said. Can someone rephrase what he just said” (Britney Pre-Observation, 9/23/15)? “What do you think a reflection is going to do? Take a moment in your head, not with your mouth, just in your head. And think about what is going to happen when you reflect a shape” (Britney Pre-Observation, 9/23/15). Moments later, Britney said, “Discuss it with your neighbor” (Britney Pre-Observation, 9/23/15). As a student was explaining what happened to the points, Britney asked clarifying questions that required the student to explain his thinking to the class. Britney said, “Discuss with your groups. Where do you think [line $x = -1$] would go” (Britney Pre-Observation, 9/23/15)? “We need to decide if it is going to be a horizontal or vertical line. Think about that one in your heads” (Britney Pre-Observation, 9/23/15). Then, Britney called on various students to share their thoughts. “Vertical [referring to the line $x = -1$], what does somebody else think? I want a couple of opinions” (Britney Pre-Observation, 9/23/15). She engaged students and allowed them to share their opinions and provide justification.
5	“Tell me how you know which way it goes” (Britney Pre-Observation, 9/23/15). “What did you get [student pulled a random name card]? Tim Callahan. In your own words, what does it mean for two figures to be congruent” (Britney Pre-Observation, 9/23/15)? “Last time they were all on one side . . . we moved them all to the other. What is different here” (Britney Pre-Observation, 9/23/15)? After a student answered with horizontal, Britney said, “Okay, why do you say that” (Britney Pre-Observation, 9/23/15)?

Table 20 continued

MTP	Evidence
6	Britney showed students how to do reflections using wax paper to reflect the figure. Britney asked a student to explain what happened to the coordinates when reflecting across the *x*-axis. The student responded by saying, “The *y*-values all stayed the same, but then the little *x*-coordinates went from negative to positive or positive to negative” (Britney Pre-Observation, 9/23/15). Britney revealed to students why the $x = -1$ is a vertical line by asking students for the *x*-value of various points on the line.
7	“You know how yesterday the trick with the ordered pairs was that you add the *x* change with the *x* and the *y* change with the *y*. Now, I am not telling you the trick” (Britney Pre-Observation, 9/23/15). Britney encouraged them to figure out the trick for reflecting. Britney gave students time to think about where the line $x = -1$ would be located. She did not tell them right away. Instead, she said, “In your heads, think about, where would I put that line? What makes sense” (Britney Pre-Observation, 9/23/15)? Britney surveyed the class to get various responses and, thus, revealed to the class that errors are a natural part of learning.
8	A student shared with the class how to reflect a figure on the board. Then, Britney used the student’s work to further explain the concept. Similarly, Britney asked another student to come to the board and reflect a different point.

Post-observation. Britney’s post-observation lesson was very similar to the pre-observation lesson with respect to structure. The class began with bell work that pressed students to make sense of a graph that represented the height of a balloon after being released from the top of a building. “What does the slope and *y*-intercept reveal about the situation? What does the slope mean in a real-world situation” (Britney Post-Observation, 11/18/15)? Britney noticed a common misconception as she circulated the room and addressed it on the board. “I would like to make a quick correction for some of you. I am going to ask you to look at the scale of the graph [which was 1000 feet]. Take a second to look at that” (Britney Post-Observation, 11/18/15). Once Britney gave students time to write sentences to describe the situation, she called on students to share their ideas (see Figure 24) and said, “Okay, talk with your neighbor. What do you think the slope would look like after it pops” (Britney Post-Observation, 11/18/15)? Britney followed this problem with another graph that displayed the number of books a student checked out by week, which was a horizontal line. She asked students to think about what the graph meant and asked, “If I were to graph their library fine owed over the weeks. What do you think that graph would look like” (Britney Post-Observation, 11/18/15)?

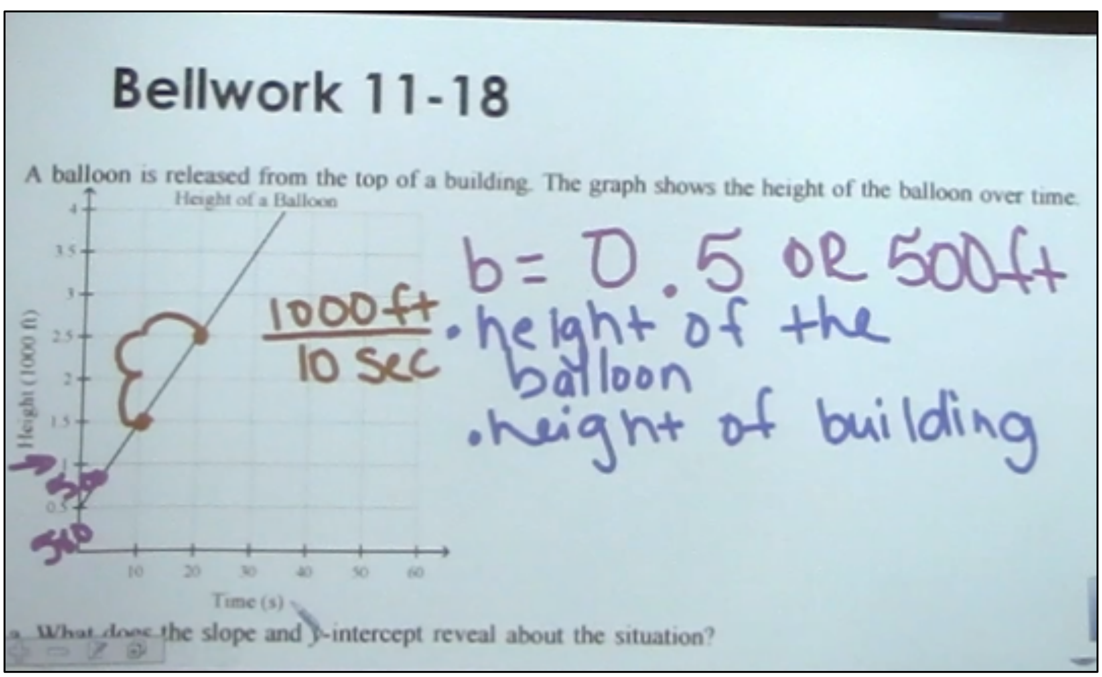

Figure 24. Britney’s bell work discussion in post-observation.

The discussion about slope in real-life situations led into the topic of the lesson, which was converting from standard form of a linear equation to slope-intercept form. Britney began with one example, $2x + 2y = 4$. After a student suggested subtracting $2x$ on both sides, Britney said, “Yes, we need to think of this as a scale, balance. So whatever we do on one side of the scale, we do the same thing to the other to keep it balanced” (Britney Post-Observation, 11/18/15). She then said, “Talk with your neighbors. Why can’t this be $2x$ [after subtracting $2x$ on the right side of the equation]” (Britney Post-Observation, 11/18/15)? After, Britney asked a student to explain why it would not be $2x$. She then added to the student’s explanation by revealing to students that 4 and $2x$ are not like terms. Britney then made a connection with students’ prior knowledge, in this case, slope-intercept form. “Yesterday when we were graphing them . . . all of the equations were already in y equals form or slope-intercept form” (Britney Post-Observation, 11/18/15).

After taking notes on a step-by-step process to convert to slope-intercept form (see Figure 25), Britney discussed another problem ($2x + y = 9$) with the class. She then addressed the same misconception: “Can someone re-explain why I can’t do $9 – 2x$” (Britney Post-Observation, 11/18/15)? After a student explained that there is a difference between 9 and $2x$, Britney finished the problem by saying, “I would prefer if you would switch it around and put the x term first. What am I trying to make it look like” (Britney Post-Observation, 11/18/15)? Britney intended for students to see the slope-intercept form in their answer by rewriting it with the linear term first.

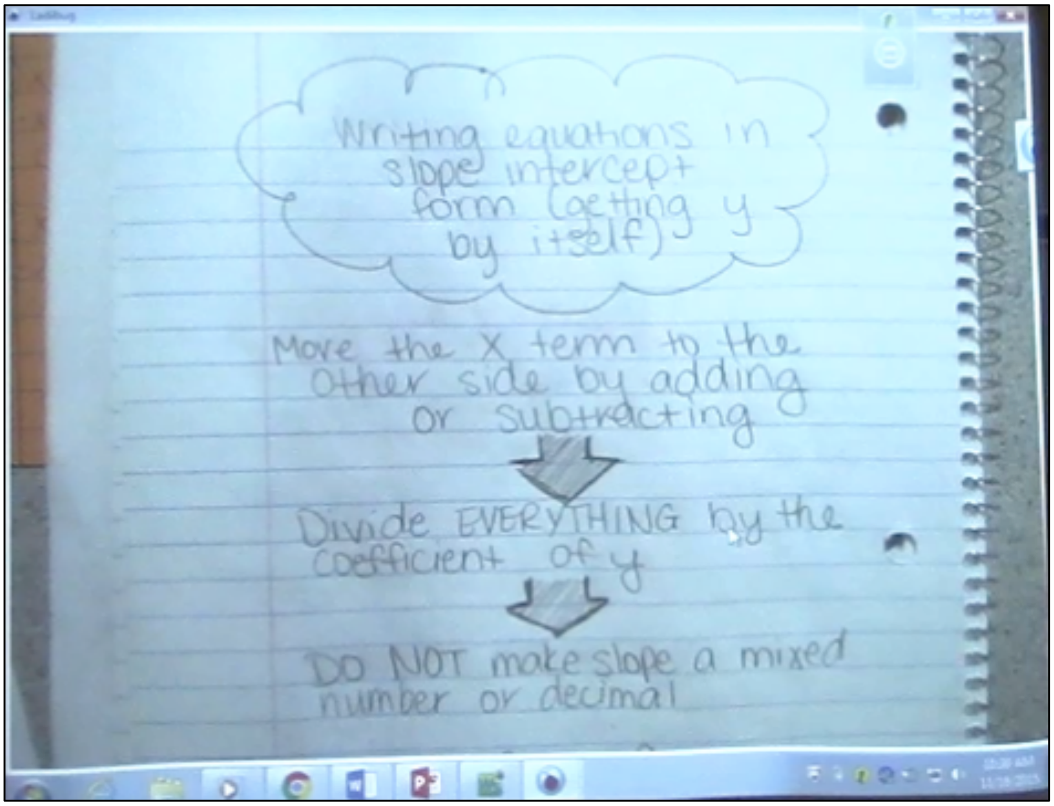

Figure 25. Britney's notes on how to convert to slope-intercept form in post-observation.

In the next problem ($10x - 3y = 5$), Britney attempted to get students to think about why the negative sign in between the terms does not indicate that they should add $10x$. She stated, "Pretend you're the teacher, and I am the student. What would you, as my teacher, say that would not make me cry, but would make me understand why that is incorrect thinking" (Britney Post-Observation, 11/18/15)? A student responded, "You wouldn't subtract 10 because the 10 is a positive 10" (Britney Post-Observation, 11/18/15). When dividing -10 by -3, Britney asked, "Can somebody explain to me why suddenly it is positive ten thirds" (Britney Post-Observation, 11/18/15)? A student noted that a negative divided by a negative is a positive.

Students then worked on practice problems until the end of class. As students worked, Britney circulated the room and helped students who were struggling. When talking with a student individually, Britney said, “Can I ask why you are dividing by -1” (Britney Post-Observation, 11/18/15)? This allowed her to understand the student’s thought process, so that she could better instruct him. As students finished their set of problems, they went to the front of the room to check their answers with the answer key.

In terms of the Mathematics Teaching Practices (NCTM, 2014) (see Table 21), Britney facilitated meaningful mathematical discourse by posing purposeful questions (MTP 5) such as, “In that one time-frame, the balloon goes from here and ends at the bottom. Is that practical” (Britney Post-Observation, 11/18/15)? The bell work tasks provided opportunities for students to think at a deep level about the concept of slope and what it means in real-life situations (MTP 3). Moreover, she addressed a common misconception and had students reason through why $9 - 2x$ is not 7 or $7x$ (MTP 7). However, the process of converting to slope-intercept form was procedural (MTP 6) and did not leave room for student reasoning and perseverance (MTP 7). In fact, Britney stated, “You know what, try it on your own. Keep in mind that Ms. [Smyth] is going to come to your rescue if you get it wrong” (Britney Post-Observation, 11/18/15). Moreover, the bell work problems were the only questions that were used to get students thinking at a deep level about mathematics.

Table 21

Britney's Post-Observation Teaching Practices

MTP	Evidence
1	Britney made a connection with prior knowledge, in this case, slope-intercept form. "Yesterday when we were graphing them . . . all of the equations were already in *y* equals form or slope-intercept form" (Britney Post-Observation, 11/18/15).
2	N/A
3	The bell work read: "A graph of the height of the balloon is shown below. What does the slope and *y*-intercept reveal about the situation? What does the slope mean in a real-world situation" (Britney Post-Observation, 11/18/15)? Britney asked students to write sentences that described the situation. Britney showed books checked out by week graph to students that was a horizontal line. She asked students to think about, "What is the graph showing me about this particular person" (Britney Post-Observation, 11/18/15)?
4	"What are some of the sentences that you wrote" (Britney Post-Observation, 11/18/15)? Britney allowed students to share their thoughts. "Talk with your neighbors. Why can't this be $2x$" (Britney Post-Observation, 11/18/15)? After, Britney asked a student to explain why it wouldn't be $2x$. Then, she rephrased the student's explanations to make them clearer. "Can someone re-explain why I can't do $9 - 2x$" (Britney Post-Observation, 11/18/15)? "Pretend you're the teacher, and I am the student. What would you as my teacher say that would not make me cry, but would make me understand why that is incorrect thinking" (Britney Post-Observation, 11/18/15)?

Table 21 continued

MTP	Evidence
5	"Think about the slope of [the height of] the balloon after it pops" (Britney Post-Observation, 11/18/15)? Britney introduced an additional question that required students to think about what the graph would look like in a different situation. Asked students to think about why the slope would not be vertical. "In that one time-frame, the balloon goes from here and ends at the bottom. Is that practical" (Britney Post-Observation, 11/18/15)? "If I were to graph their library fine owed over the weeks, what do you think that graph would look like" (Britney Post-Observation, 11/18/15)? When dividing -10 by -3, Britney asked, "Can somebody explain to me why suddenly it is positive ten thirds" (Britney Post-Observation, 11/18/15)?
6	Students copied down notes, which outlined the steps in solving for *y*. This did not build on any conceptual understanding and was very much a step-by-step list of how to perform this task. "I would prefer if you would switch it around and put the *x* term first. What I am trying to make it look like" (Britney Post-Observation, 11/18/15)? Britney wanted for students to see the slope-intercept form in their answer by rewriting it with the linear term first. Students were not given the freedom to use their own strategies.
7	"You know what, try it on your own. Keep in mind that Ms. [Smyth] is going to come to your rescue if you get it wrong" (Britney Post-Observation, 11/18/15). As a result, there was not much time for students to struggle. Britney noticed a common misconception as she was walking around and addressed it on the board. "I would like to make a quick correction for some of you. I am going to ask you to look at the scale of the graph [which was 100]. Take a second to look at that" (Britney Post-Observation, 11/18/15). Britney addressed a common misconception dealing with adding or subtracting non-like terms.
8	When working with a student individually, Britney said, "Can I ask why you are dividing by -1" (Britney Post-Observation, 11/18/15)? This allowed her to understand the student's thought process.

Comparison. The most significant difference between the two lessons occurred during the bell work of the post-observation lesson. Although there were no real-life situations in the pre-observation lesson, the bell work in the post-observation lesson included problems that pressed students to think at a deep level about the concept of slope and what it means in various real-life situations (MTP 3). The follow-up question, regarding what would happen to the graph if the balloon was popped, forced students to reason and make comparisons between the new slope and the original slope. In addition, a second graph displaying the number of books checked out per week led to a discussion about the slope of the line representing a student's library fine. During these two problems, Britney included questions that pressed students not only to explain their reasoning but also to make conjectures about mathematics (MTP 5), which was not apparent in her pre-observation. A comparison of the two lessons with respect to the Mathematics Teaching Practices (NCTM, 2014) is provided in Table 22.

Table 22

Britney's Pre- and Post-Observation Comparison

MTP	Comparison
1	Britney did not explicitly state the goal in either lesson. However, she related to students' prior knowledge in both lessons.
2	Neither lesson included a true problem-solving task that required a high level of cognitive demand.
3	Britney made comparisons with the visual of the wax paper method and the graphical representation of the coordinates in her pre-observation lesson. However, Britney included two real-life examples in her post-observation lesson that pressed students to make connections to a context in order to support students' understanding of slope.
4	Britney led meaningful class discussions and allowed students to work in groups during both lessons.
5	Britney asked questions that required explanations in both lessons. However, Britney also included questions during the bell work of the post-observation lesson that forced students to make conjectures regarding the balloon and books problems.
6	Even though the post-observation bell work included the balloon and book problems, the introduction of new content in both lessons was very procedural and did not allow for students to use their own reasoning strategies.
7	Britney gave students time to practice on their own during both lessons. However, productive struggle was limited to procedural practice.
8	During the pre-observation lesson, Britney called two students up to the board to explain how to reflect a certain figure. There was no evidence of Britney eliciting or using students' thinking in her post-observation lesson.

Changes in conception and perception. Occurrences during the lesson study, pre- and post-observations, and pre- and post-interview responses were used to identify changes that were made with respect to Britney's conception and perception of the Mathematics Teaching Practices (NCTM, 2014). Britney's major changes were related to detail in goal setting, using goals to guide instructional decisions, making connections between multiple representations, emphasizing contextual representations, requiring explanation and justification, conceptual understanding, and productive struggle. Each of these major changes will be described in detail in the paragraphs that follow along with an auditable trail that will provide support for these claims.

***Detail in goal setting*.** Britney made progress related to her conception and perception of setting goals. These changes are described in the paragraphs that follow.

Conception. Britney mentioned the learning progression in her pre-interview when describing MTP 1, but she did not discuss the detail of goal setting. However, after the Lesson 2 Debrief, Britney wrote:

> I never considered how specific I should make a learning goal. I've always thought about the big picture goal and used that to drive my teaching. Through this process I've learned that if I can narrow down my goal to specifics, I don't spend as much time wondering what I should say or which student work to encourage. (Britney Self-Reflection 5, 11/8/15)

Britney also discussed setting specific goals when she was asked what she learned by participating in this lesson study. "Setting the goals . . . knowing specifically today, here is what they need to know before they walk out. If they don't get the other things, that is okay . . . this is what I need them to get today" (Britney Post-Interview, 11/23/15). Although Britney's conception of MTP 1 before the lesson study did not provide evidence of detail in goal setting, she stated in her post-interview that MTP 1 "means focusing on the specific skills from the basic concept to each individual skill they need to be successful . . . really breaking down the general to the specific" (Britney Post-

Interview, 11/23/15). These statements indicated that Britney's conception of MTP 1 enhanced during the lesson study.

Perception. When asked about effective mathematics teaching prior to the lesson study, Britney did not mention the detail of goal setting. During the lesson study, the goal on her research lesson plan progressed from "move fluidly between visual and algebraic representations of variables in real world situations" (Britney Lesson Plan 1, 10/27/15) to "mathematical practice – develop problem-solving skills and persevere in problem solving. Content standard – develop an understanding of what the solution to a system of equations means (satisfies both stipulations) and introduce the concept of two variables" (Britney Lesson Plan 3, 11/9/15). Britney's attention to detail continued, as the lesson study group struggled to identify the goal for the research lessons. Britney shared her thoughts.

> I am wondering if my goal will be to just get them to understand that, in a system, the solution is what works for both criteria you are given . . . instead of being able to solve . . . our [goal] isn't specific, it's just about understanding of systems. I wonder if I might change it to develop an understanding of the solution to a system of equations. (Lesson 2 Debrief, 11/6/15)

Britney later wrote in her self-reflection how important goal-setting was. "Our goal was too broad. The more specific I make the goal, the better I can focus in on exactly what I want students to talk about during the lesson" (Britney Self-Reflection 5, 11/8/15). After the lesson study, Britney described the importance of goal setting.

> I think I see goal setting as more important than I did before . . . I see more of a specific goal for each lesson as being of value now. That wasn't as strong before

> the lesson study. I think that is because we had to keep changing our objective [of the Research Lesson] to be more fine-tuned. (Britney Post-Interview, 11/23/15)

Britney began to see the value of setting precise goals as she progressed through the lesson study.

Using goals to guide instructional decisions. Britney made progress related to her conception of MTP 1 related to using goals to guide instructional decisions. Although Britney mentioned the learning progression in the pre-interview, she did not describe how goals should be used to guide instructional decisions. However, she enhanced her conception of MTP 1 during the lesson study. Using goals to guide instructional decisions was an important idea throughout the lesson study, as the group worked to create a lesson for their selected goal. In Britney's case, she started with a bell work that focused on substitution, but then she "realized my bell work using substitution [referring to her Lesson Plan 1] wasn't meeting the goal we were working toward" (Self-Reflection 4, 11/8/15). After Research Lesson 1, Britney changed her mind about guiding students towards the selected goal.

> So I think that has changed my mindset . . . seeing [Mark's] today. Because I wouldn't have thrown out anything [the students] didn't say. Kind of like what you did. But I think now I am just going to be like . . . this is what I want you to see . . . see it. Try it first." (Lesson 1 Debrief, 11/2/15)

This statement indicated that Britney altered her conception of instructional decisions that guide students towards the mathematical goal of a lesson.

As she explained changes that were made to the research lesson, Britney mentioned that "we took [the Fruits and Vegetables Task (see Appendix I)] off after the first lesson because it really didn't meet the standard" (Britney Post-Interview, 11/23/15) and that "focusing the bell work on the table organization and equation writing led more toward our goal" (Self-Reflection 5, 11/8/15). Based on the goal for the lesson, Britney chose to separate the bell work task into three questions prior to her research lesson. Her rationale was that "the first and second questions combine to make the third, really bringing home the idea of what a solution to a system means" (Self-Reflection 5, 11/8/15).

During the lesson study, Britney experienced what it means to use goals to guide instructional decisions. Reflecting upon the lesson study, Britney said, "My specific goal was to know what the solution means, so I'm going to guide to that . . . so that has changed for me, throughout" (Lesson 3 Debrief, 11/9/15). Britney also began to see how to use goals to guide what is emphasized during a lesson. "We need to know what that goal is and how we are going to work towards it" (Britney Post-Interview, 11/23/15). She continued by saying, "[MTP 1] means more than just taking the standard and just knowing what they need to get to and teaching it . . . I need to have a specific skill in mind that I really want to push home" (Britney Post-Interview, 11/23/15). Taken collectively, these statements signified the alterations Britney made to her conception of MTP 1, related to using goals to guide instructional decisions.

Making connections between multiple representations. Britney made progress with respect to her perception of MTP 3 related to making connections among multiple representations. Britney referred to MTP 3 in her pre-interview by saying, "I think students learn best by seeing it different ways" (Britney Pre-Interview, 9/9/15). However, she did not mention the value of making connections among those representations.

Throughout the lesson study, Britney altered her perception of MTP 3 related to making connections. During Research Lesson 2, Britney observed Sally making connections. Britney commented that Sally made “great connections between representations” (Britney Lesson 2 Observation Protocol, 11/6/15) including real-world contexts, equations, and tables. When asked what she learned by watching Research Lesson 2, Britney said, “Focusing the bell work on the table organization and equation writing lead more toward our goal” (Self-Reflection 5). Additionally, according to Britney, the strength of Research Lesson 2 was:

> The mathematical representations . . . the connection was great, with where you had the equation written and then the girl was like, “Plug it all into the equations” and then you took that and put it into a table format. I thought that was a great connection between the representations. (Lesson 2 Debrief, 11/6/15)

In fact, Britney chose to include the same bell work task, in which her rationale was for students to “connect between verbal list and table . . . then write an equation with two variables to represent the situation” (Britney Lesson Plan 3, 11/9/15). As Britney implemented her research lesson plan in Research Lesson 3, she connected multiple representations when she displayed student work including tables, verbal descriptions, and equations. When moving from the verbal descriptions to the equations, Britney said, “Same concept here, only this particular person put them into the equations . . . so we see the same methodical counting down” (Research Lesson 3, 11/9/15). When asked about her beliefs about the Mathematics Teaching Practices (NCTM, 2014), Britney stated, “Connecting representations. I definitely saw the value of that . . . it is super important for them to see all the different connections” (Britney Post-Interview, 11/23/15).

Emphasizing contextual representation. Differences were found related to Britney's conception and perception of contextual representations. These two changes will be described in the paragraphs that follow.

Conception. Britney's first research lesson plan included a procedural bell work that asked students to solve two-step equations. As the lesson study progressed, Britney saw how real-life context could be used. "Yeah and I think that in our talks . . . like talking about different ways to re-do the tasks, have helped me see more real-world situations" (Lesson 3 Debrief, 11/9/15). After discussing the research lesson in Planning Meeting 2, Britney removed the procedural bell work problems from her research lesson plan and replaced them with the muffin task. After the lesson study, Britney stated that the strength of her post-observation lesson was "the [balloon and book tasks] brought out some great conversations about what the slope of a line means in real-world situations" (Self-Reflection 7, 11/23/15)! She continued by describing the weakness of the lesson. "I wish I had another real-world example for the kids to have evaluated with a partner, or even had them create an example" (Self-Reflection 7, 11/23/15). Britney's altered conception of MTP 3 became even more apparent in the post-interview when she emphasized "connecting a real-world situation also to a mathematical representation of that" (Britney Post-Interview, 11/23/15).

Perception. Even though Britney valued students seeing mathematics in different ways before the lesson study, her statements did not provide evidence of her valuing contextual representations specifically. However, her perception was enhanced during the lesson study. During Planning Meeting 1, Britney said, "I would love it if they are able to see Algebra realistically. I want them to see situations and think, this is a system of equation problem. I would love them to see a connection" (Planning Meeting 1, 10/14/15). She discussed this idea in terms of the Fruits and Vegetables Task (see Appendix I).

> I want them to be able to move between seeing it visually, on the scale, and seeing it algebraically – seeing this as 10 bananas but 10*b* is also the same thing…see *b* as bananas and *p* as pineapples . . . To see variables for what they are, is kind of what I am thinking . . . So part of the goal, the one I have written, is to move between visual and algebraic representations of variables for real life. (Planning Meeting 2, 10/27/15)

In her post-interview, Britney explained how she thought that "students best learn mathematics when they see the real-life application" (Britney Post-Interview, 11/23/15). As a result, she stated:

> I plan to continue that and bring in more opportunities for discussion. Even if it is just in bell work like [Post-observation, balloon task] . . . it was really a big task that they could talk about, but I still tried to bring that in in bell work . . . and I probably wouldn't have done that before the lesson study - value my bell work as something just to do as a discussion. Yes, the balloon going . . . the slope going up and then talking about it popping and talking . . . just talking about it was helpful. I wouldn't have probably done that before the lesson study. It probably would have just been, find the slope. So that was a big change. I am going to keep it up . . . because it was a good discussion. I am super excited that it went so well. (Britney Post-Interview, 11/23/15)

This statement signified Britney's alterations to her perception of contextual representations and her desire to continue this practice in her lessons.

Using questions that require explanation and justification. Britney altered her conception of MTP 5 related to using questions that require explanation and justification. When asked about MTP 5, Britney focused her pre-interview response on guiding student thinking using questioning techniques. Even though she did not emphasize pressing students to provide explanation and justification in her pre-interview, Britney discussed it when she was asked what her beliefs were about the teaching of mathematics after reading *Principles to Actions* (NCTM, 2014). She responded, "Teachers should be asking 'why' questions and forcing students to come up with explanations for their mathematical processes" (Britney Self-Reflection 3, 10/14/15). During Research Lesson 1, Britney made note of Mark's use of the phrase "show proof" (Britney Lesson 1 Observation Protocol, 11/2/15). It was so impactful, she mentioned it during the Lesson 1 Debrief. "I also liked how you said, 'show proof.' I say why a lot. I like prove it" (Lesson 1 Debrief, 11/2/15). As the discussion on questioning techniques continued, Mark asked for suggestions to improve his questioning. Britney explained her own idea.

> I like the idea, when you are asking, "What method did you use?", having another student explain what another student said. "So and so said this, can you explain what they were doing? Can you put that in your words? What are you understanding about what you see here?" Stuff like that. (Lesson 1 Debrief, 11/2/15)

Similarly, Britney appreciated the questions used in Research Lesson 2. She wrote, "Good questioning! Drew out their thoughts by asking clarifying questions" (Britney

Lesson 2 Observation Protocol, 11/6/15). This statement signified Britney's ability to identify questions that press students to further explain their reasoning.

Britney also altered her research lesson plan to include more detail about questioning. Although she included specifics about students justifying and explaining their thinking in all three of her research lesson plans, Britney wrote additional statements in her third research lesson plan. "[The teacher will] move through the room asking students to explain what they've done" and "[The student will] explain their thinking to the class, answering any questions their peers or teacher asks" (Britney Lesson Plan 3, 11/9/15). Indeed, as Britney circulated the room, she said to a student, "Prove to me" (Research Lesson 3, 11/9/15). When asked about it, she said, "I actually picked that up from [Mark] . . . because I saw him saying that, and I was like, oh I like that. I always say 'Why?' But I like 'prove it.'" (Lesson 3 Debrief, 11/9/15).

Britney's altered conception of MTP 5 was also evident in her post-observation. According to Britney, "The bell work [in the post-observation] built conceptual knowledge. I asked lots of questions . . . I even answered questions with questions. I asked why a lot, using student thinking. There was also a lot of mathematical discourse" (Britney Post-Interview, 11/23/15). When asked how the lesson study will impact her teaching, Britney said, "I try to plan questions in advance more now" (Britney Post-Interview, 11/23/15). When asked what MTP 5 meant to her, Britney responded, "Asking them to explain how it worked . . . why did that work? Why does that work every time? And posing questions that get them thinking deeper than just the procedure of it" (Britney Post-Interview, 11/23/15). These statements indicated that Britney had altered her conception of planning and using questions that require explanation and justification.

***Conceptual understanding first*.** During the lesson study process, Britney improved her perception of MTP 6 regarding the order in which conceptual understanding and procedural fluency should occur. Britney mentioned MTP 6 in her pre-interview by discussing the importance of “understanding the why” (Britney Pre-Interview, 9/9/15), but did not mention the order in which understanding and fluency occur. After reading *Principles to Actions* (NCTM, 2014), Britney selected MTP 6 as the Mathematics Teaching Practice (NCTM, 2014) she agreed with most.

> Build fluency from conceptual understanding. I believe that students will remember how to do mathematical procedures better if they understand why it's done a certain way. Memorization of algorithms may help make things go faster, but it certainly doesn't promote the real-world problem-solving skills students will need when they join the work-force. (Britney Self-Reflection 3, 10/14/15)

To build conceptual understanding, Britney planned for students to share and discuss various methods and respond to these questions during the research lesson. “How do you feel about this method? Did you think of it this way? If not, does it make sense now? Is this method similar to your group’s method? How is it similar or different” (Britney Lesson Plan 2, 11/2/15)? However, up to this point in time, she had not distinguished the order of conceptual understanding and procedural fluency. This was first distinguished in her third research lesson plan when Britney wrote that the rationale for using the muffin task during the bell work was to “introduce the concept of two variables through a real world concept. Procedural fluency through conceptual knowledge” (Britney Lesson Plan 3, 11/9/15).

Similarly, Britney described her post-observation lesson by saying, “The bell work built conceptual knowledge” (Britney Self-Reflection 7, 11/23/15). She stated that her bell work was a result of her experience in the lesson study. “Yes! My typical bell work would have been fluency practice, but I felt the need to bring in that conceptual reinforcement first. I think it was the best possible thing I could have done” (Britney Self-Reflection 7, 11/23/15). Moreover, when Britney was asked about what she would take away from this experience, she stated that she would continue to improve in this area while placing an emphasis on the order.

> More of that conceptual before fluency. I think I am still stuck in that world of -do the fluency and then we'll bring in the real-world stuff at the end when they know how to do it. Maybe flipping that more and bringing in the real-world to teach it. (Britney Post-Interview, 11/23/15)

Britney stated that conceptual understanding should come first “rather than just showing them how to do it . . . then giving them the real-world problem” (Britney Post-Interview, 11/23/15). Britney appeared to have realized the importance of building conceptual understanding prior to procedural fluency.

Productive struggle in mathematics. Britney altered her perception related to MTP 7 throughout the lesson study. Specifically, she enhanced her perception related to learning from mistakes. In her pre-interview, Britney made note of MTP 7 by describing the importance of not “telling them every time” (Britney Pre-Interview, 9/9/15) and “allowing them to do some discovery” (Britney Pre-Interview, 9/9/15). However, Britney did not discuss learning from mistakes or that mistakes are a natural part of the learning process. In fact, in the beginning of the lesson study, MTP 7 was identified by Britney as the Mathematics Teaching Practice (NCTM, 2014) that she agreed with the least. She said, “While there is a place and time for productive struggle, there is a fine line between productive struggle and frustrating struggle. There are many students who, if allowed to struggle for too long will simply give up” (Britney Self Reflection 7, 11/23/15). The group agreed with this statement and decided that developing students into problem solvers would be their long-term goal for the lesson study. When Mark asked the group, “How closely related do you feel like mathematical discourse and productive struggle are?” (Planning Meeting 1, 10/14/15), Britney explained how they can be useful when combined.

> They can be linked, because if you have a problem up and they are trying to answer it and you’re not telling them the right answer . . . like I did that with this problem on the board [referring to a problem she had worked in class that day]. For them to just argue back and forth. And let them struggle through, “Am I right or wrong?” So that's like both of those joined together. (Planning Meeting 1, 10/14/15)

When Dr. Ross asked the group what the obstacles were to providing opportunities for students to struggle, Britney said, “Right, especially with time. Time constraints. We are stressing about getting everything taught” (Planning Meeting 1, 10/23/15).

However, during the lesson study process, Britney attempted to reveal to students that mistakes are a natural part of learning. In her first two research lesson plans, Britney’s rationale for one of the tasks was to “get all students prepared to solve the problem by clearing up misconceptions that may arise” (Britney Lesson Plan 2, 11/2/15).

Britney also mentioned in a self-reflection that “it's helpful when they can find their own mistakes” (Britney Self-Reflection 7, 11/23/15).

There were two occurrences in Research Lesson 3 in which Britney helped students realize that mistakes are valuable. First, she encouraged a student who had made a mistake during a class discussion to revise her equation in front of the class. Second, when another student shared an incorrect equation, Britney did not correct the student. Instead, she said, “Let’s see if that equation matches the solutions we [have]” (Research Lesson 3, 11/9/15) and helped the students see that the solutions did not work for that equation. In her post-observation, Britney noticed a common misconception as she was circulating the room and addressed it on the board. “I would like to make a quick correction for some of you. I am going to ask you to look at the scale of the graph [which was 1000 feet]. Take a second to look at that” (Britney Post-Observation, 11/18/15). As Britney addressed this misconception, students experienced how mistakes are a natural part of learning.

Britney’s improved perception of MTP 7 was evident at the end of the lesson study. When asked how students learn best during her post-interview, Britney responded, “They learn best when they put forth effort and they aren't afraid to make the mistakes. When they let the mistakes happen and they learn from those” (Britney Post-Interview, 11/23/15). She continued by describing what teachers can do to support students in valuing mistakes.

> And in your teaching, show them that there are mistakes that happen and try to have them . . . try to make mistakes every once and a while that they try to catch

> so that they see that everybody makes mistakes . . . they can learn that way also. (Britney Post-Interview, 11/23/15)

This statement summarized Britney's changes to her perception of MTP 7, as she valued mistakes made by both teachers and students.

Major influences. It is important to consider the aspects of the lesson study that caused Britney to change. In the sections that follow, Britney's rationale for the different types of change will be provided along with evidence that supports or opposes each claim.

Implementation. In terms of her implementation of the Mathematics Teaching Practices (NCTM, 2014), Britney indicated that the changes were primarily due to discussing the research lessons. When she was asked what was most influential, she commented:

> I think our discussions and our reflections . . . that we met after each lesson, coming in and talking about what supported and what didn't support [the Mathematics Teaching Practices (NCTM, 2014)]. That helped me to be able to implement them better because I saw things that, when we were talking about what supported setting goals [for example], and the [Mathematics Teaching Practices Summary] and being like, okay, those are the things I need to do then. So that helped me to implement it in my third lesson. Seeing what worked and didn't work, or what supported and what didn't support [the practices] in the first two. (Britney Post-Interview, 11/23/15)

Considering Britney made similar changes to her implementation (MTP 3, MTP 5) as she did her conception (MTP 1, MTP 3, MTP 5), it was not surprising that discussing lessons also influenced her implementation. Even though her post-observation lesson did not provide evidence that she changed her implementation regarding using goals to guide instructional decisions, the results revealed that she altered both her conception and implementation of contextual representations and questioning strategies.

Conception. In terms of her conception of the Mathematics Teaching Practices (NCTM, 2014), Britney stated that the changes were primarily due to observing and discussing them with her colleagues. When she was asked what was most valuable, Britney commented:

> Discussing with peers, I think was the most helpful for understanding what they meant, getting their ideas and observing my peers, seeing what they did and placing that thing we filled out that had each of [the Mathematics Teaching Practices (NCTM, 2014)], and we could list things that supported that [on the Observation protocol]. To observe so and so teach, and to say, okay, that is something, where could I put that on [the Observation protocol]. That was super helpful for me understanding what they meant. And then hearing what, like when we observed Lesson 1, seeing what I put in certain categories and what [Sally] also put in those categories, and being like, okay, I didn't see that as that and now I do. That was helpful, the observation portion helped me understand them better. (Britney Post-Interview, 11/23/15)

Making connections between her observations and the Mathematics Teaching Practices (NCTM, 2014) appeared to be influential in Britney's changes in conception. Evidence from the lesson study supported this notion. During Research Lesson 1, Britney altered her conception of using goals to guide instructional decisions: "So I think that has changed my mindset . . . seeing [Mark's] today" (Lesson 1 Debrief, 11/2/15). In addition, Britney's conception of questioning strategies changed as she made connections between the research lessons and MTP 5. She noted good questioning techniques during Research Lesson 1 and Research Lesson 2. However, she stated that changes to her conception of MTP 3 were mainly a result of discussing the research lesson. "I think that in our talks, like talking about different ways to re-do the tasks, have helped me see more real-world situations" (Lesson 3 Debrief, 11/9/15). Taken collectively, observing and discussing the Mathematics Teaching Practices (NCTM, 2014) seemed to be the most prominent influence on Britney's conception.

Perception. In terms of her perception of reform-oriented practices, Britney stated confidently that the changes were due to reflections she made about Research Lesson 3. When she was asked what was most impactful, she explained:

> The lesson that I actually taught was the one that most influenced my perception about their importance . . . seeing my kids, the things I do, how they responded . . . that helped me understand how I really felt about them. Like, okay, I see this is super important. So my lesson helped the most with my perceptions towards those practices. (Britney Post-Interview, 11/23/15)

Changes to Britney's perception (MTP 1, MTP 3, MTP 6, MTP 7) were evident in her research lesson. In Research Lesson 3, Britney narrowed the learning goal (MTP 1), made connections between representations (MTP 3), included contextual representations (MTP 3), built conceptual understanding first (MTP 6), and allowed students to experience productive struggle (MTP 7).

Summary. In Britney's case, the most notable change pertained to MTP 3, as she made meaningful changes to her implementation, conception, and perception of using contextual representations. Although changes related to MTP 1 were not found in her implementation, Britney made alterations to both her conception and perception of using goals to focus learning. Finally, Britney made changes related to MTP 7, regarding her perception of mistakes being a natural part of the learning process. A summary of all major and minor changes made by Britney is provided in Table 23. With respect to changes to Britney's conception, she stated that observing the research lessons and discussing the Mathematics Teaching Practices (NCTM, 2014) were instrumental. She stated that discussions with her colleagues were also influential in her alterations to her implementation. However, Britney indicated that observing how her students responded in Research Lesson 3 was the most impactful to changes in her perception.

Table 23

Summary of Changes: Britney

MTP	Implementation	Conception	Perception
1	N/A	Using goals to guide instructional decisions Detail in goal setting	Goals were not mentioned prior to the lesson study. Each comment made in the post-interview was new.
2	There were no real-life situations in the first lesson, but the bell work in the second lesson included tasks that caused the students to think at a deep level about the concept of slope and what it means in various situations.	N/A	The main difference on this teaching practice was the emphasis on contextual situations. This was mentioned three times.
3	Contextual Representations – balloon and book problems.	Contextual representations	Prior to lesson study, Britney only used the phrase multiple ways. In the post-interview she used the word connection three times while talking about representations.
4	N/A	N/A	N/A
5	Making conjectures.	Require explanation and justification More than just gathering information to probe thinking	N/A

Table 23 continued

MTP	Implementation	Conception	Perception
6	N/A	N/A	In the pre-interview/journal, Britney only focused on making sure students understand why. After the lesson study, she commented on the order of conceptual then procedural. “Rather than just showing them how to do it . . . then giving them the real-world problem” (Britney Post-Interview, 11/23/15).
7	Britney noticed a common misconception as she was walking around and addressed it on the board. “I would like to make a quick correction for some of you. I am going to ask you to look at the scale of the graph [which was 1000]. Take a second to look at that” (Britney Post-Observation, 11/18/15).	N/A	Learn from mistakes. Mistakes are a part of the learning process.
8	N/A	N/A	N/A

Holistic Analysis

The holistic analysis took place in two forms. First, the researcher examined the lesson study group as a whole to see how the collaboratively designed lessons aligned with the Mathematics Teaching Practices (NCTM, 2014) as well as changes to the lesson that were made during the process. To identify changes made by the lesson study group as a whole, the researcher examined each of the research lessons and subsequent debriefing sessions. Second, the researcher sought to develop patterns of change among the three participants. An inductive coding analysis (Yin, 2014) was used to discern relevant concepts. This helped “bring order, structure, and interpretation to the mass of data collected” (Marshall & Rossman, 1999, p. 150). Specifically, the researcher identified cross-case themes by identifying Mathematics Teaching Practices (NCTM, 2014) that were associated with multiple participants in each domain (i.e., implementation, conception, and perception) (see Appendix J). Additional themes that emerged during the study will be described as well.

Holistic Case

As the participants progressed through the lesson study, they made meaningful changes with respect to the collaboratively designed research lessons. Specifically, the group made changes that related to detail in goal setting, task design, connecting representations, using goals to guide instructional decisions, and teaching through problem solving. These findings will be described in detail in the sections that follow.

Detail in goal setting. The group made meaningful changes related to the goal of the research lesson. The group’s initial goal was very broad: “To produce the algebraic thinking and starting to use variables” (Mark, Planning Meeting 1, 10/14/15). As the lesson study progressed, the group attempted to match the tasks with the objective and considered what students were prepared to accomplish at that moment. The goal became more and more refined as the discussions took place (see Table 24).

Table 24

Evolution of Research Lesson Objective

Phase	Research Lesson Objective
Planning Meeting 1	Produce the algebraic thinking and starting to use variables.
Planning Meeting 2	Understand what they are doing when they substitute a known quantity for something unknown.
Research Lesson 1	Introduce substitution as a method for keeping balance in equations and relate variables in systems of equations to real-world situations.
Lesson 1 Debrief	Develop the concept of systems of linear equations.
Research Lesson 2	Solve real-world and mathematical problems that develop the concept of systems of equations.
Lesson 2 Debrief	Understand what the solution to a system of equations means.
Research Lesson 3	Develop an understanding of what the solution to a system of equations means (satisfies both stipulations) and introduce the concept of systems of linear equations with two variables.

The goal was a main topic of discussion throughout the lesson study, as Dr. Ross made statements like, “We just need to make clear, what is our purpose” (Planning Meeting 1, 10/14/15) and “So next question I think is critical, so what is the specific goal” (Planning Meeting 2, 10/27/15)? At some moments during the lesson study, the group members were frustrated and uncertain about the goal. Mark stated, “I feel like we can either refine our objective or start over” (Planning Meeting 2, 10/27/15). Similarly, Britney said, “I felt confused after our talk about whether our goal matched our task” (Britney Self-Reflection 4, 11/8/15). By the end of the lesson study, however, the group was finally satisfied with their goal. "I think the goals were much more focused. I'm glad that I narrowed it down and thought okay, solution. I want them to understand the solution has to satisfy both [conditions]. I feel like that improved throughout this process" (Britney, Lesson 3 Debrief, 11/9/15). Sally also commented:

> I agree with you about the goal. I think finally that we have narrowed it down enough to really get what they are capable of at this moment, just working with two variables and being able to identify a solution. (Lesson 3 Debrief, 11/9/15)

Research Lesson 3 was the first lesson in which the term system of equations was clearly connected to a set of two equations. The data suggested that the group made enhancements with respect to establishing goals to focus learning (MTP 1). When asked how this experience might enhance their PLC, Mark commented:

> I feel like we do share quite a bit right now as far as what we do, but I feel like being able to talk about our expectations and our goals and be a little more specific there. I think that's helpful. (Lesson 3 Debrief, 11/9/15)

The group not only refined the expectations and goals of the research lesson, but they also intended to continue this practice within their PLC.

Task design. Throughout the three research lessons, the group continued to improve the design of the bell work task. Mark first introduced the group to the muffin task, which he used for bell work in his first research lesson plan (see Table 25). Mark's purpose for this task was "to get them to start thinking about exchanging" (Planning Meeting 2, 10/27/15). However, after discussing the muffin task during Planning Meeting 2, the task became more specific to systems of equations in Research Lesson 1. Specifically, the group narrowed the coins to just nickels and pennies and introduced a second stipulation restricting the number of coins (see Table 26). Again, after discussing the bell work in Lesson 1 Debrief, Sally altered the muffin task to press students to write equations to represent the situation. In Research Lesson 2, the bell work asked students to write an equation to represent the first condition (see Table 25). This created opportunities for students to begin to develop the concept of a system of equations.

Based on the group's discussion in Lesson 2 Debrief, Britney separated the muffin task into three separate questions in order to show that the equations formed in the first two questions combined to make a system of equations and that the solution to a system must satisfy both equations (see Table 25). In addition, students were asked to create the equation for the second stipulation, which did not occur in Research Lesson 1 or Research Lesson 2. Throughout the lesson study, this particular task became more focused and aligned with the goal for the lesson. As the goal changed, so did the task. This was evident in the final task, as it revealed to students that a solution to a system of equations must satisfy both stipulations presented. As Britney reflected on changes that were made to the research lesson, she stated, "We changed [the bell work task] throughout this to really drive home the systems concept . . . which I think was a huge change that made a big difference in the lesson" (Britney Post-Interview, 11/23/15). This statement signified the group's willingness to make instructional decisions in order to better align the research lesson with the selected goal.

Table 25

Evolution of Bell Work Task

Phase	Bell Work Task
Planning Meeting 2	The original problem: "I went to the store to buy a muffin. Muffins cost 25 cents each. I had a lot of change in my coin purse. I have quarters, dimes, nickels, and pennies. How many ways could I pay for the muffin? List the ways" (Mark Lesson Plan 1, 10/27/15).
Research Lesson 1	A second condition was added and given to students after students found the solutions to the first stipulation: "How could I pay for my muffin if the clerk said she could only take nine coins per purchase" (Mark Lesson Plan 2, 11/2/15)?
Research Lesson 2	A prompt was added to the end of the first question to press students to write an equation: "Write an equation to show your answer" (Research Lesson 2, 11/6/15).
Research Lesson 3	The task was separated into three problems: (1) "I only have nickels and pennies in my change purse. I want to purchase a muffin that costs $0.25. Write an equation to represent this situation. List the possible solutions" (Britney Lesson Plan 3, 11/9/15). (2) After a class discussion about the first question: "I have only nickels and pennies in my change purse. How much money could I have if there are only 9 coins total? Write an equation and list the possible solutions" (Britney Lesson Plan 3, 11/9/15). (3) After a class discussion about the second question: I return to the store with my change purse of nickels and pennies. The clerk tells me he can only accept 9 coins. How can I pay for the muffin? List the solutions" (Britney Lesson Plan 3, 11/9/15).

Connecting representations. As the research lessons progressed, the group built stronger connections among representations. During Research Lesson 1, Mark revealed solution strategies that included words, pictures, and equations for the Fruits and Vegetables Task (see Appendix I). However, strong connections were not made in order to build the concept of substitution or systems of equations. The connection became stronger in Research Lesson 2. Sally introduced students to a tabular representation of their solutions to the muffin problem and made connections to the equation students had written. However, this was only done with the first question of the muffin task, which resulted in the equation $5n + 1p = 25$.

During the discussion of the first question of the muffin task in Research Lesson 3, Britney displayed student work in order to reveal the connection among the words, pictures, and a table. She then used the tabular representation to help students build the corresponding equation. Britney also made these connections during the discussion of the second question of the muffin task. Perhaps the main difference was that Britney displayed information from the task on the board simultaneously so that students could make connections among the questions. Dr. Ross commented that:

> We call it board writing . . . so I think today you already applied that idea. Question 1, Question 2, and then [Question] 3 representations and put them together. So I think this kind of board writing design . . . I think you demonstrated this very well. (Lesson 3 Debrief, 11/9/15)

Britney's writing on the board provided opportunities for students to not only connect multiple representations, but also make connections among the various questions and summarize their learning.

Using goals to guide instructional decisions. Changes to the bell work task and emphasizing connections among representations were decisions that were made to better align the lesson with the goal. Not only did the group connect representations, but they also altered the lesson to build upon students' strategies to achieve their goal. In Research Lesson 1, students primarily used a guess-and-check strategy to find a solution to the

Umbrella and Hats Task (see Appendix H). Instead of forcing students to use other methods, the group introduced the tabular representation to aid students in their guess-and-check strategy. As a result, students were able to guess-and-check more efficiently. When asked about what aspect of Research Lesson 2 was most successful, Sally explained:

> When a student actually took what we did for bell work and made his own table for the umbrellas [task]. I thought, oh he got it, and then he could explain it to his group . . . Then when he explained it to the class after, I heard several kids say, “Oh that's easy, why didn't I think of that?” So I thought that was successful in that aspect. (Lesson 2 Debrief, 11/6/15)

Introducing the tabular representation created opportunities for students to solve the Umbrellas and Hats Task (see Appendix H) more efficiently and begin to develop the concept of a system of equations.

Other alterations were made to the research lesson to better align with the goal of the lesson. For example, the group decided to remove the Fruits and Vegetables Task (see Appendix I) because it did not align with the goal. Mark stated, “I don't feel like the fruits led that way as much to systems as it did just substituting and exchanging . . . if we want to get our goal across, I feel like the fruits [task] can go honestly” (Lesson 1 Debrief, 11/2/15). Removing the Fruits and Vegetables Task (see Appendix I) provided more time for exploration during the bell work and during the Umbrellas and Hats Task (see Appendix H). Taken collectively, these instances signified the group’s mindfulness of the goal of the research lesson as they made instructional decisions.

Teaching through problem solving. As the group used the goal to guide instructional decisions, the focus went from teaching problem solving to teaching through problem solving. In Research Lesson 1, Mark stated, "My goal for this lesson was for them to really just work on their problem-solving skills" (Mark, Lesson 1 Debrief, 11/2/15). As a result, Mark led a discussion during Research Lesson 1 about problem-solving strategies. He listed various strategies on the board and revisited the list after the Fruits and Vegetables Task (see Appendix I) so that students could add more strategies. However, Mark reflected on the lesson by saying, "I felt like that took way too long and I felt like that was a stalled moment in the lesson . . . I regret taking that long on that and not having enough time for that final task" (Mark, Lesson 1 Debrief, 11/2/15). Mark realized that too much time was spent on teaching problem solving instead of teaching through problem solving.

Dr. Ross added, "Our goal is to develop the concept through problem solving as the tool to achieve that goal. So if you all agree, then we need to adjust our task" (Lesson 1 Debrief, 11/2/15). The group realized that they could better accomplish their long-term goal of developing problem solvers by teaching through problem solving instead of teaching problem-solving skills. As the lesson study progressed, the goal shifted towards developing the concept of systems of equations through problem solving.

Cross-Case Analysis

General themes were extracted from the embedded cases to develop patterns of change among the three participants. The major themes that emerged included establishing goals to focus learning (MTP 1), using and connecting mathematical representations (MTP 3), posing purposeful questions (MTP 5), and supporting productive struggle in learning mathematics (MTP 7). Each of these findings will be described in the sections that follow. In addition, comparisons among participants will be made in regard to their participation in FormUp. Finally, the cross-case analysis identifying patterns of change will be described along with the group's corresponding growth networks.

Establishing goals to focus learning. Perhaps the most prominent change across the embedded cases was in relation to MTP 1. In fact, changes to MTP 1 were found across implementation, conception, and perception. Moreover, all three participants made at least one change related to MTP 1. The primary changes participants made with respect to MTP 1 were alterations in their conception of MTP 1. Specifically, the most prominent changes within MTP 1 were related to referencing and reflecting upon goals during a lesson and detail in goal setting (see Table 26). Overall, a consistent thread throughout the embedded cases was simply the valuing of goals throughout the entire learning process. Not only were changes associated with MTP 1 made across the embedded cases, but within the collaboratively designed research lessons as well. The participants' meaningful changes with MTP 1 aligned with the focus of the group discussions during the lesson study, as goals were a prominent theme throughout. Further, these discussions about the research lesson's goal seemed to be a primary factor in participant change in this area. For example, Mark shared, "I think just being able to talk about the goals of my lesson . . . I think that really clarified a lot of it for me and just what the purpose of the goal was" (Mark Post-Interview, 11/18/15). In summary, meaningful changes to MTP 1 was a common theme throughout the cases, which was a result of discussing the goal of the research lesson during the lesson study.

Table 26

Changes Related to MTP 1

Domain	Aspect of Change
Implementation	Referencing and reflecting upon goals throughout lesson (MG)
Conception	Referencing and reflecting upon goals throughout lesson (MG) Situating goals within learning progression (SM) Detail in goal setting (BS) Goals guide instructional decisions (BS, MG)
Perception	Referencing and reflecting upon goals throughout lesson (MG) Detail in goal setting (BS)

Note: BS = Britney Smyth, MG = Mark Gibson, SM = Sally Mills.

Posing purposeful questions. The cross-case analysis also revealed that MTP 5 was a major change made by the group. Sally made changes to her implementation and conception of MTP 5, while Britney altered her implementation and Mark enhanced his perception (see Table 27). Overall, the group made changes related to asking questions that focused student thinking, pressing students to make conjectures, and requiring explanation and justification. It appeared as if observing each other teach was crucial to change related to this teaching practice, as numerous statements were made about other teachers' questioning strategies. For example, Britney commented on Sally's use of questioning. "I thought your questioning was really, really good. I liked the questions . . . making them explain . . . what does it mean? So just a lot of questioning going on that I liked" (Lesson 2 Debrief, 11/6/15). Mark later mentioned, "I heard [Britney] say several times 'Tell me why you did that,' and so I liked that question to try to get them to explain" (Lesson 3 Debrief, 11/9/15). Taken collectively, these statements indicated that changes were made related to questioning strategies in order to better align with MTP 5. Moreover, observing each other teach the research lesson was a prominent factor in making those changes.

Table 27

Changes Related to MTP 5

Domain	Aspect of Change
Implementation	Posing purposeful questions (BS, SM)
Conception	N/A
Perception	Probe and extent student thinking (SM) Posing purposeful questions (MG)

Note: BS = Britney Smyth, MG = Mark Gibson, SM = Sally Mills.

Supporting productive struggle in learning mathematics. With respect to MTP 7, each participant made changes in either their conception and/or perception (see Table 28). A major theme that emerged across the cases was persevering through problem solving. Although Britney altered her perception of students learning from mistakes, Sally and Mark both made modifications related to persevering through problem solving. Primarily, the participants began to value struggle as natural part of the learning process. At the beginning of the lesson study, the participants' main concern was with students giving up. However, the lesson study addressed those concerns, as the group was able to teach and observe lessons in which the students persevered through the entire lesson. However, none of the participants provided high cognitively demanding tasks in the post-observation that allowed students to experience productive struggle.

Table 28

Changes Related to MTP 7

Domain	Aspect of Change
Implementation	N/A
Conception	Persevering through problem solving (SM, MG)
Perception	Persevering through problem solving (SM) Learning from mistakes (BS)

Note: BS = Britney Smyth, MG = Mark Gibson, SM = Sally Mills.

Using and connecting mathematical representations. Britney and Sally made meaningful changes related to their implementation and conception of contextual representations (see Table 29). They both included real-world situations in their post-observation lessons after not doing so in their pre-observation lessons. Moreover, they both altered their conception of contextual representations. In fact, at the end of the lesson study both participants described the importance of real-life contexts in learning. However, there was no evidence that signified change for Mark in this area, as he did not provide real-life contexts in pre- and post-observation lessons.

Table 29

Changes Related to MTP 3

Domain	Aspect of Change
Implementation	Contextual Representations (BS, SM)
Conception	Contextual Representation (BS, SM)
Perception	N/A

Note: BS = Britney Smyth, MG = Mark Gibson, SM = Sally Mills.

Patterns of Change. Based on participant responses on the major influences of change, the researcher identified general themes across the cases.

Implementation. Another consistent thread throughout the embedded cases was that observations with follow-up discussions were the most influential in changes to implementation. Again, reflecting upon what aspects of the lesson supported or did not support the Mathematics Teaching Practices (NCTM, 2014) was valuable. This result can be represented by a different growth model (see Figure 27) that displays how participants changed their implementations. In this growth model, participants began by enacting what they had discussed in the Planning Meeting 1 and Planning Meeting 2 (Arrow 1). As they reflected upon what they observed, conceptions and/or perceptions were changed (Arrow 2). These changes resulted in a debriefing session that focused on those teaching practices (Arrow 3). Finally, a combination of changes in conception and/or perception (Arrow 4a) along with discussion in the debriefing session (Arrow 4b) led to changes in implementation. In this lesson study, this pattern was repeated three times, once for each research lesson.

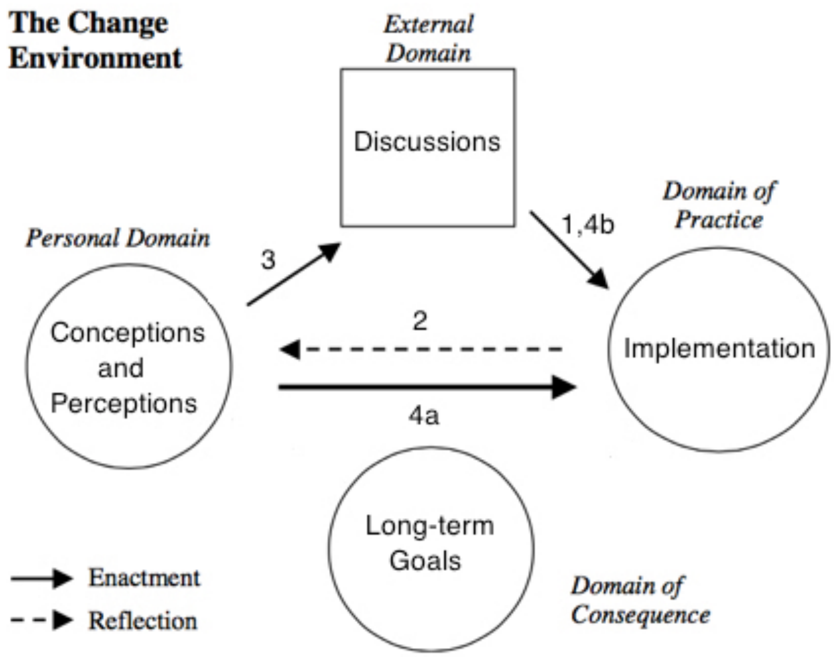

Figure 26. Implementation growth network. Adapted from "Elaborating a model of teacher professional growth," by D. Clarke and H. Hollingsworth, 2002, *Teacher and Teacher Education*, *18*, p. 951.

Conceptions. With regard to participants' conceptions, it appeared as if reading *Principles to Actions* (NCTM, 2014) and discussing the Mathematics Teaching Practices (NCTM, 2014) with their colleagues were most impactful. Discussion topics that seemed to be most helpful involved sharing opinions of the practices and reflecting on how the research lessons supported or did not support the practices.

The data suggested a particular growth network (see Figure 27) in which the participants changed their conceptions of the Mathematics Teaching Practices (NCTM, 2014). The participants began by reading *Principles to Actions* (NCTM, 2014) and altered their conception as they reflected upon the reading (Arrow 1). Reflecting upon subsequent discussions about the practices within the lesson study group further enhanced their conceptions (Arrow 2). After reading *Principles to Actions* (NCTM, 2014) and meeting to discuss the research lesson in Planning Meeting 1 and Planning Meeting 2, they altered implementation of the research lessons (Arrow 3). Participants enhanced their conception by reflecting upon how these changes to implementation in Research Lesson 1 (Arrow 4) supported or did not support the Mathematics Teaching Practices (NCTM, 2014). This process of discussion, reflection, and enactment (Arrows 3 and 4) was repeated two more times as further changes to implementation were made as a result of Lesson 1 Debrief and Lesson 2 Debrief. Overall, reading *Principles to Actions* (NCTM, 2014) formed participants' initial conceptions. Then discussing how the research lessons supported Mathematics Teaching Practices (NCTM, 2014) with their colleagues further refined their conceptions.

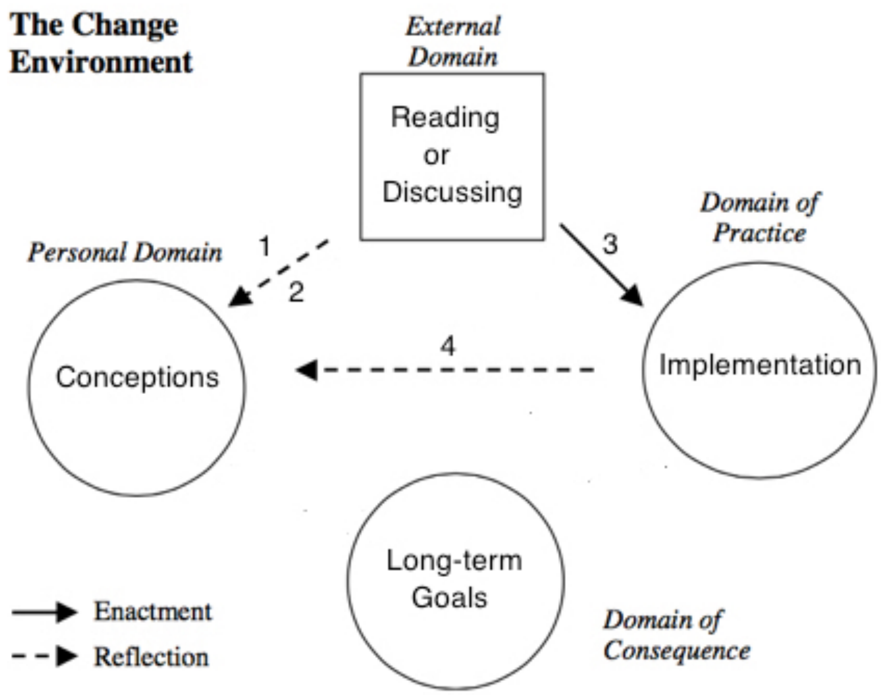

Figure 27. Conception growth network. Adapted from “Elaborating a model of teacher professional growth,” by D. Clarke and H. Hollingsworth, 2002, *Teacher and Teacher Education*, *18*, p. 951. Note: numbers 2 and 3,5,7 below the arrows from the external domain indicate reflection upon group discussions.

Perception. In terms of changes made to participants’ perceptions, observing the success of the research lessons was a powerful influence. For Sally, it was observing successes throughout all of the research lessons. However, in Britney’s case, it was her research lesson specifically that was instrumental to her change. This pattern of change is represented in the growth network found in Figure 28. Participants first enacted teaching practices discussed in *Principles to Actions* (NCTM, 2014) and lesson study meetings (Arrow 1). As participants observed students’ perseverance and engagement in the research lessons, new conclusions were drawn as participants associated these changes with certain practices that were used (Arrow 2). These conclusions then altered participants' perceptions of those practices (Arrow 3). In the lesson study, this pattern of change was repeated three times, once for each research lesson.

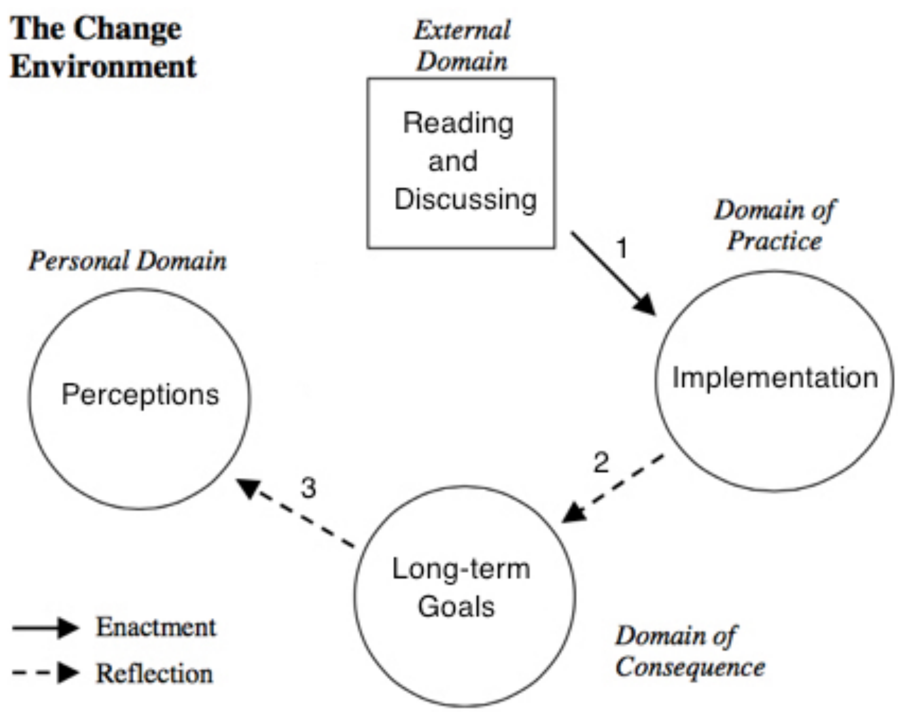

Figure 28. Perception growth network. Adapted from "Elaborating a model of teacher professional growth," by D. Clarke and H. Hollingsworth, 2002, *Teacher and Teacher Education*, *18*, p. 951. Note: number 1 below and above the arrow from the external domain indicates reflection upon both reading and group discussions.

It is important to note, however, that not all of the participants followed this growth network. Mark did not mention observations as being influential, but instead stated that talking about the practices with the group (Planning Meeting 1) had an effect on his perceptions. As noted earlier, it appeared as if Mark's transition related to questioning strategies took place over the entire lesson study process, which did not support his statement.

Summary

From a holistic perspective, the group made meaningful changes to the research lesson related to detail in goal setting, task design, connecting representations, using goals to guide instructional decisions, and teaching through problem solving. As a result, the research lesson became more focused and created rich opportunities for students to learn through problem solving. The most prominent changes across the embedded cases occurred with respect to MTP 1, MTP 3, MTP 5, and MTP 7. When assessing the influences of change, certain growth networks or patterns of change emerged for each domain of change: implementations, conceptions, and perceptions.

Chapter Summary

The purpose of this study was to explore how lesson study can be used to aid teachers in enhancing their implementations, conceptions, and perceptions of the Mathematics Teaching Practices (NCTM, 2014). To examine this, an embedded case-study design was adopted, in which each participant of the lesson study served as an embedded case within the lesson study group. The participants completed three cycles of *Keli* lesson study, in which all participants got a chance to take the lead role in teaching the lesson. Throughout the process, the researcher collected information from a variety of data sources to examine how and why the participants changed their implementations, conceptions, and perceptions, if at all.

Analysis of the data took place in two forms. First, the researcher analyzed the embedded cases to identify specific changes that were made by each participant. Second, a holistic case analysis revealed patterns within the embedded cases and allowed the researcher to investigate how the collaboratively designed lessons aligned with the Mathematics Teaching Practices (NCTM, 2014) throughout the lesson study. The results of this study revealed certain aspects of lesson study that provoked change in participants' implementations, conceptions, and perceptions related to the Mathematics Teaching Practices (NCTM, 2014). A discussion of the results of this study and implications for mathematics teacher education are discussed in the next chapter.

CHAPTER V: DISCUSSION

Introduction

This qualitative study explored the impact of lesson study in aiding teachers in conceptualizing and implementing the Mathematics Teaching Practices (NCTM, 2014) and investigated how teachers' perceptions of reform-oriented teaching change while participating in lesson study, if at all. A restatement of the research problem, a review of the methods used in this study, and a review of the results are presented first in this final chapter. Then a discussion of the results will follow. Finally, this chapter ends with areas of future research and implications for mathematics teacher education.

The Research Problem

International comparison tests have revealed that the U.S. is not adequately preparing students to complete cognitively demanding tasks (OECD, 2012). With traditional teaching methods pervasive in the U.S. (Hiebert et al., 2005), it is imperative that mathematics teacher educators and professional development leaders understand what methods result in authentic changes in classroom instruction. Although authentic change is difficult to achieve (Richardson, 1990), lesson study offers some unique advantages that can aid teachers in conceptualizing and implementing reform-oriented practices (Lee & Ling, 2013). Specifically, the lesson study model presents a promising approach to learning about reform-oriented instruction (Yoshida, 2013), as it is situated within the classroom (Takahashi et al., 2013), draws upon rich discussions about lesson development (Lewis et al., 2009), and creates opportunities for reflection upon practice (Ricks, 2011). Although the literature has shown the usefulness of lesson study (e.g., Huang & Li, 2009; Lewis et al., 2013; Lewis et al., 2006; Ricks, 2011; Takahashi et al., 2013; Yoshida, 2013), strategies of how lesson study can support teachers in conceptualizing and implementing reform-oriented instruction could be vital to the success of mathematics education locally and across the country.

Review of Methodology

An embedded case-study design (Yin, 2014) was used to explore how lesson study influenced changes in teachers' implementations, conceptions, and perceptions of the Mathematics Teaching Practices (NCTM, 2014). To achieve this goal, three participants were selected to participate in one round of *Keli* lesson study, consisting of three cycles. Multiple sources of data were collected before, during, and after the lesson study to corroborate evidence and achieve data triangulation (Creswell, 2007). Detailed case descriptions were written for each participant, in which changes that participants made throughout the lesson study process were described. Taken collectively, the participants' experiences were analyzed holistically through common themes that emerged from the research lessons and the individual cases. Together the themes extracted across the embedded cases aided in determining patterns of change, which aided the development of the various growth networks.

Review of Results

In Chapter Four, a chronological narrative of the lesson study provided a description of the major components of the lesson study including planning meetings, research lessons, and debriefing sessions. For each participant, major changes in implementation, conception, and perception of the Mathematics Teaching Practices (NCTM, 2014) were described along with the aspects of the lesson study that influenced these changes. Finally, a cross-case analysis and an analysis of the changes that were made to the research lesson were used to describe the holistic case in this study. The major findings will be summarized in the paragraphs that follow.

Each participant made changes with respect to their implementation, conception, and perception of the Mathematics Teaching Practices (NCTM, 2014). Britney's most prominent changes were related to MTP 1, MTP 3, and MTP 5. Of the three practices, Britney's greatest change was in MTP 3, as she altered her implementation, conception, and perception of using contextual representations. In addition, Britney enhanced her conception and perception of setting goals to focus learning (MTP 1), with an emphasis on the detail of goal setting. Finally, Britney progressed in her implementation and conception of posing purposeful questions (MTP 5).

Similarly, Sally made numerous changes to her implementation, conception, and perception of the Mathematics Teaching Practices (NCTM, 2014). Sally's primary change was related to facilitating mathematical discourse, as she altered her implementation, conception, and perception of MTP 4. Sally also changed her implementation and conception of using and connecting multiple representations (MTP 3), especially related to using contextual representations. Finally, the lesson study also brought about meaningful changes in Sally's implementation and perception of posing purposeful questions (MTP 5).

Although Mark made alterations during the lesson study, his changes were not as prominent. In fact, Mark's only major change was related to MTP 1, as he altered his implementation, conception, and perception of referencing and reflecting upon goals during a lesson. However, Mark, along with Sally and Britney, made other, minor changes in addition to the major changes described in this section.

From a holistic perspective, the most noticeable changes across the embedded cases occurred with setting goals to focus learning (MTP 1), using and connecting multiple representations (MTP 3), posing purposeful questions (MTP 5), and supporting productive struggle in mathematics (MTP 7). Themes also emerged with respect to the causes of change. The participants viewed reading *Principles to Actions* (NCTM, 2014) and discussing the Mathematics Teaching Practices (NCTM, 2014) with their colleagues as the most impactful to their conceptions. In addition, the participants indicated that observations with follow-up discussions had the greatest influence on their implementations. Finally, the participants stated that observing student learning during the research lessons had the most impact on their perceptions of the Mathematics Teaching Practices (NCTM, 2014).

In regards to the research lesson itself, the group made meaningful changes with respect to setting goals to focus learning, using goals to guide instructional decisions, designing the task, connecting representations, and teaching through problem solving. First, the goal became more detailed throughout the research lessons and the participants made instructional decisions that altered the lesson to better align with the selected goal. Second, the bell work task involving the purchasing of muffins was refined to make the idea of systems of linear equations visible, and participants used a table to direct students towards writing equations to represent the constraints provided. Finally, the discussion of problem-solving strategies in general and the Fruits and Vegetables Task (see Appendix I) were removed so that students could have more time to reason and problem solve. As a result, the research lesson became more focused, and thus, allowed more opportunities for students to grapple with what a solution to a system of equation was and how it could be found.

Discussion of Results

The results of this study further support and add new insights to existing theory, research, policy, and practice. The discussion points delineated in the paragraphs that follow have both theoretical and practical significance.

Previous Professional Development Experience

Britney and Mark participated in a professional development project, FormUp, during the summer prior to this study, but Sally did not. Mark and Britney's participation in FormUp provided initial exposure to the Mathematics Teaching Practices (NCTM, 2014). Although Mark made fewer changes than Britney and Sally, Britney made the most changes (seven) of the group. However, it appeared as if the varying levels of conception and perception related to the practices aided the enhancements of the group, as Britney and Mark accepted lead roles during discussions within lesson study. This result corroborated the findings of Suh and Parker (2010), who discovered that varying expertise and abilities within a group of inservice teachers led to a sense of collective efficacy within the lesson study group. Therefore, varying levels of teaching experience, educational backgrounds, and knowledge of mathematics teaching practices, in general, can aid the development of teacher learning.

Role of Knowledgeable Other

Throughout the lesson study, Dr. Ross' input directly influenced some of the shifts that occurred related to the research lesson. For instance, Dr. Ross' comments influenced the development of the muffin task, the use of tabular representation, and the goal of the research lesson. However, some of Dr. Ross' comments did not influence the instructional decisions immediately. For example, Dr. Ross made multiple comments related to narrowing the goal towards developing the concept of the solution to a system of equations beginning in Planning Meeting 2. Yet, the goal of the research lesson did not shift to understanding the solution until Research Lesson 3. This result was not surprising, given that the role of the researcher was to focus teachers' attention and not funnel them towards a particular idea. Even still, Dr. Ross influenced many of the changes that were made to the research lesson. Therefore, the results of this study supported research that suggests that knowledgeable others are vital to the success of lesson studies (e.g., Groth, 2012; Huang et al., 2011). Participants were not asked about Dr. Ross' contributions directly. Therefore, participants' views regarding the influence of Dr. Ross on their changes were not clear.

Interconnected Model

The growth networks revealed in this study relate to various frameworks described in the literature. The change in perception that emerged in this study followed the progression described by Wood et al. (1991), in which observations and reflections press teachers to resolve conflicts between their beliefs and observations in the classroom. As the participants observed successes in the research lesson, they altered their perceptions of those practices. This particular growth network was also very similar to change described by Guskey (1986). However, teachers' perceptions in this study were formed from associating practices with engagement and persistence and not necessarily increased student learning. Guskey (1986) argued that associations are formed because attitudes and beliefs about teaching are largely derived from classroom experiences.

With respect to implementation, the growth network followed by the teachers in this study suggested that changes in conceptions and perceptions of practices as well as changes in the external domain affected change in implementation. The relationship between the personal domain and the domain of practice found in this study supported the work of Cobb et al. (1990), who suggested that there is not a linear causality between practices and beliefs, but a continuous interplay between the two. Moreover, Wood et al. (1991) suggested that teachers can change their practices when given opportunities to learn within the classroom setting and observe new practices with their students. The participants in this study, especially Britney, changed as a result of relating new practices with successes in their own students' engagement and persistence.

Although changes in implementation and perception in this study could have been described by other, less dynamic, models, participants' changes in conception were authentically represented through the Interconnected Growth Model of teacher change (Clarke & Hollingsworth, 2002). Moreover, representing changes in the conception of teaching practices further informed the literature base on Clarke and Hollingsworth's (2002) dynamic model of teacher growth, as this had not been done before. This was indicated by Goldsmith et al.'s (2014) synthesis of related literature that identified six areas of teacher learning, which did not include changes to teachers' conceptions or understandings of teaching practices. In addition, this study revealed how the interconnected growth model can be used to model a group of teachers, instead of an individual. Finally, this study expanded on how this model can be used by parsing out the personal domain into conceptions and perceptions.

Reading and Lesson Study

The participants in this study appeared to develop their conception of the Mathematics Teaching Practices (NCTM, 2014) as a result of reading *Principles to Actions* (NCTM, 2014). Reflecting on which practices were supported in each research lesson and which ones were not supported appeared to further enhance their conceptions, which sometimes included referring back to *Principles to Actions* (NCTM, 2014). However, with respect to perceptions and implementations, the lesson study itself seemed to be most valuable. In Mark's case, his initial perceptions were developed by reading *Principles to Actions* (NCTM, 2014), but he commented that discussing with the lesson study group helped clarify what he was thinking. However, in Britney's and Sally's cases, it appeared as if the lesson study was the only factor that provoked changes in perception. The group indicated that they did not directly improve their implementation by reading *Principles to Actions* (NCTM, 2014). Instead, the group valued observing their peers and discussing with the lesson study group. The results suggested that reading materials about teaching practices can support the development of teachers' initial conceptions, but these ideas are further enhanced along with their implementations and perceptions through lesson study. Therefore, as was the case in this study, it is important for lesson study participants to read the teaching materials prior to the lesson study and continuously study them as they reflect upon the research lessons in order to enhance their conceptions.

Implementation

According to NCTM (2000), the primary focus of professional development should be to help teachers teach their students using reform-oriented strategies. Situated learning theorists suggest that this can occur as teachers have the opportunity to develop tools that shape their identity in such a way that members are able to transfer forms of participation to new settings (Lave & Wenger, 1991). However, there were 11 changes across the participants in this study related to conception and perception that were not associated with changes in implementation. The disconnect between the two could be explained from multiple perspectives. For instance, the content (slope) in participants' post-observation lessons was very different from the content found in the lesson study (systems of linear equations). Although the context of the classroom remained invariant, the differences in content lessened the relation between the lesson study and the post-observation lessons, which could have caused this disconnect. As situated learning theorists have argued, the amount of transfer between tasks is dependent on the degree to which the tasks share cognitive elements (Singley & Anderson, 1989).

Alternatively, it is plausible that the participants changed implementation regarding MTP 1, but these changes were not visible during the post-observation lesson. For example, the researcher did not collect data on the post-observation lesson related to goal setting and using goals to guide instructional decisions. Yet another reason could be related to the nature of the content, as converting from standard form of a linear equation to slope-intercept form is more procedurally oriented than systems of linear equations. Therefore, it may have been more challenging to implement certain Mathematics Teaching Practices (NCTM, 2014).

A final possible explanation for this disconnect is that the participants simply had difficulties implementing the practices. Richardson (1990) argued that, even with new images of reform-oriented teaching, transitioning from traditional methods of teaching to reform-oriented practices can be a very difficult task to achieve and sustain. This notion represents a viable possibility given the difficulty of implementing the Mathematics Teaching Practices (NCTM, 2014). In fact, Steve Leinwand (personal communication, February 4, 2016), a Principal Research Analyst at American Institutes for Research and a member of the *Principles to Actions* (NCTM, 2014) writing team, argued that by asking teachers to implement the Mathematics Teaching Practices (NCTM, 2014) on a daily basis “we are asking them to do the impossible.” Therefore, the difficulty of implementing the practices is an additional factor that could have caused this disconnect between changes in implementation and changes in conceptions and perceptions.

Cognitive Conflict

Cognitive learning theorists suggest that learning is the result of cognitive conflict that occurs while trying to reach a consensus with others (Kurt et al., 2014). Moreover, Forman and Cazden (1994) argued that collaboration enhances the development of logical reasoning by reorganizing knowledge brought on by cognitive conflict. The most prominent change found in this study was participants’ changes in their implementations, conceptions, and perceptions of MTP 1. The participants in this study experienced cognitive conflict related to MTP 1, as the group struggled to narrow the goal of the research lesson. Discussion about the goal of the research lesson continued throughout the entire lesson study. In fact, the goal continued to be refined between each lesson and there was not always agreement as to what the objective should be.

In regards to the interconnected growth model, the results of this study suggested that change in the external domain involving discussions of MTP 1 among participants resulted in cognitive conflict (Arrow 1) (see Figure 29). Changes related to cognitive conflict then altered participants' implementations of MTP 1 (Arrow 2). In general, cognitive conflict is caused by the conflict between what one person conceives and what others conceive (Kurt et al., 2014). Situated learning theorists would argue that once teachers experience this, they are then able to transfer the forms of participation to their teaching and other conversations (Lave & Wenger, 1991). Therefore, cognitive learning theory and situated learning theory combine to create a dynamic by which to describe the role of cognitive conflict in this study.

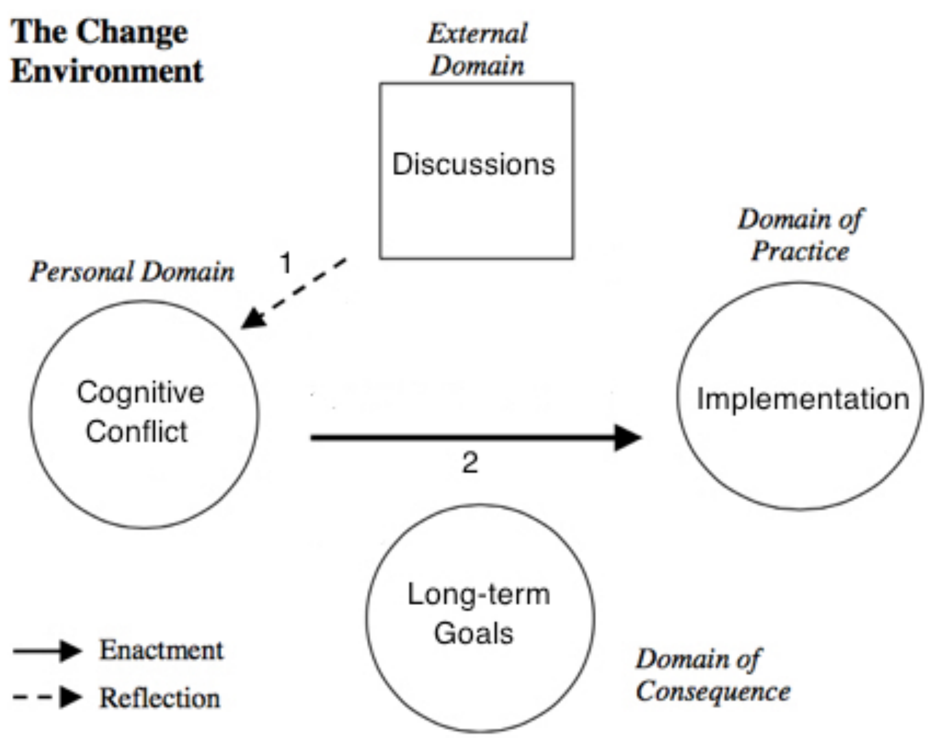

Figure 29. MTP 1 growth network. Adapted from "Elaborating a model of teacher professional growth," by D. Clarke and H. Hollingsworth, 2002, *Teacher and Teacher Education*, *18*, p. 951.

Influential Aspects of *Keli* Lesson Study

There were many aspects of *Keli* lesson study that aided participants' changes in this study. However, observation and reflection were two main aspects that influenced change. These two aspects will be discussed further in the paragraphs that follow.

Observations. Takahashi et al. (2013) argued that lesson study takes place in the context of the classroom, and thus clearly communicates what it looks like to implement reform-oriented practices. Observation of the research lessons was a prominent aspect of lesson study that was viewed by the participants as influential in their changes in this study. Indeed, observation likely enhanced participants' implementations and perceptions of the Mathematics Teaching Practices (NCTM, 2014). In terms of implementation, participants stated that observing the research lessons and examining what supported and what did not support the Mathematics Teaching Practices (NCTM, 2014) helped them better implement the practices. Moreover, participants indicated that observing students engage and persist as a result of implementing the Mathematics Teaching Practices (NCTM, 2014) enhanced participants' perceptions of the practices. The fact that salient outcomes were only necessary for the perceptions growth model further signified the importance of salient outcomes in altering teachers' perceptions of practices. These results support Yoshida (2013), who found that observing student learning during a lesson within the classroom enriches the discussion and experience of professional development.

Reflection. As teachers begin to reconceive ideas about teaching, Goldsmith et al. (2014) suggested that they need time to observe and reflect. It is through this process that teachers can consider elements of instruction that are similar or different from their own current teaching, and thus connect new reform ideas to their own practices (Takahashi et al., 2013). In this study, participants reflected upon the research lessons by attempting to match the practices in the lessons with the Mathematics Teaching Practices (NCTM, 2014). This reflection process is a natural part of the *Keli* lesson study process, in general, as the goal of each debriefing session is to identify gaps between the research lessons and ideas presented by the new reform. Across multiple research lessons, participants experience both enactment and reflection, which are key components to the interconnected model of teacher growth (Clarke & Hollingsworth, 2002). Although all lesson studies require a research lesson, *Keli* lesson study emphasizes multiple teachings that provide meaningful opportunities for reflection.

Specifically, there are two stages of reflection within *Keli* lesson study: identifying gaps between existing practices and reform-oriented practices and finding gaps between reform-oriented design and implementation of new design during research lessons. Within this dynamic between enactment and reflection, teachers can learn about new reform, create and implement lessons informed by reform documents, and reflect upon and alter their own practice (Gu & Wong, 2003). Although lesson study is not a practice-embedded professional approach in the U.S., this study supports the notion that teachers could enhance their implementation and conception of innovative ideas through the repeated process of enactment and reflection within a professional learning community (lesson study group).

Implications

The results of this study have implications for mathematics teacher education. Theoretically, this study adds to the literature base on teachers transitioning to reform-oriented practices as well as the effectiveness of lesson study. Practically, this study informs the practices of professional learning communities and professional development programs. The following paragraphs will discuss these implications.

Lesson Study as an Ongoing Process

In many cases, teachers attempt to implement practices from written documents on their own or attend professional development programs instead of participating in collaborative communities at the school level (Wei et al., 2010). However, this study revealed that reading was not enough to alter implementations and perceptions, as reading *Principles to Actions* (NCTM, 2014) seemed to alter only participants' initial conceptions of the Mathematics Teaching Practices (NCTM, 2014). Therefore, professional development leaders should create additional embedded programs to aid teachers in learning about new reforms.

As professional development leaders construct these opportunities, they need to consider its sustainability. Although the results of this study revealed that lesson study is a promising mode of professional development, the disconnect between changes in implementation and changes in conception and perception suggested that ongoing efforts are required to influence sustained changes in implementation. In addition, lesson study needs to vary with respect to core mathematical content to aid teachers in developing strategies for teaching more procedurally oriented content, with which it may be more difficult to implement certain Mathematics Teaching Practices (NCTM, 2014). Moreover, existing professional learning communities can provide the structure by which lesson study can be sustained over long periods of time. Therefore, lesson study should be embedded within teachers' professional learning communities so that it can become a routine practice that leads to authentic changes in teaching. One possible challenge, however, may be identifying knowledgeable others who can facilitate the discussion and focus teachers' attention.

Usefulness of *Keli* Lesson Study

Keli lesson study differs from lesson study, in general, by requiring multiple teachings, input of a knowledgeable other, and constant comparisons through enactment and reflection. The results of this study revealed the effectiveness of *Keli* lesson study in altering participants' implementations, conceptions, and perceptions of the Mathematics Teaching Practices (NCTM, 2014). The primary aspect of *Keli* lesson study that provoked change was the constant comparisons between the research lessons and the Mathematics Teaching Practices (NCTM, 2014). These constant comparisons were made during the lesson as well as during the debriefing sessions. Participants attempted to identify the practices within the lessons, discussed how each lesson supported the practices, and reflected upon how the changes in practices results in student learning. Making constant comparisons is not a commonly used practice in lesson studies, but it is a critical component of *Keli* lesson study. Taken collectively, these results indicated that *Keli* lesson study is a promising variation of lesson study in learning about reform-oriented practices. Therefore, PLCs and practice-embedded professional development programs should adopt *Keli* lesson study when attempting to transition teachers to reform-oriented practices.

Future Areas of Research

This study focused on the implementations, conceptions, and perceptions of teachers as they progressed through one round of *Keli* lesson study. Although studying changes in teachers' implementations, conceptions, and perceptions is valuable, research is needed to determine whether changes in these areas are sustainable over time while participating in *Keli* lesson study. This research would add to the results of this study and, in particular, further refine the interconnected growth models used in this study. Moreover, the development of an instrument to measure understanding of the Mathematics Teaching Practices (NCTM, 2014) is desirable. If developed, this instrument could be used to transcend conceptions to measure teachers' understandings throughout their participation in *Keli* lesson study. Another area of future research includes the role of content knowledge in lesson study and how it affects teacher change. This research would reveal the role of content knowledge in the lesson study process and how it can be leveraged to provoke change. Research is also needed to explore how knowledgeable others can be developed and trained to facilitate effective lesson studies. Practically, this research would equip districts and school officials with effective ways to cultivate knowledgeable others from within their organizations. Finally, although this study provided local proof of the effectiveness of *Keli* lesson study, further research of how *Keli* lesson study can be scaled up in the U.S. is needed. As new reform is disseminated, this research would be crucial given the dynamic interaction between enactment and reflection in *Keli* lesson study that leads to meaningful changes in teachers' implementations, conceptions, and perceptions of mathematics teaching.

Chapter Summary

According to the OECD (2012), PISA results have indicated that U.S. students scored lower than their international counterparts, especially on high-cognitively demanding tasks. Although reform-oriented teaching has been found to positively affect student learning and achievement (Firmender et al., 2014; Gimbert et al., 2007), the majority of lessons in the U.S. are not aligned with such practices (Jacobs et al., 2006; Silver et al., 2009; Wood et al., 2006). Therefore, it is imperative that mathematics teacher educators and professional development leaders understand what methods result in authentic changes in classroom instruction. Guided by situated (Lave & Wenger, 1991) and cognitive (Kurt et al., 2014) theoretical perspectives on teacher learning, the purpose of this qualitative study was to explore how *Keli* lesson study can be used to aid teachers in conceptualizing and implementing the Mathematics Teaching Practices (NCTM, 2014) and investigate how teachers' perceptions of reform-oriented teaching change while participating in lesson study.

The results of this study revealed the effectiveness of *Keli* lesson study in supporting teaching in transitioning to reform-oriented practices, as each participant made meaningful changes related to their implementation, conception, or perception. Moreover, this study revealed not only the aspects of professional development that impacted teacher change, but also that ongoing efforts are required to influence sustained changes in implementation. Overall, this study adds to the literature base on teachers transitioning to reform-oriented practices as well as the effectiveness of lesson study in the U.S. Furthermore, the results from this study inform current professional development and professional learning community practices, which is crucial given the need to improve mathematics teaching in the U.S. (Hiebert et al. 2005; NCTM, 2014; Stigler & Hiebert, 1999).

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APPENDICES

APPENDIX A

Mathematics Teaching Practices Observation Protocol

1. Establish mathematics goals to focus learning.

Action	Who?	When?

2. Implement tasks that promote reasoning and problem solving.

Action	Who?	When?

3. Use and connect mathematical representations.

Action	Who?	When?

4. Facilitate meaningful mathematical discourse.

Action	Who?	When?

5. Pose purposeful questions.

Action	Who?	When?

6. Build procedural fluency from conceptual understanding.

Action	Who?	When?

7. Support productive struggle in learning mathematics.

Action	Who?	When?

8. Elicit and use evidence of student thinking.

Action	Who?	When?

APPENDIX B

Mathematics Teaching Practices Summary

1. Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

- Goals should describe what mathematical concepts, ideas, or methods students will understand more deeply as a result of instruction and identify the mathematical practices that students are learning to use more proficiently.
- Evidence:
  - o Teacher: Discussing and referring to the mathematical purpose and goal of a lesson during instruction
  - o Teacher: Using the mathematics goal to make in-the-moment decisions during instruction
  - o Students: Engaging in discussions of the mathematical purpose and goals related to their current work in the mathematics classroom
  - o Students: Connecting their current work with the mathematics that they studied previously and seeing where the mathematics is going

2. Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

- These tasks encourage reasoning and access to the mathematics through multiple entry points, including the use of different representations and tools, and they foster the solving of problems through varied solution strategies.
- Evidence:
  - o Teacher: Providing opportunities for exploring and solving problems that extend students' current mathematical understanding
  - o Teacher: Posing tasks that require a high level of cognitive demand
  - o Teacher: Supporting students in exploring tasks without taking over student thinking
  - o Teacher: Encouraging students to use varied approaches and strategies to make sense of and solve tasks
  - o Students: Persevering in exploring and reasoning through tasks
  - o Students: Using tools and representations as needed to support their thinking and problem solving
  - o Students: Accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another

3. Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

- The general classification scheme for types of representations includes important connections among contextual, visual, verbal, physical, and symbolic representational forms.
- Evidence:
  - o Teacher: Allocating substantial instructional time for students to use, discuss, and make connections among representations
  - o Teacher: Introducing forms of representations that can be useful to students
  - o Teacher: Asking students to make math drawings or use other visual supports to explain and justify their reasoning
  - o Teacher: Focusing students' attention on the structure or essential features of mathematical ideas that appear regardless of representation
  - o Students: Using multiple forms of representations to make sense of and understand mathematics
  - o Students: Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations
  - o Students: Making choices about which forms of representations to use as tools for solving problems
  - o Students: Contextualizing mathematical ideas by connecting them to real-world situations
  - o Students: Considering the advantages or suitability of using various representations when solving problems

4. Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

- Mathematical discourse includes the purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication.
- Evidence:
  - o Teacher: Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations
  - o Teacher: Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion
  - o Teacher: Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches
  - o Teacher: Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning
  - o Students: Presenting and explaining ideas, reasoning and representations to one another in pair, small-group, and whole-class discourse
  - o Students: Listening carefully to and critiquing the reasoning of peers, using examples to support and counterexamples to refute arguments

- Students: Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others
- Students: Identifying how different approaches to solving a task are the same and how they are different

5. Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

- Purposeful questions allow teachers to discern what students know and adapt lessons to meet varied levels of understanding, help students make important mathematical connections, and support students in posing their own questions.
- Evidence:
  - Teacher: Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking
  - Teacher: Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification
  - Teacher: Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion
  - Teacher: Allowing sufficient wait time so that more students can formulate and offer responses
  - Students: Thinking carefully about how to present their responses without rushing to respond quickly
  - Students: Reflecting on and justifying their reasoning, not simply providing answers
  - Students: Listening to, commenting on, and questioning the contributions of their classmates

6. Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

- Teacher: Providing students with opportunities to use their own reasoning strategies and methods for solving problems
- Teacher: Asking students to discuss and explain why the procedures that they are using work to solve particular problems
- Teacher: Connecting student-generated strategies and methods to more efficient procedures as appropriate
- Teacher: Using visual models to support students' understanding of general methods
- Students: Making sure that they understand and can explain the mathematical basis for the procedures that they are using
- Students: Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems

o Students: Determining whether specific approaches generalize to a broad class of problems

7. Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

- Such instruction embraces a view of students' struggles as opportunities for delving more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions.
- Evidence:
  - o Teacher: Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them
  - o Teacher: Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles
  - o Teacher: Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems
  - o Students: Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks
  - o Students: Helping one another without telling their classmates what the answer is or how to solve the problem

8. Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

- Evidence:
  - o Teacher: Eliciting and gathering evidence of student understanding at strategic points during instruction
  - o Teacher: Making in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend
  - o Students: Revealing their mathematical understanding, reasoning, and methods in written work and classroom discourse
  - o Students: Asking questions, responding to, and giving suggestions to support the learning of their classmates.

APPENDIX C

Post-Lesson Comments Form

What were the strengths of the research lesson?

What were the weaknesses of the research lesson?

Suggestions for further improvement?

APPENDIX D

Self-Reflection Journal Questions

Self-reflection Journal 1: Prior to the Lesson Study

R1. What are your beliefs about the teaching of mathematics? (i.e., what should teachers be doing during a lesson?)

R2. What are your beliefs about the learning of mathematics? (i.e., what should students be doing during a lesson?)

Self-reflection Journal 2: After first observation

R3. What were the strengths of your observation lesson?

R4. What were the weaknesses of your observation lesson?

R5. If another teacher were to teach the same lesson tomorrow, what suggestions would you give him or her?

Self-reflection Journal 3: After reading *Principles to Actions*

R6. Which of the Mathematics Teaching Practice(s) do you agree with most?

R7. Which of the Mathematics Teaching Practice(s) do you disagree with most?

R8. What aspects of your observation lesson supported the Mathematics Teaching Practices?

R9. What aspects of your observation lesson did not support the Mathematics Teaching Practices?

R10. What are your beliefs about the teaching of mathematics? (i.e., what should teachers be doing during a lesson?)

R11. What are your beliefs about the learning of mathematics? (i.e., what should students be doing during a lesson?)

R12. Have any of your views changed since the beginning of this study? Please describe.

Self-reflection Journal 4: After first research lesson

R13. What changes did you make to your lesson plan as a result of the discussion/sharing of lesson plans? Be specific and cite the source of change.

R14. In general, what did you learn by discussing this lesson?

R15. What aspects of the first research lesson supported the Mathematics Teaching Practices?

R16. What aspects of the first research lesson did not support the Mathematics Teaching Practices?

R17. How has observing this lesson changed your lesson plan?

R18. What did you learn by observing this lesson?

Self-reflection Journal 5: After second lesson study lesson

R19. What aspects of the first research lesson supported the Mathematics Teaching Practices?

R20. What aspects of the first research lesson did not support the Mathematics Teaching Practices?

R21. How has observing this lesson changed your lesson plan?

R22. What did you learn by observing this lesson?

Self-reflection Journal 6: After third lesson study lesson

R23. What aspects of the first research lesson supported the Mathematics Teaching Practices?

R24. What aspects of the first research lesson did not support the Mathematics Teaching Practices?

R25. How has observing this lesson changed your lesson plan?

R26. What did you learn by observing this lesson?

Self-reflection Journal 7: After final lesson observation

R27. What were the strengths of your observation lesson?

R28. What were the weaknesses of your observation lesson?

R29. If another teacher were to teach the same lesson tomorrow, what suggestions would you give him or her?

R30. What aspects of your final observation lesson supported the Mathematics Teaching Practices?

R31. What aspects of your final observation lesson did not support the Mathematics Teaching Practices?

R32. Was any part of the lesson influenced by the lesson study process? In other words, did you alter the lesson at all because of your experience in the lesson study?

APPENDIX E

Before Lesson Study Interview

B1. What are your views about the teaching and learning of mathematics?
a. In other words, how can students best learn mathematics?
b. How can teachers best teach mathematics?

B2. How are your beliefs enacted in your lessons?

B3. What does a typical lesson look like in your classroom?

B4. Have you read or heard about the Mathematics Teaching Practices released by NCTM in 2014?

B5. What does it mean to establish mathematics goals to focus learning?

B6. What does it mean to implement tasks that promote reasoning and problem solving?

B7. What does it mean to use and connect mathematical representations?

B8. What does it mean to facilitate meaningful mathematical discourse?

B9. What does it mean to pose purposeful questions?

B10. What does it mean to build procedural fluency from conceptual understanding?

B11. What does it mean to support productive struggle in learning mathematics?

B12. What does it mean to elicit and use evidence of student thinking?

APPENDIX F

After Lesson Study Interview

A1. What are your views about the teaching and learning of mathematics?
a. In other words, how can students best learn mathematics?
b. How can teachers best teach mathematics?

A2. Regarding your view of mathematics learning and teaching, what changes, if any, have you made through participating in this lesson study?
a. Why did you make these changes?

A3. What are your beliefs about the Mathematics Teaching Practices?
a. Have those changed at all through the lesson study?

A4. Regarding the implementation of the lessons in the lesson study, what were the major changes that were made?
a. Why and how were these changes made? Please explain in detail.

A5. Are you satisfied with the performance in the third teaching?
a. Do you have any suggestions for further improvement? Please give reasons and suggestions if any.

A6. What does it mean to establish mathematics goals to focus learning?

A7. What does it mean to implement tasks that promote reasoning and problem solving?

A8. What does it mean to use and connect mathematical representations?

A9. What does it mean to facilitate meaningful mathematical discourse?

A10. What does it mean to pose purposeful questions?

A11. What does it mean to build procedural fluency from conceptual understanding?

A12. What does it mean to support productive struggle in learning mathematics?

A13. What does it mean to elicit and use evidence of student thinking?

A14. Which part, if any, was most helpful in understanding the mathematics teaching practices? For example, reading the book, participating in the lesson study itself, or maybe more specifically, your discussions, reflecting?
   a. Implementation
   b. Perceptions (beliefs towards or about)

A15. If you were given an opportunity to share your experience with lesson study, what would you say?

A16. What are the main things that you are going to take away from this lesson study?
a. How will it impact your teaching?

APPENDIX G

Institutional Review Board Approval

IRB
INSTITUTIONAL REVIEW BOARD
Office of Research Compliance,
010A Sam Ingram Building,
2269 Middle Tennessee Blvd
Murfreesboro, TN 37129

PROTOCOL APPROVAL NOTICE

6/30/2015

Investigator(s): Kyle M. Prince (PI), Angela Barlow and Rongjin Huang
Investigator(s) Email: kmp3f@mtmail.mtsu.edu; angela.barlow@mtsu.edu; rongjin.huang@mtsu.edu
Department: Mathematics and Science Education
Protocol Title: "Learning Within Context: Exploring Lesson Study as an Aid in Enhancing Teachers' Understandings, Implementations, and Perceptions of the Mathematics Teaching Practices "
Protocol Number: 15-335

Dear Investigator(s),

The MTSU Institutional Review Board, or a representative of the IRB, has reviewed the research proposal identified above. The MTSU IRB or its representative has determined that the study poses minimal risk to participants and qualifies for an expedited review under 45 CFR 46.110 and 21 CFR 56.110. This approval is for one (1) year from the date of this letter **for 4 (FOUR) participants**. This protocol expires **7/5/2016**.

Any unanticipated harms to participants or adverse events must be reported to the Office of Compliance at (615) 494-8918 within 48 hours of the incident. Any change(s) to the protocol must be approved by the IRB. MTSU HRP defines "researcher" as anyone who works with data or has contact with participants. Anyone meeting this definition needs to be listed on the protocol and needs to complete the required training. New researchers can be amended to this protocol by submitting an Addendum request researchers to the Office of Compliance before they begin to work on the project.

Completion of this protocol MUST be notified to the Office of Compliance. A "completed research" refers to a protocol in which no further data collection or analysis is carried out. This protocol can be continued up to THREE years by submitting annual Progress Reports prior to expiration. Failure to request for continuation will automatically result in cancellation of this protocol and you will not be able to collect or use any new data.

All research materials must be retained by the PI or the faculty advisor (if the PI is a student) for at least three (3) years after study completion. Subsequently, the researcher may destroy the data in a manner that maintains confidentiality and anonymity. IRB reserves the right to modify, change or cancel the terms of this letter without prior notice. Be advised that IRB also reserves the right to inspect or audit your records if needed.

Sincerely,

Institutional Review Board
Middle Tennessee State University

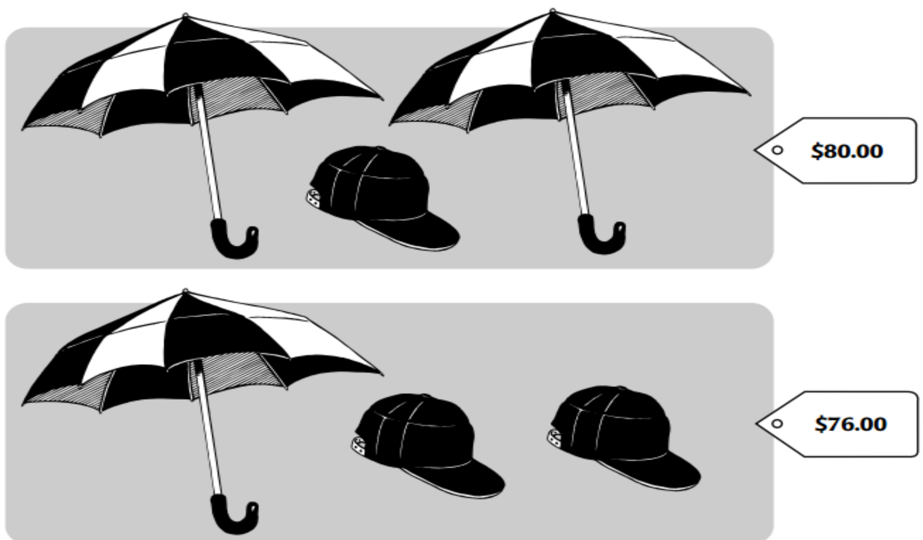
NOTE: All necessary forms can be obtained from www.mtsu.edu/irb.

APPENDIX H

Umbrellas and Hats Task

Name: ____ Date: ___________________ Period: _______

Umbrellas and Hats



What is the price of each umbrella and each hat? Explain your reasoning.

APPENDIX I

Fruits and Vegetables Task

Name: ____ Date: _____________________ Period: _________

Fruits and Vegetables

How many bananas are needed to make the third scale balance? Explain your reasoning.

APPENDIX J

Cross-Case Analysis Coding

First Iteration: Initial Codes from Embedded Cases		
RQ#1: Implementations?	RQ#2: Conceptions?	RQ#3: Perceptions?
1. Referencing and reflecting upon goals throughout lesson (MG) 3. Contextual Representations (BS, SM) 4. Mathematical discourse (SM) 5. Questions (BS, SM)	1. Referencing and reflecting upon goals throughout lesson (MG) 1. Situate goals within learning progression (SM) 1. Detail in goal setting (BS) 1. Goals guide instructional decisions (BS, MG) 3. Contextual Representation (BS, SM) 4. Mathematical Discourse (SM) 5. Questions – Conjectures, Explanation, justification (BS) 7. Persevere through problem solving (SM, MG) 8. Share student work with other students (MG)	1. Referencing and reflecting upon goals throughout lesson (MG) 1. Detail in goal setting (BS) 3. Contextual Representation (BS) 3. Connections between representations (BS) 4. Facilitate mathematical discourse (SM) 5. Questioning (SM, MG) 6. Conceptual then procedural (BS) 7. Persevere through problem solving (SM) 7. Learning from mistakes (BS)
Second Iteration: Themes Across Multiple Cases		
MTP 3 (BS, SM) MTP 5 (BS, SM)	MTP 1 (BS, MG, SM) MTP 3 (BS, SM) MTP 7 (MG, SM)	MTP 1 (BS, MG) MTP 5 (MG, SM) MTP 7 (BS, MG)

Note: BS = Britney Smyth, MG = Mark Gibson, SM = Sally Mills.