THE EFFECTS OF THE THINK INTERACTION FRAMEWORK AS AN INTERVENTION TO SUPPORT STUDENTS' ENGAGEMENT IN MATHEMATICAL DISCOURSE AND MOVEMENT TOWARD MATHEMATICAL PROFICIENCY

By

Samantha A. Stevens

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Dissertation Committee:

Dr. Sarah Bleiler-Baxter, Chair

Dr. Angela Barlow

Dr. Kyle Butler

Dr. Kim Sadler

Dr. Chris Stephens

I dedicate this research to my daughter, Casey, for teaching me how to become a better mathematics educator and to never give up on my goals and dreams. You are my inspiration. I love you.

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ABSTRACT

To prepare students to compete for jobs in the global economy, teachers must find ways to teach mathematics and higher order thinking skills using effective teaching practices that not only engage all students in inquiry-based activities that promote problem solving and reasoning skills but that also develop and encourage effective communication skills. Though research in mathematics education has shown that all students can benefit from using effective teaching practices that engage students in the learning process, educators in the field of special education assert that students who are at risk for mathematical difficulties or who are identified with a specific learning disability (SLD) may benefit from traditional instructional practices that include rote memorization, teacher modeling, and direct instruction. A review of literature revealed that both fields espouse the effectiveness of heuristics in teaching mathematics. Whereas mathematics educators use heuristics during inquiry-based activities to encourage student reasoning and collaboration, special educators use heuristics as an intervention to promote verbalization and procedural fluency. The common thread between the two types of implementation is the use of language.

Based upon Vygotsky's theory of social constructivism, this study investigated the effectiveness of heuristics as an intervention tool to promote student discourse and movement toward mathematical proficiency among students who are at risk for mathematical difficulties. Specifically, this embedded multiple case study examined the effects of the THINK interaction framework on students' discourse and movement toward mathematical proficiency in a ninth-grade mathematics intervention course. Qualitative data were collected in the form of student interviews, classroom observations, audio recordings, and student work samples. Results revealed that a change in each of the participants' exploratory talk occurred upon using the THINK framework. Further, each of the participants' demonstrated movement in various strands of mathematical proficiency. Implications for classroom practice and suggestions for future research are discussed.

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CHAPTER I: INTRODUCTION

Introduction

To remain a leader in the global economy, America must prepare its citizens for a competitive job market through helping potential job seekers develop skills in problem solving, analyzing, reasoning, and communicating with others (Baroody, 2011; Elizabeth, Anderson, Snow, & Selman, 2012). Such skills are not only needed to become mathematically literate but also to be successful in the workplace, to function in everyday life, and to serve in the scientific and technical communities (National Council of Teachers of Mathematics [NCTM], 2000). As such, it is of primary importance that mathematics curricula address the needs of all students. In *Principles and Standards for School Mathematics*, NCTM (2000) addressed the importance of educational equity in mathematics in that all students "regardless of their personal characteristics, backgrounds, or physical challenges, must have the opportunities to study - and support to learn - mathematics" (p. 12). Further, NCTM (2000) recognized that equity does not equate to equal instruction but the provision to "ensure that all students participate in a strong instructional program that supports their mathematics learning" (p. 13). In order to ensure access and equity for all students, teachers must implement high quality instructional practices that engage students in mathematical processes that support their development toward mathematical proficiency and provide sufficient learning opportunities that address the learning needs of each student in their classrooms (Doabler et al., 2012; Kroeger & Kouche, 2006; NCTM, 2014). In the following subsections, I discuss the Standards for Mathematical Practice (SMP) (Common Core State Standards Initiative [CCSSI], 2010) and the importance of facilitating mathematical discourse.

Standards for Mathematical Practice

The necessity to provide students with opportunities to increase their learning and conceptual understanding of mathematical topics requires that teachers provide instruction that is structured, sequenced, and follows a natural learning progression of how students learn mathematics (CCSSI, 2010). The Common Core State Standards for Mathematics [CCSSM] serve as a guidepost for teachers, as the standards define what all “students should understand and be able to do” (CCSSI, 2010, p. 4) by setting grade-level expectations that incorporate conceptual understanding and procedural knowledge. Although the standards do not address curriculum or pedagogy, the document incorporates the Standards for Mathematical Practice (SMP) that describe “varieties of expertise” (CCSSI, 2010, p. 6) in mathematical processes and proficiencies that all students should develop throughout their academic careers. Included in these practices, students should be able to: make sense of problems and persevere in solving them; reason abstractly and quantitatively; construct viable arguments and critique the reasoning of others; model with mathematics; use appropriate tools strategically; attend to precision; look for and make use of structure; and look for and express regularity in repeated reasoning (CCSSI, 2010). Though the SMP and CCSSM are fairly new to the literature, the processes described by the SMP are historically recognized as characteristics of mathematically proficient students and competencies that equip students to solve problems (NCTM, 2000; National Research Council [NRC], 2001) and become productive consumers of mathematics in society (NCTM, 2000; NRC, 2001).

Facilitating Mathematical Discourse

Providing opportunities to learn and making mathematics accessible to all students require that teachers be responsive to the learning needs of their students (Kroeger & Kouche, 2006; NCTM, 2014). Effective teachers implement instructional practices that help students to learn and understand related mathematical concepts (Khan, 2012; NCTM, 2014) and that engage students in mathematical processes that align to the SMP. As such, NCTM (2014) identified facilitating meaningful discourse as one of the eight effective teaching practices that teachers should use to strengthen their instruction to support all students in their learning of mathematics. NCTM (2014) recognized the importance that mathematical discussions have on students' conceptual understanding of mathematical topics when students are engaged in the learning process. "Mathematical discourse among students is central to meaningful learning of mathematics" (NCTM, 2014, p. 35). Communication allows students to develop understanding by clarifying their thinking to peers through justification and by listening critically to the ideas of others (NCTM, 2000; NRC, 2001). Webb et al. (2014) found that students who engaged in meaningful mathematical discourse (e.g., offering suggestions for improvement or challenging another student's solution) displayed higher achievement than students who participated weakly or not at all. To support students' engagement in meaningful mathematical discourse, teachers must set ground rules for interaction (Webb et al., 2014), find ways to promote meaningful mathematical discussions, and support all students in their engagement of productive mathematical talk (NCTM, 2014).

This dissertation contains a description of an explanatory embedded multiple case study that investigated the ways that an instructional tool, used as an intervention, supports students at risk for mathematical difficulties when they are engaged in discussions during activities that promote inquiry and construction of knowledge. To inform this study, this chapter presents background information, a discussion of research-based interventions, a statement of the problem, and the purpose statement. Finally, the chapter concludes with the significance of the study, definition of terms, and a chapter summary.

Background of the Study

The provision of high-quality instruction for students with a specific learning disability (SLD) and those at risk for mathematical difficulties begins with an understanding of SLD and the challenges and historical views associated with teaching students with a SLD and those at risk for mathematical difficulties. In the following sections, I present an introduction to SLD, review caveats and conventional instructional ideologies for students with a SLD, and discuss the use of heuristics as an instructional tool and research-based intervention for teaching and remediating students who are at risk for mathematical difficulties or identified with a SLD. The organization of these sections is designed to provide the reader with an understanding of differing viewpoints between mathematics educators and special educators concerning effective teaching practices for all students, as well as possible sources of struggle for students with SLD and those at risk for mathematics difficulties.

Cognitive Deficit or Cognitive Difference

In 2004, the Individual with Disabilities Education Act (IDEA) listed 13 categories of disability for which children ages three to 21 could receive services through the school system. These categories included “autism, deaf-blindness, deafness, emotional disturbance, hearing impairment, intellectual disability, multiple disabilities, orthopedic impairment, other health impairment, specific learning disorder (SLD), speech or language impairment, traumatic brain injury, and visual impairment including blindness" (National Dissemination Center for Children with Disability [NDCCD], 2012, p. 2). To inform the reader, the definition of SLD as defined in IDEA is presented below.

> SLD means a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, that may manifest itself in the imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations. (NDCCD, 2012, p. 4)

Though this definition has been the accepted definition of SLD, new research is challenging the criteria by which students with low achievement in mathematics are identified as having SLD (Lewis, 2014). Until recently, the identification of students with SLD relied on the use of the discrepancy model that determined if a discrepancy occurred between a student's intelligence quotient (IQ) and academic achievement for his or her age (Burton & Kappenberg, 2012). Recent perspectives on the framing of students with learning disabilities warrant that possible reasons for low achievement may not result from a cognitive deficit but occur as a cognitive difference due to "incompatibilities between the student's cognitive processing and the mediational tools intended to support understanding" of mathematical concepts (Lewis, 2015, p. 380). As a result, researchers suggest that students who have difficulties in mathematics receive "different types of instruction" (Lewis, 2015, p. 381) and not just more of the same instruction. Further, this recommendation includes that educators find ways to provide remediation to struggling learners using appropriate intervention tools (Lewis, 2015). In the following sections, I discuss methods of identification of SLD.

The discrepancy model. The discrepancy model, also known as the IQ-achievement discrepancy model (O'Donnell & Miller, 2011), used a standardized intelligence quotient (IQ) assessment (e.g., the WISC-IV) and an academic achievement test (e.g., Woodcock Johnson Achievement Test) to determine if a severe discrepancy occurs between the student's IQ and his or her achievement level for his age (Burton & Kappenberg, 2012). When a large discrepancy occurred between the two measures, students were identified and recommended to receive special education services as determined by an Individualized Education Program (IEP) of study (Burton & Kappenberg, 2012; O'Donnell & Miller, 2011).

The discrepancy model did not warrant the screening of students at the onset of difficulty to identify areas of mathematical weakness, the development of an intervention plan to address areas of need, or the monitoring of students' progress to determine the effectiveness of prescribed interventions (Burton & Kappenberg, 2012). Therefore, by the time a student's performance merited referral for assessment to determine eligibility for services defined by an IEP, the student's performance was below his or her current grade level in the content area of study. Since a difference between IQ and achievement levels occurred over a period of time, the discrepancy model became known as the wait-and-fail method of identification of SLD (Burton & Kappenberg, 2012; Yell, Shriner, & Katsiyannis, 2006).

Further, because identification was often sought while the child was in elementary school, "the defining characteristic [of a child with a SLD in mathematics] was considered to be insufficient automaticity of arithmetic number facts" (Lewis, 2105, p. 352). Additionally, the method for identifying the SLD did not discern whether a deficit resulted from cognitive processing issues or whether the difference in achievement and IQ scores ensued from the types of instruction received (Gutiérrez, 2013; Lewis, 2015). The use of interventions, based upon assessments to provide remediation to struggling learners, is the foundation of Response to Intervention (RTI) .

Response to intervention. Since the reauthorization of IDEA and the removal of the requirement to use the discrepancy model (Bradley, Danielson, & Doolittle, 2007), some states have changed from the discrepancy model of identifying and referring students for services identified in an IEP to the RTI model (Burton & Kappenberg, 2012). In contrast to the discrepancy model, the RTI model is a framework that utilizes tiered interventions and intense instructional practices to address the learning needs of all students, including potential students who are at risk of failing in the areas of mathematics, reading, and writing, prior to identification of SLD (Burton & Kappenberg, 2012; O'Donnell & Miller, 2011). The foundations of RTI were laid in the 1970s using "data-based program modification" (Burton & Kappenberg, 2012, p. 11) to identify skill deficits, drive instruction, and monitor progress of students (Bradley et al., 2007). Students who fail to respond to interventions as documented by numerous data points including classroom grades and scores obtained through progress monitoring assessments may then be identified with a SLD and referred to receive special education services specifically prescribed in their IEP to meet their individual learning needs (Burton & Kappenberg, 2012).

RTI first occurs in the regular classroom (i.e., Tier I instruction) with all students receiving high quality, research-based instructional practices “that meet the needs of a diverse population” (Johnson, Smith, & Harris, 2009, p. 4). Initially, all students are screened for skill deficits using a standardized assessment known as a universal screener (Gersten, Dimino, & Haymond, 2011). Using the results from the universal screener and other various data points including students’ results on state tests and classroom grades, students are identified by the RTI instructional support team as needing interventions and additional support beyond Tier I instruction for content area success (Burton & Kappenberg, 2012; Gersten et al., 2011). Designed to support students whose needs are not being met in Tier I and who are ranked in the lowest 15 to 20 % of the class, Tier II instruction at the high school level provides academic support “that targets skills and content that will support student performance in the general education class (i.e., Tier I)” (Johnson et al., 2009, p. 84). Academic strategies that promote student engagement and motivation are also implemented in a Tier II intervention setting (Johnson et al., 2009).

The RTI instructional support team meets at specified time intervals to frequently monitor and discuss student progress (Crandall, Ax, & Burton, 2012). Using benchmark tests and other assessments, the instructional support team prescribes the types of interventions needed as well as placement into or removal from either Tier II or Tier III (Burton & Kappenberg, 2012; Johnson et al., 2009). Tier III provides interventions with increased focus and duration for students whose needs were not met in Tiers I and II (Burton & Kappenberg, 2012; Johnson et al., 2009). A student’s continued lack of progress in Tier III prompts the instructional support team to refer the student for further evaluation to determine need for special education services (O’Donnell & Miller, 2011).

Although the discrepancy and RTI models serve the same purpose (i.e., identifying students with a SLD), the discrepancy model does not emphasize remediating students prior to classification whereas the RTI model requires implementing research-based interventions, tracking student progress, and collecting and analyzing data prior to referring a student for evaluation for a SLD (Burton & Kappenberg, 2012; Johnson et al., 2009).

Challenges to Teaching Students with SLDs

Although research shows that implementation of effective teaching practices promotes mathematical proficiency in students with SLDs (Fennell, 2011; NRC, 2001), there exist proposed challenges to utilization of such instructional practices with students at risk for mathematics difficulties or who are identified with SLDs. One such challenge in employing effective teaching practices in small-group settings is that students with SLDs may not exhibit appropriate and necessary social skills to work in groups (Gillies & Boyle, 2010). In addition, students who are identified with SLDs may exhibit passivity as a result of a learned helplessness due to a lack of mathematical skills (Kroeger & Kouche, 2006; Miller & Mercer, 1997). In the following section, I will expand on the challenges of instructing groups of students who are identified with a SLD or who are at risk for mathematical difficulties. Additionally, I discuss the impact of prior ineffective teaching practices (e.g., direct-instruction techniques) on struggling learners and students with SLDs.

Passivity. A possible avenue for encouraging the sharing of ideas and facilitating mathematical discourse is the placement of students into cooperative groups (NRC, 2001; Steele, 2002). Although some studies reveal positive results in using small group learning (e.g., Johnson & Johnson, 2009; Kuntz, McLaughlin, & Howard, 2001; Whicker, Bol, & Nunnery, 1997), McMaster and Fuchs (2002) reviewed 15 studies and reported that achievement outcomes were mixed. Studies by Johnson and Johnson (2009), Forman and Cazden (1985), and Kuntz, McLaughlin, and Howard (2001) found that the achievement of students with SLD increased while working in small group learning situations. However, research conducted by Slavin, Madden, and Leavey (1984) revealed that cooperative learning situations yielded no differences in student achievement but had positive effects on students with SLD attitudes toward mathematics and social acceptance.

A study by Woodward, Monroe, and Baxter (2001) revealed that students with SLD were passive in whole-group discussions. An additional study by Baxter, Woodward, and Olson (2001) found that students with SLD were also passive when working in small groups with learners of differing ability levels. Furthermore, Kroeger and Kouche (2006) revealed that continuous academic failure in mathematics has contributed to a state of learned helplessness for students with SLD. This state of learned helplessness has resulted in students with SLD and those at risk for mathematics difficulties relying and depending on other students and the teacher for continuous support and help in their mathematics learning (Wachira, Pourdavood, & Skitzi, 2013). The outcome of such continuous learned helplessness has allowed these students to become passive learners (Kroeger & Kouche, 2006; Miller & Mercer, 1997, Wachira, et al., 2013). The membership of passive learners leads to a breakdown in collaboration in small group learning (King, 1993). Gillies and Boyle (2010) noted that teachers perceived that students with SLDs need explicit training in how to work in groups as an intervention to promoting engagement.

Challenges resulting from direct instruction. Often children with SLDs are taught using a direct-instruction approach in which the teacher models procedures using a step-by-step process (Fennell, 2011; Fuchs, Fuchs, Hamlett, & Appleton, 2002; Montague, Applegate, & Marquard, 1993; Steele, 2002). The emphasis on direct instruction results in four issues. First, the implementation of direct instruction fails to provide opportunities for students to explore concepts or make connections among representations (Akyuz, Dixon, & Stephan, 2013; Fennell, 2011). Second, explicit instruction deprives students of learning skills that "promote strategic and adaptive thinking" (Fennell, 2011, p. 33). Third, many teachers believe that constant drill of procedures and rote memorization helps students with SLDs develop procedural fluency (cf. Alt, Arizmendi, & Beal, 2014; Baxter, Woodward, & Olson, 2001; Doabler et al., 2012) when in actuality such instructional techniques cause boredom and dislike for mathematics (Fennell, 2011). Although shortcuts and tricks (e.g., mnemonics) may temporarily result in mastery of mathematical facts and procedures by students with SLDs, they do not promote conceptual understanding or build procedural fluency and thus, compound the learning difficulties experienced by the student (Baroody, 2011). Additionally, such actions by the teacher assist students in co-constructing an identity that promotes learned helplessness and an academic status of mathematical failure (Gutiérrez, 2013; Heyd-Metzuyanim, 2013). Fourth, an overreliance on direct instruction robs students of studying the relevance of mathematics in real-world situations. As a result, students are deprived of opportunities to develop and apply strategies that develop problem-solving skills (Baroody, 2011).

Summary. Though effective instructional practices for all students often incorporate small group learning, there exist concerns when using this instructional approach with students identified with a SLD or who are at risk for mathematical difficulties. First, students with a SLD may exhibit passivity and fail to engage in small group activities. Second, some teachers choose to teach students with SLDs using a direct-instructional approach. The use of direct instruction often fails to support student discourse, the development of students' strategic competence, and knowledge of how to work collaboratively in small groups (cf. Baroody, 2011; NRC, 2001).

Historical Views of Instructing Students with SLDs

In considering effective instructional strategies for supporting students with SLDs, there seems to be some potential conflicting views between the fields of mathematics education and special education (cf. Doabler et al., 2002; Hiebert & Grouws, 2007; Marcus & Fey, 2003; Montague, 1997). Research in mathematics education has shown that all students can benefit from effective teaching practices that engage the student in the learning process (NCTM, 2014; NRC, 2001). Such practices include but are not limited to the implementation of cognitively demanding tasks, the facilitation of discourse, and the use of representations.

In contrast, some special educators indicate that students with a SLD may benefit from instructional practices that include direct instruction, modeling how to solve problems using algorithms or processes (Doabler et al., 2012; Montague, 1997), mnemonic devices, rote memorization, and step-by-step procedures (Fuchs, et al., 2002;

Steele, 2002). Studies conducted by Montague, Applegate, and Marquard (1993) and Fuchs, Fuchs, Hamlett, and Appleton (2002) reported that children with SLDs in mathematics “need explicit instruction in problem-solving strategies as well as guided learning experiences in mathematical problem solving” (Montague, 1997, p. 166). Such practices describe teacher-centered instruction, however, and may fail to develop proficiencies and competencies specified by the SMP (Fennell, 2011).

Regardless of the differing views espoused by mathematics educators and special educators on how students with SLDs learn mathematics, a review of literature documented that students with SLDs struggle in learning and understanding mathematics (cf. Montague, 1997; Woodward Monroe, & Baxter, 2001). However, the use of heuristics as an instructional tool to assist students in problem solving may address the recommendations of both the special education and mathematics education communities (Gersten et al., 2009; Woodward et al., 2001).

The Benefits of Using Heuristics

Students identified with a SLD often have difficulties in mathematics due to a lack of organizational skills (Steele, 2002). Heuristics can provide an organizational structure that guides students’ thinking and possibly regulates impulsive behavior (Gersten et al., 2009). In a meta-analysis conducted by Gersten et al. (2009), several instructional interventions, including the use of heuristics and student verbalizations, were found to increase the mathematical performance of students with SLDs. In the following sections, the use of heuristics as a tool to guide students’ thinking, encourage student verbalizations, and promote group discourse is discussed.

Heuristics as an Intervention Tool

A heuristic is a set of general guidelines or strategies that provides students with a generic way of solving problems (Gersten et al., 2009). For example, Polya's problem-solving process (i.e., understand the problem, devise a plan, carry out the plan, and check the solution) is an example of a heuristic used to help students solve word problems (Polya, 1945). Teachers may develop or use a curriculum purchased heuristic to use as an intervention to help students with a SLD develop necessary skills in solving word problems (Woodward et al., 2001).

As reported by Woodward et al. (2001), teachers may have to scaffold instruction to teach students how to use heuristics to meet learning objectives. Woodward et al. (2001) found that the use of a heuristic (e.g., the Problem Solving Guide from the *Everyday Mathematics* curriculum) in ad hoc tutoring sessions for small groups increased the mathematical performance on assessments for students with SLDs. The tutor's role was to explicitly help guide students through the heuristic (Woodward et al., 2001). When students were no longer making progress, the tutor made explicit suggestions for solving the problem. This ad hoc tutoring approach differed from the traditional approach of modeling processes (Montague et al., 1993) to solve word problems in that the tutoring approach encouraged communication rather than memorizing and implementing specific steps for solving problems (Woodward et al., 2001). Furthermore, in the study conducted by Woodward et al. (2001) students were prompted by a tutor to offer suggestions on how to solve a problem. Results of the study suggested that the use of heuristics increases students' verbalizations.

Another study yielded similar findings. Al-Fayez and Jubran (2012) revealed that the use of heuristics in mathematics instruction not only motivated students but engaged students to "express their mathematical solutions verbally" (p. 459). In summary, heuristics provide students with a general guide to aid in solving problems, but can be used as an intervention tool to promote verbalizations and promote discourse for all students.

Student verbalizations in mathematics learning. Often, people can be seen or heard talking aloud to themselves. Although such actions may seem frivolous, they may actually serve a purpose. Montague (1997) stated that verbalizing one's thoughts increases metacognition that "enables learners to adjust accordingly to varying task demands and contexts" (p. 165). Schunk and Cox (1986) reported that having students verbalize thinking was effective in helping students implement strategies in mathematics, though Montague (1997) found that students with disabilities verbalized less when mathematics problems became more challenging, possibly due to cognitive overload.

In addition to helping students self-regulate and focus thoughts to the task at hand (Schunk & Cox, 1986) and control impulsive behavior (Steele, 2002), Montague (1997) and Miller and Mercer (1997) revealed benefits of verbalizing for oneself in the form of thinking aloud. Though the technique of thinking aloud involves a person talking out loud, whispering, or thinking to oneself (Montague, Warger, & Morgan, 2000) the benefits of communicating one's ideas to a community of learners allows students "to test their ideas" (NCTM, 2000, p. 61) and gain experience in clarifying their justification. Additionally, Doabler et al. (2012) recognized the benefits of verbalization leading to engaging students in discourse. To help students gain communication skills, teachers should make available opportunities for students to engage in mathematical discourse with peers (Webb et al., 2014). Small group instruction using heuristics (e.g., a protocol to solicit engagement of group members) is one possible way to encourage communication during classroom activities (Thomas, 2006; Woodward et al., 2001).

Heuristics and encouraged discourse. An important element in small-group learning is promotive interaction which involves face-to-face, engaged student interactions in which students not only provide feedback to improve solutions but they may also challenge other members' reasoning of the solution path (Johnson & Johnson, 2009). Promotive interaction encourages discourse that can promote mathematical talk. For this to happen, teachers must "build a community in which students feel free to express their ideas" (NCTM, 2000, p. 61). One such heuristic that provides structure to student communications, promotes listening to the ideas of others, and encourages each student to have a voice in a mathematical discussion is the THINK interaction framework (Thomas, 2006).

Components of the THINK interaction framework consist of general directions for collaboratively engaging in a problem-solving activity: Talk, How, Identify, Notice, and Keep. In the first component (i.e., Talk) students are encouraged to talk about and identify the nature of the problem. In the second component of the framework (i.e., How), students discuss how they will solve the problem. During this stage of the framework, each student describes and justifies his proposed strategy or solution path for the problem. In the third component of the framework (i.e., Identify), the group collectively identifies a strategy for solving the problem. Throughout the problem-solving process, group members discuss the effectiveness of the strategy and whether they need to try a different strategy. During the fourth phase (i.e., Notice) each group member describes how the strategy helped him solve and understand the problem, if at all. During the last component of the framework (i.e., Keep), students continue to discuss the relevance of the answer and determine if there are alternate solution paths (Thomas, 2006). Simply stated, students "keep thinking about the problem" (Thomas, 2006, p. 88) to establish whether the solution is sensible and if there are other solution paths.

Summary. Throughout my review of the literature, I noticed that special educators and mathematics educators use heuristics differently. Whereas special educators often use heuristics as scaffolds in explicit instruction to assist students with SLDs in procedural and problem-solving processes and to promote student verbalizations (Baxter et. al, 2001; Schunk & Cox, 1986, Woodward et al., 2001), mathematics educators use heuristics to promote problem-solving processes among groups of students through inquiry-based activities (Al-Fayez & Jubran, 2012; Thomas, 2006). Furthermore, I found a lack of empirical studies that bridged this pedagogical disconnect concerning the use of heuristics for teaching students who were at-risk for mathematics difficulties. Throughout my search, I found no studies that addressed the use of heuristics to both engage students as a community of learners in inquiry-based learning activities and to scaffold instruction for students at-risk for mathematics difficulties.

Statement of the Problem

Current reforms in mathematics education call for teachers to use effective teaching practices to support all students in becoming mathematically proficient (NCTM, 2014). These practices include implementation of inquiry-based instruction using tasks to advance students' reasoning through problem solving, provision of opportunities for students to experience productive struggle, and facilitation of meaningful mathematical discourse in small group settings (Fennell, 2011; NCTM, 2014). A review of literature concerning reformed-based teaching practices concludes that all students who engage in instruction based on effective teaching practices can learn mathematics (Fennell, 2011; NCTM, 2014; NRC, 2001). However, my review of literature regarding the mathematics instruction of students identified with a SLD discerned that these students are often exposed to instructional methods different from those previously described as effective teaching practices. Unless all students are provided opportunities to learn mathematics through effective teaching practices, students at risk for mathematical difficulties may continue to lag behind their peers in their development of mathematical knowledge (Fennell, 2011; NRC, 2001). Empirical studies are needed to investigate how instructional tools used as interventions, such as heuristics, can be used to support students with SLD in an environment in which instructional methods promote the facilitation of inquiry-based activities and engagement in mathematical practices (e.g., constructing and critiquing arguments, making sense of problems, and engaging in meaningful discourse).

Purpose Statement

The purpose of this study was to understand how a specific heuristic (i.e., the THINK interaction framework) used as an intervention tool supported student discussions during problem-solving activities in a small group setting in a Tier II high school mathematics classroom. The following primary research question guided the study: How does the THINK interaction framework support students' exploratory talk and their movement toward mathematical proficiency, if at all?

Significance of the Study

This case study contributes to the existing knowledge base concerning the teaching of students at-risk for mathematics difficulties in three ways. First, the connection among implementation of effective teaching practices (NCTM, 2014; NRC, 2001), the SMP (CCSSI, 2010), and small group learning (Johnson & Johnson, 2009; Forman & Cazden, 1985) provides mathematics and special education researchers with a deeper understanding of ways to support struggling learners in their learning and understanding of mathematics. Second, this study informs researchers and practitioners of how the use of a heuristic in an intervention setting supports students in peer discussions when they are engaged in inquiry-based activities (Woodward et al., 2001). Finally, this study extends the knowledge of researchers and practitioners about the effectiveness of intervention tools used to address the proposed challenges of teaching students with passivity and learned helplessness (Kroeger & Kouche, 2006; Miller & Mercer, 1997; Prater, Bruhl, & Serna, 1998).

Definition of Terms

To assist the reader with clarity and understanding, this section provides the following definitions and terms.

Conceptual Understanding

Conceptual understanding is used to describe a student’s ability to comprehend and connect mathematical concepts, operations, and relations (NRC, 2001). Conceptual understanding is the basis for developing procedural fluency (NCTM, 2014).

Discourse

In mathematics, discourse refers to the "purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication" (NCTM, 2014, p. 29).

Discrepancy Model

The discrepancy model is a method of identification of students who show a discrepancy between their achievement level and IQ score. Because a noted difference between the two measures develops over time, the discrepancy model is also known as the wait-and-fail method of identification of students with specific learning disabilities (Burton & Kappenberg, 2012; Yell et al., 2006).

Effective Teaching Practices

This term refers to a framework of eight mathematics instructional practices, referred to as the Mathematics Teaching Practices defined by NCTM (2014) and known to strengthen the teaching and learning of mathematics.

Explicit Instruction

Explicit instruction refers to the instructional method in which the teacher directly shows students how to perform a procedure using each step or solve a problem by stating steps in the process (Doabler et al., 2012).

Heuristic

A heuristic is a general set of guidelines or strategies that provide support to students in solving problems (Gersten et al., 2009).

Intermental Activity

Intermental activity refers to the social interaction among a group of individuals where language is the catalyst for collective thinking and knowledge is constructed (Rojas-Drummond & Mercer, 2003).

Intramental Activity

Intramental activity refers to cognitive activity within an individual that results in reflection and internal dialogue (Rojas-Drummond & Mercer, 2003).

Procedural Fluency

Procedural fluency is used to describe a student's ability to flexibly and accurately use procedures to perform computations and to solve problems (NCTM, 2011; NRC, 2001).

Research-based Interventions

Research-based interventions are instructional tools and activities that have been found through research studies to be effective in providing support student learning (Gersten et al., 2009).

Response to Intervention

Response to Intervention (RTI) is a tiered framework that uses research-based interventions and targeted instructional practices to support students who are identified as having skill deficits (Burton & Kappenberg, 2012).

Specific Learning Disability

This term is used to describe students who have been evaluated and identified as having a learning disability in a specific learning discipline (NDCCD, 2012).

Students At Risk for Mathematical Difficulties

Students who are at risk for mathematical difficulties are identified as not likely to succeed in a mathematics curriculum without intervention and targeted instruction (Burton & Kappenberg, 2012).

Task

A task in this study refers to a cognitively demanding activity that requires students to use prior knowledge along with inquiry to find a solution. Additionally, the activity helps students develop conceptual and procedural understanding (Marcus & Fey, 2003).

Tier I

Tier I instruction is core instruction that uses research-based instructional methods and effective teaching practices to support students in their learning efforts (Burton & Kappenberg, 2012).

Tier II

Tier II instruction is used to describe an instructional time during the daily schedule when students identified with mathematical deficits receive targeted instruction and research-based interventions to support students in their learning endeavors (Burton & Kappenberg, 2012).

Verbalization

This term is used to describe a student's overt thinking aloud using a spoken or whispered response (Montague et al., 2000).

Chapter Summary

To prepare our students as future prospects in the global job market, teachers must find ways to teach mathematics to all students using effective teaching practices (NCTM, 2014). Such instructional practices should provide opportunities for all students to engage in inquiry-based activities that promote problem solving and reasoning abilities (Marcus & Fey, 2003; NRC, 2001). Additionally, instructional activities should allow all students to have a voice in justifying their work and participating in mathematical conversations that increase their conceptual understanding of the topics studied (Webb et al., 2014). All students, regardless of their academic abilities, must prepare for skills needed for future jobs. As such, educators should continue to declare high expectations for all students (Fennell, 2011) and use intervention tools and practices that attempt to prevent and even close gaps in students' knowledge in spite of instructional challenges due to SLDs (Burton & Kappenberg, 2012). Thus, in this study I will investigate how the use of a heuristic (i.e., the THINK interaction framework) supports students' exploratory talk and movement toward mathematical proficiency.

CHAPTER II: REVIEW OF LITERATURE

Introduction

Globalization of the economy and job market requires members to possess not only analytical, critical thinking, and reasoning skills but also skills that go beyond conversational speech (Fennell, 2011; Michaels, O'Connor, & Resnick, 2008). To prepare all students to engage in such venues, teachers should utilize effective teaching practices that provide opportunities for students to learn mathematics (Kroeger & Kouche, 2006). Such practices include experiencing productive struggle through sense-making activities (Marcus & Fey, 2003; Smith, Bill, & Hughes, 2008), collaborating in problem-solving experiences (Johnson & Johnson 2009), using multiple representations (Fennell, 2011; NRC, 2001), and exercising the type of communication skills described by SMP3 (i.e., construct viable arguments and critique the reasoning of others) (CCSSI, 2010). Effective communication skills in the classroom entail negotiation, justification of thoughts, knowledge building, and consensus building (Michaels et al., 2008). To promote effective communication skills, teachers must ensure that students are engaged in productive classroom discourse.

A framework that has allowed teachers to gauge the type of talk that occurs in their classroom is the discourse analysis framework created by Mercer, Wegerif, and Dawes (1999). The framework allows teachers and researchers to categorize student dialogue into three types of observable discourse in the classroom: disputational talk, cumulative talk, and exploratory talk (Mercer, Wegerif, & Dawes, 1999). Using Mercer et al.'s framework supplemented with the strands of mathematical proficiency (i.e., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition) (NRC, 2001), this study attempted to explain how an instructional tool (i.e., the THINK interaction framework) used as an intervention supported students' discussions about mathematics and provided evidence of talk that indicated movement toward mathematical proficiency.

Because the scope of this study was to examine whether a heuristic supported students' mathematical discourse and provided evidence of talk that indicated movement toward mathematical proficiency, it is important for the reader to have an understanding of the meaning of mathematical proficiency. To assist the reader in understanding mathematical proficiency, I first describe the strands of mathematical proficiency and provide examples of each to provide clarity. Second, I discuss the challenges of teaching students with mathematical difficulties to provide the reader with an understanding of the hurdles that students with SLDs and their teachers must overcome in helping them to move toward mathematical proficiency. Third, I review the literature concerning the effects of instructional strategies on students who are at-risk for mathematics difficulties or who exhibit a SLD in mathematics as well as the effects of teacher expectations on student learning. Fourth, I examine empirical research on the effect of language disabilities on mathematics learning, the effects of students' passiveness and reluctance to participate, and the effects of the use of heuristics on student engagement and the learning of mathematics. Fifth, I discuss Vygotsky's theory of social constructivism that focuses on the use of language as a cultural, cognitive, and pedagogical tool and establishes a theoretical framework for the study. Following, I discuss the components of Mercer et al.'s framework for discourse analysis to demonstrate the need for a tool to analyze student conversations prior to and during the use of the THINK interaction framework. Last, I conclude with an explanation of how the extant literature supports the need to investigate the THINK interaction framework, a heuristic, used as an intervention to promote mathematical discourse and movement toward mathematical proficiency chapter summary.

Strands of Mathematical Proficiency

In 2001, the NRC chose the term mathematical proficiency to describe the essence of successful mathematics learning and understanding. In doing so, the NRC deemed that mathematical proficiency has five strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. None of these strands stands alone, but rather they are intertwined and interdependent (see Figure 1).

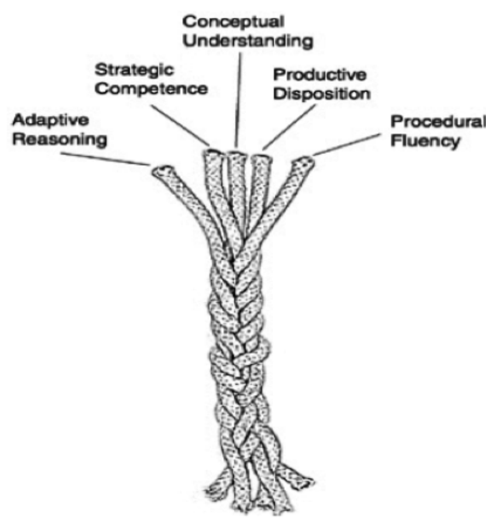

Figure 1. Intertwined strands of mathematical proficiency. Reprinted with permission from “Adding It Up: Helping Children Learn Mathematics,” (2001) by the National Academy of Sciences, Courtesy of the National Academies Press, Washington, D.C.

To provide clarity and understanding of the strands of mathematical proficiency in this section, I have included definitions adapted from *Adding It Up: Helping Children Learn Mathematics* (NRC, 2001) and examples to illustrate the context.

Conceptual Understanding

Conceptual understanding is defined as a student's ability to comprehend mathematical concepts and the relations among the various concepts (NRC, 2001; Suh, 2007). For example, a student displays conceptual understanding when he comprehends that the product of two linear factors result in a quadratic function and that the zeros of the quadratic function are the same as the x-intercepts of the graphs of the lines that produce the linear factors. To illustrate this concept, I will use the quadratic function of $f(x) = x^2 - 2x - 8$. Written in the factored form of $f(x) = (x + 2)(x - 4)$, one can see that the zeros, x = -2 and x = 4, of the quadratic function are equal to the x-intercept of the respective lines, $f(x) = x + 2$ and $f(x) = x - 4$. As Suh (2007) reported, students who acquire conceptual understanding of a mathematical topic retain knowledge about the concept and are able to transfer such knowledge to other contexts.

Procedural Fluency

Procedural fluency is the ability to accurately carry out computations and procedures flexibly and appropriately (NRC, 2001). An example of procedural fluency occurs when the student uses factoring techniques to find the factors and to write the quadratic function, $f(x) = x^2 - 2x - 8$, in an alternate way such as in its factored form of $f(x) = (x + 2)(x - 4)$. Additionally, the student may write the quadratic function in its vertex form, $f(x) = (x - 1)^2 - 9$ using procedures that require him to find the x-intercept of the vertex, x = 1, and then the value of the y-coordinate of the vertex by computing $f(1)$. Conceptual understanding supports the development of students' procedural fluency in that when students understand the meaning and relations among concepts, they are more able to use an algorithm because they understand the reasoning for using an algorithm. For example, understanding that to find the zero of a quadratic function requires similar procedures for finding the y-intercept of a line, the student is transferring his knowledge of linear functions to finding solutions to quadratic equations (Suh, 2007). Furthermore, the student is learning to use procedures flexibly by applying them to different contexts (NCTM, 2014).

Strategic Competence

Strategic competence relates to the student's ability to identify the problem, represent the problem using symbols, graphs or other representations, and solve the problem (NRC, 2001). To illustrate, a student is assigned the following problem.

> Students in Mr. Bowen's Physics class are assigned a project that requires them to design, build, and launch a rocket. Johnny built a rocket that had an initial velocity of 96 feet per second when launched from a platform that was 112 feet tall. When will the rocket reach its maximum height?

Although the problem is routine, the student must use strategic competence to understand that the rocket is a projectile whose path of flight models a quadratic function and to identify the graphic representation as a parabola. In constructing the function, the student must use conceptual understanding to realize that the coefficient of the quadratic term is -16, representing acceleration due to gravity. Also, the student must distinguish that the coefficient of the linear term is represented by the initial velocity of the rocket, 96, and is positive due to the fact that it will be launched upward. Last, the student must know that the constant term is the initial height of the rocket, 112 feet, to be successful in his construction of the equation, $f(t) = -16t^2 + 96t + 112$. Last, the student must use procedural fluency in solving the problem.

Adaptive Reasoning

Adaptive reasoning refers to the ability to reflect, explain, and justify one's thoughts (NRC, 2001). For example, if a student is asked to explain the meaning of the zeros of the above problem, he would realize that the horizontal axis represents the time traveled by the rocket. Although the zeros of the function are $t = -1$ and $t = 7$, he would have to explain and justify his reasoning as to why $t = -1$ (i.e., negative time) is not a realistic answer in the context of the problem. The student could also use a graphic representation to show that the y-intercept represents the initial height with $x = 0$ representing the initial time prior to launching the rocket. In doing so, the student can refer to the zero, $x = -1$, as an extraneous solution to the problem. Requiring students to justify and explain solutions is an example of the use of SMP3, construct viable arguments and critique the reasoning of others (CCSSI, 2010).

Productive Disposition

Productive disposition is a student's "inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (NRC, 2001, p. 5). The teacher's role in mathematics is to plan and implement mathematics instruction in a logical sequence as to develop mathematical proficiency. In doing so, the selection of tasks and use of scaffolds helps students to not only build skills but to also promote efficacy (Fennell, 2011). To help students develop a productive disposition, teachers must help students see the relevance of doing and understanding mathematics as it relates to their own lives (Suh, 2007). Additionally, the practice of engaging students in productive struggle helps students

> to build connections among ideas [i.e., conceptual understanding], to feel a sense of accomplishment [i.e., productive disposition], to apply their own knowledge [i.e., strategic competence and adaptive reasoning] and to develop a strong self-efficacy for being able to do mathematics [i.e., productive disposition] (McGatha & Bay-Williams, 2013, p. 169)

Summary

To foster children's development of mathematical proficiency, teachers should implement effective teaching practices that motivate and engage all learners (NRC, 2001). Through the use of activities that provide students with opportunities to exercise higher order cognitive skills, engage in social interaction and productive struggle, and use manipulative objects, teachers can help students to link informal knowledge with abstract ideas (NRC, 2001; Suh, 2007). As teachers work to move students toward mathematical proficiency, it is especially important to note that one cannot separate the understanding or abilities described by any one strand as the process is interwoven and requires that the skills and proficiencies described by each must be used and developed for an individual to be successful in learning and understanding mathematics.

The Challenges of Teaching Students with Mathematics Difficulties

Although reform-based mathematics calls for the implementation of effective teaching practices into daily instruction to provide all students with high quality core mathematics instruction and opportunities to learn (NCTM, 2014), teachers are faced with the challenges of teaching students who are at-risk for mathematics difficulties or who are identified with an SLD in mathematics. In the following sections, I discuss the challenges of teaching students who have identified language disabilities, exhibit a lack of social skills and passivity, or have difficulty using representations. Additionally, I discuss the differing opinions of educators and findings of researchers on what is considered appropriate instruction for students with SLDs in mathematics along with the effects of teacher expectations on student learning.

Language Disabilities

Research has shown that there exists a direct relationship between language disabilities and mathematical achievement and understanding (Alt et al., 2014; Harrison et al., 2009; MacGregor & Price, 2002; Miller & Mercer, 1997; Montague, 1997, Morin & Franks, 2010). Throughout this chapter, I refer to specific language impairments (SLI) as a specific category of a learning disability associated with language processing difficulties whereas a SLD is a specific learning disability in a certain category other than language processing. SLIs include, but are not limited to, phonological processing issues and metalinguistic, symbol, and syntax awareness difficulties (Harrison et al., 2009; MacGregor & Price, 2002). The types of SLIs listed above are discussed in the next few paragraphs but have not been explicitly identified as such. Students with SLIs often perform poorly as compared to peers without SLIs on language heavy and symbol heavy tasks (Alt et al., 2014). Students with limited language proficiency experience heavy demands on the cognitive load due to a lack metalinguistic awareness, which refers to a person's ability to structure and form a word or phrase (MacGregor & Price, 2002). Students with language disabilities often struggle with the reading demands presented by word problems including difficulties with decoding and comprehension skills (Harrison et al., 2009; Miller & Mercer, 1997). Such disabilities may interfere with the necessary proficiencies described by SMP1, make sense of problems and persevere in solving them, to be successful in solving word problems (CCSSI, 2010).

In addition to difficulties due to decoding and comprehension skills, students who exhibit problems with phonological processing often have difficulty in number sequencing as well as the retrieval of mathematical facts and vocabulary (Morin & Franks, 2010; Steele, 2002). Such difficulties present a cognitive load for students who lack prerequisite knowledge in mathematics, including basic number sense (Baxter et al., 2001). These difficulties are a result of a lack of metalinguistic awareness and affect not only phonological awareness but also symbol awareness and how students represent symbolic forms of written language (Harrison et al., 2009; MacGregor & Price, 2002). Thus, students identified as having SLIs often experience difficulties representing mathematical expressions due to using symbols (Alt et al., 2014; MacGregor & Price, 2002).

Students who exhibit a lack of metalinguistic awareness often have difficulty expressing “simple operations and relationships in algebraic notations” (MacGregor & Price, 2002, p. 111). For example, students with an SLI may not be able to translate the following problem into an algebraic expression.

> Paul has 10 less apples than Sarah. Sarah has *s* apples. Write an expression to represent how many apples Paul has.

Students with an SLI may misunderstand what the problem is requiring them to do as a result of a lack of metalinguistic awareness. This misunderstanding often affects their interpretation of the problem and ability to recognize ambiguity in the structure of the problem (MacGregor & Price, 2002; Miller & Mercer 1997). As a result, the student has difficulty in translating linguistic and numerical information contained in word problems into symbolic representations (Montague, 1997). In this example, students may symbolically represent the above scenario as $10 - s$ or $s + 10$ instead of $s - 10$.

Students with learning disabilities and a lack of language proficiency often have difficulty articulating their justification and reasoning in the processes used to solve a problem (Poch, van Garderen, & Scheuermann, 2015) and in asking for assistance to understand or solve a problem (Baxter et al., 2001). Harrison et al. (2009) explained that students with SLIs often experience challenges to initiate conversations and formulate responses when working in whole or small-group settings. Baxter et al. (2001) found that students with a SLD in mathematics require more time to respond to classmates' questions and explanations. Additionally, the requirement of a quick response time resulted in students' avoidance of participation in classroom discussions (Baxter et al., 2001). Active participation in classroom discourse is a central component in learning mathematical concepts (NCTM, 2014). Students' lack of engagement and participation avoidance presents instructional challenges (King, 1993; Kroeger & Kouche, 2006), some of which are discussed in the next section.

Passivity

Involving students in active participation in cognitively demanding tasks and engaging them in productive mathematical discussions is vital to student understanding and the development of mathematical skills (Wachira et al., 2013; Webb et al., 2014). Such opportunities to develop the varieties of expertise in mathematics processes and proficiencies described by the SMP (CCSSI, 2010) are often implemented through small-group interactions and the implementation of effective mathematical teaching practices including implementing cognitively demanding tasks, supporting productive struggle, and facilitating discourse (NCTM, 2014; Steele, 2002). Reform-based teaching practices encourage students to engage in mathematical discussions, ask questions, make conjectures, justify thinking, and listen to the arguments of other students (Akyuz et al., 2013). However, students who are at-risk for mathematical difficulties or who have been identified with an SLD, especially in the areas of speech and language, display passiveness and participate at a low level in the activities (Kroeger & Kouche, 2006; Mercer & Miller, 1997). As discussed in the previous section, students identified as having a SLD in mathematics had difficulty explaining their solutions, and thus, exhibited passive behavior (Baxter et al., 2001). This type of mathematics avoidance has been documented in as early as fifth grade (Suh, 2007).

In a study conducted by Baxter et al. (2001), students who were identified as at-risk for mathematics difficulties or having an SLD in mathematics were passive, often silent, and used avoidance measures such as lack of eye contact with the teacher to avoid participating in whole class discussions. Possible reasons for avoidance measures include lack of time to formulate a response, lack of confidence, lack of prerequisite knowledge, and communication issues (Baxter et al., 2001; Kroeger & Kouche, 2006). Additional causes of the display of avoidance measures include students' learned helplessness, repeated failure experiences, ineffective learning and teaching strategies, and learned coping mechanisms (Kroeger & Kouche, 2006; Miller & Mercer, 1997; Suh, 2007; Wachira et al., 2013).

Miller and Mercer (1997) reported that repeated failure in mathematics directly affects self-efficacy and passivity. As a result, students engage in learned helplessness as a coping mechanism to refrain from actively engaging in tasks and classroom discourse (Kroeger & Kouche, 2006; Miller & Mercer, 1997). Such behavior, reports King (1993), is a "self-preserving behavior" (p. 400). However, King (1993) found that students who were at-risk for mathematics difficulty experienced fewer opportunities to provide and receive explanations and justifications from members in their groups. Although Wachira et al. (2013) found students' "mathematical dispositions can be transformed over time by a teacher's pedagogical practices" (p. 34), many students remain resistant to reformed-based teaching practices that encourage student participation in mathematical discourse and continue to display passiveness and resistance during classroom activities. Wachira et al. (2013) attributed such resistance to traditional classroom practices, such as lecture and the use of drills that promote rote memorization of mathematical facts and procedures.

Difficulty with Visual Representations

Acquiring mathematical proficiency requires that an individual gain conceptual understanding as well as procedural fluency, strategic competence, adaptive reasoning, and strategic competence (NRC, 2001). Conceptual understanding involves understanding patterns, making connections among mathematical topics, and using various representations to help make such connections (Atl et al., 2014; NRC, 2001). In order to help students acquire conceptual understanding and the understanding needed to model with mathematics, students need opportunities in solving a variety of problems and in using various representations in differing contexts (Hiebert & Grouws, 2007; Marcus & Fey, 2003). Visual representations, including diagrams and sketches, are ways of representing mathematical concepts and can "serve as tools for mathematical communication, thought, and calculation, allowing personal mathematical ideas to be externalized, shared, and preserved" (NRC, 2001, p. 94). Additionally, diagrams are cognitive tools that require students to identify relevant information, to construct accurate figures, and to use new and prior knowledge (Poch et al., 2015).

Doabler et al. (2012) posited that core instruction (i.e., Tier I) should provide all students, including students with or who are at risk for mathematics difficulties, with opportunities to develop conceptual understanding. Additionally, Doabler et al. (2012) and Jayanthi, Gersten, and Baker (2008) claimed that teacher modeling, visual representations, and scaffolding of material help students to develop conceptual understanding across mathematical concepts and to identify relationships among various concepts. In contrast, Alt et al. (2014) stated that students with SLDs and SLIs often have difficulty using visual representations due to poor working memory. In agreement with Alt et al. (2014), Montague (1997) reported that students who exhibit language-processing deficits also have difficulty translating word problems into pictorial and graphic representations. According to Montague (1997), "the acquisition of conceptual, declarative, procedural, and strategic knowledge in mathematics may be negatively affected by developmental delays, representational and retrieval problems, and visuospatial deficits" (p. 166). Additionally, MacGregor and Price (2002) found that a lack of symbolic awareness and weak metalinguistic awareness, which are both characteristic of students with a SLI, often result in student difficulties in interpreting representations and recognizing ambiguity in structural relationships. Poch et al. (2015) concurred that using diagrams for representations requires that students accurately construct and label the diagram, skills in which students with SLDs are often weak. As a result, students with SLDs sometimes only realize one representation for a problem. A narrow focus of using only one representation limits a student's ability and strategic competence (Poch et al., 2015). Also, students with an SLI may have difficulty accurately articulating and describing the reasoning for constructing and labeling the diagram as well as justifying his reasoning for doing so (Poch et al., 2015). These limitations affect students' adaptive reasoning and possibly their productive disposition regarding how the visual representation can be useful in solving the problem (Poch et al., 2015).

Summary

Implementation of reformed-based mathematics requires teachers to engage students as active learners (NCTM, 2014). To do so, teachers must employ effective teaching practices including selecting and implementing tasks that require students to actively engage in constructing their knowledge and in communicating with other students. However, students with SLDs and SLIs not only have difficulty in communicating with others but they also have mathematical difficulties due to a lack of metalinguistic, symbol, and syntax awareness (MacGregor & Price, 2002). Such disabilities affect students' abilities to understand mathematical problems that are symbol and language heavy (MacGregor & Price, 2002) as well as visual representations (Poch et al., 2015). As a result, students exhibit passive and self-preserving behaviors (King, 1993; Kroeger & Kouche, 2006)

Instruction of Students With or At-risk For Mathematics Difficulties

Although educators and researchers agree that mathematics can be challenging for some students (Baxter et al., 2001), there seems to be conflicting views between the fields of special education and mathematics education on the types of teaching practices that are most effective for students with SLDs or who are at risk for mathematics difficulties (cf. Baxter et al., 2001; Doabler et al., 2012; Grouws, 2003; Hiebert & Grouws, 2007; Jayanthi, Gersten, & Baker, 2008). Although reform-based teaching practices encourage students to engage in mathematical discussions, ask questions, present ideas, justify thinking, and critique the ideas of others (Akyuz et al., 2013; CCSSI, 2010; NRC, 2001; Suh, 2007), Baxter et al. (2001) posed that direction and guidelines for the instruction of students with SLDs are absent from many reform-based documents. Though in agreement that students with SLDs need more time to engage in mathematical discourse and opportunities to learn (Baxter et. al, 2001; Doabler et al., 2012; Hiebert & Grouws, 2007; Montague, 1997; NCTM, 2014), many in the field of special education posit that students with SLDs need explicit and systematic instruction, teacher modeling, more sequenced instructional examples, and frequent opportunities to practice and review content to develop and promote proficiency (Alt et. al., 2014; Baxter et al., 2001; Doabler et al., 2012; Jayanthi et al., 2008; Montague, 1997; Steele, 2002).

Although mathematics educators and special educators agree that students benefit from differentiated instruction, the question remains as to what types of instructional practices benefit learners of differing abilities (Khan, 2013; Jayanthi et al., 2008; McGatha & Bay-Williams, 2013; Miller & Mercer, 1997). Tutoring, using heuristics, promoting verbalization and discourse, and scaffolding are methods of differentiated instruction reviewed in the literature. Fuchs et al. (2002) and Woodward et al. (2001) found that adult tutors were beneficial in helping students develop strategic competence. Al-Fayez and Jurban (2012) found that students who were taught how to use heuristics to solve problems improved their problem-solving abilities. Additionally, Al-Fayez and Jurban (2012) and Baxter et al. (2001) found that the success of heuristics depended upon scaffolding in the form of questioning by teacher. Scaffolding helped students to develop their verbal and communication skills.

Driscoll (2003), Thomas (2006), and Woodward et al. (2001) reported that the use of a heuristic encouraged students to think aloud and verbalize their reasoning skills. Verbalization of thoughts encouraged self-awareness and self-regulation strategies during problem-solving activities (Miller & Mercer, 1997; Montague, 1997; Montague et al., 2000). Thus, it seems reasonable that the use of heuristics to promote verbalization is an instructional practice that supports differentiated instruction. Even though the use of heuristics were reported to be a successful intervention used to promote verbalization in individual students, few studies have been conducted on its effectiveness to promote dialogue among groups of students (Thomas, 2006; Woodward et al., 2001). Thomas (2006) advocated that heuristics used in a group setting fostered student interactions and promoted mathematical discussions in small group settings. Additionally, Woodward et al. (2001) reported some group dialogue when reporting the effects of ad hoc tutoring of small groups of students using heuristics.

Along with active engagement in discourse and mathematical activity, teachers' expectations for all students remain an important component in developing students' productive dispositions and mathematics success (Grouws, 2003; NRC, 2001). Grouws (2003) suggested that it was a teacher's role to create a climate in which students feel comfortable to participate and mistakes are viewed as learning opportunities. Woodward et al. (2001) reported on the importance of a classroom climate conducive to learning, noting that the teacher failed to call on non-volunteers and students who exhibited passiveness (Woodward et al., 2001).

Instructional practices that promote inquiry-based instructional techniques that use cognitively demanding tasks to promote conceptual understanding, procedural fluency, and mathematical discourse in which all students' ideas are valued encourage productive disposition and hold all students to high expectations (Fennell, 2011; NRC, 2001). Teachers who hold low expectations for students, including those who are at risk for mathematics difficulties and those who are identified as having a SLD, often fail to implement effective mathematical teaching practices and resort to a reliance on the textbook and traditional instructional methods (Fennell, 2011). Unfortunately, the overt use of textbooks and traditional instruction limits students' opportunities to engage in meaningful mathematical discourse and to verbalize their thinking (Doabler et al., 2012). As a result, students often fail to see mathematics as useful and relevant (Baroody, 2011) and fail to develop a productive disposition (Fennell, 2011; NRC, 2001; Suh, 2007).

Empirical Research on Teaching Students with Mathematics Difficulties

This section reviews empirical studies concerning the effects of language disabilities on mathematics learning, student passivity and reluctance to participate in classroom discourse, and the effects of heuristics on student participation and learning of mathematics. In each review, I will provide an overview of the study and how the research of each topic relates to and supports the proposed study.

The Effects of Language Disabilities on Mathematics Learning

During my review of literature, I found three studies regarding the effects of language disabilities on mathematics learning. In the sections that follows, I describe the methods of each study, the findings, and how the reviewed study relates to the current study.

The relationship between mathematics and language. Alt et al. (2014) examined the relationship between language and mathematics in an attempt to identify the nature of mathematical difficulties in students with SLIs. Participants in the study included 20 students with SLIs, 20 students who were identified as English language learners, and 21 native monologue English students from 10 elementary schools. The researchers included students who had an SLI, English Language Learners (ELL), and native English learners to identify whether difficulties were due to brain-based differences or a lack of limited English proficiency. Methods included assessing students with the KeyMath3 assessment tool, a norm-referenced assessment (Alt et al., 2014), and three computer game-like mathematics assessments. The assessments varied in their language and mathematical demands.

The results of the study revealed that native English learners without a SLI performed better than students who were ELL or identified as having an SLI on assessments that were considered to be both language and symbol heavy. The assessment was considered to be language heavy and symbol heavy because it contained no modifications to the syntactic complexity and use of mathematics vocabulary (Alt et al., 2014). On the assessments that were considered to be language-light and symbol heavy, both the ELLs and native English learners scored better than students with SLIs. Other assessments that measured students' performance on the concept of patterns disclosed that students with SLIs scored less accurately than students in the other two groups. The findings showed that students with SLI exhibited difficulty in mathematical assessments and on tasks that used average language vocabulary and syntax containing symbols (Alt et al., 2014). Additionally, the study showed that students with an SLI had difficulty with using and recognizing patterns; such difficulty could be due to "heavy demands on visual working memory" (Alt et al., 2014, p. 229). Further, findings showed that there were no statistically significant differences in among the students in all three groups (i.e., ELL, SLI, and native English learners) on tasks that were both language and symbol-light (Alt et al., 2014). Assessments that were language heavy and symbol-light were not conducted in the study.

The findings in the study conducted by Alt et al. (2014) are similar to the findings by MacGregor and Price (2002) who reported that students with language impairments have difficulties with word problems and tasks that are language and symbol heavy. Although research has shown that students with SLIs often have difficulty with word problems, I am curious as to the benefits afforded to students with an SLI when working in small groups. According to Vygotsky's theory of social constructivism, students who work in groups benefit from dialogue as individuals internalize information obtained through dialogue and cognitively reorganize such knowledge in an attempt to conceptually understand the concepts of which they are studying (Churcher, Downs, Tewksbury, 2014; Mercer & Howe, 2012; Rojas-Drummond & Mercer, 2003;). Though students may have difficulty with expressive language, it stands from Vygotsky's theory of social constructivism that group work that engages all students in active learning and mathematical discourse could benefit students with SLI.

The effects of language impairments on literacy and numeracy. Harrison et al. (2009) conducted a longitudinal study to investigate the progress of students two years after they were identified with an SLI in early childhood. The participants were 3,632 children between the ages of six and seven who participated in both phases of the study. Data were collected to inform the researchers of the children's development and learning across different contexts. Data collection included interviewing the parents or caregivers and children's teachers, conducting individual children's assessments about learning and development, and collecting survey information through questionnaires completed by parents and the children's teachers. Indicators for speech-language impairment for the first phase included parental reports of both receptive and expressive speech and language concerns, use of speech-language pathology services, and low receptive language scores on the Peabody Picture Vocabulary Test-III (Harrison et al., 2009). Phase two of the study included the previously used sources for phase one except speech-language pathology services.

The results of the study confirmed that children identified as having a SLI at five years of age with continued language difficulties did not perform as well on school achievement measures as same-aged peers without a SLI or who were no longer identified as having an SLI (Harrison et al., 2009). Additionally, teachers reported that students with SLIs performed lower in literacy and mathematical reasoning and "were less well adjusted than non-impaired children" (Harrison et al., 2009, p. 400) than their same-aged peers.

As I conducted a search for literature concerning the relationship between language impairments and mathematics, I found few articles (e.g., MacGregor & Price, 2002; Pathchell & Hand, 1993) that discussed the effects of language impairments on secondary school students' mathematical achievement. As I reviewed articles, I wondered if a high school student having difficulties in justifying his answers and exhibiting passive or disruptive behavior could possibly have an undetected language disorder. According to Patchell and Hand (1993), students at the secondary level may have a language disorder that is not an expressive disorder. Instead, students at the secondary level who have a language disorder may exhibit signs that include, but are not limited to, the inability to follow instructions, disorganization, rudeness, and lack of participation in classroom discourse (Patchell & Hand, 2003).

Aspects of language proficiency and algebra. MacGregor and Price (2002) conducted two empirical studies to investigate three components of language proficiency (i.e., metalinguistic awareness of symbol, syntax, and ambiguity in mathematics structures) and student success in algebra. In the first study, 1,236 students between the ages of 11 and 16 who were enrolled in their first to fourth algebra class participated in the study that sought to investigate whether a relation existed between students' awareness of symbols and syntax and algebra learning. Using a self-constructed assessment, the authors chose items to assess symbol awareness and syntax, simple algebra, basic literacy, and arithmetic skills. The results showed mixed findings. Some students scoring high on language items scored high on algebra items. However, some students scoring high on language items also scored low on algebra items. Additionally, no student who scored low on the language items scored high on algebra items.

To investigate further, MacGregor and Price (2002) conducted a second study to assess the relation between metalinguistic awareness and potential ambiguity (i.e., that algebraic expressions may have more than one interpretation). Participants of the study included 340 bilingual students in grades eight through ten. Students were assessed using a newly constructed test that consisted of new language items and previously used algebra items. The results revealed positive correlations between language and algebra scores, indicating that students who had high language scores also had high algebra scores. Additionally, students who had low language scores had low algebra scores.

As I reviewed the findings of the second study, I was reminded of the difficulties that students with language impairments and disabilities often face in comprehending word problems and expressing themselves through the written language. Furthermore, I was curious as to the types of interventions that would prove beneficial in helping students with SLIs in their study of mathematics.

The Effects of Students' Passiveness and Reluctance to Participate

In reviewing the literature, I found three studies that discussed students' passivity and reluctance to participate in mathematical discussions. For each study, I provide a description, the results, and how the reviewed studies inform this current study.

Effects of reform-based instruction on low achievers. Baxter et al. (2001) examined the effects of reform-based instruction on low achievers in five third-grade classrooms. Participants included five third-grade teachers and 104 third-grade students. For this study, the target group consisted of participants who were classified as having mathematics disabilities. Although only seven students were identified through an IEP as having mathematical learning disabilities, a discrepancy arose when the researchers learned that other students who should have received special education services had not been referred for such services for various reasons with the participating teachers citing poor quality mathematics instruction in special education mathematics classes. To further identify participants who needed special education services but did not have an IEP, the researchers administered the Iowa Test of Basic Skills and set the benchmark of 34 percent to identify at-risk students. Nine additional students who did not have IEPs scored below the predetermined benchmark of the 34^{th} percentile. These students were placed into the target group, raising the number of students in the target group to 16 students. Data collection methods included 34 classroom observations of students in five third-grade classrooms during mathematics instruction using a reputable reformed-based mathematics curriculum over a period of an academic year along with audio-recorded teacher interviews. To illustrate the happenings of a typical day in the third-grade reform-based mathematics classrooms, I provide the following description. A typical mathematics lesson began with an open-ended investigation in which students worked individually. Next, the lesson involved an individual problem-solving activity. Students were then directed by the teacher to work with a partner on the problem-solving activity. The lesson usually concluded with students working on a worksheet to practice the skills learned. Thus, lessons were implemented using whole-class and pair work as well as discourse.

Results showed that lower-achieving students did not often volunteer to participate in class discussions or question other students' solutions. Additionally, when the teachers tried to engage the target students in whole-class discourse, the students often gave one or two word utterances. The researchers noted that in all of the classroom observations, target students used avoidance behaviors such as engaging in off-task behaviors, doodling, and failing to make eye contact with the teacher. Results indicated that the target group of at-risk and SLD students lacked confidence to engage in conversations and needed additional time and resources to construct answers. However, students engaged in conversations and activities when working with a partner. The results also indicated that a highly structured curriculum was taxing to students with less than average ability as it was paced for the average or above student. Moreover, a finding that supports the current study was that when low-achieving students were placed in ad-hoc tutoring groups with students with similar abilities, student engagement was high.

As I reviewed this study, one characteristic that stood out to me was the limited use of group work by four of the five teachers. Although each lesson involved pair work, the use of ad-hoc tutoring of groups by one teacher successfully engaged all students in the activity and learning process. Students who exhibited passiveness during whole-group activities and discussions were more engaged in small group activities where the groups consisted of students exhibiting similar abilities. Additional research is needed for the type of support structures and interventions needed for students who are at-risk for mathematics difficulties to become engaged and successful in using a reform-based curriculum.

Passivity in small group instruction. King (1993) studied 22 third graders in a large American elementary school to examine the effects of passivity during small group instruction. The class dynamics consisted of high- and low-achieving students in mathematics working in groups of four students, known as the groups-of-four variation. In addition to studying the types of learning and information processing occurring during small groups, King sought to study the nature and degree of cooperation among students in small groups as well as the frequency and quality of contributions made by low-achieving students in the groups. The method employed consisted of studying two groups of four students made up of two high-achieving students and two low-achieving students who studied in the group-of-four variation one day a week for four weeks. Data collection consisted of video-recorded lessons lasting 50 minutes in duration and transcriptions of audio-recorded interviews of eight targeted students, 10 non-targeted students, and the teacher. Results from the study provided evidence of student passivity.

Although low-achieving students preferred small group settings compared to whole group settings, results revealed that the low-achieving students found it difficult to initiate ideas as higher-achieving students dominated the group process. Additionally, low-achieving students reported having “limited ability to explain mathematical concepts” (p. 407) and labored in the recall of mathematics facts. King (1993) found that lower-achieving students experienced frustration, participated less than higher-achieving students, and withdrew from tasks even though they reported that they learned better in small group settings and that they received assistance from group members when requested. Moreover, the results indicated that students had few opportunities to explain their reasoning or to justify their solutions. As a result, low-achieving students often adopted a self-preserving behavior, such as task avoidance.

After reviewing the study, I found myself questioning whether the establishment of group norms could have prevented the dominance displayed by higher-achieving students. As a review of the literature revealed that students should be provided with opportunities to engage in the learning process (NCTM, 2001, 2014; NRC, 2001), I wondered if the use of a heuristic (e.g., the THINK interaction framework) created to promote collaboration in the problem-solving process would provide opportunities for all students to participate, allow for students to present and justify their reasoning, and decrease passivity through the promotion of social interactions. To gain information to my questions, I believe further research using observation of the implementation of a heuristic in groups of students with diverse learning abilities should be conducted.

The effects of PALS on student passivity and mathematical dispositions. Kroeger and Kouche (2006) studied 150 seventh graders with differing mathematical abilities to examine the effects of peer-assisted learning strategies (PALS) as a remediation and intervention tool for concepts in which students demonstrated mathematical difficulties. Students were paired in each class using a split-list procedure in which the high-performing students were paired with lower-performing students. Students were trained in implementing the PALS process for five days over a two-week period. The PALS process uses scripts demonstrating the roles of tutor and tutee. All pairs of students learn the process for each role, as the PALS process requires the roles to switch between the dyads (Kroeger & Kouche, 2006). The tutor reads the script word-for-word when assisting the tutee. The tutee is expected to work through each set of problems, stating aloud his thought process simultaneously. The PALS intervention occurred three times a week during regular instruction for a period of several months.

The results of the study showed that students who previously exhibited passive behavior engaged in activities as a result of the PALS implementation. Additionally, students' social skills improved overall as students had learned through the use of a script how to communicate with their partners through the PALS process. Students who had exhibited behaviors at-risk for mathematical difficulties and who were reluctant to ask for assistance engaged in asking for and offering assistance to other students. The researchers noted increased confidence, achievement, and a positive disposition toward mathematics. Although some students noted that they did not like PALS because they found the reading of the script to be repetitive, the use of PALS as a remediation and intervention tool provided access to the curriculum.

In reviewing the study, I found myself inquiring about the dynamics of PALS that promoted productive dispositions and eased passivity. Although activities were scripted and a required part of the PALS process, I related the results of the program to my review of literature concerning the effect of having students think aloud while engaged in the problem-solving process. Although PALS resulted in success in studied dyads at the middle school level, I am curious about the effectiveness of the think-aloud process when used in small group instruction in a high school setting. During my search for empirical literature concerning passivity and social skills, I found few studies related to the implementation of a process or intervention designed to engage students in the collaborative learning process for mathematics at the secondary level. As RTI becomes a mandated process for the identification of students with SLDs at all levels, future research is needed to identify methods that can be effectively used as an intervention at the secondary school level.

The Effects of Heuristics in Mathematics Instruction

During the review of literature, I found two studies that discussed the effectiveness of the use of heuristics as a tool to promote students' problem-solving processes. In the sections that follow, I will discuss the methods and the results of each study and importance of the study in supporting the current study.

Heuristic teaching methods in a high school mathematics class. Al-Fayez and Jubran (2012) examined the impact of heuristic teaching methods on Jordanian high school mathematics students. Participants consisted of 69 male students and 73 female students enrolled in the tenth grade at King Abdulla School in Irbid. The participants were divided into a control group and an experimental group for each gender. The experimental groups were taught using the heuristic method of teaching where students used a heuristic to investigate a problem and discover the solution. The teacher assumed the role of facilitator and only asked questions. The control groups were taught using a traditional methods approach. The study lasted for eight weeks. Pretest and posttest data were collected at the beginning and ending of the study.

An analysis of the pretest showed that both the male and female experimental and control groups were statistically equivalent prior to the intervention used in the study. At the end of eight weeks, an analysis of the results revealed that the use of the heuristic teaching approach was beneficial for both male and female students as compared to students who received instruction using conventional methods. The students who used the heuristic approach to investigate problems showed significant improvement as compared to their peers who received conventional methods of instruction. There was no difference in the impact of heuristic methods of teaching due to gender.

This study supports the current study as it provides empirical evidence to the effectiveness of the use of heuristics in teaching mathematics to high school students. As I reviewed the literature, I only found one study that investigated the use of heuristics at the high school level. However, this study lacked details as to whether participants worked individually, in pairs, or in groups.

Enhancing student achievement using heuristics and tutoring. Woodward et al. (2001) examined the effects of a combination of whole-class and small group problem-solving instruction. The participants in the study included seven fourth-grade teachers and 182 students at three schools. Students were categorized as average-achieving students, at-risk students, or students with disabilities. The data were collected using a pre-test and post-test format that was administered at the beginning and end of the study, respectively.

The method of the study involved the implementation of problem-solving instruction for the whole class using performance-based assessments that were modeled after the statewide assessment and included extended response items and materials from the curriculum. Students with learning disabilities participated in both whole-class performance assessments and in ad hoc tutoring sessions four days a week for thirty minutes per session. Tutors provided remediation for students with disabilities and included problem-solving instruction at least once a week. During the problem-solving instruction sessions, tutors worked on facilitating group discussions to solve the problem using a heuristic that came with the curriculum. The tutor's role was to clarify problems, keep students on track, and offer explicit suggestions when students were unable to proceed after discussion.

A comparative analysis revealed that students with disabilities who had received ad-hoc tutoring and who used a heuristic approach to solving problems displayed more growth from the pretest to the posttest than did the average achieving and at-risk groups of students. Additionally, students with disabilities exhibited organization to the way they answered questions and wrote explanations for their work. However, Woodward et al. (2001) reported that students with disabilities showed no significant difference on difficult problems from the pretest to the posttest. The authors reported that students were assisted by tutors and did not work the problems by themselves, which might explain this lack of significance.

As the study provided evidence of the success of the use of heuristics with tutoring, the study also revealed the importance of scaffolding instruction even with the use of a heuristic. The study supports the proposed study in that heuristics can be used successfully as a tool to promote problem solving through “socially mediated interactions among students” (Woodward et al., 2001, p. 37) and meaningful discourse. However, further research is needed to determine whether the use of heuristics as an intervention at the secondary level promotes the same type of success.

Significance of the Reviewed Literature to This Study

In the literature, there was a lack of empirical evidence of the use of interventions to assist high school students, who exhibit passivity and mathematical and communication difficulties, in their learning and achievement. Specifically, the review of literature resulted in a lack of studies concerning interventions to aid students in self-regulation practices, metacognition, and the promotion of effective collaboration and discourse in secondary content courses. Though the literature review provided some evidence of the effective use of heuristics as an intervention tool in helping students to problem solve and in promoting achievement at the elementary and high school levels, more research is needed to establish the effects of a heuristics in promoting communication and effective dialogue in mathematics learning at the high school level.

Theoretical Perspectives

Efforts in education reform link effective teaching practices (e.g., implementation of tasks to promote problem-solving and reasoning skills, use of mathematical representations, and support of productive struggle) to constructivist approaches of instruction (Powell & Kalina, 2009). Constructivism is grounded in cognitive theory and began with cognitive psychologist Jerome Bruner whose theory of learning placed an emphasis on discovery learning in which children make meaning from discovering relationships among objects through active learning opportunities (Lefrançois, 2006). According to Lefrançois (2006), Bruner's theory of discovery learning complemented the "conceptual change movement in education" (p. 235) that emphasized inquiry-based learning and a student's individual engagement in the learning process. Bruner believed that an individual's active engagement in inquiry-based learning opportunities led to "continual construction and reorganization of knowledge" (Lefrançois, 2006, p. 235). Further, Bruner claimed that such reorganization of knowledge helped students to realize relationships among objects, and therefore, helped them to develop conceptual understanding (Lefrançois, 2006). An extension of Bruner's theory was realized by cognitive psychologist, Lev Vygotsky, who posited his theory of making meaning using a constructivist approach and emphasized that knowledge is constructed through culture and social interaction (Powell & Kalina, 2009). Recognizing the role of communication through discourse, I will discuss Vygotsksy's theory of social constructivism.

Social Constructivism

Lev Vygostsky, founding father of social constructivism (Powell & Kalina, 2009), emphasized that education is a cultural process where "knowledge is not only possessed individually but shared amongst members of a community" (Rojas-Drummond & Mercer, 2003, p. 100). Vygotsky stressed that the co-construction of knowledge through social processes occurs as a result of the use of language and dialogue among members in a community (Churcher et al., 2014; Mercer & Howell, 2012). For the purpose of this study, this type of communication in the classroom is identified as classroom discourse. To further explain Vygotsky's teachings concerning the importance of classroom discourse, Mercer et al. (1999) and Mercer and Howell (2012) suggested that academic discourse holds three functions: a cultural tool by which members can share knowledge through the spoken word; a cognitive tool that aids individuals in processing various knowledge structures; and a pedagogical tool that provides the opportunity for instructional assistance. In the following vignette, I will provide an example of how academic discourse serves as a cultural, cognitive, and pedagogical tool.

Mrs. Smith presents the class with a situation where four people must cross a treacherous bridge in enemy territory in the middle of the night within a specified amount of time under certain conditions. Students must develop a possible solution to the problem. Working in groups, students share aloud information about what is known about the situation and suggest what is needed toward finding a possible solution. The use of language to share understanding of the meaning of the problem and to identify important information exemplifies academic discourse and functions as a cultural tool. Students then work individually for a few minutes to construct a possible solution path, talk aloud about their possible avenues toward finding a solution, and again work to construct their own solution. As such, academic discourse becomes a cognitive tool by which the students process and use the information that was shared to continue to find a solution. Last, students are asked to share and explain their solutions to group members. In this instance, academic discourse becomes a pedagogical tool in which students assist other students in understanding a solution to the problem. In conclusion, the vignette provides an example of how academic discourse can be facilitated to function as a cultural, cognitive, and pedagogical tool.

In the classroom, Vygotsky claimed that the use of language to share and listen to the ideas of others through social interaction is a cultural process and, thus, language is a cultural tool (Forman & Cazden, 1985). Following the principles of cognitive psychology, Vygotsky theorized that social interaction through dialogue promoted cognition. He viewed language as a cognitive tool where the social interaction among individuals promoted a type of intermental activity during which knowledge is constructed. According to Vygotsky, the result of such intermental activity (i.e., collective thinking) spurs intramental activity (i.e., an internal mechanism that occurs within the individual) that promotes reflection and internal dialogue (Churcher et al., 2014; Mercer & Howe, 2012; Rojas-Drummond & Mercer, 2003). Forman and Cazden (1985) concurred, “Learning consists of internalization of social interaction processes” (p. 341).

The use of language and social interaction as an instructional tool is best known in Vygotsky’s theoretical construct of the zone of proximal development (ZPD). ZPD is defined as “the distance between the actual developmental level as determined by independent problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). In this respect, students may be grouped for cooperative learning or tutoring activities in which language and communication are key to scaffolding learning (Forman & Cazden, 1985; Powell & Kalina, 2009). Scaffolding is an instructional technique that provides support and guidance to the learner in increments by a more capable peer or teacher (Lefrançois, 2006; Powell & Kalina, 2009). However, Forman and Cazden (1985) suggested that intermental activity to intramental activity may occur during collaborative activities whenever peers have equal capabilities. The support that comes with scaffolding occurs through social interaction when members have "complementary roles" (Forman & Cazden, 1985, p. 341).

Summary

In summary, social interaction and the use of language are keys to the construction of knowledge and learning (Lefrançois, 2006). Language, in the form of discourse, functions as a cultural, cognitive, and pedagogical tool when used appropriately in the classroom (Mercer & Howell, 2012). Language, used as a social and instructional tool, is a catalyst in promoting academic discourse when students collaboratively engage in an assigned task, learning is scaffolded (Powell & Kalina, 2009), and the theory of ZPD is utilized (Vygotsky, 1978). Learning occurs when knowledge is constructed through social interactions and is internalized by the individual (Vygotsky, 1978). Such internalization occurs when students support their positions through justification of reasoning (Mercer & Howe, 2012). In this respect, language serves as a cognitive tool as students internalize knowledge and restructure meaning to make sense of their world (Rojas-Drummod & Mercer, 2003).

To determine if language serves as a functional tool to promote academic discourse in the classroom, a discourse analysis framework is needed to analyze the type of talk spoken among the participants.

Discourse Analysis Framework

To guide this study, I sought a framework that identified the types of classroom talk in which students participated. In the following sections, I discuss discourse analysis frameworks that I reviewed in the literature.

The Need for a Discourse Analysis Framework

As I reviewed the literature, I found few frameworks that related to mathematics discourse (e.g., Huang, Normandia, & Greer, 2005; Mercer et al., 2009; Ryve, 2006; Yackel & Cobb, 1996). While Huang, Normandia, and Greer (2005) devised a framework that studied the different knowledge structures constructed in a secondary mathematics classroom between the teacher and the students, Yackel and Cobb's (1996) framework centered on the process of establishing sociomathematical norms in a classroom. Where Huang et al. (2005) and Yackel and Cobb's (1996) frameworks related to discourse and processes thereof between teacher and students, Ryve (2006) discussed the use of interactive flow charts to analyze student discourse of a group of students who were constructing concept maps. Ryve's framework provided an in-depth analysis based on utterances of students' remarks as they related to the constructed concept maps. However, this framework required additional analysis of student interpretations of the task and did not align with the purpose of this study.

To determine the nature of student conversations, Mercer et al.'s (1999) framework for discourse analysis offered me a tool to identify the type of talk used among students in a classroom and draws upon the work of cognitive psychologist, Lev Vygotsky, and his sociocultural theory (Powell & Kalina, 2009). Vygotsky posited that language could be used as a tool to promote collective thinking and to develop individual reasoning skills. Because Mercer's framework for discourse analysis allows the observer to classify student-to-student discourse into three categories, it provided a means to analyze the type of talk (i.e., disputational, cumulative, and exploratory) that occurs within a classroom.

For this study, I was interested in studying the discourse among ninth grade students enrolled in a mathematics intervention classroom. Although Mercer et al. (1999) used the framework to analyze the talk of students aged 9-10 years who were engaged in a teaching program designed to instruct students how to use language in a scaffolding fashion, the framework provided the means to analyze whether high school students' discussions surrounding cognitively demanding tasks change as a result of an intervention designed to promote collective problem solving. Because this study focused on a ninth grade mathematics intervention classroom, I used Mercer's framework to identify the types of talk. Whenever students' discourse resulted in exploratory talk, I attempted to identify the strands of mathematical proficiency to determine the quality of talk used during collaborative group discussions as it relates to mathematics. The identification of exploratory talk and the mathematical strands of proficiency allowed me to track whether changes occurred in student discussions as a result of the intervention.

Although the previous description provided the basis of the study's conceptual framework, I will discuss Mercer et al.'s (1999) discourse analysis framework in relevance to this study in the third chapter of this document. In the following sections, I further discuss the components of Mercer et al.'s (1999) framework and conclude with a summary.

A Discourse Analysis Framework

To discern whether discourse in the classroom is effective, teachers and researchers need a tool to discern the types of talk that occur during student activities. Mercer et al.'s (1999) discourse analysis framework allows the user to differentiate among the types of talk that occur in the classroom. These types of talk include disputational, exploratory, and cumulative talk, which are described in this section.

Disputational talk is descriptive of student conversations that are competitive and uncooperative in nature (Mercer et al., 1999). During conversations that exhibit disputational talk, students are rude, disrespectful to each other, and fail to listen to one another's ideas (Elizabeth et al., 2012). Such conflict is displayed as "counterproductive and combative" (Elizabeth et. al., 2012, p. 1219). In contrast with disputational talk, Mercer et al. (1999) defined exploratory talk as

> that in which partners engage critically but constructively with each other's ideas. Statements and suggestions are sought and offered for joint consideration. . . . In exploratory talk, knowledge is made publically accountable and reasoning is visible in the talk. (p. 97)

Such talk correlates with the expectations for student discourse set forth in SMP3. Students' critiques of each other's ideas are seen as respectful contributions to the discussion (Elizabeth et al., 2012).

Finally, cumulative talk is characterized by students contributing to the conversation in a polite manner through taking turns in adding information to the conversation (Mercer et al., 1999). Elizabeth et al. (2012) reported that caution should be used not to characterize a polite exchange of ideas as quality discourse. Unlike exploratory talk, Elizabeth et al. (2012) explained that cumulative talk among students does not exhibit traits of a critical analysis of students' comments by one another or the contribution of viable arguments about the topic.

Although facilitating meaningful discourse is noted as an effective teaching practice (NCTM, 2014), all too often researchers have noted that such meaningful academic talk observed in classrooms is rare (Elizabeth et al., 2012). However, Mercer et al. (1999) reported that the explicit teaching of the ground rules of exploratory talk often improve the quality of group work and discourse among students.

Theoretical Connections to the Study

In this section, I draw connections between Vygostsky's theory of social constructivism and how it may be useful in interpreting the findings of the literature that I reviewed to inform this study. Additionally, I explain how Mercer et al.'s (1997) framework for discourse analysis was used to inform the design of this study. Finally, I discuss how the review of literature led to my decision to examine the THINK interaction protocol as an intervention tool in a Tier II classroom setting.

In the second section of this chapter (i.e., The Challenges of Teaching Students with Mathematics Difficulties), I presented the reader with possible challenges of teaching mathematics to students who have language disabilities, who exhibit passivity, or who have difficulty with visual representations. The research showed that there exists a connection between language disabilities and mathematics skills (Alt et al., 2014; Harrison et al., 2009; MacGregor & Price, 2014; Poch, 2015). The literature review also revealed that students with language disabilities often have difficulty with decoding and comprehending word problems (Harrison et al., 2009). Additionally, the research disclosed that students develop language impairments due to a lack of metalinguistic awareness that impacts symbol and syntax awareness needed in the development of reading and mathematics skills. As such, students who are impacted by metalinguistic awareness have difficulty in constructing and using visual representations (Poch et al., 2015) that are useful in developing conceptual understanding, strategic competence, adaptive reasoning, and productive dispositions (NRC, 2001). The review of literature also revealed that students with mathematics difficulties exhibit passivity (Baxter et al., 2001; King, 1993; Kroeger & Kouche, 2006). Passivity is often a result of learned helplessness (Kroeger & Kouche, 2006) and self-preserving behavior (King, 1993). The theory of social constructivism posits that through the ZPD, students can learn from capable peers through tutoring and dialogue. Further, although students may have difficulties with comprehending the wording in problems, through dialogue students construct knowledge through interacting with a group and reorganize such knowledge through internalizing thoughts and reflection (Vygotsky, 1978). Thus, it is important that students engage in exploratory talk as described by Mercer et al.'s (1999) discourse analysis framework.

In the third section of this chapter (i.e., Instruction of Students With or At-risk For Mathematical Difficulties), I pose that there exist conflicting views between the fields of mathematics education and special education regarding how best to instruct students with or at risk for mathematics difficulties. Although researchers in the field of mathematics education indicate that effective mathematical instruction implements the Mathematical

Teaching Practices described by NCTM (2014) (Akyuz et al., 2013; CCSSI, 2010; NRC, 2001; Suh, 2007), researchers in the field of special education (Baxter et al., 2001; Miller & Mercer, 1997; Montague, 1997; Woodward et al., 2007) suggest that instructional practices that include tutoring (Woodward et al., 2007), explicit instruction (Miller & Mercer, 1997; Montague, 1997), and the use of heuristics (Woodward et al., 2001) are effective in teaching students with disabilities.

As this study attempted to explain how a heuristic (i.e., the THINK interaction framework) used as an intervention supported students' discussions about mathematics, Mercer et al.'s (1999) discourse analysis framework aided in the identification of the types of talk as I looked for evidence of talk that indicated movement toward mathematical proficiency.

Why THINK?

The review of literature presented the challenges faced by students who are at risk for or have been identified as having a SLD in mathematics. These challenges included language disabilities that affected mathematics learning, passivity, and difficulties with language heavy problems and visual representations. Even though mathematics educators and special educators espouse differing views of instructing these students, both discuss the effectiveness of using a heuristic in teaching mathematics. However, their reasons for implementation of heuristics differ.

Special educators often used heuristics to promote verbalization and procedural fluency. Mathematics educators used heuristics during inquiry-based learning activities to encourage student reasoning and collaboration. The common thread between the two styles of implementation was the use of language. This required me to explore

Vygotsky's theory of social constructivism, which provided a basis of how language could be used as a social, cognitive, and pedagogical tool. I then contemplated the effectiveness of the THINK interaction framework and how its implementation could bridge the pedagogical differences between mathematics educators and special educators in teaching students who are at-risk for mathematical difficulties or who have been identified as having a SLD. This prompted me to investigate how the THINK interaction framework, a heuristic, could be used as an intervention in a Tier II mathematics class to address passivity and mathematics difficulties and to support students' engagement in mathematical discourse to promote movement toward mathematical proficiency.

Chapter Summary

This chapter contained a review of the empirical and theoretical literature that informed this study. Additionally, a description and examples of the five strands of mathematical proficiency were discussed to provide the reader with clarity and an understanding of the meaning of mathematical proficiency as it is discussed for the analysis of data in the next chapter. Further, I provided theoretical connections between the literature and the theory of social constructivism and explained my reason for the selection of using the THINK interaction framework as an intervention tool that gave a purpose to my proposed study.

CHAPTER III: METHODOLOGY

Introduction

As students graduate from high school and post-secondary institutions, they enter an increasingly competitive job market that requires its members to possess analytical, reasoning, and communication skills (Fennell, 2011). To prepare students for such a competitive environment, teachers must provide all students with learning opportunities that engage them in activities that necessitate the use of higher-order thinking, problem-solving, and communication skills (NRC, 2001). Such activities assist teachers in implementing effective teaching practices that include facilitating inquiry-based learning, supporting productive struggle, and sharing and justifying answers in small group learning situations (NCTM, 2014). However, past studies (e.g., Baxter et al., 2001; Woodward et al., 2001) revealed that students identified with a SLD were often passive learners. King (1993) concluded that such passivity in small group learning led to a lack of collaboration during group learning activities. To support student engagement and encourage student interactions, teachers often need pedagogical tools that promote meaningful mathematical discourse (Webb et al., 2014).

Although the effect of discourse on student learning has been well investigated (NCTM, 2014; NRC, 2001; Webb et al., 2014), the use of intervention tools to promote and support mathematical discourse among students with disabilities and students at-risk for mathematical failure are limited. Therefore, the purpose of this qualitative study was to examine how a research-based instructional tool used as an intervention supported students' discussions during small group learning activities and possible movement toward mathematical proficiency.

This chapter begins with a research overview and a description of the context for the study. A description of the participants and selection process will follow. Next, a depiction of the instruments, data sources, procedures and a data collection timeline will be discussed. The chapter concludes with a description of the processes used for analyzing the data, limitations, and delimitations of the study.

Research Overview

This study employed an embedded multiple-case study design (Yin, 2014) to examine students' discourse as they participated in problem-solving activities using an intervention. The study used the method of an explanatory case study to understand how the use of an intervention tool supported students who were at-risk for mathematical difficulties in their discussions concerning mathematical topics. More specifically, the study focused on how the intervention supported students' exploratory talk in moving toward mathematical proficiency. Three important characteristics supported the choice of a case study design (Gay et al., 2009; Yin, 2014). First, the study was focused on a problem of contemporary significance (Yin, 2014) of how to engage all students in the learning of mathematics using effective mathematical practices. Because students who were at-risk for mathematical difficulties and who were identified as SLD often exhibited passivity in the classroom (King, 1993; Kroeger & Kouche, 2006), the study addressed a problem of contemporary significance (Yin, 2014). Second, the main units of analysis were well defined and included three students working collaboratively with each individual student identified as a case in the study (Yin, 2014). The benefit of conducting a multiple case study was that "the findings [are] likely to be more robust than having only a single case" (Yin, 2014, p. 164). Third, the study was guided by theoretical propositions and elements of a conceptual framework that guided data collection and analysis (Yin, 2014). Because I examined verbal interactions of students working in a small group setting, I selected Vygotsky's theory of social constructivism to show how intermental activities (i.e., collective thinking) possibly supported by the THINK interaction framework might have spurred intramental activity (i.e., reflection and internal dialogue) exhibited by the individual student through verbal utterances consisting of justification and argumentation in an effort to reach clarification of ideas. The study was also guided by Mercer et al.'s (1999) conceptual framework for discourse analysis that characterized student conversations by the type of talk used by students. In particular, I used Mercer et al.'s (1999) framework to discern the type of discourse exhibited by students and to determine if the exploratory talk addressed components of mathematical proficiency (NRC, 2001). The framework provided an avenue to guide data collection and analysis and a means to consider alternative findings.

Context

The current study took place in a ninth grade Tier II intervention class. To help the reader better understand the context of the study, I provide an overview of the school demographics, student performance in mathematics on a national and state assessment, and an introduction to Mrs. Smith (i.e., the intervention teacher). Because the study was situated in an intervention setting, the section concludes with a discussion of the school's status concerning RTI during the time of data collection.

School Demographics

The study took place in a rural high school located 50 miles from the fourth largest city in a Southeastern state. For reference purposes, the high school of the study will be referred to as Grady Mountain High School, or GMHS. The school district hosted one high school and six feeder schools serving grades pre-kindergarten through grade eight. At the time of this study, 739 students were enrolled at GMHS: 225 ninth graders, 172 tenth graders, 189 eleventh graders, and 153 twelfth graders. Student demographics at GMHS consisted of 99% White, 0.9% Hispanic or Latino, and .1% Asian. The percent of economically disadvantaged students was 82.1. The population of students identified with a SLD per class included: 20.4 % of ninth graders, 17.4 % of 10th graders, 20.1% of 11th graders, and 16.3% of 12th graders. In total, 18.8% of the student population was identified with a SLD.

Standardized Test Scores

Data from the 2014-2015 school year revealed that students scored on average 17.4 on the ACT mathematics subtest as compared to the statewide average of 19.3. Additionally, the average ACT composite score for students at GMHS was 17.6 compared to the average composite score of 19.8 for students across the state. Results on the state mandated Algebra I End-of-Course exam disclosed that students at Grady Mountain High School performed below the state average in the performance categories of advanced and proficient (see Table 1). Also, students scoring basic and below basic exceeded the state average of 18% and 12%, respectively. Compared to the state average of students with a SLD who scored proficient or above (i.e., 32.8%), 7.2 % of students with SLD at GMHS scored proficient or better on the Algebra I End-of-Course exam. Furthermore, results for the participants' class as eighth graders on the 2014-2015 state comprehensive assessment program mathematics subtest revealed that only 35% of the eighth grade class scored proficient and above as compared to 47% of eighth grade students across the state. In conclusion, compared to other ninth grade students across the state, the ninth grade students at Grady Mountain High School performed on average below their peers in the categories of proficient, advanced, and in the subgroup of SLD.

Table 1

2014-2015 Algebra I End-of-Course Performance Levels

	Performance Categories			
	% Advanced	% Proficient	% Basic	% Below Basic
GMHS	22	26.2	28.6	23.2
State	41	29	18	12

Teacher Credentials

Mrs. Smith (a pseudonym) was the teacher who agreed to let me conduct the study in her Tier II mathematics classroom. She had seven years of experience in teaching high school students and had taught Algebra I, Algebra II, Foundations of Mathematics I and II, pre-calculus, and ACT mathematics. Additionally, she had one year of experience teaching middle school mathematics. Prior to 2015-2016, Mrs. Smith had never taught in a school that implemented RTI. Her teaching career spanned two counties with her most recent two years of teaching at Grady Mountain High School. She had a Bachelor of Science in secondary education with a major in mathematics and a Master of Arts in Education in the area of curriculum and instruction. Mrs. Smith's teaching style was self-described as a mixture of lecture and discovery learning methods. She described herself to be open to new teaching methodologies.

RTI Implementation

Although the reorganization of IDEA in 2004 (Yell et al., 2006) allowed state and local education agencies to use RTI to identify students with a SLD, the state in which the study took place recognized the implementation schedule of RTI in schools as follows: 2014-2015 grades kindergarten through five; 2015-2016 grades six through eight; and 2016-2017 grades 9 through 12. To prepare for full implementation of RTI for 2016-2017, all students at Grady Mountain High School were placed into tiers and began RTI in August 2015. Ninth grade students were identified using a free software tool (i.e., the Early Warning System) provided by the state's department of education. Data sources used to identify students included: mathematics and reading language arts scores on the state-mandated comprehensive assessment program; grades for reading, English, and mathematics courses; student absences; and confirmation of services for English Language Learners. Additionally, student risk factors for behavior (i.e., suspensions, alternative educational settings, repeated behavior incidents, and possible encounters with the juvenile court system) and academics (i.e., previous tiered interventions, retention, or presence of an IEP) were also entered into the program. Upon completion of data entry, students were identified to receive tiered interventions. In mid-November, students initially identified by the Early Warning System or identified by mathematics and ELA teachers as needing interventions were given an additional assessment (i.e., a universal screener) to identify specific skills and to appropriately place students. Although it is customary for students scoring below the 25th percentile to be placed into a Tier II intervention classroom, the school's RTI team decided that students who exhibited deficits that were one and a half to two years behind their current grade level should receive extensive interventions in a Tier III setting, according to the state's RTI framework. Also, students whose needs were not being met in a Tier I setting were placed into a Tier II setting for academic support for the content that they were studying as well as skill deficits. Interventions that supported academic needs, student engagement, and motivation were used in a Tier II intervention setting (Johnson et al., 2009). An analysis of the results of the data collection tool revealed the following interventions needed by ninth grade students: 18 students were assigned to Tier II mathematics; 19 students were assigned to English Language Arts Tier II; 25 students were assigned to Tier III mathematics; and 25 students were assigned to Tier III English Language Arts.

For this study, I chose a Tier II ninth-grade mathematics class for the setting. Because the students were ninth graders, the class was most likely representative of a mixture of students from each of the six elementary feeder schools from each small community in the county or of a school outside of the school district. My selection of this classroom was due to students' lack of familiarity with one another as incoming ninth graders and because they had yet to establish deep-rooted relationships with one another. As I considered which participant group to study, I felt that a lack of familiarity among the students would expose the need for clarity in communication efforts among the participants whereas a high level of familiarity could result in dialogue less familiar to the researcher, including terminology using slang terms. The Tier II ninth grade mathematics class was comprised of students who did not necessarily have mathematics or other classes together.

Participants

Students in a Tier II classroom received an average of 50 minutes of intervention each day in addition to receiving 50 minutes of core instruction (i.e., Tier I instruction) in their assigned daily mathematics course. At Grady Mountain High School where the study took place, RTI was a school-wide endeavor and occurred during sixth period on Monday through Friday. Participants for the study were selected from a group of students enrolled in Mrs. Smith's ninth grade Tier II mathematics intervention course. Because the participants were selected from a Tier II mathematics intervention course, the participants were considered to be representative of students who were identified as needing additional support to be successful mathematics learners. As such, participant selection utilized purposive sampling, "the process of selecting a sample that is believed to be representative of a given population" (Gay, Mills, & Airasian, 2009, p. 605). To aid in the selection process I considered teacher recommendation, insight from student interviews, and information about student absences. The method of selection is described in detail in the procedures section of this chapter. A description of the participants will be provided in each case study in Chapter IV.

Instruments and Data Sources

The qualitative embedded-case study focused on the types of verbal interactions that occurred among students. More specifically, the data collected focused on verbal interactions among students in a Tier II mathematics classroom in which students identified as at-risk for having mathematics difficulties were enrolled. To strengthen the validity of the study, data were collected using a variety of sources and triangulated. "Triangulation is the process of using multiple methods, data collection strategies, and data sources to obtain a more complete picture of what is being studied" (Gay et al., 2009, p. 377). Sources used to collect data included: two self-developed individual interview protocols and a selection rubric (see Appendices A, B, and C), a group interview protocol (see Appendix D), transcriptions of video and audio recordings, artifacts (i.e., students' written work produced during small group instruction), a classroom observation protocol (see Appendix E), and a researcher journal. To provide clarity and understanding for the reader, I have provided a list of the data sources with their respective abbreviations in Appendix F. The abbreviations will be used as references to their respective data sources in reporting the results in Chapter IV. In the following sections, each of the instruments will be described.

Student Interviews

Semi-structured interviews using interview protocols (i.e., the Participant Selection Interview Protocol (PSI), the Post-study Interview Protocol (PTI), and the Group Interview Protocol (GIP)) were audio recorded and transcribed to gather information about individual students. To aid in the selection of participants, I created the Participant Selection Interview Protocol (see Appendix A) to gain insight into students' perceptions concerning cognition and learning, learning through social activities, the use of types of grouping strategies as a pedagogical tool, and the usefulness of mathematics. Since Mercer et al. (1999) suggested that academic discourse serves as a cognitive and social tool, I grouped the interview questions into these categories to aid in the selection of participants. Because Mercer et al. (1999) insisted that academic discourse can function as a pedagogical tool, I comprised questions that would provide insight into students' experiences with instructional arrangements that used small group instruction.

Also, I included questions concerning students' thoughts and disposition towards mathematics, as I selected students who had a good disposition toward the learning of mathematics and regarded the subject as a useful tool in life. Additionally, I developed the Participant Selection Interview Rubric (Appendix B) to score students' responses based upon their pre-study interview. The participants' answers to each question were scored using points that ranged from one to three. The use of the rubric and the selection process is described later in this section.

As an aid to collect information from individual students following the intervention, I created the Post-study Interview Protocol (see Appendix C). The purpose of the Post-study Interview Protocol was to gain insight into students' perceived thoughts concerning the use of the THINK interaction framework and how it supported, if at all, students' interactions through mathematical discourse. Also, group interviews were conducted throughout the study using the Group Interview Protocol (see Appendix D) to clarify and offer insight as to the group's discussion concerning written artifacts collected throughout the study.

Audio and Video Recordings

Each observed lesson was audio and video recorded to offer insight into the classroom structure and students' interactions with one another. Verbal as well as nonverbal interactions informed the researcher of types of interactions among students that occur in the natural classroom. Transcriptions of video recordings were produced to enable coding of data.

Artifacts

Participants written work produced during group activities were collected. All written artifacts were collected and corroborated with audio and video transcriptions and memos written by the researcher.

Observation Protocol

An observation protocol (see Appendix E) was used to record notes while observing the participants engaged in a group activity. The observation protocol was designed to allow the researcher to record the type of talk according to Mercer et al.'s (1999) framework, a description of the artifacts constructed by the students, and the researcher's thoughts and memos. The observation protocol was used to design specific questions for the group interview.

Researcher Journal

To aid in organizing my thoughts, I kept a researcher journal in which I recorded my thoughts, insights, and revelations that occurred during and upon completion of the data collection process. Such narrative material helped to sort "evidence more methodically to determine the strength of the empirical support for these themes and ideas" (Yin, 2014, p. 126). Patton (2015) stated that documentation of the analytic process by the qualitative researcher provides the "foundation of rigor" (p. 523).

Researcher as Key Instrument

As the researcher in this case study, I performed all data collection roles including interviewer and transcriber. As such, I became the key instrument in the data collection process (Creswell, 2007). My qualifications included six related experiences prior to conducting this study. First, I co-designed an experiment and collected and analyzed data as a doctoral student. Additionally, I co-authored an article that presented the findings of the experiment described above and was accepted to a peer-reviewed journal. Second, as a graduate assistant, I participated in the data collection and analysis in a qualitative research project. Third, I have completed 77 semester hours of course work toward a Doctorate of Philosophy degree in Mathematics and Science Education. Fourth, as a master teacher fellow participating in a large federally funded grant project over the course of a five-year period, I conducted four action research projects of qualitative and mixed-design research. Fifth, as a veteran teacher I taught high school mathematics to students with differing learning abilities for 23 years. Finally, as a curriculum and instructional supervisor for four years, I observed various instructional practices and identified academic and instructional needs of students with varied learning abilities resulting in the desire to publish my findings in a researcher journal and to add to the empirical literature in the field of teaching mathematics proven methods of intervention designed to address the learning needs of all students.

Procedures

In this section, a chronological narrative provides information concerning the planning and implementation of the study. A synopsis of the training of the classroom teacher in the use of the THINK interaction framework heuristic (Appendix G) and the structure of the study are included.

Training Mrs. Smith

Early in the study, I met with Mrs. Smith to discuss the plan of research and the use of the THINK interaction framework heuristic during the instruction of students in her Tier II mathematics intervention class. During our discussion, I shared with Mrs.

Smith the lesson plan (see Appendix H) that I had created to initially demonstrate the THINK interaction framework. To guide me in structuring the lesson, I used the Thinking Through a Lesson Plan (TTLP) protocol (Smith et al., 2008) as it helped in anticipating student misconceptions and in preparing scaffolding questions to promote student engagement for students of all ability levels. Additionally, I shared the mathematical tasks (Appendix I) that would be used during the four-day training period to allow her to familiarize herself with the problems. The tasks used during the training period were not used in the operational study. To ensure the integrity of the study, students from a tenth-grade Tier II mathematics intervention course were chosen to participate in the model lessons during the training period.

I requested and received permission from the school principal and Mrs. Lawson, the tenth-grade Tier II mathematics intervention teacher, to swap students in their Tier II mathematics intervention classes while I trained Mrs. Smith. During the four-day training session, I taught the first two model lessons while Mrs. Smith taught the remaining two model lessons. While teaching the first two model lessons, I modeled how to implement and introduce students to the THINK interaction framework in a task-based lesson. Mrs. Smith and I conferred after each lesson to address any concerns and questions that she had concerning the implementation process, the THINK interaction framework, and the lesson structure. Mrs. Smith's active participation of instructing the class for the second two model lessons allowed me to observe her instructional practices and offer suggestions for the best facilitation of lessons during the research study. After the four-day training period, I met with Mrs. Smith once more to discuss her concerns and to answer questions regarding the study. Further, I provided Mrs. Smith with the tasks

(Appendix J) that were to be implemented throughout the study to allow her to prepare in advance of the first lesson. The tasks selected for the study addressed learning goals and objectives representative of a reform-based pre-algebra and Algebra I curriculum and were designed to support and strengthen students' problem-solving skills. By having Mrs. Smith facilitate the intervention activities during the study, I enabled myself to become immersed in the observations of the participants as they worked in a group.

Structure of the Study

I spoke to students in Mrs. Smith's ninth-grade Tier II mathematics class to identify myself, explain the purpose and overview of the study, and state my desire for them to participate in the investigation. I then explained that permission granting their participation was needed from both the students and their parents prior to their engagement in the study. Next, I explained the consent and assent forms and answered students' questions. Over the next two weeks, Mrs. Smith asked students daily if anyone had returned the forms and agreed to participate in the study. After two weeks, five students submitted forms with an authorization of consent from their parents and a statement of assent for themselves. Participant selection interviews occurred over a three-day period and were scored using the Participant Selection Interview Rubric (see Appendix B). Students whose scores fell within the first and third quartile were considered for selection. Though there were no outlying scores and because only five students remitted the required paperwork, selection was based upon Mrs. Smith's recommendation. Upon conferring with Mrs. Smith about the selection of students, she recommended four of the five students to participate in the study based upon her knowledge of the students' attendance and overall disposition. Two male students (i.e., Gary and Nick) and two female students (i.e., Tonya and Darla) were selected to participate in the study. The four students selected for the study were notified, and the study began on the following day. Due to unforeseen absences, one participant (i.e., Tonya) was later dropped from the study.

The study consisted of 14 lessons that occurred over three phases. During all phases of the study, each student in Mrs. Smith's ninth-grade Tier II mathematics class participated in the lesson and was placed into a predetermined group. The participants selected for the study were placed into the same group for the remainder of the study. Mrs. Smith discussed the roles to be assigned to each member of a group. Roles were selected by group members themselves and rotated daily to ensure that each member of the group experienced the responsibilities of the roles that included the secretary, administrator, supervisor, and the director of human resources.

For each lesson in the study, audio and video recordings were made, researcher memorandums were noted, group observations were recorded, and group interviews were conducted if further information was needed to clarify thoughts concerning artifacts or participants' conversations. In the following subsections, I discuss Phases 1 through 3 of the study.

Phase 1. Phase 1 of the study consisted of Lessons 1 through 5. Data collected during this phase provided a picture of how students interacted on a typical day when solving problems during the Tier II mathematics intervention class. During this phase, students were presented with a task to solve as a group. Each day prior to the beginning of the lesson, Mrs. Smith reminded students to choose roles and inform her with the raise of a hand as she inquired who served in each role. She began each lesson by having a student read the problem aloud to the class. Mrs. Smith did not instruct the class on procedures for problem solving or assist in step-by-step instruction. On occasion, she provided instructional scaffolds for the whole class concerning the meaning of the problem when most groups exhibited difficulty as to how to begin to solve the problem and after several minutes had passed. As needed, Mrs. Smith approached groups and provided assistance through posing purposeful questions.

Phase 2. Lessons 6 through 10 composed Phase 2 of the study. During these lessons, Mrs. Smith introduced the THINK interaction framework and provided scaffolds as to how to use the heuristic. She began each lesson as she did in Phase 1, with the affirmation of student roles. For Lessons 6 through 10, Mrs. Smith presented each group with a handout that outlined the THINK interaction framework. She explained that the THINK interaction framework was a heuristic designed to help guide the group through collaborative problem solving. An example of Mrs. Smith addressing the class is presented below.

> We are introducing something new today. We are going to pass out the slips with the questions on it. Then, we are going to introduce a different way for you to do group work today. So, when you get these, I need for you to hang on just a second. Okay. Will someone read the problem to us, and then I will pass out the THINK interaction framework? (TS, 4/18) (See Appendix F)

She explained each of the components of the framework: Talk, How, Identify, Notice, and Keep. Further, she led the students through each component as the groups engaged in solving the task. Mrs. Smith began Lesson 6 with the following discussion.

> T stands for talk. Okay. So, talk is the first thing that you are going to do. You are going to read through the problem as a group. It says, what is the problem asking? What important information do you need? Talk to the members of your group and determine the information needed. So, the first thing that you are going to do is to talk through your question. I'm going to give you a little bit of time to do that. So, read through your question again and talk about it out loud with your group members. Everybody talk. I want to hear words. (TS, 4/18)

Although students were given a copy of the THINK interaction framework each day, Mrs. Smith continued to scaffold students' use of the heuristic but gradually reduced her assistance with each additional lesson. The following was an excerpt of the video transcription from Lesson 10. Mrs. Smith discussed the *How* component of the framework:

> Okay. The next step after Talk, and you have discussed the problem, and you figured out what information that you have, and what the problem is asking you to find, the second part is the How. So, each of you are going to individually find your own strategy. How are you going to find the answer? So, by yourself, without talking, come up with how am I going to solve this, and not what I think the answer is, but how. What is my strategy of finding the solution? (TS, 4/25)

Mrs. Smith reminded students that beginning with Lesson 11 that they would continue to use the THINK interaction framework and that she would be offering minimum assistance with how to use the heuristic.

Phase 3. The third phase of the study consisted of Lessons 11 through 14 in which students were provided the THINK interaction framework and reminded to use it to guide their discussions. The following is an excerpt of Mrs. Smith speaking to her class.

> Let's read it together first, and then we'll start. You all can work through the THINK interaction framework. Remember the steps. Okay. This is Problem 11. A game using nickels, dimes, and quarters requires that there must be twice as many dimes as nickels and twice as many quarters as dimes. How many coins would you need to create the largest possible amount less than $12.00? All right. I'm turning it over to you. (TS, 4/26)

Although Mrs. Smith did not assist them in using the THINK interaction framework, she continued to assist struggling groups through using questions as scaffolds and reminding the group to carefully read the task. Additionally, she reminded students who asked her for help that they were in a group and to ask group members for assistance. Between Lessons 13 and 14, state mandated assessments for this group of students occurred for two days. Throughout Lesson 14, students exhibited complacency and a lack of willingness to participate. Conversations were off topic throughout the lesson. The following was a description of the actions and statements made by one of the participants in the study. He read the problem aloud as if to himself and not his group, "Which is the smallest natural number that is divisible by the numbers one through 10, inclusive?" He yawns. "Anybody got a pencil? I did not think that I would be doing anything today" (TS, 5/4)

Upon completion of the fourteenth lesson, I thanked Mrs. Smith, the participants, and the students of the class for allowing me to conduct research in their classroom. On the following week, I conducted post-study interviews (see Appendix C).

Timeline of Events

Data collection for this study began as soon as IRB approval was confirmed and assent forms and consent forms were collected. A timeline of events for the study is presented in Table 2.

Table 2

Timeline of Events

Dates	Phase	Activities
March 28 – April 6	Pre-study	Students were informed of the study and provided with forms. Potential participants were interviewed an selected. Mrs. Smith was trained.
April 7 - April 15	Phase 1	Observations were conducted of the participant group during Lessons 1-5, prior to the implementation of the THINK interaction framework.
April 18 - April 25	Phase 2	Observations were conducted of the participant group during Lessons 6-10, using the THINK interaction framework. Mrs. Smith directed the class in the use of the components of the framework.
April 26- May 4	Phase 3	Observations were conducted of the participant group during Lessons 11-14, using the THINK interaction framework. Mrs. Smith provided minimal scaffolds.
May 6	Post-study	Participants were interviewed using the Post-study Interview Protocol (see Appendix C). Data collection for the study ended.

Data Analysis

To answer the primary research question, data analysis for the study occurred in four stages.

Stage One

Stage one of the analysis process in this embedded multiple-case study began with the transcribing of participant interviews, videos of the participants working in a group during Lessons 1 through 14, group interviews, and post-study interviews. Transcriptions were checked multiple times for accuracy. Next, I read the transcripts while listening to the audio recordings. During that time I recorded my thoughts and initial interpretations in my researcher journal.

Stage Two

Based on Patton's (2015) suggestion of using sensitizing concepts to guide observations and after reading the transcripts closely a second time and performing a textual analysis of each transcript, I first reduced data to the broad categories of the types of talk conceptualized by Mercer et al.'s (1999) discourse analysis framework. Using an initial qualitative deductive analytical approach (Patton, 2015), I looked for and initially coded transcripts of audio and video recordings using the categories of disputational, cumulative, and exploratory talk (Iteration 1) (see Table 3). During this initial analysis stage, I noticed a fourth type of talk that involved conversations with the teacher. I then created a fourth category of talk and coded it as teacher support (see Table 3). During this initial analysis stage, I looked for evidence of exploratory talk during individual interviews, student conversations during group work, and group interviews.

A subsequent textual analysis of the transcripts was conducted by reading each line of the transcripts closely. Open coding analysis (Yin, 2014) was then used to code text based upon student behaviors noticed during the initial readings of the transcripts (Iteration 2) (see Table 3). This allowed me to identify any changes in students' behavior in a chronological fashion. Next, I conducted an additional textual analysis of the categories of text coded as exploratory talk. Codes indicating the strands of mathematical proficiency were assigned to each complete statement, exclamation, question, or command (Iteration 3) (see Table 3).

To collect further information concerning the effectiveness of the THINK interaction framework, I conducted an additional textual analysis of the transcripts for Lessons 6 through 14 and coded blocks of text according to the component of the THINK interaction framework being implemented (Iteration 4) (see Table 3).

Additionally, I recorded my thoughts and insights in my researcher journal and added memos to the transcriptions as needed for reference points including dates, relevancy to the THINK interaction framework, lesson numbers, and insights. Such reference points aided in the triangulation of data and clarified attempts to identify movement toward mathematical proficiency possibly promoted by the intervention and to what Vygotsky referred to as the change process (Forman, 2003). The coded data, researcher's journal, group observations, and interviews were stored as digital files and as hard copies in binders labeled for each participant (i.e., individual case study records). Collectively taken for all three participants, the data were stored as the case study database.

Table 3

Embedded Case Codes

First Iteration: Types of Talk
Disputational Talk
Cumulative Talk
Exploratory Talk
Teacher Support
Second Iteration: Student Behavior
Lack of Self-regulation
Verbalizations
Passivity
Group Roles
Likes Traditional Instruction
Third Iteration: Strands of Mathematical Proficiency
Conceptual Understanding
Strategic Competence
Procedural Fluency
Adaptive Reasoning
Productive Disposition
Fourth Iteration: Components of the THINK Interaction Framework
Talk
How
Identify
Notice
Keep

Stage Three

This stage of data analysis involved looking for emerging themes in each individual case study record and was based on the coding schema of the transcripts, artifacts, observation protocols, notes in journal entries in the researcher journal, and memos written by the researcher. To enable me to find emergent themes, I created a database for the group and for each participant and recorded the frequency of utterances spoken during each type of talk in the following categories: student behavior, mathematical strands of proficiency, and components of the THINK interaction framework. Only lesson transcriptions from the 14 lessons were used to record the frequency of utterances. The use of the database allowed me to conduct a within-case analysis to identify key issues and themes within each individual case. Quotations from lesson transcriptions and interviews and specific artifacts were used to triangulate and support the findings. A narrative for each case record was constructed and included an interpretive analysis that described the findings of the study. The databases were inserted into individual case study records and the case study database.

Stage Four

In stage four of the analysis process, I conducted a cross-case analysis to identify any patterns or emergent themes across cases. A cross-case analysis allowed me to "aggregate findings across a series of individual studies" (Yin, 2014, pp. 164-165). A final narrative was constructed and included a holistic interpretive analysis that described the overall findings of the study.

Examples of the Types of Talk in a Mathematics Classroom

This study was guided by Mercer et al.'s (1999) discourse analysis framework that described three types of talk that occur between students during classroom instruction. The types of talk described by Mercer et al. (1999) included disputational, cumulative, and exploratory talk. While conducting a textual analysis of the transcripts, I noticed another type of talk that did not fall into any of the three categories of talk: teacher support. Dialogue excerpts coded as teacher support were conversations between the teacher and an individual student or the group of students in which the students were not the primary stakeholders of the conversation. In the following subsections, I describe each type of talk and present an example or vignette to provide clarity, understanding, and to help the reader differentiate among the different types of conversations that took place during the study.

Disputational Talk

Disputational talk occurred when students exhibited competitive conversations without justification of reasoning or explanation (Mercer, 2004). Further, student conversations were often rude and disrespectful when they were engaged in disputational talk. Elizabeth et al. (2012) described such discourse as “counterproductive and combative” (p. 1219). The following excerpt was an example of disputational talk. In this excerpt, the students discussed finding a solution for the assigned task that required them to find the number of dimples on a golf ball given that the hundreds and tens digits were the same prime number, the sum of the digits of the solution was 12, and the total number of dimples on the golf ball was divisible by seven (viz., the golf ball problem) (see Appendix J).

1 Gary: What are you thinking?

2-9 Nick: Oh, man! Ok. I just went through here and when she said that they’re not all prime that it could. I just went through here and started going, like 22, you can go like two plus two is four and then you’re going back to that -. Shut up and leave me alone [He spoke to a student in a different group who said something to him.] It’s got to be in one of those first few digits. {He spoke to himself. Two plus two is four. Six plus six is 12. No. Seven plus seven is 14. Eight plus eight is 16. No. Obviously, that one won't be it.} It's either --.

10 Tonya: Wait. So, it's --. [Nick interrupted her.]

11 Nick: It's three --.

12 Darla: It's the same number so --. [Tonya interrupted her.]

13 Tonya: Three-digit number --. [Darla interrupted her.]

14 Darla: It can be divisible by seven.

15 Nick: It's got to be --. I can't believe that's not divisible by seven. (TS, 4/8)

The above excerpt of student discourse in Lesson 2 was discerned to be an example of disputational talk due to the competitive utterances exchanged by the participants. In the excerpt, students interrupted one another (i.e., Lines 9, 11, and 12) and failed to listen to ideas of group members. Nick was disrespectful to a classmate and told him to "shut up" (i.e., Line 5) (TS, 4/18). At times, students spoke incomplete utterances and failed to explain their thoughts. For example, in Lines 10 and 13, Tonya failed to complete her sentences. The same was true for Nick (i.e., Line 11) and Darla (i.e., Line 14). Additionally, Darla and Nick failed to justify their reasoning in their statements in Lines 14 and 15, respectively. For example, when Darla stated, "It can be divisible by seven," the context of her statement was unclear as to what she meant by the word, it. The same was true about Nick's statement in Line 15. Without the context of the problem, the reader could not have discerned the meaning of most of the statements in the excerpt.

The following excerpt illustrated an additional example of disputational talk. The discussion surrounded the task in Lesson 4. Students were to find the time that a man left his house to arrive at his appointment located 50 miles away if he drove 30 miles per hour and arrived 20 minutes late (viz., the appointment problem) (see Appendix J).

1-2 Tonya: I don't know. The location is 50 miles away, so the speed limit is --. [Nick interrupted her.]

3-4 Nick: And he had to be there by one o'clock even though he was 20 minutes late. Wait a minute.

5 Gary: [Inaudible. Tonya interrupted and spoke over him.]

6-7 Tonya: If he was 20 miles --. He was 20 minutes late, and his appointment was at one --. [Nick interrupted her.]

8-11 Nick: I'm telling you. It caused him to be 20 minutes late if he was going 30. There's no way to drive 30 miles an hour and be somewhere at one o'clock. There's no way. Well, it would depend on what time that you started.

12 Gary: [Inaudible. Tonya interrupted him.]

13 Tonya: Exactly! That's what we are trying to figure out, Nick! (TS, 4/14)

In the above excerpt, the participants not only interrupted each other (i.e., Lines 2, 5, 7, and 12) but they also spoke above each other at times (i.e., Lines 5 and 12). Other times, the participant remarks were somewhat condescending (i.e., Line 13). At no point in the above conversation did the students make statements that indicated their reasoning or thought processes about the solution or solution path. As such, no form of justification occurred.

Unlike disputational talk, exploratory talk resulted when students engage in effective discourse. In the next subsection, I provide a description of exploratory talk followed by an excerpt of participants' conversations that illustrate this type of talk.

Exploratory Talk

Exploratory talk was exhibited when students were engaged in a task, and their discussions contributed to expanding the knowledge of the participants (Elizabeth et al., 2012). During exploratory talk, participants made their statements accountable through reasoning and justification (Mercer et al., 1999). The following excerpt, taken from dialogue that occurred in Lesson 12, was an example of exploratory talk. In this lesson, the students were tasked to find the player who hit the center of the target in a dart game. The students were provided with a picture of a dartboard with darts marking the score of each of the 12 throws. Although the two players received the same number of throws and the female player scored 26 points on her first two throws, both players ended the game with the same final score (viz., the dart game) (see Appendix J).

1 Gary: What is the problem asking?

2 Darla: Which players hit the center of the board?

3 Gary: What is the problem asking?

4-7 Nick: Pam and Bob each threw six darts at a dartboard, and they earned the same number of total points. Okay. The results are shown in the picture. If Pamela scored 26 on her first two throws, which player hit the center of the board?

8 Darla: I'm going to go with Bob.

9 Gary: I'm not understanding.

10 Darla: Now, if it said total points --. [Nick interrupted her.]

11-13 Nick: Wait. It says if Pamela scored 26 on her first two throws which player hit the center of the board? Well, I mean obviously she did it because if they are talking about the first two throws.

14 Darla: Her first two throws have to be a 25 and a one to get 26.

15-17 Nick: So, more than likely she was the one that hit the 25, and she hit that one out there. All right. Yeah, I'd say she had to hit the 25 and one. So --.

18 Gary: It doesn't mean that she necessarily has to. (TS, 4/28)

The above excerpt was an example of exploratory talk. The participants exhibited reasoning skills (i.e., Lines 11-18). Darla justified her reasoning to the group when she explained the results of Pam's throws on her first two attempts. Gary questioned both Darla and Nick when he critiqued their answers and suggested that the first two throws did not have to be 25 and one. The justification and display of reasoning provided the distinction between exploratory talk and cumulative talk, which Mercer et al. (1999) explained was a polite exchange of ideas among participants in a conversation. In the next section, I explain cumulative talk and provide an example.

Cumulative Talk

Cumulative talk was described as a polite exchange of words where students added information to the conversation and did not require an explanation of each other's answer or justification of one's solution (Mercer et al., 1999). Elizabeth et al. (2012) cautioned that cumulative talk was often mistaken for exploratory talk when students communicated and added politely to the conversation. However, unlike exploratory talk, cumulative talk lacked student accountability for answers (Elizabeth et al., 2012). The following excerpt from Lesson 6 was an example of cumulative talk. In this problem, students were required to find the number of different stacks of three blocks (viz., the block problem) given that there were two blue blocks, two red blocks, one purple block, and one yellow block.

1 Tonya: I have 10.

2 Nick: I do, too. I have 10. [Long pause.] How many you got, Darla?

3 Darla: I have 11.

4 Nick: Really. You got 11. I got 10.

5-6 Darla: I went through the list I have, and if I didn't have the combination, I put it down.

7 Nick: No, no, no. I'm just saying what is the one that you got?

8 Darla: Red, yellow, red, blue, purple, blue.

9-10 Gary: Don't you eventually --. Doesn't the combination repeat itself? Just in different orders?

11-12 Nick: Well, that's the point. It's going to eventually --. That's the number we are looking for. It's going to eventually do that. (TS, 4/18)

In this excerpt, the students did not interrupt each other or argue. There was a polite exchange of words as students added information to the conversation. For example, Nick explained to Darla that he asked for the combination of colors that he did not have and not for the number of combinations (i.e., Line 7). Although Darla replied to Nick with her answer (i.e., Line 8), she did not explain or justify her answer. For this reason, the talk was considered to be cumulative instead of exploratory. Throughout the study, cumulative and exploratory talk was sometimes spurred when Mrs. Smith assisted students, often in the form of questioning. The exchange of utterances between Mrs. Smith and members of the group, for this study, was coded as teacher support. In the next section, I provide examples of such conversations.

Teacher Support

Throughout the study, conversations occurred between the teacher and the students in the form of teacher support. Types of teacher supports in a classroom included questioning, hinting, coaching, giving away parts of the answer, and providing examples (Halttunen, 2003). Conversations where teacher support furthered students' thoughts and enabled the conversation among the students to continue were coded as disputational, cumulative, or exploratory talk, as the students were the primary participants of the conversation. However, some dialogue occurred that did not fit the description of any of the types of talk described by Mercer et al. (1999). These conversations were characterized by extended teacher support, where the teacher engaged in the conversation as a primary participant rather than the facilitator of the lesson. As such, these blocks of conversation were coded as teacher support. The following excerpt was an example of the teacher supporting the group through questioning, coaching, giving away parts of the answer, and engaging as a participant in the conversation. In this excerpt, the participants were tasked to find seven dates for which the year 2013 was divisible by the product of the month and day (viz., the date problem) (see Appendix J).

1-4 Mrs. Smith: I still think you all are getting hung up on the seven days thing. Really, what it's saying is at the end of the problem you are going to have seven different dates. It's not seven days. Ya'll are reading wrong into that.

5-6 Nick: Like seven different years? Like that's 2013, so two thousand fourteenish?

7 Mrs. Smith: No. The year stays the same. It's all this year. 2013.

8 Nick: I just don't --. I still don't --. I'm confused.

9 Mrs. Smith: Why does the first one work?

10 Nick: The first what? What are you talking about?

11-12 Mrs. Smith: [She pointed to the paper to November 3rd.] Why does that one work?

13 Nick: That's a good question? I don't know why.

14 Mrs. Smith: Back to what I said about this word right here.

15 Nick: Divisibility?

16 Mrs. Smith: Umm, hmm. It says that divided by that.

17 Tonya: [Excitedly.] I think I get it! (TS, 4/7)

In the above excerpt, Mrs. Smith was a primary stakeholder in the conversation that mostly occurred between her and Nick. As such, the conversation was coded as teacher support because it failed to further the conversation among the students as the primary participants. In the above excerpt, Mrs. Smith used questions to prompt student discussion (i.e., Lines 9, and 11-12). She also provided a form of the answer in Lines 2 and 3 when she told the group that the answer included seven different dates. Mrs. Smith provided hints (i.e., Lines 3 and 7) when she told the group that it was the same year (i.e., 2013) and told them that the process involved the word, divisibility.

Because Mrs. Smith was a primary stakeholder in the conversation in the above excerpt, the conversation failed to fit into Mercer's types of talk that describe conversations among students for this study.

Summary of the Types of Talk

This study was guided by Mercer et al.'s (1999) discourse analysis framework. Mercer et al. (1999) described three types of talk (i.e., disputational, exploratory, and cumulative talk) that occurred among students in the classroom. An analysis of lesson transcripts revealed that in addition to Mercer et al.'s (1999) types of talk among students, discussions between the teacher and students often occurred when students were engaged in groupwork. For the sake of clarity in the study and because the conversations between the students and teacher failed to fit into any of the three types of talk described by Mercer et al. (1999), I coded such talk as teacher support. In this section, I discussed the types of talk and provided excerpts from transcriptions of video recorded lessons to provide the reader with clarity and understanding. In the next section, I provide the reader with a description of the strands of mathematical proficiency supported with examples from the data.

Examples of the Strands of Mathematical Proficiency

The scope of this study was to examine whether a heuristic (i.e., the THINK interaction framework) supported students' mathematical discourse and provided evidence of talk that indicated students' movement toward mathematical proficiency. The strands of mathematical proficiency included conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. To provide clarity for the reader in how I decided whether a student displayed skills characteristic of a particular strand of mathematical proficiency, in the following sections I include a review of each strand, illustrated with an example or vignette, and provide reasons for my determination.

Conceptual Understanding

Conceptual understanding is defined as a student's ability to comprehend mathematical concepts, operations, and relations (NRC, 2001). Further, students are able to make connections among related mathematical concepts, representations, and procedures for using the identified concepts to execute calculations and to solve problems. For this study, displayed conceptual understanding depended upon the participants' exhibition of understanding of the relationships among mathematical concepts, representations, and mathematical operations as each related to their use in the task.

In deciding whether a participant exhibited skills that demonstrated movement toward conceptual understanding, I examined the transcriptions of dialogue for evidence of the following: understanding of mathematical concepts as illustrated through their use of definitions of mathematical terms as related to the context of the problem, relations between forms of numbers (e.g., fractions and decimals) and between mathematical concepts and representations of those concepts, and understanding of mathematical operations as related to mathematical terms used in the task or to solve the problem (e.g., sum as the total of numbers). Also, I examined participants' work samples for additional evidence of conceptual understanding (e.g., one-half and 0.5).

In Lesson 6, the participants were asked to find the number of different stacks of three blocks that a toddler could make given six blocks of four colors (see Appendix J).

In the following dialogue excerpt, Darla and Gary exhibited understanding of the relationship among the mathematical concept of combinations and a visual representation depicting the concept of combinations to solve the task.

1-2 Darla: Yeah. It says different stacks of three blocks that the toddler can make.

3-7 Nick: It says how many different stacks of three blocks can the toddler make. Well, that depends, I mean it don't say by color or anything. It just says how many different stacks. Wait a minute. You can do more than two because --. Yeah. It's like he [Gary] said, you can change the colors around.

8 Gary: You can change the colors to make more than a different stack. (TS, 4/18)

In the above example, the participants tried to identify the requirements of the problem. Darla and Nick stated that the task asked them to find the number of ways the blocks could be stacked in quantities of three. Gary added that colors could be changed to make different stacks, indicating that the stacks were unique. Gary and Darla's work samples illustrated their strategy of listing blocks using different colors (WS, 4/18). Additionally, they labeled each block in each stack by the letter of the color of block intended (e.g., *r* for red).

Procedural Fluency

Procedural fluency is defined as a student's ability to know when and how to perform mathematical procedures and operations flexibly, appropriately, and fluently (NRC, 2001). In determining whether a participant displayed movement toward procedural fluency, I examined lesson transcriptions and looked for evidence of talk that described mathematical procedures (e.g., solving an equation) and operations (e.g., add, subtract, multiply, and divide) and provided either accurate solutions or an estimation of a quantity after conducting a procedure or operation. Further, I inspected participants' work samples for evidence of students' understanding of what procedure and operation to use to solve the problem. However, the participants used calculators and evidence from work samples indicating accurate calculations or estimations might not have existed. In such cases, I referred to classroom observation notes to search for written evidence of procedural fluency.

In Lesson 8, participants were asked to find the time that it took for 53 students to exit a bus (see Appendix J). Though the task could have been solved in different ways, the following quotation demonstrated Nick's thoughts for solving the problem.

> That's your answer. I kept wondering why it wasn't 112. I just did that, and 8 times 14 is 112. Subtract 60. Get 52. Can't go over a minute. That's a minute 52 seconds. Bam! I figured this out. (TS, 4/21)

In the above excerpt, Nick described the procedure for solving the task by using the operations of multiplication and subtraction. His calculations were accurate, and his solution was correct. Further, he understood the relationship between minutes and seconds as he converted 112 seconds to one minute and 52 seconds.

Strategic Competence

Strategic competence refers to the student's ability to understand what the problem is asking, to identify important information in the problem, to represent the problem using symbols, graphs, or other representations, and to solve the problem (NRC, 2001). In determining whether a participant exhibited strategic competence, I examined lesson transcriptions and looked for evidence of the following: understanding of the requirements of the task, identifying important information within the problem, or a verbal statement naming the strategy used to solve the problem. Additionally, I looked at participants' work samples for evidence of the strategy mentioned. Because participants might not have completed the task within the allotted time frame of the lesson, I credited the participant who demonstrated evidence of some of the requirements listed above as having hints of strategic competence.

In Lesson 11, the participants were assigned a task that required them to find how many coins were needed for an amount less than $12.00, given certain restrictions on how many nickels, dimes, and quarters could be used (see Appendix J). The following excerpt illustrated Darla's strategy for solving the problem.

1-2 Nick: Does everybody got a way that we can possibly solve this solution, or solve this problem?

3 Gary: More or less.

4-6 Nick: Darla? Let's go looking for a strategy. You don't have to solve it, right now. Just a way to solve it. Like draw a picture, make a list, and stuff like that. Graph it if you have to.

7 Darla: I always start with quarters.

8 Nick: What?

9-10 Darla: Since the largest possible amount is less than $12.00, I was doing -- . I was kind of grouping by quarters.

11 Nick: So you are more or less drawing a picture?

12 Darla: Yeah. (TS, 4/28)

In Line 7, Darla admitted that her strategy for solving the task was grouping quarters. Although students did not have actual coins, Darla used a list (WS, 4/28).

Adaptive Reasoning

Adaptive reasoning refers to a student's ability to reflect, explain, and justify one's thoughts (NRC, 2001). To determine whether a student displayed adaptive reasoning skills, I examined lesson transcriptions and looked for the following: explanation of solutions, justification or explanation of procedures, reflection concerning the context of the problem, and reflective critique of another student's solutions or explanations as this demonstrated "capacity for logical thought" (NRC, 2001, p. 5). Although students may have stated their solution, unless they explained and justified their thoughts, adaptive reasoning did not occur. The following was an excerpt of Nick's reasoning from Lesson 12 (i.e., the dart game).

> She scored 26 on her first two throws, right? I'm going to put P-T-S on her first two throws. She got 26. You add 50 to that and get 76. She got that for her first two throws. All right. So, if she had hit the 50 anywhere else she would have exceeded 76 points. There were only 76 scored. There's no way that she was the one who hit the 50. So, it had to have been Bob that hit the 50 (TS, 4/28).

Nick stated that Bob hit the center of the target (i.e., a score of 50 points), and he also explained why Pam could not have been the one to hit the center of the board first.

Productive Disposition

Productive disposition is defined as a student's ability to see mathematics as useful and worthwhile. Further, students who displayed a productive disposition exhibited confidence and efficacy in their mathematical abilities (NRC, 2001). In determining whether a participant displayed skills characteristic of productive disposition, I examined transcriptions from participant selection, post-study, and group interviews and lessons. I looked for evidence of statements concerning the usefulness of mathematics and of one's positive self-efficacy concerning his mathematical abilities. In the following excerpt, Nick expressed his belief concerning the usefulness of mathematics.

> Well, I want to first of all go to a small tech school and get my industrial maintenance degree, and then go to Company A. And then you have to, before you're up for a job, you have to pass this math test because you have to measure the tires and measure the layers and all that. So, mathematics is very, very important. (PSI, 4/4)

Although Nick sometimes displayed a lack of productive disposition during lessons in confessing that he was lost, the following statement exhibited a productive disposition. Nick replied, "Yeah. He also hit the 25. But, still. I'm pretty sure I'm right" (TS, 4/28). In the previous statement, Nick demonstrated confidence and efficacy in his ability to solve the problem.

Summary

The purpose of this study was to investigate whether the THINK interaction framework supported students in their discussions concerning mathematics and provided evidence of talk that indicated movement toward mathematical proficiency, if at all. In this section, I provided a brief review of each of the strands of mathematical proficiency (i.e., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition) and illustrated each with excerpts of dialogue or quotations from lesson transcriptions of video recordings. Further, I discussed the specifications that I used to determine whether a participant displayed skills of movement toward a specific strand of mathematical proficiency as justification of my reasoning in such determinations.

Limitations and Delimitations of the Study

In the following sections, I offer limitations and delimitations of the study.

Limitations

A review of the methodology revealed possible limitations of the study. First, the process of interviewing students prior to, during, and after the intervention of the heuristic may have promoted reflection that otherwise would not have occurred with the introduction of the THINK interaction framework. Second, the selected tasks may have differed from those used in the Tier II classroom prior to the beginning of this study. As such, the tasks, irrelevant to the introduction of the THINK interaction protocol, may have promoted student discourse. However, evidence was collected and analyzed throughout the study to determine if the introduction of the THINK interaction protocol affected the types of talk divulged in student discourse as a result of its implementation.

Delimitations

In this section, I offer a description of the delimiting factors. First, the participants in the study were selected from a convenience sample of students enrolled in an RTI mathematics intervention course as opposed to a core instruction Algebra I class. Doing so provided a participant pool that was more homogenous than selection from a core Algebra I class of students exhibiting a wide range of academic skills and allowed the investigation of the context described in this study. Another delimiting factor surrounded the setting of the study. The setting of the study involved a ninth grade mathematics intervention class in a rural high school consisting of a student population of 739 students in which 82.1% were categorized as economically disadvantaged. Additionally, the state guidelines for implementation of RTI did not specify the structure or process of implementation, leaving the design and class make-up open for interpretation to administrators and RTI coordinators. For example, the population of students enrolled in mathematics intervention courses may not have been delineated by grade level but by students in grades nine through eleven. Thick descriptions were provided to support the transferability of findings. Finally, interpretation of the findings was through the lens of the researcher but grounded in data.

Establishing Trustworthiness

The researcher established trustworthiness of a study by "addressing the credibility, transferability, dependability, and confirmability" (Gay et al., 2009, p. 375) of my findings. As a researcher, I attempted to establish trustworthiness throughout the study by engaging in the following activities. First, to establish credibility, I conducted persistent observations of the participants engaged in a learning activity for 14 lessons of the study. A second way that I established credibility was through structural corroboration and referential adequacy. To build structural corroboration and coherence, I conducted group interviews to verify my field notes and any questions that I had concerning the data that I collected on site. Additionally, group interviews assisted me in establishing referential adequacy to ensure that I had accurately interpreted the meanings of the participants' written and verbal comments and artifacts in the context in which they were constructed (Gay et al., 2009).

Third, to establish credibility, transferability, and confirmability, I practiced triangulation and the collection of detailed descriptive data and produced thick descriptions of the collected data. By collecting multiple forms of data including transcriptions of video and audio recordings, artifacts, field notes, and my researcher journal that included memos and journal reflections, I demonstrated the collection of detailed, descriptive data. Also, I created case narratives that provided details of the findings as it related to the context of the study. Further, I provided a holistic narrative of the case study that offered a thick description of the case and the overall findings.

Fourth, I provided dependability by creating an audit trail that allowed for the examination of processes, collected data, journal memos, and interpretation. Digital files for each case were created to store transcriptions, researcher journal notes and memos, and artifacts. In conclusion, I attempted to establish trustworthiness throughout the study by addressing credibility, transferability, dependability, and confirmability.

Chapter Summary

As students progress through their academic careers, teachers must seek ways to engage all students in activities that require them to use analytical reasoning and problem-solving skills. Additionally, students must be provided with instructional supports and opportunities to express their thoughts and ideas and to justify their potential solution paths and solutions. To investigate the effects of the THINK interaction framework on students' mathematical discourse and movement toward mathematical proficiency, I employed an explanatory embedded multiple-case study. This chapter described the research methodology of this study. Included in this chapter was a description of the context of the study, the participants, the instruments and methods used to collect data, a timeline of events, methods of data analysis for the case study, limitations and delimitations of the study, and an account of how I established trustworthiness.

CHAPTER IV: RESULTS

Introduction

As educators prepare students for post-secondary opportunities involving academia, professional fields, career and technical fields, and the job market, they must engage students in analytical, critical thinking, and collaboration skills that reach beyond casual conversational speech (Fennell, 2011; Michaels et al., 2008). Effective teaching practices provide opportunities for all students to participate and exercise communication skills that include reasoning, justification of answers, and knowledge building (Michaels et al., 2008) such as those described by SMP3 (i.e., construct viable arguments and critique the reasoning of others) (CCSSI, 2010). Good communication skills might result from student engagement in productive classroom discourse by which students build understanding of mathematical concepts through collaboration (NCTM, 2014).

This study used an embedded case study design to investigate how a heuristic (i.e., the THINK interaction framework) used as an intervention tool supported students' exploratory talk, if at all, during small-group problem-solving activities implemented in a Southeastern high school Tier II mathematics classroom. The following research question guided the study: How does the THINK interaction framework support students' exploratory talk and their movement toward mathematical proficiency, if at all?

In this chapter, I will provide a narrative detailing a typical lesson using the THINK interaction framework to provide the reader with a clear description of how the participants interacted with each other in a group setting. Next, I will provide a summary of the types of talk with examples to provide clarity for the reader. Third, I will discuss the amount of participation by each of the participants per lesson along with the group's engagement in exploratory talk to establish an understanding of the frequency of utterances and types of engagement exhibited by each over the course of the study. Fourth, a holistic analysis of the components of the THINK interaction framework will provide a detailed description of how the THINK interaction framework, implemented during collaborative group activities, supported students' exploratory talk, if at all. Fifth, I will provide a narrative that details the findings of each of the embedded case studies. Next, I will present the findings from a cross-case analysis. The chapter will conclude with a chapter summary.

A Description of a Lesson Using the THINK Interaction Framework

The following narrative provides a description of how the group members interacted with each other and Mrs. Smith on the second day of using the THINK interaction framework (i.e., Lesson 7). This lesson was chosen to provide the reader with a clear description of the types of talk in which the participants engaged, methods used by Mrs. Smith to move the group's thinking forward when at an impasse and in using the THINK interaction framework, and the types of behaviors exhibited by participants as the lesson progressed.

Day 2 of the Introduction of the THINK Interaction Framework

Mrs. Smith began Lesson 7 by introducing the THINK interaction framework (RJ, 4/20; GOP, 4/20). The following is an example of how she guided students through each component of the framework.

> All right, guys. Okay. Remember this from Monday. T in the THINK, this spells T-H-I-N-K. T is talk. What we are trying to encourage you to do is to talk, and that means everybody talk and not just one person. So, everybody has to share

> your own individual ideas. Okay? We will walk through this, but I wanted to re-iterate and make sure that you are talking today about the problem that we are solving and not about random [things]. All right. Does someone want to read to us, today? Tonya, do you want to read the problem? (TS, 4/20)

Tonya read the problem aloud to the class. The problem required students to determine how many students were in a college mathematics class given that the mean on the class exam was 71 prior to the teacher realizing that she made a mistake in grading a question. To correct her error the teacher added one point to each student's grade, making the sum of all grades 936 (viz., the corrected score problem) (see Appendix J). Mrs. Smith reminded students to look at the bright colored piece of paper given to each group that detailed the components of the THINK interaction framework (RJ, 4/20; GOP, 4/20). She stated, "So, what you are doing is answering what the problem is asking. What important information do you need? As a group, talk through what the problem is asking and what important information you need" (TS, 4/20).

Talk. As the administrator of the group, Nick questioned the group during the Talk component of the framework. The group engaged in exploratory talk and began analyzing the task. The following excerpt demonstrated the group's interaction.

1 Nick: What is the problem asking?

2 Gary: How many students were in the class?

3 Nick: Say that again.

4 Gary: How many students were in the class?

5-6 Nick: All right. Is it talking about a specific way, or is it just how many students are in the class?

7 Gary: How many students are in the class?

8-10 Nick: Now, we just basically talk about what information --. Mean, median, and mode. I'm assuming that's what he's talking about. The mean is?

11-12 Tonya: The mean is divided by --. The sum is divided by how many students are in the class.

13-14 Nick: The class [mean] was 71. The teacher realizes --. [He was interrupted by the intercom.] I'm telling you what.

15 Darla: Is it asking how many kids was in the class?

16-17 Nick: The sum of all the grades was 936. How many students were in the class?

18 Darla: So, is the mean before she added the one point to all the grades?

19 Nick: So, the mean was 71. What did you say the mean is?

20-21 Tonya: The mean is the sum of all the numbers added together and divided by the number of students. (TS, 4/20)

The group worked to understand what the problem was asking and identified important information in the problem. In Lines 2, 4, and 7, Gary reminded the group that the question asked how many students were in the class. Nick told the group that the mean was 71 in Lines 13 and 19. Tonya displayed signs of conceptual understanding, procedural fluency, and adaptive reasoning (i.e., Lines 11-12, 20-21) when she explained to Nick the definition of mean and the process to find the mean. Darla pressed the group to clarify what the mean was prior to the teacher correcting the scores (i.e., Line 18).

After giving each group time to talk, Mrs. Smith addressed the class and asked, "As a whole group, what is the question asking you to find" (TS, 4/20)? She then called on students in a random order to provide answers to her question. Mrs. Smith then asked students what information was needed to solve the problem. Different students raised their hand to volunteer to answer her question. Darla, a shy member of the participant group answered, "The sum" (TS, 4/20). Next, Mrs. Smith prompted the group to go to the next section of the THINK interaction framework, the How section.

How. In the same fashion that she introduced the Talk component of the THINK framework, Mrs. Smith told students that during the How section that they were to work individually to find a strategy to solve the problem and would later share out with the members of their group. After time for individual exploration, the group engaged in sharing their thoughts. Nick, the administrator, led the group in the following discussion.

1 Nick: What about you?

2 Tonya: Well, that's all I got.

3 Nick: How did you get 12?

4 Tonya: I plopped in random numbers.

5 Nick: What number were you looking for?

6 Tonya: A number that was close to 71, and then she added one point.

7-8 Nick: So, she added one point. Do you think you would be looking for a number of 70?

9-10 Tonya: She added one point. It would be 72. Well, technically, every grade would have one point added.

11 Gary: Everyone added one extra point.

12-13 Nick: Okay. So, she gave them an extra point and then it equaled 71.

Right? Is that what we are saying here?

14 Darla: Is 71 before she added the one point or after?

15 Gary: Yeah, ‘cause she realized that she made a mistake.

16 Darla: That’s going to be higher than --. [Nick interrupted her.]

17-18 Nick: Was. It says was 71. The teacher realized she made a mistake. So, it was 71 before.

19 Darla: So, it’s going to be higher now.

20 Nick: It’s going to be 72.

21 Tonya: Got it! It would be 13 then. 72.

22 Nick: Hey!

23 Gary: I don’t know.

24 Tonya: Well, if it’s adding one point --. [Darla interrupted her.]

25 Darla: Wait! How many kids does she have?

26 Tonya: Thirteen if she has 72 points.

27 Darla: How are 13 kids going to get a 936?

28 Gary: Well, I mean it’s the sum of all grades.

29 Tonya: I have no idea. (TS, 4/20)

Although there were two interruptions that occurred during this excerpt of the conversation, the type of talk displayed in the above conversation hinged on exploratory talk because Gary and Nick justified their answers using textual evidence from the problem. In Lines 17-18, Nick explicitly explained his reasoning that the mean of 71 was prior to the teacher’s correction of scores by placing emphasis on the word, was, and recognizing that the tense of the word had meaning. Gary exhibited signs of strategic competence by understanding what the problem was asking and pulling information from the text when he explained to Darla (i.e., Line 27) that 936 was the sum of all of the grades (i.e., Line 28). Tonya displayed understanding of the relationship between the mathematical concept of mean and the procedures needed to find the mean in Lines 9-10 when she stated that by adding one point to every score that the mean would be 72. Yet, when pressed by Darla as to how 13 students had a sum of 936, Tonya's response indicated that she did not know. However, all of the participants failed to verbally state a strategy except for Tonya who stated that she plopped in random numbers to find a number that was close to 71. Similar to lessons in Phase 1, the students tried to solve the problem without naming a strategy. In other lessons (e.g., Lessons 6, 9-13), the participants named a strategy but failed to collectively identify the strategy that they would use to solve the problem.

Identify. Following her pattern in assisting students in using the THINK framework, Mrs. Smith explained that after each member of the group shared a strategy for solving the problem, the group members were to identify which strategy the group would use to solve the problem (GOP, 4/20; RJ, 4/20). She stated, "You are on the I part, Identify" (TS, 4/20). Later, Mrs. Smith told the class that they should write down the strategy that was agreed upon by the group to solve the problem. The participant group then engaged in a combination of cumulative and disputational talk. The following excerpt demonstrated the discourse among the participants.

1 Nick: We haven't [inaudible]. [Mrs. Smith spoke at the same time.]

2 Darla: I guess we find numbers that add together to get [inaudible].

3 Nick: Well, you can't --. I mean. If I'm right, 71 is a prime number.

4 Gary: What strategy are we going to use?

5 Nick: I don't know. [Long pause.]

6-7 Gary: What about you, Darla? (Darla and Tonya spoke quietly among themselves.)

8-9 Nick: Wait a minute. How does 936 divided by 12 equal 78? And how does 936 divided by 13 equal 12?

10 Tonya: What?

11-13 Nick: You may want to correct that. Hang on. How in the world does 936, right here, divided by twelve equal 78? How does 936 divided by 18 equal 72?

14 Tonya: I don't know. (TS, 4/20)

In Line 2, Darla offered a strategy of finding scores that yield a sum of 936. Gary assumed the role of administrator and asked Darla what strategy the group had decided to use (i.e., Line 4). The group conversation went from cumulative talk in Lines 1-7, to disputational talk in lines 8-14 when the conversation went from group discourse to conversations among pairs (e.g., Nick and Tonya in Lines 8-14). The group once more failed to collectively identify a strategy to solve the problem.

Notice. The intention of the Notice component of the framework was to provide an avenue for students to reflect individually on how their selected strategy helped them to understand and solve the problem, if at all, and to share their thoughts with the group. To begin the Notice component, Mrs. Smith reminded students that once the group had identified a strategy that they would work as a group to solve the problem. Without an identified strategy to solve the problem, the group quickly engaged in disputational talk

(GOP, 4/20; RJ, 4/20; TS, 4/20). The group discourse was competitive, argumentative, and laced with interruptions and a lack of respect for one's space to speak. The following excerpt demonstrated the group's conversation.

1-2 Tonya: [Inaudible. Nick spoke at the same time and interrupted her. His voice was louder than hers, making it difficult to hear Tonya.]

3-4 Nick: Because you're adding --. Because you add 936 both times there. Nine hundred thirty-six times 2 or 936 plus 936. That completely makes no utter sense.

5 Tonya: What are you talking about?

6-7 Nick: I don't know, but anyway that's what the calculator says. Anyway.

8 Gary: You've confused me.

9 Nick: Tonya, is this coming to you? I don't know a quick way to do it.

10 Tonya: So, it was 71 and then --. [Nick interrupted her.]

11 Nick: Then she added --. She added one point to it. It became 72.

12 Darla: Why are ya'll still on 71? That's the mean.

13-14 Tonya: I'm just saying if it was and became --. [Nick interrupted her].

15 Nick: 72. One plus 71 is 72. (TS, 4/20)

Nick dominated this part of the conversation through interrupting and by speaking louder than everyone (TS, 4/20). In Lines 1-2, 10, and 13-14, Nick interrupted Tonya and consumed her space in the conversation as she attempted to explain her reasoning and to justify her answer. Gary, who had spoken more in Lesson 7 than during lessons in Phase 1 (RJ, 4/20), admitted aloud that Nick had confused him (GOP, 4/20; RJ, 4/20). Darla questioned the group's motive when she asked why they were still discussing the mean in

Line 12. Although Darla spoke more in Lesson 7 than in Lessons 1, 2, and 4, she exhibited passivity in the above excerpt as she only spoke one utterance. Overall, Nick's domination of the conversation hindered the group's progress, as he did not allow for discussion or reasoning by other group members.

Mrs. Smith approached the participant group and asked about their progress. Nick, once again, dominated the conversation by telling Mrs. Smith that he did not understand and that he was confused in his understanding of the mean as it was used in the problem (TS, 4/20). Mrs. Smith attempted to thwart Nick's domination of the conversation and asked Darla to explain the definition of the term, mean. Darla answered, "You add all the numbers together and divide by how many numbers there are" (TS, 4/20). Mrs. Smith urged the group to write down how to find the mean and to look at the information that was provided in the problem. Nick asked Darla to repeat her statement in a louder voice (TS, 4/20). He spoke aloud to the group as he thought through the problem. Nick's comments did nothing to advance the group's thinking. Mrs. Smith came back to the group and provided support through questioning. The following excerpt demonstrated the conversation.

1 Mrs. Smith: Didn't you do one like this yesterday?

2 Nick: Yesterday?

3 Mrs. Smith: On Monday.

4 Nick: Monday? I haven't done one like this. I like this.

5 Mrs. Smith: Where we at?

6 Gary: Lost.

7 Tonya: Square one.

8-9 Mrs. Smith: What? Square one? Is that what she said? Okay. Tell me your thought process right now.

10 Tonya: I just divided a bunch of numbers and got close to 71.

11 Mrs. Smith: Okay. Darla what are you thinking?

12-13 Darla: I was thinking about since, since it was the sum of nine numbers that equal four, it was 36.

14-16 Mrs. Smith: Okay. So, let me ask you this. When something is missing in the problem, like you have a question that you are solving for, what do you use in place of that?

17 Darla: A variable.

18-20 Mrs. Smith: Umm, hmm. So, is there a way that you can use that to find the mean that you all just talked about a minute ago? Use a --. What was that word that you said a minute ago for what you're missing?

21 Darla: A variable.

22 Mrs. Smith: A variable? For what you're missing? Think so? I think so. (TS, 4/20)

Only Darla and Tonya attempted to write an equation using a variable (GOP, 4/20). Darla's equation was somewhat correct, but Tonya's equation was incorrect (see Figures 2 and 3).

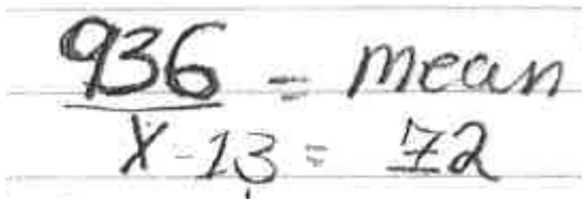

Figure 2. A sample of Darla's work from Lesson 7.

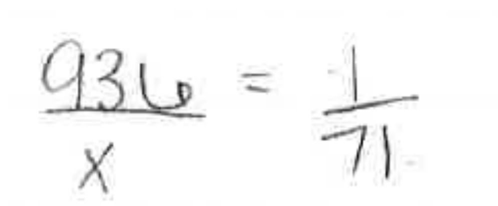

Figure 3. Sample 1 of Tonya's work from Lesson 7.

Neither Gary nor Nick had written much at this point in the lesson (GOP, 4/20; RJ, 4/20; SW, 4/20). They fiddled with their calculators instead. Though Darla and Tonya had written down an equation, they did not show in their work sample the procedure for solving the problem.

Mrs. Smith approached the group once more to see their progress. The group had all but quit working and stated that they were lost (GOP, 4/20; TS, 4/20). Tonya insisted that the answer was 13 and explained her reasoning to Mrs. Smith. Tonya replied, "Nine hundred thirty-six divided by 13 is 72. I just put a random number" (TS, 4/20). Mrs. Smith pressed her to explain her method. Tonya insisted that she put numbers in to find the closest number to 71. Tonya's work sample (see Figure 4) supported her strategy of guess and check.

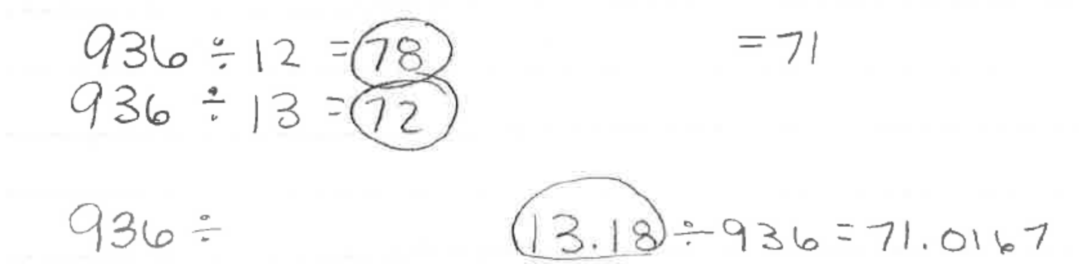

Figure 4. Sample 2 of Tonya's work from Lesson 7.

Mrs. Smith led students using questions and comments concerning the use of the variable, *x*. The following excerpt demonstrated Mrs. Smith's efforts to further the group's thinking.

1 Mrs. Smith: Okay. So, the number of grades would be the number of --?

2 Darla: Students.

3-4 Mrs. Smith: Students. Okay, and what was the number of students? What did you say the number of students was?

5 Tonya: Thirteen. 936.

6 Mrs. Smith: Umm, Umm.

7 Tonya: It's that variable.

8 Darla: The variable, *x*.

9-10 Mrs. Smith: *x*. *x* stood for? The number of students. So, instead of having 936 *x*, what should you have?

11 Nick: Nine hundred thirty-six divided by *x*.

12-14 Mrs. Smith: Yes. And then you have to think about this. The teacher realized that she made a mistake, so she added a point to each student's grade.

15 Nick: So, this would take that to 72, right?

16 Mrs. Smith: It doesn't say that.

17-18 Nick: Okay. I'm confused. That's what got me the last time, when it said that she realized that she messed up and added a point.

19-20 Mrs. Smith: Okay. Here's what I would do. At this point, maybe you should write down how to find the mean. Write down the mean equal to what? (TS, 4/20)

In this excerpt, Mrs. Smith had to prompt the students more than in previous teacher-student conversations. In Lines 9-10, Mrs. Smith led the students to the correct setup of the equation (i.e., Line 11). After many efforts, Mrs. Smith suggested that the students go back to writing down how they found the mean (i.e., Lines 19-20). Upon her return, the students insisted that the only answer that they came up with was 13 students in the class. Mrs. Smith told the group, "I'm okay with her saying this is the answer if you can say how or why. That's the next step, isn't it" (TS, 4/20)? Mrs. Smith then spoke to the class and told them that the last component in the THINK interaction framework was Keep.

Keep. In prompting students concerning the Keep component of the THINK interaction framework, Mrs. Smith told the class to explain in writing whether the solution made sense. If it did not, she urged them to go back, solve the problem again, and share and discuss the information with the group. The participant group then engaged in exploratory talk by sharing their thoughts about the procedures and the solution to the problem. The following was the concluding conversation of the lesson.

1-4 Tonya: She had the whole total, but the old mean was 71. She made a mistake. That meant that she had to go back and add a point to everyone's or whatever so, that would make it 72. If you put 72 divided by 936 would be 13 students in the class.

5 Nick: All right. Now, Darla what do you have to say?

6 Darla: I put, divided 936 by 13 got 72, which is one more than 71.

7 Nick: And, you [He pointed to Gary.].

8 Gary: Huh?

9-11 Nick: Okay, then. I did --. I did the same thing as Tonya, basically. I just took 936, divided by 72, and got 13. That's the only way that I see how to do it. I didn't see no other way.

12 Gary: That's what I got, too.

13-14 Nick: We all got --. Darla, did you do same as me or did you do it a different way? Get a different answer? What did you do?

15-16 Darla: I was just dividing by 936 until I got a whole number that is more than 71. Tonya had gotten that earlier. (TS, 4/20)

During this conversation, Tonya justified her reasoning to the group (i.e., Lines1- 4) as she explained how she got the solution. Darla exhibited signs of procedural fluency (i.e., Line 6) in her explanation of how she arrived at the answer but added that 72 was one more than 71, illustrating that she understood the context of the problem and the strategy for solving it. As for Nick and Gary, the two participants wrote a minimum amount of information as illustrated in their work samples (GOP, 4/20; SW, 4/20). Their responses were copies of the replies offered by Tonya and Darla.

Summary

The above narrative illustrated how the participants interacted with each other and their teacher, Mrs. Smith, during Lesson 7 of the study (i.e., the second day of using the THINK interaction framework). This account provided the reader with examples and vignettes of the types of talk and behaviors in which the participants engaged during group conversations. The reader was introduced to the methods used by Mrs. Smith to further students thinking in using the THINK interaction framework and in assisting students who came to an impasse. Further, student work samples were presented as evidence of student participation at the respective time during the lesson. In the following section, I discuss the frequency of student utterances (i.e., turns) and engagement in exploratory talk to provide understanding of the amount and type of participation of each participant throughout the different phases of the study.

Student Participation

This study was based on the theoretical perspective of Vygotsky's social constructivism, a cultural process where knowledge is constructed among members of a community through language and dialogue (Churcher et al., 2014; Mercer & Howell, 2012; Rojas-Drummond & Mercer, 2003). It is through such communication (i.e., classroom discourse) that ideas are shared and individual knowledge constructed through internalization and personal reflection (Churcher et al., 2014; Rojas-Drummond & Mercer, 2003). In order to narrate the findings of each case and to demonstrate whether exploratory talk occurred, if at all, as a result of the introduction of the intervention (i.e., the THINK interaction framework), I found it necessary to provide background information concerning the frequency of utterances of the participants and their engagement in exploratory talk. For the sake of this study, utterances were defined as spoken turns verbalized by the participants. Utterances ranged from a word to lengthy declarations or inquiries. In the following subsections, I discuss the frequency of utterances made by each participant and their engagement in exploratory talk throughout the study.

Frequency of Student Participation

In Table 4, the frequency of participant utterances while engaged in all types of talk is presented by lesson.

Table 4

Frequency of Utterances

Lesson	Duration	Darla	Gary	Nick	Tonya
1	45:52	34 (11.3)	20 (6.6)	145 (48.2)	102 (33.9)
2	19:03	16 (18.0)	3 (3.4)	36 (40.4)	34 (38.2)
3	42:44	56 (53.8)	48 (46.2)	-	-
4	46:11	37 (12.6)	21 (7.2)	147 (50.2)	88 (30.0)
5	26:47	-	-	55 (77.5)	16 (22.5)
6	47:01	46 (19.3)	23 (9.7)	139 (58.4)	30 (12.6)
7	46:47	43 (27.2)	21(13.3)	54 (34.2)	40 (25.3)
8	41:22	-	35 (27.1)	87 (67.4)	7 (5.4)
9	40:12	42 (29.2)	51 (35.4)	51 (35.4)	-
10	35:55	-	43 (25.3)	127 (74.7)	-
11	37:16	22 (14.4)	33 (21.6)	98 (64.0)	-
12	30:17	32 (30.8)	16 (15.4)	56 (53.8)	-
13	46:49	54 (21.4)	57 (22.6)	141 (56.0)	-
14	41:50	44 (16.3)	39 (14.4)	133 (49.3)	54 (20.0)

Note. The percentage of utterances spoken by each participant per lesson as compared to other participants is presented in parentheses. The dash indicates that the student was absent for the lesson.
[a]In Lesson 8, Tonya left the class after 15 minutes into the lesson. [b]In Lesson 9, Nick entered the classroom 20 minutes after class began.

Because Tonya only attended six lessons and left during Lesson 8 after 15 minutes into classroom activities, she was excluded from the study as an embedded case. However, to illustrate each participant's level of engagement relative to one another, I used Tonya's data concerning the number of utterances to establish a sense of each student's participation by lesson. In the following subsections, I discuss the amount of each participant's engagement in each phase of the study.

Phase 1. In Lesson 1, Nick and Tonya posed a high level of utterances (i.e., 145 and 102, respectively) compared to Gary (i.e., 20 utterances) and Darla (i.e., 34 utterances) (see Table 4). The pattern of Nick and Tonya speaking the most turns continued throughout Lessons 1, 2, and 4. During the study, Nick spoke the most frequently of all the members of the group. Compared to Nick and Tonya, Darla and Gary were the more passive members of the group. In Phase 1 of the study, the average number of utterances spoken by Darla was 36 (i.e., 16.7%) as compared to Gary who spoke an average of 23 utterances (i.e., 10.7%) per lesson and Nick who spoke an average of 96 utterances (i.e., 44.6%) per lesson (see Table 5). Tonya spoke, on the average, 60 utterances (i.e., 28.0%) per lesson during Phase 1 of the study (see Table 5). As a group, the average number of spoken utterances for Phase 1 was 171.6 utterances (see Table 6).

Table 5

Average Frequency of Utterances Per Phase of the Study

	Darla	Gary	Nick	Tonya
Phase 1	36 (16.7)	23 (10.7)	96 (44.6)	60 (28.0)
Phase 2	44 (15.6)	35 (20.6)	92 (54.6)	26 (9.2)
Phase 3	38 (19.5)	36 (18.6)	107 (54.9)	0.07 (6.9)

Note. The percentage of utterances spoken by the participant per each phase of the study is listed in parentheses.
[a]In Phase 3, Tonya was present for only one lesson.

Table 6

Average Frequency of Group Utterances by Phase of the Study

Phase	Total Utterances	Average Per Lesson
1	858	171.6
2	839	167.8
3	779	194.8

Phase 2. In Phase 2 of the study (i.e., Lessons 6 through 10) with the introduction of the THINK interaction framework, Darla spoke an average of 44 utterances per lesson as compared to 36 utterances in Lesson 1. Although Darla's average number of utterances increased, her average percentage of utterances decreased slightly in Phase 2 (i.e., 15.6%) as compared her average percentage of utterances in Phase 1 (i.e., 16.7%) (see Table 5). Gary spoke relatively more in Phase 2 than in Phase 1 of the study with an average of 35 utterances (i.e., 20.6%) per lesson (see Table 5). In Phase 2, Nick spoke an average of 92 utterances (i.e., 54.6%) per lesson as compared to an average of 96 utterances (i.e., 44.6%) per lesson in Phase 1. A possible reason for the increase in the number of utterances made by Gary during Phase 2 was the implementation of the THINK interaction framework, that when followed, encouraged collaboration through each

member sharing their thoughts and ideas. A possible reason for the decrease in Darla's relative percent of utterances in Phase 2 as compared to Phase 1 may have been the result of two absences (i.e., Lessons 8 and 10). In Phase 2 of the study, Mrs. Smith introduced each component of the THINK in Lessons 6 through 10 and emphasized that all members were to share their thoughts with the group (RJ, 4/18, 4/20, 4/21, 4/22, 4/25; TS 4/18, 4/20, 4/21, 4/22, 4/25). However, the average number of utterances spoken by the group was 167.8 utterances (see Table 6). One possible reason for the reduced number of utterances in Phase 2 was that there was one less member of the group for Lessons 9 through 13. An additional explanation for the reduced number of average group utterances in Phase 2 was that the newly introduced THINK interaction framework provided structure to the group conversation, lessening the amount of utterances classified as cumulative and disputational talk. A comparison of the types of talk will be covered in a latter section.

Phase 3. In Phase 3 of the study, Mrs. Smith provided minimal assistance in the use of the THINK interaction framework. Although the number of utterances made by Darla in Lesson 11 decreased (see Table 4), she spoke relatively more in Lessons 12, 13, and 14 than in Lessons 1, 2, and 4. On the average, Darla spoke more in Phase 3 (i.e., 19.5%) than in Phase 1 (i.e., 16.7%) and Phase 2 (i.e., 15.6%) (see Table 5). The average percent of utterances spoken by Gary increased in Phase 2 and slightly decreased in Phase 3 (see Table 5). In Phase 1, Gary spoke an average of 23 utterances (i.e., 10.7%) per lesson. The average number of utterances spoken by Gary in Phases 2 and 3 were 35 (i.e., 20.6%) and 36 (i.e., 18.6%), respectively (see Table 5). During Phase 3, Nick spoke more utterances (i.e., 54.9%) than in Phase 1 (i.e., 44.6%) and approximately the same as in Phase 2 (i.e. 54.6%). In Lessons 13 and 14, Nick spoke 141 (i.e., 56.0%) and 133 (i.e., 49.3%) utterances, respectively (see Table 4). In Phase 3, the average number of group utterances (i.e., 194.8) increased as compared to those spoken in Phase 1 (i.e., 171.6) and in Phase 2 (167.8) (see Table 6). One possible explanation for this increase was that the participants continued to follow the directions of the THINK interaction framework, given that the number of utterances decreased in Phase 2.

Frequency of Student Engagement in Exploratory Talk

In Table 7, the frequency of group utterances during exploratory conversations was presented by lesson.

Table 7

Frequency of Group Utterances During Exploratory Conversations

Lesson	Number of Utterances	Lesson	Number of Utterances
1	19 (6.31)	8	38 (29.46)
2	1 (1.12)	9	47 (32.64)
3	3 (2.88)	10	2 (1.18)
4	7 (2.39)	11	11(7.19)
5	0 (0)	12	74 (71.15)
6	50 (21.01)	13	46 (18.25)
7	46 (29.75)	14	38 (14.07)

Note. The percentage of utterances spoken during exploratory conversations per lesson as compared to the total number of utterances spoken during the lesson is presented in parentheses.

In Phase 1 (i.e., Lessons 1-5), the group spoke a total of 30 utterances during exploratory conversations (i.e., 3.5%) as compared to the utterances coded as cumulative talk (i.e., 21.8%), disputational talk (i.e., 36.1%), and teacher support (i.e., 38.6%) in Phase 1 (i.e., 858 utterances) (see Tables 6, 8, and 9). Though the group spoke 19 utterances during exploratory talk in Lesson 1, students engaged very little in exploratory talk in Lessons 2 through 5 (see Table 7). In Phase 2 (i.e., Lessons 6-10) of the study, the group spoke a total of 184 utterances (i.e., 21.9%) (see Table 8) coded as exploratory talk as compared to the percent of utterances coded as cumulative talk (i.e. 11.0%) and disputational talk (i.e., 12.6%) (see Table 9). The increase was noticeable after the introduction of the THINK interaction framework. Such an increase may also have been due to the teacher's prompting of the use of the THINK interaction framework per each component of the framework. Thus, the increase in utterances coded as teacher support (i.e., 54.5%) (see Table 9) increased in Phase 2. In Lessons 6 through 9, students frequently engaged in exploratory talk, averaging approximately 45 utterances (see Table 7). In Lesson 10, the number of utterances spoken during exploratory talk drastically reduced to two utterances as compared to those spoken in the previous four lessons (see Table 7). Possible explanations for such a reduction were the absence of two participants and the nature of the problem presented in Lesson 10. Only Gary and Nick, two opposing personalities (e.g., shy and talkative), were present for Lesson 10. Although Nick spoke 127 total utterances for the entire lesson, Gary only spoke 43 utterances (see Table 4), wagering a lopsided conversation and little opportunity for productive, exploratory conversation. Additionally, the problem in Lesson 10 (see Appendix J) though routine for non-struggling students in Algebra I, proved to be difficult for the pair of students who attempted to solve using the guess-and-check method.

During Phase 3 (i.e., Lessons 11-14) of the study, the group spoke 169 utterances (i.e., 21.7%) during exploratory conversations and with less prompting from Mrs. Smith to follow the THINK interaction framework (see Table 8). Although the percent of utterances coded as exploratory remained relatively the same as in Phase 2, the amount of utterances coded as cumulative (i.e., 16.6%) and disputational (i.e., 36.3%) talk increased in Phase 3 (see Table 9). The average number of spoken utterances (i.e., 42.3) by the group of participants in the last four lessons of the study illustrated that the group continued to engage in exploratory talk as represented in Table 8.

Table 8

Average Frequency of Group Utterances During Exploratory Conversations

Phase	Number of Utterances During Exploratory Conversations	Average Number of Utterances Per Lesson During Exploratory Conversations
1	30 (3.5)	6.0
2	184 (21.9)	36.8
3	169 (21.7)	42.3

Note. The percentage of the frequency of utterances spoken during exploratory conversations in each phase of the study is in parentheses as compared to the total number of utterances spoken per each phase.

Table 9

Percent of the Types of Talk by the Group Per Phase of the Study

Phase of Study	Exploratory Talk	Cumulative Talk	Disputational Talk	Teacher Support
1	3.5	21.8	36.1	38.6
2	21.9	11.0	12.6	54.5
3	21.7	16.6	36.3	25.4

The increase in the percent of spoken utterances during Phases 2 and 3 suggested that the implementation of the THINK interaction framework supported exploratory talk (see Table 9). Further, the decrease in the percent of utterances coded as cumulative and disputational talk decreased during Phase 2 as compared to Phase 1 (see Table 9). One possible explanation was that the THINK interaction framework provided structure for collaborative activities and group conversations. A possible explanation for the increase in disputational talk in Phase 3 might have been the timing of the last four lessons, which occurred during the same week of state-mandated exams. During that time, the participants' routines were disrupted, as they were required to take state assessments for the core subjects of Algebra I and English I on-line in a computer lab. This change in their normal daily routine could have resulted in a shift in their perceptions of how instruction would be implemented in all classes for the days for which assessments occurred.

Summary

Participants' engagement in conversations was partly gauged by the number of utterances made in each phase of the study. For example, the average number of utterances made by Gary increased throughout the study as compared to those he spoke in Phase 1. An analysis of the average number of utterances made by Darla during each phase of the study disclosed that she spoke relatively less during the implementation of the THINK interaction framework during Phase 2 of the study. The frequency of utterances disclosed in Tables 4 and 5 demonstrated that Nick dominated most conversations throughout the study. Although the average number of utterances made by Nick during Phase 2 exceeded the number of combined utterances of Darla and Gary, an examination of Table 5 revealed that Nick was less dominant in Phase 2 of the study than in Phases 1 and 3.

As a group, the participants engaged in little exploratory talk during Phase 1 of the study, speaking an average of 6 utterances (i.e. 3.5%) as compared to the average number of utterances for all conversations during Phase 1 (i.e., see Tables 6 and 8). During Phase 2 of the study, the average number of group utterances (i.e., 36.8) increased during exploratory conversations, possibly due to the introduction of the THINK introduction framework (see Table 8). As a result, the group engaged in less cumulative (i.e., 11.0%) and disputational (i.e., 12.6%) talk, indicating that the group may have stayed attentive to the task as compared to the group's behavior in Phase 1 (see Table 9). The upward trend of utterances spoken during exploratory talk (i.e., an average of 42.3) continued in Phase 3 (see Table 8), possibly due to the familiarity of the use of the THINK interaction framework.

The background information presented in this section informed the reader of the participants' level of engagement during conversations of all types of talk and especially, during conversations coded as exploratory talk. In the following section, I discuss the effectiveness of each of the components of the THINK interaction framework in promoting exploratory talk during Lessons 6-13.

The Use of the THINK Interaction Framework in Promoting Exploratory Talk and Movement Toward Mathematical Proficiency

In Table 10, the frequency of group utterances occurring during exploratory talk by component of the THINK interaction framework is presented. Because students failed to utilize the framework during Lesson 14, only utterances spoken during exploratory talk in Lessons 6-13 are reported. The percentage of utterances spoken during each component was calculated to allow comparison to the total number of utterances spoken during conversations coded as exploratory talk.

Table 10

Frequency of Utterances per the THINK Interaction Framework

Component	Number of Utterances
Talk	78 (24.76)
How	82 (26.03)
Identify	51 (16.19)
Notice	95 (30.16)
Keep	9 (2.86)

Note. The percentage of utterances spoken per component of the THINK interaction framework as compared to the total utterances spoken during exploratory talk is presented in parentheses.

The Use of the Talk Component

Analyses of the transcripts revealed that 24.76% (see Table 10) of the utterances coded as exploratory talk occurred during the Talk component of the THINK interaction framework. The directions of the Talk component supported the participants in exploratory talk as individual participants were prompted to talk to other group members, to share perceptions of the meaning of the problem, and to offer thoughts concerning the necessary information needed to solve the problem. Often, such intermental activity spurred intramental development that allowed each participant to reflect and monitor the information that he or she had given and received toward understanding the problem. As a reminder, intermental activity is characterized by the students' interactions and collective thinking, while intramental activity is often referred to an individual's reflection and internal dialogue (Mercer et al., 1999; Vygotsky, 1978). As such, the Talk component encouraged conceptual understanding in identifying and defining important mathematical concepts as related to the context of the problem. Additionally, students frequently provided assistance to each other in further defining concepts and procedures for calculating answers. Though assistance for calculating answers was sometimes offered during the Talk component, there was not enough evidence to suggest that students increased their abilities toward procedural fluency.

An example of the group engaging in exploratory talk occurred during Lesson 7 (i.e., the corrected exam score problem). The following vignette presented an account of the students' interactions.

> Assuming the role as the administrator, Nick asked members of his group to state what the problem was asking. Although Gary offered that the problem required the participants to find the number of students in the mathematics class, Nick needed further clarification and asked, "Is it talking about a specific way or just how many students are in the class" (TS, 4/20)? Upon receiving assurance from Gary that the objective of the task was to find the number of students in the class, Nick then demonstrated a lack of understanding of the mean and the procedure for finding it. Information concerning the mean score on the exam was provided in the wording of the task and essential information needed to solve the problem. Nick inquired, "Now, we just basically talk about what information --. Mean, median, and mode. I'm assuming that's what he's talking about. The mean is" (TS, 4/20)? Group members often provided assistance, if possible, in defining terms and concepts and in stating the procedure for calculating the answer. In this example, Tonya stated the procedure for finding the mean. Tonya replied, "The mean is divided by --. The sum is divided by how many students are in the class" (TS, 4/20). Realizing that the mean (i.e., 71) was stated in the problem, Nick remained confused about the procedure for finding the mean and asked Tonya to

restate the procedure.

The above vignette was fairly representative of student conversations during engagement with the Talk component of the framework, with approximately half of lessons depicting a situation where participants actively engaged in exploratory talk and shared their ideas as to what the problem was asking, clarifying definitions for group mates, and describing mathematical procedures. In summary, the Talk component of the framework supported students in voicing what the problem was asking, in identifying necessary information for solving the problem, and in asking questions for clarification of information in order to understand the task.

The Use of the How Component

Analyses of the transcripts revealed that 26.03% (see Table 10) of the utterances coded as exploratory talk occurred during the How component of the THINK interaction framework. Such an increase demonstrated that the implementation of the How component supported participants' exploratory talk. More specifically, the directions of the How component supported the participants in exploratory talk as individual participants were directed to construct a strategy and then to explain how the use of the strategy could be used to solve the problem. Participants (i.e., Darla and Gary) occasionally included a visual representation in their explanations of their strategies. Often, such intermental activity spurred intramental activity, requiring the group to revisit the Talk component to further investigate the problem. As such, the How component encouraged strategic competence by prompting students to find means of representing the problem and identifying important information in the form of a list or the use of sketch. Also, adaptive reasoning was evident in the participants' explanations of their strategies when the participants described how the strategy could be used to solve the problem or required other participants to further explain their reasoning for clarification and understanding of the processes used to solve the problem.

The following vignette is a depiction of the participants' conversation during the How component of Lesson 6. The participants were required to find the number of different stacks of three blocks that could be made from six colored blocks (i.e., the block problem).

> Nick once more assumed the role of administrator and inquired as to what strategies that his group mates proposed to solve the problem. Tonya proposed to arrange the blocks into groups without attention to color to see how many stacks of three blocks could be made. Darla took a different approach, suggesting that the order of the colors mattered. Darla stated, "I arranged them into different colors in order" (TS, 4/18). Gary interjected that more discussion should occur to determine what the problem was asking. Before Nick interrupted him, Gary stated, "It depends on what you are asking. But if you were asking how many different colors in the stacks, you put red then a couple of -" (TS/ 4/18).

The above scenario was fairly representative of student dialogue during engagement with the How component of the framework, with five of the eight lessons depicting a situation where participants actively engaged in exploratory talk and collectively participated in proposing a strategy for the group to use in finding a solution to the problem.

The Use of the Identify Component

Analyses of the transcripts revealed that 16.19% (see Table 10) of the utterances coded as exploratory talk occurred during the Identify component of the THINK interaction framework. During this component, students were to identify the strategy that they would use to solve the problem and decide whether their strategy was working or if they needed to choose a different strategy (Thomas, 2006). However, analysis of the data sources revealed that the participants only engaged in exploratory talk during the Identify component in Lessons 9 and 13. Further, in all other lessons, the participants rushed into solving the problem without collectively identifying a strategy to solve the problem (TS, 4/18, 4/20, 4/21, 4/22, 4/25, 4/26, 4/28, 4/29, 5/24). The lack of utilizing the Identify component of the framework as directed often resulted in inefficient collaboration to solve the assigned problem. Although students spoke 47 utterances during the Identify component during Lesson 9, they primarily discussed possible strategies but failed to identify a strategy until led by Mrs. Smith to do so. During Lesson 13, the group quickly identified a strategy to solve the task that required the participants to determine the number of hits in a target game at a county fair (viz., the target shooting problem) (see Appendix J). However, the participants' broad explanation of a strategy (e.g., a list) resulted in a lack of cooperation and failure to solve the problem. In conclusion, the participants weakly engaged in exploratory talk during the Identify component of the framework often resulting in failure to identify a strategy, solve the problem, and to collaborate effectively. Further, the lack of implementation of the Identify component resulted in participants' lost opportunities for engagement in rich conversations concerning why one strategy was more effective than another for solving the task. As such, opportunities for increasing their strategic competence were stifled during the Identify component.

The Use of the Notice Component

An analysis of the lesson transcriptions (TS, 4/18, 4/20, 4/21, 4/22, 4/25, 4/26, 4/28, 4/29, 5/24) for Lessons 6 – 13 revealed that students spoke the most utterances coded as exploratory talk during the Notice phase of the THINK interaction framework than during other components (see Table 10). Though the participants failed to follow the directions during the Notice component as intended by the authors of the THINK interaction framework (Thomas, 2006), productive conversations centered on mathematical talk during this component. Participants' engagement in exploratory talk spurred intermental activity as illustrated by conversations and provoked by the sharing of explanations, strategies, and difficulties experienced toward solving the problem. As a result, this engagement spurred intramental activities of monitoring and reflection and often allowed participants to redirect their thinking as exhibited in the conversations and student work discussed in the individual case narratives. As such, the Notice component encouraged adaptive reasoning and strategic competence as exhibited in participants' explanations and justifications of their solutions and strategies. Further, conceptual understanding and procedural fluency was occasionally displayed as the participants explained mathematical concepts (e.g., the meaning of congruency) and procedures for finding the answers for such concepts.

The following vignette is a depiction of the type of student engagements that occurred during the Notice component of Lesson 6.

> Although the group of participants presented individual strategies, the participants failed to collectively identify one strategy that could be used to solve the task (i.e., the block problem). As a result, Nick continued to experience difficulty as

> demonstrated in his lack of understanding of using combinations of colored blocks in his solution. Nick stated, "I got two different combinations. You can get more than two? So, you are saying that you switch them up and can get more than one? You can switch the colors up so you can have blue, red, blue" (TS/ 4/18)? After receiving confirmation from Mrs. Smith that the solution contained more than two combinations, Nick insisted that Gary explain his work. Gary justified his reasoning in his explanation to Nick. Gary explained, "So, you can have three different combinations. You can have red, blue, blue, or you can put the red on top and change the combination" (TS, 4/18).

The above excerpt represented how students engaged in exploratory talk during the Notice component of the framework in approximately half of Lessons 6 through 13. To summarize, the Notice component of the THINK interaction framework provided opportunities and encouraged participants to engage in meaningful exploratory talk including explanations, justifications, sharing of solution paths, and difficulties experienced while solving the problem. These conversations exhibited displays of conceptual understanding, adaptive reasoning, and strategic competence. Thus, the Notice component was effective in promoting individual accountability displayed as exploratory talk through explanations and justification of students' thoughts.

The Use of the Keep Component

During the Keep component, participants were to discuss alternate ways to solve the problem and whether their solution, if any, was logical; however, students rarely made it to the Keep component throughout the study. Students only engaged in exploratory talk using the Keep component during Lesson 7. In conclusion, the participants did not utilize the Keep component of the framework throughout the study, resulting in lost opportunities to further their understanding by finding alternative methods for solving problems. Thus, the Keep component of the framework, for this study, was ineffective in promoting exploratory talk and movement toward mathematical proficiency.

Summary of the Use of the THINK Interaction Framework

An analysis of the video transcriptions for Lessons 6 through 13 provided insight into the effectiveness of the THINK interaction framework. In the previous section, I discussed the effectiveness of the framework by component and presented excerpts of dialogues and vignettes as examples of occurrences of exploratory talk and movement toward mathematical proficiency or lack thereof. Implementation of the Talk component not only supported exploratory talk as student conversations often focused on what the problem was asking, but it also promoted conceptual understanding as reflected in members' responses to other participants' need for clarification of concepts, mathematical procedures, and the further explanation of how certain words provided necessary information needed to solve the problem in context.

The How component of the THINK interaction framework proved equally effective in structuring the participants' mathematical talk. The instructions for utilizing the How component of the framework promoted exploratory talk as it prompted participants to construct their individual strategies for solving the problem, to share their proposed strategies, and to explain their proposals to the other members of the group. Occasionally, the participants revisited the Talk component during this phase to better understand what the problem was asking and to search for necessary information not identified during the Talk component. As such, the How component of the framework encouraged strategic competence as the participants often used a variety of methods to represent situations and used these representations in their solution paths. The How component promoted adaptive reasoning, as participants were required to explain their solution paths and conceptual understanding as participants connected physical representations, mathematical concepts, and sometimes procedures for solving the problem.

During the Identify stage of the framework, participants failed to collectively identify a strategy for solving the problem. As such, collaboration and peer assistance was somewhat stifled due to participants working toward different solution paths. Thus, the Identify component of the framework proved to be ineffective in truly promoting productive exploratory talk.

Students engaged in exploratory talk most frequently during the Notice component of the framework (see Table 10). Though the intention of the authors of the THINK interaction framework (Thomas, 2006) was for students to discuss how their chosen strategy helped them to solve and understand the problem, the participants engaged differently in exploratory conversations. The participants instead discussed their strategies, solutions, and difficulties in solving the problem. Further, students often provided explanation and justification of their strategies and progress in solving the problem. Thus, the Notice component promoted conceptual understanding, strategic competence, and adaptive reasoning through participant conversations that evolved as a result for the need of clarification, understanding, and venting of frustrations.

Finally, students infrequently engaged in exploratory talk during the Keep component of the framework speaking 2.86% of utterances coded as exploratory talk (see Table 10). As a result, valuable opportunities to seek additional solution paths and to further the participants' mathematical content knowledge were lost. The background information presented in this section informed the reader of participants' level of engagement in each phase of the project to provide a deeper understanding of the findings within each case. In the following sections, I present three individual case study narratives.

Embedded Case Analyses

Each participant was considered a single case in this embedded case study. Data were gathered through multiple sources including participant selection and post-study interviews, audio and video recordings of lessons and group interviews, classroom observation notes, students' work artifacts, and the researcher's journal. Transcriptions of video recorded lessons were coded and analyzed to determine whether changes occurred, if any, in student discussions as a result of the implementation of the THINK interaction framework. Additionally, transcriptions were analyzed to determine if movement toward mathematical proficiency occurred as a result of the intervention. The following sections are structured to introduce each of the participants to the reader followed by an overarching theme for each case. Each overarching theme will be supported by subthemes that provide the reader with insight and background information. In Table 11, I presented the overarching theme and subthemes for each case.

Table 11

Overarching Themes with Associated Subthemes

	Darla	Gary	Nick
Overarching Theme	From Passivity to Increased Self-efficacy: Maximizing the Opportunities Provided by the Think Interaction Framework to Strengthen Skills Toward Becoming Mathematically Proficient	From Passivity to Finding a Voice: Using Opportunities Provided by the THINK Interaction Framework to Engage in Conversations and Acquire Skills Toward Becoming Mathematically Proficient	From a Lack of Behavioral Self-regulation to Improved Impulsivity: Using the THINK Interaction Framework to Provide Structure and Support to Engage in Exploratory Talk and Acquire Skills in Adaptive Reasoning
Subtheme 1	Voicing Strategies And Understanding Tasks	Finding Space in Conversations	Learning to Share Space in Conversations
Subtheme 2	Justifying Reasoning: From Explanations to Critiques	Voicing Strategies and Understanding of Tasks	Using Verbalizations When Solving Tasks
Subtheme 3	Making Connections Among Mathematical Concepts, Procedures, and Representations	Voicing Understanding of Mathematical Concepts	Exhibiting Increased Reasoning and Justification Skills in Conversations

Meet Darla

At the time of this study, Darla was a ninth-grade Caucasian girl who was enrolled in both Algebra I and a Tier II mathematics intervention class in a rural high school located in the Southeastern United States. In describing how she best learns mathematics, Darla stated “Normally, I like showing how to do it step-by-step instead of just throwing it all together” (PSI, 4/6). Darla discussed that her teacher’s instructional method of going over homework problems on the board step-by-step provided support for her learning and understanding of mathematics. Opportunities to receive help from the teacher came in the form of individualized help during class and summoned by the raising of one’s hand or tutoring received prior to the start of school in the morning (PSI, 4/6).

When asked how she liked working in groups, Darla paused before answering, “I like it ‘cause like everyone has like their own different way of how to do it, and some might be easier than others” (PSI, 4/6). Additionally in the interview, Darla admitted that group work was infrequent and that students did not understand how to participate in groups. She stated that participation in groups resulted in students solving the problem or task individually before sharing strategies or solutions. Further, she mentioned off-task behavior and failure to participate as two disadvantages of working in groups (PSI, 4/6).

Darla expressed that academic support from her peers was more of a comparison of answers to check the accuracy of her work than actual tutoring for procedural or conceptual understanding. When asked what opportunities she had to explain her ideas to other students, Darla confided that she did not provide support to others. “I don’t really help people. Normally, I’m not confident if it is right, and I don’t want to give them the wrong thing” (PSI, 4/6; RJ, 4/6). Darla’s lack of confidence was evident in Lesson 3 when she did not respond to another student who asked about her ideas. On two occasions during the lesson, Darla either shook her head to indicate her lack of participation or did not respond to her classmate (TS, 4/11; RJ, 4/11). Darla's passivity was observed in her refusal to respond to Ms. Smith when asked to explain her ideas in Lesson 1 of the study (TS, 4/7; GOP, 4/7) and in her use of a low voice that was often inaudible (TS, 4/7, 4/8; GOP, 4/8).

Although Darla expressed the usefulness of mathematics in the real world, she conveyed that the study of some mathematical topics were useless. "Some things won't help us, and I'd rather learn things that would help us than not" (PSI, 4/6). Darla stressed that she did not like to study mathematics but resorted to exerting her best effort in doing so (PTI, 5/8).

From passivity to increased self-efficacy: Maximizing the opportunities provided by the THINK interaction framework to strengthen skills toward becoming mathematically proficient (Overarching theme – Darla). Throughout the study, Darla exhibited various levels of confidence as evidenced in lesson transcriptions (e.g., TS, 4/7, 4/8/, 4/11, 4/14, 4/28), classroom observations (e.g., GOP, 4/7, 4/8, 4/14) and researcher notes (e.g., RJ, 4/8/, 4/11, 4/14, 4/20). An analysis of the aforementioned data sources revealed the overarching theme, "From Passivity to Increased Self-efficacy: Maximizing the Opportunities Provided by the THINK Interaction Framework to Strengthen Skills toward becoming Mathematically Proficient." To help the reader better understand the emergence of the overarching theme, I provide a chronological account of the noticeable gain in Darla's confidence and self-efficacy through each of the phases of the study and illustrate with excerpts of dialogue, vignettes, and student work samples. Further, I discuss the subthemes that support the findings.

Phase 1. In Phase 1 of the study, Darla's lack of confidence and self-efficacy was exhibited as passiveness in the form of a lack of utterances (see Tables 5 and 12) and through physical gestures such as shaking her head from left to right or the shrugging of her shoulders to indicate that she either did not understand or did not want to participate (GOP, 4/7, 4/8; RJ, 4/7, 4/8). In Phase 1, Darla only spoke an average of 36 utterances (i.e., 16.7%) (see Table 5) per lesson, a characteristic of her shyness and passive behavior.

Table 12

Frequency of Darla's Utterances Throughout the Study

Lesson	Number of Utterances	Lesson	Number of Utterances
1	34 (11.3)	8	-
2	16 (18.0)	9	42 (29.2)
3	56 (53.8)	10	-
4	37 (12.6)	11	22 (14.4)
5	-	12	32 (30.8)
6	46 (19.3)	13	54 (21.4)
7	43 (27.2)	14	44 (16.3)

Note. The percentage of utterances spoken by Darla per lesson with respect to all student utterances spoken during that lesson is presented in parentheses. The dash indicated an absence.

As a result of her lack of confidence and self-efficacy, Darla often provided partial answers and weak explanations to members of her group. An example of such behavior was witnessed in Lesson 1. Although Darla found six of the seven possible dates of the solution (SW, 4/7) (see Figure 5), she only offered bits and pieces of the solution without explanation or justification of her answer (TS, 4/7). Her lack of collaboration and inefficient communication hindered the group's progress as they stumbled and failed to solve the problem in its entirety (TS, 4/7; GOP, 4/7). The following excerpt illustrated Darla's passive behavior.

1-2 Tonya: I don't understand how you get those dates there. You are going to have to find numbers that go into 2013 that are dates evenly.

3 Darla: January 11th.

4 Nick: Huh?

5 Tonya: January 11th?

6 Darla: Un, huh.

7 Tonya: Does that work?

8 Nick: January 11th? It does?

9 Tonya: That is what I got. So, January 11th?

10 Darla: Uh, huh. (TS, 4/7)

In the excerpt above, Darla spoke two-word utterances and offered no explanation or justification for her answer but only replied "Un, huh" when questioned by Tonya (i.e., Lines 6 and 10).

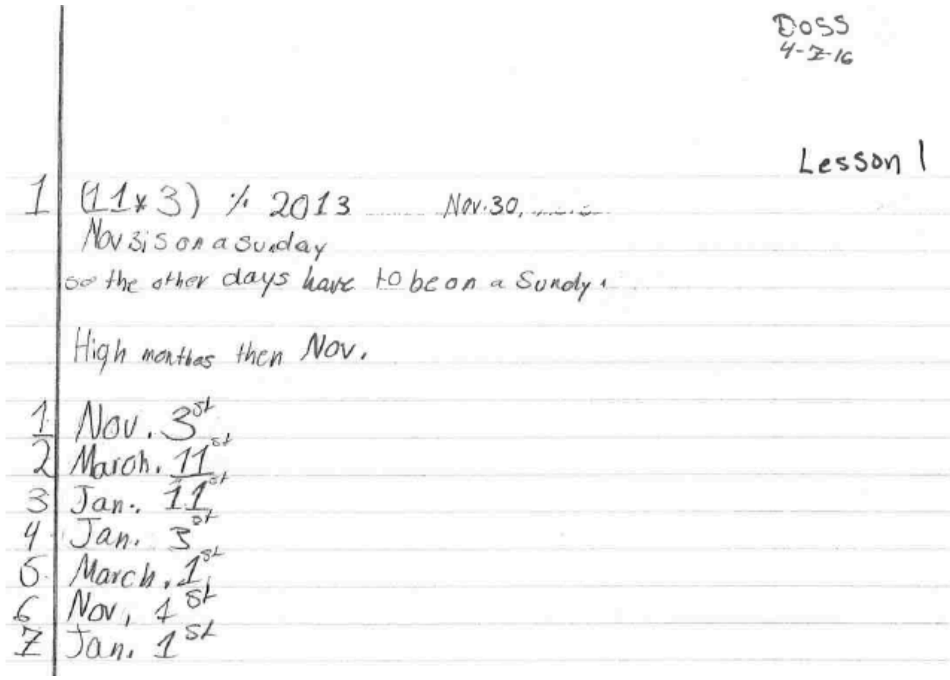

Figure 5. Darla's work from Lesson 1.

In Lesson 2, Darla only spoke 16 total utterances (see Table 4). When asked to explain the answer to the class, Darla refused and shook her head no (GOP, 4/8; RJ, 4/8). In Lesson 4, Darla continued to display low self-efficacy and confidence as she failed to share her strategy or procedures for solving the assigned problem (SW, 4/14; TS, 4/14). Again, Darla only nodded her head to reply to the teacher's question when asked if that was the procedure that she had followed. After Mrs. Smith encouraged Darla to speak up and share her thoughts with her group, Darla replied, "I did but ya'll didn't think that was right" (TS, 4/14). The other group members confided that they did not hear Darla share her solution or thoughts about the problem and encouraged her to speak up (TS, 4/14). The group members' statements indicated that they did not hear her, possibly due to Darla's speaking in a low voice.

Phase 2. Although the average percentage of utterances (i.e., 15.6%) spoken by Darla in Phase 2 slightly decreased as compared to the average percentage of utterances spoken during Phase 1 (i.e., 16.7%) (see Table 5), the frequency of her utterances when engaging in exploratory talk increased during Phase 2 as compared to Phase 1 (see Tables 13 and 14). During Phase 1, Darla only engaged in exploratory talk during Lessons 1 and 3. Although she had two absences during Phase 2, Darla spoke 22 more utterances during exploratory talk as compared to her engagement in exploratory talk during Phase 1 (see Table 14).

Table 13

Frequency of Darla's Utterances During Exploratory Conversations

Lesson	Number of Utterances	Lesson	Number of Utterances
1	7 (20.6)	8	-
2	0 (0)	9	12 (28.6)
3	2 (3.6)	10	-
4	0 (0)	11	3 (13.6)
5	-	12	26 (81.3)
6	10 (21.7)	13	14 (25.9)
7	9 (20.1)	14	0 (0)

Note. The percentage of utterances spoken by Darla during exploratory conversations with respect to the total utterances spoken by her during that lesson is displayed in parentheses. The dash indicates an absence.

Table 14

Frequency of Darla's Utterances During Exploratory Conversations Per Phase

Phase	Number of Utterances
1	9 (6.3)
2	31 (23.7)
3	43 (28.3)

Note. The percentage of utterances spoken by Darla during exploratory conversations by phase is presented in parentheses as compared to the total number of utterances spoken by her during that phase.

One possible explanation for the increase in Darla's participation was the implementation of the THINK interaction framework, beginning in Phase 2. In Table 15, Darla's utterances during exploratory talk are presented per each component of the framework.

Table 15

Frequency of Darla's Utterances per Each Component of the THINK Interaction Framework

Lesson	Talk	How	Identify	Notice	Keep
6	5	1	0	4	0
7	2	5	0	0	2
8	-	-	-	-	-
9	0	0	12	0	0
10	-	-	-	-	-
11	0	3	0	0	0
12	8	5	0	13	0
13	2	3	1	8	0
Total	17	17	13	25	2

Note. The dash indicates an absence. The group did not attend to the framework during Lesson 14.

The increase in Darla's engagement in exploratory talk during Phase 2 indicated that Darla's self-efficacy and confidence had increased as compared to her lack of display of confidence in Phase 1. Additionally, while engaged in exploratory talk, Darla spoke the most utterances during the Notice component (i.e., 25 utterances across Phases 2 and 3) of the framework, when she discussed her strategy, solution, or brainstormed with her group to determine a different strategy if unable to solve the problem. She frequently engaged in exploratory talk during the Talk and the How components (see Table 15) in deciding what the problem was asking and in determining the important information needed to solve the problem.

Unlike her participation in Lesson 1, Darla engaged in exploratory talk during the How component of the framework in Lesson 6 when she stated her strategy for finding the number of combinations for arranging colored blocks (i.e., the block problem) (see Appendix J). The following excerpt provided a sample of the conversation.

1-2 Nick: What you got?

3-4 Tonya: Arrange them into groups without the colors and see how many groups you can get.

5 Nick: Just say what you got. Just say what you are doing.

6 Darla: I arranged them into different colors in order. (TS, 4/18)

Unlike her two-word utterances for sharing her thoughts in Lesson 1, Darla stated her strategy for finding the solution (i.e., Line 6). Further, Darla's statement was unique, as she did not re-state Tonya's strategy for solving the problem. Whereas Tonya recommended arranging the blocks in order without attention to color, Darla recommended using color to distinguish the stacks with an indicated structure to her strategy. The use of the framework provided Darla an opportunity to share her strategy with members of her group.

Phase 3. In Phase 3, Darla continued to engage more frequently in exploratory conversations as noted by the increase in the number of utterances spoken during exploratory talk (i.e., 43) (see Table 14) and the type of explanations and justifications provided by her during the lesson activities (GOP, 4/28, 4/29; RJ, 4/28, 4/29; TS, 4/28, 4/29). Though her engagement in exploratory talk in Lesson 11 (i.e., 3 utterances) was weak, Darla's engagement in Lesson 12 (i.e., the dart game) demonstrated her confidence as she provided deeper explanations and critiqued the reasoning of her group mates (TS, 4/28). The following vignette illustrates the increase in Darla's self-efficacy and confidence to actively engage in conversations.

> In Lesson 12 (i.e., the dart game), the group was tasked to find the player who hit the target first (see Appendix J). Though information provided in the wording of the task specified that the players scored the same number of points, Nick stated that one of the players won. Darla interjected stating that neither player won and explained why the female player did not hit the center of the target. Darla stated, "Because she only has twenty-six [points] in her first two throws, she would have had to get a twenty-five and a one" (TS, 4/28).

Darla's response to Nick's comments exhibited her critique of Nick's reasoning. Further, Darla's explanation of the female's score on her first two attempts demonstrated her reasoning concerning why it was not the female player to hit the center of the target. The exchange of comments demonstrated Darla's increased self-efficacy as she confidently engaged in the exploratory conversation during the Talk component of the framework (see Table 15) (GOP, 4/28; RJ, 4/28; TS, 4/28). The use of the framework during the Talk component provided Darla with an avenue to verbally discern the meaning of the problem (i.e., which player hit the center of the target first) as well as the necessary information needed to solve the problem (i.e., that neither player won and that the female player scored 26 points on her first two throws).

Summary. In the beginning of the study, Darla offered few utterances, spoke in a low voice, and failed to explain or justify her answers (GOP, 4/7, 4/8, 4/14; RJ, 4/7, 4/8, 4/14; TS, 4/7, 4/8, 4/14). After Phase 2 of the study, Darla exhibited increased confidence and self-efficacy as she engaged in exploratory conversations to voice her understanding of the assigned tasks and propose possible strategies for solving those tasks, offer explanations and justifications for her utterances, critique group members' statements and thoughts, and demonstrate her understanding of the connections among various mathematical concepts, representations, and procedures (GOP, 4/18, 4/20, 4/22, 4/28, 4/29; RJ, 4/18, 4/20, 4/22, 4/28, 4/29; TS, 4/18, 4/20, 4/22, 4/28, 4/29). Possible explanations for Darla's exhibited increase in self-efficacy and confidence included increased familiarity with the members of her group and for the process of participating in a group, practice in solving tasks, and the implementation of the THINK interaction framework. In the following subsections, I discuss the subthemes that support the findings of Darla's increased confidence and self-efficacy.

Voicing strategies and understanding of tasks (Subtheme 1 – Darla). Though Darla often exhibited passivity and shyness throughout the study (GOP, 4/7, 4/14, 4/26; RJ, 4/8, 4/11, 4/14; TS, 4/7, 4/14, 4/26), she became more vocal after the implementation of the THINK interaction framework (GOP, 4/18, 4/20, 4/22, 4/28; RJ, 4/18, 4/20, 4/22, 4/28; TS, 4/18, 4/20, 4/22, 4/28, 4/29). Employment of the THINK interaction framework provided opportunities for Darla to participate, have a voice, and share ideas. To help the reader better understand the emergence of the first subtheme, I provide an account of how the components of the THINK interaction framework supported Darla in her exploratory talk and movement toward mathematical proficiency.

Talk. The Talk component of the framework provided Darla with opportunities to state and explain what the problem was asking, to identify important information needed to solve the problem, and to identify the mathematical concepts associated with the objective of the problem (TS, 4/18, 4/20, 4/22, 4/26, 4/29). Examples of each of the types of Darla's engagement in exploratory talk during the Talk component was particularly noted in Lessons 6 and 9. In Lesson 6, Darla engaged in the discussion by presenting necessary information concerning the task (i.e., the block problem) (see Appendix J). To assist Nick in his understanding of what the problem was asking, Darla emphasized that the wording of the problem provided necessary information for solving the problem. Darla stated, "Yeah. It says different stacks of three blocks that the toddler can make" (TS, 4/18). Darla not only restated information given in the problem, but she also understood that the problem was asking for the number of distinct ways that the blocks could be arranged.

An example of Darla exhibiting her understanding of the mathematical concepts related to the objective of the task transpired in Lesson 9 in which the task required the participants to divide an irregular figure into four congruent figures (viz., the congruent figure task). To assist Gary in understanding the requirements of the task, Darla confirmed her understanding of the mathematical concept of congruency. In her explanation to Gary, Darla replied, "Divide the figure [into] four congruent [figures]. Congruent means the same. I'm pretty sure. So, you are going to use the lines as the legal dividing lines. It means that you want four boxes the same" (TS, 4/22). In summary, Darla seized the opportunities to share her thoughts about the assigned tasks during the Talk component of the framework.

How. The How phase of the THINK framework provided space for Darla to reveal the strategy that she proposed and how to use necessary information to solve the problem. Examples of Darla's engagement in sharing her strategies were noted particularly in Lessons 6, 11, and 12 (TS, 4/18, 4/26, 4/28; SW, 4/18, 4/26, 4/28).

Throughout this subsection, I provide examples of Darla proposing strategies to solve assigned tasks via excerpts of dialogues, vignettes, and work samples.

When asked about her strategy for solving the task in Lesson 6 (i.e., the block problem), Darla replied, “I arranged them into different colors in order” (GOP, 4/18; RJ, 4/18; TS, 4/18). Darla’s work sample (see Figure 6) confirmed the use of her strategy as she drew sketches to solve the problem (SW, 4/18). Darla used the letters b, r, y, p, (i.e., blue, red, yellow, and purple) to represent the colors of the blocks stacked by the toddler.

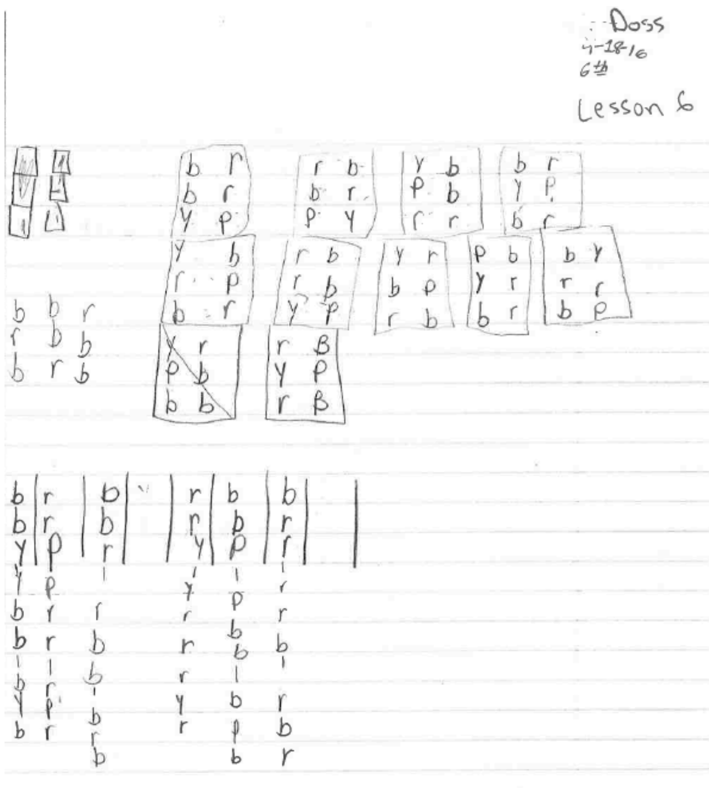

Figure 6. Darla’s work from Lesson 6.

To solve the task in Lesson 11, the participants were required to find the number of coins needed to create the largest possible amount less than $12.00 and given certain requirements (viz., the coin problem) (see Appendix J). Darla shared her proposed strategy for solving the problem. She stated, "I always start with quarters. Since the largest possible amount is less than $12.00, I was doing --. I was kind of grouping by quarters" (TS, 4/26). Darla's proposed strategy involved more than just drawing a picture. Darla intuitively understood that four quarters equal one dollar, thus beginning with the largest coin allowed per the information in the problem. However, Darla's strategy took the form of a list more than a picture (see Figure 7).

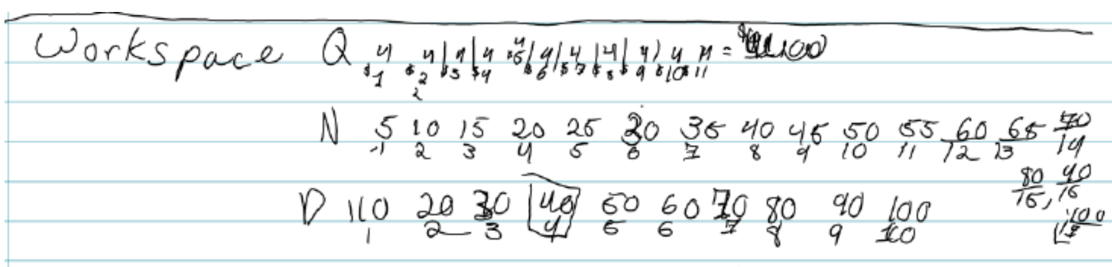

Figure 7. A Sample of Darla's work from Lesson 11.

She confirmed her strategy during the group interview after the lesson. When asked what helped her to start on the problem or get on the right path to solving the problem, Darla replied, "I started working on a list. I made a list and started grouping, like four quarters make a dollar" (GIP, 4/26).

In Lesson 12, a visual representation of a dartboard with darts was provided with the task (viz., the dart game) (see Appendix J). Although the participants quickly determined what the problem was asking, the group skipped the Talk phase and began solving the problem. Though prompted to follow the THINK interaction framework, Darla was the only group member to share out her strategy as Nick thought a strategy to be transparent since the information was provided in a visual representation. When asked about her proposed strategy, Darla replied, “I added up all the dart throws to get the total” (TS, 4/28). Darla explicitly described her strategy in detail during the Notice component of the framework when she justified her reasoning for using her proposed strategy. In summary, the How component of the framework provided Darla with an avenue for proposing and explaining her strategy during group activities.

Identify. During the Identify component of the framework, students were to identify a strategy that they would use to solve the problem. As a whole, the group only engaged in exploratory talk during the Identify component in Lessons 9 and 13, as they mostly failed to identify a strategy for the group to use but worked on individual strategies. Although Darla spoke a total of 13 utterances during the Identify component (TS, 4/29), 12 of those were spoken during Lesson 9 where the group was tasked to find four congruent figures within an irregular polygon (see Appendix J). After identifying the problem and understanding the terminology of congruence (GOP, 4/29; RJ, 4/29; TS, 4/29), Darla named a strategy for solving the problem. Though Gary remained confused and failed to name a strategy at that time, the pair agreed on Darla’s strategy of drawing lines on the sketch in an attempt to find four congruent figures (RJ, 4/29; TS, 4/29). The Identify component was not utilized to the intended capacity to generate much exploratory talk. As such, it proved to be an ineffective component of the THINK interaction framework for this study.

Notice. The Notice component of the framework proved to be the most effective component in promoting Darla’s exploratory conversations. During the Notice component, Darla often re-stated, explained or justified her proposed strategy and the process that she used to solve the problem (RJ, 4/18, 4/28, 4/29; TS, 4/18, 4/28, 4/29). She sometimes shared whether her strategy led her to a solution, re-stated the necessary information for solving or discussed any problems that she had in understanding what the problem was asking (RJ, 4/28, 4/29; TS, 4/28, 4/29).

In Lesson 6, the group worked to solve "the block problem" (see Appendix J). Although the group members proposed individual strategies during the How component, they failed to collectively identify a strategy and struggled to solve the problem. During the Notice phase, members of the group discussed where they were in the process of solving the problem and offered parts of a solution. Darla engaged in this exploratory conversation by justifying her reasoning for constructing a stack containing one blue block, one red block, and either a purple or yellow block. Darla stated, "Well, if you are using blue and red, well, without having doubled the blue or the red then you can only have one [stack]" (TS, 4/18). Although one can interpret Darla's reasoning for the construction of a stack of block containing three distinct colors, she was incorrect in stating the number of three-block stacks that contained different colored blocks.

In Lesson 12, the group skipped the How component of the framework and began solving the problem (i.e., the dart game) individually. During the Notice component, Darla explained her strategy for solving the problem and the procedures involved in the process. Darla stated, "I divided the shots up and gave Bob the fifty [points] because I knew if she [Pam] had got the fifty that she would go over the 76 [the total score for each player]" (TS, 4/28). Her work sample demonstrated her strategy for solving the problem (see Figure 8).

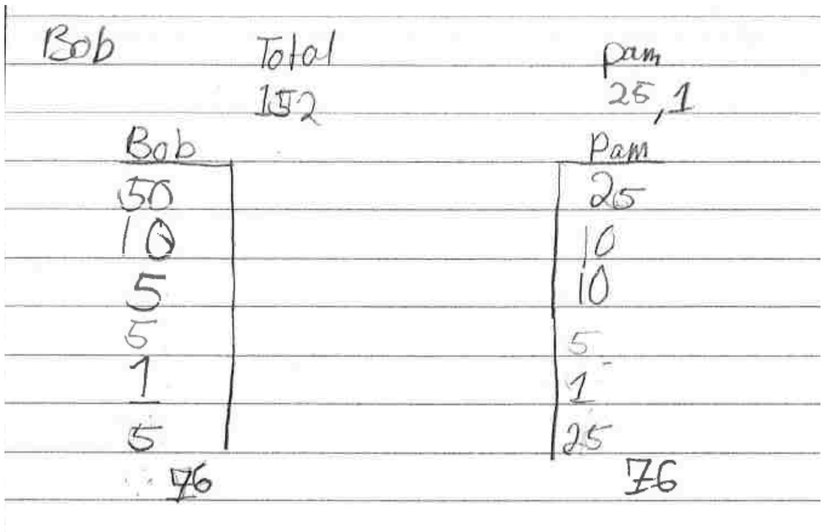

Figure 8. Darla's work from Lesson 12.

As a reminder, in Lesson 13 the group worked to solve the "target shooting" problem (see Appendix J). During the Notice component, Darla not only voiced her proposed strategy, but she also demonstrated her understanding of what the task was asking (TS, 4/29). When asked what her strategy was, Darla stated, "I did a list" (TS, 4/29). Further, Darla reinforced the information provided in the problem. The following excerpt demonstrated Darla's efforts to help the group understand what the problem was asking.

1 Nick: Oh! I'm still confused. I'm stuck.

2 Gary: Wait. What?

3 Darla: For each hit he got ten cents back. Each miss he had to pay five.

4 Nick: So, he ended up in debt? Is that what this is saying?

5-6 Gary: Oh! Okay. Okay. So, we need to figure up how many times he shot --.

7 Darla: How many times he's hit. (TS, 4/29)

In Line 3, Darla restated the necessary information given in the problem to help the group focus on what the problem was asking them to do. She reinforced the fact that John received ten cents for each hit and had to pay five cents for each miss. In Line 7, Darla quickly corrected Gary's statement (i.e., Lines 5 and 6) about the need to calculate how many times the group shot. Information was provided in the wording of the problem that informed the group that John shot 30 times. The main point of the problem was asking students to find how many hits that John made. Throughout the study, Darla remained cognizant about the information needed to solve the problem (e.g., Lessons 2, 7, 9, 11, 12, and 13) (TS, 4/8, 4/20, 4/22, 4/26, 4/28, 4/29).

Keep. The purpose of the Keep component of the framework was to provide an avenue for students to determine if their answer made sense and to find other solution paths for the task. The group only engaged in exploratory talk during the Keep component in Lesson 7 (i.e., the corrected score problem) (TS, 4/20). The effort was minimum with only a total of eight utterances (e.g., Darla spoke two utterances) spoken during this component. Darla spoke two utterances to mainly restate the strategy that she proposed to solve the problem (TS, 4/20). Therefore, the Keep component was also ineffective in promoting exploratory talk whereas students determined if their solutions made sense or if they needed to find additional strategies for solving the task.

Summary. After the implementation of the THINK interaction framework, Darla became somewhat more vocal with her increased participation during group discussions (GOP, 4/18, 4/22, 4/26, 4/29; RJ, 4/18, 4/26, 4/28, 4/29; TS, 4/18, 4/20, 4/22, 4/26, 4/28, 4/29). The components of the framework most effective in supporting Darla's exploratory talk and movement toward mathematical proficiency were: the Talk (GOP, 4/22, 4/29;

RJ, 4/22, 4/29; TS, 4/18, 4/20, 4/22, 4/26, 4/29); How (GOP, 4/18, 4/26; RJ, 4/18, 4/26; TS, 4/18, 4/26, 4/28; SW, 4/18, 4/26, 4/28); and Notice (RJ, 4/18, 4/28, 4/29; TS, 4/18, 4/28, 4/29) components.

The implementation of the Talk component supported Darla's exploratory talk by focusing her utterances on the nature of the problem as she determined the meaning of the problem in context, identified important information needed to solve the problem, and understood the mathematical concepts of the problem. Darla's expressed attention to the details in the wording of problems and the realization of the mathematical concepts demonstrated her progressive movement in her conceptual understanding in the context of problem solving.

The How component of the framework supported Darla's exploratory talk as she was required to construct a strategy for solving problems and then to explain and justify her strategy for solving the problem. In following the directions set forth in the How component, Darla not only stated and explained her strategy for solving the problem, but she often provided visual representations, including lists and sketches, to emphasize how she proposed to use information provided in the tasks. Darla's ability to formulate a strategy and to use visual representations toward solving the problem was representative of her growth toward becoming strategically competent.

The Notice component of the framework was the most effective in supporting Darla's exploratory talk. During the Notice component, Darla often re-stated, explained or justified her proposed strategy and the process that she used to solve the problem. Further, she sometimes questioned and critiqued her fellow group members' responses during this component. Such activity and reflection assisted the group in furthering their thinking about the assigned task (e.g., the dart game). Hence, Darla's engagement in exploratory talk promoted intermental activity as the group engaged in focused mathematical discussions. Such activity resulted in a display of Darla's conceptual understanding, growth in strategic competence, and increased confidence to logically communicate her thoughts, thus revealing her adaptive reasoning skills.

Justifying reasoning: From explanations to critiques (Subtheme 2 - Darla). Darla exhibited passive behavior during Phase 1 of the study in the form of low and inaudible speech (GOP, 4/7, 4/14, 4/26; TS, 4/7, 4/14/, 4/26), in her lack of responses when addressed directly by the teacher or group mates (e.g., GOP, 4/8, 4/14; RJ, 4/8, 4/11, 4/14; TS, 4/11; 4/14), and through the use of physical gestures (GOP, 4/8; RJ 4/8). Having only displayed her reasoning abilities during Lesson 1 (TS, 4/7) prior to the introduction of the THINK interaction framework, Darla exhibited her reasoning skills in Phases 2 and 3 of the study. Beginning in Lesson 6, Darla became more vocal in explaining and justifying her answers. As the study progressed, Darla displayed her intuitive reasoning abilities through her explanations and justifications of her strategies and solutions.

In the following vignette, I present an example of Darla's increased vocalization of her explanation of her thoughts to the group. In Lesson 6 (i.e., the block problem), the participants worked through the problem, discussed possibilities of the combinations of blocks in stacks of three, but had difficulty in determining the number of combinations when the stack contained three blocks of different colors (TS, 4/20). Though Darla continued to speak in a low voice (GOP, 4/20; RJ, 420), she presented her reasoning concerning her understanding of the requirements of the problem. When describing one possible answer to the solution, Darla stated, “Let me look. Well, if you are only using blue and red, well, without having doubled blue and doubled red then you can only have one [stack]” (TS, 4/18). Her reasoning was only partially correct, as six combinations of a stack of three blocks containing the colors red, blue, and purple were possible.

In Lesson 7 (i.e., the corrected exam score problem), the group engaged in cumulative talk after Mrs. Smith provided support to further the group’s thinking. In this lesson, Darla displayed her intuitive reasoning ability as she comprehended that since the teacher increased each student’s score by one point that the corrected mean score for the class increased by only one point. Thus, since the previous mean score on the class exams was 71, the new mean for the corrected exam scores was 72. Darla also used the information in the problem that the sum of the correct exam scores totaled 936 to find the number of students in the class. Darla stated, “I divided 936 to see if it’s a whole number by 13” (TS, 4/20). In justifying her reasoning, Darla stated, “If you divide [936 by 13], the closest thing that you are going to get is 72 (i.e., the new mean of the corrected scores)” (TS, 4/20).

The discussion among group members was classified as cumulative talk since most participants failed to justify their reasoning for their actions. However, Darla explained her reasoning concerning finding the mean and clarified that she divided 936 by 13 to determine divisibility by 13. Darla justified her reasoning that her answer of 72 worked because it was the next whole number greater than 71 when finding the mean of the scores after the teacher made the correction. Though shy and lacking in confidence, Darla justified her reasoning during the Notice phase of Lesson 7 (GOP, 4/20; TS, 4/20).

In Lesson 12, Darla engaged in exploratory talk several times throughout the lesson (GOP, 4/20; TS, 4/28). The assigned task concerned two people playing a dart game (see Appendix J). The task required students to find the player who hit the target first. Though Nick insisted that the female hit the target first, Darla demonstrated her reasoning about the female player's score for the first two throws. She stated, "Her first two throws have to be a 25 and a one to get 26 [points]" (TS, 4/28). Additionally, Darla critiqued Nick's reasoning concerning the final score of the match. Because Nick failed to read the text of the problem carefully, he thought that one of the players won. Nick stated, "They had six throws apiece. So, they did not have the same type of score. One of them won" (TS, 4/28). To set the record straight, Darla replied, "Well, neither won, but she wouldn't have been able to hit the 50 on her first throw" (TS, 4/28). Though the above conversation occurred in the beginning of class, the group failed to engage in the Talk phase of the THINK framework (GOP, 4/28). However, Darla used adaptive reasoning during this discussion to explain to Nick and Gary how Pam got 26 points in her first two throws (i.e., with scores of 25 and one). Darla explained to Nick and Gary how her strategy helped her to solve the problem (GOP, 4/28). Darla explained why the female player did not hit the center of the target. Darla claimed, "I divided the shots up and gave Bob the 50 [points] because I knew if she had got the 50 [points], she would go over the 76 [points]" (TS, 4/28).

Summary. Although Darla exhibited passive behavior in the beginning of the study and failed to justify her answers to the group (e.g., Lesson 1), she became more vocal in explaining her strategies and justifying her answers with each lesson in the study. Additionally, Darla's ability to express her reasoning concerning mathematical problems and concepts grew stronger as the study progressed. For example, Darla weakly explained her thoughts in the block problem in Lesson 6. However, Darla exhibited her intuitive reasoning abilities in Lesson 7 when she explained her reasoning as to why the corrected mean score for the class was 72. In Lesson 12, Darla was more vocal than in previous lessons in justifying her solution and explaining to her group. In Lesson 12, Darla spoke up, and sometimes interrupted other members of the group, to explain why the female player did not hit the center of the target. It is challenging to explain why someone's reasoning ability noticeably increased, but such an increase could be due to Darla's familiarity with the members of her group and the problem-solving process and the opportunities created by the THINK interaction framework for students to find a voice. In summary, the opportunities presented to Darla through the implementation of the THINK interaction framework supported her engagement in exploratory talk as exhibited in her construction and proposal of strategies, in display of her reasoning abilities and her critique of the reasoning of her group mates in providing clarification of a problem or solution path. As such, the THINK interaction framework supported Darla's progressive movement in strategic competence and in her increased adaptive reasoning abilities.

Making connections among mathematical concepts, procedures, and representations (Subtheme 3 – Darla). Although Darla demonstrated some connections among mathematical relationships in the beginning of the study, she exhibited progress in making connections among mathematical relationships beginning in Phase 2 of the study (SW, 4/18, 4/20, 4/26; TS, 4/18, 4/20, 4/26, 4/28, 4/29). Such understanding included the relationship between: mathematical concepts and graphical representations (e.g., Lesson

6); the relationship between mathematical concepts and processes (e.g., Lesson 7); the relationship among the context of the problem, mathematical concepts, and mathematical procedures (e.g., Lessons 7 and 12); and the relationship among the context of the problem, mathematical concepts, graphical representations, and mathematical procedures (e.g., Lesson 12).

An example of Darla making connections between mathematical concepts and graphical representations occurred in Lesson 6 when students were assigned “the block problem.” Darla understood that the problem was asking students to find a distinct combination of three blocks and that the arrangement of blocks affected the final count in the answer. Darla expressed her thoughts during the Talk component of the framework. Darla stated, “It says *different* stacks of three blocks that the toddler can make” (TS, 4/18). In her statement, Darla stressed the word different. She again asserted that the stacks were to be distinct in stating her proposed strategy during the How component. When Nick asked Darla what she had done to solve the problem, she replied, “I arranged them into different colors in order” (TS, 4/18). Darla’s work sample (see Figure 6) displayed her understanding of the concept of distinct combinations as demonstrated by the representation of the stacks of sketched blocks arranged in different combinations (SW, 4/18). In other words, Darla’s understanding was exhibited in the connections that she made between the mathematical concept of distinct combinations and the visual representation of the problem.

In Lesson 7, Darla demonstrated her ability to make connections among the context of the problem (i.e., the corrected exam score problem) and mathematical procedures and concepts. Darla understood the concept of mean and the procedure for finding the mean. When Mrs. Smith asked Darla to explain the mathematical term, mean, and the process for finding the mean score, Darla replied, “You add all the numbers together and divide by how many numbers there are” (TS, 4/20). Further, Darla exhibited her understanding of the context of the problem and what it meant to add one point to each student’s score in terms of finding the number of students in the classroom. To clarify, Darla understood that since the original mean was 71 the mean after corrections were made to students’ scores would be a whole number larger than 71 and found by dividing 936 by whole numbers that represented the students in the class. The next whole number larger than 71 was 72, which could be found by dividing 936 by 13 (see Figure 4) (SW, 4/20). In explaining and justifying her solution during the Keep component, Darla stated, “I divided 936 by 13 is 72, which is one more than 71” (TS, 4/20).

As a reminder, in Lesson 12 students were assigned “the dart game” task (see Appendix J). Darla exhibited her understanding of the relationships among the context of the problem, mathematical concepts, graphical representations, and mathematical procedures. For example, Darla demonstrated her understanding of the context of the problem and the mathematical procedures of addition and division when she explained to Gary and Nick that the female scored one point and 25 points on her first two throws to get a score of 26 points as stated in the context of the problem. Further, Darla stated that she had added up the total points and divided by two to get the final score of 76 points for each player since the two players earned the same number of total points. Darla stated, “They both scored 76 points” (SW, 4/28; TS, 4/28). Darla justified her reasoning as to why the female player could not have hit the center of the target for 50 points. Darla argued, “I divided the shots up and gave Bob the fifty (i.e., score for the center of the target) because I knew if she had got the fifty [points], she would go over the 76 [points]" (SW, 4/28; TS, 4/28). Darla's work sample (see Figure 4) demonstrated her understanding of the relationship among the context of the problem, the graphic representation of the scores for each player, mathematical processes used to solve the problem, and the mathematical concepts of the problem.

Summary. Although Darla presented some understanding of mathematical relationships in Lesson 1 (SW, 4/7; TS, 4/7), she exhibited a stronger understanding of the relationships among mathematical concepts, graphical representations, mathematical procedures and the context of the assigned problems in Phases 2 and 3 of the study (SW, 4/18, 4/20, 4/26; TS, 4/18, 4/20, 4/26, 4/28, 4/29). In Lesson 6, Darla exhibited integrated understanding of the relationship among mathematical concepts and graphical representation when she drew sketches to represent the possible combinations of blocks (SW, 4/18; TS, 4/18). In Lesson 7, Darla presented her understanding of the relationship among mathematical concepts, procedures, and the context of the problem when she explained and justified her reasoning as to why the mean of the corrected scores on the exam was 72 (SW, 4/18; TS, 4/18). Finally, in Lesson 12 Darla displayed her understanding of the relationships among the context of the problem, mathematical concepts, graphical representations, and mathematical procedures when she used the drawing of the dartboard to determine each player's total score through the mathematical procedures of addition and subtraction; the information provided in the problem concerning Pam's first two throws; and concept of the property of equality in determining which player scored what points. In summary, the THINK interaction framework supported Darla's engagement in exploratory talk as represented by her display of progress in her understanding of the relationships among certain mathematical concepts, strategies, visual representations, and procedures.

Darla's overall summary. Darla was a shy girl who admitted her lack of confidence (PSI, 4/6) and who exhibited low self-efficacy as displayed in her lack of utterances, low voice, and physical gestures indicating that she did not wish to participate (GOP, 4/7, 4/8, 4/14; RJ, 4/7, 4/8, 4/14; TS, 4/7, 4/8, 4/14). Darla's display of increased confidence and self-efficacy was most noticeable beginning in Phase 2. The THINK interaction framework supported Darla's engagement in exploratory talk, as it required her to focus her utterances and thoughts on the nature of the presented tasks. Darla understood the information needed to solve the problem and realized the mathematical concepts embedded in the objective of the tasks. Further, as Darla formulated strategies and explained them to the members of the group per the directions of the framework, she often presented physical representations that demonstrated her understanding of the connections among the mathematical concepts and procedures. Her ability to formulate representations as they related to the context of the problem demonstrated her continued growth toward obtaining strategic competence. Last, the requirements of the THINK interaction framework to share one's understanding, strategies, explanations, and reasoning revealed Darla's progression in increasing her adaptive reasoning skills. However, Darla's statements in the post-study interview (PTI, 5/8) revealed her demonstrated lack of productive disposition, possibly a result of her low self-efficacy. Additionally, Darla's use of a calculator concealed her ability to perform calculations and procedures fluently.

Though Darla showed progressive movement in her conceptual understanding, strategic competence, and adaptive reasoning abilities, she did not consistently demonstrate the "knowledge, skills, abilities, and beliefs that constitute mathematical proficiency" (NRC, 2001).

Meet Gary

At the time of the study, Gary was a ninth-grade Caucasian male who was enrolled in Algebra I and a Tier II mathematics class. When asked how he best learned mathematics, Gary stated that he learned best by studying mathematics and receiving help from others, including his teacher and peers (PSI, 4/6). The academic support that Gary received from his teacher was provided during class instruction whenever the teacher passed by his desk or if he prompted her for assistance. Gary did not elaborate whether his teacher's assistance was in the form of scaffolding, using questions, or procedural instructions but only that he received individualized help. Further, Gary's response indicated that he had ample opportunities on different occasions to receive academic support from his teacher. When asked what his teacher did to help him best learn mathematics, Gary stated, "She'll come by, or I'll ask, and she'll come help me and see if I need any help" (PSI, 4/6; TS, 4/9, 4/11, 4/14).

In the interview, Gary's response concerning working in groups indicated that he did not prefer that academic arrangement. When asked what were his thoughts about working in groups, he replied, "It's all right, sometimes" (PSI, 4/6). In discussing the structure, roles, and responsibilities of group mates, Gary responded that students have self-assigned jobs based on individual preferences. He stated, "Normally, people will have jobs, and they'll assign them to what people want. Like if somebody wanted to write it down and if somebody actually wanted to be doing the math, we would let them" (PSI, 4/6). However, his responses concerning individual accountability of all group members were contradictory. When asked if each member in the group worked on the task, he responded that they did. Yet, his explanation of how members of the group work to solve the assigned problems or tasks painted a different picture. Gary acknowledged, "If somebody thinks they know it, they work on it. If they need help, we'll see if we can help them in any way. One person will kind of do it" (PSI, 4/6). His answers supported his responses to the advantages and disadvantages of working in groups. Gary noted that the advantage to working in groups was the opportunity for peer tutoring while the disadvantage of group work was the possibility of someone "not doing their part" (PSI, 4/6). Nevertheless, Gary expressed that group work occurred infrequently and was implemented at most three times a month.

Gary discussed the opportunities to receive or provide help to his peers. In addition to any help received during group work, Gary reported that he could get help from his friends in class but often went to a family member for help. "I have a cousin that will help me if I want it" (PSI, 4/6). He explained, "A friend can normally explain it better than a teacher can" (PSI, 4/6). He reasoned that he is around friends more than his teacher and that they can explain mathematical topics in a way that he understands the content. In regards to the opportunities to help a peer, Gary replied, "I really don't. If someone asks me, I'll do my best to help them" (PSI, 4/6).

Although Gary stated that he thought mathematics was needed in everyday life, he reported that some of the topics that he studied were not necessary to be successful in life. Gary stated, "I mean you need it in everyday life, but some of the stuff they teach is not really necessary, but you know" (PSI, 4/6). Gary's attitude was reflected in his feelings about the study of mathematics. "I hate it. I don't like math one bit. I don't like it" (PSI, 4/6).

The following section describes the overarching theme, "From Passivity to Finding a Voice: Using Opportunities Provided by the THINK Interaction Framework to Engage in Conversations and Acquire Skills Toward Becoming Mathematically Proficient," that resulted from the analyses concerning Gary's participation throughout the study.

From passivity to finding a voice: Using opportunities provided by the THINK interaction framework to engage in conversations and acquire skills toward becoming mathematically proficient (Overarching theme – Gary). Throughout the study, Gary displayed various levels of engagement in conversations with the other participants of the study as evidenced in lesson observations (GOP, 4/7, 4/8) and audio transcriptions (TS, 4/7, 4/8, 4/18, 4/20, 4/28). An analysis of the researcher's journal, group observation notes, and lesson transcriptions revealed the overarching theme, "From Passivity to Finding a Voice: Using Opportunities Provided by the THINK Interaction Framework to Engage in Conversations and Acquire Skills Toward Becoming Mathematically Proficient." To provide clarity and understanding of the overarching theme, I present an in-depth account of three emergent subthemes that describe Gary's levels of participation in the group conversations, discuss the types of engagement that he displayed throughout the study, and illustrate with excerpts of dialogue, vignettes, and student work samples that support the findings. Additionally, when applicable, I note how components of the THINK interaction framework supported Gary's engagement in exploratory conversations, if at all.

Finding space in conversations (Subtheme 1 – Gary). Though Gary exhibited passive behavior throughout the study (GOP, 4/7, 4/8, 4/14, 4/18; TS, 4/7, 4/8, 4, 14, 4/18, 4/20, 4/28), he displayed differing levels of engagement during group conversations, noticeable in each phase of the study (see Table 5). In Phase 1, Gary spoke few utterances during each of the lessons and only one utterance during exploratory conversations. Further, Gary became more passive and failed to participate or productively re-engage in the conversation after members of the group interrupted him. In Phase 2, after the introduction of the THINK interaction framework, Gary became somewhat more vocal, speaking on average 35 utterances (i.e., 20.6%) per lesson as compared to 23 utterances (i.e., 10.7%) per lesson in Phase 1 (see Table 5). The THINK interaction framework provided Gary with opportunities to engage in the group's conversation and to present his ideas. Subsequently, Gary continued to participate and contribute meaningful thoughts to the group's conversation during Phase 3. To help the reader better understand the emergence of the first subtheme, "Finding Space in Conversations," I provide a chronological account of Gary's noticeable levels of engagement in each of the phases of the study and note how the components of the THINK interaction framework supported him, if at all, in finding his voice during exploratory conversations.

Phase 1. In Phase 1 of the study, Gary's passive behavior was displayed in his lack of utterances (see Tables 5 and 16) and lack of assertiveness to join group conversations when interrupted by others or when the conversation was dominated by one or two members of the group (GOP, 4/7, 4/8; RJ, 4/7, 4/8; TS, 4/7, 4/8, 4/14, 4/18). The frequency of Gary's utterances throughout the study is presented in Table 16.

Table 16

Frequency of Gary's Utterances Throughout the Study

Lesson	Number of Utterances	Lesson	Number of Utterances
1	20 (6.6)	8	35 (27.1)
2	3 (3.4)	9	51 (35.4)
3	48 (46.2)	10	43 (24.9)
4	21 (7.2)	11	33 (21.6)
5	-	12	16 (15.4)
6	23 (9.7)	13	57 (22.6)
7	21 (13.3)	14	39 (14.4)

Note. The percentage of utterances spoken by Gary per lesson as compared to the total utterances spoken in the lesson is presented in parentheses. The dash indicates an absence.

During Phase 1, Gary spoke a total of 92 utterances (see Table 4) and averaged 23 turns (i.e., 10.7%) per lesson (see Table 5). His lack of participation was especially evident in Lesson 2 (i.e., the golf ball problem), when he spoke only three times throughout the lesson (i.e., 3.4%) as compared to Darla (i.e., 18.0%), Tonya (i.e., 38.2%), and Nick (i.e., 40.4%) (see Table 4). Further, his utterances did not productively help the group advance their thinking. For example, his only utterances throughout Lesson 2 were, "I am really confused," "What are you thinking?" and "I don't know how you did that" (TS, 4/8).

Although Gary spoke the most frequently during Lesson 3 compared to Lessons 1, 2, and 4 (TS, 4/7, 4/8, 4, 14), Darla and he were the only participants present for that lesson (see Table 4). Additionally, Gary's contributions toward solving the problem were similar to his utterances in Lesson 2. In Lesson 3, the participants were assigned the task of finding how many tablespoons equal two-thirds of a cup (i.e., the measurement task) (see Appendix J). The following excerpt illustrated the types of utterances contributed by Gary in Lesson 3 during a conversation among Mrs. Smith, Darla, and himself.

1 Gary: Do you have anything?

2 Darla: [She shakes her head no.]

3 Mrs. Smith: Now, where are we?

4 Gary: Still lost.

5 Mrs. Smith: Still thinking? What does she [Darla] have over there?

6 Darla: [Inaudible]. I think that might be one tablespoon.

7 Gary: [Inaudible].

8-13 Mrs. Smith: So, if 4 is ¼ is a quarter of a cup. You have half a what? A half a cup? Is that what you mean? A half a cup is 1 tablespoon? Is that what you are thinking? I think you are thinking in the right direction as far as equating. What do you have to something else? Your conversion is not correct, but you are thinking in the right direction. See if you can fix [it]. From a fourth to a half is not the right direction.

14 Darla: I'm thinking one tablespoon equals something of a cup.

15 Mrs. Smith: What are you thinking?

16 Darla: [She pauses and then answers.] I'm lost.

17-19 Mrs. Smith: Still lost? Ok. Go back to what you got. You got 4 tablespoons equal ¼ of a cup. So, you have this much, and you really need that much?

20 Darla: So, you have to see how many tablespoons equal 2/3 cup?

21-25 Mrs. Smith: Un, huh. Yeah. You have it written out here. You got it. This many tablespoons equal that. [She points to Darla's paper.] So, uh, actually, you have it written backwards here. It would be this question mark equals 2/3 tablespoons of a cup. So, you are looking for how many tablespoons instead of how many cups. Does that make sense?

26 Darla: I really want to say six. Six tablespoons.

27 Gary: Why's that?

28-29 Darla: Because four times one is still one. I mean it's four, and then three times two equals six.

30 Mrs. Smith: Ok. Let me ask you this question? What does that mean?

31 Gary: What does that mean?

32 Mrs. Smith: That fraction.

33 Gary: It's a part of something. (TS, 4/11)

In the above excerpt, Gary spoke six utterances. Although Mrs. Smith attempted to support Gary and Darla through questioning, leading, tips, and partial answers, Gary did not attempt to engage in the conversation with meaningful thoughts. In Lines 1, 4, and 31, Gary's lack of enthusiasm and a lack of a productive disposition were displayed in his responses. In Line 7, Gary spoke in a low and inaudible voice, undetectable by the video and audio recorders (TS, 4/11).

In Lesson 4, Gary attempted to engage more frequently in the conversations among group members, contributing 21 utterances (i.e., 7.2% of the conversation) (TS, 4/14). Although, several of Gary's utterances were similar to those he spoke in Lessons 1, 2, and 3 (TS, 4/7, 4/8, 4/11), he attempted to contribute meaningful thoughts to the conversation (RJ, 4/14; TS, 4/14). However, he found difficulty finding space to speak in the conversation as he was frequently interrupted by Tonya and lacked assertiveness to be heard (TS, 4/14).

Phase 2. The average number of utterances per lesson spoken by Gary in Phase 2 (i.e., 35 or 20.6%) increased compared to the average number of utterances that he spoke in Phase 1 (i.e., 23 or 10.7%) (see Table 5). Further, his engagement in conversations coded as exploratory talk increased from one utterance (i.e., 0.0%) in Phase 1 to 51 utterances (i.e., 29.5%) in Phase 2 (see Table 18). In Lessons 6, 7, and 8, nearly half of Gary's utterances were coded as exploratory talk (see Table 17). The increase in the frequency of Gary's utterances in all conversations and especially, during exploratory conversations in Phases 2 and 3 indicated that he asserted his voice to gain space in the group's conversations. One possible explanation for the increase in Gary's engagement in the group's conversations was the introduction and implementation of the THINK interaction framework, beginning in Lesson 6. In Table 19, Gary's utterances that occurred during exploratory conversations are presented per each component of the THINK interaction framework.

Table 17

Frequency of Gary's Utterances During Exploratory Conversations

Lesson	Number of Utterances	Lesson	Number of Utterances
1	0 (0.0)	8	16 (45.7)
2	0 (0.0)	9	14 (27.5)
3	1 (0.0)	10	2 (4.7)
4	0 (0.0)	11	2 (6.1)
5	0 (0.0)	12	13 (81.3)
6	10 (43.5)	13	13 (22.8)
7	9 (42.9)	14	0 (0.0)

Note. The percentage of Gary's utterances spoken during exploratory conversations as compared to the total number of utterances spoken by him during that lesson is in parentheses.

Table 18

Frequency of Gary's Utterances During Exploratory Conversations Per Phase

Phase	Number of Utterances
1	1 (0.0)
2	51 (29.5)
3	28 (26.4)

Note. The percentage of utterances spoken by Gary during exploratory conversations by phase is in parentheses.

Table 19

Frequency of Gary's Utterances per Each Component of the THINK Interaction Framework

Lesson	Talk	How	Identify	Notice	Keep
6	4	1	0	5	0
7	3	4	0	0	2
8	5	5	0	6	0
9	0	0	14	0	0
10	2	0	0	0	0
11	0	2	0	0	0
12	8	3	0	2	0
13	3	2	1	7	0
Total	25	17	15	20	2

Note. The group did not attend to the framework during Lesson 14.

In Lesson 6, Gary explained his thoughts concerning the meaning of the problem. Although he only spoke 23 turns in the lesson, Gary spoke 10 utterances (i.e., 43.5%) during conversations coded as exploratory talk (see Table 17). As the group worked on the "block problem," Gary confirmed that the problem was asking them to find how many distinct stacks of three blocks the toddler could make and interjected that the order of the blocks was based on color and needed consideration when determining the number of distinct stacks of three blocks (TS, 4/18). He confidently stated that the blocks could be changed around based on color to make different stacks of three. Gary stated, "You can change the colors to make more than a different stack" (TS, 4/18). Unlike Gary's lack of assertiveness in Lessons 1, 2, and 7, Gary continued to participate when interrupted by a group mate.

In Lesson 8, Gary not only continued to voice his thoughts but he also began to critique the thoughts of other members of the group. For example, during the Talk component of the conversation, Nick interrupted Gary who was attempting to explain the information given in the problem. Unlike previous lessons when Gary became quiet and failed to continue to participate in the conversation (i.e., Lesson's 1, 2, and 7) (GOP, 4/7, 4/8, 4/21; RJ, 4/7, 4/8, 4/21; TS, 4/7, 4/8, 4/21), Gary questioned Nick's solution to the problem by asking him to explain how he got that answer.

The Talk component of the THINK interaction framework provided Gary with opportunities to share his thoughts and suggestions with the members of his group when determining what the problem was asking and any necessary information that could aid in solving the problem. Additionally, it provided Gary with entry points into the group's conversation and allowed him to find space to share his thoughts, as reflected in the number of utterances in lessons in Phases 2 and 3. In summary, the questions in the Talk component that prompted students to determine the important information needed to solve the problem possibly directed Gary to focus deeper on the wording of the problem and to further analyze the text of the problem.

Phase 3. In Phase 3, Gary continued to speak more frequently (i.e., 18.6%) than he did in Phase 1 (i.e., 10.7%) but somewhat less frequently than in Phase 2 (i.e., 20.6%) (see Table 5). The pattern of the frequency of utterances that Gary spoke during exploratory conversations was similar to that described in Phase 2. Gary spoke 0.0% during exploratory conversations in Phase 1, 29.5% during Phase 2, and 26.4% during Phase 3. The data presented in Table 17 revealed that Gary spoke only 2 utterances coded as exploratory talk in Lessons 10 (i.e., 4.7%) and 11 (6.1%). One possible explanation for the decrease in the frequency of utterances during Lesson 10 was that only Gary and Nick were present for the lesson, in which Gary spoke 25.3% of the time, yielding a lopsided conversation. It is unclear as to why the group had difficulty in solving the task in Lesson 11 (i.e., the coin problem) as it presented a real-world context.

In Lesson 12 (i.e., the dart game) Gary contributed 15.4% to the conversation, speaking a total of 16 utterances (see Table 4) (TS, 4/28). Of the 16 utterances that he spoke, 13 were coded as exploratory talk, with 8 utterances spoken during the Talk component of the framework (see Tables 4, 17, and 19). During the conversation, Gary assumed the role of administrator, facilitated the conversation by following the THINK interaction framework, and solicited information from both Darla and Nick. Because the group did not dissect the problem during the Talk component by discerning what necessary information was given, Gary was confused as to what the problem was asking, was interrupted by Nick, and did not contribute much after the Talk component of the conversation. Though he asked for clarification from Darla and Nick, they ignored his request as they plunged into working to solve the problem. Gary's lack of participation in the lesson was possibly due to his lack of understanding the problem and reinforced his statement in the pre-study interview concerning his seeking assistance from a family member to get help as compared to asking his classmates (PSI, 4/6).

In Lesson 13, Gary took advantage of the opportunities provided by the THINK interaction framework to engage in the group's conversation, speaking 57 utterances (i.e., 22.6%), with 13 utterances (i.e., 22.8%) occurring during exploratory conversations. During the Notice component, the group revisited the problem (i.e., the target shooting problem) to better understand what was required to solve the problem. Gary stated what information he thought was needed to solve the problem. Though Darla sometimes interrupted him prior to him completing his statements, Gary became assertive, took back his space in the conversation, and interjected his thoughts (TS, 4/29).

Summary. Gary exhibited various levels of engagement in the group conversations throughout the study (GOP, 4/7, 4/8; RJ, 4/7, 4/8; TS, 4/7, 4/8, 4/14, 4/18). During Phase 1 of the study, Gary contributed only 10.7% of the conversation (see Table 5). Of the 10.7% of the utterances that he spoke during Phase 1 (see Table 5), Gary only spoke one utterance during conversations coded as exploratory talk (see Table 17). Besides exhibiting passive behavior, Gary was often interrupted and lost his space in conversations (GOP, 4/7, 4/8, 4/14; RJ, 4/7, 4/8; TS, 4/7, 4/8, 4/14, 4/28). After the introduction of the THINK interaction framework, Gary spoke more utterances (i.e., 20.6%) during Phase 2 than in Phase 1 (i.e., 10.7%) (see Table 5). Also, the frequency of

Gary's utterances coded during exploratory conversations increased from 0% to 29.5% during Phase 2 (see Table 18). In Phase 3, Gary spoke more utterances (i.e., 18.6%) than he did in Phase 1 (10.7%) (Table 5). However, in Lesson 11 and 12, he exhibited passive and less assertive behavior than in Lessons 6, 7, and 8 (RJ, 4/18, 4/20, 4/21; TS, 4/18, 4/20, 4/21). The THINK interaction framework provided Gary opportunities to find entry points into the conversation and to express his thoughts about the problem (TS, 4/21, 4/29). The directions in the Talk component possibly helped Gary to focus on the wording of the problem as it prompted students to find the important information needed to solve the problem. The directions in the How component required Gary to construct the strategy that he proposed would solve the problem. In both components, the directions prompted students to share their thoughts with the group. Gary took advantage of the opportunities to share his thoughts and found space to actively engage in conversations.

Voicing strategies and understanding of tasks (Subtheme 2 – Gary). Throughout the study, especially in Phase 1, Gary displayed passive behavior in the form of a lack of utterances during group work and discussions (GOP, 4/7, 4/20; TS, 4/7, 4/8, 4/20). Analyses of the transcriptions from the video recorded lessons revealed that Gary's passive behavior and lack of engagement often resulted after he was interrupted or when other group members dominated the conversation (RJ, 4/7, 4/8, 4/20; TS, 4/7, 4/8, 4/14, 4/20). Further, his work samples in Phase 1 illustrated little or no understanding of the task (see Figures 12 and 13) (SW, 4/7, 4/11, 4/14). Gary's work sample from Lesson 1 demonstrated that he did not understand the task or have a strategy for solving the task. Additionally, it looked similar to Darla's work sample (see Figure 5) as if he had copied the work from her. The work sample from Lesson 3 confirmed that he drew the sketch as directed by Mrs. Smith but did not understand how to use it toward solving the problem (see Figure 10).

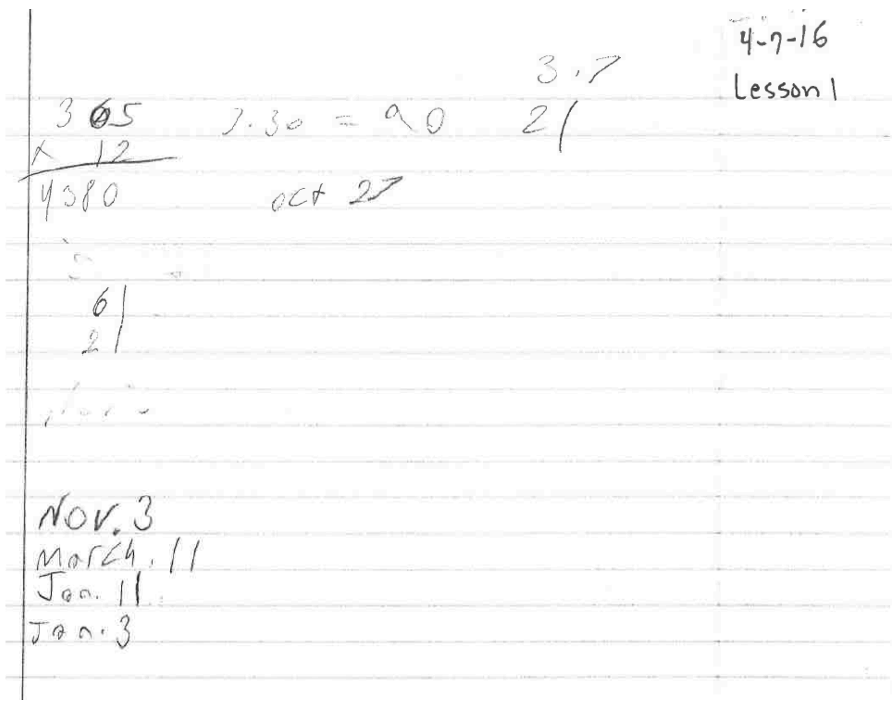

Figure 9. Gary’s work from Lesson 1.

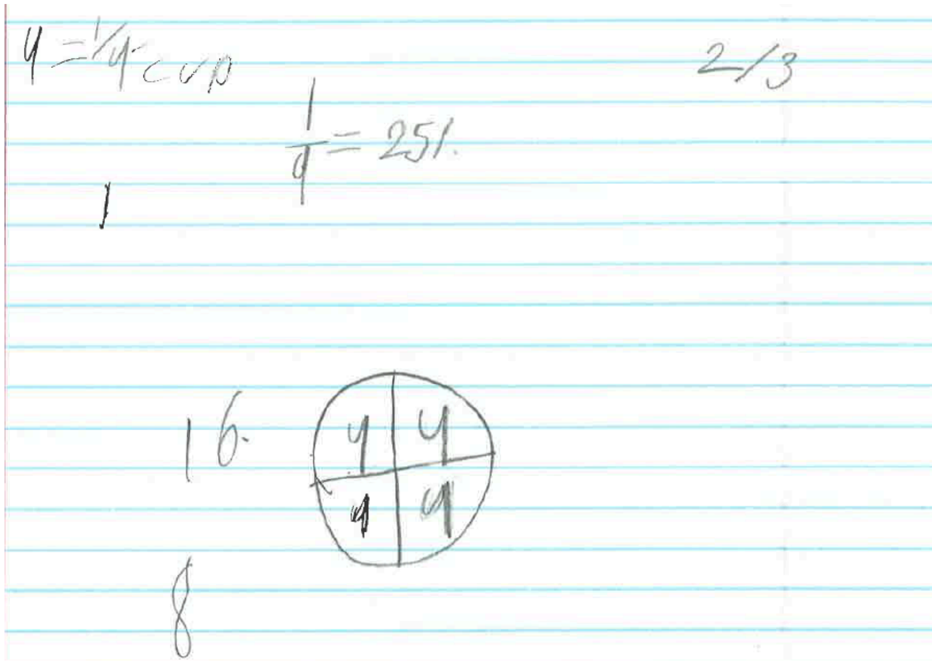

Figure 10. Gary's work from Lesson 3.

A deeper examination of the lesson transcriptions, the researcher journal, and Gary's work samples disclosed evidence of Gary's understanding of how to use strategies to solve some tasks after the introduction and implementation of the THINK interaction framework (RJ, 4/18; TS, 4/18; SW, 4/18, 4/21). In Lesson 6, the group was assigned the "block problem" (see Appendix J). Gary recognized that the solution required distinct stacks of three blocks, based on the order of the colors of the blocks. He stated, "You can change the colors to make more than [one] different stack" (TS, 4/18).

His strategy for solving the task in Lesson 6 (i.e., the toddler block problem) included making lists of possible combinations of the colored blocks (see Figure 11) (SW, 4/18). He used the letter *b* to represent the blue blocks, the letter *r* to represent the red blocks, the letter *p* to represent the purple block, and the letter *y* to represent the yellow block. Although his solution was incomplete, his work sample provided evidence of his understanding of a strategy that could be used to solve the problem (see Figure 11).

In Lesson 8, (i.e., the school bus problem), neither Gary nor Nick initially understood what the problem was asking. After Mrs. Smith suggested drawing a sketch to represent the problem, Gary used a sketch to represent the scenario and a list to represent the amount of accumulated time that it took to vacate the bus (see Figures 15 and 16). When asked which strategy helped him to solve the problem, Gary responded, “The second one helped because I could actually visualize the kids getting off the bus” (GIP, 4/21).

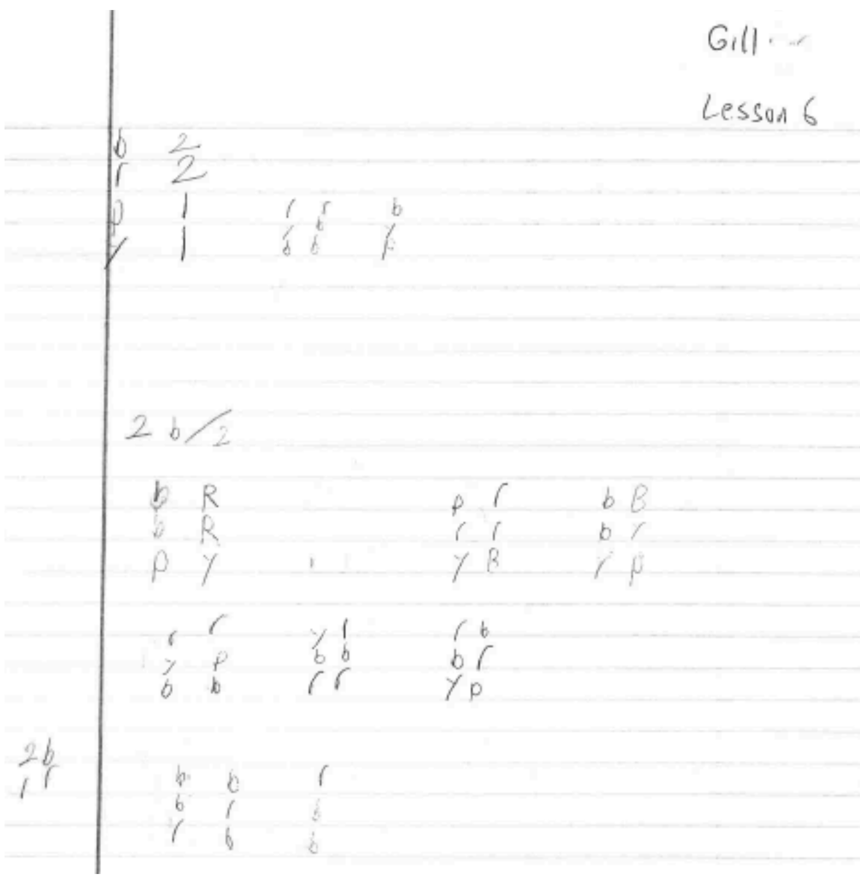

Figure 11. Gary’s work from Lesson 6.

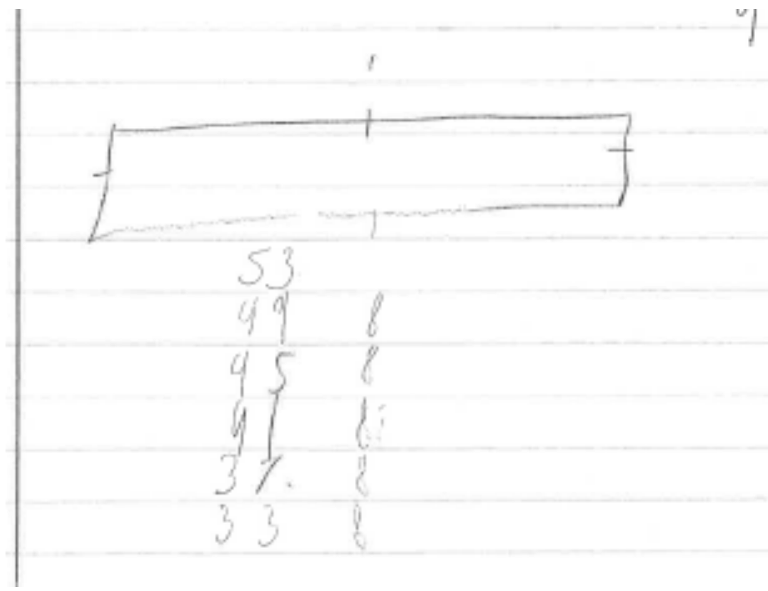

Figure 12. Gary’s work from Lesson 8, page 1.

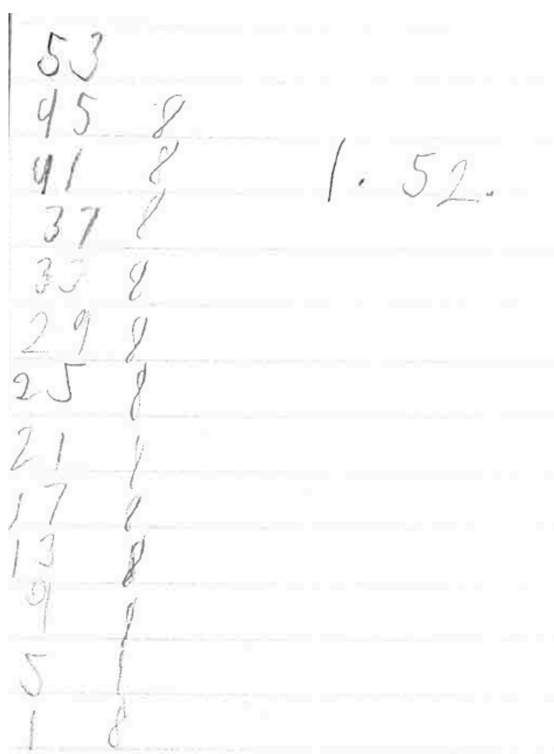

Figure 13. Gary’s work from Lesson 8, page 2.

Though he was initially confused when addressing the assigned task in Lesson 9 (i.e., the congruent figure task) (see Appendix J), Gary understood that the shape of the sketch provided in the problem could not change and that he must use the irregular polygon provided in the problem. Gary stated, “We’re trying to get four congruent figures, and I don’t think it matters how many times you do it. You have to do it on these same lines. You have to have four of the same [figures]” (TS, 4/22). Gary’s work sample demonstrated his strategy and understanding of the task (see Figure 14) (SW, 4/22). Upon finding the solution to the task, Gary explained his reasoning to Darla and Nick.

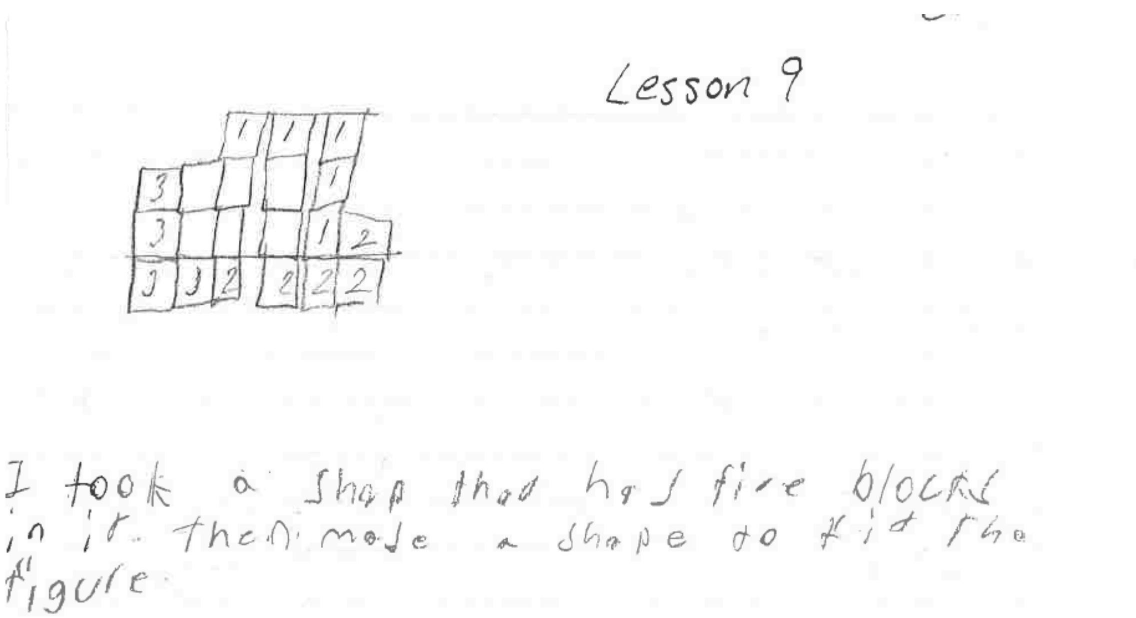

Figure 14. Gary’s work from Lesson 9.

Summary. In Phase 1 of the study, Gary exhibited passive behavior and lacked assertiveness to engage in conversations. Beginning in Lesson 6 and after the introduction of the THINK interaction framework, Gary found space in the group’s conversations and exhibited some understanding of how to use strategies to solve problems (GOP, 4/21; SW, 4/18, 4/21; TS, 4/18, 4/21). Further, Gary's understanding of the problem was often displayed in the strategies that he used to solve the problem (e.g., Lessons 6, 8, and 9). By following the prompts of the How component of the THINK interaction framework, Gary constructed strategies that he believed would help him to solve the task. Further, Gary admitted that drawing a visual representation helped him to visualize the scenario and allowed him to actively solve the task (e.g., the school bus problem). Directions in the How component of the framework also prompted students to share their strategy with group members. Not only did the THINK interaction framework provide Gary with an entry point into the conversation, it provided him with the opportunity to explain his thoughts concerning effective strategies and solution paths.

Voicing understanding of mathematical concepts (Subtheme 3 – Gary). Though Gary spoke few utterances during Phase 1 of the study, he exhibited little understanding of the mathematical concepts presented in the assigned tasks. Further, Gary failed to express any connections that he might have made among mathematical concepts, procedures, and representations. After the introduction of the THINK interaction framework, Gary voiced his understanding of mathematical concepts for some problems as they related to the tasks (SW, 4/18, 4/22; TS, 4/18, 4/22). At the beginning of Lesson 6, the group engaged in exploratory talk in discussing what the task (i.e., the block problem) required. During the Talk component of the discussion, Gary emphasized that the problem required the participants to find the number of different stacks of three blocks and again asserted, "I wonder if it is talking about the order of the colors" (TS, 4/18). Later in the lesson during the Notice component of the discussion, Gary provided an example of the different combinations that could be made with three blocks. Gary stated, “You can have red, blue, blue, or you can put the red on top and change the combination” (TS, 4/18). Gary’s understanding of the concept of combinations was exhibited in his work as he provided sketches of stacks containing three blocks. Each block in the stack was labeled with the first letter of the color (see Figure 11) (SW, 4/18).

In Lesson 9 during the Talk component, Gary and Darla discussed what the task was asking them to do (i.e., the congruent figure task). Unlike Nick and Tonya throughout the study, Gary was attentive to the wording of tasks. For example in Lesson 9, Darla explained her understanding of the task. Darla stated, “Divide the figure into four congruent --. Congruent means the same. I’m pretty sure. So, you are going to use the lines as the legal dividing lines. It means that you want four boxes to be the same” (TS, 4/22). Gary corrected Darla and stated, “Four congruent figures” (TS, 4/22). As a reminder, the figure that the group was tasked to divide into four congruent figures was an irregular polygon (see Figure 15). Gary’s understanding of the congruency of irregular polygons was confirmed during the Identify component of the framework when he stated, “I thought in the beginning we had to have four squares, but that ain’t going to happen. I think we just need four congruent figures” (TS, 4/22). Further, Gary explained the task to Nick who came to class 20 minutes late. Gary stated, “We’re trying to get four congruent figures. I don’t think it matters how many times you do it, but you have to do it on these same lines. You have to have four of the same [figures]” (TS, 4/22). Further, Gary understood that there would be four congruent figures since the sketch contained 20 squares. Unlike his lack of assertiveness in Phase 1 when he became passive after being interrupted, Gary explained to Nick that in order to solve the problem that the congruent figures were within the entire irregular polygon. For this task, Gary successfully found the solution and justified his reasoning to Darla and Nick (GOP, 4/22; RJ, 4/22; TS, 4/22).

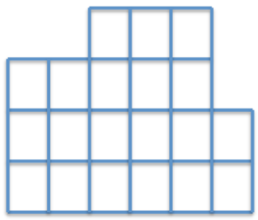

Figure 15. The figure presented in the task assigned for Lesson 9.

Summary. The Talk, How, Identify, and Notice components of the THINK interaction framework provided Gary with opportunities to voice his understanding of mathematical concepts in some of the tasks and to explain his understanding to the members of his group. This is evidenced by Gary's transition from Phase 1 to Phase 2 of the study. In particular, during Phase 1 of the study, Gary exhibited limited understanding of the mathematical concepts embedded in the assigned tasks. His limited understanding was possibly displayed in his lack of engagement in conversations. After the introduction and implementation of the THINK interaction framework in Phase 2, Gary became more assertive than in Phase 1 and voiced his understanding of the tasks.

An analysis of the lesson transcriptions, revealed that not only was Gary attentive to the wording of the tasks, but he also voiced his understanding of the mathematical concept presented in the task (e.g., the coin problem and the congruent figure task).

Gary's overall summary. Throughout the study, Gary displayed various levels of engagement in his group's conversations (GOP, 4/7, 4/8; RJ, 4/7, 4/8, 4/18, 4/26; TS, 4/7, 4/8, 4/18, 4/20, 4/28). He acknowledged that peer tutoring was one advantage to working in groups but admitted that he did not prefer that academic arrangement as he often received individual help from his teacher or a family member (PSI, 4/6). During Phase 1 of the study, Gary spoke the fewest utterances of all group members and only spoke three utterances during Lesson 2 (TS, 4/7, 4/8, 4/11, 4/14). Though he exhibited passive behavior sometimes, he became more disengaged in conversations whenever he was interrupted and lost space in the conversation (TS, 4/7, 4/8, 4/26). One possible explanation for his behavior might have been his lack of understanding of the mathematical concepts and procedures presented in the tasks.

During Phases 2 and 3 of the study, Gary's increased participation in some lessons (e.g., Lessons 6, 8, 9, 11, and 13) resulted in increased utterances, especially those coded as exploratory talk. Although he continued to sometimes exhibit passive behavior, Gary found space and inserted himself into conversations even after interruptions (TS, 4/18, 4/21, 4/22). Probable explanations for Gary's increased participation levels included his increased familiarity with the members of his group, the problem solving process, and the introduction and implementation of the THINK interaction framework. A possible reason as to how the components of the THINK interaction framework supported Gary in his exploratory talk was the wording of the prompts used in the directions for using the framework. During the Talk component of the framework, Gary was attentive to the wording of the text of the problem (e.g., the block problem and the congruent figure task). The directions of the Talk component prompted Gary to find what the problem was asking and the important information needed to solve the problem (see Appendix D). For example, in "the block problem," Gary understood that the wording of the task required students to find the number of distinct combinations of the colored blocks in order to find the solution.

In the How component of the framework, the prompts urged students to find their own strategy for solving the assigned task and then to describe their proposed strategy with members of their group. Gary's proposed strategies were often in the form of a sketch, a list, or a combination of both a sketch and a list (e.g., the block problem, the school bus problem, and the congruent figure task) (SW, 4/18, 4/21, 4/22). Gary admitted that the graphic representations helped him to visualize the scenario described in the problem (e.g., the bus problem). Further, Gary learned how to formulate and use strategies to solve the problem.

During the Identify component of the THINK interaction framework, Gary only engaged in exploratory talk during Lesson 9 when he explained his understanding of the meaning of congruency and congruency of an irregular polygon. The directions of the Identify component of the framework required students to identify the strategy as a group that they would use to solve the problem. The group used Gary's strategy for solving the problem. Gary identified and explained his strategy to Nick who was late coming to class (TS, 4/22).

Although Gary was prompted by the directions of the Notice component to explain how the chosen strategy helped him to solve the tasks, he used the opportunity provided to speak so that he could ask for clarification of the meaning of the problem or to provide understanding of the meaning of the problem and to emphasize the wording of the text of the problem in identifying important information (e.g., the block problem).

Gary sometimes used this time to vent his frustration of not understanding the strategy of the problem (e.g., the dart game).

Not only did Gary's participation levels increase but his ability to formulate strategies to solve tasks also improved. Gary began to voice the strategies that he proposed to use to solve the tasks demonstrated his understanding of how to use strategies to solve problems as demonstrated in his work samples (e.g., the block problem and the school bus problem) (SW, 4/18, 4/21). His display of these skills demonstrated his increased strategic competence. Gary often demonstrated his understanding of tasks and of the mathematical concepts associated with the tasks (e.g., the block problem, the school bus problem, and the congruent figure problem). As such, Gary often exhibited a display of his ability to make connections among the mathematical concepts and strategies used to solve the tasks.

The THINK interaction framework provided Gary with a possible sense of accountability as a member of the team and entry points into the conversation. The Talk, How, and Notice components of the framework proved to be effective in promoting Gary's increased levels of engagement in exploratory talk and movement toward mathematical proficiency. Although there was evidence of progress and movement in his abilities to formulate strategies, solve problems, and make connections among mathematical concepts and representations, Gary was not considered to be mathematically proficient at the end of the study. Though his mathematics abilities continued to evolve with the study of mathematics, Gary failed to demonstrate strong skills in procedural fluency, adaptive reasoning abilities, or a productive disposition.

Meet Nick

At the time of the study, Nick was a ninth-grade Caucasian male who was friendly, fun-loving (i.e., laughed and engaged in frivolous tricks) (RJ, 4/21) and played varsity football for a rural high school located in the Southeastern United States (TS, 4/22; RJ, 4/22). Nick was enrolled in both Algebra I and a Tier II mathematics intervention course. When asked how he best learned mathematics, Nick replied that he needed to see the teacher model the procedures by working many examples on the board (PSI, 4/4).

Nick's learning style coincided with his preference of participating as a member of the group. He stated, "I like it because it's easier. Like you can be like, you got this answer. Okay. I got this. What did you do? I did this. Okay. I see what you did, and you can correct your mistakes" (PSI, 4/4). Nick's fondness for working in groups and his reply concerning correcting one's mistakes during group work correlated with his behavior of copying other students' work (GOP, 4/11, 4/15, 5/4; RJ, 4/8, 4/26, 4/28). When asked how members of a group work to solve a task or a problem, Nick provided conflicting responses. First, he replied that the members of the group first talk about the problem, decide the steps for solving the problem, and then work it out. His response was disassociated with his behavior during some lessons of the study (e.g., Lessons 6 and 8) as he sometimes exhibited impatient behavior and tried to rush through the lesson (RJ, 4/18, 4/21). Later, Nick confessed that often a member of the group who liked mathematics would work out the problem while others copied the answer (PSI, 4/4). He referred to copying answers without understanding the process for solving the task as one disadvantage of group work (PSI, 4/4).

In the pre-study interview, Nick admitted that although he liked mathematics he struggled to understand many concepts and often received help from his teacher before and during class or from his classmates when working individually or in groups (PSI, 4/4). When asked to describe the opportunities that he had in mathematics class to explain his ideas to others, Nick replied, "I don't 'cause I'm not exactly like the teacher" (PSI, 4/4). Nick exhibited a productive disposition in his discussion of the importance of the use of mathematics. An excerpt of Nick's explanation follows.

> Mathematics is very useful. You'll use it when you graduate high school. That's when life begins. . . . Like my job that I want, I know I have to know algebra. Like they'll give you a test. I want to go to Company A. Well, I want to first of all go to a small tech school and get my industrial maintenance degree and then go to Company A. . . . Before you're up for a job, you have to pass this math test because you have to measure the tires and measure the layers and all that. So, mathematics is very, very important. (PSI, 4/4)

Although Nick expressed the necessity of knowing mathematics for post-secondary opportunities and liked the instructional arrangement of groups, he struggled with the dynamics of communication and collaboration that are essential in that setting.

From a lack of behavioral self-regulation to improved impulsivity: Using the THINK interaction framework to provide structure and support to engage in exploratory talk and acquire skills in adaptive reasoning (Overarching theme – Nick). "Self-regulation is an important personal characteristic which strongly affects one's actions and behavior" (Jakesova, Gavora, & Lalenda, 2016, p. 58). Behavioral self-regulation involves one's ability to respond in an appropriate manner as expected based on contextual factors (Montroy, Bowles, Skibbe, & Foster, 2014). In the classroom setting, one aspect of behavioral self-regulation is appropriate interactions with peers and teachers (Montroy et al., 2012). Limited self-regulatory behavior is connected with a person's lack of control in the form of impulsivity and regulation of emotions (Jakesova et al., 2016). Throughout the study, Nick exhibited a lack of behavioral self-regulation in the form of impulsivity (GOP, 4/21, 4/22, 4/26; RJ, 4/18, 4/21, 4/22, 4/29; TS, 4/7, 4/8, 4/14, 4/15, 4/18), domination of peer conversations and teacher-student interactions (GOP, 4/7, 4/8, 4/14, 4/15, 4/18; TS, 4/7, 4/8, 4/14, 4/15, 4/18), and overt verbalizations (GOP, 4/7, 4/15, 4/18, 4/21, 4/22, 4/26, 4/28, 4/29; RJ, 4/7, 4/21, 4/22, 4/25, 4/26, 5/4; TS, 4/7, 4/21, 4/22, 4/25, 4/26, 5/4) that were not part of the interactions and discussions with members of the group.

Analyses of the researcher's journal, classroom observation notes, and lesson transcriptions provided insight into Nick's behavior and participation throughout the study and revealed the overarching theme, "From a Lack of Behavioral Self-regulation to Improved Impulsivity: Using the THINK Interaction Framework to Provide Structure and Support to Engage in Exploratory Talk and Acquire Skills in Adaptive Reasoning." To help the reader better understand the emergence of the overarching theme, I provide an account of Nick's behavior and interactions with his peers and levels of engagement throughout the lesson, using subthemes to support the findings. Further, I describe how the THINK interaction framework supported Nick's engagement in exploratory talk and strengthened his adaptive reasoning abilities.

Learning to share space in conversations (Subtheme 1 – Nick). Throughout the study, Nick spoke the most frequently as compared to the members of his group (see

Tables 4 and 20). During Phase 1 of the study, Nick spoke an average of 44.6% of the total utterances as compared to Darla (i.e., 16.7%), Gary (i.e., 10.7%) and Tonya (i.e., 28.0%) (see Table 5). Nick dominated the conversation during each lesson for which he was present (see Table 4).

Table 20

Frequency of Nick's Utterances Throughout the Study

Lesson	Number of Utterances	Lesson	Number of Utterances
1	145 (48.2)	8	87 (67.4)
2	36 (40.4)	9	51 (35.4)
3	-	10	127 (75.1)
4	147 (50.2)	11	98 (64.0)
5	5 (77.5)	12	56 (53.8)
6	139 (58.4)	13	141 (56.0)
7	54 (34.2)	14	133 (49.3)

Note. The percentage of utterances spoken by Nick per lesson was presented in parentheses.
[a]The hyphen indicates an absence.
[b]In Lesson 9, Nick entered the classroom 20 minutes after class began.

Additionally, Nick interrupted members of his group and Mrs. Smith by speaking louder or at the same time to re-gain space and to dominate the conversation (see Table 21).

Table 21

The Number of Interruptions Imposed on Members of the Group by Nick per Lesson

Lesson	Darla	Gary	Tonya	Mrs. Smith	Total
Lesson 1	3	3	16	8	30
Lesson 2	2	0	2	0	4
Lesson 4	3	2	23	3	31
Lesson 5	-	-	1	1	2
Lesson 6	7	2	5	8	22
Lesson 7	1	0	2	1	4
Lesson 8	-	4	1	3	8
Lesson 9	0	3	-	3	6
Lesson 10	-	4	-	5	9
Lesson 11	3	1	-	4	8
Lesson 12	3	3	-	2	8
Lesson 13	7	4	-	3	14
Lesson 14	3	4	6	3	16

Note. The numbers in the table represent the number of times that Nick interrupted each participant per lesson. The hyphen indicates that the student was absent for the lesson.

As displayed in Table 21, Nick interrupted the conversation 30 times during Lesson 1, 31 times during Lesson 4, and 22 times during Lesson 6 (TS, 4/7, 4/8, 4/18). The following excerpt from Lesson 1 (i.e., the date problem) illustrated Nick's domination of the conversation and his lack of behavioral self-regulation in the form of interruptions.

1-2 Tonya: You would take month times day divides the year. [Nick interrupted her.]

3-4 Nick: Month times day divides the year. You would divide by six different days.

5 Gary: [Inaudible. Nick spoke over him.]

6 Tonya: So, you--. [Nick interrupted her and took over the conversation.]

7 Nick: I did it, and I did not --. [Tonya interrupted Nick.]

8-9 Tonya: So, if you find the month times that by the day, --. [Nick interrupted her and spoke over Gary.]

10-11 Gary: [Inaudible. He tried to speak, but Nick interrupted and took over the conversation.]

12 Tonya: Divided by 2013. (TS, 4/7)

In the above example, Nick interrupted Tonya in most of the conversation (i.e., Lines 1-2, 6, 8-9). Although Nick interrupted Tonya most frequently throughout the study (see Table 21), he often consumed Gary and Darla's conversational space during discussions (e.g., Lines 5, 10, and 11).

Nick not only interrupted his classmates, but he often interrupted Mrs. Smith during a supporting conversation. Further, he also dominated her attention and the conversation. The following excerpt from the lesson transcription illustrated Nick's behavior when Mrs. Smith approached the group.

1 Nick: I'm still stumped.

2-3 Mrs. Smith: Ok. Look right here. The last question, the last sentence what does it say? You need exactly --. [Nick interrupted her.]

4 Nick: Seven dates in the year.

5-6 Mrs. Smith: This is the last one. So, that means that all of the rest of them are where?

7 Nick: Behind it.

8-9 Mrs. Smith: Think about a calendar. This is the last one that works. All the other ones are where?

10 Nick: They would be in the months behind it.

11 Mrs. Smith: Yes.

12 Nick: [Inaudible.]

13 Mrs. Smith: Now, I understand what you are saying. Yes. Did that help?

14 Nick: Well, I know that I'm not going to December, now. It helped that much. (TS, 4/7)

In this example, none of the other participants had an opportunity to engage in the conversation. This excerpt represented Nick's dominance that was characteristic of many supporting conversations with Mrs. Smith throughout the study (GOP, 4/7, 4/8, 4/18, 4/26; TS, 4/14, 4/15, 4/18; 4/21, 4/25; RJ, 4/7, 4/14). Further, it illustrated how Nick interrupted Mrs. Smith (i.e., Lines 2 and 3).

Nick's continued domination of conversations often resulted in utterances that provided incorrect and confusing information, as it appeared that he did not allow time to adequately think through his thoughts in order to control the conversation. The following excerpt illustrated this observation as Nick attempted to describe the procedures for solving the task in Lesson 7 (i.e., the corrected exam problem).

1-2 Tonya: [Inaudible. She was interrupted by Nick, who spoke louder than her and at the same time.]

3-4 Nick: Because you're adding --. Because you add 936 both times there. 936 times 2. 936 plus 936. That completely makes no utter sense.

5 Tonya: What are you talking about?

6 Nick: I don't know, but anyway that's what the calculator says anyway.

7 Gary: You've confused me. (TS, 4/20)

Although he spoke only four interruptions during Lesson 7, Nick demonstrated a lack of conceptual understanding and a lack of strategic competence. In the above excerpt, he failed to accurately name the numerals that he was using in his calculations and uttered procedures without explaining his strategy to solve the task. Tonya confronted Nick about his utterances in Line 5, as she did not understand his explanation. Gary expressed his frustration toward Nick and stated that he had confused him (i.e., Line 7) (GOP, 4/20; RJ, 4/20).

Nick demonstrated other accounts of a lack of behavioral self-regulation including singing utterances during group work (GOP, 4/25; TS, 4/25, 5/4), laughing for no apparent reason during group discussions (e.g., TS, 4/14, 4/18), and overtly verbalizing his thoughts but not as part of the conversation (GOP, 4/7, 4/15, 4/18, 4/21, 4/22, 4/26, 4/28, 4/29; RJ, 4/7, 4/21, 4/22, 4/25, 4/26, 5/4; TS, 4/7, 4/15, 4/18, 4/21, 4/22, 4/26, 4/28, 4/29). For example, he sang as he worked on the assigned task in Lesson 10 (i.e., the phone plan) (see Appendix J). Nick stated, "Focus now" and sang aloud "Duh duh, batman" after replying in three utterances earlier that his head hurt, interfering in his thought processes (GOP, 4/25, 4/25). In Lesson 14, Nick sang, "Do you want to build a snowman" (TS, 5/4).

After the introduction and implementation of the THINK interaction framework, the number of interruptions spoken by Nick decreased as compared to Phase 1 of the study (see Table 22). The average number of interruptions during Phase 1 was 16.8 (i.e., 17.5%) as compared to those in Phase 2 (i.e., 9.8 or 10.7%) and in Phase 3 (i.e., 11.5 or 10.7%). A possible explanation for the decrease in interruptions was that the THINK interaction framework provided Nick with structure as he engaged in exploratory conversations with the members of the group. For example, the directions of the Talk component prompted students to talk to the members of the group to determine what the problem was asking and to identify important information to solve the problem. As a result of following the prompts provided in the directions of the THINK interaction framework, Nick began to engage in self-regulatory behavior by asking group members about their ideas and listening with fewer interruptions (see Tables 21 and 22). For example, Nick exhibited his quest for information in Lesson 7 when he asked the group to determine what the problem was asking (TS, 4/20). He continued to press the group to clarify the meaning of the problem. Nick asked, "Now. Are we just basically talking about what information [like] the mean, median, and mode? I'm assuming that's what he's talking about. The mean is [what]" (TS, 4/20)? As such, Nick engaged in exploratory talk as the group worked together to sort the necessary information found in the text of the problem. Further, he learned to regulate his behavior and better control his impulsivity.

Table 22

Average Frequency of Nick's Interruptions per Phase of the Study

Phase	Number of Interruptions	Average Number of Interruptions Per Lesson
1	67 (17.5)	13.4
2	49 (10.7)	9.8
3	46 (10.7)	11.5

Note. The percentage of the frequency of interruptions per phase spoken by Nick as compared to the total number of utterances spoken by him in each phase of the study is presented in parentheses.

During the How component, Nick continued to assume the role of the administrator in the group and urged his group members to defend their proposed strategies for solving the problem. Per the directions of the How component, students were prompted to describe their strategy for solving the problem to members of their group. An example of Nick engaging in exploratory conversations to discuss strategies for solving the problem occurred in Lesson 8 (i.e., the school bus problem). Though Tonya stated that she "plopped in random numbers" to get the answer, Nick continued to press Tonya to clarify her reasoning. He inquired as to what number she was seeking. Tonya stated that because the teacher added one point to each student's score that the new mean would be a score of 72. As he continued to seek information for clarification, Nick asked, "Okay. So, she gave them an extra point and then it equaled 71, right? Is that what we are saying here" (TS, 4/21)? The How component provided Nick with the opportunity to receive clarification in understanding possible strategies for solving the problem as it prompted students to describe and explain individual strategies to the group. However, Nick frequently failed to offer original strategies and often repeated the strategy of a group member for his own (TS, 4/7, 4/8, 4/20).

During the Notice component, Nick found opportunities to receive assistance in understanding what the problem was asking, to request clarification of strategies, and to listen to the process used to solve the problem, rather than stating how the identified strategy helped him to solve the problem. An example of Nick receiving assistance from a group member occurred in Lesson 9 (i.e., the congruent figure task) when Gary explained to Nick what the problem was asking (i.e., to divide an irregular polygon into four congruent figures using the lines of the figure provided in the task) (see Appendix J and Figure 15). Because Nick did not understand the concept of congruency of irregular polygons, Gary provided explanation as to the meaning of congruency (TS, 4/22). To summarize, the Notice component of the framework provided Nick with opportunities to receive further assistance in understanding the problem, to identify the mathematical concepts embedded in the tasks, and to understand procedures for finding mathematical calculations such as the mean of a group of numbers.

Summary. Throughout the study, Nick spoke the most frequently of the participants and dominated the conversation (see Table 4). His lack of behavioral self-regulation was exhibited through interruptions (TS, 4/7, 4/8, 4/18), singing (GOP, 4/25; TS, 4/25, 5/4), laughing (TS, 4/14, 4/18), and verbalizations (GOP, 4/7, 4/15, 4/18, 4/21, 4/22, 4/26, 4/28, 4/29; RJ, 4/7, 4/21, 4/22, 4/25, 4/26, 5/4; TS, 4/7, 4/21, 4/22, 4/25, 4/26, 5/4). To remain in control of the conversation, Nick often spoke in a loud tone to drown out the person speaking (TS, 4/7, 4/8). After the introduction and implementation of the THINK interaction framework, Nick's interruptions decreased in Phases 2 and 3 as compared to the number in Phase 1 (see Table 22). One possible reason for Nick's increased self-regulatory behavior was that the use of the THINK interaction framework provided Nick with structure to engage in exploratory conversations by presenting directions that prompted all members of the group to share their ideas about what the problem was asking them to do, to describe how their individual strategies could be used to solve the problem, and to receive additional assistance to understand the problem, strategies, or procedures used to solve the problem. By following the prompts in the THINK interaction framework, the members of the group were provided with entry points into the conversation, lessening Nick's domination of the conversation. As such, Nick began to display some self-regulation and to share space in the conversation as the group engaged in exploratory talk.

Using verbalizations when solving tasks (Subtheme 2 – Nick). After the introduction of the THINK interaction framework, Nick spoke utterances aloud and imposed fewer interruptions on group members. These verbalizations were not considered utterances in which Nick directly interrupted the members of his group or teacher during conversations but were spoken aloud or under his breath, mumbled, or whispered. In all cases, these verbalizations were spoken for Nick himself and not as part of a direct conversation with his group mates (GOP, 4/7, 4/15, 4/18, 4/21, 4/22, 4/26, 4/28, 4/29; RJ, 4/7, 4/21, 4/22, 4/25, 4/26, 5/4). The numbers of verbalizations per lesson are recorded in Tables 23 and 24.

Table 23

The Number of Verbalizations Spoken By Nick Per Lesson

Lesson	Number of Verbalizations	Lesson	Number of Verbalizations
1	4	8	7
2	4	9	4
3	-	10	9
4	4	11	16
5	6	12	5
6	3	13	13
7	0	14	16

Note. The hyphen indicates an absence.

Table 24

Average Frequency of Nick's Verbalizations per Phase of the Study

Phase	Number of Verbalizations	Average Number of Verbalizations Per Lesson
1	18 (4.7)	3.6
2	23 (5.0)	4.6
3	50 (11.7)	12.5

Note. The percentage of the frequency of Nick's verbalizations spoken during each phase of the study as compared to the total utterances spoken by him in each phase are presented in parentheses.

Nick had the least number of verbalizations during Phase 1 of the study and the most verbalizations during Phase 3 of the study (see Table 24). Verbalizations occurred in different forms. Whereas Nick verbalized many inaudible utterances, he also verbalized utterances concerning counting, performing mathematical procedures, and reading and working out the tasks.

Verbalizing counting. Often, Nick verbalized aloud as he counted. In Lesson 6, students were assigned the task (i.e., the block problem) (see Appendix J) to find the number of different stacks containing three blocks. The excerpt below demonstrated Nick verbalizing aloud as he counted the number of stacks of blocks as part of his solution.

1 Tonya: So far, nine. I'm not done.

2 Darla: She's only done one stack so far.

3 Gary: I've just done seven.

4-5 Nick: [He picks up his paper and counts aloud. *I've got one, two, three, four, five, six, seven, eight, nine, ten.*]

6 Darla: [She speaks so low it is inaudible.] I've just got [inaudible.]

7 Nick: I'm out of combinations. I can't figure out another one. (TS, 4/18)

In Lines 4 and 5, Nick counted aloud to himself the number of combinations that he had found. He did not direct his verbalization to any group member. Another example of Nick verbalizing his thoughts aloud occurred in Lesson 11 (i.e., the coin problem) (see Appendix J) when Nick used his list to count change. Nick stated, "*See, here's five, ten, fifteen, twenty. Here's twenty-five. Here's twenty-five, thirty-five, forty-five, fifty-five, sixty-five, seventy-five, eighty-five. All right, five, ten, fifteen, twenty. There's ninety*" (TS, 4/26). Nick spoke aloud to himself as he counted the amount of coins that he listed in solving the problem.

Verbalizing through procedures. Nick sometimes spoke aloud as he performed mathematical procedures. For example, in Lesson 2, the assigned task required students to find the number of dimples on a golf ball (i.e., the golf ball problem) (see Appendix J). During the discussion, Nick verbalized his steps through the procedures that he used in attempting to solve the problem. The following excerpt demonstrated the account of Nick speaking to Gary and then verbalizing the procedures aloud.

> Oh, man! Ok. I just went through here and when she said that they're not all prime that it could. I just went through here and started going, like 22, you can go like 2 plus 2 is four and then you're going back to that --. [During this moment, a student in a different group shouted something at Nick. He told another student to "shut up" and leave him alone.] It's got to be in one of those first few digits. [He spoke to himself. *Two plus two is four. Six plus six is 12. No. Seven plus seven is 14. Eight plus eight is 16. No. Obviously, that one won't be it.*] It's either --. (TS, 4/8)

In the above excerpt, Nick spoke aloud as he discussed the procedures that he used in attempting to solve the problem. However, he impulsively told another student to be quiet and to leave him alone.

Verbalizing working through the problem. In Lesson 12, students were presented with a problem of a dart game (i.e., the dart game problem) (see Appendix J) in which they were provided a sketch containing the dartboard with darts and asked to find the player who hit the center of the target. The following excerpt demonstrated Nick verbalizing aloud as he worked through the problem.

1 Darla: But, there's eleven throws on the board.

2-11 Nick: There's twelve throws. You miscounted. There's 12 [darts]. There's a total of 142 points scored. {He talks aloud to himself. *You do something with that number, I just don't know what. Seventy-one. So, they would have scored 71 points each. Okay. Back this up. Okay. No. It would have had to been him.}* It would have had to been Bob. No. Mrs. Smith. All right. She, Pamela, scored 26 on her first two throws. She scored a 26 on her first two throws, and then if she hit the 50, she would have had 76 points, but out of the combined two there were 71 points. There is no way it could have been her. (TS, 4/28)

In the above excerpt, Nick engaged in exploratory talk and uttered verbalizations as he worked through the task. In doing so, he constructed new knowledge that allowed him to solve the problem and explain his reasoning to the group and to Mrs. Smith. He exercised adaptive reasoning in justifying his thinking to the group.

Another example of Nick verbalizing aloud as he worked through the problem occurred in Lesson 13 (i.e., the target shooting problem) (see Appendix J). The following excerpt demonstrated Nick verbalizing aloud to himself as he worked through the problem.

1 Nick: Like I said --. He --. There's thirty shots.

2 Gary: Yeah.

3-11 Nick: He took all thirty of them. It ended up costing him 15 [cents]. So, it's like 30X --. [He begins to talk to himself. *Whoa! Wait a minute. Hold on. That could be --. Okay. Thirty-X equals 15. Thirty-X. Hold on. Equals 15. Hold on here. You do the opposite. You would go 30 times 15 --. Thirty times --. No. Forget that. There's no way he hit 45 shots out of 30 shots.*] There's no way that he did that. Forget that. Mark that off the board. You can do a fraction for this some how. Some way. All right. [He speaks to himself. *Thirty, 15, 10, and five.*] Somehow, them four numbers are going to give you a number. (TS, 4/29)

In Lines 4-7 in the excerpt above, Nick not only verbalizes his thinking, he also critiques his thinking aloud.

Interruptions versus verbalizations. A comparison of Nick's interruptions verses his verbalizations are presented in Table 25. During Phase 1, Nick uttered 67 interruptions (i.e., 17.5%) as compared to 18 verbalizations (i.e., 4.7%). After the implementation of the THINK interaction framework in Phase 2, Nick uttered fewer interruptions (i.e., 10.7%) but a slightly increased number of verbalizations (i.e., 5.0%) as compared to those in Phase 1. The trend of Nick speaking a decreased number of interruptions and an increased number of verbalizations continued in Phase 3 with 46 interruptions (i.e., 10.7%) and 50 verbalizations (i.e., 11.7%). A possible explanation as to why the verbalizations increased and the interruptions decreased was that the THINK interaction framework supported Nick in his exploratory talk as he engaged in conversations with the members of the group. As he followed the prompts provided in the framework in his assumed role as the group administrator, Nick used self-regulatory behavior in letting members of the group speak with a minimum number of interruptions imposed by him as compared to those spoken by Nick in Phase 1. As such, Nick dominated conversations less in Phase 2 than in Phase 1. The verbalizations may have been a result of the use of the THINK interaction framework as an intervention tool. Not only did the THINK interaction framework promote exploratory talk as the members of the group followed the prompts that encouraged all members to share their thoughts, but it also provided Nick with a personal intervention tool that supported him in self-regulatory behaviors and intramental activity. For example, Nick's engagement in intramental activity could be witnessed through his use of verbalizations after his engagement in exploratory talk with the members of his group (TS, 4/8, 4/18, 4/28, 4/29).

Table 25

Comparison of the Frequency of Nick's Interruptions and Verbalizations

Phase	Number of Verbalizations	Number of Interruptions
1	18 (4.7)	67 (17.5)
2	23 (5.0)	49 (10.7)
3	50 (11.7)	46 (10.7)

Note. The percentage of the frequencies of Nick's verbalizations and interruptions spoken during conversations in each phase of the study are presented in parentheses as compared to the total number of utterances spoken by him in each phase.

Summary. Throughout the study, Nick either interrupted the members of his group and teacher or verbalized to himself. Verbalizations often occurred when Nick counted objects (e.g. Lesson 6), performed mathematical procedures (e.g., Lesson 2), or read and worked through the problem (e.g., Lessons 12 and 13). Although he verbalized to himself aloud under his breath or in a whisper or a mumble, Nick's verbalizations were not directed to the members of his group or to Mrs. Smith (GOP, 4/7, 4/15, 4/18, 4/21, 4/22, 4/26, 4/28, 4/29; RJ, 4/7, 4/21, 4/22, 4/25, 4/26, 5/4). After the introduction of the THINK interaction framework, there was a noticeable trend in the number of utterances spoken as interruptions and verbalizations. As the number of interruptions spoken by Nick decreased per each phase of the study, the number of verbalizations increased. One possible explanation for this trend was that the THINK interaction framework provided structure for Nick to participate in conversations. Though the framework presented all members with the opportunity to engage in the conversation, it provided Nick with support in achieving better control of his impulsivity and lack of self-regulation. As the THINK interaction framework encouraged exploratory talk through prompting all students to share their thoughts as they worked through each component, it also promoted intermental activity as the group discussed what the problem was asking, the proposed strategies for solving the problem, mathematical concepts, and often the procedures for preforming calculations. As a result, Nick's engagement in intermental activity may have spurred intramental activity and a reorganization of his thoughts that could be witnessed through his spoken verbalizations meant for himself as he worked to solve tasks.

Exhibiting increased reasoning and justification skills in conversations (Subtheme 3 – Nick). Throughout the study, Nick struggled with mathematical procedures and lacked strategic competence and conceptual understanding. Nick's explanations often were a re-statement of a group member's explanation. For example, when questioned by Mrs. Smith as to what he had done to solve the task in Lesson 5 (i.e., the number card task) (see Appendix J), Nick replied that he had the same strategy as Tonya. Mrs. Smith asked Nick, "What are you doing here" (TS, 4//15). He stated, "That? I started like doing what she was doing. She got farther than I did so we just used her paper" (TS, 4/15). Another example of Nick taking credit for someone's ideas occurred again in Lesson 7. After asking all members of the group how they solved the problem, Nick admitted that he used the same strategy to solve the problem. As he again confirmed Darla's strategy, Nick stated, "We all got --. Darla, did you do the same as me or did you do it a different way? Get a different answer? What did you do" (TS, 4/20)? Nick never stated his strategy or his solution for the task proposed in Lesson 7. Until the group shared out during the Keep component of the THINK interaction framework, Nick failed to understand the problem and required more explanation from the group and Mrs. Smith.

Nick, however, displayed some adaptive reasoning abilities in Lesson 4 (i.e., the appointment problem) (see Appendix J) in discussing the task for which the participants were asked to find the time a person left his house given the distance from his home, driving speed, and the fact that he arrived 20 minutes late. The excerpt below demonstrated Nick's ability to provide explanations concerning his thoughts about the time the man left his house.

> Something isn't right. If it took two --. If he had gone 30 miles an hour and took two hours, then he went 60 miles. He is going 30, and it's 50 miles away. So, 30 divided by 50 --. He went an hour and like some odd minutes to go from wherever

> in the world that he is starting at – his house – or wherever to the appointment. Because you said, if he went two hours driving 30 [miles per hour], two times 30 is 60. If you divide the 50 and the 30 you get 1 hour and like six odd minutes. So, I guess you could take it and just backtrack it. Take off an hour and six minutes that would be your time. So it would be like 12:54, or 11:54, or something like that (TS, 4/14).

Nick used adaptive reasoning to explain his thoughts about the solution to the task. Though his reasoning was somewhat accurate, Nick lacked procedural fluency and conceptual understanding, which caused him to arrive at an incorrect answer (RJ, 4/14). The quotient of 50 and 30 is one and two-thirds. Nick, who used the decimal form of his answer as six minutes instead of 40 minutes, subtracted an hour and six minutes from the time that the man arrived at his appointment (i.e., 1:20 p.m.) to get his answer of 11:54 a.m.

As the study progressed, Nick engaged more frequently in exploratory talk. After the introduction and implementation of the THINK interaction framework, he displayed progress in exhibiting his adaptive reasoning abilities. For example, in Lesson 8 (i.e. the school bus problem), Nick and Gary worked to find how long it took 53 students to exit a bus with four exits if each student exited the bus in eight seconds (see Appendix J). Upon working through the problem using verbalizations and in discussing the situation with Gary, Nick explained his solution. In the following excerpt Nick displayed adaptive reasoning abilities.

1 Nick: One minute and 52 seconds. What do you think?

2 Gary: What did you do to get the one that you have?

3-9 Nick: I took 53, divided by four and got 13 point two five. Rounded that up to 14, times that by eight, got a 112, subtracted that by 60, which is how many seconds are in a minute. You get 52. You can't subtract 60 into that again, so you end up with a 1 minute 52 seconds. I mean, that's the best possible answer that I can possibly think. Whether or not that answer is correct, I don't know, but, it don't sound like a bad answer. [Pause.] What do you think? (TS, 4/21)

Nick used adaptive reasoning as he justified and explained his answer to Gary (i.e., Lines 3-7). Additionally, he exhibited conceptual understanding of the relationship between minutes and seconds and procedural fluency in accurately calculating the correct time.

In Lesson 12, the participants were assigned the task of determining which of two players hit the center of the target and scored 50 points (i.e., the dart game) (see Appendix J). A sketch of the dartboard containing the scored darts accompanied the task. Nick fully engaged in explaining his reasoning concerning who hit the target first. An excerpt of Nick's explanation was presented below.

> There's 152 [points]. Divide that by two. You get 76 [points]. Okay. Get 76 [points]. Yes, she could have hit --. Twenty-six. She scored 26 [points] on her first two throws, right? I'm going to put P-T-S on her first two throws. She got 26 [points]. You add that 50 [points] to that and get 76 [points]. She got that for her first two throws. All right. So, if she had hit the 50 anywhere else she would have exceeded 76 points. There was only 76 points scored. There's no way that she was the one who hit the 50. So, it had to have been Bob that hit the 50 (TS, 4/28).

In justifying his thinking, Nick displayed adaptive reasoning. His explanation as to why the female player did not hit the center of the target illustrated logical thinking.

Summary. Throughout the study, Nick lacked strategic competence, conceptual understanding, and procedural fluency as indicated in the transcribed dialogues of the conversations with the members of this group. In the beginning of the study, Nick's adaptive reasoning abilities were masked by his utterances that were merely reworded explanations previously stated by members of his group. After the introduction and implementation of the THINK interaction framework, Nick progressed somewhat in his abilities to justify and explain his reasoning for his thoughts. Possibilities for Nick's progress in his reasoning abilities included exposure to other students exhibitions of their reasoning and justifications of their thought processes and understanding of the process for the sharing of one's ideas as promoted by the structure provided by the THINK interaction framework that addresses collaboration among members of a group. As written in the directions of the components of the THINK interaction framework, students were encouraged to talk, describe, identify, and explain their thoughts and reasons with the members of the group. In summary, the THINK interaction framework provided Nick with the structure to engage in exploratory talk with members of his group and the exposure to witness other students' display of adaptive reasoning. As a result, Nick progressed in his development and display of adaptive reasoning skills.

Nick's overall summary. Nick was a friendly and outgoing young man who admitted that although he liked the study of mathematics, he struggled to understand mathematics and often received tutoring from his teacher or peers (PSI, 4/4). As such, Nick stressed that he like working in groups as he could ask a classmate for help if needed. However, Nick sometimes mistook copying of others' work as peer assistance (GOP, 4/11, 4/15, 5/4; RJ, 4/8, 4/26, 4/28). In the pre-study interview, Nick professed to the usefulness of mathematics in life, especially in post-secondary opportunities for job preparation. Nick stated, "Mathematics is very useful. You'll use it when you graduate from high school. That's when life begins" (PSI, 4/4).

Throughout the study, Nick struggled with self-regulatory behavior as he exhibited impulsivity through interruptions, dominance of the conversations, and in his use of verbalizations (GOP, 4/7, 4/15, 4/18, 4/21, 4/22, 4/26, 4/28, 4/29; RJ, 4/7, 4/21, 4/22, 4/25, 4/26, 5/4; TS, RJ, 4/7, 4/21, 4/22, 4/25, 4/26, 5/4). Further, Nick voiced the most utterances per lesson as compared to Darla, Gary, and Tonya (see Table 4) and controlled the conversation through his interruptions and exhibition of impulsive behavior. After the introduction of the THINK interaction framework, the number of interruptions by Nick decreased in Phases 2 and 3 as compared to Phase 1 (see Table 22). The THINK interaction framework provided Nick with structure to engage in conversations of exhibited exploratory talk by presenting directions in the prompts that encouraged and offered all members of the group with entry points to describe, share, explain, and talk in a matter that promoted collaboration. The effect of the THINK interaction framework as an intervention tool resulted in Nick sharing space in conversations as resolved to verbalizing utterances as he counted objects, performed procedures, and worked through problems (GOP, 4/7, 4/15, 4/18, 4/21, 4/22, 4/26, 4/28, 4/29; RJ, 4/7, 4/21, 4/22, 4/25, 4/26, 5/4; TS, RJ, 4/7, 4/21, 4/22, 4/25, 4/26, 5/4). Though the verbalizations were never directed to address members of his group or Mrs. Smith, the number of verbalizations increased after the introduction and implementation of the

THINK interaction framework. One possible reason for Nick's display of self-regulatory behavior was that his engagement in exploratory talk and intermental activity fostered a restructuring of thoughts and activity that allowed Nick to focus his statements when communicating with his group. As such, Nick demonstrated increased adaptive reasoning skills when justifying and explaining his thoughts concerning solutions and statements about the problem.

In summary, the implementation of the THINK interaction framework provided Nick with support to engage in exploratory conversations, use self-regulatory mechanisms as verbalizations, and re-organize his thoughts as displayed in his increased adaptive reasoning abilities. However, Nick was not considered to be mathematically proficient at the end of the study due to his lack of strategic competence, procedural fluency, conceptual understanding, and weak adaptive reasoning skills.

Cross-case Analysis

The current study investigated how a heuristic (i.e. the THINK interaction framework) used as an intervention tool supported students in their exploratory talk and movement toward mathematical proficiency. Using a cross-case comparison based upon the overarching themes in each case as identified by analyses of the lesson and interview transcriptions, researcher's observation notes and journal, and student work, I discuss the emergent themes of how the THINK interaction framework supported each participant's exploratory talk. Further, I discuss and compare each participant's movement toward mathematical proficiency as a result of their engagement in exploratory talk.

Structure for Group Conversations and Support for Individual Participants

A review of all data sources identified how the THINK interaction framework supported participants' exploratory talk through providing structure for engagement in conversations as a group. First, evidence from the video transcriptions revealed that the wording of the prompts in the Talk, How, and Notice components of the THINK interaction framework provided structure for the group of participants to engage collaboratively in working to solve the assigned tasks. Specifically, the wording of the prompts in the Talk, How, and Notice components encouraged the participants to talk, describe, explain, and share their thoughts with members of the group. Second, the participants began to critique each other's reasoning, strategies, and thoughts as they worked to understand and solve the problem. The critiques occurred most when the participants engaged in conversations where further clarification was needed to understand the wording of the tasks and the procedures for solving the tasks. As a result, Darla, Nick, and Gary learned how to participate collaboratively in a group setting.

Third, the directions of the prompts changed the group dynamics in that it encouraged Darla and Gary to voice their thoughts and provided entry points into the conversation. In the beginning of the study, Darla and Gary exhibited passive behavior and spoke few utterances. Often, Darla used physical gestures to indicate that she did not wish to speak during group work or in front of the class. Gary spoke few utterances and quickly disengaged from the conversation if he was interrupted. After the implementation of the THINK interaction framework, Darla shared her understanding of the problems and proposed strategies to solve them. The increase in her total utterances and in her exploratory talk provided evidence of her increased self-efficacy and mathematical abilities. Gary found space to voice his understanding of the tasks and gained confidence to interject his thoughts after experiencing interruptions. Also, Gary questioned other member's thoughts concerning their understanding of the problems.

Fourth, the introduction and implementation of the THINK interaction framework supported Nick differently than Darla and Gary. Whereas the implementation of the THINK provided Darla and Gary opportunities for engagement into conversations to voice their understandings of the problem and of mathematical concepts and to propose strategies for solving the tasks, the THINK interaction framework as a whole provided Nick with structure and support for better control of his impulsivity and lack of self-regulation. Beginning in Phase 2 of the study, Nick began exercising self-regulatory mechanisms, such as verbalizing to himself. As a result of Nick's increased verbalizations, Nick imposed fewer interruptions on Darla and Gary. In other words, as verbalizations increased interruptions decreased. In summary, the implementation of the THINK interaction framework provided structure to the group's conversations by encouraging all members to find a voice to interact collaboratively in discussions directed toward the solving of tasks. Further, the directions of the THINK interaction framework supported individual participants to share their thoughts about their understanding of the problem, to construct and propose strategies for solving problems, and to explain and justify their reasoning while also providing critiques of other members reasoning.

Conceptual Understanding

Although Darla exhibited evidence of limited conceptual understanding in Lesson 1, a review of the data sources revealed that she failed to exhibit such understanding in Lessons 2 through 5. After the implementation of the THINK interactive framework, Darla often displayed her ability to understand the mathematical concepts presented in the task as they related to the objectives of the task. The directions of the prompts in the Talk component of the framework directed Darla to question what the problem was asking and to determine the provided necessary information in the text of the problem. As such, Darla focused on the nature of the problem in the context that it was written to determine the meaning of the problem. Further, Darla was instructed to talk to the members her group about her understanding of the problem. As the study progressed, Darla displayed understanding of the relationships among the mathematical concepts in the problem, strategies that could be used to solve the problem, and how to construct visual representations of the problem.

Gary also exhibited passive behavior throughout the study, especially during Phase 1. Gary's passivity was evidenced by his total lack of utterances and exploratory talk in Lessons 1 through 5. Though it was difficult to determine his level of conceptual understanding based on lesson transcriptions, his work samples displayed limited conceptual understanding. After the implementation of the THINK interaction framework, Gary found space in conversations to sometimes explain his understanding of the assigned task. The prompts of the Talk component of the framework directed Gary to determine what the problem was asking and to identify the information needed to solve the task. Following the prompts of the Talk component, Gary expressed attention to the detail in the wording of the task is sharing his thoughts with the members of his group. His attention to such detail supported a deeper focus on the task and promoted Gary to further analyze the problem, allowing him to better understand what was needed to solve the problem.

Like Darla, Gary's display of understanding of mathematical relationships among concepts, strategies, and procedures increased throughout the study. However, my perception of Darla's understanding of mathematical relationships was different than that of Gary's. Whereas Darla had a more integrated understanding of the relationship among mathematical concepts, procedures, and strategies, Gary sometimes had a deeper understanding of a concept because of his attention to detail in the wording of the task. For example, in Lesson 6 (i.e., the block problem), Darla understood the concept of distinct combinations and that parts of the solution came from rearranging the blocks. Gary's attention to detail promoted his understanding of combinations and the procedure needed to solve the problem resulting in his statements that the order of the blocks by color was important in the solution. His understanding was displayed in his work sample where he provided labeled sketches with colored labels written on the blocks. Although Gary and Darla understood that the solution required distinct ways that the blocks could be arranged, Gary initially stated that the distinction could be explained by arranging the stacks by the color of the blocks. Also, the video transcriptions from Lesson 6 (TS, 4/18) provided evidence that Gary's statements concerning his thoughts about using color to discern the different combinations of the blocks spurred intermental activity as witnessed in the group's discussion and intramental activity based upon the participants' work samples. Unlike Darla and Gary, Nick struggled to understand the mathematical concepts presented in the tasks. Further, he expressed difficulty in understanding the procedures for calculating items such as the mean of a group of numbers. In summary, Gary and Darla exhibited an increased display of their understanding of the relationships among concepts and strategies after the introduction of the framework possibly due to the directions of the prompts incorporated into the Talk and Notice components.

Strategic Competence

Although Darla displayed knowledge of constructing strategies to solve a problem throughout the study, the lesson transcriptions revealed that Darla often proposed strategies after the group engaged in exploratory talk, beginning in Phase 2. One possible explanation for Darla's increased strategic competence was that the directions of the prompts in the Talk and How components of the THINK interaction framework delivered structure to the group's conversation. The directions required the participants to find what the problem was asking and to identify important information needed to solve the problem. The directions in the prompts of the How component required participants to construct strategies and to share their thoughts with the group. This engagement in exploratory talk to discern the nature of the problem and to identify necessary information to clarify misunderstandings promoted intermental activity that resulted in intramental activity for Darla concerning constructing and proposing strategies as evident in lesson transcriptions and in her work samples. Darla's cyclic engagement in intermental and intramental activity resulted in strengthened skills to formulate strategies and to construct graphic representations (e.g., a list or sketch) to implement the strategies. Thus, Darla exhibited increased strategic competence as the study progressed.

Unlike Darla, Gary exhibited limited strategic competence in Phase 1 of the study as evidenced by his work samples for Lessons 1 through 4. After the introduction of the THINK interaction framework, Gary found space in conversations to explain his proposed strategies in Lessons 6, 8, and 9. The directions of the prompts in the Talk components encouraged Gary to focus on the wording of the text to address the question of what the problem was asking and to identify the information needed to solve the problem. As a result, Gary's expressed attention to detail was exhibited in his proposed strategies to solve the problem. Further, Gary displayed increased understanding of how visual representations could be used to support his proposed strategies in solving problems. Though he constructed and labeled sketches in Lesson 6 (i.e., the block problem), Gary's realization of how visual representations and strategies could be used collectively to solve the problem occurred in Lesson 8. After struggling to solve the problem in Lesson 8 (i.e., the school bus problem), Mrs. Smith suggested that Gary draw a sketch to represent the problem. Upon completing the sketch, Gary immediately constructed a list to represent the time and number of students left on the bus after each round of exits. When asked which strategy helped him to solve the problem, Gary replied, "The second one helped me because I could actually visualize the kids getting off the bus" (GIP, 4/21).

Though Nick's work samples illustrate that he used visual representations and lists, Nick struggled with constructing strategies and visual representations to solve problems as evidenced by lesson transcriptions. His work samples were copies of other participants' work. In summary, the directions in the prompts of the THINK interaction framework provided support for Darla and Gary to understand the problem, to identify information needed to solve the problem, and to use such information to construct strategies and representations to solve the problem. As such, Darla and Gary's skills in strategic competence increased as a result of the implementation of the THINK interaction framework.

Adaptive Reasoning

Because Darla exhibited passive behavior and shyness throughout Phase 1 of the study, it was difficult to ascertain her adaptive reasoning abilities due to the lack of spoken utterances. Darla spoke one and two-word utterances (e.g., un huh) to explain her reasoning for her work in Lessons 1 and 2. After the introduction of the THINK interaction framework, Darla's total utterances and exploratory talk increased as evidenced by the lesson transcriptions. The directions of the prompts in the Talk, How, and Notice components provided Darla with opportunities to engage in the group's conversations as the prompts included words such as talk, describe, explain, and share. As students implemented the THINK interaction framework with some fidelity, the group dynamics began to change. Gary and Darla found entry points to share their ideas with the group while Nick became less dominant in controlling the conversation. As such, Darla not only shared her ideas, but she also began to explain and justify her reasoning for using her proposed strategy to solve the problem or to explain the procedures for solving the problem. In Lesson 12 (i.e., the dart game), Darla's increased self-efficacy resulted in the exhibition of strengthened skills to explain and justify her thoughts and her work. Further, she also committed to critiquing Nick's thoughts to keep the group focused on the objectives of the task. A possible reason for Darla's display of increased self-efficacy as displayed in her adaptive reasoning ability was that by following the directions of the prompts of the Talk, How, and Notice components, Darla better understood the problem and the relationship among the provided sketch, the mathematical concept, and the mathematical procedure needed to support her solution.

Darla's ability to explain and justify her thoughts displayed her increased adaptive reasoning abilities that were noticeable as the study progressed.

Similar to Darla, Gary's passive behavior masked his adaptive reasoning ability in Phase 1 of the study as he spoke few utterances. Though Gary often found a voice after the implementation of the THINK interaction framework to interject his thoughts concerning the meaning of the problem, his displayed adaptive reasoning skills were limited as compared to those exhibited by Darla and Nick.

Throughout the study, Nick lacked strategic competence, conceptual understanding, and procedural fluency. After the implementation of the THINK interaction framework, Nick's proposed strategies were simply re-statements of other members' strategies. Although he used a calculator, Nick lacked procedural fluency as he merely just keyed in numbers in hopes of getting the correct answer. Nick struggled with understanding of mathematical concepts and frequently asked for clarification of the meaning of concepts, procedures, and strategies. However, the directions of the prompts of the THINK interaction framework provided Nick with structure that promoted self-regulation. As a result, Nick learned how to collaboratively participate in a group arrangement and allowed the participation of other members with fewer interruptions.

Many of Nick's utterances coded as exploratory talk were the result of Nick engaged in the conversation and asking for clarification. As such, he critiqued the statements of the members of his group and urged them to explain and justify their answers and thoughts. As the study progressed, Nick learned how to justify his reasoning to explain his thoughts concerning what the problem was asking and to justify his work. For example, Nick provided detailed explanations and justifications in defending his thoughts and mathematical processes in Lesson 8 (i.e., the bus problem) and in Lesson 12 (i.e., the dart game) (see Appendix J). Nick displayed increased adaptive reasoning skills, especially in tasks where a member of the group constructed a sketch or the sketch was provided in the problem.

In summary, Darla's exhibited adaptive reasoning skills progressively increased throughout the study. This increase was possibly due to the structure provided by the directions in the components of the THINK interaction framework and her increased self-efficacy that allowed her to voice her thoughts. Though Gary's attention to detail to the wording of the text often prompted him to critique members' thinking concerning the meaning of the problem, he continued to exhibit limited adaptive reasoning abilities. Nick displayed increased adaptive reasoning abilities after the THINK interaction framework, possibly due to the structure that promoted Nick to engage in self-regulation skills and the group to better collaborative skills. Another possible reason for Nick's increased adaptive reasoning skills was that learned how to exercise such skills through exposure in exploratory conversations.

Summary

A cross-case comparison based upon the overarching themes as identified by analyses of the data sources revealed emergent themes of how the THINK interaction framework supported participants' exploratory talk and movement toward mathematical proficiency. Emergent themes included structure for group conversations and support for individual participants, increased conceptual understanding, increased strategic competence, and increased adaptive reasoning. In the above section, I provided an analysis of how the THINK interaction framework supported group conversations through the wording of the directions in each of the components of the framework. The support resulted in increased exploratory talk, in which the participants shared their thoughts about the problem and critiqued each other's reasoning. Further, the group dynamics changed as a result of the group utilizing the THINK interaction framework as Darla and Gary's engagement in exploratory talk increased and the interruptions imposed by Nick on the other two participants decreased. Consequently, Darla and Gary exhibited increased conceptual understanding and strategic competence while Darla and Nick displayed increased adaptive reasoning skills. Though the participants displayed increased abilities in certain strands, none achieved mathematical proficiency by the end of the study.

Chapter Summary

This embedded case study investigated how the use of a heuristic (i.e., the THINK interaction framework), used as an intervention tool, supported students' exploratory conversations during a ninth-grade mathematics intervention class in a Southeastern high school. Qualitative data were gathered over a period of three weeks and analyzed to provide information addressed by the research question in this study: How does the THINK interaction framework support students' exploratory talk and their movement toward mathematical proficiency, if at all? In this chapter, I provided a narrative detailing a typical lesson using the THINK interaction framework, a summary of the types of talk, a description of the amount of participation and types of engagement per each participant, and a holistic analysis of the effectiveness of each of the components of the THINK interaction framework. Finally, I provided rich narratives detailing the findings of each embedded case, present the findings from a cross-case analysis, and conclude with a chapter summary.

CHAPTER V: SUMMARY AND DISCUSSION

Introduction

This qualitative study examined how the use of a specific heuristic used as an intervention supported student discussions and movement toward mathematical proficiency, if at all, during problem-solving activities in a small group setting in a Tier II high school mathematics classroom. A statement of the problem, review of the methodology, and summary of the results will be presented followed by a discussion of the results. The chapter concludes with recommendations for mathematics educators, suggestions for future research, and a summary.

Statement of the Problem

Current reforms in mathematics education charge all teachers to use effective teaching practices to support all children towards becoming mathematically proficient (NCTM, 2014). Such practices often encourage student-centered instruction, engage students in meaningful mathematical discourse that is promoted by inquiry-based tasks, and allow students to productively struggle as they reason through the problem-solving process (Fennell, 2011; NCTM, 2014). A review of the literature concerning reform-based instructional practices concluded that children who engage in activities that promote exploration and mathematical discourse have opportunities to strengthen their mathematical skills (NCTM, 2014; NRC, 2001). However, students who are at risk for mathematical difficulties or who have been identified as having a learning disability are often exposed to instructional practices that differ from the eight Mathematics Teaching Practices described by NCTM (2014). Unless all students are provided with opportunities to learn mathematics through effective instructional practices, students who are at risk for mathematical difficulties or who have been identified with a learning disability continue to lag behind their same-age peers in their development of mathematical knowledge and proficiency (Fennell, 2011; NRC, 2001). The purpose of this embedded multiple-case study was to investigate how the use of a heuristic (i.e., the THINK interaction framework) supported students' exploratory talk while enrolled in a Tier II mathematics intervention course. Specifically, the study addressed the research question: How does the THINK interaction framework support students' exploratory talk and their movement toward mathematical proficiency, if at all?

Review of Methodology

An embedded case study design (Yin, 2014) was used to investigate how a heuristic (i.e., the THINK interaction framework) used as an intervention tool supported students' exploratory talk and movement toward mathematical proficiency. Four ninth grade students enrolled in a Tier II mathematics classroom in a Southeastern high school were initially selected to participate in the study. Due to excessive absences, one student was dropped from the study. Data were collected over three phases of the study involving 14 lessons. Multiple sources of data were collected before, during, and after the study to achieve triangulation and to provide validity of the findings. Collected data included participant interviews, classroom observations, video recordings, student work samples, and written memos in the form of the researcher's journal. Detailed case narratives were written for each participant to provide the reader with a deeper understanding of how the THINK interaction framework supported participants' exploratory talk. A cross-case analysis based on the overarching themes in each case as identified by analyses of lesson and interview transcriptions, classroom observations, the researcher's memorandums and notes, and student work was conducted. Emergent themes resulted from the cross-case analysis of the embedded cases and provided insight as to how the THINK interaction framework supported students' exploratory talk and movement toward mathematical proficiency.

Summary of the Results

In Chapter IV, rich narrative descriptions of each case were presented to introduce the participant to the reader and to provide understanding and clarity of how the THINK interaction framework supported participants' conversations and progressive movement toward mathematical proficiency. For each participant, an overarching theme and supporting subthemes were presented to identify how the THINK interaction framework supported participants' exploratory talk and movement toward mathematical proficiency. Finally, a cross-case analysis was used to describe emergent themes across the cases. The overall findings will be summarized in the following paragraphs.

Each participant exhibited changes in his or her exploratory talk. Though Darla displayed passive behavior and spoke few utterances coded as exploratory talk in Phase 1 of the study, her exploratory talk and self-efficacy increased after the implementation of the THINK interaction framework, beginning in Phase 2. The directions of the prompts for the Talk, How, and Notice components of the framework provided Darla with support to focus her utterances on the nature of the tasks and to construct and explain strategies for solving the tasks. Further, Darla exhibited an increase in her display of her knowledge of relationships among mathematical concepts, graphic representations, and procedures used to solve tasks. Also, Darla's ability to formulate strategies and to construct graphic representations to solve tasks provided evidence of her increased strategic competence.

Finally, the directions of the components of the THINK interaction framework that called for students to explain, talk, share, and describe their thoughts, strategies, and solutions provided Darla with opportunities to engage in the conversation and reveal her adaptive reasoning skills. Throughout the study, Darla exhibited increased self-efficacy as displayed in her exploratory talk. Further, Darla's strategic competence and adaptive reasoning skills progressed throughout the study.

Like Darla, Gary displayed passive behavior throughout the study. He also became quickly disengaged when he was interrupted when trying to speak. He spoke the fewest utterances throughout the study, especially in Phase 1. After the introduction of the THINK interaction framework, Gary's engagement in exploratory talk increased as he found entry points into the conversation and opportunities to voice his strategies and reasoning concerning the meaning of the problem. The directions in the Talk component prompted Gary to become attentive to the wording of the text of the problem. As a result, Gary often critiqued the ideas of other members as he drew attention to details and explained the meaning of the problem. In doing so, Gary often displayed his understanding of particular topics (e.g., congruency of irregular polygons, permutations, and combinations). The directions of the prompts of the How component required Gary to construct a strategy to solve the problem and to describe the strategy to the members of the group. As he engaged in exploratory talk concerning how to solve tasks, Gary realized how a strategy involving a list or a sketch could be used to solve the problem. As such, his ability to formulate strategies that used sketches or lists evolved and displayed his increased strategic competence. Though his mathematical understanding and ability to formulate strategies for solving problems were initially obscure, Gary's strategic competence and understanding of mathematical relationships progressed throughout the study.

Unlike Darla and Gary, Nick exhibited a dominant personality and controlled the group's conversations by overpowering other participants while talking or interrupting them and taking over their conversational space. In other words, Nick displayed a lack of self-regulatory behavior especially in Lessons 1 through 5. Though Nick professed the usefulness of mathematics, he struggled in understanding mathematical concepts and with implementing mathematical procedures. After the implementation of the framework, Nick imposed fewer interruptions and used more verbalizations as he worked through the tasks. The directions in the components of the THINK interaction framework that required students to share their answers provided Nick with structure to execute self-regulatory mechanisms, allowing for other group members to find entry points into the conversation to share and explain their thoughts. Further, the THINK interaction framework supported his engagement in exploratory talk as he often critiqued other members' thinking and urged them to explain their thoughts and reasoning about mathematical concepts, strategies, and procedures. Nick often needed such explanations for clarification and understanding in his efforts to solve the tasks. The intermental activity often spurred from such conversations resulted in intramental activity and a restructuring of Nick's thoughts. This reflection was often witnessed as Nick verbalized his thoughts aloud. Additionally, Nick began to focus his statements to explain his thoughts and reasoning when communicating with his group. As such, Nick demonstrated increased adaptive reasoning abilities by the end of the study. Though Nick lacked strategic competence, conceptual understanding, and procedural skills, he progressed in his argumentation and reasoning abilities.

A cross-case comparison revealed emergent themes across the embedded cases of how the THINK interaction framework supported exploratory talk and progressive movement toward mathematical proficiency. Common themes included how the THINK interaction framework provided structure for group conversations and support for individual participants in realizing mathematical connections among concepts, strategies, and graphic representations. The directions in the prompts of the components of the THINK interaction framework supported the participants' increased exploratory talk by which students shared, described, and explained their thoughts concerning the meaning of the tasks, mathematical concepts, proposed strategies, and mathematical procedures for completing calculations. Further, the group's dynamics changed as Darla and Gary found entry points to engage in exploratory conversations and Nick used self-regulatory mechanisms that allowed him to engage differently in the conversation as he imposed fewer interruptions and more verbalizations. Although the participants' initially displayed weak mathematical abilities, they showed progress in their problem-solving abilities and in certain strands of mathematical proficiency by the end of the study.

Discussion of the Results

Though the results of this study are not generalizable, they support existing theory and research and bring awareness to classroom practice. A discussion of the results is presented in the following paragraphs followed by recommendations for educators and suggestions for future areas of research.

Increased Exploratory Talk

In the pre-study interviews, Gary and Darla mentioned that group work occurred infrequently in both the RTI and regular Algebra I classroom, leaving one to question to what extent the participants had been taught expectations and social norms for group work. Though Gary and Darla exhibited passive behavior in their pre-study interview and throughout the study, a lack of understanding of expectations such as participation and individual accountability may have accounted for some of their exhibited passive behavior. Additionally, Nick may not have been informed of his dominance of conversations or his lack of personal accountability for making contributions to the group's final artifacts. Nevertheless, each participant's engagement in exploratory talk changed upon implementation of the THINK interaction framework. Although Mrs. Smith introduced the group and guided them through the use of the framework during Phase 2 of the study with occasional reminders during Phase 3, the support provided by the THINK interaction framework introduced the participants to a form of sociomathematical norms (Yackel & Cobb, 1996) where the participants were encouraged to explain, describe, identify, and talk to each other concerning their solutions, strategies, and thoughts about the assigned mathematical tasks. As a result, the participants often engaged in justification and explanation of their solutions, strategies, and thoughts. Finally, the participants began to engage in argumentation whereby they critiqued other members' strategies, solution paths, and explanations in working toward the solution of some tasks. Therefore, exposure to collaborative group arrangements with the aid of a heuristic such as the THINK interaction framework to guide mathematical conversations creates opportunities to teach and expose students to sociomathematical norms and to increase students' exploratory talk in mathematics classrooms.

Movement Toward Mathematical Proficiency as a Result of Peer Collaboration

Throughout the study, the participants exhibited increased skills and abilities in one or more of the strands of mathematical proficiency, especially after the implementation of the THINK interaction framework. There are several possible conclusions for this exhibited increase in skills. First, the increase in participants' adaptive reasoning and strategic competence could have resulted from repeated exposure and engagement in problem-solving activities. Second, the participants gained familiarity of the instructional arrangement of group work and with the members of their group. Third, the directions of the prompts in each of the components of the framework encouraged participants to discuss and share ideas with group members. Such discourse resulted in exploratory talk whereby intermental activity spurred intramental activity and the reorganization of knowledge, thus resulting in increased self-regulatory mechanisms (e.g., Nick's verbalizations) and skills in reasoning, strategizing, and mathematical understanding.

As such, Vygotsky's theory of social constructivism and Forman's extended view of ZPD, referred to as collaborative ZPD (Goos, Galbraith, & Renshaw, 2002), can be used to better explain why the THINK interaction framework supported exploratory talk and the progressive movement of the participants' skills toward mathematical proficiency. As a reminder, Vygotsky posited that academic discourse held three functions: a cultural tool by which members of a society shared knowledge through language; a cognitive tool that aided individuals in personal reflections and a reorganization of his or her knowledge structures; and a pedagogical tool that provided opportunities for members to receive instructional assistance from equal or more abled peers (Mercer & Howell, 2012; Vygotsky, 1978). With respect to this study, the THINK interaction framework served as a cultural tool in that it promoted exploratory talk during which participants shared their understanding of the meaning of the problem, identified important information needed to solve the problem, and shared strategies for solving the problem. Further, the THINK interaction framework functioned as a cognitive tool as it required students to share their thoughts, justifications, and critiques of other members' and promoted self-reflection and internalization of ideas that often resulted in the participants' use of visual representations or verbalizations. Finally, the THINK interaction framework worked as a pedagogical tool (i.e., a heuristic) as it guided students to work collaboratively and often resulted in explanations of thoughts, strategies, and procedures. These collaborations resulted in students working together to understand and assist one another in problem-solving attempts. As such, the students engaged in collaborative ZPD, the actions of students of equal abilities working together to gain knowledge and understanding of a topic (Goos et al., 2002). Because the participants were students placed into a Tier II RTI mathematics class based upon their score on a universal screening instrument, they were considered to have somewhat equal mathematical abilities.

Although it cannot be discerned as to why participants experienced increased adaptive reasoning, strategic competence, or display of understanding of relationships among mathematical concepts, strategies, and procedures, evidence supported positive effects of the THINK interaction framework and the instructional arrangement of collaborative groups in which exploratory talk was promoted and encouraged.

Fidelity of the Implementation of the THINK Interaction Framework

Research on the fidelity of implementation (FOI) of instructional interventions suggests that the level of adherence to implement an intervention or curriculum as intended is correlated to student outcomes (Azano, Callahan, Oh, Missett, & Brunner, 2011). Findings by Loflin (2015) reinforced the notion that the level of FOI of a research-based curriculum has statistically significant effects on students' achievement. Though Mrs. Smith guided students through each component of the THINK interaction framework during Phase 2 of the study, the participants implemented the components with differing levels of fidelity. As illustrated in Table 26, the participant group implemented the Talk, How, and Notice components but rarely implemented the Identify (i.e., Lessons 9 and 13) and Keep (i.e., Lesson 7) components of the framework.

Table 26

A Comparison of the Use of the THINK Interaction Framework

Component	Author's Intention	Participant Implementation
Talk	Students are to discuss the problem with each other, discuss what the problem is asking them to do, and to determine what information is needed to solve the problem.	Students discussed what the problem was asking, often determined the necessary information in the problem, and debated the meaning of the mathematical concepts.
How	Individual students are to propose a strategy for how to solve the problem and to ask other students how their strategy will work and why they think so.	Students often proposed individual strategies for solving the problem and asked other participants to explain their strategies.

Table 26 (cont.)

Component	Author's Intention	Participant Implementation
Identify	Students are to collectively identify a strategy to use to work the problem, to discuss how to use the strategy, and and to determine if the strategy is working or choose a different strategy.	Students failed to collectively identify a strategy to use to solve the problem in all lessons except Lessons 9 and 13.
Notice	Students are to notice and discuss how the collective strategy helps them to solve and understand the problem.	Students explained and justified their thoughts, individual strategies, and solution paths. Students expressed difficulties in working toward a solution and critiqued each other's ideas and thoughts in an effort to seek clarification.
Keep	Students are to continue to think about the problem, discuss if there are alternative solution paths, and share their ideas with the group. Also, they are to determine if the solution makes sense.	Students failed to utilize this component.

Note. The author's intention is summarized as adapted from Thomas, K. (2006). Students THINK: A framework for improving problem solving. *Teaching Children Mathematics, 13,* 86-95.

A comparison of the author's intentions (Thomas, 2006) and participant implementation of the Talk component revealed that the students not only implemented the component as directed but they also extended the directions by discussing mathematical concepts as needed for clarification and understanding (see Table 26). This extension to the directions of the Talk component was often necessary to further students' conceptual knowledge that otherwise presented a roadblock and prevented forward movement toward solving the problem.

During the How component, the participants often implemented the framework with fidelity with some students proposing individual strategies. However, they failed to collectively identify a strategy during the Identify component and to implement the framework with a high level of fidelity. This failure to collectively identify a strategy often proposed initial difficulties as students worked toward a common goal of solving the problem in different ways. Such difficulties included misunderstandings concerning mathematical procedures and the use of strategies to solve the problem (e.g., the phone plan problem). However, rich discussions concerning students' thoughts often resulted during the Notice component as some members' needed clarification and justification of other's ideas (e.g., the dart game problem).

The students implemented the Notice component differently than intended by the authors of the THINK interaction framework. Whereas the authors intended for students to reflect on and share how the selected strategy helped them to solve and understand the problem, the participant group found opportunities during the Notice component to engage in meaningful exploratory talk including explanations and justifications of thoughts, strategies, solution paths, and difficulties experienced while solving problems. Further, participants often critiqued other members' explanations as they searched for clarification of concepts, strategies, or procedures. As such, the alternative use of the Notice component often sparked rich discussions that promoted intermental and intramental activity as evidenced by student work samples and verbalizations (e.g., the block problem). Because students implemented the THINK interaction framework differently than as intended by the authors, the use of the THINK interaction framework failed to achieve collaborative problem solving. To promote the use of the THINK interaction framework with fidelity, I offer a suggestion to amend the directions for the Talk, Identify, Notice, and Keep components. The altered directions of the Talk component would require students to identify any mathematical terms that may help them to better understand the problem. The altered directions in the Identify component would require the student to only identify, state, and explain the collectively identified strategy while the directions of the Notice component would, in addition to the current directions, require students to determine if the strategy was working, to explain their difficulties in trying to solve the problem, to continue to work using the selected strategy or to choose a different strategy and explain how to use it solve the problem (see Figure 16).

Original THINK Interaction Framework Components	Suggested THINK Interaction Framework Components
"TALK about the problem with one another. Describe the situation. Explain what the problem is asking. Talk about the important information.	"TALK about the problem with one another. Describe the situation. Explain what the problem is asking. Talk about the important information" (Thomas, 2006, p. 88). **Identify any mathematical terms that may help you better understand the problem.**
HOW can the problem be solved? Have each person share ideas for how to solve the problem. Ask others how and why their plan will work.	"HOW can the problem be solved? Have each person share ideas for how to solve the problem. Ask others how and why their plan will work" (Thomas, 2006, p. 88).
IDENTIFY a strategy for solving the problem. Use it. Talk about how to use your strategy. Is your strategy working, or do you need to choose another one to solve the problem?	"IDENTIFY a [collective] strategy for solving the problem" (Thomas, 2006, p. 88). **State the strategy aloud. Explain to one another how to use the strategy for solving the problem.**
NOTICE how your strategy helped you solve the problem. Have each person share how the strategy helped him or her understand and solve the problem.	NOTICE if the **collective strategy is helping you to solve the problem. If so, share with one another** "how the strategy helped" [you to] "understand and solve the problem" (Thomas, 2006, p. 88). **If not, discuss with one another the difficulties that you are experiencing. Continue to work toward solving the problem with the selected strategy or choose a different strategy and explain how to use it for solving the problem.**
KEEP thinking about the problem. Does your answer make sense? If you can think of another way to solve the problem, share it" (Thomas, 2006, p. 88).	"KEEP thinking about the problem. Does your answer make sense" (Thomas, 2006, p. 88)? **Are there other ways to solve the problem? If so, share a strategy that can be used and solve the problem using the different strategy.**

Figure 16. A comparison of the components in the original and suggested versions of the THINK interaction framework. Developed from "Students THINK: A Framework for Improving Problem Solving," by K. Thomas, 2006, *Teaching Children Mathematics, 13,* 86-95.

The group failed to implement the Keep component, and thus, missed valuable opportunities to extend their knowledge and to make sense of the problem in terms of the validity of solutions. Though FOI for the Keep component was nonexistent, the need for students to implement this component is valuable. In order to utilize the Keep component to further student knowledge, teachers must ensure the FOI of the THINK interaction framework by modeling the use of each of the components of the framework, stating expectations during the implementation of each component, and allowing for multiple days of instruction for investigation of alternative methods to solving the problem. Therefore, the updated prompts for the Keep component would include the original directions and require students to not only share a different strategy once the problem has been solved but to also use the new strategy for solving the problem. The suggested directions for the Notice component would situate the discussion in a ZPD framework and allow peer assistance as needed.

In summary, the level of FOI poses the question of the possible impact that the THINK interaction framework could have on participants' movement toward mathematical proficiency if implemented with a high level of fidelity. Additionally, educators should consider opportunities to further student learning and outcomes often associated with high levels of FOI (Azano et al., 2011; Loflin, 2015) of an intervention such as the THINK interaction framework.

Implications for Practice

Although a review of literature revealed challenges to teaching students at risk for mathematics difficulties using effective teaching practices (Gillies & Boyle, 2010; Kroeger & Kouche, 2006; Miller & Mercer, 1997), the results of this study offer insight into how the use of the THINK interaction framework as an intervention tool supported students in their engagement in mathematical discourse and provided support during mathematical problem-solving activities. First, the THINK interaction framework provided support and structure for students who lack appropriate social skills (e.g., passivity, learned helplessness, and impulsivity) to participate in small-group activities. The prompts of each of the components of the framework provided opportunities to speak and entry points into group conversations as each member was expected to contribute to the conversation. Therefore, vigilance by the teacher is required to ensure that all students implement the THINK interaction framework as intended to promote the development of appropriate social skills. Second, the use of the THINK interaction framework promoted exploratory talk as individual students were held publically accountable for their statements and encouraged to explain their reasoning. Such communication was effective in promoting reasoning abilities and encouraging mathematical understanding. Third, the implementation of the THINK interaction framework supported students in their understanding of the use of strategies. Last, teacher modeling of the use of the THINK interaction framework provided students who are at risk for mathematical difficulties with instructional support to use the heuristic to engage in inquiry-based activities that promote productive struggle. Therefore, teachers should consider the use of the THINK interaction framework as an intervention and instructional tool during small-group, inquiry-based activities to provide support and opportunities for students who are at risk for mathematical difficulties to engage in effective mathematical practices. Efforts to engage all students in learning and understanding of mathematics using effective teaching practices promotes equity and progress toward mathematical proficiency.

Future Areas of Research

This embedded multiple case study investigated the effectiveness of the THINK interaction framework to promote exploratory talk and movement toward mathematical proficiency. Though the results revealed that the THINK interaction framework supported exploratory talk, increased reasoning and logic abilities, and improved students' understanding of the relationships among mathematical concepts, procedures, and strategies, research is needed concerning the effects of the THINK interaction framework on students' level of mathematical proficiency when the framework is implemented with fidelity over an extended period of time. Though this study disclosed that implementation of the framework showed movement toward mathematical proficiency, exposure to the framework was for a limited amount of time. A longitudinal study in which a group of students were observed over a two-to-three year period could add to the results of this study and the literature base surrounding the use of heuristics to promote student achievement and development of sociomathematical norms.

Based upon the results of the study concerning how the frequencies of the types of talk differed throughout each phase of the study (see Table 9), another area of future research is to determine the effects of teaching students the difference between the types of talk. Helping students to understand the distinction between the three types of talk coupled with the use of the THINK interaction framework could help students in their understanding of the importance that mathematical discussions have on advancing their mathematical abilities.

Finally, the results of this study were based on the fact that students were introduced to a new task requiring understanding of new mathematical concepts and strategies for solving the task in each lesson. As such, the students were able to start fresh with each lesson, possibly impacting the dialogue during group discussions. Future research is needed to ascertain the impact of the THINK interaction framework once the unit of study progresses, and students have to rely on the knowledge gained in previous lessons. As reform-based curricula are implemented and required in many states, this research could add to the literature base and provide information to teacher preparation programs.

Meta-reflection: What I Learned About the Research of Discourse in a RTI Mathematics Classroom

As the researcher of this study, I find that reflection about the process of conducting educational research concerning discourse analysis of secondary students who are at risk for mathematical difficulties is both insightful and necessary in order to bring awareness to the topic. Through this experience, I learned much and grew as an educational researcher. First, an investigation concerning student discourse among a group of students requires passion for the topic, dedication to the process, flexibility in scheduling, and a lengthy time commitment. Second, the process of transcribing video/audio recordings and using an iterative approach to the analysis of the data provided insight to each of the cases. Specifically, using multiple iterations to code and to review the lesson transcriptions and other sources of data provided new and deeper understandings of each participant. Third, I learned as a seasoned teacher with 23 years of classroom experience that it is difficult, but absolutely necessary, to remove the lens of practitioner when conducting educational research. To explain, I will describe the following difficulties.

- The language used to communicate with colleagues concerning educational initiatives, duties, and topics associated with the teaching profession is likely considered to be slang or jargon and must be correctly and explicitly stated when writing a technical report. For example, data reports detailing state-mandated assessments often use the acronym, SWD, to indicate students with disabilities. However, a review of the extant literature revealed that the term, SLD, is the appropriate acronym to use when referring to students with specific learning disabilities.
- As a result of the information gleaned from my experiences as a seasoned teacher, administrator, and graduate student, it was initially challenging to reference statements that appear to be common knowledge derived from experience. For example, the recommendation that teachers establish group norms and ensure that students are individually accountable when working in collaborative groups is information not only found in the literature but that I often provide to educators when discussing the results of an evaluation.

Fourth, I learned that the organization of all documents was crucial to my productivity and forward movement in this process. Finally, I learned that educational research is both tedious and rewarding as I gained knowledge and insight as to ways that students who are at risk for mathematical difficulties better learn mathematics.

Chapter Summary

As states adopt and require implementation of reform-based standards in mathematics education, teachers must realize and engage in best practices for teaching mathematics. Such practices were outlined in the Mathematics Teaching Practices and included the necessity of facilitating meaningful discourse among students (NCTM, 2014). Thus, teachers must provide opportunities for students to engage collaboratively in problem-solving tasks and to promote mathematical talk that furthers students' understanding of mathematical concepts and abilities. The purpose of this study was to determine how the THINK interaction framework supported students' exploratory talk and movement toward mathematical proficiency.

The results of this study revealed the effectiveness of the THINK interaction framework to increase exploratory talk, mathematical understanding, logic, and reasoning abilities in part for the participants. Further, the findings revealed how the THINK interaction framework provided support for students' mathematical discussions, increased self-efficacy for students who exhibited passivity, and provided structure for students who lacked self-regulating mechanisms. Overall, this study added to the current literature on the use of a heuristic as a tool and an intervention to promote effective mathematical discourse among students who are at risk for mathematical difficulties.

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APPENDICES

APPENDIX A. PARTICIPANT SELECTION INTERVIEW PROTOCOL

Interviewer: Thank you for participating in my study. This interview will help me gather information about how students best learn mathematics. I am interested in hearing all of your thoughts.

Cognitive Aspects of Learning Mathematics

A1: How do you best learn mathematics?

A2: What does your teacher do to help you best learn and understand the mathematics that you study in class?

Social Aspects of Learning Mathematics

A3: What are your thoughts about working in groups?

A4: Can you explain to me how each member of the group knows what they are to do?

A5: How do members of the group work to solve the problem or task?

A6: What are the advantages, if any, of working in groups?

A7: What are the disadvantages, if any, of working in groups?

Pedagogical Aspects of Group Work

A8: What types of opportunities do you have in mathematics class to receive help from the teacher?

A9: What types of opportunities do you have in mathematics class to receive help from other students?

A10: How do these opportunities support you, if at all, in learning the mathematical content that you are studying?

A11: What opportunities do you have to explain your ideas to others? (Adaptive Reasoning)

Student Disposition Toward Mathematics

A11: Explain your thoughts about the usefulness of mathematics.

A12: Explain how you feel about studying mathematics.

A13: Is there anything else that you would like to share with me?

A14: Do you have any questions for me?

Student Information

A15: Please state your name to help me keep my records clear.

Thank you for your help in assisting me with my research study.

APPENDIX B. PARTICIPANT SELECTION INTERVIEW RUBRIC

Category	**3 Points**	**2 Points**	**1 Point**
A1: How do you best learn mathematics?	The student indicates that he or she best learns mathematics through discussions and student activities.	The student indicates that he or she best learns mathematics through individual hands-on activities.	The student indicates that he or she best learns mathematics through teacher lecture.
A2: What does your teacher do to help you best learn and understand the mathematics that you study in class?	The student names group activities as the method of instruction from which he or she learns best.	The student names individual and hands-on activities as the method of instruction from which he or she learns mathematics.	The student names teacher lecture as the instructional method from which he or she learns mathematics.
A3: What are your thoughts about working in groups?	The student indicates that he or she likes to work in groups.	The student indicates no preference of whether or not they like working in groups.	The student indicates that they do not like to work in groups.
A4: Can you explain to me how each member of the group knows what they are to do?	The student indicates that students have roles and responsibilities and are held accountable for individual and group learning.	The student does not indicate that students have roles and responsibilities or that they are held accountable for individual and group learning but that they work together.	The student indicates no knowledge of the structure of group work.
A5: How do the members of the group work to solve the problem or the task?	The student indicates that the group uses exploratory talk.	The student indicates that the group uses cumulative talk.	The student indicates that the group uses dispositional talk.
A6: What are the advantages, if any, of working in groups?	The students indicates and names advantages of working in groups.	Students learn through peer tutoring.	The students indicate that there are no advantages to working in groups.

A7: What are the disadvantages, if any, of working in groups?	The student names one to two disadvantages to working in groups.	The student names three to four disadvantages to working in groups.	The student names more than four disadvantages to working in groups.
A8: What types of opportunities do you have in mathematics class to receive help from the teacher?	The student indicates that the teacher offers help through posing questions.	The student indicates that the teacher offers help through tutoring and direct instruction.	The student indicates that he or she receives no help from the teacher.
A9: What types of opportunities do you have in mathematics class to receive help from other students?	The student indicates that he or she receives help from peers through group work or tutoring activities.	The student indicates that there are a minimal number of times that he or she receives help from peers through tutoring or group work.	The student indicates that he or she does not receive help from peers.
A10: How do these opportunities support you, if at all, in learning the mathematics content that you are studying?	The student indicates that he or she understands the concepts better when working with peers.	The student indicates that he or she can complete the assignment or activity.	The student indicates that the opportunities to receive help are not beneficial.
A11: What opportunities do you have to explain your ideas to others?	The student indicates that small group work activities allow him to help others or explain his or her ideas.	The student indicates small group work activities do not occur often to allow him or her to help others or to explain his or her ideas.	The student indicates that he or she has no opportunity to explain his or her ideas to others.
A12: Explain your thoughts about the usefulness of mathematics.	The student indicates that mathematics is used in everyday life.	The student indicates that knowing mathematics is useful for school purposes only.	The student indicates that knowing mathematics is not useful.
A12: Explain how you feel about studying mathematics.	The student indicates that he or she likes mathematics.	The student indicates no strong feelings for or against the study of mathematics.	The student indicates his or her dislike for mathematics.

APPENDIX C. POST-STUDY INTERVIEW PROTOCOL

Interviewer: I want to thank you once more for participating in my study. I am interested in hearing your thoughts about some things since your first interview.

Cognitive Features of Learning Mathematics

C1: After using the THINK Interaction framework, what are your thoughts about how you best learn mathematics?

C2: In what ways, if any, did the THINK interaction framework provide you with opportunities to better learn mathematics?

Social Features of Learning Mathematics

C3: How do you feel about working in groups since using the THINK interaction framework?

C4: How did group members participate in activities during the use of the THINK interaction framework?

C5: What are the advantages, if any, of working in groups?

C6: What are the disadvantages, if any, of working in groups?

Pedagogical Aspects of Group Work

C7: How has the THINK interaction framework changed your opportunity, if at all, to participate while working in a group in this class?

C8: What opportunities, if any, did you have to be the expert and teach your group mates?

Student Disposition Toward Mathematics

C9: Explain your thoughts about the usefulness of mathematics.

C10: Explain how you feel about studying mathematics.

C11: Is there anything else that you would like to share with me?

C12: Do you have any questions for me?

Student Information

C13: Please state your name to help me keep my records clear.

Thank you for your help in assisting me with my research study.

APPENDIX D: GROUP INTERVIEW PROTOCOL

Date of observation: ___________________ Lesson:_____________________

Interviewer: I am interested in learning more about what you meant in some of the things that you all said in solving the task or problem for this lesson. Your answers will help me to clarify my notes on what I observed in person and from viewing the video.

* Questions will be specifically worded concerning to what the participants said and did as observed in person by the researcher, the video recorded lesson, and the artifact produced by the LiveScribe® Pen. Possible questions are listed below.

D1: Can you explain what you meant when you said . . .?

D2: Can you explain what you were thinking when you . . .?

D3: Why did you say or do this?

APPENDIX E. GROUP OBSERVATION PROTOCOL

Date of observation: ____________________ Lesson:______________________

Type of Talk: D, C, E	Participant(s)	Artifact (verbal, sketch, written comments)	Researcher Thoughts

APPENDIX F. LIST OF DATA SOURCES WITH ABBREVIATIONS

Data Source	Abbreviation
Participant Selection Interview Protocol	PSI
Post-study Interview Protocol	PTI
Group Interview Protocol	GIP
Group Observation Protocol	GOP
Researcher Journal	RJ
Student Work Artifacts	SW
Video Transcriptions	TS

APPENDIX G. THINK INTERACTION FRAMEWORK

Talk	**What is the problem asking? What important information do you need? Talk to the members of your group and determine the information needed.**
How	**How do YOU propose to solve the problem? Describe how you would solve the problem to your group.**
Identify	**With your group members, identify a strategy that you will use to solve the problem. Write the proposed solution strategy that your group has decided upon. Solve the problem using the strategy identified by the group.**
Notice	**Explain in writing how the strategy used helped YOU to solve and understand the problem. Each person in the group is share how the strategy helped them to understand and solve the problem.**
Keep	**Explain in writing whether or not your solution made sense. Is there another way to solve the problem? If so, explain in writing your idea. Share your ideas with your group members.**

Note. Adapted from Thomas, K. (2006). Students THINK: A framework for improving problem solving. *Teaching Children Mathematics, 13*, 86-95.

APPENDIX H. LESSON PLAN FOR MODEL LESSON

Lesson Title: A Game of Simplified Football

Setting: Tenth-grade Tier II mathematics class using predetermined heterogeneous groups

Introduction: Mrs. Stevens will project and introduce students to the THINK Interaction Framework Heuristic. She will explain students' roles and responsibilities at each step in the framework.

Materials: THINK Interaction Framework Heuristic, the worksheet with the problem, computer, LCD projector, poster paper, and markers

Learning Goals:

- Students will understand multiples of three and seven or a combination of the two.
- Students will use prior knowledge structures to aid in finding the unknown value.
- Students will work collaboratively in groups and use exploratory talk to solve the problem.

Task: Students will solve the following problem:

In a game of simplified football, a touch down receives a score of seven points and a field goal a score of three points. What is the largest game score not possible?

Completing the Task: In what ways can the task be solved?

- The students may list the numbers three through 35 and cross off multiples of three, multiples of seven, or sums of multiples of three and seven or a combination of the two numbers.
- The student may sketch possible solutions.

Evidence of Student Understanding: What will students say, do, etc. so that will ensure that they understand the context of the problem?

- Students will use vocabulary including multiples and sum.
- Students will understand that a score that is a multiple of seven or three is not the largest score or that a sum using the two in multiples are not the score.
- Students will understand that all whole numbers after 11 are possible scores.

Instructional Support – Tools, Resources: What tools will students use to gain entry into solving the problem?

- Students will work in groups and will use the THINK Interaction Framework Heuristic to guide them through the problem solving process. Students may use their books as a resource if needed.

Instructional Support – Teacher: What questions will I ask that will support students' thinking and exploration of the problem?

- The teacher will provide scaffolding to groups as needed and to the whole group if necessary through posing questions and drawing the sketch with help from the group if needed.
- How can the sketch of the figure help you in understanding the information that you have?
- What is the problem asking you to do?
- Can you provide a different sketch that might aid you in solving the problem?
- What do you know about possible scores or solutions?
- For students who don't know how to start, ask them how they could use a list to aid them.
- For students who finish early, ask them to find the largest score not possible if you could add a point after attempt for a score of one point.
- To ensure that students remain engaged in the task, ask questions that require them to explain aspects of their solution to you.

Sharing Out: Choose groups who have similar solution paths with the one that is partially correct and then the one that is correct. Choose groups that have different solution paths to present.

Connecting Responses: Remember to ask specific questions regarding the different solutions that help the students to make sense of the mathematical ideas as they relate to the goals of the lesson.

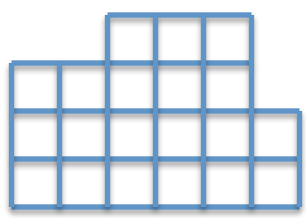
APPENDIX I: TASKS USED FOR TRAINING MRS. SMITH

Training Lesson	Problem	Answer	Source
1	Game of Simplified Football: In a game of simplified football, a touch down results in a score of a 3. What is the largest game score not possible?	11 Points	This problem was part of a problem set in MATH 6320: Mathematical Problem Solving at Middle Tennessee State University in Spring 2011.
2	A Coin Problem: Bert received a total of $5.29 from his friends when he asked them to bring in their loose change. The surprise was that each friend brought in the same amount of money, and each brought in exactly 6 coins. How many nickels did Bert collect?	46 Nickels	Mathematics Teacher, December 2013/ January 2014, p. 359.
3	A Grade Problem: The average of five quiz grades is 10. When the lowest grade is dropped and the new average is calculated, it turns out to be 11. What was the score of the dropped grade?	6	Mathematics Teacher, February 2014, p. 440.
4	A Bridge Problem: Four women must cross a treacherous bridge over a deep ravine in enemy territory in the middle of the night. The treacherous bridge will hold only two women at once and it is necessary to carry a lantern while crossing. Persons must stay together while crossing. One of the women requires five minutes for the trip across, one takes ten minutes, a third requires 20 minutes,	The women can cross successfully if on the first trip the women who can make the trek in five and ten minutes cross with the woman who can make the trek in five minutes returning. The women who cross in twenty and twenty-five minutes cross. The woman who can	This problem was part of a problem set in MATH 6320: Mathematical Problem Solving at Middle Tennessee State University in Spring 2011.

	and the last takes 25 minutes (each of them can walk slower if necessary but no faster). Unfortunately, they have only one lantern among them. How can they make the crossing if they have only 60 minutes before the bridge is destroyed?	make the trek in ten minutes goes back across into enemy territory to get the woman who can make the journey in five minutes. All women cross with not a minute to spare.	

APPENDIX J: PROBLEMS AND TASKS FOR THE STUDY

Phase	Lesson	Problem	Answer	Source
1	1	November 3 is the last of seven days of this year for which the product (month • day) divides the year (2013). Find the other six such dates for which such divisibility occurs on exactly seven dates in the year 2013.	Jan 3rd, Jan 11, March 1st, March 11, November 3, November 1	Mathematics Teacher, Nov 2013, p. 280.
1	2	Determine the number of dimples on a particular golf ball given that the hundreds digit and the tens digit are the same prime number, the sum of all three digits is 12, and the number is divisible by 7.	336	Mathematics Teacher, Nov 2013, p. 280.
1	3	We know that 4 tablespoons equal 1/4 cup. How many tablespoons will equal 2/3 cup?	32/3	Mathematics Teacher, Nov 2013, p. 280.
1	4	Jim had an appointment at 1:00 p.m. at a location 50 miles from his home. Driving at the speed limit of 30 mph, he arrived 20 minutes late. What time did Jim leave his house?	11:40 a.m.	Mathematics Teacher, Nov 2013, p. 280.
1	5	The numbers 1-9 are written on nine index cards, one number per card. Arrange the cards into three piles so that the sum of the numbers in each pile is 15.	{1, 5, 9; 3, 4, 8; 2, 6, 7} or {2, 4, 9; 1, 6, 8; 3, 5, 7}	Mathematics Teacher, February 2014, p. 440.
2	6	A toddler has 2 blue blocks, 2 red blocks, 1 purple block, and 1 yellow block. If the blocks are stacked vertically, how many different stacks of 3 blocks can the toddler make? 	42 Different Stacks. Each stack has either 3 different colors or 2 blocks of the same color and 1 block of another color.	Mathematics Teacher, April 2016, p. 600.

2	7	The mean on an exam in a high school mathematics class was 71. The teacher realized that she made a mistake in grading one question, so she added 1 point to each student's grade. The sum of all the grades became 936. How many students were in the class?	13	Mathematics Teacher, Dec 2013, p. 359.
2	8	A school bus has four exits. In an emergency, 1 child can leave the bus through each exit every 8 seconds if all the children leave their backpacks behind. If 53 children were on the bus, how long would it take for all of the children to exit the bus in an emergency?	1 min, 52 seconds	Mathematics Teacher, April 2016, p. 600.
2	9	Divide the figure into four congruent figures using only the lines shown as "legal" dividing lines. 		Mathematics Teacher, April 2016, p. 601.
2	10	One phone company charges $12 per month plus $.05 per minute after the first 500 minutes. A second phone company charges $10 per month plus $.04 per minute after the first 300 minutes. How many minutes would a subscriber have to use so that the cost of the two plans is the same for a given month?	1100 minutes	Mathematics Teacher, Dec 2013, p. 359.
2	11	A game using nickels, dimes, and quarters requires that there must be twice as many dimes as nickels and twice as many quarters as dimes. How many	63 coins (n=9, d=18, q=36)	Mathematics Teacher, Nov 2013, p. 280.

		coins would you need to create the largest possible amount less than $12.00?		
2	12	Pam and Bob each threw six darts at a dartboard, and they earned the same number of total points. The results are that 2 darts landed for 1 point each, 4 darts landed for 5 points each, 3 darts landed for 10 points each, 2 darts landed for 25 points each, and 1 dart landed in the center for 50 points. If Pamela scored 26 points on her first two throws, which player hit the center of the board?	Bob	Mathematics Teaching in the Middle School, December, 2015/ January 2016 (v21), p. 271
2	13	The following problem comes from a book, Everyday Algebra: "At a county fair John tried a game of target shooting. He received 10 cents for each hit but had to pay 5 cents each time he missed. If 30 shots cost him 15 cents, how many hits did he score?"	9	Mathematics Teacher, Nov 2013, p. 281
2	14	What is the smallest natural number divisible by each of the integers 1-10, inclusive?	2520	Mathematics Teacher, Feb 2014, p. 441
2	15	Gina has a bouquet of flowers that includes 12 irises, each with 5 petals and 12 roses each with 6 petals. She picks off the petals of each flower, alternating between the irises and the roses. If she begins with "He loves me" as she plucks a iris petal, what does Gina find out at the end?	He loves her not. The 24 flowers have a total of 12(5) +12(6) = 132 petals. She says "he loves me not" for each even-numbered petal.	Mathematics Teacher, Nov 2015, p. 278

APPENDIX K: INSTITUTIONAL REVIEW BOARD APPROVAL

IRB
INSTITUTIONAL REVIEW BOARD
Office of Research Compliance,
010A Sam Ingram Building,
2269 Middle Tennessee Blvd
Murfreesboro, TN 37129

MIDDLE TENNESSEE STATE UNIVERSITY

IRBN001 - EXPEDITED PROTOCOL APPROVAL NOTICE

Wednesday, March 15, 2017

Principal Investigator	**Samantha Stevens** (Student)
Faculty Advisor	Angela Barlow & Sara Bleiler-Baxter
Co-Investigators	NONE
Investigator Email(s)	*sas3r@mtmail.mtsu.edu; angela.barlow@mtsu.edu; sarah.bleiler@mtsu.edu*
Department	Mathematical Sciences
Protocol Title	***The effects of THINK interaction framework as an intervention to support students' engagement in mathematical discourse in movement toward mathematical proficiency***
Protocol ID	**16-2202**

Dear Investigator(s),

The above identified research proposal has been reviewed by the MTSU Institutional Review Board (IRB) through the **EXPEDITED** mechanism under 45 CFR 46.110 and 21 CFR 56.110 within the category *(7) Research on individual or group characteristics or behavior* A summary of the IRB action and other particulars in regard to this protocol application is tabulated as shown below:

IRB Action	APPROVED for one year
Date of expiration	3/17/2018
Participant Size	15 (FIFTEEN)
Participant Pool	Grundy County High School Students
Exceptions	NONE
Restrictions	**1. Mandatory informed consent** **2. Destroy/delete audio data after transcribing data** **3. Approved to recruit Grundy County high school**
Comments	Approval notice updated to new template format – MP 03/15/2017

This protocol can be continued for up to THREE years (3/17/2019) by obtaining a continuation approval prior to 3/17/2018. Refer to the following schedule to plan your annual project reports and be aware that you may not receive a separate reminder to complete your continuing reviews. Failure in obtaining an approval for continuation will automatically result in cancellation of this protocol. Moreover, the completion of this study MUST be notified to the Office of Compliance by filing a final report in order to close-out the protocol.

Continuing Review Schedule:

Reporting Period	Requisition Deadline	IRB Comments
First year report	2/17/2017	Continuing review completed in accordance with Category 8 of Expedited Procedures. The protocol is approved to continue for ONE additional year
Second year report	2/17/2018	TO BE COMPLETED
Final report	2/17/2019	TO BE COMPLETED

Post-approval Protocol Amendments:

Date	**Amendment(s)**	**IRB Comments**
None	NONE	NONE

The investigator(s) indicated in this notification should read and abide by all of the post-approval conditions imposed with this approval. Refer to the post-approval guidelines posted in the MTSU IRB's website. Any unanticipated harms to participants or adverse events must be reported to the Office of Compliance at (615) 494-8918 within 48 hours of the incident. Amendments to this protocol must be approved by the IRB. Inclusion of new researchers must also be approved by the Office of Compliance before they begin to work on the project.

All of the research-related records, which include signed consent forms, investigator information and other documents related to the study, must be retained by the PI or the faculty advisor (if the PI is a student) at the secure location mentioned in the protocol application. The data storage must be maintained for at least three (3) years after study completion. Subsequently, the researcher may destroy the data in a manner that maintains confidentiality and anonymity. IRB reserves the right to modify, change or cancel the terms of this letter without prior notice. Be advised that IRB also reserves the right to inspect or audit your records if needed.

Sincerely,

Institutional Review Board
Middle Tennessee State University

APPENDIX L: PERMISSION LETTER TO REPRINT STRANDS OF MATHEMATICAL PROFICIENCY IMAGE

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April 13, 2017 **Reference #: 04131702**

Samantha Stevens

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