Implementing Reform-Oriented Statistics in the Middle Grades: Teacher Support Structures
by
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I would like to dedicate this dissertation study to my Mrs. Linda King. Had it not been for her passion for mathematics and belief in me, none of this would have been possible.

Thank you, Mrs. King, for believing in me.

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#### Abstract

With the Common Core State Standards for Mathematics (CCSSM) (CCSSI, 2010), statistics and probability have a more influential role in the middle grades curriculum than in the past. Specifically, sixth-grade now begins middle-school students' formal exposure to and experience with statistics. However, statistics is generally not a priority in teacher preparation programs leading to teachers' poor content knowledge in statistics. Also, some teachers do not feel prepared to teach statistics as described in the CCSSM and other reform-based documents. These prove to be barriers to reform-based instruction, and some have recommended using lessons created by statistics educators as way to address these barriers. Unfortunately, simply having these lessons is not enough to ensure that students develop a conceptual understanding of the topic. In addition, even if teachers have those lessons, there is limited research on how well instruction aligns with curriculum expectations when it is implemented in the classroom. Therefore, this descriptive case study explored the implementation fidelity of a reform-oriented statistics lesson in a sixth-grade classroom. A secondary component of the study was to explore what support structures the teacher identified as necessary for implementation.


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## CHAPTER ONE: INTRODUCTION

## Introduction

People are exposed to statistical information on a daily basis, and being a critical consumer of this information is extremely important (Franklin \& Mewborn, 2008; Kader \& Mamer, 2008). Hovermill, Beaudrie, and Boschmans (2014) stated that being able to reason about statistics and having a statistically literate society would be beneficial to those in that society. However, "sound statistical reasoning skills take time to develop and cannot be honed in a single general-purpose statistics course" (Franklin \& Kader, 2010, Abstract, para. 1). Statistical reasoning skills need to be emphasized throughout elementary school, middle school, high school, and post-secondary education (Franklin \& Kader, 2010) so that the need for a statistically literate population (Sorto, 2011) can be met. Therefore, statistics education should begin early and happen frequently for all students so that they can acquire "statistical reasoning to intelligently cope with the requirements of citizenship, employment, and family and to be prepared for a healthy and productive" (Franklin et al., 2007, p. 1) life.

Consequently, statistics education is currently part of a reform effort, which began over 30 years ago. This reform is happening at the pre-K-12 level (Franklin et al., 2007); National Council of Teachers of Mathematics [NCTM], 1989, 2000) in which statistics is now a fundamental part of the curriculum. We also see the reform at the collegiate level (i.e., introductory statistics) (Garfield, Hogg, Schau, \& Whittinghill, 2002). This effort is not limited to the United States (Jacobbe \& Horton, 2012), but rather is across the globe as countries recognize the "importance of statistics in the education of its citizens" (Peck,

Kader, \& Franklin, 2008, p. 1). Efforts to improve statistics education have led to the development of many reform documents, articles, and research papers demonstrating the need for and importance of quality statistics education in the K-12 system. Some the more influential U.S. documents include: Principles and Standards for School Mathematics (NCTM, 2000), The Mathematical Education of Teachers II (MET2) (Conference Board of the Mathematical Sciences [CBMS], 2012), the American Statistical Association's Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K-12 Curriculum Framework (Franklin et al., 2007), and the Statistics Education of Teachers (SET) (Franklin et al., 2015), the companion document to MET2.

During this time of reform in statistics education, 44 states in the U.S. have adopted the Common Core State Standards for Mathematics (CCSSM) (Common Core State Standards Initiative [CCSSI], 2010), a set of standards that begins statistics education explicitly in the middle grades. Upon examination of the CCSSM, it is obvious that statistics and probability have a significant role in sixth through twelfth grade mathematics (Franklin, 2013). Specifically, "the middle grades bear a great deal of the load for developing students' statistical thinking in CCSSM" (Groth \& Bargagliotti, 2012, p. 44), and it is in this grade range that students are expected to develop an understanding of statistics that allows them to succeed in later statistics courses (Zbiek, Jacobbe, Wilson, \& Kader, 2013).

The K-12 statistics standards in the CCSSM underscore conceptual understanding (Franklin, 2013), and the middle grades are when students first experience statistics. Similarly, the GAISE framework describes statistics education conceptually. Specifically,
statistics education is described as a "two-dimensional framework model, with one dimension defined by the statistical problem-solving [sic] process" (Franklin, 2013, p. 6) and the second dimension composed of three developmental levels roughly corresponding to elementary, middle, and high school. The statistical problem-solving process described in the GAISE framework supports the content in the CCSSM. This four-component investigative process allows students to engage in statistical problem solving (Zbiek et al., 2013) and supports students in developing their statistical thinking. Research has shown students engaging in classrooms designed with the GAISE framework in mind perform better in terms of statistical reasoning and self-efficacy (Olani, Hoekstra, Harskamp, \& van der Werf, 2010).

Unfortunately, there are two primary obstacles towards meeting statistics instruction as envisioned by the CCSSM and GAISE. First, K-12 teachers have acknowledged a deficit in their content knowledge that prohibits them from efficiently answering students' questions, which might be due to the second obstacle, namely that statistics has not been historically a required part of the teacher education curriculum (Franklin, 2013; Franklin \& Kader, 2010). Many universities do not require nor offer a statistics course for pre-service teachers (Harrell-Williams, Sorto, Pierce, Lesser, \& Murphy, 2014). Furthermore, Metz (2010) reported, "Many elementary school teachers have not had the opportunity to become statistically literate themselves. In addition, they are not equipped pedagogically to provide effective instruction in statistics" (p. 1).

Although statistics educators emphasize the need for statistics education courses for pre-service teachers (Jacobbe, 2007; Kader \& Perry, 2002), these courses are unable to affect the quality of instruction for in-service teachers (Jacobbe \& Horton, 2012).

Further, researchers have identified a lack of understanding of statistical concepts (Callingham, 1997; Groth \& Bergner, 2006; Jacobbe, 2008; Leavy \& O’Loughlin, 2006; Makar \& Confrey, 2004; Mickelson \& Heaton, 2004) and a feeling of unpreparedness to teach statistics in teachers (Begg \& Edwards, 1999; Greer \& Ritson, 1994). This lack of understanding leads to misconceptions in both teachers' and students' thinking (Carnel, 1997). As a result, teachers often "focus their instruction on the procedural aspects of probability and statistics" (Meletiou-Mavrotheris \& Mavrotheris, 2007, p. 114) instead of conceptual aspects (Nicholson \& Darnton, 2003).

In a study conducted by Groth (2008), the researcher used a group of pre- and inservice teachers in a graduate level course to determine guidance needed for "statistics educators seeking to make the implementation of the Pre-K-12 GAISE guidelines a reality" (p.33). These teachers identified five primary categories of guidance, one of which addressed teacher support structures. The teachers identified support structures as supplementary materials that improved teachers' content knowledge. Unfortunately, this study was limited by the teachers' lack of experience implementing the GAISE recommendations in a classroom. Hence, an important concern of statistics educators is how best to support teachers in teaching statistics (CBMS, 2012) in a way that addresses both the GAISE framework and CCSSM.

## Problem Statement

Many K-12 mathematics teachers studied mathematics or mathematics education in college (Harrell-Williams et al., 2014) and have limited or no experience in statistics (Foley, Khoshaim, Alsaeed, \& Nihan, 2012). Unfortunately, those teachers who completed a statistics course likely received traditional instruction and are unlikely to
know that their statistics knowledge is limited (Jacobbe \& Horton, 2012; Stohl, 2005). Jacobbe and Horton (2012) stated that many in-service teachers may not be aware of their statistical misconceptions that would be revealed through experiences in a conceptuallydriven statistics course. This issue has immediate concerns for how in-service teachers are teaching statistics.

The American Statistical Association and the NCTM (2013) emphasized the importance of professional development for teachers specifically in statistics that models appropriate pedagogies for teaching statistics, as well as coursework for pre-service teachers that focuses on conceptual statistical knowledge and the key components of statistical problem solving. Unfortunately, in a study by Jacobbe and Horton (2012), the results revealed that after receiving limited professional development on statistics, teachers still harbored statistical misconceptions. With the release of the SET (Franklin et al., 2015), it is important that in-service teacher professional development "respond appropriately by expanding and adjusting curricula to address the recommendations put forth in the report" (Bargagliotti, 2014, para. 15).

Bargagliotti, Jacobbe, and Webb (2014) recommended that teachers should use "K-12 statistics lessons that have been reviewed and written by statistics education experts" (p. 11) and that this would provide, initially, appropriate teacher training for statistical content. However, no mention in this article was made about the fidelity of implementation for these lessons, which, according to O'Donnell's (2008) summary of the literature on implementation fidelity, can affect student learning outcomes significantly (i.e., high implementation fidelity was statistically significantly correlated with higher student learning outcomes). Similarly, the NCTM (2014) stated that having
these lessons is not enough to ensure "meaningful, effective, and connected lesson sequences" (p. 71). This statement aligned with findings from George, Hall, and Uchiyama (2000) that there is a significant difference between what we call researchbased curriculum and "research-verified" (p. 8). That is, curriculum that is research-based does not necessarily ensure that student achievement will take place. What would expose student learning potential are research-verified curriculum innovations that are studied at the classroom level.

It is apparent the need for in-service teachers' consistent exposure to best teaching practices such as the Mathematics Teaching Practices identified in NCTM (2014) to assist in teacher's instruction of reform-oriented (i.e., conceptually driven) statistics lessons. These teachers also need exposure to reform-oriented professional development in statistics so that students can meet the expectations of the CCSSM (CCSSI, 2010) and the GAISE framework.

## Purpose of Study

Given in-service teachers' lack of or limited preparation in statistics (Franklin, 2013) along with calls in reform-based documents for teachers to teach for deeper conceptual understanding (Franklin et al., 2007; NCTM, 2000, 2014), the purpose of this study was to examine a middle-grades teacher's implementation of a reform-oriented statistics unit, identifying support structures needed for successful implementation. Specifically, the fidelity of implementation of this unit was observed and support structures from this observation were identified.

The central research question for this study was: How does a sixth-grade teacher implement a reform-oriented statistics unit? Given this implementation, the secondary
research question was: What support structures will the participant identify as needed for the unit to be implemented with fidelity?

## Significance of the Study

The study served to inform mathematics and statistics teacher educators, administrators, and teachers as to how to prepare practicing middle-grades teachers to teach statistics so that students meet the expectations set forth in the CCSSM and GAISE framework. Considering the importance of statistics in the CCSSM and the need for teacher professional development for delivering the statistics in the CCSSM (Franklin, 2013), this study was timely and the results hold the potential to support in-service teachers in their statistics instruction in middle-grades as envisioned by the CCSSM.

## Definitions

This section defines terms used throughout the document so as to provide clarity to the reader.

## Reform-oriented Statistics

In this document, the term reform-oriented statistics is based upon the standards and principles as elaborated in the Principles and Standards for School Mathematics (NCTM ,2000) and Principles to Actions: Ensuring Mathematical Success for All (NCTM, 2014). Reform-oriented statistics requires students to learn statistics "with understanding, actively building new knowledge from experience and prior knowledge" (NCTM, 2000, p. 20). Specifically, the reform-oriented statistics unit implemented by the teacher in this study included "tasks that promote[d] reasoning and problem solving" (NCTM, 2014, p. 10), engaged students in statistical discourse, introduced students to multiple representations and their connections, encouraged students to struggle with the
statistics, elicited student thinking, included purposeful questions, and used students' conceptual understanding to build their procedural fluency (NCTM, 2014).

## Conceptual Understanding and Procedural Fluency

According to NCTM (2014), for students to develop procedural fluency, they must have a conceptual understanding of mathematics. The National Research Council (2001) defined these terms (i.e., conceptual understanding and procedural fluency) in a ways that are used in this document. Conceptual understanding requires that students have the "comprehension of mathematical concepts, operations, and relations" (p. 116). In contrast, procedural fluency involves "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (p. 116).

## Implementation Fidelity

Implementation fidelity is defined as how well the teacher's implementation of the curriculum aligns with the intended implementation (Dietz \& Holstein, 2009). In this study, implementation fidelity included two components: deviations from the intended curriculum and alignment of the enacted curriculum with the reform-oriented philosophy.

## Pre-service Teachers and In-service Teachers

Pre-service teachers are defined as those in their teacher preparation program who have not yet begun their teaching career. In-service teachers are defined as those with teacher certification who are currently teaching.

## Traditional Instruction and Reform-based Instruction

Traditional instruction in mathematics is defined as instruction that emphasizes procedures and rules through the use of lecture. In this type of classroom, the mathematical authority rests with the teacher, and the classroom is described as teacher-
centered. Alternatively, reform-based instruction in mathematics is defined as instruction that emphasizes conceptual understanding. In this classroom, students hold the mathematical authority, and the classroom is described as student-centered (Wilson \& Lloyd, 2000).

## Chapter Summary

The reform efforts in mathematics have been prevalent since the NCTM's release of Curriculum and Evaluation Standards for School Mathematics in 1989. Although this early document and subsequent documents have highlighted the importance of statistics in the mathematics curriculum, issues are still prevalent in statistics education. Three examples of these issues include the need for professional development in statistics education (Franklin, 2013), the prevalence of statistical misconceptions in teachers (Callingham, 1997; Groth \& Bergner, 2006; Jacobbe, 2008; Leavy \& O’Loughlin, 2006; Makar \& Confrey, 2004; Mickelson \& Heaton, 2004), and the lack of appropriate statistical content knowledge that teachers need to address student questions in statistics (Franklin \& Kader, 2010).

Although these issues are present, teachers are expected to teach statistics for conceptual understanding so that students are able to meet the recommendations set forth in the CCSSM (CCSSI, 2010) and GAISE (Franklin et al., 2007). Standards-based documents, such as Principles to Actions (NCTM, 2014), identify reform-based teaching practices that, if employed by teachers, should support students' successes in meeting these recommendations. However, the additional support structures that teachers need when implementing these conceptually-driven statistics lessons in their classroom is
unclear. Therefore, this study sought to illuminate the support needed by a middle grades teachers to successfully implement reform-oriented statistics in the sixth grade.

## CHAPTER TWO: LITERATURE REVIEW

## Introduction

Teachers are expected to teach statistics so that students develop a conceptual understanding of the content, yet they are faced with many challenges (ASA \& NCTM, 2013). These challenges can stem from the teachers having limited or perhaps even no experience in statistics (Foley et al., 2012); a background in statistics taught traditionally focusing on procedures instead of conceptual understanding (Garfield, 2002); or the lack of opportunities for professional development in statistics that promote statistical thinking instead of mathematical thinking (Jacobbe \& Horton, 2012). Recommendations have been made to help teachers who are being charged with teaching statistics within their mathematics classrooms, including using K-12 lessons that have been written and reviewed by experts in the field (Bargagliotti et al., 2014). Unfortunately, simply having these lessons does not ensure that lessons are taught in a meaningful way (NCTM, 2014). Therefore, the purpose of this study was to examine how a middle-grades teacher implemented a reform-oriented statistics unit within her sixth-grade mathematics classroom, identifying support structures that she recognized as necessary for implementation. To better understand the phenomenon of implementation and potential supports for high fidelity, the literature on mathematical and statistical content and pedagogical knowledge, mathematical perspectives, and implementation fidelity will be reviewed in this chapter.

## Mathematics Teachers' Knowledge

In regards to the knowledge that teachers need in their classrooms, Shulman (1986) classified types of knowledge specific to teachers that are needed to be successful
in the classroom. Referred to as pedagogical content knowledge (PCK), he described this type of knowledge as a blend of pedagogy and content in a way that is specific to the teacher's content. Inspired by this research, Ball, Thames, and Phelps (2008) expanded upon this idea by exploring the content knowledge specific for mathematics teachers and created the current mathematical knowledge for teaching framework.

Although there is no doubt that the disciplines of statistics and mathematics complement each other, Franklin et al. (2005) stated that statistics could be distinguished from mathematics because of the role of variability and the importance of context in statistics. Furthermore, Batanero and Diaz (2010) indicated that teachers' statistical knowledge has the potential to affect their instructional choices in their classrooms. Recognizing this distinction between statistics and mathematics and the importance of teachers' statistical knowledge on student results, Groth (2007) utilized the mathematical knowledge for teaching framework (Ball et al., 2008) to conceptualize a statistical knowledge for teaching framework. In this section, results from these and similar studies on all aspects of teachers' knowledge needed for teaching, including content and pedagogical knowledge, are discussed.

## Mathematical Knowledge for Teaching

Recognizing a lack of a synthesized theoretical framework for PCK, Ball et al. (2008) explored the role of content knowledge for teaching within the discipline of mathematics. Ball and colleagues used work from two projects to further understand the role of mathematical knowledge necessary for teaching. The first project "focused on the work teachers do in teaching mathematics" (p. 390), and the second project used results from this first project to develop surveys for the content knowledge for teaching
mathematics. Ball and her colleagues used the results from these two projects, which indicated that teachers' mathematical knowledge is a positive predictor for student achievement, to develop a theoretical framework for the "domains of mathematical knowledge for teaching" (p. 403). This framework differed from Shulman's (1986) framework in that it divided two of his original domains, subject matter knowledge and PCK, into six new domains: common content knowledge, horizon content knowledge, specialized content knowledge, knowledge of content and curriculum, knowledge of content and teaching, and knowledge of content and students. These domains are described in Table 1.

Table 1
Mathematical Knowledge for Teaching Framework

| Domain | Description | Example |
| :--- | :--- | :--- |
| Common Content <br> Knowledge | Mathematical knowledge <br> known commonly among <br> most people | Knowing that $1 / 2$ of 4 is $2 ;$ <br> Knowing how to multiply |
| Horizon Content | Knowledge of how content <br> relates across entire <br> mathematics curriculum | Knowing how <br> understanding linear <br> functions prepares students <br> for quadratic functions |
| Specialized Content | Mathematical knowledge <br> unique to mathematics <br> teaching | Knowing how to represent <br> data in a hat plot; Knowing |
| Knowledge | Knowledge of mathematics <br> and knowledge of <br> curriculum | Knowing how the content <br> progresses over time and |
| Knowledge of Content and |  |  |
| Curriculum | what curriculum resources <br> to use to teach particular <br> concepts |  |
| Knowledge of Content and | Knowledge of mathematics <br> and knowledge of teaching | Knowing which <br> representations to use and <br> Teaching to sequence them for |
|  |  | multiplication to deepen <br> understanding |
| Knowledge of Content and | Knowledge of mathematics <br> and knowledge of students | Knowing common mistakes <br> that students might make <br> when performing long |
| Students |  |  |

In the following two sections, I summarize the literature on mathematical knowledge for teaching and its effect on both student achievement and instructional practices in the classroom.

Mathematical knowledge for teaching and student achievement. In an earlier study, Hill, Rowan, and Ball (2005) examined the relationship between teachers' mathematical knowledge for teaching and students' achievement gains in mathematics.

The researchers collected data from both first and third grade students and teachers in 115
elementary schools over the course of two years. All but 26 of the schools were part of a school-wide reform program. The 26 schools not part of a program were used as a comparison group. The demographics of students for the schools overrepresented highpoverty elementary students; however, the teacher demographics were fairly representative of the typical elementary teacher (i.e., majority Caucasian, around 90\% certified, and over 12 years of experience). To better understand the relationship between students' mathematics achievement and teachers' mathematical knowledge for teaching, the researchers collected student assessments, parent interviews, annual questionnaires for the teachers, and a teacher log that identified the time spent on daily mathematics instruction, daily mathematics content, and the daily instructional practices for delivering that topic.

Using a linear-mixed methodology, the researchers reported three key findings. First, surprisingly, the analysis revealed that neither teacher certification nor increased experiences in mathematics content or methods courses during their teacher certification programs guaranteed "strong content knowledge for teaching mathematics" (p. 393) among the teachers in this study. Second, the analysis revealed that teachers' mathematical content knowledge was significantly related to students' mathematics achievement gains in both first and third grade. This result held even after controlling for teacher and student background information. Finally, the researchers found that first graders improved more in their mathematics achievement gains over the course of the study than their third grade counterparts. This verified the importance of mathematical content knowledge for teaching at the early elementary levels.

Similarly, Baumert et al. (2010) examined the relationship between mathematical knowledge for teaching and student achievement. In this study, the researchers explored the distinction between teachers' PCK and content knowledge and how those two distinct domains contributed to student achievement and instructional quality. The study took place over one year in Germany with Grade 10 students and teachers and was part of a study that took place at the Max Planck Institute for Human Development. To answer the research questions, the researchers collected data from both the students and the teachers. Student data included mathematics achievement scores and survey data that revealed background information and descriptions of the mathematics instruction in their classes. Teacher data included an assessment of their professional knowledge (i.e., content knowledge and pedagogical content knowledge) and a survey revealing background information, motivations, beliefs, perceptions of instruction, and self-regulation skills. A total of 181 teachers and 4,353 students participated in the study.

To analyze this data, the researchers employed a multi-level structural equation model to determine the relationships between content knowledge, PCK, and student outcomes. The analysis revealed that "the relationship between PCK and mathematics achievement was linear" (p. 162), indicating that PCK was significantly related to student progress. Examining this relationship further, the researchers found that PCK also influenced "the cognitive level, curricular level, and learning support dimensions of instructional quality" (p. 163). In this study, curricular level referred to how well the tasks aligned with the curriculum prescribed. The researchers also found that the students in the lowest socioeconomic status were most impacted by teachers with higher PCK. Further, the model revealed that PCK had greater power in explaining student outcomes
than content knowledge. The only aspect of instruction that was significantly related to content knowledge was the curricular level of the tasks (i.e., alignment to the curriculum).

Similar to Baumert and colleagues (2010), Rockoff, Jacob, Kane, and Staiger (2008) also explored how certain teacher factors could be used to predict students' mathematics achievement including mathematical knowledge for teaching. In this study, the researchers assessed 418 teachers in New York City who taught mathematics for fourth through eighth grade on several "non-traditional predictors of effectiveness" (Abstract, para. 1) including self-efficacy, mathematical knowledge for teaching, cognitive ability, personality traits, and scores from a teacher selection instrument used in New York City to hire teachers. Data were also provided from the district on teacher and student demographics, teacher certification route, teacher experience, teacher attrition, pupil-teacher ratio, grade composition, the amount of students on free or reduced lunch, the number of special education students and English Language Learners, and Title I eligibility of schools.

Similar to the previous studies described, the researchers also found that teachers' mathematical knowledge for teaching was a statistically significant predictor for students' mathematics achievement. Data analysis revealed a significant relationship between students' mathematics achievement and teachers' self-efficacy and cognitive ability. The analysis also demonstrated a modest relationship "between student achievement and several non-traditional predictors of teacher effectiveness" (Introduction, para. 5) including the teaching selection instrument.

The studies in this section all examined mathematical content knowledge including PCK and its relationship to students' mathematics achievement. However, given the unique factors examined in each of the studies, the researchers found different results pertaining to this relationship. First, Hill et al. (2005) found that teachers' mathematical knowledge for teaching was more significant for students' mathematics achievement in the first grade than the third grade. Second, Baumert et al. (2010) found that PCK was actually a more significant predictor for students' mathematical achievement than content knowledge and that teachers' PCK significantly influenced how well the in-class tasks aligned with the prescribed curriculum. Finally, Rockoff et al. (2008) found that teachers' self-efficacy and cognitive ability were also significant predictors for students' mathematics achievement. Despite these differences, similar results were found across the studies. The results from Hill et al. (2005) revealed the importance of teachers' mathematical knowledge for teaching as early as first grade for mathematics students. Similarly, Baumert et al. (2010) and Rockoff et al. (2008) found that teachers' mathematical knowledge for teaching was a significant predictor for students in high school (specifically, Grade 10) and fourth through eighth grade, respectively. Collectively, the results from the studies revealed the importance of mathematical knowledge for teaching in improving students' scores on mathematics assessments.

## Mathematical knowledge for teaching and instructional practices. Other

 research on mathematical knowledge for teaching has examined how this construct relates to the instructional practices that teachers employ in the classroom. As an example, Copur-Gencturk (2015) explored the relationship of mathematical knowledgefor teaching in 21 teachers and their mathematics instruction. The researcher followed 21 teachers, all K-8 teachers in a master's program, over the course of three years. During this time, the researcher repeatedly assessed the participants' mathematical knowledge for teaching and also collected qualitative data in the form of classroom observations and interviews. For this study, Copur-Gencturk focused specifically on the two domains of common content knowledge and specialized content knowledge, combining the two into a new domain called content knowledge for teaching. The master's program in which the participants were enrolled was designed to improve teachers' content knowledge and PCK for both mathematics and science. In terms of demographics, all but one participant was female, three participants were African American, and participants taught grades within the range of first to seventh. The researcher assessed all participants on their content knowledge for teaching; however, only six teachers were chosen for qualitative data collection.

To analyze content knowledge for teaching and the participants' instruction, Copur-Gencturk used a multilevel multivariate growth modeling method. Results from this quantitative analysis revealed that all participants began with room for improvement in terms of their content knowledge for teaching. Over the course of the study, the participants' content knowledge for teaching improved significantly, and Copur-Gencturk further found that these gains were related to the "quality of their inquiry-based lesson[s], mathematical sense-making agenda[s], and classroom climate" (p. 297). That is, as participants' content knowledge for teaching improved so did their ability to implement mathematics lessons that focused on problem solving and more closely aligned with inquiry-based practices. Of the three instructional practices (i.e., inquiry-based lessons,
mathematical sense-making agenda, and classroom climate), the weakest relationship was found between content knowledge for teaching and a mathematical sense-making agenda. That is, their content knowledge for teaching did affect their mathematical sense-making agenda, but not as significantly as it affected their inquiry-based lessons or classroom climate. Also, the participants' number of years of teaching experience was not related significantly to the instructional practices employed.

To better understand these relationships between content knowledge for teaching and the three instructional practices, Copur-Gencturk analyzed the qualitative data including classroom observations and interviews with the participants. In terms of the instructional practice of using inquiry-based lessons, the researcher found that those participants with the highest gains in content knowledge for teaching also showed the most aligned lessons in terms of the inquiry-based philosophy. These participants focused their lessons more on developing conceptual understanding than procedural fluency. Alternatively, those participants who had the lowest gains in content knowledge for teaching showed only marginal gains in creating inquiry-based mathematics classrooms. Although these participants allowed their students to spend more time in groups, the focus of the lessons remained on procedural fluency.

Similarly, in terms of the mathematical sense-making agenda instructional practice, those participants with the highest gains in content knowledge for teaching also demonstrated classroom practices that supported students in making sense of the mathematics. These participants encouraged their students to critique each other's work, required students to justify their thinking, and presented mathematical ideas more accurately and clearly. In contrast, those participants with the lowest gains in content
knowledge for teaching showed limited gains in developing mathematical sense making within their students. These participants failed to support students in making connections between mathematical ideas and did not articulate mathematical ideas clearly for students.

Finally, in terms of the classroom climate, Copur-Gencturk (2015) found that all participants demonstrated more respect for students' questions and thoughts. All participants except one encouraged students regularly to participate in class, and only three of the teachers observed made a change to their pacing (i.e., allowed students more time to work on tasks in class).

Similarly, another study conducted earlier also looked at the relationship between mathematical content knowledge and instructional practices. In 2013, Galant looked at how mathematical knowledge for teaching was related to how teachers selected and sequenced instructional tasks within the mathematics classroom. Specifically, the researcher sought to understand the specific aspects of mathematical knowledge for teaching that teachers used to make decisions about tasks. As part of this study, 46 third grade teachers in South Africa from 14 different schools participated in hour-long interviews. During the interviews, participants were presented with two instructional tasks from their national curriculum documents. Both tasks involved the concept of multiplication; however, one was for second grade students and the other was for third grade students. The connection between the two tasks was that they segued from repeated addition to multiplication. Participants in the study were unaware of the grade level for each of the tasks. In the interview, the participants reviewed both tasks and then justified how they would select and sequence the two tasks in their classrooms. The researcher
also probed the participants to gain a better understanding of the connections between the mathematical concepts within the tasks that the participants might have used in making their decisions.

The researcher found that many of the participants selected one task before another based upon the illustrations within the tasks. In doing so, some participants felt that one task was more contextual and less abstract than the other which resulted in their choice to sequence that task first. In contrast, other participants made similar remarks about the second task. Further, the analysis revealed that some participants did not view the tasks as entailing multiplication at all, but instead focused on counting despite the illustrations depicting arrays and the inclusion of the multiplication symbol in parts of one task. Only seven of the 46 participants explicitly recognized the connection between the two tasks as being repeated addition as a segue to multiplication. Overall, Galant reported that participants' weaknesses in their mathematical knowledge for teaching influenced how they selected and sequenced tasks. Specifically, the researcher noted that the participants demonstrated a poor "understanding of progression and development of mathematical concepts and processes, in this instance, multiplication" (p. 46).

The studies in this section examined the relationship between mathematical knowledge for teaching and instructional practices in the classroom. Copur-Gencturk (2015) found that as teachers' content knowledge improved so did their ability to implement mathematical tasks focused on problem solving and sense making. Similarly, Galant (2013) found that third grade teachers' mathematical knowledge for teaching affected how they selected and sequenced tasks for their classrooms. These studies
demonstrated how improved mathematical knowledge for teaching supports instructional strategies that align with reform-based expectations.

## Statistical Knowledge for Teaching

Similar to the mathematical knowledge for teaching framework developed by Ball et al. (2008), Groth (2007) described a statistical knowledge for teaching (SKT) framework based upon the foundational work for the MKT framework (e.g., Hill \& Ball, 2004; Hill et al., 2005; Hill et al., 2004). The original SKT framework included all of the same components of the mathematical knowledge for teaching framework, and Groth (2007) stated that this was not surprising given that statistics utilizes mathematics. However, later Groth (2013) re-conceptualized the SKT framework, adding key developmental understandings and pedagogically powerful ideas. According to Groth, key developmental understandings are "cognitive landmarks in the learning of fundamental ideas needed to understand content" (p. 123). Pedagogically powerful ideas were described as ideas that form from changing key developmental understandings "into ideas that facilitate students' learning of the "key developmental understandings" (p. 123). After Groth's first conception of the SKT framework in 2007, he conducted several studies, three of which are described in this section, further exploring the concept of SKT. The first two studies conducted by Groth examined statistical subject matter and pedagogical content knowledge and are described in the first section. The third examined support structures and is described in the second section. Similar research from other authors on the subject of SKT is included in the section as well.

Subject matter and pedagogical content knowledge. In the first of his three studies, Groth (2012) explored how pre-service teachers enrolled in a course designed to
improve SKT did so through the use of writing prompts. In this course, the PSTs were assigned thirteen reading assignments throughout the semester that included the writing prompts. To understand the PSTs' growth in their SKT, Groth assessed participants' writing using the Structure of the Observed Learning Outcome taxonomy (Biggs \& Collis, 1982; Biggs \& Tang, 2007). He also examined their statistical knowledge using a survey developed by the LMT project and the Comprehensive Assessment of Outcomes in a First Statistics Course (CAOS) test (delMas, Garfield, Ooms, \& Chance, 2007). Although the taxonomy was useful in providing a rubric for the writing prompts, the survey and CAOS test were better predictors for how the PSTs' SKT changed over the course of the semester. Results from the CAOS test indicated that the PSTs achieved the same level of conceptual understanding as those in a traditional introductory statistics course. Further, the survey indicated that the PSTs "gained statistical knowledge specifically required for teaching" (p.33). These results indicated that focusing on conceptual understanding and SKT within a statistics course for teachers allowed the participants to improve within both areas.

In the second study examining the concept of SKT, Groth and Bergner (2013) explored "a model for mapping cognitive structures related to content knowledge for teaching" (p. 247). In this study, 31 pre-service elementary and middle school teachers enrolled in a course designed to improve their PCK and subject matter knowledge for elementary level "nominal categorical data analysis" (p. 251). To gain a better understanding of the PSTs' SKT, the participants responded to writing prompts designed to expose their thinking about the mathematics and statistics within the prompts. PSTs also analyzed student work containing errors on the topic of nominal categorical data.

Groth and Bergner created node-link diagrams to reveal the conceptions and misconceptions held by each PST. The researchers reported five key results regarding the PSTs' knowledge for teaching the concept of nominal categorical data. First, Groth and Bergner found that ten of the 31 participants who had most of the expected knowledge on the content "did not [extract] appropriate ideas for teaching from the assigned articles to help students confront and revise errors in thinking" (p. 260). This resulted in these participants identifying direct instruction as a way to address the student errors instead of allowing the students to realize and correct their own errors. Second, seven of the participants had missing elements of basic knowledge regarding nominal categorical data. This resulted in the PSTs providing weaker "responses to PCK-related tasks" (p. 258) when they were analyzing student errors. Third, five participants held conflicting ideas regarding the content. This resulted in the PSTs hypothesizing potentially misleading ways to address students' errors. Fourth, four participants held both missing and conflicting elements of knowledge about nominal categorical data. Not surprisingly and as seen with the previous two findings, "participants were not able to apply incompletely grounded declarative knowledge to the analysis and resolution of student errors" (p. 260). Finally, five of the 31 participants held all expected knowledge about nominal categorical data, as defined by Groth and Bergner prior to the study. As an example, one of these five participants revealed that due to her understanding of the content, she was able to analyze the students' errors and hypothesize ways to address their misconceptions without direct instruction. Taken collectively, these results demonstrated how teachers with strong content knowledge are better able to identify and address student misconceptions without reverting to direct instruction.

Studies examining SKT are not limited to the work of Groth. These studies, however, have not consistently utilized the term SKT. Rather, those researchers refer to subject matter and PCK for statistics teachers. The first of these, Reston and Bersales (2008) conducted focus groups, as part of a larger study, with introductory statistics teachers at the collegiate level in the Philippines to determine "their instructional goals and classroom practices in light of the global reform movements that focus on statistical literacy" (Methods, para. 1). The overall purpose of the project was to determine the training and preparation of current statistics instructors, the activities that have been conducted to support statistics instructors, the challenges to the reform efforts, and the efforts needed to ensure "more effective institutionalization of these reform efforts" (Introduction, para. 6). The results from the focus groups revealed that some of the 44 participants had statistical misconceptions in sampling and inferential statistics procedures. Also, the researchers found that the participants lacked "the skill to design activities geared towards developing statistical literacy among students" (Results and Discussion, para. 2) and typically assessed their students with traditional, procedural based exams. Both of these findings indicated that the participants lacked the appropriate subject matter knowledge and PCK needed to teach statistics and assess their students effectively. Specifically, the participants were unable to assess and teach their students in ways that aligned with reform-based efforts and documents.

Similarly, Begg and Edwards (1999) explored subject matter knowledge of teachers, specifically K-12 mathematics teachers. This study examined 22 in-service and 12 pre-service teachers in New Zealand and their ideas of statistics. The ideas explored included teachers' beliefs about statistics and the teaching of statistics, statistical content
knowledge, and knowledge of teaching statistics. The researchers found that "a number of the teachers held an instrumental view of statistics" (p. 2). Although these authors did not describe their interpretation of an instrumental view of statistics, according to Skemp (1976), instrumental mathematics involves the learning of "rules without reason" (p. 21) as contrasted with relational mathematics in which students learn how to use a method and also why the method works (i.e., conceptual understanding). The quote from Begg and Edwards seems to imply then that teachers with an instrumental view of statistics viewed the subject procedurally instead of conceptually. The researchers explored the participants' content knowledge further and found that the participants specifically held a poor understanding of probability and measures of central tendency and viewed graphs "as communicative tools" (p. 5). Although these results were not encouraging, the researchers noted that the participants identified having had little statistics in their education and also identified statistics professional development as a low priority in comparison to other forms of professional development. These findings indicated that teachers' content knowledge was poor in statistics, and they had yet to recognize the importance of professional development and further support in statistics education.

In another study examining subject matter knowledge, Callingham (1997) examined teachers' understanding of the average in light of the Structure of Observed Learning Outcomes Taxonomy as developed by Biggs and Collis (1991). This taxonomy consists of "five modes of thinking: sensori-motor, ikonic, concrete symbolic, formal and post-formal" (p. 206). These modes of thinking become progressively more abstract and develop as one ages. In this study, Callingham (1997) surveyed 136 pre- and in-service teachers who participated in a compulsory mathematics professional development. The
survey consisted of four items designed to assess the participants' understanding of average. The results from this survey indicated that some of the participants demonstrated thinking within several of the modes described above. That is, the participants "when faced with a more difficult problem . . . moved from responses in earlier modes . . . to those demonstrating higher levels of understanding" (p. 221). Callingham stated that teachers need to experience activities in professional development that facilitates thinking among all of the five modes. This can be accomplished, according to Callingham, by presenting content in statistics with multiple representations, encouraging teachers to provide explanations and justifications for their work, and having teachers reflect on their thinking processes. Despite the age of this study, the practices recommended by Callingham align with the reform-oriented practices identified in CCSSM (CCSSI, 2010), GAISE (Franklin et al., 2007), and NCTM (2000, 2014).

The studies in this section examined SKT, specifically the statistical subject matter knowledge and PCK of pre-service, in-service, and collegiate teachers. Each of the studies revealed insight into of the relationship between the two domains of knowledge. Groth (2008) found that PSTs in a course focusing on improving SKT were able to meet the same conceptual understanding of statistics as those in a traditional statistics course while also improving their PCK. Related to the idea of teachers' PCK, Groth and Bergner (2013) found that participants in their study with poor content knowledge were more likely to address student misconceptions with direct instruction. Similarly, Reston and Bersales (2008) found that introductory statistics teachers at the collegiate level with statistical misconceptions lacked the appropriate subject matter knowledge and PCK to be able to assess students' statistical reasoning. Begg and Edwards (1999) also revealed
that K-12 teachers in their study held an instrumental view of statistics and did not recognize the importance of professional development in statistics. Finally, Callingham (1997) found that when teachers were engaged in difficult problems in a supportive way, they developed a deeper understanding of the statistics. Collectively, the results from these studies demonstrated the importance of improving teachers' subject matter and PCK.

Support structures. A third study on SKT by Groth (2008) focused on graduate students in a mathematics education research course. The purpose of the course was to expose graduate students to research within mathematics education. The participants, which included pre- and in-service pre-K-12 and collegiate teachers, created a wiki and responded to a discussion board for the class. The wiki project required participants to "design a document that could be used to evaluate statistics curriculum and textbooks" (p. 23) and that reflected the expectations in the GAISE. Groth designed the discussion board activity to instigate discussion about the GAISE document.

According to the study's findings, participants identified a desire to improve in their statistical curricula knowledge, content knowledge, and technological pedagogical statistical knowledge. Specifically, the participants stated the need for supplementary materials to the GAISE that identified "where all of the [statistics] content to be included may fit within the broader context of the entire course" (p.26) and that enhanced participants' content knowledge in statistics. Furthermore, the participants were able to identify that technology was an important aspect of statistics recommended in the GAISE; however, they were unable to discuss how technology should be used appropriately in the classroom thus demonstrating a need for improved technological
pedagogical statistical knowledge. Finally, the participants were unsure of how the content in the GAISE document aligned with current standards documents, indicating a need for improved statistical curricular knowledge. Also, the participants did not agree upon one way to improve teachers' statistical content knowledge, instead citing several different ways to achieve this. This study by Groth was limited, however, in that the participants were not able to actually implement a lesson aligned with the GAISE document and then reflect on potential support structures needed for successful implementation.

Although examining the needed support structures was limited to one study, the study in this section demonstrated the specific support structures that graduate students in mathematics education identified as necessary for statistics instruction. This included technological pedagogical statistical knowledge, statistical curricular knowledge, and content knowledge.

Overall summary. Across the studies on both mathematical and statistical knowledge for teaching, the results in this section revealed the importance of teachers' knowledge, both subject matter and pedagogical content knowledge, in supporting their success as a teacher. Improved content knowledge was shown to help teachers identify and address students' misconceptions whereas lack of content and pedagogical content knowledge hindered teachers' ability to assess curriculum and textbooks. Given that teachers' knowledge has been shown to affect students' achievement in mathematics classes (Baumert et al., 2010; Fennema \& Franke, 1992; Hill et al., 2005; Rockoff et al., 2010), these results demonstrate the importance of exploring how these components of
teacher knowledge might be a support or barrier for implementing a reform-oriented statistics unit with fidelity.

## Mathematics Teachers' Perspectives

As teachers make the transition from traditional instruction to reform-based instruction, some teachers find it difficult to "participate effectively in reforming mathematics teaching" (Simon, Tzur, Heinz, Kinzel, \& Smith, 2000, p. 579). With this in mind, Simon and colleagues attempted to identify the perspectives (i.e., the meaningmaking systems) that these teachers in transition might have that influences the instructional practices used in their classrooms. The researchers originally identified three perspectives that potentially influence teachers' instructional practices: traditional perspective, perception-based perspective, and conception-based perspective. Teachers with a traditional perspective view mathematics as existing "independently from human experience" (Simon et al., 2000, p. 593) and believe that students assume a passive role in obtaining mathematical knowledge (e.g., reading textbooks, watching others solve problems). In contrast, those teachers with a perception-based perspective view mathematics as connected, understandable, and accessible to all learners. Learning mathematics from this perspective involves students discovering mathematics for themselves. Finally, those with a conception-based perspective believe that the mathematics students learn is based upon their current understandings and previous experiences. The role of teachers with this perspective is to focus "on understanding the students' conceptions (assimilatory schemes) and determining ways to promote transformations" (p. 594) in their mathematical understanding. Literature related to this idea of mathematical perspectives is explored in this section.

Initially, Simon and colleagues (2000) explored two of the perspectives that they considered pivotal in the practices of mathematics teachers participating in reform efforts: perception-based perspective and conception-based perspective. These constructs developed from the Mathematics Teacher Development (MTD) Project, a four and a half year project focused on exploring the transition of mathematics teachers' practices as their understandings of mathematics became more aligned with a mathematics education reform philosophy. The 19 participants in the case study, including both pre- and inservice teachers, were enrolled in five MTD classes within which they engaged in "whole-group teaching experiments" (p. 581). To better understand the perspectives, Simon and colleagues used data in the form of teaching sets (including interviews around a lesson and classroom observations of a lesson) and interviews from the first four courses. The researchers also collected case-study data from the first year on three of the 19 participants.

Within one of the MTD courses, one of the researchers discussed with the participants the idea of children's development of the concept of number. Included in this discussion was the idea of manyness (i.e., that when counting objects in a set, the last number represents how many objects are in that set). Simon and his colleagues found from the data that the participants did not "conceive of a child who does not see the world in terms of number" (p. 587). According to the authors, this reflected a perception-based perspective of the world because it demonstrated that for learners to understand a concept "they must 'see' for themselves the mathematical relationships that exist" (p. 593). This was in comparison to a conception-based perspective that views the learning of mathematics as a "process in which mathematical objects and relationships are
constructed by the learner on the basis of her current knowledge and experiences" (p. 593), aligning with the constructivist philosophy. In another part of the course, Simon and his colleagues explored the understanding of long division and found similar results. Specifically, that the participants relied upon creating specific situations in which the connection between the algorithm for long division and a model representation would become apparent (i.e., a perception-based perspective). This study revealed the importance of how a teacher's perspective of mathematics can affect their instructional practice.

In 2001, Tzur, Simon, Heinz, and Kinzel further refined the perspectives framework just described. As part of the same MTD study, the researchers examined four data sets on one participant including four observations of consecutive mathematics lessons on the development of the long division algorithm and five interviews. In this specific case, Tzur and colleagues found that during the observations of the consecutive lessons, the participant was frustrated that students could not make the connection between using unifix cubes to demonstrate dividing and the algorithm for long division. The researchers identified the participant's teaching and interviews as reflecting an idea that "division could be seen by all in the distribution of physical objects into groups" (p. 242). This indicated that the participant viewed learning as seeing mathematical concepts and their connections. This view aligned with the perception-based perspective first described above.

In this study, Tzur et al. were able to further define this perception-based perspective construct as being composed of four parts: a "platonic view of mathematics" (p. 246); a view of mathematics concepts as interrelated and sensible; a view that students
learn mathematics by participating in hands-on experiences; and a view that being a mathematics teacher means to reveal these mathematical concepts to students. The results from this study allowed the researchers to further expand the conception-based perspective and refine the traditional perspective. Teachers within the traditional perspective still hold a "platonic view of mathematics" (p.246), as in the perceptionbased perspective, but rely heavily on the direct transmission of knowledge from the teacher to the student. The teacher holding a traditional perspective also believes that her role is to tell the students what they should know and then let them practice similar problems until reaching the correct answer. This contrasts with the perception-based perspective that describes teacher practices that allow students to see the mathematical ideas for themselves. Finally, the conception-based perspective entails teaching practices that acknowledge students "have no access to a reality independent of their ways of experiencing it" (p. 247). That is, students learn mathematics by building upon distinctions within mathematics that they can make based upon their current knowledge and understandings. Teachers with a conception-based perspective feel that their role is to elicit students' thinking, make sense of this mathematical thinking, and use it to guide their instruction.

After examining the instructional practices of Chinese mathematics teachers, Jin and Tzur (2011) studied 11 Chinese middle grades mathematics teachers teaching and linked their instructional practices to their mathematical perspectives through a qualitative approach, leading to the introduction of the progressive incorporation perspective. Teachers with this perspective strive to connect the students' understandings with previously learned topics and acknowledge that students have multiple ways of
thinking about mathematics. Those teachers with a progressive incorporation perspective also believe their role is to have students take the initiative in their learning and to create problems that challenge all learners. This fourth perspective differs from the traditional and perception-based perspective in that teachers with this perspective consider the conceptions that students have at that moment and use these conceptions to incorporate new knowledge. This is in contrast to the conception-based perspective in that those with this perspective view new knowledge as being transformed by each student independently into known information. Jin and Tzur (2011) noted that this fourth perspective could be viewed as a richer version of the perception-based perspective. Specifically, both of these perspectives value active learning through discovery, however, the progressive incorporation perspective places more emphasis on students' prior conceptions and knowledge. Teachers with the progressive incorporation perspective view mathematics as being "dialectically independent and dependent on the knower" (Akar, 2015, p. 4). That is, mathematical ideas have similarities regardless of the mathematics learner. What is dependent on the learner is their ability to problem solve.

Jin and Tzur observed the 11 participants teach two consecutive lessons on the same topic, taking field notes and videotaping the lessons. Before, between, and after the two lessons, the researchers interviewed the participants to get a better understanding of how their instructional practices could be attributed to their mathematical perspectives. The researchers found that all participants used a typical lesson structure in their mathematics classroom. This structure included four components: reviewing, bridging, variation, and summary. In the reviewing component, the participants reviewed a recently learned topic with the students. This would usually entail a problem similar to the
previous day's lesson or homework, but would be unique from problems they had solved thus far. Next, the bridging component involved asking students to solve a problem that was learned potentially years prior to the current lesson. This component was "designed to enable every student's learning of this day's new knowledge by posing a problem or presenting examples that connect with knowledge the teacher presumes all students know" (p. 5). The third component, variation, referred to the actual learning goals set forth for the day's lesson. Finally, the teacher engaged students in the summary of mathematics learned, emphasizing the relationship between the problems presenting in the reviewing and bridging components. Interviews with the participants revealed that they rationalized using this four-component structure for their lessons because it connected students' previous knowledge with the intended material for the day. The researchers viewed this as reflecting the progressive incorporation perspective where "learning proceeds from old to new, from familiar to unfamiliar, from easy to complex, and from specific examples to general processes" (p.31).

Although preceding the Jin and Tzur (2011) study, Escudero and Sánchez (2007) also explored the relationship between a teacher's perspective of mathematics and his or her instructional practice within the classroom. Part of a larger study focusing on teachers' knowledge and practice, this case study utilized data from one of the participants in the larger study who was chosen because he was considered representative of all the participants. The researchers interviewed the participant, video-recorded his implementation of a unit on similarity in his high school classroom, and conducted classroom observations of that unit. The participant was an experienced high school
teacher who was considered by the researchers to be transitioning from utilizing traditional practices to utilizing reform-based practices.

Observations of the classrooms and reflection on the video data revealed three general structures for the lessons: presentation of ideas, monitored practice of problems, and checking of homework. The researchers examined each of these segments for the daily lessons, identifying how the instructional practices could be attributed to the perceived mathematical perspective of the participant. During a segment on similarity with triangles, the participant questioned students as to whether triangles met the general conditions of similarity as for other polygons. When the students failed to respond, the participant altered his approach and began to explain the answer to this question with almost no feedback from the students. In a similar instance, the participant wanted students to understand Thales theorem. When his students expressed difficulties with this idea, he resorted to showing the students what he wanted them to see mathematically. This type of instruction was typical throughout the unit. Analysis of the interview data revealed that the participant viewed mathematics as interconnected and his role as making mathematics visible for students. He also discussed how mathematical learning would happen when students were able to have first-hand experiences to create connections among mathematical ideas. This view of mathematics teaching and learning aligned with the perception-based perspective described by Tzur et al. (2001). The researchers attributed the participant's tendency to "explain in detail the intended properties" (p. 102) to this perspective and concluded their work with a statement linking the importance of mathematics teachers' perspectives and their instructional practices.

In 2015, Chamberlin, Troudt, Nair, and Breitstein explored this idea of mathematics teachers' perspectives specifically for teacher leaders. In their qualitative study, the researchers wanted to know what perspectives regarding the teaching and learning of mathematics were held by mathematics teacher leaders throughout a leadership program and how those perspectives might have changed. The leadership program was designed specifically for mathematics teacher leaders to improve their leadership skills for grades K-12 mathematics. Those enrolled in the course participated in online courses during the spring and fall, face-to-face institutes in the summer, and one weekend retreat. Data for this study were collected from nine participants from the first cohort in the leadership program that included two elementary teachers, two middle school teachers, three high school teachers, a district mathematics coordinator, and a Response to Intervention coordinator. The data collected included entrance essays, a pedagogical content knowledge assignment, a reflection on instructional strategies, a report on culturally responsive teaching, and a reflection on conducting a lesson study.

Using a template analysis, the researchers found that five of the nine participants progressed along the perspectives continuum towards a conception-based perspective, three remained at the perception-based perspective, and one participant submitted work that was unable to be analyzed. Of the five participants that moved along the continuum, the researchers found that typically the teacher leaders began at a perception-based perspective and progressed to an incorporation-based perspective. The researchers attributed this move along the continuum to the assignments that the participants completed within their teacher leadership program. Specifically, the tasks were described as allowing the participants to problematize learning in a way that helped them
understand that students can view mathematical relationships differently. Overall, the researchers suggested that to move mathematics teacher leaders along the continuum towards the conception-based perspective, the teacher leaders need to be provided "with opportunities to reflect on the problematic and individualized nature of students learning mathematics" (p. 52).

The results from these studies reveal the importance of considering mathematics teachers' perspectives of teaching and learning mathematics when they are in transition towards a more reform-based style of teaching. Specifically, Simon et al. (2000), Tzur et al. (2001), and Jin and Tzur (2011) conducted studies that revealed four different mathematical perspectives that teachers hold. In another study, Escudero and Sánchez (2007) found that teachers' instructional practices with a perception-based perspective included demonstrating and telling students the mathematics that the teacher wanted them to see. In the final study, Chamberlin et al. (2015) found that for teacher leaders to move along the perspectives continuum towards a conception-based perspective that they need to experience tasks that problematize mathematics learning. These past studies influenced the current study by revealing these perspectives as a potential support or barrier to implementing with fidelity.

## Implementation Fidelity

Implementation fidelity is defined in this study as how well the teacher's implementation of the curriculum aligned with the intended implementation. Although this construct of implementation fidelity is relatively new with respect to $\mathrm{K}-12$ curriculum (O'Donnell, 2008), the literature has shown direct links between high implementation fidelity and student achievement (George, Hall, \& Uchiyama, 2000).

Such literature and others related to implementation fidelity of mathematics curricula are described in this section.

McNaught, Tarr, and Sears (2010) conducted a study that examined the implementation of curriculum in mathematics classrooms, focusing on the differences in the implementations of two different mathematics textbooks. The results from this study were part of a larger study that examined high school students' mathematical achievement and its relationship to the mathematics curriculum being used. As part of this study, over 2600 students from 113 classes and their respective teachers across five different states were examined. The two programs within the study included one program in which students followed a traditional sequence of Algebra 1, Geometry, and then Algebra 2. The second program approached mathematics from an integrated curriculum perspective. The goal of the project was to determine how the using the two different curricula affected student learning.

In regards to fidelity of implementation, the researchers examined two aspects of the written materials (i.e., textbooks) based upon previous work from Grouws, Tarr, and McNaught (2008): content implementation and presentation implementation. The former assessed what content was implemented while the latter assessed how the content was implemented. To better understand the implementation fidelity of the curriculum, McNaught et al. (2010) collected data from both the researchers' and teachers' perspectives including, self-reflection journals, teacher records of what content was taught, and observation protocols.

The researchers found that those teaching from a subject-specific textbook spent fewer days on average on topics than was suggested by the textbook authors and assigned
fewer homework problems than suggested. Those teaching from the integrated textbooks spent more days on average than was suggested but also assigned less homework problems. In regards to the content that the teachers covered, both teachers using the subject-specific and integrated textbooks covered less than $75 \%$ of the content within the text. When looking at content fidelity, the researchers found that content fidelity was high regardless of the type of curriculum used. In terms of presentation fidelity, however, McNaught et al. (2010) found that the teachers' implementation of the materials was not consistent with the curriculum authors' expectations. The researchers also found a moderate correlation ( 0.50 ) between the content and presentation fidelity indicating that those teachers with higher content fidelity had higher presentation fidelity. McNaught et al. examined two different curricula from an implementation fidelity perspective in a different way than in the current study. Specifically, neither of the curricula were described as reform-oriented thus the interpretation of implementation fidelity is different than in the current study. However, the results still inform the current study given that it reveals the nature of how curricula are implemented organically in the classroom.

As part of the same larger project, Chávez, Tarr, Grouws, and Soria (2013) continued to explore the effects of integrated and traditional curriculum on student achievement. In this study, several factors were analyzed including implementation fidelity. The researchers collected student mathematics achievement from two different assessments and data related to demographics. The teachers in the study completed two surveys at the beginning and middle of the academic school year to gather demographic information, background data (e.g., years of experience), the amount of time spent teaching with that specific curriculum, professional development history, beliefs about
the teaching and learning of mathematics, perceptions of the textbook quality, instructional practices used in the classroom, and how the curriculum was used during class time. Similar to the previously described study, the researchers asked the teachers to maintain a table of contents record that identified the source of content taught in the classroom (i.e., was the textbook the primary source of content taught). They used this data to create three different indices used to describe implementation fidelity: opportunity-to-learn, extent of textbook implementation, and textbook taught content. The participating teachers collected this implementation fidelity data and submitted it to the research team quarterly.

Nesting the students within classes and classes within schools, the researchers employed a multi-level modeling strategy to analyze the relationship between the factors described above and students' mathematics achievement. The researchers found that a majority of teachers relied primarily on the textbook for content to be taught although some teachers chose to supplement the material. Those teachers using the traditional (subject-specific) textbook, on average, discussed more of the content within the textbook than those using the integrated curriculum. In regards to the implementation fidelity indices, the researchers found that between the traditional and integrated curriculum users, there was no statistically significant difference in the extent of textbook implementation and textbook taught content indices. Further analysis revealed that students with teachers that claimed to use practices aligned with reform philosophy scored significantly better on their mathematics assessments. Similarly, those teachers who stated beliefs that aligned with reform philosophy also had students with greater achievement scores. Overall, the researchers found that those students in the integrated
curriculum scored better than those in the traditional curriculum, but that implementation fidelity as defined using the indices described above was not a significant predictor for student mathematics achievement. It is important to note that implementation fidelity, in this study, did not include an actual assessment of how the curriculum was implemented in the classroom. In addition, the teachers reported the measures used to define implementation fidelity without the researchers actually observing the classroom lessons.

In another study examining the relationship between student achievement and implementation fidelity, George, Hall, and Uchiyama (2000) examined how the implementation of a mathematics curriculum based upon the NCTM (1989) standards within a K-8 setting affected student outcomes over several years. Along with the curriculum, teachers within the school were supported during the implementation by "school district personnel ... [and] change process researchers" (p. 8). In addition, 37 of the 107 teachers within the study attended a summer program designed to prepare them to use the curriculum. The study took place within 14 schools in the Hessen District in Germany. The schools served children of U.S. military and personnel and, at the time, had recently adopted a new mathematics program aligned with NCTM (1989) standards. To assess students' learning, the researchers implemented a mathematics assessment as a pre- and post-test over the course of two years. The researchers assessed the implementation of the curriculum using the Levels of Use (Hall, Loucks, Rutherford, \& Newlove, 1975) and Innovation Configuration Mapping (Hall \& Hord, 2001) constructs.

Over the three years, the researchers found three primary results. First, the researchers found that even over the course of several years and with the option for a summer training session, not all teachers made a "paradigm shift in teaching" (p. 24) that
aligned with NCTM (1989) standards although all of them showed some degree of alignment. Second, as teachers made the transition to higher implementation fidelity, "higher levels of student learning were observed" (p. 24). Also, students whose teachers' practices more closely followed NCTM (1989) standards showed greater achievement on assessments. Finally, the researchers found that the students who started the year with the lowest test scores benefitted the most from teachers who had high implementation fidelity.

To better understand how curriculum affects mathematics teaching that aligns with reform efforts, Remillard (1999) studied how two elementary teachers interpreted and enacted a newly adopted reform-oriented textbook and how the textbook affected their teaching. The study took place over one year and included classroom observations, interviews, and written reflections. Remillard employed an interpretive, case study approach to analyzing "each teacher's curriculum development activities, including her interactions with the textbook" (p. 319). This approach allowed the researcher to describe the two teachers' "orientation toward textbook use" (p. 320) which was further used to describe each teachers' interaction with the textbook and how that affected their teaching and thinking. Through the data analysis, a model for how teachers develop mathematics curriculum emerged, composed of "three arenas . . . design, construction, and curriculum mapping" (p. 315).

The researcher described the design arena as how teachers select and design tasks in which their students can engage. In regards to this arena, Remillard found that the two teachers used the textbook differently as they selected and designed tasks for students. Specifically, the first teacher, Catherine, used the textbook as a source of activities to
implement in her classroom. The second teacher, Jackie, used the textbook as a source of inspiration for her own tasks that she created. Remillard stated that the two different uses of the textbook were influenced by the participants' (i.e., Catherine and Jackie) views of mathematics teaching and learning. That is, Catherine felt that students needed to be shown how to do the mathematical procedures and her job as their teacher was to make sure they could follow each step within the procedure. In contrast, Jackie felt that students needed to understand the mathematical relationships within the discipline, and she felt that teachers needed to use students' thinking as a guide instead of a curricular plan.

The researcher described the construction arena as that in which teachers implement tasks and then respond to how students engage with the task. Within this arena, Remillard again found different ways in which the two teachers implemented selected tasks in their classroom. The researcher found that for both teachers the textbook had little influence on how they implemented the tasks. When implementing tasks, Catherine focused on not letting the students get frustrated while they worked on finding the correct solution to a problem. Throughout her participation in the study, Catherine's opinion of how students learn mathematics changed, subsequently changing the way she implemented tasks. For example, later in the study, Remillard noticed that Catherine was cognizant of students' difficulties with a particular task and instead of following the original lesson plan rigidly, decided to offer the students the option to use the calculator to help them with the task. In contrast, throughout the entirety of the study when implementing tasks, Jackie consistently asked students to justify their answers regardless of the correctness of those answers. When implementing tasks, if she noticed that
students did not understand the concepts, she would improvise by changing her questions to reveal their thinking.

Remillard (1999) described the final arena (i.e., curriculum mapping) as including teachers' decisions for content to be addressed and how to organize it within the curriculum. The researcher further refined this to include two categories for decision making within this arena: topic and content determination. Within topic determination, the teacher determines how the mathematical topics are divided. Within content determination, the teacher determines the skills needed to be learned, how to sequence the learning, and how long to spend on topics. Aligning with previous findings, Catherine and Jackie used the textbook differently within the curriculum-mapping arena. Catherine relied heavily on the text for topic determination. She felt rushed to teach the skills on which the students would be tested and thus did not spend as long on some topics as Jackie. In contrast, Jackie relied heavily on students' mathematical thinking to guide instruction, spending more time on a topic if she realized that students were not understanding. In this study, Remillard revealed how two different teachers implemented the same curriculum differently in their classrooms. This lack of fidelity between the two teachers reveals the importance of studying implementation fidelity to better understand how to support teachers in using the curriculum effectively.

In reviewing these studies focused on implementation fidelity, it is apparent the influence of teachers' implementation fidelity on students' mathematical learning. Specifically, Chávez et al. (2013) and George et al. (2000) both found that as teachers' implementation fidelity improved so did their students' mathematical achievement. Unfortunately, when given a curriculum, not every teacher implements it in the same way
as was seen in Remillard (1999). Many factors can potentially influence the implementation including some of the ideas discussed earlier in this chapter (e.g., mathematical and statistical knowledge for teaching and mathematical perspectives). This warrants the exploration of the relationship between mathematical and statistical knowledge for teaching, mathematical perspectives, and implementation fidelity.

## Conceptual Framework

Given the literature reviewed in this chapter, I created a conceptual framework guiding the analysis of my study. As demonstrated in Figure 1, MKT and SKT as well as teachers' mathematical perspectives affect teachers' implementation of a unit. For example, those teachers with a traditional perspective of mathematics may implement a reform-oriented unit differently than those with a conception-based perspective. These three constructs (i.e., MKT/SKT, teachers' mathematical perspectives, and implementation fidelity) all have been cited as affecting students' mathematical achievement, although this was not an explicit part of this study. It is this framework that guided the analysis of my data as described in Chapter Three.


Figure 1. Conceptual framework

## Chapter Summary

In this chapter, I have provided an overview of the literature from the statistics and mathematics education community that offered a foundation for this study. Results from the implementation fidelity section create a foundation for understanding the complexities of the participant's implementation of the statistics unit in this study. This implementation fidelity is linked to the results from previous sections on teachers' knowledge and perspectives in a way that reveals how the implementation fidelity is directly affected by specific barriers or supports within these two constructs. Also, I have provided a conceptual framework based upon the literature that guided my analysis of the data.

## CHAPTER THREE: METHODOLOGY

## Introduction

Although teachers have been identified as feeling unprepared to teach statistics (Begg \& Edwards, 1999; Greer \& Ritson, 1994; Harrell et al., 2009) and as having statistical misconceptions (Callingham, 1997; Groth \& Bergner, 2006; Jacobbe, 2008; Leavy \& O’Loughlin, 2006; Makar \& Confrey, 2004; Mickelson \& Heaton, 2004), current reform documents, such as the GAISE document (Franklin et al., 2007) and the CCSSM (CCSSI, 2010), require the teaching of statistics as early as elementary school. Although professional development opportunities for in-service teachers may address these issues, Jacobbe and Horton (2012) found that teachers' misconceptions persisted despite participation in a statistics professional development. Given these concerns, it is apparent that more research is needed to better understand how statistics is being taught in the classroom and what can be done to support teachers.

This case study documented the implementation of a reform-oriented statistics unit to determine support structures deemed necessary by the teacher for implementation of statistics in the middle grades as envisioned by reform documents. This chapter describes the overall methodology of this case study. This includes the design, study context, the research participant, instruments and data sources, procedures, and data analysis.

## Overview of Research Design

This study examined the implementation of a reform-oriented statistics unit within a sixth-grade classroom. Specifically, the primary research question posed was: How does a sixth-grade teacher implement a reform-oriented statistics unit? In addition, a
second research question was posited: What support structures does the participant identify as needed for the unit to be implemented with fidelity?

The primary research question sought to describe the circumstance of implementation in the classroom and was, therefore, appropriate for utilizing a descriptive case-study approach (Yin, 2014). Yin (2014) stated that the case study method is appropriate when the behaviors being observed are unable to be manipulated, as was true in this study. This study embodied the single-case, holistic design because it included only one participant observed over several points in time, and the global nature of the implementation of the unit was examined (Yin, 2014).

## Research Context

This study occurred over eight days in a mathematics classroom in a rural middle school (Grades 6-8) located in the southeastern U.S. The school was part of a district that served a population of students that was 90.9\% Caucasian, 5.2\% Hispanic, 3.0\% African American, and $0.7 \%$ Native American/Alaskan. The total student population was 4,588 students in the 2013-2014 academic year. This district reported 59.1\% economically disadvantaged students, $13.4 \%$ disabled students, and $1.7 \%$ limited English proficient students. Per results from state testing, $48.9 \%$ of the students in this district in grades 3-8 scored basic or below basic on their mathematics assessment. The study occurred within in a single sixth-grade mathematics classroom that met daily for a duration of 46 minutes. This classroom consisted of 26 students whose make-up resembled that of the district.

## Participant

Yin (2014) stated that a case should be selected in a way that "will most likely illuminate your research questions" (p. 28). In this study, a teacher, further addressed by
the pseudonym Ms. Thomas, was selected based upon several characteristics that revealed the phenomenon of implementation, as described below.

## Selection of Participant

To identify a participant, I first attended a 10-day professional development held for K-6 teachers during the summer of 2014. The purpose of the professional development was to assist teachers in understanding how to implement effective instructional practices in their classrooms with a focus on fractions. My goal was to identify a sixth-grade teacher among the K-6 teachers in the project. It was important for me to identify someone within this project because then he/she would be familiar with the type of instruction that he/she would be expected to teach. Although this person might not yet have had experience teaching in such a way, this familiarity could have possibly eased some nervousness that he or she might have had about participating in the study. During the professional development project, I decided to ask Ms. Thomas, a new mathematics teacher, if she would like to participate in my study. Although any sixthgrade teacher willing to participate could have been selected, the choice to select Ms. Thomas was due to accessibility, her willingness, and the willingness of her administration.

As an incentive to participate in the study, I offered Ms. Thomas a resource bundle of three carefully selected books that she might find useful during the implementation of the reform-oriented statistics unit and in the future. The three books selected were: Developing Essential Understandings of Statistics for Teaching Mathematics in Grades 6-8 (Zbiek et al., 2013); Bridging the Gap Between Common Core State Standards and Teaching Statistics (Hopfensperger, Jacobbe, Lurie, \& Morena,
2012); and 5 Practices for Orchestrating Productive Mathematics Discussion (Stein \& Smith, 2011).

As part of the professional development project, Ms. Thomas completed a survey regarding her mindset (Dweck, Chiu, \& Hong, 1995). According to Dweck (2006), there are two types of mindset that people may have: fixed and growth. Those with a fixed mindset believe that an individual's qualities, such as talent and intelligence, are fixed traits that are unable to be developed. Alternatively, those with a growth mindset believe that such qualities can be developed through work and dedication. On the survey administered during the professional development, Ms. Thomas was identified as having a growth mindset in regards to students' ability to learn mathematics and moral character. She had a neutral mindset on intelligence and world view. This indicated that she viewed students' mathematical ability as being able to grow through work and commitment. I valued this characteristic because it meant that Ms. Thomas might value a reformoriented approach to teaching that emphasized student work and collaboration.

## Background of Participant

At the time of the study, Ms. Thomas was in her third year of teaching, but she was in her first year as a mathematics and science teacher. Ms. Thomas held both bachelor's and master's degrees and participated as an athletic coach at her school. She was in her mid-forties at the time of the study and grew up in the local area. At the time of this writing, Ms. Thomas had not had the opportunity to participate in any professional development specifically focused on teaching statistics. When asked about other professional development experiences, Ms. Thomas stated that she participated in a data and planning professional development, but this was not specific to mathematics content.

With regard to her background in statistics, she indicated that she had completed a onesemester course of statistics in her teacher preparation program. In reflection on this class, she described the use of the Excel software program to calculate typical statistics of quantitative data sets and to create representations for such data sets.

## Instruments and Data Sources

To explore the research questions, five sources of data were used: field notes, a Daily Observation Protocol, Interview Protocols, Participant Research Journal, and a Researcher Journal. These data sources are typical of case study research, and the use of these multiple sources allowed for "the development of converging lines of inquiry" (Yin, 2014, p. 120). That is, by having multiple data sources, I was able to provide "more convincing and accurate" (p. 120) evidence for the findings. Yin (2014) also mentioned how implementing data triangulation in this way "strengthens the construct validity" (p. 121) of the case. These five data sources are described below.

## Field Notes

Each day prior to completing my observation protocol, I took field notes of Ms. Thomas' implementation of the unit. I acted as a nonparticipant (Creswell, 2013) in that I was not participating in the lesson and took field notes away from the class without interacting with the students or teacher.

## Daily Observation Protocol

I observed Ms. Thomas daily in her classroom and then would complete the Daily Observation Protocol (see Appendix A). I completed the Daily Observation protocol during the implementation of the unit as well as one time prior to the unit implementation. This protocol aligned with NCTM (2014), CCSSM (CCSSI, 2010), and

GAISE (Franklin et al., 2007) documents, identifying instruction that embodies the key components of reform-oriented instruction.

## Interview Protocols

I interviewed Ms. Thomas prior, during, and after the unit using the semistructured Pre-, Mid-, and Post-Interview Protocols (see Appendix B). These interviews were semi-structured in that some interview questions were asked as planned, but other questions were asked that were not previously planned to potentially identify "new ways of seeing and understanding" (Cohen \& Crabtree, 2006, p. 1) the implementation fidelity as perceived by Ms. Thomas. I designed these interview questions as a means to better understand Ms. Thomas' current statistical knowledge, a potential barrier to implementation of statistics content as identified in the literature. In addition, the interview questions allowed for Ms. Thomas to raise her questions or concerns regarding the unit and its overall implementation.

## Participant Research Journal

Ms. Thomas maintained a Participant Research Journal (see Appendix C) throughout the implementation. The prompts for her journal were similar to the interview questions in that the purpose was to gain insights into the implementation, her understanding of the content, and issues with the unit. I emailed the prompts to Ms. Thomas daily after each implementation, and Ms. Thomas responded to the prompts digitally.

## Researcher Journal

I also maintained a Researcher Journal throughout the data-collection process. This journal consisted of daily reflections on implementation and did not follow a
structured format. The intent of the researcher's journal was to facilitate reflexivity (Ortlipp, 2008), the practice of making visible "to the reader the constructed nature of research outcomes" (Ortlipp, 2008, p. 695). I wrote in my researcher journal daily, after each implementation prior to our daily interview.

## Researcher as Instrument

I was the primary data collection instrument (Creswell, 2013). I relied solely on interview and observation protocols that I created and not "on questionnaires or instruments developed by other researchers" (Creswell, 2013, p. 45). I am qualified to be the primary data collection instrument because of my academic and professional experiences. I hold a Bachelor of Science and a Master of Science in Mathematics, and have completed graduate-level courses in qualitative research and educational research methods. I have participated in multiple research projects employing qualitative research methods, both at my university and as an external research assistant for another university. These professional experiences support my qualifications to serve as an instrument in this study.

## Statistical Unit

The unit was developed following the Understanding by Design (UbD) framework (Wiggins \& McTighe, 2005). This framework consists of three stages. In the first stage, one identifies the learning goals for the student. In selecting the learning goals for the statistics unit, I aligned the unit goals with the sixth grade statistics standards in the CCSSM (CCSSI, 2010) and the GAISE statistical problem-solving process (Franklin et al., 2007). In the second stage of the UbD framework, the assessments are developed that align with the learning goals identified in stage one. For this unit, the assessments
included: a formal, end-of-unit assessment; exit tickets; and a performance task to be completed after the final assessment. Finally, in the third stage, the daily lesson plan and tasks are identified. For this stage of the process, I followed the Thinking Through a Lesson Protocol (Smith, Bill, \& Hughes, 2008) in planning the daily lessons to ensure that the students were given the opportunity to engage in high-level statistics.

I selected and refined some of the daily tasks from the following sources: Browning and Channell (2003); Zbiek et al. (2013); and Revak and Williams (1999). Aimed at meeting the sixth grade statistics standards, the unit had two major goals of having students develop their understanding of variability and having them "summarize and describe distributions" (CCSSI, 2010, p. 41). To achieve this goal, the unit consisted of six tasks to be completed over a course of eight days and two assessments to be completed in the remaining two days. I designed two of the six tasks to engage students in the statistical problem-solving process in its entirety for both a quantitative and categorical data set. I designed the remaining four tasks to engage students in creating and analyzing statistics for quantitative and categorical data sets and developing appropriate graphical representations. The intended curriculum detailed by day is represented in Table 2. The enacted curriculum detailed by day can be seen in Table 3. Appendix E contains the daily tasks and their appropriate sources.

Table 2

| Day | Task | Summarized Goal |
| :---: | :---: | :---: |
| 1 | French Fry Task | Calculate mean, median, mode, and range for two quantitative data sets and interpret in context |
| 2 | Answering a Statistical Question Task | Understand, collect data to answer, and represent a statistical question |
| 3 | Construct Your Own Graph Task | Calculate interquartile range for a quantitative data set, including a representation and description of distribution |
| 4 | I Wonder What Happens If . . . Task | Understand how different statistics affect the shape of a distribution |
| 5 | Statistical Problem-Solving Process Task | Complete the statistical problem-solving process for quantitative data set |
| 6 | Statistical Problem-Solving Process Task | Complete the statistical problem-solving process for quantitative data set |
| 7 | Categorical Data Task | Complete the statistical problem-solving process for qualitative data set |
| 8 | Categorical Data Task | Complete the statistical problem-solving process for qualitative data set |
| 9 | Unit Test | Formal assessment of previous goals |
| 10 | Oreo Performance Task | Performance assessment of previous goals |

Note. See Appendix E for all tasks and expanded learning goals for each task.

Table 3
Enacted Curriculum Plan

| Day | Task | Summarized Goal |
| :---: | :--- | :--- |
| 1 | French Fry Task | Calculate mean, median, mode, and <br> range for two quantitative data sets and <br> interpret in context |
| 2 | French Fry Task <br> 3 <br> Answering a Statistical Question <br> Task | Continue from previous day <br> Understand, collect data to answer, and <br> represent the data answering a statistical <br> question |
| 4 | Construct Your Own Graph Task | Calculate interquartile range for a <br> quantitative data set, including a <br> representation and description of <br> distribution |
| 5 | Construct Your Own Graph Task <br> I Wonder What Happens If . . | Continue from previous day <br> Understand how different statistics <br> affect the shape of a distribution |
| 7 | Task Wonder What Happens If . . <br> Task | Continue from previous day |
| 8 | Oreo Performance Task | Performance assessment of previous <br> goals |

To ensure the quality of the unit, a statistician, a mathematics educator, and one external reviewer examined the unit for content and alignment with reform documents. I considered each of these reviewers qualified to review the unit. The statistician earned a Doctor of Philosophy in applied mathematics with a concentration in probability theory. She had taught college statistics courses for almost 20 years, had been supported by the National Science Foundation to develop statistics curriculum for introductory statistics, and had been asked to evaluate the effectiveness of statistics education materials with
various grants for approximately 15 years. Along with being asked to frequently review statistics textbooks, she had been recognized by the American Statistical Association with the Waller Education Award for outstanding contributions to and innovations in the teaching of introductory statistics.

The mathematics educator earned a Doctor of Philosophy in mathematics education and had taught numerous undergraduate and graduate courses in mathematics education, including a probability and statistics course for elementary and middle school teachers. She was knowledgeable of standards-based instruction in general, as evidenced by her numerous national presentations and publications on this topic, and she was knowledgeable of statistics in general through her doctoral graduate coursework as well as reviews of statistics textbooks.

The external reviewer held a Bachelor of Science in Mathematics and Master of Science in Teaching Mathematics and was pursuing a Doctor of Philosophy in mathematics and science education with a concentration in mathematics education. He had been a high-school mathematics and statistics teacher for 11years. Previously, he had participated in two research grants focused on statistics education at the K-16 level and was an Advanced Placement Statistics Reader for the College Board for four years.

I sent the reviewers a digital copy of the statistics unit. Each reviewer's comments and feedback were considered and included in the final statistics unit that was delivered to Ms. Thomas. An example of the type of feedback included a stronger emphasis on the data collection component in the statistical problem-solving process.

## Procedures

After receiving Institutional Review Board approval (see Appendix F), I gave Ms. Thomas the statistics unit (see Appendix E) in October 2014 that she would implement later that semester. Ms. Thomas took a month to review the unit, and then in early December, Ms. Thomas and I discussed the unit and what was expected in terms of the reform-oriented documents that aligned with the unit. This was an opportunity for her to ask questions regarding content, pedagogy, or the unit in general. My role during this discussion was to assist her in understanding what was expected and to help her better understand any statistical content with which she may not be familiar or understand. This discussion was the only time, until after the unit, that I answered her questions in terms of how to teach and what to teach. This discussion was audio recorded. After this discussion of the unit, on the same day, I interviewed Ms. Thomas using the Pre-Interview Protocol (see Appendix B).

In reflecting on this initial session with Ms. Thomas, I recognized that this session might be viewed as a support structure for the participant, but I deemed that it was appropriate to offer Ms. Thomas support prior to her implementation, but not during implementation, as a way to make her feel comfortable with participating in my study. I hoped that this would eliminate any temptation she might have had in removing herself from the study. I also expected that this conversation reflected typical conversation that she may have with fellow teachers or mathematics coaches prior to teaching a new unit.

In December of 2014, I observed Ms. Thomas teaching her mathematics class, maintained field notes, and completed the Daily Observation Protocol (see Appendix A). This observation was a way to gauge her typical instructional practices prior to the unit
and to aid as a comparison to her instruction during the unit. A few days later, Ms. Thomas began the implementation of the statistics unit in her classroom. Each day of implementation, I video recorded Ms. Thomas, maintained field notes, and completed the Daily Observation Protocol (see Appendix A). During this implementation, Ms. Thomas would sometimes have questions regarding whether her response to a student's question was correct or if she taught the lesson in the way it was expected. I did not answer any of these questions, as a way to remove myself as a potential support structure and maintain my role as the researcher.

After each daily lesson, immediately after her last period class, I wrote in the Researcher Journal. The time lapse between the end of her lesson implementation and our interview was approximately 55 minutes. I then interviewed her using the semi-structured Mid-Interview Protocol (see Appendix B) which lasted approximately 20 minutes. The participant emailed her journal response to me daily either immediately after I left our interview or later that evening. This process was repeated every day of implementation, a total of eight days.

On the last day of the implementation, I conducted the final interview following the semi-structured Post-Interview Protocol (see Appendix B). At this time, Ms. Thomas discussed her opinions of the unit. I used these considerations and her suggestions to improve upon the unit. I then returned the final, revised unit to Ms. Thomas for her to use in her future classes and for her to disseminate among her colleagues.

## Data Analysis

The data analysis procedures consisted of four primary steps. The first step in the data analysis procedure was to examine the data "searching for patterns, insights, or
concepts that seem[ed] promising" (Yin, 2014 p. 135). This initial step, what Yin (2014) referred to as playing with the data, can be accomplished by representing the data in various forms such as different arrays, matrices, data displays, tables including frequency of events, and chronological form (Yin, 2014). In this study, I chose to examine the data chronologically since it seemed that a subsequent lesson implementation was highly related to the previous day's implementation.

The second step in the data analysis procedure was to thoroughly examine the data using the inductive strategy described in Yin (2014). This step included assigning "various kinds of codes to the data, each code representing a concept or abstraction of potential interest" (Yin, 2014, p. 138). To assist with this coding process, qualitative analysis software, NVivo, was used. I also based the codes (see Appendix D) upon the conceptual framework provided in Chapter Two. In this second step, I read the data, and I summarized the general idea of the statement being made into a code. As an example, Ms. Thomas talked during our pre-interview about her experience in a statistics class in college and how this unit revealed more about statistics than she had anticipated. She said, "I thought [statistics] was an average. . . . It's always been an average. Nothing this in depth." These statements were summarized into a code regarding her subject matter knowledge, as defined by Ball, Thames, and Phelps (2008). Appendix D contains a complete list of codes.

The third step in the analysis was to aggregate the codes into larger themes to create a larger idea. An example of these aggregated codes included taking the quote described in the previous paragraph and merging into the larger code of barriers. These themes, which were based upon the literature, were then interpreted to determine "the
larger meaning of the data" (Creswell, 2013, p. 187) at which time the final step in the analysis was conducted. This final step included constructing a "case study report" (Yin, 2014, p. 178) following a chronological structure (Yin, 2014). During the construction of this report, I asked Ms. Thomas to review the report as a way to increase construct validity (Yin, 2014), and a "chain of evidence" (Yin, 2014, p. 127) was maintained in the report to improve reliability. From this report, I created the final case study narrative.

## Limitations and Delimitations

This case study had four limitations and one delimitation imposed upon the research. These are described below. One limitation of the study was the number of days Ms. Thomas was able to devote to the unit in her classroom. At the onset of the study, Ms. Thomas and her principal agreed to conduct the study for the full ten days in her classroom. However, other schooling priorities arose, and the unit implementation was limited to eight days. Because of this restriction, Ms. Thomas and I met and decided that we could shorten the number of days needed to for the unit by eliminating the unit test. Since student results to this test were not pertinent to this study, I deemed that eliminating the test was feasible.

A second limitation of the study was the placement of the unit in the yearlong curriculum. Originally, the unit was to be implemented during the first two weeks of the school year. I chose this placement within the curriculum for two reasons. First, by having students engaged in the statistical problem-solving process early, they would have been prepared to collect data appropriately and then represent that data using proportions, a focus of the sixth-grade curriculum. Thus, the unit would have segued students from the topic of statistics to the topic of fractions. Second, given that local teachers and
administrators have stated that statistics is traditionally taught at the end of the academic year, this placement allowed for statistics to not be discarded as an unworthy topic of instruction or delayed until later in the year. Unfortunately, due to unforeseen circumstances, the unit had to be implemented at the end of the fall semester right before the winter break. Given this timing of the unit and other end-of-the-semester obligations, Ms. Thomas was unable to devote as much planning time for the implementation of this unit as she typically would during other units.

The third limitation of the study was the hurriedness of the interviews due to the placement of the unit in the school year as described above and interruptions during our interviews. During this time of the academic year, Ms. Thomas was busy with several school and personal related items that prohibited us from having extended amounts of time for our interviews. I suspect that this hurriedness affected the interview data by not allowing Ms. Thomas adequate time to reflect or for me to ask more probing questions. Also, during several of our interviews, colleagues and students of Ms. Thomas would interrupt to ask a quick question or make a statement, often distracting both her and me.

The final limitation of the study was the failure of the camera on Day Six. Specifically, the camera failed with approximately 20 minutes remaining in the class period. This caused me to rely solely on field notes for the remainder of the period.

My selection of Ms. Thomas as the participant is considered one delimitation of the study. After talking with the participant, I found that this case would be especially interesting given her new position as a mathematics teacher and her desire to teach mathematics from a reform-oriented stance. This delimitation does not allow me to generalize her results, but this is not the goal of qualitative research. Through thick
description (Creswell, 2013) of the case, however, readers are encouraged to consider the transferability of the results from this study to similar situations.

## Chapter Summary

In this chapter, I identified the methodology for this study. The case study approach, as described by Yin (2014), was appropriate because the purpose of this study was to explore the phenomenon of implementation. This single case study utilized five primary data sources with the researcher acting as the primary data collection instrument. I analyzed these data sources using four steps outlined in Yin (2014) and resulted in a general idea that determined "the larger meaning of the data" (Creswell, 2013, p .187). I provide an analysis of this data in Chapter Four.

## CHAPTER FOUR: RESULTS

## Introduction

With current documents calling for statistics education happening early and often throughout a student's academic career (CCSSI, 2010; Franklin, et al., 2007), the issues of statistical misconceptions (Callingham, 1997; Groth \& Bergner, 2006; Jacobbe, 2008; Leavy \& O’Loughlin, 2006; Makar \& Confrey, 2004; Mickelson \& Heaton, 2004) and the feeling of being unprepared to teach statistics (Begg \& Edwards, 1999; Greer \& Ritson, 1994) become serious issues for teachers that need to be better understood. Efforts to address these issues have included professional development opportunities for in-service teachers and statistics courses in teacher-preparation programs. Despite these efforts, researchers have found that those teachers who engaged in professional development for statistics often maintained statistical misconceptions (Jacobbe \& Horton, 2012). Similarly, those who received a course in statistics often received traditional instruction and were unaware of their limited statistics knowledge (Jacobbe \& Horton, 2012; Stohl, 2005). Therefore, the need arises to gain better understanding of how teachers are implementing statistics in their classrooms to understand how best to support these teachers in meeting expectations within reform documents.

In this chapter, I present the results from my data analysis as described in Chapter Three. The goal of this descriptive case study was to explore the implementation of a reform-oriented statistics unit within a sixth-grade classroom, identifying supports and barriers that affected the implementation fidelity of the unit. Implementation fidelity, as defined in Chapter One, describes how well the teacher's implementation of the curriculum aligns with the intended implementation. This includes both the deviations
from the intended curriculum and the alignment of the enacted curriculum with the reform-oriented philosophy. Since the implementation was affected daily by the classroom experiences, the results are presented in chronological order, organized according to each day of the implementation. This allows for the reader to understand the context of the classroom, ensuring that a rich description of the study context is portrayed. In each of these sections, I describe the daily implementation as observed in the video data. Then the implementation fidelity is addressed, including both the participant's deviations from the intended curriculum and the alignment of her enacted curriculum with the reform-oriented philosophy. Each section concludes with barriers and supports.

It may be important for the reader to know that the classroom was structured similarly each day of implementation. This structure was observed during other visits to the classroom on non-implementation days. Ms. Thomas had the students seated in groups of three to five students. Students turned their desks towards one another to make a table for the group. Students in each group were designated a role within the group based upon a color on their desks. Although Ms. Thomas stated that she frequently changed the students' groups, the student groups remained fixed during the implementation of this unit. Ms. Thomas also intended to teach the same lessons each day in an earlier period. Although I did not collect data on this period, Ms. Thomas occasionally mentioned her earlier class period. From these conversations, it seemed that at first the two periods were on the same lesson plan. Towards the end of the implementation of the unit, the earlier period and the period in which I conducted my study were not on the same lesson plan due to unforeseen scheduling issues.

## Day One

In this section, I describe the implementation of Day One based upon the video data. Utilizing all data sources, I then describe the fidelity of implementation, barriers, and supports related to the implementation. On this day, Ms. Thomas started the French Fry Task (see Appendix E), a task that had the goal of having students calculate statistics for two quantitative data sets and interpret these statistics in context.

## Implementation

At the beginning of the lesson, Ms. Thomas gave students a brief description of what they could expect during the course of the statistics unit, including how to represent their ideas on chart paper.

There will be times I have to stop and look at my notes more than you normally see me do. Second of all, this is very interactive. You're expected to participate whether you're sitting in the front, or you're sitting in the back.

Following this introduction, Ms. Thomas put the warm-up for the French Fry Task on the projector and asked the students to read it silently to themselves as she handed out copies of the task. She then realized that she had forgotten to ask them an introductory question prior to having them start the French Fry Task.

I went ahead and jumped in. I want to talk to you for a minute about what do you know about statistics? Who can tell me what they know about statistics? . . . Do you all know anything about statistics?

Several students responded, and a typical response was, "10 out of 100 people." Ms. Thomas probed them further by presenting a situation about football.

Peyton Manning throws the football 15 times during a game. The first time he throws the ball, maybe he threw it, and they caught it ten yards away. The next time he threw it, maybe it was 90 yards away. The next time it was three yards away. We can add all this up, and come up with a statistic that shows the average [emphasis added] of how much he's throwing the football.

Following this football scenario, a student questioned Ms. Thomas, "Can statistics be fractions or decimals?" Ms. Thomas responded by indicating that the student would figure that out as she learned more about statistics during the unit. At this point, Ms. Thomas realized that many of the students might not have the understanding of statistics that she anticipated. "You all are kind of throwing me for a loop because, [pause] do you know what median means?" Many of the students in the class said that they had learned a song the previous year about mean, median, and mode; however, none of them could recall the song. After a few moments, a student remembered that the median would be "the middle number." Ms. Thomas checked the class for agreement with this idea, wrote this term on a piece of chart paper designated for vocabulary for the statistics unit, and stated that this idea was correct. "The median is actually the middle number. It's the middle number in a range," she verified.

After this initial conversation about statistics, Ms. Thomas put the warm-up task back on the projector, asked a student to read the task out loud, and requested that the students think about a strategy for how they could find the median of that data set. A student responded, "Line all the numbers up from least to greatest." Ms. Thomas checked for agreement and asked the students to read the numbers to her from least to greatest as she wrote them on the white board. To help students avoid an error in their calculations,
she stated that they should always verify that they have the same number of data points in their list as was in the original list. "If you leave a number out, you're data is not going to be complete." Then Ms. Thomas probed them further about the procedure for finding the median once the numbers were in ascending order. A student responded, and Ms. Thomas restated what she said, "She said her teacher said to start marking them off." To verify that the rest of her class understood the student's idea, Ms. Thomas asked them which numbers she should mark off and in what order. She demonstrated this procedure at the board once all students agreed about how to mark off the numbers, asking students what number they had just found and what it represented in the problem. The students responded that they had found the median, and she agreed, "The median because it is the number in the center. Ok, now we need to talk about mean."

Moving to the next statistic, Ms. Thomas asked her students what the mean would be and what it represented. Many of the students gave similar responses, stating that they would have to add everything in the data set. Ms. Thomas tried to get them to think a bit further about the word mean by telling them that the mean was also called by another name. Several students stated that it could also be called the average, and the class agreed that these terms were interchangeable. Ms. Thomas noticed that during this conversation, a group of girls kept trying to tell her how to find the mean. "These girls have been trying to say how you find the mean instead of what it meant so go ahead. They said it means you have to add and divide. What do you have to add and divide?" Many students had incorrect responses. For example, one student said, "You take the least and the greatest number, and you add it." Ms. Thomas rebutted, "You're on to something, but that's not the average." When a student gave the correct procedure, she asked that one student from
each group, based upon their roles from the color on their desks, use the calculators at their desks to find the sum of all of the numbers on the board and then asked another student from each group to write down the numbers in order from least to greatest on one of their task sheets. The students then told Ms. Thomas the average value that they had calculated. Ms. Thomas inquired, "Is that the average of how many fries you get each time you go [to the restaurant]?" The students disagreed, and one student said you should divide by the highest number in the set to find the average. Recognizing this mistake, Ms. Thomas stated, "Do you? Go ahead, and do this. Divide 723 by 60. You said that's the highest number." Once the students had calculated this, several shared that they found an average of 12.5. "Are you saying, then to me, that the average of fries is 12 fries? How can that be the average? We never had less than 41 ." When the students did not correct their error, Ms. Thomas went further with a suggestion. "Why don't we divide [723] by how many numbers are up here?" When students did this calculation and told her this value, she asked if that was a more reasonable value for the average number of fries. After students agreed, she had them start the task.

A student read the task out loud, and Ms. Thomas asked them to think about what strategy they would use to solve the problem. As students talked with one another, she handed out chart paper, told them to work on this problem, and to put their work on the chart paper. This work, she stated, should include their "group's decision and the mathematics to back that up." To verify students understood the task, she asked them, "Who knows what mode is?" A student responded that the mode is the number that occurred the most, and Ms. Thomas wrote this term on the vocabulary chart paper. Continuing to ensure clarity with the task, Ms. Thomas asked her class how they could
find the range. A student responded, "Take the smallest number and the biggest number, and divide it." Ms. Thomas replied, "You're close." Then Ms. Thomas walked to the back of her room where another white board was on the wall. She wrote the numbers 90, 93 , and 95 on the white board and told her students to think of these as some of her students' grades. She stated to the students,"I would take the highest number minus the lowest number. They were within a five-point range of each other. The range is the highest number minus the lowest number. Everybody feel ok about that?"

Ms. Thomas then set her timer for five minutes, asking the students to think about what strategy they would use to solve the task. After the timer went off, she asked the students to share their ideas. "I think we should try to find the mean for the first part because that worked in the warm-up [task]," a student responded. Ms. Thomas continued by asking the class how they would find the average. A student responded, and Ms. Thomas restated what she said for the whole class. Ms. Thomas then reminded the students that their poster was not about "looking pretty" but was supposed to be "mathematically correct." She set the timer again and asked them to continue with solving the task. The students worked in groups as Ms. Thomas circulated the room, stopping at groups if the students had a question. Although I was not able to hear all of these interactions, a typical interaction included the students looking for her verification of their work. At one group, Ms. Thomas told the students, "You should always check your answers twice. [inaudible] I'm just saying. Let's check our answers again." This type of assistance was typical in the Day One implementation. When the timer went off, she noticed that many students were still working on the first part of the task and had not
made it to the second part of the task. She put more time on the timer and asked the students to focus on solving the first task instead of the second.

After this time was up, Ms. Thomas selected two groups of students to share their work. The first group of students presented their procedure for finding the average. When the students gave an incorrect procedure for calculating the average, Ms. Thomas stopped them. "You divided 48 by 15 . Where did the 48 come from?" The group of students explained where they found 48 . Ms. Thomas utilized this opportunity to ask the other students in the class to tell the group what mistake they had made. She then noticed another error, "I'm a little concerned about the mode. Mode, which means that's the one that occurred the most often, is 76, but I don't see 76 in [the data set]." The presenting students retorted, "We added 43 and 48 because 43 and 48 both occurred three times. So we added them both and subtracted." Ms. Thomas asked the class to thank these students for sharing and asked the second group to present their work. Before the second group started, Ms. Thomas asked the first presenting group to pay attention to see if they could "catch on to the difference" in their work and in the work to be presented as a way to correct their mistake in finding the mode. The second group described their procedures for finding the statistics. She had the class thank the second group of presenting students and asked everyone to sit down and pay attention to her.

During this second presentation, Ms. Thomas noticed an error in the students' average calculation so she decided to show her teacher solution sheet for the task on the projector to the entire class. "This is not how we did it this summer [in the professional development], but this is how I'm going to do it because I want you all to have a little bit stronger start." Recognizing that the second group of presenting students had potentially
missed a number in their sum of all the values, Ms. Thomas again discussed checking that you have all of the numbers in your calculations as she had during the warm-up task. When you're doing statistics, it's kind of hard if you leave one number out or you add one thing wrong one time. It skews all the data so it really messes it up. So you've really got to make sure you do these [calculations] twice.

With the teacher solution sheet still on the projector, Ms. Thomas directed the students' attention to the answers for the average number of fries from the two restaurants. "So that's the average number. Was it better to get the fries from Burger King with 49 fries or McDonald's for 48 ?" The students concurred the former would be the better than the latter choice because you would get more fries. Continuing with this demonstration, Ms. Thomas showed the table that the students were asked to complete in the second part of the task.

I want you all to look at this [table]. Nobody made it to this because we don't have the days in the schedule to do this. I want you to see what I meant [by table]. You were meant to make a chart for McDonald's, and give me the mean, median, mode, and range.

Ms. Thomas pointed to the dot plot on the teacher solution sheet, explaining from where the dots and numbers on the number line came.

This is a dot plot. You've never seen one? Basically, you just put, ok, there [was] one fry that had 40 [fries]. And there were two 41 's on that plot. There's 43 , there were three. You just plot them. That's called a dot plot. Everybody understand that?

Ms. Thomas stepped to the vocabulary chart paper, asking the students to tell her synonyms for mean, median, and mode. The students answered with average, middle, and most. "Who can remember how we did the range?" she questioned. A student responded, "You take the least and the greatest." Ms. Thomas clarified, "You take the biggest number and minus your smallest number, and that's how much a range in between those two numbers." Recognizing that her class period was about to end, Ms. Thomas asked students to take a sticky note on which they would write the responses to their exit ticket. As she circulated the room collecting their chart papers, she asked them to write down "two big ideas that you learned today, and answer one thing that you want to know more about or that you didn't understand." As students wrote their answers, they posted them on a board on the wall that had a square for each student to place exit tickets. After they posted their exit tickets, the students were allowed to leave for their next class.

## Fidelity of Implementation

Within this section, I first describe the deviations of the enacted lesson from the intended lesson for Day One. Then I discuss the alignment of the enacted lesson to reform expectations as outlined in the Daily Observation Protocol.

Deviations of enacted lesson from intended lesson. In the intended lesson plan as seen in Table 2 and Appendix E, the goal of the first day was to have students engage in the French Fry task, addressing the procedures for calculating the mean, median, mode, and range through a conceptual focus and interpretation of those statistics in context. The data revealed that Ms. Thomas deviated from the intended lesson plan in three ways. First, in the lesson plan, the lesson should have opened with an introductory question, "What do you know about statistics?" However, Ms. Thomas forgot to open
with this question and instead placed the warm-up task on the projector. She quickly recognized this and asked the introductory question. The purpose of this question was, in part, to determine students' previous knowledge of statistics and also to set the norm that student responses were valued within the classroom. This question made it apparent to Ms. Thomas that the students had limited knowledge of statistics. This was evident in her statement, "Today's lesson was an eye opening [sic] experience in the fact that I did not feel my students were where they needed to be to understand the terms" (Participant Journal, 12/5/14). During this part of the lesson, Ms. Thomas deviated from the lesson plan when she introduced the football scenario. "Peyton Manning throws the football 15 times during a game," (Video, 12/5/14) she began. She continued this scenario, describing what type of statistic this scenario would provide. This scenario was not in the intended lesson plan for Day One.

Second, Ms. Thomas' realization that her students were not as prepared in statistics as she had anticipated led her to present the procedures for computing the various statistical measures more so than was in the intended lesson. Video data revealed that Ms. Thomas demonstrated how to calculate the median on the board with students' input on the procedure. Later in the lesson, Ms. Thomas also told the students how to calculate the mean. When the class discussed how to find the mean for the data set, Ms. Thomas recognized that many students were calculating the mean incorrectly, which subsequently led her to telling the students the procedure for the mean. "Let's try something different. Why don't we divide [our sum] by how many numbers are up here?" (Video, 12/5/14). This type of instruction also continued for the mode and range. When she asked the students how to calculate the range, their responses were incorrect. Ms.

Thomas responded, "I would take the highest number minus the lowest" (Video, 12/5/14). Ms. Thomas acknowledged this focus on procedures in her interview that afternoon. "I know you want them to learn all this on their own, but I could tell [that] they're going to have to be told, what is mean, median, you know?" (Interview, 12/5/14). I noted in my journal that this change in focus to procedures seemed to originate and be seen as necessary by Ms. Thomas when students "responses [were] not what she expect[ed] or [were] noted in the curriculum" (Researcher Journal, 12/5/14). This type of instruction was not in the intended curriculum for Day One.

Finally, after the students' presentations, Ms. Thomas recognized that the students had incorrect solutions. This led Ms. Thomas to project the teacher solution sheet for the task to the whole class, an action that was not in the intended lesson for the day. "Let's look at what the answer would have been," she stated (Video, 12/5/14). Ms. Thomas demonstrated the statistics for each data set and reminded students:

It's kind of hard if you leave one number out, or you add one thing wrong one time. It skews all the data. It really messes it up. So, you've really got to make sure you do these [calculations] twice. (Video, 12/514) Ms. Thomas continued this demonstration by showing students the summary statistic table and dot plot. "You were meant to make a chart for McDonald's, and give me the mean, median, mode, and range. This is a dot plot" (Video, 12/5/14). This demonstration of the procedures for the task was not in the intended lesson plan. During the interview, Ms. Thomas, as seen in the last quote of the previous paragraph, stated that this was necessary. Further, she talked about how for this group of students she had to follow the lesson plan "step-by-step-by-step. It would help me guide the first group better and let the
other group [her third period] that needs to soar, soar" (Interview, 12/5/14). This statement indicated that Ms. Thomas felt she should follow the lesson plan exactly to be able to guide her students.

Alignment of enacted lesson. Analysis of the Daily Observation Protocol for Day One revealed that Ms. Thomas' implementation fidelity for the first lesson addressed parts of three sections within this protocol. In the analysis that follows, I exclude sections of the protocol that were not evident during the lesson.

Statistical problem-solving process. Students were partially engaged in the third component of the statistical problem-solving process (i.e., analysis of the data). Video data revealed that students had multiple opportunities to practice calculating the statistics (i.e., mean, median, mode, and range) for two quantitative data sets. Although it may seem that if students were calculating statistics that they were engaged entirely in analyzing the data, however, the GAISE document (Franklin et al., 2007) described the analysis component as including a selection of appropriate methods and then using these methods to analyze the data. The nature of this task did not allow for students to select their own methods, graphical or numerical, to analyze the data, a reflection on the unit and not on Ms. Thomas.

Standards for Mathematical Practices. Students engaged in three of the Standards for Mathematical Practice. First, students persevered in solving the task. Analysis of the video data and field notes demonstrated that students had many misunderstandings about statistics. An example was first demonstrated during the introductory question. A student inquired of Ms. Thomas, "Can statistics be fractions or decimals?" (Video, 12/5/14). In my field notes, I also observed that during this
introductory conversation many students could recall that they had learned a song about calculating statistics, but none of the students could actually remember the song. This demonstrated that the students had not yet developed a "conceptual meaning for those statistics" (Researcher Journal, 12/5/14). Despite this struggle with content knowledge, the students continued working the problem using the knowledge they had to solve it. Second, the students appropriately used their calculator. The data sets for this task contained several data points that would have made for time-consuming calculations with the strong potential for error. Utilizing the calculator demonstrated efficiency and created more time for discussion. Finally, students reasoned quantitatively about statistics (i.e., addressing the meaning of the statistics contextually) that they calculated incorrectly when prompted by Ms. Thomas. For example, when the students miscalculated the average number of fries, Ms. Thomas questioned their answer, "How can that be the average? We never had less than 41 [fries]?" (Video, 12/5/14). When the students correctly arrived at the average, many students voiced that they agreed their new calculated mean seemed more reasonable, recognizing the importance of contextualizing the answer.

Mathematics Teaching Practices. My field notes and the video revealed that Ms. Thomas engaged in two of the Mathematics Teaching Practices during this lesson: posing purposeful questions and eliciting and using student work. First, she had the students' reason when they had incorrectly calculated statistics for the data set. Asking the students to reason about the incorrect answer allowed Ms. Thomas to engage the students in a discussion about their work, posing purposeful questions regarding the reasonableness of their answers (see quote in previous section). This student-work
discussion was evident during the presentation at the end of the lesson when Ms. Thomas asked the first group to focus on the second group's presentation with the goal of perhaps noticing the error in their own work. "Ok, girls. I want you to watch to see if you catch on to the difference" (Video, 12/5/14). Second, Ms. Thomas elicited and used student thinking multiple times throughout the lesson. For example, Ms. Thomas asked students to think individually and then "talk to [their] shoulder partner" (Video, 12/5/14), calling on students to share their ideas. This instructional strategy continued throughout the lesson, ending with her asking students to complete their exit ticket as described in the lesson plan.

## Barriers

Through the data, it was clear that several barriers potentially affected her implementation per her observations. Specifically, she made comments that demonstrated barriers related to a traditional perspective and subject matter knowledge.

Traditional perspective. In her participant journal after the first day, Ms.
Thomas stated, "Today's lesson was an eye opening experience [sic] in the fact that I did not feel my students were where they need to be to understand the terms" (Participant Journal, $12 / 5 / 14$ ). Ms. Thomas echoed this sentiment in her interview the day of this lesson. "I know you want them to learn all this on their own, but I could tell [that] they're going to have to be told, what is mean, median, you know?" (Interview, 12/5/14). She continued later in the interview:

I know that the goal is to try to let the student come to the conclusions and answers without as much guidance, but this group wouldn't have gotten there
without me almost giving them the definitions like I kind of ended up [doing]. (Interview, 12/5/14)

These statements aligned with a traditional perspective of the teaching and learning of mathematics as defined by Tzur and colleagues (2001). Specifically, the view of teaching mathematics as the teacher telling students what they need to know about mathematics, in this case, vocabulary, is in contrast to the conception-based perspective that emphasizes students developing their own understanding of the content.

Subject matter knowledge. Throughout the interview, Ms. Thomas mentioned the general topic of subject matter knowledge several times. When asked to describe which parts of the lesson she felt the most/least comfortable, she stated, "I'm not comfortable at all with statistics. So, it's probably going to be the hardest unit I will teach because I'm not familiar—It's going to be the content itself [that] is going to be difficult" (Interview, 12/5/14). This focus on subject matter continued when I asked what resources she used in her planning for the lesson. "The internet. I used it kind of to understand what the words, I mean, sadly, what is mean, mode, range. I looked them up to make sure" (Interview, 12/5/14). Towards the end of the interview, I asked about how she might prepare and implement this lesson in the future. She replied, "This is unfamiliar in the sense that I've lost all that I've [learned]" (Interview, 12/5/14). Although from these quotes it is unclear if Ms. Thomas specifically struggled with specialized content knowledge or common content knowledge, in general, her statements support her perceived barrier of subject matter knowledge.

Interview data also revealed the barrier of horizon content knowledge. During the interview, I asked Ms. Thomas if after this first day of teaching, she wished she had had
more professional development in statistics. She responded, "I wish I was more familiar with where our students come from, what are the standards that they had to learn in fourth and fifth grade to know that my children-What level they were prepared for this" (Interview, 12/5/14). Ms. Thomas continued to reflect on this barrier in her journal. "Today's lesson was an eye opening [sic] experience in the fact that I did not feel my students were where they need to be to understand the terms" (Participant Journal, $12 / 5 / 14)$. She discussed this barrier further in her interview when she stated that she was "shocked to see that that [topic] was [pause] It was new for them" (Interview, 12/5/14). Her desire to know more about her students' previous mathematical understandings demonstrated horizon content knowledge as a perceived barrier to her implementation.

## Supports

Towards the end of the interview, Ms. Thomas revealed two different supports: experience and practice. First, in the question described above about professional development, Ms. Thomas stated:

I think there can be nothing better than what I'm going to be doing right now. Having [the unit] in front of me and having to learn [statistics] with the kids. I think that's going to be the absolute best way for me to get where I need to with it. (Interview, 12/5/14)

I coded this statement as indicating that this experience of using the unit was a support. I did not base this code upon the literature, but I noticed that experience became a recurring theme for her throughout many of our interviews and her participant journal entries. Second, Ms. Thomas later talked about how she would prepare for this lesson in the future. "I'm actually going to go home, and I'm going to put my college kid through
this once beforehand. I'm going to try to go through each lesson beforehand." I coded this support as one of practice. Ms. Thomas valued the ability to practice the lessons before she actually taught them, either with her own children or by reading through the lessons prior to teaching.

## Day Two

In this section, I describe the implementation of Day Two based upon the video data. Utilizing all data sources, I then describe the fidelity of implementation, barriers, and supports related to the implementation. On this day, Ms. Thomas extended the Day One task from one day to two days, a choice she made at the end of Day One. This choice was recorded in her participant journal at the end of Day One in response to a question regarding how her lesson implementation would change in the future. "I would not even show the second part of the task on day one" (Participant Journal, 12/5/14). This statement was in reference to her future implementation, but her notion that the task was too long for one day was evident in her extension of it from a one-day to a two-day task. Although this change should not be viewed as negative, it was not the expectation within the original unit timeline (see Table 2).

## Implementation

Ms. Thomas began Day Two asking students, "If you have any of that paperwork from Friday, get it out." As students collected their sheets from the French Fry Task, Ms. Thomas walked to the vocabulary chart paper and stood in front of the paper so that the students could not see the words. "These are the words we learned the other day. Who can tell me one of the words we learned from the other day? [paused] Median, what does median mean?" A student responded, "The middle number." Ms. Thomas asked the
entire class if they agreed or disagreed with this definition. The entire class agreed, and she moved to the next word on the chart paper. "What does mean mean?" A student responded, "The answer. The way you work it out." Recognizing the need for clarification, Ms. Thomas probed the student further. "Elaborate on what you're trying to say. We know that median is the number in the middle of all of our numbers. What is mean? Talk there with your table partner. See if she can help you." Continuing with the vocabulary review, Ms. Thomas questioned the class about mode. "What is mode?" "The number that appears the most," a student replied. Finishing this review of the terms, Ms. Thomas asked them to tell her about range, received a student response, and started the lesson.

Any questions about what we did Friday? Now, the problem with Friday was we didn't even really get through the first part of the task. So, I'm going to put it back up. Let's re-read what we're supposed to do.

Ms. Thomas asked two students to read the first and second parts of the task to the entire class. Reminding students of the parts of the task that they did not complete, she stated, "I want a representation. [off-topic conversation to a student] You can use a dot plot. You can use some other-tell me another way that you can show data." A student answered, "A graph." Wanting further description, Ms. Thomas asked this student, "What kind of graph?" The student replied, "A bar graph." Ms. Thomas acknowledged this response and directed the conversation to the entire class.

While you do this task, I'm going to go ahead and put the numbers back up for McDonald's, and you may have to re-find your mean, median, and mode for that
because some of you didn't keep it. . . . I want you to go ahead. I'm going to set the timer.

Ms. Thomas set the timer for 15 minutes, students began working, and she circulated the room, looking at student work. Addressing the entire class after they had been working for approximately one minute, "Does anybody feel like they are lost from Friday?" A student in the back of the room had a question about finding the range of the two data sets. Ms. Thomas walked to his table to help him. "Well, we talked about range. Let's talk about it some more. Let's say, we play a game of Sorry," she responded to the student. Another student joined this student and Ms. Thomas at the back of the room to hear the conversation. Ms. Thomas continued this scenario, writing the hypothetical scores for each of three students on the white board located at the back of the room. I want to know the range, the difference. I'm going to take 12 minus seven. There are five. There's a spread of five points in this range. Because you always take your largest minus your smallest to get your range. Do you understand?

After these students agreed that they understood the procedure described, Ms. Thomas helped another group of students also calculate the range, demonstrating the procedure. Moving to another group of students, Ms. Thomas recognized that an Englishlanguage learner was having a difficult time participating in his group. She solicited help from another student who spoke the student's native language. Utilizing the bilingual student, Ms. Thomas communicated to the English-language learner the synonyms for each of the statistics listed on the vocabulary chart paper. She then moved the two students to the back of the room towards the white board and wrote a small data set on the board. She asked both of the students, with the bilingual student translating, "What
one appears the most?" She pointed to each of the numbers in the data set, stating how many times each number appeared within the entire data set. "One time. One time. One time. Two times [emphasis added]." Above this last number, she wrote the word 'mode' and asked the two students about the median. After waiting for a response but not receiving one, she demonstrated how to find the median by crossing off numbers from each end of the data set until arriving at one number remaining in the middle. She circled this number and stated, "This is what's left. Median." She wrote this term above the median number in the data set. She continued, "Range. Take the biggest number. Biggest minus the smallest. Got it? You got it? Alright."

Ms. Thomas circulated the room, addressing students' questions as she passed. I was unable to hear the students' questions during some of these interactions, but her responses were typical of other instances of her helping students. In some groups, she wrote on the students' scratch paper, demonstrating a procedure or helping them to understand what to do next. Ms. Thomas asked a group, "Where's your table?" The students in this group had not created a table the way Ms. Thomas expected, but they had stated that they were finished with the task. The students showed Ms. Thomas their table, but she questioned the group further. "Where are the numbers mean, median, and mode?" A student in this group stated how they should change their table, Ms. Thomas agreed, and she walked to a different group of students with a similar issue. They showed her their work, and she recognized an error in their table. "You have to have [a table] for both [restaurants]. You made a chart for one. I need a chart for both."

Ms. Thomas addressed questions from two more groups, and then the timer went off. Recognizing a similar struggle among groups of students, Ms. Thomas asked all of the students to pay attention to her before they could continue working.

I realize that most of you are not finished so I'm going to give you more time. But I want you to realize, this part right here [the second part of the task] is what a lot of you are not doing. It says create a table for the McDonald's and Burger King data set that demonstrates. Would someone give me another word for demonstrate?

The students responded, "present," "example," and "explain." Ms. Thomas retorted: What I was kind of thinking about [was] label. You have to label the mean, median, and range for the fries in both companies. You're going to have something that shows both companies. Then we're going to see the mean of both companies, the median of both companies, and the range.

She asked the students to verify their understanding by raising a thumbs-up or thumbsdown. Some students did not respond so she continued. "If you're really lost, meet me at the back [of the room]. Everyone else, go ahead and continue."

Ms. Thomas walked to the back of the room where four students were waiting next to the white board. She asked one of the students what they were confused about, but when the student had difficulty verbalizing their issue, she changed her tactic. "Ok. Let's have a little mini-lesson here." Ms. Thomas wrote some numbers on the board and asked the students, "What does mode mean?" A student responded to her question, and Ms. Thomas went further by counting how many times each number appeared in the data set. "Three is in here one time. Four is in here two times. Five is in here one time. Six is in
here one time." A student stated that he thought four would be the mode for that data set. "Why would four be the mode?" Ms. Thomas asked. Some of the students replied, and she restated what one had said, "Mode is the number that appears the most." Ms. Thomas continued this mini-lesson by next exploring median. "What does median mean?" she asked the group. A student gave an inaudible response, to which she replied, "No. Median means middle, and the easiest way to find the median is to do this. Get rid of this one, and this one. Get rid of this one, and this one. What number is left in the middle?" The students stated the number in the middle, Ms. Thomas verified that each student understood the procedure, and she continued with the mini-lesson.

So, the range. You take the biggest number minus the smallest number, and that's the range. How do we find the average? [pause] We add them all, and then we divide by what? By this [counted the numbers in the data set]-five numbers. So, that would be your average. [pause] So, do you all understand the terms now? Once the students voiced that they understood the procedures, Ms. Thomas moved the focus of the mini-lesson to making a table to represent the statistics for each restaurant. She demonstrated on the white board how she would make a table, including labels for each of the statistics and restaurants. She reminded them, "Now, I also told you to do a representation," which was elaborated with her demonstration of how to create a dot plot. When she had shown how to create a dot plot using five of the data points from the task, she told the students to go back to their groups and "share [their] knowledge" with their group members.

After this mini-lesson, Ms. Thomas resumed helping other groups in the classroom. One group had a question about finding the mode for the Burger King data set
if multiple numbers appeared as the mode. A student from another group had told this particular group what he thought one should do if one has multiple modes. Recognizing this opportunity for a whole class discussion, Ms. Thomas asked the class to give her their attention. "I'm going to clarify something [a student] said. What if you had two 43's and two 45 's? You can have two modes. You can have three modes!" After this message for the group, the students continued working, and Ms. Thomas asked the student who had the incorrect idea about mode to come to the front white board, where she showed him how to find the mode for the data sets in the task. After talking with this individual student, the timer went off again, and a student in the back of the room voiced a concern that he still did not understand how to find the mode. Ms. Thomas addressed the class: We are having a little bit of trouble understanding mode. The mode of a number means how many you have that occurs the most in your data set. So, if I showed you my markers [showed several markers in her hands, mostly yellow], why is yellow the mode?

The students responded to this question correctly, and Ms. Thomas added more time to the timer. While the students worked, Ms. Thomas resumed circulating the room, answering students' questions. Although inaudible, Ms. Thomas appeared to be answering similar questions to the ones she had previously answered. This appeared true because when the timer went off for the final time, Ms. Thomas addressed the whole class again, acknowledging that students had been struggling with similar issues throughout the lesson.

Let's go over our data because some you think you just have to find the data for McDonald's, and some of you think you just have to find the data just for Burger

King. In essence, you have to find them for both. This has been a real big struggle for me. According to how I'm supposed to do this, I'm not supposed to be giving you all answers and showing you all what to do, but I'm trying to give you a good foundation to start with. So, I'm going to go outside the box, and I'm going to go ahead and give you the numbers, and you're going to check your numbers. And if you're numbers are right then I'm going to help you go a step further. So, everybody check your McDonald's numbers first.

Ms. Thomas then posted the teacher solution sheet on the projector, giving students the opportunity to check their answers for the McDonald's data set to the teacher solutions. Utilizing this opportunity to talk to the entire class, Ms. Thomas continued this conversation, addressing a group of students who had incorrectly calculated the average number of fries as eight. "Sometimes you have to use your instincts of 'guesstimating'." This group recognized their mistake, and Ms. Thomas took a question from another student regarding the mean for the McDonald's data. "How'd you get 48.2?" Ms. Thomas asked the student, "Well, what does mean mean?" The student replied with the synonym 'average.' "So you took all those numbers [pointed to data set]. You add them all up. You divide them by the amount of orders [of small fries]," she answered to the student. Ms. Thomas reminded the class to make a table representing data from both restaurants, and then called on one group of students who had finished their task to share their work. The students displayed their chart paper for the class and explained the procedures that they used to solve the problems. After this presentation, Ms. Thomas asked that a member from that group go to each of the other groups within the class to help them finish the task.

After a few more minutes of working, Ms. Thomas asked that the students reconvene into their original groups. She distributed sticky notes to the groups and stated:

I'm going to come around and give you each a post-it note. And we're going to rotate, and we're going to look at the other posters. And I want you to make a comment about the poster. One comment to help clarify things to you, and one comment that maybe they could have done differently. And stick it on the poster. Ms. Thomas asked the students to rotate their group's chart paper demonstrating their work to every other group in the class. Ms. Thomas elaborated that the comments on the sticky notes should not focus on "ugly colors" or "handwriting" but on an aspect of the group's work that helped the student better understand the day's topic. At the last rotation, Ms. Thomas asked the groups to present the comments on the sticky notes to the entire class. The comments that the groups presented were not the comments on their own chart paper. Rather, they presented the comments from the last poster they analyzed. After each presentation of comments, Ms. Thomas asked the entire class to clap for that group's work and then the next group presented. This continued until all of the groups had presented. Once these presentations were complete, Ms. Thomas distributed the homework and dismissed the students.

## Fidelity of Implementation

Within this section, I first describe the deviations of the enacted lesson from the intended lesson for Day Two. Then I discuss the alignment of the enacted lesson to reform expectations as outlined in the Daily Observation Protocol.

Deviations of enacted lesson from intended lesson. Using the Day One lesson plan as a guide, Ms. Thomas deviated from this lesson plan in four ways. First, at three
distinct times during the lesson, Ms. Thomas called students to the back of the room to the white board to have a "mini lesson" (Video, 12/8/14). In two of these mini-lessons, Ms. Thomas described the procedures for finding the statistical measures in the task. In the final mini lesson, she described these procedures as well as how to create the table and dot plot for the task. These mini lessons involved Ms. Thomas writing either numbers from the task or numbers from another scenario (e.g., the Sorry scenario described previously) and using those to demonstrate the procedures for the statistics, table, and dot plot. Ms. Thomas stated that she felt these mini-lessons were important despite not being a part of the lesson plan. She stated, "I know we're not supposed to give them the answers, but some of them, if I don't show them, this is this gets this, they'll never get it" (Interview, 12/8/14). In this interview, I asked her to describe how the students had interpreted their data. In reference to this question, Ms. Thomas highlighted the incorrect student interpretations as another justification for these mini-lessons.

And, so, again [the incorrect student interpretations] helped me realize more so that maybe [the] $25 \%$ that still needed me to visually show them, which is why I took them back to the board, and we went over what each one of the words looks like with words-I mean with data. (Interview, 12/8/14)

Second, while the students were working on their task, Ms. Thomas circulated the room, helping students directly at their groups by showing them the procedures for finding the statistical measures. This type of explicit instruction was not in the intended lesson plan. My researcher journal indicated that at one of the student groups, Ms. Thomas incorrectly told the students how to calculate the median. Although I was unable to ascertain the exact type of help she gave at each of these groups, in the sessions that I
could hear, Ms. Thomas typically told students how to calculate the statistics and create the table and dot plot.

Third, towards the end of the lesson, Ms. Thomas noticed that many of the students were still asking similar questions. When the timer went off, she pulled the students together as a whole group and asked them to focus their attention at the front of the room.

I'm not supposed to be giving you all answers and showing you all what to do. But, I'm trying to give you a good foundation to start with so I'm going to go outside the box, and I'm going to go ahead and give you the numbers, and you're going to check your numbers. (Video, 12/8/14)

At this point, Ms. Thomas projected the teacher solution sheet, an action that was not in the lesson plan. Similar to Day One, Ms. Thomas used the teacher solution sheet to show the students the answers for the task hoping that this would give the students "a good foundation" (Video, 12/8/14).

Finally, Ms. Thomas deviated from the lesson plan at the end of class when she asked the students to use sticky notes to comment on each other's posters. In the unit, the end discussion was intended to be a presentation of three student solutions followed by a discussion aimed at the students' conceptual understanding. In her version of the lesson conclusion, she asked the students to rotate their posters to the other groups. On their sticky notes, she asked them to write "something that helped you, and then something that you see that you might could have done better" (Video, 12/8/14).

Alignment of enacted lesson. Analysis of the Daily Observation Protocol for Day Two revealed that Ms. Thomas' implementation fidelity for the second day of the
first lesson addressed parts of two sections within this protocol. In the analysis that follows, I exclude sections that were not evident during the lesson.

Standards for Mathematical Practice. Students were engaged in two of the Standards for Mathematical Practice during this lesson. First, students persevered in continuing to solve the task presented to them. In the video data, it was apparent that Ms. Thomas continued to give the students time to solve the problem when she realized that they were struggling more than she anticipated. She felt that this extra time for letting the students persevere was important. "I think after spending the extra time on this lesson, the students were able to get a better understanding. I know [sic] feel better about moving on" (Participant Journal, 12/8/14). Second, the students attended to precision by ensuring their tables were properly labeled and that their calculations for the statistics were calculated accurately. Ms. Thomas encouraged this precision by reminding students during the lesson about the importance of labels on their tables. She asked a group, "Where are your labels?" (Video, 12/8/14), and she later reminded the whole class, "I need a chart for both [restaurants]. . . . You need to label it" (Video, 12/8/14). The Daily Observation Protocol, video data, and research journal also indicated that Ms. Thomas pressed students for precision specifically in their calculation of the average, asking them, "How can you have [this data] and the average [of eight]?" (Video, 12/8/14).

Mathematics Teaching Practices. Analysis of the Daily Observation Protocol revealed that Ms. Thomas engaged in the practice of eliciting her students' thinking through a presentation and the sticky note poster review at the end of the lesson. Ms. Thomas asked one group of students to present their work to the entire class. After this group of students presented, she asked the group "to disperse to the other tables, and what
you all are going to do is stand in the background and [the other students] can ask you questions" (Video, 12/8/14). She used these students to help other students continue working on their task. At the end of the lesson, Ms. Thomas asked the students to comment on each other's work through the sticky-note poster review, emphasizing that they not focus on the aesthetics of the group work. After the students had an opportunity to review all the other posters, she called on students to share the comments on the posters.

## Barriers

Through the data, it was clear that several barriers potentially affected her implementation per her observations. Specifically, she made comments that demonstrated barriers related to a traditional perspective, pedagogical content knowledge, and horizon content knowledge.

Traditional perspective. The field notes revealed that Ms. Thomas provided direct instruction on how to calculate the statistics and create the table and dot plot many times throughout the lesson. This direct instruction happened either at the group level when a student would solicit help or at the back of the room in what Ms. Thomas called a "mini lesson" (Video, 12/8/14). In my journal, I also noted that Ms. Thomas approached me during the lesson, referencing how she was teaching. I wrote, "She said to me during the lesson that she knows she should not teach this way (i.e., giving [the students] answers) but [she] is going to do it anyway" (Researcher Journal, 12/8/14). This statement was echoed three times during the interview. First, Ms. Thomas stated, "I know we're not supposed to give them the answers, but some of them, if I don't show them this is this gets this, they'll never get it" (Interview, 12/8/14). Second, she commented on how
the incorrect student interpretations of the data confirmed that her mini lessons were necessary.

And, so, again [the incorrect student interpretations] helped me realize more so that maybe [the] $25 \%$ that still needed me to visually show them, which is why I took them back to the board, and we went over what each one of the words looks like with words-I mean with data. (Interview, 12/8/14)

Finally, she also made a comment about the one student who had incorrectly told another group of students how to find the mode if there were multiple modes within the data set. "I said [to the boy], please don't tell them what to do unless you know what you're doing" (Interview, 12/8/14). These comments reflected a traditional perspective of mathematics in that they emphasized that the teacher views his or her role in the classroom as being the one who transmits the knowledge to students. It also aligned with this perspective in that Ms. Thomas wanted the students to practice until they arrived at the correct answers for the task.

Pedagogical content knowledge. In terms of pedagogical content knowledge, I identified two barriers related to the knowledge of content and teaching and knowledge of content and students. In terms of knowledge of content and teaching, Ms. Thomas stated in her interview, "That is a struggle as a first time teacher of this subject. I don't know what they've got. I don't know how long to spend and make sure to hammer it" (Interview, 12/8/14). This idea of knowing how long to spend on a topic aligned with the knowledge of content and teaching. In the interview, Ms. Thomas reflected on the lesson and seemed surprised when students were unfamiliar with the topics. She stated, "I thought they would have known that [mean, median, mode]" (Interview, 12/8/14). These
statements aligned with PCK because Ms. Thomas was not aware of where the students might have difficulties and their learning progressions.

Knowledge of content and students. Knowledge of content and students also appeared as a barrier for Ms. Thomas' implementation for Day Two. In my journal, I noted, "It appeared to me that she did not know what students knew coming into her class" (Researcher Journal, 12/8/14). This statement was supported in her journal. "I feel that I have possibly started this unit without proper knowledge of what the students are capable of. I will research $5^{\text {th }}$ grade standards and make sure my students are more familiar with stats in the future" (Participant Journal, 12/8/14).

Subject matter knowledge. My researcher journal revealed that when Ms. Thomas circulated the room and helped students, she incorrectly told students how to calculate the median for their data set. Although I was unable to hear the entire conversation she would have with students, Ms. Thomas made an error describing the procedure for finding the median of an even-numbered data set. For example, if a data set had six data points, she told the students to look at the two numbers in the middle and see which one appeared the most throughout the data set. She stated that that number would be the median. If neither of the numbers appeared more than the other, she claimed there was no median.

## Supports

In analyzing the data for Day Two, one support structure was revealed: time. In her journal, Ms. Thomas talked about how she would implement this task in the future. This lesson in the future will definitely need to be broken into two days. As I stated in our interview, I would break day one into two days and definitely start it
with a day or two to revisit without a weekend break. (Participant Journal, 12/8/14)

I coded this as a support because later in her journal, Ms. Thomas referred to the benefit of this extra time spent on the task. "I think after spending the extra time on this lesson, the students were able to get a better understanding. I know [sic] feel better about moving on. Without today I think day2 [sic] would be very confusing" (Participant Journal, 12/8/14). Ms. Thomas saw this extra time spent on the task as essential for students developing an understanding of the content and preparing them to continue with the unit.

## Day Three

In this section, I describe the implementation of Day Three based upon the video data. Utilizing all data sources, I then describe the fidelity of implementation, barriers, and supports related to the implementation. On Day Three, Ms. Thomas implemented the original Day Two task: Answering a Statistical Question. As indicated in Table 3, the goal of this task was to have students understand what constitutes a statistical question, collect data for a quantitative statistical question, and represent that data with an appropriate graphical representation and statistics.

## Implementation

To start the lesson, Ms. Thomas told the class, "I want you to think about this question to yourself," as she posted the lesson's warm-up sheet on the projector. After giving the students some time to examine the two columns of questions, she stated to the entire class, "For one to two minutes, I want you to talk with your partner . . . and decide what makes a statistical question looking at that [warm-up task]. Look at the left. Look at the right." After students had time to share with their partner, Ms. Thomas asked them to
talk to their group. She then probed the entire class, "Can somebody tell me what is different about the left side versus the right side?" Although I had difficulty hearing the responses, one student indicated that the left column was "more than one person and [the right] side tells that one person." Acknowledging the student's response, Ms. Thomas wrote on the left column group and on the right column individual. Verifying what the student said, Ms. Thomas questioned, "So, you're saying this [pointing to left column] is more of a group, and this [pointing to right column] is more of an individual. Is that what I hear you saying?" The student agreed, and Ms. Thomas asked for more students to respond. A student stated, "The thing that my group discussed was that like statistics are supposed to be more than one so if you just have one, you have nothing to compare it to." Ms. Thomas restated what this student had said, wrote his response on the board, and repeated this process for another student.

With the student responses on the board for all to see, she questioned the class further, "So, what are we saying makes a good statistical question? What has to be present for it to be a good statistical question?" The students replied, "More than one," "Multiple," "Plural," and "Several." Trying to get students to use the appropriate word, she continued her questioning. "The data doesn't need to be all the same. It needs to?" A student replied, "Be different!" The teacher restated this student's response and continued to question them. "Who can come up with another name for different?" she asked. When students still did not respond with the word she wanted, she reminded them of the French Fry Task. "We were talking about the fries yesterday-the numbers 41, 45, 43, 46-It wasn't that they were unusual. They were not alike. They were?" When students still had yet to respond with the appropriate word, Ms. Thomas walked to the vocabulary chart
paper and pointed to the word vary. She used the definition of this word to segue into a review of the other vocabulary words learned so far.

Ok, so, [the numbers] were varied. So, we're going to be talking about statistical variability. [off-topic conversation to a student] We're going to be talking about the fact that you don't take statistics on things that are the same or on one thing. You have to have a variety of information to collect your data. Does everyone understand that? We need to remember [points to the vocabulary chart paper]let's have a quick review. Median?

Ms. Thomas went through each of the vocabulary words, restating students' phrases as they responded with the procedures for finding each statistic. For example, she asked, "What is mode?" When a student responded, "The most of something," she pushed him further by asking him to review the example data set she had written on the board. "Look at my data. What is the mode?" she asked this student. The student replied correctly, and she countered, "The number that appears the most." Ms. Thomas ended this review by verifying that everyone felt comfortable with the definitions for those words.

Ms. Thomas then posted the statistical questions sheet from the unit on the projector and read the last question first. After reading this question, she asked the class, "Let's just do a quick poll. How many people in here have left the state?" Every student raised his or her hand, and she replied:

So, if we were statistically speaking, this information doesn't vary because everyone has [traveled outside of the state]. What if I said how many of you have, and I listed all the states, and I said, how many of you have been to Kentucky?

Some students responded to this question by raising their hands. Ms. Thomas continued by asking how many of the students had also visited Georgia and then Mississippi. As the students responded, Ms. Thomas wrote the number of students that had visited those states on the board under the name of each state. After polling the class, she posed the questions, "Did I just collect statistical data? Did these numbers vary?" The students agreed to both of these questions. Ms. Thomas read through the rest of the statistical questions on the sheet, and when she finished reading, she told the students that they would explore the first question listed (i.e., "How many letters are in the first name of my classmates?"). She reminded the students of their roles within their groups and passed out sticky notes to each group. "You're going to put in big, bold letters or numbers, how many letters are in your first name. How many letters are in your first name? Be counting that up," she directed the class. The students began working on the task, and she reminded them, "As soon as your group's finished, put them up here. Put them up here on the board. Make sure they're bold and big if you can." A student asked for clarification regarding how many sticky notes her group should have to post on the board. "Your name is one. Her name is one. Her name is one. One post-it note per person," Ms. Thomas replied. As the group's finished, one member from each group took all the group member's sticky notes and posted them on the white board in no particular order. Ms. Thomas noticed that some students had not made their number bold enough for the entire class to see. For these students, she traced over their response in a dark marker.

When all of the students had posted their sticky notes, Ms. Thomas addressed the entire class. "So, you now have a sheet of paper that says, find the mean, median, mode,
and range of the data set for the entire class." Then recognizing that this was not the first question on their task sheet, Ms. Thomas stated to the whole class:

What do you need to do? Well, it says. The first question says: write the statistical question that you are exploring today, and describe how you collect the data to answer the question. What are we doing today? What is this [points to the statistical question still on the projector] question?

A student replied, "How many letters." Ms. Thomas agreed and stated:
The data collector in each group needs to fill out number one, and it says, describe how you collected the data to answer the question. Let's take about three minutes and get through that part because you all-you've got to work together as a team. I want to hear some conversation.

The students began working, and Ms. Thomas circulated the room, looking at students' work.

After the students worked on the task for a few minutes, Ms. Thomas asked the students again about the statistical question that they were exploring. "What is our question? What does this data represent?" she asked. When the students replied correctly, she moved to her next question. "Describe the second part. The second part of the question says to describe how you collected the data to answer that question. How did we collect this data?" A student incorrectly responded, "Add." Ms. Thomas questioned this student's response, and before this student could reply, a second student gave his answer. He stated:

We got a sticky note, and we wrote the letters of our name. And then you put them on the board so everybody can see them, and you write them down on a
piece of paper, and add them all together, and get the amount of all the classmates.

Satisfied with this response, Ms. Thomas asked students to continue with their task, finding the mean, median, mode, and range for the data set. "I don't care if you come up here. If the data collector . . . needs to come up here to look at anything [referring to the data set], that's up to you." Ms. Thomas then set the timer to 20 minutes, and the students began working. Some students went to the board to write down the data set, and others were able to copy the data set from their seats.

As students worked, Ms. Thomas walked around the room, looking at the students' work. Stopping at a group, Ms. Thomas commented, "I love that you are jumping in, but if you make one error, then it's going to be hard to go back and look. So, if I were you, I'd get all the numbers first." She had noticed that this group did not write down the data set, and instead had started entering the numbers directly into the calculator to calculate the average. After this comment, the students wrote down the entire data set and then calculated the average. Ms. Thomas made her way to another student who asked an inaudible question about the vocabulary words. Ms. Thomas responded, "Add up all the numbers, and divide by [how many there are]. What would that be?" The student incorrectly replied, "Mode." Clarifying for the student, Ms. Thomas said, "Mode is the number that appears the most. Add them all up, and divide?" When the student correctly replied with mean, Ms. Thomas affirmed her response and moved to another group of students. At this time, all of the groups had sent one student to the board to write down all the data. Recognizing an opportunity for a whole class discussion, Ms. Thomas rang a bell on her desk and asked all of the students to pay attention to her. She
queried the class, "I really anticipated somebody would come up here and go [started physically moving the sticky notes to numerical order]. Nobody felt the urge to put them in order? Because that's the first thing that got to me." Stopping before she had put all of the numbers in order numerically, she then asked the students about their work so far. "Share what you are doing now," she asked one group. "We added them all up, and then we divided it by 18 which is how many numbers there is [sic]," a member from the group replied. Ms. Thomas asked them, "What are you trying to find now?" The students replied, "The mean." Probing further, Ms. Thomas asked, "It has another word. So, if I said to the class, do you want to know what your mean is for your grades? I don't usually say that. I'll say, do you want to know what your?" A student replied, "Average is!" Satisfied with this response, Ms. Thomas agreed, asked the students to continue working, and proceeded to put the rest of the sticky notes in numerical order on the white board.

Once Ms. Thomas had arranged all of the sticky notes, she counted how many students were in the room and then addressed the entire class. "There's [sic] 19 numbers. That could make a difference." Recognizing that the group that had just shared had incorrectly counted 18 numbers instead of 19, Ms. Thomas asked this group, "Would you please divide 800 by 18 ? What is 800 divided by 18 ?" A student in the group replied, and Ms. Thomas pushed further, "Would you divide 800 by 19?" When the student gave the answer, Ms. Thomas checked for understanding, "Did it make the answer different by not using the correct amount of numbers up here?" The class agreed, and she reiterated the importance of checking their work. Recognizing this potential error, many students from various groups went to the board, checking that they had written all of the numbers correctly. While these students were at the board, another student asked Ms. Thomas
about finding the mean of the data set. Ms. Thomas asked this student a synonym for mean, and the student responded, "Average." Ms. Thomas retorted, "You add all your data on this, in this case, the letters in our first name, you add all those up, and you divide by how many data there were. That's how you find it."

At this point, a group of students verbalized that they were ready to move to the next part of the task. Ms. Thomas announced to the entire class that those groups who were ready to move forward with the task could do so, and when they were finished, they "were going to compare data." As the students worked, Ms. Thomas wrote the words mean, median, mode, and range on the white board, and a student approached her, asking how to create a dot plot. She started by asking the student, "How many four's? How many five's?" Once Ms. Thomas felt this student understood, she asked for the entire class' attention. "Let's go on and discuss the mean, median, mode, and range." Ms. Thomas asked each group to tell her what they found for the mean, writing their responses on the board next to the word mean. Once all groups responded, she asked the class, "Looking at these numbers, is there one that we think may be wrong?" The students agreed that the number 62 looked incorrect. Ms. Thomas circled this number and questioned the group who had calculated that value. "What did you forget to do? What did you do to get 62?" The students gave an inaudible response, and she continued, "Did you first add all these numbers up right here? And then you need to divide by?" After this questioning, the group recognized their mistake and gave the correct answer. Before moving to the next statistic, Ms. Thomas asked, "Do you all feel pretty good about the mean now?" When most of the class agreed, she proceeded to the median. Ms. Thomas called on two groups to share, and the second group had not calculated the median
correctly. Recognizing this error, Ms. Thomas said to the group, "Let us show you how we've been finding the median." She went to the sticky notes on the board and began removing one sticky note from each end. Describing the procedure, Ms. Thomas said, "Take the first number off and the last number. Take the first number off and the last number, and we've been doing this all the way." When Ms. Thomas had one sticky note left, a student stated, "And if we have two [numbers in the middle], we divide!" Addressing this statement, Ms. Thomas replied, "And if we have two, what is the number that appears the most?" A student replied, " 6.5 " to which Ms. Thomas retorted, "I don't even see a 6.5. It's the number that appears the most. So, it's 6 . You see how we got it? [pause] Alright, what's the mode?"

Ms. Thomas received responses from all groups, writing their numbers on the board. Once all statistics had been addressed, Ms. Thomas asked the class to examine all of the data on the board. "I want somebody to tell me if something on [the board] stands out to you," she questioned. A student replied, "A bunch of sixes!" Responding to this statement, she said, "So, we probably have a good idea of what our average is going to be around. What else stands out?" After a few responses, Ms. Thomas heard the statement for which she was looking. "Why does 11 stand out to you?" Ms. Thomas asked. The student replied, "It's the biggest, and it's the only one sitting there." Ms. Thomas restated this student's response and probed the class, "Does it look like it kind of doesn't belong?" The class agreed, and Ms. Thomas elaborated, "In statistics this is called an outlier. Which means it kind of falls outside of what we're working with. Can everyone say outlier, falls outside the range?" The students repeated what she said, and she moved to the next part of the task.

Ms. Thomas picked up one group's page on which they had completed their dot plot and projected it for the entire class to view. "Now look at her dot plot. For those of you who didn't know what a dot plot is, it is how many times that number was present," she said in reference to the student work. The students responded to how many times each number was present in the data set, and Ms. Thomas checked for understanding. "Do you all understand a dot plot now? [pause] If I ask you to draw another representation of this data, who can come up with something they could do?" The students responded to this question, and Ms. Thomas wrote all of their responses on the board. The responses included a bar chart, a bar graph, and a tally sheet. After several students shared, Ms. Thomas stated, "Ok, well, in my third period, we came up with a histogram. Anyone remember what a histogram is?" She wrote the word histogram on the board and asked the students to recall what they had learned the previous year. Several students stated that they could not recall learning how to make a histogram. Ms. Thomas responded, "I had one of my students, and I kept their work [displays the histogram fake student work on projector], and they were using this for height. Ok, so here's a histogram. What do we notice about a histogram?" Some of the students responded with, "Bars," "Looks a lot like a bar graph," and "It counts by fives." Ms. Thomas restated these student responses and wrote them on the board as they were stated. Questioning them further, Ms. Thomas asked, "What is the difference [from a bar graph]?" "They're on the side," a student responded. Recognizing that the student was referring to the numbers on the x -axis not being in the center of the bars but instead being on the edge of the bars, Ms. Thomas restated, "They're on the side. Ok, so, basically this covers a bigger range, and they touch. Don't they?" Students agreed, and she continued, "Does it always
have to be five? No, but this [the $x$-axis] is their total height. And this [the y-axis] was how often a student was that height." She then told the students that they would be creating a histogram for the data they just collected, passed out chart paper, and then read the directions formally to the entire class. "You're going to create a histogram, and you're going to use the same data that we just used, and then we're going to compare the data."

Looking at the lesson plan after saying this, Ms. Thomas realized she had skipped a part in the plan. "Before you go, how did we get this number right here [pointing to $x$ axis]? How did we get this number on the bottom? The student that did this, where did they get these numbers?" A student responded, "From the data." Ms. Thomas agreed and questioned about the origin of the numbers for the $y$-axis. She asked, "I think I told you that, and I was supposed to ask you that. Where did this set come from [pointing to yaxis]?" A student replied, "How often." Ms. Thomas agreed and asked the students to begin creating their own histogram for their data. As the students worked, Ms. Thomas circulated the room, looking at student work. When the timer went off, she asked a group to share. After the students presented, Ms. Thomas asked the group from where the numbers on their x - and y -axes came and then called on another group to show their poster to the class. When the group was standing at the front of the room displaying their histogram, Ms. Thomas also put their dot plot on the projector. She asked the class, "Let's look at their dot plot, and hold it up to the side." She called a second group to stand at the other side of the white board, also showing their histogram. She questioned the entire class, "What do we notice looking at the histogram versus the dot plot?" Students gave responses such as, "Simple," "Both the same," and "One has rectangles,
and one has dots." Utilizing the second response, Ms. Thomas queried further, "The same what? Do they kind of look the same shape?" The class all agreed, and Ms. Thomas asked the class to clap for each group that presented. She asked the students to sit down, as one student collected all the group's chart papers. She was preparing to administer the wrap-up question when the bell for the end of the period rang, and she dismissed the class.

## Fidelity of Implementation

Within this section, I first describe the deviations of the enacted lesson from the intended lesson for Day Three. Then I discuss the alignment of the enacted lesson to reform expectations as outlined in the Daily Observation Protocol.

Deviations of enacted lesson from intended lesson. Analysis of the data revealed that Ms. Thomas deviated from the intended lesson plan in four ways. First, at the beginning of the lesson, Ms. Thomas engaged the students in a review of the vocabulary, a practice not included in the unit. After the warm-up task, Ms. Thomas stated, "We need to remember [points to the vocabulary chart paper]. Let's have a quick review" (Video, 12/9/14). In this review, Ms. Thomas emphasized the procedures for each of the statistical measures listed. For example, when she asked the students about finding the mode, she stated, "The number that appears the most" (Video, 12/9/14). The second deviation happened when Ms. Thomas posted the statistical questions for the task. In the intended lesson plan, students were expected to select the statistical question from the list that they would then explore that day. In the enacted lesson, Ms. Thomas selected the first question for them. She verbalized this change during the lesson. "I decided we are going to do [the first question]" (Video, 12/9/14).

The third deviation happened after the students had put all of their sticky notes on the board for the number of letters in their first name. In my field notes, I noted that she seemed to anticipate that someone "would have put them in order" (Field Notes, 12/9/14). This statement was affirmed in the lesson when she stated, "Nobody felt the urge to put them in order? Because that's the first thing that got to me" (Video, 12/9/14). After making this statement, Ms. Thomas then organized the data in ascending, numerical order on the white board, a practice that was not in the unit. The final deviation happened at the end of the lesson. In the unit, the students were expected to address a wrap-up prompt to summarize their learning for the day. Unfortunately, Ms. Thomas ran out of time at the end of the lesson and was unable to have students respond to this prompt. In the interview, Ms. Thomas acknowledged this when commenting about needing more time, in general, for the unit. She stated, "It may be that like, for every five days we have, need to be seven or something [for the unit] because like I did not get to the end of this [lesson]" (Interview, 12/9/14).

Alignment of enacted lesson. Analysis of the Daily Observation Protocol for Day Three revealed that Ms. Thomas' implementation fidelity for the first lesson addressed parts of all three sections within this protocol.

Statistical problem-solving process. Students engaged in the second and third components of the statistical problem-solving process, collecting and analyzing the data. Students collected their own data by putting the number of letters in their first name on a sticky note. Ms. Thomas also reinforced this component in the statistical problem-solving process by asking the students how they collected the data to answer their question. "How did we collect all this stuff right here [pointing to the sticky notes on the board]?"
(Video, 12/9/14). In terms of analyzing the data, students were calculating their own statistical measures and creating and comparing representations for the data. This was supported in the video data, for example, when Ms. Thomas asked the students, "What do we notice looking at the histogram versus the dot plot?" (Video, 12/9/14). Students responded with their thoughts, and Ms. Thomas restated all students' responses.

Standards for Mathematical Practice. In terms of the Standards for Mathematical Practice, the students engaged in one of the practices, using tools appropriately. This was evident when the students used the calculator to find the statistics of their data set.

Mathematics Teaching Practices. In terms of the Mathematics Teaching Practices, Ms. Thomas engaged in two of the practices. First, Ms. Thomas elicited and used student thinking multiple times during the lesson. The video data revealed several instances of her asking students for their response, restating what they said, and also writing their responses on the board. For example, when asking students about other representations they could create, many students responded, "Bar graph," "Bar chart," and "Line graph" (Video, 12/9/14). As students said these representations, Ms. Thomas wrote them on the board. Second, Ms. Thomas connected mathematical representations twice during the lesson. She first asked the students to compare the histogram that was displayed on the projector to a bar graph. Specifically, Ms. Thomas asked, "What is the difference [from a bar graph]?" (Video, 12/9/14). I wrote in my field notes that asking what "the students . . . notice" (Field Notes, 12/9/14) aligned with the expectation of the lesson. Towards the end of the lesson, Ms. Thomas concluded "the lesson [by] comparing the dot plot and histogram" (Field Notes, 12/9/14). She questioned students about the
shape of the two representations and how they might be similar, aligning with expectations for the lesson.

## Barriers

Through the data, it was clear that several barriers potentially affected her implementation per her observations. Specifically, she made comments that demonstrated barriers related to a traditional perspective, lack of knowledge of content and students, and subject matter knowledge.

Traditional perspective. My field notes revealed that when students worked during the lesson, Ms. Thomas would often help groups of students or individuals in a direct fashion, telling the students the procedure for the specific task at hand. For example, in my field notes, I wrote, "A student later asked, 'what is a dot plot.' She took him to [the] back of the class to the back board and gave him a mini lesson [on] how to do it" (Field Notes, 12/9/14). I also observed this type of assistance in the video data. At one point in the lesson, a student had a question about the mean. Ms. Thomas first asked the student for the synonym for mean and then focused on the procedures. "You add all your data on . . . the letters in our first name. You add all those up, and you divide by how many data there were" (Video, 12/9/14). I also made note of this direct help in my research journal. I stated that Ms. Thomas "doesn't seem to like students to struggle" (Researcher Journal, 12/9/14) and subsequently helped students directly when she noticed this struggle. This aligned with a traditional perspective of mathematics because of the emphasis was on rules and procedures instead of conceptual understanding.

Knowledge of content and students. I observed one specific instance of a barrier related to knowledge of content and students. When all students put their data on the
board, Ms. Thomas was surprised that no student wanted to put the data in numerical order by rearranging the data. This surprise is evident during the lesson when she stated, "Nobody felt the urge to put them in order?" (Video, 12/9/14). My field notes also indicated that "she did not anticipate" (Field Notes, 12/9/14) the students failing to see the need to put the data in numerical order, reflecting a barrier of knowledge of content and students.

Subject matter knowledge. The barrier of subject matter knowledge appeared four times during the lesson. The first demonstration of this barrier happened when Ms. Thomas posted the statistical questions on the projector for the class to view. She asked the students, "How many people in here have left the state?" (Video, 12/9/14). She continued with this type of question by asking how many students had visited several other states. These questions are not statistical questions in that the answer to the question does not vary. The second demonstration of this barrier happened when Ms. Thomas explained to a student how to find the median by removing a sticky note from each end until she arrived at the middle number. A student added to the conversation, "And if we have two [numbers in the middle], we divide!" Ms. Thomas extended this statement, "And if we have two, what is the number that appears the most? . . It's the number that appears the most. So, it's six. You see how we got it? [pause] Alright, what's the mode?" This statement reflected a misconception of how to find the median of an even-numbered data set. The third demonstration of subject matter knowledge as a barrier happened when Ms. Thomas transcribed all of the student data to the board for the class to examine. She asked the class to describe any data that appeared different than the rest of the data. This led to a conversation about outliers. Ms. Thomas described an outlier as "falls outside the
range" (Video, 12/9/14). This reflects a misconception about outliers. The final demonstration of this barrier happened at the end of the lesson when Ms. Thomas asked the students to construct their own histogram for their data. When the students presented their work, I noted in my field notes that the "students were making a bar chart and not a histogram," and later added, "I noticed that the bin[s] were not the same length [in their histograms]" (Field Notes, 12/9/14). I echoed this error in my researcher journal, noting that "she allowed students to think the histogram was created similarly as a bar chart" (Researcher Journal, 12/9/14). These both reflected a barrier of subject matter knowledge for statistics, specifically related to the median and histograms.

## Supports

In terms of supports, the data for Day Three revealed four supports: practice, experience, curricular support, and knowledgeable others. First, in my researcher journal, I noted that Ms. Thomas "seemed more comfortable today because she had practiced this before" (Researcher Journal, 12/9/14). This statement was supported when Ms. Thomas wrote in her journal:

Going through the lesson with one of my administrators helped me prepare better.
I always go over the lesson the night before with a 2 year old and 7 year old.
Being able to go over it without rushing helped me be more prepared. (Participant Journal, 12/9/14)

Ms. Thomas also talked about the importance of practice during our interview. I asked her what would have helped improve the lesson, and she replied, "Well, I definitely think if I had sat down and read through it five times instead of two times" (Interview, 12/9/14).

During our interview, I asked Ms. Thomas about her comfort level in teaching statistics and what could be attributed to her increase in comfort level. She retorted, "I feel more comfortable with the content," and later added, "Just being exposed to [the content]. Just more and more" (Interview, 12/9/14). This comfort level was also revealed in her journal in response to a question regarding any thoughts she was having about the unit. She wrote, "I am beginning to feel more comfortable with the content" (Participant Journal, 12/9/14). I coded this support as one of experience because it demonstrated how teaching the unit (i.e., experiencing) supported her in raising her comfort level with the content. The third support, curricular support, appeared during the interview in response to a question about what she felt was the most important for her being successful in implementing the curriculum. She stated, "Just having the unit. Just having it prepared and ready to go" (Interview, 12/9/14).

The final support Ms. Thomas identified was in reference to a knowledgeable other. The day of this lesson, she had met with an administrator prior to implementation. The administrator observed Ms. Thomas teach an earlier class period, during which Ms. Thomas taught the current statistics lesson. After this observation, Ms. Thomas and her administrator reviewed her observation score and discussed the lesson. Although I do not have data on their conversation, Ms. Thomas made me aware of this meeting prior to our interview. During the interview, I asked her about this meeting and if she felt the meeting helped her in feeling prepared for the lesson. She responded, "Yes, I think it helped me. New teachers are learning every day, and there's just things [they tell you] that you think, why haven't I been doing it that way?" (Interview, 12/9/14). This sentiment is echoed later in her journal when she stated, "I had my meeting before . . . this morning, and
going through the lesson with one of my administrators helped me prepare better" (Participant Journal, 12/9/14). This idea of a knowledgeable other was also mentioned in the interview when I asked if she felt talking with someone would help her when she is teaching a new topic in general. She replied, "Definitely. If you had somebody, whether it was an administrator or a mentor teacher, to go through a lesson-plan, of course" (Interview, 12/9/14). This was the first instance in which Ms. Thomas identified a knowledgeable other as a support.

## Day Four

In this section, I describe the implementation of Day Four based upon the video data. Utilizing all data sources, I then describe the fidelity of implementation, barriers, and supports related to the implementation. On Day Four, Ms. Thomas started the Construct Your Own Graph Task. The goal of this task was for the students to calculate the interquartile range for a quantitative data set, describe the distribution, and create a representation for the data set.

## Implementation

To start the new task, Ms. Thomas first gauged whether the students were enjoying the statistics unit so far. "Is everyone enjoying the statistical unit? Is everyone enjoying the way we're approaching it?" The students agreed, and she asked:

When I stand up here, and give you notes, and we go over a problem. You take notes. I give you a problem. If you think that way makes you retain the information better, raise your hand. If you think that doing this task that we're doing is helping you retain the information better, raise your hand.

A majority of the students agreed to the latter question, and then Ms. Thomas posted the warm-up for the task asking students if a question listed was statistical. She asked the class, "Think about this for just a minute." When the class finished reading the prompt together out loud, she called on a student to share his thoughts. The student disagreed that the question in the prompt demonstrated a statistical question, and Ms. Thomas requested he explain why it was not a statistical question. "Because it's talking about only one," he replied. Questioning the class further, Ms. Thomas asked, "What else is it talking about?" Another student replied, "It's only talking about your one math teacher. Like, you need other teachers to compare, like a fraction. You can't have only one. You need both." Verifying the student's response, Ms. Thomas said, "So, what you're saying is, this is not a statistical question because it's only comparing me to what I like. So, take a minute. Talk at your table, and see how we could make this a statistical question."

After the students talked to their group for about a minute, Ms. Thomas called on a group to share their response. "All of the math teachers," a student from this group stated. Leading the students to the idea of increased sample size, Ms. Thomas prodded, "If we did all the math teachers on this floor, there are two of us. So, if we wanted a broader range of data, what could we do?" A student replied, "Ask all the math teachers in the school," and then another student added, "We could change it from only math teachers to teachers." In response to these ideas, Ms. Thomas stated, "So, instead of 14 math teachers, you could do all the teachers in the building which is 40 -ish, 50 -ish." Seeming satisfied that students understood the prompt, next she decided to do a quick review of the vocabulary words learned so far. "Yesterday, we talked about median, mode, mean, and range. If you are $100 \%$ sure you know what that means, thumbs up."

Most of the class raised their thumbs except two students who had been absent the previous days. Recognizing that those two students would likely need some help understanding the words, she directed this quick review to those two students, asking them to look at a small data set she had written on the board. "So if we had four, five, six—what is the median?" One of these students responded, "Five." She continued by adding one number, "If we had four, five, five, six, what is the mode?" The two students responded correctly, and she continued with the other statistical measures. "Now, if I wanted to find the mean, which is the average, what would I do?" she asked. "You add them all up," one of the students replied. She asked what he would do after adding all the numbers, and the student responded, "You divide by how many numbers there is." Satisfied with response, she moved to asking the students about range. "Do you know what range is?" One of the two students shook his head no. She elaborated for him, "We take our biggest number, and we subtract our smallest number, and that gives us our range."

Following this review of vocabulary and procedures, Ms. Thomas asked the entire class, "Who wants to tell [the students who were absent] . . . what does varied mean?" Two students in the class replied, "Different," and Ms. Thomas elaborated on their answer. "We"ve been talking about [how] statistical questions need to vary. If I measured everyone in here, how tall you are, are the answers going to be the same?" Most of the class stated, "No." Continuing with this refresher, "They're going to vary. So, to have good statistical questioning, you're going to have varying answers and several to choose [from]. Who can remember these things [pointed to student work on wall] we made yesterday?" Answering her question, a few students responded, "Histogram." Reminding
students of the previous day's lesson, Ms. Thomas inquired, "When we put a histogram up to a dot plot, did they have the same [pause]?" Students quickly responded, "Shape," and Ms. Thomas advanced the students further, "It's different than our dot plot, in what way?" A girl replied, "A dot plot-you make, like, the number of dots, but the other ones, you just move it up and over to how many there are." To ensure that the class understood her comment, Ms. Thomas then demonstrated how to make a histogram quickly on the board:

When we do a dot plot, we can also say [drew a dot plot on the board], this is what a dot plot looks like. Now, on a histogram [drew a histogram on the board], we want to do four to six and then six to eight [marked these as bin widths]. We can do more [emphasis added] numbers-more of these statistical numbers in each [bin]. I think the sample we looked at had five in each [bin] yesterday.

After most of the class agreed with the representations that Ms. Thomas drew on the board, she transitioned the conversation to a new representation by putting the box plot student work from the lesson plan on the projector.

Look at it, and see if you can figure out how to make this representation. Now, this goes back to the same work that we followed from my other student of the histogram—still using the height. So, let's see if you can figure out this. Take a minute quietly to yourself to think about that.

The students readily talked at their tables about the representation as Ms. Thomas circulated the room listening to students. While walking between tables, she also examined the lesson plan that she was carrying in her hands, a practice not observed in prior lessons. After approximately two minutes, Ms. Thomas solicited group responses. A
student stated, "We agreed that the box might represent, like, if you're talking about height, it might represent the height of something." Ms. Thomas acknowledged their response, "So, it would be talking about the heights of the students. Can you be more specific?" "Those are the ones that are the answers for the mean, median, mode, and range," the student responded, referring to the dots in the box plot representing the minimum, first quartile, median, third quartile, and maximum. Wanting other students to analyze this response, Ms. Thomas engaged the class, "Let's check what she said. Ok, so, we can say that, well, obviously, this one [pointed to the median] is correct. That is the mean. Yes, that's correct." Wanting that group to explain more, she asked them about the mode and pointed to the third quartile, which is what that group had identified as the mode of the data set. "But why didn't the mode go to here [pointed to the true mode of the data set on the number line under the box plot]?" Ms. Thomas asked. When this group could not respond, Ms. Thomas asked the entire class to consider this. "I think we are on to something with part of this. . . Who wants to elaborate?" A student gave an inaudible response about the mode, and Ms. Thomas was unsure what the student meant. "I'm just not following you, but you're saying it could be there. You just don't see it. [called on another student] I want your thoughts on what's going on right here [pointed to the box]," she questioned. When Ms. Thomas recognized that it was difficult for the class to make sense of the different parts of the representation, she called a specific student to come to the board and demonstrate his thinking to the entire class. The student approached the board and shared his thoughts. Pointing to the maximum and minimum respectively, he elaborated, "This right here, and this right here. The dots at the end represent as the last number. The lowest number, and the biggest number." Ms. Thomas restated to the entire
class, "He's saying the dots represent the range. One is the biggest number in this group. One is the lowest. If you agree with that, show me a thumbs-up." Ms. Thomas then asked the student to continue. The student then incorrectly identified the third quartile as "the lowest mode" and the median " 58 would be the mean." Ms. Thomas corrected him, "Median." He agreed and then stated that he was unsure about the origin of the dot representing the first quartile. "And then I was thinking, why there was a dot right here, and I don't know," he observed.

Another student shared similarly at the board, but I was unable to view what the student was referring to due to a lack of parental video consent. From the audio and field notes, it appeared that this student had an incorrect interpretation about the origin for each of the five dots on the box plot. Acknowledging this error, Ms. Thomas responded, "I like that observation, but I think we're going in a different direction. What I want you to know [is], we have definitely found out this dot [pointed to the maximum] represents what?" A student responded, "The most." Ms. Thomas corrected and continued, "The largest number on here. What does this number [pointed to the minimum] represent?" A student chimed, "The lowest." Persisting with this discussion, Ms. Thomas asked, "The lowest number, and do we all agree that this [pointed to the median] is the median?" The class agreed, and she moved to her next question.

So, what if I asked you, how many values are in this data set? This is a data set [pointed to the number line below the box plot], and each of these marks [pointed to the tick marks on the number line] is a value. Take a minute there, and see if you can come up with a value.

Giving students a minute to think, she then asked if anyone had an answer. A student replied, " 25 ," to which Ms. Thomas verified, "So, there are 25 values in this data set. Does everybody understand that?" Most students agreed to this question.

Moving towards the end of this discussion, Ms. Thomas looked at her lesson plan and then queried the class, "So, the last thing that we have to decide is, what does this box [in the representation] represent? What does the end of this represent? Just the end." She directed students to think individually for thirty seconds, and when this time ended, she asked them to talk with their table group. After the students talked for a moment, Ms. Thomas called on a group to share their ideas. A group member stated, "Me and my group saw that the box is two parts, and we thought that at the beginning of the box and the end of the box, those would be numbers that were most mentioned. Am I close?" Answering her question, Ms. Thomas stated, "You're very close. . . . [What was the] first thing you said? What did you say?" The student restated, "Well, we noticed that box was two different parts." Wanting to elaborate on this statement, Ms. Thomas stopped the student and continued, "Two parts! Let's stay with that. Now that she's pointed that out, take about thirty more seconds, and discuss with your partners if that helps bring anything out." The students discussed this idea at their tables and then were asked to share their ideas. "Well, we said that there was three numbers on one side, and then there was three numbers on the other side, and then the middle of them, the median," a student observed. Ms. Thomas expanded on this student idea:

I think we're right there on it because first of all-we know that this [pointed to the median] is the middle. So, if I cut this right in half [drew a vertical like at the
median]-look at this separately [pointed to lower half of the data], and look at this separately [pointed to upper half of the data].

A student responded to this, making an inaudible comment about drawing more lines on the box plot. Ms. Thomas asked the student to come forward and show her thinking on the board. The student went to the board; drew vertical lines at the first quartile, third quartile, minimum, and maximum; and made an indistinguishable comment about the number of sections she created with the vertical lines. Ms. Thomas excitedly asked, "There's how [emphasis added] many sections?" The student at the board stated, "Two," and Ms. Thomas asked more about those two sections on each side of the median. "There's two sections on each side [of the median]. They're broken into what?" she asked the entire class. A student in the back of the room stated unsurely, "Quarters?" Ms. Thomas replied, "Who said it?! What's a one-fourth [called]? A quarter? We're going to talk about interquartile!" Ms. Thomas thanked the student who drew the vertical lines on the box plot and asked her to sit down.

Referring to the vertical lines that the student drew, Ms. Thomas continued this idea, inserting the appropriate vocabulary.

What we need to do, once we have our data set. We have a lower set [pointed to lower half of the data set], and we have a higher set [pointed to upper half of the data set]. This is the lower quartile. This is the upper. Let's find the median for this [lower] set of numbers. Just looking up there [at the data], what do you think it is?

No students responded to this query. Ms. Thomas glanced at her lesson plan, set it on her podium, and walked to the left side of the board where the lower half of the data was
projected. "I'm looking now for the median [from] here [pointed to median of the entire data set] to here [pointed to the minimum]. What do you think it is?" she asked the class. A student guessed, "Between 50 and 52." Pointing to the lower quartile dot on the box plot, Ms. Thomas asked, "Do you think this [pointed to lower quartile] represents it?" Ms. Thomas grabbed a marker and wrote interquartile 1 above the lower quartile, erased these words, and grabbed her lesson plan. "Let me look at my papers. This is the first quartile," she corrected herself. She wrote the words first quartile above the lower quartile and, pointing to the upper quartile, asked, "What do you think this represents?" A student stated, "The second," to which she replied, "Probably not the second." Another student excitedly stated. "The third!" Ms. Thomas agreed and wrote 'third quartile' above the upper quartile. Continuing to insert this vocabulary, Ms. Thomas then asked about the name for the median in terms of quartiles. "The second," a student answered. Verifying this in her lesson plan, Ms. Thomas continued, "So, the word quartile means dividing the data into quarters. . . . That took us a while to get there, but, now, looking at [the representation], is it visibly there for you? Do you see it?" The students agreed by showing a thumbs-up, and Ms. Thomas changed the projected sheet from the box plot to the quartile student work sheet from the lesson plan. "This same person who did this [box plot] started making a quartile sheet. So, let's see what's going on here."

Ms. Thomas asked the students to think individually and then at their table group about the quartile sheet. While the students talked, she asked them to retrieve their data, dot plot, and histogram from the previous day's task. Ms. Thomas then displayed both the quartile sheet and box plot from the lesson plan on the projector simultaneously, asking students for their thoughts on the student work displayed. "Alright, let's talk about what
you see going on up here." A student asked to come to board, Ms. Thomas agreed, and the student pointed to the median on the box plot projected on the board. "This is the middle," he stated. Ms. Thomas agreed and called on another student. The student commented on the quartile sheet, "So, the middle one's obviously the medium [sic], and 64 and 65 is the mode, and I think 50 or 53 would be the mean or the range. Because he, like, boxed in those two." Recognizing that this misconception of the quartiles representing a mode had persisted, Ms. Thomas challenged that student. "I'm going to challenge you because 64 is up here three times, as well as 66 . So, I'm going to argue that point, but you are right that this is the medium [sic]." She then called on another student to share their thoughts at the board. This student began by showing how there were twelve numbers in the upper half of the data. "So, you've got 12 numbers [in the upper half], but you divide by two. That's six," he noticed. Then he described how counting from the maximum value down six places and from the median value up six places resulted in "two numbers that were boxed in." He continued:

But, there's two, [so] we have to box them both. Over here [pointed to lower half of data], there's twelve [numbers]. You've got twelve over here, and you take six [counting up from the minimum value], and it'd be this number [pointed to one of the numbers in the lower half of the data that was boxed in on the quartile sheet]. You take six here [counting down from the median], and you got that number. So, you box those two together. [inaudible] Those numbers boxed in divide the upper half in half. You take the middle of those two numbers [in the box], and that's where the line is on the box [plot].

Ms. Thomas asked the students to clap if they understood what this student shared and then continued, "This student [who created the work] said this was how they found their first and third quartile, but they didn't finish. What your job is: to determine how the student found these numbers." Ms. Thomas further asked the students to both "create a box plot for the data from yesterday" and examine "how they got the numbers" for the first and third quartiles.

The students worked at their tables for a few moments, and then Ms. Thomas stopped the class for a discussion. "When we talk about a first quartile, a second quartile, and a third quartile, this right here [drew a brace over the box in the box plot sheet] is called the interquartile. Wonder why? Why do you think that's [called] the interquartile range?" Answering her previous question regarding how the student found the first and third quartiles, a student replied, "I think you got to divide both of the numbers." Directing the students back to her current question, Ms. Thomas asked, "Interquartile range is what we're talking about. Is it because it all falls inside these marks, you think?" The students seemed unsure how to respond, and, catching on to this uneasiness, Ms. Thomas stated, "Well, I'm learning a little bit of this with you all. I'm not going to lie. This has not been in an easy task. I've not looked at statistics in about 33 years." Dismissing the conversation about interquartile range, Ms. Thomas directed the students to "go ahead and work on our box plots" for the data from the previous day. The students began working at their tables on creating a box plot as Ms. Thomas circulated the room. A student, unsure of his task, asked if they were making a histogram. Ms. Thomas answered, "No, this [pointed to the box plot projected on the board] is what we're supposed to be doing."

Before students were finished working, Ms. Thomas asked the class to pay attention to her. "I'm going to make an executive decision to move on [to the day's task] because I do want you all to get to do this." She posted the Construct Your Own Graph Task on the projector, called on a student to read the task out loud, and passed the task out to each student. "You're going to spend the first couple of minutes looking [at the task] and thinking all on your own," she directed the students. After students had some time to look over the task, she told the class that instead of choosing their own graph, as was stated in the task, they would first create a box plot. A few students voiced concern about the difficulty of creating a box plot, and Ms. Thomas retorted, "Well, we've not done a box plot. Histograms were hard until we seen [sic] one and did it, correct?" The students began to work in pairs per Ms. Thomas' direction, and she circled the room, answering student questions. Two students asked a similar question about how to get started in making the box plot. Recognizing this similar query, Ms. Thomas called on one group to share with the class their strategy for getting started on the problem. "We're lining up the numbers, and then we're getting the . . . median of all the numbers. Then we divide the extra numbers on the side by two," a student from this group shared. Continuing with this response, Ms. Thomas first asked the class, "What do you have to find to do the box plot?" A student responded, "Median," and Ms. Thomas questioned further, "Median of what?" A student correctly replied, and Ms. Thomas continued, "Find the median first which divides it in half, and then we use that to find the median of the first quartile and the third quartile." She checked that students understood what she had stated and asked the students to continue working.

As students worked, Ms. Thomas set the timer and walked to groups, helping them as they asked questions. The two groups that she had the opportunity to help during this time asked similar questions, and she provided similar assistance. Ms. Thomas directed to one of the groups, "This [pointed to the student's data set] is the median for the whole thing. Then we found the median of each, the first quartile median, and this is the second quartile median." After helping the second group, the timer for the task sounded, and Ms. Thomas asked if any group had a box plot completed that they could share with the whole class. One group replied and went to the front of the class. The group put their work on the projector and shared their process for creating the box plot, most of which was inaudible from my location in the classroom. Recognizing the limited time in the period, Ms. Thomas summarized, "Did that help anybody? We're going to pick this back up tomorrow for discussion." Ms. Thomas handed out the homework for the evening and asked the class to look at the projector for something she wanted them "to think about." Before she could post anything on the projector, the bell rang for the end of the period, and she did not have time to for the students to discuss the academic prompt serving as an exit ticket.

## Fidelity of Implementation

Within this section, I first describe the deviations of the enacted lesson from the intended lesson for Day Four. Then I discuss the alignment of the enacted lesson to reform expectations as outlined in the Daily Observation Protocol.

Deviations of enacted lesson from intended lesson. Analysis of the data revealed three deviations of the enacted lesson from the intended lesson. First, in the lesson plan, the students were expected to reach a consensus for how to find the first and
third quartiles using the student work provided in the lesson plan as the means for supporting this consensus. In the enacted lesson, it was unclear if students reached this consensus. A student responded at one time to a question posed by Ms. Thomas about the interquartile range, "I think you got [sic] to divide both of the numbers" (Video, 12/10/14). This student directed this response to a previous request from Ms. Thomas "to determine how the student found these numbers" (Video, 12/10/14) for the first and third quartiles and revealed a beginning understanding for how to find those two values. However, before students were able to reach an overall consensus for finding the first and third quartiles, Ms. Thomas asked them to create their own box plot. This was supported by a comment in my field notes, written when Ms. Thomas had asked them "to create a box plot for the data from yesterday" and examine "how they got the numbers" (Video, 12/10/14) for the first and third quartiles. I wrote at that time in the lesson, "[Students] didn't really come up with how to do it. The teacher ask[ed] them to make the box plot without finishing the idea about how to calculate [the] first and third quartiles" (Field Notes, 12/10/14).

The second deviation was related to the first deviation just described. When students were expected to reach a consensus on how to find the first and third quartiles, the expectation in the lesson was for students to use the student work within the lesson to reach this understanding and then practice the procedure for creating a box plot with the task. Ms. Thomas deviated from the intended lesson plan when she asked the students to "create a box plot for the data from yesterday" (Video, 12/10/14). Returning to the previous day's data was not an expectation within the unit. It appeared that she recognized this deviation during the lesson because shortly after asking students to do
this, she made "an executive decision to move on [to the task]" (Video, 12/10/14) before students were "done making the boxplot" (Field Notes, 12/10/14).

The final deviation occurred at the end of the lesson as the period was about to end. In the lesson plan, the expectation was for three groups of students to share their box plots, followed by a discussion of specific questions geared at developing students' conceptual understanding for the representation. Likely due to the lack of time, Ms. Thomas deviated by asking one group of students to share their work, not engaging the class in the discussion questions, and not addressing the academic prompt that served as the exit ticket. Ms. Thomas recognized this lack of time to formally conclude the lesson in our interview. "I want to finish up where we were at [today]," (Interview, 12/10/14) she commented.

Alignment of enacted lesson. Analysis of the Daily Observation Protocol for Day Four revealed that Ms. Thomas' implementation fidelity for the lesson addressed parts of two sections within this protocol. In the analysis that follows, I exclude sections there were not evident during the lesson.

Standards for Mathematical Practice. The students engaged in two of the Standards for Mathematical Practice during the lesson. First, students made sense of the problems presented in the lesson and persevered in solving those problems. An example of this was when the students analyzed the box plot representation. When Ms. Thomas presented the box plot, students readily analyzed this representation, offering their thoughts on how to create it. One student shared, "We agreed that the box might represent like, if you're talking about height, it might represent the height of something" (Video, 12/10/14). Another student noticed, "Those [dots] are the ones that are the
answers for the mean, median, mode, and range" (Video, 12/10/14). During this analysis of the representation, students all seemed "engaged in [finding] how to construct [the] box plot" (Field Notes, 12/10/14), and this engagement continued when Ms. Thomas simultaneously demonstrated the quartiles sheet and box plot on the projector. Students used the quartile sheet to make sense of the box plot, making statements such as, "The middle one's obviously the medium [sic], and 64 and 65 is the mode" (Video, 12/10/14).

Second, students constructed reasonable arguments while critiquing other's work. An example of this mathematical practice can be seen from the quotes in the previous paragraph and also during the warm-up for the task. During the warm-up, Ms. Thomas asked the students to determine if a question posted on the projector was statistical, and if not, determine how they could make it a statistical question. In this discussion, a student offered her opinion. "No . . . because it's only talking about one," (Video, 12/10/14) she stated. Another student added to this reasoning, "It's only talking about your one math teacher, like you need other teachers to compare like a fraction," (Video, 12/10/14). Utilizing the two previous students' reasoning, a third student expanded on their statements, "We could change it from only math teachers to teachers" (Video. 12/1014). I noted in my journal that during this discussion students "were being very critical . . . [and] making really insightful comments" (Researcher Journal, 12/10/14).

Mathematics Teaching Practices. In terms of the Mathematics Teaching Practices, Ms. Thomas addressed three of these practices. First, throughout the lesson, Ms. Thomas asked purposeful questions to engage students in thinking conceptually about the problem. These questions appeared when Ms. Thomas asked the students, "What else is it talking about," (Video, 12/10/14), in reference to the statistical question
in the warm-up task. This type of questioning happened again during the warm-up. A student suggested changing the question to, "all of the math teachers" (Video, 12/10/14) to which Ms. Thomas asked, "If we did all the math teachers on this floor, there are two of us. So, if we wanted a broader range of data, what could we do?" (Video, 12/10/14). Another example happened shortly after this discussion when she asked the class, "[the histogram] is different than our dot plot in what way?" (Video, 12/10/14).

Second, Ms. Thomas supported students in their procedural understanding of a box plot by developing a conceptual understanding of the structure of the representation. When the students were analyzing the box plot, Ms. Thomas asked a student to share his ideas about the box plot. The student responded, "There was three numbers on one side, and then there was three numbers on the other side. And then the middle of them, the median" (Video, 12/10/14). Ms. Thomas elaborated by drawing a vertical line at the median, revealing the data as halves. This conceptual focus continued when a second student came to the board and drew vertical lines at the first and third quartiles, minimum, and maximum values. Ms. Thomas strategically questioned the class, "There's two sections on each side [of the median]. They're broken into what?" (Video, 12/10/14). A student correctly responded, "Quarters" (Video, 12/10/14). Visualizing the box plot as quarters, the next student was able to use this conceptual understanding of the box plot to describe to the class how to find the first and third quartiles. Finally, Ms. Thomas frequently requested and used students' thinking during the lesson. The discussion described in the previous paragraph about the box plot and quartiles was a representative example of how Ms. Thomas valued and used her students' thinking to progress the students toward the goal during the lesson.

## Barriers

Through the data, it was clear that several barriers potentially affected her implementation per her observations. Specifically, she made comments that demonstrated barriers related to a traditional perspective, pedagogical content knowledge, and subject matter knowledge

Traditional perspective. At points in the lesson, Ms. Thomas demonstrated a traditional perspective of the teaching and learning of mathematics. I observed this perspective at times when Ms. Thomas circulated the room, helping students that asked questions about the task. After the warm-up task, Ms. Thomas engaged the students in a review of the vocabulary words, doing so specifically for two students who had been absent. "We're going to talk really quickly about median. We find the number in the middle," (Video, 12/10/14) she stated. This type of direct instruction continued for the rest of the statistical measures during this initial review. Towards the end of the lesson after students started working on the Construct Your Own Graph Task, Ms. Thomas stopped at a group's table, examining their work. Pointing to their work, Ms. Thomas said, "Then you find the number in the middle for this section, and then you find the number in the middle for this section" (Video, 12/10/14). I noted this direct instruction in my field notes, stating that, when students asked for help, she was "very direct in how she helps-tell[ing] them how to [find the] median" (Field Notes, 12/10/14). Similarly, I reflected in my journal that when students seemed to struggle, she "was very direct in her instruction" (Researcher Journal, 12/10/14). Ms. Thomas acknowledged this barrier during the interview. I asked Ms. Thomas if she had recommendations for a new teacher who might teach these standards. Ms. Thomas instead reflected on receiving
recommendations herself from others. She stated that she did not "want to be told how to run [her] class, but with that being said, there are some aspects [she does] want to know how to present that to get them thinking about it without drill and kill" (Interview, 12/10/14). This reflected a traditional perspective because she acknowledged that she taught procedurally. However, this statement also indicated a transition from teaching directly to a more conceptual type of instruction. It should be noted that I observed this type of direct help from Ms. Thomas to her students significantly less during the Day Four lesson than in previous lessons.

Pedagogical content knowledge. In terms of pedagogical content knowledge, the data revealed two barriers related to knowledge of content and teaching and knowledge of content and students. In terms of knowledge of content and teaching, Ms. Thomas reflected on not having her peer teachers to meet with during a co-planning period while teaching this unit. I asked her to elaborate on the type of help she received from her peers during common planning to which she replied, "I can get the answer, but I don't always feel confident [that] I'm getting the answer to the students right" (Interview, 12/10/14). In this interview, she continued by stating that she sometimes asked her peers, "Show me on the board what you're going to say to the students" (Interview, 12/10/14). This theme of knowledge of content and teaching continued in the interview when we later discussed resources she used to help her teach the unit.

Had [the unit] not said, you're trying to get the kids to get to interquartile, I wouldn't have known what I was waiting for them to say. I wouldn't have known what is it that they're trying to get at. I don't want to be spoon-fed what to say,
but I also need to know where I'm going, and what I'm looking for. (Interview, 12/10/14)

She echoed this sentiment later in the interview when she acknowledged, "There are some aspects I do want to know how to present that to them to get them thinking about it without drill and kill" (Interview, 12/10/14). These quotes demonstrated the barrier of knowledge of content and teaching because Ms. Thomas addressed the idea of how to teach the ideas within this unit. She lacked the common planning with her fellow teachers, a time when she would typically ask about how to teach the topic and what to say to the students, aligning with knowledge of content and teaching.

In terms of knowledge of content and students, Ms. Thomas addressed this topic once during the interview. I asked her how she was going to prepare for the next day's lesson. She replied:

I want to finish up where we were at [today]. [pause] But then you stand up in front of the kids, and they throw one little question out there and that makes you say, oh crap! They make sense! That makes sense! Maybe I'm wrong! (Interview, 12/10/14)

This aligned with knowledge of content and students because she was not able to anticipate the students' responses and subsequently second-guessed her understanding of the topic.

Subject matter knowledge. The barrier of subject matter knowledge appeared as both common content knowledge and specialized content knowledge in this lesson. In terms of common content knowledge, Ms. Thomas made errors with terminology during the lesson. For example, she used phrases such as "first quartile median" and "second
quartile median" (Video, 12/10/14). In terms of specialized content knowledge, when students analyzed the box plot, Ms. Thomas asked one of the discussion questions in the lesson plan. "So, what if I asked you how many values are in this data set?" (Video, 12/10/14). When Ms. Thomas asked this question, she had the box plot sheet displayed on the projector, but did not have the entire sheet projected. She focused on the representation and was not showing the data set with the student's work underneath. The intent of this question was for students to count the number of points in the data set that was listed. Instead Ms. Thomas directed students, "So, basically, we had to count the tally marks" (Video, 12/10/14), referring to the tick marks on the number line underneath the representation. This revealed the barrier of specialized content knowledge because recognizing that a box plot is a summarized representation of that data (i.e., individual data points cannot be determined as they can in a dot plot) is specialized knowledge about statistics that a teacher should know.

In my journal, I reflected on the apparent struggle Ms. Thomas had during the lesson, specifically at this moment. "She thought you could find the number of data values in a data set with a box plot" (Researcher Journal, 12/10/14). Ms. Thomas agreed with this barrier both in her participant journal and during the interview. Ms. Thomas responded in her journal to a question about how she felt the lesson went. "I didn't prepare, and not being familiar with the content, I struggled a little" (Participant Journal, 12/10/14). In regards to what had helped/hindered her during the lesson, she wrote, "Poor planning and content knowledge" (Participant Journal, 12/10/14). Ms. Thomas referenced subject matter knowledge three times in the interview. First, I asked Ms. Thomas to reflect on the lesson. She replied, "I hate that the terms gets mixed up in my mind. I even
put on the board 'interquartile' when I was talking about something else" (Interview, 12/10/14). Second, Ms. Thomas noted her lack of familiarity with this topic when I asked her about the common planning she typically does with her fellow teachers. "If I had that free time after school right now, I would be getting more familiar with it on my own, or if I could have that time during the day with them [her fellow teachers]" (Interview, 12/10/14). Finally, Ms. Thomas coupled her last comment about subject matter knowledge with knowledge of content and student. I asked about her intended preparation for the next day to which she replied, "[The kids] throw one little question out there, and that makes you say . . . That makes sense! Maybe I'm wrong! So, that's part of the problem, too" (Interview, 12/10/14).

## Supports

Analysis of the data revealed two supports identified by Ms. Thomas: time and knowledgeable others. First, Ms. Thomas mentioned time twice during the interview. At the beginning of the interview, she talked generally about how she felt the lesson went that day.

Our common planning this week has been taken up with entering test scores-I'm not getting the time I need to fully develop my own thoughts and such on this. [off-topic conversation] So, the planning time that I could really use and utilize that time to do this, even if I didn't have anyone to go to and ask them questions, is non-existent right now with the trying to catch all the kids up on their testing and getting ready for [testing]. (Interview, 12/10/14)

She again mentioned it later in the interview when I asked about her planning time. She stated:

So, if I had that free time after school right now, I would be getting more familiar with it on my own or if I could have that time during the day with [the other teachers]. I've lost the opportunity to spend some extra time after school on this, and I've lost the opportunity to spend time with my peers so I just feel very rushed. (Interview, 12/10/14)

The support of a knowledgeable other appeared four explicit times during the interview, two of which are described in the two preceding quotations. In the interview, I asked Ms. Thomas to generally talk about the lesson to which she replied, "I'll be glad when the vocabulary flows freely from me, as well, which is one of those things that I would get if I was talking about it with my other four teachers every couple of days" (Interview, 12/10/14). Recognizing the importance of common planning to Ms. Thomas, I asked her to elaborate on what she and her peers discuss in their common planning time. When we meet, if there's a concept, that I've, many times, said, you're going to have to write that out for me. I can get the answer, but I don't always feel confident [that] I'm getting the answer to the students right. So, I'll say, one of you all show me on the board what you're going to say to the students. (Interview, 12/10/14)

These instances when Ms. Thomas referred to her peers demonstrate the importance of this support for Ms. Thomas to feel successful in her classroom.

## Day Five

In this section, I describe the implementation of Day Five based upon the video data. Utilizing all data sources, I then describe the fidelity of implementation, barriers, and supports related to implementation. On Day Five, Ms. Thomas continued the

Construct Your Own Graph Task and gave students time in class to work on their previously assigned homework.

## Implementation

To start the day, Ms. Thomas asked the students to get out their paperwork from the previous day. She asked, "How many of you actually got the box plot finished?" One group raised their hands, and she conducted a quick review before letting the students finish their work from the previous day.

We're going to review a few things, and then we're going to give those who haven't finished a few moments to wrap up. But let's talk about what a box plot is. Let's talk about the information we had to have to make a box plot. Let's just throw out some numbers [wrote $4,5,6,6,7,8,8,8,9$ on the board]. What do we have to find first?

A student replied, "Median," to which she asked, "How do we find the median?" A student told her the procedure. "You cover up two numbers on each side, and keep going," he said. Ms. Thomas demonstrated this procedure on the board, drawing an arrow to the middle number and writing 'median' above it. Ms. Thomas asked if everyone agreed and then continued with her review. Pointing to the set of data below the median, she asked, "There's something down here we're going to be looking for. This is broken into what?" A student replied, "Halves," and Ms. Thomas asked again, "Not halves, but?" A student answered correctly (i.e., quarters), and Ms. Thomas elaborated, "To find the first quartile and the third quartile, we have to find the median of each half. This [drew a box around the lower half of the data] is considered the lower quartile. So, what's this [pointed to upper half of the data] considered?" Answering her question, a student said,
"The upper!" Ms. Thomas agreed and wrote 'quartile two' above the median of the entire data set.

Continuing with this review of box plot, Ms. Thomas probed, "What is the median of the lower quartile?" A student stated something inaudible about it being between the numbers five and six. Ms. Thomas nodded and drew a vertical line between the numbers five and six. "So, we would call that 5.5. That is where quartile one starts. What is the median of the upper quartile?" she continued. A student responded, "Eight," and, recognizing that students might need clarification for the specific eight to which he was referring, Ms. Thomas asked, "Eight, and how did you get that?" The student elaborated, "Because the numbers in the middle-there's two eight's." Ms. Thomas added:

There's two eight's. It's not going to be 8.5. It's just going to be eight. That was a quick reminder. Now, we have this section right here [drew a box around the interquartile]. What is entailed was the middle part of the entire data set. I want you to get out and work on your box plots. I'm going to come around and be checking them.

The students pulled out their box plots from the previous day and started to work. Ms. Thomas then stopped them to clarify finding the median based upon something she had heard in her earlier class. "Everybody stop for a minute because I do want to bring up something that they came up with morning." Ms. Thomas walked to the front white board and wrote a data set on the board.

One of my students said, 'the number I came up with [for the median] is not in the middle,' and I said, 'well, let's work through this.' So, I wrote these numbers up
here: $4,5,7,10,50,70$, and 71 . We did the same method for finding out the number in the middle.

Ms. Thomas demonstrated the procedure for finding the median and asked the class if they agreed that the median was ten. Most of the class agreed, and she questioned further, "Is ten the number in the middle between four and 71 if you were counting-would ten be in the middle?" Students shook their heads, and she continued, "You have to learn to let your head think about this in two different ways. You have to think about the numbers as a data set, but when you draw your box plot, you're going to use all the numbers." Ms. Thomas used this discussion to clarify that the median is not necessarily the middle number between the minimum and maximum value on a number line, but instead, the median is the middle number of only the values in the data set. Ms. Thomas continued to question the students before letting them work on their box plots. Clarifying how to create the box plot, she stated, "Do you have to write $4,5,6,7,8,9,10$ ? No! You can do [the number line] in increments of whatever you feel is necessary." Ms. Thomas checked that the class understood that their number line underneath their box plot could be in increments of their choosing, and when the class agreed, she asked them to start working.

The students started working with their groups, and Ms. Thomas circulated the room. A student stopped her and stated that he needed help with creating a box plot. Recognizing that the student had completed a lot of work mentally without writing it down, she stated, "I know you like to do a lot in your head, but when you're doing statistics, and you're looking for shapes of stuff, you've got to write it down." The student wrote down the data set in numerical order, and Ms. Thomas helped him by stating, "Now, find the median." The student worked about a minute individually,
calculating the lower and upper quartiles and the median of the data set. He then started to make his number line for his box plot representation and asked Ms. Thomas an inaudible question about the length of his increments. She examined his work and asked, "You're counting by three's. If you count three's for the last numbers, you have to count by three's for all of them. Did you count three's all the way across?" He shook his head and said, "No." Ms. Thomas used this student error to start a whole-class conversation:

Everybody give me your attention for a moment. If you're going to make your dot plots in increments of three, you can't go three, six, nine, and then you mysteriously want it to end on 13 [drew a number line on the board with these numbers], and then you just change it. It has to be the same increments all the way down the line. Go back to work.

The students worked about two minutes in groups when one student told Ms. Thomas that she was "lost." Ms. Thomas told the student that the whole class would "do it together." She went to the projector and posted the box plot student work from the previous day on the projector.

Boys and girls, look up here. I think what got us all sidetracked, maybe, is in my head-because again, I'm kind of learning this with you as far as how to present it, and it make sense. And in my head, I wanted this pretty little box to be in the center of my numbers, but that's not always going to be the case. It's not always going to be the case. Some of you are getting tied up on it lining up. It's not always-according to your number charts, this is not necessarily, or for most of the time, not going to fall in the middle [pointed to the box representing the interquartile in the box plot representation].

A student asked another question about the increments for the number line, and Ms. Thomas addressed the whole class, "Who has finished this and feels good about it?" One group of students raised their hands, and Ms. Thomas asked this group to demonstrate their work to the entire class at the white board at the back of the room. "I want you all to go to the back, and with really good hand writing, I want you to do your best to draw us out a box plot with these numbers." She asked the rest of the class to finish on their box plots so that they could compare it in a moment to the group at the back of the room. As the students worked, two different students solicited help from Ms. Thomas. To the first student, Ms. Thomas directed, "First of all, we have to take these numbers, and find the median, and then the median of the other two sections." When the second student asked for help, Ms. Thomas addressed the entire class. "Anybody else confused? If you're confused, come back here. Let's have a mini lesson."

From the video, I saw six students follow Ms. Thomas to another group's table in the back. She asked the group to look at one student's data set that I was unable to see. "Everybody's eyes on this paper," she directed:

We've written the twelve numbers from least to greatest. Everybody see that? Then, we're going to find the median just like we've been doing for the last week. We found 19 , do we agree? . . . If you use this 19 , it's going to throw your numbers off because now you have one, two, three, four [numbers] on this side. One, two, three, four, five [numbers] on this side. But that's not what-we have to use the middle. We know the number's 19, but we can't take that out of the equation. If you divide, you get 19 . Nineteen plus 19 divided by two is 19 . So, the number is 19. That is correct, but if she circles this first 19-what she's done is,
she's created unequal numbers on each side, and I think that's where she got confused. Does everybody understand that?

This mini lesson continued with Ms. Thomas demonstrating how to find the first and third quartile. In this demonstration, a student who was observing the mini lesson recognized that the student work being examined contained an error. "She has four [numbers on each side]. I have five. That's wrong," she stated. Ms. Thomas checked the validity of this response and agreed. The student fixed the error on her paper, and Ms. Thomas proceeded with the mini lesson. She stated, "There is your first quartile median. Here is your third quartile median, and here is your second quartile." Ms. Thomas then drew a horizontal line and created a number line for her representation that started with the number six and ending with 24, counting by two's. A student questioned, "How do you know two's?" Ms. Thomas replied:

You choose how you're going to break the data up. I'm counting by two's. That's just for my mind. Ok, everybody look. You could have counted by one's when you wrote this out. You could have counted by two's. You could have counted by five's.

Ms. Thomas then concluded this mini lesson by showing the student where the first quartile, median, and third quartile were located on the number line. "Does that help?" she asked. One student agreed, and Ms. Thomas asked the students who were there for the mini lesson to return to their seats. Addressing the entire class, she stated, "We're going to let [the students at the back board] present how they did it. We had a conflict. We are going to resolve. They're going to teach us how they got it." A student in the group explained their work, demonstrating how they found the median and first and third
quartiles. Ms. Thomas asked them to share with the class the disagreement their group had when working on their representation. Two of the students in the group both made their case. The first student stated, "He had two numbers right here, and I don't know why he would have two numbers." The second student retorted, "These numbers are separate, and then this number divides them in two." The two students were disagreeing about how to find the first and third quartiles when two numbers appeared in the middle of the lower and upper half of the entire data set. Ms. Thomas utilized this opportunity for a whole class discussion. She told the class to listen to their reasoning and "pick a side."

The second student continued to share his reasoning, "You can put 19 as the median, but it would be 11 as the median on this side, and 20 as the median on this side." Some of the students in the class nodded their head in agreement, and he continued. "But that one's twelve there because you have 11 and 13. Divide that in half, and that's 12. ." Ms. Thomas chimed in, "He has a point. Eleven and 13 divided in half is twelve. What do you have to say to that?" The first student said that she believed she was still correct. Ms. Thomas responded to her, "I'm not saying you are right or wrong. I'm just saying-on that particular step, he is right." Ms. Thomas then polled the entire class to see who agreed with the male student and with the female student. A majority of the students agreed with the male student. Ms. Thomas asked another student if she could present her work to the class, the student agreed, and Ms. Thomas placed this work on the projector. Ms. Thomas asked the class to look at this student work and state the first step for creating the box plot as a way to help them understand how to find the first and third quartiles. "Put them in order," a student stated. Ms. Thomas affirmed this step and continued, "And then we found the median." In the data set projected, two numbers fell
in the middle of the data set. Ms. Thomas put a rectangle around these two numbers, drew a vertical line between them, and continued. "If there are two-if it falls in the middle, you circle them both. That [referred to the vertical line between the two numbers] is the median." She continued by showing the same procedure for finding the first and third quartiles, and a student asked an inaudible question about how many values were in the lower and upper halves of the data based upon whether or not you count the two numbers that were in the middle of the entire data set. Ms. Thomas responded, "You have to decide whether you're going to count that one or not. We're not going to count it." Making this decision for the class, Ms. Thomas demonstrated again how to find the first and third quartiles and then summarized what she had demonstrated:

So, we got our numbers that we needed: 11, 19, and 20. So, what we did up here [pointed to the box plot that she drew on the projected student work]-we drew our numbers, and we [divided] them by two's. Then I put a little dash to represent the numbers in between. Alright, so, once we got this, I had to find the first median-it's not actually called the first median, the second quartile. We found the median. We found the number 19.

Ms. Thomas continued by pointing to the first and third quartiles and asking students what those numbers were called. "What is this called?" she asked to which a student responded, "Third quartile." She continued, "And what's this called?" to which a student answered, "First quartile." Ms. Thomas then drew a box around the interquartile range and drew lines from the first and third quartiles to the minimum and maximum data point respectively.

The last point was 23 , and the first point was seven. Some people call this a box and whiskers because it looks like whiskers. If you need to call it that to remember, do so. Do you all feel comfortable about where we just came from? Finishing this discussion of creating a box plot, Ms. Thomas then asked the students to compare the box plot to a dot plot. She asked, "Who can tell me how this was different than when we created a dot plot?" A student described, "Dot plot has dots above the numbers and just a line. It doesn't have a box. . . . A box plot-you have three quartiles and then you find the median of the middle." Ms. Thomas called on another student to add his thoughts to the discussion. "The box-it shows you the median and the histogram don't [sic]. It doesn't show the median," he noticed. Wanting to extend this student's idea, Ms. Thomas asked the whole class, "So, what do we get to see on here that we don't get to see on a histogram or a dot plot? What are we seeing here?" A student replied, "Median," and Ms. Thomas questioned further about how the box plot was broken into sections. "We see them as what?" she asked to which a student answered, "Quartiles?" Ms. Thomas affirmed his response and continued, "We're getting to see the first, second, and third quartiles this way. Which [representation] do you all like doing better?" Ms. Thomas polled the class for the dot plot, box plot, and histogram, and then summarized:

Dot plot is the easiest, and when we look at the dot plot we see something different - this data [pointed to the box plot on board] shows us the quartiles, but what do we notice about the dot plot? Or the histogram? What do we see in this particular histogram [picked one that was on chart paper on the wall]-we can actually see what? What do we see going on?

A student responded, "We can see how higher up they are. We can see they are increased." Another student made an inaudible comment, and Ms. Thomas restated her sentiment, "She said, it's actually easier to compare the numbers. So, we're actually seeing the shape of the data." Another student chimed in, "Variation!" Ms. Thomas acknowledge his response, "You see more variation. Good job!" Then Ms. Thomas asked the students to retrieve their homework that she had previously assigned.

Ms. Thomas asked how many students had completed their homework, and from what could be seen in the video data, only two students raised their hands. Ms. Thomas was astonished. "Whoa! This has never happened before. I guess I'm going to have to take grades on homework!" A student responded, "What if you were a little bit confused?" Several other students agreed that the homework was confusing and that was why they did not complete it. Recognizing that many students were confused, Ms. Thomas decided to work on the homework in class. She projected the homework and read the first problem out loud. After reading it, she stated, "Because so many people did not do it, you all need to do that right now." The students began to work, and Ms. Thomas walked around the room, examining student's work. Ms. Thomas stopped at a group in the back of the room at the request of a student. Ms. Thomas examined the student's work, acknowledged to the student that her work was correct, and stated that when the rest of the class finished, she would share her work with the class. Continuing around the classroom, Ms. Thomas stopped and examined another student's work. The student explained his work to Ms. Thomas, and she clarified for him, "First of all, the mean is 35 so, that's the average. Then you have to have at least ten points down here, and you only have four." This student corrected his mistakes, and Ms. Thomas continued
to another student. "It's the average. Look here," Ms. Thomas stated as she demonstrated on the student's paper how to calculate the average. Ms. Thomas asked this student further:

Let's say that I only had two data points, and the average has to be 35 . What would my two points be? So, one of your points would be 35 . What would your other- 35 plus what [number] equals an average of 35 ?

Trying to get the student to recognize that the average of 35 and 35 would be itself, Ms. Thomas decided to try another approach to help the student recognize this idea. "Let's do it this way. . . . Thirty-five plus 10 and divided by two," she asked. The student retrieved her calculator and entered the calculation. Looking at the calculation, Ms. Thomas explained, "That's still low. Do 35 plus 20. Guess and check until you get there." Ms. Thomas walked to another student, answered her question, and then addressed the entire class:

Some of you all don't know where to start. So, let's walk through this together. Let's talk about the things [the task is] talking about. Look up here. Here are the criteria that we have to meet. One of the things is it has to have ten data points. Ms. Thomas drew ten spaces on the board as the class watched. Then she asked for another word for the mean. The students replied, "Average," and she continued, "The mean is the average. These ten numbers have to add up to something that when they're divided by ten equals 35 ." A student gave his idea excitedly, " 350 !" Ms. Thomas acknowledged his response, and he elaborated:

I was playing around with the calculator figuring out how to get the mean of 35, and so, I was like what if I do this. So, I just added 35 to itself ten times, and I divided it by ten, and I got it.

Ms. Thomas agreed with his reasoning and asked, "Is that the only [emphasis added] correct answer?" The class disagreed, and Ms. Thomas asked the first student that she had helped to bring her work to the projector. Ms. Thomas asked if she wanted to share, but the student declined and asked if Ms. Thomas would share it for her. Ms. Thomas tried to make sense of the student's work. "We see that she has the mean [correct]. I'm guessing that she started with certain numbers, and it added to the same thing?" Ms. Thomas asked the student. The student shook her head in disagreement.

Ms. Thomas then decided it might be more beneficial to the class to show her way of solving the problem. "Ok. So, let me do it my way then," she stated. Ms. Thomas put the original homework problem on the projector and probed the class, "Somebody throw out two numbers that you can add and get 35." Many students gave their ideas, and Ms. Thomas wrote these numbers on the board. She continued to receive numbers from the students that would sum to 35 until she had ten numbers written on the board. "I want you all to figure this on your calculator, divide by ten, and tell me what the average is," she directed the class. The students replied, "17.5," and recognizing her error, Ms. Thomas questioned, "What did we do wrong if we are not getting an average of 35 ? Everybody think about that for a minute. What are we doing wrong?" A student replied that the "goal was to get 350 not 175 ." Another student added, "We were trying to get 35 , but 35 divided by two does not come out to 35 ." Ms. Thomas restated his response and asked what the sum of the two numbers should be so that the average is 35 . A student
correctly replied, "70." Ms. Thomas agreed and stated, "So, let's go back, and double everything." With the students giving her the values, Ms. Thomas erased the ten numbers she had written and wrote the data set with every value doubled. She then asked the students to calculate this average to which the students replied, " 35 !" Going back to the task, Ms. Thomas verified, "We had to have ten data points that had a mean 35. Did we do it?" The students agreed, and Ms. Thomas told the students to use the data set on the board to finish the homework problems.

As the students worked, Ms. Thomas circulated the room, helping students similar to earlier in the lesson. For example, one student had written ten 35 's as her data set and asked Ms. Thomas what her range would be. Ms. Thomas responded, "I'm assuming if you use 35 ten times in row, there is no range." Ms. Thomas gave this type of assistance to a few more students, and then a student made a comment about the wording of the homework questions being "confusing." Ms. Thomas shared this concern to the entire class. "She feels the wording of the question is confusing," she stated. Several students agreed, and Ms. Thomas turned to the task projected on the board. "Let's reword this. . . . Create a group of numbers that when added together and divided by that number equal 35. Would that have sounded better?" she questioned the class. Many students agreed that they understood the new wording better, and Ms. Thomas continued with the rest of the task, putting the task into her own words. "We could have said, you have to have at least ten numbers. Ok? Show your work to verify that you got a mean of 35 . Show your work to prove that you got an average of 35 ," she restated. The students all agreed that her new wording was less confusing, and Ms. Thomas told the students to look at the last part of the homework. "It says find the interquartile range of your data set. What is the
interquartile? When we drew the box, everything in the box was the?" she asked. Recognizing that some students were confused about this, Ms. Thomas quickly went through how to create a box plot for the data, soliciting student responses as she described the procedure. An example of this description included her asking, "Once we have it broken into two sections, what do we have to find of each section?" to which the students replied, "Median." Realizing that the class period was almost over, Ms. Thomas told the class to do a dot plot for the first part of the homework and a box plot for the second part of the homework. She told the students they could use the remaining class period to work on their homework, and then, immediately after, the bell rang.

## Fidelity of Implementation

Within this section, I first describe the deviations of the enacted lesson from the intended lesson for Day Five. Then I discuss the alignment of the enacted lesson to reform expectations as outlined in the Daily Observation Protocol.

Deviations of enacted lesson from intended lesson. Analysis of the video data revealed that the enacted lesson deviated from the intended lesson in four ways. First, Ms. Thomas decided to continue the lesson from Day Four and began with a review about how to create a box plot. She stated, "We're going to review a few things, and then we're going to give those who haven't finished a few moments to wrap up" (Video, $12 / 11 / 14$ ). This review entailed how to create a box plot for a data set that Ms. Thomas created, focusing on the procedures for the representation. For example, at one point, Ms. Thomas stated, "To find the first quartile and the third quartile, we have to find the median of each half" (Video, 12/11/14). Ms. Thomas acknowledged this deviation in the interview when I asked her about the lesson. She stated, "I tried to back up, and start over
with some of the things from yesterday" (Interview, 12/11/14). This initial review of the procedures for creating a box plot was not in the intended lesson plan for Day Five.

The second deviation happened after students had been working on their box plots for some time at the beginning of the lesson. Ms. Thomas helped a few groups with their work, describing to them how to create the box plot. Recognizing that several students were struggling to create a box plot of the data, she stated to the class, "If you're confused, come back here. Let's have a mini lesson" (Video, 12/11/14). Ms. Thomas recognized this deviation and spoke about it during the interview. I asked her how she decided to split the task into two days. She responded, "They're not getting it. I'm having to spend a lot more time at individual groups. That's why today I pulled everybody back there because I was really losing track of, at each group, who understood what was going on" (Interview, 12/11/14). Pulling students together for a mini lesson on box plots was not in the intended lesson plan.

The third deviation happened just after the previously described mini lesson. Two students in a group disagreed on how to find the first and third quartiles of the data set. Ms. Thomas asked the students to each present their case to the class and requested that students vote for the perspective with which they agreed. After the students had presented their case, Ms. Thomas directed the class, "If you believe he is right, stand up" (Video, $12 / 11 / 14)$. She then polled for the other student and told the students that the first student's idea was correct. I noted in my field notes that this practice allowed the students to "critique each other and engage in discussion" (Field Notes, 12/11/14). However, this critique of different student ideas was not in the intended lesson plan.

The final deviation happened at the end of the lesson when Ms. Thomas made the decision to let students work on their homework in class. When Ms. Thomas realized that many students had not finished their homework, she stated, "Because so many people did not do [the homework], you all need to do that right now" (Video, 12/11/14). During this time, Ms. Thomas also demonstrated to the students how to address the first part of their homework. She stated, "Ok, so, let me do it my way then" (Video, 12/11/14), and then she proceeded to direct students through the first part of the homework. Ms. Thomas elaborated on this during our interview. I asked her to reflect on the lesson, and she stated, "I also decided to do a little more modeling than I had been in the past, and I think for this group of students, they needed that" (Interview, 12/11/14). I asked her to elaborate on what she meant by modeling to which she replied, "Showing them a little bit more. Little more hand-holding than I probably-definitely more than I do with third period, but this group needs that" (Interview, 12/11/14).

Alignment of enacted lesson. Analysis of the Daily Observation Protocol for Day Five revealed that Ms. Thomas' implementation fidelity for the first lesson addressed parts of two sections within this protocol. In the analysis that follows, I excluded sections that were not evident during the lesson.

Standards for Mathematical Practice. Students were engaged in two of the Standards for Mathematical Practice during the Day Five lesson. First, students critiqued each other's and the teacher's work during two specific instances in the lesson. The first instance occurred when Ms. Thomas asked two students from a group to present their differing opinions for how to find the first and third quartiles. The second student analyzed the first student's work stating, "I disagree with her because you shouldn't have

11 and 13" (Video, 12/11/14). This student continued to critique the other student's work while the first student defended her mathematical ideas until she later recognized her mistake. When Ms. Thomas was talking to the entire class about finding the first and third quartiles, this student had her revelation, "I see my mistake!" (Video, 12/11/14). The second instance of this practice happened when Ms. Thomas created a data set on the board that had an average of 17.5 instead of the required 35 . A student quickly responded that Ms. Thomas had made an error because the sum of each pair of numbers in the data set should have been 70, an insight that led Ms. Thomas to change the data set by doubling every number. This student was able to critique the work displayed, describing the error in the reasoning.

Second, students used the calculator at specific times throughout the lesson to check their work. The video data revealed that the students did not use the calculator for every calculation, instead only when they needed to check their calculations that they had written or when they needed to quickly check a calculation per Ms. Thomas' request. An example of this type of calculator usage happened when Ms. Thomas asked a group of students to "guess and check until" (Video, 12/11/14) they found the correct data set that had an average of 35 .

Mathematics Teaching Practices. Ms. Thomas engaged in three of the Mathematics Teaching Practices during the lesson. First, Ms. Thomas engaged the students in meaningful discourse when she asked the two disagreeing students to present their case and had the class decide who they felt was correct. By asking the students to "listen to these ideas" (Field Notes, 12/11/14) so that they could "pick a side" (Video, 12/11/14), Ms. Thomas engaged the class in the opportunity to analyze and compare the
students' arguments. Second, this student disagreement demonstrated how Ms. Thomas also elicited and used student thinking in the lesson. Recognizing the disagreement among this group as being similar to other questions the class asked, Ms. Thomas made the decision to have these two students present their ideas and have the students decide who they felt was correct. Finally, Ms. Thomas helped the students see connections among the three representations that they had studied so far. Towards the end of the lesson, Ms. Thomas asked, "Who can tell me how this was different than when we created a dot plot?" (Video, 12/11/14). The students made connections such as, "Dot plot has dots above the numbers and just a line, and it doesn't have a box plot" (Video, $12 / 11 / 14$ ). This continued when other students stated, "We can see they are increased," and "Variation" (Video, 12/11/14).

## Barriers

Through the data, it was clear that several barriers potentially affected her implementation per her observations. Specifically, she made comments that demonstrated barriers related to a traditional perspective and subject matter knowledge.

Traditional perspective. Analysis of the data revealed several instances when Ms. Thomas demonstrated a barrier related to a traditional perspective of mathematics teaching and learning. In my field notes, I recognized several instances when she provided direct help to students who were struggling with a particular part of the task. For example, I noted, "As students are working on their box plots, she tells them how to increment their number lines" (Field Notes, 12/11/14). Later, I also commented on how Ms. Thomas approached helping the class with first part of the homework. She had originally asked a student to share her work to the entire class, but the student did not
want to talk in front of everyone. Ms. Thomas tried to make sense of the student's work, and when she could not, she "put up her box plot on the projector and explained to the class how she did it" (Field Notes, 12/11/14). This is supported from the video data when she stated to the class, "Ok, so, let me do it my way then" (Video, 12/11/14). Ms. Thomas also reflected on this type of help in the interview when she stated, "I also decided to do a little more modeling than I had been in the past" (Interview, 12/11/14). These actions aligned with a traditional perspective of mathematics because the emphasis was on rules and procedures instead of conceptual understanding.

Subject matter knowledge. The data revealed the barrier of subject matter knowledge during the Day Five lesson implementation. During the lesson, Ms. Thomas recognized that some students were struggling because they thought that the box in their box plot should be in the center of the data. She stated:

I think what got us all sidetracked maybe is in my head-because again, I'm kind of learning this with you as far as how to present it and it make sense. And in my head, I wanted this pretty little box to be in the center of my numbers, but that's not always going to be the case. (Video, 12/11/14)

At this moment, Ms. Thomas acknowledged subject matter knowledge as a barrier to her implementation, a sentiment she echoed during our interview:

I panicked again, and I've told third period-I've not looked at statistics in this capacity in 30 years. I never paid this much attention to statistics, or-Let me rephrase that, box plots. Because, we have to take averages and stuff, but box plots and whiskers-where they all end and quartiles? So, even though I may say,
ok, I understand it now, the minute I get up there and a child questions it, I'm questioning myself. (Video, 12/11/14)

The video data revealed that Ms. Thomas referred to the lower half of the data set as the lower quartile, reflecting a barrier of common content knowledge. She stated, "This [drew a box around the lower half of the data] is considered the lower quartile. So, what's this [pointed to upper half of the data] considered?" (Video, 12/11/14). A student replied, "The upper," (Video, 12/11/14) to which Ms. Thomas agreed. Ms. Thomas reflected on this barrier during the interview when she stated, "I've never paid this much attention to statistics, or let me rephrase that-box plots" (Interview 12/11/14).

## Supports

The data revealed that Ms. Thomas identified two supports for the Day Five implementation: curricular support and knowledgeable others. First, in terms of curricular support, Ms. Thomas responded in her journal to a prompt asking about her preparation for the lesson. She stated that she had "used a textbook for teachers of statistics" (Participant Journal, 12/11/14), referring to one of the books I supplied to her as a benefit for participating in my study. Later in the interview, Ms. Thomas also talked about the unit as being something she had used in her preparation. I asked her what specifically she did to prepare for the lesson. She replied, "Well, I did pull from this [the unit]" (Interview, 12/11/14). Second, Ms. Thomas made one reference to a knowledgeable other as a support. She stated in her journal that she had "talked to someone" (Participant Journal, 12/11/14) when she prepared for the lesson.

## Day Six

In this section, I describe the implementation of Day Six based upon the video data. Utilizing all data sources, I then describe the fidelity of implementation, barriers, and supports related to the implementation. On Day Six, Ms. Thomas started the I Wonder What Happens If . . . Task. The goal of this task was for the students to understand how different statistics can affect the shape of a distribution for quantitative data. As discussed in the Limitations section of Chapter Three, on this day the video camera failed with approximately 20 minutes left in the lesson. Therefore, I use both the video and my field notes to describe the implementation for this day.

## Implementation

Ms. Thomas started the lesson by asking the students to share how they solved their homework to their group, giving each student a prescribed amount of time to share. After the prescribed time amount for a student to share was up, Ms. Thomas asked the groups to then let the next student share. While the students shared, Ms. Thomas circulated the room, looking at the lesson plan and helping students who asked questions. For example, one student asked about creating a box plot. Ms. Thomas replied, "The median of this group is called the first quartile. Then you find the median of the upper quartile, and you mark it. Then that's your box." As the students continued to share in their groups, Ms. Thomas stopped at one and listened to the student sharing her work. Ms. Thomas listened and recognized that the student had not finished her box plot. She probed the student as the other students in the group listened, "Is this the median of the second group?" The student agreed, and Ms. Thomas continued, "Ok. So, this is your box [pointed to the student's representation]. Now, what is your first number here?" The
student replied with the minimum value for her data set. Ms. Thomas continued, "So, that's a point and a line, and what's your last number?" The student replied with the maximum, and Ms. Thomas continued to help the student and then addressed the whole group, "So, that's a point and a line. Do you all see that? Everybody understand?" When each student in the group agreed, Ms. Thomas left, turned the projector on, and looked at one more student's work. "Where's your box plot?" she asked the student. The student showed her his representation, and Ms. Thomas replied, "Ok. You did a dot plot. That's fine."

Ms. Thomas then rang a bell at her desk to get the class's attention. "I'm going to start by asking [a student] to come up here to the board, and show how he did his," she told the class. The student went to the front of the class and projected his work on the board. Ms. Thomas asked him to share his work for the second problem and he started, "I found what was equal to the third quartile. It was 25 ." He then made an inaudible statement about how he found the first quartile but seemed unconfident in his response. Ms. Thomas asked him, "You're right. Why are you second-guessing yourself?" The student continued, "It [the rest of the problem] said, 'what percentage of your data is less than or equal to the first quartile?'" He did not respond to this question, and noticing his confusion, Ms. Thomas asked, "How did you figure that out?" The student said he did not know how, and Ms. Thomas decided to elaborate on this question for the entire class. "What do we know about the interquartile range? What do we know about how much of [the data set] it is? What do we know about this section right here [pointed to the box in his box plot]?" she asked the class. One student responded something inaudible, and Ms. Thomas probed further, "So, how much does that area, in percentage, take up?" The
student correctly replied, " $50 \%$." Continuing, Ms. Thomas asked about the lower and upper quarter of the data. "This would be how much percent, and this would be how much percent?" A student stated, " 25 ," and Ms. Thomas stated, "Does everybody see how he got that? Ok, next-who did I say was sharing?"

A student went to the front of the room and stood by Ms. Thomas as the class examined her work that was projected on the board. Ms. Thomas explained the student's work. "She did the dot plot instead of the box plot, but that will give you all one more chance to look at it. Can you visually tell that this [box] is representing what percent?"

Ms. Thomas questioned. No students replied to this, and she continued, "If we cut this in half [pointed to the median], and we cut this section in half [pointed to the lower quartile]. We cut this section in half [pointed to the upper quartile]. How many sections did we have?" Several students replied, "Four," and Ms. Thomas questioned them again, "Four quarters?" Several students stated, "Quartiles." Pointing to each of the quarters of the data set, Ms. Thomas summarized, "This is a fourth. This is a fourth. This is a fourth. This is a fourth! How much is the interquartile?" A student responded incorrectly, " $75 \%$." Recognizing that many students still did not understand, Ms. Thomas explained further, "That's ok. Alright, we took our box, and we cut it in half [drew a vertical line at the median]. If this is half, what percentage is this?" She pointed to the lower half of the entire data set, and the students replied, " 50 ." Ms. Thomas wrote $50 \%$ above the lower half of the data and continued with the upper half, "What is this?" The students again said, " 50 ," and Ms. Thomas wrote ' $50 \%$ ' above the upper half of the data. She then drew vertical lines at the lower and upper quartile and asked the students, "Each section is now what?" The students stated, " 25 ," and she continued, "What does this [box] represent in
percentages?" Many students did not respond, and sensing students were still confused, Ms. Thomas elaborated more. Pointing to each of the quarters on the data set now displayed using the vertical lines, Ms. Thomas said, " $25 \% .25 \% .25 \% .25 \%$. What is this [pointed to box] section?" A student claimed, "I'd say 50\%." Wanting the student to elaborate, Ms. Thomas asked him, "Why do you think it's 50\%?" He explained, "You have two of the ends here, and you have one in the middle. Each one is 25 , right? So, you have two in the middle, and that's 25 plus 25 which is $50 . "$ Ms. Thomas asked how many people understood what the student had just explained. Many students in the class stated that they were still confused, and Ms. Thomas decided to try and explain the idea a different way.

Ms. Thomas asked the class, "If I had a dollar, and I divided it into fourths because I have four children . . What am I going to give each child?" To illustrate this idea, she drew a rectangle on the board, labeled it $\$ 1$, and sectioned it into fourths. A student responded, " 25 cents." Ms. Thomas agreed, "I'm going to give them each 25 . It takes how many 25 's to make 100 ?" A student correctly responded, and excitedly, Ms. Thomas stated:
$100 \%$ ! Now, this is 100 pennies, but we're talking about $100 \%$. So, it takes four of my fourths or quarters to make 100 . So, it takes four quarters over here [pointed to the box plot] to make $100 \%$ of the table. This table from here [pointed to the maximum] to here [pointed to the minimum] is $100 \%$ of my data. Ok, we cut my data in half [drew vertical line at median]. When we cut my data in half, we have something called the [pointed to lower half of the data]?

A student replied to the question, "Lower quartile." Ms. Thomas agreed, called the upper half of the data the "upper quartile," and continued. "If it's in half, is it $50 \%$ ? Is it? If you agree, nod," she asked. She elaborated, "If you take half of [the data set] and you divide it in half again, you now have four sections. A section is considered onefourth. Is that right? How do we know what percentage one-fourth is?" A student responded, "Four-fourths is 100 !" Not getting the answer she expected, Ms. Thomas explained the procedure for converting a fraction into a percent.

Do you need to go back to how we turn a fraction into a percent? The one jumps off [showed procedure for dividing one by four] - then how do we take a decimal and make it a percent? We moved it?

A student responded to the question, "Two places," to which Ms. Thomas asked, "To places to the?" Most students said, "Right," and Ms. Thomas continued, "Is it starting to sink in? We've got to know this before we move on." Many students nodded their heads, indicating that they understood. Ending this conversation about percentages represented in a box plot, Ms. Thomas segued into the day's task. She stated, "We've got a fun task. It looks fun to do, but what I need to talk to you about is something called distribution." Before Ms. Thomas could start a conversation about distribution, a student indicated that he wanted to share part his homework to the class.

Ms. Thomas called the student to the front of the class where he put his homework on the projector. The student read the data set that he created to the class, including the statistics for his data set that he was expected to find. Ms. Thomas asked the class if they understood his work so far, and the class agreed. The student continued, "The first quartile is 32.5 , and the second quartile is 35 . The third quartile is 37.5 ."

Recognizing that the student had not created the box for the interquartile range, Ms. Thomas drew on his box plot and stated, "Let me draw your lines. This would be your box. So, that is what percent?" The student stated, " $50 \%$." Ms. Thomas then asked the student about the percentages found in the lower and upper quarters of the entire data set to which the student replied, " 25 ." Ms. Thomas thanked the student for sharing, asked him to sit down, and continued with the topic of distribution.

Let's say I have the numbers four, eight, eight, eight, nine, ten, ten, eleven. Ok, let's say I have a dot plot [for this data]. Four happened once. Eight happened three times. Nine happened once. Ten happened twice. Eleven happened once [drew a dot plot for this data]. Now, with a dot plot we can see kind of what we called a shape. Do you remember that?

Ms. Thomas waited for students to agree and then drew rectangles around the dots on the dot plot. She explained her work, "If we did this [drew rectangles], this is a histogram. It would look like this. So, the data takes on a certain shape." Ms. Thomas then went to her computer and projected a website that had histograms representing three different distributions. One of the histograms was skewed left. One histogram was skewed right, and the final histogram on the website demonstrated a random distribution. Ms. Thomas explained these three distributions to the students. "Data can be distributed-spread out-in different ways. We notice here [pointed to the skewed-left distribution]. The data seems to climb and then drop suddenly. What do we see in this one?" she asked the class about the skewed-right distribution. A student replied, "It goes up, and then it drops." Pointing to the random distribution, Ms. Thomas continued, "Or it can be all jumbled around. I want you to understand that the distribution can take on different shapes."

After this demonstration of different distributions, Ms. Thomas asked a student from each group to get a piece of chart paper from the front of the room as she handed out the task for the day. "You're going to be using two different sets of data per group. So, just look and make sure that you have two different sets of data that you're going to use," she stated as she handed the task out. Once all groups had a task sheet and chart paper, Ms. Thomas told the students they could use their calculators, set the timer to ten minutes, and asked them to begin working.

As the students worked, Ms. Thomas circulated the room and answered their questions. One student asked, "Is this the minimum?" as she pointed to her task. Ms. Thomas took the task from the student, read over it, and told her, "That's the average, and what's the lowest number going to be? Oh! That's the minimum amount [emphasis added] of points. Ten values. So, minimum number that it can be." The student seemed confused, and Ms. Thomas reminded her of the homework question that was similar to the task. "You do just what we did for [the homework], but you use those [pointed to the task] numbers." Still confused, the student stated, "We don't have a median." Ms. Thomas clarified, making connections to the homework, "You're creating the data set. Remember how we created yesterday? For homework, it said create a data set with an average of 25 ." Still not understanding, the student stated, "There's no middle number." Ms. Thomas replied, "You won't know that until you put your data set down. You get to pick it." The student asked, "So, our first number is?" to which Ms. Thomas replied, "It said it must have at least ten values. You can have more than ten." Seeing that the student was still confused, Ms. Thomas addressed the whole class, "I want to see the groups with
thumbs up that are confused. You meet me at this table back here-the ones that are really confused today."

Several students followed Ms. Thomas to the back of the room for a mini lesson. Ms. Thomas asked one of the students to grab an extra task from her desk so that they could "do one together." First, Ms. Thomas read the task to the students, asking them about the statistics. For example, she asked, "What does mode mean? And minimum?" When the students replied, she continued by referring to the amount of values the data set was supposed to have. "Now, fifteen spots," she stated as she drew 15 lines on her piece of paper as the group of students watched. Ms. Thomas then told the students how to find a data set with a specific average. She stated, "So I'm going to use these numbers because I want each group to add up to this number." After some inaudible conversation back and forth between the students and Ms. Thomas about this procedure, Ms. Thomas continued, "So, now we have to find what's in the middle. So, we found the median. Do you agree?" When students agreed, she continued with the mini lesson. "We all see that this is $50 \%$ of the data. This is $50 \%$. Do you see that?" she asked. Ms. Thomas then created a box plot for the data, calling each part by its name. "This was the first quartile-the second quartile-the third quartile," she said. Then Ms. Thomas dismissed the students and asked them to return to their groups and continue working.

A few students remained after she dismissed the students, and Ms. Thomas asked them what questions they still had. A few of these conversations were inaudible, but one student verbalized his lack of understanding for how to create a histogram. Ms. Thomas responded, "A histogram looks like this [demonstrated a histogram], and it's the same thing. Just a different representation." These few remaining students then returned to their
desk, and Ms. Thomas circulated the room, helping other students. One student asked, "What would our range be?" Ms. Thomas retorted, "It doesn't say anything about stating a range unless it gives you a maximum and a minimum." The student stated that her task did list those statistics, and Ms. Thomas replied, "Then you take your maximum, 25, minus your minimum, and what do you get?" The student responded, "Ten." Ms. Thomas questioned her, "What does ten represent?" The student gave an inaudible response and then asked Ms. Thomas about the average. Ms. Thomas explained this procedure, "Our average is 17 , and we have ten values. Ten times 17 is what? 170 ? Ok, you can't go over 25 and 15 . How much is 25 over the average?" The student answered, "Six," to which Ms. Thomas added, "You have to take off six total points from other data points." Ms. Thomas continued to assist this student in creating a data set that met the requirements within the task.

Ms. Thomas moved to another group of students, and one student asked, "Is minimum the lowest and maximum the highest?" Ms. Thomas looked at this student's task, agreed, and questioned further, "What is one number that averages to 17 ?" The student replied, "17 and 17." Once the student found a data set that met the requirements in the task, he asked, "What do I do next?" Recognizing that many other students were still working on the first part of the two-part task and that the class period was almost over, Ms. Thomas asked the class to get to a "good stopping point." While students were still working, Ms. Thomas told the students, "We will continue this Monday. Now, we have some housekeeping things." As students put up their work and chart paper, Ms. Thomas handed out lost-and-found items and took care of similar housekeeping items. Then the bell for the end of the period rang, and Ms. Thomas dismissed the class.

## Fidelity of Implementation

Within this section, I first describe the deviations of the enacted lesson from the intended lesson for Day Six. Then I discuss the alignment of the enacted lesson to reform expectations as outlined in the Daily Observation Protocol.

Deviations of enacted lesson from intended lesson. Analysis of the data revealed that Ms. Thomas deviated from the intended lesson plan in three ways. First, in the intended lesson plan, the warm-up section of the lesson was to include a discussion of the homework. This discussion was intended to emphasize how the statistics affected the data that the students could choose for their data set and also to introduce the word percentage, to be added to the vocabulary list. Instead, Ms. Thomas chose to assist students in understanding the percentage of data that is represented in each quartile of a box plot by talking about distributing a one-dollar bill to her four children. She stated, "If I had a dollar, and I divided it into fourths because I have four children, and they all want equal parts of my money. What am I going to give each child?" (Video, 12/12/14). To help students visualize this concept, she drew a rectangle on the board representing one dollar and divided it into fourths. This scenario of a one-dollar bill as a discussion point for percentages was not included in the intended lesson plan.

Second, during the first part of the task, Ms. Thomas created a data set, wrote it on the board, and created a dot plot to represent this data. After she drew this, she reminded students, "Now, with a dot plot we can see kind of what we called a shape. Do you remember that?" (Video, 12/12/14). Ms. Thomas then continued, "If we did this as a histogram, it would look like this" (Video, 12/12/14). Ms. Thomas used these two representations to introduce the word distribution and then followed this conversation by
showing students a website that had three distributions demonstrated: skewed left, skewed right, and random. Ms. Thomas talked about this moment during our interview. I asked her what helped her implement this lesson to which she replied, "Well, I panicked over distribution. So, I 'googled' that. Sometimes the definitions of something don't seem to be what I'm looking for. I'll do some more research on distributions" (Interview, $12 / 12 / 14)$. Using the representations on the website as a way to discuss distribution was not in the intended lesson plan.

Finally, Ms. Thomas deviated from the intended lesson plan when students engaged with the task and struggled to create their data set. She asked students who were "really confused today" (Video, 12/12/14) to meet her at the back of the room for a mini lesson. During this mini lesson, Ms. Thomas showed the students how to create a data set that met the statistics listed on one of the tasks. Ms. Thomas reflected on this mini lesson during our interview:

I think I needed to model one for them-one of those tasks, a very simple task. Something simple, and let them do the harder ones. I think with my students that are struggling more, I will model a small one. (Interview 12/12/14)

Engaging students in this mini lesson focused on solving one of the more "simple" tasks was not in the intended lesson plan.

Alignment of the enacted lesson. Analysis of the Daily Observation Protocol for Day Six revealed that Ms. Thomas' implementation fidelity for the lesson addressed parts of one section within this protocol. In the analysis that follows, I exclude sections there were not evident during the lesson.

Standards for Mathematical Practice. In regards to the Standards for Mathematical Practice, students engaged in using tools appropriately. Given the nature of the task, students were constantly calculating statistics, a task that was made more efficient by using their calculators. Although Ms. Thomas instructed students to use calculators, I observed students strategically using them for more tedious calculations (i.e., the mean) rather than for simpler calculations (i.e., the range).

## Barriers

Through the data, it was clear that several barriers potentially affected her implementation per her observations. Specifically, Ms. Thomas made comments that demonstrated barriers related to a traditional perspective and subject matter knowledge.

Traditional perspective. Analysis of the data revealed instances when Ms.
Thomas demonstrated a barrier of a traditional perspective of mathematics teaching and learning. An example of this barrier happened when Ms. Thomas decided to conduct a mini lesson with one of the easier tasks. Recognizing that some students were struggling, Ms. Thomas stated, "Meet me at this table back here-the ones that are really confused today" (Video, 12/12/14). My field notes revealed that during this lesson, Ms. Thomas showed the students "how to do the task" (Field Notes, 12/12/14). Reflecting on this mini lesson during our interview, Ms. Thomas discussed why she felt this mini lesson was beneficial for students. She stated, "I think I needed to model one for them-one of those tasks, a very simple task. Something simple, and let them do the harder ones" (Interview, $12 / 12 / 14)$. This inclination to show students how to do mathematics, focusing on the procedures, aligned with the traditional perspective of mathematics teaching and learning.

Subject matter knowledge. Ms. Thomas explicitly identified subject matter knowledge as a barrier twice during our interview. First, I asked her how the implementation of the lesson could have been improved. She stated, "If I had felt more comfortable about distribution" (Interview, 12/12/14). She expanded on this barrier when she stated, "Well, I panicked over distributions" (Interview, 12/12/14), and this barrier was evident during the implementation. When Ms. Thomas discussed distribution with the class, she first demonstrated this idea with a dot plot. Next, she created rectangles around the dots in the dot plot and called this a histogram. During this moment, in my field notes, I observed that she "box[ed] in the data and call[ed] this a histogram" (Field Notes, $12 / 12 / 14$ ) and later reflected in my journal that she "had a misconception about histogram" (Researcher Journal, 12/12/14)

## Supports

Ms. Thomas discussed three supports that would have improved her implementation of the lesson for Day Six: experience, practice, and knowledgeable others. During the interview, it became apparent that Ms. Thomas felt her lack of ability to experience the lesson affected her implementation and had she had this opportunity, the implementation could have been improved. This barrier was revealed when I asked her to reflect on the lesson. She stated:
[It went] slower than anticipated. I don't know why I expected them to get through it better, and it might have ran smoother had I did it [sic] with the first period, but the first period is off. (Interview 12/12/14)

In this quote, Ms. Thomas is reflecting on missing the opportunity to teach the lesson in her earlier class before the class in which this study took place.

Ms. Thomas mentioned the second support of practice later in the interview when I asked about how the lesson implementation could have been improved. She stated, "If I had worked a few problems-that's going to be my recommendation for the teachers that teach this in the future-work every problem the night before" (Interview, 12/12/14). This quote is different from the first in that Ms. Thomas first talked about experiencing the lesson in her earlier class. The second statement instead revealed a different type of support: one of actually practicing problems within the lesson prior to teaching, hence why I coded this statement as one of practice.

The support of knowledgeable other also appeared during the interview. I asked Ms. Thomas to reflect on her preparation for the lesson as compared to how she would normally prepare for her mathematics classes. She reflected:

I don't have someone to common plan with and to go over before each day-how it's going to look. I don't have someone that's ahead of me to say, what would you have done different? I don't have someone ahead of me to say, cut that out. I don't have anybody to talk to. I'm alone. (Interview, 12/12/14)

I asked Ms. Thomas to talk more about how she works with other teachers during their common planning, and she stated, "We'd actually work problems out" (Interview, 12/12/14). It is apparent that missing these opportunities to collaborate with knowledgeable others appeared as a barrier to Ms. Thomas during this implementation. Not surprisingly, this support overlapped with the support of practice in that Ms. Thomas stated how her time spent collaborating with others including practicing problems within the lesson.

## Day Seven

In this section, I describe the implementation of Day Seven based upon the video data. Utilizing all data sources, I then describe the fidelity of implementation, barriers, and supports related to the implementation. On Day Seven, Ms. Thomas decided to continue the What Happens If . . . Task from Day Six. Although the decision to extend the task to two days should not be viewed as negative, it was not the expectation within the original unit timeline (see Table 2).

## Implementation

Ms. Thomas began the class by asking the students to retrieve the work they started on Friday. As the students pulled out their papers, Ms. Thomas returned each group's chart paper and task. When all the papers had been distributed, Ms. Thomas stated, "Let's go over a few things as a reminder." Ms. Thomas calmed down a few students who were being rambunctious and continued:
[The task] gives you certain things you need to know. What I've been doing is, if it says you have to have at least ten data points, I put my ten points [drew ten blank lines on the board]. Then, if it tells you what your median is, which is your number in the middle, I go on, and I put that there [pointed to the middle of the lines she drew]. If it tells me my range, if I had a range of 60 here, how could we find out what's going on the ends?

A student replied, "What you do is you find two numbers that are supposed to add up to-like you know how you have 76 as the mean? That could be the range, right? We would double that." Ms. Thomas asked the student to clarify, "Would that be the range? Would 76 necessarily have to be the range?" The student continued his thought, "Not the
range, the mean. You add the two together, and you have to have 76-like how you get the two numbers on the end. You just make sure they're 60 apart." Capitalizing on the student's last idea, Ms. Thomas restated, "This number [put an X at the minimum] and this number [put an X at the maximum] have to be 60 numbers apart." Ms. Thomas then pointed to the ten blank lines she drew on the board. She reminded the class about the previous homework:

We had ten blocks-ten data points that had to equal a mean of 35 ? Remember that? What did we do? We did ten times 35 to come up with the numbers that these all were going to have to total. Didn't we? That's one way to go about trying to solve these.

Ms. Thomas then ensured every student knew who their partner was and that they had their tasks and chart paper. Then she asked the students to begin working.

As the other students began working, one student asked Ms. Thomas about her data set. "It says minimum ten and maximum 100. We had ten sets of 40 . Would we have to put a ten in there somewhere?" Ms. Thomas restated what the student had said and asked her to clarify her question. The student asked, "We had all 40's. We had ten 40's. Do we need to fix our work, or is it ok as it is?" Ms. Thomas replied, "If it comes out ok, I'm ok with it. The data should support what is on this [task] card." After answering this student's question, Ms. Thomas went to her desk as the class worked in their groups. Ms. Thomas started to enter grades into her grade book when a few students approached her desk, asking how to solve the task. Ms. Thomas addressed the entire class, "If anybody is $100 \%$ lost-nobody at your group can help you-come here." One of the students at her desk asked about the second part of the task. Ms. Thomas replied, "I want you all to
brainstorm the second part of the question. Everybody brainstorm at [your] table," and she sent the student back to her group. Another student remained at her desk and asked an inaudible question about the task. Ms. Thomas probed, "Do you remember what median means? Median is the number in the?" The student replied, and then Ms. Thomas continued with another statistic, "Mode-remember most!" Then Ms. Thomas read the student's task and provided inaudible help. Some of the things that were heard included Ms. Thomas asking the student, "Middle number has to be ten, and the maximum number has to be 30 . Ok?" Ms. Thomas then proceeded to show the student how to construct a data set with the appropriate mean. "Now, 215 is the number we're shooting for. Twofifteen divided by ten. So, each one of these has to be worth that." The student replied something inaudible about the task being difficult, and Ms. Thomas continued, "If I were you, I would do these blanks, and do like I did. Add them up, and divide them by $12 . "$ The student then returned to her desk and continued working on the task.

Ms. Thomas entered a few more grades into her grade book and then circulated the room, looking at students' work. A group of students told Ms. Thomas that they were finished with their task to which Ms. Thomas replied, "So, now you're going to pick a dot plot, box plot, or histogram. Whatever you want." A student from another group asked about the types of values that were acceptable for the task. She asked, "Will you accept point eight?" Ms. Thomas answered, "I will accept point eight on this activity." When the timer went off, Ms. Thomas asked the group of students who claimed that they were finished to share their work with entire class. To get the group started with their presentation, Ms. Thomas asked, "What did you have to have [in your data set]?" A student in the group replied, "Mean of 40 , a minimum of ten." The student then stated the
rest of the required statistics, but it was inaudible on the video data. The students showed the class their chart paper with their data set and calculations. Ms. Thomas asked the class, "Does anybody see what they've done wrong? They've done a great job with the mean, but if your [minimum] number has to be 10 , do you show a ten on here?" A student in the group replied, "No," and Ms. Thomas continued, "We have to show a ten, and did you show your maximum number?" The student again stated, "No," and then she added, "I thought we could use all 40 's." Ms. Thomas replied, "You can use all 40's except your maximum and minimum." Ms. Thomas then asked the class if they saw any other errors in the work, and when the students could not find another error, Ms. Thomas asked the group to fix their error and for the rest of the class to continue working.

Ms. Thomas then walked to one student to whom she had given a task to solve individually. The exchange was inaudible because of her location, but Ms. Thomas sat with the student at a table, listened to him present his work, and then provided him with some assistance. Then Ms. Thomas rotated to another group of students. Looking at their task, Ms. Thomas asked, "What does it mean right here when it says, I have to have a minimum and a maximum?" The students replied, and Ms. Thomas provided feedback that was mostly inaudible. At one point, she asked how they could find the average of the numbers so that it met the requirements in the task. Then she moved to another group where a student asked Ms. Thomas to verify their work. Ms. Thomas agreed that her work was correct and then stopped the entire class. "I want to share something. Ok, everybody: A median of ten, a range of 40 , a minimum of zero, a mean of nine, and have exactly ten values in the set," she stated as she projected the task from which these came on the board. Ms. Thomas asked the class to think about how they "would approach the
problem" and gave them a few moments to think. Ms. Thomas added, "If you want to try it on a post-it notes, I don't care. Think how you would go after that." After making this statement, several students started working on small white boards at their desks and on sticky notes to solve the task that Ms. Thomas posted.

As the students worked, Ms. Thomas paused and asked, "Does anybody have any words of wisdom right here?" When the students did not respond, she asked them to continue working. After a few more moments of letting the students work individually, Ms. Thomas asked if any student would like to share how they approached the task. One student shared, "I would have approached it saying the minimum is zero, right? So, you would have the lowest number at zero. The biggest number as 40 -no, the highest number is $80 . "$ Ms. Thomas wrote these numbers on the board and he continued, "When you add those two together it's supposed to be [pause]" Ms. Thomas stopped the student and added, "Well, if you start at zero to 80 , what is your range?" The student answered, "40." Ms. Thomas asked again, "If you have 80 and zero, what is your range?" The student incorrectly answered again, and Ms. Thomas restated the question. Then the student said, "Oh, the range! What I would put on my paper is 40 , not 80 ." Ms. Thomas asked the student to continue. He said, "I went and put the median as ten, and I didn't get much farther. I put two ten's, and then I had two 30's up with the other numbers." Ms. Thomas wrote those numbers on the board, and then solicited thoughts from another student.

A second student shared her thoughts, "I messed around with the numbers, and I did one 50 because of the range, and I did 85 tens." Ms. Thomas asked, " 85 ten's?" The student agreed and continued, "Yeah, and it came out to a total of 900. ." Ms. Thomas
asked the student, "What was your highest number?" The student replied, "50," and then realizing that her work may be wrong, the student chose not to share anymore. Ms. Thomas then asked the students to look at some work from one group in the class that met the required statistics in the task projected. She took the students' work that was on a small white board and showed it to the entire class. "They took the zero. They put the 40. The rest of them - they plugged in with zeros and tens. Sometimes we're making it a little harder than it appears." Ms. Thomas then asked one group to come to the front of the room to share their work for their task. One of the students shared:

What we did: we lined up all of the numbers that we came up with and the range the range had to be 20 minus one number. Then we came up with eight, and then because we had to do 20 minus the small number to get [inaudible]. Then the mode is 11 , and we put four 11 's. We did the mean. We added up all the numbers and divided it by 15 . We added up all the numbers and got [inaudible]. Then we divided it by 15 .

Recognizing an error in their calculations, Ms. Thomas challenged this group, "You really need to add something somewhere because 14.2 averages out to 14 not 15 . As a challenge to you all, I want you to see if you can change one number and make it work."

Another pair of students then shared their work for their task. One of the students began, "We figured out that our mean was $1140 . "$ Ms. Thomas stopped the group and clarified, "That's actually not your mean. That's your total. Your mean is the average of that. Go ahead." The student continued, "We put down the number that if we add all this up, it would equal that and then we just found the median as 25 ." Ms. Thomas summarized their work for the class:

They took the number of data plots that they had to have, which was 15 . They knew that their mean had to be 76 . So, they did 76 times 15 . They came up with a total. Then they knew every single blank had to add up to that total. So, that's another way to approach it instead of trying to find the average of two numbers. Ms. Thomas then asked each of the four students who presented to help a group in the class and asked the class to work a bit longer.

After the students worked for a few minutes, Ms. Thomas recognized that the period was almost over and decided to conduct a summary discussion with the class. "I want everybody back to your seats. We're going to look at something. Did anybody attempt the question at the bottom that said what do you think would happen if you double the data?" A student replied something inaudible, and Ms. Thomas reminded the class, "Remember, we talked about distribution. Let's make up a quick [data set]." She proceeded to write a data set on the board and then changed her mind. Instead, she projected the teacher solution sheet for one of the tasks given to students. The solution sheet included a dot plot and box plot for the data set. Ms. Thomas began by reading the task, "They were supposed to find the mode, 11 , the range, 12 , the maximum, 20. What do you notice about-where is most of the data [in the dot plot]?" A student replied, "On the end." Ms. Thomas asked for clarification, and the student replied, "11." Ms. Thomas agreed and elaborated, "So, what does that make you think about what the average is going to be? You think the average is going to be close to that?" A student stated, "No," and Ms. Thomas challenged him by asking another student to calculate the average for the data set projected. The student quickly did the calculations and stated the average as, "12 something." Ms. Thomas then probed further, "Is it close to 11 ?" Several students
agreed, and Ms. Thomas expanded this idea, "What would have happened if we doubled this data? Would the shape continue to be the same?" A student dissented and Ms. Thomas asked him, "What would be different about the shape?" The student answered, "It would be bigger!" Ms. Thomas asked further, "It would be bigger? Would there still be more [data] in this area [pointed to the mode]?" The students agreed, and Ms. Thomas stated, "The shape would probably stay the same. It would get bigger as a unit, but I want you to look at this." Ms. Thomas then moved the teacher solution sheet to focus on the box plot, and the period for the end of class rang. Ms. Thomas dismissed the class before finishing this discussion.

## Fidelity of Implementation

Within this section, I first describe the deviations of the enacted lesson from the intended lesson for Day Seven. Then I discuss the alignment of the enacted lesson to reform expectations as outlined in the Daily Observation Protocol.

Deviations of enacted lesson from intended lesson. Ms. Thomas deviated from the intended lesson in three ways. First, during the lesson, Ms. Thomas decided to have the entire class work individually on a task that she projected on the board. She asked the students, "Think about how you would attempt that. If you want to try it on a post-it note, I don't care" (Video, 12/15/14). After students had individual time to work on the projected task, she called on students to share their work. Having the class work on this task individually and share their ideas for how to solve it was not included in the intended lesson plan.

Second, Ms. Thomas deviated from the intended lesson towards the end of the lesson during the students' presentations. In the intended lesson plan, two or three groups
were to share followed by a discussion for which questions were listed. Ms. Thomas asked two groups to share their work, but did not address the discussion questions in the intended lesson plan. I reflected on this in my journal, noting, "She did not use the discussion questions provided" (Field Notes, 12/15/14). Finally, Ms. Thomas deviated from the intended lesson plan at the end of the lesson when she projected the teacher solution sheet for one of the tasks. Specifically, Ms. Thomas asked the students to analyze the dot plot. She questioned, "What do you notice about-where is most of the data?" (Video, 12/15/14). This conversation continued by Ms. Thomas asking the students, "What would have happened if we doubled this data?" (Video, 12/15/14). Using the representations from the teacher solution sheet as a means to talk about this question was not in the intended lesson plan. In the lesson plan, students were expected to address this question individually, writing their response on paper and turning this in as an exit ticket.

Alignment of enacted lesson. Analysis of the Daily Observation Protocol for Day Seven revealed that Ms. Thomas' implementation fidelity for the lesson addressed parts of two sections within this protocol. In the analysis that follows, I exclude sections there were not evident during the lesson.

Standards for Mathematical Practice. Students engaged in one of the Standards for Mathematical Practice, using tools appropriately. As discussed in this section for Day Six, the nature of the task required students to constantly calculate different statistics and check the appropriateness to the task requirements. The students demonstrated efficiency by using the calculators to calculate more strenuous calculations (e.g., the mean) and not for easier calculations (e.g., the range).

## Barriers

Through the data, it was clear that one barrier potentially affected her implementation per her observations, a traditional perspective of the teaching and learning of mathematics. Ms. Thomas demonstrated this barrier during the lesson when she asked students who were struggling to come to her desk for assistance. While helping one of the students, Ms. Thomas directed her in how to solve the task, ending the session by stating, "If I were you, I would do these blanks, and do like I did" (Video, 12/15/14). This type of direct help happened at different groups during the lesson, an action that Ms. Thomas reflected on during the interview. I asked her about differentiating the task for students of various levels. She reflected on a student who was struggling and added, "I actually did half of it for them - to try to point out what he had to do" (Interview, $12 / 15 / 14$ ). I similarly reflected on this in my journal, describing Ms. Thomas' help as "very giving" (Researcher Journal, 12/15/14). Ms. Thomas also described this perspective in our interview in response to a question about the statistical problemsolving process. She stated:

I think that my fifth period can't be given the same freedom that my third period can. That's where it comes in hard to know how much of this type of activity you can do because some of them, if they're not handed the pencil, pen to paper, going, going, going ... [they're] not going to get [it]. (Interview, 12/15/14) This inclination to tell students how to do the mathematics in a task, focusing on the procedures, aligned with the traditional perspective of mathematics teaching and learning.

## Supports

Analysis of the data revealed that Ms. Thomas identified one support for the implementation of the lesson: curricular support. During our interview, I asked Ms. Thomas what has helped her in teaching this unit. She replied, "How it's laid out. How the lesson plan is there. How I didn't have to decide what questions to ask" (Interview, $12 / 15 / 14$ ). This statement reflected curricular support as important for Ms. Thomas during the implementation.

## Day Eight

In this section, I describe the implementation of Day Eight based upon the video data. Utilizing all data sources, I then describe the fidelity of implementation, barriers, and supports related to the implementation. On Day Eight, Ms. Thomas implemented the Oreo Task that was designed as a performance assessment for the intended goals of the unit.

## Implementation

On Day Eight, Ms. Thomas began the lesson by introducing the Oreo Task. She stated to the class, "I found this really cool complaint online. I thought we would go ahead and do this today. So, listen up." She then read the complaint from the performance assessment to the class. She explained the task further:

What you're going to do today is, you're going to address the concern of the customer, and the goal is to use the result from the experiment to improve the quality of Oreo cookies if your results say that it needs to be. Let's just all assume that you're working for the Nabisco Company, and you're considered a quality control officer. You've been asked to investigate this concern. So, what you're
going to do. You're going to come up with stuff, and you're going to go back and present it to the board of directors and the production manager. You're going to create a statistical experiment, identifying the statistical question and conducting the experiment. Remember, what is a statistical question? What makes it different from a regular question-non-statistical?

A student replied, "Having more than one to compare it to." Ms. Thomas added to her statement, "Or, the data can change. What's another word for change?" A student answered, "Vary," and Ms. Thomas agreed, "Vary. I'm going to put up the rubric, and you all are going to follow that." Ms. Thomas then walked to the projector and displayed the grading rubric for the students to see. She stated that students should use the rubric to "see [their] expectations" for the task. Ms. Thomas asked the students to take two minutes and examine the rubric. "Everybody take a moment, and read that to yourself."

As the students read the rubric, Ms. Thomas and one of her students distributed cookies (both single and double-stuffed and off-brand and name-brand) and a report form for the task to each group of students. Once students had an opportunity to read the rubric, she stated, "When we talk about statistics, we talk about what can change-what can cause the data to change." Ms. Thomas held up her package of cookies from which she had been giving students cookies and stated:

After looking at my package that I'm passing out, I think we can take this question, 'are double stuffed really double stuffed?' one step further. Because I think if we compare your data to the data in third period, I think Nabisco brand versus off-brand may show a different result. Because I'm looking at these double stuffed, and I'm not feeling it, but that doesn't mean they're not.

Ms. Thomas proceeded to explain to students how to use the digital scales they had been provided for the task, asking them to use napkins on the scales to keep them clean. Once all groups had their scales ready, Ms. Thomas instructed them, "Take one minute completely to yourself. You're going to think for one minute. How do I want to attack this?"

Once students had thought about the question, Ms. Thomas then asked them, "Spend two minutes, and share with your friends." Students talked at their groups, and then Ms. Thomas asked a student to share an idea about how to answer the question. The student shared, "We thought we could weigh the two Oreo's, and if they come out the same weight, then that double stuffed really isn't double stuffed. But if it comes out a better weight, then that means it is double stuffed." Ms. Thomas thanked the student for sharing and asked the class, "What is the reason you got two [cookies] instead of one of each?" A student replied, "You put the stuffing inside that one, and you weigh that. Then you do the double, and if they're the same, then that means they are." Clarifying his response, Ms. Thomas stated, "So, [he] wants to take the stuffing out of the single stuff, and put it on the other single stuff to see if that cookie weighs as much as the double stuff. Anybody got a different way they're going to do it?"

Another student shared his idea, "We're going to weigh both double stuffs, and if that comes out the same answer, then that means they give their double stuff like the same. And we're going to weigh both singles." Ms. Thomas acknowledged this response and asked another student to share. "I'm thinking take the whole thing apart. Weigh the cream, and then the cookie because they might scam you and just make the cookie bigger," he shared. Ms. Thomas responded, "He's thinking they might make the cookie a
little heavier so he wants to weigh the cream. I thought someone would come up with that. So instead of a knife, you can scrape it off with this." She handed each group an index card so that they could scrape the stuffing out of the cookies, set her timer to fifteen minutes, and asked the students to begin working on the task the way they felt was appropriate.

As the students worked, Ms. Thomas asked the class, "Keep your data for each cookie because if we have enough time, we're going to compare them to the brandname." Students then proceeded to work in their groups for several minutes, measuring the stuffing inside the cookies on the scales and recording their results. Ms. Thomas paused the students as they were working and stated, "Raise your hand if your group is still working on the task. . . . If you need my help, I'm here, but I'm just trying to see where you get to with the information you've been given." As the students continued to work, Ms. Thomas went to her desk and entered grades into her grade book. Two students approached her with questions about the task. The first student explained the weights for the stuffing that he found, and Ms. Thomas replied, "You're conclusion was? I need you to write that down." This student returned to his seat, and the second student asked her question about writing up their report. Ms. Thomas asked her, "State what you think your goal for this project was. What do you think it was?" The student gave her response, and Ms. Thomas probed further, "What statistical question did you answer?" Satisfied with the help, the student returned to her seat.

When the timer went off, Ms. Thomas asked the class about the name brand versus the off-brand cookies. She asked, "What do we do with them? You're going to compare. Is the double-stuffed double? If so, is it also more than the off brand? Have we
decided which has more stuffing?" Some students stated that they were still working so Ms. Thomas circulated the room, checking if students needed more cookies for their data collection. Students continued to work as Ms. Thomas entered grades and then circulated the room. Unlike the previous lessons, students did not ask Ms. Thomas as many questions about how to solve the task and instead self-directed their work.

After the time elapsed, Ms. Thomas asked the students to put away their scales, throw away any cookies that they used for data collection, and wash their hands in the restroom. When all students had cleaned their tables, Ms. Thomas asked a group of students to come to the front of the room and share their work. The students put a sheet of paper on the projector, demonstrating all of the data they collected.

First, we measured the stuffing and this is-single, double, single, double, and measured each of the stuffing's to see if they were the same number. This was six points shorter than this the first time. Mine was almost ten points shorter. Ms. Thomas clarified what the student meant by the word points. Ms. Thomas asked, "Was that grams it was measured in?" A student in the class replied, "Yes," and Ms. Thomas said, "Alright, so, ten grams [emphasis added]. Keep going." The students continued, "Then we measured just the cookies to see if they weighed more. The first time is 4.0 . The second time was $4.7 \ldots$ Then we weighed them all together, and the doubles were larger than the singulars." Ms. Thomas questioned the group, "Were they double?" A student in the group replied, "We didn't measure them, but they were more." Ms. Thomas asked about some of their values and what they should have been if it was truly double the amount of stuff. The student from the group replied, "Yeah, so, we didn't think it was double double [emphasis added]. It was just more, but not double."

Ms. Thomas asked the class to clap for the group that shared and asked a second group to share their work. The students started to share, but it was apparent they were unsure how to present their work. The group members chatted with each other at the front of the room, and recognizing that they needed a few moments, Ms. Thomas asked the class about the previous group, "The last group decided [the cookies] were not double. Is that correct?" The class agreed, and then the group at the front appeared ready to share. Ms. Thomas told the group, "Go ahead now." Two students from this group took turns sharing their results, including data on the entire cookie and just the stuffing. Ms. Thomas clarified, "Let me interrupt for just a minute. The data that includes the cookie-Is that data going to show us if the stuffing is double or not?" A student in the class stated, "No," and Ms. Thomas asked him to clarify why not. "Because the cookie could be more than the other cookie of the-than on the double. If both the tops were even, then, yes, you could do that," he shared. Ms. Thomas wanted to clarify this idea to the entire class so she asked three students (one girl and two boys) to join her at the front of the room and asked the group that was presenting to sit down. The three students stood at the front of the room but about two feet from Ms. Thomas. Ms. Thomas pointed to the two boys that had joined her at the front of the room and stated:

We're going to assume, for the sake of our argument, that they both weigh 70 pounds. We know they weigh 70 pounds. If you put [one boy] with me [one boy joined Ms. Thomas] and you put [the other boy] with her, is it fair to say that now our weight [pointed to herself and the boy next to her] is a whole lot on the scales and that [theirs] is not?

The bell rang for the end of the period, and Ms. Thomas asked the students to stay for one more moment. She asked, "Is it fair to say that you have to take the stuffing out to get a fair answer?" A few students replied, "No," to which she replied, "You do," and then dismissed the class.

## Fidelity of Implementation

Within this section, I first describe the deviations of the enacted lesson from the intended lesson for Day Eight. Then I discuss the alignment of the enacted lesson to reform expectations as outlined in the Daily Observation Protocol.

Deviations of enacted lesson from intended lesson. In this lesson, Ms. Thomas deviated from the intended lesson in two ways. First, at the end of the lesson, Ms. Thomas decided to let some students share their ideas for solving the task. After two groups shared their work, Ms. Thomas started a discussion about how to answer the question, using the scenario of weights of students to help students understand how to measure the stuffing in the cookies in a way that answered the question. She asked, "Is it fair to say that now our weight [pointed to herself and the boy next to her] is a whole lot on the scales and that [theirs] is not?" (Video, 12/18/14). Although Ms. Thomas was not able to finish this discussion because of time, this discussion was not in the intended lesson plan. Second, Ms. Thomas deviated from the lesson plan in that she did not ask students to take their data home to write their report for the project that was in the intended lesson plan. Instead, after the final discussion, Ms. Thomas dismissed the students.

Alignment of enacted lesson. Analysis of the Daily Observation Protocol for Day Eight revealed that Ms. Thomas' implementation fidelity for the lesson addressed parts of three sections within this protocol.

Statistical problem-solving process. Students were engaged in two parts of the statistical problem-solving process. First, students designed a plan for collecting their data to answer the statistical question. This component was evident at the beginning of the lesson when Ms. Thomas solicited students' ideas about how to answer the question. An example of a student response was, "We thought we could weigh the two Oreo's, and if they come out the same weight, then that double stuffed really isn't double stuffed" (Video, 12/18/14). My field notes also revealed that students were pursuing their own ideas for collecting the data once Ms. Thomas let them begin the task. I observed, "most [groups] were on-task and collecting lots of data the way they decided how to measure" (Field Notes, 12/18/14). Ms. Thomas reflected on this process twice in her journal. First, responding to a prompt about the day's lesson, Ms. Thomas wrote, "I feel the students enjoyed getting to decide 'how' to approach the question and how to analyze the data" (Participant Journal, 12/18/14). Second, Ms. Thomas reflected about what was good in the lesson. She wrote, "The students had different ways to 'weigh' the cookie or stuffing. I liked them coming up with that on their own" (Participant Journal, 12/18/14).

Second, students analyzed data as they collected it. I observed in my field notes that students were comparing results across the different brands. I noticed, "The students found out the name brand is not double compared to off brand" (Field Notes, 12/18/14). Students compared across the brands of cookies as well as across the type (i.e., single- or double-stuffed), using the statistics that they calculated. I also noted in my field notes
how one group of students calculated averages for all the different types of cookies (i.e., single- or double-stuffed and name-brand or off-brand cookie), "collecting lots of data and analyzing" (Field Notes 12/18/14). Ms. Thomas also reflected on this process in her journal as can be seen in a quote in the preceding paragraph.

Standards for Mathematical Practice. Students were engaged in one of the Standards for Mathematical Practice, using tools appropriately. Although students were asked to use the scales, some of the students also decided to measure the heights of cookies using rulers. I observed other students using "calculators as needed" (Field Notes, $12 / 18 / 14$ ) to calculate the statistical measures from their data that they felt would answer their question. Ms. Thomas reflected on this independence of the students in how they analyzed the cookies in her journal and during our interview. In her journal, Ms. Thomas stated, "I feel the students enjoyed getting to decide 'how' to approach the question and how to analyze the data" (Participant Journal, 12/18/14).

Mathematics Teaching Practices. Ms. Thomas engaged in one of the Mathematics Teaching Practices, eliciting and using student thinking. This was evident at the end of the lesson when she told students, "We're going to share our data" (Video, $12 / 18 / 14$ ) and then subsequently asked two groups to share their work. In this section of the lesson, Ms. Thomas recognized through the presentations that some students might not have understood the importance of measuring just the stuffing in the cookies. This realization was evident when she asked after a student presentation, "The data that includes the cookie-Is that data going to show us if the stuffing is double or not?" (Video, 12/18/14).

## Barriers

Analysis of the data revealed one barrier related to a traditional perspective of mathematics teaching and learning. In our interview, I asked about how she prepared for the implementation of the Oreo Task. Ms. Thomas commented:

I feel like had we gotten a little bit further in the unit, we could have done a lot more with it. . . . I hated that a lot of them didn't put together that you can't count the cookie. I didn't do a very good job trying to explain that before they left. (Interview, 12/18/14)

This barrier revealed itself again during the interview in response to a question about the statistical problem-solving process. Ms. Thomas commented on the first part of the process about formulating a question. She stated, "Formulate the question, no. They pretty much went with what I said. Is it double stuffed or not" (Interview, 12/18/14).

## Supports

Analysis of the data revealed that Ms. Thomas identified one support for the day's implementation: curricular support. In the interview, I asked Ms. Thomas what was most helpful in the implementation the lesson. She reflected:

Having the lesson plan given to me. Because like I said earlier [off-topic conversation to student] as a new teacher and as a new teacher to two subjects . . . I don't have the extra time to [make a lesson plan] so this way-by having it . . . The lesson is done. Now, I can concentrate on understanding it. (Interview, 12/18/14)

This reflection on curricular support continued later in the interview. I asked her about having the lesson plan for each task in the unit and she reflected, "My lesson's done. All I
have to do is figure out how I'm going to implement it. . . . I think having the lesson plans handed to me where I could just concentrate on delivering the content" (Interview, 12/18/14). This statement revealed that Ms. Thomas felt that having the lesson plan allowed more time for her to understand the ideas within the lesson instead of on actually creating the lesson plan itself.

## Emerging Themes

After analyzing each individual day, I looked across all eight days for emerging themes. This analysis revealed themes in four key areas: deviations of enacted lesson from intended lesson, alignment of enacted lesson with intended lesson, barriers, and supports. In this section, I discuss the themes for each of the aforementioned areas separately.

## Deviations of Enacted Lesson from Intended Lesson

In terms of deviations, I observed two common deviations from the intended lesson plan across all eight days. First, Ms. Thomas decided to have mini lessons for students who struggled with the tasks during the unit implementation. In all of these mini lessons, the structure was similar. Ms. Thomas asked students for questions, created a data set, and demonstrated the appropriate procedures for the task using the data set for the students. On Day Two, Ms. Thomas reflected on the mini lesson that she conducted that day. "I know we're not supposed to give them the answers, but some of them, if I don't show them, this is this gets this, they'll never get it" (Interview, 12/8/14). On this same day, she gave a justification for this deviation. She commented about how the students needed her "to visually show them, which is why I took them back to the board, and we went over what each one of the words looks like with" (Interview, 12/8/14).

Second, on three occasions, Ms. Thomas displayed the teacher solution sheet to students on the projector. On Day Two, she informed the students before showing the solution sheet, "I'm not supposed to be giving you all answers and showing you all what to do. But, I'm trying to give you a good foundation to start with" (Video, 12/8/14). Ms. Thomas deviated in this way on days One, Two, and Seven. It appears, from the preceding quote, that Ms. Thomas felt that showing this to the students helped them in understanding the concepts.

## Alignment of Enacted Lesson

In terms of the alignment of the enacted lessons with reform philosophy, I analyzed the Daily Observation Protocol for each of the eight days. Looking across days, overall themes emerged for each of the three components on the protocol. Next, I describe the themes for each of these three components.

Statistical problem-solving process. The four components of the statistical problem-solving process represent a process in which students are expected to engage as outlined in the GAISE document (Franklin et al., 2007). The four components include: formulate questions, collect data, analyze data, and interpret results, and at the sixthgrade level, students should engage in all four of the components. It is important to note that Franklin et al. (2007) stated that students at the earlier levels of understanding would not be expected to engage in the statistical problem-solving process at a deeper, more sophisticated level as students at later levels.

Across the eight days, the statistical problem-solving process was the least addressed portion of the Daily Observation Protocol, only being addressed on three of the eight days (see Table 4). Of the four components in the process, the implementation
addressed the second and third components. The third component was evident most often for a total of three of the eight days followed by the second component for a total of two days. Interestingly, observations of the lessons revealed that students did not have the opportunity to formulate questions or interpret results in context despite their inclusion in the unit.

Table 4
Evidence of the Statistical Problem-Solving Process
Day

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Statistical Problem-Solving Process

1. Formulate questions.
2. Collect data. X X
3. Analyze data. X X X
4. Interpret results.

Standards for Mathematical Practice. According to the CCSSM (CCSSI, 2010) the Standards for Mathematical Practice include eight mathematical practices, which should be developing in students throughout their mathematical career. In terms of the three components of the Daily Observation Protocol, only the Standards for Mathematical Practice component was addressed in some form on all eight days of implementation (see Table 5). That is, on any day of implementation, one would observe students' engagement in at least one of the Standards for Mathematical Practice within the classroom. Of these standards, students were most engaged in using tools appropriately throughout the unit. Students engaged in this standard six of the eight days. This
engagement was limited, however, as students only used the calculator and the scales as tools during the unit.

Across the unit, students also engaged in the first practice (three of the eight days), the third practice (two of the eight days), and the second and sixth practices (one of the eight days). The first practice was evident when students conjectured how to solve a task, as can be seen during the Oreo Task implementation on Day Eight. The second practice was evident when students would reason about the statistics and use the context of the task to make sense of the values, as can be seen during the French Fry Task on Day One. The third practice was evident when students evaluated each other's arguments, as can be seen during the continuation of the Construct Your Own Graph Task on Day Five. The sixth practice was evident when students carefully created tables and attended to the accuracy of their calculations, as can be seen during the continuation of the French Fry Task on Day Two. Practices four, seven, and eight were never evident throughout the unit implementation.

## Table 5

Evidence of the Standards for Mathematical Practice

|  | Day |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Standards for Mathematical Practice | 1 | 2 | 3 |  |  | 5 | 6 |  | 7 | 8 |
| 1. Make sense of problems and persevere in solving | X | X |  | X |  |  |  |  |  |  |
| them. |  |  |  |  |  |  |  |  |  |  |
| 2. Reason abstractly and quantitatively. | X |  |  |  |  |  |  |  |  |  |
| 3. Construct viable arguments and critique the reasoning |  |  |  |  |  | X |  |  |  |  |
| of others. |  |  |  |  |  |  |  |  |  |  |
| 4. Model with mathematics. |  |  |  |  |  |  |  |  |  |  |
| 5. Use appropriate tools strategically. | X |  | X |  |  | X | X |  | X | X |
| 6. Attend to precision. |  | X |  |  |  |  |  |  |  |  |
| 7. Look for and make use of structure. |  |  |  |  |  |  |  |  |  |  |
| 8. Look for and express regularity in repeated reasoning. |  |  |  |  |  |  |  |  |  |  |

Mathematics Teaching Practices. The Mathematics Teaching Practices are eight teacher practices that have been identified as essential for promoting students' deep understanding of mathematics (NCTM, 2014). Teachers who exhibit these practices help students make sense of the mathematics they are learning through interactions and communication. In terms of the Mathematics Teaching Practices, Ms. Thomas engaged in at least one of the eight practices on six days (see Table 6). On Days Four and Five, Ms. Thomas engaged in the highest number of Mathematics Teaching Practices throughout the implementation, specifically three of the practices. On Day Four, she posed
purposeful questions, helped students build procedural understanding from a conceptual understanding, and elicited and used student thinking. On Day Five, Ms. Thomas used and connected different mathematical representations for the data set, facilitated meaningful discourse around the mathematics, and elicited and used student thinking.

Three of the eight practices were never addressed: establish mathematics goals to focus the learning, implement tasks that promote both reasoning and problem solving, and support students in productive struggle. It is not surprising that Ms. Thomas did not engage in the first two of these practices as these were inherent within the unit itself and not something she would have been expected to create herself. That is, each lesson had a specific mathematics goal listed in the unit and included tasks that promoted reasoning and problem solving. I determined that students were not engaged in productive struggle given Ms. Thomas' direct help and mini lessons with her students when they had difficulty correctly calculating a statistical measure. This type of assistance does not allow for the students to productively struggle in mathematics.

Table 6
Evidence of the Mathematics Teaching Practices

|  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations. X X
4. Facilitate meaningful mathematical discourse. X
5. Pose purposeful questions. X X
6. Build procedural fluency from conceptual X understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking. $\quad \mathrm{X} \quad \mathrm{X} \quad \mathrm{X} \quad \mathrm{X} \quad \mathrm{X} \quad \mathrm{X}$

## Barriers

Data from Ms. Thomas revealed four specific barriers throughout the eight days: a traditional perspective of mathematics teaching and learning, the general area of subject matter knowledge, the general area of pedagogical content knowledge, and knowledge of content and students (see Table 7). Teachers who hold a traditional perspective of mathematics teaching and learning believe that his or her role as a teacher is to transmit knowledge to students by telling them procedures and then asking the students to practice
those procedures until they are correct (Tzur et al., 2001). Subject matter knowledge is traditionally defined as including common content knowledge, specialized content knowledge, and horizon content knowledge (Ball et al., 2008). Pedagogical content knowledge, according to Ball and colleagues (2008) includes knowledge of content and teaching, knowledge of content and curriculum, and knowledge of content and students.

When discussing barriers to implementation, Ms. Thomas' ideas were categorized as representing a traditional perspective of mathematics on each of the eight days. This barrier was often evident across multiple data sources. Ms. Thomas next identified subject matter knowledge (i.e., common or specialized content knowledge) most frequently, doing so on five of the eight days. This barrier was also typically evident across multiple data sources. Finally, she identified the specific barrier of knowledge of content and students one of eight days and the general barrier of pedagogical content knowledge on two of the eight days.

Table 7
Barriers Identified


## Supports

Ms. Thomas identified five distinct supports over the eight days of implementation: experience, practice, time, curricular support, and knowledgeable others (see Table 8). Of those five, Ms. Thomas most identified both curricular support and knowledgeable others. Specifically, she discussed those supports on four of the eight days. Next, Ms. Thomas identified the support of practice three of the eight days. Finally, she identified both experience and time two of the eight days.

Table 8
Supports Identified

| Supports | Day |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Experience. | X |  | X |  |  |  |  |  |
| Practice. | X |  | X |  |  | X |  |  |
| Time. |  | X |  | X |  |  |  |  |
| Curricular support. |  |  | X |  | X |  | X | X |
| Knowledgeable others. |  |  | X | X | X | X |  |  |

## Chapter Summary

In this chapter, I detailed the implementation of the unit for each of the eight days followed by an analysis of how the implementation deviated from and aligned with reform-based philosophy. This analysis also included barriers and supports that were revealed in Ms. Thomas' responses to the prompts in her participant journal, the video
data, and interviews. After describing each day, I then summarized the themes that were evident across all of the eight days.

## CHAPTER FIVE: DISCUSSION

## Introduction

As statistics education continues to become more pronounced in the K-12 curriculum, issues around teachers' statistical misconceptions (Callingham, 1997; Groth \& Bergner, 2006; Jacobbe, 2008; Leavy \& O’Loughlin, 2006; Makar \& Confrey, 2004; Mickelson \& Heaton, 2004) and the feeling of being unprepared to teach statistics (Begg \& Edwards, 1999; Greer \& Ritson, 1994) become issues that need to be better understood. Despite efforts to prepare in-service teachers to teach statistics as is expected, many teachers maintain statistical misconceptions after participating in professional development (Jacobbe \& Horton, 2012). Similarly, those teachers who completed a course in statistics in their teacher preparation programs often received traditional instruction and were unaware of their limited statistics knowledge (Jacobbe \& Horton, 2012; Stohl, 2005).

The purpose of this study was to examine a middle-grades teacher's implementation of a reform-oriented statistics unit, identifying support structures needed for successful implementation. Specifically, the fidelity of implementation of this unit was observed and support structures from this observation were identified. The central research question was: How does a sixth-grade teacher implement a reform-oriented statistics unit? Given this implementation, the secondary research question was: What support structures will the participant identify as needed for the unit to be implemented with fidelity? In this section, I review the methodology and results as described in Chapters Three and Four. I then discuss the relationship of these results to other results in
the literature, identifying the unique contribution of this study to the education community. Finally, I make recommendations for further research.

## Review of Methodology

The primary research question sought to describe the circumstance of implementation of the statistics unit in a middle-grades classroom. Therefore, following Yin's (2014) recommendation, I utilized a descriptive case-study approach. Yin (2014) stated that the case study method is appropriate when the researcher is unable to manipulate the behavior being observed, which pertained to the nature of this study. This study included one single case of a sixth-grade mathematics teacher observed over several instances and, therefore, embodied a single-case, holistic design observed over several points in time. The study took place in one rural, sixth-grade mathematics classroom that met daily for approximately 46 minutes.

I chose the participant in this study, Ms. Thomas, for two reasons. First, she had some experience with reform-oriented instruction due to her enrollment in a professional development project that utilized reform-oriented practices for each professional development session. Second, assessment revealed that Ms. Thomas held a growth mindset. This was enticing in terms of a participant because it revealed to me that she would be open to teaching in this manner. At the time of this study, Ms. Thomas was in her third year of teaching, but she was only in her first year of teaching mathematics and science. Prior to this position, she taught language arts in the same school. Ms. Thomas revealed that she had received no professional development in statistics and only took one course of statistics in her teacher preparation program.

To answer the research questions posed, I utilized five sources of data: field notes, a Daily Observation Protocol, Interview Protocols, Participant Research Journal, and a Researcher Journal. Using myself as the primary data collection instrument (Creswell, 2013), I interviewed Ms. Thomas prior to, throughout, and after the implementation of the unit that lasted eight days. I videotaped each lesson implementation and took field notes during the lesson. After each daily lesson, both Ms. Thomas and I wrote in our journals. Ms. Thomas specifically answered prompts that I gave her digitally, and I wrote freely each afternoon between the lesson implementation and our daily interview. I analyzed this data following four steps outlined in Yin (2014). The first step in the data analysis procedure was to examine the data "searching for patterns, insights, or concepts that seem[ed] promising" (Yin, 2014, p. 135). The second step in the data analysis procedure was to thoroughly examine the data using the inductive strategy described in Yin (2014), assigning codes to the data that revealed a theme or concept. The third step in the analysis was to aggregate the codes into larger, emerging themes. These emerging themes were then interpreted to understand "the larger meaning of the data" (Creswell, 2013, p.187) at which time the final step in the analysis was conducted. This final step included constructing a "case study report" (p. 178) following a chronological structure (Yin, 2014).

## Review of Results

In Chapter Four, I detailed the implementation of the lessons for each of the eight days. I then described how those lessons deviated from and aligned with the reformoriented philosophy of the unit. Finally, I discussed the barriers and supports that Ms. Thomas identified during the course of the study. Across the eight days, themes emerged
related to these four areas. In terms of deviations from the lessons, Ms. Thomas frequently deviated from the lesson in two ways. The first was by engaging students in what she called a mini-lesson. These mini-lessons often involved a small group of students from the larger class. During these mini-lessons, the focus was on procedures and calculations. She justified these mini-lessons during one of our interviews, stating, "I know we're not supposed to give them the answers, but some of them, if I don't show them, this is this gets this, they'll never get $\mathrm{it} \mathrm{\prime} \mathrm{\prime}$ (Interview, 12/8/14). The second common deviation happened when she projected the teacher solution sheet to the class. Similar to her justification for the mini-lessons, Ms. Thomas stated to her class, "I'm not supposed to be giving you all answers and showing you all what to do. But, I'm trying to give you a good foundation to start with" (Video, 12/8/14). These deviations can be attributed to Ms. Thomas' traditional perspective about mathematics teaching and learning. That is, her belief that mathematics learning is achieved when students can complete the procedure influenced her choice to demonstrate procedures in the mini-lessons and have students verify their procedures with the teacher solution sheet.

In terms of alignment, I specifically examined the statistical problem-solving process, the Standards for Mathematical Practices, and the Mathematics Teaching Practices. Analysis of the data revealed that the statistical problem-solving process was the least addressed component during the length of the study. When students were engaged in this process, they were engaged in either collecting or analyzing data. Students never engaged in formulating a statistical question or interpreting the results in context. In contrast, students engaged in at least one of the Standards for Mathematical Practice on each of the eight days. The most common standard involved students using
tools appropriately. Finally, in terms of the Mathematics Teaching Practices, Ms. Thomas engaged in some of the practices on six of the eight days. Most frequently, she elicited and used student thinking during the implementation.

In regards to barriers, Ms. Thomas identified five specific barriers to implementation that were classified as follows: a traditional perspective of mathematics teaching and learning, the general area of subject matter knowledge, horizon content knowledge, the general area of pedagogical content knowledge, and knowledge of content and students. Across the eight days, the data revealed that the traditional perspective of mathematics was the most common barrier cited. In regards to supports, Ms. Thomas identified five distinct supports over the eight days of implementation: experience, practice, time, curricular support, and knowledgeable others. Both curricular support and knowledgeable others were most cited as supports throughout the study.

## Discussion

The research questions for this study were twofold: how does a sixth-grade teacher implement a reform-oriented statistics unit, and what support structures will the participant identify as needed for the unit to be implemented with fidelity? The results from Chapter Four revealed that Ms. Thomas did not fully implement the unit as intended. Utilizing the conceptual framework in Chapter Two, I hypothesize that her implementation was significantly affected by both Ms. Thomas' MKT/SKT and her mathematical teaching perspective. Returning to the data supported the hypotheses made in this section.

Although it was expected that Ms. Thomas would deviate from the lesson plan to some extent, her implementation revealed two common deviations throughout the unit
including the use of mini-lessons and the projection of the teaching solution sheet. I hypothesized that these deviations directly connected to the traditional perspective barrier. Given that teachers with this perspective feel that students assume a passive role in obtaining mathematical knowledge through, for example, watching others solve problems (Simon et al., 2000), it seemed that her utilization of the mini-lessons and teacher solution sheet were directly related to her traditional perspective of mathematics. In terms of the teacher solution sheet, Ms. Thomas would use this as an alternative to the summary discussions included in the unit. Dismissing the lesson discussion prohibited the students from engaging in lesson closure. Due to Ms. Thomas' traditional perspective, her replacement of the lesson summary with the teacher solution sheet was likely due to her believing that once students achieved the answer then the lesson was complete.

In terms of her alignment, the fact that students never engaged in the statistical problem-solving process component of formulating a question also connected to the traditional perspective barrier. Within the intended unit, the expectation was for students to create and explore their own statistical questions, the first component in the process. However, during the lesson implementation, Ms. Thomas decided to guide the students by telling them what question to explore. Although she never verbalized this, it appeared that, similar to her justification for the mini-lessons, she felt that students needed this guidance to provide them with a "good foundation to start with" (Video, 12/8/14). This reflected a traditional perspective of mathematics, the most common barrier cited.

When considering the Standards for Mathematical Practice and Mathematics Teaching Practices, the students use of tools and Ms. Thomas' ability to elicit and use students' thinking seemed to be related to the supports she identified and I coded as
curricular support. Within the unit, expectations for what tools students could use were listed as well as discussion questions for specific parts of each task. For example, lesson plans addressed the materials and tools needed for each daily lesson (see Appendix E). Also, by examining the teacher lesson plans in the unit (see Appendix E), one can see the explicit attention to the Mathematics Teaching Practices. For example, throughout the unit, the lesson plan states that the teacher should randomly call on students to share their ideas. It appeared that these supports provided Ms. Thomas with the help needed to address and have students address these practices within her classroom.

Analysis of barriers revealed that Ms. Thomas consistently identified both content knowledge and PCK as barriers to implementing the unit. Despite the pre-unit discussion, the previously completed statistics course, and the available resources, subject matter knowledge proved to be a significant barrier for Ms. Thomas. This aligned with results with Groth and Bergner (2013) and Hill et al. (2005) in which it was revealed that experience in content courses does not guarantee strong content knowledge. Given that Copur-Gencturk (2015) and Galant (2013) both found that teachers' mathematical knowledge for teaching affected their instructional practices, these barriers could potentially have been the catalyst for the deviations described above. Analysis of the five supports identified by Ms. Thomas revealed the importance of experience, practice, time, curricular support, and knowledgeable others in supporting teachers' implementation of reform-oriented units. I hypothesized that had some of these supports been more pronounced during the implementation of the unit (e.g., more time for planning with knowledgeable others), the deviations and alignment of the lesson would have been different.

As demonstrated in Figure 2, I designed the unit in such a way that the statistical problem-solving process, the Standards for Mathematical Practice, and the Mathematics Teaching Practices were an explicit part of the unit, situating the unit on the right end of the perspectives continuum. On the other end of the continuum, I situated Ms. Thomas within the traditional perspectives continuum. Despite this misalignment, Ms. Thomas elicited and used students' mathematical thinking throughout the implementation of the unit. Given that this Mathematics Teaching Practice was explicitly part of the unit, it can be hypothesized that the unit supported her in utilizing these practices, indicating a transition from her traditional perspective towards a perception-based perspective (see Figure 3). As seen in Figure 3, however, Ms. Thomas is not situated entirely within the perception-based perspective. Given that I was unable to continue observing Ms. Thomas, I am unaware if she continued engaging in the Mathematics Teaching Practices after the end of the study. Hence, I am unable to determine if her perspective completely transitioned along the continuum. I hypothesized that had Ms. Thomas had more time to implement the unit, more of the Mathematics Teaching Practices would have been addressed and this shift might have been more pronounced.


Figure 2. Perspectives continuum including placement of unit and participant.


Figure 3. Perspectives continuum including placement of unit and transition of participant.

## Recommendations

In this section, I discuss recommendations for improving similar studies, conducting future research, and influencing the mathematics and statistics education communities.

## Improving Similar Studies

In terms of improving similar studies, the timing of the unit and lack of time for Ms. Thomas to plan adequately with peers was a detriment. Ensuring adequate time for
implementation should be considered and implementation of similar units should be purposefully placed in the academic year to avoid hurried times for teachers. The unit addressed several mathematical and statistical ideas that take longer than eight days for students to develop. Had the unit been implemented over a longer period of time, different results might have been observed. A second recommendation for improving similar studies would be to provide stronger support in the unit for subject matter knowledge and PCK as these both appeared as barriers for her implementing the unit. As a specific example for subject matter knowledge support, procedures for finding the statistical measures that highlight the underlying concepts for those procedures should be included as these were sometimes a challenge for Ms. Thomas. Although this type of support is readily available in other resources, an inclusion of specific PCK supports should be included to differentiate assistance in the unit from other resources. For example, stronger support should be provided in terms of students' misconceptions and expected student responses to certain questions.

A final recommendation would be to replicate this descriptive case study attending to the limitations that presented themselves. If similar results emerge, specifically that the reform-oriented unit supported the development of the teacher in transition despite her traditional perspective, then a follow-up explanatory case study should be conducted. This explanatory case study should examine whether or not there exists a relationship between the reform-oriented unit and the teacher's transitioning mathematical perspective.

## Conducting Future Research

In terms of future research, it would be beneficial to examine the implementation of the unit over a longer period of time. This could reveal how having and implementing a reform-oriented curriculum over a longer period of time affects: the teacher's mathematical perspectives and mathematical and statistical knowledge for teaching; the teacher and students' beliefs about statistics; and the teacher's instructional practices in the classroom. The support of knowledgeable others also reveals another potential research study. Given that the timing of the unit happened when Ms. Thomas was unable to meet for common planning, it would be interesting to explore how that influence of a consistent knowledgeable other affects the implementation fidelity of a reform-based unit.

## Influencing the Mathematics and Statistics Education Communities

The results of this study found in Chapter Four reveal two recommendations for the mathematics and statistics education communities. First, those providing professional development for teachers in statistics should consider how teachers' mathematical perspective affects their instructional practices. It would be interesting to explore whether mathematical perspectives are the same for those teaching statistics given the different epistemology of statistics. Also, those leading professional development for teachers should emphasize engaging teachers in the type of instruction expected of them, that is one that assumes the reform-oriented philosophy. This aligned with the results from Chamberlin (2015) who found that when teacher leaders were allowed to problematize learning, the researchers observed a transition in the teacher leaders' perspectives towards a more conception-based perspective. Second, the significant barrier of subject matter
knowledge for Ms. Thomas emphasizes that those in teacher preparation programs should continue to focus on developing and improving pre-service teachers' mathematical and statistical content knowledge. This also indicates that in-service teachers need continued support in improving their mathematical and statistical content knowledge.

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## APPENDICES

## APPENDIX A: Daily Observation Protocol

| Date: |  |  |
| :---: | :---: | :---: |
| Description of Classroom: |  |  |
| Reform-Oriented Practice Fractiose way be met ha tharir eneferty or in pare gitker rarialian puth y VIS. Net meting ary pert zera a No. | Was this practice met? | Justification |
| Students were engaged in the statistical problem-solving process (Franklin et al., 2007). <br> 1. Formulating a statixtical question <br> 2. Designing a plan for collecting uneful data, implementing the data, and collocting the data. <br> 3. Analyzing the data <br> 4. Interpreting the results |  |  |
| Students were engaged in the Standards for Mathematical Practice (CCSSI, 2010). <br> 1. Make sense of problema and persevere in solving them. <br> 2. Reawon abotractly and quantitztively. <br> 3. Construct viable arguments and eritique the rezwning of ofters. <br> 4. Model with mutbematics. <br> 5. Une approprate tools stategically. <br> 6. Attend to precixion <br> 7. Look for and make une of atructure. <br> 8. Look for and exprexe regularity in repasted reanoning. |  |  |
| The teacher practiced the following Mathematics Teaching Practices <br> (NCTM, 2014). <br> 1. Extablish mathematicx goals to foces learning. <br> 2. Implement tarks that promote reasoning and problem solving. <br> 3. Use and connect mathematical representations. <br> 4. Facilitate meaning ful mathematical diveoume. <br> 5. Pove purposeful questions. <br> 6. Build procodural fluency from conceptaal underntanding. <br> 7. Suppoet productive atruggle in learning mathematice <br> R. Elicit and une evidence of stedent thinking. |  |  |

The teacher may not engage in all of these practices during one lesson. Make note of practices that teacher is engaged in and how this was justified during the lesson.

## APPENDIX B: Interview Protocols


4. Did you feel prepared to teach the content?
5. What did you perceive as the big ideas from today's lesson?
6. How do you think students were engaged in statistical problem solving?
7. What were some pros/cons/difficulties with today's lesson?
8. What would have made the implementation better?
9. What is needed for upcoming implementation? (i.e., what supports do you need to continue this tomorrow? Any concerns about content? Pedagogy?)
10. Any suggestions for the unit so far?

1. Describe your knowledge of the GAISE document, CCSSM statistics standards, NCTM standards for statistics.
2. What do you think are the big ideas for your students to learn about statistics? Why do you think these ideas are important?
3. What is it you want students to learn about statistics in your class?
4. What helped/hindered you most during this unit?
5. What do you wish you had (for example, knowledge, professional development, books, resources, etc.) to assist you in implementing this unit?
6. How has the support you received in the past affected (good or bad) how you taught this unit?
7. Describe how your future statistics instruction might look.

## APPENDIX C: Participant Research Journal

Daily Prompts:

1. Describe how you feel about today's lesson.
2. What do you think was good/bad about today's lesson?
3. How would you change this lesson if you were to teach it again?
4. What helped/hindered you during the implementation of this lesson, be specific? This could be related to content knowledge, student discourse, classroom management, etc.
5. Please write any other thoughts pertaining to today's lesson or any part of the study thus far.

## APPENDIX D: Codes

Curricular Support<br>Experience<br>Knowledge of Content and Students<br>Knowledgeable Others<br>Pedagogical Content Knowledge<br>Perception-based Perspective<br>Practice<br>Subject Matter Knowledge<br>Time<br>Traditional Perspective

## APPENDIX E: Statistics Unit

## STAGE 1: IDENTIFY DESIRED RESULTS

Standards, Competencies and Objectives:

- $6^{\text {th }}$ grade
- 6.SP.A.1-3; 6.SP.B.4, 6.SP.B.5(a-d)
- Develop understanding of statistical variability; Summarize and describe distributions

Understanding (s):
Students will understand that:

1. statistical questions anticipate responses with variability;
2. a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape; and
3. measures of center and variation must be interpreted in context.

Knowledge:
Students will know:

1. what makes a question statistical in nature;
2. what the term distribution represents and means;
3. that the measures of center (mean and median) describe all values with one number;
4. that the measures of variation describe how all values vary with one number;
5. what you can glean about the data from a dot plot, histogram, and box plot;
6. that without interpretation within a context, the statistics calculated have no contextual meaning;
7. that a percentage represents a part to a whole; and
8. how the shape of the distribution will affect the choice for measure of center.

Skills:
Students will be able to:

1. calculate $I Q R$ with a given data set;
2. construct a data set with a given mean, median, and range;
3. write a statistical question;
4. determine unusual observations and what makes them unusual;
5. describe a distribution by its shape and unusual observations;
6. create a representation including dot plots, histograms, and box plots;
7. report the total number of observations in a data set;
8. identify the units of measurement for measures of center and variation;
9. calculate a percentage;
10. justify their choice of measure of center and variability, and describe these in context of the problem; and
11. use informal language appropriately when describing the distribution

## STAGE 2: PLANNING ASSESSMENT



## Other Evidence:

Academic Prompt(s):

1. Two data sets were collected to answer the two following statistical questions: What are the heights (in inches) of sixth grade boys at Anywhere Middle School? 55, 68, 56, 61, 60, 56, 59, 64, 65, $69,73,60,57,58,54,62,62,64$ Last year, Mrs. Johnson's students were asked what their favorite color was. They responded with the following:
blue, red, blue, yellow, yellow, red, green, black, red, red, red, pink, blue, purple, red, red, orange, red, blue. Compare and contrast these two data sets making sure to discuss the representations you might use for each set.
2. Compare and contrast the dot plot with the histogram. What are some similarities that you notice between the graphs? What are some differences? Which do you prefer, and why?
3. What is a percentage? Describe using numbers, words, and symbols.
4. Pick one of the data sets that you created from today's task. Imagine that I asked you to DOUBLE every value in the set. What would happen to your histogram? Would it change? If so, how?
5. Now we must interpret our results in the context of our problem. Our goal is to write a letter to another classmate, describing the statistical process that we did, including what question we explored, how we collected the data, how we analyzed

$\square$

| 9.6 | 5.4 | 3.3 | 6.7 | 7.0 | 5.0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.0 | 3.4 | 5.5 | 5.5 | 8.8 | 10.0 |
| 8.3 | 5.6 | 2.3 | 6.1 | 7.1 | 11 |

What is the typical commute time for these $5^{\text {th }}$ graders? What does typical mean? How do you find this value?
Round to one decimal place.
9. What do measures of center (mean and median) tell us about a data set? What do measures of variation tell us about a data set?

## Statistics Unit Test

Name: $\qquad$ Date: $\qquad$

1. Which of the following are statistical questions? Circle yes or no beside each question and state why you think it is or is not a statistical question.
a. How many days are in March? YES NO

Explanation: $\qquad$
b. How old is your dog? YES NO

Explanation: $\qquad$
c. How old are the dogs on this street? YES NO

Explanation: $\qquad$
d. Do people in this school like watermelons? YES NO

Explanation: $\qquad$
e. Do you like watermelons? YES NO

Explanation: $\qquad$
2. Write a statistical question about your school. Explain why you think this is representative of a statistical question.
Question: $\qquad$

Explanation: $\qquad$
$\qquad$
$\qquad$
3. Eli really loves chocolate chip ice cream, but his sister's favorite ice cream is vanilla. He is interested in the students at his school and what might be their favorite flavor. Write a question that Eli could ask his classmates, and describe how he might collect data to answer this question.

Statistical Question:
$\qquad$

Describe how to collect data for this question:
$\qquad$
$\qquad$
$\qquad$
4. Below are the 20 birth weights, in ounces, of all the Labrador Retriever puppies born at Kingston Kennels in the last six months.
$18,17,15,15,20,17,17,16,13,16,16,18,18,18,17,16,17,18,19,19$
Use an appropriate graph to represent these birth weights. Explain why you chose the graph that you did.
Graph:

Explanation:
$\qquad$
$\qquad$
a. Describe the distribution of birth weights for puppies born at Kingston Kennels in the last six months. Be sure to describe shape, center, and variability.

Shape: $\qquad$
$\qquad$

Center: $\qquad$
$\qquad$
$\qquad$

Variability: $\qquad$
$\qquad$
$\qquad$
b. What is a typical birth weight for puppies born at Kingston Kennels in the last six months? Include the appropriate unit of measurement. Explain why you chose this value.

Explanation: $\qquad$
c. Are there any unusual observations? If so, what are they, and why are they unusual?
$\qquad$

If not, why not?
$\qquad$
$\qquad$
5. Each of the 20 students in Mr. Anderson's class timed how long it took them to solve a puzzle. Their times (in minutes) are listed below:

| Time (minutes) | 3 | 5 | 4 | 6 | 4 | 8 | 5 | 4 | 9 | 5 | 3 | 4 | 7 | 5 | 8 | 6 | 3 | 6 | 5 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Display the data using a boxplot.
a. Find the mean and median of the data. Interpret these in context of the data.

Include the appropriate unit of measurement. Round your mean to one decimal place.
Mean:
Interpretation: $\qquad$

## Median:

Interpretation: $\qquad$
b. What is the range for this data set? What does it tell us about the data?

Range:
Interpretation: $\qquad$
6. Aidan was interested in the eye color of people at his after school club. He collected the following data from those people in his club:

| Blue | Brown | Brown | Blue | Brown |
| :---: | :---: | :---: | :---: | :---: |
| Hazel | Green | Hazel | Brown | Brown |
| Brown | Green | Blue | Blue | Brown |
| Brown | Blue | Blue | Hazel | Hazel |

Construct a bar chart with the frequencies for eye color. Interpret the results in the context of the problem.

Graph:

Interpretation: $\qquad$
$\qquad$
7. Construct a data set with the following parameters:

Mode: 25

Range: 20

Median: 30

## KEY Statistics Unit Test

1. Which of the following are statistical questions? Circle yes or no beside each question and state why you think it is or is not a statistical question.
a. How many days are in March? YES NO

Explanation: The number of days in March does not vary.
b. How old is your dog? YES NO

Explanation: The age of your dog does not vary.
c. How old are the dogs on this street? YES NO

Explanation: Each dog on this street will have a different age. The responses will vary.
d. Do people in this school like watermelons? YES NO

Explanation: Not everyone will respond with the same response. The responses will vary.
e. Do you like watermelons? YES NO

Explanation: There is no variability in this answer.
2. Write a statistical question about your school. Explain why you think this is representative of a statistical question.
Question: e.g. What color do students most like at my school?
Explanation: The answers to this question can vary. Not everyone will have the same favorite color.
3. Eli really loves chocolate chip ice cream, but his sister's favorite ice cream is vanilla. He is interested in the students at his school and what might be their favorite flavor. Write a question that Eli could ask his classmates, and describe how he might collect data to answer this question.

Statistical Question: e.g. What are my classmates' favorite ice cream flavors? Describe how to collect data for this question:
Ask everyone in the school to write their favorite ice cream on a slip of paper and turn in (as a poll/survey).
Any variation on this would be acceptable as long as it seems feasible.
4. Below are the 20 birth weights, in ounces, of all the Labrador Retriever puppies born at Kingston Kennels in the last six months.
$18,17,15,15,20,17,17,16,13,16,16,18,18,18,17,16,17,18,19,19$
Use an appropriate graph to represent these birth weights. Explain why you chose the graph that you did.
Graph:
Students could make a dot plot, histogram, or box and whisker plot. For each of these, check that students have the correct values (listed below) and number of
observations (for the dot plot). E.g. of a dot plot


Average/mean: 17
Maximum: 20
Minimum: 13
Range: 7
Median: 17
Explanation: e.g. "I chose a dot plot because it shows the shape of the distribution." "I chose a histogram because it is easy to see the shape of the distribution." "I chose a box and whisker plot because there were so many data points and this allows me to only have to plot a few statistics."
a. Describe the distribution of birth weights for puppies born at Kingston Kennels in the last six months. Be sure to describe shape, center, and variability.
Shape: e.g. response "The dots in my dot plot are centered about 17 ounces. There is one dog weight that is somewhat different than the rest. This value is 13 ounces."
Center: Students need to calculate the average. "The center of the distribution is at the average which is 17 ounces."

Variability: e.g. response "The data points are clustered together. There is not much variability with the dog weights except for one data point. That is, the weight 13 ounces seems to vary more than the rest of the data points."
b. What is a typical birth weight for puppies born at Kingston Kennels in the last six months? Include the appropriate unit of measurement. Explain why you chose this value.
e.g. response "The typical birth weight for dogs is 17 ounces. I chose this as my typical value because it is the average weight for the dogs in this sample. Average means typical."
"I chose 17 ounces because it is the median of the data set. The median of
the data set can be used a typical value." Students who talk about the median are able to use this as a typical value since the data set is not very skewed.
c. Are there any unusual observations? Yes

If so, what are they, and why are they unusual? 13 ounces e.g. response "Because it is not clustered around the mean/median like the other data points"
If not, why not?
Students might also argue that this point is NOT unusual because it is not really that far away from the mean $(17-13=5)$. Either case, the student would be right, but they need to justify it by arguing using the STATISTICS (mean, median) for the data set. Students at this level have not yet decided how to quantify "unusual" with the standard deviation, so that is why either response is ok given that students refer to the center (mean or median).
5. Each of the 20 students in Mr. Anderson's class timed how long it took them to solve a puzzle. Their times (in minutes) are listed below:

\section*{| Time (minutes) | 3 | 5 | 4 | 6 | 4 | 8 | 5 | 4 | 9 | 5 | 3 | 4 | 7 | 5 | 8 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | 3}

Display the data using a boxplot.

a. Find the mean and median of the data. Interpret these in context of the data. Include the appropriate unit of measurement. Round the mean to one decimal place.
Mean: 5.4 minutes
Interpretation: On average, students took 5.4 minutes to complete the puzzle. Median: 5 minutes

Interpretation: The middle number of the data set is 5 minutes. Half of the data will be above and half will be below this number.
b. What is the range for this data set? What does it tell us about the data?

Range: 6
Interpretation: The difference between the largest and smallest value for puzzle times is 6 minutes. This tells us how much the students' puzzle times vary.
6. Aidan was interested in the eye color of people at his after school club. He collected the following data from 20 people in his club:

| Blue | Brown | Brown | Blue | Brown |
| :---: | :---: | :---: | :---: | :---: |
| Hazel | Green | Hazel | Brown | Brown |
| Brown | Green | Blue | Blue | Brown |
| Brown | Blue | Blue | Hazel | Hazel |

Construct a bar chart with the frequencies for eye color. Interpret the results in the context of the problem.
Graph:


Interpretation: The most common eye color is brown (8 students), the least common is green (2 students).
7. Construct a data set with the following parameters:

Mode: 25
Range: 20
Median: 30
Students could have many solutions for this question. An example would be:
20, 25, 25, 30, 31, 32, 40 Mode: 25 ; Median: 30 , Range: $40-20=20$.

## Performance Task Assessment: Oreo Cookie Task

Goal: Your task is to conduct an experiment that addresses the concern of the customer. The goal is to use the results from the experiment to improve the quality of Oreo cookies if the results deem that as necessary.

Role: You work at the Nabisco cookie company as the Quality Control Officer. You have been asked to investigate a customer concern, determine if it is valid or not, and then make a decision about what to do in terms of cookie production.

Audience: Your audience is the Nabisco Board of Directors, Production Managers, and the customer who originally made the claim.

Situation: As the Quality Control Officer, you are in charge of customer complaints and concerns. Today you received an email from a customer regarding Doublestuf Oreo cookies. The email reads as follows:


Product, Performance, and Purpose: You will create a statistical experiment, identifying the statistical question and conducting the experiment. This process needs to be written as a report and should include a detailed description of your process and how you conducted the experiment (i.e., what did you do to answer the statistical question that you posed). The results of your study must be included in this report, as well as what the results mean for your company. You must also include a response to the customer as well as what you will tell your Board of Directors and Production Managers.

Standards and Criteria for Success: Your report needs to follow the attached report format and will be graded according to the attached rubric.

## Oreo Study Report Format

| Name: |
| :--- |
| Date: |
| Statistical Question |
| Data Collection and Analysis |
| 1. Restate what you think your goal was for this project: |
| 2. What statistical question did you decide to explore? |
| 3. How does this address the concern of the customer? |
| 1. Describe how you answered the question that you described above. This should <br> include what data you decided to collect and how was the data collected. Make sure to <br> identify how this data answered the question posed above. <br> 2. Please include either as attachments or written in this section any calculations and/or <br> graphical and numerical representations that you did once you collected this data. Discuss <br> why you chose the representations/calculations and how they are appropriate for <br> answering the question above. |

## Interpreting Your Data

1. Examine the calculations and representations you have above. Interpret these results in light of your statistical question and the customer's concern. Does your data address the customer's concern?

## Attachments Needed

1. Include a response to the customer. This needs to be written or typed (whichever you prefer) in any format that you choose. However, pay special attention to grammar and spelling (see Rubric for scoring guide). Also, you must clearly state the following:

- What statistical question did you design and how did this address her concern?
- How did you answer this question (what were your data collection methods)?
- What do your results tell you in light of her concern? Include at least one graphical representation in your response to demonstrate the results from your data collection that is easy to understand (see Rubric for scoring guide). Insure that you write this in a way that is easy to understand for someone who may not be familiar with statistical experiments. Use common language and write clearly.

2. Include a response to the Board of Directors and Production Managers. This will be similar to the above letter; however the difference will be in the technicality of this response. You may be more formal in this letter, using appropriate statistical and mathematical language and symbols as necessary. Also, you must include an "Action Decision" where you must decide what the company should do (if anything) in terms of Oreo production.

| Oreo Study Rubric |  |  |  |
| :---: | :--- | :--- | :--- |
| Section | 2 points | 1 point | 0 points |
| Statistical Question | The student states a <br> question that is <br> statistical AND <br> identifies one that <br> addresses the <br> customer's concern. | Either the question <br> is not statistical in <br> nature OR the <br> question does not <br> address the <br> customer's concern. | Neither the question <br> created is statistical <br> NOR does it address <br> the customer's <br> concern. |
| Data Collection | The data collected <br> and the data <br> collection method <br> are clearly stated, <br> AND the data <br> answers the <br> question posed. | Either the type of <br> data collected and <br> data collection <br> method are not <br> included OR the <br> data do not answer <br> the question posed. | Neither the data <br> collected and data <br> collection method <br> are included NOR <br> does the data answer <br> the question posed. |
| Data Analysis | Calculations and <br> representations are <br> included and <br> appropriate for the <br> question, AND <br> student makes clear <br> why those <br> calculations and <br> representations were <br> chosen. | Either the <br> calculations and <br> representations are <br> not included and/or <br> not appropriate OR <br> the student fails to <br> justify why those <br> calculations and <br> representations were <br> chosen. | Neither the <br> calculations and <br> representations are <br> included NOR does <br> the student provide <br> justification for <br> calculations and <br> representations <br> chosen. |
| Interpreting Your |  |  |  |
| Data | Data are interpreted <br> in context, AND the <br> student mentions <br> how the results <br> address customer's <br> concerns. | Either data are <br> interpreted in <br> context OR student <br> mentions how the <br> results address <br> customer's <br> concerns. | Neither the data are <br> interpreted in <br> context NOR does <br> the student mention <br> how the results <br> address the <br> customer's <br> concerns. |
| Data Analysis <br> (correctness) | 0-1 calculation <br> errors are noted. | 2-3 calculation <br> errors are noted. | More than 3 <br> calculation errors <br> noted. |
| ner |  |  |  |


| Attachments | 0-2 grammatical <br> errors are noted. | $3-5$ grammatical <br> errors are noted. | More than 5 <br> grammatical errors <br> are noted. |
| :---: | :--- | :--- | :--- |


| STAGE 3: Learning Plan |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Monday | Tuesday | Wednesday | Thursday | Friday |
| French Fry Task <br> Homework 1 (due Tuesday) | Day 2 WarmUp <br> Answering a Statistical Question Task <br> Academic Prompt 2 | Informal Check 6 <br> Construct Your Own Graph Task <br> Academic <br> Prompt 3 <br> Homework 2 <br> (due Thursday) | Homework <br> Review <br> I Wonder What <br> Happens If... <br> Task <br> Academic <br> Prompt 4 <br> Homework 3 <br> (due Monday) | Academic <br> Prompt 1 <br> Statistical <br> Problem- <br> Solving Process <br> Task |
| Review <br> Homework 3 <br> Finish <br> Statistical <br> Problem- <br> Solving Process <br> Task <br> Academic <br> Prompt 5 | Informal Check 7 <br> Categorical Data Task <br> Homework 4 <br> (due <br> Wednesday) | Informal Check 8 <br> Finish <br> Categorical <br> Data Task <br> Academic <br> Prompt 5 | Unit Test <br> Oreo <br> Performance <br> Task | Oreo <br> Performance <br> Task (due <br> following class) |

## Day 1

| Student Handouts: | Materials: |
| :---: | :---: |
| • French Fry Task | • $\quad$ Chart paper |
| • Homework 1 | • Tape |
|  | • Markers |
|  | • Projector |
|  | • Timer |

Lesson Goal: The students will calculate mean, median, mode, and range for two quantitative data sets. The students will interpret these statistics in context.

## Warm-up (10-15 minutes)

Ask students to think-pair-share about the following question:
What do you know about statistics?

Students might say things such as: mean, median, or mode. They could also talk about graphs and statistics that they see on TV. Since this is the first lesson of the unit, the goal for this warm-up is to get students thinking about statistics informally. As students talk about these ideas, it would be beneficial to have them talk about things they know about what they say. For example, if a student says something about mode, then probe them to talk about what the mode is and how they might find it.

Randomly call on 2-3 students, and have them share their ideas. Let them know that we will be using statistics and possibly some of their ideas for today's task. First, students will work as a whole group to solve a warm-up task.

Display Warm-Up task on French Fry Task sheet on projector. Read the problem or have a student read the problem aloud. Ask students to think about the Warm-Up task individually for 30 seconds. Then have them share with their partner for 30 seconds.

Finally, randomly call on 2-3 students to share what they or their partner thought about how to solve the task.

Identify a student or pair of students with a strategy for finding the average and median, and tell students that as a whole group you will use this strategy to solve the Warm-Up task. Have students tell you how to find the median using the selected strategy and demonstrate on projector. Once the whole class has found the median, ask students to think about the question in the task: Do you agree or disagree that the average number of fries is 52 fries per order and that the median number of fries is 45 ? Let students think-pair-share on the question.

Throughout this unit, maintain chart paper that has vocabulary associated with the statistics unit. After this Warm-Up task, add the words mean and median to the chart paper and randomly call on students to have them share what they think the definition for mean and median are and how they might be calculated.

## French Fry Task

## Understanding the Problem (2-3 minutes)

Have students in groups of 3-4. Distribute the task to each student.
Display Parts 1 and 2 of the French Fry Task to the class via projector. Read Parts 1 and 2 aloud. Ask students to think without speaking aloud about how they might answer these questions ( 30 seconds). Then have students share their idea with a partner (1 minute) without beginning the task. Have students write down their strategy for solving these tasks, but not actually start the task (1 minute).

## Solving the Problem ( 25 minutes)

Tell students that their goal is to write clearly on the chart paper what they would like to present to the class as a response to the question(s) in each part. They may wish to do their "scratch work" on a separate sheet and save the chart paper for their "presentation work". Tell students that they may use pictures (e.g., bar graphs, dot plots), words, and/or symbols to represent their thinking, but that these must be mathematical in nature and not
simply for aesthetics. Distribute a sheet of chart paper and two colors of markers to each pair of students.

Start the timer with 5 minutes. At the end of 5 minutes, randomly call on groups to have them share their strategy for solving the tasks and not the solution.
Allow students 10 additional minutes to work (time can be dependent on how far along students are in the tasks. It is not necessary that all groups complete all three tasks, but that students are at least making progress). Circulate among the students, looking for different solution paths. Identify three potential posters (one for each task) to be presented and ask the students if they are willing to share their poster with the class.

Ask the three groups to present. Tape each poster at the board before the group presents. After the three group's presentations, lead a discussion using some of the potential questions below.

Potential presentation questions for discussion:

1. How do you think the person who conducted this study collected the data?
2. For Part 1, how do you justify your choice with mathematics? (The whole class can be polled to see if they agree/disagree with this group's response. If there is disagreement, some of those groups/students can be called on to share their reasoning. Emphasize that they use mathematics to justify their response.)
3. For Part 2, what statistic (mean, median, mode, and range) helped you make your decision about from which restaurant to buy fries? Did someone use another statistic?
4. For Part 2, is there another representation that we could have created to represent this data? How does your representation reflect the statistics that you calculated?

Students need to be comparing the average number of fries for each restaurant in Part 1.
The restaurant with the larger average would have more fries on average than the other.

For Part 2, the students might need a refresher on how to calculate range and mode. If students struggle, ask the class if anyone remembers how to calculate a range or mode. If a student has the correct answer, suggest that they use that method for finding those statistics. If not, state that a previous student found those statistics by subtracting the lowest value from the highest value (range) and by listing the most common number of fries (mode) for each restaurant. Students can pick any of these statistics as their choice for choosing the restaurant. For example, students might say, "I picked $\qquad$ restaurant because it has the highest mode number of fries." OR "I picked $\qquad$ restaurant because it has the smallest range number of fries. " It is important for students to connect their choice to a statistic and justify what that means.

Add new vocabulary words to the chart paper (mode, range, statistic, etc.) Statistic can be defined as a number calculated from a sample of data points. Students need to know that the data points are each amount of fries counted for each restaurant. The entire pool of data points from McDonald's would be the sample of data from McDonalds (similarly from Burger King).

## Wrap-up (3-5 minutes)

Exit Ticket: Today we re-examined mean, median, mode, and range by answering questions using data about French fries from McDonald's and Burger King. Please list the following: THREE big ideas about today's task related to the mathematics, TWO questions that you still have, and ONE question that you would like to examine that could be answered by collecting data.

Students may write their exit ticket responses on a sheet of paper.
Distribute Homework 1 to be turned in the next day.
*Revised from Browning, C. A., \& Channell, D. E. (2003). Probability and statistics for elementary/middle school teachers. Dubuque: Kendall/Hunt Publishing Company.


## Warm-Up Task

Sara loves French fries, especially those from McDonald's. When she goes to the restaurant, her mom lets her get one small order of fries. Over the course of several visits, Sara notices that the number of fries in her small order is not always the same. She begins to keep track of how many fries are in her small order. She collects the following amount of fries per small order from McDonald's:

Number of Fries in Small Order from McDonald's

| 41 | 48 | 43 | 57 | 41 | 40 | 43 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 60 | 56 | 49 | 55 | 43 | 48 | 48 |
| 51 |  |  |  |  |  |  |

Sara shares this information with her brother, Joe, who states that the average number of fries per small order is 52 fries per order but that the median number of fries is 45 . Do you agree or disagree?
*Modified from Browning, C. A., \& Channell, D. E. (2003). Probability and statistics for elementary/middle school teachers. Dubuque: Kendall/Hunt Publishing Company.

## French Fry Task *



## Part 1

Sara's best friend, Tori, thinks that Burger King's fries are not only better, but that you actually get MORE fries on average in a small order than at McDonald's. To help determine if this is true, Sara and Tori collect the number of fries in a small order from

Burger King and get the following information:

Number of Fries in Small Order from Burger King

| 42 | 49 | 45 | 57 | 43 | 42 | 45 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 62 | 51 | 49 | 60 | 40 | 51 | 50 |
| 50 |  |  |  |  |  |  |

Is Tori correct? Why or why not? Justify with mathematics.

## Part 2

Create a table for the McDonald's and Burger King data set that demonstrates the mean, median, mode, and range for the number of fries in a small order of fries. From which place would you rather get a small order of fries? Why? Write about this, including mathematics as a way to justify your response. If possible, create a representation of this data (you could use, for example, histogram, dot plot, or any representation that portrays the data effectively).
*Modified from Browning, C. A., \& Channell, D. E. (2003). Probability and statistics for elementary/middle school teachers. Dubuque: Kendall/Hunt Publishing Company.

## Homework 1

Name: $\qquad$ Date: $\qquad$

1. Shannon was waiting to ride her favorite ride, the Himalaya, at the county fair when she noticed that another girl from her class was not tall enough to ride the Himalaya. You have to be at least 48 inches to ride. Shannon wondered how many people in her class were able to ride the Himalaya so she collected her entire class's height in inches.

| 48 | 50 | 45 | 42 | 57 | 51 | 51 | 52 | 53 | 49 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 47 | 48 | 48 | 43 | 55 | 50 | 49 | 42 | 55 | 50 |
| 51 | 49 | 46 | 47 | 48 | 48 | 52 | 51 | 45 | 48 |

Answer the following questions regarding the height data set.
a. What is the average height of students in Shannon's class?
b. What is the median height?
c. What is the mode for the data set?
d. What is the range for the data set?
2. At the fair, Shannon noticed the "Guess Your Weight" game. The game operator claimed he could guess anyone's weight, and if wrong, the person would get a prize. Eleven people of various ages played the game while Shannon was watching. Their weight in pounds is given in the following table.

| 134 | 100 | 167 | 98 | 175 | 183 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 110 | 145 | 201 | 155 | 105 |  |

a. What is the average weight of a person playing the "Guess Your Weight" game?
b. What is the median weight?
c. What is the mode for the data?
d. What is the range for the data?
3. Of the two data sets above, which one varied the most? How do you know?

## KEY Homework 1

1. Shannon was waiting to ride her favorite ride, the Himalaya, at the county fair when she noticed that another girl from her class was not tall enough to ride the Himalaya. You have to be at least 48 inches to ride. Shannon wondered how many people in her class were able to ride the Himalaya so she collected her entire class's height in inches.

| 48 | 50 | 45 | 42 | 57 | 51 | 51 | 52 | 53 | 49 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 47 | 48 | 48 | 43 | 55 | 50 | 49 | 42 | 55 | 50 |
| 51 | 49 | 46 | 47 | 48 | 48 | 52 | 51 | 45 | 48 |

Answer the following questions regarding the height data set.
a. What is the average height of students in Shannon's class? 49
b. What is the median height? 49
c. What is the mode for the data set? 48
d. What is the range for the data set? 15
2. At the fair, Shannon noticed the "Guess Your Weight" game. The game operator claimed he could guess anyone's weight, and if wrong, the person would get a prize. Eleven people of various ages played the game while Shannon was watching. Their weight in pounds is given in the following table.

| 134 | 100 | 167 | 98 | 175 | 183 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 110 | 145 | 201 | 155 | 105 |  |

a. What is the average weight of a person playing the "Guess Your Weight" game? 143
b. What is the median weight? 145
c. What is the mode for the data? none
d. What is the range for the data? 103
3. Of the two data sets above, which one varied the most? How do you know? Students responses will vary. The second set of data actually has the larger standard deviation. However, students are not expected to know this yet.

Acceptable responses would include, for example, "The second set varies more because the range is larger than for the first set." The students should make the connection between a large range and more variability.

## Day 2

## Student Handouts:

- Answering a Statistical Question

Teacher Handouts:

- Day 2 Warm-Up Task
- Student Work Histogram
- Academic Prompt \#2


## Materials:

- Chart paper
- Tape
- Markers
- Projector
- Timer
- Post-it Notes
- Yard sticks/tape measures

Lesson Goal: The students will understand that a statistical question is one that anticipates variability. The students will collect data to answer a quantitative statistical question. The students will represent the data with graphical representations and statistics.

## Warm-up (5-10 minutes)

Display Day 2 Warm-Up Task. Read the questions aloud. Tell students that they have 30 seconds to think individually about what might make the questions on the left statistical, but not the questions on the right. Have them share their ideas with their partner (1-2 minutes). While students are discussing, on chart paper write: What makes a question statistical?

Randomly call on 2-3 students and have them share what they or their partner had to say in regards to what would make a question statistical ( 5 minutes). Write their ideas on the chart paper. Once all ideas are on paper, have students discuss what the definition of a statistical question should be.

Students should reach the conclusion that statistical questions anticipate variability in their responses instead of a deterministic answer. "Statistical question" and "variability" can be added to vocabulary chart paper. So a definition for a statistical
question would be "A question that has answers which vary/change." A definition for variability might be, "vary, different, change"

## Answering a Statistical Question Task

## Understanding the Problem (5 minutes)

Display Statistical Questions sheet on the projector. Tell students that today we will be answering one of these questions in class. Let students vote for which question that they would like to answer today (2-3 minutes). If no consensus is reached, pick a question.

Have students in groups of 3-4 students. Tell students that they will turn in only one task sheet for their entire group so they might want to designate a recorder for the group.

Distribute the Answering a Statistical Question task (one per group), markers, and post-it notes.

Students may also need scrap paper for calculations.

## Solving the Problem (30 minutes)

If students choose the height problem, also distribute tape measures or meter/yard sticks to each group. In any case, tell the students that they should first collect data within their group (e.g., gather all the names of group members and count the letters in their names; measure every member's height). Tell students to EACH write their data value (e.g., how many letters are in their first name) on a Post-it Note with a marker in large font. One group member from each group can collect the Post-It notes and stick them to a piece of chart paper for the entire class to see.

Maintain the piece of chart paper with the data. This will be necessary for the next day's task. Tell the students that it is now their job to find the mean, median, mode, and range of the data set for the whole class, as well as represent this data with a dot plot. Randomly call on 1-2 groups to discuss their process for creating the dot plot (2-3 minutes). Let students continue working.

Circulate room while students are working, and select 2 groups to present: one to present their answers to question 3 and one for question 4. Let them know that they must emphasize the process for solving the question (e.g., "We created a dot plot by drawing a number line ..." or "We found the range by finding the minimum and maximum height. . .")
Once most groups are finished, indicate to the entire group that they have 1-2 minutes to finish their work and have their work presentation-ready. Begin group presentations on projector.

Poster questions:

1. How did we collect the data for the statistical question?
2. How do we know the question was statistical? The answers to the question varied person to person.
3. (with solution for question 3 still on projector) What do we notice about the mean and median for this dataset? (i.e., students might notice that one is higher than the other or that they are the same) One important thing to highlight with this question is that the mean is the AVERAGE point for all of the data sets. It describes the, for example, average height of a student in this class. This interpretation in context is very important for students to understand and experience. Students also need to recognize that the median separates the data set into two equal sized groups. That is, half of the data fall below the median and half fall above.
4. Are there any unusual data points in the set? (i.e., outliers. If there are unusual points, state that this is what we call an outlier and add this to the vocabulary sheet.) An outlier can be defined as a data point or observation that lies outside the overall pattern of data points.
5. (with solution for question 4 still on projector) Someone talk about the dot plot. What do you notice about how the data are distributed (this might be a new word for students. Could use the word "shaped" but eventually call this the distribution
of data) on this dot plot? Students might struggle with the word distribution. It is ok during this unit for students to talk about the shape of our data points when we put it in a graphical representation. The formal definition of a distribution is the arrangement of data points that demonstrates how frequently each point occurs.
6. Does anyone know of any other ways that we might represent this data?

Students may or may not mention a histogram. If someone does mention a histogram, have the student(s) discuss the characteristics of a histogram and make note of this on chart paper. Check for agreement among the class about the characteristics of a histogram.

If students do not mention a histogram, mention that a previous student of yours created something to represent similar data called a histogram. This student was representing data for all heights of the 25 students in her class. Put Student Work Histogram on the projector. Pose the question to the class: How do you think this student created this histogram?
Give students 30 seconds of think time. Then let them share their ideas with a partner. Share out whole group.

Discussion questions for histogram discussion:

1. How did this student get the numbers on the horizontal axis? (i.e., $45,50,55,60$, $65,70)$ How do these relate to her data set? These are called "bins" for a histogram. They represent groupings of number values that make sense. These are arbitrary, and the creator of a histogram can choose whatever range for their bins as they would like. It very well could have been 45-47, 47-49, 49-51, etc.
2. How did she get the numbers on the vertical axis? What does frequency mean?

The numbers on the horizontal axis represent the frequencies for those bin groupings. For example, the height of the first bar (4) means that there are four observations (data points) that fall between 45-50. Students need to recognize that it is up to the creator of the histogram as to whether they count, for example,
the number 50 in the $45-50$ bin or in the 50-55 bin. The typical way it will be created in statistics is to include 50 in the 45-50 bin. However, this can be changed as long as the person creating the histogram is consistent with their choice.

Next have the students create a histogram for their data (8 minutes). While groups are working, select 2 groups to present their histogram to the class. Allow 2-3 minutes for the group to present how they decided to create their histogram. Students need to mention how they selected the width of their bars (numbers on horizontal axis) and determined which numbers to put into each bar (e.g, does the number 50 go into the group 45-50 or 50-55?).

## Wrap-up (5 minutes)

Select two groups to put their dot plot and histogram on the projector, displaying both. Ask students to take out a sheet of paper for their exit ticket. Ask them to reflect on both of these displays. Write Academic Prompt \#2 on the board. Tell students to think for one minute about the question and then begin writing (3-4 minutes). Have the students turn those in when they are finished.

Students should notice that the shape of the data might be similar for both the dot plot and histogram. This is true typically if the bin widths are not too wide or narrow (this is something students will deal with in later statistics courses). However, generally, the shape of the distribution in a dot plot will be similar to one in a histogram as well as to one in a box plot. Students might notice differences such as the dot plot lists individual data points while the histogram groups data into bins.

## Day 2 Warm- Up

| Statistical Questions | Non-statistical Questions |
| :---: | :---: |
| What are the ages of all of my classmates? | What is my age? |
| How many letters are students' first name in sixth- <br> grade classes? | How many letters are in my brother's name? |
| What are the favorite colors of people at my |  |
| school? |  |$\quad$| What is my favorite color? |
| :---: |
| How many fries are in all the small orders of fries <br> for one day at McDonald's? |
| How many fries are in my order of small fries <br> from lunch today? |

## Statistical Questions for Task

1. How many letters are in the first name of my classmates?
2. How many pets do students in my class have?
3. How tall are people in my class?
4. How many siblings do my classmates have?
5. How many states in the United States have people in my class visited?

## Answering a Statistical Question



Names of Group Members: $\qquad$

Date: $\qquad$

1. Write the statistical question that you are exploring today. Describe how you collected the data to answer that question.
2. Find the mean, median, mode, and range of the data set for the entire class.
3. Now, represent this data with a dot plot.

Student Work Histogram

- Heights in inches of people in my class
$\begin{array}{llllllll}53 & 50 & 58 & 66 & 55 & 45 & 62 & 62 \\ 66 & 49\end{array}$
$\begin{array}{llllllll}55 & 47 & 58 & 54 & 64 & 63 & 66 & 48 \\ 53 & 64 & 63\end{array}$
5368645068
mean height: 58.04 inches
median height: 58 inches
mode: 64 and 66
range: $68-45=23$
data in order:



## Day 3

| Student Handouts: | Materials: |
| :---: | :---: |
| • Construct Your Own Graph | - Chart paper |
| • Homework 2 | - Tape |
| Teacher Handouts: | • Markers |
| • Informal Check \#6 | - Projector |
| • Student Work Quartiles | - Timer |
| • Student Work Box Plot | - Post-it notes |
| • Academic Prompt \#3 |  |

Lesson Goal: The students will calculate the interquartile range for a data set. The students will construct a dot plot, box plot, and histogram for a quantitative data set. The students will describe the shape of a distribution for a quantitative data set.

## Warm-up (5 minutes)

Post Informal Check \#6 on the projector. Tell students to think about this question for 30 seconds. Randomly call on 2 students to have them share out ( 2 minutes). Ask students to think about how they might turn this question into a statistical question. Call on 2 students to share out (2 minutes).

This is not a statistical question because the answer is deterministic. There is only one answer for this question. This question could be turned into a statistical question by asking the favorite ice cream flavor of all teachers in this county. This is a statistical question because the answers will vary from person to person.

## Construct Your Own Graph Task

## Understanding the Problem (15-20 minutes)

Remind students of the "previous" student's histogram that the class examined yesterday. Tell them that this same student was working in a group on her height data when a group mate created a different representation for that data set called a boxplot.

Display Student Work Box Plot sheet on the projector. Ask students to take 30 seconds to look at what this student and see if they can figure out how he made this representation. Share with a partner (2 minutes). Randomly call on 3 students and have them share out what they think the student did to create the box plot (3 minutes).

## Discussion Questions:

1. What do the dots on the ends represent? These points represent the minimum and maximum value for the entire data set.
2. How many values are in this data set? There are 25 data points in this set.
3. What does the line in the middle of this rectangle represent? What statistic is this? This is the median.
4. What do the ends of the rectangle represent? Students will most likely struggle with this part. The next student work (Student Work Quartiles) should help them with this concept. The ends of the rectangle represent the median for the lower half and upper half, respectively of the data set. To find these, one must first find the median for the entire data set. Then the data set is split into two data sets: one below the median and one above the median. Finally, one finds the median of the lower data set and upper data set. These two medians are called the first and third quartile respectively. Then the median of the entire data set would be referred to as the second quartile. The word quartile refers to dividing the data set into quarters.

Show Student Work Quartiles. It may help to have both the Student Work Quartiles and Student Work Box Plot displayed. This may entail writing the quartiles on the board and projecting the plot. Tell students that the student who created the boxplot told you that this was how he could find the ends of the rectangle, what we call the first and third
quartile, but that his work was incomplete. Their task for the day is to determine how this student found the values 51.5 and 64 as the ends of his rectangle (first and third quartiles), and then to create a box plot for the data that they collected yesterday. It may help to label the ends of this rectangle with the values 51.5 and 64 and first and third quartile, respectively.

Give students 30 seconds to think about this problem. Have them pair with their partner, and randomly call on 2-3 pairs to share out. If students struggle with how to find the first and third quartile, state that this student suggested finding the median of the lower half and upper half of numbers.

Once a consensus is reached on how to find the first and third quartile (this method is described above in the teacher notes for question 4), include these words on the vocabulary chart paper. Introduce the word interquartile range as the difference between the third and first quartile and put this on the vocabulary chart paper.

Tell students that our task today will be to use this information to help us create a box plot and other plots for a data set.

## Solving the Problem ( 15 minutes)

Distribute the Construct Your Own Graph one per pair of students. Read the task aloud. Let students begin on the task in pairs. Once most pairs begin to start their box plots, randomly call on a pair to share their process for creating a box plot (not to give the numbers for the quartiles. (Students should arrive at they need to find the median for the lower half of the data set as well as the upper half of the data set.)

While students are working, circulate room and choose 3 pairs to present.
Once most pairs appear to be finishing with their 3 graphs, let the 3 pairs present their box plots on the projector.

Discussion Questions:

1. How was creating the box plot different from creating the dot plot and histogram? Students may say that the box plot required finding the first and third quartile and the other two graphs did not.
2. How was it similar?
3. What do these graphs tell us about the data? Students should conclude that graphs give us an idea of the shape of our distribution of data.
4. Which graph do you find most useful, and why?
5. Talk about the shape of dotplot/histogram/boxplot. What do you notice about these shapes? Are the shapes similar in each graph or different? Why or why not?

## Wrap-up (5 minutes)

Post Academic Prompt \#3 on the projector, and read aloud. Ask students to think-pairshare to Academic Prompt \#3. Randomly call on 3 students to share their ideas.

Emphasize students should focus on what a percentage represents and also the procedure for finding a percentage.

Distribute Homework 2 for day 3.

Student Work Boxplot


## Student Work Quartiles


$\qquad$

## Construct Your Own Graph!



Presley loves to watch and play basketball. During the season, she noticed that not all of the players on each team were given the same amount of opportunities to play in the game. After the season was over, she was interested to see how many games each of her favorite players actually played so she collected that data from www.nba.com . Here's what she found:

| Player | Games Played During the Season |
| :---: | :---: |
| Kevin Durant | 19 |
| LeBron James | 20 |
| LaMarcus Aldridge | 11 |
| Kyle Lowry | 7 |
| David West | 19 |
| Dwyane Wade | 20 |
| Damian Lillard | 11 |
| Tony Parker | 23 |
| Jamal Crawford | 13 |
| Chris Bosh | 20 |
| JJ Redick | 13 |
| Lance Stephenson | 19 |

Presley does not know how to represent this data graphically, but she would like to be able to do this so she could present her findings in an easy-to-understand way to her family. Select a graphical representation (dot plot, histogram, box plot) and create the graphical representation in your group. State why you chose the graphical representation that you did.

## Homework 2

Name:
Date: $\qquad$

1. Create a data set that has a mean of 35 and at least 10 data points. Show your work verifying that it does have a mean of 35 , and create a dot plot of your data set. What are the median, mode, and range of your data set?
2. Find the interquartile range (IQR) of your data set above. What percentage of your data set is greater than or equal to the third quartile? What percentage of your data set is less than or equal to the first quartile? What percentage is between the first and third quartile?

## KEY Homework 2

1. Create a data set that has a mean of 35 and at least 10 data points. Show your work verifying that it does have a mean of 35 , and create a dot plot of your data set. What are the median, mode, and range of your data set?

Student answers will vary.
2. Find the interquartile range (IQR) of your data set above. What percentage of your data set is greater than or equal to the third quartile? What percentage of your data set is less than or equal to the first quartile? What percentage is between the first and third quartile?

The IQR will vary based upon students' data sets. The percentage of data points below and greater than the first and third quartile respectively is $25 \%$ each. The percentage between the two would be $50 \%$. This makes sense because if we sum the percentages below the first, between the first and third, and above the third quartiles then we would have $25 \%+50 \%+25 \%=100 \%$.

## Day 4

## Student Handouts: <br> - I Wonder What Happens If ... <br> - Homework 3

## Teacher Handouts:

- Academic Prompt 4


## Materials:

- Chart paper
- Tape
- Markers
- Projector
- Timer
- Post-it notes
- Calculators

Lesson Goal: The students will create datasets with certain statistics. The students will understand how these statistics affect the shape of the distribution.

## Warm-up (15-20 minutes)

Have students pull out their homework and put the students into groups of 3-4. Each student in the group should present their solution to their homework problems to their group. Give each student 2 minutes to present to each other and then ask them to switch.

As student work, circulate the room and identify 2-3 students with unique data sets (this could be a set of the number 35, ten times; a set with outliers; or a set that is normally distributed) and students who attempted to calculate percentages for the last question. These 2-3 representations chosen should be different from each other to highlight how statistics can affect the shape of a graph.

Call on the students chosen and ask them, "Tell me about how you calculated the percentages for the last problem." As students share their solutions, ask the entire class for agreement. Percentage can be added to the vocabulary list.

Students may struggle with how to calculate a percentage. If there is not a student who can share that has calculated the percentage correctly, then state that a student from
another class said that she thought a percentage was a part of a whole and that one could find a percentage by dividing the total number of data points by the part that we are interested in and then multiplying by 100.

Post the selected graphical representations on the projector at the same time if possible. Tell the class to look at these three graphs for 30 seconds and think about how the choice of data set affected the dot plot's shape. Give them one minute to share this with a partner, and then randomly call on 2-3 students to share out whole group. Chart their ideas on chart paper titled "How Data Affects Shape." Tell them that these ideas will help them solve today's task.

## I Wonder What Happens If . . . Task

## Understanding the Problem (3 minutes)

Tell students that today's task will ask them to construct data sets, similar to what they did for homework, and prior to creating any representations, anticipate how the data set's statistics will affect the shape of the graph.

The task uses the word "distribution". It would be helpful to have a discussion about the word distribution might mean to students. In terms of the graphical representation, the distribution is a representation of the values of a variable (for example, heights of people in the sample) demonstrating the observed or theoretical frequency of occurrence. Students might start by thinking about the distribution as a representation of the data points. The question in the task asks them to think about the different statistics affect the SHAPE of the distribution.

Each group will get a different task (task is divided into 4 sheets each with a two sets of required statistics for a data set. Each group should have a final product of two data sets and associated graphs) and a whole sheet of chart paper that has been folded in half and markers. One half of the chart paper will be for their first data set including histogram and the other half will be for the second data set. Allow students to use calculators to help with calculations.

## Solving the Problem (20 minutes)

Allow students 8-10 minutes to work on each data set.
Randomly call on 2-3 groups to present their work. Students can tape their chart paper to the wall after all groups have presented.

Discussion questions during group presentations:

1. How did you (a particular group) decide what values to put in your data set?
2. How did the required statistics affect the shape of your dot plot/box plot/histogram? In a data set, if the data are skewed this will affect the location of the mean. For example, if there are some extreme (outliers) data points (either extreme in that they are above or below the majority of the other data points) these will cause the mean to be higher or lower (if the extreme points are above or below, respectively) than if the data points were all normally distributed. The median, however, is resistant to these extreme data points.
3. What statistic made constructing your dataset most difficult? Why? (e.g., since the range was 90 but the mean was 30 , we had a difficult time determining how many values to use)
4. Did anyone notice a very interesting dot plot/box plot/histogram? What was interesting about it? How was this dot plot/box plot/histogram affected by the statistics?
5. Are the measures of center (median and mean) always going to be the same for these data sets? Why or why not? As stated in question 2 above, if the data set is skewed because of outliers, these two statistics will not be the same. However, If the data is not skewed but normally distributed, then these two data points could be the same.
6. When the mean was greater than the median (or vice versa), what did you notice about the shape of the dot plot/box plot/histogram?
7. (once all groups' chart paper is posted) What do you notice is similar or different about each of the groups' representations? Does any representation stand out as interesting to you? Why?

## Wrap-up (5 minutes)

Post Academic Prompt \#4 on the projector. Read aloud to the class and ask them to respond to the prompt on a sheet of paper. They are to turn this in as their exit ticket for the day.

By doubling each point in the data set, the new data set would shift. The shape of the distribution would not change, but the location of the mean would be doubled.

It would be helpful to provide feedback to the students and return their exit tickets to them.

What Happens If. . .


1. Create a dataset with the following statistics:

Mean: 40
Minimum: 10
Maximum: 100
Must have at least 10 values in the set

How do you think these statistics will affect the shape of the distribution? After you make a guess, create at least one representation and check your guesses.
2. Create a dataset with the following statistics:

Median: 40
Range: 20
Minimum: 32
Must have exactly 10 values in the set

How do you think these statistics will affect the shape of the distribution? After you make a guess, create at least one representation and check your guesses.


1. Create a dataset with the following statistics:

Mean: 76
Range: 90
Median: 65
Must have at least 15 values in the set

How do you think these statistics will affect the shape of the distribution? After you make a guess, create at least one representation and check your guesses.
2. Create a dataset with the following statistics:

Median: 76
Range: 90
Mean: 65
Must have at least 15 values in the set

How do you think these statistics will affect the shape of the distribution? After you make a guess, create at least one representation and check your guesses.


1. Create a dataset with the following statistics:

Mean: 17
Minimum: 15
Maximum: 25
Must have at least 10 values in the set

How do you think these statistics will affect the shape of the distribution? After you make a guess, create at least one representation and check your guesses.
2. Create a dataset with the following statistics:

Median: 10
Range: 40
Minimum: 0
Mean: 8
Must have exactly 10 values in the set

How do you think these statistics will affect the shape of the distribution? After you make a guess, create at least one representation and check your guesses.


1. Create a dataset with the following statistics:

Mode: 15
Minimum: 10
Maximum: 30
Mean: 25
Must have at least 15 values in the set

How do you think these statistics will affect the shape of the distribution? After you make a guess, create at least one representation and check your guesses.
2. Create a dataset with the following statistics:

Median: 60
Mean: 64
Range: 50
Must have at least 10 values in the set

How do you think these statistics will affect the shape of the distribution? After you make a guess, create at least one representation and check your guesses.

## Homework 3

Name: Date: $\qquad$
The amount of time that middle school children spend on homework for their classes in a given day varies considerably from one student to another. Parents have expressed to a middle school principal their concerns that students are being assigned too much homework. To address the concerns of parents, the principal decided to investigate the following statistical question: "How much time do middle-grades students spend on homework each day? **from Zbiek, Jacobbe, Wilson, \& Kader (2013)

Here are the times, in minutes, that 28 seventh-grade students reported spending on homework for one day:

| 20 | 25 | 35 | 40 | 45 | 46 | 50 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 55 | 56 | 56 | 57 | 60 | 60 | 60 |
| 62 | 62 | 63 | 64 | 65 | 65 | 65 |
| 66 | 66 | 68 | 69 | 70 | 71 | 75 |

Create a dot plot and histogram for this data (include this on a separate sheet), and use the graphs to answer the following questions:

1. Calculate the median and mean for the $7^{\text {th }}$ graders study times.
2. 

a. How many study times are above and below the median (do not count the median, but the numbers above and below)?

Below:
Above:
b. How many study times are above and below the mean (do not count the mean, but the numbers above and below)?

Below:
Above:
c. Are your answers to a and c above the same? That is, is the number of study times above the median the same as the number of study times above the mean? And is the number of study times below the median the
same as the number of study times below the mean?
3. Which statistic that measures the center of the study times (that is, the mean, median, or mode) best describes the set of study times? Why? Use your answers to question 2 and your graphical representations to help justify your response.

## KEY Homework 3

Name: $\qquad$ Date: $\qquad$
The amount of time that middle school children spend on homework for their classes in a given day varies considerably from one student to another. Parents have expressed to a middle school principal their concerns that students are being assigned too much homework. To address the concerns of parents, the principal decided to investigate the following statistical question: "How much time do middle-grades students spend on homework each day? **from Zbiek, Jacobbe, Wilson, \& Kader (2013)

Here are the times, in minutes, that 28 seventh-grade students reported spending on homework for one day:

| 20 | 25 | 35 | 40 | 45 | 46 | 50 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 55 | 56 | 56 | 57 | 60 | 60 | 60 |
| 62 | 62 | 63 | 64 | 65 | 65 | 65 |
| 66 | 66 | 68 | 69 | 70 | 71 | 75 |

Create a dot plot and histogram for this data (include this on a separate sheet), and use the graphs to answer the following questions:


1. Calculate the median and mean (round the mean to one decimal place) for the $7^{\text {th }}$ graders study times.

Median: 61, Mean: 57
2.
a. How many study times are above and below the median (do not count the median, but the numbers above and below)?
Below: 14 Above: 14
b. How many study times are above and below the mean (do not count the mean, but the numbers above and below)?
Below: 10 Above: 17
c. Are your answers to a and c above the same? That is, is the number of study times above the median the same as the number of study times above the mean? And is the number of study times below the median the same as the number of study times below the mean?

No.
3. Which statistic that measures the center of the study times (that is, the mean, median, or mode) best describes the set of study times? Why? Use your answers to question 2 and your graphical representations to help justify your response. Students' responses to this will differ. Students who base their decision on the statistics and distribution could be completion points. It is not expected that all students will recognize that the mean and median are not equal, the number of data points above and below the mean and median are not equal, and so the data are skewed. This skewness is also recognizable from the dot plot and histogram. This distribution is skewed left, therefore the mean will be lower than the median. In these cases of skewed data, the median is a better measure of center than the mean or mode. As stated above, however, not all students may reach that conclusion here. If they are basing their decision on true statements about the data and statistics, this would be the beginning of developing their understanding for this concept.

## Day 5

| Student Handouts: <br> - Statistical Problem-Solving Process Task <br> Teacher Handouts: <br> - Academic Prompt \#1 | Materials: <br> - Chart paper <br> - Tape <br> - Markers <br> - Projector/elmo <br> - Timer <br> - Calculators <br> - Candy <br> - Marbles <br> - Yard sticks/rulers |
| :---: | :---: |

Lesson Goal: The students will engage in the statistical problem-solving process for a quantitative data set.

## Warm-up (5-8 minutes)

Put the data sets from Academic Prompt \#1 on the projector/board. Tell students that they should compare and contrast the two data sets (one is quantitative and the other is categorical) and decide how they can represent the data set (i.e., what statistic and graph might we use to describe this data set? Why?).

Think-Pair-Share. Randomly call on 3 pairs of students and have them share their thoughts.

As students share their ideas of the data sets, the following discussion questions may help as probes to get students to describe one data set as categorical and one as numerical. (The teacher may need to interject these new vocabulary words as needed. The teacher may find it helpful to add these to a vocabulary list on chart paper that is posted during this unit.)

Discussion Questions:

1. What is the same/different between the two data sets? (Make sure students say/hear the words quantitative for the first data set-since the set consists of numerical values - and categorical for the second data set - since the set consists of categories of color. These words can be added to the vocabulary chart paper.)
2. How do you think the data were collected for these two data sets? Student answers will vary. Responses might include, students the school filled out a survey OR students in the classroom wrote their responses about favorite color on a piece of paper and turned it in to the teacher.
3. Can you represent these two data sets with the same statistics (e.g., can you calculate a mean for both data sets? Do both sets have a mode?) ? Why, or why not? (Include these responses on chart paper, perhaps noting which type of data set allows for certain calculations) Students cannot represent the data sets with exactly the same statistics. Quantitative data sets can have means, medians, modes, ranges, IQR, etc. Categorical data sets can only have the mode.
4. Which graph did you use to represent the data sets? Why?
5. Can you represent these two data sets with the same graph? Why, or why not? Quantitative data sets can be represented by many types of graphs (e.g., histogram, dot plot, box plot, etc.). Categorical data sets can be represented by a bar chart, pie chart, or dot plot.

## Statistical Problem-Solving Process Task

## Understanding the Problem (10 minutes)

This task will involve writing a question that is statistical, determining how to answer it, and then answering it. Prior to starting the task, ask students the following questions to help them with the day's task. Add their ideas to chart paper (7-8 minutes).

1. What are other big ideas, terms, or definitions that we have talked about and worked with this week that haven't made it to the chart paper yet? What is their
definition? (these may include mean, median, mode, range, histogram, statistic, data set, box plot, variability, dot plot, measure of center, measure of variability, etc.)
2. When we had a question that we wanted to answer, how did we know if it was statistical or not? (i.e., answer to the question varies)
3. Once we had a statistical question, how did we answer the question? (i.e., collect data)
4. Once we collected the data, what did we do? (i.e., found a statistic, represented it with a graph).

## Solving the Problem ( 25 minutes)

Distribute the Statistical Problem-Solving Process handout to each student. Tell them that their first task will be to read each step listed in the Statistical Problem-Solving Process and jot down their ideas about what it might mean and entail. Students can work in groups of 3-4.

Randomly call on a student for each of the four steps and have them share out what they thought the step might entail ( 2 minutes). On a sheet of chart paper titled "Statistical Problem-Solving Process," write down their ideas for each step. After each student gives their response ask the rest of the class (or randomly call on other students) if they want to add/remove anything from that step and finally if they agree that that is how that step should be conducted (1 minute for each step).

Now ask students in their groups to create a question that they would like to answer that can be answered within the class (these could be something like, how many hours do my classmates sleep at night? OR What is the hand-span length of my classmates?) using the materials that they have available in the class (2-3 minutes). Tell students that their question should be quantitative and not categorical in nature.

Once most groups seem to have decided on a question, have students individually think for 30 seconds about how they might collect data to answer this specific question. Let students share within their groups for 2 minutes. Randomly call on 2 groups and ask them to share what they think about how to collect the data ( 2 minutes). It is not necessary that the entire class answer the same statistical question.

Allow rest of task time for students to collect their data. Since groups may have different questions to answer, it may assist groups to have Post-It notes. One member from each group to go to the other groups and ask them to answer their statistical question on the post-it note and return to that group. (e.g., If a group is measuring the heights of all classmates, one student from this group could go to each other group and ask them to record their heights on a post-it note and return to them). To assist with the management of this part, it may be beneficial for each group to present their statistical question and have the entire class put their data to that question on a post-it note. Then this could continue throughout the groups.

After students have been working on collecting their data for 3-4 minutes, stop the entire class and ask them what issues they might have faced when collecting the data (e.g., some students put their height in inches and others put their height in feet). The important aspect to emphasize here is that data collection is not easy and the most time consuming part of the entire process. Also, students might emphasize that it is important to consider the units of measurement (vocabulary word) when collecting data.

Ask that they collect and organize the data in a table. Groups who finish early can be asked to consider appropriate statistics and representations and begin calculating and creating these.

## Wrap-up (5 minutes)

Ask students to describe the difference between categorical and quantitative data.
Students can write this on an exit ticket and submit. Categorical data involves attributes
(or categories) as a response to a statistical question. For example, what are my classmates' favorite animal? Quantitative data involves numerical data as a response to a statistical question. For example, what the heights of my classmates? Quantitative data can be summarized using many statistics including the mean, median, mode, range, and IQR. Categorical data can be summarized using the mode.

## Statistical Problem-Solving Process

1. Formulating a statistical question.
2. Data Collection.
3. Analyzing the data.
4. Interpreting the results.

## Day 6

| Student Handouts: | Materials: |
| :---: | :--- |
| • Statistical Problem Solving Process | • $\quad$ Chart paper |
| from Day 5 | • Tape |
| Teacher Handouts: | • Markers |
| • Academic Prompt \#5 | • $\operatorname{Projector~}$ |
|  | • Timer |
|  | • Calculators |
|  | • Candy |
|  | • Marbles |
|  | • Yard sticks/rulers |

Lesson Goal: The students will engage in the statistical problem-solving process for a quantitative data set. The students will interpret the results of a statistical investigation in context.

## Warm-up (15 minutes)

Ask students to take 1 minute and review their homework from Thursday night, focusing on their choice for graphical representation and the last question, emphasizing their choice for the measure of center.

Randomly call on 2 students to share (one student for representation question, one for measure of center question). Ask the whole class the following, utilizing think (30 seconds)-pair (30 seconds)-share (1 minute).

If you were analyzing this data for the principal, what would you tell him about $7^{\text {th }}$ graders homework time?

Student responses should emphasize their choice for measure of center. A good response could be: "The average time spent on homework for $7^{\text {th }}$ graders is $\qquad$ because that is my mean." Or "The most common time spent on homework for $7^{\text {th }}$ graders is $\qquad$ because that is my mode." Students might also respond about the shape of the data.

## Statistical Problem-Solving Process Task (continued from Day 5)

## Understanding the Problem (5 minutes)

Redistribute each group's chart paper from Day 5. The goal today will be to continue with the Statistical Problem-Solving Process, focusing on step 3: Analyzing the data. Have them take 30 seconds silently to review their work and think of how they might want to represent their data and what statistics they might want to calculate (may be helpful to remind them of the statistics studied thus far). Then have them share with their group ( 2 minutes) without starting to actually create the representations and statistics.

## Solving the Problem (25-30 minutes)

Students will need to create more than one representation for their data set, as well as more than one statistic.

Pause students after approximately 2 minutes and randomly call on 2-3 students to have them share the statistics and representations they are creating (2-3 minutes). Ask them to state why they chose that statistic and representation. Remind students that they must create at least two statistics and representations for their data set. While students are working, circulate room and pick groups to present that are creating different representations (box plot, histogram, dot plot) and statistics (mean, median, mode, range).

Continue to let students work for 15 minutes then call on pre-selected groups to present. Ask the following discussion questions for each group if they do not address them in their presentation: (5-7 minutes)

1. What representation was your first pick to create, and why?
2. Talk about the shape of your graphs. It may help to probe the students. The teacher could probe the students by asking them to talk about the center of their graph (mean and/or median), how spread out the graph looks (variability), the highest part of their graph (which would be the mode). As students talk about
these aspects of their graph, let them know to which statistic that aspect refers. For example, if a student comments on the highest part of the graph, the teacher could interject with, "Wonderful! That part, the highest part of the graph, would represent the MODE or the most common data point."
3. What statistic did you choose to calculate, and why?

Post the Academic Prompt \#5 on the board and read aloud. Give students 30 seconds to think about this prompt, and then let them work in groups to finish this task ( 20 minutes). They must write up, as a group, a formal response, i.e. one per group, to this prompt to turn in for the day. After students have been working for 3 minutes, stop the class and randomly call on 3 students. Have them discuss what they think is most important to include in their write up and why. Students need to emphasize or be reminded that the most important part of their write up is to talk about their statistics and graphs in context. Statistics makes sense because we can talk about values and data in context. For example, it is not enough for the student to say that the mean is 12 . The student needs to be able to verbalize that the mean is the, for example, average shoe size for a sixth grader in my classroom.

Allow students to continue working. As students work, circulate and ensure that students know to base their reasoning on both the statistics and representations that they have created.

When students are complete, the groups may turn in their response. All group members should come to an agreement on what to include in their write up.

## Wrap-up (5 minutes)

Ask students to think-pair -share about the following question:
What was the most difficult part of today's task, and why?

## Day 7

| Student Handouts: | Materials: |
| :---: | :---: |
| • Statistical Problem-Solving Process | • Chart paper |
| - new sheet | • Tape |
| • Homework 4 | • Markers |
| Teacher Handouts: | • Projector |
| • Informal Check 7 | • Timer |
| • Student Work Categorical | • Calculators |
| Representation | • Candy |
|  | • Marbles |

Lesson Goal: The students will engage in the statistical problem-solving process for a categorical data set. The students will interpret the results of a statistical investigation in context.

## Warm-up (8-10 minutes)

Post Informal Check \#7 on the projector. Read aloud, and tell students to think-pairshare. Randomly call on 3 students and have them share their partner's response and if they agree or disagree with their partner including why or why not (3 minutes).

After students share, ask the following discussion questions, randomly calling on students to respond after appropriate think time:

1. Do you think the results of this survey would be biased towards football? What does bias mean? Statistical bias refers to systematic favoritism due to how we collected the data. This can lead to misleading results. The results of this survey would be biased towards football since we surveyed people leaving a football game.
2. How could we better collect results for this survey? Students could state many ways to better collect data. The important aspect of this would be to randomly collect data that is representative our population in Nashville. This might include
randomly picking people from a list of people living in Nashville and calling them to ask the question.
3. Is this the same type of data that we collected the other day? Is it numerical? Why, or why not? This data set is not numerical because we are surveying the people's favorite sport. The responses; e.g. football, basketball, soccer; are categorical and not quantitative.

## Statistical Problem-Solving Process - Categorical Data Task

## Understanding the Problem (5 minutes)

Ask students to think-pair-share about a statistical question that would be categorical in nature, that is, the responses would not be quantitative but categories (similar to the warm up), and that could be answered in class.

If no students construct a feasible question, pose the example question (e.g., What are my classmates' favorite animal?) and have a discussion about why this is categorical and statistical.
This conversation might include that since our responses are categories instead of quantitative results, then this would be called a categorical question. Since the answers to this question can vary person to person, this question is statistical.
"Categorical" can be added to the vocabulary chart paper.

## Solving the Problem ( 25 minutes)

Students can work in groups of 3-4 for this task. It might be beneficial to change their groups from that which they had for the quantitative task. Ask students to think-pairshare on the following question:

How could we collect data to answer our categorical statistical question?

Randomly call on 3 groups and ask them to share their ideas (1-3 minutes). After all groups have presented, summarize their statements about how to collect the data and ask
if the whole class agrees. Once agreement has been made, allow the students to collect the data.

When students are concluding their data collection, ask the students how they might represent the data? Utilize think-pair-share. Then display Student Work Categorical Representation. Read the scenario aloud to students, and ask them, in their groups to discuss what this student was doing, and how he might have created the first bar chart. Give students 2 minutes to discuss. Randomly call on 3 groups and have them present their thoughts on what this student was doing ( 3 minutes).
Then show students his relative frequency bar chart and again give students 2 minutes to discuss in groups, randomly call on 3 groups, and have the groups present.

Discussion questions:

1. How did this student find the heights of the bars? The first bar chart has the frequency of answer on the y-axis. That is, this student counted how many, for example, chose dog as their favorite animal. The height of the bar for dog is how many people stated this as their favorite animal. The second bar chart has percent as the $y$-axis. The student calculated how many people chose dog as their favorite animal and then divided that number by the total number of responses to get the percent of people surveyed who chose dog as their favorite animal. The second chart is called a relative frequency graph, where relative frequency could be interchanged with percent for this class.
2. What is different/similar between the two charts? One is a frequency (or count) bar chart. The other is a relative frequency (or percentage) bar chart.
3. Was this similar to what you wanted to create for your representation? Why or why not?

Distribute chart paper and markers to students. Let students begin their representations on the chart paper, employing both strategies from the unknown student work and any other representations that they wish (pie chart, frequency table, percentage table).

After most groups are finishing their representations, tell students that they will rotate their posters to the next group. Each group will get 2 minutes to look at the other group's chart paper and write any questions or comments (related to the task) on post-it notes and stick to the chart paper. After 2 minutes, rotate to the next group. The second group can ask new comments/questions or may respond to previous comments/questions. Continue through 3 groups. Once groups get their chart paper back, give them 2 minutes to read comments and discuss within their group. Randomly call on groups to share any comments or suggestions on the post-it notes that are on their chart paper.

Wrap-up (5 minutes)
Ask students to think-pair-share on comparing and contrasting this data collection and data analysis process to the one the other day for the quantitative data. Randomly call on students to share. Students might say that the data collection was similar. You simply gathered the data to answer the question of interest from your classmates. The data analysis would be similar in terms of creating a graphical representation. However, students might say that the quantitative data analysis was more in depth because you can calculate more statistics for a quantitative data set than a categorical data set. If this discussion results in new observations about a categorical and quantitative data set, it might be helpful to add to the chart paper where these two were contrasted.

Distribute homework for Day 7.

Statistical Question to Explore: What are my classmates' favorite animals?

Data: Students from my class responded to the question above. The responses are below in a table.

| Dog | Cat | Cat | Dog |
| :---: | :---: | :---: | :---: |
| Dog | Dog | Dog | Bird |
| Cat | Bird | Dog | Cat |
| Dog | Snake | Bird | Cat |
| Cat | Dog | Cat | Dog |
| Fish | Cat | Dog | Cat |
| Dog | Dog | Cat | Cat |
| Cat | Dog | Fish | Dog |




## Homework 4

Name: Date: $\qquad$
A previous student polled his friends about their favorite color. The following were the responses that he collected:

| Blue | Green | Orange | Blue | Blue |
| :---: | :---: | :---: | :---: | :---: |
| Green | Gold | Red | Red | Green |
| Purple | Red | Blue | Green | Blue |
| Blue | Red | Red | Blue | Blue |
| Yellow | Purple | Purple | Green | Red |

1. Create a representation for this data.
2. Are we able to calculate a mean for this data? Why, or why not?
3. What statistic can we calculate for this data? Calculate it below.

## KEY Homework 4

A previous student polled his friends about their favorite color. The following were the responses that he collected:

| Blue | Green | Orange | Blue | Blue |
| :---: | :---: | :---: | :---: | :---: |
| Green | Gold | Red | Red | Green |
| Purple | Red | Blue | Green | Blue |
| Blue | Red | Red | Blue | Blue |
| Yellow | Purple | Purple | Green | Red |

1. Create a representation for this data.

Students could create a pie chart, bar chart (relative frequency or frequency), dot plot, etc.
2. Are we able to calculate a mean for this data? Why, or why not?

No because this data set is categorical.
3. What statistic can we calculate for this data? Calculate it below.

We can calculate the mode. The most common color chosen was blue.

## Day 8

## Student Handouts:

- Test Review


## Teacher Handouts:

- Informal Check \#8
- Academic Prompt \#6


## Materials:

- Chart paper
- Tape
- Markers
- Projector
- Timer

Lesson Goal: The students will engage in the statistical problem-solving process for a categorical data set. The students will interpret the results of a statistical investigation in context.

## Warm-up (6-8 minutes)

Post Informal Check \#8 on the board, and ask students to solve individually. Randomly call on students to respond to the following discussion questions.

Discussion questions:

1. What does the question mean by typical? Typical refers to the average value. If students were unable to come up with this answer, let them know this definition, add it to the vocabulary chart paper, and then ask them to calculate the average for the data set. They may want to use calculators, and they should round to one decimal place.
2. How did you find this value? Students should say that they added all of the data points and divided by the total number of data points to get the answer of 6.3.

## Statistical Problem-Solving Process - Categorical Data Task

## Understanding the Problem ( 5 minutes)

Redistribute chart paper from day 7. Ask students to pull out their responses from yesterday to other group's comments/questions for their posters. Have each student take 30 seconds to review their responses. Then tell students to talk about the concerns and their solutions in their groups for 2 minutes.

## Solving the Problem (20-25 minutes)

Tell students that their goal today will be to continue with the Statistical Problem-Solving Process, focusing on Step 4 (interpreting the data). Tell students that their goal will be to respond to Academic Prompt \#6 on the board (read aloud). They must write up a formal response to this prompt to turn in for the day. Each group can turn in one write up for the entire group.

Give students 15 minutes to work on the problem. Then randomly call on 3 groups and have each share their group response to the prompt (3 minutes).

Distribute (if not posted on wall) students quantitative statistical problem solving task that they started on Day 5 to each group. Give them 30 seconds to review their work from the previous day. Students should be comparing/contrasting the two processes, one with quantitative data and the other with categorical data. Ask them to focus on similarities/differences. Randomly call on 3-4 students to share, and follow with discussion questions:

1. What is similar/different from when we did this process for quantitative data?
a. Did we collect our data similarly or differently from when we did the quantitative data task? Explain.
b. Did we interpret our results differently? Explain. Students' responses to this might include the differences stated previously between categorical and quantitative data sets. These include the difference of type of statistics and graphs that one can use to represent the data sets. The data collection process, however, is similar regardless of the type of data.
2. What type of statistics can we calculate for categorical data? Students might at first say mean, but then realize that mode is the appropriate statistic. The goal of this question is for students to recognize that categorical data does not allow for a mean, median, or range calculation.
3. What representation did you use, and why? How is your statistic reflected in the representation? Since the only statistic that can be calculated here is a mode, the students should say that the highest part of their bar chart (if they created this) or dot plot reflects the mode.

Tell students that they will be given a test tomorrow, and that the rest of the time will be spent working in groups by looking at previous homework and tasks in class. Tell students that they should be prepared to do the following for the exam (can post on board):

## Test Review Topics

- Know what makes a question statistical. A question whose answers vary from person to person. A question that anticipates variability.
- Write a statistical question. E.g. What are girls' favorite color? What are the heights of all males under the age of 15 at my school?
- Calculate a mean, median, median, range, and interquartile range. Students need to remember the process for finding each of these.
- Create an appropriate representation for quantitative data (histogram, box plot, or dot plot).
- Create an appropriate representation for categorical data (bar chart with frequencies and/or percentages).
- Discuss the shape of a graph for quantitative data. Students need to be able to talk about the center of the graph, how much the graph is spread out (variability), and any unique, or outlying, data points.
- Discuss the center for quantitative data. Students need to talk about the mean, median, or mode including the units for those statistics.
- Discuss how the quantitative data vary. Students need to talk about how the variability of data can be seen in a graph and could be represented by the IQR or range.
- Interpret results (mean, median, mode, and/or range) in context of a problem. For example, students can not simply say that the mean is 12 and the mode is 13. Students' responses should be something to the effect, "The average number of candies in a Halloween size bag of chocolate candies is 12. The most occurring number of candies in a Halloween size bag of chocolate candies is 13 candies."


## Test Review Topics

You will want to know how to do all of the following for your test tomorrow. You can work in pairs or groups on these topics. It will be helpful to look at your homework questions.

- Know what makes a question statistical.
- Write a statistical question.
- Calculate a mean, median, median, range, and interquartile range.
- Create an appropriate representation for quantitative data (histogram, box plot, or dot plot).
- Create an appropriate representation for categorical data (bar chart with frequencies and/or percentages).
- Discuss the shape of a graph for quantitative data.
- Discuss the center for quantitative data.
- Discuss how the quantitative data vary.
- Interpret results (mean, median, mode, and/or range) in context of a problem.


## Day 9

| Student Handouts: | Materials: |
| :---: | :---: |
| - Statistics Unit Test | - Oreos |
| - Oreo Cookie Task | - Projector |
|  | - Rulers |
|  | - Scales |

Distribute Statistics Unit Test. Tell students that they can use calculators to verify their answers, but they need to show all calculations.

If students have time after the exam, the Oreo Cookie Task can be started. This would entail reading through the task as a whole class and then beginning the statistical problem-solving process with the task. However, if students do not have time to actually begin the task, then the following can be done to prepare them for the task. In the case where this is not enough time for a discussion, the students can be sent home with the task and asked to read over the whole task that evening for discussion the next morning.

Tell students that for the end of this unit, they will be asked to complete a task involving Oreos. Post Oreo Cookie Task on projector and have students read aloud the Goal, Role, Audience, and Situation sections (hide the rest). Ask students to share out what they think their purpose is for the task. Tell students that they will collect data for this task tomorrow but that they will have the weekend to work on their project.

## Day 10

| Student Handouts: | Materials: |
| :--- | :--- |
| $\bullet$ Oreo Cookie Task | • Oreos |
|  | • Projector <br>  <br>  <br>  <br>  <br>  <br>  |

## Understanding the Task ( 20 minutes)

Post Oreo Cookie Task on projector. Randomly call on 3 students to have them share what they think their purpose is for the task.

Then show the Product, Performance, and Purpose; and Standards and Criteria for Success to students. Have students read aloud. Randomly call on 2-3 students to have them share their interpretation of their goal.

Distribute Oreo Cookie Task to all students. Have students look at the Report Format. Tell students briefly about this format, and state that this will be their guideline for their final project report. Review the Oreo Study Rubric with students and ask them for questions. Let them know that this is how their final score will be calculated.

## Collecting Data

Next ask students to get into their groups, and to spend 3 minutes discussing with group how they might collect data to solve this problem. After 3 minutes, randomly call on 3 groups, and have them each share their data collection strategies.

Students might suggest measuring the height of the stuff, the weight of the stuff, or density. If students are struggling with how to collect data, make a recommendation that they measure the weight of the stuff. Students need to have a discussion about how they can use the regular and "doublestuf" Oreos to see if the "doublestuf" cookies really have double the filling. This might entail measuring both the weight of the stuff for the
regular and "doublestuf" cookies and comparing the average weight of a regular cookie to a "doublestuf" cookie. Given the openness of this task, it might be helpful to make suggestions to guide students but allow them to first discuss ways to collect the data themselves.

Distribute Oreo cookies (one regular, one "doublestuf") to each group of students. Remind them to not consume the cookies since this will be their only chance of collecting data that they will have to analyze over the weekend.

Once students begin collecting data, stop class after about 4 minutes. Randomly call on 23 groups and ask them how they are collecting data and why they chose that method.

Let students spend the rest of the class period collecting their Oreo data. Each student will need the Oreo data, so ensure that each student is collecting data measurements on their own sheet of paper to take home with them.

## Wrap-Up (5 minutes)

Tell students to think-pair-share what how they think they will proceed with their task this weekend. How will they use the data from today to solve the task?

# APPENDIX F: Institutional Review Board Approval 

Investigator(s): Natasha E. Gerstenschlager, Angela Barlow
Department: Mathematical Sciences
Investigator(s) Email: neg2d@mtmail.mtsu.edu, angela.barlow@mtsu.edu

Protocol Title: "Implementing Reform-Oriented Statistics in the Middle Grades: Teacher Support Structures "
Protocol Number: 15-016
Dear Investigator(s),
The MTSU Institutional Review Board, or a representative of the IRB, has reviewed the research proposal identified above. The MTSU IRB or its representative has determined that the study poses minimal risk to participants and qualifies for an expedited review under 45 CFR 46.110 and 21 CFR 56.110, and you have satisfactorily addressed all of the points brought up during the review. You will need to obtain approval from the school district prior to beginning your project. Please forward this permission letter to the Office of Compliance once you receive it.

Approval is granted for one (1) year from the date of this letter for 40 participants.
Please note that any unanticipated harms to participants or adverse events must be reported to the Office of Compliance at (615) 494-8918. Any change to the protocol must be submitted to the IRB before implementing this change.

You will need to submit an end-of-project form to the Office of Compliance upon completion of your research located on the IRB website. Complete research means that you have finished collecting and analyzing data. Should you not finish your research within the one (1) year period, you must submit a Progress Report and request a continuation prior to the expiration date. Please allow time for review and requested revisions. Failure to submit a Progress Report and request for continuation will automatically result in cancellation of your research study. Therefore, you will not be able to use any data and/or collect any data. Your study expires 7/30/2015.

According to MTSU Policy, a researcher is defined as anyone who works with data or has contact with participants. Anyone meeting this definition needs to be listed on the protocol and needs to complete the required training. If you add researchers to an approved project, please forward an updated list of researchers to the Office of Compliance before they begin to work on the project.

All research materials must be retained by the PI or faculty advisor (if the PI is a student) for at least three (3) years after study completion and then destroyed in a manner that maintains confidentiality and anonymity.

Sincerely,
Kellie Hilker
MTSU Institutional Review Board Member

