LOSS AVERSION AND THE EQUITY PREMIUM PUZZLE: EVIDENCE FROM QUANTITATIVE EXPERIMENTS

BY

YUANYUAN CHEN (CATHERINE)

A DISSERTATION SUBMITTED TO
THE GRADUATE FACULTY OF
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ABSTRACT

This dissertation undertakes both quantitative and empirical analyses of a Dynamic Stochatic General Equilibrium (DSGE) asset pricing model addressing an issue that has contributed to the empirical failure of these models: household inter-temporal preferences. The preferences studied have two parts: loss aversion and narrow framing. Loss aversion implies an agent's utility falls by more from a loss than it rises from an equal sized gain. Narrow framing is modelled by preferences that are defined over the differences in equity and risk-free bond returns. Towards these goals, this dissertation makes two advancements. First, a Hybrid Perturbation-Projection Method is introduced and evaluated to illustrate problems with current solution methods in estimation. Second, a prior predictive analysis is undertaken to find the distribution that describe the parameters of loss aversion and narrow framing preferences.

The Hybrid Perturbation-Projection Method, that combines in a less precise but fast perturbation method with a change of variables and projection algorithm, is shown to be an accurate and speedy mechanism well suited for structural estimation. The structural estimation conducted in a prior predictive analysis indicates that loss aversion and narrow framing are not a global solution to the Equity Premium Puzzle. That is, other theories must be incorporated with loss aversion and narrow framing for the equity premium to be reconciled.

To My Parents,

Yunmei Lu and Zhuan Chen

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CHAPTER I

INTRODUCTION

The equity premium is the observed excess of stock returns over the risk-free rate. The Equity Premium Puzzle (EPP) stems from the inability of economists to predict the magnitude of the premium using the standard neoclassical asset pricing models. These models generally predict a magnitude substantially less than the observe premium. For example, Mehra and Prescott (1985) found that the average equity premium of 6% could be reconciled with the predictions of the neoclassical asset pricing models only by making unrealistic assumptions about consumer risk aversion.

This dissertation claims that one problem with past studies of the equity premium is the form of the utility function used to explain the behavior of economic agents. In contrast to the usual assumptions, this dissertation focuses on the Loss Aversion/Narrow Framing (LANF) hypothesis advocated by Benartzi and Thaler (1995). Their work suggested LANF as a potential explanation for EPP and provided a platform for the later studies by Barberis and Huang (2001, 2004, and 2008) and Grüne and Semmler (2008).

The goal of this dissertation is to estimate the equity premium by building the LANF's effect into a Dynamic Stochastic General Equilibrium Model (DSGE). One drawback to this approach noted in the literature is the complexities introduced into DSGE by the LANF function. However, the contribution of this dissertation is to provide both a solution procedure and estimation method that overcome these difficulties. Essentially, a Hybrid Perturbation-Projection Method (HPP) is used to solve the DSGE asset pricing model and then a prior predictive analysis is undertaken to estimate the LANF's effect on the equity premium.

1.1 Equity Premium Puzzle

The quandary associated with the equity premium arises from the inability of traditional neoclassical theories of asset pricing to explain the magnitude of the excess returns on risky assets over the risk-free rates. Merha and Prescott (1985) attempted to reconcile this difference by hypothesizing a Lucas'(1978) pure exchange model where household maximized their lifetime utility of consumption and production grew according to a Markov process. Estimation of the model failed to predict an equity premium closely matching the data. Nevertheless, the Merha and Prescott study provided an impetus for further work on this important issue. Two specific points in their work are worth considering. First, they noted that the EPP might be more of an issue of why the risk-free rate is low rather than why the risky rate is so high. Second, they contended that a successful solution to the EPP requires not only matching the estimated premium to the observed data, but also being able to explain the fluctuations in the premium's value. Further studies have investigated these points.

For instance, Mankiw and Zelds (1991) examined the consumption behaviors for non-stockholders and stock-holders using Panel Study of Income Dynamics (PSID) data and found that stockholders had higher consumption volatilities than non-stockholders. They concluded that the higher consumption volatility for stockholders could explain the risk premium on equities and consequently, why stock returns were so much higher than the risk-free rates; at least in the context of standard capital asset pricing model. Dathine and Donaldson (2002) approached the issue by estimating a DSGE business cycle model that included three economic agents - workers who were not buying stocks, stockholders without labor contracts, and firms. They justified the equity premium by assuming that operating leverage increased the variability of the residual payments to the firms' owners. Nevertheless, their estimates of the equity premium could not be reconciled with previous empirical observations. In yet another approach, Boldrin, Christiano and Fisher (2001) estimated a real business cycle DSGE model with habit persistence and limited intersectional factor mobility. In their model, habit persistence accounts for technology shock's effect on household sector's ability to smooth consumption and therefore impacts asset returns. The model generated high stock returns, which is consistent with the EPP. However, the exaggerated rate of return on the risk-free asset undermined their explanation of the EPP.

In a similar study, Guvenen (2009) employed a two-agent model (labor and equity holders) with limited stock market participation and heterogenous elasticity of inter-temporal substitution in consumption. Guvenen postulated that economic agents maximized utility assuming a Cobb-Douglas functional form. Labor (non-equity holders) was assumed to have a low Elasticity of Inter-temporal Substitution (EIS) in smoothing its life-cycle income, while stockholders with high EIS demanded high expected premiums for bearing the risk associated with volatile residual returns. Estimations of the model were unable to reproduce the high equity premium conjected by the EPP. Finally, Guvenen (2009) replaced the utility function advocated by Greenwood, Hercowitz, and Huffman (GHH), but he still was not able to generate high equity premium. None of these studies discussed above have been able to reconcile EPP in the general equilibrium framework first developed by Mehra and Prescott (1985). Allowing households to have LANF preferences may be a way though this bottleneck. Why this could be the case is discussed in the section.

1.2 *Loss Aversion/Narrow Framing Preferences*

The idea of LANF preferences was developed by Kahneman and Tversky (1979) in their attempt to explain several anomalies in the decision patterns of individuals in a variety of experimental situations. In their experiments, individuals weighted probabilistic losses more than certain gains even through the expected values of the choices were the same. That is, the perspective pain of a small loss tended to more than offset a potential gain. For example, one of their experiments asks 72 respondents to choose between the following vacation trips.

(A) a 50% chance to win a three-week tour of England, France and Italy;

(B) a sure win of a one-week tour of England.

Twenty-two percent of the respondents selected option (A) and 78% chose option (B), even though the expected outcomes were identical.

Narrow framing refers to the context in which a decision is made. For example, an individual may decide to purchase a stock without considering other income-earning alternatives or the impact of the decision on portfolio diversification. The focus is on the context of the individual decision. Or, as expressed by (Barberis and Huang, 2008), a narrowly framed person gains utility directly from the outcome of a gamble even if this is just one of many potential possibilities that could affect aggregate risk. They explained narrow framing behavior in terms of either "regret" or "inaccessibility". Regret is the disutility arising from realization that an individual would be better off today had she/he not taken a particular gamble. All choices (gambles) subject individuals to the possibility of regret.

The second explanation of narrow framing - inaccessibility - was suggested by Kahneman and Tversky (1979). Accessibility refers to the distribution of the outcomes of a specific choice being more accessible than the impact of the choice on the possible distributions of other choices. Thus, the decision is "narrowly framed" around the specific choice and not its impact on, say, overall portfolio risk. Barberis and Huang (2008, p.7) summarized this point succinctly "if agent thinks about a gamble intuitively, the distribution of the gamble, taken alone, may play a more important role in decision-making than would be predicted by traditional utility functions defined only over wealth or consumption". According to LANF, individuals make decisions about stock market choices based on the narrow frame of that choice and the potential regret of that decision. Combining narrow framing with loss aversion provides a theoretical basis for the equity premium.

LANF preferences have been incorporated into some extant studies of EPP, but in a partial equilibrium framework. One important paper is Benartzi and Thaler (1995). They included a discrete loss aversion function in their partial equilibrium model. In this case, people demanded a large premium to accept return variability. Estimates of the model failed to generate high equity premium, but encouraged the authors to suggest that further improvements in the preference structure might lead to fruitful results.

Other studies adopting LANF preferences include Barberis, Huang, and Santos (2001), Barberis and Huang (2004 and 2008), and Grüne and Semmler (2008) . Barberis, Huang, and Santos (2001), Barberis and Huang (2004 and 2008), included LANF preferences in a pure endowment economy, similar to the partial equilibrium model of Benartzi and Thaler (1995). However, they were only able to generate equity premiums in models where the parameters of technology shocks and loss aversion were unreasonably high. Grüne and Semmler (2008) incorporated LANF preferences in a production (in contrast to an endowment) economy. Again their estimates of the equity premium were high but only under extreme outcomes of other parameters.

This dissertation includes LANF preferences in a DSGE asset pricing model. The goal is to determine whether high equity premiums can be generated in models with reasonable estimates of loss aversion and other parameters.

1.3 Quantitative Advancement

To facilitate the experiment, two quantitative techniques are brought forward in this dissertation. To overcome the complexities with solving DSGE asset pricing models, a Hybrid Perturbation-Projection Method (HPP) is applied in the next Chapter. First introduced by Judd (1996, 2002) and later studied by Fernández-Villaverde and Rubio-Ramirez (2006), this method has been shown to accurately solve basic neoclassical growth models. The advantages of the method are that they are accurate and fast. Indeed, Judd (1996) shows that this method can increase accuracy by two orders of magnitude in a simple problem. He also shows that one is capable of improving the quality of the final approximation at little computational cost. In this dissertation, I adopt the HPP in a DSGE asset pricing model and compare it with a popular solution method – Value Function Iteration (VFI). The DSGE asset pricing model comprises a non-linear proportion that regular first-order log-linearization method is inadequate to solve. With the combination of higher-order perturbation and projection methods, this HPP method simplifies the computation for the solution without sacrificing the accuracy of the solution. Another achievement in this dissertation is the prior predictive analysis, basically an iterative Bayesian technique, for DSGE asset pricing model estimation. The analysis takes a similar form of Geweke and Whiteman (2006) and Geweke (2010). Using this technique with reasonably specified priors I find that no combination of estimations can replicate an equity premium of 6%.

1.4 Organization of the Dissertation

The dissertation is organized as follows. Chapter 2 introduces and evaluate a new solution method for DSGE asset pricing models. Chapter 3 conducts a prior predictive analysis to estimate an asset pricing model with LANF preferences. Chapter 4 concludes the dissertation.

CHAPTER II

A HYBRID PERTURBATION-PROJECTION METHOD FOR SOLVING DSGE ASSET PRICING MODELS

2.1 Introduction

The use of Dynamic Stochastic General Equilibrium (DSGE) asset pricing models has gained increasing popularity among financial economists because they describe agents' optimizing behavior (e.g., Chen 2012; Guvenen 2009; Danthine and Donaldson 2002; Boldrin, Christiano and Fisher 2001). However, the first-order perturbation solution method typically found in the DSGE literature have long been recognized to be inappropriate in asset pricing models where uncertainty is a key determinate for difference in asset returns (Mehra and Prescott 1985). Essentially, the certainty equivalence implied by first-order perturbation washes out differences in asset returns from risk. Additionally, recent asset pricing models involve extreme non-linear parts (e.g., Chen 2012; Guvenen 2009; Danthine and Donaldson 2002; Boldrin, Christiano and Fisher 2001; Fernández-Villaverde, Binsbergen, Koijen and Rubio-Ramirez 2010; Caldara, Fernández-Villaverde, Rubio-Ramirez, and Yao 2012; Ruge-Murcia 2012) that are not easily solvable by log-linearization.

Studies of DSGE asset pricing models have relied on Value Function Iteration (VFI) methods defined over discrete grids (e.g., Guvenen 2009; Danthine and Donaldson 2002; Caldara, Fernández-Villaverde, Rubio-Ramirez, and Yao 2012). Unfortunately, this method is inefficient in a large state-space; the curse of dimensionality means that the time it takes to find a solution on a grid increases exponentially with the state space. In this Chapter, I explore the applicability of a Hybrid Perturbation-Projection Method (HPP) in solving of recent complex DSGE asset pricing models. The HPP method, first introduced by Judd (1996) for regular DSGE models, combines an imprecise N'th-order perturbation solution and a projection algorithm (via change of variables) to improve the solution's accuracy. This method, as demonstrated in Judd (1996, 2002) and Fernández-Villaverde and Rubio-Ramirez (2006), has been shown to greatly improve the precision of simple DSGE models.

In this Chapter, I study HPP's solution properties when a specific kind of complexity is introduced into a DSGE model. More specifically, I consider a DSGE asset pricing model. This model has standard non-linearities in both household preferences and firm production technologies. Additionally, the asset pricing markets introduces their own type of non-linearities; leverage can force a nonlinear wedge between asset rates. The asset pricing model is in the same spirit as the DSGE asset pricing model of Chen (2012, Chapter 3) albeit without Loss Aversion/Narrow Framing (LANF) preferences.

With these extreme non-linearities, it may be the case that HPP fails to correctly and efficiently describe the economies' equilibrium. Therefore, my experiment is to evaluate the performances of HPP relative to VFI on discrete grid states in asset pricing models. To conduct this experiment, I solve the DSGE asset pricing model using both HPP and VFI methods. I then compare the methods along two dimensions. In the first dimension, I evaluate how accurate the solution methods are in describing the equilibrium allocations and prices. This is achieved by computing the implied distribution of the Euler Equation Errors (EERs) along the simulated path. Comparing the EERs is natural since they are on average zeros in equilibrium. In addition, the errors are unit-free, which facilitates comparisons across different solution methods.

The second dimension focuses on the speed (as opposed to accuracy) of the competing solution methods. Theoretically, the HPP method should be speedier since the solution method does not involve computation of the equilibrium in a large number of points. Though computational speed is not unit-free (it is in seconds), computational speed is a standard method for evaluation algorithm efficiency in computer science.

In terms of results, I find that VFI is superior in accuracy to HPP only around the steady state. However, as the one moves to the tails of the state space, HPP is more accurate than VFI. In other words, the HPP method is more stable (lower error variance). Additionally, the VFI method is computationally slow; it takes 151.27 minutes longer on an Intel Celeron(R) 2.53 GHz Microsoft Windows XP system. By reducing the size of the grid, the VFI method can approach an equal speed of the HPP method. Unfortunately, the accuracy of VFI method in this environment deteriorates and therefore it is no longer superior to HPP at any state. In all, the results of my study show that HPP method is suitable for both DSGE and DSGE asset pricing models since it is: (i) as accuracy as VFI, and (ii) the computation time is relatively small.

The remaining part of the Chapter is organized as follows. Section 2.2 details the Hybrid- Perturbation Projection Method. Section 2.3 describes the DSGE asset pricing model with two different cases. Section 2.4 compares the results from the Hybrid-Perturbation Projection Method and from discrete grid search and value function iterations. Section 2.5 concludes the Chapter.

2.2 Hybrid Perturbation-Projection Methods

2.2.1 Perturbation with COV

To understand how COV works, consider a simple example from Judd (2002). At first, the researcher has a basic second order Taylor series expansion of a function

$f(x)$:

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{1}{2} f''(a)(x-a)^2 \tag{1}$$

where x has been expanded around a.[1] The COV is then defined by $y = Y(x)$ with an inverse function existing as $x = X(y)$. The COV finds $g(y) = f(X(y))$ at $y = b = Y(a)$. Note that $g(y)$ can be approximated with the Chain Rule by:

$$\begin{aligned} g(y) &= f(X(y)) \\ &\approx f(X(b)) + f'(X(b))X'(b)(y-b) + \ldots \\ &\frac{1}{2}(X'(b)^2 f''(X(b)) + f'(X(b))X''(b))(y-b)^2 \\ &= f(a) + f'(a)X'(b)(y-b) + \ldots \\ &\frac{1}{2}(X'(b)^2 f''(a) + f'(a)X''(b))(y-b)^2. \end{aligned}$$

2.2.2 Logarithmic COV

More concretely, suppose $a = 1$, meaning x is expanded around 1 and the COV is $y = Y(x) = \log(x)$. Then, we immediately see that $b = 0$ and the inverse function is $x = X(y) = \exp(y)$. As a result, $X'(b) = 1$ and $X''(b) = 1$. The COV expansion is thus:

$$f(a) + f'(a)\log(x) + \frac{1}{2}(f''(a) + f'(a))\log(x)^2, \; .$$

or

$$f(1) + f'(1)\log(x) + \frac{1}{2}(f''(1) + f'(1))\log(x)^2,$$

where $\{f(a), f'(a), f''(a)\}$ are presumed to be known from Euler equations.

2.2.3 Power COV

As another example, suppose $a = 1$ and the COV is $y = \alpha x^3$. That is:

[1] In a dynamic model, $f(x)$ is typically the solution for the next period x (i.e., $x_{t+1} = f(x_t)$).

$$Y(x) = \left[\begin{array}{c} y \end{array}\right] = \left[\begin{array}{c} \alpha x^{\beta} \end{array}\right].$$

The inverse function is:

$$X(y) = \left[\begin{array}{c} \left(\frac{y}{\alpha}\right)^{1/\beta} \end{array}\right].$$

As a result,

$$\begin{aligned} b &= \alpha \\ X'(b) &= \frac{1}{\alpha\beta}\left(\frac{b}{\alpha}\right)^{1/\beta-1} = \frac{1}{\alpha\beta} \\ X''(b) &= -\frac{\beta-1}{(\alpha\beta)^2}\left(\frac{b}{\alpha}\right)^{1/\beta-2} = \frac{1-\beta}{(\alpha\beta)^2}. \end{aligned}$$

We see that and the second order perturbation equation (1) for x of:

$$\begin{aligned} g(y) &= f(X(y)) \\ &\approx f(X(b)) + f'(X(b))X'(b)(y-b) + \ldots \\ &\quad \frac{1}{2}(X'(b)^2 f''(X(b)) + f'(X(b))X''(b))(y-b)^2 \\ &= f(a) + f'(a)\left(\frac{1}{\alpha\beta}\right)[y-\alpha] + \ldots \\ &\quad \frac{1}{2}\left(\left(\frac{1}{\alpha\beta}\right)^2 f''(a) + f'(a)\left(\frac{1-\beta}{(\alpha\beta)^2}\right)\right)[y-\alpha]^2 \\ &= f(a) + f'(a)\left(\frac{1}{\alpha\beta}\right)\left[\alpha x^{\beta} - \alpha\right] + \ldots \\ &\quad \frac{1}{2}\left(\left(\frac{1}{\alpha\beta}\right)^2 f''(a) + f'(a)\left(\frac{1-\beta}{(\alpha\beta)^2}\right)\right)\left[\alpha x^{\beta} - \alpha\right]^2. \end{aligned}$$

2.2.4 Non-linear Transformation COV

Besides exponential and log expansions, the original function can be more complex. Here, let us suppose $a = 0$ and the COV takes the form of $y = \left(\frac{\alpha-x}{\beta-x}\right)^{1-\gamma}$. That is:

$$Y(x) = \begin{bmatrix} y \end{bmatrix} = \begin{bmatrix} \left(\frac{\alpha-x}{\beta-x}\right)^{1-\gamma} \end{bmatrix}.$$

The inverse function is therefore:

$$X(y) = \begin{bmatrix} \frac{\alpha-\beta y^{\frac{1}{1-\gamma}}}{1-y^{\frac{1}{1-\gamma}}} \end{bmatrix}.$$

Based on the COV and supposition:

$$\begin{aligned}
b &= \left(\frac{\alpha}{\beta}\right)^{1-\gamma} \\
X'(b) &= \frac{\frac{(\alpha-\beta)}{(1-\gamma)}b^{\frac{\gamma}{1-\gamma}}}{\left(b^{\frac{1}{1-\gamma}}-1\right)^2} = \frac{\frac{(\alpha-\beta)}{(1-\gamma)}\left(\frac{\alpha}{\beta}\right)^{\gamma}}{\left(\left(\frac{\alpha}{\beta}\right)^2-1\right)^2} \\
X''(b) &= \frac{\frac{(\alpha-\beta)}{(1-\gamma)^2}b^{\frac{2\gamma}{1-\gamma}}\left(\gamma\left(b^{\frac{1}{1-\gamma}}-1\right)^2 b^{-\frac{1}{1-\gamma}}-2\right)}{\left(b^{\frac{1}{1-\gamma}}-1\right)^4} \\
&= \frac{\frac{(\alpha-\beta)}{(1-\gamma)^2}\left(\frac{\alpha}{\beta}\right)^{2\gamma}\left(\frac{(\alpha\gamma-\beta)^2}{\alpha\beta}-2\right)}{\left(\left(\frac{\alpha}{\beta}\right)-1\right)^4}.
\end{aligned}$$

Using the chain rule the perturbation of $f(x)$ is approximated as

$$
\begin{aligned}
g(y) &= f\left(X(y)\right) \\
&\approx f(X(b)) + f'(X(b))X'(b)(y-b) + \ldots \\
&\quad \frac{1}{2}(X'(b)^2 f''(X(b)) + f'(X(b))X''(b))(y-b)^2 \\
&= f(a) + f'(a)\frac{\frac{(\alpha-\beta)}{(1-\gamma)}\left(\frac{\alpha}{\beta}\right)^{\gamma}}{\left(\left(\frac{\alpha}{\beta}\right)^2 - 1\right)^2}\left[y - \left(\frac{\alpha}{\beta}\right)^{1-\gamma}\right] + \ldots \\
&\quad \frac{1}{2}\left(\frac{\frac{(\alpha-\beta)^2}{(1-\gamma)^2}\left(\frac{\alpha}{\beta}\right)^{2\gamma}}{\left(\left(\frac{\alpha}{\beta}\right)^2 - 1\right)^4} f''(a) + f'(a)\frac{\frac{(\alpha-\beta)}{(1-\gamma)^2}\left(\frac{\alpha}{\beta}\right)^{2\gamma}\left(\frac{(\alpha\gamma-\beta)^2}{\alpha\beta} - 2\right)}{\left(\left(\frac{\alpha}{\beta}\right) - 1\right)^4}\right) \times \cdots \\
&\quad \left[y - \left(\frac{\alpha}{\beta}\right)^{1-\gamma}\right]^2 \\
&= f(a) + f'(a)\frac{\frac{(\alpha-\beta)}{(1-\gamma)}\left(\frac{\alpha}{\beta}\right)^{\gamma}}{\left(\left(\frac{\alpha}{\beta}\right)^2 - 1\right)^2}\left[\left(\frac{\alpha - x}{\beta - x}\right)^{1-\gamma} - \left(\frac{\alpha}{\beta}\right)^{1-\gamma}\right] + \ldots \\
&\quad \frac{1}{2}\left(\frac{\frac{(\alpha-\beta)^2}{(1-\gamma)^2}\left(\frac{\alpha}{\beta}\right)^{2\gamma}}{\left(\left(\frac{\alpha}{\beta}\right)^2 - 1\right)^4} f''(a) + f'(a)\frac{\frac{(\alpha-\beta)}{(1-\gamma)^2}\left(\frac{\alpha}{\beta}\right)^{2\gamma}\left(\frac{(\alpha\gamma-\beta)^2}{\alpha\beta} - 2\right)}{\left(\left(\frac{\alpha}{\beta}\right) - 1\right)^4}\right) \times \cdots \\
&\quad \left[\left(\frac{\alpha - x}{\beta - x}\right)^{1-\gamma} - \left(\frac{\alpha}{\beta}\right)^{1-\gamma}\right]^2 .
\end{aligned}
$$

The examples above show that a perturbation solution can easily be modified by a COV. In the first case, the log COV was relatively straightforward. The second and third examples, though less straightforward, show how richer non-linear COVs can be incorporated into the perturbation solution. These three cases display that the resulting solution can contain parameters (e.g., α,β, $\ldots$) that define the COV. The next section, building upon Fernández-Villaverde and Rubio-Ramirez (2006), shows how to estimate these parameters using the optimality equations that define the model's asset pricing equilibrium.

2.2.5 Projecting the COV

The next step quantifies the unknown parameters of the COV; $\{\alpha, \beta, \gamma\}$, by examining of the perturbation Equation Errors (EER). Following Fernández-Villaverde and Rubio-Ramirez (2006) and Judd (2002), the COV solutions are projected onto the EER and minimized by choice of parameters α, β, and γ. Presumably, equation (1) was found by expansion of optimality equations that describe optimal behaviors of the agents; often called Euler equations in macroeconomics. These equations are denoted $EER(x_{i+1}, x_i)$. By summing up EER by element and across sets of values for $\{x_t\}$, a minimization problem can be found:

$$\min_{\{\alpha,\beta,\gamma\}} \sum_{j,x_i,y_i} |EER_j(g(y_i), x_i)| = \min_{\{\alpha,\beta,\gamma\}} \sum_{j,x_i} |EER_j(f(X(x_i)), x_i)|. \tag{2}$$

In Fernández-Villaverde and Rubio-Ramirez (2006) a grid for x is made. For example, a grid can be chosen so that $\{x_i\}_{i=1}^{N}$ crosses over 90 percent below and above the steady state. If x contains exogenous shock z, then Tauchens procedure can be used to make a grid and evaluate the expected values using the transition probability matrix P. The parameter N is chosen to control the degree of accuracy of the projection.

2.3 A DSGE Asset Pricing Model Application

The following model is an asset pricing model described by Chen (2011). The asset pricing model is comprised of two sectors: households and firms. The agents in these sectors transact in four markets: goods, stocks, bonds and labor markets.

2.3.1 Households

Infinitely-lived households enter the financial market to invest their financial wealth in both stocks and bonds. These households maximize their lifetime utility function:

$$\max_{\{c_t, l_t, s_{t+1}, B_{t+1}\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{\left(c_t^{\gamma} (1 - l_t)^{1-\gamma}\right)^{1-\rho}}{1 - \rho} \right) \right\},$$

subject to:

$$c_t + p_t^f B_{t+1} + p_t^s s_{t+1} \leq B_t + s_t \left(p_t^s + d_t\right) + w_t l_t,$$

where β is the time discount rate. The term $B_t + s_t \left(p_t^s + d_t\right) + w_t l_t$ is the total wealth that the agent possesses in period t that includes: the returns from buying bonds B_t, returns from investing stocks $s_t \left(p_t^s + d_t\right)$ with the share of the stock s_t at price of p_t^s, and labor income $w_t l_t$. Expenditures that include: current consumption c_t, the purchase of bonds $p_t^f B_{t+1}$, and stock purchase $p_t^s s_{t+1}$ cannot exceed the total wealth at the end of time t. Prices for stocks and bonds at time t are p_t^s and p_t^f, respectively, while d_t is the dividend paid to the investor by the firm.

The momentary utility function, $(c_t^\gamma \left(1 - l_t\right)^{1-\gamma})^{1-\rho} / \left(1 - \rho\right)$, follows standard neo-classical macroeconomics. This utility function, defined on consumption and labor hours l_t, has two main parameters, ρ and γ, that mutually determine the EIS (Elasticity of Inter-temporal Substitution), risk aversion, and the Frisch labor supply elasticity. With this form of Cobb-Douglas (CD) utility function, risk aversion is measured by the parameter ρ (> 0), EIS is measured by $1/\rho$, and the Frisch labor supply elasticity is $\left((1 - l)/l\right) \left(\left(1 - \gamma(1 - \rho)\right) / \rho\right)$. The quantitative magnitude of these parameters are shown to be important in the subsequent analysis.

Household optimization yields three first-order conditions:

$$1 = E_t \left\{ \beta \left(\frac{1 - l_{t+1}}{1 - l_t} \right)^{(1-\gamma)(1-\rho)} \left(\frac{c_{t+1}}{c_t} \right)^{\gamma(1-\rho)-1} \left(\frac{1}{p_t^f} \right) \right\}, \tag{3}$$

$$1 = E_t \left\{ \beta \left(\frac{1 - l_{t+1}}{1 - l_t} \right)^{(1-\gamma)(1-\rho)} \left(\frac{c_{t+1}}{c_t} \right)^{\gamma(1-\rho)-1} \times \left(\frac{p_{t+1}^s + d_{t+1}}{p_t^s} \right) \right\}, \tag{4}$$

$$0 = \frac{1 - \gamma}{(1 - l_t)} - w_t \frac{\gamma}{c_t} \tag{5}$$

Equation (3) is the inter-temporal Euler for bond purchasers, (4) is the inter-temporal

Euler for stocks and (5) is the intra-temporal Euler between consumption and labor hours.

2.3.2 Firms

The firms in this economy produce the consumption good with Cobb-Douglas technology $y_t = z_t k_t^{\theta} l_t^{1-\theta}$ in perpetuity. The level of technology evolves according to the exogenous process

$$\log\left(z_{t+1}\right) = \eta \log\left(z_t\right) + \varepsilon_{t+1},\ \varepsilon_t \overset{iid}{\sim} N\left(0, \sigma^2\right),$$

where η represents the persistence of aggregate shock with noise having an independent and identical distribution (iid) with mean of 0 and variance of σ^2. Firm value is maximized through the distribution of dividends to the agents (owners) in the household sector. The discounted value of the firm is $\sum_{j=0}^{\infty} \beta^j \Lambda_{t+j} d_{t+j}$,where Λ_{t+j} signifies the relative price of consumption; i.e., the equity owner's marginal utility of consumption. The specific maximization problem for the firm is:

$$p_t^s = \max_{\{k_{t+j}, l_{t+j}\}} E_t \left\{ \sum_{j=0}^{\infty} \beta^j \Lambda_{t+j} d_{t+j} \right\}. \tag{6}$$

The firm owns the capital k, and funds operations internally through retained earnings and externally by selling bonds. The total supply of the bonds at price of p_t^f is constant over time and equals to $\chi \bar{k}$, where χ is a constant representing the leverage ratio. The capital stock follows a law of motion

$$i_t = k_{t+1} - (1 - \delta)\, k_t.$$

In order to create a wedge between the return to physical capital and the return to financial capital, Danthine and Donaldson (2002) introduced a cost that adjusts the firm's capital stock from its current level in macroeconomy in their study of

EPP. More specifically, Basu (1987) highlights the importance of this adjustment cost to determining such financial variables as stock prices and long term real interest rates. Existence of this cost implies diminishing returns to augmenting the quantity of capital in the economy and therefore capital stock tends to adjust in a sluggish manner with respect to any productivity shock. Following previous studies (Guvenen 2009; Danthine and Donaldson 2002), an adjustment cost of investment is presumed to be concave and given as:

$$g(k_t, i_t) = \left(\frac{\phi}{2}\right)\left(\frac{1}{k_t}\right)(i_t - \delta k_t)^2 .$$

After retaining capital for future use, paying out the net interest to bondholders, making labor payment to employees, and including the capital adjustment cost, the firm maximizes its value subject to a dividend constraint:

$$d_t = y_t - w_t l_t - i_t - (1 - p_t^f)\chi \overline{k} - g(k_t, i_t).$$

Solving the maximization problem in (6) yields the first order conditions for a typical firm

$$\begin{aligned} 0 &= E_t \left\{ \begin{array}{c} \beta \Lambda_{t+2} \left(\begin{array}{c} \theta z_{t+1} k_{t+1}^{\theta-1} l_{t+1}^{1-\theta} + (1-\delta) \\ + \frac{\phi}{2}\left(\frac{k_{t+2}-k_{t+1}}{k_{t+1}} + 1\right)^2 - \frac{\phi}{2} \end{array} \right) \\ -\Lambda_{t+1}\left(1 + \phi \frac{k_{t+1}-k_t}{k_t}\right) \end{array} \right\}, \quad (7) \\ w_t &= (1-\theta)\, z_t k_t^{\theta} l_t^{-\theta}. \quad (8) \end{aligned}$$

Equation (7) is an inter-temporal Euler equation for the firm. Since it is assumed that the households are both owners and workers, this Euler equation is equivalent to the households' inter-temporal Euler equation when they own the stock of capital and rent it to the firm. In this model, ownership by the firm enables equity to have value as it is a claim to the returns from that capital stock. Ultimately, however, the predictions in either case are the same. Equation (8) is the standard intra-temporal Euler equation that equates the marginal product of labor with the wage rate.

2.3.3 Equilibrium

Equations (3), (4), (5), (7), and (8) are the necessary conditions that describe optimal behavior of the agents in the general equilibrium model. Equilibrium is formally defined in Equation (9).

$$y_t - g(k_t, i_t) = c_t + i_t. \tag{9}$$

Equation (9) shows that in general equilibrium, the total consumption for the representative households plus investment cannot exceed the total production minus the adjustment cost. All variables in equilibrium are represented in aggregate quantities. The reason why we label the aggregate level in the same fashion as the individual level (i.e., c_t, i_t) is that the representative agent is assumed to be distributed uniformly over $[0, 1]$.

The clearing of the bond market requires (10):

$$B_{t+1} = \chi \bar{k}, \tag{10}$$

and the clearing of the stock market requires (11):

$$s_{t+1} = 1. \tag{11}$$

Equations (10)-(11) characterize the aggregate levels for bonds and stocks. The law of motion for bonds requires that in equilibrium, the total bonds supplied (issued) by the firms are equal to the total demand for the bonds by households. As mentioned in section 2.2, the total supply for the bonds is $\chi \bar{k}$, while the total demand for the uniformly distributed bonds is B_{t+1}. Similarly, the shares of stock are a uniformly distributed among the populace where the total supply for the stocks s_{t+1} is inelastically set to 1 throughout time. Note that the firm is not issuing new shares and therefore the price of equity is changing solely due to demand.

In equilibrium, the return to equity is

$$r_t^e = \frac{p_t^s + d_t}{p_{t-1}^s},$$

the risk-free rate is

$$r_t^f = \frac{1}{p_t^f},$$

and the average equity premium is

$$r^{ep} = \sum_{t=1}^{T} \frac{\left(r_t^e - r_t^f\right)}{T}.$$

The equity premium is approximated by:

$$r^{ep} \approx E_t[r_t^e - r_t^f].$$

2.3.4 Solving the Asset Pricing Economy

2.3.4.1 The Base Case

The solution method here makes use of Taylor series expansion with changes of variables (Judd 1996, 2002; Fernández-Villaverde and Rubio-Ramirez 2006). Every policy function (i.e., l_t, k_{t+1}, etc.) is first approximated by a perturbation solution:

$$f(z,k,\sigma) \approx \sum_{i,j,m} \frac{1}{(i+j+m)!} \left.\frac{\partial^{i+j+m} f(z,k,\sigma)}{\partial z^i \partial k^j \partial \sigma^m}\right|_{\{0,kss,0\}} z^i (k - kss)^j \sigma^m. \qquad (12)$$

The solution in (12) is, presumably, accurate around $\{z,k,\sigma\} = \{0, kss, 0\}$. Then, k is replaced by a change of variables (COV) defined by a polynomial in itself. More specifically, let the COV be;

$$\begin{aligned} y_{1,t} &= z_t \\ y_{2,t} &= k_t^{\alpha_1} \\ y_{3,t} &= \sigma. \end{aligned}$$

To get a better understanding of how COV works, consider a simple example from Judd (2002). At first, the researcher has a basic second order Taylor series expansion of a function $f(x)$:

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 \tag{13}$$

where x has been expanded around a. The COV is then defined by $y = Y(x)$ with an inverse function existing as $x = X(y)$. The COV finds $g(y) = f(X(y))$ at $y = b = Y(a)$. Note that $g(y)$ can be approximated with Chain Rule at second order by:

$$\begin{aligned} g(y) &= f(X(y)) \\ &\approx f(X(b)) + f'(X(b))X'(b)(y-b) + \dots \\ &\frac{1}{2}(X'(b)^2 f''(X(b)) + f'(X(b))X''(b))(y-b)^2 \\ &= f(a) + f'(a)X'(b)(y-b) + \dots \\ &\frac{1}{2}(X'(b)^2 f''(a) + f'(a)X''(b))(y-b)^2. \end{aligned}$$

More concretely, suppose $a = 1$ and the COV is $y = Y(x) = \log(x)$. Then, we immediately see that $b = 0$ and the inverse function is $x = X(y) = \exp(y)$. As a result, $X'(b) = 1$ and $X''(b) = 1$. The COV expansion is thus:

$$f(a) + f'(a)\log(x) + \frac{1}{2}(f''(a) + f'(a))\log(x)^2,$$

where $\{f(a), f'(a), f''(a)\}$ are presumed to be known from Euler equations.

In this study, the proposed transformation is:

$$Y(x_t) = \begin{bmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \end{bmatrix} = \begin{bmatrix} z_t \\ k_t^{\alpha_1} \\ \sigma \end{bmatrix},$$

where $x_t = [z_t, k_t, \sigma]'$. The inverse function is thus:

$$X(y_t) = \begin{bmatrix} Z(y_{1,t}) \\ K(y_{2,t}) \\ \Sigma(y_{3,t}) \end{bmatrix} = \begin{bmatrix} y_{1,t} \\ (y_{2,t})^{1/\alpha_1} \\ y_{3,t} \end{bmatrix}.$$

And, suppose that an initial second order perturbation gave equation (12) for k_{t+1} of:

$$\begin{aligned} k_{t+1} &= \mathcal{K}(z_t, k_t, \sigma) \\ &\approx \mathcal{K}_{<0,0,0>} + \mathcal{K}_{<1,0,0>} z_t + \mathcal{K}_{<0,1,0>}(k_t - kss) + \mathcal{K}_{<0,0,1>}\sigma + \ldots \\ &\quad \mathcal{K}_{<2,0,0>} {z_t}^2 + \mathcal{K}_{<0,2,0>}(k_t - kss)^2 + \mathcal{K}_{<0,0,2>}\sigma^2 + \ldots \\ &\quad 2\mathcal{K}_{<1,1,0>} z_t(k_t - kss) + 2\mathcal{K}_{<1,0,1>} z_t\sigma + \ldots \\ &\quad 2\mathcal{K}_{<0,1,1>}(k_t - kss)\sigma. \end{aligned}$$

where

$$\mathcal{K}_{<i,j,m>} = \frac{1}{(i+j+m)!} \left.\frac{\partial^{i+j+m}\mathcal{K}(z,k,\sigma)}{\partial z^i \partial k^j \partial \sigma^m}\right|_{\{0,kss,0\}}.$$

Applying the COV gives:

$$
\begin{aligned}
k_{t+1} &= \mathcal{K}\left(Z(y_{1,t}), K(y_{2,t}), \Sigma(y_{3,t})\right) \\
&\approx \mathcal{K}_{<0,0,0>} + \mathcal{K}_{<1,0,0>} y_{1,t} + \dots \\
&\quad \mathcal{K}_{<0,1,0>} K_{<1>} \left(y_{2,t} - \bar{y}_2\right) + \dots \\
&\quad \mathcal{K}_{<0,0,1>} y_{3,t} + \dots \\
&\quad \mathcal{K}_{<2,0,0>} y_{1,t}^2 + \dots \\
&\quad \left(\mathcal{K}_{<0,2,0>} K_{<1>}^2 + \mathcal{K}_{<0,1,0>} K_{<2>}\right) \left(y_{2,t} - \bar{y}_2\right)^2 + \dots \\
&\quad \mathcal{K}_{<0,0,2>} y_{3,t}^2 + \dots \\
&\quad 2\mathcal{K}_{<1,1,0>} K_{<1>} y_{1,t} \left(y_{2,t} - \bar{y}_2\right) + \dots \\
&\quad 2\mathcal{K}_{<1,0,1>} y_{1,t} y_{3,t} + \dots \\
&\quad 2\mathcal{K}_{<0,1,1>} K_{<1>} \left(y_{2,t} - \bar{y}_2\right) y_{3,t}.
\end{aligned}
$$

where

$$
K_{<i>} = \left. \frac{\partial^i K\left(y_2\right)}{\partial y_2^i} \right|_{\{\bar{y}_2\}},
$$

My choice of COV gives $K_{<1>} = 1/\alpha_1 \left(\bar{y}_2\right)^{1/\alpha_1 - 1}$, and $K_{<2>} = 1/\alpha_1(1/\alpha_1 - 1) \left(\bar{y}_2\right)^{1/\alpha_1 - 2}$.

2.3.4.2 Projection Methods

Given the COV transformations for the policy solution set: $\{c_t, l_t, k_{t+1}, p_t^s, p_t^f\}$, the next step in the solution method quantifies the unknown parameter of the COV; $\{\alpha_1\}$, by examining of the Euler Equation Errors (EER). Following Fernández-Villaverde and Rubio-Ramirez (2006) and Judd (2002), the COV solutions are projected onto the EER and minimized by choice of parameters.

In the next step, using (8), the optimality equations (3), (4), (5), (7), and (9) are evaluated using the COV perturbation solutions for any given set of states and unknown parameters: $\{k_t, z_t, \sigma, \alpha_1\}$. These equations are stacked into a vector that is denoted $EER(k_t, z_t, \sigma, \alpha_1)$. By summing up EER by element and across sets of values for $\{k, z\}$, the minimization problem is:

$$\min_{\{\alpha_1\}} \sum_{j,k_i,z_i} |EER_j(k_i, z_i, \sigma, \alpha_1)| . \tag{14}$$

A grid of 40 productivity points for z is found by employing Tauchen's procedure given a calibrations for η and a drawn σ from the prior. The grid $\{z_i\}_{i=1}^{40}$ has a Markov transition matrix, P, that is used to compute the expectations in equation (14). For the grid on capital, I follow Tauchen (1990) by first solving and simulating the model (by second-order perturbation). The simulated capitals are then used to form an unevenly spaced grid using percentile grouping {1%-25%, by 2.5}, {26%-45.5%, by 1.5}, {46%-54%, by .5}, {54.5%-74%, by 1.5}and {75%-99%, by 2.5}, where {x%-y%, by z} implies the percentile ranges from x to y increasing by z basis points. In the end, this forms a grid of 70 points, $\{k_i\}_{i=1}^{70}$. This is very similar to Fernández-Villaverde and Rubio-Ramirez (2006) where they form a grid so that it crosses over 90 percent below and above the steady state capital.

2.3.4.3 The Alternative: Value Function Iteration on a Grid

A common approach to solve DSGE models is to conduct Value Function Iteration (VFI). To understand how this mechanism works in a stochastic atmosphere, I now present several steps to describe VFI in the model:

1. Make a grid for the values of the time varying states in the model, $\{k_j, z_j\}_{j=1}^{N}$ where $N = 40 \times 70$. This is constructed using the same procedure as presented above.

2. Set $t = 0$.

3. For each $j \in [1, \ldots, N]$ states, guess at the initial value functions, bond prices, and stock prices: $\{V_t(k_j, z_j) = 0, p_t^f(k_j, z_j) = \bar{p}^f, p_t^s(k_j, z_j) = \bar{p}^s\}$.

4. For all realizations of $\Omega_{t,j} = \{V_t(k_j, z_j), p_t^f(k_j, z_j), p_t^s(k_j, z_j)\}$, find the allocations $\{c(\Omega_{t,j}, k_i'), l(\Omega_{t,j}, k_i'), i(\Omega_{t,j}, k_i'), y(\Omega_{t,j}, k_i'), d(\Omega_{t,j}, k_i')\}$ for all $i \in [1, \ldots, 70]$ using the intra-temporal Euler equation, resource constraint, and investment

$$
\begin{aligned}
0 &= \frac{1-\gamma}{(1 - l(\Omega_{t,j}, k_i'))} - (1-\theta) z_j k_j^\theta (l(\Omega_{t,j}, k_i'))^{-\theta} \frac{\gamma}{c(\Omega_{t,j}, k_i')} \\
0 &= z_j k_j^\theta (l(\Omega_{t,j}, k_i'))^{1-\theta} - g(k_j, i(\Omega_{t,j}, k_i')) - c(\Omega_{t,j}, k_i') - i_t, \\
i(\Omega_{t,j}, k_i') &= k_i' - (1-\delta) k_j, \\
y(\Omega_{t,j}, k_i') &= z_j k_j^\theta (l(\Omega_{t,j}, k_i'))^{1-\theta} \\
d(\Omega_{t,j}, k_i') &= y(\Omega_{t,j}, k_i') - w(\Omega_{t,j}, k_i') l(\Omega_{t,j}, k_i') - \ldots \\
& \quad i(\Omega_{t,j}, k_i') - (1 - p_t^f(\Omega_{t,j}, k_i'))\chi \bar{k} - g(k_j, i(\Omega_{t,j}, k_i'))
\end{aligned}
$$

5. Find the allocations $\{k'\}$, the prices $\{p_{i+1}^f(k_j, z_j), p_{i+1}^s(k_j, z_j) d(\Omega_{t,j}, k_i')\}$, and value function $V_{i+1}(k_j, z_j)$ by solving the Bellmans equation:

$$
V_{i+1}(k_j, z_j) = \max_{\{k_j'\}} \left\{ U(c(\Omega_i, k_j'), l(\Omega_i, k_j')) + \beta P V_i(k_j', z_j') \right\}.
$$

6. Find the asset prices $p_{t+1}^f(k_j, z_j)$ and $p_{t+1}^s(k_j, z_j)$ using equations (3) and (4).

7. Define a criterion ε small enough that if $\|V_{t+1}(k, z) - V_t(k, z)\| < \varepsilon$, then a solution has been found and stop; else set $i = i + 1$ and go to step 4.

2.3.5 The Experimental Design: Model Calibration

The parameters are calibrated based on the estimations found in other studies (Abel 1980, Danthine and Donaldson 2002, Jermann 1998, Grüne and Semmler 2008, Guvenen 2009). The capital share in output is set at $\theta = 0.3$, the same value as chosen by Kydland and Prescott (1982), Jermann (1998) and Grüne and Semmler (2008). This selection conforms with the labor elasticity suggested in the data during the period studied by Mehra and Prescott (1985).The discount rate β is set to 0.99 according to a steady state return on capital of 4%. Both Danthine and Donaldson. (2002)

and Guvenen (2009) assumed this rate of return in their quarterly estimates in correspondence with a quarter period. The utility power parameter ρ – the relative risk aversion – is set equal to 4 following Danthine and Donaldson (2002). For the model, consumption's share in utility is set to $\gamma = 0.395$. This is chosen to follow Guvenen (2009) to match the average time devoted in market activities (0.36 of discretionary time). The leverage ratio χ is assumed to be 0.15 which lies in the historical range of 0.13 to 0.44 (Jermann, 1998).

Another important parameter is the cost of adjustment constant ϕ which measures the elasticity of investment. But studies that incorporate adjustment cost functions have used a varying range for ϕ. Danthine and Donaldson (2002) and Jermann (1998) both states that the value of ϕ is set to maximize model's ability to match a set of moments of interest; too large value of ϕ leads to low volatility of investment. For example, Abel (1980) picked ϕ in the range of $[0.27, 0.52]$, Jermann (1998) estimated it as 0.23, and Guvenen (2009) calibrated it as 0.40. The way ϕ is picked in this Chapter is to match the adjustment cost not "too large" (Danthine and Donaldson, 2002) and to pursue the goal of smoothing the capital stocks. Therefore, this constant is set to $0.35\exp(k_{ss})$ to be able to replicate Tobin's Q values. All of the calibrations are detailed in table 1.

Table 1: Parameterizations for Experimental Model

θ	Capital Share	0.30
δ	Depreciation Rate	0.02
β	Time Discount Factor	0.99
χ	Leverage Ratio	0.15
γ	Consumption Share	0.395
ρ	Relative Risk Aversion	4
ϕ	Elasticity of Investment	$0.35 \times \exp(k_{ss})$
η	Persistence of Aggregate Shock	0.95

2.4 The Results

Table 2 reports the results of the computational experiments. Column two and three represent the statistics for the two competing solution methods, respectively. The performances of these two solution methods are compared in two dimensions: accuracy and speed. In terms of accuracy, I consider the average and the standard deviation of the logged (to base 10) absolute values of inter-temporal Euler equations errors that are defined in equations (3) and (4). For both methods, the errors are computed at every combination of capital and productivity shock. For each combination, the optimal current and future allocations are computed using the solutions rules found from both methods; the HPP and VFI methods. Then, these allocations are substituted into the Eulers to find the *ex-post* errors that represent how mistaken the agents decisions are. Small errors that deviate little, which are expected from rational household behavior, imply the solution method is good.

Table 2 shows that the HPP yields a mean of the $\log_{10}|Euler| = -1.0572$ for the bond price Euler. Alternatively, the VFI gives $\log_{10}|Euler| = -0.954$. Also, the standard deviations, defined by $STD(\log_{10}|Euler|)$, are 0.0029 and 0.0594 for the HPP and VFI methods, respectively. These statistics imply that the HPP method is, on average, more accurate than VFI method for the bond pricing equation; the HPP method is, therefore, superior along both dimensions with describing bond holdings.

Table 2 shows that the HPP yields a mean of the $\log_{10}|Euler| = -1.2394$ for equity pricing Euler defined in equation (4). The VFI gives mean error of $\log_{10}|Euler| = -1.4498$. In terms of equity pricing accuracy, the VFI method does a better job than HPP method. The standard deviations of the errors, defined by $STD(\log_{10}|Euler|)$, show a different story. The HPP method is more stable than the VFI method; as the states space changes the error variance of the HPP doesn't significantly change.

Figures 1 and 2 reinforce the findings of Table 2. The comparisons for bond price

Table 2: Solution Method Performance

Method:	HPP	VFI
Estimate	$\alpha_1 = 1.1075$	-
Mean ($\log_{10}\lvert Euler\rvert$): Bond	-1.0572	-0.954
Mean ($\log_{10}\lvert Euler\rvert$): Equity	-1.2394	-1.4498
Standard Deviation ($\log_{10}\lvert Euler\rvert$): Bond	0.0029	0.0594
Standard Deviation ($\log_{10}\lvert Euler\rvert$): Equity	0.0013	0.0846
Computational time (secs.)	17.438	9.0932e+003

and stock price Euler errors are displayed correspondingly. In figure 1 the green dot represents the performance for VFI method and the blue dot shows how HPP method captures the accuracy for bond price Euler. Even though the VFI achieves the lowest EER at one point of state, it is clear that HPP method yields lower errors than VFI. Note that the HPP Euler errors form almost a horizontal line. In other words, as capital changes the HPP accuracy stays relatively stable. This in contrast to the VFI method where the errors increase in the tails of the state space.

Figure 2 displays the error patterns for the HPP and VFI methods with respect to the equity pricing Euler. Most of the logged errors for VFI method are lower than HPP method. It is plausible that VFI method reduces the impreciseness better than HPP method. Nevertheless, the deviations are not statistically big enough to deny the performance of HPP method. The lowest logged errors (at base 10) the VFI method obtains 0.0971 unit lower than HPP method does and in some state space, e.g., when capital increases to 2.32or higher, the EERs generated by VFI are larger than HPP does. However, this figure shows that the HPP method is more stable than the VFI method.

The last row of table 2 shows the computation time, or speed, of the two competing methods. The table shows that HPP is very fast relative to VFI; the HPP

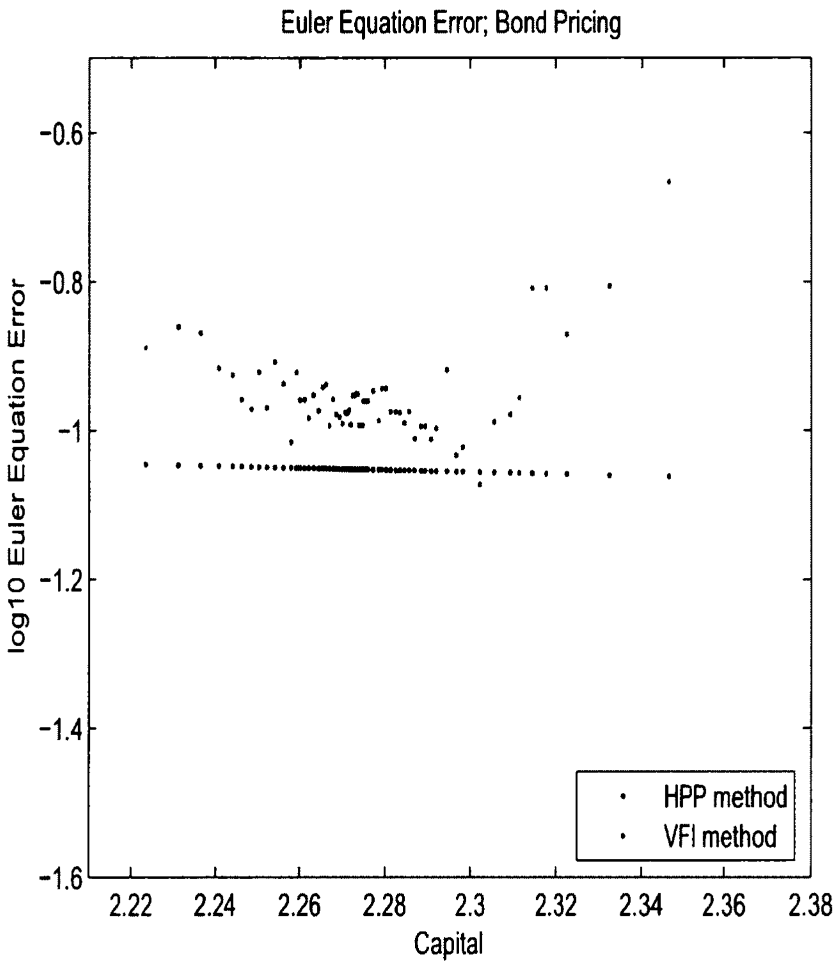

Figure 1: Relationship Between Euler Errors and the State Space (Bond Pricing)

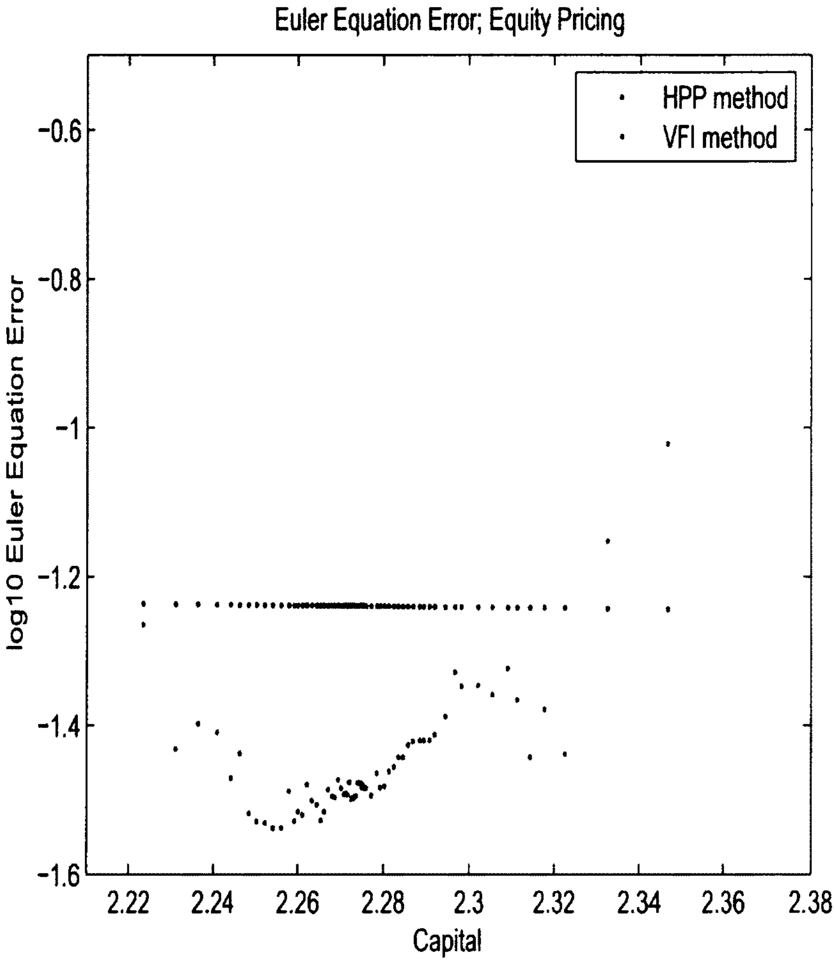

Figure 2: Relationship Between Euler Errors and the State Space (Equity Pricing)

speed is 151.25 minutes (2.52 hours) faster! Speedy solution methods are desirable in DSGE models since solutions are often required during structural estimation analysis. In fact, Chapter 3 of this dissertation combines the HPP method with a Bayesian structural estimation method. It is evident that VFI methods would be inoperable in structural estimation settings.

As an ancillary result, I ask do the solution methods produce similar equilibrium outcomes for the economy's allocations? Figures 3 and 4 display the histograms for investment i, consumption c, labor hours l, output y, bond price p^f and gross return for equity $1/p^s$ for 15,000 simulated time periods. Comparing the means and distributions for these allocation can we find that the HPP method and VFI method generate similar patterns. This is important because it is generally believed that VFI method can produce acceptable results for the allocations of an economy in a DSGE model. Consequently, HPP method, which maintains the accuracy in the EERs, does not distort the main outcomes of the DSGE asset pricing model.

2.5 Conclusion

In this Chapter, I conducted a quantitative experiment to evaluate a Hybrid Perturbation-Projection Method for solving DSGE asset pricing model. This experiment was operated on reasonable calibrated macro-based asset pricing model similar to the studies by Guvenen (2009), Danthine and Donaldson (2002), Barberis, Huang and Santos (2001), Barberis and Huang (2004 and 2008), and Grüne and Semmler (2008). This model was solved and simulated using both the proposed HPP and the competing VFI methods.

These two solution methods were compared along several dimensions. In the first dimension, I assessed the accuracy of the solution methods. This was achieved by comparing the averages and the standard deviations of the logged (to base 10)

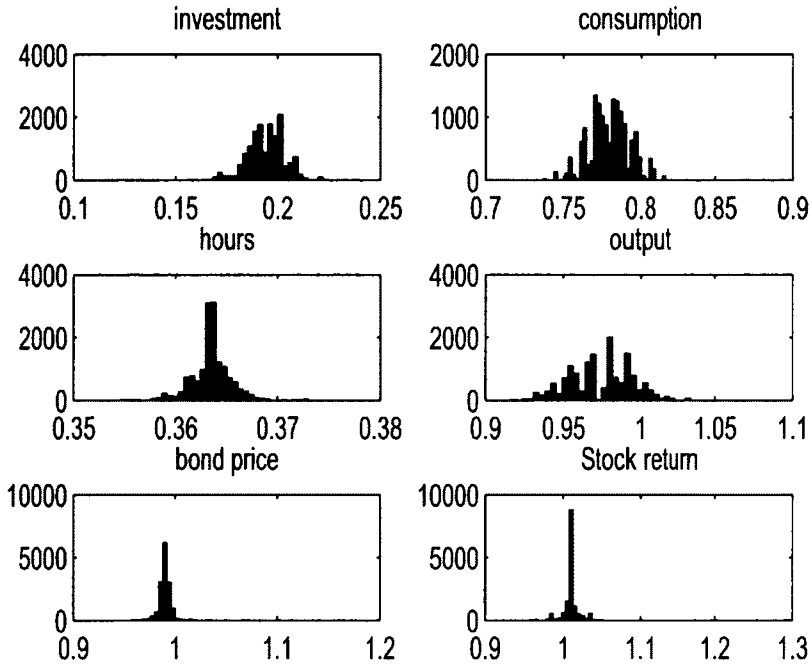

Figure 3: Histogram of Simulated Allocations for the VFI Method

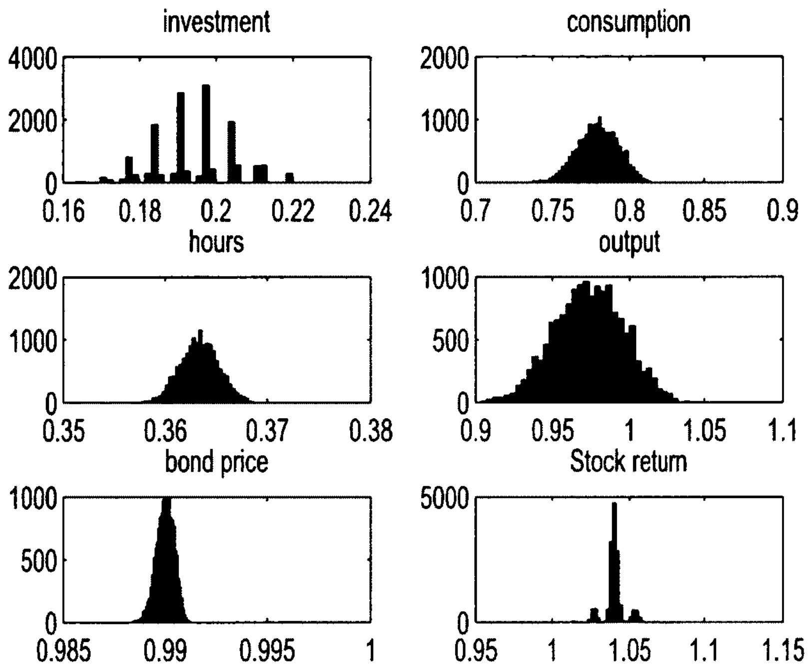

Figure 4: Histogram of Simulated Allocations for the HPP Method

absolute values of inter-temporal Euler equations errors of the Models by Hybrid Perturbation-Projection Solution Method and by Value Function Iteration Method. The second dimension is to evaluate how speedy the HPP method is with comparison to the VFI method.

The results of the experiments show that the HPP method was far superior in computational time to VFI method. It was hours faster. This is important in modern macroeconomics that typically combines estimation methods with solution methods. The HPP method was also as accurate as the competing VFI method. It was shown that the HPP method produced very stable Euler Equation Errors that did not fluctuate with respect to the state space. Alternatively, the VFI method, the more accurate around the steady state, was less accurate in the tail of the state space.

In the next Chapter, I employ the HPP method with a Bayesian prior predictive analysis in attempt to uncover the structural parameters of a similar asset pricing model. the results shown here suggest there should be a high level of confidence in these estimations.

2.6 *Appendix: Sample Matlab Code for Solving DSGE Asset Pricing Models via HPP methods*

hybrid3.m

```
%
% matlab file to simulate from the solutions of guvenen6.map
% where lambda = 0. Then; dump distribution of capitals
% to an output file
%

% ----- save output to file ----- %

clear all
diary hybrid3.mout
diary off
delete hybrid3.mout
diary hybrid3.mout

% ----- set seed of random number generator ----- %

randn('state',0)
rand('state',0)

% ----- display date and time of computation ----- %

%format long
date
time0 = clock;

% ----- control vars ----- %
```

```
iters = 15000;
drop  = 1;

% ----- define global variables ----- %

global eta theta delta beta mu chi zbar gamma phi b0 rho1 rho2
global csss lsss pfss lnnss kss psss cnss
global d1k d2k d3k d4k d1cs d2cs d3cs d4cs d1cn d2cn d3cn d4cn d1ls d2ls d3ls d4ls
global d1lnn d2lnn d3lnn d4lnn d1ps d2ps d3ps d4ps d1pf d2pf d3pf d4pf
global d11k d12k d13k d14k d22k d23k d24k d33k d34k d44k d11ls d12ls d13ls
global d14ls d22ls d23ls d24ls d33ls d34ls d44ls
global d11lnn d12lnn d13lnn d14lnn d22lnn d23lnn d24lnn d33lnn d34lnn d44lnn
global d11cs d12cs d13cs d14cs d22cs d23cs d24cs d33cs d34cs d44cs
global d11cn d12cn d13cn d14cn d22cn d23cn d24cn d33cn d34cn d44cn
global d11pf d12pf d13pf d14pf d22pf d23pf d24pf d33pf d34pf d44pf
global d11ps d12ps d13ps d14ps d22ps d23ps d24ps d33ps d34ps d44ps

% ----- define the parameters ----- %

eta    = 0.65/0.35;
theta  = 0.3;
delta  = 0.02;
beta   = 0.99;
mu     = 1;
chi    = 0.15;
zbar   = 0;
gamma  = 0.395;
phi    = 0.95;
b0     = 0.0;
rho1   = 4;
sigma  = 0.0035;
sigma  = 0.0045;
lambda = 0;


% ---- coefficients from guvenen1.map -----%

pfss = -.1005034e-1; csss = -.2492224; kss = 2.272306; psss = 2.111551; lsss = -1.012299;
d3cs = 0.; d3k = 0.; d3ls = 0.; d3pf = 0.; d3ps = 0.; d1cs = .6518841; d1k = .7604424e-1;
d1ls = .3997644; d1pf = -.1335594e-1; d2ps = 1.053457; d2k = .9772048; d2pf = .1914496e-1;
d1ps = .3949578; d2cs = .3546303; d4cs = 0.; d4k = 0.; d4ls = 0.; d4pf = 0.; d4ps = 0.;
d2ls = -.6273559e-1; d44ls = -0.; d13ps = -0.; d23ls = -0.; d11ls = -.2993506; d22ps = -.8329112e-1;
d12ls = .1331670; d11pf = .1050269e-1; d33ls = 3.911562; d22pf = -.8960692e-2; d12ps = -.2389474;
d24ls = -0.; d34ps = 0.; d44ps = -0.; d33pf = .2534220; d22ls = -.3408408e-1; d33ps = 2.801036;
d11ps = .3021013; d11k = .7433601e-1; d22k = .1528821e-1;  d11cs = .1173844;  d13pf = -0.;
d12k = -.3724910e-1;  d12cs = -.9347639e-1;  d22cs = .2615120e-1;  d14ls =-0.;  d14pf = 0.;
d13ls = -0.;  d23ps = 0.;  d23pf = 0.;  d34ls = -0.;  d34pf = 0.;  d24ps = 0.;  d24pf = 0.;
d44pf = 0.;  d23k = 0.;  d13k = -0.;  d33k = .5483740;  d33cs = -3.406198;  d12pf = .3293599e-2;
d14k = -0.;  d34k = -0.;  d24k = 0.;  d24cs = 0.;  d13cs = 0.;  d23cs = 0.;  d44k = -0.;
d14ps = 0.;  d34cs =0.;  d14cs = 0.;  d44cs = 0.;

% ----- import states ----- %

Omega = load ('hybrid1a.dat');
P     = load ('hybrid1b.dat');
k0    = load ('hybrid1c.dat');
z0    = load ('hybrid1d.dat');

[r1,c1] = size(Omega);
[r2,c2] = size(k0);
[r3,c3] = size(z0);

% ----- make grids of states ----- %

X0 = z0 ;
X1 = gridmake(z0,k0) ;
Y1 = gridmake([1:r3]',k0) ;

% ----- solve for the change of variable coefficients ------ %
% - set options - %
%options = optimset('Display','iter','MaxIter',1500,'MaxFunEvals',1500,'TolX',1e-06,'TolFun',1e-06);
options = optimset('Display','iter');
ALPHA0  = [1];

ALPHA1(1,:) = fminsearch('hybridFF',ALPHA0,options,r2,r3,sigma,X0,X1,Y1,P)

% ----- compute eulers at every state -----%

[EULER1,EULER2]=hybridFFFF(ALPHA1,z0,k0,Omega,sigma) ;

% ----- simulate a markov chain ----- %

[z1,ttz1] = markov(P,iters+3,1,z0') ;
z1 = z1' ;

% ----- use solution to sumulate capitals ----- %
```

```
k1 = median(k0) ;

% ----- resimulate the model with loss aversion ----- %

[X01,X02] = hybridFFF(ALPHA1,z1,sigma,iters,drop) ;

inv1 = X01(:,3) ;
c1 = X01(:,2);
l1 = X01(:,6);
y1 = X01(:,1);


Pf1 = X01(:,10);

Re1 = X02(:,1)+1;



figure(1)
 subplot(4,2,1), hist(inv1,38), title('investment')
 subplot(4,2,2), hist(c1,38), title('consumption')
 subplot(4,2,3), hist(l1,38), title('hours')
 subplot(4,2,4), hist(y1,38), title('output')
 subplot(4,2,5), hist(Pf1,38), title('bond price')
 subplot(4,2,6), hist(Re1,38), title('Stock return')

print -dpng hybrid3a
print -depsc hybrid3a




% ----- read in euler erros from hybrid2 ----- %
XX = load ('hybdrid2a.dat');

EULER11 = XX(:,2);
EULER22 = XX(:,3);

% ----- plot in same graph ----- %

figure(3)
 plot(k0,EULER1,'.',k0,EULER11,'.'), ...
     legend('HPP method','VFI method','Location','SouthEast'), title('Euler Equation Error; Bond Pricing'), ...
   xlabel('Capital'), ylabel('log10 Euler Equation Error'), ...
    axis([2.21 2.38 -1.6 -0.5])
print -dpng hybrid3c
print -depsc hybrid3c

figure(4)
 plot(k0,EULER2,'.',k0,EULER22,'.'), , ...
     legend('HPP method','VFI method','Location','NorthEast'), title('Euler Equation Error; Equity Pricing'), ...
     xlabel('Capital'), ylabel('log10 Euler Equation Error'), ...
     axis([2.21 2.38 -1.6 -0.5])
print -dpng hybrid3d
print -depsc hybrid3d

mean(EULER1)
mean(EULER2)

std(EULER1)
std(EULER2)


% ----- exit ----- %

comptime = etime(clock, time0)
diary off
```

hybrid3.m

hybridFF.m

```
function [q] = hybridFF(ALPHA,statesk,statesz,sigma,X0,X1,Y1,P)

global eta theta delta beta mu chi zbar gamma phi b0 rho1 rho2
global csss lsss pfss lnnss kss psss cnss
global d1k d2k d3k d4k d1cs d2cs d3cs d4cs d1cn d2cn d3cn d4cn d1ls d2ls d3ls d4ls
global d1lnn d2lnn d3lnn d4lnn d1ps d2ps d3ps d4ps d1pf d2pf d3pf d4pf
global d11k d12k d13k d14k d22k d23k d24k d33k d34k d44k d11ls d12ls
global d13ls d14ls d22ls d23ls d24ls d33ls d34ls d44ls
global d11lnn d12lnn d13lnn d14lnn d22lnn d23lnn d24lnn d33lnn d34lnn
global d44lnn d11cs d12cs d13cs d14cs d22cs d23cs d24cs d33cs d34cs d44cs
global d11cn d12cn d13cn d14cn d22cn d23cn d24cn d33cn d34cn d44cn d11pf
global d12pf d13pf d14pf d22pf d23pf d24pf d33pf d34pf d44pf
```

```
global d11ps d12ps d13ps d14ps d22ps d23ps d24ps d33ps d34ps d44ps

alpha1  = ALPHA(1,1);

sigma10 = sigma ;
sigma11 = sigma ;
sigma12 = sigma ;
sigma13 = sigma ;
sigma14 = sigma ;

% ----- derivatives of transformations ----- %

Kss = kss^(alpha1) ;
Kp  = 1/alpha1*Kss^(1/alpha1-1) ;
Kpp = 1/alpha1*(1/alpha1-1)*Kss^(1/alpha1-2) ;

% ----- define states from imported data ----- %

z0 = X0(:,1) ;
z1 = X1(:,1) ;
k1 = X1(:,2) ;

[r1,c1] = size(X1);

% ----- initialize values --- %

phi2   = 0.35*exp(kss);

% ----- compute time t allocations ----- %

k1(:,2) = kss + d1k*(z1(:,1)-zbar) + ...
                [d2k*Kp]*(k1(:,1).^alpha1-Kss) + ...
                d3k*sigma10 + ...
                d11k/2*(z1(:,1)-zbar).^2 + ...
                [d12k*Kp]*(z1(:,1)-zbar).*(k1(:,1).^alpha1-Kss) + ...
                d13k*(z1(:,1)-zbar).*sigma10 + ...
                [d22k*Kp^2+d2k*Kpp]/2*(k1(:,1).^alpha1-Kss).^2 + ...
                [d23k*Kp]*(k1(:,1).^alpha1-Kss).*sigma10 + ...
                d33k/2*sigma10.^2  ;

cs1(:,1) = csss + d1cs*(z1(:,1)-zbar) + ...
                  [d2cs*Kp]*(k1(:,1).^alpha1-Kss) + ...
                  d3cs*sigma11 + ...
                  d11cs/2*(z1(:,1)-zbar).^2 + ...
                  [d12cs*Kp]*(z1(:,1)-zbar).*(k1(:,1).^alpha1-Kss) + ...
                  d13cs*(z1(:,1)-zbar).*sigma11 + ...
                  [d22cs*Kp^2+d2cs*Kpp]/2*(k1(:,1).^alpha1-Kss).^2 + ...
                  [d23cs*Kp]*(k1(:,1).^alpha1-Kss).*sigma11 + ...
                  d33cs/2*sigma11.^2 ;

ls1(:,1) = lsss + d1ls*(z1(:,1)-zbar) + ...
                  [d2ls*Kp]*(k1(:,1).^alpha1-Kss) + ...
                  d3ls*sigma12 + ...
                  d11ls/2*(z1(:,1)-zbar).^2 + ...
                  [d12ls*Kp]*(z1(:,1)-zbar).*(k1(:,1).^alpha1-Kss) + ...
                  d13ls*(z1(:,1)-zbar).*sigma12 + ...
                  [d22ls*Kp^2+d2ls*Kpp]/2*(k1(:,1).^alpha1-Kss).^2 + ...
                  [d23ls*Kp]*(k1(:,1).^alpha1-Kss).*sigma12 + ...
                  d33ls/2*sigma12.^2  ;

lambda1(:,1) = 0 ;

ps1(:,1) = psss + d1ps*(z1(:,1)-zbar) + ...
                  [d2ps*Kp]*(k1(:,1).^alpha1-Kss) + ...
                  d3ps*sigma13 + ...
                  d11ps/2*(z1(:,1)-zbar).^2 + ...
                  [d12ps*Kp]*(z1(:,1)-zbar).*(k1(:,1).^alpha1-Kss) + ...
                  d13ps*(z1(:,1)-zbar).*sigma13 + ...
                  [d22ps*Kp^2+d2ps*Kpp]/2*(k1(:,1).^alpha1-Kss).^2 + ...
                  [d23ps*Kp]*(k1(:,1).^alpha1-Kss).*sigma13 + ...
                  d33ps/2*sigma13.^2  ;

pf1(:,1) = pfss + d1pf*(z1(:,1)-zbar) + ...
                  [d2pf*Kp]*(k1(:,1).^alpha1-Kss) + ...
                  d3pf*sigma14 + ...
                  d11pf/2*(z1(:,1)-zbar).^2 + ...
                  [d12pf*Kp]*(z1(:,1)-zbar).*(k1(:,1).^alpha1-Kss) + ...
                  d13pf*(z1(:,1)-zbar).*sigma14 + ...
                  [d22pf*Kp^2+d2pf*Kpp]/2*(k1(:,1).^alpha1-Kss).^2 + ...
                  [d23pf*Kp]*(k1(:,1).^alpha1-Kss).*sigma14 + ...
                  d33pf/2*sigma14.^2  ;

l1(:,1)   = mu*exp(ls1(:,1)) ;
```

```
w1(:,1)   = (1-theta).*exp(z1(:,1)).*exp(k1(:,1)*theta).*(l1(:,1).^(-theta));

y1(:,1)   = exp(z1(:,1)).*exp(theta*k1(:,1)).*(l1(:,1).^(1-theta)) ;

inv1(:,1) = exp(k1(:,2)) - (1-delta)*exp(k1(:,1)) ;

g1(:,1)   = phi2/2*1./(exp(k1(:,1))).*(inv1(:,1)-delta*exp(k1(:,1))).^2 ;

d1(:,1)   = y1(:,1)-w1(:,1).*(l1(:,1))-inv1(:,1)-(1-exp(pf1(:,1))).*chi*exp(kss)-g1(:,1) ;

ee1(:,1)  = y1(:,1) - inv1(:,1) - g1(:,1) - exp(cs1(:,1));

% ----- make new grid out of states ----- %

X2 = gridmake(z0,k1(:,2)) ;

z2 = X2(:,1) ;
k2 = X2(:,2) ;

[r2,c2] = size(X2);

% ----- compute time t+1 allocations ----- %

k2(:,2) = kss + d1k*(z2(:,1)-zbar) + ...
                [d2k*Kp]*(k2(:,1).^alpha1-Kss) + ...
                d3k*sigma10 + ...
                d11k/2*(z2(:,1)-zbar).^2 + ...
                [d12k*Kp]*(z2(:,1)-zbar).*(k2(:,1).^alpha1-Kss) + ...
                d13k*(z2(:,1)-zbar).*sigma10 + ...
                [d22k*Kp^2+d2k*Kpp]/2*(k2(:,1).^alpha1-Kss).^2 + ...
                [d23k*Kp]*(k2(:,1).^alpha1-Kss).*sigma10 + ...
                d33k/2*sigma10.^2  ;

cs2(:,1) = csss + d1cs*(z2(:,1)-zbar) + ...
                  [d2cs*Kp]*(k2(:,1).^alpha1-Kss) + ...
                  d3cs*sigma11 + ...
                  d11cs/2*(z2(:,1)-zbar).^2 + ...
                  [d12cs*Kp]*(z2(:,1)-zbar).*(k2(:,1).^alpha1-Kss) + ...
                  d13cs*(z2(:,1)-zbar).*sigma11 + ...
                  [d22cs*Kp^2+d2cs*Kpp]/2*(k2(:,1).^alpha1-Kss).^2 + ...
                  [d23cs*Kp]*(k2(:,1).^alpha1-Kss).*sigma11 + ...
                  d33cs/2*sigma11.^2  ;

ls2(:,1) = lsss + d1ls*(z2(:,1)-zbar) + ...
                  [d2ls*Kp]*(k2(:,1).^alpha1-Kss) + ...
                  d3ls*sigma12 + ...
                  d11ls/2*(z2(:,1)-zbar).^2 + ...
                  [d12ls*Kp]*(z2(:,1)-zbar).*(k2(:,1).^alpha1-Kss) + ...
                  d13ls*(z2(:,1)-zbar).*sigma12 + ...
                  [d22ls*Kp^2+d2ls*Kpp]/2*(k2(:,1).^alpha1-Kss).^2 + ...
                  [d23ls*Kp]*(k2(:,1).^alpha1-Kss).*sigma12 + ...
                  d33ls/2*sigma12.^2;

lambda2(:,1) = 0;

ps2(:,1) = psss + d1ps*(z2(:,1)-zbar) + ...
                  [d2ps*Kp]*(k2(:,1).^alpha1-Kss) + ...
                  d3ps*sigma13 + ...
                  d11ps/2*(z2(:,1)-zbar).^2 + ...
                  [d12ps*Kp]*(z2(:,1)-zbar).*(k2(:,1).^alpha1-Kss) + ...
                  d13ps*(z2(:,1)-zbar).*sigma13 + ...
                  [d22ps*Kp^2+d2ps*Kpp]/2*(k2(:,1).^alpha1-Kss).^2 + ...
                  [d23ps*Kp]*(k2(:,1).^alpha1-Kss).*sigma13 + ...
                  d33ps/2*sigma13.^2  ;

pf2(:,1) = pfss + d1pf*(z2(:,1)-zbar) + ...
                  [d2pf*Kp]*(k2(:,1).^alpha1-Kss) + ...
                  d3pf*sigma14 + ...
                  d11pf/2*(z2(:,1)-zbar).^2 + ...
                  [d12pf*Kp]*(z2(:,1)-zbar).*(k2(:,1).^alpha1-Kss) + ...
                  d13pf*(z2(:,1)-zbar).*sigma14 + ...
                  [d22pf*Kp^2+d2pf*Kpp]/2*(k2(:,1).^alpha1-Kss).^2 + ...
                  [d23pf*Kp]*(k2(:,1).^alpha1-Kss).*sigma14 + ...
                  d33pf/2*sigma14.^2  ;

l2(:,1)   = mu*exp(ls2(:,1)) ;

w2(:,1)   = (1-theta).*exp(z2(:,1)).*exp(k2(:,1)*theta).*(l2(:,1).^(-theta));

y2(:,1)   = exp(z2(:,1)).*exp(theta*k2(:,1)).*(l2(:,1).^(1-theta)) ;

inv2(:,1) = exp(k2(:,2)) - (1-delta)*exp(k2(:,1)) ;

g2(:,1)   = phi2/2*1./(exp(k2(:,1))).*(inv2(:,1)-delta*exp(k2(:,1))).^2 ;
```

```
d2(:,1)    = y2(:,1)-w2(:,1).*(l2(:,1))-inv2(:,1)-(1-exp(pf2(:,1))).*chi*exp(kss)-g2(:,1) ;

ee2(:,1)   = y2(:,1) - inv2(:,1) - g2(:,1) - exp(cs2(:,1)) ;

% ----- reshape next periods vars ---- %

K1  = reshape(k1(:,1),1,r1) ;
Z1  = reshape(z1(:,1),1,r1) ;
CS1 = reshape(cs1,1,r1) ;
LS1 = reshape(ls1,1,r1) ;
PS1 = reshape(ps1,1,r1) ;
PF1 = reshape(pf1,1,r1) ;
EE1 = reshape(ee1,1,r1) ;

K2  = reshape(k2(:,1),statesz,r1) ;
Z2  = reshape(z2(:,1),statesz,r1) ;
CS2 = reshape(cs2,statesz,r1) ;
LS2 = reshape(ls2,statesz,r1) ;
PS2 = reshape(ps2,statesz,r1) ;
D2  = reshape(d2,statesz,r1) ;
K3  = reshape(k2(:,2),statesz,r1) ;
EE2 = reshape(ee2,statesz,r1) ;

id = reshape(Y1(:,1),1,statesz(1,1)*statesk(1,1)) ;

% ----- compute eulers at every possible state ----- %

% - marginal utility - %

for i = 1:r1
 for j = 1:statesz
  u2(j,i) = beta*gamma*(exp(CS2(j,i)*(gamma*(1-rho1)-1))).*(1-exp(LS2(j,i))).^((1-gamma)*(1-rho1)) ./ ...
              (gamma*(exp(CS1(1,i)*(gamma*(1-rho1)-1))).*(1-exp(LS1(1,i))).^((1-gamma)*(1-rho1)) ) ;
 end
end
% - invoke loss aversion - %

for i = 1:r1
 for j = 1:statesz
  temp = ((exp(PS2(j,i)) + D2(j,i))./exp(PS1(1,i)) - 1./exp(PF1(1,i))) ;

    euler(j,i)  = 1 - u2(j,i)*(exp(PS2(j,i))+D2(j,i))./exp(PS1(1,i)) - b0*u2(j,i)*temp ;
    euler1(j,i) = (1+phi2*((exp(K2(j,i))-exp(K1(1,i)))/exp(K1(1,i)))) - ...
                     u2(j,i)*((theta)*exp(Z2(j,i))*exp(K2(j,i)*(theta-1))*exp(LS2(j,i)*(1-theta))+1-delta+ ...
                               phi2/2*((exp(K3(j,i))-exp(K2(j,i)))/exp(K2(j,i))+1)^2-phi2/2 )  ;
    euler2(j,i) = (1-gamma)/gamma*exp(CS1(1,i))/(1-exp(LS1(1,i))) - ...
                     (1-theta)*exp(Z1(1,i))*exp(K1(1,i)*(theta))*exp(LS1(1,i)*(-theta)) ;
    euler3(j,i) = 1 - u2(j,i)*1./exp(PF1(1,i)) ;
    euler4(j,i) = EE2(j,i);


 end
end

% - compute conditional expectation of Euler - %

for i = 1:r1
 error(i,1)  = P(id(1,i),:)*euler(:,i)  ;
 error1(i,1) = P(id(1,i),:)*euler1(:,i) ;
 error2(i,1) = P(id(1,i),:)*euler2(:,i) ;
 error3(i,1) = P(id(1,i),:)*euler3(:,i) ;
 error4(i,1) = P(id(1,i),:)*euler4(:,i) ;
end
% ----- add up Eulers ----- %

%q = [error;error1;error2;error3]'*[error;error1;error2;error3] ;
q = [sum(error);sum(error1);sum(error2);sum(error3);sum(error4)]'*...
    [sum(error);sum(error1);sum(error2);sum(error3);sum(error4)] ;
%q = abs(sum(error))+ abs(sum(error1))+abs(sum(error2))+abs(sum(error3))+abs(sum(error4));

end
```

hybridFF.m

hybridFFFF.m

```
% matlab file to simulate from the solutions of guvenen6.map
%

function [EULER1,EULER2] = hybridFFFF(ALPHA,z0,k0,Omega,sigma)

global eta theta delta beta mu chi zbar gamma phi b0 rho1 rho2
global csss lsss pfss lnnss kss psss cnss
```

```
global d1k d2k d3k d4k d1cs d2cs d3cs d4cs d1cn d2cn d3cn d4cn d1ls d2ls d3ls d4ls
global d1lnn d2lnn d3lnn d4lnn d1ps d2ps d3ps d4ps d1pf d2pf d3pf d4pf
global d11k d12k d13k d14k d22k d23k d24k d33k d34k d44k d11ls d12ls d13ls
global d14ls d22ls d23ls d24ls d33ls d34ls d44ls
global d11lnn d12lnn d13lnn d14lnn d22lnn d23lnn d24lnn d33lnn d34lnn d44lnn
global d11cs d12cs d13cs d14cs d22cs d23cs d24cs d33cs d34cs d44cs
global d11cn d12cn d13cn d14cn d22cn d23cn d24cn d33cn d34cn d44cn d11pf
global d12pf d13pf d14pf d22pf d23pf d24pf d33pf d34pf d44pf
global d11ps d12ps d13ps d14ps d22ps d23ps d24ps d33ps d34ps d44ps

alpha1  = ALPHA(1,1);

sigma10 = sigma ;
sigma11 = sigma ;
sigma12 = sigma ;
sigma13 = sigma ;
sigma14 = sigma ;

k000 = Omega(:,1);
z000 = Omega(:,2);

[r1,c1] = size(Omega);
[r2,c2] = size(k0);
[r3,c3] = size(z0);

% ----- derivatives of transformations ----- %

Kss = kss^(alpha1) ;
Kp  = 1/alpha1*Kss^(1/alpha1-1) ;
Kpp = 1/alpha1*(1/alpha1-1)*Kss^(1/alpha1-2) ;

% ----- initialize values --- %

phi2   = 0.35*exp(kss);

lss    = mu*exp(lsss) ;
wss    = (1-theta)*exp(zbar)*exp(kss*theta)*(lss^(-theta)) ;
yss    = exp(kss*theta)*(lss^(1-theta)) ;
invss  = exp(kss) - (1-delta)*exp(kss) ;
gss    = phi2/2*(1/exp(kss))*(invss-delta*exp(kss))^2 ;
dss    = yss - wss*lss - invss - gss - (1-exp(pfss))*chi*exp(kss) ;

rb(1,1) = 1/exp(pfss) ;
rs(1,1) = 1 + dss/exp(psss) ;
rk(1,1) = rs(1,1) ;

% ----- start to iterate ----- %

% - pure states - %

for i=1:1:r1

 k100(i,1) = kss + d1k*(z000(i,1)-zbar) + ...
                   [d2k*Kp]*(k000(i,1)^alpha1-Kss) + ...
                   d3k*sigma10 + ...
                   d11k/2*(z000(i,1)-zbar)^2 + ...
                   [d12k*Kp]*(z000(i,1)-zbar)*(k000(i,1)^alpha1-Kss) + ...
                   d13k*(z000(i,1)-zbar)*sigma10 + ...
                   [d22k*Kp^2+d2k*Kpp]/2*(k000(i,1)^alpha1-Kss)^2 + ...
                   [d23k*Kp]*(k000(i,1)^alpha1-Kss)*sigma10 + ...
                   d33k/2*sigma10^2  ;

end;

% - endogenous vars - %

for i=1:r1

 cs000(i,1) = csss + d1cs*(z000(i,1)-zbar) + ...
                   [d2cs*Kp]*(k000(i,1)^alpha1-Kss) + ...
                   d3cs*sigma11 + ...
                   d11cs/2*(z000(i,1)-zbar)^2 + ...
                   [d12cs*Kp]*(z000(i,1)-zbar)*(k000(i,1)^alpha1-Kss) + ...
                   d13cs*(z000(i,1)-zbar)*sigma11 + ...
                   [d22cs*Kp^2+d2cs*Kpp]/2*(k000(i,1)^alpha1-Kss)^2 + ...
                   [d23cs*Kp]*(k000(i,1)^alpha1-Kss)*sigma11 + ...
                   d33cs/2*sigma11^2 ;

 ls000(i,1) = lsss + d1ls*(z000(i,1)-zbar) + ...
                   [d2ls*Kp]*(k000(i,1)^alpha1-Kss) + ...
                   d3ls*sigma12 + ...
                   d11ls/2*(z000(i,1)-zbar)^2 + ...
                   [d12ls*Kp]*(z000(i,1)-zbar)*(k000(i,1)^alpha1-Kss) + ...
                   d13ls*(z000(i,1)-zbar)*sigma12 + ...
                   [d22ls*Kp^2+d2ls*Kpp]/2*(k000(i,1)^alpha1-Kss)^2 + ...
```

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                  [d23ls*Kp]*(k000(i,1)^alpha1-Kss)*sigma12 + ...
                  d33ls/2*sigma12^2 ;

 ps000(i,1) = psss + d1ps*(z000(i,1)-zbar) + ...
                  [d2ps*Kp]*(k000(i,1)^alpha1-Kss) + ...
                  d3ps*sigma13 + ...
                  d11ps/2*(z000(i,1)-zbar).^2 + ...
                  [d12ps*Kp]*(z000(i,1)-zbar).*(k000(i,1)^alpha1-Kss) + ...
                  d13ps*(z000(i,1)-zbar).*sigma13 + ...
                  [d22ps*Kp^2+d2ps*Kpp]/2*(k000(i,1)^alpha1-Kss).^2 + ...
                  [d23ps*Kp]*(k000(i,1)^alpha1-Kss).*sigma13 + ...
                  d33ps/2*sigma13.^2  ;

 pf000(i,1) = pfss + d1pf*(z000(i,1)-zbar) + ...
                  [d2pf*Kp]*(k000(i,1)^alpha1-Kss) + ...
                  d3pf*sigma14 + ...
                  d11pf/2*(z000(i,1)-zbar)^2 + ...
                  [d12pf*Kp]*(z000(i,1)-zbar)*(k000(i,1)^alpha1-Kss) + ...
                  d13pf*(z000(i,1)-zbar)*sigma14 + ...
                  [d22pf*Kp^2+d2pf*Kpp]/2*(k000(i,1)^alpha1-Kss)^2 + ...
                  [d23pf*Kp]*(k000(i,1)^alpha1-Kss)*sigma14 + ...
                  d33pf/2*sigma14^2  ;

 % - other endo vars - %

 l000(i,1)   = mu*exp(ls000(i,1)) ;
 w000(i,1)   = (1-theta)*exp(z000(i,1))*exp(k000(i,1)*theta)*(l000(i,1)^(-theta));
 y000(i,1)   = exp(z000(i,1))*exp(theta*k000(i,1))*(l000(i,1)^(1-theta)) ;
 inv000(i,1) = exp(k100(i,1)) - (1-delta)*exp(k000(i,1)) ;
 g000(i,1)   = phi2/2*1/(exp(k000(i,1)))*(inv000(i,1)-delta*exp(k000(i,1)))^2 ;
 d000(i,1)   = y000(i,1)-w000(i,1)*(l000(i,1))-inv000(i,1)-(1-exp(pf000(i,1)))*chi*exp(kss)-g000(i,1) ;
 ccs000(i,1) = y000(i,1) - inv000(i,1) - g000(i,1) ;
 muc000(i,1) = gamma*exp(cs000(i,1)*(gamma*(1-rho1)-1))*(1-l000(i,1))^((1-gamma)*(1-rho1));


end;

% - next periods endo vars - %

for i = 1:r1
 for j = 1:r3

  k200(i,j) = kss + d1k*(z0(j,1)-zbar) + ...
                  [d2k*Kp]*(k100(i,1)^alpha1-Kss) + ...
                  d3k*sigma10 + ...
                  d11k/2*(z0(j,1)-zbar)^2 + ...
                  [d12k*Kp]*(z0(j,1)-zbar)*(k100(i,1)^alpha1-Kss) + ...
                  d13k*(z0(j,1)-zbar)*sigma10 + ...
                  [d22k*Kp^2+d2k*Kpp]/2*(k100(i,1)^alpha1-Kss)^2 + ...
                  [d23k*Kp]*(k100(i,1)^alpha1-Kss)*sigma10 + ...
                  d33k/2*sigma10^2  ;
 end
end



for i=1:r1
 for j = 1:r3

 cs100(i,j) = csss + d1cs*(z0(j,1)-zbar) + ...
                  [d2cs*Kp]*(k100(i,1)^alpha1-Kss) + ...
                  d3cs*sigma11 + ...
                  d11cs/2*(z0(j,1)-zbar)^2 + ...
                  [d12cs*Kp]*(z0(j,1)-zbar)*(k100(i,1)^alpha1-Kss) + ...
                  d13cs*(z0(j,1)-zbar)*sigma11 + ...
                  [d22cs*Kp^2+d2cs*Kpp]/2*(k100(i,1)^alpha1-Kss)^2 + ...
                  [d23cs*Kp]*(k100(i,1)^alpha1-Kss)*sigma11 + ...
                  d33cs/2*sigma11^2 ;

 ls100(i,j) = lsss + d1ls*(z0(j,1)-zbar) + ...
                  [d2ls*Kp]*(k100(i,1)^alpha1-Kss) + ...
                  d3ls*sigma12 + ...
                  d11ls/2*(z0(j,1)-zbar)^2 + ...
                  [d12ls*Kp]*(z0(j,1)-zbar)*(k100(i,1)^alpha1-Kss) + ...
                  d13ls*(z0(j,1)-zbar)*sigma12 + ...
                  [d22ls*Kp^2+d2ls*Kpp]/2*(k100(i,1)^alpha1-Kss)^2 + ...
                  [d23ls*Kp]*(k100(i,1)^alpha1-Kss)*sigma12 + ...
                  d33ls/2*sigma12^2 ;

 ps100(i,j) = psss + d1ps*(z0(j,1)-zbar) + ...
                  [d2ps*Kp]*(k100(i,1)^alpha1-Kss) + ...
                  d3ps*sigma13 + ...
                  d11ps/2*(z0(j,1)-zbar).^2 + ...
                  [d12ps*Kp]*(z0(j,1)-zbar).*(k100(i,1)^alpha1-Kss) + ...
```

```
                   d13ps*(z0(j,1)-zbar).*sigma13 + ...
                   [d22ps*Kp^2+d2ps*Kpp]/2*(k100(i,1)^alpha1-Kss).^2 + ...
                   [d23ps*Kp]*(k100(i,1)^alpha1-Kss).*sigma13 + ...
                   d33ps/2*sigma13.^2  ;

 pf100(i,j) = pfss + d1pf*(z0(j,1)-zbar) + ...
                   [d2pf*Kp]*(k100(i,1)^alpha1-Kss) + ...
                   d3pf*sigma14 + ...
                   d11pf/2*(z0(j,1)-zbar)^2 + ...
                   [d12pf*Kp]*(z0(j,1)-zbar)*(k100(i,1)^alpha1-Kss) + ...
                   d13pf*(z0(j,1)-zbar)*sigma14 + ...
                   [d22pf*Kp^2+d2pf*Kpp]/2*(k100(i,1)^alpha1-Kss)^2 + ...
                   [d23pf*Kp]*(k100(i,1)^alpha1-Kss)*sigma14 + ...
                   d33pf/2*sigma14^2  ;

 % - other endo vars - %

 l100(i,j)   = mu*exp(ls100(i,j)) ;
 w100(i,j)   = (1-theta)*exp(z0(j,1))*exp(k100(i,1)*theta)*(l100(i,j)^(-theta));
 y100(i,j)   = exp(z0(j,1))*exp(theta*k100(i,1))*(l100(i,j)^(1-theta)) ;
 inv100(i,j) = exp(k200(i,j)) - (1-delta)*exp(k100(i,1)) ;
 g100(i,j)   = phi2/2*1/(exp(k100(i,1)))*(inv100(i,j)-delta*exp(k100(i,1)))^2 ;

 d100(i,j)   = y100(i,j)-w100(i,j)*(l100(i,j))-inv100(i,j)-(1-exp(pf100(i,j)))*chi*exp(kss)-g100(i,j) ;
 ccs100(i,j) = y100(i,j) - inv100(i,j) - g100(i,j) ;
 muc100(i,j) = gamma*exp(cs100(i,j)*(gamma*(1-rho1)-1))*(1-l100(i,j))^((1-gamma)*(1-rho1));

end;
end


% - eulers at every state - %
for i = 1:r1
 euler1000(i,:) = log10(max(abs(beta*muc100(i,:)/...
                 muc000(i,1)./ ...
                  exp(pf000(i,1)) - 1)));

 euler2000(i,:) = log10(max( abs(beta*muc100(i,:)/...
                 muc000(i,1).* ...
                  [exp(ps100(i,:))+d100(i,:)]/exp(ps000(i,1)) - 1) ));

end


% - find the max - %
for i = 1:r2

  EULER1(i,1) = max(euler1000( Omega(:,1)==k0(i,1),:)) ;
  EULER2(i,1) = max(euler2000( Omega(:,1)==k0(i,1),:)) ;

end
```

hybridFFFF.m

CHAPTER III

A PRIOR PREDICTIVE ANALYSIS OF THE EFFECTS OF LOSS AVERSION/NARROW FRAMING IN A MACROECONOMIC MODEL FOR ASSET PRICING

3.1 Introduction

The study of asset pricing is an important topic in monetary economics. As Sargent (2010) explains:

> *Important parts of modern macro are about understanding a large and interesting suite of asset pricing puzzles – puzzles about empirical failures of simple versions of efficient markets theories.*

Sargent's point is that while asset pricing theories provide logical frameworks for understanding how monetary policy is channeled through to the real economy, his interpretation is complicated by the existence of unresolved empirical anomalies. One of the most challenging of these anomalies is the Equity Premium Puzzle (EPP). The Equity Premium, the persistent excess of stock returns over the risk-free rate, is enigmatic in the sense of not being reconcilable with the predications of the neoclassical asset pricing models. For example, an early study by Mehra and Prescott (1985) estimated an equity premium of 6% which could only be reconciled with the neoclassical asset pricing model by assuming that consumers had extreme and unrealistic aversion to risk.[1] Mehra and Prescott's paper fostered a series of studies (e.g., Kocherlakota 1996, Benartzi and Thaler 1995, Mankiw and Zeldes 1991) that attempted to explain the existence of the Equity Premium by determining how households derive utility and form expectations. The present essay is predicated on the Loss-Aversion/Narrow-Framing (LANF) hypothesis of Benartzi and Thaler (1995). Their paper is one of the earliest studies to hypothesize LANF preferences as a basis for resolving the EPP paradox. Later literature (Barberis, Huang and Santos 2001, Barberis and Huang 2004 and 2008, and Grüne and Semmler 2008) extended the Benarti and Thaler findings using a similar loss-aversion framework by including LANF component into specific household's maximization problem with different utility functions.

Preferences with LANF are fundamentally different from the typical risk aversion assumption. Investors with LANF preferences have greater sensitivity to losses than to acquiring gains. A numerical example from DellaVigna (2009) based on the experimental evidence of Kahneman and Tversky (1979) illustrates the difference between the two behavioral assumptions:

> *There are two experiments. In the first one, subjects are asked to choose between: (A) a 50% chance to gain \$1,000 and a 50% chance to gain nothing; or (B) a sure gain of \$500, given that they have been given \$1,000. The other experiment asks to choose between: (C) a 50% chance to lose \$1,000 and a 50% chance to lose nothing; or (D) a sure loss of \$500, given that they have been given \$2,000.*

Most individuals (84%) in the first group accepted (B) whereas most people (69%) in the second group accepted (C). Risk aversion preferences cannot explain these choices

[1] Mehra and Prescott (1985) observe that, over the ninety-year period 1889-1978, the average real annual yield on the Standard and Poor 500 Index is 7%. Whereas the average yield on short-term debt is less than 1%. The data they used for the risk-free rate includes asset returns on the ninety-day Treasury bill, Treasury certificate, and sixty-day to ninety-day commercial paper.

given that (B) and (D) are statistically identical lotteries. The actual mechanism that generates greater sensitivity to losses – even small losses – is achieved, in this Chapter, by positing a kinked utility function with the slope of the loss proportion of the function steeper than the gain proportion. Assuming a kinked preference function fundamentally alters theory's predictions about the relationship between asset demand and expected rates of return.

The works of Barberis et al. (2001), Barberis and Huang (2004 and 2008) and Grüne and Semmler (2008) have shown that LANF preferences can generate high equity premiums. Nevertheless, their works lead to a question much like the one originally faced by Mehra and Prescott (1985); namely, are the calibrated LANF preferences that generates the equity premium reasonable? To address this issue, the present Chapter conducts a *prior predictive analysis* and subsequent model evaluation (e.g., Geweke and Whiteman 2006, Geweke 2010). In the prior predictive analysis, prior distributions are defined along three key dimensions; the variance of aggregate uncertainty, the elasticity of labor supply, and the degree of LANF. Although a wide array of prior distributions are initially chosen (i.e., encompassing all reasonable parameters), a major contribution of the present Chapter is to develop a unique set of appropriate priors for a Dynamic Stochastic General Equilibrium (DSGE) model with LANF preferences. Presumably, the priors established in this Chapter will have further applicability in the full structural estimations.

To be more specific, the estimation method of prior prediction is an iterative Bayesian approach (Canova 1994, Geweke and Whiteman 2006, Geweke 2007) encompassing the following steps. First, the priors are defined for the parameters of interest. Second, the priors are then used to determine the model's parameters. Third, given the parameters determined in step two, the model is numerically solved for its equilibriums values. Performing these operations many times generates the statistical distributions of interest ; i.e., the equity premium, Sharpe ratios, and other volatility measures. In the final step, the actual data is compared to the distributions generated by the model. The theory of LANF is supported when estimates from the actual data fall within these distributions.

The research of Barberis et al. (2001), Barberis and Huang (2004 and 2008) and Grüne and Semmler (2008) have shown that as the margins for wealth smoothing are increased, the effects of LANF diminish. Or, stated somewhat differently, moving from the pure endowment economies proposed in Barberis et al (2001, 2004, and 2008) to the production economy hypothesized in Grüne and Semmler (2008) shows that the equity premium declines. What this means is that the extra margins of choice in the production economy (i.e., particularly precautionary savings from capital) encourage the households to smooth away from the kink in the households preference function. The model economy developed in this Chapter is similar to recent work of Danthine and Donaldson (2002) and Guvenen (2009) in that it adds labor choice as another margin of household smoothing. Though labor effort might reduce the equity premium, it is reasonable to believe that it may strengthen LANF's predictions for equity premium given that more realistic volatilities may drive the economy near the kink. Ultimately, however, the effects are to be discovered in the quantitative analysis.

The model in the present Chapter posits two types of agents: households and firms. The firm makes decisions about labor, capital investment, and borrowing. The firm owns the capital stock and employs internal and external funds to finance investment. External funds take the form of fixed rate bonds that are paid back with certainty at the end of each period. This is the risk-free asset in the model with a price that is equal to the inverse of the return - which is determined by the household's marginal rate of substitution between two periods. The household supplies labor and purchases assets produced by the firm. Equities represent claims to dividends and their value are derived from how well the firm makes investment choices. Furthermore, by assuming that the firm maximizes the expected value of investment choices, the equity values can be directly related to the value of the firm's capital stock.

Loss aversion brings discrete elements into the agents' optimization problems (decision process). This adds complexities to the model and may be one reason LANF has not been studied more thoroughly in a general equilibrium framework. However, the present Chapter develops a new method that is better suited to the assumption of LANF preferences. This new method, called a hybrid Perturbation-Projection method, combines perturbation and projection techniques, first introduced by Judd (1996) and recently applied by Fernández-Villaverde and Rubio-Ramirez (2006). The noteworthy aspect of this algorithm is that it exploits the beneficial properties of both methods in the presence of bifurcated functions.

The results from the application of this new method revealed that contrary to the hypothesis of Benartzi and Thaler (1995), the introduction of LANF preferences does not explain equity premium under reasonable assumptions about labor supply elasticities. Only when the labor supply elasticities are unreasonably low, by the standards of the extant literature, can LANF preferences generate an equity premium. Alternatively, when the elasticity coefficients are more realistic, LANF preferences fail to generate a premium. The conclusion from these experiments is that explaining the equity premium in terms of LANF preferences depends on the assumptions made about labor choices. Another important finding is that the hybrid perturbation-projection algorithm is a robust technique for analyzing the EPP.

The Chapter is organized as follows. Section 3.2 presents the general equilibrium model. Section 3 introduces the method used to find the dynamic properties of the model. Section 3.4 discusses the main findings. Section 3.5 concludes with suggestions for future work. The appendix details how the solution method works to simulate the results.

3.2 Structure of the Model

The LANF asset pricing model is comprised of two sectors: households and firms. The agents in these sectors transact in four markets: goods, stocks, bonds and labor markets.

3.2.1 Households

Infinitely-lived households enter the financial market to invest their financial wealth in both stocks and bonds. These households maximize their lifetime utility function:

$$\max_{\{c_t,l_t,s_{t+1},B_{t+1}\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\begin{array}{c} \frac{\left(c_t^{\gamma}(1-l_t)^{1-\gamma}\right)^{1-\rho}}{1-\rho} + \\ \beta b_0 \gamma \overline{c}_{t+1}^{\gamma(1-\rho)-1} \left(1-\overline{l}_{t+1}\right)^{(1-\gamma)(1-\rho)} \overline{\upsilon}\left(G_{s,t+1}\right) \end{array} \right) \right\},$$

subject to:

$$c_t + p_t^f B_{t+1} + p_t^s s_{t+1} \leq B_t + s_t \left(p_t^s + d_t\right) + w_t l_t,$$

where β is the time discount rate. The term $B_t + s_t\left(p_t^s + d_t\right) + w_t l_t$ is the total wealth that the agent possesses in period t that includes: the returns from buying bonds B_t, returns from investing stocks $s_t\left(p_t^s + d_t\right)$ with the share of the stock s_t at price of p_t^s, and labor income $w_t l_t$. Expenditures that include: current consumption c_t, the purchase of bonds $p_t^f B_{t+1}$, and stock purchase $p_t^s s_{t+1}$ cannot exceed the total wealth at the end of time t. Prices for stocks and bonds at time t are p_t^s and p_t^f, respectively, while d_t is the dividend paid to the investor by the firm.

The first half of the momentary utility function, $(c_t^{\gamma}\left(1-l_t\right)^{1-\gamma})^{1-\rho}/\left(1-\rho\right)$, follows standard neoclassical macroeconomics. This utility function, defined on consumption and labor hours l_t, has two main parameters, ρ and γ, that mutually determine the EIS (Elasticity of Inter-temporal Substitution), risk aversion, and the Frisch labor supply elasticity. With this form of Cobb-Douglas (CD) utility function, risk aversion is measured by the parameter ρ (> 0), EIS is measured by $1/\rho$, and the Frisch labor supply elasticity is $((1-l)/l)\left((1-\gamma(1-\rho))/\rho\right)$. The quantitative magnitude of these parameters are shown to be important in the subsequent analysis.

The second half of the expected utility function is adapted from the framework outlined in Barberis, et al. (2001) and Barberis and Huang (2008) which includes the LANF component discounted in the period $t+1$.

$$\beta b_0 \gamma \bar{c}_{t+1}^{\gamma(1-\rho)-1}\left(1-\bar{l}_{t+1}\right)^{(1-\gamma)(1-\rho)}\left[\overline{v}\left(G_{s,t+1}\right)\right],$$

The term $\overline{v}(G_{s,t+1})$ represents the gain or loss in the value of financial wealth.

$$G_{s,t+1} = p_t^s s_{t+1}\left(\frac{p_{t+1}^s + d_{t+1}}{p_t^s} - \frac{1}{p_t^f}\right),$$

$\overline{v}(G_{s,t+1})$ governs the equity premium trajectory and $\overline{v}(x)$ generates the kink in preferences - i.e., the loss-aversion element of preferences.

$$\overline{v}(x) = \begin{cases} \lambda_L x & \text{for } x \geq 0 \text{ where } \lambda_L = 1. \\ \lambda_H x & \text{for } x < 0 \text{ where } \lambda_H \geq 1. \end{cases},$$

Households value the gain or loss by the function $\overline{v}(x)$ which demonstrates that agents are more sensitive to losses ($x \leq 0$) than to gains ($x \geq 0$). The parameters λ_L and λ_H ,where $\lambda_L < \lambda_H$,determine how sensitive households are to gains related to losses. More specifically, if the return on stocks is less than risk-free rate, the agent's utility is reduced more than otherwise. This behavioral assumption is in accordance with prospect theory (Kahneman and Tversky, 1979) which postulates that a consumer's utility is defined over the domain of losses or gains; in this case, the losses or gains result from purchases of equities.

Additionally, prospect theory implies that households narrowly frame their choices. Previous studies (Benartzi and Thaler 1995, Barberis et al. 2001) postulate that households narrowly frame both cross-sectionally and temporally by a fraction of the marginal utility of consumption. The term $b_0\beta\gamma\bar{c}_{t+1}^{\gamma(1-\rho)-1}\left(1-\bar{l}_{t+1}\right)^{(1-\gamma)(1-\rho)}$ represents how investors frame outcomes. Here $\{\bar{c}_t, \bar{l}_t\}$ denote the aggregate per capita consumption and labor hours for a typical participating household, b_0 is a scaling factor to signify the degree of narrow framing (Grüne and Semmler 2008, Barberis et al. 2001), and the remaining term is the marginal utility of consumption. Setting $b_0 = 0$, eliminates narrow framing, recreating the standard asset pricing model. Thus, multiplying by b_0 adjusts the function for the overall importance of utility from gains and losses in financial wealth relative to utility from consumption.

Household optimization yields three first-order conditions:

$$
\begin{aligned}
1 &= E_t\left\{\beta\left(\frac{1-l_{t+1}}{1-l_t}\right)^{(1-\gamma)(1-\rho)}\left(\frac{c_{t+1}}{c_t}\right)^{\gamma(1-\rho)-1}\left(\frac{1}{p_t^f}\right)\right\}, && (15)\\
1 &= E_t\left\{\begin{array}{c}\beta\left(\frac{1-l_{t+1}}{1-l_t}\right)^{(1-\gamma)(1-\rho)}\left(\frac{c_{t+1}}{c_t}\right)^{\gamma(1-\rho)-1}\times\\ \left[\left(\frac{p_{t+1}^s+d_{t+1}}{p_t^s}\right)+b_0\overline{v}\left(\frac{p_{t+1}^s+d_{t+1}}{p_t^s}-\frac{1}{p_t^f}\right)\right]\end{array}\right\}, && (16)\\
0 &= \frac{1-\gamma}{(1-l_t)}-w_t\frac{\gamma}{c_t} && (17)
\end{aligned}
$$

Equation (15) is the inter-temporal Euler for bond purchasers, (16) is the inter-temporal Euler for stocks and (17) is the intra-temporal Euler between consumption and labor hours. The LANF preferences are embodied in equation (16). Note that if $b_0 = 0$, then equation (16) would result in the standard asset pricing model of Danthine and Donaldson (2002). Equation (16) differs from Barberis and Huang (2008) and Grüne and Semmler (2008) in its reference to a general equilibrium environment. As the households maximize their utilities with respect to both consumption and labor, our Euler has an additional labor component not included in the model of

Barberis and Huang. The extension to general equilibrium also adds equation (17), the condition for intra-temporal substitution between consumption and leisure.

3.2.2 Firms

The firms in this economy produce the consumption good with Cobb-Douglas technology $y_t = z_t k_t^{\theta} l_t^{1-\theta}$ in perpetuity. The level of technology evolves according to the exogenous process

$$\log(z_{t+1}) = \eta \log(z_t) + \varepsilon_{t+1},\ \varepsilon_t \overset{iid}{\sim} N\left(0, \sigma^2\right),$$

where η represents the persistence of aggregate shock with noise having an independent and identical distribution (iid) with mean of 0 and variance of σ^2. Firm value is maximized through the distribution of dividends to the agents (owners) in the household sector. The discounted value of the firm is $\sum_{j=0}^{\infty} \beta^j \Lambda_{t+j} d_{t+j}$, where Λ_{t+j} signifies the relative price of consumption; i.e., the equity owner's marginal utility of consumption. The specific maximization problem for the firm is:

$$p_t^s = \max_{\{k_{t+j}, l_{t+j}\}} E_t \left\{ \sum_{j=0}^{\infty} \beta^j \Lambda_{t+j} d_{t+j} \right\}. \tag{18}$$

The firm owns the capital k, and funds operations internally through retained earnings and externally by selling bonds. The total supply of the bonds at price of p_t^f is constant over time and equals to $\chi \bar{k}$, where χ is a constant representing the leverage ratio(Jermann 1998, Danthine and Donaldson 2002, Guvenen 2009). The capital stock follows a law of motion

$$i_t = k_{t+1} - (1 - \delta) k_t.$$

In order to create a wedge between the return to physical capital and the return to financial capital, Danthine and Donaldson (2002) introduced a cost that adjusts the firm's capital stock from its current level in macroeconomy in their study of EPP. More specifically, Basu (1987) highlights the importance of this adjustment cost to determining such financial variables as stock prices and long term real interest rates. Existence of this cost implies diminishing returns to augmenting the quantity of capital in the economy and therefore capital stock tends to adjust in a sluggish manner with respect to any productivity shock. Following previous studies (Guvenen 2009, Danthine and Donaldson 2002), an adjustment cost of investment is presumed to be concave and given as:

$$g(k_t, i_t) = \left(\frac{\phi}{2}\right)\left(\frac{1}{k_t}\right)(i_t - \delta k_t)^2 .$$

After retaining capital for future use, paying out the net interest to bondholders, making labor payment to employees, and including the capital adjustment cost, the firm maximizes its value subject to a dividend constraint:

$$d_t = y_t - w_t l_t - i_t - (1 - p_t^f)\chi\bar{k} - g(k_t, i_t).$$

To be consistent with other DSGE models studying EPP, I follow other literature (Guvenen 2009, Jermann1998), to include the financial leverage χ . Jermann(1998) emphasizes that financial leverage is helpful in moving up the equity premium by 0.63%-1.72%.

Solving the maximization problem in (18) yields the first order conditions for a typical firm

$$0 = E_t \left\{ \begin{array}{c} \beta\Lambda_{t+2}\begin{pmatrix} \theta z_{t+1} k_{t+1}^{\theta-1} l_{t+1}^{1-\theta} + (1-\delta) \\ +\frac{\phi}{2}\left(\frac{k_{t+2}-k_{t+1}}{k_{t+1}}+1\right)^2 - \frac{\phi}{2} \end{pmatrix} \\ -\Lambda_{t+1}\left(1+\phi\frac{k_{t+1}-k_t}{k_t}\right) \end{array} \right\}, \tag{19}$$

$$w_t = (1-\theta)\, z_t k_t^{\theta} l_t^{-\theta}. \tag{20}$$

Equation (19) is an inter-temporal Euler equation for the firm. Since it is assumed that the households are both owners and workers, this Euler equation is equivalent to the households' inter-temporal Euler equation when they own the stock of capital and rent it to the firm. In this model, ownership by the firm enables equity to have value as it is a claim to the returns from that capital stock. Ultimately, however, the predictions in either case are the same. Equation (20) is the standard intra-temporal Euler equation that equates the marginal product of labor with the wage rate.

3.2.3 Equilibrium

Equations (15), (16), (17), (19), and (20) are the necessary conditions that describe optimal behavior of the agents in the general equilibrium model. Equilibrium is formally defined in Equation (21).

$$y_t - g(k_t, i_t) = c_t + i_t. \tag{21}$$

Equation (21) shows that in general equilibrium, the total consumption for the representative households plus investment cannot exceed the total production minus the adjustment cost. All variables in equilibrium are represented in aggregate quantities. The reason why we label the aggregate level in the same fashion as the individual level (i.e., c_t, i_t) is that the representative agent is assumed to be distributed uniformly over $[0, 1]$.

The clearing of the bond market requires (22):

$$B_{t+1} = \chi \bar{k}, \tag{22}$$

and the clearing of the stock market requires (23):

$$s_{t+1} = 1. \tag{23}$$

Equations (22)-(23) characterize the aggregate levels for bonds and stocks. The law of motion for bonds requires that in equilibrium, the total bonds supplied (issued) by the firms are equal to the total demand for the bonds by households. As mentioned in section 2.2, the total supply for the bonds is $\chi\bar{k}$, while the total demand for the uniformly distributed bonds is B_{t+1}. Similarly, the shares of stock are a uniformly distributed among the populace where the total supply for the stocks s_{t+1} is inelastically set to 1 throughout time. Note that the firm is not issuing new shares and therefore the price of equity is changing solely due to demand.

In equilibrium, the return to equity is

$$r_t^e = \frac{p_t^s + d_t}{p_{t-1}^s},$$

the risk-free rate is

$$r_t^f = \frac{1}{p_t^f},$$

and the average equity premium is

$$r^{ep} = \sum_{t=1}^{T} \frac{\left(r_t^e - r_t^f\right)}{T}.$$

The equity premium is approximated by:

$$r^{ep} \approx E_t[r_t^e - r_t^f].$$

3.3 Solution, Calibration, and Estimation Methods

3.3.1 Solution

A feature of all three models (defined by different calibrations) is that their steady states equity premiums are all equal to zero. That is, the risky asset and the risk-free asset will have the same returns in the economies with no uncertainty. It is known that linear solutions (certainty equivalence) do not account for uncertainty and therefore will still get simulated equity premiums of zero. In this Chapter, second order solutions, known as perturbations are used to account for uncertainty. Perturbation methods, first introduced by Judd (1996), solve dynamic programming problems via higher order approximations. More specifically, the central idea of perturbation is to solve for a finite set of coefficients by taking repeated derivatives of the optimality equations (15), (16), (17), (19), (20), and (21). These coefficients define the second order approximations for the allocations and prices of the model that are defined over the set of states $\{k_t, z_t, \sigma, \lambda\}$ where $\lambda = \{\lambda_L, \lambda_H\}$.

Typically, the perturbation expansion occurs around steady states defined where the model uncertainty and distortions are zero (*i.e.*, $\sigma = 0$, $\lambda = 0$). Expansions of σ and λ are taken around zero as well. This solution method is presumably only accurate for when σ and λ are near zeros. Unfortunately, in this model, for LANF preference to be important, the uncertainty parameter σ and LANF parameter λ must be nontrivial. Furthermore, λ is changing depending on the sign of the difference of the returns; it's either λ_L or λ_H.

Therefore, the perturbation solution is modified by redefining σ and λ (change of variables). This modification follows the work of Judd (1996, 2002) and Fernández-Villaverde and Rubio-Ramirez (2006) where σ and λ are estimated by a projection of the perturbation solutions back onto the optimality equations (15)-(17) and (19)-(21). The solution to this hybrid perturbation-projection method is found by minimization of the optimality equations defined over a grid of points. The grid amounts to 70 points that are intended to cross over 90 percent below and above the steady state capital. The productivity states are approximated by a 40 point grid using Tauchens procedure[2]. Details for this method are discussed in the appendix.

[2]See Fernández-Villaverde and Rubio-Ramirez (2006) for details

3.3.2 Calibrations

To facilitate this analysis, some parameters are calibrated based on the estimations found in other studies (Abel 1980, Danthine and Donaldson 2002, Jermann 1998, Grüne and Semmler 2008, Guvenen 2009). From these studies, three main parameterization are defined: (i) the baseline model with Cobb-Douglas preferences (Baseline CD); (ii) the CD baseline model where the Frisch elasticity is set to zero (CD Zero Frisch), and (iii) the CD Zero Frisch model where adjustment costs of investment are zero and capital fully depreciates (CD zero Frisch $\phi = 0$, $\delta = 1$).

For all models the capital share in output is set at $\theta = 0.3$, the same value as chosen by Kydland and Prescott (1982), Jermann (1998) and Grüne and Semmler (2008). This selection conforms with the labor elasticity suggested in the data during the period studied by Mehra and Prescott (1985).The discount rate β is set to 0.99 according to a steady state return on capital of 4%. Both Danthine et al. (2002) and Guvenen (2009) assumed this rate of return in their quarterly estimates in correspondence with a quarter period. The utility power parameter ρ – the relative risk aversion – is set equal to 4 following Danthine and Donaldson (2002). For the CD model, consumption's share in utility is set to $\gamma = 0.395$. This is chosen to follow Guvenen (2009) to match the average time devoted in market activities (0.36 of discretionary time). For the Zero Frisch models, $\gamma = 1$ and labor hours l_t are set to 1. The leverage ratio χ is assumed to be 0.15 which lies in the historical range of 0.13 to 0.44 (Jermann, 1998).

Another important parameter is the cost of adjustment constant ϕ which measures the elasticity of investment. But studies that incorporate adjustment cost functions have used a varying range for ϕ. Danthine et al. (2002) and Jermann (1998) both states that the value of ϕ is set to maximize model's ability to match a set of moments of interest; too large value of ϕ leads to low volatility of investment. For example, Abel (1980) picked ϕ in the range of $[0.27, 0.52]$, Jermann (1998) estimated it as 0.23, and Guvenen (2009) calibrated it as 0.40. The way ϕ is picked in this Chapter is to match the adjustment cost not "too large" (Danthine and Donaldson, 2002) and to pursue the goal of smoothing the capital stocks. Therefore, this constant is set to $0.35\exp(k_{ss})$ to be able to replicate Tobin's Q values. All of the calibrations are detailed in 3.

Table 3: Parameterizations for CD Baseline

θ	Capital Share	0.30
δ	Depreciation Rate	0.02
β	Time Discount Factor	0.99
χ	Leverage Ratio	0.15
γ	Consumption Share	0.395
ρ	Relative Risk Aversion	4
ϕ	Elasticity of Investment	$0.35 \times \exp(k_{ss})$
η	Persistence of Aggregate Shock	0.95

3.3.3 Estimation

This Chapter employs a prior predictive analysis (Canova 1994, Geweke 2007) to estimate the model's volatilities and equity premiums. The analysis is conducted in four steps. First, for the unknown parameters σ, b_0, and λ_H, prior distributions are assumed. The priors are threefold: σ is an inverse-gamma distribution, b_0 is a discrete uniform distribution, and λ_H follows an uniform distribution. Second, random realizations are drawn from these priors. Third, the model is solved for the equilibrium allocations and prices. Finally, the economy is simulated for a set of allocations and prices that are meant to mimic economy's volatilities and equity premiums.

The Bayesian method described above was implemented for all of the models according to the following procedure:

1. Assume prior distributions for σ, b_0, and λ_H.

2. Draw $i = 1 : 2500$ realizations for σ, b_0, and λ_H.

3. For each draw i, solve the model for endogenous output y, consumption c, investment i, capital k, price of equity p^s, and price of bond p^f.

4. For each draw of the random variables, simulate the economy for 500 quarters and form i estimates of $\sigma(y)$, $\sigma(c)/\sigma(y)$, $\sigma(i)/\sigma(y)$, $\sigma(l)/\sigma(y)$, $E(r_t^e)$, $E(r_t^f)$, $E(r_t^{ep})$ and $E(r_t^{ep})/\sigma(r^{ep})$.

5. Plot the distributions for $\sigma(y)$, $\sigma(c)/\sigma(y)$, $\sigma(i)/\sigma(y)$, $\sigma(l)/\sigma(y)$, $E(r_t^e)$, $E(r_t^f)$, $E(r_t^{ep})$ and $E(r_t^{ep})/\sigma(r^{ep})$ with their corresponding actual data built in respectively. The data were reported in previous research (Boldrin, Christiano and Fisher 2001, Danthine and Donaldson 2002, Guvenen 2009).

To obtain the most accurate results, the prior distributions are carefully picked. That is, the prior distributions should not only cover a large range of possibilities but also be consistent with economic intuition by having the correct signs. Barberis and Huang. (2008) adopt separate values for b_0 within the range of $[0, 0.1]$. Grüne and Semmler (2008) indicated that the degree of narrow framing can vary from 0 to 3. This Chapter assumes a prior for b_0 of a discrete uniform distribution from the set $[0.3, 1, 3, 10, 50, 100]$. This choice is designed to encompass the ranges studied in Barberis and Huang (2008) and Grüne and Semmler (2008). Following other asset pricing works (i.e., Jacquier, Polson and Rossi, 1994), this Chapter assumes σ in an inverse-gamma distribution. This special distribution allows the uncertainty centering above zero. The Baseline CD model assumes a shape parameter of 2.75 and a scale parameter $1.75 * 0.0045$ (this roughly matches the volatility of output). Previous studies also allow λ_H to have different values. To determine how loss aversion can impact the equity premium, this study assumes λ_H takes the form of a uniform distribution $[1, 200]$. The choice is designed to encompass the range studied in Barberis and Huang (2008) and Grüne and Semmler (2008) where λ_H falls in the range of $[1, 10]$ or the set of $[3, 5, 10, 20]$ respectively.

Following the five step procedure listed above will reveal how well the hypothetical economy captures the actual distributions of the macroeconomic volatilities and equity premiums. If the hypothesis is supported by the analysis; i.e., that agent have LANF preferences, then the data should fall within the predictive densities of the model. These predictive densities are accomplished, for each model estimate, by a non-parametric kernel smoother. The inverse cumulative distribution values for the actual data are derived from the predictive densities.

3.4 Results

This section analyzes how the LANF preferences can generate an equity premium and related economic volatilities. The results are represented by comparing the three models (Baseline CD model, CD Zero Frisch model and Zero Frisch $\phi = 0, \delta = 1$ model) with the actual data[3] and with the performances from the extant literature (Guvenen, BCF[4] and DD[5]). This comparison is shown in Table 2. To demonstrate how the three models capture the actual data via these volatilities and equity premium, Figure 1, 3 and 4 indicate the prior predictive densities for these three models respectively. The blue curves depict the prior predictive densities (estimated by a non-parametric kernel) and the red dashed line represents the actual data. Note that if the red line is within the distribution of the volatility, the model performs satisfactorily. On the other hand, if the red line falls outside the distribution, the model fails

[3]Data source: Boldrin, Christiano, and Fisher 2001, Danthine and Donaldson 2002, Guvenen 2009.

[4]Boldrin, Christiano, and Fisher (2001).

[5]Danthine and Donaldson (2002).

to describe the specific statistic. Table 3 maps these features into the prior cumulative distribution functions (C.D.F.) for the three models together. The conclusions are based on a two-sided 95% confidence level.

3.4.1 Baseline CD Model

4 shows that Baseline CD model generates satisfactory volatility of output; 1.84 compared to 1.89 found in the data. The model also accurately predicts the relative volatility of consumption to output; 0.65 compared with 0.7. The value $\sigma(i)/\sigma(y)$ in the table shows, a good match for the baseline model and the actual data. In fact, the Baseline CD model performs as well as any other model tested (Guvenen, BCF[6] and DD[7]).

Even though the baseline model is able to predict volatilities that match the actual data, it is unable to explain the following statistics: relative volatility of labor to output, equity premium, and Sharpe ratio. As shown in 4, the volatility of labor to output $\sigma(l)/\sigma(y)$ for the baseline model is 0.26 which compares poorly to the 0.8 found in the actual data. The baseline model also fails to explain the main focus of this study – the equity premium. The baseline model's equity premium is 1.46 percent compared to 6.17 percent for the actual data. The Sharpe ratio displays the households' return for their investment expressed as the ratio of equity premium to its standard deviation. The Baseline CD value of 4.62 is much higher than the 0.32 found in the actual data.

?? and 5 jointly describe how the Baseline CD model performs in explaining the equity premium puzzle and other economic volatilities. Evidently, distributions of volatilities of output, consumption and investment fully encompass the actual data whereas the actual data lines for labor, equity premium and Sharpe ratio either fall

[6]Boldrin, Christiano, and Fisher (2001).

[7]Danthine and Donaldson (2002).

Table 4: Model Performance

	Data	Model			Guvenen	BCF	DD
		CD Baseline	CD Zero Frisch	CD Zero Frisch $\phi = 0,\ \delta = 1$			
$\sigma(y)$	1.89	1.84	1.74	2.05	1.95	1.97	1.77
$\sigma(c)/\sigma(y)$	0.70	0.65	0.59	0.96	0.78	0.69	0.82
$\sigma(i)/\sigma(y)$	2.39	2.57	2.80	1.10	1.76	1.67	1.72
$\sigma(l)/\sigma(y)$	0.80	0.26	0	0	0.50	0.51	–
$E(r^f)$	1.94	3.78	3.34	3.63	1.42	1.34	3.98
$E(r^{ep})$	6.17	1.46	2.59	2.82	4.21	6.63	4.25
$E(r^{ep})/\sigma(r^{ep})$	0.32	4.62	4.81	2.67	0.24	0.36	0.21

outside of or lies peripheral to the corresponding distributions at 95% level. More precisely, the inverse C.D.F. does not fall within the range of $[0.025, 0.975]$. Column two shows the performance for Baseline CD model.

Even though the Baseline CD cannot explain the high equity premium, ?? offers insights into the influences of the prior predictive distributions of technology shock σ, loss aversion parameter λ_H and the narrow framing parameter b_0 on the equity premium. The uppermost panel shows that the technology shock parameter influences the equity premium to some extent. The LANF components, loss aversion λ_H and narrow framing b_0, were analyzed separately. Neither parameter displayed a close relationship with the equity premium in the baseline model. These weak results provide little support for the Benartzi and Thaler hypothesis in models with labor supply elasticities in the baseline ranges. The implication of these findings is that inclusion of a labor choice in the LANF preference model does not guarantee the high equity premium observed in actual data. Moreover, this model fails to explain the Sharpe ratio. For the measures the actual data falls outside of the predictive

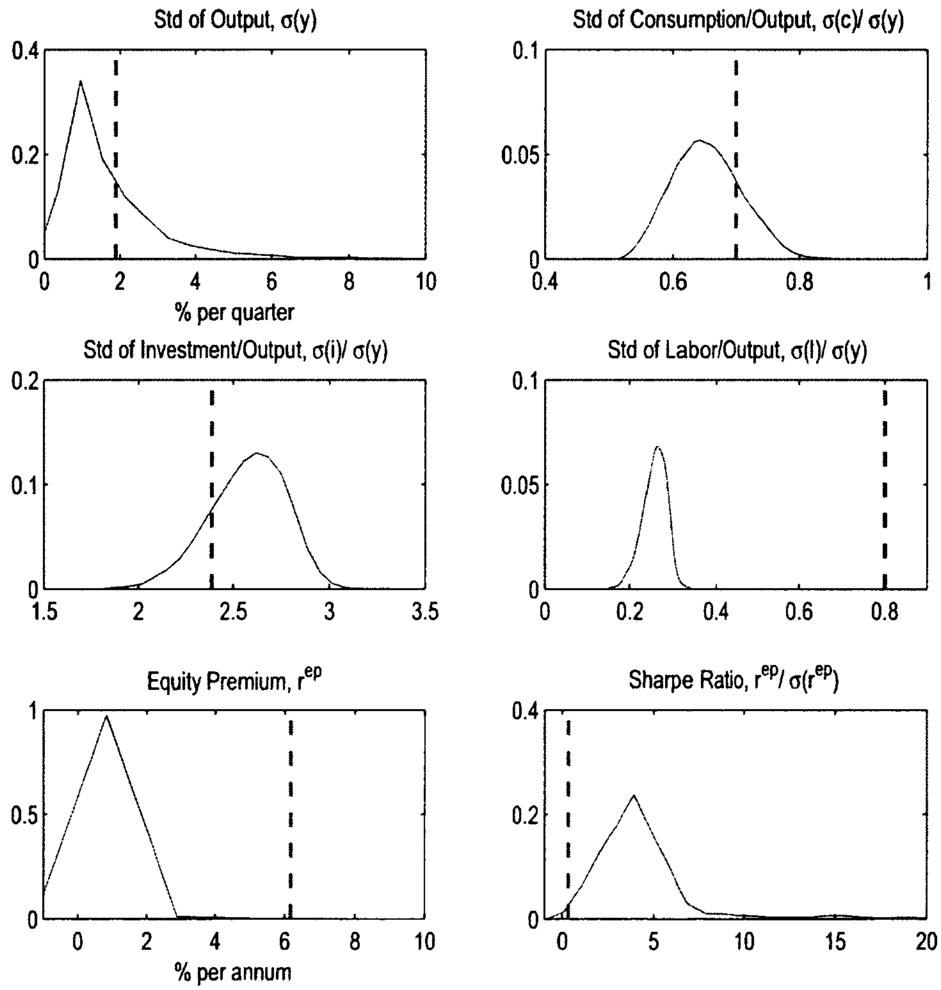

Figure 5: Prior Predicitive Densities for CD Baseline Model (dashed line is actual data).

Table 5: Inverse Prior Cumulative Distribution Function and Features

Feature	Inverse C.D.F. at Data CD Baseline	CD Zero Frisch	CD Zero Frisch $\phi = 0$, $\delta = 1$
$\sigma(y)$	0.683	0.714	0.636
$\sigma(c)/\sigma(y)$	0.797	0.969	0.000
$\sigma(i)/\sigma(y)$	0.185	0.033	1.000
$\sigma(l)/\sigma(y)$	1	1	0
$E(r^{ep})$	0.989	0.954	0.980
$E(r^{ep})/\sigma(r^{ep})$	0.005	0.007	0.016

densities. Hence, a conclusion drawn from the Baseline CD model is that LANF preferences alone cannot resolve the equity premium puzzle. Additionally, LANF parameters have no effect on the equity premium; it is mainly determined by the variance of the technology.

3.4.2 Inelasticity of Labor

The Baseline CD model's results are in sharp contrast to the previous research of Barberis et al. (2001), Barberis and Huang (2008), and Grüne and Semmler (2008). A fundamental reason for this difference is that these studies assumed some combination: (i) inelastic labor; (ii) zero investment costs; and (iii) full depreciation. Presumably, as noted in Grüne and Semmler (2008), the elimination of smoothing margins and/or increased volatilities makes fluctuations in asset prices more costly thereby generating a higher equity premium. Two additional experiments are conducted to test this hypothesis. The first test is denoted as the CD Zero Frisch model in 4. In this model $\gamma = 1$ implying that households cannot smooth consumption by substituting labor for leisure. The second experiment sets $\delta = 1$ and $\phi = 0$ in order to study the effects

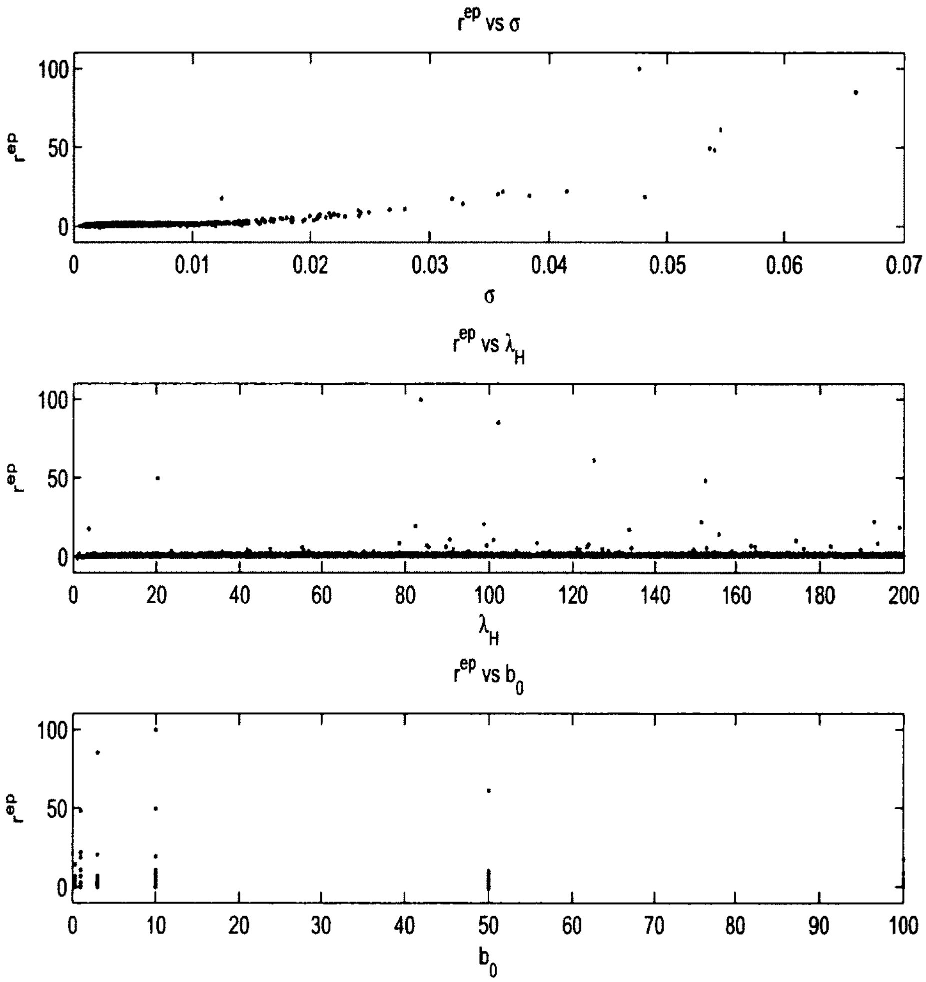

Figure 6: Relationship Between Equity Premium and Variable Estimates (CD Baseline Model).

of full depreciation and the inclusion of investments costs. This is denoted as Zero Frisch $\phi = 0, \delta = 1$ model.

Column four in 4 presents the result for the CD Zero Frisch model. The volatilities generated from this model are close to the actual data as well as those in the Baseline CD model. In terms of equity premium, changing the assumptions about the elasticity of labor alters the results significantly. Specifically, the equity premium is now 2.59 compared to 1.46 in Baseline CD model, a 72.6% increase whereas the Sharpe ratio remains high. The prior predictive densities are illustrated in 7. The densities in the first three panel do not change notably, but there is a slight improvement in the estimated magnitude of the equity premium shown in the bottom-left panel. However, because the labor is supplied perfectly inelastic, the predictive distribution in the fourth panel of 7 for the volatility of $\sigma(l)/\sigma(y)$ poorly describes the actual data (the straight blue line). The inverse C.D.F.s for this presented in 5, show a similar story to 7. First, the volatilities of output and consumption capture the actual data at 95% level. Alternatively, the volatility of $\sigma(l)/\sigma(y)$ and the Sharpe ratio cannot be explained by this model. Secondly, the distribution of investment shifts to the right. An inverse C.D.F. of 0.033 in Zero Frisch model compared to 0.185 in CD Baseline, a change not evident in Figure 3. Finally, the improvement of equity premiums is illustrated by the change in the inverse C.D.F from 0.989 in Baseline CD model to 0.954 in this model.

Danthine and Donaldson (2002) consider adjustment cost as a necessary component in a production economy since these costs drive a wedge between the return to physical capital and financial capital. They also argue that without this cost, the marginal value of capital equals the price of the investment good, a fact not supported by the data. An example offered by Huffman and Wynne (1999) illustrates the importance of this adjustment cost. Inputs used to produce computers cannot easily

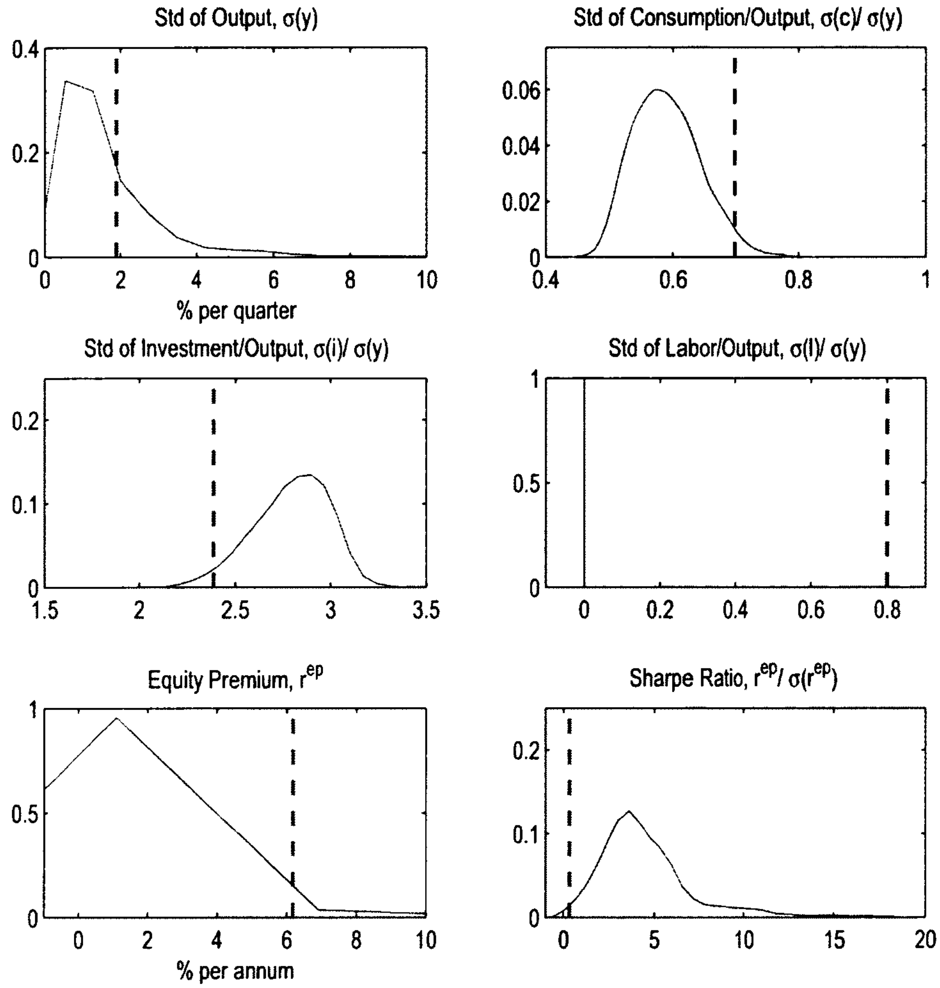

Figure 7: Prior Predicitve Densities for CD Zero Frisch Model (dashed line is actual data).

and swiftly be converted into the physical capital such as equipment or skilled labor that are needed to produce heavy industrial equipment. Therefore, the adjustment cost is designed to feature the difficulty of reorienting the production of new capital goods from one specific sector to another. Another parameter, the depreciation rate δ, reduces the return to investment.

Column five of 4 presents the results of a simulation of Zero Frisch $\phi = 0, \delta = 1$ model. The result shows that with zero adjustment costs and full depreciation the volatility of output and relative volatility of investment decrease drastically, but that the relative volatility of consumption increases as well as the volatility of output. This is inconsistent with theory. Nevertheless, this specification improves the estimates of the equity premium when compared to the Baseline CD and Zero-Frisch models. The Sharpe ratio decreases compared to the previous two models, but stays higher than the actual data. Moreover, 8 shows that the actual data lie within the long-tail of the simulated densities. Unfortunately, we see the positive results of LANF are generated at the expense of the macroeconomic performance of the model. Combining LANF preferences with the assumption of a perfectly inelastic labor supply (Grüne and Semmler 2008, Barberis et al. 2001, and Barberis and Huang 2008) generates a sizeable equity premium. However, assuming $\delta = 1$ distorts the predictions of other volatilities, such as the consumption/output ratio and investment/output ratio.

3.5 Conclusion

Benartzi and Thaler (1995) suggest LANF preferences as a possible explanation for the equity premium puzzle. Barberis,Huang, and Santos (2001), Barberis and Huang (2004 and 2008) and Grüne and Semmler (2008) include LANF preferences in a partial equilibrium model in an attempt to clarify the EPP. Their findings support the Benartzi and Thaler hypothesis. Alternatively, Danthine and Donaldson (2002) and

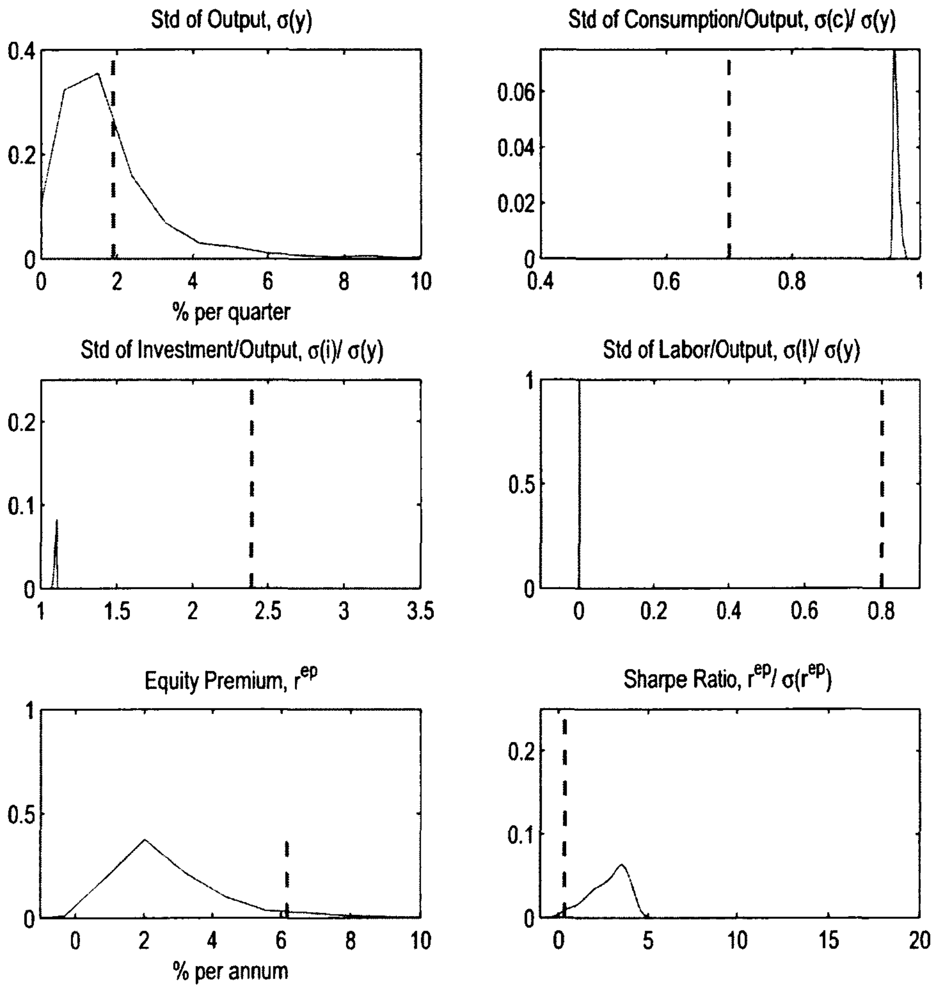

Figure 8: Prior Predicitve Densities for CD $\phi = 0$, $\delta = 1$ Model (dashed line is actual data).

Guvenen (2009) include labor choice in their general equilibrium models by assuming that households obtain utilities not only from consumption as in partial equilibrium, but also from leisure/labor component (DSGE model). The present Chapter tests the Benartzi and Thaler hypothesis in the context of a DSGE model. Conducting the tests entailed two steps: (i) solving the DSGE model with an uncertainty component, and (ii) estimating the model with non-smooth elements – the LANF preference function. Step (i) was completed by using the hybrid perturbation-projection method. To overcome the difficulties of implementing step (ii), this Chapter used prior predictive analysis; an estimation method that employs an iterative Bayesian approach.

The fundamental finding of the Chapter is that LANF preferences cannot explain the equity premium under reasonable assumptions about the standard deviations of important macroeconomic variables. When the model accurately predicts these macroeconomic volatilities, it does not produce an equity premium commensurate with past empirical findings. Only by including both unrealistic labor elasticities and depreciation rates could the model generate a reasonable equity premium.

Other studies have used alternative (non-LANF) assumptions to explain the EPP. For example, in column 7 of Table 2, Boldrin, Christiano, and Fisher (2001) generated an equity premium, 6.63% under the assumption of habit persistent preferences, which actually exceeds most empirical estimates of the EPP. Danthine and Donaldson (2002) and Guvenen (2009) generate equity premiums of 5.23% and 4.21%, respectively, by using imperfect risk sharing mechanisms. However, the results of these models are also undermined by their estimates of the standard deviation of labor which are not reconcilable with actual empirical observations. Thus, the paradox is that models with LANF preferences fail to improve the predictions of the equity premium, just like all other theories, because the equilibrium volatility for labor hours is unreasonable.

From these results it appears that the Benartzi and Thaler hypothesis needs to be modified to include other dimensions about utility function. One possible modification is to extend the work of Guvenen (2009) who utilizes GHH preferences (Greenwood, Hercowitz and Huffman, 1998). In this case, the utility function allows for separation of risk aversion and labor supply elasticity. Another avenue is the work of Cho and Cooley (1994) which focuses on the intensity of hours worked and the elasticity of labor supply.

3.6 Appendix

3.6.1 Details for Hybrid Perturbation-Projection Method

3.6.1.1 Perturbation with COV

The solution method here makes use of Taylor series expansion with changes of variables (Judd 1996, 2002; Fernández-Villaverde and Rubio-Ramirez 2006). Every policy function (i.e.,

l_t, k_{t+1}, etc.) is first approximated by a perturbation solution:

$$f(z,k,\sigma,\lambda) \approx \sum_{i,j,m,n} \frac{1}{(i+j+m+n)!} \left. \frac{\partial^{i+j+m+n} f(z,k,\sigma,\lambda)}{\partial z^i \partial k^j \partial \sigma^m \partial \lambda^n} \right|_{\{0,kss,0,0\}} z^i (k-kss)^j \sigma^m \lambda^n \tag{24}$$

where $\lambda_L = \lambda_H = \lambda = 0$ (i.e., no LANF). The solution in (24) is, presumably, accurate around $\{z,k,\sigma,\lambda\} = \{0,kss,0,0\}$. Then, λ and σ are replaced by a change of variables (COV) defined by either a constant or a polynomial in the states. More specifically, let the COV be;

$$\begin{aligned} y_{3,t} &= \tau_0 \sigma, \\ y_{4,t} &= \alpha_0 \lambda + \alpha_1 z_t + \alpha_2 (k_t - kss). \end{aligned}$$

To get a better understanding of how COV works, consider a simple example from Judd (2002). At first, the researcher has a basic second order Taylor series expansion of a function $f(x)$:

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 \tag{25}$$

where x has been expanded around a. The COV is then defined by $y = Y(x)$ with an inverse function existing as $x = X(y)$. The COV finds $g(y) = f(X(y))$ at $y = b = Y(a)$. Note that $g(y)$ can be approximated with Chain Rule at second order by:

$$\begin{aligned} g(y) &= f(X(y)) \\ &\approx f(X(b)) + f'(X(b))X'(b)(y-b) + \ldots \\ &\frac{1}{2}(X'(b)^2 f''(X(b)) + f'(X(b))X''(b))(y-b)^2 \\ &= f(a) + f'(a)X'(b)(y-b) + \ldots \\ &\frac{1}{2}(X'(b)^2 f''(a) + f'(a)X''(b))(y-b)^2. \end{aligned}$$

More concretely, suppose $a = 1$ and the COV is $y = Y(x) = \log(x)$. Then, we immediately see that $b = 0$ and the inverse function is $x = X(y) = \exp(y)$. As a result, $X'(b) = 1$ and $X''(b) = 1$. The COV expansion is thus:

$$f(a) + f'(a)\log(x) + \frac{1}{2}(f''(a) + f'(a))\log(x)^2,$$

where $\{f(a), f'(a), f''(a)\}$ are presumed to be known from (25).

In this study, the proposed transformation is:

$$Y(x_t) = \begin{bmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \\ y_{4,t} \end{bmatrix} = \begin{bmatrix} z_t \\ k_t - kss \\ \tau_0 \sigma \\ \alpha_0 \lambda + \alpha_1 z_t + \alpha_2 (k_t - kss) \end{bmatrix},$$

where $x_t = [z_t, k_t, \sigma, \lambda]'$. The inverse function is thus:

$$X(y_t) = \begin{bmatrix} Z(y_{1,t}) \\ K(y_{2,t}) \\ \Sigma(y_{3,t}) \\ \Lambda(y_{1,t}, y_{2,t}, y_{4,t}) \end{bmatrix} = \begin{bmatrix} y_{1,t} \\ y_{2,t} + kss \\ y_{3,t}/\tau_0 \\ (y_{4,t} - \alpha_1 y_{1,t} - \alpha_2 y_{2,t})/\alpha_0 \end{bmatrix}.$$

And, suppose that an initial second order perturbation gave equation (24) for k_{t+1} of:

$$\begin{aligned} k_{t+1} &= \mathcal{K}(x_t) \\ &\approx \mathcal{K}_{<0,0,0,0>} + \mathcal{K}_{<1,0,0,0>} z_t + \mathcal{K}_{<0,1,0,0>}(k_t - kss) + \ldots \\ &\quad \mathcal{K}_{<0,0,1,0>}\sigma + \mathcal{K}_{<0,0,0,1>}\lambda + \ldots \\ &\quad \mathcal{K}_{<2,0,0,0>} {z_t}^2 + \mathcal{K}_{<0,2,0,0>}(k_t - kss)^2 + \ldots \\ &\quad \mathcal{K}_{<0,0,2,0>}\sigma^2 + \mathcal{K}_{<0,0,0,2>}\lambda^2 + \ldots \\ &\quad 2\mathcal{K}_{<1,1,0,0>} z_t(k_t - kss) + 2\mathcal{K}_{<1,0,1,0>} z_t\sigma + \ldots \\ &\quad 2\mathcal{K}_{<1,0,0,1>} z_t\lambda + 2\mathcal{K}_{<0,1,1,0>}(k_t - kss)\sigma + \ldots \\ &\quad 2\mathcal{K}_{<0,1,0,1>}(k_t - kss)\lambda + 2\mathcal{K}_{<0,0,1,1>}\sigma\lambda, \end{aligned}$$

where

$$\mathcal{K}_{<i,j,m,n>} = \frac{1}{(i+j+m+n)!} \left. \frac{\partial^{i+j+m+n}\mathcal{K}(z,k,\sigma,\lambda)}{\partial z^i \partial k^j \partial \sigma^m \partial \lambda^n} \right|_{\{0,kss,0,0\}}.$$

Applying the COV gives:

$$
\begin{aligned}
k_{t+1} &= \mathcal{K}\left(Z(y_{1,t}), K(y_{2,t}), \Sigma(y_{3,t}), \Lambda(y_{1,t}, y_{2,t}, y_{4,t})\right) \\
&\approx \mathcal{K}_{<0,0,0,0>} + \left(\mathcal{K}_{<1,0,0,0>} + \mathcal{K}_{<0,0,0,1>}\Lambda_{<1,0,0>}\right) y_{1,t} + \ldots \\
&\quad \left(\mathcal{K}_{<0,1,0,0>} + \mathcal{K}_{<0,0,0,1>}\Lambda_{<0,1,0>}\right) y_{2,t} + \ldots \\
&\quad \left(\mathcal{K}_{<0,0,1,0>}\Sigma_{<1>}\right) y_{3,t} + \ldots \\
&\quad \left(\mathcal{K}_{<0,0,0,1>}\Lambda_{<0,0,1>}\right) y_{4,t} + \ldots \\
&\quad \left(\mathcal{K}_{<2,0,0,0>} + 2\mathcal{K}_{<1,0,0,1>}\Lambda_{<1,0,0>} + \mathcal{K}_{<0,0,0,2>}\Lambda^2_{<1,0,0>}\right) y^2_{1,t} + \ldots \\
&\quad \left(\mathcal{K}_{<0,2,0,0>} + 2\mathcal{K}_{<0,1,0,1>}\Lambda_{<0,1,0>} + \mathcal{K}_{<0,0,0,2>}\Lambda^2_{<0,1,0>}\right) y^2_{2,t} + \ldots \\
&\quad \left(\mathcal{K}_{<0,0,2,0>}\Sigma^2_{<1>} + \mathcal{K}_{<0,0,1,0>}\Sigma_{<2>}\right) y^2_{3,t} + \ldots \\
&\quad \left(\mathcal{K}_{<0,0,0,2>}\Lambda^2_{<0,0,1>}\right) y^2_{4,t} + \ldots \\
&\quad 2\left(\mathcal{K}_{<1,1,0,0>} + \mathcal{K}_{<1,0,0,1>}\Lambda_{<0,1,0>} + \mathcal{K}_{<0,1,0,1>}\Lambda_{<1,0,0>}\right) y_{1,t}y_{2,t} + \ldots \\
&\quad 2\left(\mathcal{K}_{<0,0,0,2>}\Lambda_{<1,0,0>}\Lambda_{<0,1,0>}\right) y_{1,t}y_{2,t} + \ldots \\
&\quad 2\left(\mathcal{K}_{<1,0,1,0>}\Sigma_{<1>} + \mathcal{K}_{<0,0,1,1>}\Sigma_{<1>}\Lambda_{<1,0,0>}\right) y_{1,t}y_{3,t} + \ldots \\
&\quad 2\left(\mathcal{K}_{<1,0,0,1>}\Lambda_{<0,0,1>} + \mathcal{K}_{<0,0,0,2>}\Lambda_{<1,0,0>}\Lambda_{<0,0,1>}\right) y_{1,t}y_{4,t} + \ldots \\
&\quad 2\left(\mathcal{K}_{<0,1,1,0>}\Sigma_{<1>} + \mathcal{K}_{<0,0,1,1>}\Sigma_{<1>}\Lambda_{<0,1,0>}\right) y_{2,t}y_{3,t} + \ldots \\
&\quad 2\left(\mathcal{K}_{<0,1,0,1>}\Lambda_{<0,0,1>} + \mathcal{K}_{<0,0,0,2>}\Lambda_{<0,1,0>}\Lambda_{<0,0,1>}\right) y_{2,t}y_{4,t} + \ldots \\
&\quad 2\left(\mathcal{K}_{<0,0,1,1>}\Sigma_{<1>}\Lambda_{<0,0,1>}\right) y_{3,t}y_{4,t},
\end{aligned}
$$

where

$$
\begin{aligned}
\Lambda_{<i,j,n>} &= \left.\frac{\partial^{i+j+n}\Lambda\left(y_1, y_2, y_4\right)}{\partial y_1^i \partial y_2^j \partial y_4^n}\right|_{\{\bar{y}_1, \bar{y}_2, \bar{y}_3, \bar{y}_4\}}, \\
\Sigma_{<i>} &= \left.\frac{\partial^i \Sigma\left(y_3\right)}{\partial y_3^i}\right|_{\{\bar{y}_1, \bar{y}_2, \bar{y}_3, \bar{y}_4\}}
\end{aligned}
$$

My choice of COV gives $\Sigma_{<1>} = 1/\tau_0$, $\Sigma_{<2>} = 0$, $\Lambda_{<1,0,0>} = -\alpha_1/\alpha_0$, $\Lambda_{<0,1,0>} = -\alpha_2/\alpha_0$, and $\Lambda_{<0,0,1>} = 1/\alpha_0$ for example.

3.6.1.2 Projection Methods

Given the COV transformations for the policy solution set: $\{c_t, l_t, k_{t+1}, p_t^s, p_t^f\}$, the next step in the solution method quantifies the unknown parameters of the COV; $\{\lambda, \tau_0, \alpha_0, \alpha_1, \alpha_2\}$, by examining of the Euler Equation Errors (EER). Following Fernández-Villaverde and Rubio-Ramirez (2006) and Judd (2002), the COV solutions are projected onto the EER and minimized by choice of parameters. To reduce the dimension of the estimation set, α_0 and τ_0, normalized to one leaving the set $\{\sigma, \lambda, \alpha_1, \alpha_2\}$ to be found.

In the next step, using (8), the optimality equations (3), (4), (5), (7), and (9) are evaluated using the COV perturbation solutions for any given set of states and unknown parameters: $\{k_t, z_t, \sigma, \lambda, \alpha_1, \alpha_2\}$. These equations are stacked into a vector that is denoted $EER(k_t, z_t, \sigma, \lambda, \alpha_1, \alpha_2)$. By summing up EER by element and across sets of values for $\{k, z\}$, the minimization problem is:

$$\min_{\{\sigma,\lambda,\alpha_1,\alpha_2\}} \sum_{j,k_i,z_i} |EER_j(k_i, z_i, \sigma, \lambda, \alpha_1, \alpha_2)| \,. \tag{26}$$

Following Fernández-Villaverde and Rubio-Ramirez (2006) a grid for k is made by a grid of 70 points intended so that $\{k_i\}_{i=1}^{70}$ crosses over 90 percent below and above the steady state capital. A grid of 40 productivity points for z is found by employing Tauchens procedure given a calibrations for η and a drawn σ from the prior. The grid $\{z_i\}_{i=1}^{40}$ has a Markov transition matrix that is used to compute the expectations in equation (26).

3.6.1.3 Evaluation of Solution Method

If the Hybrid Perturbation-Projection method is an improvement, then the Euler equation errors (EERs) evaluated at the solutions should be in magnitude smaller than the EERs evaluated at the regular perturbation solutions represented in (24). 9 shows the relationship between the relative EERs (the ratio of the EERs under the hybrid method to the EERs under the regular perturbation method) and realized variables σ, λ_H, and b_0. We see that the relative EERs are all less than one implying that the Hybrid Perturbation-Projection method reduces the computational modeling error. Also, as expected, the relative EERs are related to σ, λ_H, and b_0. Low realizations for σ, λ_H, and b_0 reduce the importance of LANF preferences and technology shocks. This economy, with small distortions, is described well by the regular perturbation method. In total, the evidence suggests successful minimizations of the EERs using the proposed Hybrid Perturbation-Projection method.

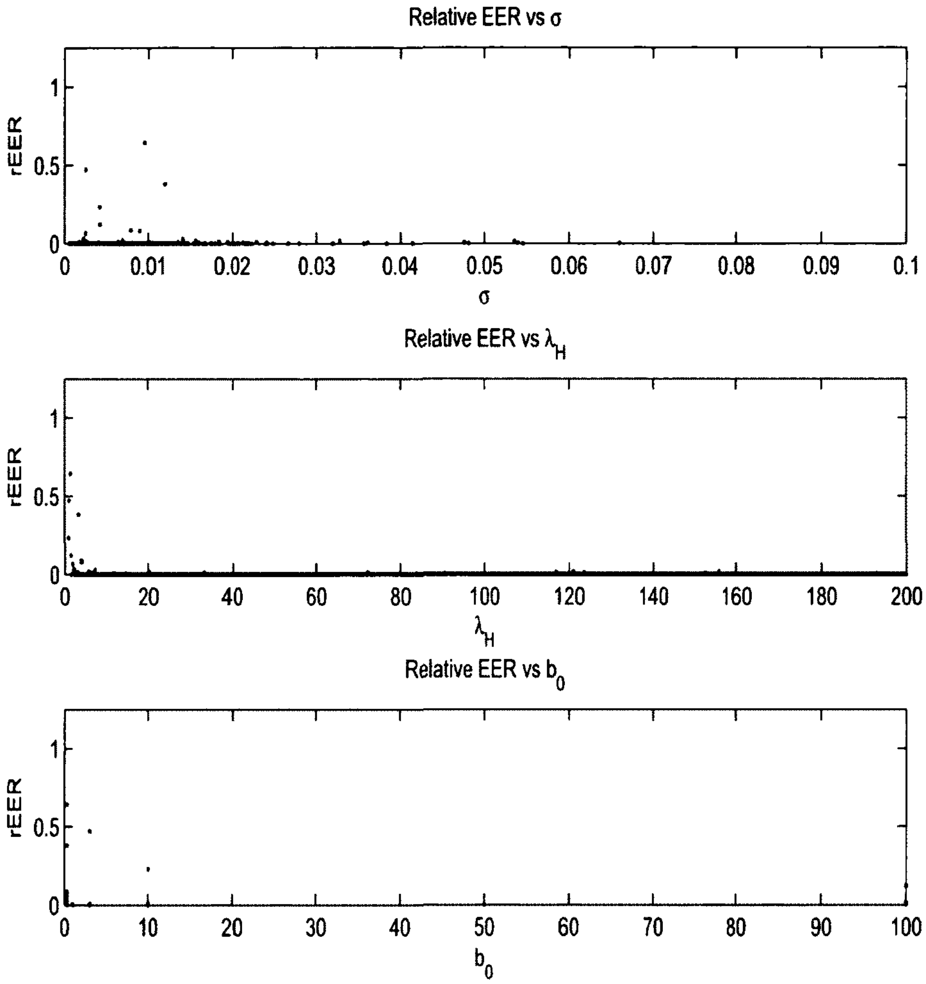

Figure 9: Relationship Between Euler Errors and Variable Estimates (CD Baseline Model).

CHAPTER IV

CONCLUSION

This dissertation consists of three main Chapters. Chapter 1 briefly discusses the background and the primary goal of this dissertation, that is, to evaluate how Loss Aversion/Narrow Framing preference can affect the Equity Premium. Chapters 2 and 3 conduct quantitative experiments that are dedicated to making this evaluation possible.

The experiment in Chapter 2 is to show the feasibility of a new method called Hybrid Perturbation-Projection Method. This method, first introduced by Judd (1996) and then followed by Judd (2002); Fernández-Villaverde and Rubio-Ramirez (2006) combines a straightforward perturbation method with a change of variables and projections. This experiment was operated using a DSGE asset pricing model similar to Guvenen (2009), Danthine and Donaldson (2002) and the one in Chapter 3. The results of the HPP method ware compared with one of the most commonly used method – Value Function Iteration. The comparison between the competing methods results in three main conclusions: (i) the HPP method is accurate relative to VFI; (ii) the HPP method has greater stability relative to the VFI method; and (iii) computational time is significantly improved when using the HPP method.

In Chapter 3, I then utilize the HPP method in solving a DSGE model with LANF components to conduct further quantitative experiments. A prior predictive analysis, that is a structural Bayesian estimation method, shows that LANF preferences cannot fully explain the EPP. Although the LANF DSGE asset pricing model does not predict an acceptable equity premium, relaxing the elastic labor supply and unrealistically changing depreciation rates can improve the equity premium. Additionally, recent studies have provided possible ways in which my model can be adjusted for further analysis. One of them is to replace the current Cobb-Douglas utility function with GHH (Greenwood, Hercowitz and Huffman, 1998) utility that separate risk aversion parameter and EIS. Another way is to specify the labor hours by considering intensity of hours worked and the elasticity of labor supply (Cho and Cooley, 1994).

REFERENCES

[1] Abel, A.B. 1980. Empirical Investment Equations: An Intergrative Framework, Carnegie-Rochester Conf. Ser. on Public Policy 12, 39-91.

[2] Barberis, N., M. Huang, and T. Santos. 2001. Prospect Theory and Asset Prices. The Quarterly Journal of Economics 116, 1-53.

[3] Barberis, N., and M. Huang. 2004. Preferences with Frames: A New Utility Specification That Allows for The Framing of Risks. Working paper, Yale University.

[4] Barberis, N., and M. Huang. 2008. The Loss Aversion/Narrow Framing Approach to The Equity Premium Puzzle. In: Rajnish Mehra, Editor, 2008. The Handbook of the Equity Risk Premium, North-Holland, Amsterdam.

[5] Basu, P. 1987. An Adjustment Cost Model of Asset Pricing. International Economic Review 28, 609-621.

[6] Benartzi, S., and R. H. Thaler. 1995. Myopic Loss Aversion and The Equity Premium Puzzle. The Quarterly Journal of Economics 110, 73-92.

[7] Boldrin, M., L. Christiano, and J. Fisher. 2001. Habit Persistence, Asset Returns, and the Business Cycle. American Economic Review 91, 149-166.

[8] Canova, F. 1994. Statistical Inference in Calibrated Models. Journal of Applied Econometrics 9, S123-S144.

[9] Caldara, D., J., Fernández-Villaverde, J., Rubio-Ramirez, and W., Yao. 2012. Computing DSGE Models with Recursive Preferences and Stochastic Volatility. Finance and Economics Discussion Series, Divisions of Research & Statistics and Monetary Affairs, Federal Reserve Board, Washington, D.C.

[10] Cho, J., and T.F. Cooley. 1994. Employment and Hours over The Business Cycle. Journal of Economic Dynamics and Control 18, 411-432.

[11] Danthine, J., and J. Donaldson. 2002. Labor Relations and Asset Returns. The Review of Economic Studies 69, 41-64.

[12] DellaViglla, S. 2009. Psychology and Economics: Evidence from the Field. Journal of Economics Literature 47, 315-372.

[13] Drèze, J.H. 1981. Inferring Risk Tolerance from Deductibles in Insurance Contracts. Geneva Papers on Risk and Insurance 20, 48-52.

[14] Fernández-Villaverde, J., and J. Rubio-Ramirez. 2006. Solving DSGE Models with Perturbation Methods and A Change of Variables. Journal of Economic Dynamic and Control 30, 2509-2531.

[15] Fernández-Villaverde, J., J. V., Binsbergen, R.S.J. Koijen and J. Rubio-Ramirez 2010. The Term Structure of Interest Rates in a DSGE Model with Recursive Preferences. Working Paper 1-49.

[16] Geweke, J., and C. Whiteman. 2006. Bayesian Forecasting. In G. Elliott, C., W. J. Granger and A., Timmerman, The Handbook of Economic Forecasting. Amsterdam: North-Holland.

[17] Geweke, J. 2007. Models, Computational Experiments and Reality. Working paper, University of Iowa.

[18] Geweke, J. 2010. Complete and Incomplete Econometric Models. Princeton University Press. Princeton, New Jersey.

[19] Greenwood, J. Z. Hercowitz and G. W., Huffman. 1998. Investment, Capacity Utilization, and the Real Business Cycles. American Economic Review 78, 402-417.

[20] Grüne, L., and W. Semmler. 2008. Asset Pricing with Loss Aversion. Journal of Economic Dynamics & Control 32, 3253-3274.

[21] Guvenen, F. 2009. A Parsimonious Macroeconomic Model for Asset Pricing. Econometrica 77, 1711-1750.

[22] Huffman, G.W. and M.A.Wynne. 1999. The Role of Intra-temporal Adjustment Costs in A Multisector Economy. Journal of Monetary Economics 43, 317-350.

[23] Jermann, U. 1998. Asset Pricing in Production Economies. Journal of Monetary Economics 41, 257-275.

[24] Jacquier, E., N. G. Polson, and P.E. Rossi. 1994. Bayesian Analysis of Stochastic Volatility Models. Journal of Business & Economic Statistics 12, 371-389.

[25] Judd, K.L. 1996. Approximation, Perturbation, and Projection Solution Methods in Economics. In: Amman, H., Kendrick, D. and Rust, J., Editors, 1996. Handbook of Computational Economics, North-Holland, Amsterdam.

[26] Judd, K.L., and S.Guu. 1997.Asymptotic Methods for Aggregate Growth Models. Journal of Economic Dynamics and Control 21,1025-1042.

[27] Judd, K.L. 2002. Perturbation Methods with Nonlinear Changes of Variables. Working paper, Hoover Institution, Stanford University.

[28] Kahneman, D., and A. Tversky. 1979. Prospect Theory: An Analysis of Decision under Risk. Econometrica 47, 263-292.

[29] Kocherlakota, N.R. 1996. The Equity Premium: It's Still A Puzzle. Journal of Economic Literature 34, 42-71.

[30] Kydland, F.E. and E.C. Prescott. 1982. Time to Build and Aggregate Flutuations. Econometrica 50, 1345-1370.

[31] Ljungqvist, L., and T. Sargent. 2004. Recursive Macroeconomic Theory. The MIT Press. Cambridge, Massachusetts.

[32] Mankiw, N.G., and S.P. Zeldes. 1991. The Consumption of Stockholders and Nonstockholders. Journal of Financial Economics 29, 97-112.

[33] Lucas, R.E., Jr., 1978. Asset prices in An Exchange Economy, Econometrica 46, 1429-1445.

[34] Masulis, R.W. 1988. The Debt/Equity Choice. Ballinger, Cambridge, MA.

[35] Mehra, R., E. Prescott. 1985. The Equity Premium : A Puzzle. Journal of Monetary Economics 15, 145-161.

[36] Ruge-Murcia, F. 2012. Estimating Nonlinear DSGE Models by the Simulated Method of Moments: With An Application to Business Cycles. Journal of Economic Dynamics and Control 36, 914-938.

[37] Sargent, T. 2010. Interview with Thomas Sargent. The Region September, 27-39.