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The use of mathematics in teaching the principles of economics

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THE USE OF MATHEMATICS IN TEACHING THE PRINCIPLES OF ECONOMICS

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THE USE OF MATHEMATICS

IN TEACHING THE PRINCIPLES OF ECONOMICS

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ABSTRACT

THE USE OF MATHEMATICS IN TEACHING THE PRINCIPLES OF ECONOMICS

by William M. Lay, Jr.

This study endeavored to identify personal and institutional characteristics that significantly contribute to the use of mathematics in teaching the principles of economics. The study defined nine personal and ten institutional characteristics which were hypothesized to contribute to the instructor's choice to use a mathematical approach in teaching the principles course. To obtain data, nineteen questions were prepared and placed in a questionnaire which was used in a national survey of 500 colleges and universities having an economics program. The sample was selected from more than 970 institutions and yielded 378 useable responses, which were analyzed using an ordinary least squares regression.

The study resulted in a predictive model in which each of five explanatory variables had a significance level of at least 0.10. Of these, four involved personal characteristics of the instructor, including the number of mathematical problems (selected from a list) used in teaching the principles course, the percent of lecture time in which mathematics was used in teaching the course, the year the

instructor received the doctorate, and whether the instructor viewed the course as a tool for eliminating potentially unsuccessful students from the major. Only one of the model's variables involved the institutional environment in which the instructor taught the course. This variable was the number of students in the economics program. Although it bore an unanticipated statistical sign, the variable completed a model capable of explaining, with a small error margin, the factors impinging on the choice of mathematics usage in an economics course.

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LIST OF ABBREVIATIONS

AEA — American Economics Association

AER — *American Economic Review*

IJT — *International Journal of Math, Education, Science, and Technology*

JEB — *Journal of Education for Business*

JEE — *Journal of Economic Education*

JEH — *Journal of Economic History*

JEL — *Journal of Economic Literature*

QJE — *Quarterly Journal of Economics*

RES — *Review of Economics and Statistics*

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CHAPTER I

INTRODUCTION

Significance of the Topic

In an increasingly interdependent world economy, the effective communication of the principles of economics is of crucial importance. Today, economics understanding and mathematics are vitally linked. Over the last thirty years, the use of mathematics in economics has greatly increased as the discipline has become more formalized.[1] During the same period, the desire to increase the use of mathematics in teaching economics has been advocated as a way to more effectively impart economics information.[2] If mathematics

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[1]There are many relevant articles. Several of the more significant include: Oskar Morgenstern, "Limits to the Uses of Mathematics in Economics," in Mathematics and the Social Sciences, ed. James C. Charlesworth, (Philadelphia, The American Academy of Political and Social Science, 1963), 22-23; L. R. Klein, "The Contributions of Mathematics in Economics," RES 34 (November 1954): 360; James T. Bennett and Manuel H. Johnson, "Mathematics and Quantification in Economic Literature: Paradigm or Paradox?" JEE 10 (Fall 1979): 40-42; and Herbert G. Grubel and Lawrence A. Boland, "On the Efficient Use of Mathematics in Economics: Some Theory, Facts, and Results of an Opinion Survey," Kyklos, 39 (1986): 419-441.

[2]Significant articles include the following: D. V. T. Bear, "The Use of Mathematics in the Teaching of the Principles Course," in New Developments in the Teaching of Economics, ed. Keith Lumsdem (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1967), 149-181; William E. Becker, "The Use of Mathematics in the Teaching of Economics," in The Principles of Economics Course: A Handbook for Instructors, ed. Phillip Saunders and William B. Walstad (New York: McGraw-Hill Publishing Company, 1990), 180-190; and William

does significantly aid in learning economics, it would be helpful to college educators to have some understanding as to the extent to which mathematics is used in teaching the principles of economics today and some insight concerning the attitude of college educators on the subject. Therefore, this study attempts to determine the extent to which mathematics is currently being used in collegiate level instruction of the principles of economics and seeks to provide an understanding of those factors that contribute to a more mathematical approach.

There are several references in the literature to the use of mathematics in economics education in general. For example, the three "languages" in which economic theory can be taught (i.e., verbal, graphical, and mathematical) is widely recognized. However, defining the extent to which mathematics is presently being used in teaching the principles of economics course has not been discussed. In addition, most studies of the use of mathematics in economics education have only dealt with theories about what *should* be done. Constraints on the implementation of these theories have not been considered. This study, therefore, seeks to provide insight into the personal and institutional factors that influence college instructors in their decision

J. Field, Daniel R. Wachter, and Anthony V. Catanese, "Alternative Ways to Teach and Learn Economics: Writing, Quantitative Reasoning, and Oral Communication," JEE 16 (Summer 1985): 213-217.

concerning the extent of mathematics used in teaching the principles of economics course.[3]

Statement of the Problem

The overall problem addressed in this study is how a college instructor might take the complex and detailed information of economics and present it to freshman and sophomore undergraduates in a comprehensible manner. Part of this process, the communication of quantitative concepts through computational mathematics, is the problem addressed. Specifically, what are the personal factors or attributes of the instructor that contribute to the use of mathematics in communicating economics concepts in the principles of economics course? In addition, how does the environment of the instructor relate to his/her choice concerning the degree of a mathematical treatment of economics? That is, does the actual decision come from the instructor's preferences or must the instructor restrict the use of mathematics because of the student's mathematical limitations or some other environmental constraint?

The desire to employ a pedagogical process that may be effective in communicating economics principles is assumed to be important to economics instructors. Economists are

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[3]The fact that the principles of economics course can be taught in either one or two semesters is recognized and will be a part of the analysis of this survey. However to avoid making this distinction every time the principles of economics course/s is/are referenced, the singular is used.

interested in utility and in the most efficient use of scarce resources. If, for example, mathematics can reduce the time required in teaching economics principles, it might be to the instructor's advantage to use mathematics whenever its use would make the instructional process more efficient. Hence, the purpose of this study is to provide an understanding of the extent to which mathematics is currently being used in teaching the principles of economics course and to get some sense of how personal and environmental influences contribute to the use or non-use of mathematics in economics instruction at the principles level.

This study thus analyzes two significant areas that potentially contribute to the use of mathematics. First, it attempts to discriminate according to the individual instructor's attributes, such as: whether or not the instructor has his/her doctorate in economics, when the degree was granted, whether the instructor has published in a refereed economics journal, whether the instructor feels that mathematics is an effective economics communication vehicle, whether the instructor prefers the encompassing or comprehensive survey approach or the more abbreviated topical approach in developing the economics course, and the instructor's view of the function of the principles course (*e.g.*, whether or not it is used to eliminate potentially unsuccessful students from the economics major). Second, this study seeks to discriminate according to significant environmental factors, such as: whether the principles course is taught within a school of business/economics, whether the college is accredited by the American Assembly of Collegiate Schools of Business (AACSB), whether the course is taught at the sophomore level, whether the course has a mathematics prerequisite, whether the school is publicly funded, and whether the instructor feels constrained by the mathematical sophistication attained by the students. The environmental consequences of the institution's average ACT score is also examined.

Approach

The study begins by defining the dependent variable. This variable is the percentage of the student's grade in the principles of economics class that is derived from his/her ability to use mathematics in solving economics problems. Such a variable is more controllable than some less quantifiable value such as "the percentage of time in which mathematics is used in the overall course, including the instructor's use of mathematics in illustrating and explaining economic concepts and theory."[4] The study then attempts to determine the impact of the two classes of

[4]Such a secondary dependent variable is also available. A question was presented in the survey (described later) to determine the percentage of lecture time in which mathematics was used in teaching economics concepts and theory. The purpose of this question was to serve as a validation check for the primary dependent variable and to also provide additional summary information.

independent variables on this dependent variable. As noted, the first class of independent variables subsumes the personal characteristics and disposition of the instructor, while the second class covers the institutional constraints upon the instructor.

In anticipation of this study, a pilot study was performed in 1991. The purpose of that study was to analyze the extent of the use of what was defined as "quantitative methods" by a group of colleges, of which Bryan College, the college at which the author teaches, is a member. This group, the Christian College Coalition, is composed of some 80 colleges in the United States and Canada. It was selected to provide an understanding of some of the characteristics of the schools with which Bryan College competes and to provide insight into the relative effectiveness of the rationale of the survey process and the survey instrument (the questionnaire) itself. In other words, it was hoped that candidate variables for the present study would emerge.

The questionnaire used in the pilot study incorporated eighteen questions (see Appendix A). The focus of the questions was suggested by a survey of literature on the pedagogy of economics which seemed to potentially contribute to the use of mathematics in the instruction of the principles of economics course. Several questions were included to provide descriptive summary data as well.

The pilot study questionnaires were mailed, without specific names, to the "Heads of the Business/Economics Departments" at each of the coalition colleges, and were accompanied by an introductory letter (see Appendix B) that explained the purpose of the survey and specified the time within which the responses should be returned. Seventy useable questionnaires were returned. This resulted in an institutional response rate of 67.5 percent and a large enough sample to permit the use of an ordinary least squares (OLS) regression with sixteen independent variables.

However, an analysis of the completed questionnaires revealed several weaknesses in the survey instrument. Distinct problems arose from preparing the questionnaire for an audience that was too broad and included state institutions. For example, the first question asked if the principles course was taught in a "School of Business." The intent was to distinguish the size of the business program by distinguishing institutions with a school of business from those with merely a business department or where economics was being taught under the social science division. However, since most institutions were very small, the respondents frequently interpreted the question to refer simply to a department of business. Since this was not the original intent of the question, and because there was an interpretive problem with the question, it was not used in the analysis. The fourth question also posed a problem. It asked if the institution was accredited by the AACSB. Since the population of choice consisted of small, private schools, the question was universally answered negatively and did not contribute to the regression. Hence it was not used. The same was true for several other questions including the question asking if the school was publicly or privately funded--all of the institutions in the population were privately funded.

A regression was performed with the twelve remaining independent variables, and the results are reported in Table 4 in Appendix C. A survey of these data suggests that in the pilot study, several predictors contributed very little. Perhaps data gathered from a more broad spectrum of institutions, as in this study, will provide a sufficiently different sample and will result in a greater statistical significance than was observed for some of the weaker coefficients in the pilot study. Since that model provided only modest results (*e.g.*, an R^2 of 0.177, an adjusted R^2 of 0.004, and an F-value of 1.02), several variables (questions) are modified and others added to the present model to improve the results. What emerges for the current study are nine personal characteristics variables and ten environmental variables.

Hypothesis

Nine ***personal characteristics*** are included in this study and they are hypothesized to individually and collectively contribute to the use of mathematics in the teaching of the principles of economics. In other words, they will significantly explain the extent that the instructor weighs a student's facility with mathematics in solving economics problems in grade determination.

1. The number of the mathematical approaches to problem-solving, commonly associated with the principles course, which are currently used by the professor (MTHPROBS). Choices of mathematical approaches on the survey are somewhat arbitrary. Nevertheless, a large number of mathematical approaches used by an instructor in teaching a principles course should indicate a propensity toward a mathematical approach. As a result, a positive association is expected.

2. The percentage of the instructor's lecture time in which he/she uses a mathematical approach in teaching economics concepts or theory (PCTLECT). The use of a mathematical approach in teaching the principles course should serve as an effective predictor of the mathematical requirements of the student. Therefore, a positive relationship is assumed.

3. Whether the instructor has won an award for excellence in teaching economics (EXAWRD). The degree of mathematics used by an instructor who has been determined to be an excellent teacher either by his/her students or by an outside organization may provide significant insight into

the relative effectiveness of mathematics in economic instruction. Use of mathematics reflects a progressive attitude toward instruction and increases the likelihood of receiving such recognition. A positive relationship is thus hypothesized.

4. Whether the instructor has published an article on economics in a refereed economics journal (PUBJNL). It is assumed that the majority of individuals who publish in such journals are comfortable with the mathematics of economics. Therefore, it would seem reasonable that such individuals would hold that economics is effectively communicated through mathematics. A positive association is expected.

5. When the instructor received his/her doctorate (YRDCTR). Since economics as a discipline is being increasingly mathematically formalized, it is projected that the more recent the degree, the more mathematical the instructor's economics education will have been, and the more mathematical the instructor is expected to be. Hence a positive relationship is assumed.

6. Whether the instructor adheres to the short-list approach to the content in teaching the principles course (SHTLST). "The long-list" and the "short-list" approach reflects the debate that exists as to the best way to teach the principles class. Should the instructor use a long-list approach and try to expose the student to many or most of the topics in the more popular, encyclopedic principles

textbooks or use the short-list approach and spend more time on selected, more important topics so that the student learns each more fully? Using the short-list approach, an instructor would have more time to develop each concept mathematically and provide time for the students to acquire greater proficiency with each mathematical concept. Therefore a positive relationship is expected.

7. <u>Whether the instructor feels that the principles of economics course is a tool for the elimination of potentially unsuccessful students from the economics or business major (ELMSTU)</u>. If an instructor has this attitude about the course, it seems reasonable that his/her use of mathematics would be more extensive and rigorous to test the student's ability to handle the quantitative side of the discipline. What the instructors' attitudes will be is uncertain, but a positive relationship is expected.

8. <u>Whether the instructor feels that mathematics is an effective economics communications vehicle (MTHCOM)</u>. Since economists are assumed to be maximizers, it is expected that if they perceive mathematics to be an effective communications tool, they will utilize mathematics in the communication of economics principles. Whether instructors perceive mathematics to be an efficient tool of economics instruction is uncertain. Therefore the relationship will be determined by empirical analysis.

9. <u>Whether the instructor has recently increased, or is planning to increase the mathematical demands of the principles course (CHGMTH)</u>. If the instructor is increasing the use of mathematics, it stands to reason that he is doing so because of either a perceived need to do so or because he/she is being coerced to do so. In either case, the mathematical demands before and after the adjustment will probably be low. As a result, a negative relationship is expected.

Ten ***environmental or institutional characteristics*** are proposed for the study and they are also hypothesized to individually and collectively contribute significantly to the understanding of the use of mathematics in the teaching of the principles of economics. In other words, they will significantly explain the extent the instructor weighs a student's facility with mathematics in solving economic problems in grade determination.

1. <u>Whether the principles of economics course is taught from a Business/Economics sponsorship (SCHBUS)</u>. When economics is taught from a business/economics department, it is assumed that there would be more emphasis on problem solving, and it would be more quantitative in approach than when taught from another area such as social science. Hence a positive relationship is expected.

2. <u>Whether the duration of the course is two-semesters (DURATN)</u>. When economics is taught in a

two-semester module, the institution is more committed to economic understanding. With this approach, it seems that there would be more time for a mathematical development and, if mathematics is perceived to be an effective method of economics instruction, a more mathematical approach would be used. Therefore a positive relationship is anticipated.

3. Whether the principles course is addressed to students in their sophomore year or higher (UPRLVL). Some institutions orient the principles of economics course to the students' freshman year. When this is done, it would be expected that such a course would be less rigorous, and therefore less mathematically demanding. Hence a positive relationship is assumed.

4. Whether the program is accredited by the AACSB (AACSB). If the program is accredited by the AACSB, a more quantitative approach to the major is mandated. As a result, it is reasonable to assume that the principles course would be taught in a more mathematical manner. Hence a positive relationship is expected.

5. Whether mathematics prerequisites are specified for the principles course (PRERQS). While many institutions have found that mathematical prerequisites for lower level courses tend to discourage students, it is expected that those institutions that do require mathematics prerequisites would utilize more mathematics in the principles course. Therefore a positive relationship is anticipated.

6. Whether the institution is publicly funded (PUBFND). Privately funded institutions, as opposed to publicly funded institutions, may be more concerned with the retention of students and particularly with the success of those students who are related to supporters of the institution. Such an orientation may increase the probability that privately funded institutions would be relatively less mathematical. Thus a positive relationship is assumed.

7. The number of students enrolled in the business/economics program/major (NMSTUS). Larger programs would have more economics options and majors. Therefore it is assumed that in such institutions, the instructors would feel more compelled to ensure that students in the principles course learn more economics, which would argue for a greater use of mathematics. Hence a positive relationship is expected.

8. Whether the microeconomics portion is taught first (MICFST). Much has recently been written on the greater success rate of students who begin their economics course of study with microeconomics. It is expected then, that those institutions that structure the principles course accordingly would also be aware of the arguments for the use of mathematics in teaching economics. Therefore a positive relationship is thought to exist.

9. Whether the instructor feels constrained to use less mathematics because of the lack of mathematical

sophistication of the students attracted to his/her institution (CSTMTH). Frequently, instructors, who would prefer to utilize a mathematical presentation or method of teaching, feel forced into using less mathematics because many of their students are not well equipped in the area of quantitative skills. So to avoid frustration and student dissatisfaction (e.g., bad student evaluations), the instructor uses a less quantitative approach. A negative relationship is thus expected.

10. Whether the average SAT mathematics score of the institution contributes (SAT_M). If the institution attracts students with unusually high average SAT_M scores, it is reasonable to assume that the principles course would be taught in a more mathematical manner. Hence a positive relationship is expected.

Both individually and collectively these characteristics of the instructors and the institutional environment are hypothesized to contribute significantly to an understanding of the degree that mathematics is used in teaching the principles of economics course. They will significantly explain the extent that the instructor considers the student's ability in math to solve economic problems in assigning grades (PCTMATH). A linear ordinary least squares regression (multiple regression) is used to estimate the relative importance of the characteristics. The following functional equation illustrates the anticipated sign for each factor:

$$\begin{aligned}\text{PCTMATH} = f(&\overset{+}{\text{MTHPROBS}},\ \overset{+}{\text{PCTLECT}},\ \overset{+}{\text{EXAWRD}},\ \overset{+}{\text{PUBJNL}},\ \overset{+}{\text{YRDCTR}},\\ &\overset{+}{\text{SHTLST}},\ \overset{+}{\text{ELMSTU}},\ \overset{?}{\text{MTHCOM}},\ \overset{-}{\text{CHGMTH}},\ \overset{+}{\text{SCHBUS}},\ \overset{+}{\text{DURATN}},\ \overset{+}{\text{UPRLVL}},\\ &\overset{+}{\text{AACSB}},\ \overset{+}{\text{PRERQS}},\ \overset{+}{\text{PUBFND}},\ \overset{+}{\text{NMSTUS}},\ \overset{+}{\text{MICFST}},\ \overset{-}{\text{CSTMTH}},\ \overset{+}{\text{SAT}_M}).\end{aligned}$$

Many of the independent variables are specified as binomial variables and are structured on the questionnaire in such a manner. Other questions are binomial, but are structured on the questionnaire to provide a range of values, because data on the range of opinion are of significance.

A question is also placed in the questionnaire with a list of mathematical economics processes. From the list each respondent is asked to check those that he/she is currently using. This arrangement has a dual function. First, it serves as a check on the dependent variable, in the sense that a more mathematical approach should also be characterized by the use of a greater number of the more difficult mathematical processes. Hence it is also used as an independent variable. Second, this arrangement serves to illustrate to the respondent what is meant by the term "mathematics" in the questionnaire (e.g., such things as those specifically mentioned). The question reads as follows: "From the following list of mathematical approaches to economics problem solving that are commonly associated with the principles of economics course at different levels, check the ones that you currently use." It is then followed by a list of mathematical economics processes in random order. An added benefit of this and other questions in the survey is that the data provided can be tabulated and presented separately to provide a summary statement of the current state of economics instruction on the principles level in the United States.

The last question on the questionnaire is included to determine if the respondents feel that the questionnaire is biased. Obviously if the instrument appears biased, it will influence the results. This question reads "Do you feel that this questionnaire is biased? (If so, please indicate how on the back, thanks.)" Responses to this question should provide important information on the validity of the instrument.

Definition of Terms

In this study, several terms are used in the abstract. Those more frequently used are defined here.

Axiom. "A statement that needs no proof because its truth is obvious."[5]

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[5]Webster's New World Dictionary, College ed. (1966), *s.v.* "axiom."

Axiomatic Formulation (Axiomatization Process). In economics, the reduction of economic theory to a set of assumptions that give formal mathematical expression to the theory. Morgenstern states that

> The axiomatic method consists of formulating a set of propositions which must fulfill certain conditions. They must, in particular, be free of contradictions, and the deductions derived from them must contain our knowledge of the field and, beyond this, hopefully, lead to new insights.[6]

Such a set of axioms are self-evident and together, are cumulatively and logically compelling.

Econometrics. The application of statistical methods to economics. Ramanathan states that

> . . . it is concerned with (1) estimating economic relationships, (2) confronting economic theory with facts and testing hypotheses involving economic behavior, and (3) forecasting the behavior of economic variables.[7]

Formalization (Mathematical Formalization). The use of mathematics in economics to state, test, and prove economic theory. It results in the axiomatic formulation of economic theory. Katzner defines *formalism* in the following statement.

> To economists, a formalist analysis consists of identifying (usually quantifiable) variables, assuming the existence of relations among them, assuming that the relations exhibit certain properties or characteristics, and working out at length the implications of all of these assumptions. Such analyses may also concern themselves with maximization, and with solutions and proofs

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[6]Morgenstern, "Mathematics in Economics," 23.

[7]Ramu Ramanathan, Introductory Econometrics with Applications, 2d ed., (New York: The Dryden Press, 1992), 3.

of existence of solutions for some system of (simultaneous) relations.[8]

(General) Equilibrium. "A state of balance or equality between opposing forces."[9] This concept reveals the heritage economics owes to the physics model. In economics, *equilibrium* refers to a distribution of goods to consumers, and a set of price ratios, in which demand equals supply in all markets at those prices[10] and where there is no net tendency to change.

Macroeconomics (Macroeconomic Theory). That aspect of economics that addresses aggregate variables such as gross domestic product, government spending, unemployment, inflation, the levels of taxation, the balance of payments, and economic growth and development.[11]

Marginalist Revolution. The transformation of conventional economics during the last half of the nineteenth century resulting in a theory that differed sharply from classical economics. Spiegel states:

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[8]Donald W. Katzner, "The Role of Formalism in Economic Thought, with Illustration Drawn from the Analysis of Social Interaction in the Firm," in The Reconstruction of Economic Theory, ed. Philip Mirowski (Boston: Kluwer-Nijhoff Publishing, 1986), 137.

[9]Webster's, *s.v.* "equilibrium."

[10]Brian R. Binger and Elizabeth Hoffman, Microeconomics with Calculus (Glenview, IL: Scott, Foresman and Company, 1988), 181.

[11]Ralph T. Byrns and Gerald W. Stone, Economics, 4h ed. (Glenview, IL: Scott, Foresman and Company, 1989), 822.

> The labor theory of value was shed, and with the help of a new unifying principle there was accomplished the integration of the theories of the consumer and of the firm, as well as the integration of the theories of value and distribution. . . . The unifying principle, which was now at hand, was the marginal one. . . . it pointed the way toward the establishment of theoretical optimum positions, or equilibria, at which consumers and producers would maximize such magnitudes as satisfaction or net revenue.[12]

The new theory permitted mathematical modes of thought and marked the beginning of modern (neoclassical) economic analysis.

Mathematization (of Economics) (or Mathematical Economics). [Used somewhat synonymously with *formalism*, above.] The incorporation of a mathematical mode of thinking in economic theory. Gerard Debreu, in The New Palgrave: A Dictionary of Economics, states that

> . . . as a formal model of an economy acquires a mathematical life of its own, it becomes the object of an inexorable process in which rigor, generality[,] and simplicity are relentlessly pursued.[13]

When it is properly used, according to Morgenstern, it

> . . . yields the same insights which can be obtained from common sense. . . [but thereafter gives] results which go far beyond common sense, results which common sense could never even guess at. . . .[It is] a higher state in the development of a science . . . [which is] only attainable by means of mathematics.[14]

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[12]Henry William Spiegel, The Growth of Economic Thought, 3d ed. (Durham, NC: Duke University Press, 1991), 505.

[13]Eatwell, John, Murray Milgate, and Peter Newman, eds. The New Palgrave: A Dictionary of Economics (New York: The Stockton Press, 1987), *s.v.* "Mathematical Economics," by Gerard Debreu.

[14]Morgenstern, "Mathematics in Economics," 22-23.

Methodological Monism. An approach in which only one common methodology is applied to both natural science (as in physics) and social sciences (as in economics). It is antithetical to "methodological dualism," which holds that the social sciences cannot utilize the same methodology as the natural sciences.[15]

Microeconomics (Microeconomic Theory). That aspect of economics that addresses both individual economic units such as consumers, firms, and governments, and individual decision making.[16]

Neo-Austrian School. The return by economists such as Ludwig von Mises to Austrian economics, in repudiation of the Keynesian revolution and the mathematical revolution (neoclassicalism). It espouses a free market, individualism in economics and politics, *laissez faire*, and the rejection of public policies and socialism.[17]

Neoclassical School. A school of economists, in the marginalist revolution, which successfully "synthesized the classical analysis of production and supply with a marginal utility-based theory of demand."[18] By appropriating a

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[15]Mark Blaug, The Methodology of Economics or How Economists Explain (Cambridge: Cambridge University Press, 1990), 46-47.

[16]Byrns and Stone, Economics, 823.

[17]Spiegel, Economic Thought, 670.

[18]Lila J. Truett and Dale B. Truett, Economics (St. Louis: Times Mirror/Mosby College Publishing, 1987), 519.

mathematical equilibrium model for the economy, the school discovered how to analyze the economy as a simultaneous general equilibrium of all the labor, land, and product markets.[19] This is the basis for modern, mainstream economic theory.

New Classical School. "Modern theories that extend classical theories of competitive markets, normally resulting in underpinnings for *laissez faire* macroeconomic policies."[20] This school applies microeconomic analysis to all economic problems because it believes that economics is the study of rational behavior. The school only accepts equilibrium models, in which each economic agent seeks to maximize its choices, and attempts to establish all macroeconomic theories on microeconomic foundations.[21]

New Keynesian School. A school whose common feature is the existence of disequilibrium in macroeconomic markets. This school seeks to place the older Keynesian theory on a firm microeconomic foundation and feels that prices do not adjust instantaneously in labor, product, or credit markets. As a result, realized output and employment are the outcome

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[19]Paul A. Samuelson and William D. Nordhaus, Economics, 14h ed. (New York: McGraw-Hill, Inc., 1992), 378.

[20]Byrns and Stone, Economics, 825.

[21]Joachim Zeitz, Unpublished class notes given in Advanced Macroeconomics at Middle Tennessee State University during the Fall 1991 semester, 108.

of agents optimizing their positions subject to the constraints that are external to each economic agent.[22]

Pedagogy. An educational term that refers to "the art or science of teaching; especially, instruction in teaching methods."[23]

Theoretical Core (Hard Core). A technical term in the methodology of Imre Lakatos, with which he describes the purely metaphysical beliefs that unite the adherents of a school of thought (referred to as a *scientific research program*). It is treated as irrefutable by its protagonists. The hard core is surrounded by the *protective belt* of testable hypotheses or theories.[24]

Constraints in the Study

The study is limited to four-year colleges and universities within the continental United States. On the one hand, various types of institutions, including private, religious, and state institutions are included in the survey. However, junior colleges, community colleges, and vocational schools are specifically excluded, because they are a much more heterogeneous group whose members do not offer senior-level degrees.

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[22]Zeitz, Class Notes, 123.

[23]Webster's, *s.v.* "pedagogy."

[24]Blaug, Methodology, 36-37.

The study also is performed at one specific time, that is, during the Fall semester of the 1992-1993 academic year. As such, it provides cross-sectional data, or a measurement of the opinion of economics educators at a moment in time. The study intends to capture and describe the attitudes currently held by these educators in various colleges and universities in the United States. The scope is more limited than a longitudinal study, which would describe the changes in opinion over some period of time. Such a study, however, is beyond the scope of this project, but it could provide a significant area for further research in the future.

Another characteristic of this project is that it focuses on the foundational level of economics education--the principles of economics course, and not on other courses in economics theory. The reason for this focus is that the principles course has a great impact on college instruction. Many college students with different majors will study economics on the principles level without pursuing economics further. For these students, the principles course provides a significant level of economics education that will prepare them to serve as informed citizens. On the other hand, various social science, business, and economics majors will continue in their economics education. For these students the principles course equips them for additional economics coursework. Since this course has such a vital role, it is the subject of this study. A consideration of the use of mathematics in teaching other economics courses, therefore, is beyond the scope of this research but it would perhaps form the basis for additional research in the future.

As indicated previously, the precise definition of the term "mathematics" as used here is crucial. It is limited to the mathematics that is relevant to a principles level course and yet includes a broad range of processes from the simplest mathematical calculations, as in the computation of average and marginal revenues, to a more advanced level of mathematics that uses calculus, for example, to find marginals and maximizations. Also, since this study differentiates between a mathematical and a graphical approach to the presentation of economics, graphical manipulation is specifically excluded.

Procedures for Collecting and Treating the Data

In order to collect the data, a one page questionnaire is prepared, which obtains the instructors' characteristics and institutional factors that are hypothesized to contribute to the use of mathematics in the principles of economics course. A copy of the questionnaire is found in Appendix D. This questionnaire is mailed to a random sample of 500 institutions throughout the United states. Because of the problem imposed by job mobility, each survey is mailed to the economics/business department head rather than to individual instructors. As in the pilot study, several copies of the survey accompany an introductory letter (see Appendix E), which requests that the department head have "a faculty member, or preferably two," who currently teach the principles course complete the survey. To encourage participation, an offer is made to share the findings of the survey with those respondents that request it. Two directories, The College Blue Book, published by Macmillan Publishing Company and The College Handbook, published by the College Entrance Examination Board, are used to select the survey institutions.

Each questionnaire is numbered and keyed to a master file, so that upon return, the responding institutions may be identified. Once they are identified, the average SAT_M sub-score for each institution is determined for analysis. The data are then entered into a spreadsheet program for summary analysis. The file that is created is then loaded into an econometrics program for statistical study.

Organization of the Study

Chapter one describes the purpose and significance of the study. It provides information about a pilot study that was performed, along with the hypothesis, definitions, constraints, and procedures for collecting and treating the data.

Chapter two reviews relevant literature on the historical mathematization and formalization of economics as a science and summarizes the current debate on the validity of that formalization.

Chapter three reviews relevant literature pertaining to the use of mathematics in the teaching of economics.

Chapter four presents the methodology of the study and applies the statistical techniques which are used in the evaluation and analysis of the data.

Chapter five presents the summary, conclusions, and implications of the study.

CHAPTER II

THE MATHEMATICAL FORMALISM OF ECONOMICS

Introduction

A review of the literature on the mathematical development of economics provides perspective on the use of mathematics in teaching economics. If mainstream economics has developed as a rhetorical or verbally based science, so that the institutions, theories, and deductive processes essential to its practice can be fully utilized without resorting to mathematics, then its use would be purely pedagogical and somewhat suspect. If, on the other hand, it has developed into a mathematical science, the use of mathematics in teaching economics would be necessary. The overwhelming evidence points in the latter direction.

Evidence of the Mathematization of Economics

The fact that economics has been quantified is hardly a matter for debate. Regularly, the critics and opponents of increasing mathematization deliver papers and produce articles expressing their frustration over the deluge of equations and advanced mathematics appearing in economic journals and the increasing degree of quantification both in the economics profession and in higher economics education. Yet the mathematization of economics seems to be firmly entrenched, as judged by the outward symbols of success.

These symbols include the number of articles in the journals, the volume of quantitative scholarly contributions, the numbers of courses offered, and student attendance in such courses.

Mathematization in the Journals

Even a casual observation of economics journals reveals that a much greater emphasis on mathematization or economic quantification exists now than earlier in the century. At least two studies have sought to describe this phenomenon. The first was that of Bennett and Johnson, which counted the occurrence of pages with equations, tables, and graphs in nine economics journals.[25] They found that from a base year, which ranged from 1934 to 1968, for six of the nine journals the percent of pages with equations increased by more than one percentage point a year. The results for graphs and tables were more diverse; however, they had also increased in rigor, but at a slower rate.[26]

The other study was published by Grubel and Boland, in which they formally analyzed the American Economic Review.[27] The authors examined all the pages in each issue from 1950 through 1983 and found that the proportion of pages that contain one, two, three, or more equations has risen

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[25]Bennett and Johnson, "Mathematics and Quantification," 40.

[26]Ibid., 42.

[27]Grubel and Boland, "Efficient Use," 425.

continuously and at a sharp rate throughout the period. Specifically, they found that whereas in 1951 only 2.2 percent of the pages contained at least one equation, by 1978 the proportion had risen to 44 percent.[28] In an analysis of other journals, they simplified their evaluation procedure by defining articles as consisting of pure mathematical models if they contained several equations but no empirical data. They concluded that the older, more established journals had increased the space devoted to mathematical articles continuously during the postwar years to the extent that approximately half of all journal space was devoted to pure mathematical models. Grubel and Boland also found that total journal space devoted to this purpose had increased further because of the publication of several new journals devoted exclusively to mathematical articles.[29]

Mathematization of the Profession

Virtually every area of modern microeconomic or macroeconomic theory has been touched by mathematization. Mathematical and econometric tools have been successfully incorporated into the microeconomic subfields of labor, public finance, and antitrust and government regulation, as well as into macroeconomic model building and forecasting of

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[28] Ibid.

[29] Ibid., 429.

national income and employment.[30] As a result, Francis observed that a greater number of economists are complaining about what is perceived to be an "overemphasis" on mathematics, especially in economics instruction in the nation's graduate schools.[31] A similar concern surfaced in a survey of professional economists by Grubel and Boland, in which a quarter of the respondents felt that there was too much emphasis on mathematics even in graduate economics education.[32] An interesting correlation that the study discovered was that dissatisfaction with the increasing mathematization of economics was an increasing function of age.[33] This can most probably be explained as the result of a lack of mathematical training of older economists. The study also found that "distinguished" economists (*i.e.*, Nobel laureates, *American Economics Association* executive officers, and those in Who Is Who in Economics) were more negative on the use of mathematics than other respondents.[34] This concern of many economists dramatically highlights the

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[30]Robert B. Ekelund, Jr. and Robert F. Hébert, A History of Economic Theory and Method, 3rd ed., (New York: McGraw-Hill Publishing Company, 1990), 604.

[31]David R. Francis, "Are Math Formulas Washing Common Sense Out of Economic Theory?" The Christian Science Monitor, 2 April 1985, 35.

[32]Grubel and Boland, "Efficient Use," 433.

[33]Ibid., 438.

[34]Ibid.

increased mathematization that represents a marked change in the direction of professional economics today.

Students, on the other hand, do not seem to be intimidated by the new mathematized economics. A recent study described economics as a popular major, representing about two percent of majors nationally.[35] Increasingly, economics Ph.D.'s are earned in more quantitative areas. Many recent graduates are receiving their degrees in mathematical subfields such as econometrics.[36] The majority of university professors seem to be forming a positive consensus about mathematical economics. Through their survey, Grubel and Boland found that, while professors teaching at the leading universities value the use of mathematics more highly than those at less well-known schools,[37] approximately 88 percent of the total number of professionals surveyed felt that the resources devoted to teaching mathematical economics to university undergraduates ranged from too little to adequate, and 71 percent held a similar consensus about the resources devoted to teaching mathematical economics to graduate students.[38]

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[35]John J. Siegfried *et al.*, "The Status and Prospects of the Economics Major," JEE 22 (Summer 1991): 198.

[36]Francis, "Math Formulas," 35.

[37]Grubel and Boland, "Efficient Use," 439.

[38]Ibid., 434.

Mathematical Formalization

The economics profession has changed. It is much more quantitative today than it has been in the past. However, there is a difference between quantification and mathematical formalization. Much of economic reasoning is mathematical because it deals with quantities, costs, and profits. Many problems may be solved in economics using basic mathematics, such as algebra or graphical depiction. This is the generally accepted, practical side of the science.

Mathematical formalization, however, is quite different. According to Mirowski, it is the "imposition of logical rigor upon the loose and imprecise common discussion of economic phenomena."[39] Mathematical formalization is not simply the use of mathematics to solve problems of quantity and cost, it is, rather, the use of mathematics to state, test, and prove economic theory. Such formalization culminates in the axiomatic formulation of a discipline's contents. Many economic theorists feel that the discipline of economics cannot become truly scientific until its theory has been "forged in the logic of mathematics" and verified as reliable.[40]

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[39]Philip Mirowski, ed., "Mathematical Formalism and Economic Explanation," in The Reconstruction of Economic Theory (Kluwer-Nijhoff, 1986), 181-182.

[40]Ekelund and Hébert, History, 605.

Much of the mathematical writing that has appeared in the economics journals involves this latter type of mathematical application. The process utilizes the more recently developed tools and concepts such as matrix algebra, set theory, difference equations, stochastic processes, and the axiomatic method,[41] in addition to calculus in which problems are cast in the now familiar constrained maximization/minimization framework. Through the use of these tools, economic theory is being formalized. However, this process has resulted in making much theory inaccessible to many economists with only a moderate ability in mathematics.[42]

The Development of the Mathematization of Economics

The Origins of the Mathematical Method

From the earliest attempts at economic analysis, economists have looked to mathematics to display and illustrate their ideas. For example, Aristotle relied on mathematics (e.g., mean and proportion) to discuss his conception of exchange, and Sir William Petty formulated "Political Arithmetick" to explain an early national income accounting system.[43] The use of mathematics has had an

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[41]Tjalling C. Koopmans, "On the Use of Mathematics in Economics," RES 34 (November 1954): 377.

[42]Ibid., 215.

[43]Ekelund and Hébert, History, 583, note 2.

intuitive appeal since economics deals with quantities and prices. In addition, the deductive reasoning used by Adam Smith, David Ricardo and other early pioneers of the science actually encouraged later economists to more fully use mathematics.

The formalization of economics began later with the development of mathematical economics and the marginal utility revolution. Mirowski has produced a significant analysis of the origins of mathematical economics and observes that economic histories such as Blaug's have denied that there was any unified, self-conscious movement in this direction.[44] For instance, concerning the development of the marginal revolution, Blaug wrote:

> . . . to try to explain the origin of the marginal utility revolution in the 1870's is doomed to failure: it was not a marginal *utility* revolution; it was not an abrupt change, but only a gradual transformation in which the old ideas were never definitively rejected; and it did not happen in the 1870's.[45]

Instead, the marginalists describe a gradual accretion of economic writings which had as their only common denominators the dual concepts of diminishing marginal utility and utility-determined prices. Since neither notion was unique in the 1870's, it follows that there was no real discontinuity of thought during the period, so that

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[44]Philip Mirowski, Against Mechanism: Why Economics Needs Protection from Science, (Totawa, NJ: Rowman and Littlefield, 1988), 11.

[45]Mark Blaug, Economic Theory in Retrospect, 3d ed. (Cambridge: Cambridge University Press, 1978), 322.

economics gradually and consistently developed or evolved during the period.

Mirowski finds three problems with this concept of the continuity of economic development during the period. First, not all of the economists in the marginalist revolution would have agreed with such a view. He states:

> One cannot read the letters and published works of Stanley Jevons, Léon Walras, Francis Edgeworth, Irving Fisher, Vilfredo Pareto, and others without repeatedly encountering assertions that their work represented a fundamental break with the economics of their time.[46]

Second, the most disruptive aspect of the marginalist revolution was not the concept of the utilitarian theory of value, it was rather the use of mathematics to derive theory. Mirowski notes that these economists recognized each other primarily as *mathematical theorists* and spent much of their time defending the mathematical method.[47] In his History of Economic Analysis, Schumpeter provides a description of the essence of this method.

> Numerical or algebraic formulations and numerical calculations had occurred of course in the earlier stages of economic analysis . . . but the use of figures . . . or of formulae . . . or even the restatement in algebraic form of some result of non-mathematical reasoning does not constitute mathematical economics: a distinctive element enters only when the reasoning itself that produces the result is explicitly mathematical.[48]

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[46]Mirowski, Against Mechanism, 12.

[47]Ibid.

[48]Joseph A. Schumpeter, History of Economic Analysis (New York: Oxford University Press, 1954), 954-5.

Augustin Cournot understood the significance of his use of mathematics for theory and made it clear in his Mathematical Principles:

> I have said that most authors who have devoted themselves to political economy seem also to have had a wrong idea of the nature of the applications of mathematical analysis to the theory of wealth. They imagined that the use of symbols and formulas could only lead to numerical calculations, and as it was clearly perceived that the subject was not suited to such a numerical determination of values by means of a theory alone, the conclusion was drawn that the mathematical apparatus, if not liable to lead to erroneous results, was at least idle and pedantic. But those skilled in mathematical analysis know that *its object is not simply to calculate numbers, but that it is also employed to find the relations between magnitudes which cannot be expressed in numbers and between functions whose law is not capable of algebraic expression* (italics added).[49]

The third problem with the continuity concept involves the question of the scientific nature of economics. The major developers of this new economic theory were careful to "differentiate their handiwork from previous political economy on the explicit ground that it was of a scientific character."[50] This desire to make economics "scientific," especially by W. S. Jevons and those who followed, may be explained by the parallel developments in physics in the mid-nineteenth century.

The Physical Science Model and the Use of Calculus

During the 1870's and 1880's, the new economics of the

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[49]Augustin Cournot, Researches into the Mathematical Principles of the Theory of Wealth, trans. Nathaniel. T. Bacon (New York: Augustus. M. Kelley, 1960), 2-3.

[50]Mirowski, Against Mechanism, 12.

marginal revolution experienced a distinctive break with the economic tradition of the day because it consciously incorporated the metaphor and mathematics of physical equilibrium into its theoretical core. The new theory that developed, formed the basis of what is today regarded as Neoclassical theory.

The evidence that such a deliberate infusion took place is found in the published works of the first neoclassicals and in a description of the physics of the time which shows the parallels.

Evidence from published works. It is interesting to find that all of the major contributors to the marginalist revolution stated in their published works that the metaphor which they used in their theories was found in the physical sciences. William Stanley Jevons excelled both as a theorist and as a mathematician and as such, was, more than Léon Walras or Carl Menger, the premiere pioneer of modern (Neoclassical) economics. Jevons did not hesitate to describe the source of the metaphor he used. Mirowski states the following:

> Jevons wrote that his equation of exchange does ". . . not differ in general character from those which are really treated in many branches of physical science." He then proceeds to compare the equality of the ratios of marginal utility of two goods and their inverted trading ratio to the law of the lever, where in equilibrium the point masses at each end are inversely

> proportional to the ratio of their respective distances from the fulcrum.[51]

This application of the metaphor of the physical sciences is not at all surprising when one realizes that Jevons wrote extensively on the logic and philosophy of science. In his work The Principles of Science, Jevons suggests that the notion of a hierarchy in the sciences justifies a mathematical approach throughout. He states "If we are to apply scientific method to morals, we must have a calculus of moral effects, a kind of physical astronomy investigating the mutual perturbations of individuals."[32] Concerning this comment, Mirowski states that in Jevon's thought,

> The reduction of social processes to simple utilitarian considerations is compared to the reduction of meteorology to chemistry and thence to physics, implying that there is only one scientific methodology and one mode of explanation--that of physics--in all human experience.[33]

As a result, it is clear that Jevons held to some type of methodological monism for all science, and incorporated the physics metaphor in his scientific theory of economics.

Spiegel observed that the theoretical contributions that Léon Walras made went well beyond those of Jevons and Menger. He pushed to develop the concept of general

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[51]Ibid., 13.

[32]W. S. Jevons, The Principles of Science: A Treatise on Logic and Scientific Method. (New York: Dover Publications, Inc., 1958), 760.

[33]Mirowski, Against Mechanism, 14.

equilibrium for economics and its expression in the form of a series of simultaneous equations.[34] He was just as specific as to his motives in his published writings in economics. In his work, The Elements of Pure Economics, he stated that "this pure theory of economics is a science which resembles the physico-mathematical sciences in every respect."[35] To Walras, a pure science was only concerned with the relationships between blind forces. According to Mirowski, "Such 'pure' relationships . . . *demand* the application of the *same* mathematical techniques as those deployed in mid-nineteenth century physics . . . ."[36]

This methodological monism that required the same technique in both physics and economics is described in Walras's article, "*Économique et Mécanique*." Its significance is summarized by Mirowski:

> In this article he develops the two favorite metaphors of neoclassical economists, the rational mechanics of the equilibrium of the lever and the mathematical relations between celestial bodies; he also asserts that the "physico-mathematical science" of his Elements uses *precisely* the identical mathematical formulae.[37]

Another prominent proponent of marginalism and "mathematical physics" is Vilfredo Pareto. He used a more

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[34]Henry William Spiegel, The Growth of Economic Thought, 3d ed., (Durham: Duke University Press, 1991), 549.

[35]Léon Walras, Elements of Pure Economics, trans. William Jaffé (New York: Augustus M. Kelley, 1969), 71.

[36]Mirowski, Against Mechanism, 14.

[37]Ibid.

belligerent approach but held an essentially identical position:

> Strange disputes about predestination, about the efficacy of grace, etc., and in our day incoherent ramblings on solidarity show that men have not freed themselves from these daydreams which people have gotten rid of in the physical sciences . . . . Thanks to the use of mathematics, this entire theory, as we develop it in the Appendix, rests on no more than a fact of combinations between which the individual is indifferent. The theory of economic science thus acquires the rigor of rational mechanics.[38]

Pareto's methodological monism is hinted at in Schumpeter's History:

> Pareto . . . was trained (and throughout his prime practiced) as an engineer. This means more than that he had a good training in mathematics. His powerful mind roamed far beyond the precincts of applied science into the realm of the pure concepts that are perfectly general: few people can ever have realized with such intensity as did he that, ultimately, *all exact sciences or parts of sciences are fundamentally one* (italics added).[39]

Because of his obvious proclivity toward what Schumpeter called "intense realizations," Pareto was one who stated the physical metaphor more aggressively and openly than his predecessors. Thus, he was the first of the neoclassicals to be placed in a position in which he had to defend himself from attacks by both mathematicians and physicists.[40]

These are by no means the only statements concerning

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[38]Vilfredo Pareto, Manual of Political Economy, trans. Ann S. Schwier, and ed. Ann S. Schwier and Alfred Page (New York: Augustus. M. Kelley, Publishers, 1971 [1906]), 36, 113.

[39]Schumpeter, History, 859.

[40]Mirowski, Against Mechanism, 14.

the approach adopted by the early neoclassicals. Such passages are also found in the writings of such neoclassicals as Fisher, Antonelli, Laundhardt, and Auspitz. Mirowski concludes:

> In fact, the explicit appropriation of this specific physical metaphor is present in every major innovator of the marginalist revolution, with the single exception of the Austrian school of Carl Menger. The adoption of the "energetics" metaphor and framework of mid-nineteenth century physics is the birthmark of neoclassical economics, the Ariadne's thread which ties the protagonists, and which can lead us to the fundamental meaning of the neoclassical research program.[41]

Evidence of Mid-Nineteenth Century Physics Parallels. Chroniclers of economic thought have long been aware of the close resemblances between mid-nineteenth century physics and neoclassical economic theory.[42] It has been suggested, however, that the reason that these observations have not been noticed is that the significance of the relationship has not been presented from the perspective of physics.[43]

It is possible that the development of mid-nineteenth century physics prepared the way for neoclassical economic

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[41]Ibid., 15-16.

[42]See for example Gregor Sebba, "The Development of the Concepts of Mechanism and Model in Physical and Economic Thought," AER, Papers and Proceedings 43 (May 1953), 259-271; A. Lowe, "On the Mechanistic Approach in Economics," Social Research 18 (1951): 403-434; Frank Knight, "Statics and Dynamics" in On the History and Methodology of Economics (Chicago: University of Chicago Press, 1956); and H. Thoben, "Mechanistic and Organistic Analogies in Economics Reconsidered," Kyklos 35 (1982): 292-305.

[43]Mirowski, Against Mechanism, 16.

theory. The physics of this period had changed the subject matter, techniques, and methodological prescriptions from the earlier Newtonian approach. The development included the Newtonian calculus of the rate of continuous change in variable quantities and the English fondness for geometrical argument, but it also included changes in the acceptable standards of theory formulation, including: an increasing refusal to specify the underlying nature of the phenomena being described mathematically and a tendency to accept the "usefulness" of a model as a form of proof. There was also a subtle transformation of vague "forces" into undefined "energy" (energetics), which could only be discussed clearly through the intermediary of mathematics.[44] According to Mirowski,

> The rise of energetics in physical theory induced the invention of neoclassical economic theory, by providing the metaphor, the mathematical techniques, and the new attitudes toward theory construction. Neoclassical economic theory was appropriated wholesale from mid-nineteenth century physics; utility was redefined so as to be identical with energy.[45]

The parallel is identified by Tjalling Koopmans in his Three Essays on the State of Economic Science, in which he comments: "A utility function of a consumer looks quite similar to a potential function in the theory of

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[44]Ibid., 17.

[45]Ibid.

gravitational or electrical fields."[46] However, it was not until the work of Nicholas Georgescu-Roegen that this link was explored.[47] Mirowski has been insistent upon laying open the parallels in order to reinvestigate the appropriateness of the use of the metaphor. In his work, The Reconstruction of Economic Theory, he specifies the relationship between mid-nineteenth century physics and economics.

To illustrate the parallel, the physics model must first be specified. Because of the technical nature of the subject matter, the following extended citation was provided to describe the relevant portion of the mid-nineteenth century physics model:

> Suppose we are describing a mass-point moving in a three dimensional Euclidean space from point A to point B (by a force vector *F*). The conventional method of describing this motion, developed in the early- to mid-nineteenth century, would postulate a 'force' decomposed into its orthogonal components, multiplied through by the spatial displacement of the mass-point, also suitably decomposed. In order to incorporate cases of non-linear displacement and acceleration, the 'work' done in the course of the motion from A to B was defined as the summation of the infinitesimal forces multiplied times their displacements:
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> $$T = \int_A^B (F_x dx + F_y dy + F_z dz) = (\tfrac{1}{2})mv^2 \Big|_A^B$$

[46] Tjalling Koopmans, Three Essays on the State of Economic Science (New York: McGraw-Hill, 1957), 176.

[47] Note his two works: Nicholas Georgescu-Roegen, The Entropy Law and the Economic Process (Cambridge: Harvard University Press, 1971) and Energy and Economic Myths (Elmsford, N.Y.: Pergamon Press, 1976).

> The writings of Lagrange, and more importantly of Hamilton, argued that the total energy of this system also depended in a critical way upon the *position* of the mass-point. This was subsequently clarified in the following manner: suppose that the expression $(F_x dx + F_y dy + F_z dz)$ was an exact differential, which implies that there exists a function $U(x,y,z)$ such that
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> $$F_x = (\delta U/\delta x); \quad F_y = (\delta U/\delta y); \quad F_z = (\delta U/\delta z)$$
>
> The function $U(x,y,z)$ defines a gravitational field, which later was identified as 'potential energy'. The sum of potential and kinetic energies, $T + U$, was then understood as being conserved within the confines of a closed system. This conservation law, in turn, clarified and encouraged the employment of constrained maximization techniques (such as the Principle of Least Action, Lagrangean multipliers, and the Hamiltonian calculus of variations) in the description of the path of the mass-point under the influence of the impressed forces.[48]

The similarity of the calculus in this model and in neoclassical economics is clear if "forces" is redefined as prices and "displacements" as infinitesimal changes in the quantities of individual goods x,y,z. The integral, $F * dq = T$, is defined as total expenditure.. If the function is integrated as an exact differential, then again the scalar function of goods can be defined as $U = U(x,y,z)$, which is interpreted as the "utilities" of the goods. When this has been done, the constrained maximization (or minimization) of a quantity in this model fixes the permissible configuration of prices. Thus, in exact parallel to the original concept of potential energy, the calculus and form may be shown to be the same in both instances.[49]

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[48]Mirowski, Reconstruction, 195-186.

[49]Ibid., 187; and Mirowski, Against Mechanism, 18-19.

It is interesting to note that the early neoclassicals openly admitted the similarities between physics and economics. Walras, for example, insisted that his *rarité* (scarcity) equations were the same as those of the physical sciences in every regard.[50] A few years later, Irving Fisher described the parallels in a table in which he demonstrated the correspondences between physics and economics labels for variables in the same mathematical model.[51]

Once this source is acknowledged, the necessity of mathematics for neoclassical economic theory is clear. Mirowski, indeed, thinks that the mathematics of the physics model is the distinctive part of the "hard core" of the neoclassical research program.[52] If this is the case, it would explain why neoclassical economic theory has been so inextricably united with mathematical formalism since its inception in the 1870's.

Directions and Dangers of Mathematical Formalism. One area that developed quickly was a method to quantify and measure empirical data in a way that contributes to neoclassical mathematical formalism. The almost separate field in which this has developed is that of econometrics, involving

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[50]Ibid.

[51]Irving Fisher, Mathematical Investigations in the Theory of Value and Prices (New Haven, CT: Yale University Press, 1926), 85-86.

[52]Mirowski, Against Mechanism, 24.

the use of statistics and regression analysis. Its origin can be traced to 1933. In that year, a group of economists founded the Econometric Society and began the publication of Econometrica. The society and its journal was established to develop the empirical side of neoclassical economics and over the years it has pursued the goal of developing the theory and method of testing economic ideas.[53]

Formalist mathematical and econometric tools have effectively spread through the microeconomic subfields of labor, public finance, and antitrust and government regulation, to mention only a few. The same is true on the macroeconomics side. Model building and the forecasting of national income and employment would be impossible without these tools. As a result, courses in calculus and statistics form the basis of most graduate curricula in economics at major and minor universities around the world.

Assuming the mathematical formalization of the neoclassical program is legitimate, then, according to the historians, Ekelund and Hébert,

> A meaningful evaluation must rest on the costs and benefits, the advantages and deficiencies, of this development as it relates to some concept of *progress* in economics. The chief argument for continued formalization (i.e., mathematization) of economics is that the discipline cannot become truly scientific until it attains the rigor and completeness of science--in other words, until its fundamental propositions have been tested and proved. . . . *Facts without theory forged in*

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[53]Ekelund and Hébert, History, 604.

the logic of mathematics are meaningless (italics added).[54]

The Current Debate on the Validity of Mathematical Formalization

Opposition to the Mathematatization of Economics

The mathematical formalization of economics, as previously noted, is not the use of mathematics to solve economic problems, but to state, test, and prove economic theory. Opposition to the mathematization of economics has existed from the beginning. As early as 1803, the efforts of Nicolas François Canard to quantify market relationships with algebra was criticized by the Edinburgh Review. Attempts such as this, the article stated, ". . . ought most severely to be censured [for] divert[ing] an instrument [algebra] from its proper use, and [for] attempt[ing] to remove those landmarks by which sciences are bounded from each other."[55]

Criticism on the grounds of methodology. Later in the nineteenth century, John Elliot Cairnes, in an attempt to defend the classical system, provided a rational statement as to why he was opposed to the mathematization of economics:

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[54]Ekelund and Hébert, History, 605.

[55]Cited by Leonid Hurwicz, "Mathematics in Economics: Language and Instrument," in Mathematics and the Social Sciences, ed. James C. Charlesworth (Philadelphia: The American Academy of Political and Social Science, 1963), 2.

> So far as I can see, economic truths are not discoverable through the instrumentality of Mathematics. If this view be unsound, there is at hand an easy means of refutation--the production of an economic truth, not before known, which has been thus arrived at; but I am not aware that up to the present [1875] any such evidence has been furnished. . . .
>
> Unless it can be shown either that mental feelings admit of being expressed in precise quantitative forms, or, on the other hand, that economic phenomena do not depend on mental feelings, I am unable to see how this conclusion can be avoided.[56]

Several critics of economic formalism utilize similar arguments today. Economists like the neo-Austrians, contend that the subjective nature of social science, and hence economics, makes any type of exact or quantifiable measurement and testing impossible. Part of the reason for this involves the insufficiencies in the data being sampled and in the random errors inherent in the measurement of variables. Ekelund and Hébert provide insight into this "quality of data" problem with the formalization program:

> Modern economic techniques are, generally speaking, most appropriate when data samples are large; yet in many instances, large-sample data do not exist. Thus, the quantities and *qualities* of economic data are often insufficient to the task. In contrast to conditions in the physical and natural sciences, the collection of most economic data is not predetermined or predesigned to fit tests of economic theory. Indeed, most economic data are collected by government agencies for far less specific purposes, often for purely political reasons. While inadequate theory and poor data are not sufficient reasons in themselves to reject quantitative methods, some critics argue that the design costs and the collection costs necessary to secure high-quality data are prohibitive.[57]

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[56]Cited by Hurwicz, "Mathematics," 3.

[57]Ekelund and Hébert, History, 605.

Concerning the issue of quality of data, Wassily Leontief argues that, over time, as economists pursued more elaborate quantification, they were forced to turn to "less accessible, more specialized information" in the form of government statistics.[58] The problem is that such information was designed for administrative purposes and not for scientific usage. As a result, government statistics are not compiled with the harsh rigor necessary to make them the accurate figures necessary for economic quantification.

Leontief also suggests that economists have a penchant for sloppy deductive reasoning, with the result that they tend to avoid systematic empirical inquiry. He thinks that this may be observed:

> . . . in the devices they employ to avoid or cut short the use of concrete factual information. Instead of constructing theoretical models capable of preserving the identity of hundreds, even thousands, of variables needed for the concrete description of a modern economy, they first of all resort to "aggregation."[59]

The schools of economic thought that most vocally oppose the mathematical formalization of economics are the neo-Austrian and the institutional schools. They argue that the attempt to make economics a science through mathematical formalization and empirical testing is futile. According to Ekelund and Hébert,

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[58]Wassily Leontief, "Academic Economics," Science 217 (July 9, 1982): 104.

[59]Ibid.

> In the opinion of these critics, the fruits of decades-long intellectual investment in mathematical and statistical techniques have been small, if not negative. . . . these futile attempts at rendering economics scientific has *[sic]* engendered wide-spread distrust of the economic pronouncements of policymakers and an almost total breakdown in communication between economists and other social scientists. Even worse, mathematics and calculation in the hands of those equipped with tools but no ideas can lead economists away from the basic truths about markets and market functioning.[60]

More recently, the formalization of economics has been criticized by historical economists, on the basis of its origin. Mirowski, has led the attack with several major works.[61] His main line of reasoning, as has already been noted, involves the fact that the derivation of the constrained maximization model is a direct appropriation of the gravitational theory in physics.[62] He concludes that the adoption of this model forced the originators to make subtle changes in economic theory to make the theory fit the model. Referring to his book More Heat than Light, Mirowski states:

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[60]Ekelund and Hébert, History, 605.

[61]Three of the first works by Philip Mirowski are "Macroeconomic Instability and Natural Fluctuations" JEH 44 (1984): 345-354; "Physics and the Marginalist Revolution" Cambridge Journal of Economics 8 (1984): 361-379; and "The Role of Conservation Principles in 20th Century Economic Theory" Philosophy of the Social Sciences 14 (1984): 461-473. A chapter in a work edited by Mirowski was "Mathematical Formalism and Economic Explanation" in The Reconstruction of Economic Theory, ed. Philip Mirowski, (Kluwer and Nijhoff, 1986). He then authored and published two major works: Against Mechanism; and More Heat than Light: Economics as Social Physics, Physics as Nature's Economics (Cambridge: Cambridge University Press, 1989).

[62]Mirowski, "Mathematical Formalism," 185.

> . . . the fact of the appropriation of mathematical techniques from the physics of the nineteenth century has clearly influenced both the content and the mode of research in economics. These clinging associations are the residue of the projection of a metaphor from physics onto the sphere of the economy, where aspects of economic experience were then subject to reinterpretation. An enumeration of the myriad ways in which the mathematical model smuggled a hidden agenda into political economy would be a very substantial undertaking in the intellectual history of the discipline.[63]

In a later work, Mirowski goes even further and questions the validity of continued formalization using a model of physical theory that has already been discarded in the study of physical science.[64]

Negative Practical Effects

Many economists critical of economic formalization feel that an emphasis on quantification results in several negative, practical effects. It is argued that one possible negative effect is that an emphasis on mathematics can result in individuals being trained in the use of tools but with limited ideas, leading them away from the basic truths about markets and the economic process.[65] While not arguing against the use of mathematics, Siegfried *et al.*, suggests that an overemphasis on mathematics in economics

> . . . tempts instructors to emphasize algebraic manipulation at the expense of intuitive explanations of economic behavior. It is relatively easy to teach

[63] Ibid., 188.

[64] Mirowski, Against Mechanism, 24.

[65] Ekelund and Hébert, History, 605.

> formal tools and techniques, but it is difficult to provide students with the capacity to use these tools.[66]

The tendency clearly exists, on the part of some instructors, to emphasize formalism too early in the economic curriculum. Debrew cautions against what he refers to as the "seductiveness" of mathematical tools by which researchers "may be tempted to forget economic content and to shun economic problems that are not readily amenable to mathematization."[67] On the undergraduate level, Siegfried *et al.*, assert that a preoccupation with formalism, rather than a focus on rigorous analysis of economic issues, is one of the contemporary practices that works against the effectiveness of intermediate theory courses.[68]

Evidence exists that employment opportunities for the highly mathematically trained economist is limited. In summarizing the results of a recent survey, Grubel and Boland suggest that while

> . . . non-mathematical economists have opportunities for employment in private industry, the media[,] and policy oriented government agencies[,] . . . mathematical economists have practically no other employment opportunities outside of universities and a very small number of specialized institutes.[69]

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[66]Siegfried *et al.*, "Status and Prospects," 204.

[67]Eatwell, Milgate, and Newman, eds. The New Palgrave: A Dictionary of Economics, *s.v.* "Mathematical Economics," by Gerard Debreu.

[68]Siegfried *et al.*, "Status and Prospects," 204.

[69]Grubel and Boland, "Efficient Use," 422.

A majority of the respondents to their survey on the mathematization of economics agreed with their assessment. Around 60 percent felt that mathematical economics leaves students inadequately prepared for non-academic work.[70] It is also possible that such training leaves graduate students poorly equipped to serve in academic settings other than research universities. Francis reported that liberal arts colleges are experiencing difficulty in getting young economics faculty who are broad based and trained in policy issues, rather than in narrowly focused mathematical economics.[71]

Finally, there is the danger that mathematical formalism, which results in highly technical analysis, can be self-perpetuating. Those who have large human-capital investments in mathematical and econometric techniques would have a strong incentive to perpetuate the *mystery of the discipline* even if the process did little to achieve progress. The formalization of economics can lead to the erection of significant barriers to entry--whether it is in graduate curriculum requirements, university hiring standards, journal editorial policies, or professional recognition--to all but those disciplined in the mathematical techniques.[72]

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[70] Ibid., 438.

[71] Francis, "Math Formulas," 35.

[72] Ibid., 606.

What is more damaging is that this ongoing process tends to further divide those who regard economics as a significant, though less-quantifiable, social science and those who regard it almost as a branch of applied mathematics. It is interesting to observe that the formalization process can also serve to place *the science of economics* beyond the grasp of the uninitiated. Isaac Newton admitted that he recast his *Principia* into its mathematical format making it "designedly . . . abstruse" in order "to avoid being bated by little Smatterers in Mathematicks . . . ."[73] Since such a sociological role admittedly exists for the mathematization of a discipline, it has been suggested that the formalization of economics is a tool in the creation of both a well-behaved audience and "an orthodoxy which automatically serves to exclude dissension."[74] Obviously, if years are invested in mastering the necessary mathematics and learning how to translate it into the symbols of economic orthodoxy, this would discourage the skeptical and reinforce the self-esteem of the willing student.

The Defense of the Mathematization of Economics

The mathematization of economics clearly began at the hands of the neoclassical school. The resulting

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[73]Cited in R. S. Westfall, Never at Rest: A Biography of Isaac Newton (Cambridge: Cambridge University Press, 1980), 459.

[74]Mirowski, Reconstruction, 192.

quantitatively based theory is that which now constitutes the core of undergraduate and graduate economics education in the United States. However, it is not the only school of economics to seek quantification. The new classical economists are dedicated to the practice of theoretical rigor. Most of their arguments are highly formal and their writings are, therefore, very mathematical.[75] In addition, the new Keynesian school has succeeded in making a greater use of mathematics.[76] The implication here is that the use of mathematics is not the exclusive tool of any particular school of economic thought, but is rather a new way of thinking, developing, and expressing economics, started by the early neoclassicals.

Legitimization of the Discipline. The primary argument in favor of mathematical formalization is that the discipline cannot become truly scientific until it has attained the rigor and completeness of science. This process involves the axiomatic formulation of its theories, so that they can be tested, and demonstrated to be valid, logically. Morgenstern described this process, focusing on the expected practical result in an article written in 1963:

> The new theory also shows clearly a feature that must be expected in any true mathematization; the mathematical theory must first yield the same insights which

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[75]"Schools Brief: What it all adds up to," The Economist 293 (December 15, 1984): 72.

[76]Ibid.

> can be obtained from common sense. Thereafter it will give results which go far beyond common sense, results which common sense could never even guess at [*sic*]. When this happens, some of the new results may be translatable into ordinary language, but still for others, this will become impossible. They will remain in mathematical symbolism. When that point has been passed, a higher state in the development of a science has been reached. It is only attainable by means of mathematics.[77]

The axiomatization process, as described by Morgenstern, involves the formulation of a set of propositions which have to fulfill specific conditions. For example, they cannot contain contradictions and the deductions derived from them should clearly demonstrate the knowledge of the discipline. In addition, axioms, themselves, do not contain any truth superior to that of the implications of the deductions upon which they are based. They are simply used for convenience, because they are intuitively clear and even aesthetically appealing. They are not, however, proven, self-evident truths as the old view of axioms asserted; such a level cannot be attained in any science.[78] The axiomatic method is a method of summarizing knowledge in a field of study and of discovering additional knowledge deductively. Frequently this process involves logical-mathematical operations that are extremely complex and which take time to develop. However, when a satisfactory state of

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[77]Morgenstern, "Mathematics in Economics," 22-23.

[78]Robert Dorfman, "A Catechism: Mathematics in Social Science" RES 34 (November 1954): 375.

axiomatization of a particular discipline has been achieved, as in physics, mathematics is indispensable.[79]

Expansion of Economic Understanding. The value of the formalization of any discipline lies not only in the fact that it establishes the field scientifically, but that it also results in increased understanding. Interestingly, there are different levels of understanding that are affected. Klein argues that formalization provides a replicative understanding:

> Much of mathematical economics deals with the formulation of, and the rediscovery of solutions to, problems that are already familiar parts of literary economics. Is this wasted effort? I answer no, because our subject deals with inherently difficult problems which should be considered from many different points of view in order to gain new insights.[80]

The process of replicating economic theory into mathematics is complicated, but the effort often results in new insights. For example, the work of Pareto, Slutsky, Hicks, and Allen, in quantifying utility theory, provided tremendous insight into its essential elements, by forcing it to be analyzed mathematically and therefore logically. Indeed, were this mathematization process not accomplished, the concept of the Slutsky equation (which uses calculus to derive the change in demand due to the change in the rate of

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[79]Morgenstern, "Uses of Mathematics," 24.

[80]L. R. Klein, "The Contributions of Mathematics in Economics," RES 34 (November 1954): 360.

exchange between two goods), referred to as the "fundamental equation of value theory," would not have been developed.[81]

A substantial contribution of mathematical formalization is the resolution of theoretical problems and the expansion of economic knowledge. This is accomplished as theory is specified mathematically and as problems which surface in the process are resolved. For example, in the 1870's Walras sought to formalize the law of supply and demand in the physico/competitive-equilibrium model. In the process, there were questions that he posed that just could not be answered at the time. It was not until the necessary mathematical tools, such as fixed-point theorems and Liapunov's stability-analysis methods, were developed, that the questions could be answered. Through the work of Wald in the 1930's followed by that of Arrow, Debrew, and McKenzie, new information is now available as to the conditions under which competitive-equilibrium is or is not attainable.[82] Clearly, a major purpose of mathematization, then, is in the solution of critical theoretical problems in economics.

A related contribution of the formalization process is the condensation and clarification that it provides to economic thought. Fisher described this contribution almost poetically in 1892:

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[81] Ibid.

[82] Hurwicz, "Mathematics in Economics," 8-9.

> The effort of the economist is to see, to picture the interplay of economic elements. The more clearly cut these elements appear in his vision, the better; the more elements he can grasp and hold in mind at once, the better. The economic world is a misty region. The first explorers used unaided vision. Mathematics is a lantern by which what before was dimly visible now looms up in firm, bold outlines. The old phantasmagoria disappear. We see better. We also see further.[83]

Klein explains that literary contributions to economic theory tend to be expansive and imprecise. When such theory is formalized into mathematical logic, wordy volumes and lengthy manuscripts are frequently compressed into a few understandable pages. Hence, clarity of thought tends to characterize mathematical economic theory.[84] Grubel and Boland feel that this attribute of formalization contributes to the logical rigor of the discipline.[85]

Augmentation of Instructional Effectiveness. Mathematics in economics provides the additional advantage of contributing to the preparation and training of economists. Grubel and Boland admit that the mathematization of economics

> . . . has increased the efficiency of the education process and served as an effective screening since mathematical abilities appear to be useful in the study of economics.[86]

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[83]Fisher, Mathematical Investigations, 119.

[84]Klein, "Contributions of Mathematics," 360.

[85]Grubel and Boland, "Efficient Use," 420.

[86]Ibid.

Hence, learning appears to be enhanced, because economic theory that has been mathematized tends to be clear and easy to explain, and learning is made more efficient, because it is now easier to screen students for success.

Since education theory stresses the importance of a student's active participation in facilitating his/her learning, quantitative involvement is yet another way that mathematization may improve student understanding. DePauw University is one institution that has sought to utilize student involvement to increase their learning. One area in which the school involves its students is that of quantitative reasoning. This engages students in an economics course that stresses the development and application of quantitative reasoning skills to develop competence in "independent reasoning, problem solving, interpretation, and critical assessment involving quantitative models and information."[87] In this program, learning is facilitated because mathematical problems are easy to structure and students actively generate ideas and solutions to economic theory.

Perspective on the Debate

As a result of an increasing awareness of the difficulties and potential problems of the mathematization of economics, there have been requests for a reduction of

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[87]Field, Wachter, and Catanese, "Alternative Ways," 214.

mathematical literature.[88] Possibly, in response, there is some evidence of a slowdown in the production of such material. Grubel and Boland found a reversal in the growth of quantitative articles in several journals beginning around 1982.[89]

Criticism of the mathematization of economics does have its purpose. As more thought is put into a consideration of the process and purpose of mathematization, an awareness of the strengths and weaknesses of the current state of the science becomes apparent. In addition, the danger of an overly technical focus on the science as a substitute for the study of economic reasoning is observed. Ekelund and Hébert summarize the implications of this fact:

> . . . Some skepticism is undoubtedly healthy, and yet the complete and substantial abandonment of mathematical formalization would be a bigger mistake than its uncritical acceptance. . . . An appreciation of the limits to mathematics and econometric technique fosters an understanding of their correct and useful place in economic science.[90]

Obviously, the mathematization of economics is here to stay. However, the economist feels that the debate as to the appropriateness of economic mathematization will

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[88]The most recent articles are the following: Grubel and Boland, "Efficient Use;" and Gary S. Becker, "Competition Among Pressure Groups for Political Influence" QJE 98 (1983): 371-400.

[89]Grubel and Boland, "Efficient Use," 425-432. *Cf.* also Debreu, "Mathematical Economics," in The New Palgrave, Figure 1, p. 401.

[90]Ekelund and Hébert, History, 606.

ultimately be settled using market forces.' Solow, in an article in 1954, referred to the law of the *Invisible Hand* which would finally resolve such differences. He asks:

> . . . Why is economic theory becoming more and not less mathematical? Why do so many graduate students want to include a little mathematics in their education? Why do editors of journals find themselves receiving a growing number of papers couched at least partly in symbolic terms? Why is the line between mathematical economists and general economic theorists growing fuzzier and vaguer? . . . I suspect it is because a large and growing faction of what is interesting and valuable in the economic theory of the last twenty years . . . has been produced by theorists who do at least some of their theorizing in mathematics. Survival in the literature is a test of fitness, if an imperfect one. If mathematical techniques continue to produce good economics then, . . . I predict that . . . most people interested in economic theory will as a matter of course learn some mathematics.[91]

If this were the situation nearly four decades ago, how much more is it today? No tool is perfect, nor will it always be perfectly applied. Yet mathematical economics has clearly demonstrated its usefulness. The problem facing today's economists is not to determine whether or not economics should be quantified--that has already been determined. The problem today is whether or not it is possible to capture "the benefits of mathematical formalization without making economics moribund or irrelevant--the ultimate potential cost."[92]

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[91]Robert Solow, "The Survival of Mathematical Economics," <u>RES</u> 34 (November 1954): 373-374.

[92]Ekelund and Hébert, <u>History</u>, 606.

Conclusion

In this chapter, the fact that mainstream economics has been firmly ensconced upon a mathematical formalism has been established. First, the studies noting the growth of mathematics in the economic literature concluded that contemporary economic discussions have a far greater emphasis on mathematics than ever before. Second, a survey of the development of neoclassical economic theory notes that the reason for the extent of today's mathematization is that mainstream, neoclassical theory was developed using the mathematics of the mid-nineteenth century physics model. In the words of Debreu, "Economic theory is fated for a long mathematical future."[93]

While it is entirely proper to appropriate and use a model from another science to explain similar data in a separate field, some historical economists have questioned whether this adaptation for economics was entirely justifiable. Assuming, however, that the mathematics of neoclassical economic theory is valid and appropriate, the next question to be addressed is whether a mathematical approach to teaching assists in the pedagogy of economics. In the following chapter, literature is surveyed that relates to the effectiveness of mathematics in teaching an increasingly mathematical economics.

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[93]Eatwell, Milgate, and Newman, eds. The New Palgrave: A Dictionary of Economics, *s.v.* "Mathematical Economics," by Gerard Debreu.

CHAPTER III

THE PEDAGOGICAL USE OF MATHEMATICS IN TEACHING THE PRINCIPLES OF ECONOMICS

Introduction

A survey of the literature on the use of mathematics in teaching economics reveals little that is directly related to the effectiveness that a mathematical approach offers pedagogy. However, much has been written that offers indirect help. In order to understand whether and how a mathematical approach may be useful in pedagogy, first it is helpful to understand the requirements of mathematics in economics education. If mathematics is an essential element in economics in general, and in the principles course in particular, the issue is moot. If, however, mathematics is an optional method of expressing truth, the issue of its effectiveness in pedagogy becomes significant. To assist in this analysis, a consideration of examples of mathematics used in teaching economics is provided.

A second contribution to understanding the significance of using mathematics in teaching is to determine if learning theory recognizes the value of a mathematical approach. A role for mathematics in pedagogy could be ascertained if it can be demonstrated that there is a place for mathematics either in the theory of learning or in the factors that govern the learning process.

A third area which offers perspective is a discussion of the abilities and current limitations of students in mathematical understanding. If students find mathematics difficult to understand because of poorly developed skills, any benefit from a mathematical exposition might be lost.

This chapter therefore begins with an examination of relevant literature in the first area, namely the mathematical requirements in an economics curriculum. The chapter continues with a review of research concerning pedagogical theory relating to the effectiveness of the use of mathematics. Concluding the chapter is a more abbreviated survey of literature concerning the mathematical limitations of contemporary students.

The Requirements of Mathematics in Economics Curriculum

The principles of economics course is the foundation of most students' economics education. Today, the principles course is required for many students who are majoring in a wide variety of disciplines and fields in addition to economics. Enrollment in the principles of economics course typically exceeds one million undergraduate students a year,[94] making it among the highest of any

[94]M. Jane Barr Sweeney, *et al.*, "The Structure of the Introductory Economics Course in United States Colleges," JEE 14 (Fall 1983): 68.

discipline in higher education.[95] One aspect of the importance of this course is that since only a small fraction of these undergraduates typically continue with their economics education,[96] the course is the only significant exposure to the subject of economics that many of them will ever receive. Another factor that makes the principles course important is that the strength of the analytical foundation that the economics major acquires in the principles course plays a significant role in how well the student is able to perform in intermediate level economics courses.[97] As a result, it is critical that the principles of economics course be effectively taught.

Most principles of economics instructors would like their students to think clearly about economics problems and issues years after the course is completed. The typical response is for instructors to focus on analytical and mathematical problem solving ability in the principles course. Such a goal is stated by Bach:

> Students completing a good principles course should:
> [1] Exhibit an awareness of, and a continuing interest

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[95]Phillip Saunders and William Walstad, "Teaching the Principles of Economics: An Introduction and Summary," in The Principles of Economics Course: A Handbook for Instructors, eds. Phillip Saunders and William B. Walstad (New York: McGraw-Hill Publishing Company, 1990): 1.

[96]John J. Siegfried, *et al.*, "The Status and Prospects of the Economics Major," JEE 22 (Summer 1991): 198.

[97]Butler, *et al.*, "Is More Calculus Worth It?" Unpublished working paper, Vanderbilt University (October, 1992): 11.

> in, the major economic problems of modern society. [2] Demonstrate a firm grasp of the few basic principles and analytical concepts necessary to think intelligently about economic problems for themselves. . . . [3] Develop an independent ability to apply these analytical tools in thinking independently about economic problems. (This involves placing major stress on the *process of applying* economic concepts and principles in thinking about economic issues.)[98]

At times, when alluding to *analysis* and *analytical tools*, economists are referring simply to institutional concepts, verbal models, and chains of deductive reasoning. However, most frequently these terms relate to the use of mathematics and mathematical models in problem-solving. Siegfried, *et al.*, describe the quantitative aspect of this practice in a description of the skills that economics students need to acquire:

> . . . the economic approach emphasizes *decision-making* techniques, perspectives on how choices are made, and the consequences of these choices. This approach orients economists toward (1) the examination of *trade-offs*, a comparison of alternatives; (2) measuring the costs of one choice in terms of the foregone benefits of another, or *opportunity costs*; (3) formulations involving *constrained maximization* with carefully specified constraints (necessitated by scarcity); and (4) issues relating to *efficiency*, getting the most our of limited resources. Finally, while all economic problems involve normative issues, a strong bias exists toward an analytical approach that abstracts from and downplays "value" issues.[99]

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[98]G. L. Bach, "What Should A Principles Course in Economics Be?" in The Principles of Economics Course: A Handbook for Instructors, eds. Phillip Saunders and William B. Walstad (New York: McGraw-Hill Publishing Company, 1990), 39-40.

[99]Siegfried, *et al.*, "The Economics Major," 200.

Not every instructor would insist that these goals would be entirely appropriate for the principles course, but they do serve to illustrate the quantitative nature of economics in general.

The Mathematical Nature of the Economics Major

Economics is a rather large major that attracts over two percent of all undergraduates nationally. Approximately 32,000 undergraduate students graduate annually from over 900 colleges and universities with a major in economics. The number of students in the major has grown steadily since 1976 at an annual rate of approximately 3.5 percent, "reflecting a belief that majoring in economics will improve a graduate's job prospects."[100]

As to what happens to the economics major after graduation, a recent study by Siegfried, *et al.*, reports the following:

> Most economics majors plan to continue their education beyond the baccalaureate level, but fewer than half actually do. Of those who do continue their education, about half pursue a masters of business administration, and most of the rest enroll in law school; less than 3 percent enroll in economics Ph.D. programs. Those who enter the labor force directly after graduation go into a variety of occupations in a diverse set of industries, government agencies, and not-for-profit organizations. Few describe their employment as an "economist."[101]

In determining the purpose of the economics major, such information must be taken into consideration. If the

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[100] Ibid.

[101] Ibid.

major is to train "economists" on the undergraduate level, it would seem that such a purpose is out of focus, because few actually become economists. If the major is to educate students and enable them to "think like economists," that is, to develop the analytical, problem-solving skills, characteristic of economists, that can be utilized in a wide array of interests and professions, then one can readily see the importance of acquiring and utilizing mathematical skills. In their recent report on the economics major, Siegfried, *et al.*, argue for this purpose, stressing the quantitative nature of these skills:

> Understanding economic relationships is the central goal. This involves formulating hypotheses to explain these relationships, constructing models that capture their essential features, assembling empirical observations bearing on these relationships, and testing the hypotheses using *quantitative techniques*. Such testing not only increases the understanding of economic phenomena but also promotes ever more effective predictions of the consequences of changes in our evolving world. In essence, this is the form of scientific method that is used in many disciplines, but economists usually must conduct their hypotheses tests without the luxury of controlled experiments.(italics added)[102]

Several studies point to the increasingly quantitative nature of the economics major in the United States.[103] For example, as a result of a 1980 survey of economics

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[102]Ibid.

[103]See, for example, Donald P. Cole, "Economics and the Liberal Arts: A Comment," JEE 11 (Winter 1980): 49-50. John J. Siegfried and James T. Wilkinson, "The Economics Curriculum in the United States: 1980," AER 72 (May 1982): 127-135; Siegfried, *et al.*, "The Economics Major," 199-205; and J. S. Butler, *et al.*, "Is More Calculus Worth It?", 1-4.

curricula over the previous 25-year period, Cole provided the following generalizations:

> Curricular changes occurring over the 25-year sample period suggest two major developments: (1) a strengthening of theoretical and quantitative offerings and (2) increased coverage of certain subjects which in 1955 occupied a relatively less important position in the curricula of the survey schools. During the 1960's most survey schools augmented the theoretical component of general economics, primarily through the introduction of intermediate-level courses in macro- and microeconomic analysis. *Courses in quantitative methods (e.g., econometrics and mathematical economics) also took on relatively greater importance during the sample period; by 1980 this field ranked second in the curricula of the schools surveyed* (italics added).[104]

The conclusion to be drawn from such information is that the economics undergraduate major is becoming increasingly quantitative in nature, requiring greater mathematical sophistication from students today than from students in the past. This process was anticipated in the 1960's by Bear, who began asking questions about how this related to the principles course:

> . . . if my forecast is correct and mathematics, together with mathematical statistics, become interwoven in the fabric of upper-division economics, where does this leave the freshman and sophomore principles course? Can it continue to serve, as it does today, the dual purpose of introducing nonmajors to economic analysis and policy while at the same time preparing majors for their eventual field of concentration?[105]

Bear was concerned about the need for a principles course that would have sufficient mathematical depth to prepare the major for advanced undergraduate economics study. This led

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[104]Cole, "Economics and the Liberal Arts," 49.

[105]Bear, "Use of Mathematics," 151.

him to suggest an extremely mathematical approach to the principles course, which seems to have garnered few followers in the instructional arena. However, this line of reasoning raises questions about the principles course as it relates to the non-major.

The Increasing Mathematical Requirements for the Non-Major

To evaluate the usefulness of mathematics in teaching economics to the non-economics major, it will be helpful to first understand the function of the principles course to these students.

The so-called "citizenship argument" is that the course provides benefits to non-majors as future citizens. It increases their "sensitivity to the political, economic, and social system of which they are a part and increases the intelligence with which they participate in it."[106] McKenzie argues that to be effective, basic economics education must also win over the convictions of the student:

> . . . Finally, economic education must overcome the tendency of people, in spite of what they know about the economic merits of legislation, to vote their own private interests.[107]

Hence, the principles course for non-majors should be adequate to help students sufficiently understand economic

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[106]John J. Siegfried and Rendigs Fels, "Research on Teaching College Economics: A Survey," JEL 17 (September 1979): 934.

[107]Richard B. McKenzie, "Where Is the Economics in Economic Education?" JEE 8 (Fall 1977): 10.

issues so that they can make proper economic decisions.

This ability would be based on their understanding of economic arguments and analyses in the popular press (including newspapers such as the Wall Street Journal; periodicals like U. S. News and World Report, Forbes, Fortune, and The Economist; broadcasts such as "The Leherer Report"; and discussions of economic issues in the evening news). Such knowledge would clearly include a facility with minimal quantitative concepts such as indexes, the national accounts, and the interrelationships between supply and demand, prices, interest rates, quantity, unemployment, and inflation.

Bear argues that to differentiate too sharply between what the non-major and the major needs in the principles course, because of a perceived difficulty on the part of the non-major to understand the more quantitative aspects of economics would result in

> . . . a great deal of confusion between the content of the introductory course and the average quality of the student taking it. If the students are mediocre, so the argument goes, then the principles course must be "watered down" to take account of that fact. I agree, but it is the nature of that watering down that is crucial. A reduction in difficulty does *not* mean that the essential topics are deleted from the course, for to do so would result in serving neither the citizenship nor specialist functions.[108]

He then proposes that many poorer students would benefit from a mathematical exposition because it provides some

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[108]Bear, "Use of Mathematics," 151-152.

"advantages over the diagrammatic method."[109]

A further benefit of a good principles of economics course is that if it provides an accurate presentation of the subject matter, it might dissuade some from pursuing and persuade others to pursue the economics major. Hence, if sufficiently quantitative, the course might serve to eliminate from the major those students who would find the course of study discouraging later in the program. On the other hand, for those non-majors who are superior in mathematical ability, the principles course might stimulate interest and effectively recruit majors into the field.

More importantly, the principles course should accurately and adequately survey the field. McConnell, an author of one of the more successful principles textbook, specifies the close relationship between the quantification of the economics field with the increasingly quantitative nature of the principles course:

> The second trend is the evident dramatic increase in the scope and sophistication of economics in the past twenty-five or thirty years, and this has been reflected in the principles course. . . . The level of analysis has been upgraded as the quantitative tools of the profession have been sharpened and extended. In brief, both the scope and level of analysis of today's principles course is more complex than it was a generation ago.[110]

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[109] Ibid.

[110] Campbell R. McConnell, "Economics 101: Where Do We Stand?" JEE 11 (Winter 1980): 16.

However, as early as 1971, Mandelstamm alerted the economics community to the danger of incorporating too much material in the principles course. Especially problematic was the inclusion of what was perceived to be excessive higher mathematics:

> Most of us are simply giving too many "principles." Every year, more and more concepts, which previously had been reserved for the intermediate theory or even the advanced theory sequences are being taught in principles courses. . . . We find ourselves enmeshed by envelope curves and saddle points. We are caught up by accelerators and even LaGrange multipliers and set theory. Then we spend the rest of the undergraduate program, and a good deal of the graduate program, repeating the same material. The results are predictable. The poor students are hopelessly swamped in the principles course, and the best students are turned off in later courses when they find that they are getting very little new material.[111]

Thus, there is a danger to the over-mathematization of the principles course. Yet, from a consideration of the purpose and function of the principles course for the non-major, two conclusions emerge. First, the principles course does not have to be non-quantitative because of the limitations of the non-major to effectively serve its intended purpose. Second, the principles course must be quantitative, to some extent, to fully present the principles of an increasingly quantitative field of study.

Limits on the Mathematics that Are Necessary to the Economics Curriculum

As noted earlier, in the late 1960's, Bear advocated

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[111]Allan B. Mandelstamm, "The Principles Course Revisited," JEE 3 (Fall 1971): 43.

the necessity of using a more mathematical approach in teaching the principles course. In the first of two articles on the principles course, he argued that a mathematical treatment was more straight-forward:

> With a little care--primarily quick review of how to solve one linear equation in two or more variables for one of those variables in terms of the others--the mathematics of macroeconomics should not constitute an obstacle to mastery of the material. In fact, the algebra is so straightforward that, as an expository device, it puts the corresponding geometry to shame!
>
> . . . Of course this is not the case with microeconomics, where differential calculus is essential to a mathematical treatment of the utility-maximizing consumer and the profit-maximizing firm.[112]

Then, in the second article, he argued for the use of reduced form equations, calculus, and Cramer's Rule to determine the signs of coefficients, as well as statics and dynamics analysis:

> Of course, explicit mathematical treatment of the simple cobweb does require that the student learn how to solve a linear, first-order difference equation with constant coefficients. . . . I do not buy the argument that this result is "too advanced" for the "average" student . . . .[113]

But as Becker has pointed out, such a mathematical approach to teaching the principles course has failed to materialize in most colleges and universities:

> Books that have attempted to incorporate differential calculus in the main body of the text have failed or

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[112]Bear, "Use of Mathematics," 155.

[113]D. V. T. Bear, "More on the Use of Mathematics in the Teaching of the Principles Course, in <u>Recent Research in Economics Education</u>, ed. Keith Lumsdem (Englewood Cliffs, NJ: Prentice-Hall, Inc., 1970), 180.

> been revised, with calculus relegated to footnotes and appendices. . . .
>
> The lack of extensive mathematical treatments in principles texts today may reflect a resource misallocation arising from consumer sovereignty in education, as alleged by Bear twenty years ago. That is, a Gresham's law of textbooks may have caused the mathematical treatment to be driven from the market, as students and their professors select the cute, easy, verbal texts over the good, hard, mathematical ones. I doubt that this is the case, however.[114]

One of the possible reasons for the failure of such an approach is that while the study of economics does require some knowledge of mathematics and statistics, the effective instruction of the principles course can be achieved largely without an undue reliance on mathematics.

Becker argues from the fact that even professional economists prefer a less mathematical approach:

> That the American Economic Association has found it necessary to introduce two journals to convey and review ideas in a nontechnical manner suggests that its elected executive committee recognized that most of the AEA membership prefer to learn without the excess baggage of specialized mathematics. . . .
>
> How can an instructor of economics justify the use of a mathematical treatment to teach freshmen and sophomores when colleagues are relying on _The Journal of Economic Perspectives_ and _The Journal of Economic Literature_ to stay current in areas in which they may not specialize?[115]

If much in the economic principles course can be taught verbally, and even the professionals find a verbal presentation preferable, instructors need only utilize a

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[114]Becker, "Use of Mathematics," 180-181.

[115]_Ibid_., 181.

quantitative approach when it is absolutely necessary to clearly explain an economic concept.

Becker identifies three of the most basic mathematical concepts that are important to undergraduates in economics. They include functional notations, such as when consumption (c) is described as a function of income (y), and written as $c = f(y)$; simple linear algebraic relationships in the slope-intercept form, such as $c = a + by$; and regression equations in the form $c = \beta_0 + \beta_1 x + \epsilon$. Other needed mathematical tools are included in or based on these general forms.[116]

Interestingly, Becker presents these mathematical concepts as those necessary for complete undergraduate economic understanding. The amount of mathematics necessary to understand the principles of economics course could be appreciably less. How can one determine what is absolutely necessary on the principles level? Becker suggests that the answer can be found in the mathematics used in the common periodicals that economists and businessmen read:

> If articles in The Wall Street Journal, Business Week, Forbes, Barrons, and the like do not require readers to have specialized mathematical knowledge and skill, then students in an introductory economics course should not be expected to know or learn such things.
>
> Many of the articles in current periodicals that deal with economic issues and policy rely on a knowledge of percentages, indices, expected values, and distributions. Some articles require a knowledge of elasticities, standard deviations, functions, and even the marginal propensity to consume. An ability to read

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[116] Ibid., 182-187.

> charts and tables is taken for granted. Clearly these are quantitative concepts and skills students of economic principles should know on completion of their study.[117]

Thus, while a minimal amount of mathematics is necessary in teaching the principles course, a large body of optional mathematical approaches and illustrations exists for the instructor's pedagogical use.

Pedagogical Theory Relating to the Use of Mathematics

Pedagogy, the science of teaching methods, defines *learning* as the effects of formal instruction. Saunders offers the following definition:

> Learning is the acquisition and retention of knowledge and habits of thought in a way that permits them to be employed in a useful way after the initial exposure has been terminated.[118]

Morrill and Spees observe that everyone has the natural desire to learn. As people develop, they first learn *what*, then ask *how*. The earliest learning occurs on a personal, interactive basis. However "as education becomes more structured and more complex, students begin to *resist*--and appear to contradict their natural desire to learn."[119] As a

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[117] Ibid., 188.

[118] Phillip Saunders, "Learning Theory and Instructional Objectives," in The Principles of Economics Course: A Handbook for Instructors, eds. Phillip Saunders and William B. Walstad (New York: McGraw-Hill Publishing Company, 1990), 63.

[119] Paul H. Morrill and Emil R. Spees, The Academic Profession: Teaching in Higher Education (New York: Human Sciences Press, Inc., 1982), 49.

result, a philosophy of teaching-learning is necessary on the college level to overcome this tendency to resist.

Memory Development Theory

One of the earliest theories of learning was the *mental discipline/faculties development* theory. Those who held this view thought that there was developmental value in certain subjects. They proposed "that the mind and person was best developed by the study and discipline of ideas and abstract knowledge. . . , the liberal arts."[120] Bransford, *et al.*, provide specific examples of thought from this theory:

> Many have focused on the need to develop "mental discipline" by subjecting students to the rigors of learning difficult subjects such as mathematics and Latin. Mann cited Plato's arguments: "Arithmetic stirs up him who is by nature sleepy and dull, and makes him quick to learn, retentive, and shrewd. He makes progress quite beyond his natural powers." Similar ideas were espoused by Sir Francis Bacon, who favored the study of mathematics as a remedy to students' lack of attention. In the 1800's, many educators argued that the study of Latin would develop the mental discipline necessary to learn any domain.[121]

In other words, since subjects like mathematics were considered difficult, they were taught for the purpose of exercising or developing the mind. Students were prepared for the decision-making requirements of life by developing

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[120]Ibid., 51.

[121]John Bransford, *et al.*, "Teaching Thinking and Problem Solving," American Psychologist 41 (October 1986): 1078.

their powers of reasoning as a result of studying these subjects.

According to this line of thinking, if this works for life, it ought to work in the field of economics. Studies, in fact, have been conducted in which a positive effect was discovered for students who were required to study quantitative methods simultaneously with economics.[122] Payne and Skoro suggest a plausible explanation:

> There are at least two reasons for believing that mathematical ability influences student performance in introductory economics courses. The first is that *both math and economics use formal, abstract, deductive reasoning*. The second is that so many economic concepts can be expressed in mathematical notation.(italics added)[123]

The conclusion that is easily reached from this approach is that since economics draws upon a common set of aptitudes, then the study of mathematics prepares or develops the student for the study of economics. From a study investigating this relationship, Payne and Skoro offer the following observation:

> The ability to perform certain simple mathematical operations, such as working with decimals and fractions, evaluating simple formulas, figuring the slope of a line, reading a graph, and understanding the elements in

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[122]George G. Dawson, "Special Report: An Overview of Research in the Teaching of College Economics," JEE 7 (Spring 1976): 114.

[123]Richard D. Payne and Charles L. Skoro, "Mathematical Preparation/Aptitude and Performance in Principles of Economics Courses," JEB (April 1988): 307.

> a linear equation, is strongly correlated with test scores in principles of economics.[124]

Dawson also points out that mathematical ability as measured by "grade point averages, SAT scores, etc., turn out to be highly significant predictors of the extent to which gains will be made on economics tests."[125] Hence there seems to be a natural positive relationship between the development of mathematical skills and learning economics.

Cognitive Developmental Theory

Research in the area of cognitive developmental theory has been performed in the area of "*how we reason, think*, or *make meaning of our experiences*."[126] The basic unit of cognitive development is a *structure*, which is a set of assumptions which defines how a person perceives, organizes, and evaluates experiences and events in the learning process.

Rodgers presents a helpful summary of this theory in a discussion of Selman's approach:

> Hence, Selman claims there are *levels of cognitive* structure which underlie the more surface or content oriented cognitive developmental theories such as interpersonal and moral reasoning. That is, logically prerequisite structures (*e.g.*, Piaget and Selman) are deeper level structures and may develop before or with

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[124] Ibid., 310.

[125] Dawson, "Overview of Research," 112.

[126] Robert F. Rodgers, "Theories Underlying Student Development," in Student Development in Higher Education: Theories, Practices, and Future Directions, ed. Don G. Creamer (Cincinnati, OH: ACPA Media, 1980), 13.

> more surface structures (interpersonal or moral reason), but they cannot develop after more surface structures.[127]

He then presents a figure in which the order of structures is described. Briefly these concepts are described in the following order. The first order of analysis includes Fowler's Faith Development and Piaget's Logico-Mathematical Stages of Cognition. The second order of analysis includes Selman's Social Cognition or social perspective-taking levels. The third order of analysis includes Isaacs' Interpersonal Relationships, which include the independent domains of person, dyadic, groups, and institutional; Kohlberg's Moral Reasoning, which includes the independent domains of cheating, theft, and value life; and Perry's Intellectual Development, which uses philosophy, history, physics, and English, to relate to the independent domains of career, spouse, religion, and politics.[128]

The point of Selman's hierarchy that is germane to this study is that in education "it is easier to stimulate development in the surface domains closest to everyday experience (third level theories such as those of Kohlberg and Perry) than in the [higher level] domains of Selman, Piaget, or Fowler."[129] Therefore, it would seem, at first glance, that greater learning in economics would result from

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[127]Ibid., 23-25.

[128]Ibid., 24.

[129]Ibid.

relating theory to "real-life"-everyday experience (third level domain) than in explaining it abstractly and mathematically (first level, logico-mathematical).

Yet, when the theory is more deeply considered, the logico-mathematical domain is the underlying, categorical base, upon which the structures are built. This foundational level can also be developed independently of the domains closest to everyday experience. As a result, it would seem, the capacity to understand in the surface domain would be expanded as the logico-mathematical base is developed. This means that studying and using mathematics in the exposition of economic concepts to increase pedagogical effectiveness is consistent with this theory. It could also be hypothesized that while the development of this logico-mathematical domain would not be accomplished as easily as the domains closest to everyday experience are developed, it should have the greater effect because it develops the underlying base of understanding.

Factors which Govern the Cognitive Processes

In two separate studies, Bach and Saunders list three cognitive processes of learning (*i.e.*, acquisition, retention, and transfer) and the factors that govern or affect these processes.[130] Concerning the reliability of this list,

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[130]G. L. Bach, "Student Learning in Basic Economics: An Evaluated Experimental Course," in New Developments in the Teaching of Economics, ed. Keith Lumsdem (Englewood Cliffs, NJ: Prentice-Hall, Inc., 1967), 74-88; and

Bach states: ". . . we are convinced that the propositions [listed] have more empirical validity than the usual casual empiricism on which most teachers build their courses and teaching behavior."[131]

General Propositions. It has been generally accepted today by most psychologists that there are certain clear limitations on the learning process. These limitations include the following: (1) that the human mind has a limited ability to process information; (2) that it is important to have a hierarchical arrangement of thought, known as the "learning set," upon which to build in learning; (3) that the student's motivation is extremely important in determining the amount of learning that occurs; and (4) that redundant forms of the same information assist in learning.[132]

While several important implications for economics education may be garnered from an understanding of these limitations, the final observation (4) in this series may have a direct relationship to the use of mathematics in teaching economics. Bower and Hilgard observe that from a large number of learning experiments "imaginal or pictorial" representations of information assist memory

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Saunders, "Learning Theory," 62-83.

[131]Ibid., 76.

[132]Saunders, "Learning Theory," 67-72.

significantly.[133] There is almost a hierarchy in which pictures, images, and then concrete words are remembered in that order. The theoretical process that explains this is the "dual trace" hypothesis:

> So a word (or word pair) that is imaged or a picture that is named has the advantage of having two, redundant copies of the memory trace laid down. The redundancy prolongs memory in comparison to abstract items, since the second, imaginal trace is likely to survive after the initial, verbal trace has decayed. That is, not only are there two traces, but the one in the imaginal system seems more resistant to forgetting.[134]

Clearly if visual aids are effective in increasing memory, the salient development of a precise mathematical example, in conjunction with a verbal description and a graphical presentation, could be a quintessential visual aid.

<u>Acquisition of Learning</u>. When a student achieves mastery over new facts, processes of thought, or relationships, knowledge has been acquired. Briefly, the following several strategies facilitate this process: (1) providing students with "advanced organizers" to let them know exactly what they are to learn, (2) increasing the "meaningfulness" of the material to be learned by imposing structure on the material, (3) providing prompt, accurate feedback or "knowledge of results" to correct erroneous learning, and (4) involving students in active learning, as opposed to

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[133]Gordon H. Bower and Ernest R. Hilgard, <u>Theories of Learning</u>, 5h ed., (Englewood Cliffs, NJ: Prentice-Hall, 1981): 440.

[134]<u>Ibid</u>.

passive learning.[135] Each of these factors demonstrate that the acquisition of learning is not a passive process.

There are many ways to secure the involvement of the student in the learning process. In the principles of economics, one of the most effective methods may be having the students solve mathematical problems in economics. Saunders, long an advocate of the use of a workbook in the principles course, provides a suggestion that integrates several of these factors:

> . . . that we should give our students every opportunity to "respond" and "get involved" by having them complete homework problems and other assignments that are carefully graded and promptly returned with constructive comments and suggestions for improvement. . . . [In addition,] after students have a broad overview and sense of perspective on the entire course, perhaps a "lecture-work example-let student work a different example" sequence may be useful in the early stages until the students have accumulated enough "tools" to work for themselves. But at a later stage of the course it is probably important to put students in problem-solving situations on their own. At this stage, we must develop patience and resist the temptation to "tell them the answer." We must let them mull it over and work it out for themselves.[136]

The relevance of a mathematical approach in the acquisition of learning in the principles course is obvious. Students are assisted in learning when structure has been imposed on the material, when there is prompt feedback, and when they are actively involved in the process. A mathematical approach imposes such a structure, permits feedback when the

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[135] Saunders, "Learning Theory," 73.

[136] Ibid., 74.

problems are corrected and returned, and actively involves the student in learning.

Retention of Learning. Evidence exists that those factors that advance acquisition also advance retention of knowledge. Saunders recognizes this, but also suggests that retention is based on the *degree of initial learning*:

> Indeed, one cannot remember what one has not learned in the first place, and most tests of acquisition require the use of memory. The single most important thing in retention, therefore, is the *degree of initial learning*. Material learned by rote is not remembered as well as "meaningful" or "organized" material, and material that is used and applied is remembered much better than material that is not.[137]

Again, a mathematical approach to problem-solving seems appropriate to promote the retention of learning.

Since the key to the *degree of initial learning* is the organization and use or application of material, examples of specific mathematical problems that illustrate "real-life" solutions that economists perform would effectively tie theory to application. Abstract concepts discussed verbally and graphically could instantly become relevant as economics problems are developed and worked-out mathematically. Through mathematics, economic concepts can become more meaningful, so that retention is increased.

Transfer of Learning. Bach notes that the empirical evidence available on learning transference is

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[137] Ibid., 75.

unsatisfactory.[138] But it is known that it is a cognitive process involving the ability to transfer learning from one situation to another. One positive finding is that transfer seems to take place more readily when the general process of problem solving is presented rather than just a particular technique. Saunders emphasizes that "transfer is facilitated when the initial learning can be formulated in terms of general principles applicable to new learning."[139] In other words, if students learn "what to look for" as they approach various types of problems, there is greater overall success in the learning process.

This finding has relevance to the manner in which mathematical problems are presented in economics. Saunders states that they must involve the process of interpretation:

> The studies of transfer of learning have implications not only for how problem-solving material is presented but also for how homework or practice problems are designed. Practice problems that emphasize computation rather than interpretation and application not only may encourage the student to relate the material to a rote learning set rather than a meaningful learning set but also may have limiting effects on the future study behavior of the student. If we want to develop in our students the capacity for meaningful (as opposed to rote) understanding and broad (as opposed to narrow) transfer, we must design out instruction and homework problems accordingly.[140]

Mathematical economics problems serve as a base from which the process of transfer can take place. Problems are

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[138]Bach, "Student Learning," 78.

[139]Saunders, "Learning Theory," 76.

[140]Ibid., 77.

presented in such a way as to illustrate the general problem-solving nature of each technique, then students are allowed to apply that technique in different situations.

This survey of the literature on pedagogical theory focuses on that portion which relates to the use of mathematics in teaching economics. As has been demonstrated, while there is some theory that has offered some insight to the topic, it has been indirect. Further studies concerning the direct effectiveness of a mathematical presentation of economics theory would be extremely helpful.

The Mathematical Limitations of Students

There is little question that a mathematical approach is useful in the undergraduate economics curriculum. Most colleges and universities in the United States require at least one course in either calculus or statistics.[141] However, mathematical relationships are also important in understanding the basics of economics as found in the principles course. The concepts of percentages, incremental and average values, and the solution of simple, linear equations are pervasive at this level. While only high school algebra is typically used in the principles course, the mathematical difficulty of the course has been increasing steadily over the past few decades.

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[141]Siegfried and Wilkinson, "Economics Curriculum," 130-131.

McConnell thinks that the increase in the quantitative requirements of the principles course is a definite trend that is negatively related to another more serious problem:

> Dissatisfaction with Economics 101 may also emanate from . . . evidence that the academic capabilities of high school students are diminishing. College-entrance examinations for the past ten or fifteen years [the article is dated 1980] reflect apparent declines in the communication and quantification skills of college-bound students.[142]

The Scholastic Aptitude Test (SAT) offers a vivid example of this decline. Despite those who criticize the instrument, the SAT is being used by an increasing number of colleges because it provides such an accurate assessment of the basic skills needed for college-level work. The validity of the evidence is clearly stated by Singal:

> The SAT also has the virtue of having a rock-steady scoring system: it is calibrated, by the College Board, so that a score earned in 1991 will represent almost exactly the same level of performance it did in, say, 1961. Thus, by tracking the percentage of students coming in above the benchmark of 600 on the College Board scale (which runs from 200 to 800), one can get a good sense of how the country's most capable students have fared over the years.[143]

When the scores of our nation's college-bound students are reviewed in this manner, the results are clearly in line with McConnell's statement:

> In 1972, of the high school seniors taking the SAT, 11.4 percent had verbal scores over 600; by 1983, the number had dropped to 6.9 percent, and despite modest gains in the mid-1980's, it remains in that disheartening

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[142]McConnell, "Economics 101," 16.

[143]Daniel J. Singal, "The Other Crisis In American Education," <u>The Atlantic Monthly</u> (November 1991): 60.

> vicinity. ***That's a decline of nearly 40 percent.*** The decline since the mid-1960's has probably been closer to 50 percent, . . . The math SAT presents a somewhat different story. Though the percentage scoring over 600 dropped from 17.9 in 1972 to 14.4 by 1981, it has climbed back up to 17.9 in 1991. However, an influx of high-scoring Asian-American students (who now make up eight percent of those taking the test, as compared with two percent in 1972) has apparently had much to do with this recent upsurge (italics added).[144]

What does this mean for the typical top-quartile senior? Singal proposes that he/she would score approximately fifty to sixty points lower on the verbal section and twenty-five points lower on the math section than he or she would have in 1970.[145]

The point of this data is that college professors are faced with the problem of effectively communicating concepts of a more quantitatively demanding subject to students whose capabilities on the average are declining. The encouraging thing is that mathematical skills seem to have declined less than verbal. Therefore, the necessity of providing a mathematical presentation is perhaps greater than ever.

Conclusion

In this chapter, the usefulness of a mathematical approach to the pedagogy of the principles of economics course has been suggested. First, it was demonstrated that the curriculum for the economics major in the colleges and universities of the United States, reflecting the changes in

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[144] Ibid., 61.

[145] Ibid.

the field, has been increasingly quantified over the past several decades. Then, it was shown that the principles course has also become more highly quantified and sophisticated. The encouraging factor, however, is that the mathematical tools required in most principles courses are no more rigorous than high school algebra.

Next, literature involving the pedagogical theory of using mathematics in teaching economics was surveyed. While very little theory related directly to the use of mathematics in teaching economics, it did suggest that the appropriate use of both a mathematical presentation and mathematical problem-solving could be helpful in advancing learning. However, in the literature, there appeared to be no research specifically concerning the use of mathematics in teaching the principles of economics. Thus, questions remain as to the relative effectiveness of a mathematical approach compared to other approaches, such as verbal or graphical, and a question also remains concerning the extent of the mathematical approach in current use. The last question is addressed in the next chapter. To what extent is a mathematical approach presently used in the principles of economics course and what factors are significant in making that determination?

CHAPTER IV

RESEARCH METHODOLOGY AND STATISTICAL ANALYSIS OF THE DATA

Research Methodology

Introduction

The approach used in this study involves three aspects of research: (1) the review and synthesis of literature relating to the rise and proliferation of mathematical economics and the pedagogical use of mathematics in teaching the principles of economics; (2) the development and design of the experiment, including specifying the dependent and possible independent variables, the creation of the survey questionnaire, and the conception of a plan for the collection of a random sample from the research population; and (3) the collection and analysis of the data. The first part of the study (number 1 above) was presented in the second and third chapters; a description of the second and third parts of the study is the focus of this chapter.

The development of the experiment design took more than a year and involved a pilot survey using an experimental questionnaire. The design was carefully developed, involving the assistance of several professors, econometricians, and research specialists. Because of the lack of information about the current use of mathematics in teaching the principles course, as noted in the literature review, the most significant aspect of this research is the collection and analysis of the data.

Collection and Coding of the Data

Selection of the Sample from the Population. The population for this study consists of economics professors from colleges and universities in the United States who hold their doctorate in economics and who teach the principles of economics course. The College Blue Book was used to identify the target population. The book categorizes colleges and universities by degrees offered in various subjects. Under the subject description, *economics*, all of the colleges and universities in the United States that offer an economics major are listed by state.[146] Since community or junior colleges are outside of the population parameters for this study, they were eliminated. Of the remaining institutions, 971 were numbered serially and a sample of 500 was randomly selected using a *random numbers table*.[147]

In order to obtain completed questionnaires from as many college professors as possible, two copies of the

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[146]The College Blue Book: Degrees Offered by College and Subject, 22d ed., (New York: Macmillian Publishing Company, 1989), 534-538.

[147]William Mendenhall and Terry Sincich, A Second Course in Business Statistics: Regression Analysis, 3d ed. (San Francisco, CA: Dellen Publishing Company, 1989), 778-780.

survey instrument were mailed to each of the selected institutions, addressed to the economics/business department head. The questionnaires (see Appendix D) accompanied an introductory letter (see Appendix E) which requested that the department head have one or several faculty members who currently teach the principles course complete the survey. A total of 380 completed questionnaires were received from 275 different institutions. This represents a 38 percent questionnaire response rate and a 55 percent institutional response rate which is quite high for a mail survey.

Several factors could explain the high response rate. First, every effort was made to personalize the letter and make the questionnaire attractive. Each was carefully composed and laser printed in an attractive font. Then, copies were cleanly reproduced and each letter was individually signed in blue ink. Finally, the addresses were individually typed on gummed labels using a Word Perfect merge program. Second, each letter contained a self-addressed, stamped envelope to facilitate return. Third, to further encourage participation, an offer was made to share the findings of the survey with those who would request it. In response to this offer, twenty-three requests were received for the survey results.

Response and Data Coding. The survey instrument contained twenty-one questions which could be divided into three categories. The first category consists of nine questions that collect information about the instructor's *personal characteristics*. In this category, the questionnaire included an additional question used only to determine if the respondent has a doctorate in economics. This question does not contribute a variable for the regression equation. The second category included nine questions that address the *environmental* or *institutional characteristics* in which the instructor works. A tenth variable in this category is the average SAT math sub-score for the institution, which is obtained by research from several directories.[148] The third category included two questions: a question designed to collect the data on the dependent variable and a question that was added to obtain the respondent's perception of bias in the instrument.

The nine questions concerned with personal characteristics are in a variety of forms. Six of the questions ask for responses that are not measured on a quantitative scale. Four ask for a yes or no answer. Two others seek to determine the same thing (*e.g.*, whether or not the instructor uses the short-list approach and whether or not the instructor has or plans to increase the mathematical demands of the course), but do so in a manner that is designed to clarify the intent of the question (*i.e.*, for one question,

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[148]Lovejoy's College Guide, eds. Charles T. Straughn II and Barbarasue Lovejoy Straughn, 21st ed. (New York: Prentice Hall, 1992); and Barron's Profiles of American Colleges, 19th ed. (Hauppauge, NY: Barron's Educational Series, Inc., 1992).

does the professor use the long or short list approach, and for another question, has the instructor increased, decreased, or not changed the mathematical demands of the course). One question in this category serves only to select the qualified respondents and is not used in the equation. Therefore, five of the six question responses are binary in form, or dummy variables. The result is that the "yes" responses are coded as *1's* and the "no" responses are coded as *0's*.

The other three questions describing personal characteristics collect quantitative information. Specifically, one question requests the year the respondent received his/her doctorate, another question asks for the number of mathematical approaches to economic problem-solving that the respondent uses from a list of thirteen, and yet another asks for the percentage of lecture time using mathematics in teaching economics concepts or theory.

The ten questions concerning environmental or institutional characteristics are also varied. Eight of the questions are qualitative in nature. Five ask for a simple "yes" or "no" answer and three others require a similar binomial response. Of these questions, one seeks to determine if the institution offers economics in a two-semester/quarter sequence. Selection of the response *two semesters/quarters* is a "yes;" selection of any other is a "no." Another seeks to discover if the institution teaches the principles course at the sophomore level or higher. If the respondent selects *sophomore*, or *other*, the answer is interpreted as "yes;" if he/she selects *freshman*, the answer is "no." One question requests information on whether or not the respondent feels constrained from using mathematics because of students' lack of mathematical sophistication. If the instructor selects *very constrained* or *moderately constrained*, the answer is "yes;" if he/she makes any other response, the answer is "no." As a result, eight responses are coded as binary, dummy variables, with all "yes" responses coded as *1's* and all "no" responses coded as *0's*.

Unlike the eight qualitative variables, the two remaining variables in the environmental category are quantitative and they are determined in different ways. The first is obtained from a question which asks for the number of students with an Economics major at that institution. The second is the students' average SAT mathematics sub-score. It is assumed that instructors would generally not know this information, nor would they take the time to make this determination; therefore, each questionnaire was numbered so that the institution could be identified when the questionnaire was returned. Then, as the questionnaires were returned, the average SAT mathematics sub-score was determined for each institution from college directories.[149]

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[149] Lovejoy's College Guide and Barron's Profiles of American Colleges.

The question for the dependent variable is also quantitative in nature. It asks what percentage of the student's final grade in the principles course is derived from his/her use of mathematics to solve economics problems. Because this variable uses continuous, rather than discreet data, the analysis can be accomplished using the more familiar ordinary least squares regression model, instead of logit or probit models.

Analysis of the Raw Data

In the first phase of analysis, the raw data responses were entered into a Lotus 1-2-3 spreadsheet file with the rows representing the individual responses and the columns denoting the individual questions. The data were entered into a spreadsheet file for two purposes. First, the institutional name is entered to identify the responses of each row's data. The data can then be sorted by various data elements without losing the institutional identification. Second, entry of the data into Lotus 1-2-3 permits the relevant data to be transferred to the econometrics program in a single file transfer.

The summary analysis of the raw data includes determining the mean, standard deviation, minimum and maximum responses, and the number of responses in the sample. These data are presented in Table 1 (each variable is listed and defined in Table 5 in Appendix F).

TABLE 1

SUMMARY OF RAW DATA ANALYSIS

Question/Variable	Mean	Std Dev	Max	Min	Number
Independent Variables					
SAT Mathematics Sub-score--SAT_M	486.50	74.85	735	330	359
Number of math approaches used of 13--MTHPROBS . .	8.55	2.76	13	0	375
Percent of use of math in lecture--PCTLECT . . .	18.95	13.86	95	0	375
Course taught in dept of business--SCHBUS	0.78	0.41	1	0	375
Course taught in two semesters--DURATN	0.86	0.35	1	0	375
Taught on sophomore level and above--UPRLVL	0.49	0.50	1	0	375
Taught in accredited department--AACSB	0.31	0.46	1	0	375
Require mathematics pre-requisites--PRERQS . . .	0.21	0.41	1	0	375
Institution publicly funded--PUBFND	0.45	0.50	1	0	375
Student number majoring in economics--NMSTUS . .	88.38	161.55	1,800	0	374
Require micro portion first--MICFST	0.30	0.46	1	0	375
Won award in teaching economics--EXAWRD	0.31	0.46	1	0	375

Table Continued

TABLE 1 (Continued)

SUMMARY OF RAW DATA ANALYSIS

Question/Variable	Mean	Std Dev	Max	Min	Number
Published econ article in journal--PUBJNL . .	0.68	0.47	1	0	375
Have or are getting the doctorate	0.92	0.28	1	0	375
Year doctorate was received--YRDCTR . . .	78.77	10.81	94	0	347
Short-list view of teaching--SHTLST . . .	0.52	0.50	1	0	375
Eliminate unsuccessful students--ELMSTU . . .	0.36	0.48	1	0	375
Math effective teaching economics--MTHCOM . . .	0.84	0.37	1	0	375
Constrained from using math--CSTMTH	0.72	0.45	1	0	375
Increased math demands in course--CHGMTH . . .	0.14	0.35	1	0	375
Think questionnaire is not biased?	0.83	0.38	1	0	374
<u>Dependent Variable</u>					
Percent students' grade from math--PCTMATH . .	19.88	14.16	100	0	375

Several of the statistics in Table 1 contribute to an understanding of the current educational status of the principles of economics course as taught in institutions of higher learning in the United States. The average institutional SAT mathematics sub-score (SAT_M) is 486.5 out of a possible 800. Since a sub-score of 600 is frequently used as a benchmark for well-prepared students, the score seems rather low. It is even lower than the median between the highest and lowest scores. On the other hand, the mean percentage of lecture time in which mathematics is used to teach economics concepts or theory (PCTLECT) is a substantial 18.95 percent, as shown in Table 1.

These two factors signal a dilemma. While our students seem poorly equipped in mathematics, as reflected in their average SAT mathematics sub-score, a rather large portion of the lecture time (18.95 percent) is spent using mathematics to illustrate or explain economic concepts. A simple conclusion emerges. Either the large amount of time spent on mathematics includes a significant level of foundational work developing the students' mathematics skills so that they can understand the economics, or much that is taught in the form of mathematics is not being understood.

Such results could suggest the need for a mathematics prerequisite, which is another variable in Table 1 (PRERQS). The mean for the responses to this question is only 21 percent. This indicates that only 21 percent of the institutions have a mathematics prerequisite for the principles of economics course. One reason for this low percentage may be that courses with mathematics prerequisites are intimidating for college students, prompting them to take the courses in a less threatening environment, such as the community college, and transferring the credit. Observation of this phenomenon could cause colleges to eliminate the mathematics prerequisite.

Reference to Table 1 indicates that the division from which the principles of economics course is taught (SCHBUS) is a department of Business or Economics in 78 percent of the responses. This means that only 22 percent of the institutions house the principles course in the Social Science division, where it would be taught less quantitatively and more frequently in the one-semester format.

Consistent with research findings that students learn more economics in two semesters than in one,[150] the two-semester sequence for the principles course (DURATN) was designated as important in 86 percent of the total responses. The significance of the two-semester sequence is evident. However, a relationship may be suggested between this and the previously discussed variable. While the 14 percent of institutions that only require a one-semester sequence is slightly smaller than the percentage of the institutions which teach the course from the Social Science division; the similarity of proportions is enough to at least suggest that a consistent relationship exists.

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[150]Dawson, "Overview of Research," 113.

Two other sets of responses in Table 1 also suggest a curiosity that exists in the contemporary principles classroom. When asked if mathematics was considered effective in teaching economics (MTHCOM), 84 percent responded affirmatively. On the other hand, nearly three quarters of those surveyed (72 percent) stated that they felt constrained in using mathematics because of a lack of mathematical sophistication in their students (CSTMTH).

Only 31 percent of the economics professors who responded had received any kind of an award or recognition for teaching excellence (EXAWRD), yet 68 percent had successfully published an article on economics in a refereed journal (PUBJNL). Such a contrast suggests that the typical institution offering economics instruction places more importance on publishing than on teaching excellence.

Surprisingly, with studies showing increased learning when the principles course is sequenced so that the principles of microeconomics is taught first,[151] only 30 percent of the respondents indicated that they required this sequence (MICFST). By implication, 70 percent of the institutions either require macroeconomics first, or do not specify a sequential order. Thus in most cases, the econom-

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[151]William B. Walstad and Michael Watts, "The Principles of Economics Textbook: History and Content," in The Principles of Economics Course: A Handbook for Instructors, eds. Phillip Saunders and William B. Walstad (New York: McGraw-Hill Publishing Company, 1990), 146-147.

ics curriculum is not in conformance with what research indicates is sound pedagogical practice.

Other responses which contribute to an understanding of the current status of the economics course may be briefly summarized from Table 1. Only 31 percent of the respondents teach economics from a division that is AACSB accredited (AACSB). Less than half (45 percent) of the institutions that responded were publicly funded (PUBFND). The average number of students in each institution majoring in economics (NMSTUS) is over 88 (88.38). Of the respondents, 92 percent either had their doctorate in economics or were completing their studies leading to the doctorate and the average year the doctorate was received (YRDCTR) was 1979 (78.77).

Further reference to Table 1 will reveal that a majority of economics instructors (52 percent) utilize a short-list approach in teaching the principles course (SHTLST), apparently in an attempt to provide depth in selected areas. Only 14 percent had recently increased mathematics demands in the course (CHGMTH). However, 36 percent thought of the principles course as a vehicle for eliminating potentially unsuccessful students from the economics major (ELMSTU). These independent variables when taken together thus provide a current profile of the circumstances under which principles of economics is taught and by whom.

The dependent variable, the average percent of students' final grade derived from the usage of mathematics in solving economics problems (PCTMATH), was approximately 20 (19.88). If ± 2 standard deviations are added to obtain a range, in most instances (approximately 92 percent, if the data are normally distributed) between 0 and 48 percent of the students' grade will be based on their use of mathematics to solve economics problems.

A final observation can be made from Table 1. Eighty three percent of those responding to the survey thought that the questionnaire was free from bias. This high statistic along with the high return rate contributes to a confidence that the questionnaire was not offensive and collected accurate data. The possibility of unintentional bias was registered, however, by several comments from those who answered in the negative. Several respondents felt that the questionnaire had a pro-mathematics slant.

Summary of Raw Data Analysis

From the raw data analysis several additional observations emerge. The intent was to summarize the information gained in the survey to help describe the state of economics education as it relates to the principles course. The dilemma over the perceived effectiveness of using mathematics in teaching economics, on the one hand, and the lack of mathematical sophistication of students, on the other, was observed with two sets of variables. The variables

PCTLECT and MTHCOM in different ways report the perceived need for the use of mathematics, whereas SAT_M and CSTMTH differently suggest a lack of mathematical sophistication of today's students.

The average professor responding to the survey with the doctorate in economics received his/her degree in 1979 (YRDCTR). Hence, he/she should be familiar with the mathematical approach which developed most rapidly in the 1970's.[152] This is confirmed by the large percentage of instructors who feel that mathematics is effective in teaching economics (MTHCOM).

Finally, generalizations about the categories of variables may be informative.[153] The nine variables concerned with the personal characteristics of the instructors teaching the principles course reveals an individual, who, on average: uses a mathematical approach in approximately 19 percent of class time, has not won an award for excellence in teaching, has published an article on economics in a refereed journal, received the doctorate in 1979, uses the short-list approach, does not feel that the principles course is to eliminate potentially unsuccessful students from the major, and has not recently increased the mathematical demands in the course.

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[152]Grubel and Boland, "Efficient Use," 425-427.

[153]Binomial variables with results near the 50 percent range are not included in the generalization.

The ten variables that describe the characteristics of the institutions with an economics major reveal a college or university which, on average: attracts students with lower SAT mathematics sub-scores, with the result that its instructors feel constrained in the use of mathematics in teaching; offers a two-semester principles course from the division of business; is not AACSB accredited; does not require a mathematics prerequisite for the course; has approximately 88 economics majors; and does not require that the microeconomics portion be taught first.

Analysis of the Initial Regression Results

The Statistical Model

The data collected for this study may be utilized in analyzing the current status of economics education as it relates to the principles course in two ways. First the data may be summarized by category. Such an approach provides general observations about the status of economics education relating to the principles course, insight into the average economics professor teaching the principles course and the typical environment of an institution with an economics department that offers the course. This has been accomplished in the above section. The second way the data may be treated is through multiple regression analysis. With this method, the relative influence that each of the personal and institutional characteristics has on the use of mathematics in the principles course may be determined. If a characteristic is influential, it would be of interest to know the direction and magnitude of that relationship in influencing the decision to use mathematics.

Thus an initial ordinary least squares regression model is used to provide an estimate of the relative influence of each of the nineteen characteristics, which are the independent variables in the study.[154] This is followed by a more formal and focused predictive model containing only statistically significant variables.

In general terms, the following ordinary least squares equation is designed to describe a linear functional relationship in which the values of the personal and institutional characteristics (independent variables) are used to

[154]While authors such as Becker and Spector and Mazzeo have criticized the over-use of the ordinary least squares approach in economics education research, the basis of the criticism is that the dependent variable is all too often qualitative in nature. With a qualitative dependent variable, a logit or probit model is more appropriate. However, in this study, while several independent variables are qualitative, the dependent variable is quantitative and continuous in nature, so that ordinary least squares is the appropriate model. See William E. Becker and Donald M. Waldman, "The Probit Model," in Econometric Modeling in Economic Education Research, eds William E. Becker and William B. Walstad (Boston: Kluwer Nijhoff Publishing, 1987), 135-140; and Lee C. Spector and Michael Mazzeo, "Probit Analysis and Economic Education," JEE 11 (Spring, 1980): 37-44.

predict the extent of the use of mathematics in teaching the principles course (dependent variable):[155]

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ . \ . \ . + \beta_{19} x_{19} + \epsilon$$

where y is the percentage of mathematics utilization as determined by the independent variables, β is the coefficient of the independent variables, x is the independent variable, and ϵ is the value of the random error term. In this initial model, y is assumed to be linearly related to the characteristics specified in this study according to the following:

$$\begin{aligned} PCTMATH = {} & \beta_0 + \beta_1 MTHPROBS + \beta_2 PCTLECT + \beta_3 EXAWRD + \beta_4 PUBJNL + \\ & \beta_5 YRDCTR + \beta_6 SHTLST + \beta_7 ELMSTU + \beta_8 MTHCOM + \beta_9 CHGMTH + \\ & \beta_{10} SCHBUS + \beta_{11} DURATN + \beta_{12} UPRLVL + \beta_{13} AACSB + \beta_{14} PRERQS \\ & + \beta_{15} PUBFND + \beta_{16} NMSTUS + \beta_{17} MICFST + \beta_{18} CSTMTH + \beta_{19} SAT_M \\ & + error. \end{aligned}$$

The dependent variable, PCTMATH, is the percent of the student's grade in the principles class that is derived from his/her ability to use mathematics in solving economics problems. The variable serves as a proxy for the extent of the use of mathematics in teaching the principles class. PCTMATH, then, is related to both the personal and institutional characteristics in the regression. As part of the analysis, the directional effects of these measures are also

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[155]Harry H. Kelejian and Wallace E. Oates, Introduction to Econometrics: Principles and Applications, 3d ed. (New York: Harper & Row, Publishers, 1989), 136-138.

assessed. The data are regressed using the Econometrics Toolkit, Version 2,[156] and the results are summarized in Table 2.

<u>Regression Analysis</u>

The analysis of the initial regression results is facilitated by reference to Table 2. The fit of the equation, indicated by the R^2 of 0.5456, is very acceptable for a cross-sectional regression of this type and the global F-test of 19.65 is significant at the 0.00 level. These results indicate that at least one of the model coefficients is statistically significant and that the model has predictive potential.

While several coefficients in the initial regression are statistically insignificant, a number of interesting results emerge. First, all of the coefficients had the expected signs except for four, DURATN, UPRLVL, AACSB, and NMSTUS, which had negative signs when positive signs were anticipated. Second, strong *t*-values suggest that an increase in the following variables: the number of types of problems required (MTHPROBS), the use of mathematics in the lecture to explain economics (PCTLECT), whether or not an institution is publicly funded (PUBFND), the year the professor received his/her doctorate (YRDCTR), and the perception that the principles course is a tool for the

[156]<u>The Econometrics Toolkit</u>, Version 2, is a general statistics and econometrics software package.

TABLE 2

ESTIMATED INITIAL EQUATION
FOR USE OF MATHEMATICS (PCTMATH)

Variable	Coefficients	Expected Sign
Constant	-5.11 (-0.74)	
SAT_M	0.003 (0.30)	(+)
MTHPROBS	0.39 (2.32)***	(+)
PCTLECT	0.69 (17.35)***	(+)
SCHBUS	0.06 (0.04)	(+)
DURATN	-0.95 (-0.59)	(+)
UPRLVL	-0.78 (-0.65)	(+)
AACSB	-1.56 (-1.23)	(+)
PRERQS	1.33 (0.96)	(+)
PUBFND	1.86 (1.42)*	(+)
NMSTUS	-0.009 (-2.27)***	(+)
MICFST	0.92 (0.77)	(+)
EXAWRD	0.94 (0.83)	(+)
PUBJNL	0.84 (0.71)	(+)

Table Continued

TABLE 2 (Continued)

ESTIMATED INITIAL EQUATION
FOR USE OF MATHEMATICS (PCTMATH)

Variable	Coefficients	Expected Sign
YRDCTR	0.08 (1.42)*	(+)
SHTLST	0.37 (0.35	(+)
ELMSTU	2.55 (2.32)***	(+)
MTHCOM	0.31 (0.20)	(?)
CSTMTH	-0.09 (-0.07)	(-)
CHGMTH	-1.16 (-0.74)	(-)
	N	331
	Std.Err.	9.2617
	R^2	0.5456
	Adj.R^2	0.5178
	F-Test	19.6496

Notes: The dependent variable is the percentage of the students' grade in the principles of economics class that is derived from the use of mathematics in solving economics problems (PCTMATH). Variable definitions are given in Appendix F. *T*-values are reported in parenthesis.

*** Significant at $P \leq .05$ * Significant at $P \leq .15$

elimination of potentially unsuccessful students from the economics major (ELMSTU), will result in an increase in the use of mathematics in teaching the principles course. A third regression result is that increasing the number of students in the economics major (NMSTUS) might result in a reduction in the use of mathematics in teaching the principles course.

The Personal Characteristics Variables. All of the personal characteristics variables had the expected positive sign and four were statistically significant. By referring to Table 2 it may be observed that MTHPROBS, while significant, has a relatively small coefficient. Hence, one may anticipate that more of the student's final grade will be determined by the use of mathematics in solving economics problems for every additional type of problem used in the classroom. Such an increase is entirely appropriate, because it would be illogical for instructors to include a mathematical technique without requiring the student in some way to utilize the technique and then tying it to his/her grade.

The next variable, PCTLECT, is the most statistically significant (at the 0.00 level). It represents an estimate of the proportion of the professor's lecture time that is devoted to the use of mathematics in teaching economic concepts or theory. That this variable would be the most highly correlated with the dependent variable was anticipated, since such a practice would reflect a conviction. If an instructor uses a mathematical approach in teaching, it is a reasonable assumption that he/she would make the use of mathematics a substantial part of the student's grade.

Reference to Table 2 reveals that while EXAWRD shows little statistical significance (*t*-value of 0.83), it is positively related to the mathematics requirement in solving economics problems in the principles course. This indicates that instructors who are recognized as excellent have a tendency to require their students to work some mathematical economics problems. Excellence in economics instruction, therefore, takes advantage of the enhancement that a mathematical approach provides.

The next variable, PUBJNL, has a coefficient and *t*-value similar to the previous variable. It is not significant and has a positive sign as anticipated. This suggests that instructors who have published in refereed economics journals are familiar with a mathematical approach and utilize it in their instruction.

By referring to Table 2, it may be observed that the year the instructor received his/her doctorate (YRDCTR) is significantly related to the amount of mathematics required in the principles course. The relationship is positive as anticipated, and indicates that a small fractional increase in the student's grade from mathematics usage will result from each more recent year the instructor received his/her doctorate. The increasing quantification of economics is being translated into an increasing emphasis in a mathematical approach.

Table 2 also demonstrates that SHTLST has the expected sign (positive), but it is statistically insignificant. Accordingly, this provides some evidence that when an instructor spends more time on fewer selected topics, he/she will, to some extent, utilize a more mathematical approach.

The variable, ELMSTU, stands out as one of the more significant in the category. With a *t*-statistic of 2.32, it is highly correlated with the dependent variable. It also has one of the largest coefficients. This variable tells us that when an instructor thinks that the principles course is a tool for the elimination of potentially unsuccessful students from the economics major, more of the student's grade will be determined by his/her use of mathematics in solving economics problems. Again, this was expected, since this variable reflects a mind-set about the quantitative nature of economics.

The last variable in the personal characteristics classification is CHGMTH, or the recent increase of the use of mathematics in the principles course. While it is statistically not significant, it does have the expected negative sign, indicating that those who have recently increased mathematical usage in their approach to the principles course generally have a less quantitative approach than average. Either they have a perceived need to use more mathematics or are being coerced into doing so.

<u>The Environmental or Institutional Variables.</u> When the institutional variables are considered, Table 2 indicates that four of the ten have an unanticipated negative sign. These variables are analyzed first. Initially, it was hypothesized that the course with the longer duration (DURATN) would have more time for a mathematical approach, yielding a positive sign. The outcome, however, indicates that when the course is for a shorter period, the reliance on mathematics increases. Perhaps the time constraint prompts teachers to apply more mathematics in their instruction.

The original hypothesis for the next variable, UPRLVL, was that when economics is taught in the freshman year it will be taught less rigorously, and thus less mathematically. Therefore, when taught on the sophomore level or higher, a positive sign was anticipated. However, if mathematics is effective in presenting difficult economic concepts, it would be logical that instructors would tend to rely on its use more heavily for freshmen, who would need a more concise and clear-cut explanation than any other group. In this case, then, while not statistically significant, the unexpected negative sign can be explained.

Reference to Table 2 reveals that for the variable, NMSTUS, both an unexpected sign and statistical significance exists. It was felt that in larger programs instructors would feel more compelled to ensure that students in the principles course learn more economics, and therefore would use more mathematics. Thus a positive sign was expected. However, the negative sign indicates that as the size of the economics program increases, the use of mathematics in the principles course decreases. The constant is very small, indicating a very slight change; but the statistical significance of the variable indicates that it is a reliable indicator. A possible explanation is that some of the larger programs are more descriptive and institutional and less quantitative in their approach to teaching economics.

Another, more plausible explanation, is that the institutions with the greater number of students in the economics major feel less pressure to maximize economics learning in the principles course. To verify this, the response data were sorted in a spreadsheet program by the number of students in the economics major. For the top 5 percent of institutions (by number of students), the mathematics requirements (PCTMATH) averaged only 16 percent, as opposed to the sample mean of 19.7 percent. Such a difference could explain the unexpected change. Perhaps the explanation is as simple as the existence of larger classes in such institutions causing instructors to reduce the number of problems students work to ease their grading demands. Whatever is suggested, the pedagogical explanation for this phenomenon, *ceteris paribus*, remains a mystery.

AACSB also has a negative sign when a positive sign was initially anticipated. It was felt that economics programs housed in an AACSB approved business program would be more quantitatively demanding. In this instance, the problem could be with the survey question and the data rather than the hypothesis. As the data were being assessed, comments were noted on several questionnaires that AACSB does not accredit economics departments. The question was intended to determine if the economics department was a part of a business program that was accredited by AACSB. The question read "is your *economics department* accredited by the AACSB?" This may have been confusing and could have resulted in unreliable data.

Another factor may have contributed to the unanticipated negative signs on the four variables (*e.g.*, DURATN, UPRLVL, NUMSTUS, and AACSB). When a correlation matrix for the regression was analyzed, higher than average multicollinearity was observed among the variables (with values ranging from .21 to .38; see Table 6 in Appendix G). Fortunately, such collinearity did not occur to any appreciable extent among the other variables.

A review of Table 2 indicates that the remaining six variables have the expected sign. The SAT mathematics subscore, SAT_M, is insignificant, but positive. It was anticipated that in general, using a mathematical approach would be facilitated by greater student skill in mathematics. But as observed previously with respect to the UPRLVL variable, if a mathematical approach is an effective means of teaching economics, it may be to the weaker student's advantage to have the material presented mathematically. This may be the reason why the variable is not statistically significant and has such a small coefficient.

SCHBUS has the poorest showing of any of the variables with a *t*-value of 0.04. Although it has the expected sign, it is statistically not significant. But there is an indication that when principles courses are taught from the division of business, they tend to be taught more mathematically.

Table 2 also reveals that the variable, PRERQS, is insignificant but has the anticipated positive sign. Hence, while few institutions utilize mathematics prerequisites for the principles course, there is some indication that when they are required, a more mathematical approach is utilized in its instruction and in evaluating student performance.

Reference to Table 2 indicates that if an institution is publicly funded (PUBFND) it will require greater usage of mathematics by its students in the principles course. This was anticipated, because it was hypothesized that privately funded institutions are more concerned about the retention of students. If typical students are poorly prepared in mathematics and find mathematics more difficult, an institution might be inclined to reduce the mathematical nature of the principles class in an attempt to retain students. Further analysis using the spreadsheet was utilized to verify this reasoning. The data were sorted by PUBFND to separate institutions into publicly and privately funded categories. The average use of mathematics by publicly funded institutions was 21 percent, compared to 19 percent by privately funded institutions.

Still there is the possibility that the reasoning used for this PUBFND hypothesis is not valid. Again the spreadsheet was used to determine the mean SAT mathematics subscores for the public and private groups. The average subscore for the publicly funded institution was 458, whereas it was 510 for the privately funded institution. The data may indicate that if a mathematical approach is more helpful in teaching economics principles, the publicly funded institutions could be emphasizing the mathematics of economics to assist their less skilled students. This would more appropriately reflect sound pedagogical evidence.

By referring to Table 2, one can observe that while the variable, MICFST, is not significant, it does have the anticipated positive sign. Thus there appears to be some tendency for those institutions that structure the principles course with microeconomics first, to make greater use of mathematics.

The last environmental variable, CSTMTH, refers to the constraint felt by instructors whose students are less capable mathematically. While this variable was anticipated to be extremely relevant, it was one of the least significant. However, its sign is negative, as anticipated: meaning that instructors who would prefer to utilize a mathematical method of teaching do feel forced into using less mathematics when their students are not well equipped mathematically. The lack of significance might be a matter of degree. If instructors believe that a mathematical approach facilitates learning economics, little constraint may be felt in the use of simple mathematics. Consequently, instructors would ignore whatever constraint is felt, realizing that the mathematics they use will facilitate learning. On the other hand, significant constraint might be felt in instances in which an instructor would like to take his/her students into more complex mathematics, but is prohibited from doing so because the students' lack of mathematical ability.

Summary of Initial Regression Results

The results from the initial regression suggest several conclusions. The variables relating to personal characteristics produced no surprises with respect to the signs. Out of the nine variables proposed, four were statistically significant at the 0.15 level or better. Thus it can be concluded that the use of mathematics is influenced by the following: the number of types of mathematical approaches used in the classroom (MTHPROBS), the percent of lecture time devoted to mathematics (PCTLECT), the year the instructor received the doctorate (YRDCTR), and the instructor's perception that the principles course is a tool for the elimination of potentially unsuccessful students (ELMSTU). All contribute to an explanation of an increased usage of mathematics in teaching the principles course and its usage in evaluating student performance.

The group of variables describing institutional characteristics did not perform as well in the analysis. There were four unanticipated sign changes among the variables, one of which was statistically significant (NMSTUS) and could be explained. Only one other variable in this category (PUBFND) had the expected sign, was statistically significant, and shown to contribute to an understanding of an increased use of mathematics in the principles course. Considering both sets of variables, clearly there was more statistical significance among the personal characteristics variables than among the environmental variables.

Analysis of the Predictive Model

The last aspect of this research is more focused and designed to present a regression model with significant predictive ability in each of its variables. A model meeting this requirement is found in Table 3. From among the 19 original variables, the criterion for inclusion in the model is that no variable should have a significance level of less than 0.10.

TABLE 3

ESTIMATED PREDICTIVE EQUATION
FOR USE OF MATHEMATICS (PCTMATH)

Variable	Coefficients	Expected Sign
Constant	-3.40 (-0.83)	
MTHPROBS	0.41 (2.67)***	(+)
PCTLECT	0.71 (18.78)***	(+)
NMSTUS	-0.006 (-1.91)***	(+)
YRDCTR	0.08 (1.64)**	(+)
ELMSTU	2.33 (2.25)***	(+)
N		347
Std.Err		9.1887
R_2		0.5438
Adj.R_2		0.5371
F-Test		81.2860

Notes: The dependent variable is the percentage of the students' grade in the principles of economics class that is derived from the use of mathematics in solving economics problems (PCTMATH). Variable definitions are given in Appendix F. *T*-values are reported in parenthesis.

*** Significant at $P \leq 0.05$ ** Significant at $P \leq 0.10$

Of the five variables meeting this criterion for inclusion, four involved personal characteristics and only one involved institutional characteristics. When grouped into the regression equation, the signs remained the same, while the coefficients changed only minimally. With an adjusted R_2 of 0.54 and an F-statistic of over 81, the model has strong predictive capability.

Reference to the equation in Table 3 indicates that 0.41 of a percent more of the student's final grade will be determined by the use of mathematics in solving economics problems (MTHPROBS) for every additional type of problem used (from the list of thirteen mathematical approaches to economics problem-solving on the questionnaire); that for every one-percent increase of the use of mathematics in a professor's lecture time (PCTLECT), there will be a corresponding increase of 0.71 percent in the student's use of mathematics in solving economics problems; that for every additional student in an institution's economics program (NMSTUS), there will be a very slight decrease (0.006 of a percent) in the student's use of mathematics; that for each more recent year that an instructor receives his/her doctorate (YRDCTR), there is a slight increase (0.08 of a percent) in the student's use of mathematics; and that for professors who think that the principles of economics course is a tool for eliminating potentially unsuccessful students from the economics major (ELMSTU), there is a corresponding 2.3 percent increase in the student's use of mathematics.

To summarize, characteristics of professors such as the number of types of mathematical problems used, percent of mathematics in lecture time, when the doctorate was received, and the conviction that the principles course eliminates potentially unsuccessful students are significantly related to the mathematics usage in the economics course. In contrast, the institutional characteristic of larger numbers of students in the economics program has a statistically negative effect on the use of mathematics in the principles course.

CHAPTER V

SUMMARY, CONCLUSIONS, AND IMPLICATIONS

Summary

The purpose of this study was to identify personal and institutional characteristics that significantly contribute to the use of mathematics in teaching the principles of economics course. A secondary objective was to obtain descriptive information on the current status of mathematics usage in teaching the principles course in colleges and universities in the United States. Thus the primary intent was to provide information that would enhance economics instruction by gaining insight into the usefulness of mathematics in teaching the principles of economics course.

The study began by presenting nine personal characteristics which were hypothesized to influence the choice of mathematics usage in teaching the principles of economics. In addition, ten institutional characteristics were selected and hypothesized to impinge on the instructor's choice in utilizing mathematics. A questionnaire designed to gather this information was employed to obtain data concerning these characteristics.

The questionnaire was used in a national survey of 500 colleges and universities with economics programs. A random number generator was used to select the sample from more than 970 institutions. From the sample, 378 usable responses were received and analyzed by using both summary data analysis and ordinary least squares regression. The regression was then refined and presented as a statistically significant predictive model. Specifically, the data were subjected to three stages of statistical analysis, each progressively more focused and rigorous. First, the data were summarized and presented as a profile of mathematics' role in economics instruction. The presentation was primarily descriptive. Second, a regression was run which involved a more rigorous treatment of the data. Finally, the data analysis was more narrowly focused involving the development of a predictive model containing only statistically significant variables.

Conclusions

The summary data analysis was completed which provided a description of the characteristics of the average professor and the average institution responding to the survey, thus providing insight concerning the current status of economics education. In addition, the dilemma over the perceived effectiveness of using mathematics in teaching economics and the lack of mathematical sophistication of students was observed. The significance of this apparent difficulty was also highlighted by the additional observed contrast between the perceived instructor's need to use a mathematical approach in teaching the principles course and the inadequate mathematical background of students.

Following the descriptive analysis an initial regression was run which included all of the variables. This regression had fairly strong overall results, with several statistically significant variables bearing the anticipated sign. These variables were the number of different mathematical approaches (from a selected list) used in class by the professor, the percent of lecture time in which a mathematical approach is used in teaching the course, whether the school is publicly funded, the number of students in the economics program, the year the instructor received the doctorate, and whether the instructor viewed the principles course as a tool for eliminating potentially unsuccessful students from the economics major.

As noted, the most focused and refined analysis involved developing a predictive model in which no variable would have a significance level of less than 0.10. Five of the original 19 variables met this criterion. Of these, four pertained to the instructor's personal characteristics. They included the number of mathematical problems (from the list) used in teaching the course, the percent of lecture time in which mathematics was used, the year the instructor received the doctorate, and whether the instructor viewed the course as a tool for eliminating potentially unsuccessful students. The fifth variable was institutional, involving the instructor's environment. Specified as the number of students in the program, the variables had an unexpected negative sign. Although it bore an unanticipated sign, the variable completed a statistical model with the ability to predict with a small margin of error the propensity to use mathematics in economics instruction.

Implications and Further Study

From this study, several implications emerged that could serve to assist the instructor in the classroom and as the basis for further research in the use of mathematics in teaching economics.

Implications for the Classroom

1. The use of a mathematical approach in teaching the principles of economics course is currently perceived to be effective in communicating economic content. Even though many students are poorly prepared in mathematics, it seems possible that the use of simple mathematics both in teaching and in problem-solving activities could enhance economics learning.

2. The use of mathematics in teaching the principles of economics course is not a matter of chance. Personal and environmental characteristics that influence the use of mathematics have been isolated and identified; therefore, mathematics usage in this level of economics education is a rational choice by rational agents. This provides a fundamental rationale for the utilization of mathematics as a

tool in classroom instruction and in evaluating student performance.

Implications for Further Research

The implications for the classroom from this study are very tentative due to the general nature of the research. More specific inferences could result from additional investigation. Thus, the most pertinent implications from the study have to do with further research.

1. A next logical step in research on the use of mathematics might be to conduct a comparative study between students of comparable ability in which a test group received a mathematical approach and a control group received a verbal presentation of the same course material. The purpose would be to determine if the test group acquired greater economics knowledge.

2. The importance of a student's basic mathematics ability when exposed to a mathematical approach in economics instruction could be an enlightening research topic. Research could be conducted to determine if a mathematics approach is beneficial to students who are poorly prepared in mathematics. This would be of great value to instructors who are faced with students having a lack of mathematical sophistication.

3. As the questionnaires in this study were analyzed, respondents' comments were noted which indicated that the instrument did not ask all of the pertinent questions to

make the determination sought. While this model was based on two years' research, its variables statistically explained only a portion of the variation. Therefore, research might be undertaken to ascertain other explanatory variables that could improve the model.

4. Replication of this study using the existing or an improved questionnaire format would likely yield superior results if a larger or a more accurate sample were collected. Obvious misunderstandings of one question by the respondents were noted in the study. In addition, a disproportionate number of doctoral level professors, as opposed to graduate teaching assistants, reported teaching the course. Personal interviews to clarify misunderstandings and to gain a better perspective of exactly who teaches the course would improve the study.

5. The survey in this study was taken during one specific time, that is, the Fall semester of the 1992-1993 academic year. As such, it provides cross-sectional data and captures the attitudes currently held by educators in various colleges and universities in the United States. Since economics education research is ongoing, it seems very likely that things are rapidly changing in this field. Therefore it would be helpful to have the results of a longitudinal study, which would capture the changes in opinion as to the use of mathematics in economics education over time.

6. The focus of this study has been completely upon the principles course. Further research might be warranted to determine whether a mathematical approach improves economics learning on an advanced undergraduate or graduate level. Since most graduate economics education is conducted in a highly mathematical format, the results of this research could have a dramatic impact upon the future of economics education.

APPENDICES

Appendix A

QUESTIONNAIRE

INSTRUCTIONS: Please answer each question carefully and thoughtfully. Either check the number of the response which most nearly applies or fill in the blank.

1. Is your Principles of Economics course taught in a School of Business?
(1) yes _____, (2) no _____.

2. What is the duration of the Principles of Economics course for business and/or economics majors?
(1) one semester _____, (2) two semesters _____.

3. At which level is the Principles of Economics course taught?
(1) freshman _____, (2) sophomore _____.

4. Is your business/economics department accredited by the American Assembly of Collegiate Schools of Business (AACSB)?
(1) yes _____, (2) no _____.

5. Do you have math prerequisites for the Principles of Economics course?
(1) yes _____, (2) no _____.

6. Most of the students in your Principles class are pursuing which degree?
(1) B.A. _____, (2) B.S. _____, (3) B.B.A. _____.

7. In what type of institution do you teach the Principles course?
(1) publicly funded _____, (2) privately funded _____.

8. For your Principles of Economics class, does each professor choose his/her own text book or is it chosen by a committee for common use?
(1) each professor chooses _____, (2) committee chooses _____.

9. How many students do you have enrolled in your business/economics program?

Number of students:__________.

10. In sequencing the Principles course, is the macro or micro portion taught first?
(1) macro first _____, (2) micro first _____.

11. Have you won an award for excellence in teaching economics or business?
(1) yes _____, (2) no _____.

12. What is your age? __________(years).

13. Do you have a doctorate (Ph.D, D.A., D.B.A) in economics?
(1) yes _____, (2) no _____.

14. In what year did you receive the above degree? Year: __________.

15. Do you adhere to the short list view of teaching the Principles course (i.e., teaching only a few concepts thoroughly) or the long list view (i.e., teaching a more complete survey)?
(1) short list _____, (2) long list _____.

16. From the following partial list of quantitative approaches to economic problem solving that have been associated with the Principles Course/s at various levels, check the ones that you currently require your students to use in solving problems.

_____ Calculation of investment multiplier.
_____ Use of multiplier to calculate Y.
_____ Use of indexes in deflating time series data.
_____ Use of consumption equation to calculate C.
_____ Calculation of APC and MPC.
_____ Use of dynamic cobweb analysis.
_____ Comparative static analysis using total and partial derivatives and Cramer's rule.
_____ Graphical manipulation or reading.
_____ Calculation of elasticity of demand.
_____ Calculation of average and marginal costs.
_____ Calculation of equilibrium price and quantity.

(Please list any other quantitative approaches you currently require your students to use on the back of this form.)

17. What percentage of the student's final grade in your Principles of Economics course is derived from his/her ability to use quantitative approaches (as described in the list in number 16 above) to solve economic problems? (Consider the percentage and weight given to quantitative approaches on homework, quizzes, and exams.)

Percentage:__________.

18. Have you recently changed, or are you planning to change, the quantitative demands of your Principles of Economics course/s? If so, in what direction?
(1)_____ increased use of quantitative methods, (2)_____ decreased use of quantitative methods, (3)_____ no change.

______________________________ Signature ______________________________ Date

______________________________ Name of Your Institution

Appendix B

William Jennings

BRYAN COLLEGE

Christ Above All

William M. Lay, Jr.
Bryan College Box 7678
Box 7000
Dayton, TN 37321-7000
October 11, 1991

Dear Economics/Business Professor/Department Head:

By way of introduction, I am a professor of a Christian Coalition College (Bryan), and also a doctoral student at Middle Tennessee State University. My area of study is economics in the College of Business. In preparation for my dissertation research, and in the light of the recent concern over declining academic standards and rigor, I am doing a pilot study on the use of quantification in teaching the Principles of Economics course/s in the Coalition Colleges. Later, this will be compared to national standards at state and other universities. I hope this will result in some significant information which can be reported to the Coalition at a later date.

Please assist me in this investigation, if you would, by having each person at your college who currently teaches the Principles of Economics course/s complete one of the attached questionnaires. The questionnaires are very brief, and will take little time to finish. A self-addressed stamped envelope is provided in which to return the questionnaires.

Since this study is a Fall semester project, I would like to encourage you to return the completed questionnaires by **October 31.**

Thank you for your help in this project!

Cordially yours,

William M. Lay, Jr.

William M. Lay, Jr.
Assistant Professor of Business

Bryan Hill, Dayton, TN 37321-7000, (615) 775-2041

Appendix C

TABLE 4

ESTIMATED PILOT STUDY EQUATION
FOR USE OF MATHEMATICS (PCTQUANT)

Full Equation PCTQUANT (*Y*)		With Doctorate PCTQUANT (*Y*)		Without Doctorate PCTQUANT (*Y*)	
ONE	0.46 (3.45)	ONE	17.15 (1.29)	ONE	0.43 (1.52)
DURATN	0.05 (0.63)	DURATN	0.08 (0.83)	DURATN	-0.06 (-0.06)
UPRLVL	0.03 (0.63)	UPRLVL	-0.02 (-0.26)	UPRLVL	0.11 (1.38)
PREREQS	0.12 (1.81	PREREQS	0.14 (1.57)	PREREQS	0.12 (0.94)
TYPDEGR	-0.04 (-0.76)	TYPDEGR	0.06 (0.77)	TYPDEGR	-0.04 (-0.57)
TXTCHOCE	0.03 (0.56)	TXTCHOCE	0.04 (0.57)	TXTCHOCE	0.06 (0.46)
NUMSTUDS	-0.00 (-0.62)	NUMSTUDS	-0.00 (-0.07)	NUMSTUDS	-0.00 (-1.22)
MICROFST	-0.03 (-0.62)	MICROFST	-0.08 (-1.26)	MICROFST	-0.11 (-1.12)
EXCAWRD	-0.00 (-0.07)	EXCAWRD	-0.10 (-1.41)	EXCAWRD	0.15 (1.48)
AGE	-0.01 (-2.35)	AGE	-0.02 (-2.28)	AGE	-0.00 (-0.81)
DOCTOR	-0.01 (-0.25)	YRDEGRE	-0.01 (-1.23)		
SHORTLST	0.04 (0.82)	SHORTLST	0.05 (0.82)	SHORTLST	0.05 (0.67)
CHNGQUAN	0.02 (0.27)	CHNGQUAN	-0.13 (-1.62)	CHNGQUAN	0.13 (1.11)
Std.Err.	0.16	Std.Err.	0.143	Std.Err.	0.178
R^2	0.177	R^2	0.425	R^2	0.302
Adj.R^2	0.004	Adj.R^2	0.125	Adj.R^2	-0.047
F-Test	1.02	*F*-Test	1.42	*F*-Test	0.87

Notes: *T*-values reported in parenthesis.

Appendix D

QUESTIONNAIRE

INSTRUCTIONS: Please answer each question carefully and thoughtfully. Either check the blank by the response which most nearly applies or fill in the blank.

1. Is your Principles of Economics course taught in a department/school/college of Business/Economics rather than under the Social Science or other sponsorship?
   (1) yes____, (2) no____.

2. What is the duration of the Principles of Economics course for business and/or economics majors?
   (1) one semester/quarter____, (2) two semesters/quarters____, (3) other (specify)______________________.

3. At which level is the Principles of Economics course taught?
   (1) freshman____, (2) sophomore____, (3) other (specify)______________________.

4. Is your Business/Economics department accredited by the American Assembly of Collegiate Schools of Business (AACSB)?
   (1) yes____, (2) no____.

5. Do you have mathematics prerequisites for the Principles of Economics course?
   (1) yes____, (2) no____.

6. Is your institution publicly funded (as opposed to privately funded)?
   (1) yes____, (2) no____.

7. How many students does your institution have enrolled in your Business/Economics program/major?
   Number of students________.

8. In sequencing the Principles of Economics course, do you require that the students take the micro portion first?
   (1) yes____, (2) no____.

9. Have you won an award (either from your students or an outside agency) for excellence in teaching economics?
   (1) yes____, (2) no____.

10. Have you published an article on economics in a refereed economics journal?
    (1) yes____, (2) no____.

11. What is your age? ________(years).

12. Do you hold or are you currently working toward a **doctorate** (Ph.D., D.B.A., Ed.D, or D.A.) **in <u>economics</u>**?
    (1) yes____, (2) no____.

13. In what year did you receive the above doctorate? ________(year).

14. Do you adhere to the short-list view of teaching the principles course (i.e., teaching only a few concepts thoroughly) or the long-list view (i.e., teaching a more complete survey)?
    (1) short list____, (2) long list____.

15. Do you feel that the Principles of Economics course is a tool for the elimination of potentially unsuccessful students from the economics or business major?
    (1) yes____, (2) no____.

16. From the following list of mathematical approaches to economics problem solving that are commonly associated with the Principles of Economics course at different levels, **check the ones that you currently use.**
    ____Calculation of investment multiplier.
    ____Use of multiplier to calculate Y.
    ____Calculation of TR, AR, and MR.
    ____Calculation of Profit/Loss.
    ____Use of consumption equation to calculate C.
    ____Calculation of APC and MPC.
    ____Calculation of average and marginal product.
    ____Comparative static analysis using derivatives and Cramer's Rule?
    ____Calculation of elasticity of demand.
    ____Calculation of GNP, NNP, and NI.
    ____Calculation of *Laspeyres* and *Paasche* indexes.
    ____Calculation of price elasticity.
    ____Calculation of least-cost and profit-maximization.

17. To what degree do you feel that the use of mathematics in teaching economic theory is effective or necessary?
    (1) very effective____, (2) somewhat effective____, (3) somewhat ineffective____, (4) very ineffective____.

18. What percentage of the students' final grade in your Principles of Economics course is derived from his/her **<u>use of mathematics</u>** (as opposed to a purely verbal or graphical explanation) **<u>to solve economics problems</u>**? *(Consider the percentage and weight given to the use of mathematics on homework, quizzes, and exams.)*
    Percentage________.

19. To what degree do you feel constrained from using a mathematical approach in teaching the Principles of Economics course because of the lack of mathematical sophistication of your students?
    (1) very constrained____, (2) moderately constrained____, (3) slightly constrained____, (4) no constraint felt____.

20. Have you recently changed, or are you planning to change, the mathematical demands of your economics courses? If so, in what direction?
    (1) increased the use of mathematics____, (2) decreased the use of mathematics____, (3) no change____.

21. Do you feel that this questionnaire is biased? *(If so, please indicate how on the back, thanks.)*
    (1) yes____, (2) no____.

Appendix E

Economics and Finance Department
Middle Tennessee State University
Murfreesboro, Tennessee 37132

William M. Lay, Jr.
Bryan College, Box 7678
Box 7000
Dayton, TN 37321-7000

Dear Economics Professor/Department Head:

By way of introduction, I am an assistant professor of business at Bryan College, and a doctoral student at Middle Tennessee State University. My area of study is economics in the College of Business.

In the light of the recent debate over the increasing quantification of the discipline of Economics and the current concern over declining academic standards and rigor, I am conducting my dissertation research on the use of mathematics in teaching the Principles of Economics course. This is a nationwide cross-sectional study that I hope will result in some significant information on the contemporary state of economic education that can be later reported in an appropriate journal.

Please assist me in this investigation, if you would, by having a faculty member, or preferably two, at your college/university, who currently teach the Principles of Economics course/s, complete the attached questionnaire/s. The questionnaires are brief, involving only one page, and will take little time to finish. A self-addressed stamped envelope is provided in which to return the questionnaires.

Since this study is to be concluded in the Spring of 1993, I would like to encourage you to return the completed questionnaires by November 30. I would also be delighted to share my findings with you upon completion of the study should you request it.

I would be very grateful for your help in this project!

Cordially yours,

William M. Lay, Jr.

William M. Lay, Jr.
Assistant Professor of Business

Appendix F

TABLE 5

DESCRIPTION OF REGRESSION VARIABLES

Variable	Description
The Dependent Variable	
PCTMATH	The percentage of the students' grade in the principles class that is derived from the use of mathematics in solving economics problems.
The Independent Variables (Personal)	
MTHPROBS	The number from a list of thirteen mathematical approaches to principles level problem-solving, which are used by the professor.
PCTLECT	The percentage of the instructor's lecture time in which a mathematical approach is used in teaching economics concept or theory.
EXAWRD	Whether the instructor has won an award for excellence in teaching economics.
PUBJNL	Whether the instructor has published an article on economics in a refereed economics journal.
YRDCTR	When (year) the instructor received the doctorate.
SHTLST	Whether the instructor adheres to the short-list approach to the content in teaching the principles course.
ELMSTU	Whether the instructor feels that the Principles of Economics course is a tool for the elimination of potentially unsuccessful students from the economics major.
MTHCOM	Whether the instructor feels that mathematics is an effective economics communications vehicle.
CHGMTH	Whether the instructor has recently increased, or is planning to increase the mathematical demands of the principles course.

Table Continued

TABLE 5 (Continued)

DESCRIPTION OF REGRESSION VARIABLES

Variable	Description
The Independent Variables (Institutional)	
SCHBUS	Whether the principles of economics course is taught from a Business/Economics sponsorship.
DURATN	Whether the duration of the course is two semesters.
UPRLVL	Whether the principles course is addressed to students in their sophomore year or higher.
AACSB	Whether the program is taught from an AACSB accredited department.
PRERQS	Whether mathematics prerequisites are specified for the principles course.
PUBFND	Whether the institution is publicly funded.
NMSTUS	The number of students enrolled in the business/economics major.
MICFST	Whether the microeconomics portion is taught first.
CSTMTH	Whether the instructor feels constrained to use less mathematics because of the lack of mathematical sophistication of the students attracted to the institution.
SAT_M	Whether the average SAT mathematics sub-score of the students attracted by the institution contributes.

Appendix G

TABLE 6

INITIAL REGRESSION CORRELATION TABLE

NAME	SAT_M	MTHP	PCTL	SCHB	DURA	UPRL	AACS	PRER	PUBF	NMST
SAT_M	1.00	-.00	-.01	-.11	-.31	-.38	-.09	-.13	-.37	.29
MTHP	-.00	1.00	.21	.06	.00	.13	-.06	.07	.05	.03
PCTL	-.01	.21	1.00	.07	-.05	.07	.07	.07	.01	.05
SCHB	-.11	.06	.07	1.00	.06	.11	.18	.13	-.06	.10
DURA	-.31	.00	-.05	.06	1.00	.21	-.03	.06	.14	-.12
UPRL	-.38	.13	.07	.11	.21	1.00	.09	.29	.17	-.24
AACS	-.09	-.06	.07	.18	-.03	.09	1.00	.10	.36	.07
PRER	-.13	.07	.07	.13	.06	.29	.10	1.00	.08	.04
PUBF	-.37	.05	.01	-.06	.14	.17	.36	.08	1.00	.22
NUMS	.29	.03	.05	-.10	-.12	-.24	.07	.04	.22	1.00
MICF	.19	.07	-.02	-.04	-.13	-.01	.08	.19	-.11	.05
EXAW	.13	.07	.01	-.05	-.02	-.06	.03	.03	-.10	.00
PUBJ	.06	.01	-.08	-.05	-.00	-.11	.13	-.09	.10	.05
YRDC	.06	-.06	.04	-.03	-.12	-.01	.01	.00	-.08	-.09
SHTL	-.07	-.01	.03	.03	-.07	.09	-.08	.04	-.04	-.04
ELMS	.04	.06	-.02	-.04	.12	-.05	.05	.05	.02	.01
MTHC	-.01	.22	.22	.10	-.03	.04	.06	.18	.09	.05
CSTM	.20	-.10	-.07	-.04	-.16	-.06	-.03	-.12	-.06	.21
CHGM	.05	.12	.07	-.04	-.02	-.01	-.10	.09	-.04	-.05

Table Continued

Note: The names of the coefficients have been reduced to the first four letters to facilitate a more compact matrix.

TABLE 6 (Continued)

INITIAL REGRESSION CORRELATION TABLE

NAME	MICF	EXAW	PUBJ	YRDC	SHTL	ELMS	MTHC	CSTM	CHGM
SAT_M	.19	.13	.06	.06	-.07	.04	-.01	.20	.05
MTHP	.07	.07	.01	-.06	-.01	.06	.22	-.10	.12
PCTL	-.02	.01	-.08	.04	.03	-.02	.22	-.07	.07
SCHB	-.04	-.05	-.05	-.03	.03	-.04	.10	-.04	-.04
DURA	-.13	-.02	-.00	-.12	-.07	.12	-.03	-.16	-.02
UPRL	-.01	-.07	-.11	-.01	.09	-.05	.04	-.06	-.01
AACS	.08	.03	.13	.01	-.08	.05	.06	-.03	-.10
PRER	.19	.03	-.09	.00	.04	.05	.18	-.12	.09
PUBF	-.11	-.10	.10	-.08	-.04	.02	.09	-.06	-.04
NUMS	.05	.00	.05	-.09	-.04	.01	.05	.21	-.05
MICF	1.00	-.01	.06	.10	.01	.05	.10	-.03	.12
EXAW	-.01	1.00	.04	.01	-.04	-.03	-.04	-.01	-.01
PUBJ	.06	.04	1.00	-.13	-.02	.06	-.07	-.00	.00
YRDC	.10	.01	-.13	1.00	.19	-.16	-.05	.02	.04
SHTL	.01	-.04	-.02	.19	1.00	-.06	-.05	-.02	.00
ELMS	.05	-.03	.06	-.16	-.06	1.00	.11	-.05	.06
MTHC	.10	-.04	-.07	-.05	-.05	.11	1.00	-.22	.09
CSTM	-.03	-.01	-.00	.02	-.02	-.05	-.22	1.00	-.11
CHGM	.12	-.01	.00	.04	.00	.06	.09	-.11	1.00

Note: *The names of the coefficients have been reduced to the first four letters to facilitate a more compact matrix.*

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SELECTED BIBLIOGRAPHY

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