A STUDY OF PRESERVICE ELEMENTARY TEACHERS LEARNING MATHEMATICS THROUGH PROBLEM-BASED LEARNING AND PROBLEM SOLVING

by

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This dissertation is dedicated to Caroline, my wife and best friend since the fifth grade.

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ABSTRACT

This dissertation reports a mixed-methods study of preservice elementary teachers learning mathematics through problem-based learning (PBL) and problem solving that was conducted during the fall 2012 semester at Middle Tennessee State University. The study used grounded theory data analyses to investigate preservice elementary teachers learning mathematics through PBL and problem solving. Comparative analyses between two control groups and an experimental group were conducted to investigate the possible benefits of learning mathematics through PBL and problem solving. In addition to preservice elementary teachers' mathematics content knowledge, special attention was given in data collection and analyses to the preservice elementary teachers' mathematics anxiety, attitudes toward mathematics, and their role in learning mathematics in a PBL environment.

The results of the grounded theory analyses found main categories that preservice elementary teachers perceived as influential to their learning mathematics through PBL and problem solving. The categories that emerged from the data were working with others to solve problems, relying on self for learning, creating problems, and teacher mini-lectures and other teacher facilitation.

Preservice elementary teachers' perceptions of the impact of learning mathematics through PBL and problem solving on their mathematics anxiety and attitudes toward mathematics are reported.

The results of the comparative analyses on mathematics content knowledge, mathematics anxiety, and attitudes toward mathematics are reported and connections are made to the qualitative results.

This report concludes with a summary and discussion of the results. Implications of the findings, connections to the literature, and suggestions for additional research are specified.

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CHAPTER I

INTRODUCTION

This dissertation reports a mixed-methods study of preservice elementary teachers learning mathematics through problem-based learning (PBL) and problem solving that was conducted during the fall 2012 semester at Middle Tennessee State University. The study used grounded theory data analyses to investigate preservice elementary teachers learning mathematics through PBL and problem solving. Comparative analyses between two control groups and an experimental group were conducted to continue to explore the possible benefits of learning mathematics through PBL and problem solving. In addition to preservice elementary teachers' mathematics content knowledge, special attention was given in data collection and analysis to the preservice elementary teachers' mathematics anxiety, attitudes toward mathematics, and their role in learning mathematics in a PBL environment. This first chapter presents the background to the study, specifies the problem of the study, describes its significance, and presents an overview of the methodology used. The chapter concludes by defining key terms, and presenting an overview of the dissertation.

Background of the Study

Student Achievement in the United States

Since the early part of the 20th century, reports document that American students' performance in mathematics is deficient (Brownell, 1947). More recently, American students' mathematics performance has been compared on an international level showing

American students being outperformed by many Asian and European countries (McGee, Polly, & Wang, 2013; Stigler & Hiebert, 2009). The concern over U.S. students underperforming on international comparisons is compounded when the demands of a twenty-first century workforce are considered. In the present technological age in which the world is becoming increasingly connected, the international students outperforming U.S. students in school will also be competing for the same jobs in their adulthood (Dochy, Segers, Van den Bossche, & Gijbels, 2003). Many of these jobs will likely require a college degree. However, many American students are graduating from high school unprepared for college level courses (Carnevale, Smith, & Strohl, 2010; National Center for Education Statistics, 2011).

There are many factors that may contribute to American students' underachieving in mathematics. One possible explanation is the type of teaching in American mathematics classrooms and the type of mathematical knowledge that American students are learning (Skemp, 1976; Stigler & Hiebert, 2009).

Reform Efforts in Mathematics Classrooms

For over two decades, the National Council of Teachers of Mathematics (NCTM) in the United States has promoted the learning of mathematics through problem solving (NCTM, 1989, 2000). One implication of the *Principles and Standards in School Mathematics* (*PSSM*) (NCTM, 2000) is that teachers are to ensure that students engage in process standards. The process standards describe the ways that students should encounter mathematics in their classes on a daily basis. The process standards are problem solving, communication, connections, representation, and reasoning and proof (NCTM, 2000). The *PSSM* raises the expectations of all teachers of mathematics, including elementary teachers.

Currently, the Common Core State Standards for Mathematics (CCSSM) (Common Core State Standards Initiative (CCSSI), 2010) have been adopted by many states and are beginning to be implemented. These standards will require teachers to focus deeper on the content standards instead of covering a multitude of topics without depth. The CCSSM contain more than recommendations for mathematics content. They also have Standards for Mathematical Practice, which include elements of NCTM's process standards and the National Research Council's (NRC) strands for mathematical proficiency (CCSSI, 2010; NCTM, 2000; NRC, 2001). The Standards for Mathematical Practice, stated in the CCSSM, are: make sense of problems and persevere in solving them; reason abstractly and quantitatively; construct viable arguments and critique the reasoning of others; model with mathematics; use appropriate tools strategically; attend to precision; look for and make use of structure; and look for and express regularity in repeated reasoning (CCSSI, 2010, pp. 6-8). In order for new teachers to be able to design lessons that engage their students in these practices, the teachers themselves will need to be proficient in these mathematical practices, and have a deep knowledge of the mathematics content (Conference Board of the Mathematical Sciences (CBMS), 2010).

The teacher's role, when teaching according to these standards, is different from the teacher's role in a traditional mathematics classroom. Those who attempt to teach according to the standards need help understanding this new role which may be different from their conceptualization of teaching (CBMS, 2010).

Preservice Elementary Teachers

It is well documented that elementary education majors in the United States have high scores on mathematics anxiety rating instruments, indicating high levels of anxiety about anything related to mathematics (Ashcraft & Krause, 2007; Lang & Lang, 2010; Malinsky, McJunkin, Pannells, & Ross, 2004). Teachers with high mathematics anxiety often pass on negative attitudes toward mathematics to their students because they are less willing to address students' questions (Ashcraft & Krause, 2007). Students with high anxiety usually have low performance in mathematics (Lang & Lang, 2010), meaning that high mathematics anxiety can result in poor performance on mathematics tasks. Lang and Lang (2010) found that high confidence and low mathematics anxiety were related to good performance in mathematics.

Also documented is the claim that many elementary teachers in the United States have negative attitudes toward mathematics, high anxieties about studying mathematics, low confidence in their mathematics ability, and have a record of poor performance in mathematics (Ashcraft & Krause, 2007; Gresham, 2007). Instructional effectiveness is partially determined by the teacher's attitude toward mathematics (Malinsky et al., 2004). These attributes may largely be the result of the mathematics instruction they have experienced during their elementary and secondary school years: instruction that reflects mathematics as a collection of facts and processes to be memorized rather than understood and practiced only in the classroom and not in life-based contexts (Sakshaug

& Wohlhuter, 2010). These rigid views of mathematics are continued in college if the student's view of mathematics is not challenged. Often these views are strengthened rather than challenged by general education mathematics classes and even many mathematics classes in a student's teacher education program.

Elementary teachers typically lack experiences in which mathematics is learned through problem-solving activities. As a result, their mathematics classrooms are likely to be traditional (Sakshaug & Wohlhuter, 2010; Timmerman, 2004). For elementary teachers to teach in a problem-solving environment that challenges students to reason, think, and solve problems in a variety of contexts, they need to first experience this learning themselves (Jacobbe & Millman, 2009; Sakshaug & Wohlhuter, 2010). Because of their lack of exposure to mathematical problem solving, elementary teachers are often unprepared to help struggling students (Gresham, 2010).

Situations like the ones described above create a cycle, "future teachers pass through elementary schools learning to detest math . . . then return to teach a new generation to detest it" (Polya, 1945, p. ix). Similarly, Schoenfeld's (1992) summary states that students "abstract their beliefs about formal mathematics – their sense of their discipline – in large measure from their experiences in the classroom" and that "students' beliefs shape their behavior in ways that have extraordinarily powerful (and often negative) consequences" (p. 359).

Interventions are needed to reverse the negative cycle that seems to pervade the mathematical preparation of elementary school teachers. These interventions should improve future teachers' attitudes and beliefs about mathematics, reduce their anxieties toward mathematics, and increase their content knowledge before they enter the classroom.

Problem-based Learning

"Problem-based learning (PBL) is an instructional method in which students learn through facilitated problem solving" (Hmelo-Silver, 2004, p. 235). The format for problem-based learning experiences in this study consisted of posing a scenario (an event that occurs in everyday life and/or a workplace task) that includes multiple problems to be solved. It is the student's responsibility to determine what problems to solve by creating problems related to the scenario posed, to decide what information is needed to solve the problem, and to identify where the information can be obtained. Students who are successful in a PBL experience must become responsible for their own learning (Barrows, 2002). In addition, students work in collaborative groups, requiring them to distribute responsibilities, communicate reasoning, justify their solutions, recognize that problems can be solved in different ways, and determine when different solution paths are valid. Through creating and solving problems, students learn or review concepts, skills, and generalizations with the intent to construct knowledge that will be long lasting (Schmidt, 1993).

PBL has been used for decades in the United States in medical and nursing schools (Strobel & Barneveld, 2009). More recently, PBL has been used in a variety of subjects (Savery & Duffy, 1995; Walker & Leary, 2009). However, little is known about the use of PBL in mathematics instruction for preservice elementary teachers.

The Potential for PBL to Make a Change

The effectiveness of the use of PBL in various disciplines has been investigated through a number of meta-analyses (Albanese & Mitchell, 1993; Strobel & Barneveld, 2009; Walker & Leary, 2009). PBL has been shown to be more effective than traditional instruction for long-term retention of knowledge, skill development, and satisfaction of students (Strobel & Barneveld, 2009; Walker & Leary, 2009). In addition, medical students learning through PBL have scored as well or better on clinical exams than those students learning through traditional instruction (Albanese & Mitchell, 1993).

In the first study on the use of PBL for preservice elementary teachers in Australia, PBL instruction resulted in an improvement of the preservice teachers' attitudes toward science (Etherington, 2011). In Slovenia, 9-year-old students learning mathematics through PBL were more successful in solving difficult mathematical problems than students learning how to master algorithms (Cotic & Zuljan, 2009). In the United States, California students learning mathematics through a problem-based curriculum were found to perform as well or better on the mathematics portion of the Scholastic Aptitude Test (SAT) when compared to students learning mathematics through a traditional curriculum (Clarke, Breed, & Fraser, 2004). In the same study, the students learning mathematics through a problem-based curriculum were more confident, valued communication in mathematics, and viewed mathematics as useful (Clarke et al., 2004).

The lack of information on the use of PBL in the mathematical preparation of preservice elementary teachers indicates that beneficial knowledge will be gained through the study of preservice elementary teachers learning mathematics through PBL. The positive results on the use of PBL and the emphasis that PBL places on problem solving suggest that learning mathematics through PBL may be beneficial to preservice elementary teachers. In particular, preservice elementary teachers should be given the chance to engage in the type of learning that they will be expected to provide under current reform recommendations (CCSSI, 2010). Learning mathematics through PBL can give preservice elementary teachers the opportunity to learn mathematics content differently.

Pilot Studies

To form a basis of research on preservice elementary teachers' ability to learn mathematics through PBL and problem solving, two pilot studies were conducted during the two semesters prior to this study. The results from these pilot studies informed this study, including the design of the class, the data collected, and the methods used to analyze the data.

During the fall semester of 2011, the first pilot study was conducted to examine the potential benefits and drawbacks of teaching through PBL. Comparative analyses were conducted between students from both a traditional number concepts course (control group) and students from a number concepts course taught using PBL experiences and problem solving (experimental group). The Mathematics Anxiety Rating Scale (MARS) (Suinn & Winston, 2003) was used as a pretest and posttest instrument for both groups to determine the change in mathematics anxiety from the beginning of the semester to the end. A copy of the MARS can be found in Appendix A. A researcher-designed content knowledge test was given as a pretest and posttest instrument to measure differences in construction of content knowledge for both groups. The Attitudes Toward Mathematics Inventory (ATMI) (Tapia & Marsh, 1996) was given as a pretest and posttest instrument to measure changes in attitudes toward mathematics for both groups. A copy of the ATMI can be found in Appendix B. An instructor-created problem-solving assessment was used at the end of the semester to compare the groups' problem-solving abilities.

Pretests administered at the beginning of the pilot study showed that the groups were not significantly different with respect to content knowledge, attitudes toward mathematics, and mathematics anxiety (Banes & Miller, 2012). A posttest for content knowledge showed that the groups were not significantly different with respect to content knowledge (Banes & Miller, 2012). That is, PBL can be used as an instructional strategy without interfering with students' construction of content knowledge. There were no significant differences in either group's mathematics anxiety based on their ratings on the MARS (Banes & Miller, 2012). The experimental group experienced a significant decrease in their ATMI (attitudes) ratings (Banes & Miller, 2012). The instructor-created problem-solving assessment showed the problem-solving abilities of the experimental group were significantly higher than the control group at the conclusion of the semester (Banes & Miller, 2012).

A second pilot study was conducted in the spring of 2012. In this second study, a number concepts course was taught to a small group of honors students. Student reflections, responses to writing prompts, and student work were collected in addition to measuring content knowledge, mathematics anxiety, and attitudes toward mathematics.

This second pilot study was used as an opportunity to refine the PBL scenarios created for this research, to improve the implementation of the PBL scenarios, and to trial the use of writing prompts for collecting information about students' perceptions of learning through PBL and problem solving. The results of this study were that the students were able to learn concepts and structure of elementary mathematics through PBL and problem solving, the students' attitudes toward mathematics did not decrease, and their mathematics anxiety did not change.

Research Questions

This study sought answers to questions that the pilot studies did not. The pilot studies that provided a foundation for this study found that preservice elementary teachers can learn mathematics through PBL and problem solving. However, the data collected in the pilot studies did not provide any detail about how the preservice elementary teachers constructed their mathematics content knowledge nor did it reveal their perceptions about learning mathematics through PBL and problem solving. The pilot studies also gave no explanation beyond responses to Likert-scales about the influence of learning mathematics through PBL on mathematics anxiety and attitudes toward mathematics. Consequently, the neutral results on mathematics anxiety and mixed results on attitudes toward mathematics from the pilot studies cannot be explained.

These unanswered questions from the pilot studies led the researcher to the following qualitative research questions.

1. How do preservice elementary teachers learn mathematics through PBL?
2. What features of PBL do preservice elementary teachers perceive as contributing to their learning of mathematics?
3. What features of PBL do preservice elementary teachers perceive as influencing their mathematics anxiety?
4. What features of PBL do preservice elementary teachers perceive as influencing their attitudes toward mathematics?

As stated previously, there is little known about teaching mathematics to preservice elementary teachers through PBL and problem solving. Thus, to continue to establish and investigate the benefits of teaching mathematics to preservice elementary teachers through PBL and problem solving on mathematics content knowledge, mathematics anxiety, and attitudes toward mathematics, this study sought to answer the following quantitative research questions.

1. Do preservice elementary teachers learning mathematics through PBL improve their mathematics content knowledge more than preservice elementary teachers taught using a teacher-centered approach or a mixture of methods approach?
2. Do preservice elementary teachers learning mathematics through PBL decrease their mathematics anxiety more than preservice elementary teachers taught using a teacher-centered approach or a mixture of methods approach?

3. Do preservice elementary teachers learning mathematics through PBL improve their attitudes toward mathematics more than preservice elementary teachers taught using a teacher-centered approach or a mixture of methods approach?

Significance of the Study

The significance of this study is multifaceted. First, it informs teacher educators about the use of PBL and problem solving in mathematics courses designed for preservice elementary teachers. Second, this study extends the research base on the use of PBL in mathematics classrooms. Lastly, the study contributes to teacher educators' knowledge about student perceptions of the effective components of learning mathematics through PBL and problem solving.

Overview of the Methodology

A brief overview of the methodology used in this study is provided to give a general explanation of the design of the study, the participants in the study, and the data collected. A full explanation of the methodology of the study is discussed in Chapter 3.

Research Perspective and Type

This research was a qualitative-quantitative, mixed-methods study. A grounded theory methodology was used to analyze the qualitative data. The quantitative data were evaluated using comparative analyses between two control groups and an experimental group.

Participants and Participant Selection

All groups consisted of preservice elementary teachers enrolled in a course titled "Concepts and Structure of Elementary School Mathematics" at Middle Tennessee State University. The experimental group was taught by the researcher using PBL and problem solving as the primary method of instruction. One control group was taught by a tenured, associate professor in the Department of Mathematical Sciences who used teacher-centered methods of instruction (primarily lecture). The second control group was taught by a graduate teaching assistant who used a combination of teacher-centered and student-centered methods. Participant selection was determined by the section of the class in which the participant was enrolled as was the designation to experimental group, control group one, and control group two.

Data Collected

Qualitative data that were collected from the experimental group consisted of the preservice elementary teachers' work, responses to reflective writing prompts (see Appendix C), audiotaped structured interviews (see Appendix D), and video and audio-taped class meetings focused on a single collaborative group per class meeting. Research journal writings and notes made by the researcher were artifacts that guided the collection and analyses of the qualitative data.

Both control groups were observed periodically throughout the semester to document the type of instruction being used. The observations were audio-taped or video-taped and field notes were taken during the observations. The instructors of the control groups provided their course syllabi and a description of the assignments for the course. These artifacts confirm the researcher's descriptions of the classroom environments of the control groups used in this study.

Pretest and posttest data were collected from both control groups and the experimental group. A researcher-designed mathematics content knowledge test (see Appendix E) was administered to compare the control and experimental groups' content knowledge at the beginning of the semester and to compare their mathematics content knowledge at the end of the semester. The mathematics content knowledge test used for this study was different from the mathematics content knowledge test used in both pilot studies. An explanation of the development of the mathematics content knowledge test used in this study and the validation of it is provided in Chapter 3. As in the two pilot studies, the MARS (Suinn & Winston, 2003) and the ATMI (Tapia & Marsh, 1996) were given as a pretest and posttest to both groups. The MARS and the ATMI have both been shown to be valid and reliable (Frank & Suinn, 1972; Tapia & Marsh, 2004).

Implementation of PBL for the Study

The duration of each PBL cycle used in the study was four to six class meetings, the equivalent of two to three weeks of class meetings. The PBL cycle began by participants being given a written scenario (see Appendix F for an example PBL scenario used in this study). Each scenario was a story that was either a real event or a realistic event. In collaborative groups, participants first discussed the scenario and made a list of things they know, things they do not know, and resources that could give them information about what they did not know. After the discussion, the groups shared items from their list with the whole class. Next, participants were asked to create problems related to the PBL scenario (See Appendix F for examples of participant-created problems from this study) and to solve them together in their groups. They submitted their created and solved problems as part of a PBL report. Implementing PBL in this way is consistent with what Xia, Lu, Wang, and Song (2007) referred to as situated creation and problem-based instruction (SCPBI). SCPBI involves creating situations, posing problems, solving problems, and applying mathematics (Xia et al., 2007). In addition, participants were asked to write about what they learned, what they found challenging about the assigned tasks, and how it was related to learning mathematics. At the conclusion of the PBL lesson, participants were given a set of non-routine problems created by the instructor that were related to the PBL scenario (See Appendix F for examples of instructor-created problems used in this study). These problems were solved in collaborative groups before participants presented solution paths to the class. An emphasis was placed on participants being able to communicate their thoughts about a problem rather than only presenting the solution. After the solution paths to the problems were presented, the participants completed a quiz, created by the instructor and related to the PBL scenario. A full discussion of the creation of the PBL scenarios and the implementation of the PBL cycle for this study is given in Chapter 3.

Definitions of Key Terms

Attitudes Toward Mathematics

Attitudes toward mathematics is a construct that describes the emotions, associations, expectations, and values one has toward mathematics (Aiken, 1974; Hannula, 2002; Ma & Kishor, 1997). Attitudes toward mathematics includes the way one reacts to mathematics and is composed of values, enjoyment, motivation, and self-confidence (Aiken, 1970, 1974; Fennema & Sherman, 1976; Hodges & Kim, 2013; Ma & Kishor, 1997; Tapia & Marsh, 2004).

Collaborative Group Learning

Collaborative group learning is a relatively unstructured student-centered learning environment in which students are in charge of determining the goals, problems, and procedures to produce knowledge that is constructed socially (Cooper & Mueck, 1990; Wiener, 1986).

Grounded Theory

Charmaz (2012) defined grounded theory as

> A method of conducting qualitative research that focuses on creating conceptual frameworks or theories through building inductive analyses from the data. Hence, the analytic categories are directly 'grounded' in the data. The method favors analysis over description, fresh categories over preconceived ideas and extant theories, and systematically focused sequential data collection over large initial samples. This method is distinguished from others since it involves the researcher in data analysis while collecting data – we use this data analysis to inform and shape further data collection. Thus, the sharp distinction between data collection and analysis phases of traditional research is intentionally blurred. (pp. 187 – 188)

Mathematics Anxiety

Mathematics anxiety is a cognitive and psychological construct (Harper & Dane, 1998; Bursal & Paznokas, 2006) that is composed of physical reactions to doing mathematics (Fennema & Sherman, 1976), feelings and fears experienced in mathematical situations (Tobias, 1978), worry about mathematics (Wigfield & Meece, 1988), and mental disorganization when confronted with mathematical tasks (Richardson & Suinn, 1972).

Problem-Based Learning (PBL)

There is not one single form or definition of PBL, and different approaches have been used to support teaching goals and meet specific needs (Bot, Gossiaux, Rauchs, & Tabious, 2005; Hmelo-Silver, 2004, 2012; Jerzembek & Murphy, 2013; Ravitz, 2009; Schettino, 2012). PBL is student-centered with students determining what they need to learn while in collaborative groups under the guidance of a tutor or instructor (Barrows, 1996, 2002; Hmelo-Silver, 2004; Jonassen, 2011; Schmidt, 1993). The role of the tutor or instructor is to facilitate the students as they work to solve authentic problems prior to any study or instruction (Barrows, 1996, 2002; Hmelo-Silver, 2004; Jonassen, 2011; Schmidt, 1993). The scenarios presented in PBL are ill-structured and should generate multiple problems and multiple solution paths (Barrows, 1996, 2002; Hmelo-Silver, 2004). The role of problems in PBL is to help students construct content knowledge and problem-solving skills through self-directed learning (Barrows, 1996, 2002; Hmelo-Silver, 2004; Jonassen, 2011; Schmidt, 1993). The goals of PBL extend beyond learning content and include developing flexible knowledge, effective problem-solving skills, self-directed learning skills, collaboration skills, and intrinsic motivation (Hmelo-Silver, 2004; Schmidt, 1993).

Problem-Based Learning (PBL) and Problem Solving

PBL and problem solving is the name given to the PBL cycle that the researcher used in this study. PBL and problem solving is briefly described above and with a more detailed description in Chapter 3.

Problem Posing

Problem posing is an important aspect of learning mathematics through problem solving that involves the problem solver producing problems to solve instead of the problem being provided for her (Goldenberg & Walter, 2003; Kilpatrick, 1987; Silver & Cai, 1996). Creating story or word problems, a part of the PBL cycle used in this study, is one of many ways that students may pose problems (Rudnitsky, Etheredge, Freeeman, & Gilbert, 1995; Winograd, 1992).

Problem Solving

Problem solving is defined as solving problems in which there are no rules or memorized procedures to solve the problems and in which the direct path to solution is not known immediately (Brownwell, 1942; Kilpatrick, 1985; McLeod, 1985; Polya, 1945; Van de Walle, 2003).

Teacher-Centered Instruction

Teacher-centered instruction refers to instructional methods in which the teacher lectures while students silently listens and takes notes. In a teacher-centered classroom the teacher holds the knowledge and does the majority of the verbalization about content (Torp & Sage, 2002).

Teaching through Problem Solving

Teaching through problem solving is an instructional method in which students are presented with a meaningful problem that serves as a “vehicle for introducing or developing mathematical concepts” (D’Ambrosio, 2003, p. 48).

Overview of the Chapters of the Study

In this first chapter, the research study was introduced to provide an overview of the study and to provide a context for the study. The background to the study, significance of the study, a brief overview of the methodology, and definitions of key terms were discussed. Chapter 2 provides a review of the related literature for this dissertation study of preservice elementary teachers learning mathematics through PBL and problem solving while working in collaborative groups. PBL, problem solving, problem posing, mathematics anxiety, attitudes toward mathematics, learning in collaborative groups, and the mathematical preparation of preservice elementary teachers are discussed. Chapter 3 explains the methods used to conduct the study and data analyses performed. Chapter 4 presents the results of both the grounded theory analyses and the quantitative comparative analyses. Chapter 5 concludes the dissertation with a brief overview of the study, a summary of the results, and a discussion of the results.

CHAPTER II

REVIEW OF THE LITERATURE

This chapter reviews the related literature for this dissertation study of preservice elementary teachers learning mathematics through PBL and problem solving while working in collaborative groups. This study also examined the effects of learning mathematics through PBL and problem solving in collaborative groups on the preservice elementary teacher's mathematics anxiety and attitudes toward mathematics. The design of the PBL lessons used in this study required the preservice elementary teachers to participate in mathematical problem solving and problem posing. Thus, this chapter reviews the relevant literature on PBL, problem solving, problem posing, mathematics anxiety, attitudes toward mathematics, learning in collaborative groups, and the mathematical preparation of preservice elementary teachers.

Search Process

This literature review was developed through a systematic search process using the Walker Library at Middle Tennessee State University, Beaman Library at Lipscomb University, Internet searches, and the researcher's personal collection of literature. The literature review process began with the pilot studies conducted during the fall 2011 and spring 2012. The literature continued to be reviewed following the pilot studies to develop a working proposal for this dissertation study. Following the completion and approval of the working proposal, data were collected, analyzed, and the results were written prior to conducting a full literature review which is supported by grounded theory literature (Charmaz, 2012; Strauss & Corbin, 1990). Although this literature review appears in this dissertation in the second chapter, chronologically it was the last completed chapter of this dissertation. Thus, the results of the grounded theory analyses presented in chapter four of this dissertation are authentically from the data and not preconceived ideas from the literature. Consequently, connections made to the pre-existing literature in chapter five of this dissertation are more genuine because of the chronology of completion of this dissertation.

Problem-Based Learning

Definitions

PBL was developed in the 1960's at McMaster University (Spaulding, 1991). The developers of PBL were influenced by the case study method used at Harvard University and were dissatisfied with the results of traditional medical education (Schmidt, 1993). There is not one single form or definition of PBL, and different approaches have been used to support teaching goals and meet specific needs (Bot et al., 2005; Hmelo-Silver, 2004, 2012; Jerzembek & Murphy, 2013; Ravitz, 2009; Schettino, 2012). A short definition of PBL is "an instructional method in which students learn through facilitated problem solving" (Hmelo-Silver, 2004, p. 235). Savery (2006) defined PBL as a "learner-centered approach that empowers learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem" (p. 12). Schmidt's (1993) definition of PBL emphasized that students solve problems while a tutor facilitates their learning. Schettino (2012) defined PBL as

> An instructional approach of curriculum and pedagogy where student learning and content material are constructed through the use, facilitation, and experience of contextual problems in a decompartmentalized, threaded topic format in a discussion-based classroom setting where student voice, experience, and prior knowledge are valued. (p. 347)

PBL is student centered with students determining what they need to learn while in collaborative groups under the guidance of a tutor or instructor (Barrows, 1996, 2002; Hmelo-Silver, 2004; Jonassen, 2011; Schmidt, 1993). The role of the tutor or instructor is to facilitate the students as they work to solve authentic problems prior to any study or instruction (Barrows, 1996, 2002; Hmelo-Silver, 2004; Jonassen, 2011; Schmidt, 1993). The scenarios in PBL are ill-structured and should generate multiple problems and multiple solution paths (Barrows, 1996, 2002; Hmelo-Silver, 2004). The role of problems in PBL is to help students construct content knowledge and problem-solving skills through self-directed learning (Barrows, 1996, 2002; Hmelo-Silver, 2004; Jonassen, 2011; Schmidt, 1993). The goals of PBL extend beyond learning content and include developing flexible knowledge, effective problem-solving skills, self-directed learning skills, collaboration skills, and intrinsic motivation (Hmelo-Silver, 2004; Schmidt, 1993).

Just as there are many definitions of PBL, there are also different ways that PBL can be implemented (Hmelo-Silver, 2004). The PBL cycle can be viewed as having three phases: initial problem analysis, self-directed learning, and reporting (Hmelo-Silver, 2004). Alternatively, Ram (1999) suggested five stages of the PBL process: introduction, inquiry, self-directed study, revisiting the hypothesis, and self-evaluation. Lastly, Delisle's (1997) seven-step approach to implementing PBL includes setting the climate, setting up the structure, visiting the problem, revisiting the problem, producing a product or performance, and evaluating performance.

Theoretical Lens of PBL

The use of PBL to learn content and problem-solving skills is supported by research and psychological theory (Hmelo-Silver, 2004; Schmidt, 1993). PBL has roots in rationalist traditions that contend that people engage their prior knowledge when comprehending information, the amount of prior knowledge available determines the extent to which something can be learned, and a lack of prior knowledge makes learning new material more difficult (Schmidt, 1993). Additionally, PBL aligns with recommendations from Dewey, cognitive psychology, constructivist theory, and social learning theory (Hmelo-Silver, 2004; Hmelo-Silver, Duncan, & Chinn, 2007; Schmidt, 1993). PBL has the following cognitive effects on student learning (Schmidt, 1993):

1. activation of prior knowledge;
2. elaboration of prior knowledge;
3. restructuring of knowledge;
4. increased retrieval of knowledge through contextual learning; and
5. intrinsic motivation.

Zull (2002) described, in *The Art of Changing the Brain*, the way learning occurs as it relates to the biology of the brain. Prior knowledge is identified as an important factor associated with learning (Zull, 2002). Zull said all learners have prior knowledge, it is persistent, and it is the beginning of new knowledge. The recommendation is that instruction begins with students' prior knowledge even if their prior knowledge is incorrect or inadequate (Zull, 2002). The learner will have no reason to improve or change their prior knowledge unless they realize that their prior knowledge is not useful to them (Zull, 2002). Furthermore, Zull suggested that instruction should begin with the tangible before the abstract because without an experience upon which to associate the abstract then the content is meaningless to the students.

When students begin with a problem in PBL, they must use their prior knowledge to solve the problem (Schmidt, 1983). Discussing problems helps students elaborate on their prior knowledge, see that there is a lack of prior knowledge, and thus supplies intrinsic motivation for self-directed learning (Schmidt, 1993). Working in collaborative groups may be the most influential component of PBL for determining student learning because the verbalization of ideas that occurs during group discussions promotes active construction of knowledge (Hmelo-Silver, 2004; Hmelo-Silver et al., 2007; Yew & Schmidt, 2012). Defending conjectures and discussing problems with the group promotes the construction of knowledge (Koschmann, Myers, Feltovitch, & Barrows, 1994; Vye, Goldman, Voss, Hmelo, & Williams, 1997). Additionally, working in collaborative groups decreases the demands on students' working memories (Pea, 1993; Salomon, 1993). Flexible knowledge is developed through reflection because it helps students relate new knowledge to prior knowledge, identify gaps in their prior knowledge, and develop transferrable knowledge (Hmelo-Silver, 2004; Solomon & Perkins, 1989).

In conclusion, PBL is an instructional approach that is supported by psychological theories (Hmelo-Silver, 2004). Activation of prior knowledge and working in collaborative groups to solve problems serve as catalysts for learning, increasing intrinsic motivation and improving self-directed learning (Schmidt, 1993). According to theory, students will learn well in a PBL environment; however, the effectiveness of PBL has been debated since its early days of implementation (Albanese & Mitchell, 1993).

Benefits and Challenges of PBL

The majority of the research on the effectiveness of PBL has been conducted in medical education (Hmelo-Silver, 2004). During the last four decades there has been a significant amount of debate about the effectiveness of PBL for use in medical education (Albanese & Mitchell, 1993; Walker & Leary, 2009). There have been numerous meta-analyses on the comparative benefits of PBL in Medical education (Albanese & Mitchell, 1993; Berkson, 1993; Colliver, 2000; Dochy, Segers, Van den Bossche, & Gijbels, 2003; Gijbels, Dochy, Van den Bossche, & Segers, 2005; Kalaian, Mullan, & Dasim, 1999; Newman, 2003; Norman and Schmidt, 1992; Strobel & Barneveld, 2009; Vernon & Blake, 1993; Walker & Leary, 2009). The results of these meta-analyses arrived at similar conclusions about the comparative benefits of PBL and traditional medical education curricula. In general, medical education students that learned through PBL

1. enjoyed learning through PBL (Albanese & Mitchell, 1993; Berkson, 1993; Strobel & Barneveld, 2009; Vernon & Blake, 1993);

2. performed better than traditional students on assessments of clinical skills (Albanese & Mitchell, 1993; Dochy et al., 2003; Kalaian et al., 1999; Strobel & Barneveld, 2009; Vernon & Blake, 1993);
3. retained their knowledge (Dochy et al., 2003; Norman & Schmidt, 1992; Strobel & Barneveld, 2009);
4. valued understanding over memorization (Berkson, 1993; Vernon & Blake, 1993);
5. increased their intrinsic motivation (Albanese & Mitchell, 1993);
6. increased their self-directed learning skills (Albanese & Mitchell, 1993);
7. developed transferable knowledge (Albanese & Mitchell, 1993); and
8. scored lower on examinations that assessed factual knowledge (Albanese & Mitchell, 1993, Berkson, 1993, Dochy et al., 2003; Kalaian et al., 1999; Newman, 2003).

The meta-analyses have reported mixed findings about the claim that PBL is superior for developing clinical and problem-solving skills. Some meta-analyses reported that there were no differences on assessments of skill between students that learned through PBL and students that learned in a traditional curriculum (Berkson, 1993; Colliver, 2000; Norman & Schmidt, 1992). Similarly, some meta-analyses have concluded that students in a traditional curriculum do no better on measures of factual knowledge and over time, students that learned through PBL scored higher than traditional students on measures of factual knowledge (Dochy et al., 2003; Gijbels et al., 2005).

PBL in K-12 Education

There is less research on the effectiveness of PBL outside medical education, but PBL is becoming a topic of growing interest in all disciplines and internationally because of its alignment with current educational reform (Barrows, 2000; Gallagher, Stepian, & Rosenthal, 1992; Hmelo-Silver, 2004, 2012; Savery, 2006; Torp & Sage, 2002). Studies conducted outside medical education have come to similar conclusions as the meta-analyses from medical education: that is, PBL can be used to teach content with small or no differences in content knowledge learned, students that learn through PBL develop skills beyond the content, and students prefer learning through PBL.

PBL has been used to teach a variety of subjects in elementary, middle, and high school. Jerzembek and Murphy (2013) reviewed studies involving school-aged children and found that PBL could have a positive influence on students' academic development. Chinese researchers used a PBL design that consisted of creating situations, posing problems, solving problems, and applying mathematics to teach mathematics to Chinese primary and secondary school students (Xia et al., 2007). They found that students' interest in mathematics increased, their ability to pose problems improved, and they performed significantly better on a posttest that was representative of the mathematics content advocated by the Chinese national standards.

Researchers in Slovenia studied nine-year-old students' cognitive performance and attitudes while learning mathematics through PBL (Cotic & Zuljan, 2009). Using a control group for comparison, they found that students learning through PBL were more successful in solving more difficult mathematics problems, there were no differences on students' ability to compute, and no statistically significant difference on attitudes.

In Pakistan, PBL has been used to teach mathematics to elementary students (Ali, Akhter, & Khan, 2010). In this study, they found significant differences in favor of the use of PBL for mathematics achievement when compared to students learning through traditional instruction. Ali, Akhter, Shahzad, Sultana, and Ramzan (2011) reported on further research on Pakistani elementary students learning through PBL. They compared the students learning mathematics through PBL to a group learning through traditional methods and studied the relationship between motivation and achievement. The findings were that motivation played a larger role for the group learning through PBL and indicated the importance of motivation in PBL.

Pederson (2003) examined sixth grade students' intrinsic and extrinsic motivation when learning science through PBL and compared those students to a typical sixth grade science class. The results of this study indicated that students were more intrinsically motivated when learning through PBL. Others have studied the effects of learning middle school science through PBL and found that PBL positively affected students' academic achievement, their attitudes toward science, and conceptual development (Akinoglu & Tandogan, 2007).

PBL has been successfully implemented for students with special needs (Belland, Ertmer, & Simons, 2006; Hmelo-Silver, 2004; Walker & Leary, 2009). Belland et al. (2006) used PBL to teach middle school students with special needs. They found that

PBL resulted in increased student engagement, development of social skills, and experiences of success for students.

Gordon, Rodgers, Gavula, and McGee (2001) studied low-income middle school students learning health sciences through PBL. The PBL activities were used as enrichment activities to supplement the existing curriculum. Analyzing students' report cards, they found that students received better marks for behavior and had increased science achievement.

Researchers in Canada completed a two-year action research project that used cross-curricular PBL scenarios in which seventh- and eighth-grade students had access to their own laptop for learning mathematics, science, and language (Freiman, Beauchamp, Blain, Lirett-Pitre, & Fournier, 2011).

Perhaps the earliest form of PBL used to teach mathematics was the *Adventures of Jasper Woodbury*, a video-based set of problem scenarios, developed for middle school mathematics courses (CGTV, 1997). The videos use anchored instruction, a variation of PBL (Hmelo-Silver, 2004). Hickey, Moore, and Pellegrino (2001) found that students who learned mathematics in the fifth grade using the *Jasper Series* were better problem solvers than fifth graders learning with traditional curricula.

At-risk middle school females learned both mathematics and science through PBL and were interviewed to learn about their perceptions of changes in their learning processes and self-efficacy (Cerezo, 2004). The at-risk middle school females perceived themselves to be more confident and able to take control of their learning. As a result, they experienced positive changes in their self-confidence. Group interaction was also found to be beneficial component of learning through PBL.

Ferreira and Trudel (2012) implemented PBL in a high school chemistry class at an all-male Catholic high school. They found that PBL resulted in a significant increase in student attitudes toward science and problem-solving skills. The students in the study benefitted from working as a community, and they enjoyed the learning environment created by PBL.

Tenth grade biology students were taught a unit on the human excretory system using PBL and were compared to a control group that was taught with traditional methods (Sungur, Tekkaya, & Geban, 2006). The students that learned through PBL had significantly higher posttest scores for both academic achievement and performance skills.

Mergendoller, Maxwell, and Bellisimo (2006) compared a group of high school students learning macroeconomics through PBL to a group of high school students learning macroeconomics through traditional instruction. They found that PBL was more effective than traditional instruction for teaching macroeconomics.

PBL in Postsecondary Education

Researchers at Emory University taught chemistry labs using PBL (Ram, 1999). The students worked on a problem about water quality in Atlanta, and through student postings to an online bulletin board, the researchers found that students developed responsibility for their learning, learned to work with others, and the students had positive feelings about the use of PBL to learn chemistry (Ram, 1999).

Using PBL to teach a module on international relations, researchers studied students' self-efficacy, knowledge, attitudes, and behaviors (Brown et al., 2003). Analyzing the data by gender, they found that males and females had similar responses on the self-efficacy scale. Both males and females had significantly higher scores on the posttest; however, males' scores on pretest and posttests were significantly higher than the females' scores. Both genders also had a significant increase in academic skills but males were higher than females. The researchers found no significant differences in attitudes.

Students selected from a variety of courses in psychology completed a questionnaire about their perceived differences between PBL and traditional instruction (Hays & Vincent, 2004). The students responded that PBL was better at promoting student interaction, developing critical thinking, research skills, and presentation skills.

Illustrating the wide range of disciplines that are beginning to use PBL, Lyon and Teutschbein (2011) used PBL to teach a hydrology course. The researchers were interested in assessments that reflect the intended learning outcomes of the PBL environment. The researchers found that the students scored higher on assessments that reflected the goals of PBL when compared to students that were assessed with a traditional exam. The researchers concluded that alternate forms of assessment should be used in PBL courses.

Chernobilsky, DaCosta, and Hmelo-Silver (2004) studied attitudes of ninety male students in a problem-based human relations course. The students' attitudes were not consistent throughout the semester. Students felt that their group discussions, stimulation of thought, and participation decreased over the course of the semester.

Technology is being used to support the PBL environment (Duncan, Smith, & Cook, 2013; Lou, Shih, Tseng, Diez, & Tsai, 2010; Savin-Baden, 2007). The effectiveness of technology supported PBL was compared to PBL without the use of technology in a marketing course (Arts, Gijselaers, & Segers, 2002). In the technology supported course, the students solved problems collaboratively and shared information through electronic communication. The researchers used a pretest and posttest to measure content knowledge and found that the students' test scores in the technology supported PBL improved significantly more than the scores of students that learned through PBL without the support of technology (Arts et al., 2002).

Kampen, Banahan, Kelly, McLoughlin, and O'Leary (2004) changed from teaching a lecture-based introductory thermal physics course to teaching the course through a PBL curriculum. They created six PBL problems that covered the content objectives of the course and students worked in groups of four or five to solve the problems prior to any instruction. They found that the students' motivation increased, they were more responsible for their own learning, and positive effects could be seen the following semester as they applied the skills they had acquired (Kampen et al., 2004).

Sahin (2010) taught calculus-based physics to university students in Turkey and compared the students to a group of students learning through traditional methods. The group that learned through PBL showed significantly higher conceptual gains than students learning through traditional instruction. There were no significant differences found on measures of beliefs about physics.

Pease and Kuhn (2011) conducted two studies to determine benefits of learning physics through PBL and to identify the effective components of PBL. The first study found that learning physics through PBL was more effective than learning through a lecture format. The second study found that engagement with the problem was more influential than collaboration. Additionally, the benefits of PBL were evidenced by an assessment given nine weeks following the implementation of PBL.

Capon and Kuhn (2004) studied students in a Master of Business Administration program. Two economics concepts were taught with one group learning the first concept through PBL and a second concept through lecture and discussion. The other group learned the concepts in reverse methodology. The results of the analyses found that there were no differences between groups for learning the first concept and significant differences in favor of the lecture format for the second concept, but the group that learned through PBL was able to integrate new information with their prior knowledge better than students who experienced lecture and discussion.

Goldstein, Burke, Getz, and Kennedy (2011) used PBL in a pilot course of intermediate algebra. They compared the students learning algebra through PBL to students learning through the traditional intermediate algebra curriculum and found no differences in student performance and satisfaction. The researchers followed both groups of students the following semester to their college algebra class and compared the success rate of the two groups of students. The group that learned intermediate algebra through PBL was more successful in college algebra than the students that learned in the traditional course (Goldstein et al., 2011). The results of their study indicated that the positive effects of PBL may not evidence themselves immediately.

Shore, Shore, and Boggs (2004) implemented PBL in developmental mathematics classes using applications from allied health. The students learning through PBL were compared to students learning developmental mathematics through a traditional curriculum. The students learning through PBL scored significantly better on a posttest content knowledge assessment than the students learning through a traditional curriculum, and they rated what they were learning to be more useful on a posttest questionnaire.

Duda and Ross (2012) used a PBL unit in which students had to mathematically model a zombie outbreak similar to the outbreak in the extremely popular TV show *The Walking Dead*. Qualitative analyses of students' narrative course evaluations, surveys, and interviews demonstrated that the students valued, enjoyed, and learned from the PBL. The students, when compared to the students from the previous semester that did not learn through PBL, had better scores on an assessment of modeling using differential equations.

Vidic (2011) studied engineering students learning introductory statistics through PBL and investigated their statistical knowledge and problem-solving abilities. The students learning through PBL were compared to students learning through traditional methods. The students that learned through PBL performed better on statistical problems than the students that learned through a traditional approach. Research conducted in

Denmark illustrated the variety of advanced mathematics subjects that can be taught through PBL (Christensen, 2008). Christensen (2008) used PBL to teach matrices, differential equations, and other higher mathematics content to engineering students.

Teaching Preservice Teachers with PBL

Some of the most positive results on the use of PBL have been from studies of teacher education (Ravitz, 2009; Walker & Leary, 2009). If future teachers are going to use PBL in their classrooms, they need to have experienced learning through PBL (Lehman, George, Buchanan, & Rush, 2006).

Bilgin, Senocak, and Sozbilir (2009) reported on the use of PBL to teach preservice elementary teachers about gases. The researchers compared the group learning about gases through PBL to a group of students learning through traditional instruction. They found that the PBL group performed better on measures of conceptual knowledge, but the groups were not significantly different in their ability to solve quantitative problems about gases.

In a science course for preservice elementary teachers in Turkey, students learned the science content of the course, enhanced their communication skills, and performed significantly better on a problem-solving assessment than students that learned through traditional methods (Koray, Presley, Koksal, & Ozdemir, 2008). PBL is being used to teach science to preservice teachers in other countries. In the first attempt of an Australian undergraduate teacher education program to incorporate PBL to teach science, researchers found that PBL had a positive impact on preservice elementary teachers' motivation to teach science within a real-world context, and concepts were learned in

PBL that would not have been learned in a traditional classroom setting (Etherington, 2011). Etherington (2011) also found that the preservice elementary teachers in the study needed a period of time to adjust to being active learners.

Positive results were found on the use of a hybrid model of PBL to teach preservice teachers in an educational psychology course (Hmelo-Silver, Derry, Bitterman, & Hatrak, 2009). The class consisted of an online and face-to-face PBL format. The preservice teachers learning through the hybrid PBL learned more content, as measured by their performance on a posttest video-analysis task, than preservice teachers that learned educational psychology through traditional methods.

Similar results have been found on the use of PBL to teach preservice teachers problem-solving skills in an educational psychology class (Simone, 2008). Comparative analyses were conducted between two groups of students. One group of preservice teachers learned through PBL, and the second group learned through a traditional approach. The results showed that the group that learned through PBL was significantly better at constructing the central problem, elaborating on the problem, relating their solutions to the problem, and using multiple resources.

Thirty-five preservice elementary teachers in a special education course completed a technology supported PBL unit about a hypothetical case of a student facing suspension for possessing drugs at school (Ochoa, Gottschall, & Stuart, 2004). The discussions of the preservice elementary teachers were video-taped, and they filled out a satisfaction survey. The results of the analyses were that some students dominated the group discussions instead of there being an equal amount of contribution from all the group members.

Prospective secondary mathematics teachers in Turkey were studied while learning statistics through PBL (Canturk-Gunhan, Bukova-Guzel, & Ozgur, 2012). Using qualitative methods, they found that the preservice teachers held a positive view of PBL, but they worried about being able to feasibly implement PBL in Turkish schools due to limited resources.

An examination of the studies summarized above shows that PBL is being used in the United States and abroad to teach middle school and secondary subjects (Jerzembek & Murphy, 2013), university classes, and preservice teachers at the elementary and secondary level (Canturk-Gunhan et al., 2012; Simone, 2008). Closer examination by subject type and teaching classification will show that there is a need for studies that describe and evaluate the implementation of PBL in teaching mathematics to preservice elementary teachers.

Problem Solving

Definitions

Problem solving is defined as solving problems in which there are no rules or memorized procedures to solve the problems and in which the direct path to solution is not known immediately (Brownwell, 1942; Kilpatrick, 1985; McLeod, 1985; Polya, 1945; Van de Walle, 2003). Schoenfeld (1989) adds that the problem should engage students, and the students should have the necessary skills to solve the problem. Some define problem solving as the actions of the solver when the solver does not know what to do to solve the problem (Wheatley, 1984).

Problem-Solving Advocates in Mathematics Education

Problem solving has a long history in school mathematics and has been used to achieve a variety of goals that have changed over time (Stanic & Kilpatrick, 1989). Dewey's (1910) *How We Think* called for reflective thinking in education and marks one of the earliest recommendations for problem solving in school mathematics (White, 1978). Similarly, Kilpatrick (1918) advocated educational projects that included problem solving. Perhaps the most prominent supporter of problem solving in the early part of the twentieth century was George Polya whose iconic book *How to Solve It* (1945) advocated the use of heuristics to develop students' problem-solving ability. Polya's (1945) four-step problem-solving process consists of understanding the problem, choosing a strategy, implementing the strategy, and looking back at the problem, is included in most mathematics textbooks today (NCTM, 2000). The National Council of Supervisors of Mathematics (1977) stated that learning to solve problems is the principal reason for learning mathematics. Halmos (1980) wanted teachers to train their students to be better problem solvers by exposing them to more than facts. Charles and Lester (1982) advocated the use of problem solving as a means of developing the ability to deal with problems that arise in daily life and viewed problem solving as a skill that is in increasing demand. Others have advocated problem solving as a means of developing the brain (Jensen, 1998). The National Council of Teachers of Mathematics (NCTM) articulated the need for mathematics education reform documents such as *Agenda for Action* (1980) and *PSSM* (2000) in which they called for an increased emphasis on problem solving in school mathematics. Problem solving remains to be advocated for by mathematics educators with publications like *Teaching Mathematics through Problem Solving: Grades Prekindergarten – Grade 6* (NCTM, 2003) and *Teaching Mathematics through Problem Solving: Grades 6 – 12* (NCTM, 2003). This can be seen in more recent documents such as the Common Core State Standards for Mathematical Practice (CCSSI, 2010) and *Implementing the Common Core State Standards through Mathematical Problem Solving: High School* (NCTM, 2012).

Teaching through Problem Solving: A Description

The preceding discussion shows that problem solving has been a part of school mathematics throughout the past century; however, the role of problem solving has changed throughout the years. Stanic and Kilpatrick (1989) give three ways problem solving has been used in mathematics education: problem solving as a context for mathematics, problem solving as a skill, and problem solving as an art.

George Polya (1945) changed the focus of problem solving instruction to heuristics, marking a new era of problem solving (Schoenfeld, 1987). Heuristics, Schoenfeld (1987) concluded, are not enough to develop problem-solving skills in students. Students need to be able to think about what they are doing and regulate their progress (Schoenfeld, 1987). Though heuristics are valuable skills, students need to become good thinkers and sense-makers (Schoenfeld, 1992).

Problem solving is not merely a goal of learning mathematics but also a way to learn mathematics (Goldenberg & Walter, 2003; Hiebert, 2003; Lambdin, 2003;

Schroeder & Lester, 1989; Van de Walle, 2003). Problem solving can stimulate learning new mathematics knowledge as students' existing mathematics knowledge is strengthened and extended through solving good problems (NCTM, 2000). Teaching mathematics through problem solving is the current focus of problem solving in mathematics education, and it embodies the sentiments expressed by Schoenfeld (1987, 1992), NCTM (2000, 2012), and CCSSI (2010), and has been a focus in mathematics education in the past decade (Cai, 2003). Teaching through problem solving, much like PBL, begins with a meaningful problem that the teacher presents to the class, and the students investigate the problem, solve the problem, and lastly the students reflect on the mathematics in the problem (Kahan & Wyberg, 2003). Teaching through problem solving is beneficial because 1) it helps students understand that sense-making is the nature of mathematics; 2) students' understandings of the mathematical concepts involved in the problem are deepened; and 3) students' interests are increased when working problems that they find meaningful (Kahan & Wyberg, 2003).

Studies Related to Teaching Mathematics through Problem Solving

Learning mathematics by solving problems promotes achievement for all students (Cobb et al., 1991). Gooya (1992) found that beliefs about mathematics and past learning experiences with mathematics were related. Undergraduate students enrolled in a mathematics course that was taught through problem solving were surveyed at the beginning and end of the semester about their beliefs about the nature of mathematics and the purpose of problem solving. The students demonstrated a significant change in their beliefs, and their views about problem solving were more aligned with problem solving to learn mathematics.

In a comparative study of sixth grade students learning mathematics over a nine week period, Ridlon (2009) found that students learning through problem solving significantly improved on measures of achievement and attitudes toward mathematics. Low-achieving students benefitted the most from the problem-solving approach; indicating that learning mathematics through problem solving promotes equity (Ridlon, 2009).

Samuelsson (2008) examined Swedish middle school children learning arithmetic through traditional, independent, and problem-solving methods of instruction and found no differences in the effectiveness of the different approaches for teaching arithmetic. Problem solving was found to be more effective for developing intrinsic motivation, self-directed learning skills, and students' self-concept (Samuelsson, 2008).

Clark et al. (2004) reported on the use of teaching through problem solving in three high schools in California as part of the Interactive Mathematics Program (IMP). They compared the students in the IMP to students learning through a traditional curriculum and found that the students learning mathematics through problem solving had a much different view of mathematics from the students learning through traditional curricula. The students in the IMP saw mathematics as part of everyday life, they were more confident, valued communication more, and performed as well as the students learning through traditional curricula on mathematics SAT scores.

Model-eliciting activities (MEAs) are a type of teaching through problem solving (Bostic, 2012). An MEA is a rich task, often presented as a scenario, which involves multiple concepts, is open-ended, and allows for multiple solution paths (Bostic, 2012). Mathematical modeling is an activity advocated for in the Common Core State Standards for Mathematical Practice (CCSSI, 2010). The five features of an MEA are students develop an initial model, there are multiple models, students must translate between different mathematical representations to make decisions, students' initial models can be improved, and students must communicate their models (Bostic, 2012). MEAs have been used to teach middle school mathematics (Bostic, 2012), high school mathematics (Magiera & Zawojewski, 2011), calculus (Yoon, Dreyfus, & Thomas, 2010), and to identify mathematically gifted students (Chamberlin & Moon, 2005).

Model-eliciting activities and PBL are closely related approaches to instruction (Hjalmarson & Diefes-Dux, 2008). The similarities between the two approaches include their beginning with a realistic and meaningful context, placement of responsibility on the student, and construction of knowledge and acquisition of skills through solving problems (Hjalmarson & Diefes-Dux, 2008). Differences between the two approaches include, the amount of research required to produce a model in an MEA may require less research than in PBL, and MEAs are more focused on procedures and processes that contribute to the mathematical model created by the students (Hjalmarson & Diefes-Dux, 2008).

Problem Solving to Teach Mathematics to Preservice Teachers

Teacher educators, researchers, and professional organizations have recommended that preservice teachers learn mathematics through problem solving to develop their own mathematical understanding. The goals of these recommendations are to influence preservice teachers' abilities to teach mathematics through problem solving, and influence future teachers' affective characteristics.

It is important for teachers to learn to teach mathematical problem solving (LeBlanc, 1982). Research in preservice teacher education is needed to determine how to help preservice teachers develop skills for teaching through problem solving (Ernest, 1996). Teachers have a tendency to teach the way they were taught, and the way they were taught is often different from teaching through problem solving (Battista, 1994). Thus, teachers need to experience learning mathematics through problem solving and to develop heuristics for solving problems for them to teach through problem solving and help their students develop heuristics of their own (Hughes, 1976; Thompson, 1985).

There is good reason to believe that there are positive effects of preservice elementary teachers learning mathematics through problem solving. For example, Hembree's (1992) meta-analysis of 487 studies on problem solving found that teachers who had been prepared to use problem-solving heuristics had a strong impact on their students' performance in problem solving. Emenaker (1995) studied preservice teachers' beliefs about mathematics after taking a course in which they learned mathematics through problem solving. Through a Likert-scale survey and student interviews, the researcher found that the preservice teachers' beliefs about mathematics changed in four ways. First, students believed that there are multiple ways to solve problems. Second, they valued conceptual understanding instead of mere memorization. Third, they believed that learning mathematics is not determined by ability. Fourth, they changed the way they believed they would teach (Emenaker, 1995).

A teacher's view of mathematics is tied to many of the decisions he or she makes regarding instruction, including how problems will be used in the classroom (Hoyles, 1992; Karp, 1991; Thompson, 1984). Teachers that view mathematics as problem-centered have been found to view problem solving as a way to teach mathematics, and teachers that view mathematics as a collection of facts and procedures view problem solving as the concluding activity after learning some mathematical fact and procedures (Lerman, 1983).

Thompson (1984) found that teachers' instructional behaviors are influenced by their beliefs about mathematics. Similarly, Hoyles (1992) found that teachers' beliefs about mathematics affected what methods of instruction they used. Karp (1991) studied fourth and sixth grade teachers' attitudes toward mathematics and their instructional practices and found that the teachers' attitudes and instructional practices were related. Specifically, teachers with negative attitudes toward mathematics used instruction that emphasized procedures, and teachers with positive attitudes used problem solving and student-centered investigations (Karp, 1991).

However, teachers' beliefs about mathematics are not necessarily tied to their instructional practices. Raymond (1997) found that teachers who believed in NCTM's recommendations were still more likely to use instructional practices similar to the way they were taught mathematics.

Emenaker (1993) studied preservice elementary teachers participating in a mathematics course that was taught through problem solving. The preservice teachers increased their understanding of mathematics and their self-confidence about their problem-solving abilities. Additionally, pretest and posttest measures of the preservice teachers' beliefs about problem solving showed significant improvements. Others have found that preservice teachers learning mathematics through problem solving changed their beliefs about mathematics (Schram, Wilcox, Lanier, & Lappan, 1988).

Although positive results have been found, experiencing problem-solving instruction may not result in a positive change in teachers' beliefs about mathematics. Schwartz and Riedesel (1994) studied in-service and preservice teachers and found the participants that viewed mathematics as a collection of facts and procedures did not change their beliefs after learning mathematics through problem solving. Similarly, Abell and Pizzini (1992) found that teachers' attitudes toward problem solving were unchanged after experiencing mathematics instruction through problem solving.

Problem Posing

Definitions

Problem posing is a way to learn mathematics and is an important part of learning mathematics through problem solving (Goldenberg & Walter, 2003). Problem posing involves the problem solver producing problems to solve instead of the problem being provided for him (Kilpatrick, 1987; Silver & Cai, 1996). Problems can be posed by students in many different ways (Brown & Walter, 1983; Goldenberg & Walter, 2003; Silver, 2013; Silver & Cai, 1996; Walter, 1993). Students may pose problems by extending problems (Walter, 1993), by asking "what if not" questions (Brown & Walter, 1993), by creating story problems (Rudnitsky, Etheredge, Freeeman, & Gilbert, 1995; Winograd, 1992), and asking questions that arise from other problems (English, 1997a, 1997b).

Problem-Posing History

The idea of problem posing dates back to Polya (1945) when he suggested students create a simpler problem. Brown and Walter (1983) wrote about promoting problem posing by using the "what if not" strategy in which students pose problems by changing information in the original problem. Kilpatrick (1987) was an early advocate of students posing problems. He believed that students should develop the ability to formulate problems and that problem posing should be a part of their mathematics curriculum (Kilpatrick, 1987). Mathematics educators turned their attention to problem posing following several recommendations from the National Council of Teachers of Mathematics (NCTM) that problem posing be included in mathematics classrooms (NCTM, 1989, 1991, 2000). NCTM's *PSSM* (2000) suggested that students should be able to pose problems from a variety of scenarios, be able to investigate mathematical conjectures, and extend problems through posing additional questions. More recently the Standards for Mathematical Practice (CCSSI, 2010) advocated students being able to pose problems in a variety of situations.

Benefits of Problem Posing

The research literature on problem posing has addressed both benefits of students posing problems (Akay & Boz, 2008, 2010; English, 1997a, 1997b, 1997c, 1998; Stoyanova, 1999) and determining the capability of students to pose problems (Silver & Cai, 1996).

Mestre (2000) suggested a benefit of problem posing is students taking on the role of expert. Problem posing gives all students a chance to participate in mathematics that they find enjoyable and satisfying (English, 1997). Problem posing has been associated with the following:

1. mathematics talent (Silver, 1993);
2. developing a sense of ownership and autonomy (Brown & Walter, 2005; Grundmeier, 2003);
3. improving creativity and reflection (Cunningham, 2004; Nohda, 1995);
4. promoting discussion (Schloemer, 1994);
5. improving problem-solving abilities (English, 1997a, 1997b, 1998; Stoyanova, 1999);
6. producing flexible knowledge (English, 1997b; Hashimoto, 1987);
7. mediating mathematics anxiety by taking away the fear of being incorrect (Brown & Walter, 2005);
8. fostering student engagement and interest (Cunningham, 2004; Perrin, 2007);
9. changing false beliefs about mathematics (Borasi, 1993); and
10. increasing motivation (Brown & Walter, 1993; Cai & Brook, 2006).

Some research has found that students can pose problems and that their posed problems led to more mathematics content being discussed. Friel and Gannon (1995) gave students a word problem, and through class discussion and problem posing, the students engaged in mathematics beyond the mathematics of the initial problem.

Winograd (1992) used problem posing to teach fifth grade mathematics. The fifth grade students were asked to write and solve their own original mathematics story problems and share their solutions in groups. The students were able to pose challenging mathematics problems, and their solutions involved mathematics content that went beyond their grade level.

Silver and Cai (1996) examined the results of 509 middle school students posing problems. Students posed problems, and the students' posed problems were examined. The results of this study suggested that prior experience posing problems does not determine a student's ability to pose problems. That is, students with no prior experience posing problems were able to pose their own problems.

In summary, problem posing has been shown to promote a variety of positive effects on students learning mathematics and many students are able to pose problems with little prior experience.

Problem Posing and Preservice Teachers

When they become teachers, current preservice teachers will be expected to create classroom environments in which students have the opportunity to pose problems (CCSSI, 2010). However, for teachers to effectively implement this type of instruction they need to experience being problem posers in their preservice mathematics classes

(Kilpatrick, 1987; Silver, 1994). There have been several studies that investigated preservice teachers' ability to pose problems and the positive effects of problem posing (Akay & Boz, 2008, 2010; Gonzales, 1998; Grundmeier, 2002; Lavy & Shriki, 2010; Silver, Mamona-Downs, Leung, & Kenney, 1996; Toluk-Ucar, 2009).

Silver et al. (1996) examined the problem-posing abilities of 53 middle school teachers and 28 preservice secondary school teachers by having participants solve and pose problems related to the geometry of billiards. They found that the middle school teachers and preservice teachers were able to pose their own problems.

Gonzales (1994) implemented problem posing in a mathematics methods course for preservice elementary and middle school teachers by using heuristics. The preservice teachers were able to pose problems that were novel or extensions to a related problem. Gonzales (1994) suggested that problem posing should be used in mathematics content courses for preservice elementary teachers.

Comparative benefits have been found between preservice elementary teachers that engaged in problem posing and those in a traditional class. Akay and Boz (2008) found that preservice elementary teachers that learned calculus through problem posing were better on a problem-solving and problem-posing task.

Toluk-Ucar (2009) used an experimental and control group to investigate the benefits of preservice elementary teachers learning about fraction concepts through problem posing. The control group made small improvements on their understanding of fractions and abilities to pose problems involving fraction concepts. The experimental group engaged in problem-posing activities about fractions, and they increased their conceptual understanding of fraction concepts and improved their ability to pose word problems involving fraction concepts.

Instruction involving problem-posing activities has been found to positively influence measures of affect for preservice elementary teachers. Grundmeier (2002) found that problem-posing instruction for preservice elementary and middle school teachers resulted in increased attitudes toward mathematics. Other researchers have found similar results when using problem-posing activities for the mathematics instruction of preservice elementary teachers (Akay & Boz, 2010). For example, Akay and Boz (2010) found improvement on preservice teachers' self-efficacy beliefs.

In summary, problem posing is an instructional activity that is closely related to learning through problem solving and is a natural result of PBL (Hmelo-Silver, 2004, Jonassen, 2011). Its use has been advocated by mathematics educators and researchers for decades (Kilpatrick, 1987). Using problem-posing activities has been found to result in positive changes on problem-solving ability, problem-posing ability, understanding of concepts, and measures of affect (Akay & Boz, 2008; Silver & Cai, 1996). These positive results have been experienced by preservice elementary teachers that engaged in problem posing in their mathematics preparation courses (Lavy & Shriki, 2010; Silver, Mamona-Downs, Leung, & Kenney, 1996; Toluk-Ucar, 2009). Lastly, preservice teachers will be expected to foster problem-posing environments as teachers and thus they need to experience instruction that engages them in this activity (Kilpatrick, 1987; Silver, 1994).

Collaborative Group Learning

Collaborative Groups vs. Cooperative Groups

There are many different forms of small-group learning. Collaborative group learning and cooperative group learning are two forms of small-group learning that are widely used (Springer, Stanne, & Donovan, 1999). Although similar, these two forms of small-group learning are different. Cooper and Mueck (1990) defined collaborative learning as relatively unstructured because students are in charge of determining the goals, problems, and procedures to produce knowledge that is constructed socially. In contrast, cooperative group learning involves structure and a common, shared goal that is not determined by the group (Cooper & Mueck, 1990). The distinguishing characteristic of collaborative group learning is that the groups must come to agreement about the goals and methods to achieve those goals on their own (Wiener, 1986).

Benefits of Collaborative Group Learning

A meta-analysis of collaborative group learning in undergraduate science, mathematics, engineering, and technology classes from 1980 to 1999 found that collaborative group learning resulted in greater academic achievement, more favorable attitudes toward learning, and increased persistence through STEM courses and programs (Springer et al., 1999). Other research has found that collaborative group learning helps improve self-directed learning skills (Barnes, 1992), develop professional discourse (Bruffee, 1984), develop students' ability to generate and test hypotheses, and develop better communication skills (Barnes & Todd, 1995). In PBL, students learn in collaborative groups, which is significant in the theoretical basis for learning in a PBL environment (Schmidt, 1993).

Common Student Objections to Group Learning

Theory and research show the benefits of collaborative group learning; however, many students are resistant to this method of learning (Shimazoe & Aldrich, 2010). Students are resistant to collaborative group learning because of their concern about grades and unequal work distribution among the group, confusion about expectations, and negative past experiences with collaborative group learning (Shimazoe & Aldrich, 2010).

In summary, collaborative group learning is distinguished from cooperative group learning by the amount of autonomy students are given in collaborative learning environments. Collaborative group learning has potential to benefit learning, attitudes, self-directed learning skills, and communication skills. Despite the potential advantages of collaborative group learning, many students are resistant, and measures should be taken to address their concerns.

Mathematics Anxiety

Definitions

Many definitions of mathematics anxiety are given in the literature. These definitions focus on different aspects of mathematics anxiety. In general, these definitions emphasize physical reactions to doing mathematics (Fennema & Sherman, 1976), feelings and fears experienced in mathematical situations (Tobias, 1978), and worry about mathematics (Wigfield & Meece, 1988). Mathematics anxiety is defined as both a cognitive and psychological construct (Harper & Dane, 1998; Bursal & Paznokas, 2006).

Mathematics anxiety is the feelings of tension, helplessness, and mental disorganization one has when required to do mathematical tasks (Richardson & Suinn, 1972). Hembree (1990) defined mathematics anxiety as a learned condition that includes a fear of doing any mathematical activity. Tobias (1978) defined mathematics anxiety as a fear of mathematics as well as the burden of negative feelings and thoughts when performing tasks related to mathematical topics. Tobias (1981) described math anxious individuals as those who are cautious about their problem-solving abilities. These individuals experience a high level of stress when called upon to use perform mathematics in public and attempt to avoid mathematics (Tobias, 1981). Some have described mathematics anxiety as an irrational fear of mathematics (Gresham, 2004). Others contend that mathematics anxiety is not irrational but based on past experiences of failure in mathematics (Miller & Mitchell, 1994).

Development of Mathematics Anxiety

Many researchers have attempted to determine how mathematics anxiety develops in individuals. Levine (1995) found the four most common causes of mathematics anxiety were lack of support by teachers and parents, lack of confidence, fear of giving an incorrect response in a classroom environment that values correct answers, and the influence of their mathematically anxious teachers. Others have found that students' past school experiences with mathematics were fundamental in the development of mathematics anxiety (Baloglu & Zelhart, 2007; Cates & Rhymer, 2003; Harper & Daane, 1998; Reys, Linquist, Lambdin, & Smith, 2007). Instructional methods that emphasize rules, memorization, and correct answers are characteristics of early classroom experiences that contribute to the development of students' mathematics anxiety (Trujillo & Hadfield, 1999).

Effects of Mathematics Anxiety

There is a well-established relationship between high mathematics anxiety and low achievement in mathematics (Brady & Bowd, 2005; Ma, 1999; Seegers & Boekaerts, 1996; Steele & Arth, 1998). Mathematics anxiety reduces achievement by limiting students' working memory capacity (Ashcraft & Kirk, 2001). Faust, Ashcraft, and Fleck (1996) found that subjects with high mathematics anxiety took longer to complete arithmetic problems that required regrouping. Their study showed that mathematics anxiety interferes with students' working memory, and that interference may cause further increase in mathematics anxiety.

High mathematics anxiety has also been correlated with students' avoidance of mathematics and taking fewer mathematics courses (Brady & Bowd, 2005; Hembree, 1990; Ma, 1999). Additionally, Scholfield (1981) found negative teacher attitudes toward mathematics correlated with the teachers having high mathematics anxiety.

These negative effects create a self-fulfilling prophecy for students of mathematics in which their anxiety has negative effects on their performance, attitude, and engagement with the discipline of mathematics and in turn, those negative effects become the source of continued and increased mathematics anxiety. Thus, decreasing mathematics anxiety is an important topic for educators of all students, but even more important for educators of preservice teachers that will one day teach mathematics to future generations.

Mathematics Anxiety and Preservice Elementary Teachers

Gender differences in mathematics anxiety are an important consideration when studying preservice teachers because elementary teaching is a profession in which females are the majority. Early research on mathematics anxiety found that gender differences existed between female and male students and that females had higher levels of mathematics anxiety (Hembree, 1990; Zettle & Houghton, 1998). However, these differences do not appear until upper elementary and middle school (Levine, 1995; Moulds, 1986; Tobias, 1993). Explanations for the gender differences have been the perception of mathematics as a male domain (Tobias & Weissbrod, 1980), lower perceived levels of mathematics ability in females (Meece, Wigfield, & Eccles, 1990), and the fear that females will be unpopular if they are successful in mathematics (Tobias & Weissbrod, 1980). Although gender differences have been found, research has found differences in gender and mathematics anxiety to be insignificant (Felson & Trudeau, 1991; Hyde, Fennema, Ryan, Frost, & Hopp, 1990; Wood, 1988).

Many university students have high levels of mathematics anxiety (Iossi, 2007; Lazarus, 1974; Malinsky et al., 2006). Preservice elementary teachers have the highest occurrence of mathematics anxiety among university students (Battista, 1986; Burton, 1979; Bursal & Paznokas, 2006; Gresham, 2007, 2008; Harper & Daane, 1998; Haylock, 2001; Kelly & Tomhave, 1985; Lazarus, 1974; Trujillo & Hadfield, 1999; Wood, 1988).

A significant amount of research has found that teachers with high levels of mathematics anxiety can pass their anxieties on to their students (Bulmahn & Young, 1982; Hembree, 1990; Kelly & Tomhave, 1985; Larson, 1983; Lazarus, 1974; Martinez, 1987; Sloan, Daane, & Geisen, 2002; Tobias, 1981; Vinson, 2001). Thus, many researchers have doubts about the effectiveness of teachers with high mathematics anxiety (Ball, 1990; Bush, 1989; Hembree, 1990; Kelly & Tomhave, 1985; Larson, 1983; Sovchik, Meconi, & Steiner, 1981; Teague & Austin-Martin, 1981). Teachers that display a high level of mathematics anxiety have been found to use more traditional teaching methods that concentrate on skills, the same type of instruction that has been given credit for fostering mathematics anxiety for many elementary teachers (Trujillo & Hadfield, 1999).

Elementary teachers should have lower mathematics anxiety and better attitudes toward mathematics than other people because they have a responsibility to create an excitement for mathematics in their students. They will be unable to create such an environment if they have high levels of mathematics anxiety (Wood, 1988).

Educators of preservice teachers have the responsibility to reduce preservice elementary teachers' high levels of mathematics anxiety (Wood, 1988). Mural and Lynda (2006) recommended that courses should be designed in ways that all preservice teachers can lower their mathematics anxiety and improve their attitudes toward mathematics. There are many examples of teaching interventions that have been successful in reducing preservice elementary teachers' mathematics anxiety. Some of them are discussed in the following section.

Reducing Mathematics Anxiety in Preservice Teachers

Researchers have called for studies to determine interventions that will reduce preservice elementary teachers' mathematics anxiety (Hembree, 1990; Ma, 1999). Mural and Lynda (2006) found that preservice teachers with lower mathematics anxiety had higher confidence to teach mathematics. Similar findings have been found by others, which provide hope that lowering mathematics anxiety will have positive effects in other areas (Harper & Daane, 1998; Hembree, 1990; Sloan et al., 2002). Harper and Daane (1998) found that mathematics courses can have a significant impact on preservice teachers' ability to cope with their mathematics anxiety.

Many attempts have been made to reduce preservice teachers' mathematics anxiety through classroom interventions. The successful attempts have used mathematics instruction that focuses on conceptual knowledge before procedural knowledge, authentic activities to make mathematics more meaningful, students' active participation in their learning, open class discussions, student journal reflections, and manipulatives (Gresham, 2007).

Alsup (2004) used problem solving and a constructivist teaching style to teach mathematics content to preservice elementary teachers. The preservice elementary teachers' mathematics anxiety was significantly reduced (Alsup, 2004). Other researchers have called for a shift in teaching approaches that focus less on memorization and more on understanding (Miller & Mitchell, 1994), constructivist pedagogy (Tobias, 1993), the use of manipulatives (Vinson, 2001), and small-group learning (Bono, 1991).

Using a combination of field experiences, lecture, discussion, and small-group learning activities in a mathematics methods course for preservice elementary teachers resulted in a decline in their mathematics anxiety (Battista, 1986). Similar instructional interventions have resulted in reduced mathematics anxiety. Dodd (1992) found that the use of small-group learning reduced the mathematics anxiety of the participants. Sovchik et al. (1981) found the use of an active learning approach in a mathematics methods course for preservice elementary teachers resulted in reduced mathematics anxiety from pre- to posttest.

Other studies have found successful psychological interventions to lower mathematics anxiety and consequently improve mathematics performance (Hebert & Fumer, 1997; Hembree, 1990; Zyl & Lohr, 1994). Wadlington, Austin, and Bitner (1992) were successful in lowering the mathematics anxiety of preservice elementary teachers using desensitization techniques.

There is a lack of studies that investigate the relationship between students' mathematics anxiety and learning mathematics through PBL. Many successful instructional efforts to reduce mathematics anxiety have used problem solving, problem posing, and small group learning (Alsup, 2004; Battista, 1986). Learning through PBL includes these effective instructional methods, and it is likely that PBL could be effective at reducing mathematics anxiety.

Attitudes Toward Mathematics

Definitions

Attitudes toward mathematics have been a topic of interest to educational researchers since the 1970's (Hannula, 2002; McLeod, 1994). Aiken (1970) considered attitude to be a learned response to an object. Similar to mathematics anxiety, attitude is often the result of students' past experiences (Allport, 1935) and is fairly stable (McLeod, 1992). Attitude consists of beliefs, behavior, and emotions (Hannula, 2002). Enjoyment and value of mathematics are two aspects of attitudes toward mathematics (Aiken, 1974). Others have considered students' confidence about learning mathematics to be an aspect of attitudes toward mathematics (Fennema & Sherman, 1976). A simple definition for attitude toward mathematics is the "emotional disposition toward mathematics" (Hannula, 2002, p. 26). Ma and Kishor's (1997) definition of attitudes toward mathematics included students' affective responses to the importance or unimportance of mathematics. Hannula (2002) described attitudes toward mathematics as emotions, associations, expectations, and values about mathematics. Hodges and Kim (2013) defined attitudes toward mathematics the way one reacts to mathematics. Tapia and Marsh (2004), creators of the ATMI, considered attitudes toward mathematics to be composed of values, enjoyment, motivation, and self-confidence. Those affective characteristics are areas of attitude that have been found to be related to mathematics achievement (Samuelsson & Granstrom, 2007).

Effects of Poor Attitudes Toward Mathematics

Attitudes toward mathematics are important to study because they are often related to achievement (Hannula, 2002). Poor attitudes toward mathematics are positively correlated with low self-confidence and are demonstrated by a consistent lack of success in mathematics (Tobias, 1978). Many college students have developed negative attitudes toward mathematics (Walmsley, 2000), and relationships between attitudes toward mathematics and achievement in college mathematics courses have been documented (Ma & Kishor, 1997).

Tapia and Marsh (2000) administered the ATMI to 545 students and found that self-confidence and grades in mathematics classes were related. They also found that the highest grades were achieved by students with the highest self-confidence and students that made failing grades had the lowest self-confidence and motivation.

Thorndike-Christ (1991) illustrated the benefits of a positive attitude toward mathematics. They found that students with a positive attitude toward mathematics were more likely to choose mathematics related careers, and students' attitudes toward mathematics predicted final grades.

Evaluating preservice elementary teachers' attitudes toward mathematics is important because their classroom environment, closely related to their own attitudes toward mathematics, will influence the way their students view mathematics (Carter & Norwood, 1997; Ford, 1994; Garofalo, 1989). Additionally, problem-solving instruction depends on the beliefs and attitudes of the teacher (Woods, 1989).

Some research on attitudes toward mathematics has focused on gender differences (Hyde et al., 1990; Sherman, 1979). Gender differences are prevalent for students in high school and college, and females have been found to have more negative attitudes toward mathematics than males (Hyde et al., 1990). Considering that preservice elementary teachers are predominantly female, studying and attempting to improve their attitudes toward mathematics is needed. Wadlington et al. (1983) showed through students' statements about their previous mathematics teachers that teachers' attitudes influence students' attitudes toward mathematics, providing further motivation to study and improve future teachers' attitudes toward mathematics prior to their entering the classroom.

Attitudes toward mathematics and mathematics anxiety are two constructs that are negatively correlated. Hembree's (1990) meta-analysis on mathematics anxiety found that students with positive attitudes toward mathematics had lower mathematics anxiety. Bessant (1995) made similar statements about the relationship between mathematics anxiety and attitudes toward mathematics. Negative attitudes have also been associated with high levels of mathematics anxiety among preservice elementary teachers (Alkhateeb & Taha, 2002; Teague & Austin-Martin, 1981).

Philippou and Christou (1998) provided evidence that attitudes can be influenced in their study of preservice teachers during a three-year program to teach mathematics concepts and methodology. They found that the preservice teachers began the study with negative attitudes toward mathematics. Mathematics preparation courses give educators an opportunity to positively influence students' attitudes toward mathematics. However, students' attitudes toward mathematics are difficult to change because they are influenced by prior experiences in mathematics and are stable once formed (Hannula, 2002; McLeod, 1994). There have been many attempts to improve attitudes toward mathematics by altering classroom instruction, but many have been unsuccessful (McLeod, 1994). Many unsuccessful interventions have used a conceptualization of attitude that is too simplistic (McLeod, 1994; Hannula, 2002). However, student-centered instructional methods have been effective at improving attitudes toward mathematics (Boaler, 1997a, 1997b, 1998; Ridlon, 1999). Instructional changes focusing on increasing motivation have had a positive influence on students' attitudes toward mathematics (Hodges & Kim, 2013). Phelps (2010) found that improving preservice teachers' self-efficacy and motivation improved their achievement. Additionally, Dweck (2007) argued that improving students' confidence can be achieved by developing the skills of students that will help them persevere to achievement.

Schwartz (2006) suggested that students, when engaging in problem solving, be able to persevere through problems and be comfortable making mistakes and asking questions. Students' ability to persevere through problems is influenced by their self-concept and feelings about mathematics (Schoenfeld, 1985). Perseverance can be strengthened by giving students opportunities to struggle with problems (Schoenfeld, 1985). Many researchers have recommended studying students' attitudes toward mathematics and other affective measures while they are solving non-routine problems, providing support for this study in which students' attitudes and mathematics anxiety were studied while they learned mathematics through PBL and problem solving (Hannula, 2002; McLeod, 1988).

The Mathematical Preparation of Preservice Elementary Teachers

Relearning Mathematics

In *Relearning Mathematics: A Challenge for Prospective Elementary School Teachers,* Zazkis (2011) wrote about her experiences teaching number concepts to preservice elementary teachers. One of Zazkis' main arguments was that preservice elementary teachers are not learning mathematics for the first time in these courses, but they are relearning mathematics. Relearning presents some particular challenges because the preservice teachers bring with them various preconceptions and misconceptions that were constructed long ago (Zazkis, 2011). Teacher educators must re-structure preservice elementary teachers' mathematics knowledge by providing opportunities for their incomplete understandings and false beliefs to be challenged (Zazkis, 2011).

Preservice Elementary Teachers' Beliefs and Understandings

The past experiences, beliefs, and mathematical understandings of preservice elementary teachers do not match well with what will be demanded of them as teachers (Ball, 1990; CBMS, 2010). Many preservice elementary teachers have an understanding of mathematics that is limited to memorized rules and lacks understanding of the procedures that they teach (Ball, 1990; Ma, 1999). Additionally, they have trouble producing problems that illustrate the concepts they will be expected to teach (Ball, 1990; Ma, 1999). Preservice elementary teachers may have the following misconceptions about their mathematics preparation (CBMS, 2010, p. 10).

1. They learned all the mathematics they need to know during elementary, middle, and high school.
2. Doing mathematics means following the rules given by the teacher.
3. Knowing mathematics means remembering and applying the correct rule when the teacher asks a question.
4. Mathematical truth is determined by the teacher.

The past experiences of many preservice elementary teachers learning mathematics focused primarily on learning procedures. Doing mathematics, according to the Common Core Standards for Mathematical Practice (CCSSI, 2010), may not be in their catalog of experiences.

Mathematics Instruction for Preservice Teachers

It is unlikely that preservice teachers will be able to create classroom environments that foster a healthy view of mathematics unless their mathematics content and methods courses address the large array of false beliefs and misunderstandings of the discipline of mathematics. Cotic and Zuljan (2009) said, "What is lacking from the process cannot be present in the result" (p. 297). Ball (1990) concluded that preservice elementary teachers' precollege mathematics is not enough for preservice elementary teachers to develop the knowledge needed to teach mathematics. Even students with beliefs that align with reform-oriented mathematics are anxious about teaching due to a lack of examples of what mathematics classes should look like (Yazici, Peker, Ertekin, & Dilmac, 2011).

Preservice teachers' views about mathematics and its teaching are influenced by their own learning experiences in mathematics (Grossman, Wilson, & Shulman, 1989; McGowen & Davis, 2001). Teachers must have experiences learning mathematics in classroom environments that require them to communicate, collaborate, reason, make sense of the mathematics they will teach, and develop habits of mind for them to be prepared to use instructional methods that align with current educational reform recommendations (CBMS, 2001, 2010; NCTM, 2000). Courses for preservice elementary teachers should help them learn more than procedures, make connections between concepts, give them an opportunity to validate their own answers, have opportunities to make conjectures, engage in mathematical justification, interpret and appraise others' ideas, and challenge incorrect ideas (Ball, 1990).

The Use of Reflective Writing Prompts

In the 80's the use of writing for learning began to increase (Miller, 1992). Many educational researchers became convinced of the benefits of writing in education across all subject areas (Bangert-Drowns, Hurley, & Wilkinson, 2004). Research on the effectiveness of the use of writing to learn has found mixed results (Miller, 1992). A meta-analysis conducted in 2004 found that writing to learn is effective when students are given prompts that require them to reflect on their current knowledge, learning processes, confusions, and feelings about a topic (Bangert-Drowns et al., 2004). Writing prompts are more effective when they are short in length, do not use complicated language, and are given over a period of time (Bangert-Drowns et al., 2004).

Research also shows that teachers benefit from reading students' responses to writing prompts (Miller, 1992). Writing prompts give teachers information about the understandings of their students, change teachers' instructional practices, and improve the attitudes of the students and teachers toward each other (Miller, 1992). Additionally, there is evidence that the information gained from students' writing prompts is not usually shared by the students in whole class discussions (Miller, 1992).

This dissertation study used participants' responses to reflective writing prompts as a form of qualitative data to investigate preservice elementary teachers learning mathematics through PBL and problem solving, to examine changes in their mathematics anxiety and attitudes toward mathematics, and to understand their experiences learning in collaborative groups while working through a PBL lesson. The preceding discussion indicates that the participant in this study may have constructed knowledge by engaging in reflective writing prompts. Thus, in this study writing prompts can be considered both a form of data and part of the instructional design.

Summary

PBL is not a new instructional strategy, but the use of PBL in mathematics content classes for preservice elementary teachers is new. Problem solving has a long history of use in mathematics classrooms and has evolved to be an instructional strategy on its own (NCTM, 2003). Problem posing is a fundamental activity in mathematics; however, many students have only had the opportunity to solve problems that have been posed by the teacher or a textbook (Silver, 2013). These three instructional strategies, (i.e., PBL, teaching through problem solving, and problem posing) have many similarities in form and theoretical background (Jonassen, 201; Schmidt, 1993; Silver, 2013). The design of the PBL lessons used in this study created opportunities for participants to be engaged in problem solving and problem posing.

Preservice elementary teachers are known to have high levels of mathematics anxiety and poor attitudes toward mathematics (Hembree, 1990). Anxiety and poor attitudes can inhibit learning and affect teaching practices (Hembree, 1990; Ma, 1999). There is evidence that PBL, teaching through problem solving, and problem posing are instructional strategies that can reduce mathematics anxiety and improve attitudes toward mathematics (Brown & Walter, 2005; Ridlon, 2009).

In conclusion, PBL instruction for the mathematics preparation of preservice elementary teachers has the potential to positively influence their construction of content knowledge, mathematics anxiety, attitudes toward mathematics, give them the opportunity to learn mathematics in the ways that they will be expected to teach, and better prepare them to become elementary teachers who can implement mathematics instruction according to the CCSSM (CCSSI, 2010). The next chapter explains the methodology for this mixed-methods dissertation study.

CHAPTER III

THE METHODOLOGY

Chapter I provided a brief report of two pilot studies on the use of PBL to teach mathematics to preservice elementary teachers. The type of data collected in the pilot studies and the type of data analyses conducted in the pilot studies were insufficient to determine how the preservice teachers were learning mathematics through PBL, their perceptions about learning mathematics through PBL, and their perceptions of the factors of learning mathematics through PBL affecting their mathematics anxiety and attitudes toward mathematics. Additionally, results from the pilot studies gave mixed results on the improvement in students' attitudes toward mathematics and showed no comparative differences in mathematics anxiety. Thus, the research questions for this study sought to provide explanations for the results from the pilot studies. This study had the following qualitative research questions.

1. How do preservice elementary teachers learn mathematics through PBL?
2. What features of PBL do preservice elementary teachers perceive as contributing to their learning of mathematics?
3. What features of PBL do preservice elementary teachers perceive as influencing their mathematics anxiety?
4. What features of PBL do preservice elementary teachers perceive as influencing their attitudes toward mathematics?

This study had the following quantitative research questions.

1. Do preservice elementary teachers learning mathematics through PBL improve their mathematics content knowledge more than preservice elementary teachers taught using a teacher-centered approach or a mixture of methods approach?
2. Do preservice elementary teachers learning mathematics through PBL decrease their mathematics anxiety more than preservice elementary teachers taught using a teacher-centered approach or a mixture of methods approach?
3. Do preservice elementary teachers learning mathematics through PBL improve their attitudes toward mathematics more than preservice elementary teachers taught using a teacher-centered approach or a mixture of methods approach?

Prior to conducting the pilot studies, the researcher had no experience teaching mathematics using PBL nor did the researcher have any experiences learning mathematics through PBL. Thus, the pilot studies gave the researcher the opportunity to gain experience teaching mathematics through PBL and create many of the materials that were used in this study. Additionally, the researcher had the opportunity to make corrections and improvements to the materials used in this study to teach mathematics through PBL.

Hence, the two pilot studies were very influential to the researcher in making decisions about the methodology used in this study. The design of the class used to teach preservice elementary teachers mathematics through PBL in this study, the research questions, the types of data collected, and the analyses of the data are decisions that the researcher made based on his experiences participating in the pilot studies. This chapter explains the methods used in conducting this study.

The Research Perspective

The study was a qualitative-quantitative, mixed-methods study. Grounded theory data analyses were completed on the qualitative data to investigate preservice elementary teachers learning mathematics through PBL (Strauss & Corbin, 1990). Comparative analyses were conducted between an experimental group and two control groups to determine the benefits of teaching mathematics to preservice elementary teachers through PBL and problem solving.

The experimental group learned mathematics through PBL and problem solving. One control group was taught mathematics using teacher-centered methods of instruction, and the second control group was taught mathematics by using both teacher-centered and student-centered methods of instruction.

The Design of a PBL Cycle

There were a total of six PBL cycles completed in this study. Each PBL cycle lasted for two to three weeks of class. The class meetings were on Tuesday and Thursday and lasted for one hour and twenty-five minutes. Four of the PBL scenarios used in this study were created by the researcher and the co-investigator of the pilot studies, L. D. Miller, during the summer and fall of 2011. The scenarios were refined after first implementation in fall of 2011 and again after their second implementation during spring of 2012. Two video-based PBL scenarios from the Jasper Woodbury Series, produced at

Peabody College at Vanderbilt University, were used in the study (Cognition and Technology Group, 1997). A display of the PBL cycle used in this study is given below in Figure 1.

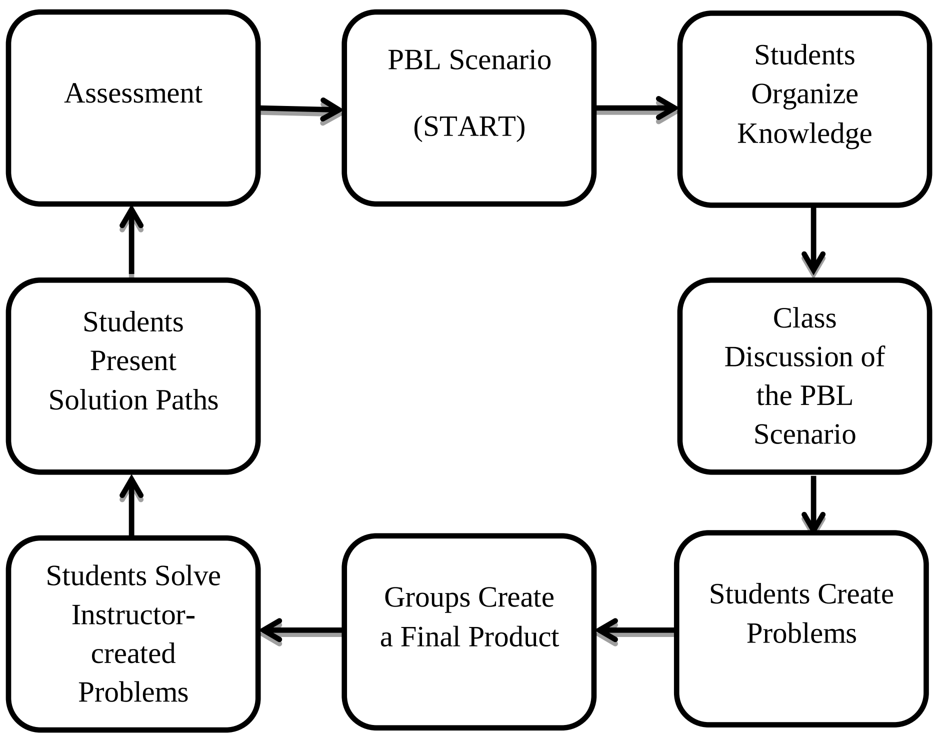

Figure 1. The Problem-based Learning Cycle Used in this Study. This cycle was developed by B. C. Banes and L. D. Miller, and is similar to the cycle developed by the Cognition and Technology Group at Vanderbilt (CGTV, 1997).

The PBL Scenario

The PBL cycle began with a PBL scenario being given to students in collaborative groups. The PBL scenario is in story form. The scenarios are real events or realistic events. The scenario is written about events that students will find interesting and relevant. This provides intrinsic motivation for the students to investigate the problem.

Students Organize Knowledge

The students begin to organize their knowledge by discussing the PBL scenario in their collaborative groups. The students discuss the information that is given in the problem (what they know), the information that is missing from the problem (what they need to know), and the resources that could provide them with the information needed (where to get it).

Class Discussion of the PBL Scenario

Following the discussion of the PBL scenario in collaborative groups, the groups would share their thoughts and questions about the PBL scenario with the whole class. The class discussion of the scenario allows students to continue to think about the scenario as they hear different perspectives.

Students Create Problems

The students, using the previous discussions about the PBL scenario, are asked to create and solve their own story problems related to the PBL scenario. The students are told that their created problems and solutions will be part of a group product that will be submitted for a grade. Appendix G provides the scoring rubric used to score student-created problems.

Groups Create a Final Product

Each group was given the responsibility of determining the created problems that would be included in their final group product. Every group member solved their group mates' created problems to determine which problems to include in the group product and to ensure that the solution given in the final group product was correct. An individual component to the group product was included for each PBL scenario. The individual component was a writing assignment that varied from scenario to scenario.

Students Solve Instructor-created Problems

The instructor created non-routine problems related to the PBL scenario that the students would solve in their collaborative groups after the completion of their group product. The instructor-created problems were developed to ensure that students would engage in the mathematics content objectives for the course and to develop the students' problem-solving skills.

Students Present Solution Paths

The students presented their solution paths to the class once all groups had completed the instructor-created problems. When there existed multiple solution paths, each solution path was presented to the class.

Assessment

After the conclusion of students' presentations of solutions, the students were given a three to four item quiz containing non-routine problems related to the PBL scenario. The students solved the problems individually and in class. This served as an assessment of individual student learning and was the final stage of the PBL cycle used in this study.

The Role of the Instructor

The instructor played the role of facilitator during each PBL cycle. While the students were creating and solving problems, he circulated around the classroom listening to the groups' discussions and helping them think about their problems. At times the instructor gave a mini-lecture when he noticed the groups struggling with the same concept. The mini-lectures were short in duration. During student presentations of solutions, the instructor observed, emphasized multiple solution paths by selecting different students to present their solution paths, asked questions to prompt the students to think further about a problem, and provided additional explanations about some of the mathematics content involved in the problems.

Creation of the PBL Cycle Used in this Study

The researcher and the co-researcher for the pilot studies, L. D. Miller, attended a PBL workshop at MTSU in the summer 2011. The workshop was led by Dr. Mark A. Serva from the University of Delaware. Two of the PBL scenarios used in this study were first created by the researcher and L. D. Miller at the workshop. The remaining PBL scenarios were developed after the PBL workshop.

The researcher and the co-investigator of the pilot studies identified all of the content objectives for the course titled "Concepts and Structure of Elementary School Mathematics." Then, PBL scenarios were written to align with the content objectives. The PBL scenarios were developed by identifying real events that the students would potentially find interesting. To help ensure that the mathematics content objectives were encountered by the students, the researcher and the co-investigator of the pilot studies decided to include non-routine instructor-created problems in the PBL cycle. Thus, this PBL cycle is referred to as PBL and problem solving to acknowledge that the students solve non-routine problems that are given to them by the instructor.

To verify the implementation of PBL described above, an independent reviewer examined a selection of recorded class meetings. The independent reviewer was an associate professor in the College of Education at Lipscomb University and director of the Master of Education programs in English Language Learners, Reading, Instructional Technology, Collaborative Professional Learning, and Teaching, Learning and Leading. The recordings examined by the independent reviewer were selected so that each component of the PBL process used in this study could be observed. The reviewer confirmed that the implementation of PBL described above is accurate and did not find any deviations from this description.

The Research Context

The study was conducted at Middle Tennessee State University (MTSU) during the fall 2012 semester. Data collection began on August 27th and concluded on December 13th. MTSU is located in Murfreesboro, Tennessee, is a member of the

Tennessee Board of Regents, and is accredited by the Commission on Colleges of the Southern Association of College and Schools (SACS) to award baccalaureate, masters, and doctorate degrees (Office of Institutional Effectiveness, Planning, and Research, 2011). MTSU has a diverse student body consisting of students from Tennessee, students from around the United States, and many international students from a variety of foreign nations (Office of Institutional Effectiveness, Planning, and Research, 2011). The student body is 54% female and 12% minority. Student enrollment at MTSU is more than 26,000, and the University employs more than 900 faculty positions (Office of Institutional Effectiveness, Planning, and Research, 2011).

The Research Participants

The participants in the study were preservice elementary teachers enrolled in a course titled "Concepts and Structure of Elementary School Mathematics," the first of two mathematics courses required at MTSU for elementary education majors. The content of the course is problem-solving strategies, set theory, functions, number theory, and examinations of number systems from counting numbers to irrational numbers. The participants completed their general education mathematics requirement prior to this study because the number concepts course used in this study does not satisfy the participants' general education mathematics requirement.

The group that learned mathematics through PBL and problem solving consisted of a single section of the course. The meeting times for this course were 11:20 a.m. until 12:45 p.m. on Tuesday and Thursday. The group began with 40 participants. There were

36 (90%) females. Five (12.5%) of the females were African American and 31 (77.5%) of the females were Caucasian. There were four (10%) males. One (2.5%) of the males was African American and three (7.5%) of the males were Caucasian. The average age for participants in this group was 23 years old.

The group that learned mathematics through teacher-centered methods consisted of a single section of the course. The meeting times for this course were 8:00 a.m. until 9:25 a.m. on Tuesday and Thursday. The group began with 28 participants. There were 26 (93%) females. Three (11%) of the females were African American, one (3.5%) of the females was Asian, and 22 (78.5%) of the females were Caucasian. There were two (7%) Caucasian male participants. The average age for participants in this group was 22 years.

The group that learned mathematics through both teacher-centered and student-centered methods consisted of two sections of the course taught by the same instructor. The meeting times for one of the sections were 8:00 a.m. until 8:55 a.m. on Monday, Wednesday, and Friday. The meeting times for the other section were 12:40 p.m. until 1:35 p.m. on Monday, Wednesday, and Friday. Both sections consisted of 19 participants. One section consisted of only females. Three (16%) were African American and 16 (84%) were Caucasian. The other section consisted of 18 (95%) Caucasian females and one (5%) Caucasian male. The average age for participants in this control group was 22.

Instructors of the Control Groups

The instructor of the control group learning mathematics through teacher-centered methods was a tenured, associate professor of mathematics with 36 years of teaching experience. He had a Ph.D. in Mathematics Education from the University of Tennessee. The instructor of the control group learning mathematics through both teacher-centered and student-centered methods was a graduate student at MTSU. This instructor had taught middle school mathematics for one year, and six years of postsecondary mathematics classes. He had a Bachelor of Science in Mathematics, Master of Science in Teaching Mathematics, and an Ed.S. in Curriculum and Instruction. All of his degrees were from MTSU. He was working toward finishing a doctoral program in mathematics and science education at MTSU during this study.

Both instructors of the control groups had previous experience teaching the number concepts course used in the study. However, the instructor using teacher-centered methods had more experience teaching the course than the instructor of the second control group.

The Instruction of the Control Groups

The control groups were observed throughout the semester and their course materials were collected. The instruction that was observed from the teacher-centered control group one was routine and predictable. The instructor would walk in the classroom and write the objectives on the board, call the roll, and go over the answers to the previous homework to begin the class. The students were quiet when going over the homework. They spoke when the instructor asked for an answer and when a student did not get an answer correct an explanation was given by the instructor. The instructor performed all of the calculations and solved all of the problems during the homework review. Following the homework review, the instructor presented a lesson and the students took notes. Homework was assigned at the conclusion of the instructor's presentation. All the assignments were from the textbook and the tests consisted of problems very similar to the homework problems.

The instruction observed from control group two (i.e., the group learning through both teacher-centered and student-centered methods) varied. The instructor lectured some, however, he engaged the students in discussion during the lesson. The discussion that the instructor had with his students was focused on understanding and less about the answer. The students were free to speak during the lecture and ask questions. The instructor did more than lecture. The students worked in groups with manipulatives, solved non-routine problems in groups, presented solutions to the class, and discussed various ways to represent concepts. The assignments varied too. The instructor used an online homework system that is aligned with the textbook for the class. He also gave the students projects that required them to write about a concept and present their results to the class. The tests for this class consisted of problems similar to the homework as well as problems that asked students to represent ideas in different ways, explain concepts, and relate the mathematics they were learning to their future as an elementary teacher.

The Researcher-Instructor

The researcher in the study was also the instructor for the group of preservice elementary teachers learning mathematics through PBL and problem solving (experimental group) and thus was a participant-observer in the study (Creswell, 2007). This is a role that can be viewed as having advantages and limitations.

Qualitative researchers study phenomena in their natural settings; however, the researcher as instructor was recognized as a potential cause for concern. The researcher as instructor increased the chance of bias and could have resulted in data reported by the participants that was untrue. To help ensure the validity of the research, qualitative data triangulation was used to give the researcher multiple views of the phenomenon being studied, and multiple types of data were collected to add richness and depth to the study (Denzin & Lincoln, 2011).

Another issue with being both the instructor and the researcher was that the research design included participant interviews. There was potential for bias and for participants to avoid honest answers for fear of an adverse effect on their grade or fear of some resulting negative classroom event. In order to avoid ethical concerns and bias, the researcher-instructor did not conduct any of the participant interviews. Instead, interviews were conducted by two people unrelated to the research project. The researcher denied himself access to the interview data until after the fall 2012 semester ended and grades were submitted.

Two undergraduate students were recommended by faculty at MTSU to conduct the participant interviews. One of the interviewers was a psychology major, and the other student was a mathematics education major. The psychology major conducted the majority of the interviews. The researcher designed the interview protocols and then gave them to the interviewers. Although the issue of bias and ethics was resolved, another issue arose. Specifically, the psychology major interviewer did not have the knowledge about the study or knowledge about the mathematics involved in the class to ask participants to clarify statements or to follow up a participant's response with questions that were not on the interview protocol. Thus, some of the interview data was lacking and was less helpful to the researcher in discovering how the participants were learning mathematics through PBL.

Researcher journal writings, notes, transcribed interviews, transcribed group interactions, writing prompts, and participant work were used to triangulate the data, make the process of learning mathematics through PBL visible, and to explain that process in a way that was representative of the data collected (Denzin & Lincoln, 2008; Strauss & Corbin, 1990).

Being the researcher-instructor provided the researcher with a unique view of the participants as they learned mathematics through PBL. The researcher was able to observe the participants, in the moment, while they learned and attempted to learn mathematics through PBL. The researcher wrote notes after class meetings which informed many of the emergent choices involved in the qualitative component of the research design. The researcher made decisions about the content of writing prompts, participants selected to be interviewed, and groups chosen to be recorded during class based in part on his research notes and experiences as the instructor of the class.

Data Collected

The data collected in this study reflect the qualitative and quantitative design of this study. Both qualitative and quantitative data were collected from the experimental group and control groups. All data were collected to help answer the research questions. Qualitative data collected from the experimental group included researcher journal entries, notes, reflective writing prompts, transcriptions of recorded class meetings, transcribed participant interviews, and participant work. Qualitative data collected from the control groups included the course syllabi, course assignments, field notes from class observations, and audio- and video-taped class meetings.

Pretest and posttest data were collected from both control groups and the experimental group. The researcher-designed content knowledge test, MARS (Suinn & Winston, 2003), and ATMI (Tapia & Marsh, 1996) were given as pretest and posttests.

Instruments

The researcher collected all of the qualitative data with the exception of the participant interviews. Thus, the researcher is an instrument in this study (Creswell, 2007). The researcher's qualifications include two years teaching high school mathematics, six years teaching postsecondary mathematics at two universities, he has his Bachelor of Science in Mathematics and Mathematics Education from Lipscomb University, and a Master of Science in Mathematics from Middle Tennessee State University. Furthermore, he had completed all coursework to obtain a Doctor of Philosophy in Mathematics and Science Education, which included both qualitative and quantitative research methods. Finally, he had conducted two pilot studies investigating preservice elementary teachers learning mathematics through PBL and problem solving.

Writing prompts and interview protocols (see Appendix C and Appendix D) were used to collect data in an attempt to answer the qualitative research questions. These instruments were created based on previous data that included the participants' work, writing prompts, and researcher notes. Two undergraduate students administered the participant interviews. They are described earlier in this chapter.

The mathematics content knowledge test consisted of 40 items (see Appendix E). The items were all open response. The content knowledge test was constructed by collecting the most current final examinations from six instructors that teach the number concepts course at Middle Tennessee State University. The problems from these exams were sorted into the following categories: properties of number, bases and place value, number theory, rational numbers, algebra, and problem solving. The problems in these categories were further sorted by problem type so that problems reflecting the same concept were put in the same sub-category. Problems selected for the content knowledge test were based on the concepts that occurred most often in these categories. Other problems were included because the researcher decided they were quality problems that emphasized important number concepts from the course. After constructing the content knowledge test, the researcher sent it to all of the instructors of the number concepts course and asked them to review it to determine if it was representative of the content knowledge from the number concepts course and to contribute their suggestions for change. None of the instructors offered suggestions for change, four of the instructors affirmed that the mathematics content knowledge test was representative of the content of the number concepts course, and two instructors did not respond.

The Mathematics Anxiety Scale (MARS) is a 30 item Likert-scale survey that assesses mathematics anxiety (see Appendix A). A score of 30 indicates the lowest mathematics anxiety and a score of 150 indicates the highest mathematics anxiety. Scores above 75 indicate a mathematics anxiety level high enough to warrant an intervention to lower mathematics anxiety (Suinn & Winston, 2003). The MARS has been found to be both valid and reliable (Frank & Suinn, 1972).

The Attitudes Toward Mathematics Inventory (ATMI) is a 40 item Likert-scale survey that assesses attitudes toward mathematics (see Appendix B). A score of 40 indicates the poorest attitudes toward mathematics, a score of 120 indicates neutral attitudes toward mathematics, and a score of 200 indicates the most positive attitudes toward mathematics. The ATMI score is broken into four subscales. Self-confidence, value, enjoyment, and motivation are the four subscales for the ATMI. The scores for self-confidence range from 15 to 75, the scores for value range from 10 to 50, the scores for enjoyment range from 10 to 50, and the scores for motivation range from 5 to 25. The ATMI is both valid and reliable (Tapia & Marsh, 2004).

Procedures Used

Participant Selection

Participant selection and designation to group was determined by the class section in which the participant was enrolled. The instructors of each of the groups were assigned to their class section by the Chair of the Department of Mathematical Sciences.

Participants that registered during the previous spring semester did not know which instructors they would have because the instructor had not been assigned. Though the participant selection was non-random, there were random events that led to participant selection. It is acknowledged that participants that registered for the course after the instructor's names were posted would possibly register based on any of the instructors' reputations.

Pretest and Posttest Procedures

On the first day of class for the fall 2012 semester, the researcher-designed content knowledge test, MARS, and ATMI, were administered to all of the participants in the study. The pretests were administered to the experimental group and one of the control groups by the researcher. The instructor of the second control group administered the pretests to his class. The Arithmetic Proficiency Test (APT), an arithmetic test that preservice elementary teachers at MTSU are required to pass, was administered to the experimental group. The results of the APT (see Appendix H) were used by the researcher to form collaborative groups among the participants in the experimental group. This process is described below.

The experimental group and one of the control groups took the posttests on the day of the final exam for their respective classes. The second control group took their posttests on the final class meeting of the semester. The posttests were the researcher-designed content knowledge test, the MARS, and the ATMI.

Qualitative Procedures for the Control Groups

Both control groups were observed by the researcher throughout the semester to verify the description of the instructional methods used by the instructors of each control group. Field notes, course assignments, and course syllabi corroborate the descriptions of the instructional methods used in the control groups.

Researcher Journal and Notes

Throughout the duration of the study, the researcher, also instructor of the experimental group, kept a research journal and wrote notes after class meetings. These data were used to make decisions about the collection of new data for the duration of the study.

Creating the Collaborative Groups

The preservice elementary teachers that were learning mathematics through PBL and problem solving were put into collaborative groups after the pretests were given. The groups were mixed ability based on their pretest scores on the Arithmetic Proficiency Test (APT), an arithmetic test that all preservice elementary teachers at MTSU must pass to advance to the next mathematics course in their program of study. The participants' APT scores on were put into quartiles, and a participant was randomly selected from each quartile to form a group of four participants. Ten mixed-ability collaborative groups were created this way.

Recorded Class Meetings

Each class meeting was either video-taped or audio-taped. The recordings focused on a single group working collaboratively to create and solve problems from a

PBL scenario. Initially, different groups were recorded for variety. As data collection continued, the selection of groups to record was informed by emerging themes from the data.

Participant Work

Various forms of participant work were collected in the study. Group products from a PBL scenario, individual solutions to problems, quizzes, and tests were collected for the duration of the study and collected from all participants in the experimental group. Individual interview protocols were created based on participants' collected work.

Writing Prompts

Writing prompts were given as classroom opening activities, an end of the class exercise, and were included as part of a PBL product. The writing prompts were collected from all participants in the experimental group. Every writing prompt focused on a theme that the researcher believed to be affecting their learning mathematics through PBL. The researcher used initial data, participant work, and his researcher journal to develop the reflective writing prompts.

Interviews

Interview protocols were created by the researcher. The interviews were conducted by two undergraduate students at MTSU, as described earlier in this chapter. The initial interviews were general in nature and each successive interview became more directed around themes that were emerging from the data. Not all participants were interviewed. Some participants were not interviewed because of scheduling conflicts between the participant and the interviewers, some participant requested to not be interviewed, and some participants were not selected for interviews. The researcher created the interview protocols by examining the initial data and looking for themes that were developing.

Data Analysis

Data Organization

Strauss and Corbin (1990) described the excitement of collecting data and then the overwhelming feeling of a researcher who is analyzing this large amount of data. To begin making sense of the data, Strauss and Corbin (1990) suggested that the researcher first develop an organizational scheme that puts the data into a format that makes later analyses easier to conduct.

The qualitative data were organized by type of data and then in chronological order. All interviews and writing prompts were transcribed and put into tables to be analyzed. Video and audio recordings of the participants working in their collaborative groups were initially transcribed completely. The first ten recordings were fully transcribed, and field notes were written for the remaining six recordings. The decision of the researcher to transcribe the recordings selectively is supported in the literature on grounded theory (Strauss & Corbin, 1990). The transcriptions and field notes of the recorded groups were put into tables along with the other transcriptions. The quantitative data were scored, and the results were saved electronically.

Qualitative Data Analyses

The researcher approached the data analyses with the intention of producing an explanation, true to the data collected, which allowed the relevant characteristics of learning mathematics through PBL to be illuminated, as is the purpose of grounded theory (Strauss & Corbin, 1990).

When data were collected and organized, the researcher began the coding process described by many grounded theory researchers (Charmaz, 2012; Strauss & Corbin, 1990). Open coding was the first level of analyses conducted (see Appendix I for a list of the codes from each stage of coding). Line-by-line open coding was performed on all of the class and interview transcriptions and writing prompts. These initial codes allowed the researcher to restructure the data in a way to think analytically about the data. As recommended by Charmaz (2012), the researcher wrote memos about the codes and developed them throughout the data analyses. A second level of coding was performed once all the data had been initially coded. During this level of coding, the researcher began to compare the open codes to the data and condense the number of codes by consolidating similar codes (Charmaz, 2012). Following the second round of coding, there remained an abundance of codes. The researcher decided to continue to compare the codes with one another and with the data to further consolidate the number of codes. The codes that remained after the third round of coding were then grouped into the main categories that are reported in Chapter 4.

Quantitative Data Analyses

The quantitative data, consisting of the pretest and posttests for the mathematics content knowledge test, MARS, and ATMI, were scored and stored electronically without any identifying information recorded. In addition to the pretest and posttests, a change score was created for each of the three instruments. The change score was calculated by subtracting each participant's pretest score from their posttest score.

Descriptive statistics were calculated for all pretests, posttests, change scores, and the subscales of the ATMI. Comparative analyses were conducted to determine any significant differences between the groups' mean pretest scores. Comparative analyses were also conducted on all change scores to determine any significant differences between the groups' mean change scores.

One-Way ANOVAs were used to compare the groups' means when assumptions for a One-Way ANOVA were met. The Sharpiro-Wilk test was conducted to check the assumption of normality. The non-normal data were analyzed using the nonparametric Kruskal-Wallis test to compare the groups' medians. Levene's Test was used to check the assumption of homogeneity of variances. Welch's *F*-Test was conducted to compare the groups' mean scores when the homogeneity assumption was not met.

The alpha value used for analyses of the pretests, the Shapiro-Wilk test, and Levine's test was .05. The researcher decided, a priori, to use a conservative alpha-value for all comparisons that involved a change score because there were multiple comparisons of the groups. The conservative alpha-value for the One-Way ANOVA, Welch's *F*-Test, and the Kruskal-Wallis test was calculated by dividing .05 by three which resulted in an alpha-value of .0167. The post hoc group comparisons used an alpha-value that was computed by dividing .0167 by three which resulted in an alpha-value of .0056.

Participant selection, as described earlier in this chapter, was non-random. However, this study used four of the six sections of the number concepts course offered during the fall 2012 semester, and the demographics of the participants in this study were similar from group to group. Their common demographics were 90% or more were female, 10% or less were male, 80% or more were Caucasian, and 16% or less were African American. These common demographics of the three control groups are shared with all of the sections of the course offered during fall 2012. Additionally, the five sections of the course being offered during fall 2013 also have similar demographics. The overall demographics for the fall 2013 sections of the course are 85% female, 15% male, 79% Caucasian, and 20% African American. The sections of the course used in this study may be considered representative, in gender and race, of the population of preservice elementary teachers taking the number concepts course at MTSU and the use of inferential statistics may be used (Smith, 1983).

Ethical Issues

Prior to beginning the study, approval for conducting the study was obtained from the institutional review board (IRB). All participants signed consent forms prior to data collection. Participants were informed by the researcher that they could withdraw their consent at any time without penalty. Furthermore, the participants were informed that not providing consent would have no effect on their course grade. Participants interviewed were given an interview consent form. Participants were not required to do anything different from being a student in the class, with the exception of those participants chosen for interviews. The IRB approval letter, the consent form, and the interview consent form can be found in Appendices J, K, and L respectively.

The researcher made every effort to ensure that no identifying information about the participants was recorded. To ensure the participants' anonymity, the researcher coded each participant's name and saved the key in an electronic file. This file was only available to the researcher. All data were recorded using the code. All audio and video data were transcribed not only for data analysis procedures but also to ensure anonymity.

Data security was a concern for the researcher. First, all recorded data were backed up onto an external hard drive. Second, the hard copy data, after being transcribed and recorded, were locked in the office of the researcher's advisor. They will remain there for three years according to the policy of the IRB.

Limitations

A possible limitation to the interpretation of the results of the analyses on the change variables is that the teacher-centered control group had fifteen participants that did not come to take the posttests. Efforts were made to obtain these posttest scores but the large dropout still occurred. In analyzing the data, all participants that took a pretest were included in the pretest analyses and only participants who took both a pretest and posttest were included in the analyses of change.

Another possible limitation to the study is that a participant from the control group that learned through both teacher-centered and student-centered methods was removed from the posttest analyses and analyses of change from pre- to posttests due to the participant's extreme behavior displayed on the day of the posttests. The participant communicated her angst about not being able to use her calculator on the posttest which the instructor had decided to use as the final exam for the class. The participant came to take the posttest thinking a calculator would be allowed and was very upset upon learning that she could not use one, and the participant slammed the door when leaving the classroom. The decision was made to remove the participant's posttest scores for fear that the participant's scores were affected by emotions.

A limitation of the quality of the transcriptions of the class meetings was audio issues that occurred early in the study. In the beginning of the research study, the participants were video-taped while working in their collaborative groups. The main purpose was to hear what the participants were saying to each other in their groups as they worked problems, but the visual image of the participants working in their groups proved to be useful data in addition to the audio. Once filmed, the video was downloaded onto the researcher's computer and transcribed along with notes about what the participants were doing. The video had poor audio and resulted in some transcriptions having sections in which participant were talking, but it is inaudible. The researcher switched to recording the groups using a digital voice recorder and the audio was transcribed without audio issues afterward.

Summary of the Methodology

In summary, this chapter presented the details of the methodology used in the study. The case was made that the pilot studies influenced the methodology chosen for this study. The design of the class used to teach mathematics through PBL was outlined. The research context, participants, and instructors of the classes were described. The data collection, types of data, organization of the data, and the analyses of the data were shared. Ethical issues and issues with audio recordings were discussed. The next chapter presents the results of the data analyses.

CHAPTER IV

THE RESULTS

Results of the data analyses on the qualitative and quantitative data are presented in this chapter. This chapter begins with a report on the history of the preservice elementary teachers' prior experiences with mathematics, including mathematics anxiety and attitudes toward mathematics. Then, results of the qualitative data analyses are presented. Additional qualitative data are reported on the participants' acceptance of PBL as a way of learning mathematics. The quantitative results are presented last followed by a summary of the results and a preview of Chapter 5.

Past Experiences with Mathematics

The presentation of the qualitative data begins with a description of the past mathematical experiences of the preservice elementary teachers that learned mathematics through PBL and problem solving.

Prior Experience with Problem Solving in Mathematics

None of the participants reported having previous experiences solving problems in mathematics that went beyond the word problems in the back of their text or having a "more challenging problem" at the beginning of the class. The participants reported a collective history with mathematics that was categorically traditional. In a writing prompt one participant said, "In prior math classes, most of the problem solving was done either by calculator or by the professor" (September 4, 2012). Another participant, responded, "During high school, problem solving is really not a main focus in math" (September 4, 2012). Another participant responded, "Every math class I have taken focuses on the actual formulas and gives you the problems already set up" (September 4, 2012). Thus, this class was their first experience with problem solving in mathematics.

Past Experiences with PBL

As reported above, the participants had no prior experience with problem solving in mathematics. However, five out of forty or 12.5% of the participants reported having an education class the previous semester that used PBL as the primary method of instruction and learning. Their experience with PBL in their education class seemed to be positive and they recognized the benefit of learning through PBL in contrast to their previous learning experiences. One participant wrote,

> I completed an education class that was strictly PBL. . . . My education before took the "one way for all" approach when really we each go about things differently. . . . I have found that it honestly makes you learn and grow and even prepares you for problems that may arise in your career. In math, I don't remember going through any problem-solving processes. I feel that it was always about the answer. We were given one formula to solve different types of problems and that was it. (September 4, 2012)

Views of Ability

The preservice elementary teachers' views of their ability to do mathematics was mixed, with participants reporting positive beliefs in their ability to do mathematics and participants reporting negative beliefs about their ability to do mathematics. Beyond believing that they are either "good" or "bad" at mathematics, the participants' views of their ability to do mathematics appeared to be heavily influenced by their past learning experiences in traditional mathematics classrooms. When the participants were asked if they were good problem solvers, the majority responded with two-part answers. One part of their response was about solving problems in the "real world" and the second part was about solving problems in mathematics. Consider the following participant's response: "In social or real life situations, I can come up with a decent solution in almost any case. Unfortunately, that is not the same when it comes to solving problems in math" (September 4, 2012). Another participant wrote,

> Overall I am ok with solving problems in the real world. But the more detailed and math oriented the problem is, the harder it is for me. I usually end up getting frustrated when I feel like I'm not solving the problem quick enough.
>
> Although my problem solving skills in life are good I must admit math is a struggle for me. I get freaked out by it. (September 4, 2012)

There is evidence in the previous quote that this participant believed solving problems in mathematics should be quick. Other participants reported similarly by saying that they "give up" or "find a non-sense formula" to quickly end work on a problem.

The influence of their traditional experiences with mathematics could also be seen by the participants' belief in needing to be shown how to solve problems. This belief could be seen throughout the semester for some participants, and it may have contributed to their being less successful learning through PBL. One participant responded in a writing prompt,

> I enjoy problem solving when I feel secure in the method. When I am not sure of how to solve a problem I tend to get frustrated. I have never really been taught how to solve problems with a distinct method. (September 4, 2012)

In the same writing prompt another participant said, "When it comes to solving math problems, I believe I am not a good problem solver. But, once I am taught the steps on how to solve the problem it becomes natural" (September 4, 2012).

Mathematics Anxiety and Attitudes toward Mathematics

Many of the preservice elementary teachers that learned mathematics through PBL and problem solving began the course with high mathematics anxiety and a poor attitude toward mathematics. Initial data were collected from participants through writing prompts and interviews. Some participants did not have high mathematics anxiety and others reported neutral attitudes toward mathematics. A few participants communicated positive attitudes toward mathematics. A considerable number of participants reported that they had experienced mathematics anxiety. For example, one participant wrote, "Well, my past experiences solving problems were extremely anxiety filled. I hate math and am fine using a calculator my whole life" (September 4, 2012). Another participant mentioned anxiety in the following writing.

> As for math problems, I've always struggled with the more complex problems. Especially word problems. The more figures in the problem the more anxiety I get. . . . To be honest, math gives me anxiety and instead of pushing forward in an attempt to understand and learn more, I typically only do the minimum amount of work I have to. Math gives me terrible anxiety. (September 4, 2012)

Evidence that the participants' previous learning experiences influenced their anxiety is illustrated by one participant that reported that when there is pressure to perform, her anxiety increases and overwhelms her. Other participants reported that they did not know how to stay calm if they "get stuck" on a problem; possibly indicating that their past experiences in mathematics had not given them enough opportunities to overcome these feelings and persevere through problems.

Other participants reported negative attitudes toward mathematics due to previous experiences in mathematics classes. This is illustrated by the following participant's response to an interview question about her attitude toward mathematics when she replied, "Discouraged [attitude] because of an experience in an algebra class" (September 27, 2012). Another participant said in an interview, "It [math] is scary and I dread it" (September 27, 2012). In summary, responses to writing prompts and interview questions showed that some of the participants had high mathematics anxiety and a poor attitudes toward mathematics.

Qualitative Results

Qualitative data were collected during the fall 2012 semester from the preservice elementary teachers that learned mathematics through PBL and problem solving. In this section the results of conducting grounded theory data analyses on transcriptions of participant interviews, transcriptions of recorded class sessions, writing prompts, researcher journal writings and notes, and participant work are reported. The qualitative results are presented in order of the research questions

.

1. How do preservice elementary teachers learn mathematics through PBL?
2. What features of PBL do preservice elementary teachers perceive as contributing to their learning of mathematics?
3. What features of PBL do preservice elementary teachers perceive as influencing their mathematics anxiety?
4. What features of PBL do preservice elementary teachers perceive as influencing their attitudes toward mathematics?

How They Learned

The qualitative data collected was insufficient to answer this research question.

Participant Perceptions of Effective Components of PBL

Grounded theory data analyses were conducted to determine the preservice elementary teachers' perceptions of the effective components of PBL. The main categories that emerged from the grounded theory data analyses were working with others to solve problems, relying on self for learning, creating problems, and teacher mini-lectures and other teacher facilitation. Working with others to solve problems was a category with three sub-categories. The sub-categories were discussing problems, teaching and learning from others, and evaluating mathematics.

Working With Others to Solve Problems

Problem posing and problem solving were the main activities of the preservice elementary teachers in the classroom. Thus, solving and posing problems were the mechanism for learning mathematics in the classroom. Through solving and posing problems created by the teacher, themselves, or one of their peers, the participants participated in four main activities with each other. These activities were discussing problems, teaching others, learning from others, and evaluating mathematics. Although presented separately here, these activities occurred interchangeably while participants worked to pose and solve problems. The transcription of a collaborative group discussion below illustrates how participants engaged in all of these activities while working to create numbers in base five.

> E: Ok if this was our regular number system and we had 43, and we wanted to find out how many that was, it would be four times ten and three times one. But since our base is five we do four times five and three times one.
>
> R: Oh you're converting it.
>
> E: I understand how to do the two-digit numbers but not the three.
>
> Z: Well why did you do three times one?
>
> E: Because there are three ones.
>
> [E and Z have a discussion about raising ten to the zero power equaling one. R listens in and says, “Oh that helps.”]
>
> Z: So then for a three-digit number it would be the same thing.
>
> E: Oh yeah it would be one times five squared.
>
> R: Are you doing base ten first.
>
> Z: So is it 37? It's 37 right?
>
> E: Is it?
>
> Z: Cause it's one times five squared plus two times five to the first plus two times five to the zero.

E: Yea, that makes sense. See we do that because we are not always going to be able to count. That formula is doing the counting for us.

(November, 15, 2012)

The participants were engaging in these activities from the beginning of the semester. After coding the audio from the first PBL lesson, the following note was made about the activities in which the participants were engaging during the lesson.

During this lesson every participant in the group had the opportunity to make sense of the mathematics they were doing, discuss the problems they were working, communicate their reasoning with their group mates, learn from their group mates, evaluate the reasoning of others, and work independently.

Discussing problems. If the main activity of the class was solving problems, then discussing problems with others was the main activity participants engaged in while solving problems. In fact, the noise level of the classroom often made transcribing audio data difficult. Most groups began working on problems independently and as questions about the problems arose, they asked questions and discussed the problems within their group. The following transcription of a collaborative group discussion reveals the verbal exchange that one group engaged in while working to solve a problem.

E: We need to know how much weight is on the plane, right? So we know that the pilot is 187 pounds and we know how much the fuel weighs, right? Is Emily attending?

E: It's a five hour walk. How do you figure out this?

D: It’s 65 miles from where the eagle is and where it needs to be.

E: Oh ok. I didn't get that part.

S3: Where?

D: [Repeats herself].

E: He said in the beginning um,

D: He can go 15 miles a gallon.

E: Yes, 15mpg and it is a 30 pound gas tank when full. 65 miles.

D: It can hold five gallons of gas. Yeah, it was a five gallon gas tank.

E: So, it can hold five gallons.

D: So he can go 75 miles on a full tank.

E: Yes. So it's going to be 65 plus 65 because he has to go there and back and so you're going to have to carry gas with you. So you have to count that in too. 75 miles on a full tank.

D: So you have to do 65 plus 65 if he did the full way.

E. Yea.

(October, 9, 2012)

At other times, participants would discuss problems after completing them individually to check their answers. When the participants had different answers, they discussed the problem and their solution paths to resolve the difference in solutions. The following transcription of a collaborative group discussion shows two participants discussing a problem in this way.

C: For number four did you get 36?

S: I haven't done that yet? [She reads the problem]. What did you get?

C: Thirty-six, I just multiplied. I thought that was right.

S: I did multiply. 2x3x3x3.

C: 2x3x3x2. You are forgetting the cheese.

S: No. 2x3x3x3.

C: Where are you getting the three? Two types of

S: [Interrupts and reads all the toppings off].

C: Yea, Mozzarella and Italian are two types of cheese.

S: Oh, I don't know what I was looking at.

(November 6, 2012)

Another group discussed a problem about gas mileage in a similar way as the following transcription reveals.

S4. So what did you do with his gas mileage?

S3: The number of miles per week divided by his 15 mpg so he is using 5.74 gallons per week and then times that by the cost per gallon and so that will tell you how much he is paying for gas a week and then divide the 386 by that,

S4: [Interrupts] and it will tell you how many weeks he has to quit driving around. Ok.

S2: So I just divided this by this [pointing to her paper].

(September 13, 2012)

The transcription of a collaborative group discussion below further illustrates that the participants discussed problems to resolve conflict with a problem.

S: It is supposed to be three carbs to one protein.

C: Yea, I got that.

S: He has 200 carbs to 220 protein. You have to multiply this [pointing to her paper], it has to be three times that. Do you see?

C: Oh, ok ok. So it's 200 times three.

S: No no. The ratio is three carbs to one protein so it should be 220 times three.

C: Why would it be 220 times three?

S: This number has to be three times this number [pointing to her paper]. So 660 divided by three has to be 220. That's what we need.

C: So it's 660, 660 - 200 because it says how many more will he need.

S: Yea.

C: Four-hundred sixty, he needs 460 more grams. More carb grams. Well, more carbs.

(November 8, 2012)

The participants perceived discussing problems and working with their group mates as helping them learn. One participant replied to a writing prompt with the following, “Working in groups has helped me because if one person does not know how to do something another person usually does. I also like discussing problems with my group because it makes me think more” (October 11, 2012).

Teaching and learning from others. Participants perceived that they learned mathematics through teaching their group mates, which resulted in their group mates learning from them. The participant teaching the group communicated her mathematical reasoning, her group mates evaluated the mathematics presented, and then used the mathematics to continue to solve problems. One participant summarized this experience in an interview by saying,

> When we are in groups we get to understand how different people approach a problem. I might see it one way and another person might say, "Well you can do this too," so it is teaching you, while you are teaching others. You are learning and teaching. (October 4, 2012)

Another participant, speaking of the benefit of teaching their group mates, said,

> It is a different process. I feel like it slows me down because I have to explain to others in my group repeatedly how I got the answer. At the same time, I feel like explaining it to others helps me have a better understanding. (October 4, 2012)

Further evidence of participants' perceptions of teaching and learning from others was documented in another participant's interview response,

> It's been like, especially with our class we have got to watch, if you are kind of stumped you get to watch the other participants work it on the board and it will kind of like, you can see where you went wrong and you can also do the same for your group members. If they can't figure something out then you can explain to them your work on the problem. (October 25, 2012)

Participants would often discuss problems with groups around them as they worked to solve problems. This resulted in their teaching and learning from other groups but also gave the participants the opportunity to apply their understanding in different contexts. For example, when participants were working on a number base PBL, all groups had different bases. The following field note shows that the participant had an adaptable knowledge of number bases and that she was able to teach it to others. The field note says,

> A neighboring group asks them [the group being recorded] a question about subtraction [in base four] even though they are using a different base [base five]. She explains to them how to count and subtract in base four. She teaches them by drawing them a number line and begins counting in base four even though she has been using base five with her own group. (November 15, 2012)

Participant presentations of solutions to the whole class provided another opportunity for teaching and learning from others. Participant presenters explained to the class how they solved their problems and provided their reasoning for the solution paths they chose. The teacher ensured that participants with different solution paths were selected to present. This resulted in the rest of the class being exposed to other ways to solve the problem. The following transcription is an example of a solution path that was different from the other groups' paths. Through the presentation of this solution path to the whole class, the participants were able to see an alternative solution using percentages.

> P: When it says that the female cocker spaniel weighs between 9 and 11 kg we just averaged it and said 10. We converted that to pounds so the average one weighs 22 pounds [presenter continues to write her work on the board].
>
> P: So this was what we considered the average and then we converted it to pounds so its 22 pounds. Then how much the dog weighed in kg times 2.2 so she

is 14.96 pounds. Then we just divided that by the 22 pounds to get the percent. So that's how much weight, how do you say that like

Teacher: That's how much of her normal weight she is?

P: Right, so then we subtracted that from 100% to see that she is 32% underweight. Then for the male we did the same thing. The 12.5 is the average. Then multiply by 2.2, so the average would be 27.5 pounds, multiply his weight times the 2.2 so that is 19.8 pounds. So the same thing, divide and by 27.5 to get 72% then subtract that from 100% to get 28% underweight. (September 20, 2012)

The participant in the previous transcription approached the problem differently by calculating the percentage of the dog's normal weight and then subtracting that from 100% to find the normal weight. Other participants approached the problem by finding the percentage underweight without the extra step of subtracting. Through presenting both approaches, the participants were able to compare the two approaches and the mathematics involved within each approach.

When a group member taught their group mates, the group mates perceived that they had learned. This statement is supported by a writing prompt in which 88% of the participants listed their group mates as a main resource for learning mathematics in the class. Perhaps group members were able to explain mathematics in ways that their peers understood. One participant supported this in the following writing prompt.

> I feel like working in my group has helped me re-learn some math skills. I get to see how they have done it and for some reason it is easier to learn from my peers and kind of just sit down and have a conversation about it. (October 16, 2012)

Another participant similarly wrote,

> I knew [percents] when I was younger but I could not always do it in my head like a lot of people can do when they are paying a bill or looking at a sales rack. . . . A couple of my group members have been able to explain it in their own words. Which is closer to what I would need to hear. (October 2, 2012)

Some participants showed a deeper understanding through their explanations given while teaching others. The following transcription of a collaborative group discussion shows a participant that has progressed past procedural knowledge into a conceptual understanding of division as making groups.

> Z: Ok, what about 12 (base 5) divided by 4 (base five)? That's going to be hard.
>
> E: Ok.
>
> EA: Wow this is hard, I don't know.
>
> E: So you start by asking how many fours are in one. That's zero. I am going to write it out. Well can we do 13 (base 5) divided by 4 (base five)? Cause I know the answer to this one. It is two.
>
> Z: Four times two would be 13 (base 5).
>
> E: Oh wait I lied, 13 (base 5) divided by 2 (base 5) would be 4 (base 5).
>
> EA: Wouldn't that just be the opposite then? [The group laughs at their realization in a light hearted way.]

EA: So 13 (base 5) divided by 4 (base 5) is still 2 (base 5).

E: Yea.

Z: How?

E: I am just looking at the number line. I am counting; 1, 2, 3, 4; 1, 2, 3, 4; Basically 4 times 2 is 13 (base 5). I used multiplication.

(November 15, 2012)

Other participants benefited from their strongest member in their group. One participant responded to a writing prompt with, "One thing that has helped me re-learn some things would be my group members. One of them is really good at math and teaching math" (November 6, 2012).

Below is a transcription of a collaborative group discussion with researcher field notes in which one participant in the group teaches the rest of the members of the group how to solve a problem and she goes further to teach them about percentage problems in general.

S2: [Makes the point that since the question is asking about a percentage then they could approach the problem differently. She leans in and points to her paper as she explains to S1].

S1: Right, Ok so we didn't even need to do that thing we just did.

S4: What are we doing?

S2: [Explains to the group what she had explained to S1. She moves in closer to the others while explaining and points to her paper as she talks. S3 and S4 do not appear to understand.]

S2: So what percentage of the teacher's average salary is going to equal the tuition increase. So what percent times the teachers' average salary divide by 100 [she is showing them on her paper that she is setting it up as a proportion]. (September 11, 2012)

Not all of the words of the participants were able to be transcribed in the previous excerpt due to audio difficulties. However, the field notes show that one participant taught the rest of her group about percentages. Her group mates all revealed through interviews and writing prompts that they perceived this moment to be influential to their learning. One participant wrote in a writing prompt, “I re-learned that [percentages] because a girl in my group [participant provides name] taught our group” (September 20, 2012). This was further confirmed by another participant in the group who wrote in a writing prompt, “I relearned how to do percents from a group mate [participant provides name]” (September 20, 2012). Another participant from the same group said in an interview, “A group mate, [participant provides name], taught me percents by creating examples and us working them out together” (October 4, 2012).

Evaluating mathematics. While participants discussed problems in their groups, listened to their group mates explain their reasoning, and listened to their peers presenting their solutions to the class, they were constantly evaluating mathematics. The participants evaluated different solution paths, checked their solutions for reasonableness, evaluated information in problems they were solving, and evaluated problems created by their group mates.

One way in which the participants evaluated mathematics was by evaluating information in the problems that they were solving. The participants began to try to understand the problems they were solving as opposed to merely putting numbers into formulas to solve “cookie cutter” problems. They developed a need for reasonableness in the information given in their problems and would protest to the teacher about the reasonableness of a problem or when they felt there was not enough information given within the problem to solve it. The following transcription of a collaborative group discussion illustrates how participants evaluated information in the problems they were solving.

> S1: It says that the money she makes pays her bills with very little to spare. But we need to know how much her bills are to know how much she has to spare.
>
> S3: Are we using her entire check? Assuming that she spends all of her check to pay bills? She needs plus, she's spending that and then she needs extra.
>
> S1: So just assume she needs a whole other pay check.
>
> (September 13, 2012)

Another group had a similar exchange about fuel capacity and fuel consumption,

> E: Ok we have a gas problem here. We have a 180 pound pilot, 30 pound gas tank and then the 15 pounds is 225 pounds. We can only have 220 so we can't fill the gas tank up.
>
> D: So she'll have to fly.
>
> (November 9, 2012)

At other times participants would evaluate others' solution paths and detect errors either in their own work or their group mates' work. The transcription of a collaborative group discussion below shows a group mate evaluating mathematics to find why they have different solutions.

D: Did you get 247 on number two?

C: I got 39. I mean I got 15%. I just multiplied 250 by 0.15.

D: I forgot the one. Did you get 221 too on number two? I got 39 then I subtracted.

C: I forgot to subtract. Did you get 161?

D: No, 221.

C: Did you add 39 or something? 260 minus 39 is 161.

D: 260 minus 39 right? I got 221. That's what my thing [calculator] says.

C: 260 minus 39, oh wait, I 'm stupid, I can't subtract. Yeah, I got 221. Wait, why is it 221? I thought it was just 39 pounds.

D: That's how much he increased his maximum bench press.

(December 4, 2012)

Participants would evaluate the reasoning of others and detect errors in their reasoning. The illustration that follows shows a group of participants discussing a percentage problem. Through their discussion, one participant begins to detect an error in her group mate's reasoning.

S4: Like the way you looked at this, I would have never thought of that.

C: Did you add up all the percents? That's the stuff I understand, it's like everything else I don't.

S: Five [percent] for the first two months, two [percent] for the third and fourth and one [percent] for the fifth. Did you add all those up?

D: You get 15%?

S4: See I have to do each month or it doesn't make sense to me. See I got a different answer than you. Will you tell me what I did wrong?

S: Thirty-nine was the increase right?

C: I just added up the percents.

D: Yea, just add up all the percents.

S4: Then why does mine come out different from yall’s?

D: I don't know.

S4: Here is the 1% for the first month so you subtract it. Here is the 2%.

D: I don't know what you are doing. That's too complicated.

S4: Wait, wait, wait! I just want to know what I am doing wrong so that way when I go to do it your way I will know what I am doing.

S4: I think because there are two here so I have to do it twice.

S: You have to either do 2% twice or do 4%.

C: Yea, we just added it up.

S4: So you can just add up percents even if they aren't the same number? For some reason I am thinking that you can't do that.

(November 27, 2012)

Participants created their own mathematics problems to solve and the usual routine used by the groups was to create problems individually, solve the problems they created, and then work each other's problems. Through solving the problems created by their group mates, they evaluated the reasonableness of the problems, determined if adequate information was provided, and evaluated the correctness of the solution provided by the creator of the problem. The transcription of a collaborative group discussion below shows a group mate pointing out some flaws in the problems created by her group mate.

S1 to S4: Ok, this one doesn't make sense. It's $3.39 per gallon and she lives 2.3 miles away. So how many gallons is she using? [She repeats herself]. There is no information to find the gallons. Does that make sense?

S4 to S1: [inaudible].

S1 to S4: Right but that's the number of miles. You can't multiply the number of miles by the cost per gallon to find the number of gallons.

[The participant continues to point out issues with another problem created by S4]

S1 to S4: On this one are you saying with the 15.9 pound bag, are you saying that each dog is going to be supplied a 15.9 pound bag?

S4 to S1: [inaudible].

S1 to S4: Yea, I get that, but in that case you would be saying that you are feeding each dog like one pouch out of the case, but in this case you don't know how much you are feeding each dog from the bag. I had a problem similar to it; I said each dog was getting fed twice a day, at five ounces of food per feeding. So that way I know how much each dog is eating and that way I know how much one bag is going to feed. Does that make sense?

S4 to S1: Yea, on the dog food I am going to put in one cup of dog food per dog.

S1 to S4: Well keep in mind that the dog food bag is talking about how many pounds not how many cups. Cups and pounds, cups is a quantity and pounds is a weight so you can't convert cups and pounds.

S4 to S1: What should I say?

S1 to S4: You should say like how many ounces are in a bag or some information on how many pounds each dog is going to be fed. You know what I mean? Just because say, each dog is getting a pound per feeding. That means this bag is going to feed fifteen dogs. And then also, how much does he want to donate? Does he want to donate for just one feeding?

S4 to S1: [inaudible].

S1 to S4: Yea so the dogs can eat one time. So make sure you put that in there and also make sure you put how many pounds each dog is eating.

(September 27, 2012)

Lastly, participants were evaluating different solution paths that were presented by their group mates and presented by their peers during whole class presentations of solutions. One participant said in an interview on November 6, 2012, “Most of the time we all found different ways to work out a problem, so that helped me in finding the way that is best for me.” Another participant interview further documented the benefit of evaluating their peers’ solutions to the class.

> Well, we do the work, he lets people come up and give their answers [explain how they arrived at the answer] and some people are wrong and you can see where they went wrong [by evaluating the mathematics presented] and some people are right and they can explain their whole coming about their answers [the participants must evaluate to determine that the explanation presented is correct]. You can see different ways [to solve a problem]. (November 6, 2012)

Relying on Self for Learning

Learning mathematics through PBL and problem solving required the participants to become responsible for their own learning. Participants had to make sure they understood for themselves the problems that their group was working on and every participant contributed to the creation of problems. In groups, participants worked independently prior to talking with their group mates about a problem, and the participants were also required to solve and create problems outside of class. The teacher facilitated groups while they worked to solve problems, but the solution paths came from the participants’ own thinking. Thus, it was not surprising that 100% of the participants listed either themselves or other self-directed learning as a main resource for learning mathematics in the course, and 48% of the participants listed themselves as the main resource for learning mathematics in the course. One participant wrote, “Everything that

I have learned or re-learned comes from my own thinking. I typically stare at a problem and then try methods that I think will work and do that until it all comes back to me" (November 15, 2012). In an interview another participant said, "It [PBL] gave me a chance to go over them [problems] myself. It helped me to do it over and over on my own" (October 4, 2012). Another participant said in a writing prompt, "PBL is ok. It gets my brain going and helps me figure problems out for myself" (October 16, 2012).

The participants in the class experienced the difference between being shown how to solve a mathematics problem and figuring out a solution to a mathematics problems on their own. One participant summarized the difference in response to a writing prompt.

> Problem solving has helped me learn or re-learn the material. By me figuring out the problems on my own the material sticks longer. Being shown something once, as in my other math classes didn't help because I could not remember how to do it when I got home. (October 25, 2012)

Creating Problems

In a few ways, the participants perceived that their created problems contributed to their learning. They created problems that were related to the PBL scenarios for each PBL, they also created problems to give extra practice on skills they were learning or re-learning, and they created problems to teach their group mates and others mathematics.

Problems created for the PBL lessons had to be related to the PBL scenario. Participants' created problems showed that they had developed the view that their everyday experiences can be problematized and solved mathematically. The participant-created problems below are provided to establish their learning of the applicability of mathematics to their personal lives.

1. I am currently working part time while attending classes at MTSU. I work 8:00 – 2:00 on Monday, Wednesday, and Friday earning $10.00 per hour. I am able to leave work at 2:00 each day to pick up my daughter, Lauren, from elementary school. My employer has offered to let me work until 5:00 if I want to earn extra money. Lauren would have to attend after-school care on MWF, which is available until 6:00 each day. The after-school care is $55.00 a week regardless of the number of days per week she attends. How much more or less will I make a week if I accept my boss’s offer and pay to send Lauren to after-school care? (September 20, 2012)
2. When you have a newborn baby, you spend a lot of money on diapers. A new baby will wear newborn diapers for around 2 months. One pack of newborn diapers costs $30 for a 96 count. A newborn baby will, on average, go through 11 diapers a day. What is the cost of each diaper in a box? How many diapers will a newborn go through in a week and how much will the cost be for the diapers for the week? How many diapers will the newborn go through in a month? (September 20, 2012)

Through creating their own problems to solve, the participants organized mathematics content into a problem scenario. Below are two participant-created problems to illustrate this point.

1. For desert on Thanksgiving, I made apple pie. To make the crust, I mixed 4 cups of flour with 4 ounces of butter. When I mixed it together, it was very dry and crumbly. So I looked up the recipe online and found that I needed 2.5 cups of flour and 8 ounces of butter. How much butter do I need to add to make the ratio correct? (November 15, 2012)
2. Sally is hosting Thanksgiving dinner this year. She wants to make sure there is plenty of food for all. She has 25 guests. The number of dinner rolls to guest ratio is 3 to 1. How many rolls are there total? How many rolls will each guest get? (November 15, 2012)

Participants created problems while solving problems given to them by the instructor. The problems they created in this context were specifically to help them understand the mathematics involved in the problem they were solving. While transcribing a class meeting in which the participants worked on problems in different number bases, the following field note was written.

> Each time they go to do another example they are actually creating their own problem [not given by the instructor]. So even here they are involved in creating their own problems. They are also posing questions about the material and then seeking to answer them together.

Participant interviews and responses to writing prompts suggested that creating problems required the participants to think about the mathematics required to solve the problem. One participant supported this in the following writing prompt.

> I had never tried to make my own questions before and it is not as hard as I thought it would be. I have had to revisit a lot of the skills like order of operations and percentages and things like that I didn't feel comfortable with before, but now I think I can actually do them and I could probably teach someone else how to do it. (October 25, 2012)

Another participant said in a writing prompt, "It [creating problems] kind of helped me remember how to work certain problems because you know, we made them ourselves and then went over it together" (October 16, 2012). Another responded in an interview,

> It was interesting coming up with math problems. It was also interesting seeing the other questions people came up with. The hardest part was making sure the math was correct in the problem and also making sure the problem could be solved. (November 6, 2012)

Participants shared their solution paths with each other, and this resulted in one participant teaching the rest of the group mates their path. The participants often created problems to illustrate their method, and their group mates worked them. One participant speaking of her group mate teaching the group said in an interview, "She [participant gives name] created examples for us to do and then we all worked them together" (October 9, 2012).

Some participants credited creating problems for developing their ability to reason and critically think. One participant wrote in a writing prompt, "I think my reasoning skills have improved drastically since the beginning of the semester because of writing our own problems" (October 25, 2012).

Teacher Mini-Lectures and Other Facilitation

The teacher's role during class was that of a facilitator. The teacher circulated the classroom and questioned them to help them think through their problems. When the teacher realized a common difficulty among the groups, he gave a "mini-lecture" to help them along. During participant presentations, the teacher questioned the class about the solution paths presented and helped them to make connections between different solution paths and to the mathematics involved in solving the problems. The amount of class time spent with the instructor conducting mini-lectures was very small. Thus, a surprising result was that mini-lectures and other teacher facilitation emerged from the data as a main category for how the participants learned mathematics. The surprising nature of this result is discussed in the next chapter. This section provides evidence from the data that mini-lectures and other teacher facilitation was perceived by the participants to be an effective component of learning through PBL and problem solving.

A writing prompt was given to the participants asking them to list their three main resources for learning or re-learning mathematics in the class and discuss how they used them. The responses from the participants were put into three categories. The categories were group mates, teacher, and myself and other self-directed learning. Forty-four percent of the participants listed the teacher as one of their main resources for learning. The reasons participants gave for listing the teacher as a resource for learning were mini-lectures, going over problems in class and emphasizing different ways to solve the problems, helping groups when they were having trouble with a problem, and the PBL materials that were provided to the participants by the teacher.

When participants presented their solutions to the problems, the teacher ensured that different ways of solving the problems were presented, and he helped the participants continue to think about the solution by questioning the class or presenting more information. Below is a transcription with field notes of a participant presentation to the class that illustrates the teacher's role during participant presentations.

> R: First you are going to add up all the vaccinations to get the total cost. We need to find 75% of 121 so, 0.75 times 121 equals 91. You are going to take the total cost of the vaccinations and the percentage of the dogs that get all the vaccinations and you get $9,362.10. Is that what everybody else got? [The groups agree]. Yay!
>
> Teacher: What if we included taxes for the vaccinations?
>
> R: [Writes on the board 9.75 times 9362.10.]
>
> Teacher: [Reminds R that sales tax is a percentage, and so she needs to convert 9.75 to a decimal in her calculation.]
>
> R: Oh, I didn't know that [she converts to .0975].
>
> Teacher: Now what is that calculation telling us?
>
> R: It tells us how much to add to it.
>
> S2: Couldn't you do 1.0975 times 9,362.10?
>
> Teacher: Yes. [Note that the teacher had not told them this before; now the participants are showing the class something new.]

R: [Makes the change that was recommended by her classmate. There is a bit of an "ah ha" from the class as if they are realizing this for the first time.]

(October 2, 2012)

In the preceding transcription with field notes, the teacher facilitated the discussion about calculating tax in a more efficient way. This discussion came about in an impromptu way, and so the reader may wonder if the participants perceived this discussion as influential. A participant, being interviewed about her work on a quiz, provided some evidence she thought the discussion about the more efficient method was influential. She said in an interview, "I normally would have done it like that [figuring a tip and then adding it back to the bill] but I just did it like he showed us on the board. Multiplying by 1.2, I had never heard of that before" (November 1, 2012).

At other times during participant presentations, the teacher ensured that the method being presented was understood by the class. Consider the following participant presentation to the class in which her solution strategy was to think about common multiples to determine the best price for puppy training pads.

E: This one I really just looked at it and figured out it's pretty easy. Because it's variables of 30, 60, and 120. Thirty is half of 60 and 60 is half of 120. But I guess you write down all your facts so you know that like 120 count is $64.99 and then two 60 counts you would multiply um ok, two 60 counts which equals one-hundred twenty you would multiply the $39.99 (cost of the 60 count package) by two which gives you $79.98 which happens to be more than the 120 count which is $64.99. So really your best deal is, I forgot what the question was, [participant reads the question]. So what's the better buy is paying for the 120 count package.

Teacher: What about the 30 count? Did you use the same reasoning?

E: Yes, multiply the 30 [cost of the 30 count] by two and that gives you 60 for $39.98 and then you multiply by two which is 70 something.

Teacher: Which is still more than the $64.99?

E: Right.

Teacher: Do y'all understand how she is thinking about the problem? She is approaching the problem from an intuitive type of approach. You probably didn't even write anything down?

E: Not at first. Not until the $19.99 times two, I wrote that down. I just looked at it. If the numbers were different like it if it was like 31 and 65 counts like that then you would have to write a lot of math down but if you just look at it and see 30, 60, and 120 then it's pretty obvious.

[Note again that the participant is teaching the class a new way to look at a problem and she is even teaching them when her strategy is and is not useful.]

Teacher: Now what would be the calculation?

S: You would divide the $19.99 by thirty and see per, the price per piddle pad. Do that for each of them and see which one is cheapest.

(October 2, 2012)

The teacher attempted to help participants make connections between different solution paths as illustrated in the following participant presentation to the class.

E: The first dog is 122 pounds and then the other dog is four.

Teacher discusses how to write the ratio in different ways and elicits the class to volunteer ways to write a ratio.

E: I first divided each by two and got 61 to two and 61 can't be divided anymore so that's your ratio for the weight. To find the percent I did two divided by 61and I set it up as “is over of equals x out of 100.” X comes out to be 3.28%. Then, the next ratio is the height of the dogs. The first dog is thirty-two, I think it’s inches, yes, and the other dog was six inches. So again you could do it in fraction form like this. It probably makes more sense to do it as a fraction since you are going to reduce it. Can you do 32 divided by six or six divided by 32?

Teacher: It depends on how the question is asked. Here we are asked for the bigger weight to the smaller weight. So we want 32 divided by six. If you were asked for the smaller weight to the larger weight it would be the other way around. [Many oh's come from the class.]

E: Then, and then, it asks are the heights and weights of the two dogs proportional. So I took my simplified fractions and it is pretty easy to come to the conclusion that they are not equal and so they are not proportional.

Teacher: Let's go through how you could come to that conclusion. You reasoned that they were not equal? Take us through your thoughts.

E: Um, my thoughts were that they are both completely reduced. 61 is not a multiple of 16. I guess if you wanted to be sure you could come up with a common denominator and make them have the same bottom number so you would multiply this [pointing to the board] by two over two and this one by three over three.

S: [Reports the fractions that you get after multiplying.]

E: So this way you can be sure because they are not the same.

E: If they were proportional they would be exactly the same.

Teacher: Right, you communicated that they are not proportional by showing that they are not equal. To be proportional means that two ratios are the same. What's another way we could have determined if these ratios are proportional? It is kind of a big idea. [A participant from the class volunteers that the cross products should be equal. The teacher goes through that calculation and discusses some other ratios that are proportional.]

(October 2, 2012)

The teacher, circulating the classroom, questioned the participants to help them think through their problems. Occasionally, as the following transcription reveals, the participants would approach a problem differently than the teacher expected. These interactions were beneficial because the participants explained their reasoning to the teacher until he was convinced of their method or until they realized a mistake in their reasoning. Consider the transcription below.

Teacher: So you did 22.5% of what?

S3: That was the total increase in money and we figured if we added in how much they needed to sell then we could do it from there.

Teacher: What is the 22.5% talking about?

S3: They are going to keep that much. So we need to sell 100% of what we need and then sell 22.5% more.

Teacher: Hm, I never thought about it like that. Does that work? So you are going to figure out what they need to sell and 1.225 more. How do you do that mathematically?

[Participants explain to teacher and realize a mistake in their reasoning.]

(September 13, 2012)

The data presented above illustrates the teacher's role as a facilitator during participant presentations of solutions. Writing prompt and interview data document that participants perceived the teacher's role as an effective component of learning mathematics through PBL and problem solving.

Participant Perceptions of PBL and Mathematics Anxiety

The qualitative data indicate that the PBL group experienced anxiety while working on their PBL lessons. Writing prompt and interview data indicated that working in groups and the overall classroom atmosphere mediated their anxiety.

Many participants' anxiety was mediated by seeing their group mates struggle with the problems just as they were. One participant wrote in a writing prompt, "Learning this way almost makes me feel less stressed about revisiting all these elementary skills because everyone else is having the same troubles that I am" (October 2, 2012). Other aspects of the group setting mediated the mathematics anxiety of some participants. One participant said in an interview, "Me and my group got along really well and tried to help each other not be so stressed and anxious about the problems" (October 4, 2012). Another participant said in an interview, "I think having a group around me helps me have less anxiety because I can work with them" (November 1, 2012).

The focus of the class was another reason given by participants for mediating participants' anxieties. The teacher focused the participants on the processes taken to solve a problem instead of the solution to the problem. One participant acknowledged this focus in an interview as the reason for her anxiety not increasing. She said in an interview, "No [anxiety increasing] due to group work. We are not concerned with getting the correct answer. The focus is on the process taken to arrive at the answer" (October 9, 2012). Another participant responded to a writing prompt similarly,

> I really enjoy working with a group on each of these problems. It allows us to find and share different ways to solve the problems. My anxiety skyrockets when I see fractions and word problems, but working with the group calms me down because the setting is less formal. (October 11, 2012)

These participants were not alone in recognizing the difference in the focus of the class. One participant said in an interview, "The class is a 'chill enough' setting where other math classes are stressful" (September 27, 2012).

Participants were also aware that the class setting was helping to mediate their mathematics anxiety. One participant said in an interview, "Before they [anxieties] were a lot higher than they are now. We have gone over a lot of different ways you can go into problems and different ways you can solve them" (October 9, 2012). Another participant wrote in a writing prompt on November 8, 2012, “I guess the class sort of helps [with anxiety] because you are learning how to break it down. . . . I’m a lot better about it right now but I still get kind of freaked out by math problems.”

Many participants learned to rely on themselves for learning mathematics. Their views about their mathematics ability may have improved leading to a decrease in the anxiety they have about mathematics. The data presented above show that the classroom environment in which participants worked with others to solve problems with an emphasis on the solution paths rather than the answer mediated the participants’ mathematics anxiety. The participants’ decrease in mathematics anxiety is discussed in detail in the following chapter.

Participant Perceptions of PBL and Attitudes

The qualitative data revealed some evidence that the participants’ attitudes toward mathematics improved in some ways. In particular, the qualitative data showed that some participants improved their confidence, increased their interest in the mathematics they did in the class, and experienced the joy of solving problems on their own. The following presentation seeks to highlight those results; however, many participants did not reveal any positive changes in their attitudes toward mathematics in the qualitative data.

Some participants revealed an increase in their confidence to do mathematics and create their own mathematical problems. One participant said in a writing prompt, “I enjoyed stepping out of my comfort zone and creating our own problems. Now I know that I am actually capable of writing my own work, which is a skill I will need as a teacher" (November 15, 2012). Another participant said in an interview, "I am a little more confident on these types of problems" (November 15, 2012).

Many participants cited an increased interest in the mathematics they were doing as the reason for their attitude change. One participant said in an interview, "The problems are interesting" (October 4, 2012). Another participant said in an interview, "Problem solving has given me a broader mind set" (November 1, 2012). Being interested in solving problems was new to the following participant who wrote, "I don't think any other math class had made me question things like this. However, these are real-life problems that are important to be able to solve" (November 29, 2012).

Lastly, participants experienced joy from solving problems on their own. One participant said to her group mates upon completion of a problem involving numbers in base five, "I actually got the addition for that one. I was like, 'Oh my God, I understand.' I felt like a genius." Another participant wrote, "The joy I got from these problems was the feeling of success once I had completed them" (November 8, 2012).

The Process to Acceptance of PBL

The research questions for this study did not address participants' acceptance of PBL, however, the qualitative data revealed a common path to acceptance of PBL as a way of learning mathematics. The participants were first hopeful about learning mathematics through PBL. During their first PBL lesson, their feelings changed to concern and confusion. Throughout the semester, their feelings of concern and confusion went away as they began to perceive benefits of learning mathematics through PBL. Consider the following response to a writing prompt administered on the last day of class,

> I really did not enjoy this class in the BEGINNING; I felt that it was very stressful. Towards the middle and the end, my group really came together and learned how to work together, study, and to ask questions. I feel that the PBL's and learning to formulate our own problems will be beneficial to me. I enjoyed this class and I felt it has caused positive feelings about mathematics.
>
> I am slowly warming up to the idea of being able to work problems out on my own and actually believe that it will be beneficial to me. (December 4, 2012)

The time from being concerned and confused to seeing the benefit was different for each participant. Some participants reported preferring traditional instruction in mathematics at the end of the semester and some participants never progressed past being concerned and confused about PBL.

Hopeful about PBL

The participants began the semester hopeful about learning mathematics though PBL and problem solving. The first class session following the administration of the pretests was used to define PBL and problem solving and to give the participants an overview of the way class would be conducted. On the following class session, they were given a writing prompt designed to learn their initial views of learning mathematics through PBL and problem solving. One participant wrote:

> I am very excited to hear that this math class is going to be different from other classes I have taken in the past . . . I hope to learn a lot and in turn pass down these skills as a teacher someday. (August 30, 2012)

Another participant responded to the writing prompt with a similar hopefulness about learning mathematics through PBL.

> I think I have the potential to be a better problem solver at the end of this semester. It's all just very new. Also, I think this type of learning could really help me retain the skills I will need as a learner and educator. In the past I've been taught to approach problems with a focus on whatever skill we were currently learning instead of accessing the broad, collective skillset I've been building since elementary school. I like the idea of trying new ways to solve for an answer instead of being given one path to follow to reach an answer. (August 30, 2012)

Other participants' hopefulness about PBL was attributed to learning in a group. One participant wrote, "I think PBL may be helpful for me at math because I am not very good at math so having a group that may have several people in it may help with my math skills" (August 30, 2012).

Concern and Confusion

When the participants began to work on the first PBL lesson, their feelings toward PBL changed. Concern and confusion were the feelings that emerged. PBL requires the participants to make sense of messy problems, and in this class they also had to create their own problems to solve. Thus it is not surprising that many of the participants reported feeling "stressed out" by PBL. One participant summarized her stressed feelings in response to a writing prompt.

> Doing mathematics through PBL is really stressing me out. There are a lot of concepts I don't remember how to do and when that is combined with solving word problems, it makes it worse. . . . I'm mostly concerned about how we are going to create our own problems. I feel that I can hardly work the problems we are given, much less come up with new problems. (September 20, 2012)

Another participant said in an interview, "PBL is stressful. It is sink or swim" (September 27, 2012). Still another participant wrote of her experience with the first PBL lesson, "I was very stressed. I did not care to be thrown into a group of people who also did not understand our work" (September 20, 2012).

Another source of concern was the noise level of the classroom. There were forty participants in the classroom working in groups of four. Verbal communication within the groups was necessary to work through the problems presented and as a result the classroom was noisy. One participant responded to a writing prompt,

> Initially I was very interested about this and thought that I would really enjoy it. Now that we have begun the process, I am so concerned. I am the type of person that needs quietness to truly focus on work, especially math. I am afraid that I will not be able to work efficiently. (September 20, 2012)

Participants were also concerned about their grades. Some disliked their grades being affected by the work of their group members, felt rushed to finish their PBL products, worried that their future as a teacher would not benefit from the class, and were confused by the participant presentations.

Group interactions were a source of concern for some participants. Poor communication was an issue for some groups. A participant wrote, “One concern I have is that the group based learning becomes difficult when communication between the group is poor” (September 20, 2012). Another issue faced by some groups was that group mates that were not contributing. Some group mates failed to contribute to the group by being absent or not carrying out the duties assigned to them by their group mates. One participant wrote, “I found the most difficult was dealing with people not pulling their weight. It was hard to pick up their slack” (September 20, 2012). Discussing her experience with the first PBL, a participant wrote, “My first experience was terrible. I asked everyone to e-mail me their two problems (all four of mine were used) by Friday and I only received one person’s. I had to create an additional four problems to make up for their laziness” (September 20, 2012). Some groups seemed to rely on their strongest member when doing mathematics. This was a cause of concern for the participants being relied upon. One participant wrote, “I feel like I am just helping others get through easier things and I’m not learning” (September 20, 2012). This feeling was shared in a writing prompt by a member of another group,

> The only thing I am concerned about right now is my group. It seems as they are depending on me to answer all the questions and explain it to them instead of us working as a group to work it out. (September 20, 2012)

Lastly, one group reported that one of their group mates was a distraction to their learning. One of the group members wrote, “One member of our group is also a big contributor to the problem. She talks too much and is lost before we have even read the problem as a whole" (September 20, 2012). Another member of the same group wrote, "Another issue is [a fellow group mate]. Somehow she always tends to make obstacles. Solutions to her problems did not make sense and she didn't know how she got them" (September 20, 2012).

The participants, after they had experienced the first PBL lesson, began to desire more traditional, teacher-led instruction. One participant seemed to devalue learning from others. She said in an interview, "In this situation, it kind of makes me feel like the learning is not productive because I am learning from my classmates" (October 4, 2012). Similarly, another participant reported in response to a writing prompt,

> I still feel a little concerned because I do not like having to figure problems out alone, or with other people who do not understand without guidance or a lesson. I just think it would be easier with a lesson and guidance rather than just trying to figure it out alone as a group. (October 4, 2012)

Other participants also wanted to be shown how to solve the problems. One participant wrote, "I learn math best when I am taught step by step how to work out the problems" (September 20, 2012). Another wrote, "I would much rather do work out of the back and be taught the methods" (September 20, 2012). Other participants found the emphasis on different ways to solve problems to be problematic. Likely a result of their past mathematics instruction, some participants wanted to be shown the way to solve the problem. One participant wrote, "I think there are too many ways to work each problem. I would prefer to have a formula or do algebraic equations" (October 4, 2012).

The instructor imposed a no calculator rule on the participants in the class. The decision to ban calculators was made to help the participants re-learn how to perform calculations by hand. The no calculator rule is discussed fully in the next chapter; however, it appears here as a source of concern. One participant summarized the reason for concern about not using a calculator when she wrote, “I am so used to a calculator and now I cannot use one. I’m really concerned about learning this way and if I will do well or not” (September 20, 2012).

The participants were new to learning mathematics through PBL and it took them a few lessons before they understood what was expected of them when learning this way. One participant said in an interview, “I don’t see what PBL really is” (October 4, 2012). During the first PBL lesson the participants expressed confusion about learning mathematics through PBL. Another participant said in an interview, “The concept [of PBL] is difficult to understand” (October 4, 2012).

Seeing the Benefit of PBL

Participants began to see the benefit of learning mathematics through PBL as the semester progressed. Some participants saw the benefit after the first PBL lesson while others took much of the semester before they were aware of the benefits of learning mathematics through PBL.

The participants eventually saw that learning through PBL resulted in them strengthening skills they need to be teachers. One participant said in their response to a writing prompt,

> Before this semester, well, I don't like working in groups to begin with so I came in with a prejudice but before this semester I would just not talk about it you know what I mean. I would just keep my work to myself and I would work for however long it took to reach the conclusion and now after this class it is easier to exchange ideas with peers and probably helps me come to the conclusion faster. So I feel like I am actually learning stuff that is going to help me in my career as a teacher. (November 29, 2012)

Another participant said in an interview,

> Um, communicating mathematics is really hard because you can use your finger and go along your paper where you have written and it completely makes sense but if you are not using the correct terminology you are saying it backwards. Like, you are dividing one thing by another; you could be saying that completely wrong even though it is written correctly on paper. I think that is one thing that this class has really done for me. Like, um get the correct lingo down and I think that is going to be really beneficial as a teacher because if you say it one way and write it another then you are asking two different questions. (October 25, 2012)

Other participants believed that their experience with PBL would help them to facilitate and conduct their classrooms in a similar way. One participant wrote, "So definitely I think it has helped. I will be a better facilitator in groups but also understand what it is like to be in a group and that will help my teaching methods" (November 29, 2012). Another participant wrote,

> So I guess group based I have learned how to deepen my understanding and everything and I think like through seeing how, like for me at least it doesn't work with the whole class [referring to whole class instruction] I can use that when I go into the classroom to do more group based stuff during math.
> (November 29, 2012)

Over time, the participants also became aware of the benefit of PBL in developing their skills in mathematics and their ability to solve problems. One participant summarized the increase in her mathematics skills by writing, "I have relearned basically everything in this class" (November 29, 2012). Another participant wrote, "I have been solving problems in this class unlike any class before" (October 11, 2012). Other participants liked learning mathematics through PBL because they had to think more than in other mathematics classes they had taken. Lastly, many participants felt PBL was beneficial. The following participant's statement reinforces the progression that many participants took in their acceptance of PBL. She wrote,

> The first PBL started off really confusing but as time went on it got much easier. I like this style of learning now that I understand what we are doing. I liked creating the questions on my own and seeing others' questions as well. . . . I think overall PBL was beneficial. (December 4, 2012)

Learning from group mates is a part of the design of the PBL lessons that were implemented in this study. Many participants seemed to view learning from their group mates while working on a PBL lesson as something different from learning mathematics through PBL. Therefore, they often gave credit for learning to the group and revoked the credit to PBL. In response to the question, "Do you believe that the problem-based learning approach has/has not been a factor in your change in math skills?" One participant said in an interview, "Group mates are a factor (in learning) but as a whole no" (October 9, 2012). Another participant contributed her success in the class to the group when she said in an interview, "I think the group helped more than anything else helped rather than the class. Probably the group work helped, working within our own little section" (November 1, 2012). The perceived impact of the group on learning is further confirmed by a participant that wrote, "Collaborative groups were essential to my relearning and new learning" (December 4, 2012). Thus, working in groups, which is part of the PBL design, was perceived by the preservice elementary teachers as a major contributor to learning mathematics through PBL and problem solving.

Some Never Accepted PBL

By the end of the semester most of the participants were able to see benefits in learning mathematics through PBL even if they still preferred to learn mathematics in traditional ways. However, some participants never accepted PBL as a way of learning. Commonalities of these participants were sought but no common characteristics emerged from the data. Participants of varied ability levels, mathematics anxiety, attitudes toward mathematics, gender, and race were found to both accept and not accept PBL as a way of learning mathematics. It appeared that a single group mate could have a significant impact on the experience of the whole group. For example, one participant seemed to always be annoyed with the class. In a writing prompt, "I was thinking about how irrelevant and dumb that the math problems and PBL are as a whole. This is the only math class I have absolutely dreaded going to" (November 13, 2012). His negative feelings about PBL did not develop over the semester but he seemed to have them early in the semester. He said after the first PBL lesson, "The first PBL I feel was horrible as a concept. . . . I am very good at math and have always enjoyed attending my math classes but it almost irritates me to come to this class" (September 20, 2012). What effect did his negative attitude have on the rest of his group? Consider his group mate's response to the final writing prompt of the semester in which she was asked to provide a description of her experience with learning mathematics through PBL.

> I found it to be difficult to work in groups and attempt to complete a PBL project. There were times when communication lacked; therefore, it made completing projects difficult. Group work was the most difficult part of the class. (December 4, 2012)

Consider also the words of his other group mate in a writing prompt, "I had an awful experience with PBL. I did not enjoy anything about PBL. To be honest lack of group work causes stress and difficulty when trying to complete this project" (September 20, 2012). Evidence of this one participant's impact on his group mates was found in the contrast in the recordings of the class meetings on two separate days. One day, he and his group mates were recorded. On this day they did not speak to each other. The field notes from that class meeting tell the story as the negative participant walks into the classroom. The field notes say, "Participant huffs and leans back in his chair. He seems very frustrated to be there." Later in the class the field notes say, "No one has talked with S3 [the negative participant] and S3 has not talked with anyone else." Now consider another day, a class meeting in which S3 is not present and his two other group members are recorded while working with another group. The field notes from this day say, "It is interesting how different the two guests are acting now that they are with a different group. They are talking." Thus, in the same way that a good group member can contribute to overall success with PBL so can a negative group member contribute to negative experiences with PBL for the whole group.

Quantitative Results

To aid in the clarity of the reporting of the quantitative results, the group that learned mathematics through PBL will be referred to as the PBL group, the group that learned mathematics through teacher-centered methods of instruction will be referred to as Control 1, and the group that learned mathematics through both teacher-centered and student-centered methods will be referred to as Control 2. The quantitative results will be presented in order of the quantitative research questions for this study.

1. Do preservice elementary teachers learning mathematics through PBL improve their mathematics content knowledge more than preservice elementary teachers taught using a teacher-centered approach or a mixture of methods approach?
2. Do preservice elementary teachers learning mathematics through PBL decrease their mathematics anxiety more than preservice elementary teachers taught using a teacher-centered approach or a mixture of methods approach?
3. Do preservice elementary teachers learning mathematics through PBL improve their attitudes toward mathematics more than preservice elementary teachers taught using a teacher-centered approach or a mixture of methods approach?

Pretest Analyses

The descriptive statistics for the groups' pretest scores on the mathematics content knowledge test, MARS, ATMI, and ATMI subscales are displayed in Tables 1 and 2, respectively. Comparative analyses were conducted on all the pretests to determine if the groups were similar at the beginning of the study.

Table 1.

Descriptive Statistics for the Pretests

	Pretest Content Knowledge				Pretest MARS			Pretest ATMI		
Group	*n*	*M*	*SD*	*SEM*	*M*	*SD*	*SEM*	*M*	*SD*	*SEM*
PBL	40	7.7	3.9	0.62	77.7	19.3	3.1	122	33.3	5.3
C1	28	6.8	3.6	0.67	70.9	22.8	4.3	126	32.7	6.2
C2	38	9.1	3.4	0.55	79.5	25.3	4.1	120	34.1	5.5
Total	106	8.0	3.7	0.36	76.5	22.6	2.2	122	33.2	3.2

Table 2.

Descriptive Statistics for the Pretest ATMI Subscales

	Self-Confidence				Value			Enjoyment			Motivation		
Group	*n*	*M*	*SD*	*SEM*	*M*	*SD*	*SEM*	*M*	*SD*	*SEM*	*M*	*SD*	*SEM*
PBL	32	43	15.1	2.6	35	7.1	1.3	29	9.1	1.6	13	6.8	1.2
C1	12	49	16.3	4.7	33	3.6	1.0	13	3.5	1.0	13	3.5	1.0
C2	32	43	15.2	2.7	39	5.7	1.0	13	4.3	0.8	13	4.4	0.8
Total	76	44	15.2	1.7	36	6.4	0.7	13	5.4	0.6	13	5.4	0.6

Pretest Mathematics Content Knowledge

The pretest mathematics content knowledge scores for Control 2 failed the normality assumption for conducting a One-Way ANOVA, *SW* (38) = 0.94, *p* = .041. The nonparametric Kruskal-Wallis test was performed. The median mathematics content knowledge score was significantly different between the groups, $\chi^2(2, 106) = 8.142$, *p* = .017. Post hoc analyses revealed statistically significant differences in the median pretest mathematics content knowledge score between Control 1 (*Mdn* = 6.38) and Control 2 (*Mdn* = 8.38), *p* = .016 but not Control 1and the PBL group (*Mdn* = 7.38), *p* = .804 and not between Control 2 and the PBL group, *p* = .191. Therefore, the experimental groups' mathematics content knowledge could be considered to not be significantly different from either control groups.

An additional analysis was conducted on the pretest because of the significant differences that were found on median scores of the content knowledge pretests. The Kruskal-Wallis test was conducted with only those participants from the groups that were included in the content knowledge change analyses, that is, only the participants that took both a pretest and a posttest. The results of the analysis found no significant differences between the median pretest mathematics content knowledge scores between the three groups, $\chi^2(2, 79) = .671$, *p* = .798.

Pretest MARS

To determine if there were any significant differences between the groups' mean mathematics anxiety, a One-Way ANOVA was performed on the pretest scores of the

MARS. No significant differences were found in the mean MARS score between the groups, $F(2, 103) = 1.260$, $p = .288$.

Pretest ATMI

An ATMI score of 120 indicates a neutral attitude toward mathematics and Table 1 shows that all groups began the study with a neutral attitude toward mathematics. To determine if the groups' mean ATMI scores were significantly different at the beginning of the study, a One-Way ANOVA was performed on the groups' scores of the pretest ATMI. The groups' mean pretest ATMI scores were not significantly different, $F(2, 103) = .314$, $p = .731$.

Pretest ATMI Subscales

The descriptive statistics for the pretest subscales on the ATMI which include self-confidence, value, enjoyment, and motivation are displayed in Table 2. The pretest value scores and the pretest motivation scores for the PBL group failed the normality assumption for conducting a One-Way ANOVA, $SW(32) = 0.93$, $p = .035$ and $SW(32) = 0.83$, $p < .001$, respectively.

Pretest self-confidence. To determine any significant differences between the groups' mean pretest self-confidence scores a One-Way ANOVA was conducted. The analysis showed that the groups were not significantly different on their mean pretest self-confidence score, $F(2, 73) = .721$, $p = .490$.

Pretest value. To determine any significant differences between groups' median pretest value scores, the nonparametric Kruskal-Wallis test was conducted. The analysis showed that the groups' medians were significantly different, $\chi^2(2, 76) = 8.268$, $p = .016$.

Post hoc analyses revealed no statistically significant differences in the medians of the pretest value score between the PBL group (*Mdn* = 31.5) and Control 1 (*Mdn* = 33.5), *p* = .488, the PBL and Control 2 (*Mdn* = 38), p = .206, and Control 1 and Control 2, *p* = .018.

Pretest enjoyment. To determine if significant differences existed in the groups' mean pretest enjoyment scores a One-Way ANOVA was performed. No significant differences were found between the means of the groups' pretest enjoyment scores, $F(2, 73) = .165$, $p = .848$.

Pretest motivation. To determine if significant differences existed in the groups' median motivation scores the nonparametric Kruskal-Wallis test was performed. The median pretest motivation scores of the groups were not significantly different, $\chi^2(2, 76) = .380$, $p = .827$.

Change Score Analyses

The descriptive statistics for the groups' change scores on the mathematics content knowledge test, MARS, and ATMI are displayed in Table 3, and the descriptive statistics for the groups' change scores on the ATMI subscales are displayed in Table 4. Comparative analyses were conducted to determine if the groups improved their mathematics content knowledge, mathematics anxiety, and attitudes toward mathematics differently.

The researcher decided, a priori, to use a conservative alpha-value for all comparisons that involved a change score because there were multiple comparisons of the groups. The conservative alpha-value for the One-Way ANOVA, Welch's *F*-Test, and the Kruskal-Wallis test was calculated by dividing .05 by three which resulted in an alpha-value of .0167. The post hoc group comparisons used an alpha-value that was computed by dividing .0167 by three which resulted in an alpha-value of .0056.

Table 3.

Descriptive Statistics for the Change Scores

	Content Knowledge Change				MARS Change				ATMI Change			
Group	*n*	*M*	*SD*	*SEM*	*n*	*M*	*SD*	*SEM*	*n*	*M*	*SD*	*SEM*
PBL	36	14.0	6.1	1.0	33	-12.5	17.1	3.0	32	-1	16.5	2.7
C1	11	6.3	4.1	1.2	12	9.0	12.4	3.6	12	-22	25.1	7.2
C2	32	10.7	5.3	.94	32	3.4	20.9	3.7	32	-13	24.8	4.4
Total	79	11.6	6.1	.69	77	-2.6	20.1	2.3	76	-9	22.5	2.5

Table 4.

Descriptive Statistics for the ATMI Subscale Change Scores

	Self-Confidence				Value			Enjoyment			Motivation		
Group	*n*	*M*	*SD*	*SEM*	*M*	*SD*	*SEM*	*M*	*SD*	*SEM*	*M*	*SD*	*SEM*
PBL	32	-0.2	8.7	1.5	2.6	5.7	1.0	-0.4	6.2	1.1	-0.9	5.4	1.0
C1	12	-11	12.4	3.6	-.67	5.6	1.6	-3.3	6.3	1.8	-1.8	2.9	0.8
C2	32	-6.4	12.2	2.2	-4.7	6.5	1.2	-4.9	6.5	1.1	-1.5	5.1	0.9
Total	76	-4.5	11.5	1.3	-1.1	6.8	0.8	-2.8	6.6	.76	-1.3	4.9	0.6

Mathematics Content Knowledge Change

The mathematics content knowledge change scores were analyzed using a One-Way ANOVA. The groups mean mathematics content knowledge change scores were significantly different, $F(2, 76) = 8.902$, $p < .001$. Tukey post hoc analyses found the difference in the mean improvement of the PBL group and Control 1 to be significant, 7.31, 99.44% CI [1.60, 13.86], $p < .001$. The difference of the mean improvement of the PBL group and Control 2 was not significantly different, 3.37, 99.44% CI [-0.95, 7.70], $p = .039$ and the difference in the mean improvement of Control 1 and Control 2 was not significantly different, -4.4, 99.44% CI [-10.58, 1.86], $p = .070$.

MARS Change

The MARS change scores for Control 2 failed the normality assumption for conducting a One-Way ANOVA, $SW(32) = 0.92$, $p = .021$. To determine if there were any significant differences between the groups' median change in mathematics anxiety, the nonparametric Kruskal-Wallis test was conducted. The median of the groups' MARS change scores was significantly different between the groups, $\chi^2(2, 77) = 17.750$, $p < .001$. Post hoc analyses revealed statistically significant differences in the median of the MARS change scores between the PBL group (*Mdn* = -12) and Control 1 (*Mdn* = 9.5), $p = .001$, and the PBL group and the Control 2 (*Mdn* = 3), $p = .003$, but not between Control 1 and the Control 2, $p = .705$.

ATMI Change

The variances of the change scores for the ATMI were not homogenous, $F(2, 73) = 3.41$, $p = .038$. The mean ATMI change scores were analyzed using Welch's *F*-Test.

The mean ATMI change was significantly different between the groups, $F(2, 27.76) = 5.06$, $p = .013$. Post hoc analyses found no significant differences in the mean difference between the PBL group and Control 1, 21.0, 99.44% CI [-8.01, 50.01], $p = .041$, the mean difference between PBL and Control 2, 11.448, 99.44% CI [-5.35, 28.25], $p = .078$, and the mean difference between Control 1 and Control 2, -9.552, 99.44% CI [-39.59, 20.49], $p = .509$.

ATMI Subscales Change

The descriptive statistics for the ATMI subscale change scores are displayed in Table 4. The value change scores for Control 1 and Control 2 were not normally distributed, $SW(12) = 0.83$, $p = .021$ and $SW(32) = 0.91$, $p = .012$, respectively. The enjoyment change scores for Control 1 were not normally distributed, $SW(12) = 0.85$, $p = .033$, and the motivation change scores for the PBL group were not normally distributed, $SW(32) = 0.68$, $p < .001$.

Self-confidence change. The mean change in the groups' self-confidence scores was analyzed using a One-Way ANOVA, and significant differences were found $F(2, 73) = 5.171$, $p = .008$. Post hoc comparisons found the mean differences between the PBL group and Control 1, 10.8, 99.44% CI [-0.95, 27.74], the PBL group and Control 2, 6.22, 99.44% CI [-2.53, 14.97], and Control 1 and Control 2, -4.68, 99.44% CI [-16.52, 7.17] were all nonsignificant, $p = .012$, $p = .065$, and $p = .419$ respectively.

Value change. The medians of the value change were analyzed using the nonparametric Kruskal-Wallis test. The analysis found that the medians of the groups value change scores were significantly different, $\chi^2(2, 76) = 18.858$, $p < .001$. Post hoc analyses revealed statistically significant differences in the median value change scores between the PBL group (*Mdn* = 3) and Control 2 (*Mdn* = -3), $p < .001$ but no significant differences between the median value change scores of the PBL group and Control 1 (*Mdn* = 2.5), $p = .52$ and no significant differences between the median value change scores for Control 1 and Control 2, $p = .197$.

Enjoyment change. The change in enjoyment scores were analyzed using the nonparametric Kruskal-Wallis test. The analysis found that the medians of the enjoyment change scores were significantly different among groups, $\chi^2(2, 76) = 9.11$, $p = .010$. Post hoc analyses revealed no statistically significant differences in the medians of enjoyment change scores between the PBL group (*Mdn* = 1) and Control 1 (*Mdn* = .-2.5), $p = .718$, no significant differences in the medians of the enjoyment change scores between the PBL group and Control 2 (*Mdn* = -5), $p = .008$, and no significant differences between the medians of the enjoyment change scores between Control 1 and Control 2, $p = .865$.

Motivation change. The groups' median motivation change scores were analyzed using the nonparametric Kruskal-Wallis test to determine if the groups' median change in motivation was significantly different. No significant differences were found between the groups, $\chi^2(2, 76) = 1.778$, $p = .417$.

Quantitative Support for Qualitative Findings

The qualitative data analyses did not reveal how participants learn mathematics through PBL and problem solving but did provide the participants' perceptions about learning mathematics through PBL and problem solving. The quantitative results of this study show that the participants did learn mathematics and thus provides some strength to the participants' perceptions of learning.

Similarly, the quantitative data showed that the participants that learned mathematics through PBL decreased their mathematics anxiety as determined by their change in the MARS from pretest to posttest. Participant s' statements about their mathematics anxiety decreasing, reported in the qualitative results section, have an additional support because their MARS scores did decrease.

There exists less quantitative support for the qualitative discussion of participants' attitudes since the PBL group's attitudes, as measured by the ATMI, decreased on average one point from pretest to posttest. The change in the PBL group's value from pretest to posttest did increase. The qualitative data showed that participants valued the mathematics that they did in the PBL classroom and that statement now has additional support from the quantitative results.

Summary of the Results

In this chapter the results of the qualitative and quantitative data analyses were reported. Grounded theory data analyses found that preservice elementary teachers' perceptions of the effective components of learning mathematics through PBL and problem solving were working with others to solve problems, taking responsibility for their learning, creating problems, and teacher mini-lectures and other teacher facilitation. Working with others to solve problems included the sub-categories discussing problems, teaching and learning from others, and evaluating mathematics.

The qualitative data showed that the participants perceived the classroom environment of learning mathematics through PBL and problem solving as influential in mediating their mathematics anxiety. This finding was further supported by the quantitative results. The data also showed that some participants that learned through PBL and problem solving increased their confidence and their interest in the mathematics they did in the class.

The participants went through a progression of acceptance of PBL as a way of learning. First they were interested in learning mathematics through PBL and problem solving, but once they began work on their first PBL lesson, they became confused and frustrated. As the semester progressed, the confusion about PBL went away for most participants, and they began to see the benefits of learning through PBL and problem solving. Some participants never accepted PBL as a way of learning mathematics, and the data showed that they had a negative influence on their group mates.

The quantitative analyses showed that the participants learning mathematics through PBL improved their mathematics content knowledge as measured by the mathematics content knowledge test. Furthermore, the improvement in their performance on the mathematics content knowledge test was shown to be significantly different from the control group that learned through teacher-centered methods. Thus, preservice elementary teachers can learn mathematics through PBL and problem solving, and they may improve their performance on assessments of content knowledge more than preservice elementary teachers that learn mathematics through teacher-centered methods of instruction.

The PBL group decreased their mathematics anxiety as measured by their scores on the MARS. The PBL group was the only group to decrease their MARS scores and their change (decrease) in mathematics anxiety as measured by the MARS was significantly different from the change (increase) of both control groups' mathematics as measured by the MARS. Therefore, it is reasonable to conclude that preservice elementary teachers that learn mathematics through PBL and problem solving will likely reduce their mathematics anxiety.

The groups in this study had a neutral attitude toward mathematics as measured by their scores on the ATMI and all groups experienced a decrease in their ATMI score from pretest to posttest. Though significant differences were found between groups on their ATMI change scores, post hoc comparisons found no significant differences. The descriptive statistics in Table 3 show that the PBL group had the smallest decrease in their ATMI, however, their attitudes remained neutral.

A summary of the study and the results, followed by a detailed discussion of the findings from this study, are presented in the next chapter.

CHAPTER V

SUMMARY & DISCUSSION

This final chapter begins with an overview of the information presented in previous chapters of the dissertation. It restates the research problem, reviews the methodology used in the study, and summarizes the major results. The implications of the results from the study are discussed following the summary.

Statement of the Problem

To form a basis of research on preservice elementary teachers' ability to learn mathematics through PBL and problem solving, two pilot studies were conducted during the two semesters prior to this study. The results from these pilot studies informed this study from the design of the class, the data collected, and the methods used to analyze the data.

During the fall semester of 2011, the first pilot study was conducted to examine the potential benefits and drawbacks of teaching through PBL. Comparative analyses were conducted between students from both a traditional number concepts course (control group) and students from a number concepts course taught using PBL scenarios and problem solving (experimental group). The MARS (Suinn & Winston, 2003) was used as a pretest and posttest instrument for both groups to determine the change in mathematics anxiety from the beginning of the semester to the end. A researcher-designed content knowledge test was given as a pretest and posttest instrument to measure differences in construction of content knowledge for both groups. The ATMI (Tapia & Marsh, 1996) was given as a pretest and posttest instrument to measure changes in attitudes toward mathematics for both groups. An instructor-created problem-solving assessment was used at the end of the semester to compare the groups' problem-solving abilities.

Pretests administered at the beginning of the pilot study showed that the groups were not significantly different with respect to content knowledge, attitudes towards mathematics, and mathematics anxiety (Banes & Miller, 2012). A posttest for content knowledge showed that the groups were not significantly different with respect to content knowledge (Banes & Miller, 2012). That is, PBL can be used as an instructional strategy without interfering with students' construction of content knowledge. There were no significant differences in either group's mathematics anxiety based on their ratings on the MARS (Banes & Miller, 2012). The experimental group experienced a significant decrease in their ATMI (attitudes) ratings (Banes & Miller, 2012). The problem-solving abilities of the experimental group were significantly higher than the control group at the conclusion of the semester (Banes & Miller, 2012).

A second pilot study was conducted in the spring of 2012. In this second study, a number concepts course was taught to a small group of honors students. Student reflections, responses to writing prompts, and student work were collected in addition to measuring content knowledge, mathematics anxiety, and attitudes toward mathematics. This study was used as an opportunity to refine the PBL scenarios created for this research, to improve the implementation of the PBL scenarios, and to trial the use of writing prompts for collecting information about students' perceptions of learning through PBL and problem solving. The results of this study were that the students were able to learn number concepts through PBL and problem solving, the students' attitudes toward mathematics did not decrease, and their mathematics anxiety did not change.

This study sought answers to questions that the pilot studies did not. The pilot studies that provided a foundation for this study found that preservice elementary teachers can learn concepts and structure of elementary mathematics through PBL and problem solving. However, the data collected in the pilot studies did not provide any detail about how the preservice elementary teachers constructed their mathematics content knowledge nor did it reveal their perceptions about learning mathematics through PBL and problem solving. The pilot studies also gave no explanation beyond responses to Likert-scales about the influence of learning mathematics through PBL on mathematics anxiety and attitudes toward mathematics. Consequently, the neutral results on mathematics anxiety and mixed results on attitudes toward mathematics from the pilot studies could not explained. These unanswered questions from the pilot studies led the researcher to the following qualitative research questions.

1. How do preservice elementary teachers learn mathematics through PBL?
2. What features of PBL do preservice elementary teachers perceive as contributing to their learning of mathematics?
3. What features of PBL do preservice elementary teachers perceive as influencing their mathematics anxiety?
4. What features of PBL do preservice elementary teachers perceive as influencing their attitudes toward mathematics?

As stated previously, there is little known about teaching mathematics to preservice elementary teachers through PBL and problem solving. Thus, to continue to establish and investigate the benefits of teaching mathematics to preservice elementary teachers through PBL and problem solving on mathematics content knowledge, mathematics anxiety, and attitudes toward mathematics, this study sought to answer the following quantitative research questions.

1. Do preservice elementary teachers learning mathematics through PBL improve their mathematics content knowledge more than preservice elementary teachers taught using a teacher-centered approach or a mixture of methods approach?
2. Do preservice elementary teachers learning mathematics through PBL decrease their mathematics anxiety more than preservice elementary teachers taught using a teacher-centered approach or a mixture of methods approach?
3. Do preservice elementary teachers learning mathematics through PBL improve their attitudes toward mathematics more than preservice elementary teachers taught using a teacher-centered approach or a mixture of methods approach?

Review of the Methodology

A brief overview of the methodology used in this study is provided to give a general explanation of the design of the study, the participants in the study, and the data collected.

This study was a qualitative-quantitative, mixed-methods study. Grounded theory data analysis methods were used to analyze the qualitative data. The quantitative data were analyzed to compare measures between two control groups and an experimental group.

All groups consisted of preservice elementary teachers enrolled in a course titled "Concepts and Structure of Elementary School Mathematics" at Middle Tennessee State University. The experimental group was taught by the researcher using PBL and problem solving as the primary method of instruction. One control group was taught using teacher-centered methods of instruction by a tenured, associate professor in the Department of Mathematical Sciences. The second control group was taught by a graduate teaching assistant using both teacher-centered and student-centered methods of instruction. Participant selection was determined by the section of class in which the participant was enrolled as was the designation to experimental group, control group one, and control group two.

Qualitative data that were collected from the experimental group consisted of participants' work, responses to reflective writing prompts, audiotaped structured interviews, and video and audio-taped class meetings focused on a single collaborative group per class meeting. Research journal writings and notes of the researcher were artifacts that guided the collection and analyses of the qualitative data.

Both control groups were observed periodically throughout the semester to confirm the type of instruction being used. The observations were audio-taped or video-taped, and field notes were taken during the observations. The instructors of the control groups provided their course syllabi and a description of the assignments for the course. These artifacts documented that the descriptions of the classroom environments of the control groups used in this study were accurate.

Pretest and posttest data were collected from both control groups and the experimental group. A researcher-designed content knowledge test was administered to compare the control and experimental groups' content knowledge at the beginning of the semester and to compare their content knowledge at the end of the semester. The content knowledge test used for this study was different from the content knowledge test used in both pilot studies. An explanation of the development of this content knowledge test and the validation of it was provided in Chapter 3. The MARS (Suinn & Winston, 2003) and the ATMI (Tapia & Marsh, 1996) were given as a pretest and posttest to both control groups and the experimental group. The MARS and the ATMI have both been shown to be valid and reliable (Frank & Suinn, 1972; Tapia & Marsh, 2004).

Summary of the Results

Qualitative Results

Grounded theory data analyses found that preservice elementary teachers perceived working with others to solve problems, relying on self for learning, creating problems, and teacher mini-lectures and other teacher facilitation as effective components of learning mathematics through PBL and problem solving. Working with others to solve problems included the sub-categories of discussing problems, teaching and learning from others, and evaluating mathematics.

The qualitative data showed that the classroom environment of learning mathematics through PBL and problem solving was perceived by the participants to mediate their mathematics anxiety, a finding that agreed with the quantitative results. The qualitative data indicated that learning mathematics through PBL and problem solving caused positive changes in participants' attitudes toward mathematics by increasing their confidence and interest in the mathematics they did.

The participants went through a progression of acceptance of PBL as a way of learning. First, they were interested in learning mathematics through PBL and problem solving, but once they began work on their first PBL lesson they became confused and frustrated. As the semester progressed, the confusion about PBL went away for most participants, and they began to see the benefits of learning through PBL and problem solving. Some participants never accepted PBL as a way of learning mathematics, and the qualitative data showed that their attitude about PBL had a negative influence on their group mates.

Quantitative Results

The quantitative analyses showed that the participants learning mathematics through PBL improved their mathematics content knowledge as measured by the mathematics content knowledge test. Furthermore, the improvement in their performance on the mathematics content knowledge test was shown to be significantly different from the control group learning through teacher-centered methods. Thus, preservice elementary teachers can learn mathematics through PBL and problem solving, and they may improve their performance on assessments of content knowledge more than preservice elementary teachers that learn mathematics through teacher-centered methods of instruction.

The PBL group decreased their mathematics anxiety as measured by their scores on the MARS. The PBL group was the only group to decrease their MARS scores and their change (decrease) in mathematics anxiety as measured by the MARS was significantly different from the change (increase) of both control groups' mathematics anxiety as measured by the MARS. Therefore, it is reasonable to conclude that preservice elementary teachers that learn mathematics through PBL and problem solving will likely reduce their mathematics anxiety.

The groups in this study had a neutral attitude toward mathematics as measured by their scores on the ATMI and all groups experienced a decrease in their ATMI score from pretest to posttest. Though significant differences were found between groups on their ATMI change scores, post hoc comparisons found no significant differences. The descriptive statistics in Table 3 show that the PBL group had the smallest decrease in their ATMI, however, their attitudes remained neutral.

Discussion of the Results

Comparisons to the Pilot Studies

Content knowledge. The pilot studies found that PBL can be used to learn mathematics; however, there were no significant differences between groups. This study found significant differences between the group learning mathematics through PBL and the group learning through teacher-centered instruction. None of the data collected in these studies can completely answer why this study found different results; however, possible explanations for the differences in results on content knowledge are discussed below.

One possible explanation for the differences in results on content knowledge between the first pilot study and this study could be that the instructor-researcher's implementation of PBL improved after using PBL for two semesters prior to conducting this study. The role of the instructor is very important to the success of learning through PBL. Instructors must learn how to teach with PBL (Dalrymple, Wuenschell, & Shuler, 2006; Etherington, 2011). It is possible that better facilitation by the instructor helped to lead to increased participant learning.

Another possible explanation is that the instrument used to assess content knowledge was different in this study. It could be speculated that the new instrument for assessing content knowledge contained items that reflected the learning objectives of a PBL classroom and thus put the PBL group in a better position to increase their scores on the instrument from pre- to posttest. This explanation seems unlikely because the content knowledge test was designed to reflect the mathematics content of the course instead of assessing other learning goals such as problem-solving skills. To investigate the possibility of a difference in the content knowledge, assessments from the pilot studies and this study were examined to determine the percentage of problems that required problem-solving skills. Both content knowledge assessments had five problem-solving items out of the forty items on the assessments, representing 12.5% of each of the assessments. These results imply that the assessments of content knowledge used in the pilot studies and this study were similar in the type of knowledge that was being assessed.

A third possible explanation is the difference between instructors of the control groups. It is possible that the instructor of the control group in the first pilot study was more effective at promoting participant construction of knowledge using teacher-centered pedagogies than the instructor of the control group in this study.

Mathematics anxiety. Another result that differed from the pilot studies was the effects of learning mathematics through PBL on the participants' mathematics anxiety. This study found that participants learning mathematics through PBL significantly decreased their mathematics anxiety from pre- to posttest whereas the pilot studies found no significant changes in mathematics anxiety.

The qualitative data presented in chapter four showed that certain aspects of the PBL environment helped to mediate participants' mathematics anxiety and thus provides an explanation for the decrease in mathematics anxiety in this study. Why did those same components of PBL not mediate the mathematics anxiety of the participants in the first two pilot studies? A possible explanation is that the first pilot study was the first time PBL was used as an instructional strategy by the instructor-researcher. In the second pilot study, the participants did not have high mathematics anxiety at the beginning of the study, and so it is possible that an instructional intervention using PBL would not change the anxieties of those that do not have high mathematics anxiety.

Interpretation of the findings

One implication of the findings is that preservice elementary teachers can learn mathematics content through PBL and problem solving while working in collaborative groups. In this study they outperformed both control groups based on their mathematics content knowledge scores. These results seem to refute the argument that there is not enough class time in a semester to use student-centered methods like PBL for the sake of needing to "cover" the content.

A second implication of this study is that learning mathematics through PBL and problem solving can mediate mathematics anxiety. In this study, participants learning through either teacher-centered practices or a mix of methods increased their mathematics anxiety. Mathematics anxiety is experienced by preservice elementary teachers and countless others. It has the power to stifle preservice elementary teachers' performance in mathematics, and results in avoidance of mathematics altogether. Educators of preservice elementary teachers want to help decrease mathematics anxiety, and the good news of this study is that they can help them by giving them more control of their learning by using PBL and problem solving.

What They Learned

The participants learned the mathematics content of the class through PBL and problem solving. The comparative benefit to their content knowledge gains was shown in chapter four and summarized again earlier in this chapter. What else did the participants learn through PBL and problem solving and could they have learned these things from the instructional methods used in the two control groups?

Participant-led Discussion of Mathematics

The PBL and problem solving class offered the participants the opportunity to focus on solution paths and make sense of several ways to solve a single problem. Many times this meant that the participants would produce a solution path that the instructor had not thought about or would not have taught if using a teacher-centered approach. A memo about participant-produced strategies written after the first round of open coding says, "I cannot overstate the importance of participant-produced strategies for solving problems. They are using mathematics and approaching the problems in ways that would have never been taught to them through lecture."

Some examples of participants producing their own, unique solution paths to problems are using proportions to solve percentage problems, ways of thinking about ratios, various geometric ideas, making number lines for numbers in bases other than ten, and writing basimal numbers.

Participants solved problems in ways that the instructor did not anticipate and would not have presented to the class if he were the only one presenting solutions. Giving the participants the responsibility of communicating their solution paths and creating their own problems resulted in a larger variety of mathematics being discussed, mathematics that was brought to the class by the participants.

Creating Problems

This class was the first time that the participants created their own problems to solve. There were no negative responses from the participants about creating their own problems and the grounded theory analyses revealed that creating problems was perceived to be an effective component of learning mathematics through PBL and problem solving. Creating problems can also be viewed as something the participants learned.

Knowing how to create problems is a skill that preservice elementary teachers will need when they become teachers in their own classrooms. The ability to formulate a problem in which the target mathematics content is embedded within an interesting scenario is also a skill that will set them apart from other elementary teachers.

Creating problems put the participants in a different role than the role taken when solving problems. They were fully in control of the mathematics being discussed within their groups when creating their own problems.

Communicate Mathematical Reasoning

Discussing problems was perceived by participants to help them learn mathematics while working in collaborative groups through their PBL lessons. Discussing their problems required them to learn to communicate mathematics.

What are the implications of the participants communicating mathematics to others? They control the mathematical discussion; they own their learning of mathematics. Through verbal and written communication with others, they became doers of mathematics that no longer passively receive mathematics.

Learning to communicate mathematics with others has a strong connection to the skills needed to teach mathematics. When the participants communicated within their groups, they had to explain so that others could understand them. Often this meant that they had to adapt their explanation when others did not understand them. Similar skills will be needed when they begin to teach elementary school mathematics.

The participants had to evaluate the mathematical communication of others. When explanations were incorrect it was their responsibility to detect the errors of their group mate's reasoning and offer the correction. As teachers, they will need to be able to listen to their students' explanations which will often be different but possibly correct. They will have to detect their students' incorrect reasoning and offer counterexamples to it as well as recognize the students that offer correct alternative solutions.

If these preservice elementary teachers take their ability to communicate and evaluate mathematics into their classrooms as previously described, then what will be the effect on their students? Will they see mathematics as a set of rules and procedures in which problems are solved only in the classroom and there is only one correct way to solve them? Will they leave the classroom feeling that mathematics does not make sense? The opposite of these is more likely to occur. Imagine the joy that a student will feel when they have solved a problem differently from everyone else and can explain their reasoning to the class. Teaching preservice elementary teachers through PBL and problem solving developed their ability to communicate mathematics and the potential benefits to their own teaching and their future students are inspiring.

A Different View of Mathematics

Learning mathematics through PBL and problem solving had an impact on the way some participants viewed mathematics. At the conclusion of the study, these participants saw mathematics as a tool that can be used to describe the world around them. One participant responded in a writing prompt,

> I have thought [outside of class] about possible problems I could make from real life experiences. I have found that after this class I see math everywhere and often think "I could make a math problem out of that." In other classes I've never done that because I really don't like math but this class made it more real life applicable. (November 13, 2012)

The participant's reference to creating problems out of everyday experiences indicates that creating problems was influential in helping her see the applicability of mathematics. Furthermore, the participant contrasted her experiences in the class with her other experiences in previous mathematics classes. Another participant responding to the same writing prompt said,

> When I'm on campus and I'm trying to create my own problems for the PBL, just walking around. I have not really done this in previous math classes. It was usually all numbers and calculations. (November 13, 2012)

Another participant wrote about using mathematics outside of class,

> I have been able to use the problems [problems from her PBL lessons] outside of class. For example, I was in Toys 'R' Us with my kids and I used some math we have learned in class to find the discount for my kids. I never used math from class before this class. (November 13, 2012)

When participants worked towards the solution to a difficult problem or worked to create their own problems, they often found that they "could not stop thinking about the problems." One participant supported this by writing, "With this base PBL I have not been able to stop thinking about how to figure it out" (November 13, 2012). One of the PBL lessons required the participants to solve problems involving fuel consumption. One participant, revealing how the mathematics from the PBL was making its way into her everyday life, wrote,

> After "Pay My Bills" I began to wonder what my gas mileage was and how much I pay weekly for gas (I commute). It's always been something I HAVE to have. I didn't even want to know how much it was costing me. I did end up calculating because of this class. (November 13, 2012)

Another participant's writing prompt response said,

> Throughout the semester (walking to classes on campus) sometimes I think about the current problems we are doing. I also think about how I could present an answer to a complicated problem on the board if asked. I lay out all the info in my mind and try to think of alternative solutions or other ways to reach the correct answer. I haven't ever really done this in past math classes. (November 13, 2012)

The participant's statement indicates that her views about mathematics had changed due to creating her own problems and presenting her solutions. She indicates that she believes there are multiple ways to solve a problem and also different solutions possible.

Some participants indicated that they were able to help a close friend, roommate, or boyfriend with their mathematics because of the class. One participant wrote,

> My friend was asking what percent out of a hundred he made when he made nineteen out of thirty. I thought back to this class and was able to help him. In my past math courses, this never happens because I try to not think about math. (November 13, 2012)

Like the previous participant, many participants reported that prior to learning mathematics through PBL and problem solving they avoided thinking about math. In this class however, participants could not help but think about the problems they were working. One participant wrote,

> I've done this [thought about mathematics outside of class] while helping my boyfriend with his homework. We were solving one of his problems … it just hit me that I should work the problem [a problem from class] a different way. I went back to it when we got done and I got the right answer. (November 13, 2012)

The most noteworthy statement from the previous participants was "I have never done this in other math classes." It speaks to both the benefits of PBL and the need for student-centered pedagogies in mathematics classes from K-12 and in higher education. Learning mathematics through PBL resulted in the participants learning more than the content, and these participants, with their teacher-centered past experiences learning mathematics, "had never done this before."

Unanticipated Findings and Possible Limitations

Participants perceived the instructor as one of the primary resources for learning mathematics through PBL and problem solving was a result of the study that was unanticipated by the researcher. The activities of the class required the participants to work together to solve and create problems. The instructor acted as a facilitator while the groups discussed problems, helped further mathematical discussions while participants presented their solutions, and held mini-lectures when it was clear that the participants were all struggling with the same concept. The percentage of time the instructor spent conducting mini-lectures was small. How is it possible that the instructor was a main resource for learning mathematics?

One possible explanation of this result is that the participants listed the instructor because they still believed that the teacher is the “deliverer of knowledge” and must be listed a resource for learning. However, the data did not reveal any information that confirmed this hypothesis. Another possible explanation is that the participants knew the instructor would see their responses and wanted to avoid offending the instructor for fear of negative consequences. However, in interviews, which the participants knew the instructor would not see until after grades were posted at the end of the semester, participants credited the instructor for helping them learn.

Another possible explanation for this result is that the participants needed information while solving problems, and the instructor gave that information at the moment in which the participants needed the information the most. Through group-to-group facilitation, the instructor helped the participants to advance their work on problems when they were stuck. Teacher mini-lectures were given when the instructor saw many groups struggling with the same concept and thus the participants were primed and ready to receive the information. The participants used the information that they learned to continue to solve the problem, and used it to solve and create other problems. This may have made their learning longer lasting and consequently, the instructor's influence more memorable.

If the previous explanation has merit, the implications are intriguing. It would imply that teacher lectures, when used in conjunction with PBL and problem solving, are perceived as more effective than lecturing in isolation. When a teacher gives control of the learning to the students, the teacher's interactions with the students are perceived to be more productive for learning. This would imply that teachers wanting to help students learn content should find or design problematic situations around their content objectives and deliver instruction to students when they need it the most.

Low sample size for Control 1. A possible limitation to the interpretation of the statistical results is that Control 1 had a large number of participants that took the pretests but did not take the posttests. This resulted in the sample size for Control 1 being quite lower than the other groups. The instructor teaching Control 1 requested that the posttests be given on the last day of class for the semester instead of on the day of the final exam. Many of the participants did not come to class on that day. In an attempt to get all of the participants to take the posttests, the researcher went to Control 1's final exam and asked the participants to take the posttests following their final exam. The researcher could not stay for the duration of the final exam and thus none of the participants stayed to take the posttests.

Participant interviews. A necessary precaution taken due to the researcher being the instructor of the PBL and problem solving group was that he did not conduct any of the participant interviews nor did he have access to the interview data until after the semester was completed and grades were submitted. This limited the quality of the interview data in two ways. First, the researcher was unable to develop interview protocols from the previous interview, data which is recommended by many grounded theorists (Charmaz, 2012; Strauss & Corbin, 1990). Second, the interview data was limited in that the main interviewer lacked the knowledge of mathematics required to know questions to ask based on the participants' responses to questions from the interview protocol. The main interviewer was also too far removed from the goals of the research to know when to ask follow up questions. It is possible that some aspects of learning mathematics through PBL and problem solving were not uncovered due to this limitation.

No calculator rule. The Arithmetic Proficiency Test (APT), an arithmetic test that covers hand calculations with fractions, decimals, integers, and order of operations, was administered after the pretests at the beginning of the semester. The University requires the preservice elementary teachers to pass the APT with 90% accuracy before taking the next mathematics course for their program of study. Only 5% of the participants passed the APT on the first attempt. All of the participants had already passed their general education mathematics course, so the instructor was surprised at this result. The participants reported that they had been allowed to use calculators to perform calculations since middle school and had forgotten how to perform calculations by hand. Thus, the instructor decided to not allow participants to use a calculator in the class.

A possible limitation to the study was the "no calculator" rule imposed on the PBL and problem solving group. The purpose of the rule was to ensure that the preservice elementary teachers learned the arithmetic procedures that they will be responsible for teaching when they become teachers. When solving problems, a calculator can reduce the cognitive load on the participants and allow them to focus more on the problems than on arithmetic. Thus, it is possible that this rule limited participants' ability to think through problems and make connections to mathematics. Another possible adverse effect of this rule could be on affective measures. It is possible that the participants had increased mathematics anxiety related to performing calculations by hand or their attitudes toward mathematics were influenced by their dislike of performing calculations by hand.

This possible limitation was realized prior to the conclusion of the semester after a writing prompt was administered to determine if the rule had any negative effects on the participants. The writing prompt asked participants about their feelings toward the rule and if they found ways to break the rule. Some participants reported that they were slowed down by doing calculations by hand and that they preferred to use a calculator. However, all of the participants reported that they found ways to get around the rule. Many of the participants used their phone calculators when the professor was not looking. The recorded class sessions revealed many groups using their calculators in secret. One group member said to her group mates, "If he says anything about it [they are using their phones as calculators] just say you are using it to check your answers." Other participants reported that they saved their calculations to do at home so they could use their calculator.

Many participants reported that they felt the rule helped them re-learn the procedures they had forgotten. One participant said, "One thing that helped me relearn is doing division without a calculator. It had been so long since I have done long division without a calculator that I forgot how to do it." Another participant said of doing arithmetic, "I used to just use the calculator but now I can do it [calculations] by hand." Lastly, a participant credited the rule for helping her learn when she wrote, "Having to work out problems without a calculator was a bit distracting at times, but I believe that it was also beneficial in helping me learn."

Distracting classroom environment. The classroom in which the participants worked through their PBL lessons in collaborative groups was filled to its maximum capacity of forty participants. The groups discussed the PBL lesson and their problems resulting in a classroom filled with communication. The communication that filled the room was a double-edged sword. During every class meeting, there were approximately forty participants communicating with each other about solving problems and the mathematics in the problems. On the other hand, the noise level was so high that it caused a distraction for many participants. One participant said it best in a writing prompt,

> The same thing that has helped me in this class is the same thing that has hindered me as well. The fact that there are 40 people in here is helpful in the sense that

> there are a lot of minds going and giving input. It is helpful when you can have a classmate explain their answer and route they took to get it. The same way the brains get going, so do the mouths. I sometimes find it difficult to focus working so close with others. (November 8, 2012)

The noise that filled the classroom made it difficult to hear group discussions when transcribing the video data and made getting the class' attention more difficult. Often, the instructor would have to give instructions more than once because participants were still working on their problems. At first the instructor was frustrated by this, but as the semester continued and the instructor reflected on the situation, he began to see it as a positive. He wrote about it in his research journal,

> It is a good problem to have. That is, to have a classroom full of students who so desperately want to solve a problem that they will not listen to the instructor giving announcements. When was the last time a lecturer could not stop his students from discussing mathematics and working on problems?

The noise was still an issue and so the instructor looked for ways to minimize the noise so the participants were able to think with fewer distractions and also to ensure that the class recordings could be transcribed. To limit the noise in the classroom, the instructor asked participants to take a few minutes to think about the problems before working on them together. To ensure that the class recordings could be transcribed, he switched from videoing groups to recording them with a voice recorder. Both of these interventions helped. The classroom was still loud but as the semester progressed, participants seemed to adapt to it and their mention of it as a distraction waned.

Participants getting off task. Participants were not always on-task all of the time. Their conversations would detour from the PBL lesson they were working to personal topics and then back to the PBL lesson. It is uncertain if the participants getting off task negatively influenced their learning.

Communicating incorrect mathematics. Another aspect of the PBL classroom that is both a benefit and possible drawback to participant learning is the participants' communicating their solutions to each other. The benefit is that participants must evaluate the solution paths of others and make sense of the mathematics that is communicated to them. However, a possible drawback is that incorrect mathematical reasoning may be provided and accepted by the entire group. One buffer to the communication of incorrect mathematics was that the participants presented their solutions to the class, and this was a time for evaluating all solutions and solution paths. Thus, participants did receive corrective feedback from their peers about correct and incorrect solution paths.

Participants relying on a group member too much. The design of the class required participants to make sense of problems on their own along with the help of their group mates. A few of the groups had one member that was stronger in mathematics than the others. It seems that the strongest group mate was used as a deliverer of knowledge at times. This may not be a drawback of learning mathematics through PBL and problem solving but some participants were able to remain in their comfortable mode of receiving information from a "teacher" instead of thinking for themselves. Teachers that implement PBL and problem solving should be aware of this possibility and look for ways to ensure that participants are not relying on one group mate for all the information.

Poor participant presentations. The participants' presentations of solution paths were poor at the beginning of the semester. Many of the participants simply wrote their solution on the board and others offered only a little explanation. They appeared to consider the solution of a problem to be most important rather than the path to the solution. That belief was in direct contrast with the goals of the class. A writing prompt was given after the first participant presentations, and it asked, "What is the most important thing when solving a problem?" All the participants responded that the solution was most important or they responded by giving steps they use when solving problems like "look for important information." The researcher decided to have a discussion with the class about the purpose of presenting their solution paths to the class and how to properly present their solution paths to their classmates. The conversation that took place at that time should have taken place prior to the participants' first presentations, and a similar conversation is recommended to all teachers planning on using pedagogies that involve students presenting their solution paths.

Connections to the Literature

Uniqueness of this study. There are two ways this study is set apart from the reviewed literature examining participants learning through PBL. First, this study is the only study that has examined the use of PBL to teach a mathematics content course to preservice elementary teachers. Secondly, this study is first to examine the effect of learning mathematics through PBL and problem solving on participants' mathematics anxiety.

Support for this research in which participants' attitudes and mathematics anxiety were studied while they learned mathematics through PBL and problem solving is provided by recommendations from researchers for studies of students' attitudes toward mathematics and other affective measures while they are solving non-routine problems (Hannula, 2002; McLeod, 1988). There is evidence in the literature that teacher education may be one of the disciplines in which PBL is most effective (Walker & Leary, 2009). Many preservice elementary teachers have not experienced mathematics instruction that is very different from teacher-centered instruction but they need learning experiences that require them to communicate, collaborate, reason, make sense of the mathematics they will teach, and develop habits of mind for them to be prepared to use instructional methods that align with current educational reform recommendations (Ball, 1990; CBMS, 2001; CCSSI; 2010; NCTM, 2000).

Instructor's influence on participant learning. A surprising result that is supported in the literature is the impact of the instructor's mini-lectures on participant learning. PBL is a student-centered pedagogy that aligns itself with constructivist theory, and the role of the instructor as that of a facilitator. However, many researchers have argued that knowledge construction can, at times, be achieved through lecture depending upon the cognitive actions taken by the learner (Ausubel, 1968; Cobb, 1994; Edelson, 2001; Ravitz, 2009; Schwartz & Bransford; 1998). The mini-lectures that were used in this study were given at times when participants needed the information to continue solving their problems or in conjunction with participant presentations of solutions. This use of mini-lectures may have been effective because they were given at times when the participants understood the need for the information (Edelson, 2001) and the participant's prior knowledge had been activated through solving problems prior to the mini-lectures (Ausubel, 1968; Schwartz & Bransford, 1998).

Students need time to accommodate to learning through PBL. The participants in this study, although intrigued by PBL in the beginning, went through a progression of acceptance to learning through PBL. Many studies have found similar results about students' need for an adjustment period when learning through PBL for the first time. The students need time to understand the goals of PBL (Hmelo-Silver, 2004; Jonassen, 2011; Savery, 2006), to change their ways of learning (Hmelo-Silver, 2004; Jonassen, 2011; Savery, 2006), to develop self-directed learning skills, and learn to work collaboratively (Hmelo-Silver, 2004).

Influence of the group. The group dynamic was found to be perceived as an integral part to participant learning and a mediator of the participant's mathematics anxiety. Collaboration within the group was both a learning process and a learning outcome. They learned through elaboration and sharing of knowledge, communicating within the group (Barnes & Todd, 1995) and responding to the ideas of their group mates (Hmelo-Silver, 2002). These results supplement previous statements in the literature about the importance of the group on learning in a PBL classroom (Barnes, 1992; Barnes & Todd, 1995; Hmelo-Silver, 2004; Ferreira & Trudel, 2012).

Attitudes. The participants that learned mathematics through PBL did not significantly change their attitudes toward mathematics in this study. Previous research on attitudes has found that attitudes are difficult to change, especially in a semester (McLeod, 1994; Sahin, 2010). The results of this study agree with the Cotic and Zuljan's (2009) study that found that learning mathematics through PBL did not improve student's attitudes toward mathematics. Other studies that have focused on improving attitudes toward mathematics through problem solving have found the same result (Abell & Pizzini, 1992; Schwartz & Riedesel, 1994). However, there have been many studies that found problem solving to be effective at improving student's attitudes toward mathematics (Stein, Boaler, & Silver, 2003). The results of this study contrast results of previous studies that found PBL to be effective at improving student's attitudes toward other disciplines (Akinoglu & Tandogan, 2007; Ferreira & Trudel, 2012).

Content Knowledge. This study found that participants who learned mathematics through PBL increased their content knowledge significantly more than participants that learned through teacher-centered instruction. Similar results have been found in many disciplines (Hmelo-Silver et al., 2009; Mergendoller et al., 2006; Pease & Kuhn, 2011; Sungur et al., 2006) and specifically in mathematics (Ali et al., 2010; Sahin, 2010; Shore et al., 2004; Vidic, 2011; Xia et al., 2007). These results challenge past comparative results between PBL and traditional methods of instruction (Albanese & Mitchell, 1993; Berkson, 1993; Capon & Kuhn, 2004; Colliver, 2003; Dochy et al., 2003; Kalaian et al., 1999; Newman, 2003).

Mathematics anxiety. This study found that a student-centered instructional method can lower preservice elementary teachers' mathematics anxiety. These results agree with other studies that have used instructional changes to positively influence preservice elementary teachers' mathematics anxiety (Alsup, 2004; Battista, 1986; Harper & Daane, 1998).

Self-directed learning skills. One of the goals of PBL is to develop students' self-directed learning skills (Hmelo-Silver, 2004; Schmidt, 1993). This study supports the effectiveness of PBL for developing self-directed learning skills. The preservice elementary teachers in this study learned mathematics by relying on their own abilities, prior knowledge, and resources that they located. These results build on other studies that have found that learning through PBL develops self-directed learning skills (Cerezo, 2004; Kampen et al., 2004; Norman & Schmidt, 1992).

Participant produced solutions and strategies. The participants that learned through PBL in this study produced their own solutions and created problems that involved mathematics content that would not be encountered in a teacher-centered classroom. Similar results have been found in a science course for preservice elementary teachers through PBL (Etherington, 2011).

Suggestions for Additional Research

The use of PBL in higher education is increasing. Teachers at the K-16 level will begin looking for teaching methods that can help them implement new standards that focus on more than memorizing facts and learning procedures. As they do this, PBL will likely be a teaching approach that is considered. Before mathematics professors and mathematics teachers begin using PBL in their classrooms, more research should be conducted on the effectiveness of learning mathematics through PBL. There are many suggestions for continued research on the use of PBL for learning mathematics; however, this dissertation focuses on those most closely related to this study.

For the purposes of the comparative analyses used in this study, PBL and problem solving were used as the primary mode of instruction during every class meeting for the duration of the semester. The control group that used a mixture of teacher-centered practices and student-centered practices never used PBL and problem solving during instruction. Thus, none of the groups used a mixture of teaching methods that included PBL and problem solving. Future research should focus on implementing PBL and problem solving along with other student-centered teaching practices and should compare its effectiveness to using PBL and problem solving exclusively.

Similar studies to the one reported in this dissertation should be conducted in different mathematics courses at the university level. The content of the course used in this study was number concepts that the participants were exposed to during their K-12 mathematics curriculum, and the goal of the course was to help them remember, re-learn, and learn it more constructively. There are many mathematics courses at the University level in which the students are learning the material for the first time. A study using PBL and problem solving in one of those classes will provide more information on the applicability of PBL across the University curriculum.

This study's primary focus was on preservice elementary teachers learning mathematics through PBL and problem solving. The bulk of the data was collected to attain information to help answer this question. However, a second focus was the effect of learning mathematics through PBL and problem solving on mathematics anxiety and attitudes toward mathematics. The two pilot studies and this study have differing conclusions on these affective measures. Thus, future research studies should primarily focus on affective measures of students learning mathematics through PBL and problem solving.

Preservice elementary teachers at Middle Tennessee State University take two mathematics courses other than their general education mathematics course. The first one is the number concepts course used in this study. The second is a course in geometry. In chapter four, it was established that there is a progression to acceptance of PBL as a way of learning mathematics, and many of the participants in this study took more than half the semester to assimilate. Future research at MTSU should study preservice elementary teachers learning mathematics through PBL and problem solving in both their number concepts and geometry course.

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APPENDICES

APPENDIX A

Mathematics Anxiety Rating Scale (MARS)

NAME________________________________Total Score__________

MATHEMATICS ANXIETY RATING SCALE: SHORT VERSION

The items in the questionnaire refer to things that may cause fear or apprehension. For each item, place a check in the box under the column that describes how much you are frightened by it nowadays. Work quickly but be sure to consider each item individually.

	Not at all	A little	A fair amount	Much	Very Much
1. Taking an examination (final) in a math course.					
2. Thinking about an upcoming math test one week before.					
3. Thinking about an upcoming math test one day before.					
4. Thinking about an upcoming math test one hour before.					
5. Thinking about an upcoming math test five minutes before.					
6. Waiting to get a math test returned in which you expected to do well.					
7. Receiving your final math grade in the mail.					
8. Realizing that you have to take a certain number of math classes to fulfill the requirements in your major.					

9. Being given a “pop” quiz in a math class.					
10. Studying for a math test.					
11. Taking the math section of a college entrance exam.					
12. Taking an examination (quiz) in a math course.					
13. Picking up the math textbook to begin working on a homework assignment.					
14. Being given a homework assignment of many difficult problems which is due the next class meeting.					
15. Getting ready to study for a math test.					
16. Dividing a five digit number by a two digit number in private with pencil and paper.					
17. Adding up 976 + 777 on paper.					
18. Reading a cash register receipt after your purchase.					
19. Figuring the sales tax on a purchase that costs more than $1.00.					
20. Figuring out your monthly budget.					
21. Being given a set of numerical problems involving addition to solve on paper.					

22. Having someone watch you as you total up a column of figures.					
23. Totaling up a dinner bill that you think overcharged you.					
24. Being responsible for collecting dues for an organization and keeping track of the amount.					
25. Studying for a driver's license test and memorizing the figures involved, such as the distances it takes to stop a car going at different speeds.					
26. Totaling up the dues received and the expenses of a club you belong to.					
27. Watching someone work with a calculator.					
28. Being given a set of division problems to solve.					
29. Being given a set of subtraction problems to solve.					
30. Being given a set of multiplication problems to solve.					

APPENDIX B

Attitudes Toward Mathematics Inventory (ATMI)

*Name*_______________ *School* *Teacher*______________________________

Directions: This inventory consists of statements about your attitude toward mathematics. There are no correct or incorrect responses. Read each item carefully. Please think about how you feel about each item. Enter the letter that most closely corresponds to how each statement best describes your feelings. Please answer every question.

PLEASE USE THESE RESPONSE CODES:
A – Strongly Disagree
B – Disagree
C – Neutral
D – Agree
E – Strongly Agree

1.	Mathematics is a very worthwhile and necessary subject.	
2.	I want to develop my mathematical skills.	
3.	I get a great deal of satisfaction out of solving a mathematics problem.	
4.	Mathematics helps develop the mind and teaches a person to think.	
5.	Mathematics is important in everyday life.	
6.	Mathematics is one of the most important subjects for people to study.	
7.	High school math courses would be very helpful no matter what I decide to	
8.	I can think of many ways that I use math outside of school.	
9.	Mathematics is one of my most dreaded subjects.	
10.	My mind goes blank and I am unable to think clearly when working with	
11.	Studying mathematics makes me feel nervous.	
12.	Mathematics makes me feel uncomfortable.	
13.	I am always under a terrible strain in a math class.	
14.	When I hear the word mathematics, I have a feeling of dislike.	
15.	It makes me nervous to even think about having to do a mathematics problem.	
16.	Mathematics does not scare me at all.	
17.	I have a lot of self-confidence when it comes to mathematics.	
18.	I am able to solve mathematics problems without too much difficulty.	
19.	I expect to do fairly well in any math class I take.	
20.	I am always confused in my mathematics class.	
21.	I feel a sense of insecurity when attempting mathematics.	
22.	I learn mathematics easily.	
23.	I am confident that I could learn advanced mathematics.	
24.	I have usually enjoyed studying mathematics in school.	
25.	Mathematics is dull and boring.	
26.	I like to solve new problems in mathematics.	
27.	I would prefer to do an assignment in math than to write an essay.	

28.	I would like to avoid using mathematics in college.	
29.	I really like mathematics.	
30.	I am happier in a math class than in any other class.	
31.	Mathematics is a very interesting subject.	
32.	I am willing to take more than the required amount of mathematics.	
33.	I plan to take as much mathematics as I can during my education.	
34.	The challenge of math appeals to me.	
35.	I think studying advanced mathematics is useful.	
36.	I believe studying math helps me with problem solving in other areas.	
37.	I am comfortable expressing my own ideas on how to look for solutions to a	
38.	I am comfortable answering questions in math class.	
39.	A strong math background could help me in my professional life.	
40.	I believe I am good at solving math problems.	

APPENDIX C

Writing Prompt Protocols

Writing Prompt 1

Take a few minutes to respond to the following:

a. Describe your past experiences solving problems.

b. Do you consider yourself a good problem solver?

c. Is problem solving easy for you?

d. How have you learned to solve problems in the past?

e. Share anything else you want me to know about your problem-solving skills and experiences.

Writing Prompt 2

Please describe your initial impressions about learning mathematics through problem-based learning. What concerns do you have about learning mathematics in this way?

Writing Prompt 3

What is the most important about solving problems? Why is that most important?

Writing Prompt 4

Describe your experience with our first PBL, Pay My Bills. What did you find most difficult about the PBL and why? What did you enjoy about the PBL and why? What mathematics did you learn or re-learn?

Writing Prompt 5

Please describe one thing that has helped you learn mathematics in this class and describe one thing that you are having trouble learning.

Writing Prompt 6

In this class, what have been your three main resources for learning/re-learning mathematics how have you used them to learn/re-learn mathematics?

Writing Prompt 7

During the semester have you thought about problems (either problems that I have given you to work, creating your own problems, or solving problems that you have created) outside of class and separate from just completing an assignment? (Example: I am driving in my car and I think about a problem from class). If so, please describe what you were thinking about. In your past math classes have you done this?

Writing Prompt 8 (Given Multiple Times During the Semester)

List three mathematics content objectives that you have learned or relearned in our class. Please describe how you learned or relearned it. Be specific about how our class activities helped you to learn or relearn the mathematics.

Writing Prompt 9

At the beginning of the semester we made a rule to not use calculators in class.

a. Please describe how this rule made you feel about the class and about mathematics.
b. Do you feel that performing calculations by hand was a distraction when solving context-based problems like the ones we solved during class?
c. Did you find ways to get around the no calculator rule? If so, please describe what you did.

Writing Prompt 10

One objective of my research is to describe how you (students in MATH 1410) learn or re-learn mathematics through collaborative group PBL and problem solving. Please take a moment and describe the ways that you have learned (or re-learned) mathematics in this class (please be specific). Also, describe the aspects of this class that have been a distraction, caused anxiety, or cause negative feelings about the class (or about mathematics). Again, please be specific. Treat this writing prompt as an opportunity to help me paint a picture of learning in this class.

APPENDIX D

Interview Protocols

Interview Protocol 1

1. Prior to entering this course, what was your attitude towards math?
2. Prior to entering this course, describe your interest in math.
3. Has your attitudes toward math changed since participating in this class? If yes, what has changed?
4. Do you find that problem-based learning has changed your attitude towards math? Please Explain.
5. Has your interest in math changed since participating in this class? How so?
6. Did you feel any anxiety/stress toward math?
7. What symptoms of anxiety/stress do you feel (if any)?
8. Did your anxiety/stress increase or decrease as you entered the classroom? Please explain.
9. Do you continue to have anxiety towards math after participating in this class? Why or why not?
10. If so, do you still have the same feelings/symptoms of anxiety? Please describe.
11. Does your anxiety increase/decrease as you enter the classroom even after participating in this class? How so?
12. Do you find the problem-based learning approach to math as easy/difficult to comprehend? How so?

13. Do you find that you are leaning at a faster/slower pace through problem-based learning? Please explain?
14. Do you believe your math skills have increased/decreased? How so?
15. Do you believe that the problem-based learning approach has/has not been a factor in your change in math skills? Please explain.
16. During the first PBL did you encounter any mathematics that you did not know/remember? How did you come to learn/re-learn it?
17. What did you learn about working in collaborative groups during the first PBL? Describe any difficulties you had with your group.
18. Did you teach any of your group mates mathematics during the first PBL?
19. The first PBL required you to solve many problems. Do you feel more comfortable solving problems now than before the class began?

Interview Protocol 2

1. State your name.
2. Think about your experiences in our class and complete the following statement: When I encounter mathematics content that I do not know (or remember) I . . .
3. Our work this semester has required you to solve many complex, context-based, problems. Tell me about your anxiety about these types of problems before this semester and how they (your anxieties) have changed this semester.
4. You have had to pose (or create) problems throughout this semester. Tell me what mathematics you have learned/relearned by creating problems. Do you feel

that you will be better at posing problems to your own students when you teach now that you have had these experiences?

5. Our class has required you to communicate your mathematical thinking with you group mates and to the class. Tell me about your experiences this semester communicating mathematics. Have you learned or deepened your understanding through communicating mathematics? Do you think that the experiences communicating this semester will help you as a teacher?
6. Tell me about a specific mathematics topic that you have learned (or re-learned) this semester and tell me how you learned (or re-learned) it.

Interview Protocol for an Individual Student 1

1. What role did the PBL environment (group problem solving, context-based problems, communication within collaborative groups, ect.) play in your correct work on quiz 2?
2. Did you have any anxiety before taking the quiz? If so, please describe your anxiety.
3. Did you feel any relief (of your anxiety) when you were able to solve the problems on the quiz?
4. Do you feel more confident about solving problems (like the problems on the quiz) now than you did before taking the quiz? Please explain.

Interview Protocol for an Individual Student 2

1. In problem 2 on quiz 2 you found the weight of 1.5 cups of dog food to equal 12 ounces. Please explain how you arrived at this conclusion. Did anything you

experienced in this class influence your solution? Can you now correctly solve this problem? If so, what role did this class (problem based learning, collaborative group problem solving, ect.) play in your learning how to solve this problem?

Interview Protocol for an Individual Student 3

1. Your responded correctly to all of the problems on the quiz. What role did our classroom activities contribute to your thinking about this problem?
2. Did you feel anxious about taking the quiz? If so, please describe what you were anxious about.
3. Did you feel a relief of anxiety after you solved the problem? Did you know that you had solved the problem correctly?

Interview Protocol for an Individual Student 4

1. In problem 1 you correctly divided 44.1 by 2.2 and interpreted this quotient correctly. What role did your experiences in our class contribute to you correctly solving this problem?
2. Please describe your feelings about solving this problem correctly.
3. On problem 2 you made some errors. Please explain your thoughts about this problem. Can you now correctly solve this problem? If so, how did you correct your thinking about this problem? Specifically, what role did our classroom activities play in you coming to know how to solve the problem ?

Interview Protocol for an Individual Student 5

1. In problem 3 you did 4.5(7) + 4.5/.75. Why divide in one place and multiply in the other? Do you now know what to do? How did you come to know what to do? Specifically, what role did Problem-based learning play in the development of your content knowledge?

Interview Protocol for an Individual Student 6

1. How did our class activities contribute to your solving the problems on quiz 2 correctly?
2. In problem 3 you did 4.5(7) + 4.5/.75. Why divide in one place and multiply in the other?
3. Do you now know what to do? How did you come to know what to do? Specifically, what role did our classroom activities play in the development of this knowledge?

Interview Protocol 3

In this interview, each student interviewed was given two past assessments in which they had displayed that they learned mathematics content by answering an item(s) incorrectly on the first assessment and answering a similar item(s) correctly on the second assessment. Students were asked to explain how they learned the mathematics content from the first assessment to the second assessment. In particular, students were asked to focus on features of the class that helped them to learn the mathematics content.

Interview Protocol for an Individual Student 7

1. Have you figured out how to solve problem A? Can you solve it now (give a chance to solve)? If you were able to solve it, how did you come to know how to solve it?
2. Do you now understand the problem and the solution?

Interview Protocol for an Individual Student 8

1. Can you describe the overwhelming feelings you have about these problems?
2. Do our class activities stimulate these overwhelming feelings often?
3. What have you done in this class to overcome (or attempt to overcome) these overwhelming feelings?
4. Have you been successful in overcoming your initial overwhelming feelings while I this class?

Interview Protocol for an Individual Student 9

1. Please attempt to solve the following problem similar to problem A.

   Joe the bodybuilder has been hitting the weights very hard. Each of the last three weeks he has increased his maximum bench press by 10%. His maximum bench press is now 366 pounds. What was his maximum bench press three weeks ago?
2. If you were able to solve the previous problem, what about our class helped you to be able to think about problems like this one?

Interview Protocol for an Individual Student 10

1. Have you reflected on the solutions to the problems since I explained them to you?

2. Please try to solve the following problem, similar to problem A.

   Joe the bodybuilder has been hitting the weights very hard. Each of the last three weeks he has increased his maximum bench press by 10%. His maximum bench press is now 366 pounds. What was his maximum bench press three weeks ago?

Interview Protocol for an Individual Student 11

1. Your response to the prompt indicates that you have some negative beliefs about your mathematics ability. How long have you had negative beliefs about your mathematics ability?
2. Has our class changed your confidence in your ability to do mathematics? Please explain in detail.

Interview Protocol for an Individual Student 12

1. Your approach to part A was different from others' approaches, but it was correct. How did you come up with this approach to the problem? Was there anything about our class that helped you?
2. Do you consider your method as valid as the method that was demonstrated in class?
3. In part B you said that you thought about the problem to solve it. Can you explain how you thought about it?

Interview Protocol for an Individual Student 13

1. You say that you feel uncomfortable with word problems (from a previous interview). Can you describe what about word problems make you uncomfortable?

2. You described your initial thoughts as anxiety (anxious thoughts). Are you able to overcome your anxieties to think clearly about a problem? Has our class activities contributed in any way to help you overcome anxious feelings?
3. After work is show to you on the board or by your group mates do you feel that you understand?
4. Your work on part B is correct. How were you able to come to think correctly about this problem?

APPENDIX E

Mathematics Content Knowledge Test

Name________________________

Date_________________________

MATH1410—Mathematics Content Knowledge Test

Directions: Please answer the following questions to the best of your ability. No calculators are permitted on this exam. Please show all your work.

1. What is the additive inverse of -5? Why is your answer the additive inverse?

2. True or False: If a is any number other than 0, then $1 \div a$ is less than 1. Please explain your answer.

3. What is the multiplicative inverse of $\frac{4}{7}$? Why is your answer the multiplicative inverse?

4. True or False: If a and b are natural numbers, then a – b will also be represented by a natural number? Please explain your answer.

5. True or False: The set of integers, {…, -2, -1, 0, 1, 2, …}, is closed with respect to subtraction. Please explain your answer.

6. The product 8×22 can be found by doing 8(20 + 2) =160 + 16 = 176. What property is being used to find the product?

7. True or False: 5 $\div$ 0 = 0. Explain your answer.

8. The fact that 2 + 3 = 3 + 2 is an example of what property of addition?

9. For each of the following, list whether the number is a natural number, whole number, integer, rational number, irrational number, or real number (You could have more than one listed).

   1/5:____

   1/3:____

   0.333…: ____

   12:____

8:____

10. Rewrite the following as a base-ten, decimal number;

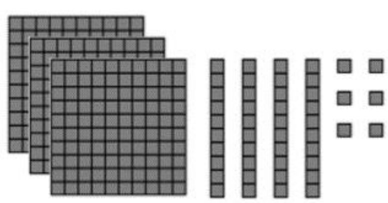
$9\times10^5+9\times10^2+9\times10^0+8\times10^{-2}+8\times10^{-3}$

11. What is the difference: $100_{(\text{base } 5)}-44_{(\text{base } 5)}$?

12. What is 329 converted to base six?

13. What is $142_{(\text{base } 5)}$ in base ten?

14. What numeral is represented by the blocks below?



15. The 5 in $13542_{(\text{base } 8)}$ is in what place value position?

16. Round 428.29981 to the nearest thousandth.

17. True or False: All numbers in this set are prime numbers: {47, 61, 73, 83, 97}.

Please explain your answer.

18. What number is both a factor and a multiple of 23?

19. Find the prime factorization of 5544.

20. Find the LCM(108, 360).

21. Find the GCF(8, 24, 52).

22. List all of the divisors of 175.

23. 2,646 is divisible by which of the following numbers (circle all that apply)?

2, 3, 4, 5, 6, 8, 9, 10, 11, 39

24. Are $\frac{5}{6}\ and\ \frac{100}{120}$ equal? Explain your answer.

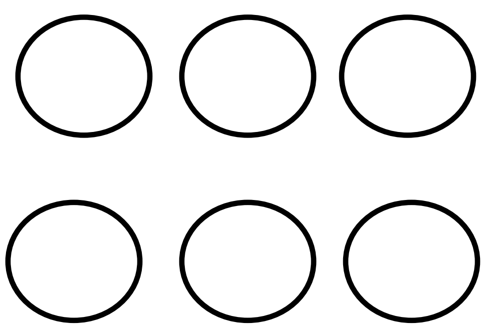
25. Place the following numbers in order from least to greatest:

1.42, -.61, $-\frac{1}{2}$, $\frac{7}{8}$, $1\frac{2}{5}$, 1.420001, $\frac{6}{7}$, $\sqrt{5}$

26. Convert 1.29% to a decimal.

27. Which container is the closest to being full, a 20 ounce coffee cup with 12 ounces of coffee in it; or, a 200,000 gallon water tank with 98,000 gallons of water in it? Please explain your answer.

28. Use the circles below to model (and find the sum) of the following: $2\frac{1}{2}+1\frac{1}{4}=?$



29. 20 is what percent of 160?

30. Find a rational number between $\frac{2}{5}$ and $\frac{1}{2}$.

31. Convert $-3\frac{3}{4}$ to an improper fraction.

32. Find the 9th term of the following sequence: 1, 3, 9, 27, …

33. Find the value of -2^4.

34. Simplify: $\frac{xy}{z} \bullet \frac{z^2 a}{x^3 y^2}$.

35. Solve for x: -3(x+5) = -4(x+5)

36. Write an equation for the pay, P, for t - hours of work if you get paid $8 an hour.

37. I have a gallon of paint that was mixed as 1 part blue to 15 parts white. I want a deeper blue paint. How much blue paint would I have to add to the original gallon to make the paint be 1 part blue to 10 parts white?

38. Sally has 12 pairs of shoes, 8 skirts, 10 blouses, and 7 pocketbooks. Assuming an outfit contains at least one pair of shoes, one skirt, one blouse, and one pocketbook, how many different outfits could Sally put together? Show your work below.

39. David has three times the money Rick does. Together, they have $400. How much money does each have?

40. The taxes on my house have increased by 20% each of the past two years. These increases have resulted in my current taxes being $2,160. What were my taxes two years ago, before either increase?

APPENDIX F

Example PBL Scenario

Pay My Bills PBL Scenario

Below is one of the PBL scenarios given to students in this study. Following group and whole class discussion of the PBL scenario, the students created and solved their own problems related to this PBL scenario. Included in this appendix are three student created problems related to this PBL scenario.

Title: Pay My Bills

Author(s): Brandon Banes

Learning Objectives:

1. The students will gain proficiency in calculating percentages and percent change.
2. The students will use inductive reasoning, linear equations and/or a variety of mathematical skills to make predictions.
3. The students will be able to calculate and interpret averages.
4. The students will learn to use estimation skills effectively and appropriately.

Target Population:

Preservice elementary teachers taking Concepts and Structure of Elementary School Mathematics

Problem Statement:

TBR students started the 2011-2012 academic year with a 6.1% increase in tuition over the 2010-2011 academic year (http://www.murfreesboropost.com/mtsu-tuition-to-climb-again-cms-17748) . Based upon the current economy, forecasts about the country's economic recovery are grim. Parents and students can anticipate a continuation of tuition increases over the next five years.

Focus Questions:

1. Using the information at http://nces.ed.gov/collegenavigator/?q=Middle+Tennessee+State+University&s=TN&l=5&ct=1&id=220978#expenses determine the cost of tuition for the 2011-2012 academic year.
2. Who are TBR students?
3. Discuss with your group the possible reasons that the tuition is increasing.
4. What are some ways that you could estimate/predict the cost of tuition for each of the remaining school years until you graduate? What mathematics have you used in the past to make predictions?
5. What bills are you currently responsible for paying? What resources contribute to your income? Are you concerned about being able to pay for school?
6. How much money will your education cost in total? Will you have student loans to repay? How much money will you owe for student loans?

7. In what county do you anticipate teaching when you finish school? What is the beginning pay for teachers in that county?
8. How much money will your monthly loan payment be when you get out of school?
9. What changes will be made to your monthly budget once you graduate (buy a house, new car, professional attire, etc.)? What will be your new monthly budget?
10. Will your income support your financial needs?

Resources:

http://nces.ed.gov/collegenavigator/?q=Middle+Tennessee+State+University&s=TN&l=5&ct=1&id=220978

http://www.ed.gov/news/press-releases/us-department-education-releases-college-cost-lists-help-students-make-informed-

http://www.murfreesboropost.com/mtsu-to-see-another-tuition-increase-cms-27632

http://www.murfreesboropost.com/tbr-approves-mtsu-tuition-increase-cms-23616

http://www.murfreesboropost.com/mtsu-tuition-to-climb-again-cms-17748

http://www.edtrust.org/dc/publication/priced-out

http://www.usatoday.com/news/education/2011-07-27-College-student-fees-budget-cuts-funding_n.htm

Student-Created Problems

1. I am currently working part time while attending classes at MTSU. I work 8:00 – 2:00 on Monday, Wednesday, and Friday earning $10.00 per hour. I am able to leave work at 2:00 each day to pick up my daughter, Lauren, from elementary school. My employer has offered to let me work until 5:00 if I want to earn extra money. Lauren would have to attend after-school care on MWF, which is available until 6:00 each day. The after-school care is $55.00 a week regardless of the number of days per week she attends. How much more or less will I make a week if I accept my boss's offer and pay to send Lauren to after-school care?
2. Rebecca wants to get an apartment of her own now that she is a senior in college. She works at Wal-Mart making $9.00 an hour and she works 40 hours a week. In order for Rebecca to qualify for an apartment she needs to make more than three times the monthly rent. Can Rebecca qualify to rent an apartment for $975.00 a month? If not, what price range would Rebecca need to search for to ensure that she qualifies to rent the apartment?
3. Sarah is an education major and is required to observe a total of 64 hours at an assigned school. The school she has been assigned is 12 miles from where she lives. Gas has recently increased in price and she will have to pay $3.60 per gallon. She drives a Honda Accord that gets 25 miles to the gallon. How much will gas be for the entire semester if she has to do 4 hours per week and goes only two days a week?

Instructor-Created Problems

1. The SGA comes up with an idea to prevent the increase in tuition and still bring in the additional revenue for MTSU. They suggest that every student participate in a donut fund raiser. The donuts will cost $5 a dozen. The donut company will keep 22.5% of all the sales and the university will get the rest of the proceeds. How many donuts will need to be sold so that students do not have to pay a higher tuition than last year? How many donuts would each student need to sale? Is that a reasonable amount of donuts to expect a student to sale?

2. Rod normally drives around Murfreesboro for fun and social interaction. His truck gets 15mpg while driving in town. In August, Rod learned of the increase in tuition. He decided to stop driving around town to save gas and thus save money. He will put this saved money toward the increase in tuition. If Rod's normal loop around town is 4.1 miles and he drives this loop 3 times a night, then how long will he have to quit driving around town? In other words, when will Rod save enough money to pay the extra tuition?

APPENDIX G

Creating Word (Story) Problems Related to PBL: Scoring Rubric

All problems must have more than one step; that is, a solution strategy must include more than one step or the problem asks more than one question where a solution to one question is used in answering another question.

Criteria (3)	0 Points	0.5 points
<u>Creativity</u>	Routine Problem Not related to PBL scenario	Non-routine, meaning no solution strategy is immediately known. Related to PBL scenario
<u>*Level of difficulty</u> Steps required Mathematics content Strategies required Reading level Length	 Single step Below or beyond expectations of MATH 1410 students Below or beyond expectations of MATH 1410 students Below or beyond expectations of MATH 1410 students Below or beyond expectations of MATH 1410 students	 Multiple steps Appropriate for MATH 1410 Ss Appropriate for MATH 1410 Ss Appropriate for MATH 1410 Ss Appropriate for MATH 1410 Ss

Solution

0 points	0.5 points	1.0 points
None submitted or work submitted is meaningless	Sufficient detail demonstrates that the student is approaching a rational solution, but major errors obstruct reaching a valid solution	Appropriate method is applied to yield a valid solution

*Level of difficulty is measured by five attributes: 1. Steps required; 2. Mathematics content; 3. strategies required; 4. reading level; and, 5. length. There is no point in creating a problem that requires mathematics or strategies that are inaccessible to students, problems that are beyond their reading level, or problems that take longer to solve than is reasonable.

APPENDIX H

Arithmetic Proficiency Test (APT)

There are five "categories" of arithmetic problems on the APT. There are order of operations problems, basic operations with whole numbers, basic operations with fractions, basic operations with decimals, and basic operations with integers. A mastery of these kinds of problems is currently expected of all eighth grade students in the state of Tennessee.

Give yourself a time limit of 30 minutes to complete these twenty problems. Show your work (use additional paper if necessary) and record your final answers in the blanks on the right. Do not use a calculator.

1. 4 + [(6 + 2) *x* 3 - 6 + 6] __________

2. (7 x 3 + 4)(24 + 4 - 5) __________

3. 8 + 6 – (4 x 2 + 1) __________

4. 19 - (10 + 5) + 3 __________

5. 378 + 129 __________

6. 3,087 - 538 __________

7. 285 x 21 __________

8. 1116 + 18 __________

9. 2u2/3 + 3u3/4 __________

10. l0ul/2 – 7u5/8 __________

11. 3u2/3 x 4u l/2 __________

12. 5u3/8 + 2u l/2 __________

For problems 13 through 16, **round your final answers to the nearest hundredth.**

13. 18.2+ 9.78 + 0.409 + 14 __________

14. 29.01 - 11.025 __________

15. 12.4 x 1.102 __________

16. 108.06 + 7.5 __________

17. (-16) + 21 + (-7) __________

18. (-6) - (-10) - 20 __________

19. (-3) x 6 x (-l) x 2 __________

20. (-18) + (-3) + 3 __________

APPENDIX I

Coding Progression

Open Codes	Second Level Codes	Third Level Codes	Main Categories with Codes
Working independently Checking with others Asking questions Answering questions Communicating needs Doubting abilities Checking with others Seeking Understanding Explaining to others Seeking Understanding Collaborating as a group Working Independently Discussing Problems Collaborating as a group Working Independently Explaining to others Asking questions Answering Questions Explaining to others Asking Questions Seeking Understanding Answering questions Explaining to others Explaining to others Teaching others Working independently Checking with others Seeking help of others Explaining to others Strategizing about expectations Asking Questions Answering others Explaining to others Making sense of answers Asking Questions	Creating problems together (with other groups) Distributing tasks Talking about personal things Communicating outside of class Evaluating problems Verifying information Strategizing about expectations Distracting atmosphere Talking off topic Monitoring others Intending on unproductivity Communicating non-verbally Breaking rules Evaluating a problem Discussing with teacher Sharing information Checking work Planning ahead Checking answers Exchanging (sharing) leadership responsibilities Recognizing a different path Working (creating problems) independently Working independently Checking with others Asking questions Answering questions Communicating needs Doubting abilities Seeking understanding Explaining to others Collaborating as a	Not communicating with group Not communicating Communicating incorrect mathematics Distributing tasks Talking about personal things Communicating outside of class Strategizing about expectations Distracting atmosphere Talking off topic Intending on unproductivity Breaking rules Planning ahead Exchanging (sharing) leadership responsibilities Doubting abilities Strategizing about expectations Not communicating Not participating Selecting strategies Talking off topic Disliking PBL Changing behaviors Talking unproductively Distracting questions Recognizing mathematics being learned Breaking rules Getting off topic Planning inadequately Waiting on others Being frustrated Organizing the group	**Working With Others to Solve Problems** **Evaluating Mathematics** Learning by solving others' problems Evaluating others mathematics Evaluating problems (and solutions) Evaluating others' mathematics Monitoring others' understanding Offering alternative strategies Making sense of calculations Recognizing alternative strategies (more productive) Looking over work Evaluating others' problems Evaluating others' explanations Doing it differently Getting different results Working others' problems Evaluating others' mathematics Realizing mistakes Evaluating others Checking with others Verifying information Making sense of answers Realizing mistakes Checking answers Recognizing differences Sharing Information Checking with others Recognizing a different path Verifying information Checking answers Evaluating a problem Sharing information Checking work Evaluating problems Monitoring others **Discussing Problems** Benefiting from group Contributing success to group dynamic Difficulties with communication Communicating mathematics

Working Independently Asking Questions Answering Questions Working independently Communicating Needs Collaborating as a group Working Independently Assessing group progress by teacher Questioning by the teacher Asking questions Asking questions Checking answers Explaining to others Questioning by the teacher Explaining to others Questioning by the teacher Explaining to others Questioning by the teacher Explaining to others Questioning by the teacher Answering questions Realizing mistakes Realizing mistakes Collaborating as a group Working Independently Asking questions Explaining to others Explaining to others Explaining to others Working Independently Checking answers Realizing mistakes Explaining to others Recognizing differences Thinking together Collaborating Thinking together	group Discussing problems Teaching others Seeking help of others Strategizing about expectations Making sense of answers Assessing group progress by teacher Questioning by the teacher Realizing mistakes Checking answers Recognizing differences Thinking together Sharing Information Not communicating Not participating Scaffolding by the teacher Selecting strategies Talking off topic Disliking PBL Changing behaviors Talking unproductively Distracting questions Realizing mistakes Creating problems Evaluating others Checking with others Teaching others Recognizing mathematics being learned Communicating needs Breaking rules Avoiding distractions Getting off topic Planning inadequately Communicating incorrect mathematics Seeking understanding Evaluating others' mathematics Seeking Help Communicating needs Doing it differently Getting different results	Learning something new Talking unproductively Strategizing about problems Alleviating anxiety through PBL Not understanding PBL Learning slowly Perceiving an increase in mathematics knowledge Unchanging perceptions of ability Positive previous experiences with mathematics Unchanging perceptions of attitude toward mathematics Uncahnging interest in mathematics Feeling anxious with PBL Increasing anxiety Decreasing anxiety Becoming compfortable Increasing perception of ability Learning with PBL Remembering mathematics Distracting classroom environment Changing perceptions of ability Negative previous feelings about mathematics Changing attitudes toward mathematics Changing anxiety Learning slowly Perceiving PBL as helping Difficulties with	Asking questions for understanding Offering help to others Persuading others Communicating needs Thinking together Collaborating as a group Discussing problems Seeking help of others Asking questions Answering questions Communicating needs Communicating non-verbally Asking questions Verifying information Seeking understanding Seeking others' help Seeking understanding Seeking Help **Teaching and Learning from Others** Learning by teaching others Learning from others Presenting to others Teaching others Explaining to others **Creating Problems** Difficulty creating problems Enjoying creating problems Creating problems Creating problems together (with other groups) Learning by creating problems **Teacher Mini-lectures and Other Facilitation** Learning from the teacher led mini-lectures Scaffolding by the teacher Facilitation by the teacher Questioning by the teacher Assessing group progress by teacher Discussing with teacher **Relying on Self for Learning** Relying on self Working (creating problems)

Explaining to others	Persuading others	group members	independently
Asking Questions	Presenting to others	Positive previous	Working independently
Answering questions	Working others’	perceptions of ability	Using previous knowledge to
Explaining to others	problems	Indifference toward	solve problems
Thinking together	Not communicating	mathematics (prior)	Using resources for learning
Working	Waiting on others	Low previous	
Independently	Being frustrated	interest in	
Checking with others	Evaluating others’	mathematics	
Recognizing mistakes	problems	Changing attitudes	
Explaining to others	Evaluating others’	toward mathematics	
Breaking Rules	explanations	Changing interest in	
Working	Seeking others’ help	mathematics	
Independently	Organizing the group	Being challenged by	
Working	Not communicating	PBL	
Independently	with group	Positive view of	
Asking Questions	Looking over work	group	
Checking answers	Learning something	Unchanging	
Evaluating a problem	new	perception of ability	
Discussing with	Offering help to others	Positive previous	
Teacher	Offering alternative	attitude toward	
Discussing a problem	strategies	mathematics	
Collaborating as a	Making sense of	Positive previous	
group	calculations	interest in	
Explaining to others	Recognizing	mathematics	
Explaining to others	alternative strategies	Perceiving PBL as	
Asking questions	(more productive)	having no effect	
Answering questions	Learning from others	Unchanging interest	
Sharing information	Talking	in mathematics	
Asking questions	unproductively	Perceiving PBL as	
Sharing information	Strategizing about	easy	
Sharing information	problems	Perceiving PBL as	
Collaborating with	Asking questions for	helpful for teaching	
group	understanding	Mediating anxiety	
Asking questions	Monitoring others’	Learning not	
Answering questions	understanding	memorizing	
Explaining to others	Alieving anxiety	(adaptable	
Sharing information	through PBL	knowledge)	
Asking questions	Not understanding	Mediating anxiety	
Answering questions	PBL	Needing to be shown	
Explaining to others	Learning slowly	Difficulties in	
Questioning by	Perceiving an increase	classroom setting	
teacher	in mathematics	Not previously	
Recognizing mistakes	knowledge	experiencing PBL	
Explaining to others	Unchanging	and PS	
Questioning by	perceptions of ability	Successful with	
teacher	Positive previous	traditional	
Recognizing mistakes	experiences with	mathematics	
Checking work	mathematics	Differentiating	
Planning ahead	Unchanging	between math and	
Planning ahead	perceptions of attitude	real world	
Asking questions	toward mathematics	Liking the idea of	
Verifying information	Unchanging interest in	PBL	

Answering questions Working independently Checking answers Asking questions Explaining to others Asking questions Explaining to others Verifying information Checking answers Exchanging leadership responsibilities Explaining to others Explaining to others Explaining to others Explaining to others Asking questions Answering questions Checking answers Asking questions Explaining to others Checking answers Asking questions Answering questions Asking questions Discussing a problem Recognizing a different path Teaching others Teaching others Working Independently Explaining to others Asking questions Working (creating) independently Planning ahead Working (creating) independently Creating problems together (with other groups) Asking a question Answering a question Explaining to others Answering a question Working (creating) independently Planning ahead Distributing tasks Planning ahead Talking about	mathematics Feeling anxious with PBL Increasing anxiety Decreasing anxiety Becoming comfortable Increasing perception of ability Learning with PBL Remembering mathematics Distracting classroom environment Changing perceptions of ability Negative previous feelings about mathematics Changing attitudes toward mathematics Changing anxiety Learning slowly Perceiving PBL as helping Difficulties with group members Positive previous perceptions of ability Indifference toward mathematics (prior) Low previous interest in mathematics Changing attitudes toward mathematics Changing interest in mathematics Being challenged by PBL Using resources for learning Positive view of group Unchanging perception of ability Positive previous attitude toward mathematics Positive previous interest in mathematics Perceiving PBL as having no effect	Previous experience with PBL Past PS in a superficial way Traditional experiences in mathematics Having anxiety Improving perceptions of ability Picking up others' slack Viewing PBL as beneficial for teaching Feeling of not learning Initial concerns Feeling stressed Feeling anxious Desiring traditional instruction Mediating anxiety by the group Not understanding PBL Unsure of benefits of PBL Devaluing learning from others Positive initial views of PBL Wanting to contribute to the group Feeling accomplished Feeling frustrated Worried about grade Feeling rushed Liking PBL Discussing group expectations Distractions from teacher Distractions from group members Feeling confused	

personal thing Planning ahead Planning ahead Distributing responsibilities Communicating outside of class Communicating outside of class Distributing responsibilities Planning ahead Communicating outside of class Evaluating problems Asking questions Planning Evaluating problems Asking questions Answering questions Verifying information Collaborating as a group Verifying information Asking questions Evaluating problems Evaluating problems Explaining to others Discussing with others Planning ahead Planning ahead Asking questions Planning ahead Explaining to others Discussing problems Strategizing about expectation Distracting Atmosphere Distracting Atmosphere Planning ahead Talking off topic Questioning by teacher Questioning by teacher Questioning by teacher Explaining to others Questioning by teacher	Unchanging interest in mathematics Perceiving PBL as easy Relying on self Using resources for learning Perceiving PBL as helpful for teaching Communicating mathematics Evaluating others' mathematics Mediating anxiety Difficulties with communication Learning not memorizing (adaptable knowledge) Learning by creating problems Learning from others Mediating anxiety Contributing success to group dynamic Using previous knowledge to solve problems Evaluating problems (and solutions) Using resources for learning Needing to be shown Difficulties in classroom setting Learning from the teacher Evaluating others mathematics Not previously experiencing PBL and PS Successful with traditional mathematics Differentiating between math and real world Liking the idea of PBL Previous experience with PBL		

Explaining to others Planning ahead Talking off topic Working Independently Monitoring others Asking questions Distracting Atmosphere Asking Questions Asking questions Explaining to others Asking questions Explaining to others Realizing mistakes Asking questions Explaining to others Explaining to others Asking questions Explaining to others Facilitation by the teacher Facilitation by the teacher Intending on unproductivity Communicating non-verbally Facilitation by the teacher Not communicating Facilitation by the teacher Facilitation by the teacher Not participating Facilitation by the teacher Not communicating Verifying information Scaffolding by the teacher Asking questions Discussing problems Discussing a problem Sharing Information Discussing a problem Sharing Information Discussing a problem Sharing Information Discussing a problem Sharing Information	Past PS in a superficial way Traditional experiences in mathematics Having anxiety Benefiting from group Enjoying creating problems Improving perceptions of ability Picking up others' slack Viewing PBL as beneficial for teaching Feeling of not learning Initial concerns Feeling stressed Feeling anxious Desiring traditional instruction Mediating anxiety by the group Not understanding PBL Unsure of benefits of PBL Devaluing learning from others Positive initial views of PBL Wanting to contribute to the group Learning by solving others' problems Difficulty creating problems Feeling accomplished Feeling frustrated Worried about grade Feeling rushed Liking PBL Discussing group expectations Learning by teaching others Distractions from teacher Distractions from group members Feeling confused		

Discussing a problem Sharing Information Asking questions Discussing a problem Sharing Information Asking questions Discussing a problem Sharing Information Discussing a problem Sharing Information Discussing a problem Sharing Information Selecting strategies Talking off topic Discussing a problem Sharing Information Asking questions Discussing a problem Sharing Information Disliking PBL Verifying Information Asking Questions Answering Questions Asking questions Discussing a problem Sharing Information Recognizing mistakes Discussing a problem Sharing Information Asking questions Disliking Asking Questions Answering Questions Talking off topic Changing behaviors Discussing a problem Discussing a problem Discussing a problem Sharing information Verifying information Asking questions Answering questions Asking questions Answering questions Discussing a problem Asking questions Discussing a problem Discussing a problem Verifying information Verifying information Discussing a problem Sharing information			

Explaining to others Sharing information Discussing a problem Explaining to others Talking off topic Distracting questions Verifying Information Realizing mistakes Explaining to others Explaing to others Working Independently Verifying information Asking questions Verifying information Discussing a problem Verifying information Talking off topic Sharing information Discussing a problem Asking questions Answering questions Sharing information Verifying information Talking off topic Asking questions Asking questions Discussing a problem Asking questions Explaining to others Talking unproductively Talking unproductively Asking questions Talking unproductively Desiring real problems Discussing the problem Talking unproductively Discussing a problem Explaining to others Creating problems Evaluating others Explaining to others Teaching others Explaining to others Explaining to others Evaluating methods			

Evaluating others Creating problems Checking with others Teaching others Asking questions Explaining to others Teaching others Realizing mistakes Planning ahead Planning ahead Facilitation by the teacher Strategizing about expectations Asking questions Recognizing mathematics learned Strategizing about expectations Strategizing about expectations Talking off topic Communicating needs Breaking Rules Talking off topic Breaking Rules Talking off topic Working independently Avoiding distractions Talking off topic Taking off topic Getting off topic Planning inadequately Breaking rules Talking off topic Explaining to others Working Independently Checking with others Explaining to others Realizing mistakes Talking off topic Checking with others Realizing mistakes Checking with others Asking questions Explaining to others Communicating incorrect mathematics Checking with others Realizing mistakes			

Asking questions Explaining to others Seeking Understanding Seeking Understanding Discussing a problem Explaining to others Checking with others Evaluating others' mathematics Explaining to others Asking questions Seeking Understanding Explaining to others Verifying information Verifying information Asking questions Asking questions Explaining to others Explaining to others Evaluating others' mathematics Evaluating others' mathematics Asking questions Explaining to others Realizing mistakes Seeking help Communicating Needs Asking questions Explaining to others Checking with others Doing it differently Seeking help Explaining to others Getting different results Explaining to others Seeking help Explaining to others Evaluating others' mathematics Persuading others Talking off topic Facilitation by the teacher Presenting to others Explaining to others Facilitation by the			

teacher Asking questions Explaining to others Facilitation by the teacher Asking questions Creating problems Working others' problems Not communicating Working others' problems Waiting on others Breaking Rules Evaluating others' problems Becoming frustrated Questioning by teacher Evaluating others' problems Evaluating others' problems Explaining to others Asking questions Asking questions Verifying information Asking questions Evaluating others' problems Explaining to others Evaluating others' problems Explaining to others Evaluating others' explanations Explaining to others Asking questions Evaluating others Evaluating others' problems Explaining to others Seeking others' help Explaining to others Asking questions Evaluating others' problems Explaining to others Evaluating others' problems Explaining to others Explaining to others			

Planning ahead Evaluating others problems Planning ahead Planning ahead Planning ahead Planning ahead Organizing the group Planning ahead Planning ahead Not communicating with group Organizing the group Organizing the group Working Independently Working Independently Not communicating Asking questions Working independently Asking questions Explaining to others Working independently Asking Questions Explaining to others Asking questions Explaining to others Verifying information Verifying Information Working Independently Explaining to others Asking Questions Working independently Breaking Rules Asking questions Discussing problems Not communicating Seeking help Not communicating Verifying information Asking questions Explaining to other Seeking the help of others Explaining to others Looking over work Looking over work			

Realizing mistakes Sharing information Explaining to others Explaining to others Discussing a problem Offering help to others Asking questions Asking questions Realizing mistakes Explaining to others Working independently Discussing problem Evaluating others' work Presenting to others Explaining to others Facilitation by the teacher Facilitation by the teacher Learning something new Questioning by Teacher Answering questions Offering alternative strategies Making sense of calculations Recognizing alternative strategies Explaining to others Facilitation by the teacher Presenting to others Explaining to others Answering questions Explaining to others Facilitation by the teacher Recognizing different strategies Explaining to others Facilitation by the teacher Recognizing different strategies Recognizing different (more productive) strategies Explaining to others			

Presenting to others Questioning by the teacher Explaining to others Recognizing alternative (more productive) strategies Teaching to others Recognizing alternative strategies Explaining to others Recognizing different strategies Explaining to others Recognizing alternative strategies Questioning by the teacher Questioning by the teacher Asking questions Explain to others Presenting to others Explaining to others Facilitation by the teacher Explaining to others Facilitation by the teacher Realizing Explaining to others Explaining to others Recognizing alternate strategies Asking questions Recognizing alternative strategies Facilitation by the teacher Teaching others Teaching others Discussing a problem Verifying information Asking questions Discussing a problem Explaining to others Discussing a problem Communicating needs Asking questions Communicating needs Communicating needs Waiting for teacher's			

help Communicating needs Planning ahead Explaining to others Communicating needs Asking questions Explaining to others Discussing a problem Teaching others Learning from others Discussing a problem Asking questions Answering questions Verifying information Explaining to others Learning from others Explaining to others Verifying information Facilitation by the teacher Asking questions Asking questions Discussing a problem Asking questions Answering questions Explaining to others Verifying information Checking with others Discussing a problem Asking questions Discussing a problem Asking questions Asking questions Explaining to others Verifying information Strategizing about expectations Asking questions Discussing a problem Asking questions Discussing a problem Explaining to others Recognizing different strategies Discussing a problem Critiquing others' mathematics Checking with others Recognizing different strategies Asking questions Communicating needs			

Asking questions Explaining to others Asking questions Waiting for teacher Explaining to others Discussing a problem Asking questions Discussing problems Explaining to others Explaining to others Creating problems Evaluating others' mathematics Creating problems Discussing problems Discussing problems Verifying information Asking questions Learning from others Creating problems Creating problems Verifying information Explaining to others Evaluating others' mathematics Discussing problems Explaining to others Discussing with teacher Creating problems Evaluating mathematics Asking questions Explaining to others Creating problems Discussing problems Recognizing alternative strategies Explaining to others Creating problems Learning from others Evaluating mathematics Explaining to others Creating problems Evaluating problems Talking off topic Asking question Explaining to others Explaining to others Creating problems Explaining to others			

Verifying information Explaining to others Evaluating others' mathematics Discussing problems Explaining to others Recognizing alternate strategies Evaluating problems Working independently Creating problems Working independently Discussing problems Discussing problems Explaining to others Discussing problems Explaining to others Explaining to others Discussing problems Explaining to others Explaining to others Discussing problems Waiting for teacher Waiting for teacher Talking off topic Asking questions Explaining to others Talking unproductively Waiting for teacher Talking off topic Strategizing about problems Explaining to others Explaining to others Strategizing about expectations Teaching others Asking questions (for understanding) Teaching others Asking questions (for understanding) Evaluating others' mathematics Explaining to others Discussing with teacher Asking questions Asking questions			

Checking with others Monitoring others' understanding Teaching others Talking off topic Talking off topic Facilitation by the teacher Discussing with the teacher Discussing problems Monitoring others' understanding Talking off topic Explaining to other Asking questions Explaining to others Strategizing about expectations Creating problem Working independently Discussing problems Working independently Strategizing about expectation Discussing problem Asking questions Answering questions Discussing problems Explaining to other Discussing problems Evaluating others' mathematics Explaining to others Discussing problems Discussing problems Working independently Evaluating others' (her own) mathematics Explaining to others Evaluating others' mathematics Explaining to others Working independently Talking off topic Facilitation by the teacher			

Negative previous experiences Disliking “traditional” math Unchanging attitude Preferring PBL over “traditional” Unchanging interest in mathematics Feeling anxiety Alleviating anxiety through PBL Not understanding PBL Learning slowly Perceiving an increase in mathematics knowledge Learning from others Figuring out mathematics Facilitation by the teacher Explaining to others Unchanging perceptions of ability Positive previous experiences with mathematics Unchanging perception of attitude toward mathematic Unchanging interest in mathematics Feeling anxious with PBL Increasing anxiety Decreasing anxiety Learning from others Becoming comfortable Increasing perception of ability Learning with PB Remembering mathematics Distracting classroom environment Teaching others Changing perceptions of ability Negative previous			

feelings about mathematics Changing attitude toward mathematics Changing attitudes toward mathematics Unchanging interest in mathematics Feeling anxious Changing anxiety Not understanding PBL Learning slowly Increased perception of ability Perceiving PBL as helping Remembering mathematics Difficulties with group members Teaching others Increasing perceptions of ability Positive previous perception of ability Indifference toward mathematics (prior) Low Previous interest in mathematic Changing attitude toward mathematics Changing interest in mathematic Being challenged in PBL Increasing perceptions of ability Perceiving PBL as helping Using resources for learning Positive view of group Teaching others Unchanging perception of ability Positive previous attitude toward mathematics Positive previous interest in mathematics			

Unchanging attitudes toward mathematics Perceiving PBL having no effect Unchanging interest in mathematics Perceiving PBL as easy Increasing perception of ability Relying on self Using resources for learning Positive view of group Recognizing different strategies (more productive) Explaining to others Improving perception of ability Relying on self Relearning mathematics Recognizing alternative strategies (more productive) Explaining to others Facilitation by the teacher Explaining to others Perceiving as helpful to teaching Learning mathematics Learning from others Asking questions Challenging (PBL) Relearning mathematics Discussing problems Recognizing alternative strategies Communicating mathematics Viewing PBL as helpful for teaching Learning from others Feeling anxious Relying on self Learning from other Recognizing alternative strategies Recognizing			

alternative strategies Learning from others Learning and Teaching (possible in vivo code) Asking questions Perceiving PBL as helpful Learning from others Recognizing alternative strategies (more productive) Relying on self Perceiving improved ability Recognizing alternate strategies Learning from others Teaching others Viewing PBL as helpful to teaching Not recognizing the mathematics Facilitation by the teacher Discussing a problem Asking teacher Relearning mathematics Benefitting from group instruction Understanding deeper Viewing PBL as helpful for teaching Facilitation by the teacher Being anxious Discussing a problem Unchanging anxiety Benefitting from group Not viewing PBL as helpful for teaching Communicating mathematics Viewing PBL as helpful for teaching Relearning mathematics Learning from others Feeling frustrated by problems			

Increasing anxiety Relying on self Mediating anxiety Difficulties with verbal communication Showing work Showing work Recognizing alternative strategies Learning mathematics Recognizing alternative strategies (more productive) Different past experiences Communicating with others Viewing PBL as helpful for teaching Deepening understanding Explaining to others Viewing PBL as helpful for teaching Learning not memorizing (adaptable knowledge) Learning by creating problems Learning from others PBL mediating anxiety Increasing perceptions of ability (confidence) Different past experiences Relearning through PBL Increasing perceptions of ability Contributing success to group dynamic Using previous knowledge to solve problems Evaluating problems Working independently Contributing success to group Evaluating problems (and solutions)			

Learning from others Relying on self Relying on self Evaluating problems (and solutions) Using previous knowledge to solve problems Learning from others Contributing success to group dynamic Using resources for learning Using previous knowledge to solve problems Learning from teacher Contributing success to the group Learning from others Needing to be shown Difficulties in classroom setting Learning from the teacher Difficulties with groups Needing to be shown Difficulties with groups Learning mathematic Contributing success to group work Contributing success to group work Relearning mathematics Relying on self Feeling Anxious Strategizing to mediate anxiety Feeling Anxious PBL mediating anxious feelings Relearning mathematics Using resources for learning Learning from the teacher Improving perceptions of anxiety			

Previous negative perceptions of ability Unchanging perception of ability Learning from the teacher Recognizing alternative strategies (more productive) Recognizing alternative strategies (more productive) Improving anxiety Evaluating others mathematics Explaining to other Learning from others Recognizing different strategies Not previously experiencing problem solving and PBL Successful with traditional mathematics Differentiating between math and real world Not experiencing problem solving and PBL History of anxiety Indifference Differentiating between math and real world Negative Prior views of ability Differentiating between math and real world Liking the idea of PBL Differentiating between math and real world Previous experience with PBL (not in mathematics) Liking the ideas of PBL Differentiating			

between math and real world Negative prior perceptions of ability Giving up quickly Believing in intrinsic math ability vs working hard Past PS in a superficial way Positive previous perceptions of ability Not previously experiencing PBL and problem solving Past experiences "traditional" Not previously experiencing PBL and problem solving Negative perceptions of ability Perceiving PS as beneficial to teaching Liking the idea of PBL Recognizing alternative strategies Not previously experiencing PBL and PS Not previously experiencing PBL and PS Not previously experiencing PS and PBL Past experience PS Differentiating between math and real world Getting anxious when stuck Previous experience with PBL (not in mathematics) Positive previous attitude toward PS Positive previous perceptions of ability Past experience with PS			

Not experiencing PBL and PS in previous math classes Experiencing PBL in other classes Valuing PS Differentiating between math and real world Previous PS experiences Not experiencing PS Negative perceptions of ability Previous PS experiences in math Feeling anxious Previous experiences with PS in math Positive perception of ability Traditional experiences in mathematics Persevering with problems Traditional previous math experiences Traditional previous math experiences Differentiating between math and real world Recognizing alternative strategies Not experiencing PS and PBL in math classes Previous experiences with PBL (not in math) Not experiencing PS in previous math classes Not experiencing PS in previous math classes Getting frustrated by PS Differentiating between math and real life			

Having anxiety Having anxiety Positive perceptions of ability Getting frustrated Traditional experience in mathematics Getting frustrated Having anxiety Traditional previous experiences in math Positive previous perceptions of ability Positive previous perceptions of ability Not experiencing PS in math Traditional previous experiences in math Liking the idea of PBL Differentiating between math and real world Previous experiences with PBL (not in math) Distracting classroom environment Teaching others Feeling of not learning Initial concerns Initial concerns Feeling stressed Initial concerns Feeling stressed Initial concerns Desiring traditional instruction Feeling stressed Desiring traditional instruction Initial concerns Benefitting from the group Recognizing alternative strategies Mediating anxiety by the group Feeling stressed Initial concerns Liking the idea of			

PBL Initial concerns Recognizing different strategies Viewing PBL as helpful for teaching Benefitting from the group Initial concerns Initial concerns Initial concerns Recognizing alternative strategies Desiring traditional instruction Initial concerns Explaining to others Initial concerns Initial concerns Feeling stressed Initial concerns Desiring traditional instruction Not understanding PBL Unsure of benefits of PBL Initial concerns PBL mediating anxiety Desiring traditional instruction Devaluing learning from classmates (where should knowledge come from) Feeling stressed Feeling anxious Desiring traditional instruction Initial concerns Viewing PBL as beneficial Initial concerns Positive initial views of PBL Benefitting from group Wanting to contribute to group Relying on self			

Liking the idea of PBL Desiring traditional instruction Viewing PBL as potentially beneficial Initial concerns Desiring traditional instruction Benefitting from group Initial concerns Initial concern Picking out information Looking for the most productive strategy Evaluating answers Picking out information Answering all questions Evaluating your solutions Being organized Getting the answer Being organized Being organized Evaluating your answer Evaluating your answer The process is most important The answer Knowing what to do Getting the answer Picking up others’ slack Learning by solving others’ problems Difficulty creating problems Feeling accomplished Feeling frustrated Worrying about grade Feeling overwhelmed and frustrated Challenging Desiring traditional instruction Difficulties with			

group Picking up others' slack Difficult to create problems Difficult to create problems Learning from others Learning from others Difficult to create problems Benefitting from group Difficult creating problems Feeling rushed Benefitting from group Benefitting from group Feeling rushed Benefitting from group Benefitting from group Liking PBL Understanding PBL Discussing group expectations Benefiting from group Strategizing about expectations Distracting classroom environment Benefitting from group Breaking rules (or misunderstanding) Feeling success Difficulty with group Benefitting from group Learning by teaching others Enjoying creating problems Evaluating problems Benefitting from group Feeling rushed			

Enjoying creating problems Feeling rushed Distractions from teacher Distracting group member Feeling confused Benefiting from group Learning from others Feeling stressed Feeling rushed Picking up others' slack Evaluating others' mathematics Benefiting from group Enjoying creating problems Picking up others' slack Viewing PBL as beneficial Difficulty with group Benefiting from group Enjoying creating problems Improving perceptions of ability Viewing PBL as beneficial for teaching Group difficulties Disliking PBL Disliking PBL Disliking PBL Difficulties with group Picking up others' slack			

APPENDIX J

IRB Approval

MIDDLE TENNESSEE STATE UNIVERSITY

July 31, 2012

Brandon Banes and Dr. Diane Miller

Department of Mathematics

bcb3u@mtmail.mtsu.edu ; Diane.Miller@mtsu.ed

Protocol Title: "A Study of How Preservice Elementary Teachers Learn Mathematics through Problem-Based Learning and Problem Solving: Working Toward a Grounded Theory"

Protocol Number: 13-020

Dear Investigator(s),

I have approved your study at the exempt level. The exemption is pursuant to 45 CFR 46.101(b) (1 and 2). This is because your study consists of educational type survey materials, and information is obtained in such a manner that human subjects cannot be identified.

You will need to submit an end-of-project report to the Office of Compliance upon completion of your research. Complete research means that you have finished collecting data and you are ready to submit your thesis and/or publish your findings. Should you not finish your research within the three (3) year period, you must submit a Progress Report and request a continuation prior to the expiration date. Please allow time for review and requested revisions. Your study expires on July 31, 2015.

Any change to the protocol must be submitted to the IRB before implementing this change. According to MTSU Policy, a researcher is defined as anyone who works with data or has contact with participants. Anyone meeting this definition needs to be listed on the protocol and needs to provide a certificate of training to the Office of Compliance. Once your research is completed, please send us a copy of the final report questionnaire to the Office of Compliance. This form can be located at www.mtsu.edu/irb on the forms page.

Also, all research materials must be retained by the PI or faculty advisor (if the PI is a student) for at least three (3) years after study completion. Should you have any questions or need additional information, please do not hesitate to contact me.

Sincerely,

Emily Born

Compliance Officer

615-494-8918

APPENDIX K

IRB Consent Form

Principal Investigators: Brandon Banes and Diane Miller (Major Professor)

MTSU IRB Approved

Date: 7/31/2012

Study Title: A Study of How Preservice Elementary Teachers Learn Mathematics through Problem-Based Learning and Problem Solving: Working Toward a Grounded Theory

Institution: Middle Tennessee State University

Name of participant: ____Age: __________ The following information is provided to inform you about the research project and your participation in it. Please read this form carefully and feel free to ask any questions you may have about this study and the information given below. You will be given an opportunity to ask questions, and your questions will be answered. Also, you will be given a copy of this consent form.

Your participation in this research study is voluntary. You are also free to withdraw from this study at any time. In the event new information becomes available that may affect the risks or benefits associated with this research study or your willingness to participate in it, you will be notified so that you can make an informed decision whether or not to continue your participation in this study.

For additional information about giving consent or your rights as a participant in this study, please feel free to contact the MTSU Office of Compliance at (615) 494-8918.

1. Purpose of the study:

You are being asked to participate in a research study because the researchers want to examine the potential benefits of teaching mathematics to preservice elementary teachers using Problem-Based Learning (PBL) and problem solving as the mode of instruction and to determine how students develop mathematical knowledge through Problem-Based Learning.

2. Description of procedures to be followed and approximate duration of the study:

If you choose to participate in this study, the duration will be the entire fall 2012 semester. Depending on the section of MATH 1410 you are enrolled, you will either be taught using traditional modes of instruction or you will be taught through PBL and problem solving. You will be given three pretests at the beginning of the study to assess your mathematics anxiety, attitude toward mathematics, and mathematics content knowledge. Throughout the semester, the researchers may collect your responses to instructor designed writing prompts. There will be no identifying information on the writing prompts. At the end of the semester, you will be given the same three tests to assess your mathematics anxiety, attitude toward mathematics, and mathematics content knowledge again. Additional data will be collected from participants learning through PBL. These data are collected work, responses to writing prompts, interviews, and video taping of class meetings. If interviewed, you will sign a separate interview consent form.

3. Expected costs:

None

4. Description of the discomforts, inconveniences, and/or risks that can be reasonably expected as a result of participation in this study:

There will be no discomforts, inconveniences, or risks from participating in this study beyond the discomforts, inconveniences, or risks from participating in a college level mathematics class. Compensation in case of study-related injury:

N/A

5. Anticipated benefits from this study:

a) The potential benefits to science and humankind that may result from this study are

1) Knowledge of the use of PBL in the mathematics preparation of preservice elementary teachers. 2) This study will inform mathematics educators about the use of PBL. 3) This study will help the mathematics community to better prepare elementary teachers to teach mathematics, improve their mathematical content knowledge, lower their anxiety toward mathematics, and improve their attitude toward mathematics. 4) Bettermathematics instruction

at the elementary level will help to improve the performance in mathematics of our youngest students, which will reduce the likelihood of those students having negative attitudes towards mathematics and mathematics anxiety.

b) The potential benefits to you from this study are

1) You will receive mathematics instruction that will potentially help you to be better at mathematics, lower your mathematics anxiety, and improve your attitude toward mathematics.

2) You will gain knowledge that you will use when teaching elementary mathematics.

6. Alternative treatments available:

The activities involved in this research are part of the normal classroom activities. There are no alternative treatments available.

7. Compensation for participation:

The activities involved in this research are part of the normal classroom activities. Thus, there will be no compensation for participation.

8. Circumstances under which the Principal Investigator may withdraw you from study participation:

You may ask to be withdrawn from the study at any time, although assignments are still part of the regular classroom activities and are still required for class purposes.

9. What happens if you choose to withdraw from study participation:

These activities are part of the regular class requirements and still must be completed, but if you choose you may opt out of having your data used as part of the research.

Contact Information. If you should have any questions about this research study or possible injury, please feel free to contact Brandon Banes at 615-218-8360 or my Faculty Advisor, Dr. Diane Miler at 615-898-5472.

10. Confidentiality. All efforts, within reason, will be made to keep the personal information in your research record private but total privacy cannot be promised. Your information may be shared with MTSU or the government, such as the Middle Tennessee State University Institutional Review Board, Federal Government Office for Human Research Protections, if you or someone else is in danger or if we are required to do so by law.

11. STATEMENT BY PERSON AGREEING TO PARTICIPATE IN THIS STUDY

I have read this informed consent document and the material contained in it has been explained to me verbally. I understand each part of the document, all my questions have been answered, and I freely and voluntarily choose to participate in this study.

Date: ____________________

Signature of patient/volunteer: _____________________

Consent obtained by: _____________________________

Date: ____

Signature: ______________________________________

Printed Name and Title: ___________________________

APPENDIX L

IRB Interview Consent Form

MTSU IRB Approved
Date: 7/31/2012
Principal Investigators: Brandon Banes and Diane Miller (Major Professor)
Study Title: A Study of How Preservice Elementary Teachers Learn Mathematics through Problem-Based Learning and Problem Solving: Working Toward a Grounded Theory
Institution: Middle Tennessee State University
Name of participant: ____ Age: ____________

The following information is provided to inform you about the interview and your participation in it. Please read this form carefully and feel free to ask any questions you may have about this interview and the information given below. You will be given an opportunity to ask questions, and your questions will be answered. Also, you will be given a copy of this consent form. Your participation is voluntary and you are also free to withdraw at any time.

You are being asked to participate in this interview because your responses will allow the researchers to understand your experiences with Problems Based Learning activities in the mathematics classroom. Your responses will be audio taped and transcribed. No identifying information will be collected during the interview.

You may get emotional when sharing your experiences. We can pause to rest at any time during the interview or stop if you choose to do so. However, this is an opportunity to share your story helping to preserve the past and hopefully enjoy yourself as well.

All efforts, within reason, will be made to keep the personal information in your research record private but total privacy cannot be promised. Your information may be shared with the MTSU Institutional Review Board, the Office of Human Research Protections, if you or someone else is in danger or if we are required to do so by law.

If you should have any questions about this interview please feel free to contact Brandon Banes at 615-218-8360 or my Faculty Advisor, **Dr. Diane Miller at 615-898-5472.** For additional information about giving consent or your rights as a participant in this interview, please feel free to contact the Office of Compliance at (615) 494-8918.

STATEMENT BY PERSON AGREEING TO PARTICIPATE IN THIS INTERVIEW

I have read this informed consent document and the material contained in it has been explained to me verbally. I understand each part of the document, all my questions have been answered, and I freely and voluntarily choose to participate in this interview.

____

Date Signature of Interviewee

Consent obtained by:

____

Date Signature of Interviewer(s)

____

Printed Name and Title