# A Cluster Analysis of Precalculus Student Performance on Function Translation Fluency Test Items

by

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This research is dedicated to Ed Gale, who showed me the incredible beauty inherent in mathematical functions and changed my world forever.  Thank you.

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# ABSTRACT

To determine the existence of individual and group differences in 199 precalculus students' abilities to interpret graphs of mathematical functions and relationships between graphical and algebraic function representations, two parallel cluster analyses were performed on the results of a ten item dichotomous test of mathematical function literacy and translation fluency. Cluster means from two hierarchical methods, Ward's ($N$ =199) and average linkage ($N$ =187), were used as seed values for two separate nonhierarchical k-means cluster analyses. Four clusters emerged from the Ward's based nonhierarchical solution and three clusters emerged from the average linkage based nonhierarchical solution. Both nonhierarchical solutions were externally validated using Likert-type scale items measuring mathematics anxiety, attitudes toward the precalculus teacher, visualization skills, and gender. Gender was associated with cluster membership for both nonhierarchical solutions. Student perception that the precalculus teacher skipped steps in demonstrating problems in lecture predicted cluster membership for both nonhierarchical solutions.

TABLE OF CONTENTS

## LIST OF TABLES

Page

Page

LIST OF FIGURES

# CHAPTER I: INTRODUCTION

Precalculus is the gateway college mathematics course for students pursuing degrees in science, technology, engineering, or mathematics (STEM) fields. The strictly procedural algebraic reasoning many precalculus students apply to analyzing mathematical functions is insufficient to provide deep understanding of functions as representations of rate of change, but function graphs show rates of change directly (Carlson, Oehrtman, & Engelke, 2010; Knuth, 2000). Mastery of functions, graphical interpretation, and rate of change have been identified as critical prerequisite skills for success in calculus (Carlson et al., 2010; Knuth, 2000). Student attrition from college-level mathematics courses and STEM majors is influenced by over-emphasis of procedural methods at the expense of graphical methods (Carlson et al., 2010; Herman, 2007). Differences in precalculus students' understanding of rate of change and mathematical functions may be related to their preference for the particular method they use to solve problems that involve mathematical functions. Precalculus students who use methods that exploit graphical properties and graphical interpretation of mathematical functions may have an advantage over students who use other methods of problem solving.

## Purpose of the Present Study

There appear to be two primary methods of approaching precalculus problems that involve mathematical functions: a static, procedural, and algebraic approach versus a dynamic, conceptual, and graphical approach. The second method includes the ability to mine the graphical representation of a function for information about its algebraic properties; in contrast, the first method does not acknowledge that the graphical representation provides any information about the function's algebraic properties. The key concept that differentiates the two approaches is translation between graphical and algebraic representations of mathematical functions. For the purposes of the present research, therefore, function translation fluency is defined as recognizing, interpreting, and predicting the behavior and properties of mathematical functions that are shown in their graphical representations together with the ability to extract graphical information from written algebraic representations of mathematical functions in order to solve problems.

It is thought that precalculus students will show individual differences in function translation fluency on a set of mathematical function problem solving tasks and that profiles of their problem solving methods can be developed. It is also thought that precalculus student function translation fluency is influenced by visualization ability, mathematics anxiety, student perceptions of precalculus teachers, and gender. Cluster analysis will be used to examine individual differences in function translation fluency and to create groups based on similarities in fluency and strategies used on the mathematical function tasks.

**Individual Differences in Mathematical Problem Solving Methods**

Prior studies have used cluster analysis to develop profiles of students based on the methods those students use to solve problems. Farrington-Flint, Vanuxem-Cotterill, and Stiller (2009) administered a set of addition and subtraction tasks to British first and second grade students and observed or asked each student for a verbal self-report of the strategy used to solve the problem; the strategies were coded in accordance with prior work. The four strategy categories: retrieval, counting, finger modeling, and other, together with the student's score on each task, were subjected to hierarchical cluster analysis. The results of the cluster analysis showed that the students could be classified into three distinct groups based on their performance and strategy used on the addition and subtraction tasks. The highest performing group used retrieval and counting strategies. The second group was not as accurate on task performance as the first, and also used counting strategies, but substituted finger modeling for retrieval. The third group had the poorest performance and almost exclusively used finger modeling or other methods to solve the problems. Individual differences in problem solving strategy choice accounted for 65% of the variability in the students' scores on the addition and subtraction tasks.

Hallett, Nunes, and Bryant (2010) developed tests of conceptual and procedural knowledge of fractions in order to examine how different types of students combined the two methods in solving fraction problems. The results of their cluster analysis showed that students could be classified into five distinct groups based on the extent to which they successfully used conceptual and procedural knowledge to solve fraction problems. Only one group appeared to be comfortable applying both types of reasoning to problems involving fractions; of two the remaining groups that successfully solved fraction problems, one clearly demonstrated a preference for conceptual methods and an aversion to procedural methods, with the reverse situation for the other group. Two distinct groups of students who had difficulty with fractions also emerged: one group struggled with the procedures that relate to fractions while the other struggled with the concept of fractions. The researchers concluded that these classification techniques should benefit students in two major ways:

> First, if children can be classified into different clusters of combining conceptual and procedural knowledge, teaching approaches can be tailored to match their current profile of conceptual and procedural understanding. Knowing the cluster to which a child belongs could help teachers identify weaknesses in their understanding, and these weaknesses could then receive some additional attention (i.e. children from the lower conceptual cluster would need some help in their conceptual understanding). Conversely, clusters could help identify strengths in different children's understanding, so instruction in new material might play to these strengths. *Thus, knowing more about individual differences could help teachers target instruction in ways that would be more suited to any given child* [emphasis added]. (p. 404).

The studies described above used cluster analysis to differentiate groups of students based on performance on mathematical tasks and, fundamentally, the student's way of thinking about the task. A cluster analysis of student performance on graphical and algebraic mathematical function translation tasks will provide meaningful classifications that could ultimately be used in the development of teaching methods and materials tailored to the needs of each group. Improving student function translation fluency is important for mastering the concept of function and therefore for success in STEM, as discussed in the following section.

**The Concept of Function and the Advantage of Graphical Representations**

Carlson et al. (2010) assert that "...the concept of function [is] the central conceptual strand of the mathematics curriculum, from algebra through calculus" (p. 115). A mathematical function is a mapping from an input set to an output set, where the mapping is the path taken to get from input items to output items. Barnier and Feldman (2000) offer the following conceptualization:

> The idea of a function plays a central role in mathematics and many other related fields. Intuitively, a function is a rule that assigns, to each given object in one set, a unique object in another set. For example, consider the rule that assigns, to the real number $x$, the real number $x^2$. If the given real number is 3, then the real number assigned to it is $3^2$, or 9. *Functions of this type are encountered in calculus* [emphasis added]. (p. 117)

The rule is expressed as an algebraic equation. It is this equation that students think of as *the* function; the importance and meaning of the input and output sets are ignored. Functions are introduced in the elementary grades of school mathematics, but the concept is not formalized with unique notation and meaning until high school algebra. In algebra and subsequent mathematics courses the notational, symbolic forms of functions and algebraic manipulations are heavily emphasized (Carlson et al., 2010; Gagatsis & Shiakalli, 2004; Herman, 2007; Knuth, 2000). This procedural emphasis may contribute to student attrition from college mathematics courses and STEM majors (Carlson et al., 2010; Herman, 2007).

When a function is presented in its graphical form, the input and output sets are instantly revealed simultaneously with the mapping between them. Students have been found to prefer symbolic manipulation to graphical representation and to struggle with translation between symbolic and graphical forms. This may indicate a limited understanding of the concept of functions and a bias toward procedural mathematics, most likely imprinted upon them by the formats modeled and preferred by their teachers (Gagatsis & Shiakalli, 2004; Herman, 2007; Knuth, 2000). This reliance on symbolic forms may hinder students' choice of problem solving strategies. Students may choose to use a more difficult symbolic strategy for a problem that is more easily solved through a graphical approach (Herman, 2007; Knuth, 2000). By selecting a more difficult approach, the student unintentionally places barriers of frustration around mathematical problem solving and develop strong negative attitudes toward mathematics. Enabling students to avoid this path by using graphical representations is worthwhile.

Graphical representations of functions offer a direct route to understanding a function as a relationship that expresses relative change between the input and output sets, and to meaningful interpretation of the function, that can be quite difficult to obtain from symbolic manipulation alone (Knuth, 2000). Students progressing through college precalculus and calculus courses absolutely need to have the ability to reason about a function as a dynamic model rather than a static procedure (Carlson et al., 2010). Graphical representations encapsulate this dynamism and directly show the critical aspect of rate of change.

If a student does not develop function translation fluency in precalculus, the properties and implications of rate of change can be very difficult to master. Because rate of change is the central concept in calculus and higher mathematics those students without fluency in function translation are not set up for success in those courses. Failure to develop function translation fluency may strongly increase the likelihood of student attrition from STEM majors. Factors that could negatively impact student mastery of graphical representations may include spatial visualization skills, mathematics anxiety attitudes toward mathematics, and gender differences (Ashcraft & Moore, 2009; Tolar, Lederberg, & Fletcher, 2009; Turner et al., 2002).

**Visualization Ability**

Spatial ability has been broadly defined as a combination of visualization and orientation. Visualization is the ability to mentally model the movement of objects in two- and three-dimensional space. Orientation is the ability to understand relationships between static and moving objects in two- and three-dimensional space (Ozer, 1987). The difference is that visualization deals with singular objects. Predicting where an object will move to when it is rotated is a visualization skill. Orientation skills are roughly equivalent to physics. Strong orientation ability is necessary for a student to develop a solid understanding of the mathematical concept of functions (Tolar et al., 2009). This is likely because mathematical functions are fundamentally descriptions of how to move from one set to another. The graphs of functions cannot be well interpreted without some sense of the relative orientation of the sets in space.

Visualization would seem to be a stepping stone to orientation. Tasks that assess visualization ability may provide some prediction of function translation fluency. Stavridou and Kakana (2008) defined a modified concept of spatial ability for learning mathematics called graphic ability. Graphic ability comprises three-dimensional mental representation of objects and mental translation between two- and three-dimensional views of objects. Problems that ask what a side view of a three-dimensional shape would look like assess graphic ability. Graphic ability is positively correlated with mathematical skill as well as positive student orientation toward mathematics (Stavridou & Kakana, 2008).

Tolar et al. (2009) also found that three-dimensional visualization ability is a predictor of performance on the SAT mathematics exam. The mathematics SAT includes non-procedural mathematics questions that cannot be solved by algebraic processes alone, so the use of graphical representations and translations is necessary. The cognitive aspects of visualization may also influence mathematical skill; Miller and Bichsel (2004) found that visual working memory was a predictor of performance on both simple and complex mathematical tasks.

Understanding and extending the two-dimensional relationships in graphs of functions as sets and mappings may be related to differences in overall visualization ability and visual translation ability. Function translation fluency may also be influenced by visualization skills since strong visualization is a predictor of mathematics performance as well as enjoyment. Since visualization skills are linked to enjoyment, there may be a relationship between visualization and mathematics anxiety as well.

**Mathematics Anxiety and Attitudes**

Mathematics anxiety levels influence grades, number and type of mathematics classes taken, and choice of college major (Ashcraft & Moore, 2009). Ashcraft and Moore (2009) define mathematics anxiety as "…a person's negative affective reaction to situations involving numbers, mathematics, and mathematical calculations." Mathematics anxiety, in general, is negatively correlated with performance on a wide variety of mathematical tasks across demographic and age groups. It has been shown that mathematics anxiety lowers mathematics performance; the effect is more pronounced as the difficulty level of the mathematical problems increases. Some explanations of this effect are based in cognitive psychology. One mechanism could be the result of the demands anxiety places on working memory; there are fewer resources available for problem solving because resources are diverted to serving the students' anxiety (Ashcraft & Krause, 2007; Ashcraft & Kirk, 2001). Another mechanism by which mathematics anxiety may impact mathematical performance is through interference with specifically visual working memory (Miller & Bichsel, 2004). Since problems requiring graphical interpretation of functions are among the more difficult problems in precalculus and involve visual abilities, it may be expected that students' anxiety levels increase when they are asked to solve function translation problems. Students with preexisting high levels of mathematics anxiety also likely achieve lower scores on function translation problems.

Student perceptions of mathematics also influence their levels of mathematics anxiety. Meece, Wigfield, and Eccles (1990) found that self-ratings of the importance of mathematics, mathematical ability, and student expectations of achievement in mathematics courses were predictors of mathematics anxiety. Precalculus students may self-identify as high achievers in mathematics with correspondingly high expectations of achievement in this new area of mathematics. Such expectations could lead to mathematics anxiety or mathematics avoidance as these students realize that the level of performance required in college is significantly higher than they may have expected. Failure to experience the immediate success in mathematics they have become accustomed to could negatively influence students' assessments of their own ability. Mathematics aversion and anxiety can also influence test performance, grades, and desire to remain in a STEM field for precalculus students.

**Gender Differences**

Gender differences in mathematics performance, mathematics anxiety, attitudes toward mathematics, and spatial thinking have been observed. Haynes, Mullins, and Stein (2004) found that test anxiety and ACT mathematics score influenced the mathematics anxiety levels of both male and female college students. As test anxiety increased, mathematics anxiety increased, but the directionality of the association between ACT mathematics score and mathematics anxiety was different for men than for women. Female students with higher mathematics ACT scores had higher mathematics anxiety; males with lower ACT scores had higher mathematics anxiety. In addition, for women, mathematics anxiety was negatively associated with both perception of mathematics ability and perception of high school mathematics teachers; neither was associated with mathematics anxiety for men.

Gender differences in emotional attitudes toward mathematics have also been observed. Frenzel, Pekrun, and Goetz (2007) examined the relationships between mathematics achievement and five emotional indicators: enjoyment, pride, anxiety, hopelessness, and shame, in a sample of fifth grade students in Germany. When girls and boys at comparable levels of mathematical achievement expressed their feelings about mathematics, boys reported higher pride, higher enjoyment, lower anxiety, lower hopelessness, and lower shame than girls. Girls also reported significantly lower perceptions of mathematical ability than boys at the same achievement levels.

Differences in spatial thinking ability between females and males have also been widely reported (Casey, Nuttall, & Pezaris, 2000; Geary, Saults, Liu, & Hoard, 2000; Miller & Bichsel, 2004; Quaiser-Pohl & Lehmann, 2002), with both skill and affective factors associated with these differences. Geary et al. (2000) examined the skill factors related to gender differences in college students' arithmetical reasoning and found that males were more skilled at solving arithmetical problems than females. Arithmetical reasoning was directly influenced by spatial and computational skill and indirectly influenced by gender. Gender also had direct effects on spatial and computational skill. Casey et al. (2000) also found gender differences among eighth grade students; there was a self-confidence advantage and a spatial ability advantage in problem solving for males; females did not show the same advantages. It seems that females are disadvantaged in problem solving tasks by both skill deficits and emotional factors, and at various ages. The correlation between perception of mathematical ability and performance for female college students was also seen by Quaiser-Pohl and Lehmann (2002).

**Review of Cluster Analysis**

As described by Hair, Anderson, Tatham, and Black (1998),

> Cluster analysis is the name for a group of multivariate techniques whose primary purpose is to group objects based on the characteristics they possess. Cluster analysis classifies **objects** (e.g., respondents, products, or other entities) so that each object is very similar to others in the cluster with respect to some predetermined selection-criterion. The resulting clusters of objects should then exhibit high internal (within-cluster) homogeneity and high external (between-

> cluster) heterogeneity. Thus, if the classification is successful, the objects within clusters will be close together when plotted geometrically, and different clusters will be far apart. (p. 473.)

The above description highlights the primary mathematical measure used in cluster analysis: similarity measures that can be interpreted geometrically. Similarity between objects in cluster analyses is measured in terms of the distance between them (Everitt, Landau, & Leese, 2001). Commonly used similarity measures include the Euclidean distance, the Minkowski distance, the simple matching coefficient, the Pearson correlation coefficient, and the Mahalanobis distance (Everitt et al., 2001). The similarity measure used for a specific cluster analysis is influenced by the type of data, categorical or continuous, (Everitt et al., 2001), and the extent of collinearity among the variables (Hair et al., 1998).

The choice of a similarity measure must be guided by the nature of the variables used to form the clusters (Everitt et al., 2001). For symmetric binary data, in which the response pairs (1,1) and (0, 0) should both be counted as matching pairs, one widely used similarity measure is the simple matching coefficient for binary data (Gan, Ma, & Wu, 2007). This coefficient can be thought of as the fraction of identical responses between two observations or between two people on a dichotomous test. More formally, the simple matching coefficient for symmetric binary data is defined by Gan, et al. (2007) as

$$s(x,y) = \frac{\sum_{i=1}^{d} x_i y_i + \sum_{i=1}^{d}(1-x_i)(1-y_i)}{d}, \tag{1}$$

where $\boldsymbol{x}$ is the binary response vector for person $x$, $\boldsymbol{y}$ is the binary response vector for person $y$, and $\boldsymbol{d}$ is the dimension of the response vector. However, if the clustering variables are highly correlated, the Mahalanobis distance may be a more appropriate similarity measure for symmetric binary data.

After an appropriate similarity measure is chosen, the next step in a cluster analysis is the selection of a procedure for forming the clusters. Two main types exist: hierarchical, which is subdivided into agglomerative and divisive methods, and nonhierarchical (Hair et al., 1998). At the start of all agglomerative hierarchical methods, there are $n$ clusters, one for each object. At each succeeding step, the two clusters with the highest degree of proximity are merged until only one cluster of size $n$ remains (Hair et al., 1998). Divisive methods begin with all objects in a single cluster and split into $n$ clusters based on the lowest degree of proximity (Hair et al., 1998). Commonly used agglomerative procedures include single linkage, complete linkage, average linkage, centroid linkage, median linkage, and Ward's method (Everitt et al., 2001). Since hierarchical procedures produce multiple numbers of clusters, the researcher must choose the ultimate number of clusters and the specific solution(s) to be submitted to external validation. The selection of a cluster solution is most often a heuristic process (Aldenderfer & Blashfield, 1984). Examination of the cluster means, $\chi^2$ tests, dendrograms, cubic clustering criterion (CCC), and practical factors can be used to determine the final number of clusters (Everitt et al., 2001; Hair et al., 1998). The pseudo $t^2$ and pseudo $F$ statistics are also frequently used.

Nonhierarchical procedures, in contrast, require the researcher to specify the ultimate number of clusters that will be formed *a priori*. The clusters are then formed around seed values, which may also be specified *a priori* or generated by the nonhierarchical clustering algorithm; commonly used nonhierarchical cluster analysis procedures include sequential threshold, parallel threshold, and optimization (Hair et al., 1998). Hierarchical methods may be used to produce sets of seed values that are sent to subsequent nonhierarchical procedures. A complete example of this combined procedure can be found in Hair et al. (1998).

Two of the more commonly used agglomerative hierarchical procedures are average linkage and Ward's method. These procedures use different methods to build the clusters, potentially generating very different cluster means solutions. The average linkage method, illustrated in Figure 1, calculates the mean distance between every pair of observations in every existing pair of distinct clusters and combines the clusters with the smallest mean distance.

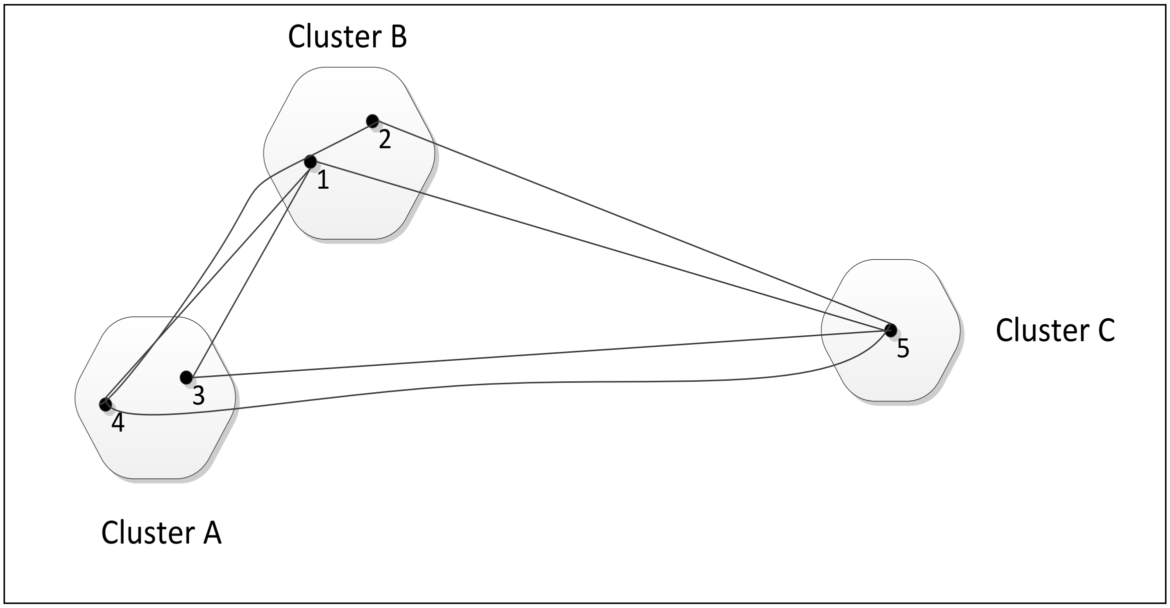



*Figure 1.* Average linkage clustering. Clusters A and B will be combined.

Average linkage combines clusters with small variances and is not as sensitive to outliers as Ward's method, but tends to create clusters that all have similar variances (Everitt et al., 2001; Hair et al., 1998). In contrast to average linkage, Ward's method is sensitive to outliers, and tends to produce clusters that are all roughly spherical and of similar size (Everitt et al., 2001; Hair et al., 1998). In Ward's method, as described by Everitt et al. (2001) the clusters are combined by "… [minimizing] the increase in the total within-cluster error sum of squares … [which] is proportional to the squared Euclidean distance between the centroids of the merged clusters" (p. 60-61). Ward's method of combining the clusters considers the cluster means, while average linkage does not. The within-cluster error sum of squares used in Ward's method is shown below (Everitt et al., 2001)

$$E \sum_{m=1}^{g} E_m \ , \qquad (2)$$

where

$$E_m = \sum_{l=1}^{n_m} \sum_{k=1}^{p} (x_{ml,k} - \bar{x}_{m,k})^2 \,, \qquad (3)$$

And where

$$\bar{x}_{m,k} = \left(\frac{1}{n_m}\right) \sum_{l=1}^{n_m} x_{ml,k} \,, \qquad (4)$$

where $x_{ml,k}$ is the score on the *k*th variable for the *l*th object in the *m*th cluster and $\bar{x}_{m,k}$ is the mean of the *m*th cluster for the *k*th variable.

The selection criteria for determining the number of clusters is based on visual inspection of the dendrograms, examination of the reasonableness of the cluster means, practical considerations, $\chi^2$ tests, the CCC, the pseudo *F,* and the pseudo $t^2$ statistics. For both the CCC and the pseudo *F* statistic, relatively larger values are indicative of good cluster solutions (SAS Institute, 2011). Peaks on a plot of the CCC versus the number of clusters where the CCC is larger than 2 or 3 are indicative of a good cluster solution; large negative values of the CCC are indicative of outliers (SAS Institute, 1983.) A large change in the value of the pseudo $t^2$ statistic between one number of clusters and the next lowest number of clusters also indicates a good solution (SAS Institute, 2011, "PROC CLUSTER"). Agreement among these statistics and well-defined cluster means should ultimately determine the final number of clusters selected.

Some researchers validate the hierarchical solution. Hair et al. (1998), however, recommend that the hierarchical cluster means be used as seed values in a nonhierarchical cluster analysis. Once the number of clusters is selected, the cluster means can be used as seed values for *k*-means clustering. *K*-means clustering uses an optimization algorithm that

> … allows for reassignment of objects. If, in the course of assigning objects, an object becomes closer to another cluster that is not the cluster to which it is currently assigned, then an optimizing procedure switches the object to the more similar (closer) cluster (Hair et al., 1998, p. 497).

The final steps in a cluster analysis are the interpretation and subsequent external validation of the cluster solution. Hair et al. (1998) suggest first that interpretation of the cluster solution be based on natural and meaningful descriptions of differences in the cluster means on the clustering variables, and second that this descriptive interpretation of the actual cluster solution be compared to the *a priori* theoretical groupings developed in the research hypothesis.

External validation of the cluster solution is the process of determining the strength and nature of relationship between cluster membership and the variables that were used to create the clusters or other external variables. If the researcher is interested in whether cluster membership can be predicted from the external variables, either discriminant analysis or logistic regression, statistical methods for predicting the classification of objects (Tabachnick & Fidell, 2007), may be used. An alternative method of external validation, if prediction is not of interest, may be to use ANOVA or MANOVA to determine if there are statistically significant group differences on the mean(s) of the external variable(s) by cluster membership. External validation is often performed with the goal of demonstrating that the cluster solution will generalize to the larger population or has criterion or predictive validity (Hair et al., 1998.)

**Objectives of the Current Research**

First, this study sought to develop profiles of precalculus students' function translation fluency based on their performance on 10 mathematical function problem solving tasks. This work built on the tasks and definitions in the work of Stavridou and Kakana (2008) and Gagatsis and Shiakalli (2004) and had similar intentions as in the research of Farrington-Flint et al. (2009) and Hallett et al. (2010); developing profiles of precalculus student function translation fluency may eventually contribute to individually tailored teaching and, it is hoped, retention of STEM students.

Second, this study also sought to investigate the extent to which function translation fluency group membership could be predicted by visualization ability, mathematics anxiety, student perceptions of precalculus teachers, and gender. Therefore, items from Likert type scales measuring mathematics anxiety, attitudes toward mathematics, teaching effectiveness, perceived teacher investment in students, and visualization ability self-ratings were used for external validation of the clusters. One difference between this study and the previously reviewed work is that a self-reported measure of visualization ability was used rather than an objective test. The reasons for the use of a self-report of visualization ability were that prior research indicated significant affective influences on actual visualization skills (Casey et al., 2000), and that the intent of the present research was that the only objective measures were items requiring function translation fluency skills.

It was thought that precalculus students would show individual differences in function translation fluency on a set of mathematical function problem solving tasks and that profiles of their problem solving methods could be developed. It was also thought that precalculus student function translation fluency is influenced by visualization ability, mathematics anxiety, student perceptions of precalculus teachers, and gender. Cluster analysis was used to examine individual differences in function translation fluency and to create groups based on similarities in fluency and strategies used on the mathematical function tasks.

The main research hypothesis was that at least two interpretable clusters of function translation fluency would emerge and that some of the clusters may have been predominantly male while others may have been predominantly female. The prediction of at least two clusters was based on the originally postulated two fundamental approaches to mathematical function problem solving, the algebraic method and the graphical method. Dominance of some of the clusters by a single gender was predicted based on the previously reviewed work which demonstrated that gender differences in mathematics task performance exist at various task content levels and across age groups. It was hoped that more than two clusters emerged and that some clusters showed more gender heterogeneity. Two secondary research hypotheses were dependent on the emergence of at least two interpretable clusters. One secondary research hypothesis was that profiles of students' function translation fluency could be developed from the results of the cluster analysis; the other was that cluster membership could be predicted by students' visualization ability, mathematics anxiety, student perceptions of precalculus teachers, and gender. It was hoped that items from at least two of these constructs would be predictors of function translation fluency group membership.

# CHAPTER II: METHOD

## Participants

All participants were volunteers selected from sections of precalculus classes being taught in the Fall 2011 semester at a mid-sized public university in the southeastern United States. A total of 209 students (94 female, 114 male, 1 unidentified) provided responses. No monetary compensation or course credit was provided. No demographic or identifying information was collected other than self-reported gender and age. All participants were at least 18 years of age. Ten respondents were dropped due to missing or invalid responses. The final sample consisted of 199 students (89 female, 110 male) active in precalculus at the time of the survey administration.

## Materials

Three polytomous measures developed by Rostorfer and Mateleska (2011) were used: attitude toward precalculus, student perception of precalculus teacher attitude and teaching ability, and self-rated visualization and spatial thinking skills. Each measure used a 5-point Likert-type scale, where 1 indicated “Strongly Agree” and 5 indicated “Strongly Disagree”. A dichotomous test of some properties of graphical and algebraic representations of mathematical functions was also used (Rostorfer & Mateleska, 2011).

All four measures were developed and pilot tested on a college student population of students enrolled in precalculus courses. Reliability estimates were derived from results obtained from a sample of 199 college level precalculus students, with 89 females and 110 males (Rostorfer & Mateleska, 2011).

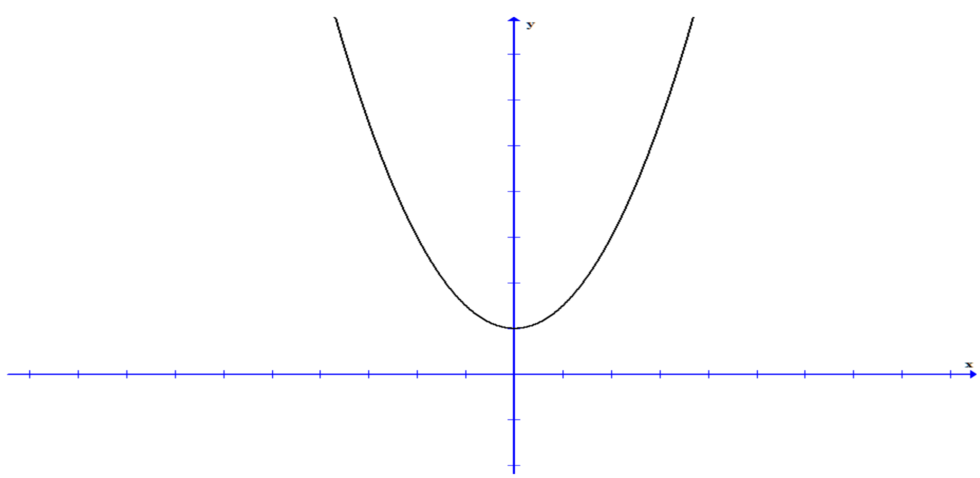
**Attitude toward precalculus.** The Precalculus Attitude Rating Scale (PARS) was used to measure student attitudes toward precalculus. The PARS is an eight item precalculus-specific self-rating of student mathematical ability, enjoyment, and anxiety. Sample items from the PARS include “I think precalculus is an easy class” and “I am nervous while taking a math test”. The Cronbach’s alpha reliability of the PARS was .85 (Rostorfer & Mateleska, 2011).

**Student perception of precalculus teacher attitude and teaching ability.** The Perception of Precalculus Teacher Scale (PTTS) was used to measure student perception of precalculus teacher attitude and teaching ability. The PTTS is a 13 item scale that asks students to rate their precalculus instructors’ attitude toward students and teaching effectiveness. Sample items from the PTTS include “I think my precalculus teacher enjoys teaching” and “I feel like my precalculus teacher makes the class difficult on purpose because he/she wants to “weed out” people”. The Cronbach’s alpha reliability of the PTTS was .91 (Rostorfer & Mateleska, 2011).

**Self-rated visualization and spatial thinking skill.** The Visio-Spatial Self-Assessment for Precalculus Students Scale (VSSA) was used to measure self-rated visualization and spatial thinking skill. The VSSA is a nine item scale that includes abstraction statements about two- and three-dimensional visualization ability (“I am good at mentally rotating 3-dimensional objects”), estimation problem statements (“I can tell by looking if a 2-dimensional object is symmetric”), and one specific precalculus situational statement (“When I have a math problem about transition points in a function, I can picture those points on the graph of that function"). The Cronbach's alpha reliability of the VSSA was .81 (Rostorfer & Mateleska, 2011).

**Graphical and algebraic representations of mathematical functions.** The Function Literacy and Translation Fluency Test was used to measure understanding of properties of graphical and algebraic representations of mathematical functions, i.e. mathematical function translation fluency. The FLTF is a 10 item dichotomous mathematical functions test composed of four content areas: Graphical Literacy (items 1 and 5), Graphical Interpretation (items 2, 7, and 10), Slope Recognition (items 3 and 6), and Function Property Line Tests (items 4, 8, and 9). Six items show graphical representations of functions; an example is shown in Figure 2.

What type of function does this graph show?




(a) cubic (b) linear (c) quadratic (d) rational

*Figure 2*. FLTF Graphical Literacy item.

The remaining four items on the FLTF are algebraic, written items that ask the student to interpret an essentially graphical property of a function, such as "Which function increases faster, $x^2$ or $x^3$, on the interval (0, 1)?". Higher scores on the FLTF indicate that the student is more fluent in function translation and more literate in the basic properties of mathematical functions commonly seen in college precalculus. The Cronbach's alpha reliability for the FLTF was .35 (Rostorfer & Mateleska, 2011).

**Procedures**

The four scales were administered together as a single survey during class periods in multiple sections of precalculus in October and November 2011. The scales were only administered to students in precalculus class sections whose instructors had previously agreed to allow the researcher 25 minutes of class time to administer the survey. Participants completed the survey on a standard Scantron sheet. Missing data accounted for at most 1.5% of cases; as the percentage of missing data was less than 5% all missing data was deleted listwise. From prior analyses it is known that the data did not significantly violate the assumptions of multivariate normality (Rostorfer, 2011). An alpha of .05 was used for all statistical tests unless otherwise indicated. Data analysis was done using SAS software.

# CHAPTER III: RESULTS

Two parallel cluster analysis procedures were performed. In both procedures, the cluster means were extracted from a hierarchical cluster analysis solution and used as the seed values to a nonhierarchical (k-means) clustering. The nonhierarchical cluster solution was interpreted with respected to the quality of its separation of the clusters and its meaning with respect to the FLTF test. Last, stepwise logistic regression was used to determine which items from the PARS, PTTS, and VSSA, along with gender, could be used to predict cluster membership for that nonhierarchical solution.

The simple matching coefficient for binary data, shown in Equation 1, was chosen as the similarity measure for both hierarchical cluster analyses since the responses to the FLTF items were coded as “1” indicating a correct response and “0” indicating an incorrect response. Complete flowcharts of the hierarchical and subsequent nonhierarchical cluster analyses appear in Figures 3 and 4, respectively.

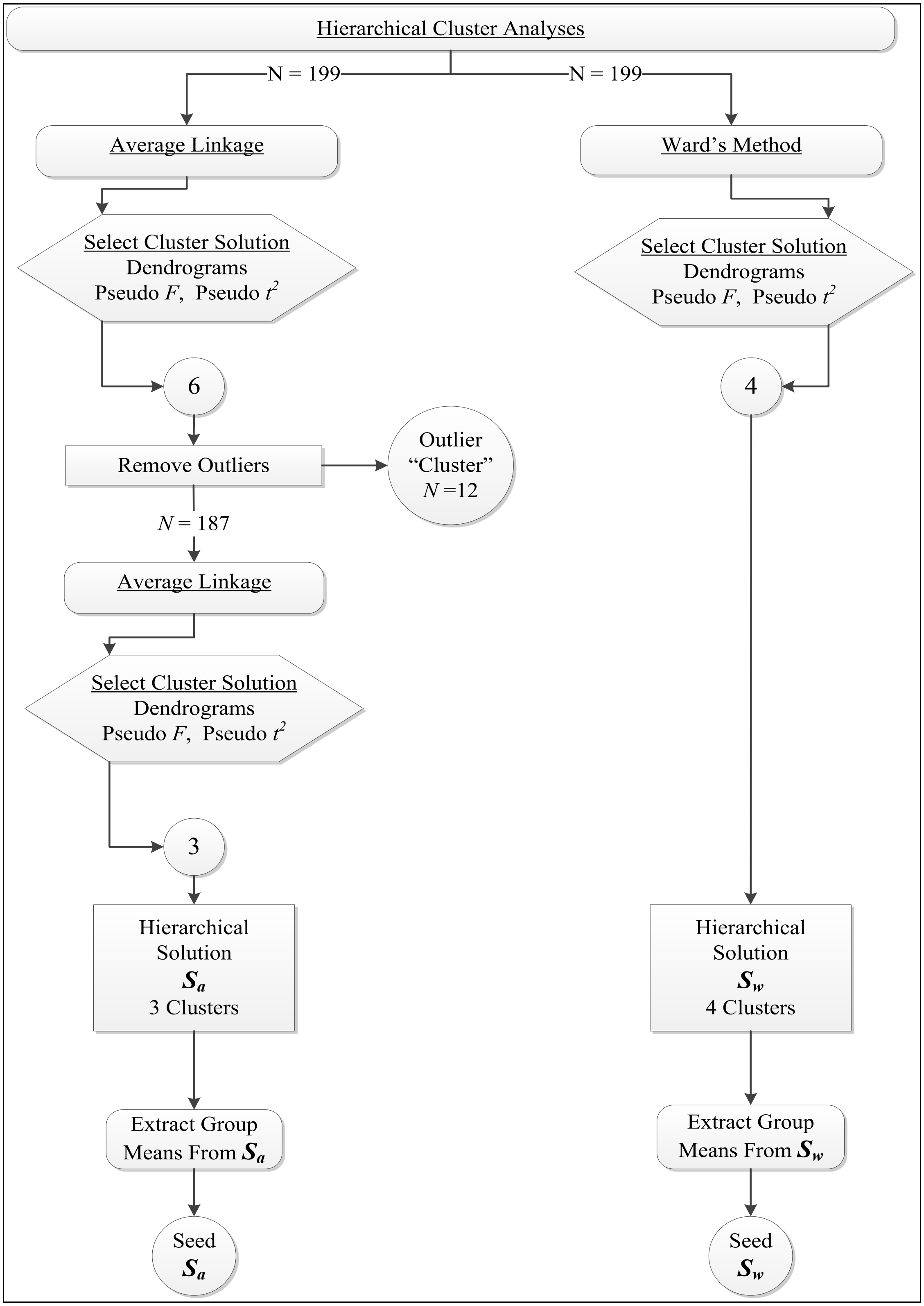



*Figure 3.* Hierarchical cluster analyses flow chart.

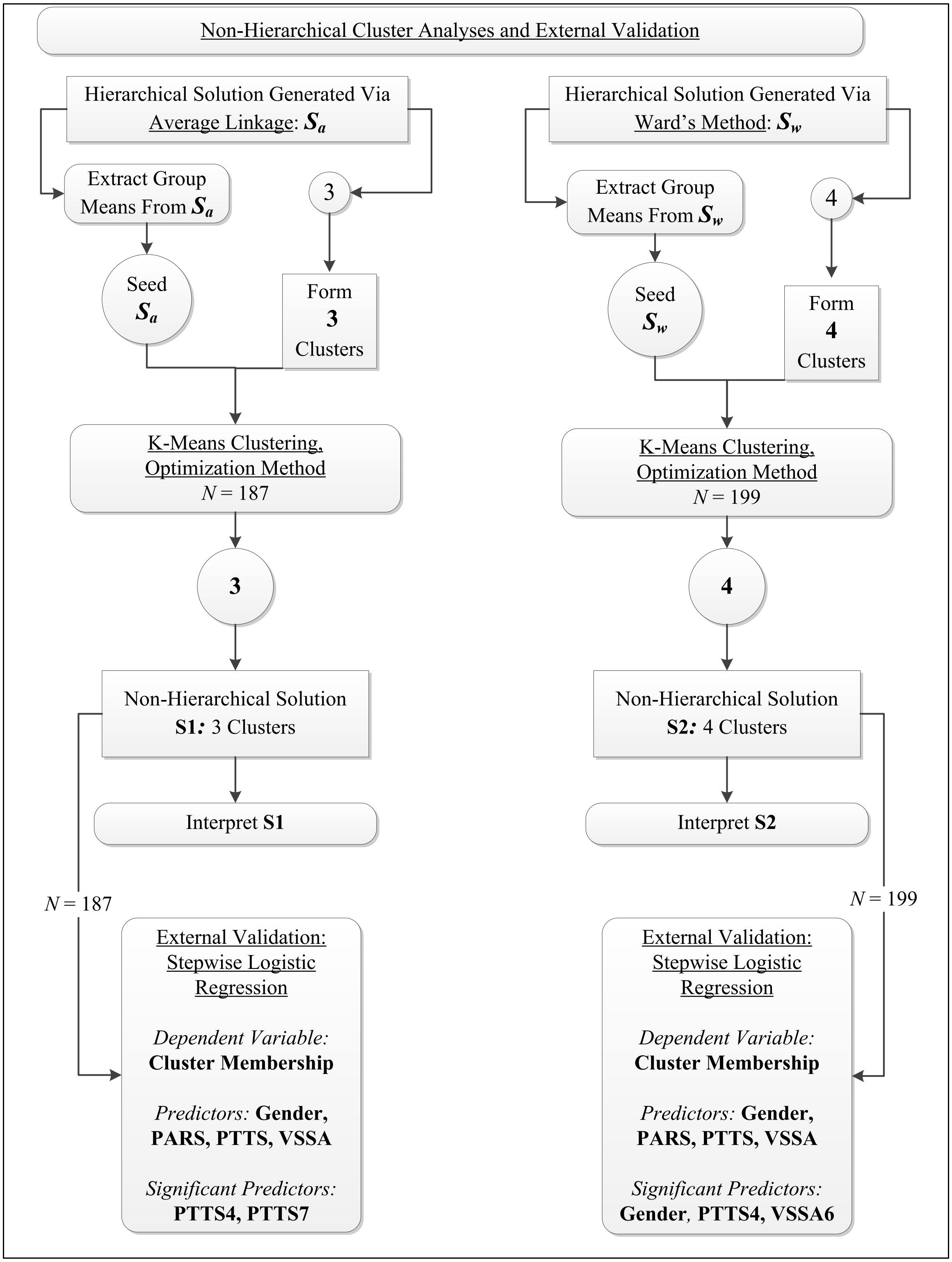



*Figure 4.* Nonhierarchical cluster analyses and external validation flow chart.

The correlation matrix for the FLTF items is shown in Table 1.

Table 1

*Correlations Among FLTF Items*

| Item | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | — | | | | | | | | | |
| 2 | .09 | — | | | | | | | | |
| 3 | **.17** | .01 | — | | | | | | | |
| 4 | -.05 | -.03 | .06 | — | | | | | | |
| 5 | .07 | .06 | -.04 | .03 | — | | | | | |
| 6 | **.15** | .01 | **.30** | -.02 | -.07 | — | | | | |
| 7 | .13 | .05 | .02 | .03 | .11 | -.06 | — | | | |
| 8 | .10 | .06 | .09 | .03 | **.18** | .02 | .05 | — | | |
| 9 | .03 | **.15** | -.10 | .05 | .09 | .07 | -.05 | **.28** | — | |
| 10 | -.01 | **.16** | -.01 | -.04 | .02 | -.04 | -.08 | -.10 | .13 | — |
| Note. | ***p* < .05** | | | | | | | | | |

Although the correlations between several items were statistically significant at the .05 level, none of the correlations among the FLTF items were above a predetermined cutoff of |.50|. Since multicollinearity among the clustering variables was not a concern, the simple matching coefficient for binary data defined in Equation 1 was used as the similarity measure between student response vectors. The distance matrix used in all hierarchical cluster analyses was a transformation of the simple matching coefficients into equivalent Euclidean distances, DMATCH, generated by the SAS DISTANCE procedure.

In the remainder of the results section, the Ward's Method based cluster analyses results and the Average Linkage method based results will be discussed separately.

**Ward's Method Based Analyses**

A four cluster solution was selected from the hierarchical cluster analysis that used Ward's Method to build the clusters, based on agreement between the pseudo $F$ statistic, the pseudo $t^2$ statistic, and the dendrogram. Both the pseudo $F$ and pseudo $t^2$ statistics indicated that four cluster and three cluster solutions were plausible, based on mutual agreement in local peaks of plots of those values. A four cluster solution was chosen over a three cluster solution because the cluster means for four clusters (Table 2) made more theoretical sense than did those of the three cluster solution.

Table 2

*Proportion of Correct Responses for Ward's Hierarchical Solution*

| FLTF Skill | Item | Cluster 1 ($n$ = 47) | Cluster 2 ($n$ = 32) | Cluster 3 ($n$ = 72) | Cluster 4 ($n$ = 48) | Overall ($n$ = 199) |
|---|---|---|---|---|---|---|
| Graphical | 1 | .91 | .88 | .86 | .67 | .83 |
| Literacy | 5 | .79 | .97 | .92 | .83 | .87 |
| *Mean* | | .85 | .92 | .89 | .75 | .85 |
| Graphical | 2 | .32 | .16 | .47 | .35 | .36 |
| Interpretation | 7 | .11 | .53 | .21 | .06 | .20 |
| | 10 | .51 | .03 | .68 | .75 | .55 |
| *Mean* | | .31 | .24 | .45 | .39 | .37 |
| Slope | 3 | .89 | .91 | .88 | .88 | .88 |
| Recognition | 6 | 1.00 | .91 | 1.00 | .94 | .97 |
| *Mean* | | .95 | .91 | .94 | .91 | .93 |
| Function | 4 | .02 | .88 | .64 | .92 | .60 |
| Property | 8 | .43 | .84 | .74 | .21 | .55 |
| Line Tests | 9 | .02 | .03 | .94 | .00 | .35 |
| *Mean* | | .16 | .58 | .77 | .38 | .50 |
| *Grand Mean* | | .50 | .61 | .73 | .56 | .62 |

Since the hierarchical solution resulted in four clusters, the k-means algorithm was restricted to optimize to four clusters. The cluster means from Table 2 were used as the seed values for the k-means clustering performed via the SAS FASTCLUS procedure. Table 3 shows the proportion of correct responses by cluster and FLTF skill for the nonhierarchical solution. The cluster profiles for the nonhierarchical solution (Table 3) exhibit a high degree of homogeneity with the cluster profiles from the hierarchical solution (Table 2), which indicates the stability of Ward's Method for clustering the

FLTF responses. See Appendix B for further details on the similarities between the cluster profiles for the hierarchical and nonhierarchical solutions.

Table 3

*Proportion of Correct Responses for Ward's Nonhierarchical Solution*

| FLTF Skill | Item | Cluster | | | | |
|---|---|---|---|---|---|---|
| | | 1 ($n$ = 46) | 2 ($n$ = 52) | 3 ($n$ = 34) | 4 ($n$ = 67) | Overall ($n$ = 199) |
| Graphical | 1 | .72 | .85 | .88 | .87 | .83 |
| Literacy | 5 | .85 | .81 | .94 | .91 | .87 |
| *Mean* | | .78 | .83 | .91 | .89 | .85 |
| Graphical | 2 | .35 | .33 | .18 | .48 | .36 |
| Interpretation | 7 | .09 | .12 | .53 | .18 | .20 |
| | 10 | .78 | .54 | .06 | .66 | .55 |
| *Mean* | | .41 | .33 | .25 | .44 | .37 |
| Slope | 3 | .91 | .87 | .94 | .85 | .88 |
| Recognition | 6 | .98 | .96 | .94 | .99 | .97 |
| *Mean* | | .95 | .91 | .94 | .92 | .93 |
| Function | 4 | 1.00 | .00 | .88 | .64 | .60 |
| Property | 8 | .22 | .38 | .82 | .78 | .55 |
| Line Tests | 9 | .02 | .04 | .00 | 1.00 | .35 |
| *Mean* | | .41 | .14 | .57 | .81 | .50 |
| *Grand Mean* | | .59 | .49 | .62 | .73 | .62 |

Referring to Table 3, it is clear that Ward's nonhierarchical solution did achieve appropriate groupings of students by their performance on the FLTF items. Cluster 2 contained the lowest performing students; their results on each of the four skill areas tested by the FLTF were below the overall mean. Cluster 4 contained the highest performing students. Clusters 1 and 3 were similar in overall performance; Cluster 1 was stronger in Graphical Interpretation, while cluster 3 was stronger in both Graphical Literacy and Function Property Line Tests. Figure 5 shows a three-dimensional separation of the nonhierarchical clusters; despite the few misclassified members of clusters 1 and 2, the four cluster nonhierarchical solution did achieve a reasonably high degree of discrimination between clusters along three dimensions.

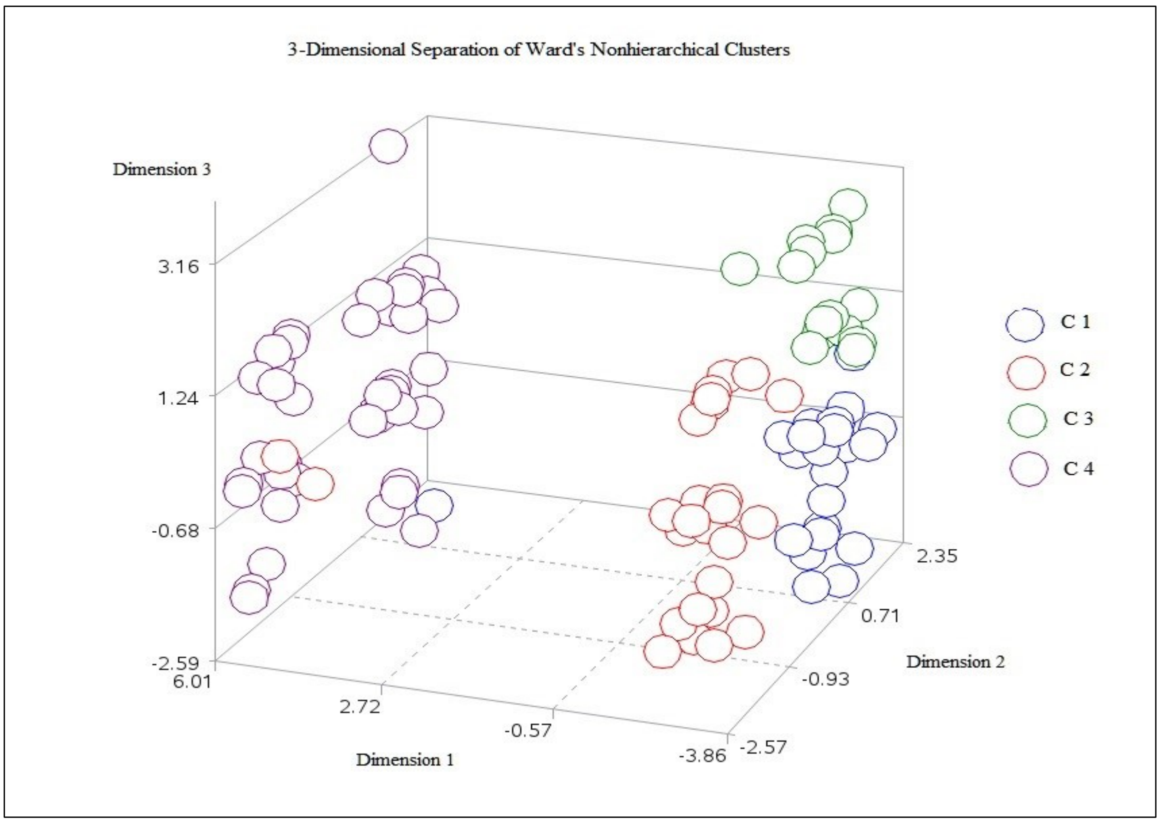



*Figure 5.* Three-dimensional separation of Ward's nonhierarchical clusters.

External validation of the Ward's nonhierarchical solution results indicated that a student's cluster membership could be predicted by one item from the PTTS, one item from the VSSA, and gender. In the stepwise logistic regression, item PTTS4 was added in Step 1, Wald $\chi^2(3, N = 192) = 16.40$, $p = .001$; item VSSA6 was added in Step 2, Wald $\chi^2(3, N = 192) = 9.03$, $p = .029$; gender was added in the final step, Step 3, Wald $\chi^2(3, N = 192) = 8.33$, $p = .040$. Cluster 3, as shown in Table 3, was used as the reference group. The logistic regression model is shown in Table 4.

Table 4

*Stepwise Logistic Regression Model for Predicting Cluster Membership From Ward's Nonhierarchical Solution*

| Parameter | Cluster | $\beta$ | *SE* | *p* | Odds Ratio | 95% CI for Odds Ratio | |
|---|---|---|---|---|---|---|---|
| | | | | | | LL | UL |
| Intercept | 1 | 2.24 | 0.97 | .0203 | | | |
| | 2 | 1.16 | 0.98 | .2361 | | | |
| | 4 | 0.08 | 0.99 | .9353 | | | |
| ptts4 | 1 | 0.18 | 0.18 | .3317 | 1.20 | 0.83 | 1.72 |
| | 2 | 0.39 | 0.18 | .0321 | 1.47 | 1.03 | 2.10 |
| | 4 | 0.65 | 0.18 | **.0003** | 1.92 | 1.35 | 2.73 |
| vssa6 | 1 | -0.78 | 0.26 | **.0029** | 0.46 | 0.27 | 0.77 |
| | 2 | -0.42 | 0.26 | .1062 | 0.66 | 0.40 | 1.09 |
| | 4 | -0.43 | 0.25 | .0892 | 0.65 | 0.40 | 1.07 |
| sex | 1 | -0.58 | 0.52 | .2630 | 0.56 | 0.20 | 1.54 |
| Male vs | 2 | -1.34 | 0.51 | **.0087** | 0.26 | 0.10 | 0.71 |
| Female | 4 | -0.41 | 0.50 | .4087 | 0.66 | 0.25 | 1.76 |

Note. $N = 192$. Reference Cluster = Cluster 3. $\boldsymbol{p < .0167}$.

Odds ratio comparisons for the three significant predictors of cluster membership are shown in Table 5. The odds ratio comparison between clusters 4 and 3 for PTTS4, 1.92, 95% C.I. [1.35, 2.73], indicates that the odds of a student belonging to the higher performing group, cluster 4, were 1.92 times greater for every one point increase in disagreement with the statement “It is hard for me to understand my pre-calculus teacher in lecture because I feel like he/she skips steps in demonstrating problems.” Similarly, the odds ratio comparison between clusters 1 and 4, 1.61, 95% C.I. [1.19, 2.18] indicates that the odds of a student belonging to the higher performing group, cluster 4, were 1.61 times greater for every one point increase in disagreement with the statement “It is hard for me to understand my pre-calculus teacher in lecture because I feel like he/she skips steps in demonstrating problems.” The odds ratio comparison between clusters 1 and 3 for VSSA6, 2.19, 95% C.I. [1.31, 3.66] indicates that the odds of a student belonging to cluster 3 rather than cluster 1 increase by a factor of 2.19 for every one point increase in disagreement with the statement “I am good at estimating the sizes of angles inside shapes.” Finally, although gender was a significant predictor of cluster membership as determined in the stepwise logistic regression, none of the individual odds ratios were significant at the required alpha cutoff for the clusterwise comparisons; however, the odds ratio comparison between clusters 2 and 3 for sex did approach significance.

Table 5

*Odds Ratios for External Validation of Ward's Nonhierarchical Solution*

| | PTTS4 | | VSSA6 | | SEX (M* vs F) | |
|---|---|---|---|---|---|---|
| | OR | *p* | OR | *p* | OR | *p* |
| C1 vs C3 | 0.84 | .3317 | **2.19** | **.0029** | 1.78 | .2630 |
| C2 vs C3 | 0.68 | .0321 | 1.52 | .1062 | 3.82 | .0087 |
| C3 vs C4 | **1.92** | **.0003** | 0.65 | .0892 | 0.66 | .4087 |
| C1 vs C2 | 1.232 | .1839 | 1.44 | .1010 | 0.47 | .0939 |
| C4 vs C2 | 0.77 | .0714 | 1.01 | .9557 | 0.40 | .0265 |
| C1 vs C4 | **1.61** | **.0023** | 1.43 | .0999 | 1.18 | .7044 |

Note. Reference cluster = cluster on left. *Reference group is Males. PTTS4 = It is hard for me to understand my pre-calculus teacher in lecture because I feel like he/she skips steps in demonstrating problems; VSSA6 = I am good at estimating the sizes of angles inside shapes; 1 = Strongly Agree, 5 = Strongly Disagree; ***p* < .0083.**

Desciptive statistics for the significant predictors of Ward's nonhierarchical cluster membership are shown in Table 6. Given the significant odds ratios (Table 5), these relatively large differences in mean scores were expected.

Table 6

*Descriptive Statistics for External Variables From Ward's Nonhierarchical Solution*

| Variable | | Cluster 1 ($n$ = 46) | Cluster 2 ($n$ = 52) | Cluster 3 ($n$ = 34) | Cluster 4 ($n$ = 67) |
|---|---|---|---|---|---|
| PTTS4 | *M* | 2.85 | 3.06 | 2.47 | 3.67 |
| | *SD* | 1.59 | 1.36 | 1.16 | 1.30 |
| VSSA6 | *M* | 2.39 | 2.75 | 3.00 | 2.60 |
| | *SD* | 0.95 | 1.01 | 1.04 | 0.95 |
| Sex | *% Female* | 43.48 | 61.54 | 41.18 | 34.33 |
| | *% Male* | 56.52 | 38.46 | 58.82 | 65.67 |

Note. PTTS4 = It is hard for me to understand my pre-calculus teacher in lecture because I feel like he/she skips steps in demonstrating problems; VSSA6 = I am good at estimating the sizes of angles inside shapes; 1 = Strongly Agree, 5 = Strongly Disagree.

**Average Linkage Method Based Analyses**

Because the initial average linkage hierarchical cluster analysis indicated the presence of outliers, this analysis proceeded in two steps (Figure 3). A six cluster solution was selected from the first average linkage method hierarchical cluster analysis, based on agreement between the pseudo $F$ statistic, the pseudo $t^2$ statistic, and the dendrogram. However, this six cluster solution contained four clusters with six or fewer observations in each cluster, for a total of 12 outlier observations. These 12 outliers were subsequently removed from the data set and a second hierarchical cluster analysis using average linkage was performed on the reduced data set of 187 observations. A three cluster solution was selected from this second hierarchical average linkage method cluster analysis, again based on mutual agreement between the pseudo $F$ statistic, the pseudo $t^2$ statistic, and the dendrogram. Descriptive statistics for the outliers removed prior to the second analysis, $N = 12$, are shown in Tables A4 through A12 (Appendix A); the proportion of correct responses for the outlier group are shown in Table A13. The cluster means for the three cluster solution are shown in Table 7.

Table 7

*Proportion of Correct Responses for Average Linkage Hierarchical Solution*

| FLTF Skill | Item | Cluster 1 ($n$ = 94) | Cluster 2 ($n$ = 35) | Cluster 3 ($n$ = 58) | Overall ($n$ = 187) |
|---|---|---|---|---|---|
| Graphical Literacy | 1 | .82 | .94 | .91 | .87 |
| | 5 | .83 | .97 | .97 | .90 |
| *Mean* | | .83 | .96 | .94 | .89 |
| Graphical Interpretation | 2 | .02 | .97 | .52 | .35 |
| | 7 | .20 | .29 | .17 | .21 |
| | 10 | .37 | .60 | .83 | .56 |
| *Mean* | | .20 | .62 | .51 | .37 |
| Slope Recognition | 3 | .96 | .97 | .83 | .92 |
| | 6 | .98 | 1.00 | 1.00 | .99 |
| *Mean* | | .97 | .99 | .92 | .95 |
| Function Property Line Tests | 4 | .60 | .60 | .62 | .60 |
| | 8 | .45 | .57 | .78 | .57 |
| | 9 | .13 | .03 | .91 | .35 |
| *Mean* | | .39 | .40 | .77 | .51 |
| *Grand Mean* | | .54 | .69 | .75 | .63 |

Since the hierarchical solution resulted in three clusters, the k-means algorithm was restricted to optimize to three clusters. The cluster means from Table 7 were used as seed values for the k-means clustering performed via the SAS FASTCLUS procedure. Table 8 shows the proportion of correct responses by cluster and FLTF skill for the nonhierarchical solution. The cluster profiles for the nonhierarchical solution (Table 8) are not as strongly homogeneous with the cluster profiles from the hierarchical solution (Table 7) as in the Ward's Method based analyses, most notably with regard to the Function Property Line Tests section of the FLTF. See Appendix B for further details on the similarities and differences between the cluster profiles for the hierarchical and nonhierarchical solutions.

Table 8

*Proportion of Correct Responses for Average Linkage Nonhierarchical Solution*

| FLTF Skill | Item | Cluster | | | |
|---|---|---|---|---|---|
| | | 1 (*n* = 64) | 2 (*n* = 88) | 3 (*n* = 35) | Overall (*n* = 187) |
| Graphical Literacy | 1 | .88 | .84 | .94 | .87 |
| | 5 | .95 | .85 | .91 | .90 |
| *Mean* | | .92 | .85 | .93 | .89 |
| Graphical Interpretation | 2 | .48 | .00 | 1.00 | .35 |
| | 7 | .17 | .23 | .23 | .21 |
| | 10 | .70 | .42 | .63 | .56 |
| *Mean* | | .45 | .22 | .62 | .37 |
| Slope Recognition | 3 | .86 | .94 | .97 | .92 |
| | 6 | 1.00 | .98 | 1.00 | .99 |
| *Mean* | | .93 | .96 | .99 | .95 |
| Function Property Line Tests | 4 | .64 | .60 | .54 | .60 |
| | 8 | .77 | .43 | .57 | .57 |
| | 9 | 1.00 | .02 | .00 | .35 |
| *Mean* | | .80 | .35 | .37 | .51 |
| *Grand Mean* | | .75 | .53 | .68 | .63 |

Referring to Table 8, it is clear that the average linkage nonhierarchical solution did achieve appropriate groupings students by their performance on the FLTF items. Cluster 2 contained the lowest performing students; their results on three of the four skill areas, Graphical Literacy, Graphical Interpretation, and Function Property Line Tests, were below the overall mean. The students in cluster 2 did score above the overall mean in Slope Recognition, but averaged lower scores on this skill than did the students in cluster 1. Cluster 1 contained the highest overall mean performance, and those students far outscored the students in clusters 2 and 3 on Function Property Line Tests, but had the lowest mean score on item 3 of Slope Recognition. The students in cluster 3 achieved the highest performance on Graphical Interpretation and were similar to those in cluster 1 on Graphical Literacy. Figure 6 shows a two-dimensional separation of the nonhierarchical clusters. Although there are two members of cluster 2 that are misclassified into cluster 1, no members of cluster 3 are misclassified; there is a bimodal distribution of the members of cluster 1, which may indicate anomalies in this cluster with respect to a particular FLTF item response.

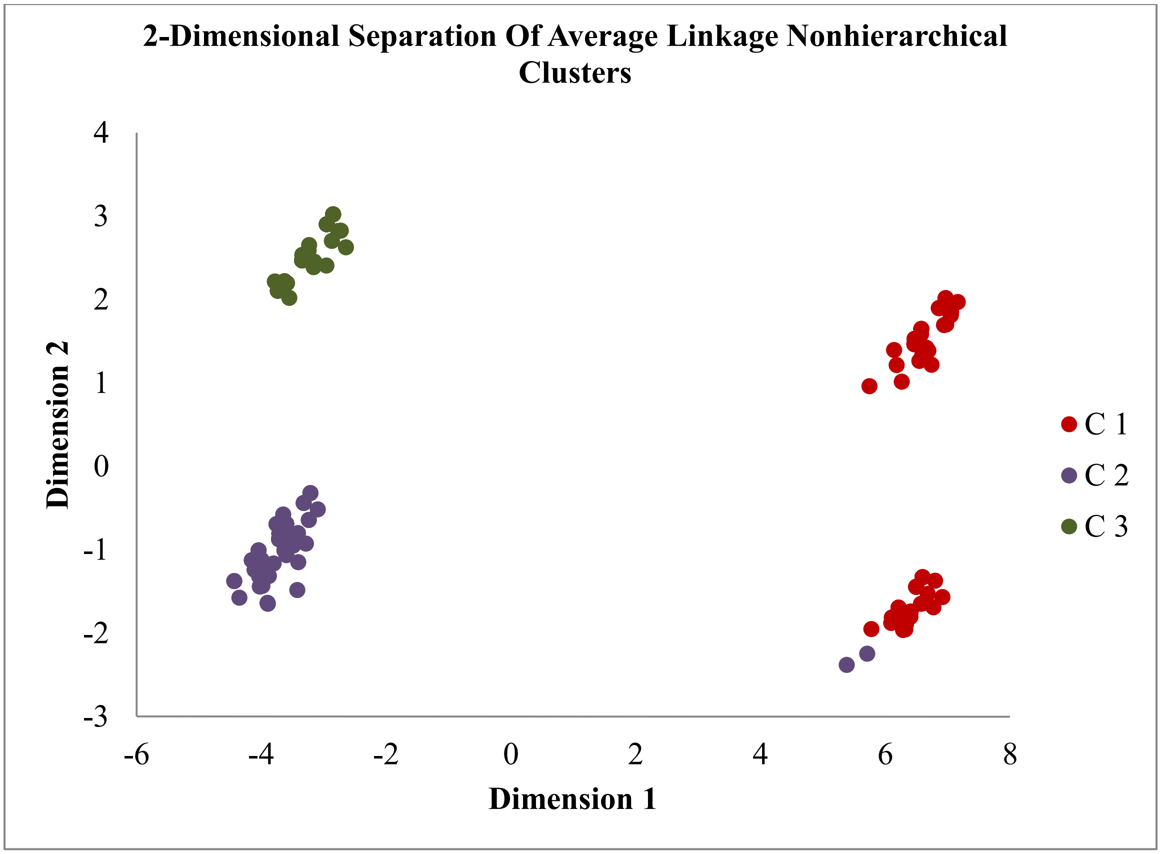



*Figure 6.* Two-dimensional separation of Average Linkage nonhierarchical clusters.

External validation of the average linkage nonhierarchical solution results indicated that a student's cluster membership could be predicted by two items from the PTTS. In the stepwise logistic regression, item PTTS4 was added in Step 1, Wald $\chi^2(2, N = 181) = 15.76$, $p < .001$; item PTTS7 was added in Step 2, Wald $\chi^2(2, N = 181) = 6.45$, $p = .040$. Cluster 3, as shown in Table 8, was used as the reference group. The logistic regression model is shown in Table 9.

Table 9

*Stepwise Logistic Regression Model for Predicting Cluster Membership From Average Linkage Nonhierarchical Solution*

| Parameter | Cluster | $\beta$ | *SE* | *p* | Odds Ratio | 95% CI for Odds Ratio | |
|---|---|---|---|---|---|---|---|
| | | | | | | LL | UL |
| Intercept | 1 | -0.67 | 0.64 | .2909 | | | |
| | 2 | 0.37 | 0.58 | .5189 | | | |
| ptts4 | 1 | 0.35 | 0.18 | .0516 | 1.42 | 1.00 | 2.02 |
| | 2 | -0.24 | 0.17 | .1502 | 0.79 | 0.57 | 1.09 |
| ptts7 | 1 | 0.04 | 0.18 | .8226 | 1.04 | 0.73 | 1.48 |
| | 2 | 0.37 | 0.18 | .0332 | 1.44 | 1.03 | 2.02 |

Note. $N = 181$. $p = .025$. Reference Cluster = Cluster 3.

Odds rato comparisons for the two significant predictors of cluster membership are shown in Table 10. The odds ratio comparison between clusters 2 and 1 for PTTS4, 1.80, 95% C.I. [1.35, 2.41], indicates that the odds of a student belonging to the highest performing group, cluster 1, were 1.80 times greater for every one point increase in disagreement with the statement “It is hard for me to understand my pre-calculus teacher in lecture because I feel like he/she skips steps in demonstrating problems.” Despite the significance of item PTTS7 as a predictor of cluster membership as determined by the stepwise logistic regression, none of the other odds ratio comparisons were significant at the required alpha cutoff for the clusterwise comparisons.

Table 10

*Odds Ratios for External Validation of Average Linkage Nonhierarchical Solution*

| | PTTS4 | | PTTS7 | |
|---|---|---|---|---|
| | OR | *p* | OR | *p* |
| C1 vs C3 | 0.71 | .0516 | 0.96 | .8226 |
| C2 vs C3 | 1.27 | .1502 | 0.69 | .0332 |
| C2 vs C1 | **1.80** | **<.0001** | 0.72 | .0342 |

Note. Reference cluster = cluster on left. PTTS4 = It is hard for me to understand my pre-calculus teacher in lecture because I feel like he/she skips steps in demonstrating problems, PTTS7 = I feel like my pre-calculus teacher makes the class difficult on purpose because he/she wants to "weed out" people; 1 = Strongly Agree, 5 = Strongly Disagree; $\boldsymbol{p < .0167}$.

Descriptive statistics for the significant predictors of the average linkage nonhierarchical cluster membership are shown in Table 11. Given the significant odds ratios (Table 10) the relatively large differences with respect to PTTS4 and the homogeneity with respect to PTTS7 were expected. Gender is included in Table 11 for comparison with the results of the Ward's nonhierarchical analysis and also since cluster weighting by gender was a research variable of interest. Additionally, despite the fact that gender did not emerge as a significant predictor of cluster membership for the average linkage nonhierarchical clusters in the stepwise logistic regression, the $\chi^2$ test of independence was significant, $\chi^2(2, N = 187) = 6.84$, $p = .033$, indicating that average linkage nonhierachical cluster membership and gender are related.

Table 11

*Descriptive Statistics for External Variables From Average Linkage Nonhierarchical Solution*

| | | Cluster | | |
|---|---|---|---|---|
| | | 1 | 2 | 3 |
| Variable | | ($n$ = 64) | ($n$ = 88) | ($n$ = 35) |
| PTTS4 | *M* | 3.64 | 2.80 | 2.97 |
| | *SD* | 1.30 | 1.43 | 1.40 |
| PTTS7 | *M* | 2.58 | 2.72 | 2.74 |
| | *SD* | 0.91 | 1.02 | 0.98 |
| Sex | *% Female* | 29.69 | 50.00 | 48.57 |
| | *% Male* | 70.31 | 50.00 | 51.43 |

Note. PTTS4 = It is hard for me to understand my pre-calculus teacher in lecture because I feel like he/she skips steps in demonstrating problems; PTTS7 = I feel like my pre-calculus teacher makes the class difficult on purpose because he/she wants to "weed out" people; 1 = Strongly Agree, 5 = Strongly Disagree.

# CHAPTER IV: DISCUSSION

The two primary research hypotheses of this study were that at least two clusters of function translation fluency would emerge and that some clusters would be dominated by a single gender. Both were supported. The four clusters that emerged from the Ward's method based analysis showed dominance of cluster 2 by females and cluster 4 by males but heterogeneous gender distributions in the other clusters (Table 6). The three clusters that emerged from the average linkage based analysis showed dominance of cluster 1 by males but heterogeneous gender distribution in clusters 2 and 3 (Table 11).

The two secondary research hypotheses of this study were that profiles of precalculus students' function translation fluency could be developed based on the results of the FLTF clustering and that students' FLTF cluster membership could be predicted by their visualization ability, mathematics anxiety, perceptions of their precalculus teachers, and gender. The first was strongly supported by the results. Profiles of students' function translation fluency were developed and externally validated in both the Ward's method and average linkage based analyses.

The remaining secondary research hypothesis was less well supported, in that the not all four constructs predicted FLTF cluster membership. Of those that did, only specific aspects of those constructs were predictors. Mathematics anxiety and attitudes toward mathematics were not predictors of FLTF cluster membership; no PARS items were significant in either external validation. Gender predicted FLTF cluster membership for the Ward's method nonhierarchical clusters, both when controlling for other predictors in the model and at the univariate level, but only at the univariate level for average linkage FLTF cluster membership.

The specific aspects of visualization ability and student perception of the precalculus teacher that predicted FLTF cluster membership varied by cluster solution. Visualization ability did not predict average linkage nonhierarchical cluster membership. Self-rated ability to visually estimate angle sizes within shapes (item VSSA6) predicted Ward's nonhierarchical cluster membership. Student perception that the precalculus teacher increased the difficulty of the class in order to "weed out" students (item PTTS7) predicted average linkage nonhierarchical cluster membership. Student perception that the precalculus teacher skipped steps while presenting solutions to mathematical problems in lecture (item PTTS4) predicted FLTF cluster membership for both nonhierarchical cluster solutions.

The observed gender differences in FLTF performance are in accordance with the work of Geary et. al (2000), Casey et. al (2000), Miller and Bischel (2004), and Quaiser-Pohl and Lehmann (2002). In both cluster solutions the highest overall mean scores on the FLTF were achieved by the students in the male dominated clusters; in the Ward's method solution, the lowest overall mean scores were achieved by the students in the female dominated cluster. Differences between the results of the current study and those obtained by Geary et. al (2000) and Casey et. al (2000) did appear. In both of these prior studies, males had direct advantages in spatial ability on a variety of mathematical problem solving tasks. However, the single spatial visualization ability predictor of FLTF cluster membership, item VSSA6, was significant only for the Ward's nonhierarchical clusters; the odds ratios (Table 5) for item VSSA6 were also significant only for comparing the two clusters that were heterogeneous by gender. Therefore, the results of this study did agree with prior research in that there were overall performance differences by gender on the FLTF, but the influence of the single specific visualization item as a predictor of cluster membership was not common to both cluster solutions and was not associated with gender.

The results of this study also differed from the work of Ashcraft and Moore (2009), Meece et al. (1990), Haynes et al. (2004), and Frenzel et al. (2007) in that mathematics anxiety, as measured by items from the PARS, was not a predictor of cluster membership. However, specific student perceptions of the precalculus teacher were predictors of cluster membership for both cluster solutions. This is somewhat similar to the results obtained by Hayes et al. (2004), although their results showed an association between lower mathematics anxiety and perceptions of high school mathematics teacher only for females.

Student perceptions that precalculus teacher is purposely making the course more difficult in order to dissuade students from pursuing further mathematics courses is also similar to results observed by Turner et al. (2002), in which negative mathematics teacher affect influenced student tendencies to avoid mathematics. The results observed for item PTTS4, which addresses algebraic manipulation of mathematical functions, are also in agreement with prior work. As discussed by Carlson et al. (2010), Gagatsis and Shiakalli (2004), Herman (2007), and Knuth (2000), a precalculus teacher focused on classroom demonstrations of procedural algebraic solutions over graphical demonstrations may be directly influencing student function translation fluency. This conclusion is also supported by the odds ratio comparisons and overall FLTF performance results as well as the Graphical Interpretation and Function Property Line Tests item scores for both cluster solutions.

**Limitations**

Limitations of this study include the use of a convenience sample, unequal sample sizes for the Ward's method and average linkage method based analyses, time taken to collect data, and length of the original survey. The generalizability of the results of this study is influenced by the sample from which the students were drawn, a regional university in the South; therefore, generalizability may be limited to similar samples.

The Ward's method based analyses were run on the full sample of 199 students as no outliers were identified by this method. In contrast, the average linkage hierarchical clustering identified 12 outliers in its first run. Thus, the average linkage based analyses were completed on a subset of 187 students. The 12 outliers removed from the average linkage hierarchical analysis could indicate the presence of another subgroup that was not able to be well identified by the average linkage method.

The remaining limitations of this study are associated with the measures used and the time span associated with data collection. Data collection for this study began in October of 2011 but was not completed until mid-November of 2011. It is possible that some students had not yet been exposed to some of the material on the FLTF in their precalculus classes and that this could have influenced FLTF scores. The measures used in this study and the survey design also had certain limitations. All of the measures used for external validation were subjective rather than objective; it would have perhaps been more relevant to include an objective measure of visualization skills for closer comparison with prior work rather than a subjective self-assessment of those skills. The original instrument used to collect all of the data for this study contained 70 items, with the 10 FLTF questions at the end of the survey. It is therefore possible that the FLTF scores may have been influenced by students' fatigue. Finally, since identifying data for the instructors was not collected, this potential source of variability was not controlled for in the analyses.

**Suggestions for Future Research**

Suggestions for future research include revision of the FLTF and further development of the construct of function translation fluency. The low Cronbach's alpha for the original FLTF and the observed results from this study indicate that function translation fluency is not a unidimensional construct; therefore, creation and validation of a truly multidimensional version of the FLTF to explore the extent of the multidimensionality of function translation fluency would be worthwhile. Further refinement of the conceptual definition of function translation fluency should also be considered in future research.

Alternative constructs could also be considered for use as predictors of FLTF cluster membership. These include race, socio-economic status, school system factors, and objective measures of both student strength of mathematical background and of visualization ability. Specific characteristics of precalculus teacher background could also be considered in future research, such as course design, years of teaching experience, and mathematical preparedness. Future research could also incorporate self-report instruments for precalculus teachers to address the issues of "weeding out" students and of skipping steps in demonstrating problems in lecture. The specific aspects of the classroom dynamic that lead students to believe that the precalculus teacher is attempting to "weed out" also merit further investigation. In addition, it should be determined whether or not precalculus teachers actually do skip steps in demonstrating problems, or if this issue is truly related to student perception rather than teacher in-class behavior.

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# APPENDICES

# APPENDIX A

## Descriptive Statistics

Table A1

*Descriptive Statistics and Response Frequencies for PARS Items*

| Item | *M* | *SD* | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|---|---|
| pars1 | 2.82 | 1.28 | 33 | 55 | 53 | 30 | 28 |
| pars2 | 2.84 | 1.07 | 19 | 62 | 63 | 42 | 13 |
| pars3 | 2.54 | 1.21 | 14 | 29 | 58 | 49 | 49 |
| pars4 | 3.36 | 1.17 | 14 | 33 | 57 | 57 | 38 |
| pars5* | 2.99 | 1.29 | 25 | 60 | 31 | 53 | 29 |
| pars6* | 2.80 | 1.22 | 15 | 51 | 46 | 51 | 35 |
| pars7 | 2.28 | 1.02 | 43 | 91 | 40 | 17 | 8 |
| pars8* | 2.27 | 1.14 | 6 | 28 | 41 | 61 | 62 |

Note. $N$ = 199 unless otherwise indicated; *$N$ = 198.

Table A2

*Descriptive Statistics and Response Frequencies for VSSA Items*

| Item | *M* | *SD* | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|---|---|
| vssa1 | 2.23 | 1.00 | 50 | 81 | 44 | 20 | 4 |
| vssa2 | 2.47 | 1.02 | 40 | 60 | 69 | 26 | 4 |
| vssa3** | 3.64 | 0.87 | 33 | 79 | 68 | 16 | 1 |
| vssa4* | 2.64 | 1.08 | 36 | 48 | 74 | 32 | 8 |
| vssa5* | 1.96 | 0.89 | 67 | 86 | 32 | 12 | 1 |
| vssa6 | 2.66 | 1.00 | 28 | 57 | 72 | 39 | 3 |
| vssa7* | 3.28 | 1.27 | 21 | 35 | 50 | 52 | 40 |
| vssa8* | 2.76 | 0.98 | 18 | 63 | 73 | 37 | 7 |
| vssa9*** | 1.84 | 0.80 | 71 | 93 | 25 | 6 | 1 |

Note. $N$ = 199 unless otherwise indicated; *$N$ = 198, ** $N$= 197, ***$N$ = 196.

Table A3

*Descriptive Statistics and Response Frequencies for PTTS Items*

| Item | *M* | *SD* | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|---|---|
| ptts1 | 2.46 | 1.34 | 61 | 53 | 42 | 19 | 24 |
| ptts2 | 2.58 | 1.27 | 45 | 64 | 41 | 28 | 21 |
| ptts3 | 4.50 | 0.85 | 132 | 44 | 16 | 4 | 3 |
| ptts4 | 3.11 | 1.43 | 42 | 52 | 28 | 41 | 36 |
| ptts5 | 3.06 | 1.18 | 19 | 64 | 49 | 44 | 23 |
| ptts6 | 3.80 | 1.09 | 54 | 90 | 29 | 15 | 11 |
| ptts7 | 3.47 | 1.35 | 53 | 64 | 31 | 25 | 26 |
| ptts8 | 2.09 | 0.99 | 60 | 85 | 35 | 14 | 5 |
| ptts9 | 2.52 | 1.20 | 45 | 62 | 50 | 27 | 15 |
| ptts10 | 2.46 | 1.28 | 58 | 56 | 35 | 35 | 15 |
| ptts11 | 2.31 | 1.06 | 42 | 90 | 41 | 15 | 11 |
| ptts12 | 3.33 | 1.25 | 30 | 84 | 33 | 27 | 25 |
| ptts13 | 3.15 | 1.49 | 50 | 45 | 33 | 28 | 43 |

Note. $N$ = 199.

Table A4

*Descriptive Statistics and Response Frequencies for PARS Items for Outliers Removed From Average Linkage Based Analysis*

| Item | *M* | *SD* | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|---|---|
| pars1 | 2.67 | 1.23 | 2 | 4 | 3 | 2 | 1 |
| pars2 | 2.92 | 1.31 | 2 | 2 | 5 | 1 | 2 |
| pars3 | 2.00 | 1.35 | 6 | 3 | 1 | 1 | 1 |
| pars4 | 3.58 | 1.51 | 2 | 1 | 1 | 4 | 4 |
| pars5 | 3.08 | 1.31 | 1 | 4 | 2 | 3 | 2 |
| pars6 | 2.92 | 1.31 | 2 | 3 | 2 | 4 | 1 |
| pars7 | 2.00 | 0.95 | 4 | 5 | 2 | 1 | 0 |
| pars8 | 2.83 | 1.34 | 2 | 3 | 4 | 1 | 2 |

Note. $N = 12$ unless otherwise indicated.

Table A5

*Descriptive Statistics and Response Frequencies for VSSA Items for Outliers Removed From Average Linkage Based Analysis*

| Item | *M* | *SD* | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|---|---|
| vssa1 | 2.50 | 1.24 | 3 | 3 | 4 | 1 | 1 |
| vssa2 | 2.58 | 1.24 | 3 | 2 | 5 | 1 | 1 |
| vssa3 | 3.50 | 0.90 | 0 | 1 | 6 | 3 | 2 |
| vssa4 | 2.50 | 1.45 | 4 | 2 | 4 | 0 | 2 |
| vssa5 | 2.08 | 1.00 | 4 | 4 | 3 | 1 | 0 |
| vssa6 | 2.42 | 1.38 | 5 | 0 | 5 | 1 | 1 |
| vssa7 | 2.92 | 1.56 | 4 | 0 | 3 | 3 | 2 |
| vssa8 | 2.25 | 0.75 | 2 | 5 | 5 | 0 | 0 |
| vssa9* | 1.82 | 0.87 | 5 | 3 | 3 | 0 | 0 |

Note. $N = 12$ unless otherwise indicated; $*N = 11$.

Table A6

*Descriptive Statistics and Response Frequencies for PTTS Items for Outliers Removed From Average Linkage Based Analysis*

| Item | *M* | *SD* | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|---|---|
| ptts1 | 2.42 | 1.73 | 6 | 1 | 2 | 0 | 3 |
| ptts2 | 2.83 | 1.47 | 3 | 2 | 3 | 2 | 2 |
| ptts3 | 4.50 | 0.80 | 0 | 0 | 2 | 2 | 8 |
| ptts4 | 3.08 | 1.44 | 2 | 3 | 1 | 4 | 2 |
| ptts5 | 2.92 | 1.24 | 1 | 4 | 4 | 1 | 2 |
| ptts6 | 4.00 | 0.74 | 0 | 0 | 3 | 6 | 3 |
| ptts7 | 3.33 | 1.83 | 4 | 0 | 1 | 2 | 5 |
| ptts8 | 2.00 | 1.41 | 7 | 1 | 2 | 1 | 1 |
| ptts9 | 2.92 | 1.38 | 2 | 3 | 3 | 2 | 2 |
| ptts10 | 2.50 | 1.38 | 4 | 2 | 3 | 2 | 1 |
| ptts11 | 2.33 | 1.15 | 2 | 7 | 1 | 1 | 1 |
| ptts12 | 3.00 | 1.28 | 2 | 2 | 3 | 4 | 1 |
| ptts13 | 3.42 | 1.73 | 3 | 1 | 1 | 2 | 5 |

Note. $N = 12$ unless otherwise indicated; $*N = 11$.

Table A7

*Descriptive Statistics and Response Frequencies for PARS Items for Outliers Removed From Average Linkage Based Analysis for Females Only*

| Item | *M* | *SD* | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|---|---|
| pars1 | 2.778 | 1.202 | 1 | 3 | 3 | 1 | 1 |
| pars2 | 3.333 | 1.118 | 0 | 2 | 4 | 1 | 2 |
| pars3 | 1.444 | 0.726 | 6 | 2 | 1 | 0 | 0 |
| pars4 | 4.111 | 1.269 | 1 | 0 | 0 | 4 | 4 |
| pars5 | 2.778 | 1.093 | 1 | 3 | 2 | 3 | 0 |
| pars6 | 2.444 | 1.130 | 2 | 3 | 2 | 2 | 0 |
| pars7 | 2.000 | 1.000 | 3 | 4 | 1 | 1 | 0 |
| pars8 | 2.444 | 1.236 | 2 | 3 | 3 | 0 | 1 |

Note. $N = 9$.

Table A8

*Descriptive Statistics and Response Frequencies for VSSA Items for Outliers Removed From Average Linkage Based Analysis for Females Only*

| Item | *M* | *SD* | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|---|---|
| vssa1 | 2.889 | 1.167 | 1 | 2 | 4 | 1 | 1 |
| vssa2 | 3.000 | 1.118 | 1 | 1 | 5 | 1 | 1 |
| vssa3 | 3.444 | 0.882 | 0 | 1 | 4 | 3 | 1 |
| vssa4 | 2.556 | 1.236 | 2 | 2 | 4 | 0 | 1 |
| vssa5 | 2.333 | 1.000 | 2 | 3 | 3 | 1 | 0 |
| vssa6 | 2.889 | 1.269 | 2 | 0 | 5 | 1 | 1 |
| vssa7 | 2.889 | 1.537 | 3 | 0 | 2 | 3 | 1 |
| vssa8 | 2.556 | 0.527 | 0 | 4 | 5 | 0 | 0 |
| vssa9* | 2.000 | 0.926 | 3 | 2 | 3 | 0 | 0 |

Note. $N = 9$ unless otherwise indicated; $*N = 8$.

Table A9

*Descriptive Statistics and Response Frequencies for PTTS Items for Outliers Removed From Average Linkage Based Analysis for Females Only*

| Item | *M* | *SD* | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|---|---|
| ptts1 | 2.889 | 1.764 | 3 | 1 | 2 | 0 | 3 |
| ptts2 | 3.333 | 1.323 | 1 | 1 | 3 | 2 | 2 |
| ptts3 | 4.333 | 0.866 | 0 | 0 | 2 | 2 | 5 |
| ptts4 | 2.667 | 1.414 | 2 | 3 | 1 | 2 | 1 |
| ptts5 | 3.111 | 1.269 | 0 | 4 | 2 | 1 | 2 |
| ptts6 | 3.889 | 0.782 | 0 | 0 | 3 | 4 | 2 |
| ptts7 | 2.778 | 1.787 | 4 | 0 | 1 | 2 | 2 |
| ptts8 | 2.333 | 1.500 | 4 | 1 | 2 | 1 | 1 |
| ptts9 | 3.333 | 1.323 | 1 | 1 | 3 | 2 | 2 |
| ptts10 | 2.444 | 1.236 | 3 | 1 | 3 | 2 | 0 |
| ptts11 | 2.556 | 1.236 | 1 | 5 | 1 | 1 | 1 |
| ptts12 | 2.556 | 1.130 | 2 | 2 | 3 | 2 | 0 |
| ptts13 | 2.889 | 1.691 | 3 | 1 | 1 | 2 | 2 |

Note. $N = 9$.

Table A10

*Descriptive Statistics and Response Frequencies for PARS Items for Outliers Removed From Average Linkage Based Analysis for Males Only*

| Item | *M* | *SD* | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|---|---|
| pars1 | 2.33 | 1.53 | 1 | 1 | 0 | 1 | 0 |
| pars2 | 1.67 | 1.15 | 2 | 0 | 1 | 0 | 0 |
| pars3 | 3.67 | 1.53 | 0 | 1 | 0 | 1 | 1 |
| pars4 | 2.00 | 1.00 | 1 | 1 | 1 | 0 | 0 |
| pars5 | 4.00 | 1.73 | 0 | 1 | 0 | 0 | 2 |
| pars6 | 4.33 | 0.58 | 0 | 0 | 0 | 2 | 1 |
| pars7 | 2.00 | 1.00 | 1 | 1 | 1 | 0 | 0 |
| pars8 | 4.00 | 1.00 | 0 | 0 | 1 | 1 | 1 |

Note. *N* = 3.

Table A11

*Descriptive Statistics and Response Frequencies for VSSA Items for Outliers Removed From Average Linkage Based Analysis for Males Only*

| Item | *M* | *SD* | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|---|---|
| vssa1 | 1.33 | 0.58 | 2 | 1 | 0 | 0 | 0 |
| vssa2 | 1.33 | 0.58 | 2 | 1 | 0 | 0 | 0 |
| vssa3 | 3.67 | 1.15 | 0 | 0 | 2 | 0 | 1 |
| vssa4 | 2.33 | 2.31 | 2 | 0 | 0 | 0 | 1 |
| vssa5 | 1.33 | 0.58 | 2 | 1 | 0 | 0 | 0 |
| vssa6 | 1.00 | 0.00 | 3 | 0 | 0 | 0 | 0 |
| vssa7 | 3.00 | 2.00 | 1 | 0 | 1 | 0 | 1 |
| vssa8 | 1.33 | 0.58 | 2 | 1 | 0 | 0 | 0 |
| vssa9 | 1.33 | 0.58 | 2 | 1 | 0 | 0 | 0 |

Note. *N* = 3.

Table A12

*Descriptive Statistics and Response Frequencies for PTTS Items for Outliers Removed From Average Linkage Based Analysis for Males Only*

| Item | *M* | *SD* | Strongly Disagree | Disagree | Neutral | Agree | Strongly Agree |
|---|---|---|---|---|---|---|---|
| ptts1 | 1.00 | 0.00 | 3 | 0 | 0 | 0 | 0 |
| ptts2 | 1.33 | 0.58 | 2 | 1 | 0 | 0 | 0 |
| ptts3 | 5.00 | 0.00 | 0 | 0 | 0 | 0 | 3 |
| ptts4 | 4.33 | 0.58 | 0 | 0 | 0 | 2 | 1 |
| ptts5 | 2.33 | 1.15 | 1 | 0 | 2 | 0 | 0 |
| ptts6 | 4.33 | 0.58 | 0 | 0 | 0 | 2 | 1 |
| ptts7 | 5.00 | 0.00 | 0 | 0 | 0 | 0 | 3 |
| ptts8 | 1.00 | 0.00 | 3 | 0 | 0 | 0 | 0 |
| ptts9 | 1.67 | 0.58 | 1 | 2 | 0 | 0 | 0 |
| ptts10 | 2.67 | 2.08 | 1 | 1 | 0 | 0 | 1 |
| ptts11 | 1.67 | 0.58 | 1 | 2 | 0 | 0 | 0 |
| ptts12 | 4.33 | 0.58 | 0 | 0 | 0 | 2 | 1 |
| ptts13 | 5.00 | 0.00 | 0 | 0 | 0 | 0 | 3 |

Note. $N = 3$.

Table A13

*Proportion of Correct Responses for Outliers Removed From Average Linkage Analysis Compared to Overall Average Linkage Nonhierarchical Solution*

| FLTF Skill | Item | Females ($n$ = 9) | Males ($n$ = 3) | Overall ($n$ = 12) | Solution ($n$ = 187) |
|---|---|---|---|---|---|
| Graphical Literacy | 1 | .22 | .00 | .17 | .87 |
| | 5 | .44 | .67 | .50 | .90 |
| *Mean* | | .33 | .33 | .33 | .89 |
| Graphical Interpretation | 2 | .33 | .67 | .42 | .35 |
| | 7 | .11 | .00 | .08 | .21 |
| | 10 | .33 | 1.00 | .50 | .56 |
| *Mean* | | .26 | .56 | .33 | .37 |
| Slope Recognition | 3 | .33 | .33 | .33 | .92 |
| | 6 | .67 | .67 | .67 | .99 |
| *Mean* | | .50 | .50 | .50 | .95 |
| Function Property Line Tests | 4 | .44 | .67 | .50 | .60 |
| | 8 | .33 | .00 | .25 | .57 |
| | 9 | .44 | .00 | .33 | .35 |
| *Mean* | | .41 | .22 | .36 | .51 |
| *Grand Mean* | | .37 | .40 | .38 | .63 |

APPENDIX B

Comparison of Hierarchical and Nonhierarchical Cluster Solutions

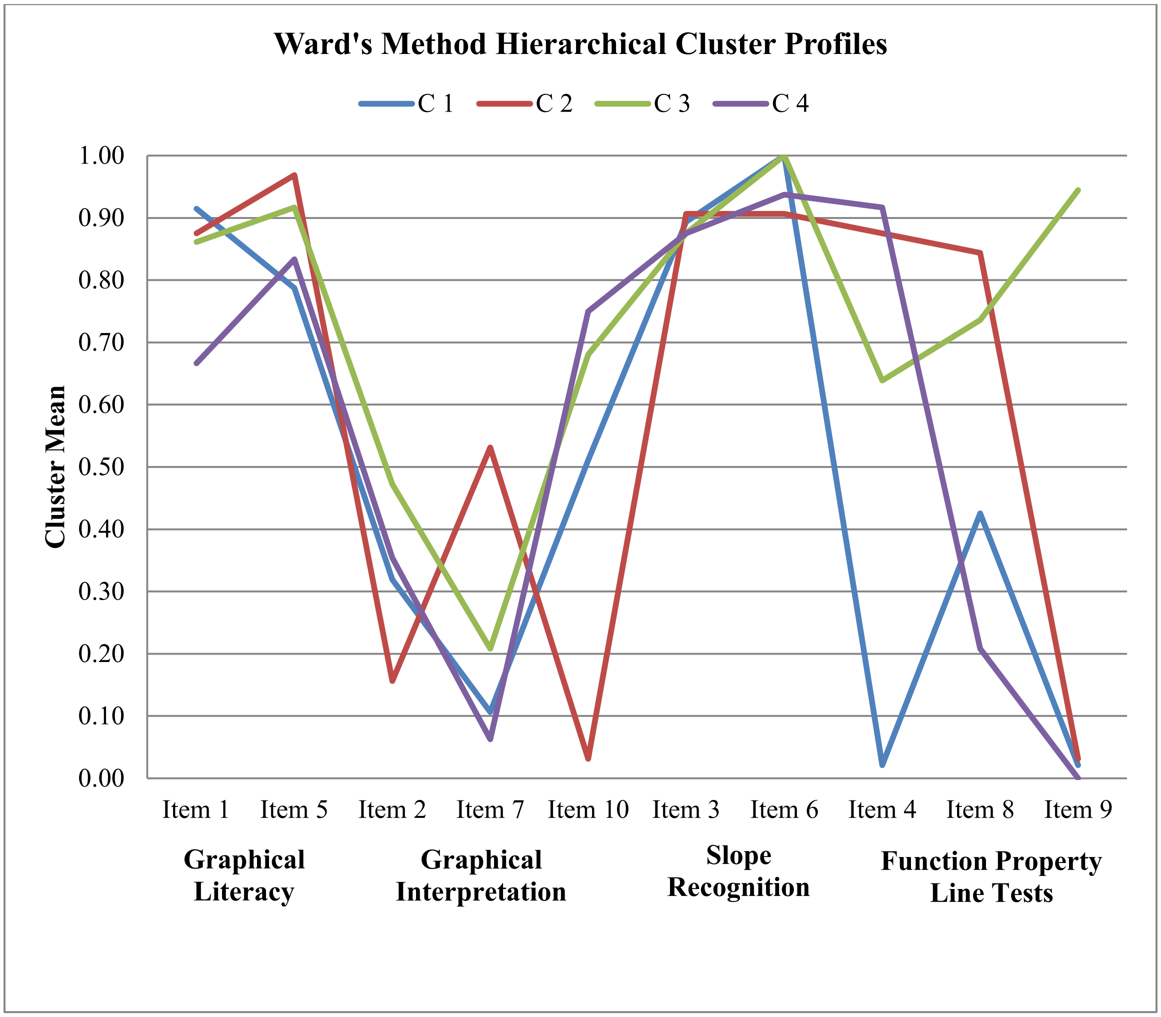


*Figure B1*. Ward's Method hierarchical cluster profiles. See Figure B2 for comparison to Ward's Method nonhierarchical cluster profiles.

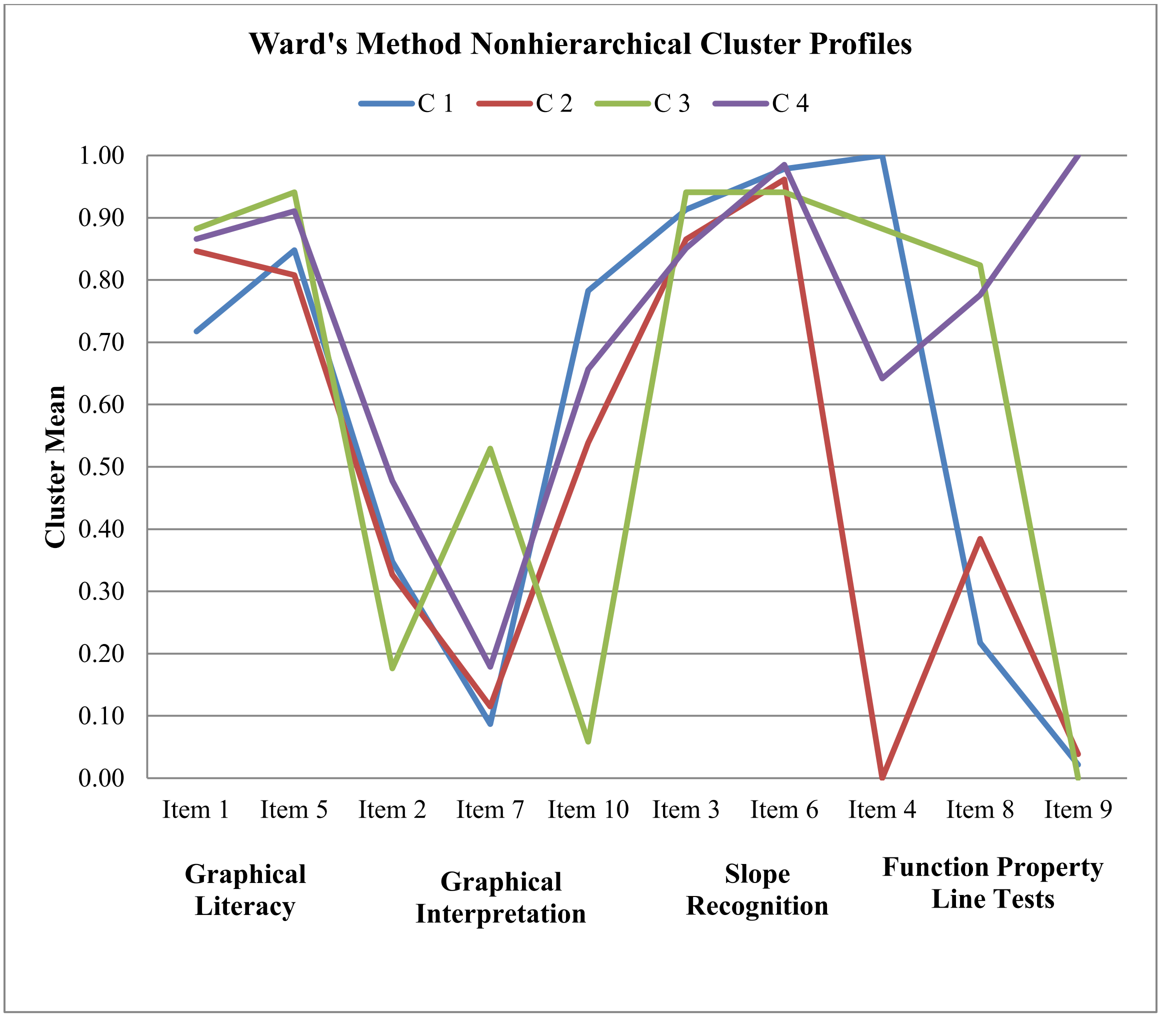


*Figure B2*. Ward's Method nonhierarchical cluster profiles. See Figure B1 for comparison to Ward's Method hierarchical cluster profiles.

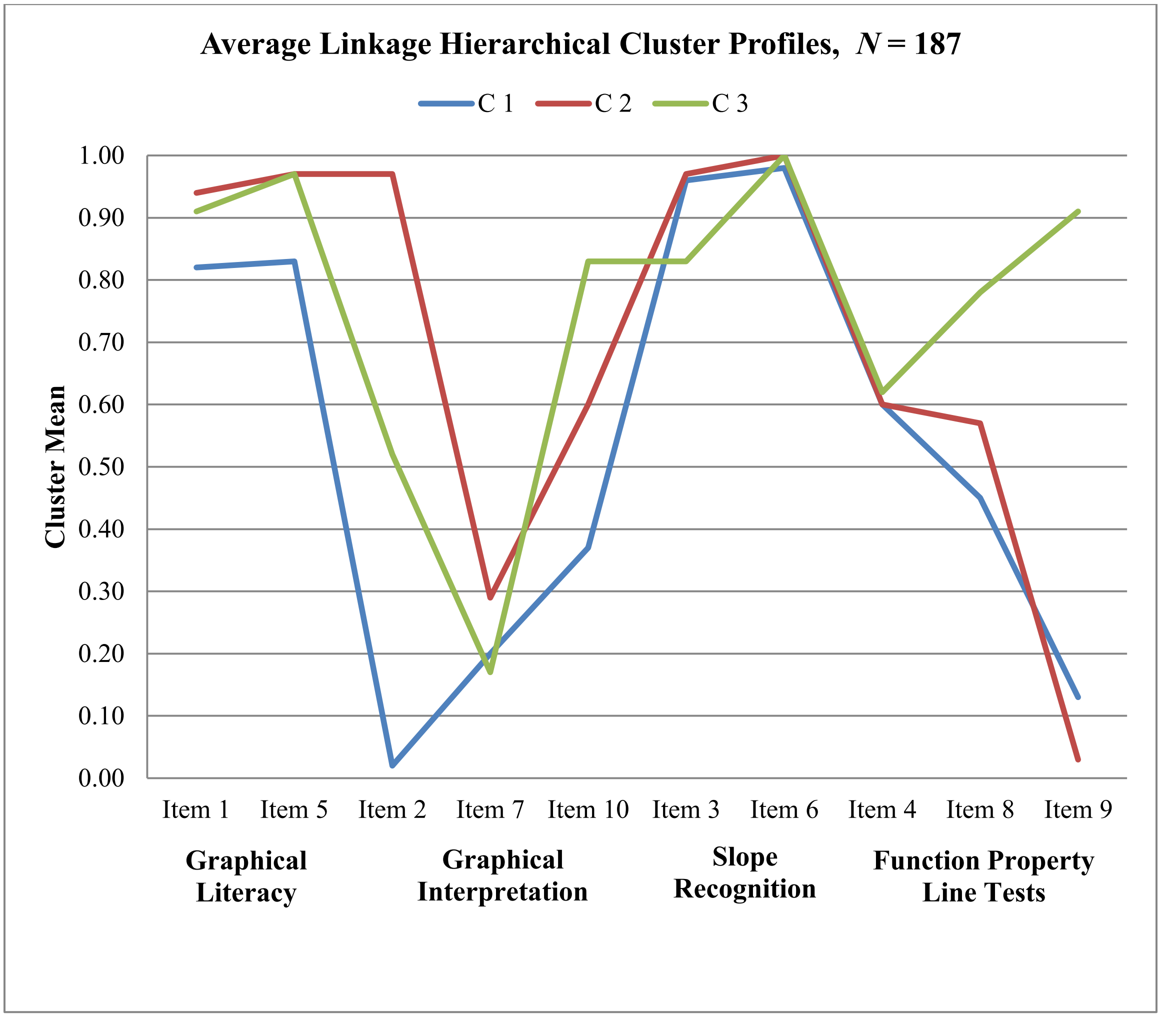


*Figure B3*. Average Linkage hierarchical cluster profiles. See Figure B4 for comparison to Average Linkage nonhierarchical cluster profiles.

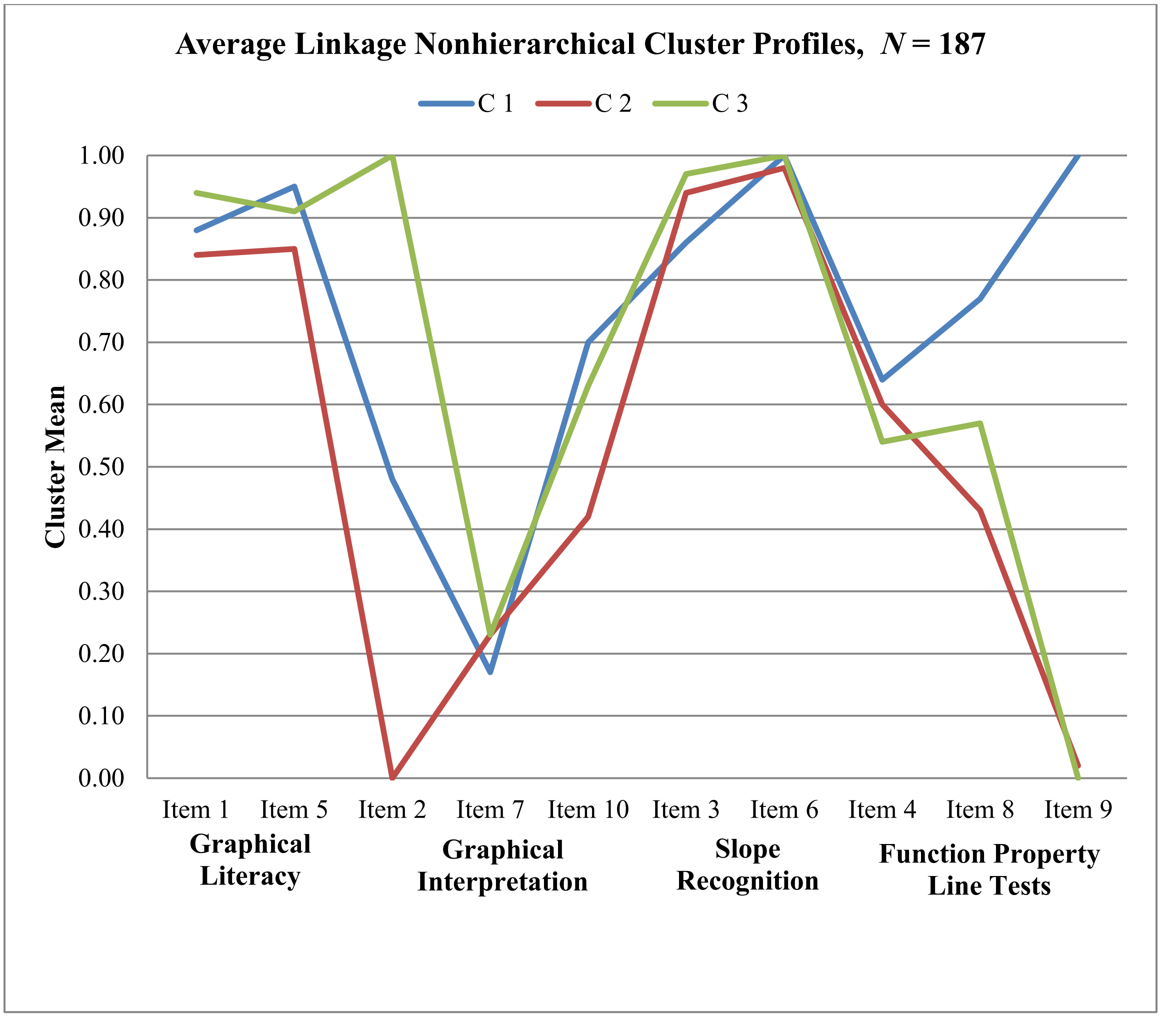


*Figure B4*. Average Linkage nonhierarchical cluster profiles. See Figure B3 for comparison to Average Linkage hierarchical cluster profiles.

# APPENDIX C

## IRB Approval Letter

MIDDLE TENNESSEE STATE UNIVERSITY

October 11, 2011
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**Protocol Title**: "Validation of Affective Influences on Function Translation Instrument"

**Protocol Number: 12-082**

Dear Investigator(s),

I found your study to be exempt from Institutional Review Board (IRB) continued review. The exemption is pursuant to 45 CFR 46.101(b) (2). This is because your study involves the use of educational tests and survey materials, and information is obtained in such a manner that human subjects cannot be identified.

You will need to submit an end-of-project report to the Office of Compliance upon completion of your research. Complete research means that you have finished collecting data and you are ready to submit your thesis and/or publish your findings. Should you not finish your research within the three (3) year period, you must submit a Progress Report and request a continuation prior to the expiration date. Please allow time for review and requested revisions. Your study expires on **October 11, 2014.**

**Any change to the protocol must be submitted to the IRB before implementing this change.** According to MTSU Policy, a researcher is defined as anyone who works with data or has contact with participants. Anyone meeting this definition needs to be listed on the protocol and needs to provide a certificate of training to the Office of Compliance. **If you add researchers to an approved project, please forward an updated list of researchers and their certificates of training to the Office of Compliance before they begin to work on the project.** Once your research is completed, please send us a copy of the final report questionnaire to the Office of Compliance. This form can be located at www.mtsu.edu/irb on the forms page.

Also, all research materials must be retained by the PI or **faculty advisor (if the PI is a student)** for at least three (3) years after study completion. Should you have any questions or need additional information, please do not hesitate to contact me.

Sincerely,

Emily Born
Compliance Officer
615-494-8918