ELEMENTARY MATHEMATICS TEACHERS' FEEDBACK PRACTICES: A MULTIPLE CASE STUDY

by

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A Dissertation Submitted to the Faculty of the College of Graduate Studies at Middle Tennessee State University in Partial Fulfillment of the Requirements for the Degree of Doctorate of Philosophy in Mathematics and Science Education

Middle Tennessee State University

December 2020

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This work is dedicated to my husband Andrew and my two beautiful daughters,

Tabytha and Zelaney.

Without your support and love, I absolutely could not have done this!

ACKNOWLEDGEMENTS

First and foremost, I want to thank my family and friends for dealing with me throughout this crazy but amazing journey. I would not have been able to survive without your constant encouragement, questions, words of wisdom, eye rolling, nagging, and above all, love. I especially want to thank my husband, Andrew Hartland, and my sister, soon-to-be Dr. Cynthia Tiongco, who have selflessly supported me through very difficult times in my life and helped me to see the light at the end of the tunnel. Additionally, my daughters Tabytha and Zelaney who believed in me from the very beginning and supported me with hugs and kisses. Remember, you can do anything you put your heart and mind to. I love you.

I would like to thank Dr. Angela Barlow and Dr. Alyson Lischka for continuing to support me, even when I seemed to disappear for months at a time, or when I failed to listen to your feedback (which, yes, is ironic, as my study focused on feedback). Thank you for your continued guidance and counseling throughout my doctoral journey and the array of opportunities you have provided me. I am the amazing mathematics educator because of you, and I am forever indebted to you both. I would also like to thank my committee members, Dr. Nancy Caukin, Dr. James Hart, and Dr. Dovie Kimmins. Thank you for providing valuable feedback and believing in me throughout this process.

To my colleagues at MTSU, thank you for all the long hours of intellectual conversations, collaborating, laughing, yelling, crying, eating, drinking, and believing in Vanilla Jesus, even when no one was around. I am profoundly grateful for your friendships and support over the years. This is especially true of Dr. Matt Duncan, Dr. Tasha Frick, Dr. Natasha Gerstenschlager-Wilson, Dr. Lucy Watson, Dr. Chris

Willingham, and soon-to-be Dr. Candice Quinn. I absolutely could not have survived without you!

To everyone who has helped me throughout my educational journey, I want to thank you for the time and effort you have invested in me. Thank you for the multitude of phone calls, Zoom meetings, and accountability checkups which kept me on track. Finally, to the two participants in my study who welcomed me into their classrooms and allowed me to observe their mathematics instructional practices, I will be forever thankful.

ABSTRACT

Feedback is essential in the mathematics classroom for conveying information to the learner about their actions intended to make a connection between what students understand and what is meant for them to understand. In fact, one of the most beneficial things teachers can do in the mathematics classroom to improve student achievement is to provide feedback that identifies the goal, assesses students' current understandings, and addresses discrepancies in the learning process. Feedback is essential for helping students move forward in their learning, and the beliefs teachers hold could potentially affect the way they provide information to their students. However, the ways in which teachers provide feedback during mathematics instruction and their own implicit beliefs are often overlooked as contributors to the various types of feedback they provide.

The purpose of this study was to examine the ways in which elementary teachers provide feedback during mathematics instruction. I used a multiple case study to explore two elementary teachers' feedback practices by type and level during their mathematics instruction. The results of this study revealed that (1) although both participants ascribed to an incremental theory, they demonstrated varying commitments to providing self-level feedback, (2) one participant provided all three types of feedback within one classroom observation on multiple days, and (3) both participants provided little to no feedback directed at the process and self-regulation levels overall. The implications of these results are provided.

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CHAPTER I: INTRODUCTION

Background of the Study

Over the past three decades, policy in education has focused on improving mathematics scores of American children (National Assessment of Educational Progress [NAEP], 2015). In 2015, NAEP showed that mathematics scores minimally increased from 1990 to 2013, with approximately 33% of eighth-grade students in the United States at or above the national average by 2013. During that time, there was a national emphasis on preparing students for success after secondary school by incorporating technology, emphasizing skills, and providing opportunities for peer collaboration (Kloosterman, 2010). Around the same time, common mathematical standards were introduced, stressing conceptual understandings and mathematical reasoning skills needed to solve real-world problems (e.g., Common Core State Standards Initiative [CCSSI], 2010). Since 2013, however, the percentage of students who are at or above the national average has not changed, nor has the national average increased (NAEP, 2015).

Despite efforts to improve these scores, students in the United States have shown an overall weakness for solving mathematical problems and engaging in cognitively demanding tasks (Organisation for Economic Co-operation and Development [OECD], 2013a). In 2012, over a quarter of the students in the United States struggled with applying the basic skills and procedures learned in the classroom to a mathematical context appropriately (OECD, 2013a). Thus, it is critical that the mathematics taught in schools aligns with the mathematics needed to manage everyday mathematical challenges (National Research Council [NRC], 2001; U.S. Department of Education, 2014) and to meet the mathematical demands of the workplace (Boaler, 2015; Tariq, Qualter, Roberts, Appleby, & Barnes, 2013).

The number of post-secondary jobs in the United States that call for higher mathematical skills is rapidly growing (Achieve, 2013). Many organizations continue to require large-scale data analysis and mathematical modeling skills to meet their current needs and future potential as organizations (Frejd & Bergsten, 2016). Thus, large numbers of people with the necessary experience in mathematics-related fields and subfields are needed to satisfy the demand for these sophisticated mathematical job requirements (Duchhardt, Jordan, & Ehmke, 2015; Hoyles, Wolf, Molyneux-Hodgson, & Kent, 2002; National Council of Teachers of Mathematics [NCTM], 2000). Engagement with mathematical concepts in the classroom is essential for this type of generalization of learning and application to a variety of contexts (NCTM, 2000). If students are unable to meaningfully engage with these concepts in the classroom, they are less likely to apply them outside of the classroom where they are required to make their own mathematical decisions in a less structured environment (OECD, 2009).

Current teaching standards for mathematics in the K-12 mathematics classrooms exemplify years of collective reorganization of standards that support students in building the mathematical knowledge necessary for post-secondary work (Hoyles et al., 2002; NCTM, 2007; OCED, 2009; Yin & Lu, 2014). In 1991, NCTM released the *Professional Standards for Teaching Mathematics* that were intended to provide "major shifts in the environment of mathematics classrooms that are needed to move from current practice to mathematics teaching for the empowerment of students" (p. 3). These standards for teaching mathematics emphasized the mathematics-learning environment, including the role of teachers and students, worthwhile mathematical tasks, tools for enhancing discourse, and the analysis of teaching and learning. Within the analysis standard, teachers were encouraged to observe what and how students were learning in order to "help teachers, on the spot, tailor their questions or tasks to provoke and extend students' thinking and understanding" (NCTM, 1991, p. 63). In this regard, the *Professional Standards for Teaching Mathematics* highlighted ways in which teachers could respond to students' actions by using strategic questioning to assess their understanding (NCTM, 1991).

In a continued effort to improve the overall teaching and learning of mathematics, NCTM (2007) released a set of mathematical teaching standards in *Mathematics Teaching Today*. This document extended the previous teaching standards (NCTM, 1991) by adding, among other things, further assessment of students' learning and essential instructional practices that support the mathematical learning needs of students. Teachers were encouraged to gather an assortment of information, ask purposeful questions, and make immediate decisions as a means of assessment (NCTM, 2007). In doing so, teachers could begin to scaffold students' new ideas about the mathematics, thus allowing students to build a greater conceptual knowledge base (Jung, Diefes-Dux, Horvath, Rodgers, & Cardella, 2015).

More recently, NCTM (2014) published the Mathematics Teaching Practices for the purpose of making a connection between the Common Core State Standards for Mathematics (CCSSI, 2010) and other mathematics standards for students to the instructional practices teachers use to implement the student standards in their classrooms. Derived from a long history of experience and research, NCTM designed the Mathematics Teaching Practices to provide teachers with a "core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics" (NCTM, 2014, p. 9). The eight Mathematics Teaching Practices focus on the practices in which teachers engage before, during, and after a mathematics lesson, enabling students to build a stronger mathematical foundation. Mathematics Teaching Practices include various methods of eliciting student understanding through the facilitation of discourse, posing purposeful questions, and supporting productive struggle to adjust instruction according to the learning needs of students (NCTM, 2014).

Embedded within the Mathematics Teaching Practices (NCTM, 2014) and previous standards (NCTM, 1991, 2007) is an emphasis on providing information related to student understandings and performance. This is often described as feedback. Feedback is the information conveyed to learners about their actions (Hattie & Timperley, 2007; Shute, 2008) that is intended to make a connection between what students understand and what is meant for them to understand (Sadler, 1989). The information educates students as to how they are doing relative to the learning goals of the lesson (Brookhart, 2008) and how they might modify their work to reach these goals (Jung et al., 2015). Collectively, these ideas suggest that feedback is the informational response given by teachers that supports student learning.

One of the most beneficial things teachers can do to provide a foundation for student learning is to provide feedback that answers the questions: "Where am I going?

How am I going? and Where to next?" (Hattie & Timperley, 2007, p. 102). When teachers provide feedback in this way, they begin by assessing students' understanding relative to the learning goals and then make decisions as to what information must be conveyed to the students to reach these goals (Hattie & Timperley, 2007; Jung et al., 2015). Many studies have shown feedback to be one of the most influential factors on student achievement; however, there are conflicting results and inconsistent patterns among these studies (Black & Wiliam, 1998; Hattie & Timperley, 2007; Kluger & DeNisi, 1996; Shute, 2008). These inconsistencies may be attributed to the different ways feedback is given in the classroom as well as the teacher's reasoning for why they provide feedback in a particular way (Shute, 2008). For example, a teacher may provide written feedback in the form of a letter grade or a lengthy explanation orally on how students could fix their mistakes. In addition, a teacher may provide feedback according to their own learning experiences or their initial judgement of the students' ability levels (Dweck, 2006; Education Week Research, 2016).

Personal experiences, cultural contexts, attitudes, or implicit beliefs held by teachers about learning mathematics may also contribute to a teacher's judgment when providing feedback (Brown, Lake, & Matters, 2011). The "different assumptions people may make about the malleability of personal attributes" (Dweck, Chiu, & Hong, 1995, p. 267) are known as a person's implicit beliefs. A person's implicit beliefs can impact interpretations or judgments of others and reactions to specific events. In fact, Boaler (2016) claimed that out of all contributing factors, teachers' implicit beliefs may have the most influence on the information conveyed in the classroom. Often teachers believe they have little influence on their students' mathematical ability (Dweck, 2006; Dweck et al., 1995). As a result, teachers may only provide feedback that focuses on steps and procedures to get a solution, rather than the process which led the students to the solution (NCTM, 2014). Teachers' underlying assumptions of students influence their decisions to provide feedback, even though it is unclear how or why (Dweck et al., 1995).

The implicit beliefs held by individuals typically fall in two categories: entity and incremental. A person who holds an entity theory believes that a person's attributes are non-malleable or cannot be improved, whereas a person who holds an incremental theory believes that a person's attributes are malleable and can be improved (Dweck et al., 1995). Teachers who hold an entity theory, often called a fixed mindset, believe that students can learn new mathematical concepts; however, the student's level of mathematical ability or performance will stay the same (Dweck et al., 1995). In contrast, teachers who hold an incremental theory, often called a growth mindset, believe that a student's mathematical ability or performance can be developed through hard work and effort. Although several empirical studies have examined students' implicit theories (e.g., Blackwell, Trzesniewski, & Dweck, 2007; Calarco, 2014; Good, Aronson, & Inzlicht, 2003), there is a lack of empirical studies that examine the role of teachers' implicit theories when approaching their pedagogical practices (Rattan, Good, & Dweck, 2012).

In a collection of studies conducted by Rattan et al. (2012), researchers surveyed undergraduate students and graduate teaching assistants in mathematics-related fields. Participants reported the types of feedback they believed that they would provide if they were seventh-grade mathematics teachers. Using various online surveys, article readings, and hypothetical situations, the authors concluded that when participants had a “fixed view of intelligence, [it] led them to express their support and encouragement in unproductive ways” (Rattan et al., 2012, p. 736). These studies, however, were limited to participants who were not actual mathematics teachers.

Problem Statement

Given the need for students to have the mathematical problem-solving skills necessary for success in today’s workplace (Achieve, 2013; Hoyles et al., 2002; OECD, 2009; U.S. Department of Education, 2014), students’ thinking must be extended to explore new ways of understanding. To be successful, students must be given opportunities to engage with meaningful mathematics and be given feedback to help them move forward in their learning and understanding (Boaler, 2015; Tariq et al., 2013). Feedback in the mathematics classroom educates students as to how they are doing relative to the learning goals of the lesson (Hattie & Timperley, 2007) and how they might modify their work to reach these goals (Jung et al., 2015). Unfortunately, there is little research that focuses on the feedback practices provided by teachers in the mathematics classroom (Jung et al., 2015) or the individual differences that may contribute to the way they provide feedback (Rattan et al., 2012). Although implicit theories have been shown to be a mediator of students’ actions in the classroom (Blackwell et al., 2007; Good et al., 2003; Rattan et al., 2012), teachers’ implicit theories have been overlooked as individual differences that may contribute to the various ways they provide feedback (Rattan et al., 2012).

Purpose of Study

Given the potential for teachers' implicit theories to have an impact on the type of feedback they provide to students (Hattie & Timperley, 2007; Shute, 2008), the purpose of this study was to closely examine whether teachers' implicit theories act as a mediator of the types of feedback given during mathematics instruction. To this end, the following research question is offered: How does an elementary mathematics teacher's implicit theory mediate the feedback given during mathematics instruction, if at all?

Significance of the Study

This study was significant in two ways. First, the study focused on the ways elementary mathematics teachers naturally provide feedback in their classrooms. Research conducted in a realistic setting creates a social environment where students and teachers naturally interact with one another and students can socially construct their mathematical knowledge. The natural classroom setting provided a necessary context for understanding how teachers provide feedback during mathematics instruction in that setting (Patton, 2015).

Second, the study focused on the individual differences which may contribute to the various ways elementary teachers provide feedback during mathematics instruction. With little existing research on why teachers provide different types of feedback (Rattan et al., 2012), this study provided insight into factors that may attribute to the different ways teachers provide feedback. The results serve to inform mathematics teacher educators who design professional development aimed at supporting teachers' feedback practices.

Theoretical Frameworks

There were two theoretical frameworks that combined to form the conceptual framework for the study. Models of feedback and implicit theories were used as theoretical frameworks to understand the distinct teacher feedback practices in two elementary mathematics classrooms. Observed feedback was categorized according to Hattie and Timperley's (2007) Model of Feedback to Enhance Learning including the types (i.e., feed up, feed back, and feed forward) and levels (i.e., task, process, self-regulation, and self) in which the feedback was directed. The second model, implicit theories (Dweck et al., 1995), was used to select the participants and served as a potential basis for understanding the individual differences that may contribute to the ways elementary mathematics teachers provide feedback.

Definitions

This section includes important terms I used throughout the dissertation study.

Classroom Environment

The term "classroom environment" was used to describe the interrelated physical, teaching, learning, and motivational environments in the classroom (Peng, 2016). The physical environment included the organization of the seats, desk, lighting, and bulletin boards, while the teaching environment included teacher-student interactions and relationships, as well as the implementation methods used (Peng, 2016; Yin & Lu, 2014). Implementation methods were comprised of the materials used for implementation, such as a white board, calculators, and manipulatives (Peng, 2016). The learning and motivational environment included the enthusiasm and willingness of all individuals to participate in the classroom environment (Peng, 2016).

Classroom Instruction

The term “classroom instruction” was used to describe the instructional approach during the time between when the teacher starts and ends the mathematics lesson for the day. Types of classroom instruction included, but were not limited to, direct lecture, large groups, small groups, independent work, and centers (Brookhart, 2008).

Cognitively Demanding Task

The term “cognitively demanding task” will be used to describe a task that was intended to elicit a variety of student responses and provide opportunities for student learning (NCTM, 2014). The cognitive demand of a task can be classified according to the demand it has on students’ thinking (NCTM, 1991). Tasks with a lower cognitive demand involve memorization or following a set of pre-determined procedures. Tasks with a higher cognitive demand are tasks that engage students in active inquiry and support students in using procedures that are meaningful and help build conceptual understanding (NCTM, 1991, 2014).

Feedback

The term “feedback” was used to describe the information given by teachers to the learners about their actions (Hattie & Timperley, 2007; Shute, 2008) that is intended to make a connection between what students understand and what they are intended to understand (Hattie & Timperley, 2007). Feedback can be conveyed in various modes including written, oral, and gestures (Brookhart, 2008), and directed at different levels

(Hattie & Timperley, 2007). Feedback can also be classified by types (i.e., feed up, feed back, and feed forward), which answer major questions to help students progress towards the learning goal (Hattie & Timperley, 2007). For the purposes of my study, feedback was used to describe the general form of feedback provided to students.

Feed up. "Feed up" answers the question, "Where am I going?" (Hattie & Timperley, 2007, p. 88), describing the information given to students about what the learning goal is and what the result looks like once they arrive at the goal.

Feed back. "Feed back" answers the question, "How am I going?" (Hattie & Timperley, 2007, p. 89), related to a "task or performance goal, often in relation to some expected standard, to prior performance, and/or to success or failure on a specific part of the task" (Hattie & Timperley, 2007, p. 89). This is different from the general form of feedback.

Feed forward. "Feed forward" answers the question, "Where to next?" (Hattie & Timperley, 2007, p. 90), describing the information given to students that completes the feedback cycle requiring students to apply the feedback they previously received.

Chapter Summary

This chapter included an overview of the need for students to engage with mathematics at higher levels, the significance of feedback in the mathematics classroom that supports students in doing so, and the need for empirical studies that examine the role of teachers' implicit theories on their feedback practices. The following chapter will provide a review of the literature that was essential for supporting the study.

CHAPTER II: LITERATURE REVIEW

Introduction

Learning is driven by what teachers and students do in the classroom (Black & Wiliam, 1998) where students build mathematical knowledge by discussing their thinking with peers and receiving meaningful feedback from teachers (Hudson, Miller, & Butler, 2006). However, teachers are frequently unaware of the ways in which they provide feedback in the classroom and whether their feedback is meaningful (See, Gorard, & Siddiqui, 2016). Given the potential for various factors to have an impact on the type of feedback offered to students (Boaler, 2015, 2016; Rattan et al., 2012; Shute, 2008), the purpose of this study was to examine how elementary teachers provide feedback during mathematics instruction. To this end, the following literature review includes an overview of the learning goals of mathematics instruction including how students learn mathematics and the supporting instructional practices. Various definitions of feedback and a model of feedback containing three types and four levels designed by Hattie and Timperley (2007) will also be examined. The ways in which the model of feedback aligns with reform-based instructional practices and different modes of feedback will also be discussed. Finally, a summary of the empirical research on students' and teachers' implicit theories will be reviewed.

Foundations for Mathematical Learning

In 1989, NCTM released the *Curriculum and Evaluation Standards for School Mathematics,* which placed an emphasis on building a conceptual understanding of mathematics for students. At the time, the level of mathematical rigor in the United States was not competitive with other countries, and the mathematics curriculum was not focused on supporting students' understanding of how mathematics could be applied to the world around them (Darling-Hammond, 2010; NCTM, 2009; Schoenfeld, 2002). In a continued effort to address these issues, NCTM (2000) released a revised set of mathematics standards titled *Principles and Standards for School Mathematics* (*PSSM*). The *PSSM* introduced the Process Standards which described the processes in which students should engage to learn meaningful mathematics (NCTM, 2000; Schoenfeld, 2002). The highlighted processes included problem solving, reasoning and proof, communication, connections, and representation. To support students in engaging in these processes, NCTM shifted their focus to the instructional strategies needed to successfully implement the Process Standards. Thus, in 2014 NCTM released *Principles to Actions: Ensuring Mathematical Success for All* that detailed fundamental components of mathematics instruction. These reform-based instructional practices, referred to as Mathematics Teaching Practices (NCTM, 2014), provided a framework for teachers that would further support students learning mathematics through the ideas embedded within the *PSSM* (NCTM, 2000). Together, the Process Standards (NCTM, 2000) and the Mathematics Teaching Practices (NCTM, 2014) represented a collective mission supported by years of research regarding how best to support students learning mathematics.

The following section details how students learn mathematics by engaging in the Process Standards (i.e., problem solving, reasoning and proof, communication, connections, and representation; NCTM, 2000). The five strands of mathematical proficiency (NRC, 2001) will also be discussed regarding how they relate to the process standards. This will be followed by the instructional practices that support students in achieving mathematical proficiency which include designing cognitively demanding tasks, posing purposeful questions, creating a classroom environment that focuses on communication, and supporting students in the productive struggle of learning mathematics. Finally, connections will be made between the reform-based instructional practices and theoretical perspectives from cognitive psychology, sociocultural theory, and constructivism.

How Students Learn Mathematics

Successful mathematics learning involves students actively engaging in problem-solving processes and mathematical discourse which allows them to construct mathematical meaning (NCTM, 1989, 2000, 2014). These actions help students to develop mathematical proficiency that will enable them to face mathematical challenges with confidence (NRC, 2001). The following sections will describe the Process Standards (NCTM, 2000), or the processes in which students must engage to learn mathematics meaningfully. The Process Standards include the processes of problem solving, reasoning and proof, communication, connections, and representation. The strands of mathematical proficiency (i.e., conceptual understanding, procedural fluency, strategic competency, adaptive reasoning, and productive disposition; NRC, 2001) and their relation to the Process Standards (NCTM, 2000) will also be discussed.

Problem solving. Students learn mathematics by actively engaging in the problem-solving process and developing habits of mind for making sense of problem situations for which students are not aware how to solve in advance (CCSSI, 2010; Levasseur & Cuoco, 2003; NCTM, 2000). In doing so, students represent the problem situation mathematically, use their mathematical knowledge to obtain results, and interpret their results in terms of the original context (OECD, 2013b). Throughout this cycle, students become familiar with using multiple tools and strategies for solving current and future problems, thus allowing students to have multiple entry points to the mathematics within the problem (CCSSI, 2010; NRC, 2001). This provides students the opportunity to struggle productively with the mathematical meaning of the problem rather than to obtain a surface-level understanding of a solution (Hiebert & Wearne, 2003). When students engage in the problem-solving process, new mathematical habits of mind develop through the connections they are able to examine, the mistakes they make along the way, and the new strategies they discover for finding a solution (Hiebert & Wearne, 2003; Levasseur & Cuoco, 2003).

Reasoning and proof. Students learn mathematics when given the opportunity to reason through their ideas and arrive at a solution based logically on prior connections and initial evidence (NCTM, 2000, 2009). Rather than following a pre-determined procedure, students should be able to choose the steps they follow as influenced by their initial ideas and the mathematical connections they make (NCTM, 2009). Students can then determine if their solution is reasonable or not within the context of the problem (OECD, 2013b) and be able to support their conjectures with logical thought (NRC, 2001). Students engage in the process of proof at the elementary level by making conjectures based on a logical sequence of reasoning and describing how they view their conjectures to be true (NCTM, 2000).

Reasoning provides students the opportunity to gain a greater understanding of why the mathematics works by justifying or proving why their conjectures are true or why possible mistakes have occurred (NCTM, 2000, 2014; NRC, 2001). This transpires most often through argumentative discourse among students or whole class discussions in which students can present their thinking to their peers (Forman, 2003; NCTM, 2000, 2014). Communication among students allows different mathematical explanations and strategies to be considered and each student the opportunity to reflect on their own solution process (NCTM, 2014; OECD, 2013b).

Communication. In addition to problem solving and formulating reasons for why the relations occur, students learn mathematics by discussing their ideas with other students (NCTM, 2000, 2014). To obtain a deeper understanding of mathematics, students should be given the opportunity to communicate with their peers, ask questions regarding others' ideas, and answer questions directed at their own ideas (CCSSI, 2010; Forman, 2003; Grouws, 2003; NCTM, 2000). This includes conversations between the student and teacher as well as among the students themselves as new ideas are developed and mathematical arguments are further explored (NCTM, 2000; Skemp, 1976). Students develop a deeper understanding when they can communicate these ideas clearly to each other and reflect on the ideas of others (CCSSI, 2010; NCTM, 2000, 2014; NRC, 2001).

Connections. Students build a strong understanding of mathematics when they can make connections between mathematical ideas and recognize how the ideas "build on one another to produce a coherent whole" (NCTM, 2000, p. 200). Students can deepen their understanding of one concept while learning another concept simultaneously (e.g., learning fractions while learning measurement). To do this, students should be responsible for determining how new mathematical ideas can be woven into their prior knowledge to make sense of the new ideas (NCTM, 2000). Once these connections are made, students can easily recognize patterns among new ideas which are now seen as extensions of the mathematical ideas learned previously (NCTM, 2000; NRC, 2001; Sidney & Alibali, 2015). With this new understanding that mathematical ideas are not isolated, but rather a network of connected ideas in a web-like fashion, students can approach new problems with confidence knowing that they can make connections between ideas and use these connections to understand the new material (NCTM, 2000).

Additionally, students build a strong understanding of mathematics when they can make connections between new mathematical ideas and real-world contexts within other disciplines (Grouws, 2003; Lefrançois, 2006; NCTM, 2000, 2009; NRC, 2001). By connecting mathematics to contexts outside of the classroom, students can see how mathematical ideas are used to solve and explain complex real-world applications (NCTM, 2000). Students are more likely to become motivated to make more connections between the processes involved and develop a greater understanding of the utility of mathematics learned in the classroom (Gainsburg, 2008; NCTM, 2000).

Representation. As students build a conceptual understanding of mathematics, they begin to represent problems in different ways and can determine which representations are more efficient in different situations (NCTM, 2000; NRC, 2001).

Mathematical representations can be referred to as external, such as visual, symbolic, verbal, contextual, and physical (Tripathi, 2008), or internal, such as abstract mathematical ideas developed from the connections made externally (Pape & Tchoshanov, 2001). For example, students can externally identify a function on a graph visually or explain the properties of functions orally. Students may represent functions symbolically in the form of an equation or physically manipulate points to represent various types of functions and explain appropriate contexts for their functions (Tripathi, 2008). When students can relate multiple representations of a concept to one another internally, they develop holistic views and much deeper understandings of the concept as they move towards more abstract stages of thinking (NCTM, 2014; Tripathi, 2008). This gives students the internal means to reason through how and why the representations are connected, helping to build their mathematical proficiency overall (NCTM, 2014).

Mathematical proficiency. The previously examined Process Standards (NCTM, 2000) support students in developing mathematical proficiency necessary to "cope with the mathematical challenges of daily life and enable them to continue their study of mathematics" (NRC, 2001, p. 116). Consisting of five interwoven strands (i.e., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition), mathematical proficiency is a comprehensive way to describe successful mathematics learning. The following section will discuss the five strands of mathematical proficiency and their connections to the Process Standards (NCTM, 2000).

Conceptual understanding. The first strand of mathematical proficiency is conceptual understanding which refers to an "integrated and functional grasp of mathematical ideas" (NRC, 2001, p. 118). Students build conceptual understandings by taking previously learned isolated concepts and organizing them to create a coherent whole (NRC, 2001). This allows students to make sense of new information by connecting the new ideas to previously learned material. Conceptual understandings may not be explicit as students make these connections in different ways (e.g., representing mathematical ideas depending on what is most useful and appropriate for the situation at hand). Instruction that attends to this strand of mathematical proficiency connects to the Process Standards (NCTM, 2000) by engaging students in the processes of connections, problem solving, and reasoning and proof. By seeing the deeper connections between concepts, students are more likely to successfully solve problems by assigning new ideas to previous clusters of knowledge. Additionally, when students build a conceptual understanding, they reason through how the ideas are connected and can justify their reasoning to others (NRC, 2001). Students who have a conceptual understanding are more likely to build procedural fluency from their connected knowledge.

Procedural fluency. The second strand of mathematical proficiency is procedural fluency which is the "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (NRC, 2001, p. 121). Instruction that attends to this strand of mathematical proficiency connects to the Process Standards (NCTM, 2000) by engaging students in the processes of problem solving and connections. Students build procedural fluency to support their conceptual understanding such as becoming fluent in multiplication to build an understanding of division and a variety of methods to divide (Hiebert & Grouws, 2007; NRC, 2001). It is easier for students to work through a procedure when they understand the underlying concepts and connections. In addition, when students forget the procedural steps or are faced with a different computational situation with future problems, they are more likely to exhibit flexibility as a result of their deep understanding (NRC, 2001).

Strategic competence. The third strand of mathematical proficiency is strategic competence which refers to the "ability to formulate mathematical problems, represent them, and solve them" (NRC, 2001, p. 124). Students build strategic competence by constructing models that represent problems rather than simply extracting numbers and keywords to perform an arithmetic operation. These models can be constructed symbolically, numerically, graphically, or verbally when describing how the variables of a problem are related (NRC, 2001). Instruction that attends to this strand of mathematical proficiency connects to the Process Standards (NCTM, 2000) by engaging students in the processes of problem solving and representations. Strategic competency also involves flexibility where students can model and work through non-routine problems (i.e., the solution method is not immediately known), requiring students to build on their conceptual understanding of the variables involved and their procedural fluency in solving routine problems (NRC, 2001). Students who build strategic competence also tend to pose their own problems about the world around them, trying to understand how math is used in the context of their own lives.

Adaptive reasoning. The fourth strand of mathematical proficiency is adaptive reasoning which is the "capacity to think logically about the relationship among concepts and situations" (NRC, 2001, p. 129). Instruction that attends to this strand of mathematical proficiency connects to the Process Standards (NCTM, 2000) by engaging students in the processes of connections and reasoning and proof. When students exhibit adaptive reasoning, they can reason through how the concepts and ideas within mathematics fit together and make sense. Students can then rely on their own reasoning to justify why their solution is valid rather than seeking the teacher's approval. Adaptive reasoning develops over time where students can practice making connections to previously learned material often by determining the efficiency of their strategies and reasoning through why their strategy is suitable for the problem (NRC, 2001).

Productive disposition. The last strand of mathematical proficiency is productive disposition which refers to the "tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (NRC, 2001, p. 131). Instruction that attends to this strand of mathematical proficiency connects to the Process Standards (NCTM, 2000) by engaging students in the processes of problem solving, communication, and reasoning and proof. When students view mathematics as important and valuable, they are motivated to work through challenging problems with the understanding that their success is a result of their effort (NRC, 2001). Students who develop a productive disposition view mathematical ability as malleable and challenging problems as opportunities to learn. They are confident in what they know and strive to communicate their ideas with others so that they can learn even more (NRC, 2001).

A productive disposition helps students to develop the other strands of mathematical proficiency where students have a positive perception of doing mathematics. This develops as students build their conceptual understanding and strategic competence when working through non-routine problems (NRC, 2001). Students become capable of learning and doing mathematics as they build their procedural fluency and can think logically about the conceptual relationships (NRC, 2001). Students who develop a productive disposition ultimately perceive themselves as capable to understand mathematics and confident in what they know. They believe that "mathematics is both reasonable and intelligible and believe that, with appropriate effort and experience, they can learn" (NRC, 2001, p. 133).

Summary. Engaging in the five Process Standards provides students a means for learning mathematics (NCTM, 2000). Through problem solving, reasoning and proof, communication, connections, and representations, students begin to develop mathematical proficiency that will enable them to face new mathematical challenges with confidence (NCTM, 2000; NRC, 2001). Mathematical proficiency takes time to develop; however, instructional practices that engage students in the Process Standards provide students the opportunity to make progress in each of the strands, which are interconnected and promote meaningful learning of mathematics (NCTM, 2000, 2014). The following section will discuss the instructional practices that can support students in building mathematical proficiency.

Instructional Practices that Support Mathematical Proficiency

Instructional practices based on the Process Standards provide students the opportunity to develop the mathematical proficiency necessary for understanding how and why mathematical concepts are related (NRC, 2001; Skemp, 1976). The type of instruction that allows for these opportunities to occur is referred to as reform-based mathematics instruction (Hiebert & Grouws, 2007). In reform-based mathematics instruction, teachers provide students with meaningful opportunities to build mathematical understanding and to view mathematics as an interactive process (Hudson et al., 2006). The following instructional actions support students' learning of mathematics: (a) implement cognitively demanding tasks; (b) pose purposeful questions; (c) create a classroom environment that allows students to communicate with each other; and (d) allow students to struggle productively in learning mathematics. The following section will describe these four instructional actions that support students in building mathematical proficiency.

Cognitively demanding tasks. Student learning is best supported in classrooms where mathematical tasks are cognitively demanding, challenging, and focused on the mathematical goals (Boston & Smith, 2009; NCTM, 2000, 2014; Warshauer, 2011). Based on the 1991 NCTM standards, Smith and Stein (1998) designed a framework for classifying mathematical tasks based on the cognitive thinking required to solve them. Tasks with a lower-level demand on student thinking consist of students learning facts and procedures through memorization. Tasks with a higher-level demand on student thinking actively engage students in inquiry and making meaningful connections between concepts and procedures with understanding (Boston & Smith, 2009; NCTM, 2000; Smith & Stein, 1998; Stein, Grover, & Henningsen, 1996). Furthermore, tasks with higher-level demand provide opportunity for students to struggle productively with mathematical ideas, access the mathematics from multiple entry points, and develop a deep conceptual understanding (Grouws, 2003; NCTM 2000, 2014; NRC, 2001; Stein et al., 1996; Wilhelm, 2014).

It is the teacher's role to select appropriate tasks, assist students while working through the tasks, and assess the students' understanding through their use of strategies (NCTM, 2000). Teachers should emphasize that perseverance is important as students productively struggle through cognitively demanding tasks to help students get into the habit of persevering when faced with difficult problems, in or out of the classroom (NCTM, 2000). As students engage with these types of tasks, teachers should ask questions encouraging students to explain their ideas, revise their strategies, and consistently develop new strategies based on their prior knowledge (NCTM, 2000). By presenting these types of tasks to their students, teachers help students to build their problem-solving skills and procedural fluency necessary for facing new mathematical challenges (Hiebert & Grouws, 2007; NCTM, 2000, 2014; NRC, 2001).

Purposeful questions. While students are working through cognitively demanding tasks, teachers must pose purposeful questions in reform-based classrooms to gather information, assess students' current understandings of mathematical concepts or relationships, and advance students' reasoning skills and thinking about essential mathematical connections (NCTM, 2014; NRC, 2001). When teachers see students struggling, they should ask advancing questions regarding students' ideas, strategies, and reasoning, giving students the opportunity to develop new strategies based on their prior knowledge (NCTM, 2000). Teachers must refine their questioning techniques to guide the learning direction and to start the conversations among students (NCTM, 2000) by focusing their questions on what the students are thinking rather than leading students to a procedure or result (Grouws, 2003; NCTM, 2014). Teachers should ask purposeful questions that advance students' thinking by allowing students to justify their responses in multiple ways and giving students adequate time to respond (NCTM, 2000, 2014; NRC, 2001). By posing purposeful questions that are directly related to students' understanding and support students in struggling productively through the mathematics, teachers maintain a student-centered classroom environment where students take an active role in their learning and become diligent in problem solving (NCTM, 2014).

Classroom environment. In order to support students in learning mathematics, teachers must establish a mathematically rich classroom environment where students are encouraged to discuss and defend their ideas with one another rather than limit their discussions between only the teacher and the students (CCSSI, 2010; Hiebert & Grouws, 2007; NCTM, 2000; NRC, 2001). Teachers facilitate discussions and classroom interactions around the room, allowing students to ask questions and reason through the mathematics with their peers (Hiebert & Grouws, 2007; NCTM, 2000). As a facilitator in a mathematically rich classroom environment, the teacher initially manages class discussions by strategically selecting which students' ideas to share and in what order to share them (NCTM, 2000, 2014). Teachers then take a step back, allowing students to guide the inclusive discussion where students are “responsible both for articulating their own reasoning and for working hard to understand the reasoning of others” (NCTM, 2000, p. 191).

Students situated within the classroom environment learn more as individuals when they are given the opportunity to communicate with their peers and develop the skills necessary for solving mathematical tasks (Greeno, 1998, 2003; NCTM, 2000; NRC, 2001). Students develop a deeper understanding in classroom environments that: (a) include participation in mathematical practices that help develop their own identity as knowers and learners of mathematics (Greeno, 2003; Lave & Wenger, 1991; Lerman, 2000; Ticknor, 2012), (b) allow teachers and other students to build relationships through their interactions with concepts and methods for solving problems (Greeno, 2003), and (c) immerse students in the context for learning where participation and interactions within the social setting enable learning (Lave & Wenger, 1991). Teachers can orchestrate meaningful mathematical discourse by creating a mathematically rich classroom environment that allows students to discuss and defend their ideas with their peers.

Productive struggle. Reform-based mathematics instruction supports students in moving toward a high level of mathematical understanding and sense making by allowing students to productively struggle (Carter, 2008; Hiebert & Grouws, 2007; NCTM, 2014). Struggle is productive when students can work through problems that are not too difficult or too easy and have multiple entry points to the underlying ideas (NCTM, 2014). When students appear to be frustrated or give up easily, teachers often find this as a sign of unproductive struggle and rush to their aid by breaking down the steps of problems for them (NCTM, 2014; Warshauer, 2011). In doing so, the cognitive demand of the problem lowers, and students are deprived of the opportunity to struggle productively with and make sense of essential mathematical ideas (Hiebert & Grouws, 2007; NCTM, 2014; Warshauer, 2011). Thus, students who struggle productively work harder to make their own connections, construct multiple solution paths, and restructure their pre-existing knowledge (Carter, 2008; Hiebert & Grouws, 2007; NCTM, 2000; NRC, 2001).

One way that teachers can support students as they productively struggle is by exposing students to a variety of problems, strategies, and representations (NCTM, 2000). Additionally, teachers should facilitate discussions of why certain strategies and representations are more efficient than others in different situations (NCTM, 2000; NRC, 2001). In doing so, when faced with difficult problems, students are more likely to make connections within the problem and be able to reason through the most efficient processes (NCTM, 2000; NRC, 2001). Teachers can also support students as they productively struggle by respecting and valuing all students' ideas (NCTM, 2000). As a result, students begin to take "intellectual risks by raising questions, formulating conjectures, and offering mathematical arguments" (NCTM, 2000, p. 185) on their own when facing new problems. By allowing students to productively struggle, teachers can help students extend their conceptual understanding of mathematics and their willingness to persevere when faced with future problems (Carter, 2008; NCTM, 2000; NRC, 2001).

Summary. Students must have opportunities to engage with mathematics through applications, examine mathematics from a critical eye in order to develop the mathematical proficiency necessary for understanding how and why mathematical concepts are related, and communicate their mathematical ideas with one another (NCTM, 2000; NRC, 2001; Skemp, 1976). Teachers can provide students with these opportunities through reform-based mathematics instruction by: (a) implementing cognitively demanding tasks; (b) posing purposeful questions; (c) creating a classroom environment that allows students to communicate with each other; and (d) allowing students to struggle productively when learning mathematics. The following section will discuss the theoretical perspectives that support students in learning mathematics in these ways.

Theoretical Perspectives

Reform-based instructional practices actively engage students in the learning process and promote a deep conceptual understanding of mathematics (Stecher et al., 2002). Developed from years of educational research, reform-based instructional practices focus on both the mathematical content and the needs of the student (NCTM, 1989, 2000, 2014; Sfard, 2003). The following sections explore the connections between reform-based instructional practices and theoretical learning perspectives from cognitive psychology, sociocultural theory, and constructivism.

Cognitive science. The cognitive learning perspective focuses on mental processes such as solving problems, making decisions, processing new information, and thinking (Lefrançois, 2006; Simon, 2009). In mathematics education, cognitive science is the study of the internal mental processes of how students learn mathematics and retain its meaning (Cobb, 2007; Siegler, 2003). Students who are told specifically how to solve problems are not able to process why they are doing so (Siegler, 2003). However, students who are given the opportunity to solve problems where the steps or solution are not immediately known begin to build their own processing schema (Greeno, 1998). Through problem solving, students can use their former knowledge to make connections to new ideas, discover new strategies for solving problems, and process and retain new information in more efficient ways (Cobb, 2007; Greeno, 1998). Siegler (2003) noted that from an early age, students naturally learn to use a variety of strategies with varying levels of difficulty when they approach problems. Thus, students can develop new ideas and concept structures when given the opportunity to struggle productively with new problems and build a deeper understanding of the mathematics overall (Cobb, 2007; Siegler, 2003).

The cognitive science perspective in mathematics education also addresses how students develop numerical reasoning skills (Cobb, 2007). Numerical reasoning, or number sense (Greeno, 1998), involves the “conceptual manipulation of mathematical objects” (Cobb, 2007, p. 19) where mental tools and resources are easily accessible (Siegler, 2003). Thus, students become more sophisticated in numerical reasoning as they interpret new ideas, analyze incorrect solutions, and attempt to complete problems through the facilitation of ideas (Cobb, 2007; Siegler, 2003). Although cognitive science research provides mathematics education with valuable information on how students process and reason with new information, the interactions within the classroom environment play an important role in how students learn as well (Greeno, 1998).

Sociocultural theory. Whereas cognitive science research highlights the learning process internally, the sociocultural perspective focuses on the learning process influenced by the social environment (Cobb, 2007; Forman, 2003; Lefrançois, 2006). The culture or classroom environment provides a lens for looking at how students use mathematical resources to develop logical thinking and reasoning skills (Cobb, 2007; Lefrançois, 2006; Simon, 2009). Teachers determine goals within the classroom environment based on how students build conceptual understandings (Greeno, 1998), what it looks like when students achieve the goals (Lefrançois, 2006), and how students can be supported in learning self-regulation skills by using their curiosity and ingenuity to achieve the goals (Forman, 2003). In doing so, teachers can take on a facilitative role, allowing students to build understanding within the social environment (Cobb, 2007).

From the sociocultural perspective, students can construct their own meaning of mathematics within and because of the classroom community (Cobb, 2007; Forman, 2003; Greeno, 1998). When students are able to interact with their peers, they begin to see how other students use mathematical tools, devise strategies for problem solving, make connections between multiple representations, and access the mathematics in different ways (Cobb, 2007; Forman, 2003; Lave & Wenger, 1991). Additionally, social interactions provide students the opportunity to reason with one another, justify their ideas, and modify their own thinking as a result (Cobb, 2007; NCTM, 2014). Students are then able to apply what they observed and learned by justifying and reasoning with their peers to future problems, discovering which strategies are more efficient for solving a variety of problems (Forman, 2003). Thus, students are given the opportunity to construct knowledge in a meaningful way because of these social interactions (Greeno, 1998).

Constructivism. Similar to both the cognitive science and sociocultural perspectives, constructivism focuses on how students construct mathematical knowledge while being influenced by the social aspects of the classroom community (Boaler, 2000; Cobb & Bowers, 1999; Cobb, Yackel, & Wood, 1992; Simon, 1995). However, constructivism has a greater focus on how students construct and organize this information into schemas or patterns (Anderson & Piazza, 1996; Steffe & Kieren, 1994). Students construct schemas by revising their pre-existing knowledge based on new knowledge resulting in cognitive development (Anderson & Piazza, 1996). Instructional practices that introduce students to problems that involve multiple representations and real-world contexts help students to make explicit connections and construct their knowledge in meaningful ways (Anderson & Piazza, 1996; Cobb et al., 1992). Additionally, instructional practices that pose problems with multiple solutions or no solution, create an imbalance of students' current knowledge (Anderson & Piazza, 1996). Learning occurs when students can resolve this imbalance.

Summary. The reform-based instructional practices are supported by strong evidence provided by theoretical perspectives of cognitive science, sociocultural theory, and constructivism detailed in this section (NCTM, 1989, 2000, 2014; Sfard, 2003). These theoretical perspectives provide insight into how teachers can support students in developing a conceptual understanding of mathematics (Cobb, 2007). As discussed in the previous section, the interactions between teachers and students are critical to learning mathematics. Hattie and Timperley (2007) found that the most common and influential interaction that occurs in the mathematics classroom is the feedback provided by the teacher to the student. The following section details the literature on feedback: the information teachers give to students that supports students in learning mathematics.

Feedback

Students develop mathematical problem-solving skills through interactions with teachers as well as other students (Jung et al., 2015; Slavin, 1996). Through these interactions students gradually increase their understanding of the content as well as their ability to solve problems (Jung et al., 2015). This information transferred between teachers and students, particularly through questions asked and informational responses, is known as feedback and can help advance students' mathematical knowledge towards a conceptual understanding of the mathematics (Brookhart, 2008; Hattie & Timperley, 2007; Slavin, 1996). The following section will review definitions of feedback and the impact of different types of feedback on student learning. Additionally, studies will be discussed that explore the most common mode of feedback and both the types and levels of feedback as described by Hattie and Timperley (2007).

Definitions of Feedback

Feedback has been defined in a variety of ways and situations (cf. Black & Wiliam, 1998; Cueto, Ramirez, & Leon, 2006; Fyfe, Rittle-Johnson, & DeCaro, 2012; Hattie & Timperley, 2007; Kluger & DeNisi, 1996; Li, Cao, & Mok, 2016). For the purposes of my study, the following discussion will focus on three main definitions of feedback: an intervention to student struggle (Kluger & DeNisi, 1996), a tool for formally assessing students' learning and modifying student engagement (Black & Wiliam, 1998), and information regarding one's performance or understanding (Hattie & Timperley, 2007).

Kluger and DeNisi (1996) defined feedback as an intervention for modifying students' performance or prior knowledge to extend the learning process. Among the studies examined in their meta-analysis, Kluger and DeNisi found that feedback was viewed as an intervention to student struggle and, at that time, produced mostly negative effects on student performance. As a result of the inconsistent findings, Kluger and DeNisi designed a variety of feedback intervention rubrics to help teachers decide the most effective way they could respond to students to enhance their motivation, process, and learning related to a task. The feedback intervention rubrics consisted of a series of paths teachers could follow depending on the student's response. For example, the teacher provides the class with details of a task. If a student's response indicated that the task was well known then the teacher would interrupt the anticipated discussion with an additional task to extend their thinking (Kluger & DeNisi, 1996). However, if a student's response indicated that the task was not well known then the teacher should initiate a pattern of generating and testing hypotheses related to the task, similar to reform-based instruction. As a result of Kluger and DeNisi's feedback intervention rubrics, the authors found changes in students' efforts and performance related to the task. When students' efforts or performance increased because of the feedback intervention, the authors predicted a positive effect on student learning (e.g., higher student achievement). In contrast, when students' efforts or performance decreased because of the feedback intervention, the authors predicted a negative effect on student learning (e.g., students being less motivated or not attempting the task at all; Kluger & DeNisi, 1996).

When teachers followed the feedback intervention rubrics, students' performances on tasks were shown to be greater when the teacher provided praise, offered positive feedback that did not threaten the students' self-esteem, or gave students thinking tasks rather than tasks that required students to follow a specific procedure (Kluger & DeNisi, 1996). However, the authors concluded that when following the feedback intervention rubrics, student achievement depended solely on how feedback was given (i.e., oral, written, computerized, etc.) which varied among teachers and students. Additionally, the feedback intervention was designed with little attention to students' conceptual understandings and not specific to the individual student needs (Kluger & DeNisi, 1996).

Rather than viewing feedback as an intervention, Black and Wiliam (1998) viewed feedback as a tool for formally assessing students' learning. Black and Wiliam (1998) defined feedback as the information given by the teacher used to modify how the students were engaged or how to correct mathematical errors. By providing feedback in this way, teachers could assess students' thinking and modify the learning activities to help students reach their academic goals. With a focus on students' learning on tasks taught in all subject areas, Black and Wiliam (1998) discussed various situations in which feedback could have negative effects on student learning, particularly when only a letter or number grade is given. Although a majority of the studies examined in their literature review focused on students as active members of the classroom community, there was a strong emphasis on test scores as a way of assessing student growth in response to feedback (Black & Wiliam, 1998). This does not align with reform-based instructional practices where teachers provide students with meaningful opportunities to build mathematical understanding and an environment where students are encouraged to discuss and defend their ideas with one another (Hiebert & Grouws, 2007; Hudson et al., 2006; NCTM, 2000; NRC, 2001).

Feedback can also be defined as the "information provided by an agent (e.g., teacher, peer, book, parent, self, and experience) regarding aspects of one's performance or understanding" (Hattie & Timperley, 2007, p. 81). Through their meta meta-analysis, including the studies of Kluger and DeNisi (1996) and Black and Wiliam (1998), Hattie and Timperley (2007) focused on the importance of all types of feedback during the time that students are constructing their knowledge. Hattie and Timperley (2007) found that, in the studies they examined, feedback was most often classified as a consequence to a child's performance, primarily given by the instructor or computer-assisted program, intended to assist students in adapting their behavior. Among the studies, feedback most often consisted of correct or final solutions, informational cues given to guide students toward the goal, and additional information provided when students were not able to progress toward the goal (Hattie & Timperley, 2007).

From their meta meta-analysis, Hattie and Timperley (2007) found that most of the studies examined had little focus on students' conceptual understandings or variation of the level of student thinking. Additionally, the number of times the instructor provided feedback within the classroom was extremely low and praise was the most common form of feedback (Hattie & Timperley, 2007). Hattie and Timperley (2007) found that feedback in the form of praise was less productive for enhancing student learning, particularly when it was directed at the student as a person. In response, Hattie and Timperley (2007) developed a framework "to assist in identifying the circumstances likely to result in the more productive outcomes" (p. 87). The framework first described three major questions that effective feedback must answer: "Where am I going?, How am I going?, and Where to next?" (Hattie & Timperley, 2007, p. 86). These questions correspond to the types of feedback: feed up, feed back, and feed forward, respectively. Hattie and Timperley (2007) further described four levels (i.e., task, process, self-regulation, and self) in which feedback could be directed that could be used to increase student effort, build error-detection skills, and develop the processes associated with achieving the desired goals. When feedback is directed at students at the type and level appropriate for their individual needs, students can make the necessary connections to progress in their learning (Hattie & Timperley, 2007).

Student Learning and Feedback

Although the results of feedback on student learning have shown to be effective under various conditions (Hattie & Timperley, 2007; Wisniewski, Zierer, & Hattie, 2020), inconsistent patterns of results have led to further studies on feedback (Shute, 2008). In her analysis of feedback, Shute (2008) found that, among the studies that were considered, there were various ways to provide feedback and the inconsistency among the studies could be attributed to teachers' individual differences such as motivation, beliefs, self-efficacy, perseverance, and knowledge base of the students. In response, Shute (2008) suggested a variety of ways that feedback could be effective in the classroom. First, effective feedback must be specifically related to how students could improve their proposed solutions. This method does not focus on students' strategies or methods for solving problems and is simply guiding them to the correct solution. Second, effective feedback should not be too long or complex as students tend to lose interest and the original intent of the feedback may be diluted. Third, effective feedback should be directed at students' progress towards an appropriate goal. Fourth, effective feedback should be immediate and should recognize the ability levels of students. Although Shute (2008) suggested these methods of providing effective feedback, the author noted that feedback should depend on the individual learner and learning outcomes.

When assessing how students respond to feedback, teachers must consider each individual student's prior knowledge not only of the content but also of the process of solving problems (NCTM, 2000, 2014; Sidney & Alibali, 2015). Fyfe et al. (2012) conducted two studies in second- and third-grade mathematics classrooms to examine the effects of different types of feedback on student learning. The study focused on teachers providing feedback to students with various levels of prior knowledge during an exploratory tutoring session prior to formal instruction. The authors referred to the person who provided tutoring as the experimenter (Fyfe et al., 2012). The experimenter was not the students' regular teacher, nor was the tutoring provided in the students' natural classroom environment. In the context of mathematics, Fyfe et al. (2012) described two types of feedback which included outcome feedback (i.e., accuracy of the student's response) and strategy feedback (i.e., assessment of how the student arrived at that solution). The authors found that neither outcome nor strategy feedback had any effect on students' understanding; however, prior knowledge did determine the impact that feedback had on the process of students' learning. Students with little prior knowledge of the subject showed greater procedural understanding when provided feedback as they had not been as exposed to using multiple strategies or efficient methods for solving problems prior to the study (Fyfe et al., 2012). Similarly, students with moderate prior knowledge showed lesser procedural understanding when provided feedback as they had already been exposed to strategies effective for problem solving and chose not to apply the feedback to their work (Fyfe et al., 2012). This study did not focus on feedback given during regular classroom instruction or by a regular mathematics teacher; however, the authors emphasized the importance of recognizing and modifying activities according to students' prior knowledge when providing feedback.

Summary. The previous section explored a variety of ways feedback could be defined as well as an overview on various studies and meta-analyses that examined the effectiveness of feedback on student learning. Among these studies was little clarification of how the results were analyzed (e.g., student achievement on tests or surveys) or elaboration on whether feedback given in these ways allowed students to build a conceptual understanding of the material (Shute, 2008). These are important to attend to as not all learners learn in the same way, and the enhancement of student understanding should be the goal. Thus, it is also important to attend to the various modes in which feedback can be provided to ensure that all students have the opportunity to learn. The following section will describe these modes of feedback and extend on the most common mode of feedback provided in the mathematics classroom, in addition to the empirical research that supports this mode of feedback.

Modes of Feedback

As shown previously, feedback can be defined in numerous ways, provided from multiple perspectives, and effectively given to students at various times (Black & Wiliam, 1998, 2009; Fyfe et al., 2012; Hattie & Timperley, 2007; Shute, 2008). These broad perspectives of feedback provide insight into how feedback can be provided and in what ways feedback can be effective for supporting students' learning of mathematics (Shute, 2008). Oral feedback can provide students with immediate feedback where teachers must quickly determine how to respond in a meaningful way (Brookhart, 2008). Written feedback provides teachers additional time to think about what students understand; however, the feedback may not always be meaningful for the students (Bijami, Pandian, & Singh, 2016; Black & Wiliam, 1998). This is especially true in the mathematics classroom where teachers are more likely to give letter or number grades for correctness (Bangert-Drowns, Kulik, Kulik, & Morgan, 1991; Black & Wiliam, 1998; Hattie & Timperley, 2007). Gesture feedback is referred to as a nonverbal response used to convey information such as a facial expression, posture, eye motion, body motion, or hand gesture (Goldin-Meadow, Kim, & Singer, 1999; Knapp, 1978; Sfard, 2009). Goldin-Meadow et al. (1999) found that the information conveyed through gestures did not always reinforce what the teachers were trying to say verbally; however, it has been shown to enhance students' mathematical understanding when used in conjunction with verbal communication (Goldin-Meadow et al., 1999; Ng, 2016; Sfard, 2009).

Among the three modes of feedback previously described, oral feedback is the most commonly used mode in the mathematics classroom (Brookhart, 2008; Cardelle-Elawar, 1990; Li et al., 2016). Oral feedback is defined as feedback given orally to an individual or group of students and is most commonly directed at accomplishing a task (Hattie & Timperley, 2007; Li et al., 2016). With an emphasis on the ways teachers provide oral feedback, Li et al. (2016) examined the role of oral feedback with 24 middle school mathematics teachers in China. The study focused on teachers' attitudes towards providing oral feedback. Oral feedback was analyzed using a previously designed set of feedback codes to note whether teachers' feedback included a negative response, a neutral response, an acknowledgment of the answer, or an application of the student's solution into their teaching. Through classroom observations, the authors found that "teachers' attitudes [could] be affected by the correctness of answers" (Li et al., 2016, p. 2470). For example, if a student's solution was incorrect, a negative oral response would be for the teacher to criticize the solution (e.g., "No, that is wrong"). However, in response to an incorrect solution, the teacher could give a neutral response allowing other students to explain why the answer is incorrect or to examine their mistake (e.g., "That's interesting. Who would like to respond to this idea?"; Boaler, 2016; Li et al., 2016). A neutral response such as this aligns with the practices of reform-based mathematics instruction by providing students the opportunity to build knowledge conceptually.

Oral feedback allows teachers to extend student thinking by providing an immediate response (Hattie & Timperley, 2007). Among the studies reviewed by Hattie and Timperley (2007), teachers' oral feedback had positive effects on student learning when the feedback itself was positive and focused on the extension of students' thinking rather than only on the correctness of a solution (Boulet, Simard, & deMelo, 1990; Kluger & DeNisi, 1996; Li et al., 2016). However, oral feedback does not always give teachers adequate time to attend to the individual needs of the students due to the immediate nature of oral feedback (Brookhart, 2008). In her study of four mathematics teachers who provided oral feedback to sixth-grade bilingual students, Cardelle-Elawar (1990) found that the teachers became better mediators by "using feedback to stimulate students' cognitive processes" (p. 173). The author found that oral feedback was necessary to teach students to think for themselves when solving mathematical problems, rather than immediately verbalizing the correctness of their solution (Cardelle-Elawar, 1990).

In addition to the three modes in which feedback can be classified (i.e., oral, written, and gestures), feedback can also be classified by types and levels (Hattie & Timperley, 2007). The following section will examine these two classifications of feedback (i.e., types and levels) which compose Hattie and Timperley's (2007) model of feedback.

Types of Feedback

Based on numerous observations and analyses of hundreds of studies on feedback, Hattie and Timperley (2007) found that in order for feedback to be effective, it must answer three questions: "Where am I going?, How am I going?, and Where to next?" (Hattie & Timperley, 2007, p. 86). These questions correspond to the three types of feedback: feed up, feed back, and feed forward, respectively (see Figure 1).

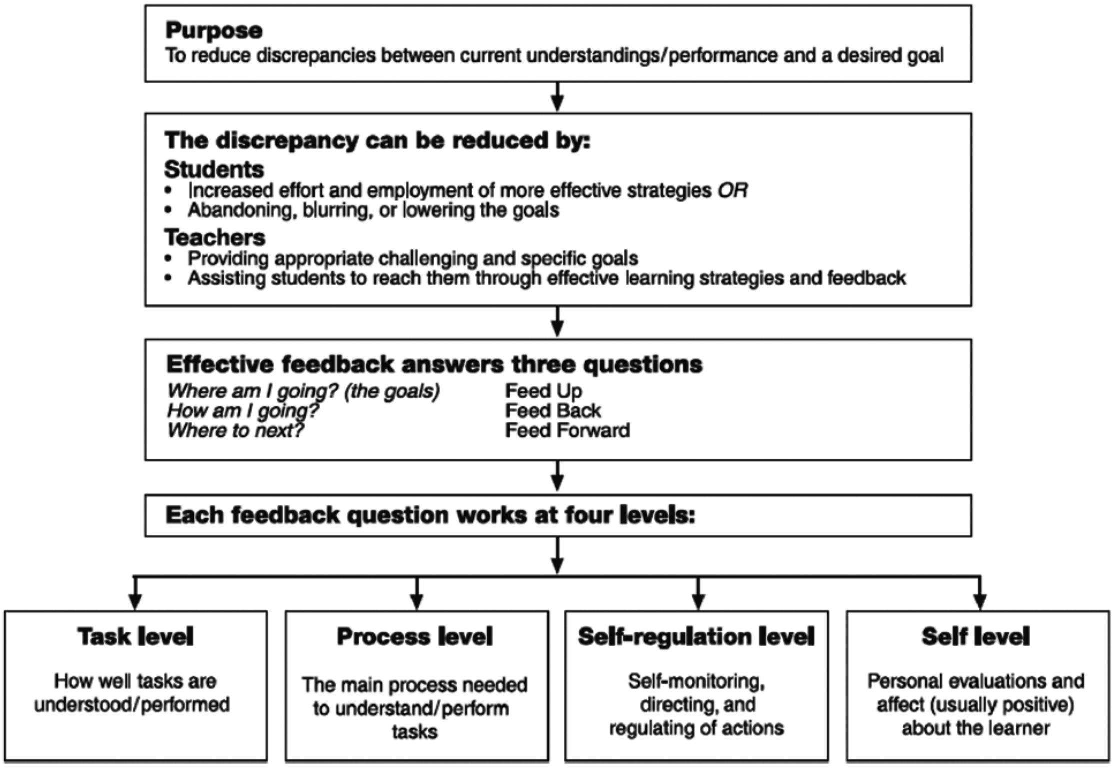

Figure 1. Model of feedback to enhance learning. From Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of Educational Research, 77,* 87.

Teachers and students who pursue the answers to each of the three questions establish an ideal learning environment where both the teacher and student are learning from each other (Hattie & Timperley, 2007). This ideal learning environment allows teachers to engage in reform-oriented teaching directed toward students' attainment of the learning goals (Hiebert & Grouws, 2007). The following section will describe these three types of feedback (i.e., feed up, feed back, and feed forward) within Hattie and Timperley's (2007) model of feedback.

Feed up. Feed up answers the question, “Where am I going?” (Hattie & Timperley, 2007, p. 88), describing the information given to students about what the learning goal is and what the result looks like once they arrive at the goal. Teachers provide effective feed up by indicating the nature of the goal and what students can do to achieve that goal (Hattie & Timperley, 2007). When teachers do not address how students can achieve the goal, a gap is created between what students are currently learning and what they are intended to learn. This results in students being unable to reduce the gap and failing to arrive at their goal (Hattie & Timperley, 2007). Feed up can be specific (e.g., “That’s what we are working on right now. How can these two numbers be added together?”) or broad (e.g., “You need to try and find an improper fraction.”) and can identify the end result (e.g., “Remember, we are modeling two fractions that are the same. Here is an example of what your model may look like.”; Brooks, Carroll, Gillies, & Hattie, 2019). When goals are clear and appropriately challenge students, students are more likely to understand the criteria for their success when provided feed up (Hattie & Timperley, 2007).

Feed back. Feed back describes the information given to students related to a “task or performance goal, often in relation to some expected standard, to prior performance, and/or to success or failure on a specific part of the task” (Hattie and Timperley, 2007, p. 89). By answering the question “How am I going?” (Hattie & Timperley, 2007, p. 89), effective feed back provides information on students’ progress and/or how to advance towards the goal. Feed back can indicate the correctness of a solution (e.g., “Yes, the answer is 5.”) or strategy (e.g., “Your application of that proof is not correct.”). However, feed back is more effective when given with additional details about the students’ solutions (e.g., “That is true. Every whole number can be represented with a fraction.”), as well as ways they can correct their errors (e.g., “You are on the right track but do not have the correct answer. To really understand it, you might have to draw a model first.”; Black & Wiliam, 1998; Hattie & Timperley, 2007).

Feed forward. Feed forward describes the information given to students that completes the feedback cycle requiring students to apply the feedback they previously received (Brooks et al., 2019; Hattie & Timperley, 2007). This type of feedback addresses the question “Where to next? . . . providing information that leads to greater possibilities for learning” (Hattie & Timperley, 2007, p. 90). Feed forward may consist of additional strategies for students to use while working on tasks (e.g., “Why don’t you try taking your model and compare it to your expression.”), extra challenges to extend their thinking (e.g., “Think about how you can explain your work in terms of a fraction with a different denominator.”), or techniques for self-regulation (e.g., “You should be thinking what can I take from this problem to help me solve problems in the future.”). Feed forward is an essential stage of the feedback process as it reduces the gap between where students currently are and where they need to be (Brooks et al., 2019). This allows students the opportunity to reflect on all the feedback provided (Quinton & Smallbone, 2010). Ultimately, feed forward has the potential to have the greatest impact on student learning (Hattie & Timperley, 2007).

Within each type of feedback (i.e., feed up, feed back, and feed forward) is a level at which it can be given (Hattie & Timperley, 2007). The following section will discuss these four levels (task, process, self-regulation, and self) at which feedback can be directed within each of the three types.

Levels of Feedback

From their meta meta-analysis, Hattie and Timperley (2007) identified four feedback levels at which each type of feedback can be given. The levels are intended to help facilitators provide "specific feedback to individual learners dependent upon their learning needs" (Brooks et al., 2019, p. 19). The feedback model shown in Figure 1 displays the four levels at which the feedback can operate, including feedback at the task, process, self-regulation, and self levels. The following sections will explore the four levels of feedback as described by Hattie and Timperley (2007) and will describe the ways in which they align with reform-based instructional practices.

Task level. Task-level feedback is defined by Hattie and Timperley (2007) as feedback regarding "how well a task is being accomplished or performed" (p. 91). Commonly known as corrective feedback, task-level feedback includes information directed at the completion of a task or about the correctness of a solution, students' errors, or other criteria related to completing the task (Hattie & Timperley, 2007; Shute, 2008). In addition, task-level feedback may be used to acquire additional or different information about the task (Hattie & Timperley, 2007). For example, a teacher may say to a student, "Your graph does not look complete. Is there something you could add?" Providing task-level feedback in this way increases the likelihood that students remember where their errors occurred and can develop conceptual understanding as a result (Hattie & Timperley, 2007). Task-level feedback may be more effective when directed at students' misinterpretations of the task rather than their lack of information, building a stronger surface understanding (Brooks et al., 2019; Hattie & Timperley, 2007). Additionally, too much task-level feedback can lead to students using more trial-and-error strategies and less mathematically rich strategies to advance their thinking (Hattie & Timperley, 2007).

Process level. Process-level feedback, as described by Hattie and Timperley (2007), is feedback "specific to the processes underlying tasks or relating and extending tasks" (p. 93). Process-level feedback focuses on the processes needed to complete the task including the use of strategies and the understanding of how concepts and skills are related (e.g., "Try to use a different strategy you have learned for solving this problem that connects to what we learned yesterday."). Process-level feedback can help students develop error-detection strategies such as choosing a more effective strategy to use if their original strategy is ineffective (Hattie & Timperley, 2007). Thus, process-level feedback is more effective for enhancing a deeper understanding than that of task-level feedback (Brooks et al., 2019).

Self-regulation level. Self-regulation level feedback, as described by Hattie and Timperley (2007), is feedback that is the "interplay between commitment, control, and confidence" (p. 93). Feedback at the self-regulation level is directed at ways students can learn to self-assess their work, make an effort to struggle productively, and be confident when arriving at a solution they believe to be correct (e.g., "Other students say that the solution is five. How can you determine if your solution of six is correct or their solution of five is incorrect?"; Brooks et al., 2019; Hattie & Timperley, 2007). Having the potential to foster independent students (Hattie & Clarke, 2019), feedback at the self-regulation level supports students in monitoring and assessing their own learning often leading to the detection of their own errors (Hattie & Timperley, 2007). This type of feedback can help students build the self-assessment skills necessary for evaluating where they are, where they are going, and how they are going to get there when solving mathematical problems (Hattie & Timperley, 2007), thus furthering their learning and conceptual understanding (Brooks et al., 2019). In addition, self-regulation level feedback is effective for developing students' own autonomy and self-monitoring skills (Carless, Salter, Yang, & Lam, 2011).

Self level. Self-level feedback, as described by Hattie and Timperley (2007), is personal feedback usually in the form of praise. Self-level feedback is often unrelated to the task and rarely leads to student engagement (Hattie & Timperley, 2007), student commitment to working through the task (Good & Grouws, 1975), or improved self-efficacy (Skipper & Douglas, 2012). Students who are given self-level feedback directed at their ability to arrive at the correct solution (e.g., "You are so smart. Great Job!"), are not likely to persevere when faced with challenges (Hattie & Timperley, 2007). This can reduce students' performance on future tasks and shift their attention to irrelevant factors, ultimately having a detrimental impact on student learning (Brooks et al., 2019; Good & Grouws, 1975; Hattie & Timperley, 2007; Hattie & Yates, 2014; Skipper & Douglas, 2012). However, students who are given self-level feedback directed at their efforts and ways they could self-appraise their errors on a variety of tasks (e.g., "You are very diligent in working through this problem.") are more likely to seek additional feedback from others to further their own understanding, demonstrating an increase in intrinsic motivation (Cameron & Pierce, 1994; Hattie & Timperley, 2007). Additionally, praise can be helpful for establishing relationships between the teacher and students in a positive classroom where the teacher provides praise in moderation (Hattie & Yates, 2014).

Feedback Framework. The levels of feedback, designed by Hattie and Timperley (2007), provide a unique framework for describing four levels at which feedback can operate. The levels of feedback align in many ways with reform-based instructional practices that support students in learning mathematics. Teachers can provide task-level feedback by posing purposeful questions related to the task that help students discover and examine errors within their solutions allowing students the opportunity to justify their thinking and reasoning (NCTM, 2000, 2014). Process-level feedback aligns with reform-based instructional practices as it is directed at the process students go through to solve problems (NCTM, 2000, 2014). Feedback at the process level is directed at the strategies students use to solve problems providing them with the opportunity to reflect on the feedback given and use it to modify their thinking (Hattie & Timperley, 2007). Considering that each student approaches problems in different ways, process-level feedback is most often customized for individual students allowing multiple entry points to problems depending on the level of understanding (NCTM, 2000, 2014). Thus, feedback at the process level supports students in devising new strategies, determining the strategies that will be most efficient for the problem, and developing a conceptual understanding throughout the problem-solving process (NCTM, 2000, 2014).

Feedback at the self-regulation level supports students in productively struggling through the problem-solving process when teachers pose purposeful questions that help students to monitor and assess their own learning (NCTM, 2014). Students can then use the feedback teachers provide to extend their thinking to their current and future problems. Most often in the form of praise, self-level feedback is focused on the student as a person rather than on the problem-solving process (Hattie & Timperley, 2007). Although self-level feedback can have a detrimental impact on student learning by focusing the attention away from the learning goals (Hattie & Timperley, 2007; Hattie & Yates, 2014), self-level feedback directed at students' efforts to solve problems (e.g., "Good effort") may encourage students to productively struggle on the task as a result of positive praise in moderation (Hattie & Timperley, 2007; Hattie & Yates, 2014).

There are many ways, however, that Hattie and Timperley's (2007) levels of feedback do not support reform-based instructional practices. Task-level feedback may be in the form of giving hints about the task or helping students arrive at a correct solution which may hinder their ability to build a conceptual understanding by taking away the productive struggle (Hattie & Timperley, 2007; NCTM, 2000, 2014). Fostering a classroom environment where self-assessment skills are valued takes time (Peng, 2016). As a result, students may view feedback at the self-regulation level as an avoidance of help from the teachers when they are provided feedback that focuses on their self-assessment skills and not toward the solution of the problem they are currently solving (Hattie & Timperley, 2007). Additionally, self-level feedback directed at the student as a person (e.g., "Good boy"; Hattie & Timperley, 2007) may hinder the student's productive struggle when failing to receive encouragement in a similar way on future problems (Boaler, 2016).

Summary

The previous section reviewed various studies and meta-analyses on the information transferred between teachers and students known as feedback. The modes of feedback were also examined with a focus on oral feedback, the most commonly used mode of feedback. The three types (i.e., feed up, feed back, and feed forward) and four levels (i.e., task, process, self-regulation, and self) of feedback that resulted from Hattie and Timperley's (2007) meta meta-analysis, provided a lens for examining ways in which feedback can operate and align with reform-based instructional practices. Although the previous section focused on how teachers provide feedback, it is important to look at the individual differences in teachers that may attribute to the various ways they provide feedback (Shute, 2008). The following section will examine the literature on students' and teachers' implicit theories and the effects of their own implicit theories on their success in the mathematics classroom.

Implicit Theories

Research has shown that people have implicit beliefs regarding the way they view themselves and others (Dweck et al., 1995; Dweck & Leggett, 1988; Good et al., 2003). When people do not confront obstacles or limit goal attainment by avoiding challenges, they are less likely to struggle productively through difficult tasks (Dweck et al., 1995). They are said to hold an entity theory: an implicit belief that a person's attributes are non-malleable or cannot be improved (Dweck et al., 1995). In contrast, when people are willing to be challenged and sustain engagement to reach valued goals, they see struggle as a gateway to success (Dweck & Leggett, 1988). They are said to hold an incremental theory: an implicit belief that a person's attributes are malleable and can be improved (Dweck et al., 1995).

Students who hold an entity theory often view mathematics as content that must be memorized, drilled, and tested repeatedly to make sense (Boaler, 2016). However, students who hold an incremental theory see mathematics as a flexible set of relationships and new ideas that can be manipulated to make sense (Boaler, 2016; Dweck & Leggett, 1988). Thus, students' own implicit beliefs regarding mathematics can influence the way they face challenges and struggles in the mathematics classroom, as well as their overall success (Aronson, Fried, & Good, 2002; Chen & Pajares, 2010; Good, Rattan, & Dweck, 2012; Rattan et al., 2012).

The way teachers view mathematics and their students within the mathematics classroom environment is also important when it comes to student success (Rattan et al., 2012). Rattan et al. (2012) found that the overall beliefs teachers hold and the level of expectations they have could potentially affect the way teachers provide information to their students. Considering that feedback is one of the most influential factors on student achievement (Hattie & Timperley, 2007), it is essential to understand both students' and teachers' implicit beliefs. The following section will contain a summary of the literature on the effects of students' own implicit theories on their success in the mathematics classroom as well as the influence that teachers' implicit theories have on their instructional practices.

Students' Implicit Theories

Students are often judged and categorized in school by their grades on standardized tests and efforts to do work in the classroom (Blackwell et al., 2007; Good et al., 2003; Good et al., 2012). Typically, students who hold an incremental theory earn higher grades in most courses and through their efforts are able to recover quickly after receiving a low score (Aronson et al., 2002; Blackwell et al., 2007; Dweck & Leggett, 1988). They believe that opportunities to learn and grow occur through perseverance and struggle (Dweck, 2006). In contrast, students who hold an entity theory may not earn high grades in all subjects as they do not believe they are smart enough to try again when faced with negative outcomes (Good et al., 2003). They may see themselves as not having the ability to do well and abstain from struggle when faced with challenges or a low score on a test (Blackwell et al., 2007; Dweck, 2006; Good et al., 2003).

Although various studies have examined the effects of students' implicit theories on student success in the mathematics classroom (e.g., Blackwell et al., 2007; Good et al., 2003), researchers continued to explore additional factors that may influence students' implicit theories (e.g., Good et al., 2012; Yeager & Dweck, 2012). These factors include praise (Gunderson et al., 2013;), grouping methods (Dweck, 2006), and teacher-provided help in the classroom (Boaler, 2016; Rattan et al., 2012). Praise given by both teachers and parents can attribute to student success in the classroom (Boaler, 2016; Gunderson et al., 2013; Kamins & Dweck, 1999). When students are given person praise (i.e., praise that focuses on only their intelligence, abilities, or worthiness), they begin to attribute their successes primarily to how smart they are or are not (Kamins & Dweck, 1999).

Thus, when students fail, they believe that they are not smart and often give up. In contrast, when students are given process praise (i.e., praised for their strategies or effort), they are more likely to try harder when they fail (Kamins & Dweck, 1999).

The grouping methods that teachers use have also been shown to be a factor for fostering students' implicit theories (Boaler, 2016; Dweck, 2006; Good et al., 2012). Students who are randomly grouped, rather than on their perceived intellectual ability, are more likely to become active members of the group where each student is able to participate equally and encouraged to do so (Good et al., 2012). The way teachers provide help to their students can also foster students' implicit theories (Rattan et al., 2012). Teachers who provide help to students by breaking down the steps and guiding them to a solution deprive students of the productive struggle by simply giving students the solutions (NCTM, 2014). This may demotivate students who hold an entity theory indicating to them that students do not necessarily have to struggle to succeed (Rattan et al., 2012). Similarly, students who hold an incremental theory do not have the opportunity to struggle or to show their efforts and thus cannot build a greater understanding (Good et al., 2003; Rattan et al., 2012; Yeager & Dweck, 2012). Students' implicit beliefs and teachers' actions that foster these beliefs have become the focus of numerous studies; however, there is little empirical research on the implicit beliefs of teachers (Rattan et al., 2012).

Teachers' Implicit Theories

There are a variety of potential factors that influence teachers' instructional practices, one of which is their implicit beliefs (NCTM, 2014; Rattan et al., 2012;

Willingham, 2016). Within the mathematics classroom, teachers' implicit beliefs can be viewed in two primary ways: the way teachers view their students' learning process (e.g., whether teachers believe their students' mathematical ability can change) or the way teachers view their own learning process (e.g., the teachers' ability to learn by struggling through the mathematics themselves; Rattan et al., 2012). In a national survey examining teachers' views of implicit beliefs in the K-12 classroom, Education Week Research (2016) found that teachers' beliefs varied significantly as to whether student success could be attributed to the teachers' own implicit beliefs. Praise was perceived as the most effective way to foster an incremental theory, yet the most significant challenge teachers faced in the classroom was connecting with struggling, apathetic, or resistant students (Education Week Research, 2016). Teachers' implicit beliefs were not scored as part of the survey and although there were questions that addressed what they believed, there was no way to know the factors that influenced their answers to the questions.

Teachers may not actually know that they hold implicit beliefs about themselves or their students in the mathematics classroom (Gleason, 2016; Rattan et al., 2012). To address how teachers view implicit beliefs in the mathematics classroom, Sun (2014) administered a survey to 40 middle school mathematics teachers and found that teachers were more likely to view students' mathematical ability based on whether they got the correct solution and were less likely to view mistakes as opportunities for learning. Additionally, teachers' implicit theories about mathematics predicted students' implicit theories at the end of the year. Thus, teachers who held an incremental theory at the beginning of the year tended to have more students with incremental theories at the end (Sun, 2014).

Teachers may also ascribe to a particular implicit theory but describe their teaching practices differently from the implicit theory to which they ascribe (Gleason, 2016). In her study of 75 elementary mathematics teachers, Gleason (2016) found that 76% of the teachers ascribed to an incremental theory of intelligence, 19% ascribed to an entity theory of intelligence, and 5% were neutral. Of the 10 teachers interviewed, Gleason (2016) found that teachers' implicit beliefs determined by an initial survey did not always align with how they described their own teaching practices. The author concluded that "teachers have more of [an incremental theory] for their own learning, but more of [an entity theory] for their teaching" (p. 76).

It is critical to understand how teachers' instructional practices are influenced by their implicit beliefs regarding mathematical ability (Willingham, 2016). In an exploratory case study, Willingham (2016) examined how a teacher's implicit beliefs influenced her interpretations of a professional development program and her instructional practices as a result. The author found that the teacher shifted towards more reform-based instructional practices after participating in a four-year professional development program. The teacher held an incremental theory throughout the period of professional development. Her implicit theory was evident in her instructional practices through her attention to the learning goals and students' understandings and explicit discussions about implicit theories with her students. In addition, the teacher began to understand that for children to build a conceptual understanding of mathematics, the children needed to be engaged in the thinking process. Willingham (2016) concluded that the teacher's implicit belief structure served as a mediator for the conceptual understanding and instructional practice changes in her classroom.

The inferences that teachers make about students' mathematical ability are also critical for understanding teachers' instructional decisions (Rattan et al., 2012). Rattan et al. (2012) surveyed 136 undergraduate students and 41 graduate teaching assistants in mathematics-related fields to explore the types of feedback they would provide if they were seventh-grade mathematics teachers. Using various online surveys, article readings, and hypothetical situations, the authors concluded that teachers "have the opportunity to play a critical role in leading students to persist and maintain their engagement" (Rattan et al., 2012, p. 736) depending on their presumed implicit theory. Although the authors attempted to address "how implicit theories of ability play out in the pedagogical practices that instructors use" (p. 731), there were no actual seventh-grade teachers nor actual pedagogical practices observed. Considering that feedback is one of the most influential factors on student achievement (Hattie & Timperley, 2007), it is essential to understand teachers' implicit beliefs and how their implicit beliefs influence the feedback they provide in the mathematics classroom.

Summary

The studies discussed in this section illustrated that implicit theories played an important role in students' success in the mathematics classroom as well as teachers' instructional practices. Various studies examined the effects of students' own implicit theories on how they approached opportunities to learn and face challenges (Aronson et al., 2002; Blackwell et al., 2007; Dweck & Leggett, 1988; Kamins & Dweck, 1999). Additionally, I examined teachers' implicit beliefs and how their implicit beliefs influenced the way they viewed mathematics and their own instructional practices and decisions (Education Week Research, 2016; Rattan et al., 2012; Sun, 2014; Willingham, 2016).

Conceptual Framework

There were two theoretical frameworks that combined to form the conceptual framework for my study. I used the models of feedback and implicit theories as theoretical frameworks to understand the distinct teacher feedback practices in two elementary mathematics classrooms. I categorized the observed feedback according to Hattie and Timperley's (2007) Model of Feedback to Enhance Learning including the types (i.e., feed up, feed back, and feed forward) and levels (i.e., task, process, self-regulation, and self) of feedback. I also used the implicit theories model (Dweck et al., 1995) to select the participants and potentially serve as a basis for understanding the individual differences that contributed to the various ways teachers provided feedback during mathematics instruction.

Chapter Summary

Given the potential for various factors to have an impact on the type of feedback offered to students (Boaler, 2015, 2016; Rattan et al., 2012; Shute, 2008), the purpose of this study was to examine how elementary teachers provide feedback during mathematics instruction. To this end, the previous chapter included an overview of the learning goals of mathematics instruction including how students learn mathematics and the supporting instructional practices. I also examined various definitions of feedback and a model of feedback containing three types and four levels designed by Hattie and Timperley (2007). I discussed the ways in which the model of feedback aligns with reform-based instructional practices and different modes of feedback. In addition, I reviewed a summary of the empirical research on students' and teachers' implicit theories. The following chapter will provide details of the methodology based upon the literature review described in this chapter.

CHAPTER III: METHODOLOGY

Introduction

Although there are many factors that contribute to students learning mathematics, teacher feedback has been established as one of the most important influences on student achievement (Hattie & Timperley, 2007; Hattie & Yates, 2014; Hubacz, 2013; Wisniewski et. al., 2020). By providing feedback, teachers can assess students' understandings and address discrepancies in the learning process (Hattie & Timperley, 2007). However, little research focuses on the types of feedback teachers give or how the feedback may be influenced by their implicit theories (Rattan et al., 2012). As the research study progressed, it became evident that the original purpose of this study (i.e., to closely examine whether teachers' implicit theories acted as a mediator for the types of feedback given during mathematics instruction) needed to be changed. The following chapter includes a description of how I modified the research question, an overview of the research design, research context including participant information, participant-selection process, instruments and data sources, procedures, data analysis, and reliability of the multiple-case study.

Modifying the Research Question

The study utilized a multiple-case design (Yin, 2014) exploring two elementary mathematics teachers' feedback practices. Originally, the study focused on the elementary teachers' underlying beliefs regarding the feedback they provided to their students during mathematics instruction. Specifically, the initial research question was: How does an elementary mathematics teacher's implicit theory mediate the feedback given during mathematics instruction, if at all? Prior to data collection, I planned for the two participants to have opposing implicit theories, and I hypothesized that their implicit beliefs would be evident in the feedback practices that I observed during mathematics instruction.

However, the potential participants who ascribed to an entity theory did not agree to participate in the study. As a result, both of my participants ascribed to an incremental theory. Although I was not able to attribute the participants' implicit belief to their feedback practices, their implicit beliefs helped to build a holistic picture of their feedback practices during mathematics instruction. With the dynamic nature of all qualitative inquiry, Creswell (2013) explained that various design components may change and emerge as qualitative research is conducted. Thus, after analyzing the data collected during my study, the modified research question emerged: In what ways do elementary teachers provide feedback during mathematics instruction? I used this qualitative question to focus my work in observing and analyzing patterns in the types of feedback elementary mathematics teachers give and the levels in which they provide feedback.

Overview of the Research Design

Yin (2014) explained that a multiple-case study can be designed by deliberately selecting two significant cases that offer contrasting conditions or are contained within the same subgroup that explain a particular condition. In doing so, the findings can "support the hypothesized contrast [where] the results represent a strong start toward theoretical replication – strengthening [the] findings compared to those from a singlecase study alone" (Yin, 2014, p. 64). My initial intent was for the two participating teachers to offer contrasting conditions (i.e., one who held an incremental theory and one who held an entity theory); however, due to the unavoidable issues during the participant selection process, both of my participants ascribed to the same subgroup (i.e., an incremental theory). Continuing my study with two participants who ascribed to the same implicit theory was beneficial to see whether there were similarities in the feedback practices within the subgroup of elementary teachers who ascribe to an incremental theory. To this end, I used an exploratory multiple-case study with two elementary mathematics teachers within the same subgroup: one who ascribed to a strong incremental theory and one who ascribed to a weak incremental theory (Dweck et al., 1995).

Research Context

The study focused on how two elementary mathematics teachers naturally provided feedback in their classrooms in one elementary school located in a southeastern part of the United States during the spring of 2018. The following sections provide detail regarding the state, district, school, classrooms, and participants.

State

I conducted the study in a southeastern state of the United States that served 975,222 students consisting of 62.1% Caucasian, 24.0% African American, 10.9% Hispanic, 2.4% Asian/Pacific Islander, and 0.6% Native American/Alaskan students in 2017-2018. The state reported 34.9% economically disadvantaged students, 4.6% limited English-proficient students, and 13.5% students with disabilities. At that time, the state had 1,819 schools and employed 69,531 classroom teachers. Although the students' mathematics scores on the NAEP (U.S. Department of Education, 2017) had continued to improve since 2011 in the selected state, they were still at or below the national average.

District

I conducted the study in a school district with 12 schools. In 2017-2018, the district had 630 teachers and 8,385 students comprised of 50.4% Caucasian, 31.3% African American, 13.6% Hispanic, 4.3% Asian/Pacific Islander, and 0.5% Native American/Alaskan students. Similar to the state demographics, the district reported 32.0% economically disadvantaged students, 6.9% limited English-proficient students, and 14.7% students with disabilities. Of the students in grades 3-6 who took the statewide comprehensive assessment program mathematics exam during the 2016-2017 school year, 13.3% scored at the advanced level, 31.4% scored at the proficient level, 21.9% scored at the basic level, and 23.4% scored at the below basic level. These results were slightly higher than the state average.

School

The study took place in an urban elementary school (Byron Elementary, a pseudonym) that served approximately 661 students from pre-kindergarten through sixth grade. Byron Elementary School employed 53 instructional faculty, 38 support personnel, and two administrators. The school served 46.4% African American, 28.6% Caucasian, 23.5% Hispanic, 1.0% Asian/Pacific Islander, and 0.4% Native American/Alaskan students from 2017-2018. Of the 661 students, 59.3% were economically disadvantaged, 11.6% were limited English proficient, and 14.5% were reported as having a disability.

Classrooms

The study took place in a third-grade classroom and a fifth-grade classroom during mathematics instruction. The third-grade classroom, taught by Ian Smith (pseudonym), consisted of 19 students. Mathematics instruction occurred 30 minutes before and 60 minutes after lunch. The fifth-grade classroom, taught by Ellie Jones (pseudonym), consisted of 24 students. Mathematics instruction occurred in the morning for approximately 50 minutes prior to science instruction. Both participants will be described in the following section.

Participants

The first participant was Ian Smith, an African American male in his late twenties, who ascribed to a strong incremental theory. Ian held a bachelor's degree in early childhood education and a master's degree in curriculum and instruction. At the time of the study, he was certified to teach kindergarten through sixth grade. Ian was in his fourth year of teaching third grade, having taught two years at Byron Elementary School and two years previously in a different district. Within his first year at Byron Elementary School, Ian took on the role of third-grade team leader. His responsibilities included communicating between the administration and the third-grade teachers and navigating his team through targeted instruction for specific groups of students as well as best practices based on common formative assessment data.

The second participant was Ellie Jones, an African American female in her mid-forties, who ascribed to a weak incremental theory. Ellie grew up in Kenya where she earned her bachelor's degree in music and taught music during her student teaching internship. While in college, she taught high school mathematics during the breaks because she enjoyed mathematics. Ellie took a position as an administrative assistant after graduating and went back to school to earn her master's degree in business administration. After working in marketing at a telecommunications company and moving to the United States, she earned her master's degree in curriculum and instruction. At the time of the study, she was certified to teach kindergarten through sixth grade. Ellie was in her eighth year at Byron Elementary School where she taught fourth grade for two years and fifth-grade mathematics and science for six years. At the time of the study, Ellie taught fifth grade mathematics and science. Prior to this study, both participants engaged in professional development on mindset at Byron Elementary School, but the extent of the professional development was unknown.

Instruments and Data Sources

I used multiple data sources and data collection methods to explore how elementary teachers provide feedback during mathematics instruction. Sources of data included an implicit theories survey, observational protocol, audio recordings, video recordings, semi-structured interviews, participant reflective journals, and the researcher as an instrument. The following sections include a description of the instruments and data sources used throughout the study.

Implicit Theories Survey

To measure participants' implicit theories, I used the Implicit Theories Survey (see Appendix A). The Implicit Theories Survey was a modified version of Dweck et al.'s (1995) Implicit Theories Measures, which measured the implicit beliefs about a person in the domains of intelligence, morality, and the world as a whole. The Implicit Theories Survey contained an additional domain of mathematical ability (Willingham, Barlow, Stephens, Lischka, & Hartland, in press).

Each domain consisted of three items with scores ranging from one to six for each item. The scores for each domain are averaged, and the averages for each domain are averaged for an overall implicit theory score (Dweck et al., 1995). According to Dweck et al. (1995), scores 3.0 and below are classified as an entity theory and 4.0 or above are classified as an incremental theory, with a higher score indicating a stronger incremental theory. For the purposes of my study, I will refer to Ian as a strong incremental theorist with a score closer to six and Ellie as a weak incremental theorist with a score closer to four (see Figure 2). The survey also included questions about teachers' backgrounds for use in the initial participant selection process.

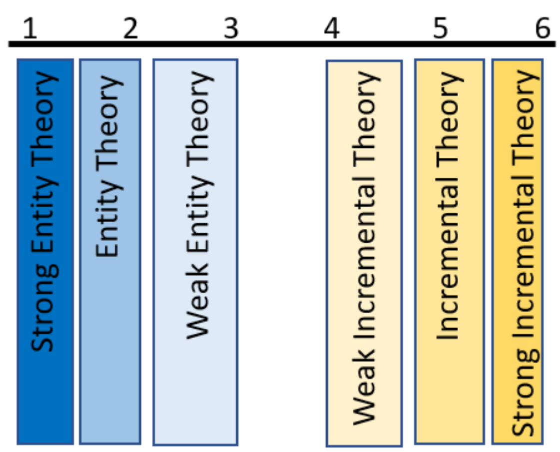

Figure 2. Implicit theory overall average score classifications ranging from a strong entity theory to a strong incremental theory. Adapted from Dweck, C. S., Chiu, C., & Hong, Y. (1995). Implicit theories and their role in the judgments and reactions: A world from two perspectives. *Psychological Inquiry, 6*, 267 – 285.

Observational Protocol

I designed the Observational Protocol (see Appendix B) to include two columns: one for descriptive field notes to record factual events and one for observer comments. Notes consisted of descriptions of participants' actions depending on the types of feedback the participants gave, in addition to possible questions that could be asked during the daily interviews following the classroom observations. The Observational Protocol notes were recorded using a Livescribe pen with an embedded computer and digital audio recorder. The Livescribe pen allowed me to synchronize notes written in a Livescribe notebook with the audio recorded at the exact moment. The use of the Livescribe pen and notebook allowed me to easily locate incidents of feedback that were used for reflection during the interview process.

Audio Recording of Participant Instruction

A portable audio recording device consisting of a portable recorder and a microphone attached to each participant's shirt allowed for mobility around the classroom. The audio device provided clear audio of the oral feedback given by the participants that was not heard by the researcher during the actual instruction.

Video during Mathematics Instruction

I video recorded each participant's mathematics instruction time on a stationary camera to provide an additional source of validation of the feedback provided by the participants. I positioned the video camera on a tripod or on a shelf in the back of the room. At times, I positioned the camera at the side of the room allowing me to focus on small group discussions or activities. The video camera captured the participants' and students' movements from a broad perspective, in addition to items displayed on the interactive boards.

Semi-structured Interviews

I used semi-structured interviews throughout the study to address the participants' perceptions of feedback "while leaving space for participants to offer new meanings to the study focus" (Galletta, 2013, p. 24). All interviews were audio recorded using the audio recording device and Livescribe pen while I took notes using the Livescribe notebook. I transcribed all interviews and cross-checked with the Livescribe notes for verification. The following sections will describe the types of interviews that I utilized throughout the study.

Participant selection interview. I designed the Participant Selection Interview Protocol (see Appendix C) to examine teachers' responses on the Implicit Theories Survey (see Appendix A) and for determining the two participants for the remainder of the study (i.e., one who ascribed to an entity theory and one who ascribed to an incremental theory).

Initial interview. I designed the Initial Interview Protocol (see Appendix D) to further explore the mathematical implicit theories of the participants and their initial interpretations of how they provided feedback to their students.

Daily interviews. I designed the Daily Interview Protocol (see Appendix E) as a guide to explore how the participants perceived their own methods of providing feedback to their students during the study timeline and the reasoning behind their decisions.

Additional questions that were recorded in the Livescribe notebook during mathematics instruction were also used. Recorded instances from that day or from previous days were occasionally reviewed for clarification and for participants to critically reflect on how and why they provided feedback during mathematics instruction.

Final interview. I designed the Final Interview Protocol (see Appendix F) to capture the participants' overall thoughts on the feedback they provided throughout the study and to revisit any topics that were relevant to the study.

Participant Reflective Journals

I asked both participants to respond to various reflective journal writing prompts (See Appendix G) through email twice during the proposed study. The writing prompts were intended for the participants to reflect on specific instances where they provided feedback and to delve further into the reasoning behind their decisions. In addition, I encouraged the participants to email me at any time to reflect on previous journal entries. Journal emails were reviewed throughout the study and participants were asked to elaborate on their entries during the daily interviews or in writing. All journal emails served as data sources, and participants were given pseudonyms for de-identification purposes.

The Researcher as an Instrument

By collecting data through observations and interviews (Creswell, 2013), I became a primary instrument of the study. At the time of the study, I held a Bachelor of Science and a Master of Science in Mathematics and had 11 years of K-12 mathematics teaching experience. I had completed two years of doctoral coursework in mathematics education, including qualitative research and educational research methods courses. Additionally, I had participated in multiple qualitative research projects related to implicit theories and teacher feedback. These professional experiences qualified me as an instrument of the study.

Procedures

Once the Institutional Review Board approved the study (see Appendix H), I began collecting data in four phases. The following section describes the procedures associated with each phase.

Phase I: Participant Selection

I proposed the study to the school principal of Byron Elementary School on December 1, 2017, and received her approval to start collecting data that month. She informed me that all faculty and staff were required to attend an upcoming professional development to be held during after-school hours. Teachers were required to attend one of two days that the professional development was available which provided me the opportunity to interact with all teachers face-to-face. I administered all teachers at Byron Elementary School the Implicit Theories Survey (see Appendix A) at the beginning of the professional development in December 2017 (see Figure 3). The school principal asked that I make it clear to the teachers that the principal would not be receiving the results of their Implicit Theories Survey. I did this explicitly to ensure a more accurate representation of the teachers' implicit theories.

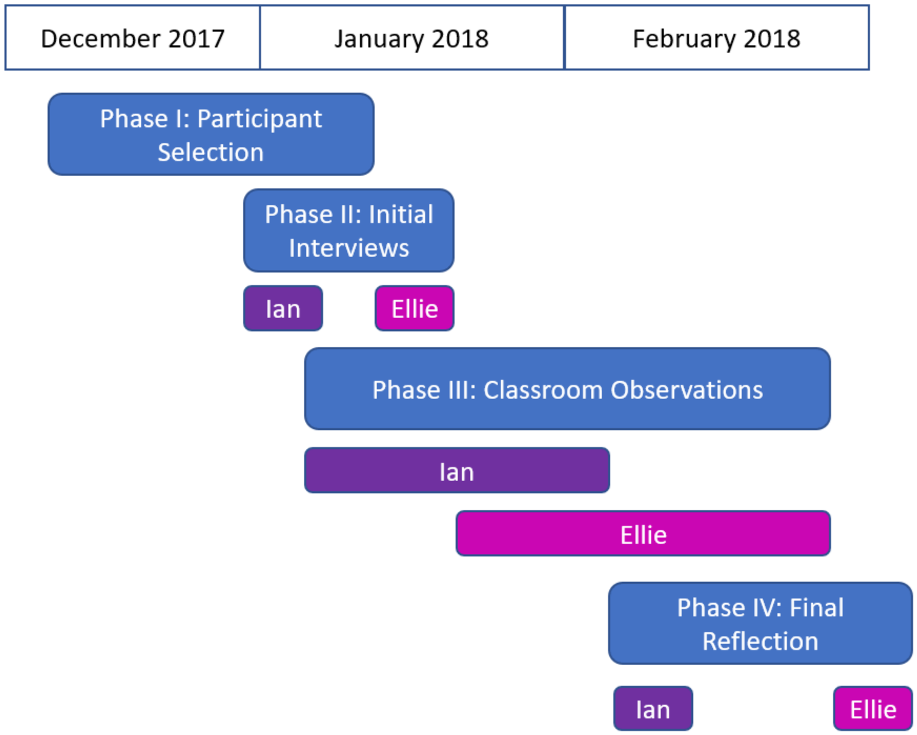

Figure 3. Study timeline phases.

The Implicit Theories Survey (see Appendix A) also included questions about teachers' backgrounds. I used this information for the final participant selection process to ensure the participating teachers currently taught mathematics and at different times during the school day. I scored the surveys and chose two teachers who, according to the survey results, ascribed to a strong incremental theory (i.e., a higher score on the Implicit Theories Survey; see Figure 2) and two teachers who ascribed to an entity theory (i.e., a lower score on the Implicit Theories Survey). I interviewed a total of five teachers from Byron Elementary School using the Participant Selection Interview Protocol (see Appendix C).

After reviewing the teachers' responses during the participant selection interviews, I invited one teacher who ascribed to a strong incremental theory and one teacher who ascribed to an entity theory to participate in the study. After the teacher who ascribed to an entity theory declined to participate, the second teacher previously interviewed who ascribed to an entity theory was asked to participate. This teacher also declined to participate after the participant selection interview and one classroom observation.

I then re-examined the results of the Implicit Theories Survey (see Appendix A) and selected a third teacher who ascribed to a weak incremental theory to interview based on her survey scores and grade level taught. It was important to keep the grade levels taught by the two participants close because teaching styles would vary significantly between a kindergarten teacher and a fifth-grade teacher, making it difficult to compare the participating teachers' feedback practices. I interviewed the third teacher who taught fifth grade and ascribed to a weak incremental theory in mid-January 2018 (see Figure 3) using the Participant Selection Interview Protocol (see Appendix C). She accepted the invitation to participate the same day. The data analysis focused only on the final two participants who accepted the invitation to participate in the study: a third-grade teacher who ascribed to a strong incremental theory (Ian) and a fifth-grade teacher who ascribed to a weak incremental theory (Ellie).

Phase II: Initial Interview

On the first day of school in January 2018 (see Figure 4), I interviewed Ian using the Initial Interview Protocol (see Appendix D) during his non-instructional time. The interview lasted approximately 18 minutes, and I planned to begin classroom observations the next day. A week after Ellie agreed to participate in the study, I conducted the initial interview with Ellie during her lunch time (see Figure 4). The interview lasted approximately 10 minutes, and arrangements were made to begin classroom observations the next week. During the initial interviews, I thanked both participants for participating in the study and answered any routine questions the participants had about the study.

January 2018					February 2018				
8 I: Initial Interview	9 I: Observe, Interview	10 I: Observe, Interview E: Selection Interview	11 I: Observe, Interview	12 No School	5 E: Observe, Interview	6 E: Observe, Interview	7 E: No Math Instruction Journal	8 I: Final Interview E: Observe, Interview	9 Researcher Absence
15 No School I: Journal	16 No School	17 No School	18 No School	19 No School	12 E: Observe, Interview	13 E: Observe, Interview	14 E: No Math Instruction	15 E: Absent	16 E: Absent
22 I: Observe, Interview E: Initial Interview	23 I: Observe, Interview	24 I: Observe, Interview	25 I: Observe, Interview	26 I: Observe, Interview	19 No School Holiday	20 E: Observe, Interview	21 E: Observe, Interview	22 E: Observe, Final Interview, Journal	23
29 I: Observe, Interview E: Observe, Interview	30 I: Observe, Interview E: Observe, Interview	31 I: Observe, Interview, Journal E: Observe, Interview							

Figure 4. Calendar of data collection including interviews, observations, and journal entries.

Phase III: Classroom Observations

Following the initial interviews as indicated in Phase II, I observed the participants' daily mathematics instruction time using the Observational Protocol (see Appendix B). During each classroom observation, participants wore a portable audio recording device to capture what they said during their mathematics instruction. I also video recorded the participants using a stationary device situated either in the back or side of the room. All recordings occurred simultaneously during the observations. I observed Ian's class for 11 days intermittently due to weather cancelations. This occurred from January 9, 2018, until January 31, 2018. During this time, Ian covered the instructional unit of fractions. I observed Ellie's class for 11 days intermittently due to school activities planned during mathematics time, teacher absences, my own personal absences, days where students reviewed previous material, or days where there was no mathematics instruction. This occurred from January 29, 2018, until February 22, 2018. During this time, Ellie covered the instructional units of multiplying and dividing fractions, coordinate points, and volume.

Following each observed class period, I interviewed the participants separately for no more than 10 minutes with questions chosen from a sample of questions found in the Daily Interview Protocol (see Appendix E). I interviewed Ian during his planning period immediately following the classroom observations and Ellie during her lunch period approximately an hour after her classroom observations. The interviews focused on instances where the participants provided feedback from that day's lesson or previous days to better understand how the participants' implicit theory influenced their feedback practices, if at all. Video and audio recordings were periodically reviewed with the participants for this purpose. In addition, participants responded to email writing prompts twice during the study using prompts from the Reflective Journal Writing Prompts (see Appendix G) or prompts that were intended to further their thinking about their feedback.

Phase IV: Final Reflection

Following the last day of classroom observations (see Figure 4), I interviewed each participant for approximately 12 minutes. I asked the participants questions using a sample of questions from the Final Interview Protocol (see Appendix F). The final interview was intended to capture the participants' final thoughts about the study and to discuss how the participants changed the way they viewed feedback throughout the process, if at all. I conducted Ian's final interview during his planning period a week after his final classroom observation and Ellie's final interview immediately following her daily interview on the last day of her final classroom observation (see Figure 4). The following section will discuss the steps I took to analyze the data collected during the procedures previously described.

Data Analysis

To find patterns and meaning in the data (Gay, Mills, & Airasian, 2014), I took multiple steps to organize, interpret, and analyze the data for both participants. Following the completion of the Participant Selection Interview transcriptions, I transcribed all audio recordings and interviews of the observed 11 days of mathematics instruction for Ian. I then watched the video recordings of Ian's mathematics instruction and created narratives from the corresponding transcribed audio recordings by adding in the actions of Ian and the students to the transcriptions. This step was crucial for verifying what I saw and heard and to align with the observational protocols and interview notes. I then wrote a detailed description for each day, making connections between the classroom observations, interviews, and journal entries.

I started step one of the coding process by first assigning codes to Ian's narratives of instances of how Ian interacted with his students during each part of his mathematics instruction (ex. whole group, small group, individual, etc.) in ATLAS.ti (see Table 1). Additionally, I assigned codes that described the context or content covered during each part of his mathematics instruction (ex. fractional understanding, halving and doubling, polygon review, volume, games, etc.). I coded all classroom observation narratives in this way.

Table 1

Descriptions of the Steps Taken in the Coding Process and Data Pieces Used in Each Case

Step	Description	Data Piece
1	Coded interactions (ex. whole group, small group, individual) and context/content (ex. fractional understanding, polygon review, volume)	Narratives
2	Coded for Hattie and Timperley's (2007) levels of feedback (i.e., task, process, self-regulation, and self)	Narratives
3	Openly coded classroom observation narratives	Narratives
4	Openly coded interviews and journals	Interviews & Journals
5	Created networks in ATLAS.ti to look for patterns across the data	Narratives, Interviews, & Journals
6	Restructured: Deleted all the levels of feedback codes created in step two but kept all open codes. Coded all classroom observations according to Hattie and Timperley's (2007) types of feedback (i.e., feeding up, feeding back, and feeding forward)	Narratives
7	Assigned a level of feedback (i.e., task, process, or self-regulation) to each instance coded as feed up, feed back, or feed forward. Coded self-level feedback independently of the types of feedback.	Narratives
8	Composed a case description for each case	Narratives, Interviews, & Journals

Step two of the coding process (see Table 1) consisted of coding Ian's narratives for instances where Ian provided feedback using a fixed list of codes according to Hattie and Timperley's (2007) descriptive framework of the levels of feedback (i.e., task, process, self-regulation, and self). In addition to coding the classroom narratives in this way, I openly coded the narratives in step three by highlighting the phrase(s) and creating new codes in ATLAS.ti to describe what was happening in the classroom. The first round of open codes described Ian's actions during his mathematics instruction time. I recorded the open codes along with examples and explanations of each in my reflective journal shown in Table 2.

Table 2

Open Codes for Step Three While Coding Classroom Observations

Open Code	Example/Explanation
Agree	Teacher agreed with the student's reply by either saying "yes" or "I agree."
Ask Class	Teacher asked the class if they agree, disagree, or neither
Ask Class Tweak	Teacher asked the class what they would tweak about a student's solution
Ask to Show Thinking	Teacher asked students to show their thinking either by demonstrating on paper, verbally, or with manipulatives
Behavior	Teacher responded to students directed at their behavior
Continue Student Ideas	Teacher elaborated on the student's reply with no indication of correctness
Correctness	Teacher indicated that the student was correct or incorrect
Different Example	Teacher responded by providing the student with a different example
Disagree	Teacher disagreed with the student's reply by either saying "no" or "I disagree"
Extension Question	Teacher responded with an extension question
Hmmm	Teacher responded by saying "Hmmm"
I Don't Know	Teacher responded to the student's reply by saying "I don't know"
I-Can Statement	Teacher mentioned the I-Can statement in some way
Intentional Mistakes	Teacher made an intentional mistake and asked students to find the mistake
Let's Check	Teacher responded in a way that allowed the students to check their solution rather than tell them whether it was correct

(continued)

Open Code	Example/Explanation
Multiple Choice	Teacher mentioned how multiple-choice problems were or were not used in their classroom
Not Give Answers	Teacher specifically said that they will not give the students the answer
Ok/All right	The first word in the response was “Ok” or “All right”
Oral Emotion	Teacher responded orally with emotion (ex. “Oohh, ok, ok, I got too excited!”)
Question	Teacher questioned the student’s reply, usually by restating or clarifying the student’s reply in the form of a question
Responsibility on Students	Teacher responded by putting the responsibility on the students (ex. “I’ll let you explain to the class and they’ll tell you if you got it right”)
Restate/Clarify	Teacher repeated or clarified the student’s reply in their own words
Reward	Teacher rewarded students in general
Reward Correctness	Teacher rewarded student for solutions that were correct
Reward Effort	Teacher rewarded students for their effort
Reward If/Then	Teacher responded by saying “I’ll give you a reward if you do...”
Reward Strategy	Teacher rewarded students for using different strategies or thinking outside of the box
Silence	When teacher was silent either before or as they responded (the amount of time they were silent was indicated)
SR Pit	Teacher referenced The Pit which fell into the category of self-regulation but specifically about The Pit

Following the open-coding process for Ian’s classroom observations in step three (see Table 2), I openly coded all of Ian’s interviews including the Participant Selection, Initial, Final, and Daily interviews, in addition to his two journal entries. Although I continued to use all codes previously mentioned, I found that new codes needed to be generated to explain Ian’s ideas stated in the interviews and his journal entries. Additionally, the open codes derived from the interviews and journals helped to illustrate

Ian’s view of teaching and learning mathematics. I recorded the new open codes in step four along with examples and explanations of each in my reflective journal shown in Table 3.

Table 3

Open Codes for Step Four While Coding Interviews

Open Code	Example/Explanation
Assess from Student Thinking	Teacher described ways to use evidence of student thinking to assess where to go next
Challenging Students	Teacher described ways that they challenged students in their classroom and/or the need for students to be challenged
Changing Regular Routine	Teacher mentioned how they changed their routine to address students’ needs
Choices Made	Teacher discussed the reasoning behind why they made certain choices
Clarification	Teacher clarified about content or teaching methods
Classroom Management	Teacher discussed the rules and/or some aspect of their classroom management
Correct Answer	Teacher mentioned something about correct answers
Curriculum	Teacher mentioned the curriculum or scope and sequence
Done Differently	Teacher mentioned what they would have done differently in their classroom
Finding Mistakes/ Misconceptions	Teacher mentioned when and where to focus on finding mistakes and/or misconceptions
Goal	Teacher described the goal of the lesson
Grades	Teacher mentioned grades, either letter or number
Grouping	Teacher mentioned grouping methods and/or why
Hands-on	Teacher mentioned students using manipulatives or doing hands-on activities (also coded in classroom observations when students did hands-on activities)
High/Low	Teachers referred to students as high achieving or low achieving
Mathematical Ability	Teacher mentioned mathematical ability or the ability to do mathematics

(continued)

Open Code	Example/Explanation
Memorization	Teacher mentioned memorization
Mindset	Teacher discussed some aspect of mindset, either of the students or their own
Motivation	Teacher described how they motivated students
Move Towards Abstract	Teacher mentioned moving from concrete to abstract ways of thinking
Multiple Representations	Teacher described representing mathematics in multiple ways
Not Correct Answer	Teacher emphasized that it is not all about the correct answer
Own Experience	Teacher described their own experience when learning mathematics
Pace of Students	Teacher mentioned the pace of the students and/or what their pace depended on
Partner Students	Teacher described how they partnered students
Perseverance	Teacher mentioned students persevering
Posing Initial Question	Teacher mentioned posing the initial question to help student thinking
Productive Struggle	Teacher mentioned student productive struggle (good or bad)
Questioning	Teacher discussed types or ways of questioning students
Reflection	Teacher reflected on some aspect of the lesson
Self-Assessment	Teacher assessed their own teaching or mentioned ways they could improve
Self-Confidence	Teacher described what they did in their classroom to help students' self-confidence
Student to Student Learning	Teacher mentioned students learning from each other
Testing	Teacher mentioned testing in some way
Understand	Teacher addressed students understanding
Validation	Teacher mentioned when they did or did not validate the correct answer or thinking
Variation Response	Teacher described how they varied the way they responded to students
Verbal Cues	Teacher mentioned how they provided verbal cues
Why	Teacher discussed the importance of understanding why things happen in mathematics

In step five I created networks using ATLAS.ti according to Hattie and Timperley's (2007) levels of feedback (i.e., task, process, self-regulation, and self), but found it difficult to identify patterns based on the vast amount of data within each level of feedback and the lack of clarity when organizing. As a result, I decided to restructure my initial coding scheme to focus on the types of feedback first rather than the levels of feedback to better reflect how the participant provided feedback. I did this by deleting all the levels of feedback codes (i.e., task, process, self-regulation, and self) created in step two, but keeping all open codes from steps three and four. Thus, I continued step six of the coding process by coding instances where Ian provided feedback according to Hattie and Timperley's (2007) types of feedback (i.e., feeding up, feeding back, and feeding forward). I only coded the classroom observation narratives in this way to look for evidence of how Ian provided feedback during mathematics instruction.

In step seven I assigned a level of feedback (i.e., task, process, or self-regulation) to each instance coded as feed up, feed back, or feed forward depending on where the type of feedback was directed. Given that self feedback could have a negative impact on learning and rarely leads to changes in a student's performance (Dweck, 2006; Hattie & Timperley, 2007; Kluger & DeNisi, 1996), I chose to code self-level feedback independently of the types of feedback. I repeated the same process for Ellie except I eliminated steps two and five, using the same open codes as previously generated when coding Ian's data. Table 4 shows the additional open codes generated while coding Ellie's case due to the ways in which Ellie was different from Ian. Context codes also generated but not included in Table 4 were Problem of the Day, Game, Video in Class, and Computer Problems.

Table 4

Additional Open Codes for Step Eight While Coding Ellie's Data

Open Code	Example/Explanation
Behavior	Teacher addressed the behavior of a student or students
Connections	Teacher mentioned making connections between concepts
Easy/Hard	Teacher indicated that some aspect of the lesson was easy or hard
Good Job/Good	Teacher responded to students by saying "Good job" or "Good"
Name on Board	Teacher mentioned the students' names on the board or putting students' names on the board
No Time for Response	Teacher did not allow time for students to respond after asking a question
Outside of Class	Teacher indicated the influence of some aspect outside of the classroom
Practice	Teacher mentioned that students needed to practice
Punishment	Teacher referenced some type of punishment
Shhh	Teacher told the class or certain students to be quiet by saying "shhh"
Some Will Not	Teacher described that some students will not be able to understand the material
Teach to Test	Teacher mentioned teaching to the test

After all data were coded in this way, I analyzed and created a case description for each participant which will be described in Chapter IV. I then conducted a cross-case analysis between the two cases by considering each individual case as a distinct study (Yin, 2014). I completed this by looking for similarities and differences across the cases, specifically in their learning experiences, observed teaching practices, types of feedback, and levels of feedback. I then developed generalizations (i.e., three key findings) from the comparison which will be discussed in Chapter V.

Reliability of the Study

With all educational qualitative research there are factors that affect the reliability of the methods used to collect data (Gay et al., 2014). It is important to identify the limitations and delimitations that influenced the study when considering future studies conducted in the same manner. The following section will describe the limitations, delimitations, and the steps taken to support trustworthiness of the qualitative study.

Limitations

Throughout the study, there were various factors out of my control that may have affected the results of the study (Gay et al., 2014). First, the third-grade teacher who held a stronger entity theory declined to participate in the study. If she would have agreed to participate, both participating teachers would have taught the same grade level and covered the same material; thus, allowing for a better comparison between feedback practices. Second, many of the teachers at Byron Elementary School scored higher on the Implicit Theories Survey (revealing stronger incremental theories), which made it difficult for me to select additional teachers to interview after the first teacher (third grade) and second teacher (fourth grade) who held a stronger entity theory declined to participate. As a result, Ellie, the fifth-grade teacher who agreed to participate in the study, ascribed to a weak incremental theory. Third, while the study ultimately took place in a third-grade classroom and a fifth-grade classroom, there were differences in mathematical curriculum covered and instructional routines. As a result, it was difficult to compare the participants' feedback practices side by side, having one teacher who covered more material and focused more on the procedures than the other. The fourth limitation consisted of interruptions in the daily data collection. Various weather cancellations including an entire week due to snow, various absences due to schedule conflicts, unannounced functions during school hours, and fire drills resulted in, at times, several days to a week between classroom observations. Because of this, the participants had to modify their lesson plans, which may have altered the way they provided feedback to their students by rushing through the curriculum to ensure they covered the material. Students often had trouble recollecting the content previously covered due to the lapse of time between instruction as well.

Lastly, both participants had visitors in their classrooms during observations, which may have affected the way they interacted with their students at that time. Ellie had five visitors with whom she spoke at the side of the room or outside in the hall for no more than two minutes. Ellie stopped what she was doing each time to address the needs of the visitors. Ian had several visitors observe his class and interact with students during the classroom observations. He had a student from a local university observe his mathematics instruction on the fifth day of observations. Ian had an intervention assistant on the sixth and seventh day of observations who walked around the classroom and helped students as they worked independently on a mathematics task. On the sixth, ninth, and eleventh day of classroom observations, Ian had a student teacher from a local university during mathematics instruction. The student teacher observed on the sixth day, helped students as they worked independently on a mathematics task the ninth day, and taught half the students on the carpet on the eleventh day while Ian worked with a small group of students at the small-group table.

Delimitations

Given the qualitative nature and the scope of the study, there were delimitations to the focus and research design that I controlled. First, I designed the study to focus only on feedback in two elementary mathematics classrooms at Byron Elementary School. I deliberately selected two cases (Yin, 2014) from the same school to minimize the variability in the school environment. Second, I designed the study to have the participants respond to journal prompts only two times throughout the study. I did this so that the participants would not feel overwhelmed with the classroom observations and interviews in addition to their daily responsibilities with teaching. Third, I chose not to let the fact that both Ian and Ellie had participated in a professional development about mindset influence my data collection or analysis. I did this because I was not informed about the professional development until the end of my data collection, nor did I know additional details other than that it occurred.

Trustworthiness

The trustworthiness of a study encompasses the balance, fairness, and diligence in taking multiple perspectives, experiences, and interests into perspective (Patton, 2015). To ensure the trustworthiness of the qualitative study, I used multiple strategies which addressed the credibility, dependability, transferability, and confirmability. Credibility refers to the “researcher’s ability to take into account all of the complexities that present themselves in a study and to deal with patterns that are not easily explained” (Gay et al., 2014, p. 345). To build overall credibility in my study, I collected classroom observation and interview data including audio recordings, video recordings, semi-structured interviews, participant reflective journals, and implicit theories surveys over a period of two months which supported the triangulation of my data. I also debriefed with my fellow doctoral colleagues daily to help me reflect on the data collection process. Additionally, my doctoral supervisors supported the credibility of my study by continuing to engage in thinking about the analysis process, ensuring that my analyses accurately reflected the data.

Dependability refers to the stability or reliability of the data (Gay et al., 2014). To ensure the dependability of my study, I maintained a reflective journal which included a daily log, the time and date of activities related to the study, a record of the methodological decisions I made, and my personal reflections on what occurred during the study (Creswell, 2013). The reflective journal provided a clear and accurate representation of the events that occurred during the study, which minimized the overall subjectivity (Gay et al., 2014; Yin, 2014).

Transferability refers to the extent in which the results of my study could be applicable in other contexts (Gay et al., 2014). To support the transferability of my study, I provided extensive detail of the context of my study including state, district, school, participants, and classroom data. By providing a thick description of the participants' classrooms and lessons in the context of my study, researchers can "establish the degree of similarity between [this qualitative study] and the case to which findings might be transferred" (Patton, 2015, p. 685).

Confirmability refers to the “neutrality or objectivity of the data collected” (Gay et al., 2014, p. 345). To build the confirmability of my study, I used multiple data sources including my own reflective journal to provide a more accurate representation of the events and to support the triangulation of the data (Gay et al., 2014; Yin, 2014). I practiced reflexivity in describing the overall methodology in this chapter and continued an audit trail throughout. All steps mentioned were critical for appropriately describing the participants’ actions as well as the overall trustworthiness of how the data was interpreted (Gay et al., 2014; Yin, 2014).

Chapter Summary

This chapter described the methodology used in the study for the purpose of exploring how elementary mathematics teachers provide feedback. The research context, participant information, instruments and data sources, procedures, data analysis, and boundaries of the study were also described. The exploratory case study was qualitative in nature and utilized multiple data sources including an implicit theories survey, observational protocol, audio recordings, video recordings, semi-structured interviews, participant reflective journals, and the researcher as an instrument. The following chapter will discuss the results of the study developed from the methodology described in this chapter.

CHAPTER IV: RESULTS

Introduction

An effective classroom environment occurs when teachers provide feedback that answers the questions “Where am I going? How am I going? and Where to next?” (Hattie & Timperley, 2007, p. 102). When teachers provide feedback that answers all three questions, they emphasize the learning goal so students know where they are going, assess students’ understanding relative to the learning goals so students know how they are doing, and then decide how to respond to the students so students know what they must do to reach these goals (Hattie & Timperley, 2007; Jung et al., 2015). In Hattie and Timperley’s (2007) model of feedback, the three questions are categorized into three types: feeding up, feeding back, and feeding forward, respectively. Additionally, the level (i.e., task, process, self-regulation, and self) at which the feedback is directed is also important for determining the effectiveness of the feedback (Hattie & Timperley, 2007).

Although feedback has been shown to be one of the most influential factors on student achievement (Black & Wiliam, 1998; Hattie & Timperley, 2007; Kluger & DeNisi, 1996; Shute, 2008), the ways teachers provide feedback and the individual differences that contribute to these ways greatly varies (Shute, 2008). Boaler (2016) claimed that out of all factors that may contribute to the ways teachers provide feedback, teachers’ implicit beliefs may be the most influential. Given the potential for teachers’ implicit theories to have an impact on the ways teachers provide feedback to students (Hattie & Timperley, 2007; Shute, 2008), the purpose of this study was to closely examine how elementary mathematics teachers provide feedback. The results from this multiple-case study with two participants, Ian (ascribed to a strong incremental theory) and Ellie (ascribed to a weak incremental theory), will be described in this chapter along with a cross-case comparison. Each case will be described separately, and throughout the cases, statements within quoted dialogue that are pertinent to the discussion will be italicized.

Ian

Ian Smith, an African American male in his late twenties, was in his second year of teaching third grade at Byron Elementary school and in his fourth year of teaching third grade in his career. At the time of the study, Ian held the position of third-grade team leader, working closely with the third-grade teachers and administration. The following sections will describe Ian's learning experience, implicit theory as shown on the Implicit Theories Survey (see Appendix A), and views of how students learn mathematics and his own instructional practices. A description of Ian's classroom structure and how he responded to students who did not answer the questions "Where am I going? How am I going? and Where to next?" (Hattie & Timperley, 2007, p. 102), including self-level feedback, will also be provided. Additionally, the ways in which Ian provided feedback that answered the three questions (i.e., feed up, feed back, and feed forward) directed at the task, process, and self-regulation levels will be described.

Ian's Learning Experience

Ian grew up in the United States and described his experience learning mathematics as always having to memorize formulas:

> It was all memorization, all of it. . . . That's the only way I studied, remember the formula, remember the process. I mean that's it, and it got me through. Luckily I could grasp it enough, I guess, on that tier-one level that I see why it makes sense. I see why the process gets me to the answer, but I don't see why it makes sense. (Interview, January 22, 2018)

Ian felt lucky that he was able to understand the procedures on his own growing up even though he did not know why it made sense at the time. As a result of having a focus solely on memorization, Ian was rewarded only for correct answers and feared that his answers would be incorrect:

> I was rewarded for memorization and correct answers. Although, I was confident to raise my hand in class by memorizing the process but if truly challenged, I would not have for fear of being incorrect. I want to stop my students from feeling the same way. (Journal Entry, January 31, 2018)

Although Ian was confident to raise his hand in class as a child, he continued to fear that his answers would be incorrect; especially because he was not being challenged in the classroom. Thus, Ian wanted to prevent his students from feeling the same way he did growing up.

Ian recognized that memorization played a role in his current mathematics classroom when teaching students how to study, even though he indicated that strictly memorizing formulas was not the best way to learn mathematics:

> Even now [I use memorization] when I teach [my students] how to study, I always show them [ways to memorize]. I will say my brain is weird, I have to

> come up with something. Like today for our number talk. They had polygons so it went from triangle all the way to a decagon so I made an acronym like The Quiet Parrot Had Hailed. (Interview, January 22, 2018)

Ian used memorization in his classroom to help his students to study by designing a memorizational tool just as he had growing up. Ian claimed that because he was only taught by memorizing formulas, he never fully understood why the formulas worked (Interview, January 22, 2018). Through professional development programs, conferences, and his first year of teaching, Ian said that he "knew more now about why stuff works than [he] ever did at any level of [his] education" (Interview, January 22, 2018).

Ian's experiences learning mathematics influenced his descisions regarding the way he taught mathematics in his classroom. In addition to his learning experiences, Ian's implicit theory may have influenced his teaching decisions in the mathematics classroom. The following sections will discuss Ian's implicit theory in addition to his views on how students learn mathematics and how he provided feedback in his classroom.

Ian's Implicit Theory

Prior to participant selection, Ian completed the Implicit Theories Survey (see Appendix A). The results showed that Ian held a strong incremental theory (see Figure 2) on the implicit theory continuum (see Table 5) with an overall average of 5.75.

Table 5

Ian's Scores on the Implicit Theories Survey (see Appendix A)

Domain	Average	Implicit Theory
Intelligence	6.00	Strong Incremental
Morality	6.00	Strong Incremental
World	5.00	Incremental
Mathematical Ability	6.00	Strong Incremental
Overall	5.75	Strong Incremental

Ian strongly disagreed (i.e., a six on the survey) with all statements on the survey except for three statements about the world, which he disagreed with (i.e., a five on the survey). During his initial interview, Ian discussed his choices on the survey and his views of learning and teaching in the mathematics classroom. The following sections will describe the results of these discussions including Ian's view of how students learn mathematics and his instructional practices during mathematics instruction.

Ian's view of how students learn mathematics. Ian viewed learning as a student's ability to "take what [they] know and apply it to new information" (Interview, January 24, 2018). Identifying this as a student's mathematical ability, Ian indicated that all students have the ability to learn mathematics:

> As long as you don't have some type of chronic illness, disease, [or] deficiency, and you can take [what you] know about beads and [you] can choose beads to add, you have the ability to do math. I might have to dig deep to find what you do know, but I think as far as in my class, every kid has the ability to learn math, just not at the same pace. (Interview, January 24, 2018)

Ian described an example where being able to understand and choose beads to add indicated that the student had the ability to do math. Ian continued by saying that students may not have the same level of achievement or learn at the same pace, but in his classroom, all students had the ability to learn mathematics.

Although Ian claimed that all students were born with the ability to learn mathematics and that traits, such as their intelligence, were inherited genetically from their parents, Ian indicated that a student's mathemtical ability could change when put into a classroom where mistakes and growth mindsets were valued:

> Every student that has come through [my class] has ended by saying that math is their favorite subject. And for them to say that shows that [their mathematical ability] can change because one, I'm more passionate about math than reading, so they feel my energy. They understand the energy. They feel the success has been broken down simple enough that they can understand it and saying they "hate math" or they're "not good at math" is just saying "I don't understand it" or "I get frustrated with it" which is a fixed mindset. And as long as you can push them out of a fixed mindset in any subject area, then they'll approach it with being able to accept mess-ups and accept mistakes because they see, you know, they'll finally see the final picture of I pushed through it and this was the end result. (Participant Selection Interview, December 19, 2017).

As a result of Ian's energy and passion for mathematics, Ian described how students could change their mathematical ability by focusing on smaller successes and viewing mathematics from a growth mindset. By emphasizing the importance of mistakes along the way, Ian explained that students were more likely to value the result of their productive struggle.

Showing growth in understanding. In addition to having the mathematical ability to do mathematics, Ian described how all students could show growth in their learning by demonstrating their understanding in the mathematics classroom:

> There really is no cap to math. . . . There hasn't been any kid that really [had] a gap that has not been filled, maybe because of time. If I can get it down as concrete as possible and as basic as possible, there is understanding there. So if there was a gap and we pour out manipulatives and I go super slow and give you some success, there's already a growth from where you came in at. So there's no cap for that person with the major gap, and there's no cap for the person that has already progressed along. And even if they have you know, mastered all of the standards in that grade, reasoning and logic inside of correct answers is a much deeper step, so they still haven't reached that cap. (Participant Selection Interview, December 19, 2017)

Ian explained that no matter how small or large the gap may be between what students know and what they need to know, all students could demonstrate an understanding of the material and show success in some way. Moving beyond the correct answer, Ian indicated that the cap on a student's learning in mathematics was infinite.

Making multiple connections. Ian also described how students in his classroom were able to demonstrate their understanding and show growth in their learning by making connections and communicating their ideas. Ian claimed that even though he identified his students at different levels (i.e., higher grade-level of understanding, English as a Second Language, gifted, mid-level, and special needs), all students were gifted in his “regular classroom” (Participant Selection Interview, December 19, 2017). Thus, Ian viewed giftedness as a student’s ability to demonstrate their learning in a variety of ways:

> When I look at giftedness, it’s not what the district looks at as gifted, I look at multiple connections being made, which a lot of that is background knowledge. But multiple connections being made, how quickly the connections can be made without scaffolding, creativity, and thinking outside of the box. Thinking abstractly, those things along with how you perform to some degree on assessment. Communication, speaking, [and] listening, but kind of those things are what I, when I really start listening. That connection he just made was from a show he watched three years ago, and he just tied it to fractions. Like that, to me is like ok, you’re not just listening [to] what I’m saying and just putting it into practice. (Participant Selection Interview, December 19, 2017)

Ian claimed that all of his students were gifted based on their ability to demonstrate how they made multiple connections, used their creativity, thought abstractly, and performed on assessments. Additionally, Ian explained that when students were able to communicate their thinking in these ways, students learned the material at a deeper level by not only following a procedure. Classroom environments that encourage students to demonstrate their giftedness, as described by Ian, create the best settings for students to learn mathematics.

Ian's view of how students learn best. Ian described that students learned mathematics best when they felt comfortable in the classroom and confident in their ability to work through problems sucessfully:

> Students learn best when they feel safe to fail and to keep trying; [where] they feel confident in the process rather than the product. This is the exact opposite from how I learned. (Journal Entry, January 31, 2018).

Emphasizing that this was not the way he learned mathematics, Ian explained that students learned best when they felt safe enough in the classroom to keep trying even when their solution was incorrect and are confident in the mathematical processes they use rather than focusing only on the product.

Student struggle. Ian recognized that it was his responsibility to create a classroom environment where students felt safe and confident, and where their learning needs were being met in the mathematics classroom. If Ian did not meet their needs (e.g., provide information as concrete as possible or demonstrate using manipulatives; Participant Selection Interview, December 19, 2017), students would struggle to demonstrate their level of understanding effectively (Participant Selection Interview, December 19, 2017). Thus, Ian attributed students' struggle in mathematics to gaps in the teacher's content knowledge resulting in the students' needs not being met:

> I think students struggle in math, all of it comes back to the teacher, and I think [teachers] don't want to take ownership. . . . Math, I can do that. I can go all the way concrete and work back up. But if I can't do that, then of course I can't meet the students' needs and I just keep pushing them along, teaching them one

> strategy and one way to do every standard and if they don't get it, I just kind of get frustrated and push them along. (Participant Selection Interview, December 19, 2017)

Ian explained that students struggled in mathematics because teachers lacked the mathematical content knowledge needed to teach the content thoroughly. Using himself as an example, Ian indicated that he needed to spend additional time to build his own content knowledge of mathematics so that he could meet the needs of his students. When teachers do not take ownership and build their content knowledge, Ian emphasized that students would struggle due to the teacher pushing them through the material.

Ian described his view of how students learned mathematics in his classroom including how they demonstrated their mathematical ability, showed growth in their understanding, made multiple connections, and learned best. The way teachers view their instructional practices also plays an important role in supporting students in learning mathematics in these ways. The following section will discuss Ian's view of his instructional practices including his view of how he provided feedback in his mathematics classroom.

Ian's view of his instructional practices. Ian described his role when working with all students as building a strong, concrete foundation to help them move forward in their understanding. Ian explained that his role with students who he described as low achieving was to "build confidence and to build a strong foundational concrete understanding of the standard" (Participant Selection Interview, December 19, 2017). With students who he described as middle achieving, Ian wanted to "build a strong concrete a little quicker, move them to abstract, and then really start trying to move them into logic and reasoning" (Participant Selection Interview, December 19, 2017). With students who he described as high achieving, Ian explained that he would "start out concrete [and] quickly move to abstract [and then] to misconceptions and then mistakes" (Participant Selection Interview, December 19, 2017). Ian explained that he tried to focus on his questioning techniques for all of his students because his questioning was what got his students to reason between every problem (Participant Selection Interview, December 19, 2017).

Questioning. To help students move forward with their thinking during mathematics instruction, Ian indicated that he would ask questions to find a baseline of the student's understanding. He did this when students appeared to be struggling and were not making progress towards the learning objective (Final Interview, February 8, 2018). By asking questions, Ian explained that he could use the student's responses to find their misconceptions, if any:

> First question I asked them is where they're at, what are they struggling with. First I would say you need to get going, just to see if it's an effort being lazy thing. Then if I find that they don't understand it, then it's what don't you understand? Where are you starting at? What don't you understand? The numbers, the problem, how to start, where to begin, what a fraction is? And that just puts some of the ownership in their hands to not just to say that to be lazy. And I think once I get that out, I can pretty much start anywhere. I mean once you can tell me where [you're] at or just work the problem and let me see . . . I can find the

> misconception in there. I can find the mistake or what he or she is lacking. (Initial Interview, January 8, 2018)

Ian described how he would question students who appeared to be struggling until he was able to find their misconception or mistake. Ian indicated that he would use this line of questioning to make a mental note that he needed to fill the student's gap in their understanding at a later time in the lesson (Initial Interview, January 8, 2018).

If students continued to struggle after asking initial questions, Ian said that he would partner the student with other students "just a little bit above them for the sake of the day and then tomorrow . . . I need to either put [them] in a group where I'm going to go a little slower and meet that gap or I need to pull [them] to a group that I really need to go super slow" (Initial Interview, January 8, 2017). This partner would act like a coach to help scaffold the student through the problem until Ian was able to work with the student.

Responding to students. In addition to questioning and partnering students, Ian also indicated that he responded to students in a neutral way to support students in productive struggle. By responding in a neutral way, Ian explained that he encouraged students to focus on the processes which led them to the answer rather than immediately giving them the correct answer (Interview, January 26, 2018). He did this by trying not to provide validation to the students:

> I'm trying to get out of them receiving validation from me . . . so I just try to ask questions for everything or let the kids give me a consensus for every problem. Where [are] we at? If we agree or disagree, then that child is explaining it and not me. (Interview, January 10, 2018)

Ian stated that he tried to respond to students in a neutral way by asking additional questions or asking for a consensus from the students rather than providing validation. This allowed students to have more control of their learning and become active members in the discussion.

Ian reflected on how he responded to students in this way and explained how he changed or modified the way he responded based on students' individual needs:

> I think I respond based off what I feel the student needs. I think some need my validation to reassure them they are in the right direction. Others need nothing from me so they can contradict themselves which hopefully leads to defending their own thinking. (Journal Entry, January 31, 2018)

Although Ian emphasized that he tried not to give any validation to his students, he explained that some students needed more validation than others to help them move towards the learning goal. Additionally, some students needed little or no validation to allow them to reason through their ideas on their own.

Ian explained that he started to provide his students with no validation at all. "Some of those kids are dying for something out of me. Like begging for just a tap on the back or something. . . . I feel like I'm too much now. It's nothing from me, nothing from me" (Final Interview, February 8, 2018). As a result, Ian sought to focus on techniques for motivating his students rather than validating their responses.

Motivation. To motivate his students during mathematics instruction, Ian described how he used an individual behavior chart where students moved their clip up or down and a friendly competition among groups. Ian indicated that when he saw an area where students were struggling (e.g., making transitions smoother), he asked the students to come up with solutions allowing them to collaborate and feed ideas off each other (Initial Interview, January 8, 2018). Resulting from these collaborative conversations, Ian made changes that addressed their concerns such as a light for each group to indicate when they were ready for instruction and team captains to monitor the groups.

Ian also described how he handed out small candies for individual motivation (e.g., for correctness of a solution or strategy) and filled a jar with marbles as motivation for the whole class (e.g., the effort of all students in the class; Initial Interview, January 8, 2018). Ian emphasized that every student needed some form of motivation; however, he indicated that motivation, in some ways, could be a hindrance to students:

> I do feel that every situation is conditional. Sometimes it's a party. Sometimes it's to be number one. Every kid needs different stuff so I think you have to have different [motivations]. . . . I think that even though it is needed sometimes, I think to tell children what they're about to get for doing something, is in some way a hindrance and it is different for every child. But to be like, "If you don't give up on this problem, you'll get a [candy]." I feel like that is a hindrance to some degree but then some kids do need that so that's digression. (Initial Interview, January 8, 2017)

Ian emphasized that all students needed some form of motivation depending on the students' needs; however, some students may need to be motivated more than others. Ian also indicated that motivation could be a hindrance when using it as a cause and effect method with students. In addition to Ian motivating students using a behavior chart, marbles, and candy, Ian also described how he used a self-motivation tool to help students move forward in their thinking.

Self-motivation. Ian also claimed that self-motivation played an important role in helping students to think deeper about the mathematical goals. One approach that Ian described had changed how he helped students to self-motivate was by referencing The Pit (see Figure 5).

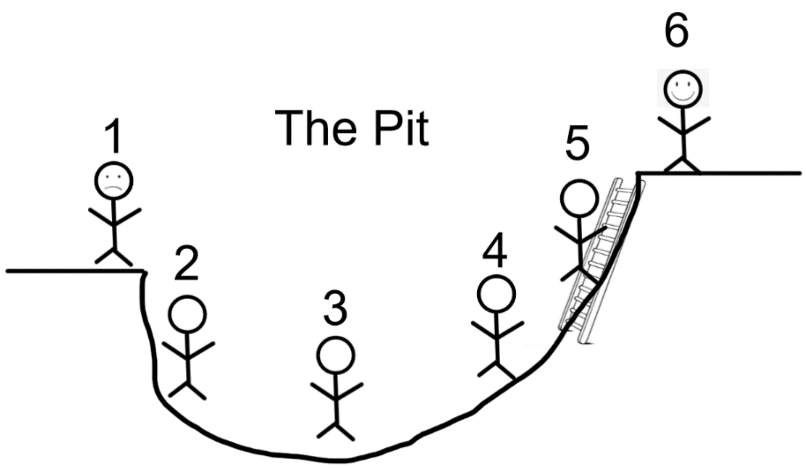

Figure 5. A representation of The Pit model with stick figures.

Rather than directly telling students that they were struggling, Ian explained that he used The Pit as a self-motivating tool by having students decide where they were (i.e., what number in The Pit) and where they needed to be relative to the learning goal:

> You can say to a kid you're struggling all day; you're struggling, you're not getting it. But to say that you are in The Pit is saying you're struggling and you're not getting it. It's saying you have two options: from this Pit you're either not going to give any effort or you're going to do this. And I think my excitement motivates them. (Initial Interview, January 8, 2018)

Ian explained that rather than telling students directly that they are struggling, he referred to The Pit to help students think deeper about where their current understanding was and the effort they needed to give to achieve the mathematical goal (i.e., get out of The Pit).

The previous section described Ian's view of how students learn mathematics and his instructional practices in his own classroom environment. The following section will describe the structure of Ian's classroom which encompassed these ideas during the 11 days of classroom observations.

Ian's Classroom Structure

Ian's third-grade class consisted of 19 students arranged in groups of six with one individual desk off to the side (see Figure 6). Mathematics instruction occurred daily for 30 minutes before lunch and one hour after lunch. Ian provided instruction to students as a whole group either at their seats or on the carpet in front of an interactive whiteboard. Occasionally, Ian divided students into two groups where one group would work on the carpet with the teacher and the other on individual computers at their desks. Although Ian frequently asked students to discuss their ideas with a neighbor both at their desks and on the carpet, written work was completed individually.

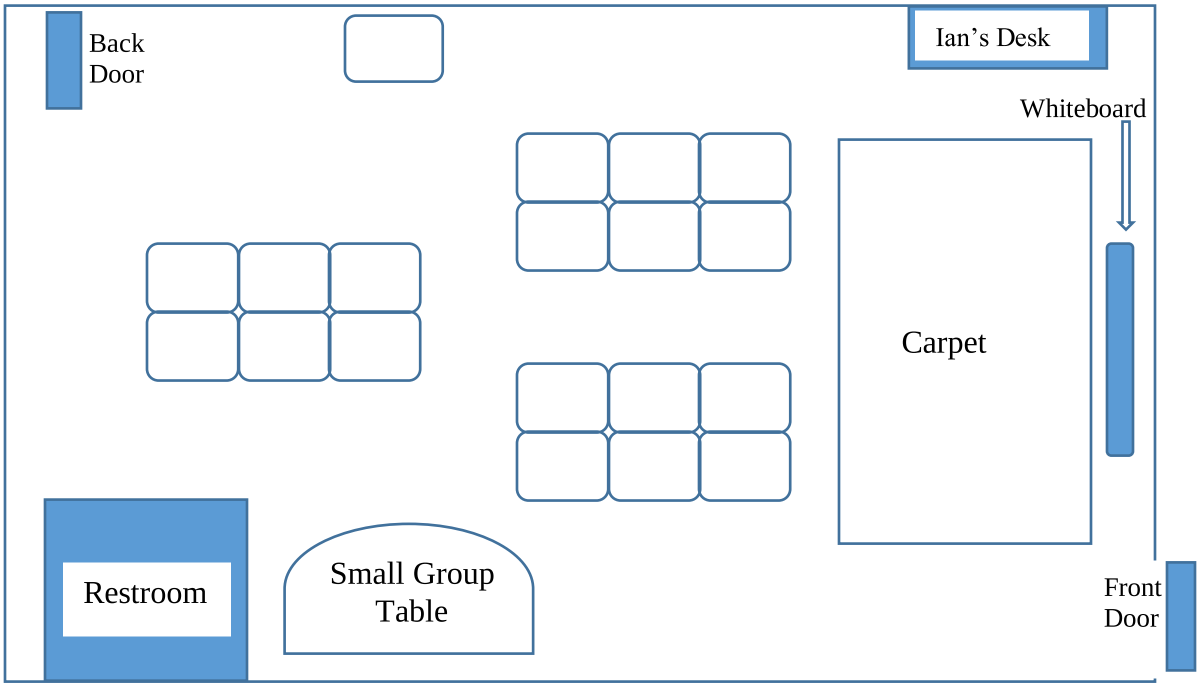

Figure 6. Ian's third-grade classroom arrangement.

Warm-up activities. Before lunch on each of the 11 days of classroom observations, Ian engaged students in one, two, or three warm-up activities (see Table 6) as an introduction to mathematics instruction time. The warm-up activities were unrelated to the learning goal of the current fraction unit except for the small-group discussions where Ian worked through fraction problems with students.

Table 6

Description of Introductory Activities Prior to Lunch in Ian's Mathematics Classroom

Activity	Description
Geometry Tae Bo	Students standing at their desks and creating figures/gestures with their arms based on the terms called out by Ian or the definition displayed on the interactive board Terms included: right, acute, and obtuse angles; parallel, perpendicular, and intersecting lines; point, line, line segment, and ray
Name That Polygon	Students standing at their desks shouting out names of polygons based on the number of sides or angles called out by Ian or displayed on the interactive board Terms included: triangle, quadrilateral, pentagon, hexagon, heptagon, octagon, nonagon, and decagon
Pattern Block Exploration	Students sitting at their desks or on the carpet exploring how to model fractions with pattern blocks or cut-out quadrilaterals
Displayed Student Work	Students sitting on the carpet engaging in a whole-group discussion of student work from the previous day displayed on the interactive board
Computation Practice	Students working at their desks through five problems consisting of either addition, subtraction, multiplication, or division displayed on the interactive board Five minutes to complete Followed by a whole group discussion in front of the interactive board with students sitting on carpet
Small Group Discussion	Small group of students sitting at the Small Group Table working through problems with Ian while the rest of the class either working at their desks on an assessment or sitting on the carpet with the student teacher

Primary instructional activities. After lunch, Ian either continued the activity from before lunch, engaged students in a new mathematical task, or continued with a task from the previous day (see Table 7).

Table 7

Ian's Daily Activities Including Learning Goal Per Day of Classroom Observations

Day	Warm-Up Activity	Learning Goals (LG) and Instructional Activities (IA)
1	Geometry Tae Bo, Computation Practice	LG: Represent and add unit fractions IA: Pattern block exploration, review student work from computation practice
2	Geometry Tae Bo, Computation Practice	LG: Partition shapes and label the unit fraction. IA: Partition activity on carpet (divide a shape into equal parts and find pieces used and total pieces), individual computer practice
3	Geometry Tae Bo, Computation Practice	LG: Represent the whole using a number line. Partition a number line and label unit fractions. IA: Draw a number line pre-assessment, number line activity on carpet, individual computer practice
4	Displayed Student Work from Halving and Doubling	LG: Review due to a week break for winter weather IA: Fraction strip exploration, application of previous learning goals including partitioning and labeling a number line, individual computer practice
5	Pattern Block Exploration	LG: Model improper fractions and equivalent fractions using pattern blocks IA: Pattern block exploration, first two questions on the 'Sixth of a Mile' task
6	Computation Practice	LG: Place all fractions on a number line including improper fractions and mixed numbers IA: Continue 'Sixth of a Mile' task
7	Name That Polygon, Computation Practice	LG: Label unit fractions and improper fractions on a number line IA: Number line noticing activity and labeling a number line task

(continued)

Day	Warm-Up	Tasks & Fraction Unit Learning Goals
8	Geometry Tae Bo, Name That Polygon, Computation Practice	LG: Label unit fractions and improper fractions on a number line IA: Warm up problem, look at student work of warm up problem, continue 'Sixth of a Mile' task, fraction assessment with small-group intervention
9	Geometry Tae Bo, Name That Polygon	LG: Represent and prove two fractions are equivalent IA: Discussion on characteristics of polygons (e.g., Is a square a rectangle? and What is the difference between a rhombus and a square?), numerator and denominator review, hands-on equivalent fraction investigation with index cards
10	Small Group Review (6 students), Name That Polygon	LG: Represent and prove two fractions are equivalent IA: Discussion on characteristics of polygons, numerator and denominator review, examination of student examples of equivalent fractions with index cards
11	Small Group Review, Pattern Block Exploration (with cut-out quadrilaterals)	LG: Application of previous learning goals IA: Numerator & denominator review, writing prompt on the 'Licorice task' with small-group intervention, examine student examples of writing prompt, equivalent fractions task

In his initial interview, Ian described the mathematical goal of the current fraction unit intended to be covered during the first week of classroom observations.

> The ultimate goal would be for them to understand what numerator and denominator is. Identify, be able to say it, speak about it, using it in their language, getting them to use it as much as possible. And then the ultimate goal, or really a subset goal, would be for them to truly understand unit fractions which is shooting for the stars. (Initial Interview, January 8, 2018)

Ian continued describing the mathematical goal of the current fraction unit for the second week of classroom observations.

> Next week we'll go into a number line. This week is kind of like one and two, maybe even day three Wednesday, will be unit fractions showing it with the blocks. Showing that it took six pieces but eventually showing me three-sixths. What would three-sixths look like on your blocks if this was the whole? Show me three-sixths. So, they have to choose what is six total and then choose three of those to show three-sixths and then move that onto a pictorial where they have to draw a box. (Initial Interview, January 8, 2018)

Throughout the classroom observations, the fraction unit learning goals (see Table 7) were either specified in an interview with Ian or selected from the I-Can statements. The I-Can statements were displayed in a PowerPoint on the interactive board at the beginning of the fraction-unit portion of the class. Ian read the I-Can statement out loud, and the students repeated each phrase out loud responsively on the first three days of observations. The I-Can statement was not displayed anywhere around the room except in the PowerPoint shown on the interactive board which Ian occasionally referred to on other observed days. The following section will discuss how Ian responded to students during the daily activities described in Table 7.

How Ian Responded to Students

During the mathematics instruction portion of the 11 classroom observations, Ian responded to students in three ways (i.e., posing additional questions, asking students if they agree or disagree, and asking students to prove their thinking) that did not answer the questions "Where am I going? How am I going? and Where to next?" (Hattie &

Timperley, 2007, p. 102). By not addressing the three questions, the following examples of Ian's responses to students were not categorized as feedback.

Posing additional questions. One way Ian responded to students was by posing additional questions after the student replied to his initial question. Ian would first say "Ok" or "All right" and then ask an additional question:

> Student: I'm just saying what my gut is saying.
>
> Ian: Ok.
>
> Student: I'm just saying a rhombus could be a square turned sideways but it can also be a parallelogram, too.
>
> Ian: *Ok, why do you say that?* (Classroom Observation, January 29, 2018).

In this instance, Ian first responded to the student by saying "Ok," and when the student continued to explain what they were thinking, Ian responded by saying "Ok" and asking an additional question.

In his interview on the fifth day of classroom observations, Ian explained that he tried not to give students validation when responding to them. Instead, he chose to respond to students by asking additional questions:

> Just most of it all stems from no validation from me. So it might be repeating, it might be [a] question. . . . Sometimes I'll say, "Is that your final answer?" just to see if they're like, final answer? . . . Just whatever comes to mind at the time. Just no validation for me, and you'll see at the end with another student explaining or something if that's correct or not. (Interview, January 23, 2018)

Ian stated in his interview that he might repeat a question or repeat what the student said to not provide validation to his students. As a result of not providing validation, he indicated that often students would end up explaining the problem themselves and realizing on their own whether their answer was correct. Thus, Ian responded to questions by asking additional questions because he did not want to validate the students' replies.

Ask if students agree or disagree. Another way Ian responded to students who did not answer the three questions was by asking the other students in the class if they agreed or disagreed with the student's reply. Ian used hand gestures to indicate the responses while asking the class if they agreed or disagreed, and students mimicked the gesture of either agree or disagree.

> Ian: Ok, start from zero. We're going to one, one whole. And how many total jumps were there?
>
> Student: Four.
>
> Ian: *Four? Agree or disagree?*
>
> Students (speaking at the same time while gesturing): Agree, disagree, agree, agree.
>
> Ian: Agree. (Classroom Observation, January 22, 2018)

In this instance, Ian did not provide validation to the student. Instead, he repeated the student's reply in the form of a question and then asked the class if they agreed or disagreed. Following the students' replies, Ian either elaborated on the majority vote or asked certain students to explain why they agreed or disagreed.

Ask students to prove their thinking. One additional way Ian responded to students who did not answer the questions “Where am I going? How am I going? and Where to next?” (Hattie & Timperley, 2007, p. 102) was by asking students to prove or show what they were thinking rather than validate their reply.

Ian: How many of these would it take to make one whole?

Student: Would it be 10?

Ian: *Prove it to me. Prove it. Prove it to me that 10, 10 of these make one whole.*

(Classroom Observation, January 23, 2018)

Rather than indicate the correctness of the student’s reply, Ian asked the student to prove that it took 10 of the pieces to make one whole. When the student replied by questioning his own answer, Ian responded by telling the student to prove whether his own answer was correct.

Summary. From the previous examples, Ian responded to students in three ways that did not answer the questions “Where am I going? How am I going? and Where to next?” (Hattie & Timperley, 2007, p. 102). Ian posed additional questions, asked the class if they agreed or disagreed with a student’s response, and asked students to prove or show what they were thinking. In addition to responding to students in the previous ways that did not answer the three questions, Ian also responded to students by directing his responses at the students themselves. This type of response was categorized as self-level feedback and did not answer the questions “Where am I going? How am I going? and Where to next?” (Hattie & Timperley, 2007, p. 102). Self-level feedback will be described in the following section in addition to how Ian provided feedback in this way.

Ian's Self-level Feedback

Feedback at the self level is defined as feedback that is directed at the person themselves and is often isolated from a person's performance on a task (Hattie & Timperley, 2007). Although several researchers found self-level feedback to have little to no effect on student learning (Good & Grouws, 1975; Hattie & Timperley, 2007; Skipper & Douglas, 2012), others found it necessary to increase student intrinsic motivation and establish valuable relationships between the teacher and the students (Brophy, 1981; Cameron & Pierce, 1994; Hattie & Yates, 2014). Thus, it was important to consider how the participants provided self-level feedback to draw connections between their implicit theory and how they provided feedback in this way. Ian provided self-level feedback 23 times during the 11 classroom observation days including praising students for their intelligence, bravery, confidence, thinking, and actions. The following section will describe five instances where Ian provided feedback in these ways.

Praising the intelligence of students. On the second day of classroom observations, Ian provided self-feedback to a small group of students on the carpet while the rest of the class worked independently on laptops at their desks. Ian displayed a rectangle on the interactive board and proceeded to partition the rectangle incorrectly into thirds (See Figure 7).

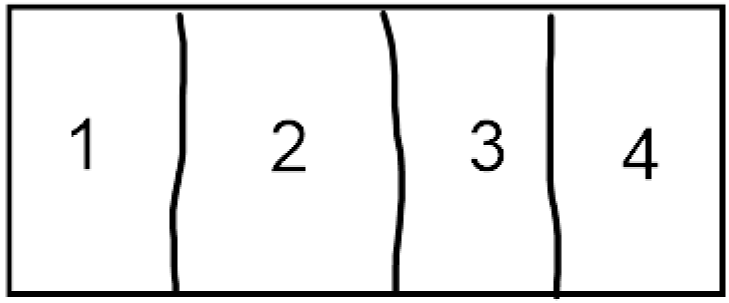

Figure 7. Example of a student partitioning a rectangle into thirds incorrectly.

As he was drawing the three lines, he explained that he wanted to make sure each piece was even. After a small discussion where the students disagreed with the way Ian partitioned the whole into three equal sections, he turned to the class to see their response:

Student: You're wrong!

Ian: Why am I wrong?

Student: Break the whole into equal thirds.

Ian: What did I break it into?

Student: Fourths!

Ian: But you saw me go one, two, three (motioning the three lines he wrote). You saw me do that. Three equal parts. One, two, three.

Student: You should have like one, two (motioning two lines). That way there's three.

> Ian: Hmmm, I'm not counting lines? Ok, ok, let me see. Will you go ahead and separate into thirds? Go ahead and label it then. Go ahead. *You all are just so fancy. You all are just so smart, so smart.* (Classroom Observation, January 10, 2018)

Ian provided feedback at the self level by praising the intelligence of the students (i.e., the students were so fancy and smart) for successfully identifying his intentional mistake. When the students continued to explain to Ian how he unsuccessfully partitioned the rectangle into thirds, Ian asked a student to go to the interactive board and correct his mistake. The self-level feedback Ian provided was directed at the intelligence of the students rather than helping the students' progress towards the learning goal of partitioning shapes into equal pieces.

Praising the bravery of students. A second example where Ian provided self-level feedback was on the seventh day of classroom observations to a student who explained why they were unsure if a problem was correct or not. After a student labeled three improper fractions on a number line (see Figure 8), Ian asked the class if they agreed, disagreed, or were neutral with how the student labeled the tick marks on the number line.

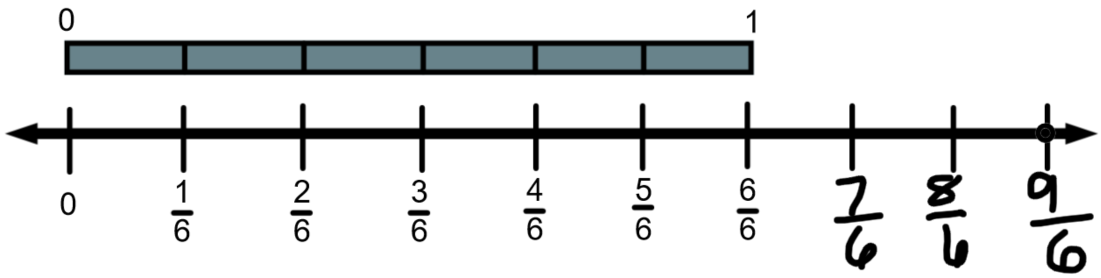

Figure 8. Number line partitioned into sixths with three additional tick marks after one labeled by a student on the interactive white board.

After Ian noticed that several students indicated that they were in the middle (i.e., they were not quite sure if they agreed or disagreed), Ian called on one student to explain their thinking:

> Ian: All right, why do you say in the middle, J?
>
> Student J: Well, because, I'm not sure if that's right but I think it's right at the same time.
>
> Ian: Ok, I like that answer. *I like that you were brave enough to admit that you were not sure but you kind of think it might be.* (Classroom Observation, January 25, 2018)

In this instance, Ian provided self-level feedback directed at student J's bravery when admitting that he was unsure as to whether the problem was correct. The praise that Ian provided was not about the student's effort or the student's intelligence. Rather, it was directed at the bravery of the student when answering and was unrelated to how the student's efforts could help progress towards the learning goal of labeling improper fractions on a number line.

Praising the confidence of students. Similarly, on the second day of classroom observations, half of the students were seated on the carpet discussing which models displayed on the interactive board were partitioned into equal parts (see Figure 9; Classroom Observation, January 10, 2018).

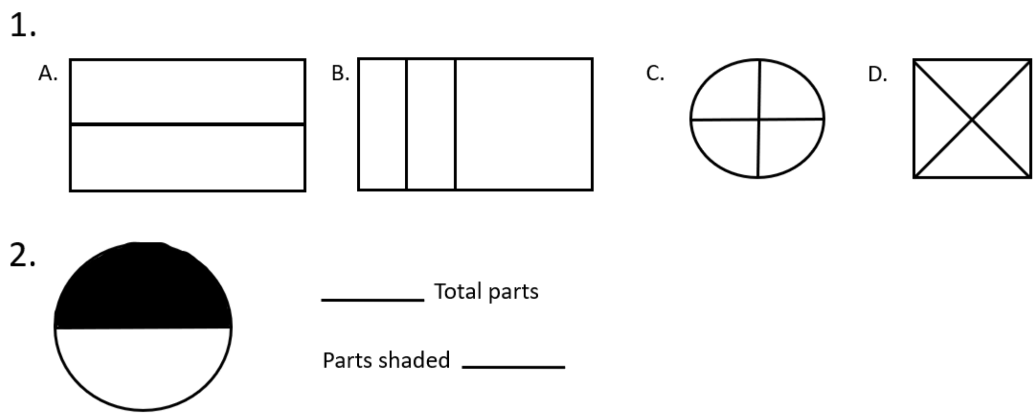

Figure 9. Two problems displayed on the interactive board with shapes partitioned into equal and unequal parts and parts shaded.

Ian called on students to answer different questions related to the models, emphasizing the correct use of fraction vocabulary (i.e., numerator instead of number on top and one-fourth instead of one out of four). The second problem showed a circle with a line drawn horizontally in the middle and the top half shaded in:

Ian: So, number two. How many total parts do I have? [Let me] see hands, how many total parts do I have right here? K?

Student K: Two.

Ian: Is that my numerator or denominator?

Student K: That's your denominator.

Ian: *I like it. I like how confident you were*. (Classroom Observation, January 10, 2018)

In this instance, student K replied to Ian's questions in a confident manner stating that the two parts were the denominator. In response, Ian provided feedback directed at the student's display of confidence when answering the question (i.e., I like how confident you were). Ian directed his feedback at how the student answered the question, addressing the student's self-efficacy when speaking about fractions. This growth praise was unrelated to helping the student move closer to the learning goal of partitioning shapes and labeling unit fractions.

Praising students for their thinking. Ian also provided feedback at the self level when reacting to how a student modeled and described his thinking using Cuisenaire rods. On the sixth day of classroom observations, Ian asked a student to explain how he determined that nine-sixths was equal to one and one-half using Cuisenaire rods (see Figure 10) on the document projector to the whole class seated on the carpet.

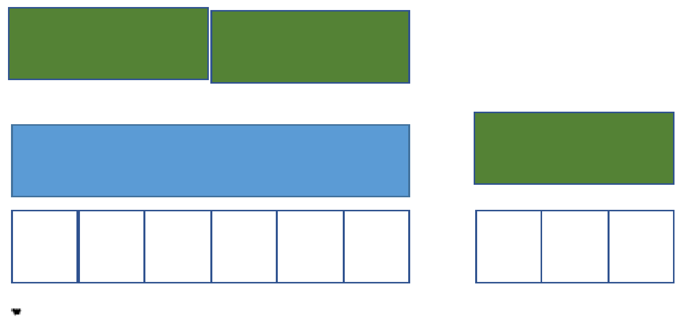

Figure 10. Example of a student modeling the Sixth of a Mile task using Cuisenaire rods displayed on the document projector.

As the student began speaking and moving the rods around, Ian stood to the side of the interactive board:

Student (manipulating the Cuisenaire rods): She stopped nine times, so she already stopped a mile and there's three more left (pointing to the individual cubes). So, this is the first time, well they're not half, they're all equal to threes. So, if it takes this, three and three, this is just one three (pointing to the green rod) so it must be (moving around the pieces). So, these two little tiny green pieces are both equal to a mile and one of these is half of it. [The green piece] is half of a mile.

Ian (bringing his hand to the top of his head): Man, *I usually don't like to show a reaction on my face, but that's so much thinking going on right there*, S. Holy cow. (Classroom Observation, January 24, 2018)

In this instance, Ian reacted to the student by bringing his hands to the top of his head and stating that there was so much thinking going on. Ian provided self-level feedback by praising the student's thinking when explaining how he progressed from an improper fraction to a mixed fraction using the Cuisenaire rods. By announcing that there was so much thinking going on, Ian did not validate the student's thinking or comment on the mathematical content of the student's explanation. Thus, Ian provided self-level feedback by acknowledging that the student demonstrated a lot of thinking.

Praising students for their actions. Ian also provided feedback at the self level on the fifth day of classroom observations. All students sat on the carpet facing the interactive board at the end of mathematics instruction time. Ian stood at the front of the class and announced to the class that there were two students whom he was most impressed with that day. He began by talking about the first person and then explained why he was most impressed with the second person:

> The second person I'm impressed with is (long pause) H. (The principal walks in the room unannounced and walks to the back of the room). Because it is not about getting the answer correct. *H never gave up the entire math lesson. Never gave up. Came up here and explained. Didn't run away from explaining, even if he wasn't for sure if he knew it. He came up here and gave it a shot.* (Classroom Observation, January 23, 2018).

In this instance, Ian provided feedback at the self level to student H regarding his actions during the mathematics lesson. Ian praised him for never giving up or running away from explaining what he was thinking, even if he was not confident in doing so. Ian directed his feedback at the student's persistence and bravery throughout the lesson by describing to the class why he was impressed with the student.

Summary. Ian provided self-level feedback to his students during mathematics instruction by praising their intelligence, bravery, confidence, thinking, and actions as shown in the previous sections. Although this feedback did not answer the questions "Where am I going? How am I going? and Where to next?" (Hattie & Timperley, 2007, p. 102), it was important to consider to draw connections between his implicit theory and how he provided feedback in this way.

It was also important to consider the ways in which Ian provided feedback that did address the three questions (i.e., feed up, feed back, and feed forward). The following sections will describe how Ian provided feedback in this way and evidence of how Ian directed each type of feedback at the task, process, and self-regulation levels.

How Ian Provided Feedback

Ian responded to students by providing feedback that answered the questions "Where am I going? How am I going? and Where to next?" (Hattie & Timperley, 2007, p. 102) in three ways: feeding up, feeding back, and feeding forward (see Table 8). The following sections will describe how Ian provided feed up, feed back, and feed forward to his students directed at the task, process, and self-regulation levels during the 11 days of classroom observations.

Table 8

Type of Feedback Occurrences for Ian Per Day of Classroom Observation

Feedback Type	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	Total
	Classroom Observation Day											
Feeding Up	3	1	3	1	1	1	1	2	0	0	0	13
Feeding Back	19	16	9	12	12	2	6	8	3	11	4	102
Feeding Forward	1	1	2	0	2	1	0	1	0	2	0	10

Feeding up. Feed up describes the information given to students about what the learning goal is and what the result looks like once they arrive at the goal. Ian provided feed up a total of 13 times (see Table 8) within the first eight classroom observation days directed at the task, process, and self-regulation levels. The instances in which Ian provided feed up in these ways will be described in the following sections.

Task level. Feed up at the task level answers the question "Where am I going?" (Hattie & Timperley, 2007, p. 88) by referring to the goal of the task and the surface information needed to successfully complete the task (Hattie & Clarke, 2019). Ian provided feed up at the task level five times during 11 classroom observation days by clarifying the goal for successfully completing the task, displaying the correct solution to a problem, and emphasizing the completed objectives within the learning goal. Three instances of how Ian provided feed up in these ways will be discussed in the following sections.

Clarifying the goal for successfully completing the task. The first instance where Ian provided feed up at the task level was on the first day of classroom observations. Towards the end of the mathematics instruction time, Ian displayed a large rectangle and a small rectangle representing a whole chocolate bar and one piece of the bar (see Figure 11) on the interactive board.

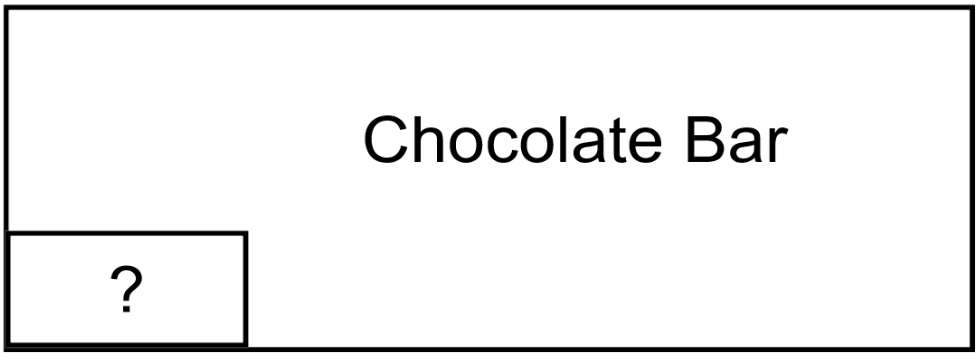

Figure 11. The unit fraction of one piece of a whole chocolate bar.

Half of the students sat on the carpet while the other half worked independently on computers at their desks. Ian asked the students on the carpet to model what the unit fraction of one piece of this chocolate bar would look like. As students worked both on the carpet and at the board to solve the task, one student started to complete the task by counting the number of rows:

> Student: I was thinking how many rows.
>
> Ian: Ok, it makes sense if you are doing rows, *but it's not asking for rows. It's asking for pieces. How many pieces make up that whole bar?* (Classroom Observation, January 9, 2018)

In this instance, Ian responded to the student by first recognizing that counting rows made sense in this instance, but that the goal of the task was not to count rows. Ian then provided feed up at the task level by clarifying that the goal for successfully completing the task was to find the total number of pieces. Thus, Ian reminded the student of the direction that he should be going towards the learning goal.

Displaying the correct solution to a problem. The third day of classroom observation was guided by three learning objectives which included representing the whole using a number line, partitioning a number line, and labeling unit fractions (Classroom Observation, January 11, 2018). Immediately after lunch, Ian asked the students to draw a number line and partition it into sixths. Ian asked his students to do this prior to any direct instruction on the topic and walked around the room assessing their initial understanding.

After the students completed the problem, Ian reviewed the learning objectives responsively with the class. He then displayed the problem on the interactive board:

> Now, we're going to jump back to our problem. This is what I asked you to do. Draw a number line from zero to one and partition it into sixths and label each unit fraction. *This is what I was looking for* (flipping to the next slide on the board revealing the correct way to model the problem; see Figure 12). . . . This is what we're kind of going to be doing today. *I asked you to partition it and label unit fractions. This is what I was looking for.* (Classroom Observation, January 11, 2018)

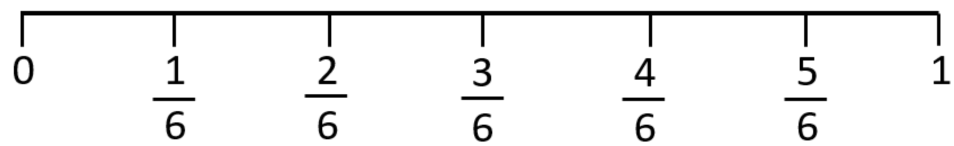

Figure 12. Solution to correctly draw a number line from zero to one and partition it into sixths and label each unit fraction.

In this instance, after Ian assessed the students' initial understanding related to the learning goal, he reviewed the problem that he posed to the students and displayed the correct solution. Ian provided feed up at the task level by revealing the correct solution to the initial assessment question (i.e., partitioning a number line and labeling each unit fraction) and indicating that he was looking for a model like the one displayed on the interactive board. In doing so, Ian demonstrated what the result would look like when the learning goal was reached.

Emphasizing the completed objectives within the learning goal. A third instance of feed up at the task level was on the first day of classroom observations where Ian emphasized the completed objectives within the learning goal. After lunch, students sat at their desks exploring how pattern blocks could represent fractions. Ian asked the students, "How many green triangles are in one blue rhombus?" and "How many green triangles are in one red trapezoid?" (Classroom Observation, January 9, 2018). As students continued to find the solutions to these questions, Ian then asked what the unit fraction would be for one green triangle to one red trapezoid (see Figure 13).

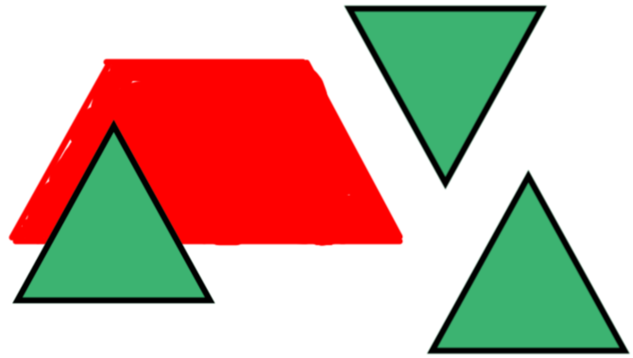

Figure 13. Example of pattern blocks representing how many green triangles are in one red trapezoid.

Ian called on a student to answer what the unit fraction for one green triangle in this situation would be:

Student: I know what it is. I just … one out of three.

Ian: One out of three or?

Student. One-third.

Ian: One-third. *So, going back to the I-Can statement. We said that we were going to represent [unit fractions] and (with emphasis) add.* Ok, so that's what we have for our trapezoid, correct? (Classroom Observation, January 9, 2018)

In this example, Ian provided feed up at the task level by referring to the learning goal (i.e., represent unit fractions) and emphasizing the completed parts within the learning goal. The class appeared to have successfully completed the first part of the goal, so Ian emphasized that the I-Can statement also indicated that they needed to add unit fractions. Thus, Ian referred to the learning objective as a reminder that they had not completed the entire learning objective.

The previous examples demonstrated how Ian provided feed up directed at the task level. Feed up can also be directed at the processes that students need to be engaged with to reach the learning goal. The following section will discuss how Ian provided feed up in this way.

Process level. Feed up at the process level addresses the question “Where am I going?” (Hattie & Timperley, 2007, p. 88) by focusing on the learning goal and the processes used to move towards that goal. Ian provided feed up by referring to the process described in the learning objective and clarifying a strategy for successfully completing a task. During the 11 days of classroom observations, Ian provided feed up at the process level a total of five times. Two instances of how Ian referred to the process and clarified a strategy will be discussed in the following section.

Referring to the process described in the learning objective. One instance where Ian provided feed up at the process level was on the third day of classroom observations where half the students worked individually on computers at their desks while the other half worked with Ian on the carpet. Ian finished working through a problem with half the students on the carpet, where four blue squares represented the whole and a number line was drawn to represent the whole (see Figure 14).

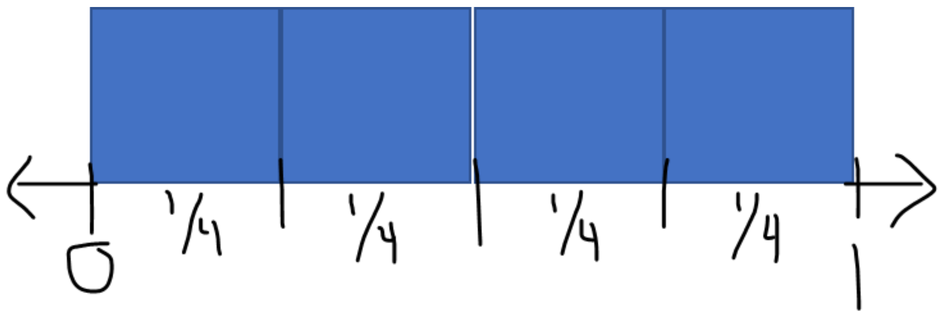

Figure 14. Four blue squares representing a whole using a number line with each square representing one-fourth.

Ian demonstrated on the number line that a tick mark should be drawn after each ¼ piece and demonstrated that a jump was the distance between tick marks. Ian then asked the students to discuss with their partners, “If I’m labeling my number line, why should I count jumps and not ticks?” (Classroom Observation, January 11, 2018). As the students began discussing with their partners, Ian walked around the carpet and stopped at a pair of students:

> Student: The ticks sometimes if you are doing a number line, there’s more ticks.
>
> Ian: You should count ticks? *My question is, I’m saying count jumps not ticks and I want to know why. Why should I?* (Classroom Observation, January 11, 2018)

In this instance, Ian introduced a concept of counting jumps and not ticks on a number line by asking students to discuss this question with their partners. The student indicated that there were more ticks so ticks should be counted. Ian provided feed up at the process level by referring to the process (i.e., counting jumps and not ticks) described in the learning objective (i.e., to correctly partition a number line). In doing so, Ian emphasized a step in the direction of the goal involving partitioning a number line and labeling unit fractions correctly.

Clarifying a strategy for successfully completing a task. A second instance where Ian provided feed up at the process level was on the eighth day of classroom observations. Three days prior, the students worked through this problem: “Sally is walking down the street and decides that every sixth of a mile, she will stop to take a photo and rest a little. If she stopped at the ninth stop, what fraction of the mile, at that point, had Sally walked?” (Classroom Observation, January 23, 2018). On the eighth day, Ian displayed the original problem on the interactive board at the front of the room and read the last part of the problem that was not discussed previously: “After lunch she continued walking and stopping until she finished at her final twelfth stop. What fraction of a mile had she walked at the end of her walk?” (Classroom Observation, January 26, 2018).

As students started to take out mathematical tools on their desks, Ian continued, “Two steps, two steps, what fraction did she walk, and could you write this fraction another way? So two steps to this problem” (Classroom Observation, January 26, 2018). Ian walked around the room looking at students’ work and stopped at a student who appeared to have completed the first part of the task:

Ian (pointing to the student’s paper): Can you write this another way?

Student: Can we do like an equation? Like a plus, a plus, a plus?

Ian: *Another fraction, another fraction that would show the same thing as this.*

Student: So, can we write an equation?

Ian: Not an equation, no. *So, for example . . . I know that 6/6 is equal to one whole. I know that ½ is equal to 3/6* so *I want to see . . . what does this mean? Can you write it another way?* (Classroom Observation, January 26, 2018)

In this example, Ian responded to the student's comment about an equation by clarifying what it meant to write the fraction in a different way. When the student asked again if he could write an equation, Ian provided the student with two examples of the strategy used for finding equivalent fractions and then restated the question. Ian provided feed up at the process level by clarifying the strategy for successfully completing the second half of the task to help the student move forward in their thinking.

To help students to monitor their own processes as they move towards the learning goal, Ian provided feed up at the self-regulation level. The following section will discuss how Ian provided feed up in this way.

Self-Regulation level. Feed up at the self-regulation level provides information about the learning goal and ways in which students can monitor their own actions to move towards the learning goal. Ian provided feed up at the self-regulation level a total of three times during three separate days of classroom observations by reminding students of the learning goal to help them reflect on their own work and helping students to self-reflect on their progression towards the goal. Two instances of how Ian provided feed up in these ways will be discussed in the following section.

Reminding students of the learning goal to help them reflect. One instance where Ian provided feed up at the self-regulation level occurred on the fifth day of classroom observations after lunch where students explored the connection between pattern blocks and fractions. As students walked in from lunch, they immediately sat on the carpet facing the figure displayed on the interactive board at the front of the room (see Figure 15).

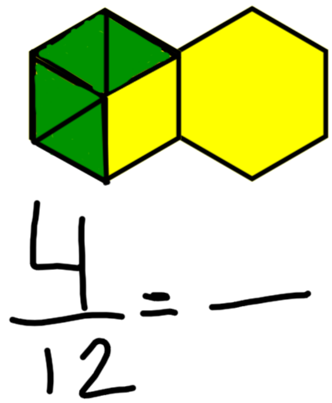

Figure 15. Two hexagons with four triangles displayed on the interactive board to demonstrate equivalent fractions.

Ian reviewed where the students left off prior to lunch saying, “Two hexagons being our whole . . . and we found that four triangles would be 4/12 . . . So after we found 4/12, the challenge . . . was can you find an equivalent fraction using the shapes to equal 4/12?” (Classroom Observation, January 23, 2018). One student stood next to the document projector and demonstrated how he solved the problem by changing two triangles into one rhombus. As the student completed the equivalent fraction by writing a six in the denominator and a two in the numerator, Ian addressed the whole class:

> Same fraction taking up the same space. *We just wrote it two different ways, that’s what we’re working on. Right now, you should be thinking where did I go wrong?*

Where did I go wrong? Was I correct? (Classroom Observation, January 23, 2018).

Ian provided feed up by first reminding the students that the learning goal was to write a fraction in two different ways and then by indicating that the students should use the learning goal to reflect on their work. This included detecting errors and indicating the correctness of their work.

Helping students to self-reflect on their progression towards the goal. A second instance where Ian provided feed up at the self-regulation level occurred after lunch on the third day of classroom observations. Half the class sat at their desks working independently on the computers while the other half sat on the carpet in front of the interactive board. Ian demonstrated on the interactive board how to represent a whole using a number line with a black rectangle (see Figure 16). After demonstrating the process for them, he then displayed a blue rectangle, asked the students to do the same on their whiteboards, and then demonstrated how to draw the number line on the interactive board.

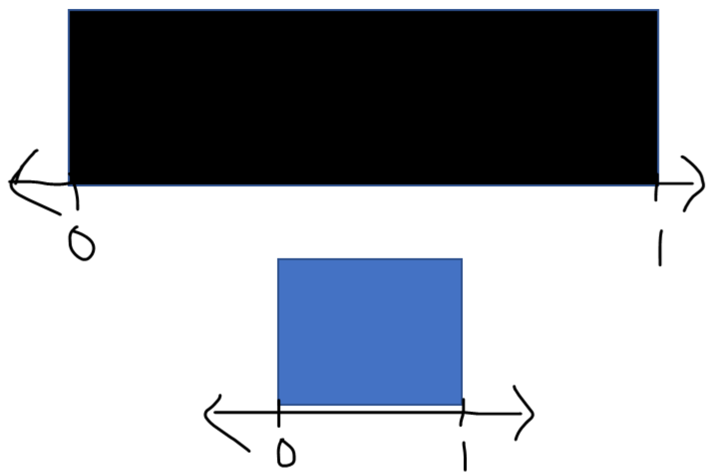

Figure 16. Black and blue rectangles displayed on the interactive board representing a whole using a number line.

Once Ian finished drawing the number line under the blue pattern block, he announced:

Ian: So let’s go back to the I-Can statement. *I can represent the whole using a number line.*

Students on the carpet: I can represent the whole using a number line.

Ian: What do you rate yourself [in The Pit]? *Right now, with this skill, do you think you could do this on your own? If I gave you a whole, could you write a number line? Could you make a number line if I gave you the whole?*

(Classroom Observation, January 11, 2018)

In this instance, Ian referred to the learning goal (i.e., the I-Can statement) and then asked the students on the carpet to rate themselves according to The Pit (see Figure 5) referring to whether the students thought they could do the skill on their own. Ian provided feed up at the self-regulation level by referring to the learning goal and having students self-reflect on their progression towards the goal.

Ian provided feed up at the task, process, and self-regulation levels by answering the question “Where am I going?” (Hattie & Timperley, 2007, p. 89). Ian also provided feed back which indicated to the students how they were currently doing relative to the learning goal. Feed back will be described in the following section in addition to how Ian provided feed back in this way.

Feeding Back. Feed back describes the information given to students related to a “task or performance goal, often in relation to some expected standard, to prior performance, and/or to success or failure on a specific part of the task” (Hattie & Timperley, 2007, p. 89). This type of feedback addresses the question “How am I going?” (Hattie & Timperley, 2007, p. 88), informing students about their current progress and/or how to continue the correct path towards the learning goal. During the 11 classroom observation days, Ian provided feed back a total of 102 times (see Table 8). The ways in which Ian provided feed back at the task, process, and self-regulation levels will be described in the following sections.

Task level. Feed back at the task level answers the question “How am I going” (Hattie & Timperley, 2007, p. 88), including how successful the student is at completing the task and/or whether the task is correct or incorrect. Ian provided feed back at the task level 50 times during the 11 classroom observations by revealing whether some aspect of a task was correct and by explaining why a student’s solution was incorrect. Ian also provided feed back by indicating to students how they were doing relative to the task such as "You're getting a little confused" (Classroom Observation, January 23, 2018), and by restating the student's reply with an "Ok" or "All right." The following section will discuss four instances of how Ian provided feed back at the task level in these ways.

Revealing the correctness of one aspect of a task. One instance of how Ian provided feed back at the task level was on the first day of classroom observations. Towards the end of the mathematics instruction time, Ian separated the class into two groups where half of the students worked independently on laptops at their desks and the other half sat on the carpet for small-group instruction. The students on the carpet discussed how to represent and add unit fractions. The first student labeled the unit fractions on the interactive board as one-third, two-thirds, and three-thirds, and the second student corrected the first student by labeling each piece as one-third (see Figure 17).

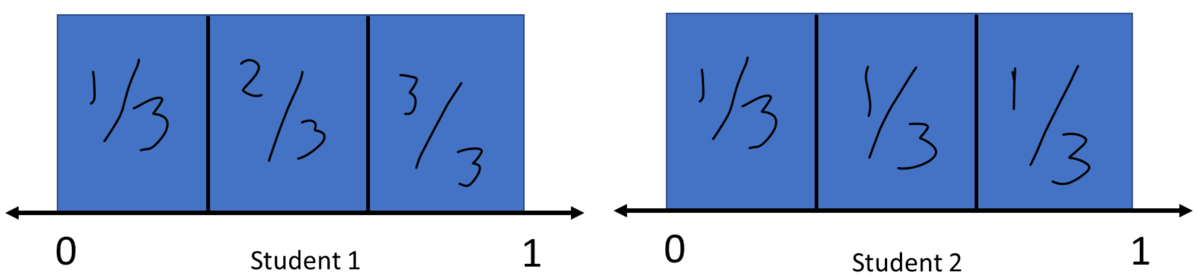

Figure 17. Student work labeling unit fractions incorrectly and correctly displayed on the interactive board.

Once both students were seated, Ian addressed the second student:

Ian: Why did you choose one each time?

Student Two: Because if you did do one, that means one shaded in. If you did a fraction one again, that's one shaded in and if you do one again, that's one shaded in.

Ian: *All right, that is correct*. (Classroom Observation, January 9, 2018)

In this instance, the second student explained why he wrote one-third on the second and third pieces considering that each piece shaded in represented one. Ian provided feed back at the task level by indicating to the student that he was correct in labeling the unit fractions as one-third in the task. This feed back indicated to the student the correctness of his thinking relative to the learning goal of representing unit fractions.

Although Ian explained in an interview that he tried not to give students any validation (Interview, January 22, 2018), he did clarify why he would respond to a student by indicating whether they were correct or not rather than ask the other students if they agreed or disagreed:

> There's nothing behind it. A lot of times I do the agree/disagree, but I don't know. Sometimes just like, I'm thinking of a response that I want and sometimes they really hit it on the head. Sometimes it's close, and then I agree and disagree to kind of see where the whole class is at and sometimes, I know I didn't do this because it was a low group. But sometimes I want it to be concrete. I don't want it to be a debate for the class to see. I wanted it to be his response, listen to his response, this is exactly what I was thinking . . . sometimes they say the exact thing and I'm like, that's exactly what I wanted to say. (Interview, January 10, 2018)

In the interview, Ian indicated that one reason why he would tell a student that they were correct rather than ask the class if they agreed or disagreed was because he wanted the students to have a concrete answer, particularly with the students Ian perceived to be low performing. Ian also stated that sometimes the student said exactly what Ian was going to say so, in response, Ian stated that the student was correct.

Explaining why a student's solution was incorrect. A second instance where Ian provided feed back at the task level was on the seventh day of classroom observations. Towards the end of mathematics instruction time, Ian walked around the room with a clipboard assessing students' work on a task where students were labeling fractions on a number line (see Figure 18). On the clipboard, Ian marked an "arrow going down, a hyphen if they've somewhat got it, and a check if they have [the indicated skill] already immediately" (Interview, January 11, 2018).

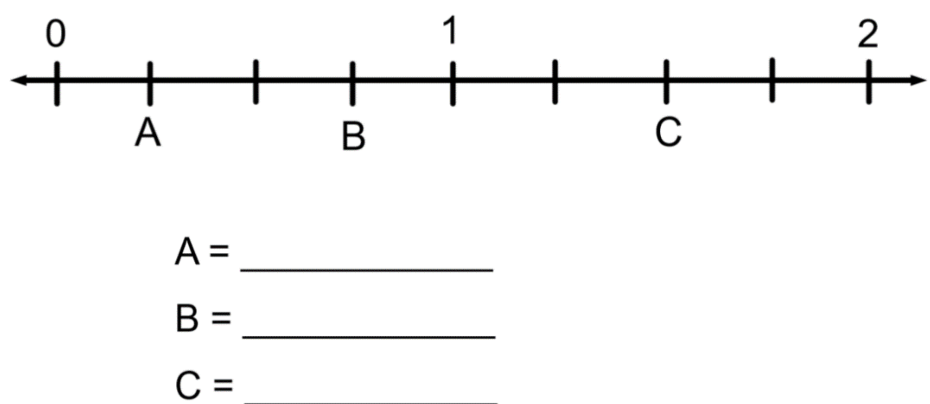

Figure 18. Fractional representation problem solving for A, B, and C on a number line between zero and two.

As Ian walked around the room, he stopped at a student's desk, picked up the student's pencil, and pointed to the student's notebook:

Ian: You said B is three, well [the number line] ends at two.

Student: I meant like jumps. Can I just put one jump?

Ian: But it says fractions.

Student: Wait, what? Where?

Ian: *You're labeling the fraction. B can't be three because three doesn't come before one. It can be a fraction; it just can't be three*. (Classroom Observation, January 25, 2018)

Ian provided feed back at the task level by first clarifying that the student needed to label the fractions, not jumps, as stated in the task, and then by explaining to the student why his answer of three could not be correct. Ian's response was directed at the student's current progress towards the learning goal of labeling fractions on a number line.

Indicating a student's progression relative to the task. Another instance where Ian provided feedback at the task level was on the fifth day of classroom observations where students worked independently at their desks with pattern blocks. Displayed on the interactive board were three problems (see Figure 19) where Ian asked the students to answer each problem one at a time.

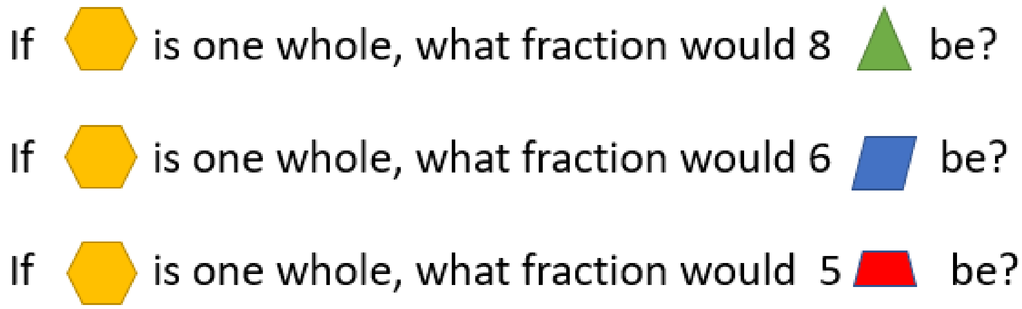

Figure 19. Three problems about improper fractions displayed on the interactive board that students solved using pattern blocks.

While students worked on answering the first problem, Ian walked around the room observing students and addressing questions. One student raised his hand, and Ian approached him by asking questions about his work. Ian discussed the problem with the student while drawing examples with a dry-erase marker on the student's desk:

Ian (pointing to the 8/6 drawn on the desk): Can you explain this to me?

Student (pointing to six): Ok, this is a whole.

Ian: *Uh huh.*

Student: It took six triangles to (pause).

Ian: *Uh huh.*

Student: To fill up the hexagon.

Ian: So, is that your numerator or denominator?

Student (pausing before he speaks): Numerator?

Ian: *These need to switch. Numerator is pieces used; denominator is total. Ok, so was it your numerator or denominator?* (Student looks at the desk, starts to

> answer, and then looks up at Ian. Ian chuckles) *Ok, you're getting a little confused*. . . . This is total pieces to make up the whole and this is pieces used. (Classroom Observation, January 23, 2018)

In this instance, Ian continued to agree with the student by saying "uh huh" and then asking more questions. Once the student answered a question incorrectly (i.e., the six was the numerator), he did not answer the following question posed by Ian and looked up. In response, Ian commented that the student was getting a little confused and proceeded to tell the student the correct way. Ian provided task-level feed back to the student by first indicating the correctness of the student's replies and then by indicating to the student how he was doing relative to successfully completing the problem.

Restating the student's reply with an "Ok" or "All right." Ian also provided feed back at the task level on the fourth day of classroom observations. Ian worked with a small group of students on the carpet while the rest of the class worked independently on computers at their desks. Displayed on the interactive board were two statements where Ian asked the students on the carpet to draw a number line that represented the first statement. After giving the students a minute to work, Ian displayed a whole partitioned into thirds and number line under the first statement (see Figure 20).

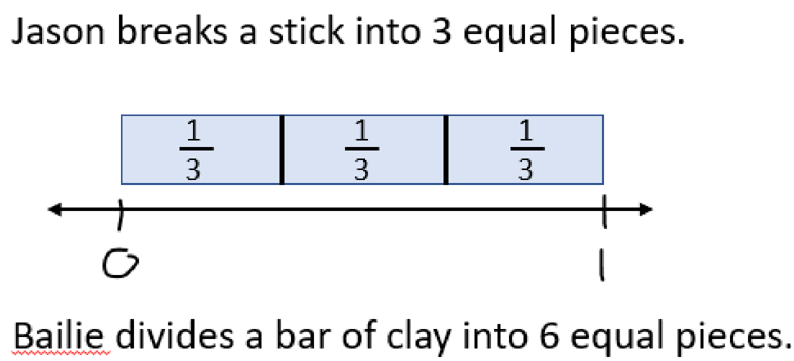

Figure 20. Two statements displayed on the interactive board and a whole partitioned into thirds. A number line is displayed below labeled with 0 and 1.

Ian labeled the zero and one on the number line and using his hand to represent the width of each partition, continued explaining the problem:

Ian: And now, just like you've modeled, you want to split your number line into three equal partitioned sections. So, as best as you can, you're going to try and split yours into three sections. Are you counting jumps or ticks?

Student: Jumps.

Ian (nodding): *Jumps, Ok. So, you want to make sure you have three total jumps in your whole*. (Classroom Observation, January 22, 2018)

In this instance, Ian directed his feedback to the student who replied to his question by repeating the correct answer, nodding, and then saying Ok. Ian provided task-level feed back by repeating what the student said, saying "Ok," and then stating what the students should do next with the information provided by the student.

Although Ian provided feed back at the task level to help students know how they were progressing in relation to the current learning goal, he also provided feed back aimed at the processes needed to successfully complete the task. The following section will discuss four instances of how Ian provided feed back at the process level during the observed classes.

Process level. Feed back at the process level answers the question “How am I going” (Hattie & Timperley, 2007, p. 88) in relation to the processes, strategies, thinking, or skills involved in a task (Brooks et al., 2019). Ian provided feed back at the process level 29 times during mathematics instruction on the 11 observation days by commenting on students’ strategies, pointing out the efficiency of a strategy, emphasizing the way students recognize patterns, and indicating the correctness of students’ strategies. The following section will discuss four instances of how Ian provided process-level feed back in these ways.

Commenting on students’ strategies. One instance where Ian provided feed back at the process level was towards the beginning of mathematics instruction on the tenth day of classroom observations. Ian worked with half of the class on the carpet while the rest of the class sat at their desks working independently on computers. A sample of student work on an index card comparing the fractions ¼ and 2/8 (see Figure 21) was displayed on the interactive board.

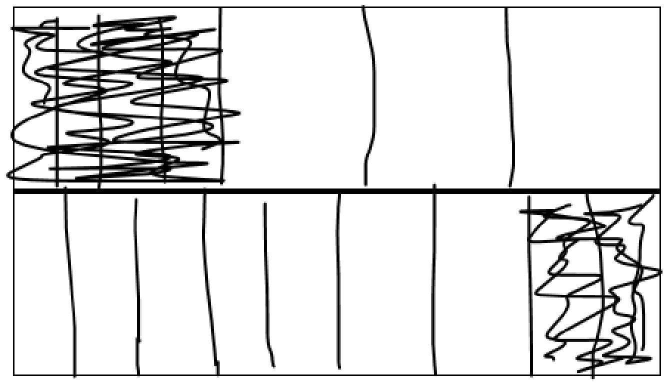

Figure 21. Example of student work on an index card comparing 1/4 and 2/8 displayed on the interactive board.

As the students on the carpet began to discuss what they would tweak or add to the example, one student asked to go to the board to show what he was thinking. Ian asked him to explain his thinking in terms of a simpler problem (i.e., 1/3 and 2/6):

Student (while drawing on the board): This is going to be 1/3 (partitioning the rectangle on the bottom into thirds; see Figure 22a). Well, like they could have split one-half and then another. So the sixths, they could have split in the middle and did it again, then did it again (splitting the top rectangle in half and then each half into thirds; see Figure 22b), and then did that. Like trying to do something sort of like that so it would be even.

Ian: *Ok, good strategy*.

Student: And not like that (drawing two rectangles with uneven partitions).

Ian: *Ok, so that's a good strategy*. *So, what he's saying is like right here for the four* (pointing to the original example), *if they started here in the middle, then* *they could have just split this in two, one, two. That might have helped them get even pieces*. (Classroom Observation, January 30, 2018)

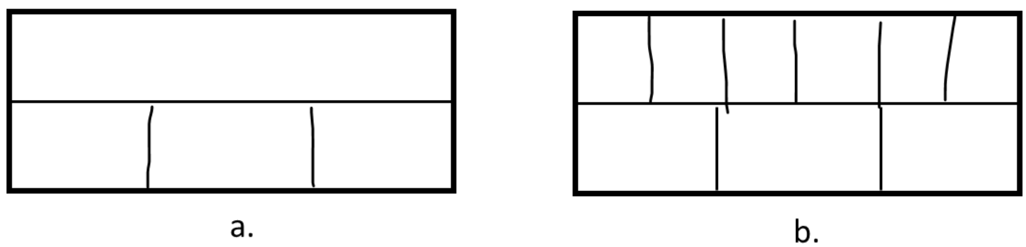

Figure 22. Student work on interactive board describing how to compare one-third and two-sixths.

In this instance, Ian responded to the student's explanation by indicating that the student had a good strategy for partitioning two wholes into even pieces. Ian then continued by explaining in his own words what the student said to the rest of the students on the carpet. Ian provided feed back at the process level by indicating that the student had a good strategy for partitioning a whole into equal pieces and addressing how the student was progressing in relation to the learning goal of representing and proving two fractions were equivalent.

Pointing out the efficiency of a strategy. A second instance where Ian provided feed back at the process level was on the first day of classroom observations where half of the students sat on the carpet and the other half worked independently on computers at their desks. Towards the end of the mathematics instruction time, Ian displayed on the interactive board a large and small rectangle representing a whole chocolate bar and one piece of the bar (see Figure 11). Ian asked the students on the carpet: "What is the unit fraction for one piece of chocolate?" (Classroom Observation, January 9, 2018). Ian walked around the carpet monitoring students as they worked both at the board and on the carpet to solve the problem.

As he approached a student on the carpet, Ian bent down and started to draw on the student's paper:

> So, if I draw this box and this row takes three, then (pauses as he stops writing). It's the strategy. *Your equation is not matching. Your strategy works [but] your numbers in the equation do not match it because you're saying whatever your denominator is, is total pieces. Yes, you're saying this is three pieces*. *But your denominator is always how many total pieces we took, and it did not take three pieces to fill this whole thing up.* (Classroom Observation, January 9, 2018).

In this instance, Ian drew an example on the student's paper, stopped to reflect on the student's work, and then pointed out that the student's strategy and equation did not match. Ian provided feed back at the process level by indicating how the student was doing relative to the efficiency of the process and strategy used by the student. Ian then clarified his feed back by further explaining what the denominator was and how the student could use this to move towards the learning goal of representing unit fractions.

Emphasizing the way students recognize patterns. Ian also provided process-level feed back after lunch on the seventh day of classroom observations after he displayed a number line on the interactive board with three questions listed below it (see Figure 23).

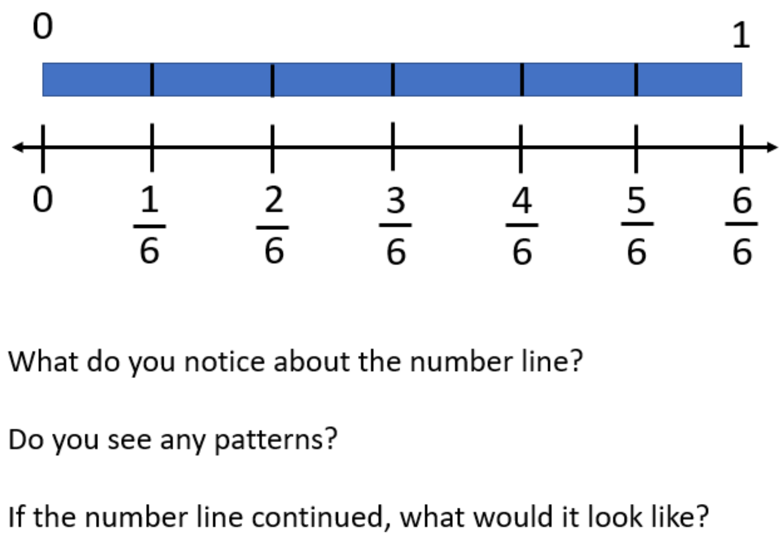

Figure 23. A number line partitioned into sixths with three questions about the number line displayed on the interactive board.

Ian read the three questions out loud to the class seated at their desks and asked them to discuss the questions with a partner. After three minutes of discussions, Ian asked two students what they noticed about the number line. The first student noticed that "every time the numerator keeps on adding itself by one" (Classroom Observation, January 25, 2018). Ian agreed with the first student and then asked the second student what he noticed:

> Student: What I noticed is that the denominator never changed because it's the whole.

Ian: Ok, *I like how you went ahead and answered where I was going next. He said the denominator. He's noticing a pattern here.* The denominator never changes. (Classroom Observation, January 25, 2018).

In this instance, Ian responded to the student by indicating that he liked how the student noticed a pattern in the denominators on the number line which also answered the second question on the board. Ian provided feed back at the process level by telling the student how he was doing relative to recognizing a pattern in the denominators of fractions on a number line.

Indicating the correctness of students' strategies. An additional instance where Ian provided process-level feed back was on the eighth day of classroom observations. While all the students sat on the carpet facing the interactive board, one student stood at the document projector to the right of the board and explained his thinking about a problem. Using a dry-erase marker on the desk, the student explained how he applied a previous example (i.e., 9/6 was equal to one and a half) to the current problem.

After the student erased and rewrote what he was trying to explain, Ian began to ask him questions about his thinking:

Ian: So, now you're saying 9/6 plus 3/3 equals 12/6?

Student: Well, yeah.

Ian: Ok, only thing about that is that your denominator is changing so you're saying it took six jumps, but then you switched to three jumps for your whole.

Student: But that's because (pause) but the one that we did the last time (pause) but somebody else said one whole and a half.

Ian: So, if I have 3/3, what is it equal to?

Student: A whole.

Ian: *A whole, so it can't be a half if I have 3/3.* (long pause) *So, you're onto something, you're onto something. We're going to keep going here because you're actually ahead of where we're going*, but you're onto something. (Classroom Observation, January 26, 2018)

Ian responded to the student in this instance by asking him questions about his thinking and providing reasons why the strategy that the student used may be incorrect. Ian provided process-level feed back by indicating the correctness of the student's strategy (i.e., 3/3 is not equal to one half), and how the student should proceed (i.e., you are ahead of where we're going and are onto something).

Ian provided feed back directed at the processes involved to help students know how they were doing relative to the current learning goal. Ian also provided feed back aimed at the way students could self-regulate their actions to successfully complete the task. The following section will discuss four instances of how Ian provided feed back in this way during the observed classes.

Self-regulation level. Feed back at the self-regulation level addresses how students are doing relative to the ways "students monitor, direct, and regulate [their] actions towards the learning goals" (Hattie & Timperley, 2007, p. 93). Ian provided feed back at the self-regulation level 23 times during mathematics instruction by addressing how students could monitor their progress towards the learning goal, learn from their mistakes, maintain their speed when working through a task, and progress on the right track towards the learning goal. The following section will discuss these instances of how Ian provided feed back at the self-regulation level.

Addressing how students could monitor their progression. In the middle of mathematics instruction on the fourth day of classroom observations, Ian provided feed back at the self-regulation level to half of the students seated on the carpet while the other half worked independently on computers at their desks. One student on the carpet demonstrated how the number line displayed on the interactive board (see Figure 24) could be partitioned into fourths. Ian followed this demonstration by asking students to discuss with their neighbor: "Which one of these is the best answer for how this number line is partitioned?" (Classroom Observation, January 22, 2017).

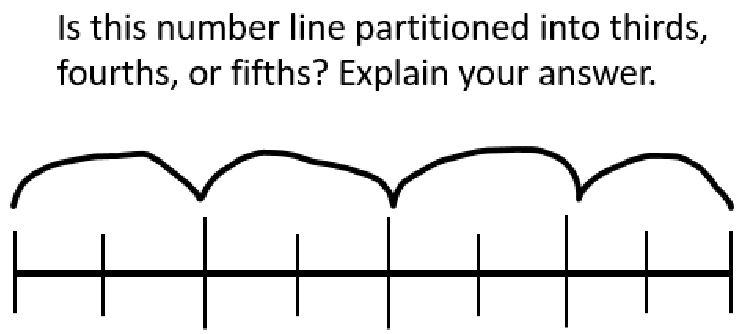

Figure 24. Open-ended problem displayed on interactive board asking if the number line is partitioned into thirds, fourths, or fifths with a student's work displayed above the number line demonstrating how the number line could be partitioned into fourths.

After 30 seconds of discussion time, Ian asked student J to come to the board to explain her thinking:

Student J (standing to the side of the interactive board): Even though it can be fourths, it can also be partitioned into thirds . . .

Ian: Can you represent your thinking?

Student J (starting to draw jumps on the number line; see Figure 25): Wait, that's not right. Wait (looking over at Ian).

Ian (slightly giggling): *But you tried it. Sometimes you have to try some stuff to realize that you were closer or how far you were*. *So, thirds were kind of close.* (Classroom Observation, January 22, 2018)

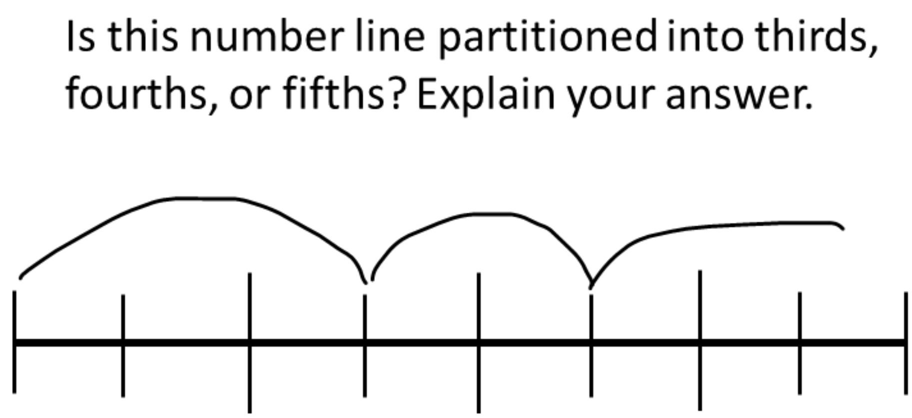

Figure 25. Jumps drawn by student J on a number line attempting to represent how the number line could be partitioned into thirds.

In this instance, student J first explained that the number line could be partitioned into thirds, attempted to show how on the interactive board, and then realized what she was saying was not correct. Ian provided feed back at the self-regulation level by addressing the student's current progress (i.e., she tried and that her solution of thirds was close) relative to how the student's actions could help her monitor her progress towards the learning goal (i.e., trying different solutions to regulate how close or far you are from a solution). Although Ian directed the feed back at student J, he said it out loud in front of the students seated on the carpet allowing them to benefit from the feedback as well.

Indicating how students could learn from their mistakes. A second instance where Ian provided feed back at the self-regulation level was on the second day of classroom observations where half of the students sat at the carpet and half sat at their desks working on computers. Ian displayed a worksheet with two rectangles on the document projector, each with the word "whole" written on it (Classroom Observation, January 10, 2018). He proceeded to partition one of the rectangles into thirds and shaded in the middle rectangle (Figure 26).

Figure 26. A whole rectangle partitioned into thirds with the center piece shaded in and the two outside pieces labeled as one-third.

Once the whole was colored in as shown in Figure 26, Ian asked the students on the carpet to write a number sentence to represent how much of the figure was unshaded. As students began writing and discussing the problem, Ian walked around looking at students' work:

> I want to see if you learned from yesterday. [Student A] got it correct. So did [student B]. . . . Let's see your equation for unshaded. (The student remains seated as Ian looks down at her paper) *We learn from our mistakes, there's nothing to be nervous about. If you don't get it today, you'll get it tomorrow.* Come on, I'm rooting for you. (Classroom Observation, January 10, 2018)

In this instance, Ian indicated that two students got the problem correct and asked to see the work of a third student. When the student was hesitant to go to the board to show her equation, Ian responded by indicating that the student had nothing to be nervous about (i.e., if she didn't get it today, she would get it tomorrow). After looking on the student's paper, Ian provided feed back at the self-regulation level by addressing how the student was doing and how she could learn from her mistakes to move towards the learning goal.

Emphasizing how students could maintain their speed when working through problems. One instance where Ian also provided feed back at the self-regulation level was on the last day of observations. While most of the students worked at their desks on a mathematics writing prompt with the help of the student teacher, Ian worked with four students at the Small Group Table (see Figure 6) on an assessment previously assigned. The assessment focused on using models to compare fractions. So, Ian started to work through a problem with all four students together by first setting up a bar model to represent 2/5. One student sitting directly in front of Ian immediately drew a rectangle, a line horizontally in the middle of the rectangle, and began drawing more lines. Ian laid his hand on the student's paper:

Ian: *You're going too fast. You're going too fast*. (The student put his hands on his head and face in a frustrated manner. Ian paused). *And you're not wrong but when you go fast, that's when errors get made. So, pump the brakes.* Go ahead and draw your rectangle. (Classroom Observation, January 31, 2018)

In this instance, Ian indicated to the student that he was going too fast when beginning to work on this problem and although the student was not wrong in what he was doing, errors could be made when working too fast through the problem. Ian provided feed back at the self-regulation level by informing the student that he was going too fast even though he was not wrong. This indicated to the student how he was performing. Ian then addressed how the student could regulate his actions (i.e., pump the brakes) to avoid future errors in his work.

Explaining how the student is progressing towards the learning goal. Towards the end of the mathematics instruction time on the first day of classroom observations, Ian displayed a large and small rectangle representing a piece and a whole chocolate bar (see Figure 11) on the interactive board. Half of the students sat on the carpet while the other half worked independently on computers at their desks. Ian asked the students on the carpet to model the unit fraction of one piece of chocolate, and the students on the carpet began working. One student tried measuring from his seat on the carpet and then at the board using his pencil as a measurement tool. Once the student sat back down on the carpet, Ian approached him:

Student: I tried to use measurement to try and count. I tried my best because I know if I tried my best.

Ian: *Ok, you're on the right track*.

Student: What am I supposed to do?

Ian: *You're on the right track. You might have to check it by taking what you started here and looking real close to that model up there. If you need to make marks up there, go right ahead*. (Classroom Observation, January 9, 2018)

In response to the student explaining that he tried his best, Ian provided feed back at the self-regulation level by indicating how the student was progressing (i.e., on the right track) and how he could monitor his actions (i.e., going back and looking closer at the model). Ian also suggested that the student make marks on the board to help him progress towards successfully completing the task.

The previous section described how Ian provided feed back during mathematics instruction by indicating to the students how they were doing relative to the learning goal. To help students to continue moving forward, they must also know where they need to go next in the learning progression. This type of feedback is described as feed forward. The following section will discuss how Ian provided feed forward (i.e., where to next) in this way.

Feeding forward. Feed forward describes the information given to students that completes the feedback cycle requiring students to apply the feedback they previously received (Brooks et al., 2019). This type of feedback addresses the question "Where to next? . . . providing information that leads to greater possibilities for learning" (Hattie & Timperley, 2007, p. 90). During the 11 classroom observation days, Ian provided feed forward a total of 10 times (see Table 8). The ways in which Ian provided feed forward at the task, process, and self-regulation levels will be described in the following section.

Task level. Feed forward at the task level answers the question "Where to next" (Hattie & Timperley, 2007, p. 88), including the direction that students should go to successfully complete the task and how students could transition from a concrete to an abstract view when completing a task. Ian provided feed forward one time at the task level during the 11 classroom observation days by indicating the direction that the students were going relative to the fraction learning goal. The following section will discuss this instance of how Ian provided feed forward at the task level.

One instance where Ian provided feed forward at the task level was on the third day of classroom observations where half the students sat at the carpet while the other half sat at their desks working independently on computers. Ian asked the small group to first represent the whole with the number line and then continued to ask questions about jumps versus tick marks referring to the figure displayed on the interactive board (see Figure 27).

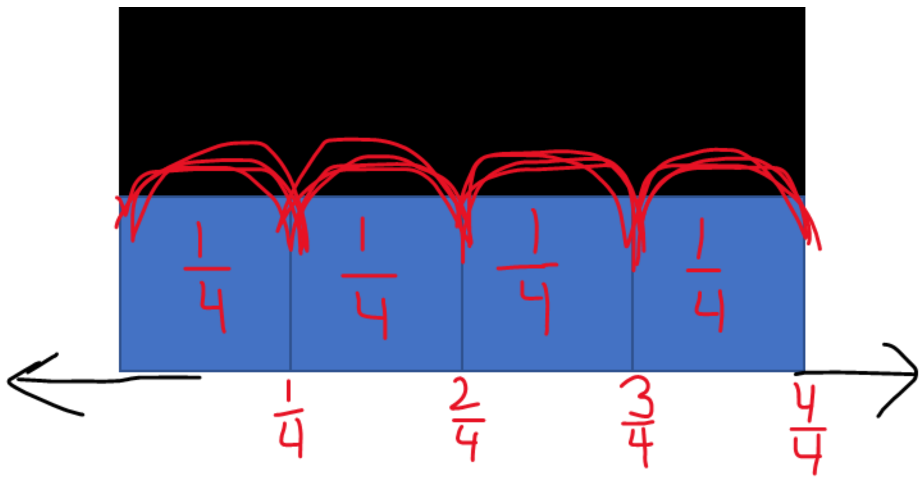

Figure 27. Image displayed on interactive board with black rectangle representing the whole and blue rectangles representing four equal partitions. Jumps are represented by half circles in the middle and a number line is drawn below.

Ian reviewed what the small group discussed regarding representing the whole using a number line, labeling the unit fractions, and by counting jumps instead of ticks. As Ian drew half circles that represented jumps on the figure (see Figure 27), he responded to the small group:

> Remember earlier I said I'll give you credit for this (drawing jumps on the board) ¼, ¼, ¼, ¼. *But eventually I want you to see this is ¼ jumps, this is my second ¼ jumps, this is my third ¼ jumps. So, we're moving from here* (pointing to the labeled unit fractions and jumps), this is what we did yesterday. *And we're moving now to here* (pointing to the number line and the fractions under the number line). (Classroom Observation, January 11, 2018)

In this instance, Ian reiterated that each individual piece of the whole represented a unit fraction of ¼ and that each jump represented an addition ¼ of the distance each time. Ian then emphasized that eventually he wanted students to see the connection between the unit fractions (i.e., jumps) and how to label the total number of jumps on the number line. Ian provided feed forward at the task level to the small group by indicating the direction that the students were going relative to the fraction learning goal of partitioning a number line and labeling unit fractions.

In addition to providing feed forward at the task level, Ian provided feed forward relative to the processes that students engage in. The following section will describe how Ian provided feed forward directed at the process level during the classroom observations.

Process level. Feed forward at the process level answers the question "Where to next?" (Hattie & Timperley, 2007, p. 88) in relation to the processes, strategies, thinking, and skills involved in a task (Brooks et al., 2019). Ian provided feed forward at the process level twice during mathematics instruction on the 11 observation days by suggesting efficient strategies and making the connection between a concrete and abstract process to help students move towards the learning goal. The following section will discuss these two instances of how Ian provided feed forward in this way.

Suggesting sufficient strategies. On the second day of classroom observations, half the class sat on the carpet in front of the interactive board while the other half sat at their desks working independently on computers. Ian displayed a worksheet with two rectangles on the document projector, each with the word "whole" written on it (Classroom Observation, January 10, 2018). As a group, the students worked through several problems partitioning the wholes into equal pieces and labeling each unit fraction.

As Ian erased the board which displayed a whole partitioned into thirds (see Figure 26), a student claimed, “I’m confused. My brain hurts” (Classroom Observation, January 10, 2018). Ian chuckled and responded to this student by indicating that they had more to practice. Ian then asked the students on the carpet to partition the next whole into fifths. As students began partitioning the next whole on their papers into fifths, Ian stood in front of the interactive board and put his hand upon the board to indicate the partitions:

> Now [student], hold on. Before you begin, it’s easier, *just a little strategy. It’s easier to start partitioning a little smaller than what I think because if I have room at the end, I just know that I can start smaller*, then I can come out a little bit more. *Don’t start too big and run out of room. But you might have to erase to get it right.* (Classroom Observation, January 10, 2018)

In this instance, Ian responded to the student who voiced this confusion by describing a strategy for partitioning the whole in an easier manner. Using his hand as a measurement tool, Ian demonstrated to the students how to initially draw each partition smaller so that they did not run out of room within the whole. Ian provided feed forward at the process level by suggesting an efficient strategy to help students move towards the learning goal of partitioning a whole into equal pieces.

Making connections to help students progress. A second instance where Ian provided feed forward at the process level was on the tenth day of classroom observations. Five students sat on the carpet working with Ian while the rest of the class worked independently on computers at their desks. Students on the carpet worked in their notebooks comparing fractions using two rectangles (see Figure 28) while Ian walked around the carpet monitoring them.

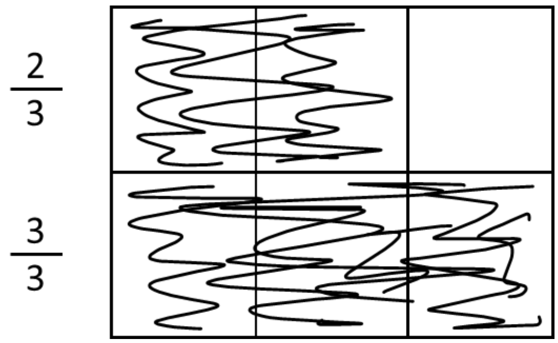

Figure 28. Example of student work in notebook comparing fractions 2/3 and 3/3 using rectangles.

Ian sat in a chair next to a student on the carpet and picked up the student's notebook displaying the drawings of 2/3 and 3/3 (see Figure 28):

Ian: Ok, now tell me how you know they're not equivalent.

Student: Hmm, because this one (pointing to the picture in the notebook) only had two shaded in and these got all of them shaded.

Ian: *Ok* (nodding)*, so I'm going to keep working with you too. Eventually we're going to take this model away* (covering the rectangles with his hand), *and I want you to look at these two fractions.* I want you to just look at them and say, I'm going to ask you, how do you know they're not equivalent? *And you can picture this in your head.* (Classroom Observation, January 30, 2018).

In this instance, Ian asked the student to indicate how he knew from the picture that the two fractions were not equivalent. After the student explained, Ian responded by indicating that eventually the model would be taken away and the student would have to determine if the two fractions were equivalent without the model (i.e., in his head). Ian provided feed forward at the process level by indicating to the student the explicit connection between a concrete (i.e., a model with two rectangles) and an abstract (i.e., a picture in your head) process for determining whether two fractions were equivalent. This indicated to the student where he would be going next in relation to the processes involved with the learning goal.

In addition to providing feed forward at the process level to help students move forward in their thinking, Ian also provided feed forward at the self-regulation level. The following section will describe how Ian provided feed forward in this way.

Self-regulation level. Feed forward at the self-regulation level answers the question "Where to next?" (Hattie & Timperley, 2007, p. 90) by providing information that helps students regulate their own actions beyond the current learning goal. Ian provided feed forward at the self-regulation level seven times during mathematics instruction on the observed days by focusing on maintaining a growth mindset while learning from others' ideas and suggesting ways to apply ideas from previous problems to future problems. The following section will discuss two instances of how Ian provided feed forward at the self-regulation level in these ways.

Focusing on maintaining a growth mindset while learning from others. On the fifth day of classroom observations, students explored how to represent improper fractions using pattern blocks at their desks. During this time, the students gathered once around a student's desk to observe how he solved a problem and four times on the carpet where Ian chose a student to demonstrate their work on the document projector. Students continued going back and forth between exploration at their desks and whole-group discussions on the carpet. Once students settled on the carpet for the third time, Ian asked the students which side of the mindset wall they felt they were on (fixed or growth). As students said their answers out loud, Ian responded:

> Now, since I'm on the [growth mindset] side of the wall, I'm only thinking *what I can take from H that's going to help me solve problems in the future*. That's what I'm thinking. *What strategies can I learn from?* (Classroom Observation, January 23, 2018)

In this instance, Ian responded to the students by stating that because he has a growth mindset, he was thinking about what information he could take from the H (the student who was going to share his work) that could help him to solve problems in the future. Ian provided feed forward at the self-regulation level by using his own mindset as an example of how students could actively listen to other students' ideas and learn from them to solve future problems. Ian addressed the way students could monitor their own actions to move towards becoming self-regulated learners.

Suggesting ways to apply ideas from previous problems to future problems. A second instance where Ian provided feed forward at the self-regulation level was towards the end of mathematics instruction on the eighth day of classroom observations. Ian worked with six students at the Small Group Table while the other students worked at their desk on a fraction assessment. Ian continued to ask one student sitting directly in front of him multiple questions about three problems including why she chose her answer each time (Classroom Observation, January 26, 2018). After Ian verified the student's last answer by counting out loud with her, he replied:

> You just answered three problems for me. *I want you to take what you just said to me, and I want you to go back to this problem because you just solved this problem*. When you did all this (pointing to the student's previous problems discussed), you were solving this problem (pointing to a problem not solved). *What you were just thinking through, if you'll take that same thinking to this problem, you'll solve it.* (Classroom Observation, January 26, 2018)

Once Ian worked through three problems with the student, he responded by indicating that she needed to apply the same thinking she applied on the last three problems to the next problem. Ian provided feed forward at the self-regulation level by indicating to the student how she could use the previous problems to help her solve future problems. Ian addressed how the student could monitor the way she thought about future problems by providing her the opportunity to invest more effort as she moved closer to reaching the learning goal.

The previous sections described how Ian responded to students by providing feedback that answered the questions "Where am I going? How am I going? and Where to next?" (Hattie & Timperley, 2007, p. 102). These questions addressed three types of feedback: feed up, feed back, and feed forward, respectively. Evidence of how Ian provided feedback at the task level, process level, and self-regulation levels within each type of feedback was also discussed.

Case Summary

The previous sections described the case of Ian who held a stronger incremental theory on the implicit theory continuum (see Table 5) with an overall average of 5.75 out of 6.00. First, Ian's learning experience and his view of how students learn mathematics and his own instructional practices were examined. Second, a description of Ian's classroom structure was described including the warm-up and primary instructional activities during the classroom observations. Third, a description of how Ian responded to students who did not answer the questions "Where am I going? How am I going? and Where to next?" (Hattie & Timperley, 2007, p. 102), was provided. Fourth, the ways in which Ian provided self-level feedback were discussed to draw connections between his implicit theory and how he provided feedback in this way. Finally, the ways in which Ian provided feedback during mathematics instruction were described with instances categorized by type (i.e., feed up, feed back, and feed forward) and by the level in which they were directed (i.e., task, process, and self-regulation).

The following section will describe the case of Ellie, the second participant of the study who ascribed to a weak incremental theory. The results from her case will be discussed in a similar format as Ian's.

Ellie

Ellie Jones, an African American female in her mid-forties, was in her sixth year of teaching fifth-grade mathematics and science at Byron Elementary School and eighth year of teaching in her career. Prior to teaching fourth and fifth grade, Ellie taught high school mathematics and music, in addition to working as an administrative assistant at a telecommunications company. The following sections will describe Ellie's learning experience, implicit theory as shown on the Implicit Theories Survey (see Appendix A), and views of how students learn mathematics and her own instructional practices. A description of Ellie's classroom structure and how she responded to students who did not answer the questions "Where am I going? How am I going? and Where to next?" (Hattie & Timperley, 2007, p. 102), will also be provided. Additionally, the ways in which Ellie provided feedback that answered the three questions (i.e., feed up, feed back, and feed forward) directed at the task, process, and self-regulation levels will be described.

Ellie's Learning Experience

Ellie grew up in Kenya, Africa, and described her experience learning mathematics as a problem-solving community where students voiced their questions and explained problems in a variety of ways:

> In my math classes the teacher would introduce [the material]. For the most part it was just like debates, discussions. Why is it this one? Why is it not? And why do you say that? So, you have a different answer, how does he know if I didn't get it that way? Okay, tell us. . . . Math class for me was the top. Just, it was problem solving, very oral and verbal, and everybody presented it the way they see it. (Initial Interview, January 22, 2018)

Ellie explained that when she was in school, the teacher first introduced the material and then allowed the students to engage in discussions about the material. Through questioning, Ellie indicated that the mathematics classroom was where students could problem solve and present their ideas to each other.

Once she moved to the United States, Ellie explained how she chose to teach mathematics and science, her two favorite subjects that she excelled in, rather than English while she was still trying to learn the language herself:

> [Mathematics and science] are my two favorite subjects that I really, really enjoy. Those are two subjects that I've really always done well in, and I thought there's no way I'm going to teach English here. I'm still learning English and the language. [Teaching mathematics and science] to me was the best option for me. (Interview, February 13, 2018)

Ellie indicated that when she moved to the United States, she chose to teach mathematics and science as they were her favorite subjects that she excelled in. Ellie chose not to teach English as she was learning the subject herself.

Ellie explained that her own experiences learning mathematics and science led her to engage students in more explorations together as a class in her classroom. However, Ellie claimed that sometimes she needed to be more of a model which was not her natural tendency:

> Sometimes it's very obvious that even when we try to explore, [students] do not know because [they] do not have it. Then it changes, reinforces me to be the model which naturally that's not my tendency. I like us to explore together and this is something that I've had to learn being a teacher that sometimes you actually have to step out there and teach them. . . . Sometimes I have to be a teacher and stand up there and do it. Let's practice it, let's do it again. . . . For me as a teacher, it's been a learning process. To stop as in, I don't know [if] they

> don't know it. Let me show them how to do it. (Final Interview, February 23, 2018)

Ellie viewed teaching as a learning experience, yet when she tried to teach mathematics through exploration, the students did not always understand the material. As a result, Ellie explained that she became more of a model by standing in front of the class, teaching them, showing them how to work problems, and providing them with practice problems.

Ellie's experiences learning mathematics influenced her decisions regarding the way she taught mathematics in her classroom. In addition to her learning experiences, Ellie's implicit theory may have influenced her teaching decisions in the mathematics classroom. The following section will discuss Ellie's implicit theory in addition to her views on how students learn mathematics and how she provided feedback in her classroom.

Ellie's Implicit Theory

Prior to participant selection, Ellie completed the Implicit Theories Survey (see Appendix A). The results showed that Ellie held a weak incremental theory (see Figure 2) on the implicit theory continuum (see Table 9) with an overall average of 4.60 out of 6.00.

Table 9

Ellie's scores on the Implicit Theories Survey (see Appendix A)

Domain	Average	Implicit Theory
Intelligence	4.67	Weak Incremental
Morality	5.00	Incremental
World	3.67	Weak Incremental
Mathematical Ability	5.00	Incremental
Overall	4.59	Weak Incremental

Ellie disagreed (i.e., a five on the survey) with all statements on the survey except for three statements: somewhat disagreed (i.e., a four on the survey) with one statement about intelligence, somewhat disagreed (i.e., a four on the survey) with one statement about the world, and agreed (i.e., a two on the survey) with one statement also about the world. During her initial interview, Ellie discussed her choices on the survey and her views of learning and teaching in the mathematics classroom. The following sections will describe the results of these discussions including Ellie's view of how students learn mathematics and how she provided feedback in the classroom during mathematics instruction.

Ellie's view of how students learn mathematics. Ellie viewed learning mathematics as a student's ability to "do math problems, solve, [and] work with numbers" (Participant Selection Interview, January 10, 2018). She referred to this as a student's mathematical ability, as well as a science and an art. Ellie indicated that most students have the ability to do mathematics but questioned whether they were born with this ability:

> Is it [that] kids [are] born with the ability to process numbers or do they need to be exposed through teaching and drills and practice to enhance the ability? . . . Not necessarily mathematical ability, but can, do they have a brain that helps them see things and process things? And then depending on what we put in, program in, are they able to pick this up and make their brain work? . . . I think for the most part, yeah. This may be wrong. If you are born with the ability to process and make connections and just make sense out of things, then you're able to see numbers and process and problem solve. But to what extent are you able to do that? (Participant Selection Interview, January 10, 2018)

Ellie explained that for the most part, students were born with the ability to do mathematics if they were able to see and process mathematical ideas, make connections, and reason through mathematical processes. However, Ellie questioned how much of this ability was innate or dependent on external influences such as teaching, drills, and practice.

Whether students were born with the ability to do mathematics or developed the ability over time, Ellie claimed that some students may not have the ability to understand mathematics at all:

> Do all my students understand [the material]? No. This guy right here. . . . He is so, so, so, so, so, so low. Even his numeric ability is kind of like on first grade. [He] has been tested at that grade. He's been getting intervention at that grade. So, when he comes and you have fifth-grade standards and expectations, this poor kid cannot do it. He really can't, and it's a shame. (Interview, February 13, 2018)

Ellie explained how one student, who she described as low achieving, did not have the numeric ability to do grade-level mathematics and as a result, would not be able to understand the material.

Although Ellie indicated that some students may not have the ability to do mathematics, she emphasized that a student's mathematical ability could change over time. Ellie attributed her own change in mathematical ability to good teachers helping her to grasp the fundamentals of mathematics and setting her brain in motion (Participant Selection Interview, January 10, 2018). This was why Ellie emphasized that a student's past experiences could contribute to a change in their mathematical ability:

> Experience. Past experiences with the child determine their place that the ability is and depending on what you do. Let me put it this way, a lot of kids have been exposed to math in certain ways and that makes them be aware of whether they are good or bad in math. I think, depending on what has been given to them, depending on the style that they got it given to them, Ok, so they are kind of are labeled. . . . So yeah, kids will be at a certain level depending on what they have been told or labeled to be or what they know. That's their experiences, but can that be changed? Yeah, for the better or for the worse, I think. (Participant Selection Interview, January 10, 2018)

Ellie described how a student's mathematical ability could change for better or worse depending on how they were labeled or the way they were exposed to mathematics. Additionally, Ellie indicated how her experiences with good teachers helped her to realize that mathematics was fun and ultimately changed her view of mathematics overall.

Connections between contexts. In addition to having the mathematical ability to do mathematics, Ellie described how students who "are able to problem solve naturally, are at a better place to problem solve naturally everywhere" (Participant Selection Interview, January 10, 2018). Thus, Ellie emphasized the importance of making connections and applying new information to other contexts to build students' natural problem-solving skills:

> I also want them to be able to apply whatever we're learning across different skills because what we learned today, you will probably be able to use in a question [with] a task that has several different elements in it. So [students] should be able to just pull it out and make those connections. . . . If you look at the questions that they give now on the test, you have several skills within a question so you cannot just learn one thing in isolation. [Students] should be able to connect them all together. (Interview, February 5, 2018)

Ellie explained that students should learn to make connections naturally between different contexts, especially as test questions often encompass multiple skills and ideas. Classroom environments that encourage students to make these connections and share their ideas with their peers create the best settings for students to learn mathematics.

Ellie's view of how students learn best**.** Ellie described that students learned mathematics best when they are able to openly discuss their ideas and value the strategies of others:

> [Students learn mathematics best with] open discussions, critiquing ideas, thoughts, [and] sharing different ways they each see possible solutions to problems. Why? [It] allows everybody's voice to be heard. I always tell my students that there is never one way to solve a math problem and all ideas/strategies are welcome. I encourage them to use strategies they feel the most comfortable with. (Journal Entry, February 7, 2018)

Ellie emphasized that students learn best when they are able to share multiple strategies and ideas with each other, which was the same way she learned mathematics growing up (Journal Entry, February 7, 2018). By encouraging students to use strategies that they feel most comfortable with, Ellie explained that she wanted to establish a classroom environment where students could reason through and support their strategy of choice.

Ellie also explained that students learn best in a learning environment where their ideas are respected and can explain their thinking to each other without being afraid to make mistakes:

> I want the students to be able to justify their answers. I want to see their thought process, connections, misconceptions. This creates an environment where all thoughts are respected and students are not afraid to make mistakes in the process of learning. (Journal Entry, February 22, 2018)

In order to see students' ideas and misconceptions, Ellie emphasized that students need to be able to justify their ideas in a respected environment. Ellie indicated that students learn mathematics best when they are given the opportunity to demonstrate their understanding in a learning environment where students are allowed to engage in class discussions, share their ideas openly, have their ideas respected, and not be afraid to make mistakes in the process.

Student struggle. Ellie recognized that students struggled in mathematics because of their own beliefs and preconceived notions that the material was too difficult for them to be successful:

> The first thing I truly believe [why students struggle in mathematics] is their own beliefs. . . . A lot of kids [have] trouble with math because they've been told it's hard, parents have told them it's hard, teachers have told them it's hard, a lot. So you're going to this thing already defeated. You're going to this thing already knowing I'm not going to make it [and] shut down, and that's a problem. (Participant Selection Interview, January 10, 2018)

Having viewed this problem through different lenses (i.e., as a student, intern, student teacher, and teacher; Participant Selection Interview, January 10, 2018), Ellie explained that students approach mathematics already defeated after repeatedly being told that mathematics is hard. Ellie indicated that students struggle with their own beliefs after being told this and often give up before they are given the opportunity to succeed.

As a result, when students entered her fifth-grade class claiming that mathematics was their worst subject, Ellie described how she responded by telling them the opposite:

> I told them math is the best subject ever. In fact, I told them that math is the easiest subject, and I will tell them it's easy because it really is. So, I think that it's just the fear of it, lots of fear on what they've been told. (Participant Selection Interview, January 10, 2018)

Ellie explained her efforts to try and help students to understand that they should not fear mathematics by telling them that mathematics was easy and the best subject ever.

Ellie described her view of how students learn mathematics in her classroom including how students demonstrated their mathematical ability, made connections between contexts, and learned best. The way teachers view their instructional practices also plays an important role in supporting students in learning mathematics in these ways. The following section will discuss Ellie's view of her instructional practices including her view of how she provided feedback in her mathematics classroom.

Ellie's view of her instructional practices. Ellie described her role when working with all students as helping them to master the scope and sequence by a certain time and that achieving this was difficult for students with different levels of understanding (Participant Selection Interview, January 10, 2018). Ellie explained that her role with students who she described as low achieving was to fill in the gaps in their understanding through small-group interventions (Participant Selection Interview, January 10, 2018). With students who she described as high achieving, Ellie explained that her role was to emphasize fluency and help students make real-world connections (Participant Selection Interview, January 10, 2018). Ellie explained that she tried to focus on preparing all her students to be successful on the test:

> This is how I see it: we ultimately teach towards the test. We do not teach the tests. We don't have it. We don't see it. We teach towards the test, but we prepare them and equip them to be able to be successful. (Interview, February 5, 2018)

Ellie indicated that her role with all her students was to teach towards the test and in doing so, prepare them to be successful overall.

Review, question, and move forward. To help students who were struggling to move forward with their thinking during mathematics instruction, Ellie indicated that she would first review the important concepts with them and then allow them to receive extra help during intervention when someone would be able to work with them individually (Initial Interview, January 22, 2018). In addition, Ellie explained that she would also ask questions or model the problem in a different way to help struggling students to move forward with their thinking:

> I think in some way that area model is kind of hard for them. To me, I think a visual would be easier to see. . . . When modeling them, ask questions [like] “What are you thinking?”, “What do you see?”, “How do you want us to write it down?”, “If I write it this way, is this the right way to do it?”, or “Why do you think it’s not right?” (Initial Interview, January 22, 2018)

Ellie described how she would question students who appeared to be struggling while referencing a visual or model. Ellie indicated that she would question students in this way to “probe just to see where they are” (Initial Interview, January 22, 2018). If students continued to struggle, Ellie said she would respond by asking, “You really don’t see it? [You] don’t get it?”; commenting further “I see that with some skills because some kids will not [understand] it at all, some skills” (Initial Interview, January 22, 2018). Ellie described that when working with struggling students, she would review the material, ask questions, and move on. However, she emphasized that despite her efforts, some students will not be able to understand certain skills.

In addition to reviewing the material and questioning struggling students to help them move forward with their thinking, Ellie explained that sometimes she needed to move forward and revisit the material later:

> [My colleagues and I] try all kinds of interventions for these kids. We do peer tutoring. We re-teach. We sit by them and probe and lead them towards [the solution]. We give them calculators. Some of them might need manipulatives but still some do not get it despite all this. So, you think, well, maybe they're just not developmentally ready for this yet. Ok, move on, step aside, move on and when I spiral back, hopefully from the other exposure from all these other skills that we've learned, maybe they will help them pick that skill up. We do it all the time [but] we just don't really call it that. I just say, "Go on and we'll come back." (Interview, January 24, 2018).

Although Ellie and her colleagues provided struggling students with different interventions (i.e., peer tutoring, re-teach, probe, lead them towards the solution, give them calculators or manipulatives), Ellie described that sometimes she needed to move on to new material and revisit the material students were struggling with later.

Responding to students. In addition to reviewing important concepts, questioning students further, and moving on with the intent of revisiting the material later, Ellie indicated that she responded to students by "probing a little further, saying 'yes', 'Ok', 'not really' [and giving] check marks, tickets, etc." (Journal Entry, February 7, 2018).

Ellie explained that she responded to students in these ways because she wanted to "hear their thinking, misconceptions, encourage them, and motivate them" (Journal Entry, February 7, 2018). In addition, Ellie indicated that there were certain situations where she modified the way she responded or chose not to respond at all to students who need extra help for fear of losing the focus of the other students in the class (Interview, January 29, 2018). One way she modified the way she responded was by writing on students' papers, rather than responding out loud where other students may hear:

> [I] quickly write [my feedback] down, and I'll walk around to check it to see what everybody's doing. [The] reason I do that is because if it's something I really want to see, I'll give feedback about performance: that they're learning it. Are they getting it. If I call on names then it makes it easy for others to just quickly get the answer or it just stops other people's thinking because they know somebody will do it for [them]. (Final Interview, February 23, 2018)

Ellie explained that she observed how students were performing and quickly responded to students' work by writing feedback on their paper rather than out loud which may have disturbed other students' thinking. Furthermore, Ellie viewed the discourse between her and her students as intrinsic motivation to help students "want to keep going [and] keep building" (Interview, January 24, 2018).

Motivation. To motivate her students during mathematics instruction, Ellie described how she engaged students in discussions to help them make real-world connections:

> We talk a lot about problems, about thinking. I like to read exactly what they know, your real-life situation, time is money. Things like that. Just giving them the opportunity to do the why. Why is this what it is? We're not just "gonna [sic] solve this." What is the whole here? How many pieces am I getting out there? You know, just making them [make] that real connection. (Initial Interview, January 22, 2018)

Ellie explained that she motivated students by talking about and making real-world connections (i.e., time and money) during mathematics instruction. Ellie stated that she motivated students in this way because it was similar to the way she grew up learning mathematics where her teacher would say, "If you don't understand it that way, show me how you understand it" (Initial Interview, January 22, 2018).

Ellie also explained how she motivated students by making mathematics exciting, fun, and challenging so students wanted to keep building their knowledge (Interview, January 24, 2018). Ellie explained that motivation was good for students in her classroom; however, she claimed that motivation could be a hindrance depending on the kind of motivation:

> I think for the most part motivation is good for students. I think the only time that motivation would hinder is depending on what kind of motivation you're giving students. If you just want to give your kids candy for trying, for doing anything then they will only work for that. It's very extrinsic. It's not anything that is intrinsic for them, they do not want to work for it. But if you motivate them and

help them want to do it, then I think that's a kind of motivation that works. (Interview, January 24, 2018)

Although Ellie stated that motivation played an important role in her classroom, she explained that extrinsic motivation could be a hindrance without the teacher's support.

The previous sections described Ellie's view of how students learn mathematics and her instructional practices in her own classroom environment. The following section will describe the structure of Ellie's classroom which encompassed these ideas during the 11 days of classroom observations.

Classroom Structures

Ellie's fifth-grade classroom consisted of 24 students arranged in two groups of 10 and four individual desks between the groups (see Figure 29). Midway through data collection, the desks were arranged in a large U-shape facing an interactive whiteboard (see Figure 30). Mathematics instruction occurred in the morning for approximately 50 minutes prior to science instruction. Students worked individually in their assigned seats during instruction and would stand either at their seats or in two lines near the interactive whiteboard when playing a game led by the teacher. Additionally, students were occasionally assigned groups in which to work and would gather in various places around the classroom.

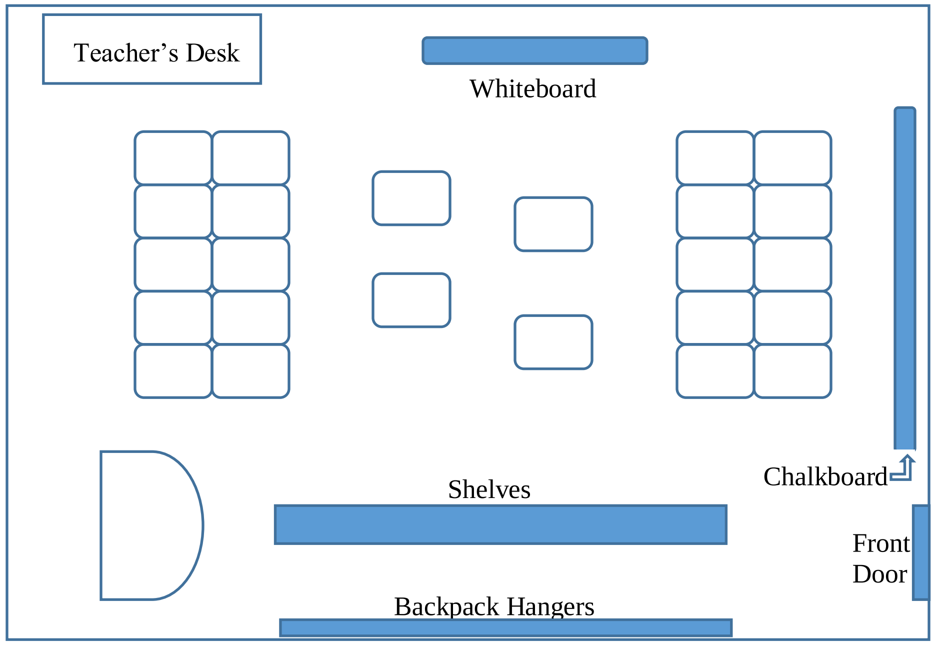

Figure 29. Fifth-grade classroom arrangement at the beginning of data collection.

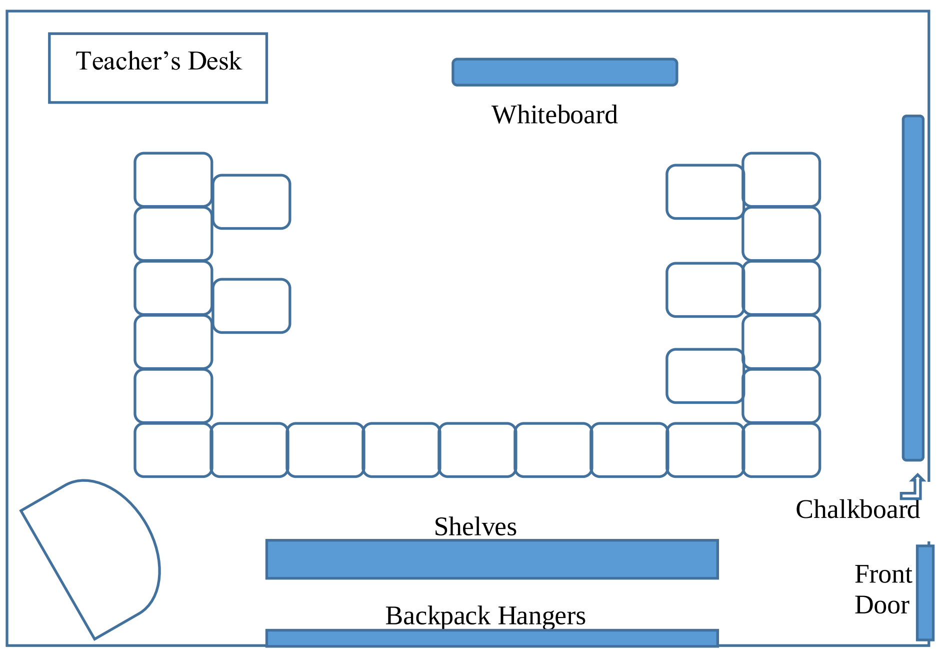

Figure 30. Fifth-grade classroom arrangement second half of the study.

Warm-up activities. At the beginning of each of the 11 days of classroom observations, Ellie engaged students in one or two introductory activities (see Table 10) as a review of previously learned material. Most introductory activities were unrelated to the current learning goal except for the ninth day of classroom observations when the Problem of the Day had a related question on it. This problem was categorized as Problem of the Day and not a primary instructional activity.

Table 10

Description of Introductory Activities in Ellie's Classroom

Activity	Description
Problem of the Day/ Daily Warm-Ups	Students working at their desks on problems printed on a worksheet or displayed on the interactive board. Students either write directly on the worksheet, on a separate sheet, on an index card handed out by Ellie, or on a small dry erase board.
Mathematics Game	Students either sitting at their desks or standing in two rows at the front of the class engaged in multiplication fluency practice.
Number Talk	Students working in groups or independently at their desks on a problem displayed on the interactive board. Ellie holds large-group discussions about each problem after small-group discussions and announces when the Number Talks begin.
Video	Students sitting at their desks or on the floor in front of the interactive board where the video is displayed. Either the whole video is played, or Ellie periodically pauses the video for a whole-class discussion.
Assessment Review	Students sitting at their desks while Ellie reviews problems on a previously assigned assessment.

Primary instructional activities. After the initial warm-up activity, Ellie either engaged students in an additional warm-up activity, introduced a new mathematical unit, or engaged students in a new mathematical task (see Table 11).

Table 11

Ellie's Daily Activities Including Learning Goal Per Day of Classroom Observations

Day	Warm-Up Activity	Learning Goals (LG) and Instructional Activities (IA)
1	Problem of the Day Mathematics Game	LG: Understand what a reciprocal is and how it works when dividing fractions IA: Vocabulary discussion, Dividing Fractions video, dry-erase board practice
2	Problem of the Day Number Talk Mathematics Game	LG: Use reciprocals to divide fractions IA: Review online test practice report, KFC bucket practice
3	Problem of the Day Number Talk Mathematics Game	LG: Use reciprocals to divide fractions IA: Direct instruction and workbook practice
4	Problem of the Day Number Talk Mathematics Game	LG: Identify the key parts of the coordinate plane IA: Direct instruction and quick world map exploration
5	Daily Warm-Up Exit Ticket Review Problem of the Day Video	LG: Identify and plot key points on the coordinate plane IA: Practice graphing points with a partner and Sailboat Graphing Worksheet
6	Problem of the Day Mathematics Game	LG: Identify and plot points on the coordinate plane IA: Levels of Thinking discussion for Problem of the Day, Video, Big Dipper group task
7	Problem of the Day Mathematics Game	LG: Identify the length, width, and height of regular shapes. Find volume then simplify. IA: Volume discussion and Sticky note/Venn diagram group activity
8	Problem of the Day Mathematics Game	LG: Identify the length, width, and height of regular shapes. Find volume then simplify. IA: Direct instruction, Finding the Volume of Boxes group activity, student presentations
9	Problem of the Day (Volume)	LG: Identify the length, width, and height of regular shapes. Find volume then simplify. IA: Direct instruction, individual volume practice, and Unifix cubes exploration

(continued)

Day	Warm-Up Activity	Learning Goals and Instructional Activities
10	Problem of the Day	LG: Identify the length, width, and height of regular shapes. Find volume then simplify. IA: Direct instruction, practice problems on interactive board, practice worksheet with partner, and small group of 4 students at front of room with Ellie
11	Problem of the Day	LG: Identify the length, width, and height of regular shapes. Find volume then simplify. IA: Direct instruction and dry-erase board practice

Throughout the classroom observations, Ellie covered three mathematical units including fraction division, the coordinate plane, and volume (see Table 11). The mathematical learning goals for each observed day were displayed on the whiteboard to the left of the interactive board at the front of the room. In her interview following the seventh day of classroom observations, Ellie explained that she did not deviate from the direction of her daily objectives:

> The reason is because there's a lot of observations that we get all the time from administration. We have that so, you kind of have anytime anybody walks in they have to ask the student "What [are] you learning?", "What's your goal?" They tell us stuff like that. If I come to your room, I should talk to any student about [what] you're going to do. What's the focus? What's the I-Can statements? Your kids should be able to know, so we kind of really, really stick to that. (Interview, February 12, 2018)

On two occasions, Ellie read the mathematical learning goals out loud in addition to having them displayed on the white board. The following section will discuss how Ellie responded to students during the daily activities.

How Ellie Responded to Students

During the mathematics instruction portion of the 11 classroom observations, Ellie responded to students in three ways (i.e., repeating or clarifying a student's response, writing down a procedure and posing additional questions, and asking the class if they understand) that did not answer the questions "Where am I going? How am I going? and Where to next?" (Hattie & Timperley, 2007, p. 102). By not addressing the three questions, the following examples of Ellie's responses to students were not categorized as feedback.

Repeating or clarifying student's response. One way Ellie responded to students was by repeating or clarifying a student's response to a question or comment:

> Ellie: Now let's look at the three values we have up there. We have five-two, right?
>
> Student: So, five and crawl.
>
> Ellie: *Crawl five.*
>
> Student: Walk up two.
>
> Ellie: *Walk up two.* (Classroom Observation, February 5, 2018)

In this example, Ellie demonstrated how to plot points on a coordinate grid using the example of the point (5,2). After stating the point five-two, Ellie clarified the student's first response (i.e., crawl five) and then repeated the student's second response (i.e., walk up two). Ellie repeated or clarified students' responses 93 times during the 11 classroom observations.

Write down procedure and pose additional questions. Another way Ellie responded to students was by writing down the procedure suggested by the student and then posing additional questions:

> Ellie: So, now we have [the equation] in division format. Now what do I keep? A?
>
> Student A: Keep the first one.
>
> Ellie (*writing down the first fraction*): *What do I do next?*
>
> Student A: Change the division to multiplication.
>
> Ellie (*writing the multiplication sign*): *What do I do next?*
>
> Student A: Switch the four and the two.
>
> Ellie (*switching the four and two in the second fraction*): *What's my answer?*
>
> Student A: Your answer would be eight and 64. (Classroom Observation, January 29, 2018)

In this instance, Ellie introduced a division problem with fractions and asked Student A questions about the steps to solve the problem. Student A continued to answer Ellie's questions while Ellie wrote what the student said on the whiteboard. Ellie did not indicate whether the student's reply was correct or incorrect, but rather wrote down the steps suggested by the student and continued to ask questions.

Asking the class if they understand. One additional way Ellie responded to students who did not answer the questions "Where am I going? How am I going? and Where to next?" (Hattie & Timperley, 2007, p. 102) was by asking the class if they understood another student's statement:

Ellie (holding up a cube made of Unifix cubes): Now, the counting part sometimes is tricky because you have to be able to kind of see what's going on back there. See how many layers, how many rows you have? Yes, how would you count?

Student (modeling how to count the Unifix cubes in the cube): You could go two, four, six, eight, 10 . . . 20, 22.

Ellie (directed at the rest of the students): *Do you get that? Did you get that?* (Classroom Observation, February 21, 2018)

In this instance, Ellie held up a cube made of Unifix cubes and asked a student to demonstrate how they would count the number of cubes. After the student modeled how they would count the Unifix cubes, Ellie validated the process in which the student was counting by asking the rest of the class if they understood what the student was saying.

Summary. From the previous examples, Ellie responded in three ways that did not answer the questions "Where am I going? How am I going? and Where to next?" (Hattie & Timperley, 2007, p. 102). Ellie responded to students by repeating or clarifying a student's reply, writing down students' replies while posing additional questions, and asking the class if they understood another student's statement. In addition to responding to students in the previous ways that did not answer the three questions, Ellie could also have responded to students by directing her response at the students themselves. This type of response was categorized as self-level feedback and did not answer the questions "Where am I going? How am I going? and Where to next?" (Hattie & Timperley, 2007,

p. 102). Self-level feedback will be described in the following section in addition to how Ellie provided feedback in this way.

Ellie's Self-level Feedback

Feedback at the self level is defined as feedback that is directed at the person themselves and is often isolated from a person's performance on a task (Hattie & Timperley, 2007). Throughout the 11 classroom observations, there was no evidence of where Ellie provided self-level feedback during mathematics instruction.

The previous sections discussed how Ellie responded to students who did not answer the three questions "Where am I going? How am I going? and Where to next?" (Hattie & Timperley, 2007, p. 102). It was also important to consider the ways in which Ellie provided feedback that did address these questions (i.e., feed up, feed back, and feed forward). The following sections will describe how Ellie provided feedback in these ways and evidence of how she directed each type of feedback at the task, process, and self-regulation levels.

How Ellie Provided Feedback

Ellie responded to students by providing feedback that answered the questions "Where am I going? How am I going? and Where to next?" (i.e., feed up, feed back, and feed forward, respectively; Hattie & Timperley, 2007, p. 102) The following sections will describe how Ellie provided feed up, feed back, and feed forward (see Table 12) to her students directed at the task, process, and self-regulation levels during the 11 days of classroom observations.

Table 12

Feedback Occurrences for Ellie Per Day of Classroom Observation

	Classroom Observation Day											Total
Feedback Type	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	
Feeding Up	1	1	0	0	0	0	0	0	0	0	0	2
Feeding Back	9	5	5	1	14	5	7	13	9	10	10	88
Feeding Forward	0	0	1	0	0	0	0	0	0	0	0	1

Feeding up. Feed up describes the information given to students about what the learning goal is and what the result looks like once they arrive at the goal. During the 11 classroom observation days, Ellie provided feed up one time at the task level and one time at the process level within the first two days (see Table 12). There was no evidence found during the 11 observed classes where Ellie provided feed up at the self-regulation level. The instances in which Ellie provided feed up in these ways will be described in the following sections.

Task level. Feed up at the task level answers the question "Where am I going?" (Hattie & Timperley, 207, p. 88) by referring to the goal of the task and the surface information needed to successfully complete the task (Hattie & Clarke, 2019). Ellie provided feed up at the task level one time on the second classroom observation day by referring to the learning objective or the steps required to complete the task.

On the second day of classroom observations, Ellie asked students in pairs to create four original computational problems involving division with fractions on notecards. When finished, students were to put their notecards in a bucket and exchange buckets with a different pair for them to solve the problems. Once the students solved all four problems, students exchanged the buckets with another group to evaluate the correctness of the division problems. As students exchanged buckets after solving the four problems, Ellie responded to a student who asked a question regarding the format of the solutions:

> I'm not looking at simplifying or changing it into a mixed number yet. Now *I'm just looking that we can keep the first fraction, we can flip the last one, and we can change the sign and actually multiply correctly* because you need to be able to multiply correctly. *That's really what I'm looking for today*. (Classroom Observation, January 30, 2018)

In this instance, Ellie repeated the procedure for dividing fractions which was embedded within the learning goal of using reciprocals to divide fractions. Ellie provided feed up at the task level by referring to the procedure students needed to follow to successfully complete the task.

In addition to providing feed up directed at the task level, Ellie also provided feed up directed at the processes with which students need to engage to reach the learning goal. The following section will discuss how Ellie provided feed up in this way.

Process level. Feed up at the process level addresses the question "Where am I going?" (Hattie & Timperley, 2007, p. 88) by focusing on the learning goal and the processes used to move towards that goal. Ellie provided feed up by focusing on the definition component of the process for finding the reciprocal of a fraction.

On the first day of classroom observations, Ellie provided feed up at the process level by referring to the definition of a reciprocal which was one part of the learning goal that day. At this time, Ellie showed a video on the interactive board about reciprocals and fraction division and paused the video to have a discussion with the whole class. After asking several students for the definition of a reciprocal, one student replied:

> Student: It's when you have a fraction, and you switch the bottom and the top and it gives you [the reciprocal].
>
> Ellie: In other words, what do we do to our fractions?
>
> Student: Swap, switch.
>
> Ellie (making a switching motion up and down with her arms): *We switch, we swap, or we flip, right? Now, that is what a reciprocal is.* (Classroom Observation, January 29, 2018)

In this instance, Ellie responded to the student by indicating that the student's explanation was the definition of a reciprocal. Ellie provided feed up at the process level by clarifying the definition of a reciprocal which was the first part of the learning goal.

Although Ellie provided feed up at the task and process levels by answering the question "Where am I going?" (Hattie & Timperley, 2007, p. 89), there were no instances where Ellie provided feed up at the self-regulation level. Ellie also provided feed back which indicated to the students how they were currently doing relative to the learning goal. The following section will describe feed back and how Ellie provided feedback in this way.

Feeding Back. Feed back describes the information given to students related to their performance when completing a task. This type of feedback addresses the question "How am I going?" (Hattie & Timperley, 2007, p. 88), informing students about their current progress and/or how to continue on the correct path towards the learning goal. During the 11 classroom observation days, Ellie provided feed back a total of 88 times (see Table 12). The ways in which Ellie provided feed back at the task, process, and self-regulation levels will be described in the following section.

Task level. Feed back at the task level answers the question "How am I going" (Hattie & Timperley, 2007, p. 88), including how successful the student is at completing the task and/or whether the task is correct or incorrect. Ellie provided feed back at the task level 63 times during the 11 classroom observations by agreeing with the student and then following that with why the student's reply was correct, or disagreeing and then following that with why the student's reply was incorrect. Ellie also provided task-level feed back by indicating why the student's work was correct or incorrect or emphasizing that they were doing a good job. The following section will discuss four instances of how Ellie provided feed back at the task level in these ways.

Agreeing with the student's statement and explaining why they were correct. One instance of how Ellie provided feed back at the task level was on the ninth day of classroom observations. Students sat at their desks working individually on a worksheet where students were asked to find the volume of a three-dimensional object when given the area of its base and its height. Ellie walked around the room answering questions and collecting worksheets from students who finished. After asking a row of students if they were done, one student asked Ellie a question:

Student: I have [this]. So, when we're doing this, do we just multiply the numbers?

Ellie (nodding up and down): *Uh huh because that's the area. That's your length times width already done for you.* (Classroom Observation, February 20, 2018)

In this instance, Ellie agreed that the student needed to multiply the two numbers and then explained that the area of the base had already been computed for them. Ellie provided feed back at the task level by agreeing with the student's statement and then explaining why the student's statement was correct.

Disagreeing with the student's statement and explaining why they were incorrect. A second instance where Ellie provided feed back at the task level was on the eighth day of classroom observations. After reviewing area and volume with the class, Ellie had students work in groups of four either at their desks or on the floor to find the volume of different-sized boxes. Using a ruler, students measured the side lengths of the boxes, calculated the volume, and recorded their results on large graph paper. As Ellie walked around the room observing the students, she stopped at a group at the front of the room trying to calculate the volume of a box in the shape of a cube:

Ellie: A cube is all the sides are the same. So, now we're going to do seven times seven times seven.

Student: Looks like I picked an easy one.

Ellie (looking on the group's paper): *No, it's not 21*.

Student: It's seven times seven times seven.

Ellie: *It's not seven times three*.

Student: Oh.

Ellie: *Seven times itself. Then 49 times seven.* (Classroom Observation, February 13, 2018)

In this instance, Ellie reviewed the procedure for finding the volume of the cube with a side length of seven. After looking at the group's work, Ellie responded that the solution was not 21. She then proceeded to explain why seven times seven times seven was not seven times three but rather seven times itself three times. Ellie provided task-level feed back by disagreeing with the student's solution and explaining why their solution was incorrect.

Indicating the correctness of the student's solution and explaining how to fix the mistake. A third instance where Ellie provided task-level feed back was on the last day of classroom observations. The students sat at their desks or on the carpet in front of the interactive board which displayed a three-dimensional composite figure (see Figure 31).

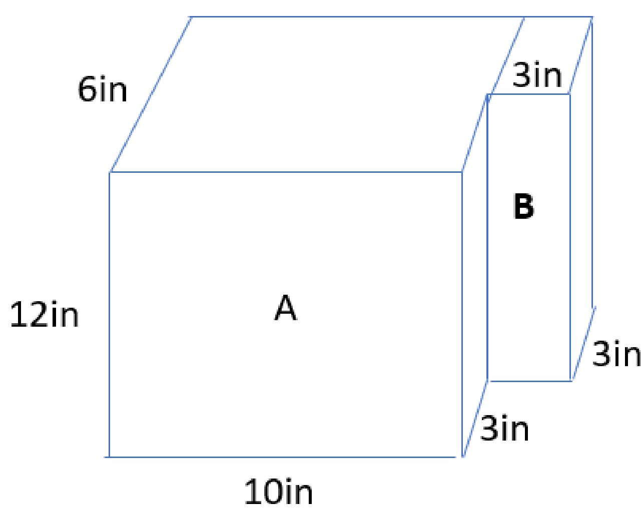

Figure 31. Example of a three-dimensional composite figure displayed on the interactive board.

Ellie instructed the students to find the volume of both A and B and display their solutions on individual whiteboards with dry-erase markers. As Ellie walked around the carpet observing the work on their whiteboards, she responded to a student seated on the carpet:

> *B is correct, but fix A. Eight times six is what? You only need three. You have six times three times 10 times six. You only need three measurements*. (Classroom Observation, February 22, 2018)

In this instance, Ellie responded to the student's work by indicating that the volume for figure B was correct but the volume for figure A was not. Ellie provided feed back at the task level by indicating the correctness of the student's solution, explaining why the solution for figure A was incorrect, and describing how the student could fix their mistake.

Emphasizing that the student did a good job. The last instance where Ellie provided feed back at the task level was on the sixth day of classroom observations. Students worked in groups on the floor plotting points on large graph paper that resulted in a graph of the Big Dipper. After announcing to the class that one group had already finished, Ellie leaned over to observe the work of a group at the front of the room:

> Quickly. You don't have to write all that, just write real quickly. Label all your stars. *Good job, very nice. And then connect the last ones. Yeah, perfect*. Label all your stars, good job! (Classroom Observation, February 8, 2018)

In this instance, Ellie responded to the student making the final edits to their group's chart paper by indicating that they needed to quickly label the stars on their graph and connect the last points. Ellie provided task-level feed back by emphasizing that the work displayed was very nice and the group did a good job relative to the learning goal of plotting points on a coordinate plane.

Although Ellie provided feed back about the task to help students know how they were doing in relation to the current learning goal, she also provided feed back aimed at the processes needed to successfully complete the task. The following section will discuss four instances of how Ellie provided feed back in this way.

Process level. Feed back at the process level answers the question "How am I going" (Hattie & Timperley, 2007, p. 88) in relation to the processes, strategies, thinking, and skills involved in a task (Brooks et al., 2019). Ellie provided feed back at the process level 23 times during mathematics instruction on the 11 observed days by indicating the correctness of a student's process when working through a task and the effectiveness of a student's strategy. Additionally, Ellie provided feed back at the process level by indicating that the student should choose the most efficient strategy or the easiest strategy that worked best for them. The following section will discuss four instances of how Ellie provided feed back at the process level.

Indicating the correctness of a student's process. On the second day of classroom observations, students worked in pairs to create four computational problems involving division with fractions on notecards, exchange the problems with a different pair of students, and then solve the problems. Ellie walked around the classroom answering questions and reminding students of the procedure for dividing fractions. While circulating the room, Ellie stopped at a pair of students seated next to each other at their desks. After acknowledging that one student in the pair solved the problem correctly, she turned to the second student to address her question:

> Student: I went to the bottom right, but my denominators are not the same, so?
>
> Ellie: *No, this way [is] dividing fractions. It doesn't, we don't have to make our denominators the same.* All we need to think about is keep, flip. Keep this fraction the way it is (pointing to the student's paper), that one, and then change the sign because we are dividing fractions. (Classroom Observation, January 30, 2018)

In this instance, Ellie indicated that the strategy the student was using to divide fractions was incorrect and explained the correct process. Ellie provided feed back at the process level by indicating the correctness of a student's process when working through a task and then by explaining how the student could proceed to successfully complete the task.

Describing the effectiveness of a student's strategy. A second instance where Ellie provided feed back at the process level was on the sixth day of classroom observations. Students sat on the floor working in groups of four, five, or six students plotting the coordinates of a constellation on large graph paper. Ellie walked around the room answering questions and supervising the groups' progress. As students started discussing how they planned on first labeling the axes of the graph, a group of six students seated on the floor in front of the interactive board caught the attention of Ellie who walked over to address the group:

> Student A (out loud to her group): It seems like the highest it goes is (inaudible).
>
> Student B: What is she doing?

Ellie (walking up to the group and leaning down to speak): You know what I'm hearing? The highest number it goes to. *That's a good strategy. Figure out which is the highest number, so you'll be able to subdivide it.* (Classroom Observation, February 8, 2018)

In this instance, Ellie responded to student A's comment by indicating that the student had a good strategy for determining how to label the axes on the graph. Ellie then continued by explaining how the group could use the highest number. Ellie provided feed back at the process level by indicating the effectiveness of student A's strategy for labeling the axes of a graph, addressing how the student was doing relative to successfully completing the task of plotting points on the coordinate plane.

Indicating that a student should choose the most efficient strategy. A third instance where Ellie provided feed back at the process level was on the eighth day of classroom observations. After reviewing area and volume with the class, Ellie instructed students to work in groups of four either at their desks or on the floor to find the volume of different-sized boxes. Using a ruler, students measured the side lengths of the boxes, calculated the volume, and recorded their results on large graph paper. While walking around the room observing the groups, Ellie stopped to answer a student's question:

Student (pointing to the edges of the box): If this is your height, can that be your height?

Ellie: *If you want it to be, just depends on how you want your box to sit.* Do you want your box to sit down this way (laying the box flat on the floor), or do you want your box up this way (standing the box up on its side)?

Student: But aren't you, that's the width and then this is the?

Ellie: *Just know you have to get used to seeing those three different dimensions. Don't do the same one two times*. (Classroom Observation, February 13, 2018)

In this instance, Ellie responded to the student by indicating that the student could choose which side he wanted to represent the height depending on how he wanted the box to sit. Ellie provided feed back at the process level by indicating that the student should choose the most efficient strategy that was best for them with the understanding that they needed to be careful not to count the same side twice.

Indicating that a student should choose the easiest strategy. The last instance where Ellie provided feed back at the process level was also on the eighth day of classroom observations. As students continued to measure the side lengths of different boxes, calculate their volume, and record their work on chart paper, several students asked questions about how to record measurements that were not whole inches (e.g., 3.5 inches or 6.75 inches). Ellie stopped by one group to answer a student asking this question:

Ellie: Approximate it to the whole number which is (picks up the ruler and measures the box). Do you want it to be 22 or 21?

Student: Maybe 22?

Ellie: *So, make it 22 then make that one 10 and make that one three because that makes it easy to multiply, Ok?* (Classroom Observation, February 13, 2018)

In this instance, Ellie responded to the student's question by telling them to approximate the measurements to the nearest whole number (i.e., 22 or 21). Ellie provided feed back at the process level by indicating that the student could choose the easiest strategy that worked best for them. Ellie further explained that the student could also round the other two side lengths (i.e., 10 and three) to make it easier to multiply when finding the volume of three-dimensional figures.

Ellie provided feed back directed at the processes involved to help students know how they were doing in relation to the current learning goal. Ellie also provided feed back aimed at the way students could self-regulate their actions to successfully complete the task. The following section will discuss two instances of how Ellie provided feed back at the self-regulation level.

Self-regulation level. Feed back at the self-regulation level addresses how students are doing in relation to the ways students "compare and adjust their work in relation to the required standards, criteria, or intent" (Brooks et al., 2019, p. 6). Ellie provided feed back at the self-regulation level two times during mathematics instruction by indicating how students could regulate their actions by trusting in themselves and by explaining how students could identify and correct their mistakes to successfully complete a task. The following section will discuss one instance of how Ellie provided feed back in this way.

Indicating how students could regulate their actions by trusting in themselves. One instance where Ellie provided feed back at the self-regulation level was during mathematics instruction on the fifth day of classroom observations. At this time, students worked independently on the floor or at their desks on a worksheet instructing them to plot points on a coordinate grid and connect the points to create a picture of a sailboat. While circulating the room, Ellie approached a student sitting in his desk at the back of the room who had his hand raised. As Ellie started talking to him about the worksheet, she picked up the student's pencil and started writing directly on his paper:

> Ellie (pointing with the pencil after marking the student's paper): Ok, that's your dot.
>
> Student: Yeah? Oh yeah, that's 18.
>
> Ellie (Drawing a line on the student's paper): And then 24, 18, then it goes up.
>
> Student: Yeah, yeah. I thought there was an 18, four, seven.
>
> Ellie: Then you have 23, 19 so you connect that (nodding as the student drew on his paper). *And just trust yourself, keep going, keep going. As long as you're going in order*. (Classroom Observation, February 6, 2018)

In this instance, Ellie drew on the student's paper while walking him through the steps on how to connect two points on the worksheet. Once the student connected the points on his own, Ellie encouraged him to keep going in order and to trust himself. Ellie provided feed back at the self-regulation level by nodding as the student continued to work on the task, indicating to the student how he was progressing. Ellie then addressed how the student could regulate his actions (i.e., trust yourself and keep going in order) as he moved towards successfully completing the task.

In her first journal entry, Ellie said that the reason she felt students were struggling with the sailboat worksheet was because they "were not sure they were doing the right thing" (Journal Entry, February 7, 2018). Ellie continued to explain why she chose to respond to students by telling them to trust what they were doing:

> I think because this task requires a long process before the final outcome. . . . The last picture that shows up is the sailboat and I think that they wanted to make this out right away. Waves, birds, [and] the sun show up first. They want to see the outcome before the process. (Journal Entry, February 7, 2018)

Ellie explained that the reason she responded to students by telling them to keep going and to trust themselves was because she wanted students to commit to the task even when they did not know the end result.

Explaining how students could identify and correct their mistakes. A second instance where Ellie provided self-regulation level feed back was also on the fifth day of classroom observations while students continued to work independently on the floor or at their desks on a worksheet instructing them to plot points on a coordinate grid and connect the points to create a picture of a sailboat. As Ellie walked around the room answering students' questions, she stopped to respond to a student who asked why the point (24,17) appeared twice:

> Ellie (pointing to the student's paper): Because from the previous [point], you will go back to it. . . . The easy thing to do is every time you plot [a point], you join them.
>
> Student: Do I have to decide? Because I did it this way?
>
> Ellie: *No, you're fine. Just keep going down [and] ignore this for right now.* You're going all the way down (pointing to the column of points on the

> student's paper) and then will come up there. *Then, when you come here, you will realize where you went wrong and [will] be able to fix it.* (Classroom Observation, February 6, 2018)

In this instance, Ellie responded to the student by explaining why the point appeared twice, that the easiest thing to do was to plot a point and join the point to the previous point, and that the student should keep going rather than start over. Ellie provided feed back at the self-regulation level by indicating how the student could regulate their actions by continuing to plot points as directed. Ellie indicated that as a result, the student would be able to identify and correct her mistakes to successfully complete the task.

The previous section described how Ellie provided feed back during mathematics instruction that indicated to the students how they were doing relative to the learning goal. To help students to continue moving forward, they must also know where they need to go next in the learning progression. This type of feedback is described as feed forward. The following section will discuss how Ellie provided feed forward (i.e., where to next) in this way.

Feeding forward. Feed forward describes the information given to students that requires students to apply the feedback they previously received (Brooks et al., 2019). This type of feedback addresses the question "Where to next?" (Hattie & Timperley, 2007, p. 90) leading to greater opportunities for learning. During the 11 classroom observation days, Ellie provided feed forward at the process level one time during mathematics instruction on the 11 observation days by connecting a previously learned process to the current learning goal (see Table 12). There was no evidence found during the 11 observed classes where Ellie provided feed forward at the task or the self-regulation levels.

Ellie provided feed forward on the third day of classroom observations. Ellie reviewed how to divide fractions on the interactive board with the whole class and after asking the class what would happen if a fraction was divided by a whole number and working the problem for the class, one student sitting on the carpet in front of the interactive board reflected on the process:

Student: That's easy.

Ellie (nodding her head): That's very simple.

Student: It's the same way as multiplying.

Ellie (nodding her head): *As long as you know how to multiply whole numbers with fractions and you know the rules that apply to that, then you can transfer it here. As long as you know to keep your first number, flip the last one, and then change the sign.* (Classroom Observation, January 31, 2018)

In this instance, the student seated on the carpet commented that fraction division was similar to multiplication. In response, Ellie nodded her head and explained how the fraction multiplication rules could be transferred to fraction division. Ellie provided feed forward at the process level by connecting the previously learned process for multiplying whole numbers with fractions to help students to think further about dividing whole numbers and fractions. Although Ellie directed the feedback at the student seated on the carpet, she said it out loud in front of the students seated at their desks allowing them to benefit from the feedback as well.

The previous sections described how Ellie responded to students by providing feedback that answered the questions “Where am I going? How am I going? and Where to next?” (Hattie & Timperley, 2007, p. 102). These questions addressed three types of feedback: feed up, feed back, and feed forward respectively. Evidence of how Ellie provided feedback at the task level, process level, and self-regulation levels within each type of feedback was also discussed.

Case Summary

The previous sections described the case of Ellie who held a weak incremental theory on the implicit theory continuum (see Table 9) with an overall average of 4.60 out of 6.00. First, Ellie’s learning experience and her view of how students learn mathematics and her own instructional practices were examined. Second, a description of Ellie’s classroom structure was described including the warm-up and primary instructional activities during the observed mathematics instruction. Third, a description of how Ellie responded to students who did not answer the questions “Where am I going? How am I going? and Where to next? (Hattie & Timperley, 2007, p. 102), was provided. Fourth, a description of self-level feedback was provided; however, there was no evidence of where Ellie provided self-level feedback during the observed mathematics instruction. Finally, the ways in which Ellie provided feedback during mathematics instruction were described with instances categorized by type (i.e., feed up, feed back, and feed forward) and by the level in which they were directed (i.e., task, process, and self-regulation).

Cross-Case Analysis

In the previous sections, I described the feedback practices of two individual cases: Ian who ascribed to a strong incremental theory and Ellie who ascribed to a weak incremental theory. In the following sections, I will share the results of the cross-case analysis to show any patterns, similarities, or differences across the two cases (Yin, 2014). First, I will briefly review the learning experience and implicit theory for both participants. Second, I will describe the ways in which each participant provided self-level feedback. Finally, I will compare the ways in which each participant provided feedback that answered the questions "Where am I going? How am I going? and Where to next?" (i.e., feed up, feed back, and feed forward; Hattie & Timperley, 2007, p. 102) directed at the task, process, and self-regulation levels.

Learning Experience and Implicit Theory

Based on their descriptions, Ian and Ellie grew up with different learning experiences, where Ian was taught to memorize formulas and Ellie was taught to problem solve and share ideas openly in a verbal environment (see Table 13).

Table 13

Learning Experience and Implicit Theory Cross-Case Analysis

Variable	Ian	Ellie
Location growing up	United States	Kenya, Africa
Learning experience	Memorize formulas, rewarded for correct answers, fear of being incorrect	Problem-solving community, able to voice her questions, problems explained in a variety of ways
Implicit theory	5.75 Strong incremental	4.59 Weak incremental

Both participants were classified as incremental theorists (see Figure 2) based on their overall average choices on the Implicit Theories Survey (see Appendix A). However, Ian ascribed to a strong incremental theory while Ellie ascribed to a weak incremental theory. Both participants' choices averaged the lowest in the world measure with Ian's choices averaging a 5.0 out of 6.0 and Ellie's choices averaging a 3.7 out of 6.0 within the world measure. Of the 12 questions in the survey, Ian chose 6.0 (strongly disagree/strong incremental theory) for nine questions, whereas Ellie chose 6.0 for zero questions.

Self-Level Feedback

During the observed mathematics instruction, Ian provided self-level feedback 23 times by praising students for their intelligence, bravery, confidence, thinking, and actions. In his interviews, Ian explained that students learn best when they feel safe to fail, confident in the process rather than the product, and brave enough to make mistakes. Ian also indicated that students could show growth in their understanding by demonstrating their thinking and explained that he tried to push his students out of a fixed mindset by utilizing The Pit to motivate students in his classroom. His responses in the interviews provided a basis for why Ian provided self-level feedback in these ways. There was no evidence of where Ellie provided self-level feedback during the observed classes.

Feedback

During the 11 days of classroom observations, both participants responded to students by answering the questions "Where am I going? How am I going? and Where to next?" (i.e., feed up, feed back, and feed forward; Hattie & Timperley, 2007, p. 102). In the following sections I will provide the results of the cross-cases analysis describing how Ian and Ellie provided feedback, first by type and then by level.

Feedback by type. A graphical representation of the daily feedback occurrences by type for both Ian and Ellie is provided in Figure 32. This graphical representation is a combination of the data from Ian's daily feedback occurrences (see Table 8) and Ellie's daily feedback occurrences (see Table 12) by type.

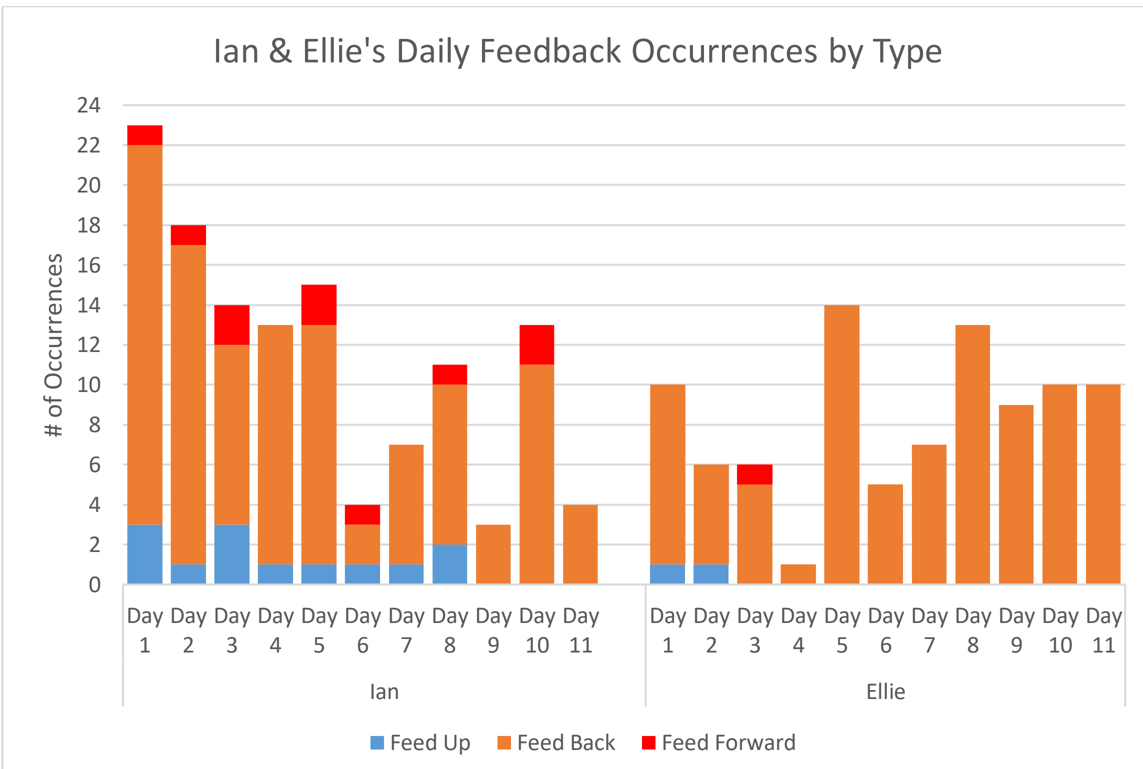

Figure 32. Ian and Ellie's daily feedback occurrences by type.

Ian provided feedback a total of 125 times with the most instances of feedback occurring on the first observed day and the least instances of feedback on the ninth observed day. Ellie provided feedback a total of 91 times with the most instances of feedback occurring on the fifth observed day and the least instances occurring on the fourth observed day (see Figure 32). Ian provided feed up at least once on each of the first eight observed days, while Ellie provided feed up once on each of the first two observed days. Ian provided feed forward on seven of the observed days, while Ellie provided feed forward only once on the third day of observations. Both participants provided feed back a similar number of times.

Additionally, the results from the cross-case analysis showed that Ellie had no instances where she provided all three types of feedback (i.e., feed up, feed back, and feed forward) in one day. However, as shown in Figure 32, Ian provided all three types of feedback on six of the 11 observed days. Considering that both Ian and Ellie ascribed to an incremental theory, and only one participant provided feedback in this way on multiple days, I conducted further examination of Ian’s feedback on the days where he provided all three types of feedback. Table 14 shows the levels that Ian directed each of the three types of feedback on the six days.

Table 14

Breakdown of Ian's Feedback Occurrences When All Three Types Were Provided

	Type		
Level by Day	Feed Up	Feed Back	Feed Forward
Day 1			
Task	2	10	0
Process	**1**	**6**	**0**
Self-Regulation	**0**	**3**	**1**
Day 2			
Task	0	11	0
Process	**1**	**2**	**1**
Self-Regulation	**0**	**2**	**0**
Day 3			
Task	1	7	1
Process	**1**	**1**	**0**
Self-Regulation	**1**	**0**	**1**
Day 5			
Task	0	5	0
Process	**0**	**3**	**0**
Self-Regulation	**1**	**4**	**2**
Day 6			
Task	0	0	0
Process	**0**	**2**	**0**
Self-Regulation	**1**	**0**	**1**
Day 8			
Task	0	1	0
Process	**2**	**3**	**0**
Self-Regulation	**0**	**3**	**1**

Note. The numbers in bold represent where Ian provided either process or self-regulation level feedback within feed up, feed back, and feed forward.

The breakdown of Ian’s feedback occurrences on the six days where he provided feedback within all three types reveals that on these six days, Ian directed his feed up, feed back, and feed forward at least one time at either the process or self-regulation levels. This was not the case with task-level feedback even though Ian provided feed back at the task level numerous times on five of the days.

Feedback occurrences by level. Both Ian and Ellie provided feed back a significant number of times more than both feed up and feed forward (see Figure 33).

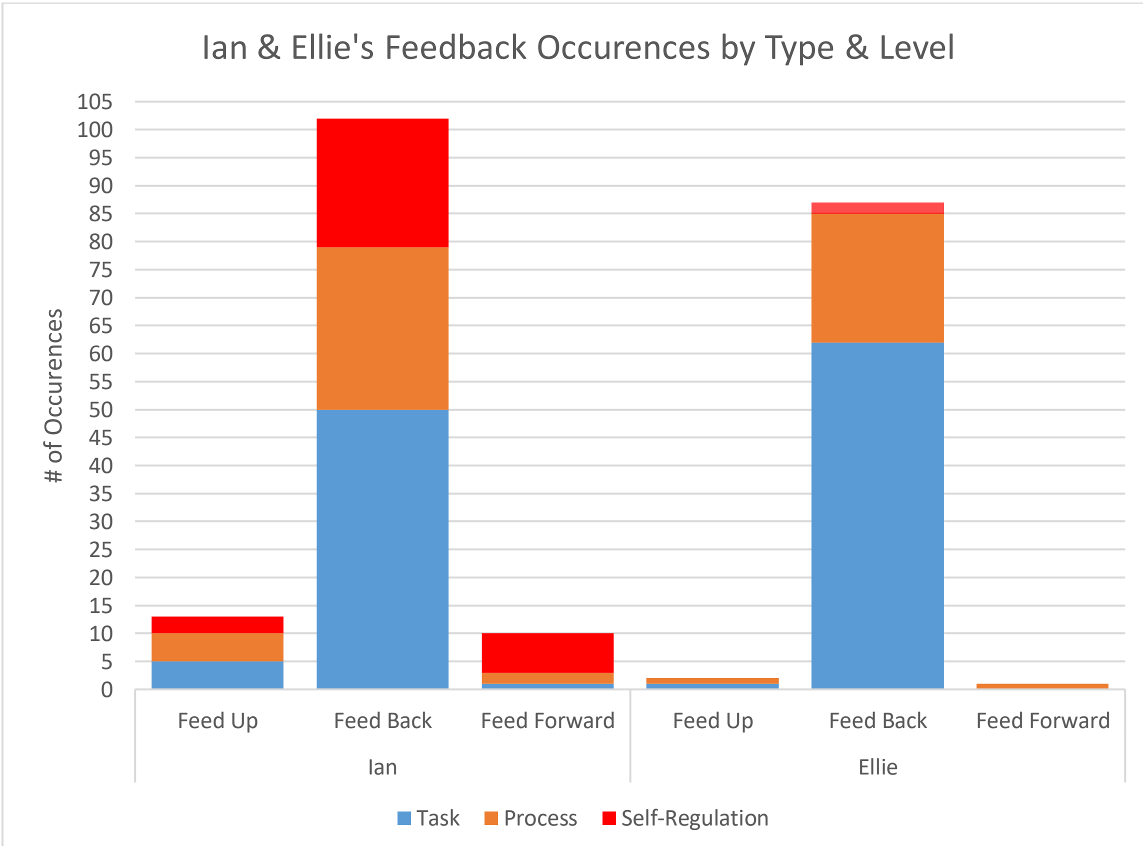

Figure 33. Ian and Ellie's Feedback Occurrences by Type and Level

From Figure 33 one can see that Ian provided more self-regulation feedback within all three types, compared to Ellie who only provided feedback at the self-regulation level within the feed back type. Within the feed back type, Ian also provided a significant amount of self-regulation level feedback more than Ellie. Additionally, Ellie provided more feed back at the task level than Ian. Ellie only directed her feed back at each of the levels (i.e., task, process, and self-regulation); however, she only provided feed up twice, once at the task and process levels, and feed forward once at the process level. Ian provided feed forward directed at all three levels with the most occurrences at the self-regulation level.

Ian provided more feedback than Ellie of each type and level (see Table 15), except for feed back at the task level (Ian 50 times and Ellie 63 times). Additionally, Ian directed his feedback at the self-regulation level a total of 33 times, whereas Ellie provided feedback at the self-regulation level a total of two times. It is important to note that Ian provided an equivalent amount of feed up directed at the task, process, and self-regulation levels; however, he provided more feed forward at the self-regulation level than at the task or process levels.

Table 15

Number of Feedback Occurrences for Ian and Ellie by Type and Level

Type & Level	Ian	Ellie
Feed Up		
Task Level	5	1
Process Level	5	1
Self-regulation Level	3	0
Feed Back		
Task Level	50	63
Process Level	29	23
Self-regulation Level	23	2
Feed Forward		
Task Level	1	0
Process Level	2	1
Self-regulation Level	7	0

Chapter Summary

In this chapter, I described the results from interviews and classroom observations of Ian who ascribed to a strong incremental theory and Ellie who ascribed to a weak incremental theory. First, I briefly reviewed the learning experience and implicit theory of both Ian and Ellie. Second, I provided a description of the ways in which each participant provided self-level feedback. Last, I compared the ways in which each participant provided feedback that answered the questions “Where am I going? How am I going? and Where to next?” (Hattie & Timperley, 2007, p. 102), first by type and then by level. In Chapter V, I will provide a summary and discussion of these results, including connections to prior research, theoretical and practical implications, and recommendations for future research.

CHAPTER V: SUMMARY AND DISCUSSION

Introduction

To be successful in the mathematics classroom, students must be given opportunities to engage with meaningful mathematics and feedback to help them move forward in their learning and understanding (Brooks et al., 2019; Hattie & Timperley, 2007; Tariq et al., 2013). Feedback, defined as the information conveyed to learners about their actions (Hattie & Timperley, 2007; Shute, 2008), is intended to make a connection between what students understand and what is meant for them to understand (Sadler, 1989). This information educates students as to how they are doing relative to the learning goals of the lesson (Hattie & Timperley, 2007) and how they might modify their work to reach these goals (Jung et al., 2015). Feedback is essential for helping students move forward in their learning (Brooks et al., 2019), and the beliefs teachers hold could potentially affect the way they provide information to their students (Rattan et al., 2012). Thus, additional studies are necessary to help teachers to be aware of their individual differences which may contribute to providing feedback and how they can use this knowledge to provide effective feedback in their classrooms (See et al., 2016).

The purpose of this study was to examine how elementary mathematics teachers provide feedback. To this end, the research question was: In what ways do elementary teachers provide feedback during mathematics instruction? In this final chapter, I will include a brief description of the research problem and a review of the methodology used throughout the study. This will be followed by a discussion of the results including connections to prior research, theoretical and practical implications, and recommendations for future research.

The Research Problem

Although there are many factors that contribute to students learning mathematics (NCTM, 1989, 2000, 2014; NRC, 2001), teacher feedback has been established as one of the most important influences on student achievement (Hattie & Timperley, 2007; Hattie & Yates, 2014; Hubacz, 2013; Wisniewski et al., 2020). By providing feedback, teachers can assess students' understandings and address discrepancies in the learning process (Brooks et al., 2019; Hattie & Timperley, 2007). However, the ways in which teachers provide feedback during mathematics instruction (Brooks et al., 2019) and their own implicit theories (Rattan et al., 2012) are often overlooked as contributors to the various types of feedback they provide. As a result, I utilized a multiple case study to explore how elementary teachers provided feedback during mathematics instruction. The methodology of my study will be described in the next section.

Review of Methodology

This study utilized a multiple case design (Yin, 2014) exploring two elementary teachers' feedback practices during mathematics instruction. The two participants consisted of Ian Smith, a third-grade teacher in his late twenties who ascribed to a strong incremental theory, and Ellie Jones, a fifth-grade teacher in her mid-forties who ascribed to a weak incremental theory. I gathered data from multiple sources to fully describe each case: an implicit theories survey, observational protocol, audio recordings, video recordings, semi-structured interviews, and participant reflective journals. I interviewed the participants individually following each classroom observation, and all data were transcribed and coded for analysis based on the analytical framework of Hattie and Timperley's (2007) characterizations of feedback types and levels. I conducted a cross-case analysis of the two participants to determine any patterns, similarities, or differences across the cases (Yin, 2014). The following sections contain a discussion of the findings that arose from this research. The discussion will include connections to prior research, theoretical and practical implications, and recommendations for future research.

Discussion of the Findings

The results in Chapter IV demonstrated how two elementary teachers provided feedback during mathematics instruction. For this discussion, I will organize the analysis of the data according to my three key findings. First, although both Ian and Ellie were identified as holding an incremental theory, there were varying commitments to providing self-level feedback during the observed mathematics instruction. Second, while both participants provided all three types of feedback (i.e., feed up, feed back, and feed forward) throughout the 11 days of classroom observations, Ian provided all three types of feedback within one classroom observation on multiple days. Last, feedback at the process and self-regulation levels are necessary for helping students to move towards the Goal of Mathematical Proficiency (NRC, 2001); however, the data showed that Ian and Ellie provided little to no feedback in these ways.

Self-Level Feedback

Although Ian and Ellie were classified as incremental theorists (see Table 13) based on their overall average choices on the Implicit Theories Survey (see Appendix A), they demonstrated varying commitments to providing self-level feedback (i.e., praise) during mathematics instruction. Ian, who ascribed to a strong incremental theory, provided self-level feedback directed at the person by praising students for their intelligence, bravery, confidence, and actions, and at the process by praising students for their thinking. He explained that by providing feedback in these ways, he wanted to instill self-confidence and a growth mindset in his students to build a safe and comfortable classroom community of learners. Ian’s actions aligned with Hattie and Yates’s (2014) idea that some praise could be valuable for establishing relationships within the classroom. These relationships are particularly essential for teachers who ascribe to an incremental theory for building a supportive learning community (Willingham, 2016).

However, Hattie and Yates (2014) also noted that praise could become problematic when it fails to provide information which helps students move forward in their thinking. Many studies have shown that self-level feedback can be detrimental to student learning and encourage a fixed mindset in students (Brooks et al., 2019; Hattie & Timperley, 2007; Kamins & Dweck, 1999; Skipper & Douglas, 2012). In fact, Brooks et al. (2019) chose not to observe self-level feedback in their study claiming that self-level feedback had potentially negative effects on student learning. Skipper and Douglas (2012) found that self-level praise directed at the student’s process may be just as sufficient as no praise when encouraging a growth mindset in students. In my study, Ellie, who ascribed to a weak incremental theory, did not provide verbal self-level feedback during the observed mathematics instruction. Considering that neither student achievement nor students' mindsets were observed in this study, I cannot assess whether the self-level feedback, or lack thereof, helped students' thinking to progress.

Both participants ascribed to an incremental theory; thus, it is important to try and understand why one participant provided self-level feedback multiple times and one did not provide self-level feedback at all during the observed mathematics instruction. First, one might question the effectiveness of the Implicit Theories Survey (see Appendix A) for accurately determining a person's implicit theory. Gleason (2016) questioned the survey in a similar way, concluding that "teachers who identified with a global mindset on a survey may not actually translate to teaching pedagogical practices . . . particularly in the area of math" (p. 73). Although Gleason (2016) only looked at the intelligence measure of Dweck et al.'s (1995) Implicit Belief Survey, this might explain why both participants ascribed to an incremental theory yet had varying commitments to providing self-level feedback.

Second, it is important to understand how a teacher's implicit theory influences the way they provide self-level feedback in the mathematics classroom. Rattan et al. (2012) showed that students, who were given simulated self-level praise, associated the self-level praise with teachers who held an entity theory. Thus, with this connection between self-level feedback and a teacher's implicit theory, one cannot assume that teachers who provide self-level feedback will ascribe to an entity theory or conversely, teachers who ascribe to an entity theory may not always provide self-level feedback.

Last, considering there were only two participants in this study, additional research should include more teachers, potentially from different levels on the implicit theory continuum. As discussed in Chapter III, it was difficult for me to select a second participant when most of the teachers ascribed to an incremental theory. Gleason (2016) found similar results in her study where 76% of the teachers surveyed held a growth mindset and 19% held a fixed mindset. With the understanding that more teachers ascribe to an incremental theory rather than an entity theory (Willingham et al., in press), additional studies with teachers who ascribe to different implicit theories would help to determine how their implicit beliefs influence the way they provide self-level feedback in the mathematics classroom, if at all.

Feedback Type

My second key finding was that although both participants provided all three types of feedback (i.e., feed up, feed back, and feed forward; see Figure 32) throughout the 11 days of classroom observations, Ian provided all three types of feedback within one classroom observation on multiple days. This aligns with Hattie and Timperley's (2007) model of feedback to enhance learning where the authors described effective feedback as feedback which answers all three questions (i.e., Where am I going? How am I going? and Where to next?). Effective feedback provided in this way should help students move forward from their current understanding to their desired goal and increase student achievement overall (Hattie & Timperley, 2007). However, Hattie and Timperley (2007) also noted that the degree of effectiveness depended on the level at which effective feedback should be directed and described the specific details of effective feedback between the types and levels as "fuzzy" (p. 103).

Due to the lack of clarity between the types and levels needed to achieve effective feedback, I examined the levels of feedback Ian provided on the six days to see whether any patterns emerged (see Table 14). The data showed that on all six days, Ian directed his feed up, feed back, and feed forward at the process or self-regulation levels, which Hattie and Timperley (2007) stated were “powerful in terms of deep processing and mastery of tasks” (p. 90). Ian was not consistent at providing task-level feedback within all three types of feedback on the six days. The results from this closer examination could contribute to the theory of what effective feedback looks like in terms of the levels in which feedback should be directed when answering all three questions (i.e., feed up, feed back, and feed forward) during a mathematics lesson. One area of future research would be to explore whether effective feedback consists of directing feedback at a combination of both process and self-regulation levels or just one of the levels on days where the teacher answers all three questions. Are there other combinations between the levels of feedback that might lead to more effective feedback as described by Hattie and Timperley (2007)? Additionally, I did not examine whether Ian answered all three questions within each teacher-student conversation (i.e., talk between the teacher and the student about a task), but rather within each of the mathematics lessons on each of these days. Thus, another area of future research would be to closely examine the types of feedback within each teacher-student conversation to determine the effectiveness of each interaction according to Hattie and Timperley’s (2007) effectiveness measure.

It is important to note that although Ian answered all three questions on multiple days, Ellie had no instances where she answered all three questions (i.e., provided feed up, feed back, and feed forward) within one classroom observation. This result, however, only shows that Ellie did not provide effective feedback during the 11 classroom observations according to Hattie and Timperley's (2007) definition of effective feedback. In her meta-analysis, Shute (2008) identified features of formative feedback that were most effective in promoting learning and found that effective feedback depended on whether the student needed it, whether the student had time to use it, and whether the student was willing and able to use it. Similarly, Fyfe et al. (2012) found that students' prior knowledge determined the effectiveness of teacher feedback and had the greatest impact on student learning. Thus, the results from these studies were centered around the ways in which students used the feedback provided to them (Fyfe et al., 2012; Shute, 2008) which was not addressed in my study. An area of future research would be to closely examine the feedback practices, particularly the types and levels of feedback, of more mathematics teachers in their own classrooms and the impact on student learning as a result of the observed feedback to determine the effectiveness of providing feedback of various types and levels in a natural setting and the direct impact on student learning.

Feedback Level

The last key finding from my data showed that Ian and Ellie provided little to no feedback directed at the process and self-regulation levels overall. When breaking down how each participant provided feedback by level, the results showed that Ellie provided task-level feedback (63 instances) approximately twice as many times as she provided process (25 instances) and self-regulation (two instances) levels combined. Ian provided the most feedback at the task level (56 instances); however, he provided more feedback at the process level (36 instances) and self-regulation level feedback (33 instances) combined. Brooks et al. (2019) showed similar results in an upper primary English classroom where feedback was directed most often at the task level and promoted mainly surface-level thinking. However, the authors found that with a majority of the teacher's feedback directed at the task level, there was little time for feedback directed at the process or self-regulation levels (Brooks et al., 2019). The result that Ellie provided little feedback at the process and self-regulation levels is important given that students use feedback at these levels to build a deeper understanding and take more control of their learning (Hattie & Clarke, 2019; NRC, 2001). Although task-level feedback is the most common form of feedback and necessary in the mathematics classroom to build a strong surface-level understanding (Brooks et al., 2019; Hattie & Clarke, 2019), it is not sufficient for helping students move towards the Goal of Mathematical Proficiency (NRC, 2001).

From their study on the feedback practices of 10 award-winning teachers in Hong Kong, Carless et al. (2011) found that students needed self-regulation feedback to help them develop effective strategies for monitoring and evaluating their own learning. Although the authors listed multiple effective feedback practices which focused on developing student autonomy and self-monitoring skills, they only collected interview data and did not conduct any actual classroom observations (Carless et al., 2011). Considering the results from Chapter IV of my study, one area for future research would be to compare the self-reported feedback practices to the actual feedback practices of teachers.

The result that Ian provided more process-level and self-regulation level feedback is important as feedback directed at the process and self-regulation levels are essential for fostering independent students and helping students in moving forward with their thinking (Hattie & Clarke, 2019; Hattie & Timperley, 2007). In addition, self-regulation level feedback supports students in monitoring and assessing their own learning, and often leads to the detection of their own errors (Hattie & Timperley, 2007). Although feedback at the process and self-regulation levels are essential in the mathematics classroom for supporting students, teachers do not often recognize what "higher levels of feedback look like, and are, thus, unable to use them to enhance learning" (See et al., 2016, p. 69). Thus, the results of my study suggest the need for preservice/in-service training to address the different types and levels of feedback in the mathematics classroom.

Conclusion

This study examined how two elementary mathematics teachers, both who ascribed to an incremental theory, provided feedback during mathematics instruction. Teacher feedback has been established as one of the most important influences on learning and student achievement (Hattie & Timperley, 2007; Hattie & Yates, 2014; Hubacz, 2013; Wisniewski et al., 2020). However, the ways in which teachers provide feedback during mathematics instruction (Jung et al., 2015; Li et al., 2016) and their own implicit theories are often overlooked as contributors to the various types of feedback they provide (Rattan et al., 2012). Considering the need for students to have the mathematical problem-solving skills necessary for success in today's workplace (Hoyles et al., 2002; Achieve, 2013; OECD, 2009; U.S. Department of Education, 2014), students must be given opportunities to engage with meaningful mathematics through effective feedback that supports their efforts in moving forward with their learning and understanding (Boaler, 2015; Tariq et al., 2013).

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APPENDICES

Appendix A: Implicit Theories Survey

Name: ___________________________________

Gender (Circle One):   M   F

What degrees do you currently hold?

Bachelors: ________________________   Masters:________________________

PhD:_____________________________   Other:________________________

How many years have you been teaching? ______________________________________

____

Of those years, how many including teaching mathematics?

____

Which grade do you currently teach?

Pre-K   K   1   2   3   4   5   6   Other:_______________

Which of the following best describes your role?

Regular Classroom Teacher   RTI Coach

Regular Classroom Teacher not teaching Mathematics

Administration   Special Area Teacher

Interventionist   Educational Assistant

If you currently teach mathematics, what time is your mathematics instruction?

____

For each of the following statements, rate how strongly you agree or disagree with the statement.	**Strongly Agree**	**Agree**	**Somewhat Agree**	**Somewhat Disagree**	**Disagree**	**Strongly Disagree**
You have a certain amount of intelligence and you really can't do much to change it.	**1**	**2**	**3**	**4**	**5**	**6**
Your intelligence is something about you that you can't change very much.	**1**	**2**	**3**	**4**	**5**	**6**
You can learn new things, but you can't really change your basic intelligence.	**1**	**2**	**3**	**4**	**5**	**6**
A person's moral character is something very basic about them and it can't be changed much.	**1**	**2**	**3**	**4**	**5**	**6**
Whether a person is responsible and sincere or not is deeply ingrained in their personality. It cannot be changed very much.	**1**	**2**	**3**	**4**	**5**	**6**
There is not much that can be done to change a person's moral traits (e.g. conscientiousness, uprightness, and honesty).	**1**	**2**	**3**	**4**	**5**	**6**
Though we can change some phenomena, it is unlikely that we can alter the core dispositions of our world.	**1**	**2**	**3**	**4**	**5**	**6**
Our world has its basic and ingrained dispositions, and you really can't do much to change them.	**1**	**2**	**3**	**4**	**5**	**6**
Some societal trends may dominate for a while, but the fundamental nature of our world is something that cannot be changed much.	**1**	**2**	**3**	**4**	**5**	**6**
A person has a certain amount of mathematical ability and they really can't do much to change it.	**1**	**2**	**3**	**4**	**5**	**6**
A person's mathematical ability is something about them that they can't change very much.	**1**	**2**	**3**	**4**	**5**	**6**
A person can learn new things about mathematics, but they can't really change their basic mathematical ability.	**1**	**2**	**3**	**4**	**5**	**6**

Source: Willingham, J. C., Barlow, A. T., Stephens, D. C., Lischka, A. E., & Hartland, K. S. (2020). *Mindset regarding mathematical ability in K-12 teachers.* Manuscript submitted for publication.

Appendix B: Observational Protocol

Teacher: Date:

Description of Topic Covered:

Time	Descriptive Notes	Observer Comments

Appendix C: Participant Selection Interview Protocol

Name: Date:

- Tell me a little about your teaching experiences, where you've taught, grade levels, etc.
- Looking over your survey, what overall theme do you notice?
- Can you describe students in your classroom that may illustrate the themes you noticed?
- Looking at the last three questions, can you describe what you think mathematical ability is?
- Do you believe that people are born with mathematical ability? Why or why not?
- Do you think it is possible to change a student's mathematical ability? How?
- Suppose you have a students at different levels, one who is low achieving, medium achieving, and high achieving (depending on their focus), tell me what your roles are with those student and what effect you have on those students.
- If you have two students that have the same mathematical achievement initially, but one student is more mathematically able, what happens to them over time in school? One has a natural math ability, yet on a test they both have similar scores, where do they end up?
- Overall, why do you think students struggle in mathematics?

Appendix D: Initial Interview Protocol

Name: Date:

A sample will be chosen from, but not limited to, the following questions:

1. What are the mathematical goals of the current unit you are teaching in mathematics?
2. How do you anticipate giving students information to help them move forward with their thinking about the mathematical goals?
3. Why do you think you give students information in that way as opposed to a different way such as ________________?
4. How do you modify or change this information in different situations?
5. Do you feel that giving this information during mathematics instruction is important? Why or why not?
6. Suppose you gave your class a mathematical problem to work on and you see a student who does not immediately begin working. You approach the student and see that they simply do not understand the problem. How would you respond to them?
7. Once this student begins working on the problem, you notice that they continue to struggle. What would you say to the student to support them in productively struggling through the problem?
8. Overall, how do you motivate students during mathematics class time to think deeper about the mathematical goal?
9. Why do you think you motivate them in this way?
10. Can you think of any ways of motivating students that may hinder their ability to think deeper about the mathematical goal?

Appendix E: Daily Interview Protocol

Name: Date:

A sample will be chosen from, but not limited to, the following questions:

1. What were the mathematics goals for today's lesson?
2. Do you feel that your students understood the mathematical goals as the lesson progressed? Why or why not?
3. During the lesson I observed an instance where you provided information to _______________ in this way. Here is a clip of the video from today. Please describe what is happening.
4. Please explain what you were thinking when you gave this type of information to this student.
5. Looking back on this instance, if you were given the chance to provide information to that student in a different way, what would you change and why?
6. Please describe some of your best math students. As a mathematics teacher, how do you provide information to those students?
7. Please describe some of your students who struggle with mathematics. As a mathematics teacher, how do you provide information to those students?

Appendix F: Final Interview Protocol

Name: Date:

A sample will be chosen from, but not limited to, the following questions:

1. For the past few weeks I have asked you a lot of questions around how you give information back to your students, so I was wondering if you could talk a little about whether or not your practices in that area have changed any over the past few weeks.
2. During our initial interview, I asked you about how you might anticipate giving students information to help them move forward with their thinking about the mathematical goals? You had mentioned ________________. Please describe any additional ways you were able to move students forward with their thinking in the past few weeks.
3. What ways do you feel you give feedback to your students most often during your mathematics instruction?
4. Are there different situations where you might provide more feedback? Please explain.

Appendix G: Reflective Journal Writing Prompts

Ian

1. Suppose one of your students understands how to properly demonstrate how to partition a number line into fifths and is able to show you how even before you present it to the class. What type of information (or response) might you provide to this student to extend their thinking?
2. Why do you think you respond to students in a very neutral way?
3. Do you feel you change or modify the way you respond to students in different situations?
4. In what situations do you feel students learn mathematics best? Why? Is this the same or different from the way you learned mathematics?

Ellie

1. When you are grading papers/work/tests, what are you specifically looking for and what determines their overall grade?
2. Many of the problems you present to the class are multiple choice. What is your purpose in doing so and have you thought about why you pose these problems as opposed to open-ended questions?
3. What do you think students were struggling with in the sailboat task? You said a few times, “Trust what you’re doing.” What were you thinking when you said this?
4. During your mathematics instruction time when students answer questions, how do you feel you most often respond to them?
5. Why do you think you respond in this way?
6. Do you feel you change or modify the way you respond to students in different situations?
7. In what situations do you feel students learn mathematics best?

Appendix H: IRB Approval Letter

IRB
INSTITUTIONAL REVIEW BOARD
Office of Research Compliance,
010A Sam Ingram Building,
2269 Middle Tennessee Blvd
Murfreesboro, TN 37129

MIDDLE TENNESSEE STATE UNIVERSITY

IRBN001 - EXPEDITED PROTOCOL APPROVAL NOTICE

Wednesday, November 22, 2017

Principal Investigator	**Kristin S. Hartland** (Student)
Faculty Advisor	Alyson E. Lischka and Angela Barlow (Univ Ctrl Ark)
Co-Investigators	NONE
Investigator Email(s)	*kristin.hartland@mtsu.edu; alyson.lischka@mtsu.edu; abarlow@uca.edu*
Department	Mathematics and Science Education
Protocol Title	***Teacher perspectives on feedback: A comparison between implicit theories***
Protocol ID	**18-2076**
Funding	*NONE*

Dear Investigator(s),

The above identified research proposal has been reviewed by the MTSU Institutional Review Board (IRB) through the **EXPEDITED** mechanism under 45 CFR 46.110 and 21 CFR 56.110 within the category *(7) Research on individual or group characteristics or behavior* A summary of the IRB action and other particulars in regard to this protocol application are tabulated below:

IRB Action	APPROVED for one year from the date of this notification
Date of expiration	**11/30/2018**
Participant Size	2 (TWO)
Participant Pool	General Adults (18+ of age) - K-6 Teachers
Exceptions	1. Permitted to record identifiable information to administer the study. 2. Approved to record videos for data collection.
Restrictions	**1. Mandatory signed informed consent; the participants must be provided with a copy of informed consent signed by the PI/FA.** **2. Video/audio data must be destroyed after data processing.** **3. Identifiable information must be deleted after data collection has ceased.**
Comments	NONE

This protocol can be continued for up to THREE years (**11/30/2020**) by obtaining a continuation approval prior to **11/30/2018**. Refer to the following schedule to plan your annual project reports and be aware that you may not receive a separate reminder to complete your continuing reviews. Failure in obtaining an approval for continuation will automatically result in cancellation of this protocol. Moreover, the completion of this study MUST be notified to the Office of Compliance by filing a final report in order to close-out the protocol.

IRBN001 Version 1.3 Revision Date 03.06.2016

Continuing Review Schedule:

Reporting Period	Requisition Deadline	IRB Comments
First year report	10/31/2018	TO BE COMPLETED
Second year report	10/31/2019	TO BE COMPLETED
Final report	10/31/2020	TO BE COMPLETED

Post-approval Protocol Amendments:

Date	**Amendment(s)**	**IRB Comments**
NONE	NONE	NONE

The investigator(s) indicated in this notification should read and abide by all of the post-approval conditions imposed with this approval. Refer to the post-approval guidelines posted in the MTSU IRB's website. Any unanticipated harms to participants or adverse events must be reported to the Office of Compliance at (615) 494-8918 within 48 hours of the incident. Amendments to this protocol must be approved by the IRB. Inclusion of new researchers must also be approved by the Office of Compliance before they begin to work on the project.

All of the research-related records, which include signed consent forms, investigator information and other documents related to the study, must be retained by the PI or the faculty advisor (if the PI is a student) at the secure location mentioned in the protocol application. The data storage must be maintained for at least three (3) years after study completion. Subsequently, the researcher may destroy the data in a manner that maintains confidentiality and anonymity. IRB reserves the right to modify, change or cancel the terms of this letter without prior notice. Be advised that IRB also reserves the right to inspect or audit your records if needed.

Sincerely,

Institutional Review Board
Middle Tennessee State University

Quick Links:

Click here for a detailed list of the post-approval responsibilities.
More information on expedited procedures can be found here.