THE THEORY OF A DISCRIMINATING MONOPOLIST FACING UNCERTAIN DEMAND IN ONE MARKET

by

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ABSTRACT

This dissertation is to provide a theory of inventory investment (change in stocks) that is capable of capturing the transmission of international economic shocks. Theories of inventory accumulation abound in the international macroeconomics literature, but this investigation provides one additional theory, one that is based on a well-known model of *discriminating monopoly* in microeconomics. This dissertation extends Joan Robinson's analysis of a discriminating monopolist to cover the case where the monopolist faces *uncertain demand* in one market, including possibly a foreign market for the same commodity.

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Chapter 1

Introduction

The concept of *change in inventory* is a useful concept in economics and finance for explaining a particular kind of investment. The purpose of my dissertation is to provide a theory of inventory investment (change in stocks) that is capable of capturing the transmission of international economic shocks.

There are many ways to accomplish this task. Theories of inventory accumulation abound in the international macroeconomics literature. This analysis provides one additional theory based on a very well known model of *discriminating monopoly* in microeconomics.

The practice of charging different prices for the same or virtually the same commodity because it is not transferable across markets has become more prevalent than it was in 1933 when Joan Robinson provided her classical analysis of a discriminating monopolist in the now famous *Economics of Imperfect Competition* (1933). This dissertation can also be seen as extending Joan Robinson's analysis to the case where the monopolist faces *uncertain demand* in one market, including possibly a foreign market for the same commodity.

This is an important theoretical development for two sets of reasons. First, it is important because there exist such interconnected international markets. It is valuable therefore to investigate this observed phenomenon. There are many examples of such market structures.

There are strong empirical grounds to suggest the existence of this market interconnection. One such example is the pharmaceutical industry, where global companies like GlaxoSmithKline, Plc, Merck & Co and Johnson & Johnson have monopoly power in patented-drugs markets both at home and abroad. They not only compete with each other for substitutable drugs, but also with illegal producers of these drugs in China, India, and other low cost production countries. In these countries, patent and copyright laws are not entirely respected. This is clearly a case of a monopolist facing certain demand in the home market, but uncertain demand in the foreign market.

Another example is in the electricity industry in the United States. A look at the stages of deregulation in the various states clearly points to the fact that states like Tennessee, Kentucky, and Georgia, to name a few, will likely be among the last to completely deregulate. If the states surrounding Tennessee deregulate, the Tennessee Valley Authority could find itself having a certain demand for electricity at home but confronting stochastic demand in the out-of-state market, since that market would be open to competitors like independent cooperative suppliers in the overlapping Kentucky market. There arises the need for the microeconomic theory of this phenomenon to explain what the effect would be on electric prices and output levels.

This investigation is also valuable on theoretical grounds. This analysis adds to the microeconomic theory literature that investigates the behavior of a discriminating monopolist faced with imperfect information. Blair and Heggestad (1977) analyzed the behavior of a discriminating monopolist in the presence of *price* uncertainty. Sandmo (1971) and Batra and Ullah (1974) investigated the behavior of the competitive firm under the assumption of price uncertainty. This investigation deals with the behavior of a discriminating monopolist in the presence of *demand*, not price, uncertainty, specifically the case in which the monopolist faces certain demand in one market and uncertain demand in the other.

Chapter 2 provides a brief survey of the literature on price discrimination under uncertainty, the details of decision-making under uncertainty in general (in the form of expected utility analysis), and its application to the theory of the firm under price uncertainty in particular. Chapter 3 explores the case of price discrimination in the absence of any uncertainty. Chapter 4 investigates the case of a discriminating monopolist in the presence of price uncertainty in one market. And, Chapter 5 analyzes the case of a discriminating monopolist who faces uncertain demand, and in some circumstances finds that there is accumulation or de-accumulation of inventories, which is the more realistic case. Chapter 5 also contains some closing remarks.

Chapter 2

Risk, Preferences, and the Theory of the Firm

2.1 Introduction

In the early part of the development of economics as a social science, economists referred to "utility" as an indication of a consumer's overall well-being or as a measurement of the happiness an individual derived from the consumption of a good. The problem with this idea is that economists did not spell out how this inherent utility can be quantified. Due to this inherent problem, economists have abandoned the view of cardinal utility and have adopted ordinal utility as a way of describing consumers' preferences.

It is generally accepted that the most essential aspect of utility is that it permits a comparison of two distinct commodity bundles by a consumer. A utility function is a way to rank every conceivable consumption bundle of goods.

We assume that a consumer is faced with set *S*, the collection of possible nonnegative consumption bundles from which a decision-maker will pick the best, but on what criterion? The decision-maker will maximize an objective function subject to a budget constraint, for example, $u = f(x)$, where x is a vector of non-negative amounts of a finite number of commodities, n.

2.2 Preferences

The consumer is assumed to have preferences regarding the consumption bundles in *S*. The expression $x \mathcal{R} y$, defines a *binary* preference relation that "the consumer thinks bundle *x is at least as good as* bundle *y*." [1] To assure that preferences follow some order for the set of bundles in S, it is necessary to make certain that these preferences that are specified are not unrealistic. This condition requires that preferences satisfy certain standard properties.

The property of *Completeness* is defined a: For all x and y in *S*, either $x \mathcal{R} y$ or $y \mathcal{R} x$ or both, meaning that any two or more given bundles can be compared.

The property of *Reflexivity is*: For all *x* in *S*, $x \mathcal{R} x$, states that bundle *x* is at least as preferred as itself. This condition is the requirement for "sanity" in ranking.

The *Transitivity* property ensures that the consumer can pick the best bundle out of a given set, a necessity for a discussion of preference maximization. It requires that for all *x*, *y*, and *z* belonging to *S*, if $x \mathcal{R} y$ and $y \mathcal{R} z$, then $x \mathcal{R} z$. If $\mathcal{R}$ defined on *S* satisfies the above three properties of completeness, reflexivity, and transitivity, then there exists an ordering. A stronger and stricter preference may also be specified. Simply put, $x P y$ means that "*x is strictly preferred to y*." Similarly the notion of indifference is given by $x I y$ if and only if $x \mathcal{R} y$ and $y \mathcal{R} x$. The concept of *Continuity*, which is necessary to rule out certain discontinuous behavior, states that if *y* is strictly preferred to *z* and *x* is a

[1] For more on preferences read Varian 1992, p. 94.

bundle that is close enough to *y*, then *x* must be strictly preferred to *z*. *Strong Monotonicity* states that if $x \geq y$ and $x \neq y$, then $x\ P\ y$. "Strong monotonicity says that at least as much of every good, and strictly more of some good, is strictly better." (Varian, 1992 , p. 96)

If a decision-maker has preferences that are (i) representable by a *binary relation*, which provides a *complete ordering* of all alternatives in *S*, (ii) *continuous*, and (iii) *strongly monotonic*, then a consumer's preferences can be summarized by means of a utility function, that is, $u: S \rightarrow R$ such that $x\ \mathcal{R}\ y$ if and only if $u(x) \geq u(y)$. In addition, the requirement of convexity of preferences provides a real valued, ordinally measurable utility function that is strictly quasi-concave. A strictly quasi-concave utility function is the most general function that guarantees a solution to a linearly constrained maximization problem.

2.3 Theory of Firm Behavior under Uncertainty

The considerable work that has been done on the behavior of the firm under uncertainty spans from Sandmo's (1971) contribution to the more recent work of Dana (2001) and Laffont (2002, forthcoming). None of these papers covers the case of a monopolist firm selling a single product in two markets, one of which has a random demand function.

The assumption is that in the first market the monopolist faces non-stochastic demand, and in a second market the monopolist's demand function is random. An example might be the pharmaceutical company, mentioned above, that has a patent monopoly in the domestic market but an international market subjected to regulation and illegal production. If such a situation occurs, what are the splash-back effects of exogenous changes in the uncertain foreign market on the certain demand in the domestic market? This analysis seeks to answer such question.

Blair and Heggestad (1977) have shown that a *risk-averse*, quantity-setting, discriminating monopolist will produce less than the monopolist who does not face uncertainty. Throughout this work it is assumed that the monopolist is a risk averter who exhibits decreasing absolute risk-aversion tendencies.[2] This investigation like most papers dealing with market structures is theoretical in nature. The pharmaceutical industry experiences demand uncertainty at home and abroad, but there is less uncertainty in the home market, so it is assumed that there is zero uncertainty at home and positive abroad.

Sandmo (1971) systematically analyzed the theory of a competitive firm under price uncertainty and risk aversion. The analysis was conducted with the assumption that the firm's attitude toward risk can be represented by a *von Neumann-Morgenstern utility function*. The question he raises is how the introduction of price uncertainty affects the optimal output of the competitive firm as opposed to the well-known solution under certainty. Using additive and multiplicative shift parameters, Sandmo proves that under

[2] On decreasing absolute risk aversion, see Arrow (1965) and Pratt (1964).

price uncertainty, output is smaller than that under certainty if the firm is risk averse, equal when the firm is risk neutral, and greater when the firm is a risk-taker.

Leland (1972) takes the analysis a step further by constructing a formulation of random demand, providing a rigorous analytical framework applicable to different behavioral modes of the firm. Leland shows that, with uncertain demand, inventories can develop, and as a result the issues of holding inventory and the associated costs arise. These phenomena do not arise under price uncertainty, as clearly shown by Sandmo (1971).

After deriving the conditions necessary and sufficient to determine the impact of uncertainty on a firm's decisions, Leland (1972) analyzed the effect of uncertainty on the optimal output level of the quantity-setting firm. Three essential factors determine the firm's behavior toward risk under demand uncertainty. First, the behavior mode of the firm is affected by the choice of *ex ante* controls. Second, the firm's attitude toward risk is a reflection of its utility function over profit; and third, the change in the firm's profit riskiness results from the change in *ex ante* controls.

Blair and Heggestad (1977) attempted to analyze the impact of demand uncertainty on a price-discriminating monopolist with a finite number of separate markets. They proved, instead, that under price uncertainty, price differences might occur as a result of differences in the riskiness of the various markets and differences in demand elasticities of each market. Further, using Arrow's definition of risk premium, they showed that identical demand functions and equal marginal production cost can still lead to different expected prices if the marginal impact of risk premiums is different for the different markets.

2.4 Measurement of Risk Aversion

It is common to argue that individuals exhibit aversion to risky undertakings. Arrow (1965) and Pratt (1964) explored how the concept of risk aversion tends to explain many observed phenomena in economic theory and in real-world occurrences. The measurement of risk aversion in conjunction with the expected-utility hypothesis has been applied extensively to explain and derive quantitative results in economic theory.

A risk averter is one who, given a certainty value, would be unwilling to take a bet on an uncertain outcome even if the probability of winning the bet is fair. Consider an individual with a fixed amount of wealth who is offered a chance to win or lose a fixed amount at fair odds. A risk averter would prefer a smaller certain amount than the expected value. The difference between the certain amount and expected value is one's risk premium. A risk premium is, therefore, the amount that the risk averter is willing to forgo to avoid being exposed to risk.

Both Arrow (1965) and Pratt (1964) identified two measures of risk aversion as a function of wealth. The measurement of these behaviors is of great importance in the prediction of the effects of uncertainty on economic agents who encounter stochastic decision choices. These hypothesized measures are decreasing absolute risk aversion and increasing relative risk aversion. The former explains the willingness to invest in risky assets as individuals become wealthier. That is, it measures the individual's unwillingness to take a bet that is less than fair and his/her insistence for a greater probability of winning than losing the bet, at least when the amount at stake is small. The increasing relative risk aversion hypothesis is one in which the bet is not considered in absolute terms but in proportion to wealth. The assertion is that, if the size of a bet and wealth are proportionately increased, the unwillingness of an individual to accept the bet increases.

Two principles can be applied to the special models of choice involving both risky and secure assets. The secure asset could simply be represented by money carrying no risk and no return. In a more generalized form, the secure asset can be seen as a guaranteed instrument like a government Treasury bond, with little or negligible risk but yielding some sort of return. Most risk averters avoid involvement in any unfair undertaking or unfair gamble, but for sufficiently small amounts at risk, the decreasing absolute risk aversion hypothesis explains that the individual will take the bet if the odds of winning are at least better than even.

2.5 Pedagogic Role

Most introductory economics courses never touch on certain aspects of uncertainty in decision-making. It is important that the presence of uncertainty in explaining some economic concepts should be covered in courses beyond the principles of economics level. For instance, the conditions for the existence of a competitive market are the presence of many buyers and sellers, homogeneity of the goods offered by the various sellers, and free entry and exit in the market. As the result of these conditions, the action of any one particular buyer or seller in the market would have a negligible impact on the market price. Each buyer and seller takes the market price as given, so the market demand curve is horizontal.

There is a puzzle in the theory of the perfectly competitive industry. If the individual firms have horizontal demand curves, how can a finite number of these flat demand curves add up to a downward-sloping industry demand curve? This puzzle may be explained by the introduction of price uncertainty, which can lead to an aggregation of a finite number of flat demand curves, resulting in a downward-sloping market demand curve. Thus, the presence of uncertainty is worth mentioning when treating the theory of the competitive firm.

The conventional method of most intermediate microeconomics textbooks begins with a discussion of the "scope and methods" of economics, dealing with the development of simple economic models of social phenomena. This introduction is followed by a discussion of markets, dealing with demand, supply, market equilibrium, and the analysis of prices as various aspects of market change. The analysis involves comparing two static equilibria, an exercise called comparative statics. A Walrasian auctioneer moves the market from one equilibrium to another. It also tackles short-run and long-run equilibrium of the market.

The next topics that must be addressed include the budget constraint, the properties of the budget set, and the effect of changes in the budget line. Economic policy analysis frequently applies tools such as taxes, subsidies, and rationing that affect a consumer's budget constraint. Another kind of policy that comes into play is the use of a lump-sum tax or subsidy by governments to affect the consumption of certain goods and also to encourage and boost production and employment in certain industries.

After modeling the market and dealing with the budget line and constraints, the next topic will typically be the theory of consumer behavior. This discussion can be formulated in terms of *binary* preferences satisfying the axioms of completeness, reflexive and transitive, which together are tantamount to the binary preference's generating a complete *ordering* of the set of all commodity bundles. One can add on some few key assumptions like *continuity* and *strong monotonicity* to make the ranking tighter. Graphically, these *binary* preferences can be represented by an indifference map. An indifference curve is the collection of all commodity bundles that a decision-maker likes equally. An Indifference *map* is the collection of all indifference curves of an individual. The additional axiom of convexity of preferences produces an ordinal utility function that is strictly quasi-concave, the most general objective function that a consumer can maximize subject to his/her budget constraint.

With preferences well covered, one can proceed to address the topic of utility functions, a way of assigning a number to every possible bundle of goods under consideration by a decision-maker, such that preferred bundles get assigned larger numbers. Combining the budget set and the theory of preferences makes it possible to examine the optimal choices of a decision-maker. The *theory of consumer behavior* postulates that people choose the best bundle of goods taking into consideration their budget constraints. For instance, a consumer can choose an optimal bundle of two goods at some set of prices, given a particular income level. In general, when prices and income change so does the consumer's optimal or maximized choice of the combination of the two goods. The function that relates the optimal choice of quantities of each good to the different prices and income is called the demand function. Once these topics of markets, budget, preferences, utility, and consumer choices are discussed, the analysis can proceed to the topics related to imperfect information or uncertainty, where the theory developed in this dissertation fits into the microeconomics literature.

Alternatively, after modeling the markets, budget, preferences, utility, and consumer theory, the discussion can go directly to producer theory, and then all aspects of uncertainty can be completely discussed. A topic like a monopolist facing uncertain demand in one market can be treated after perfectly competitive firms and monopolists have been discussed. It is important to let students know there are hybrids of the main market structures that traditionally are not considered. The introduction of uncertainty brings in reality and drastically affects the behavior of the firm. The presence of uncertainty in one market changes the attitude of the monopolist, since profit maximization (under certainty) depends on the equality of the marginal revenues in both markets to the aggregate marginal cost curve of the firm. From prior references, it is known that the presence of uncertainty, and the assumption of risk aversion, generates results from the certainty case. Therefore, the presence of demand uncertainty in one market would definitely have a splash-back effect on the profit level of the monopolist and hence on his/her terminal wealth.

2.6 History

Using geometrical representation, Robinson (1933) showed convincingly that if a constant-cost, simple-price monopolist serving two markets decides to discriminate, the total output will remain unchanged if both markets have linear demand curves. On the other hand, if the demand curves are nonlinear, the output of the simple monopolist, as compared to that of the discriminating monopolist, will depend on the concavity of the market demand curves. "If the more elastic demand curve is concave and the less elastic demand curve is straight line or convex, the increase in the market in which output is increased will be greater than the reduction of output in the second market and the discriminating monopolist output will be greater than that of the simple monopolist" (Robinson 1933, p.193).

Edwards (1950) produced a simpler result, using graphs, but pointed out that the situation is only applicable if the variation between the simple monopolist's profit and the discriminating monopolist's profit is very small. Building on Robinson (1933) and Edwards (1950), Silberberg (1970) presented an extensive and rigorous analysis of output under simple monopoly as compared to that under discriminating monopoly. He showed that the "adjusted concavity" theorem is valid not only in a situation where the simple and discriminating monopolists' profits are very close, but also in the entire range between the simple and discriminating monopolist points. Like his predecessors, Silberberg concluded that the direction of the change in output using the "adjusted concavity" theorem is ambiguous.

As an alternative to the "adjusted concavity" theorem, Smith (1981) used the simple monopolist's marginal revenues and a discriminating monopolist's marginal revenue curves (SMR-DMR criterion, hereafter) to solve the problem of the ambiguity raised by Robinson and Edwards. The SMR-DMR criterion relies on the convergence of the marginal revenue curves of the simple monopolist (SMR) and the discriminating monopolist (DMR). The SMR-DMR criterion states that when price discrimination is possible, equating the marginal revenues in each sub-market results in an increase, decrease, or unchanged output as the DMR curve at the simple monopolist output is above, less than, or equal to that of the SMR curve. Smith also showed graphically for the first time that the sign of the change in output is independent of the slope of the marginal cost curves, except in the case of a vertical marginal cost curve.

Pigou (1920), in his classic book *The Economics of Welfare*, lists the taxonomy of various degrees of price discrimination. He describes first-degree price discrimination (also called perfect price discrimination) as the type in which the seller knows each potential buyer's reservation price. In other words, the seller knows exactly how much each customer is willing to pay and therefore can successfully extract the entire consumer's surplus. Such a degree of price discrimination is unlikely unless each buyer demands only one unit of the commodity or the price per unit is based on the amount of utility derived from each unit.

The second degree is the type of price discrimination in which the monopolist divides the market into several price ranges (price steps), such that the product is sold at the lowest price in each price range to all buyers whose demand price falls within that range. In effect, the monopolist extracts the entire consumer surplus falling below each step. Therefore, as the number of prices offered by the monopolist increases, the second degree approaches the first degree. Second-degree price discrimination, though theoretical, resembles the modern-day online Dutch auction with a minimum price level at the end of the bid period, all winning bidders get to buy the item(s) at the price of the lowest winning bidder.

2.7 Third-Degree Price Discrimination

Third-degree price discrimination, which is the more realistic case and the subject of this dissertation, occurs when a monopolist divides the market into two distinct markets where the lowest price in the less elastic market exceeds the highest price in the more elastic market.

Price discrimination arises out of some degree of market imperfection. The following conditions have to be present in some form or shape for this condition to prevail.

(1) There has to be some form of market power. It could be buyers' unwillingness to switch from one seller to another, markets where no competition exists, or markets where there appears to be some sort of collusion among rival sellers.

(2) The individual sellers must have the ability to identify distinct markets to segment these markets directly or indirectly. This may occur when the monopolist is dealing with markets that are geographically separated tariff barriers exist that render

arbitrage opportunities unprofitable. This type of discrimination leads to cheaper prices of the good in the export market and higher prices in the domestic market. Such practices are prevalent in the pharmaceutical market, where the same drugs are "dumped" into less developed countries at lower prices than the prices at which the manufacturers sell in the home-country market. Local utilities like electricity and water providers separate the market by charging different rates for domestic and commercial users for services produced under the same cost conditions.

(3) The monopolist must have the ability to curb or completely stop arbitrage between the two markets. Usually sales of direct personal services with are very slim, or resale is impossible, fulfill this criterion. For example, most non-profit medical institutions discriminate by discounting surgical rates based on the patient's ability to pay. Wealthy patients pay the full surgery fee, while poor patients pay less, based on their income. This kind of practice is acceptable because medical services are not resalable and the wealthy cannot get the service at a cheaper rate anywhere.

2.8 Market Structure and Price Discrimination

Price discrimination can be classified into two broad categories, namely intrapersonal and interpersonal. Armstrong and Vickers (2001) discuss the differences and similarities between intrapersonal and interpersonal and how in some cases the distinction becomes blurred. Intrapersonal price discrimination is the type where the price-marginal cost ratios varies across batches of the goods purchased by a given customer. Monopolists who can determine the nature of the demand curve for an individual buyer can vary the marginal price per unit if the buyer's demand curve is downward sloping. This takes the form of quantity discounts or the "buy-one-get-next-at-half-price" type of discounts. On the other hand, in interpersonal price discrimination, the price-marginal cost ratios across consumers vary. Interpersonal price discrimination is the type that attracts the most attention from economists due to its welfare and political implications. A hybrid of inter-intrapersonal price discrimination occurs if a monopolist offers a particular group (for example, senior citizens) a discount or two-part pricing.

Stokey (1979) described situations where the use of time as a medium for price discrimination is feasible. Many products such as computers, computer programs, information, and movies are more expensive at the introductory offer price, but less expensive over time. Books usually first appear in hardcover at more expensive prices, and thereafter less expensive paperback becomes available. Similarly, new movies usually run at a more expensive box-office rate when premiering, and subsequently lower rates when the initial rush of impatient moviegoers has subsided, and the more patient moviegoers who were unwilling to pay the high premiere rates then attend. Such inter-temporal price discrimination arises because the monopolist can separate the market by how customers differ from one another, or by the urgency at which they are willing to pay to immediately satisfy their needs. Armed with that information and market power, the monopolist can separate the market by this criterion and practice price discrimination.

The difficulty with intertemporal price discrimination is that the decline in price over time may not necessarily be due to discrimination, but may be caused by other economic factors such as fear of competition or economies of scale. In most cases, differences in prices are due to exploitation of differences in the customers' reservation prices, falling prices may be due to declining cost of production or the introduction of competition by products that may be close substitutes.

Stole (2001) extends the theory of price discrimination to the case of market structures that are perfectly competitive or oligopolistic in nature. He also looks at the effect of a monopolist's price restructuring in response to anticipated competition in the general setting or all three levels of discrimination. This work provides some useful insight into uncertainty, but unfortunately has little to do with the issues raised here.

Chapter 3

Discrimination under Certainty

Suppose that the monopolist is able to identify two distinct markets. Also assume that the consumers in each market are unable to resell the good due to restrictions such as those on imported goods, for example, drugs. This latter restriction satisfies the conditions necessary to avoid arbitrage opportunities that, when present, would make discrimination difficult, if not impossible.

The objective of the firm is to maximize the expected utility of profit. Let $p_1(x_1)$ and $p_2(x_2)$ denote the inverse demand curves for market 1 and market 2, respectively, and $[C(x_1 + x_2) + B]$ be the total cost of producing the output, then the profit maximization problem is

$$(1) \qquad Max\ \pi(x_1 + x_2) = p_1(x_1)x_1 + p_2(x_2)x_2 - C(X) - B,$$

where $x_1 + x_2 = X$. The first-order conditions for the maximization problem are

$$(2) \qquad p_1(x_1)x_1 + p_1'(x_1)x_1 = C'(X) \text{ and}$$

$$(3) \qquad p_2(x_2)x_2 + p_2'(x_2)x_2 = C'(X).$$

Since both marginal revenues in market 1 and market 2 are equal to the marginal cost,

$$(4) \qquad MR_1(x_1) = MR_2(x_2) = MC(X).$$

If the marginal revenues in$_2$ the two markets are not equal, the monopolist will adjust sales to ensure that marginal revenue obtained in market 1 equals that of market 2.

In this circumstance, decreasing sales in the least demand-elastic market and increasing sales in the more elastic market can result in profit optimization. The profit maximization output level is attained when the aggregate cost of producing the output level equals the marginal revenues in each of the separate markets.

Applying the standard marginal revenue formula for elasticity, the profit-maximization condition can be written as

$$(5) \qquad p_1(x_1)\left[1-\frac{1}{|\xi(x_1)|}\right]=MC(X)$$

and

$$(6) \qquad p_2(x_2)\left[1-\frac{1}{|\xi_2(x_2)|}\right]=MC(X).$$

where $\xi_1(x_1)$ and $\xi_2(x_2)$ represent the elasticity of demand in the respective markets evaluated at the profit-maximizing output level.

If price $p1 > p_2$, then

$$(7) \qquad \left[1-\frac{1}{|\xi_1(x_1)|}\right]<\left[1-\frac{1}{|\xi_2(x_2)|}\right].$$

This implies that

$$(8) \qquad \frac{1}{|\xi_1(x_1)|} > \frac{1}{|\xi_2(x_2)|}$$

so that

$$(9) \qquad |\xi_2(x_2)| > |\xi_1(x_1)|.$$

Thus, the market with the lower elasticity of demand must have the higher price, and the market with the higher elasticity would have the lower price. In other words the monopolist will set a lower price for the more price-sensitive market and higher price for the less price-sensitive market. This condition ensures that the monopolist maximizes his/her overall profit.

Chapter 4

Price Uncertainty

Assume that the objective of the firm is to maximize the expected utility of profit.[3] Further, assume that the utility function is cardinally measurable (i.e., is unique up to a positive affine transformation), continuous, twice differentiable, and strictly concave[4] such that

(10) $u'(\pi) > 0 \quad and \quad u''(\pi) < 0$

The cost function of the firm is given by

(11) $F(X) = C(X) + B$

where X is output, $C(X)$ is the variable cost, and B is the "fixed cost." The firm's profit function can be defined as

(12) $\pi(X) = p_1 x_1 + p_2 x_2 - C(X) - B$

where p_i for i = $1, 2$ represents the demand functions for market 1 and market 2, respectively and $X = x_1 + x_2$ is the output where price p is assumed to be a subjective random variable with density function $f(p)$ and expected value $E[p] = \mu$, $V(p) = \sigma^2 > 0$, and price is assumed to be positive so that $p > 0$.

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[3] Sandmo (1971) pointed out, that the assumption that the firm maximizes expected utility of profit might be a strong one, because a group decision does not necessarily conform to individual preference on which the transitive axiom, required for the existence of the utility function, is based. However, there are firms whose mission ensures that preferences among decision-makers are sufficiently similar to justify the existence of group preference.

[4] The assumption that the utility function is everywhere differentiable and strictly concave is stringent, but it ensures a unique solution to the optimized output level.

The monopolist's expected utility of profits can be written as

(13) $U(\pi) = E[u(p_1 x_1 + p_2 x_2 - C(X) - B)]$,

where E is the expectations operator. The discriminating monopolist's decision problem is to choose x_1 and x_2 to maximize (13).

Differentiating with respect to x_1 and x_2 for each market, to obtain the first order conditions necessary for utility maximization. The first-order conditions (FOC) are

(14) $U_1 = E[u'(\pi)(p_1 - C'(x))] = 0$ and

(15) $U_2 = E[u'(\pi)(p_2 - C'(x))] = 0.$

The second-order conditions (SOC) are that

(16) $U_{11} = E\{u''(\pi)[p_1 - C'(x)]^2 - u'(\pi)C''(x)\} < 0,$

(17) $U_{22} = E\{u''(\pi)[p_2 - C'(x)]^2 - u'(\pi)C''(x)\} < 0,$

and

(18) $D = U_{11} U_{22} - U_{12} U_{21} > 0,$

Differentiating (15) with respect to B, to obtain

(19) $$\frac{\partial x_2}{\partial B} = \frac{1}{U_{22}} E[u''(\pi)(p_2 - C'(x))].$$

One of the basic results of the competitive firm under certainty is that, once a positive output level is selected, an infinitesimal increase in "fixed cost" has no effect on output level, but the case is different under price uncertainty as proven by Sandmo (1971). The assumption of decreasing absolute risk aversion is a necessary and sufficient condition for $\partial x_2/\partial B$ to be negative.[5] Market 2 in this model can be viewed as the foreign market in which there is uncertainty in the price of the second good.

As mentioned above, this is a striking result. It contrasts with what is known about the behavior of a competitive firm under price certainty. A change in "fixed cost" has a negative relationship with the output level in market 2.

In a competitive market, each individual firm is a price-taker, and the market forces determine the market price for all the firms in the market. In effect, each firm views price as a random variable, so it is unintuitive to speak of the effect of an "increase in price." Similar to Sandmo (1971), the discussion focused on the issue of the increase in the mathematical expectation of price, which has a finite and constant mean and variance. The result is used to evaluate the effect of an increase in the mathematical expectation of the price for a monopolist's uncertain market. One can write the price faced in the foreign market by the monopolist as $p + \varepsilon$ where ε is an additive shift parameter. Increasing ε means taking some of the probability weight from the center and adding it to the tails in such a way as to leave the mean unaltered.

Differentiating (15) with respect to B and evaluating as in Sandmo and rearranging, to obtain

[5] For proof of the calculation, refer to Sandmo (1971, p 69).

$$(20)\qquad \frac{\partial x_2}{\partial \varepsilon} = -x\frac{1}{U_{22}}E[u''(\pi)(p_2 - C'(x)] - \frac{1}{U_2}E[u'(x)].$$

Or substituting (15) into (18) to get,

$$(21)\qquad \frac{\partial x_2}{\partial \varepsilon} = -x\frac{\partial x}{\partial B} - \frac{1}{U_{22}}E[u'(x)].$$

Next it is shown that the expression on the right side of (21) is positive, assuming decreasing absolute risk aversion. If the firm's expected utility derived from terminal wealth $W = W_0 + \pi$, where W_0 is initial wealth, π is random profit. The monopolist chooses x_1 and x_2, which are the outputs earmarked for sale in markets 1 and 2, so as to maximize his/her expected utility derived from terminal wealth. That is, the monopolist maximizes

(22) $U(W) = E[U(W_0 + \pi)] = E[U(W_0 + p_1 x_1 + p_2 x_2 - C(X) - B)]$,

where E is the expectation operator, p is price of output, $X = x_1 + x_2$, and U is the von Neuman-Morgenstern utility function. This function is unique up to a positive linear transformation. In other words, $U(W)$ is cardinally measurable. At an interior maximum of (1), the following first-order conditions (FOC) are satisfied:

(23) $U_1 = E[u'(\pi)(p_1 - C'(X))] = 0$,

and

(24) $U_2 = E\,[u'(\pi)\,(p_2 - C'(X))] = 0.$

Further, the following second-order conditions (SOC) are satisfied:

(25) $U_{11} = E\,\{u''(\pi)\,[p_1 - C'(X)]^2 - u'(\pi)C''(X)\} < 0,$

(26) $U_{22} = E\,\{u''(\pi)\,[p_2 - C'(X)]^2 - u'(\pi)C''(X)\} < 0,$

and

$D = U_{11}\,U_{22} - U_{12}\,U_{21} > 0.$

To find the sign of $\partial X/\partial B$, differentiate (24) with respect to B and rearrange

$\partial E\,[u'(\pi)\cdot\pi'(X)] = 0.$

(27) $Eu'(\pi)\cdot\pi''(X)\cdot\partial X/\partial B + E\,\pi'(X)\cdot u''(\pi)\cdot\partial\pi/\partial B = 0.$

Deriving $\partial\pi/\partial B$ and substituting into (27) to obtain

$$Eu'(\pi)\cdot\pi''(X)\cdot\partial X/\partial B + E\,\pi'(X)\cdot u''(\pi)\cdot[p\cdot\partial x_i/\partial B - C'(X)\cdot\partial X/\partial B - 1] = 0.$$

Rearranging

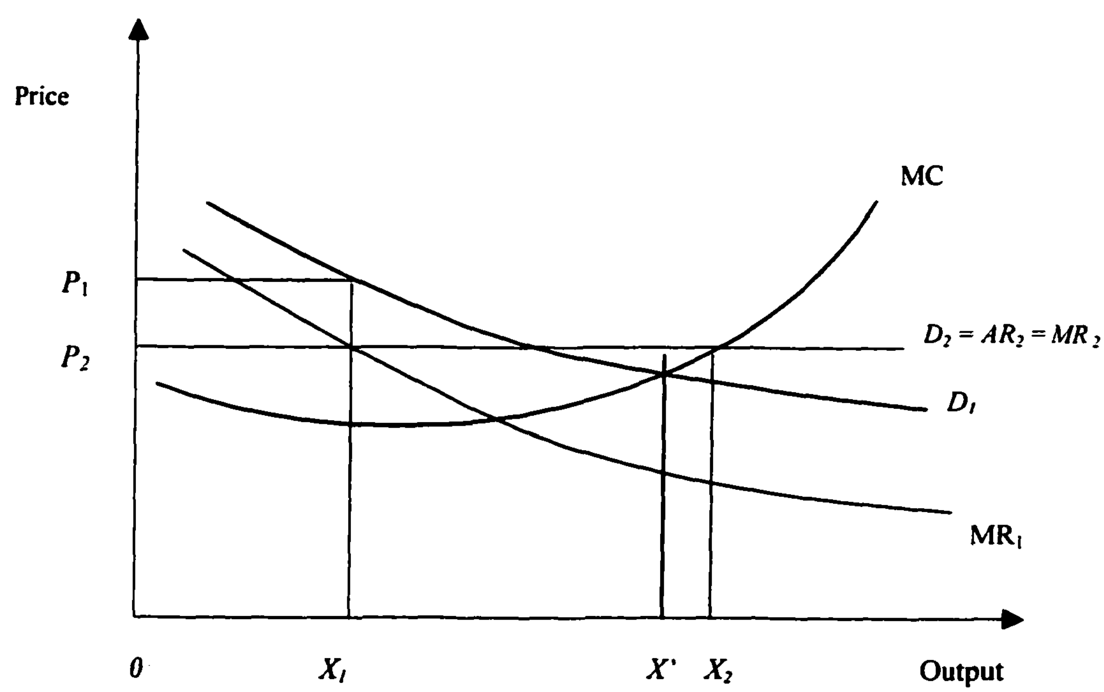
$$E[u'(\pi)\cdot\pi''(X) + \pi'(X)\cdot u''(\pi)(p_i - C'(X)]\,\partial X/\partial B = E\pi'(X)\cdot u''(\pi)$$

$$(28) \qquad \frac{\partial X}{\partial B} = \frac{1}{U_{22}} E\pi'(X).u''(\pi).$$

The diagram below explains the issues raised above. This discussion is less technical and more intuitive.

Consider Figure 1 below.

Figure 1



A monopolist who faces a random demand function in the second market can be treated as similar to a perfectly competitive firm, or a hybrid market structure between a perfectly competitive firm, and a monopolistic firm facing uncertain prices in that market. From Figure 1, $0X_1$ is the output level for market 1, and the output level for market two

is $X_2 - X_1$, which will fall between X_1 and X_2. The marginal revenue for market 2 is assumed to equal the competitive price in that market. This graph is similar to the one produced by Robinson (1933),[6] but the analysis in the current study takes into consideration the effect of price uncertainty in market 2. The effect of price uncertainty and the assumption of risk aversion are the driving forces that transmit the shock of a change in market 2 and it is consequently its splash-back effect in market 1.

If the monopolist expected the price in market 2 to fall, the risk-averse monopolist would sell less in market 2. This would result in a lower marginal cost in market 2 and consequently reduce total output level from X_2 *to* X'. This requires output in market 1 to increase in order to equate the marginal revenues in both markets for profit to be maximized. If the price in market 2 falls below the marginal cost curve and the marginal revenue curve for market 2, then there will be no output sold in market 2, the foreign market.

The RHS of (28) is negative. Thus, decreasing absolute risk aversion is a necessary and sufficient condition for $\partial x_2/\partial B$ to be negative.

From figure 2 below, let π_0 be the profit at the point where $p - C'(X) = 0$. With the assumption of decreasing absolute risk aversion, it implies that as profit increases from π_0 to π_1, the measure of decreasing absolute risk aversion (RA) decreases from R_{A0} to R_{A1}.

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[6] Figure 62, p.184.

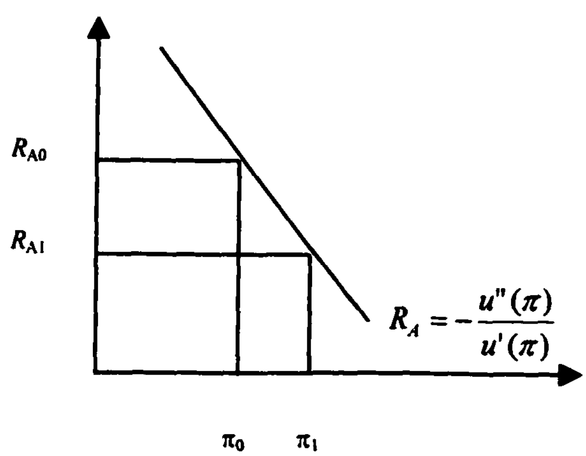
Therefore,

(29) $R_{A1}(\pi_1) \leq R_{A0}(\pi_0)$ *for* $p \geq C'(X)$.

Applying Arrow's definition of decreasing absolute risk aversion, then

$$(30) \qquad -\frac{u''(\pi)}{u'(\pi)} \leq R_A(\pi_0).$$

Figure 2



If $p > C'(X,)$ then

(31) $-u''(\pi_0)(p - C'(X)) \leq 0$, since marginal utility is positive.

Multiplying both sides of (30) by the left hand side(LHS) of (31) produces

(32) $u''(\pi_1)(p - C'(X)) \geq -R_A(\pi_0)\, u'(\pi)(p - C'(X))$.

The above holds for all levels of p, for even if $p \leq C'(X)$ the inequality in (29) and (30) would each be reversed and everything else remains the same.

Taking the expected value of (32), and keeping in mind that $R_A(W_0 + \pi_0)$ is a scalar and not a random variable, one obtains

$$E[u''(\pi_l)(p - C'(X))] \geq -R_A(\pi_0)E[u'(\pi)(p - C'(X))].$$

Applying the first-order condition (24), the right hand side(RHS) equals zero, and the left-hand side is automatically positive. Applying that to (28) and taking into consideration the fact that U_{22} is negative, that $\partial X/\partial B$ is negative has been proven.

It can be deduced that a change that makes p_2 less than the expected value of price p_2, would lead to x_2 being less than under price certainty, and quantity in market 1 must adjust accordingly to equate the marginal cost in the two markets to the total cost of producing the output level.

Chapter 5

Conclusions Pertaining to Demand Uncertainty

Consider a general model of a discriminating monopolist. Let the *p*'s, *s*'s and *x*'s be the prices, sales (number of units sold by the discriminating monopolist), and the outputs of a single commodity in market 1 and market 2. The commodity can be transferred only between the two markets at a prohibitive cost. Let W_0 be the discriminating monopolist's initial wealth, $W = W_0 + \pi$, where π is the firm's profit. Let *u(W)* be the von Neuman-Morgenstern cardinally measurable strictly convex, continuous utility function. Assume that there is certainty in market 1, but demand is uncertain in market 2. The demand function in market 1 is $s_1 = h_1(p_1)$. In market 2, however, the demand function is

(33) $s_2 = s_2(p_2) + \varepsilon$,

where $E(\varepsilon) = 0$, $V(\varepsilon) = \sigma^2 \geq 0$, $E(p_2) = h(s_2) = \mu$ is positive mean demand, σ^2, is greater than 0.

The discriminating monopolist's objective is to choose the values of the two prices and their outputs, so as to maximize

(34) $\overline{U}(W) = E(W_0 + \pi) = \int [W_0 + p_1 x_1 + p_2 s_2 - C(x(p_1, \mu, \varepsilon, a)) - B]\, dF(\varepsilon)$,

where C(x) is the cost function, F(ε) is the cumulative distribution, and *a* is the value of the measure of absolute risk aversion. Note that $s_1 = x_1$ and $x = x_1 + x_2$.

Further, to simplify some notation, define a vector of parameters of interest, $\omega = (\mu, \varepsilon, a, B)$. The objective is to obtain the signs of the total derivatives $\frac{dU}{d\mu}, \frac{dx_1}{d\mu}$ and $\frac{dp_1}{d\mu}$.

The model whose details are spelled out here is quite different from that of Chapter 4. There the quantity to be sold by a monopolist in the two markets is fixed by the firm, and the price in Market 2 is determined by the vagaries of life, weather, war, political changes, and so on in the economy. However, some of the formal mathematical structure, but with a completely different interpretation, remains largely the same. Since in the present model the monopolist decides on the price level in both markets, and lets the uncertain influences determine how much gets sold, which is the more realistic though unexplored case, equations (10) through (18) remain intact, but with some modifications.

Assume that the objective of the firm is to maximize the expected utility of profit.[7] Further, assume that the utility function is cardinally measurable (i.e., is unique up to a positive affine transformation), continuous, twice differentiable, and strictly concave such that[8]

(10a) $u'(\pi) > 0$ and $u''(\pi) < 0$

[7] Sandmo (1971) pointed out, that the assumption that the firm maximizes expected utility of profit might be a strong one, because a group decision does not necessarily conform to individual preference on which the transitive axiom, required for the existence of the utility function, is based. However, there are firms whose mission ensures that preferences among decision-makers are sufficiently similar to justify the existence of group preference

[8] The assumption that the utility function is everywhere differentiable and strictly concave is stringent, but it ensures a unique solution to the optimized output level.

The cost function of the firm is given by

(11a) $F(X) = C(X) + B$

where X is output, $C(X)$ is the variable cost, and B is the "fixed cost." The firm's profit function can be defined as

(12a) $\pi(X) = p_1 s_1 + p_2 s_2 - C(X) - B$

where p_i for i = $1, 2$ represents the demand functions for market 1 and market 2, respectively, and $X = x_1 + x_2$.

The monopolist's expected utility from terminal wealth is still given in (34) above. But the decision variables for the monopolist are the two prices not the quantities to be sold. Equation (13) is replaced by (34)

(34) $\bar{U}(W) = E(W_0 + \pi) = \int [W_0 + p_1 x_1 + p_2 s_2 - C(x(p_1, \mu, \varepsilon, a)) - B] \, dF(\varepsilon)$,

where E is again the expectations operator. The discriminating monopolist's decision problem is to choose s_1 and s_2 to maximize (34).

Differentiating with respect to s_1 and s_2 for each market, the first order conditions (FOC) necessary for an interior maximum are

(14a) $U_1 = E[u'(\pi)(s_1 - C'(x))] = 0$ and

(15a) $U_2 = E[u'(\pi)(s_2 - C'(x))] = 0.$

The second-order conditions (SOC) are that

(16a) $U_{11} = E\{u''(\pi)[s_1 - C'(x)]^2 - u'(\pi)C''(x)\} < 0,$

(17a) $U_{22} = E\{u''(\pi)[s_2 - C'(x)]^2 - u'(\pi)C''(x)\} < 0,$

and

(18) $\mathcal{D} = U_{11}U_{22} - U_{12}U_{21} > 0,$

without placing and sign restrictions on $U_{12} = U_{21}$. To derive insights in to the economic forces driving the determination of cross-market effects, the objective is to obtain the signs of the total derivatives $\frac{dU}{d\mu}, \frac{dx_1}{d\mu}$ and $\frac{dp_1}{d\mu}$.

These terms capture the effect of an increase in mean market 2 price on the discriminating monopolist's expected utility, output in market 1, and the domestic price set by the discriminating monopolist in market 1. Leland (1972) utilized a quadratic utility function to obtain sharper results. This analysis employs the constant-absolute-risk-aversion-measure utility function $u(W) = -ke^{aW}$. This function has several advantages, not the least of which is that, with a complex problem that involves two markets, the calculations become a lot easier to obtain and display. This analysis employs the same method used by Leland (1972) for the case in which the monopolist sets both the price and the output. In this case, the discriminating monopolist sets two outputs and two prices. The result converts Leland's two-dimensional optimization problem into our four-dimensional problem.

In conclusion, the results derived in this dissertation are that an increase in mean demand in market 2 will have the following effects.

Proposition 1: Prices in the foreign country, market 2, will increase.

Proposition 2: When the discriminating monopolist sells more in the foreign country, less is available for sale in the home country, market 1.

Proposition 3: The price of the product in the home country will rise.

Proposition 4: Higher prices in both countries will make the discriminating monopolist produce more, but as the discriminating monopolist increases total output, his/her inventory cost component will rise.

Proposition 5: If inventory-holding costs are relatively high, the discriminating monopolist will barely increase production. A new price *higher than the original* will be set in market 1. This linkage provides the effects of events in market 2 to induce changes in the market 1. In international trade this phenomenon is known as international shock on prices at home.

Proposition 6: Fundamentally, the *cost of inventory holdings* plays a significant role in the international transmission of higher prices between markets. *Higher inventory costs in the United States, for instance, will cause the price to rise more both at home and abroad.* The entire magnitude of international transmission of shock turns on the cost of inventory holdings at home.

The central conclusion of the research reported here is simple but profound. It is commonly thought that the international transmission of economic shocks is a phenomenon that occurs at an economy-wide, sometimes also called, macroeconomic level. The conclusion reached here is that while that is true, there are strong microeconomic forces, operating at the level of individual firms, which also bring about such international transmission, and therefore, require a serious investigation, especially in the environment of uncertainty.

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