Mexican Drug Cartel Strategy:
The Evolving Dynamics of the Illicit Drug Trade

By

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A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Arts in Economics

Middle Tennessee State University

November, 2023

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”Poverty is the mother of crime.”

- *Marcus Aurelius*

ACKNOWLEDGEMENTS

In carrying out this research, I was very fortunate to have the guidance of Dr. Wisarut Suwanprasert. His advice was instrumental in shaping the direction and quality of my work. I would also like to thank Dr. Stuart Fowler for his constructive comments and feedback and to Dr. Adam Rennhoff for serving on my committee. A big thanks to the MTSU Economics faculty I had the privilege of learning from. Certainly, all errors are my own.

I am grateful to my parents for their love and encouragement. A special thank you to my mother for taking me to her native Sinaloa, which would spark my interest in this topic. I would like to thank my sister, Leonela, for her invaluable advice in life and for instilling in me a passion for learning. I also extend my gratitude to my family and friends, whose support and presence have enriched my life significantly.

ABSTRACT

This research investigates the extent to which Mexican Drug Cartels are diversifying their operations. At this aim, a Hidden Markov Model is employed to analyze drug seizure data and introduce a marijuana legalization index as a covariate to consider the broader impacts of marijuana policy changes on demand structure. The empirical findings suggest a transition in drug trafficking patterns, indicative of the Cartels' diversification efforts. Concurrently, a rational choice model is considered that explores how product composition and law enforcement efforts influence the operational strategy within illicit drug markets; of particular interest is the introduction of fentanyl into the drug portfolio of Mexican Drug Cartels. Given fentanyl's low production cost and high potency, enabling adulteration, it is posited that the Cartel can sustain or expand drug supply even when faced with substantial drug seizures, thereby negating the cost imposed by law enforcement interdiction efforts. Comparative statics are employed to analyze the qualitative effects of variations in drug seizures on the Drug Cartel's strategic decisions. Through an empirical analysis and theoretical framework, this study comments on the evolving dynamics of the illicit drug trade peddled by Mexican Drug Cartels.

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1 Introduction

Deaths resulting from drug overdoses have reached record levels in the United States, underscoring the gravity and pervasiveness of the illicit drug trade. Central to this proliferation is Mexican Drug Cartels, also referred to as Mexican Transnational Criminal Organizations (TCOs), which work in conjunction with other criminal operatives to wholesale large quantities of cocaine, fentanyl, heroin, marijuana, and methamphetamine. Annual revenues for Mexican TCOs are estimated to be between \$19 to \$29 billion per the U.S. Department of Homeland Security. The United Nations Conference on Trade and Development estimates that Mexican inward illicit financial flows associated with the trafficking of heroin, methamphetamine, and cocaine generated approximately \$12.08 billion on average per year between 2015 and 2018. These profits translate to an estimated \$193 billion in social and economic damages from illicit drug misuse, as reported by the U.S. Department of Health and Human Services.

Prevention, treatment, and enforcement are the leading policy positions in the nation's efforts to control illicit drug markets. In response to the escalating drug crisis, the federal control budget allocated over \$39 billion in fiscal year 2022. Drug enforcement initiatives seek to disrupt all facets of the drug network, which includes crop eradication, interdiction, prosecution, and diversion of chemical precursors. Over time, compensating for drugs lost to enforcement will make it more costly for the drug network to provide the market with a given quantity, resulting in higher prices passed on to users. The price elasticity of demand, generally accepted to be somewhat inelastic, determines the extent of the market's impact Olmstead et al. (2015).

This essay will explore how recent changes in marijuana policy have shifted drug trafficking patterns and the introduction of fentanyl in Mexican TCO's operations. State-level efforts to transition into licit marijuana markets have largely supplanted demand for Mexican marijuana. Post-legalization, the strategic setting of prices is imperative to prevent the sustenance of illegal markets through price differentials with their legal counterparts, Ouellet et al. (2017). Due to the evolving landscape, Mexican TCOs now face greater difficulty in influencing marijuana prices. They perhaps would need to increase the quality of their product to compete with domestic producers. Anecdotally, some may suggest that legalization has effectively weakened the position of Mexican Drug Cartels, as evidenced by the 97% decline in the quantity of marijuana seized in the Southwestern border region since 2011. However, in response to these market shifts and as profit-maximizing agents, Mexican TCOs have adapted their operations to circumvent potential losses. One development is the growing intertwinement of drug markets, specifically involving fentanyl, which is increasingly becoming used as a cutting agent due to its low production costs and high potency.

Figure 1 below depicts the total quantity seized in pounds and the count growth rate at the Southwestern border region across the five drug categories: cocaine, fentanyl, heroin, marijuana, and methamphetamine. Notably, heroin and fentanyl exhibit interdependencies. While a decline is observed in the quantity of heroin seized, coupled with a decrease in heroin-related overdose rates by 20% in 2018, there has been a concurrent 12% increase in heroin-related overdoses that involved fentanyl NDTA (2020). This juxtaposition demonstrates a nuanced complexity in the consumption patterns of these substances. As a result, user risk increases due to the added layer of uncertainty regarding drug potency and quality. Notably, the adulteration of drugs with fentanyl is primarily observed at the retail level NDTA (2020). Although fentanyl adulteration at the wholesale level is currently considered rare, in the future, if there were to be an increase in the extent of cutting at the wholesale level coupled with additional cutting at the retail level, this would certainly warrant serious public health concerns.

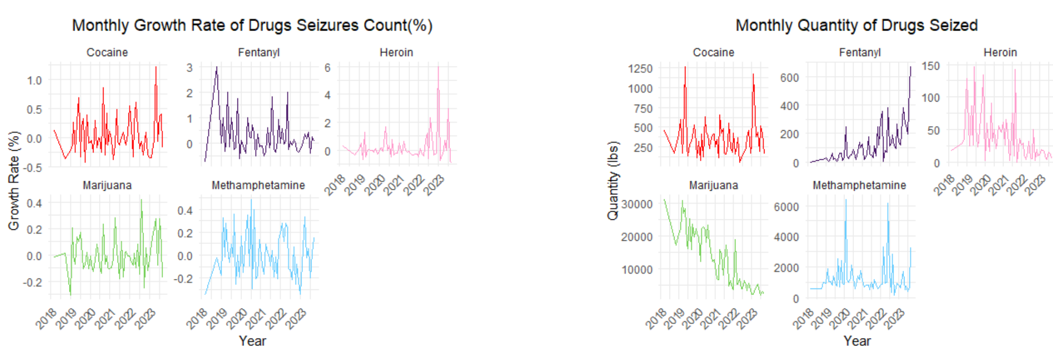

Figure 1: Quantity and Count Growth Rate (October 2018-August 2023)

A Hidden Markov Model (HMM) is introduced, which operates under the assumption that the actual state within a sequence is seldom directly observable; given this element of uncertainty, data observed with noise can be refined as an indicator of the true state. The analysis is based on the premise that drug seizure data is an indicator of drug trafficking patterns; the supply side of the drug market can be postulated since the total quantity of drugs trafficked is never fully observed. By using an HMM, we can analyze the drug seizure data to unveil underlying trends, and the model is further extended to include a marijuana legalization index as a covariate to discern how the impacts of legalization influence the

Cartels' strategies. Our HMM indicates that Mexican Drug Cartels have adaptive shifts in drug trafficking patterns, with drugs like heroin and fentanyl becoming central to drug trafficking efforts, while drugs such as cocaine, methamphetamine, and marijuana exhibit more deterministic patterns.

Furthermore, a theoretical model is employed to explore the strategic behavior of Mexican TCOs, particularly in response to law enforcement seizures. Of interest is how production methods and product composition in illicit drug markets can impact market structures. Previous research has demonstrated how drug enforcement efforts impose a cost on the drug network Reuter and Kleiman (1986). To illustrate, if large quantities of drugs are seized, the distribution system will respond by replenishing the stock, resulting in higher prices passed on to users to compensate for the risk. With the introduction of fentanyl into Mexican TCO's drug portfolio, the supply of drugs can be sustained or stretched through adulteration with fentanyl. This adaptation to seizures frames the Cartel's trade-off between the quantity of drugs to produce and the likelihood that a shipment will be seized to maximize revenue. Although not explicitly stated within the model, the uncertainty of drug quality and potency changes the risk profile for users, which can indirectly impact demand and, consequently, Cartel behavior. These dynamics could result in demand being more inelastic to dependent users and more elastic amongst occasional users, who may be more risk averse.

2 Literature Review

Informed by rational choice theory as presented in Becker (1968), our study explores Mexican Drug Cartels' strategic responses to law enforcement efforts. Like Becker's potential offenders who weigh the benefits of crime against the cost of punishment, an attempt is made to capture the continuous efforts of the Cartel to calibrate their operations to maximize profits amidst losses from law enforcement scrutiny. In this context, the framework is extended by incorporating drug adulteration as an adaptive mechanism employed by drug operatives, which serves as a tactic to mitigate losses from heightened drug seizures. Research has demonstrated how interdiction efforts increase the risks and costs associated with drug trafficking, behaving much like a tax Reuter and Kleiman (1986) and Caulkins and Reuter (2010). The price increase theoretically reduces demand, contributing to law enforcement's broader objective of mitigating drug abuse. However, the potential rise in drug prices could further incentivize Drug Cartels to adulterate their products, enabling them to maintain or sell at a lower price while sustaining quantity and potency. This emphasizes the need for a comprehensive understanding of the drug network's strategies, especially regarding the implications on public health. Regarding the legal dimensions of drug policy, Mahamad and Hammond (2019) demonstrates the impacts of recreational cannabis legalization on the illicit drug market, particularly in pricing and availability. They show that regulation and licit control of cannabis production significantly affect the illicit cannabis market, manifesting in reduced prices.

A Hidden Markov Model (HMM) is utilized to capture temporal patterns and analyze the extent of shifts in Mexican Drug Cartel operations due to legalization efforts in the United States; to the best of my knowledge, this approach is novel. Established literature has applied the use of HMMs to effectively model the behavior of market cartels and impacts associated with changes in competition law Hyytinen et al. (2018). In the context of modeling drug trafficking patterns over time, the covert nature of Drug Cartels can be modeled in a manner that captures external factors that may influence their operations. Studies present various methods to estimate Hidden Markov Models (HMMs), noting that introducing covariates tends to increase the model's complexity Bolano (2020) and Bartolucci et al. (2015). A common approach is the Expectation-Maximization (EM) algorithm, and the Bayesian Information Criterion (BIC) can be used in conjunction to balance model fit and complexity. However, the EM algorithm employs an iterative approach and can be slow to converge; therefore, it is essential to monitor the log-likelihood of the data given the parameters Altman (2007).

3 HMM Model

3.1 Methodology

This section introduces a two-state Hidden Markov Model (HMM) to distinguish between the contractive and expansive states of drug trafficking over time. Under these assumptions, the state of drug trafficking in a given period is predominately influenced by the preceding period, reflecting the response of drug trafficking dynamics to recent changes in external factors such as policy changes or user demand. To illustrate, if a drug smuggler successfully traffics a quantity of drugs on a particular route today without detection of seizure, they will likely use the same route in the next period. This demonstrates how a strategy's immediate success or failure tends to influence the immediate future.

3.1.1 Data Description

The study utilizes a panel data set from October 2018 to August 2023 derived from the United States Customs and Border Patrol (CBP). The dataset captures the incidence of drug seizures, presenting a monthly count of such events at the Southwest Border region by the United States Border Patrol (USBP). USBP is the most conducive component of CBP for drug smuggling from Mexico, as it encompasses the vast areas between the points of entry.

The dataset consists of drug seizures across five key drug types: cocaine, fentanyl, heroin, marijuana, and methamphetamine. These drugs represent the most commonly trafficked substances by Mexican TCOs. The drug counts are a direct measure of the frequency of USBP's interception efforts, serving as a proxy for the intensity of drug trafficking.

Additionally, the model is extended to include a covariate, a lagged Marijuana Legalization Index denoted by $Legalization_t$, to reflect changes in marijuana laws for recreational use in the United States. The index is calculated as the ratio of U.S. states that have enacted laws permitting the recreational use of marijuana in a given year, with data obtained by ballotpedia.org. The incorporation of the index serves to discern how policy modifications may impact drug trafficking patterns.

Table 1: Summary Statistics for the Monthly Count of Drugs Seized

Drug_Type	Mean	SD	Var	Min	Max
Cocaine	29.4	6.58	43.3	14	44
Fentanyl	18.2	10.6	112	1	42
Heroin	12.2	7.59	57.6	1	31
Marijuana	273	79.2	6272	143	432
Methamphetamine	73.8	25.3	642	36	157

n = 59 observations

The summary statistics are found above in Table 1. Notably, substances like marijuana and methamphetamine show higher maximums. This could be attributed to the fact that these drugs occupy more volume compared to their weight, perhaps making the occurrence of seizures more likely. In contrast, powdered substances like cocaine and fentanyl are more compact and potentially easier to smuggle, thereby posing more significant challenges for law enforcement detection.

3.1.2 HMM Structure and Estimation

A Markov process is predicated on the assumption that the future is independent of the past given present. A further extension of this concept is the Hidden Markov Model (HMM), where the system's states are not directly observable but can be inferred through the observable parameters. The HMM deals with two stochastic processes: the hidden state x and the observable process y. The model is characterized by transition probabilities, $P(x_{t+1}|x_t)$, which determines the likelihood of transitioning from one state to another as we move forward in time, and the observation probabilities, $P(y_t|x_t)$, which indicates the likelihood of some observation given the hidden state. The following equation represents the joint probability distribution of both sequences over time:

$$P(X,Y) = P(x_1)\prod_{t=1}^{T-1} P(x_{t+1}|x_t)\prod_{t=1}^{T} P(y_t|x_t) \tag{1}$$

Here, $P(x_1)$ represents the initial state distribution, which indicates the probability of the system starting in a particular state. The products of the transition and observation probabilities reflect the Markov chain dynamics and the hidden states' influence on the observable outcomes.

A two-state Hidden Markov Model (HMM) with Gaussian emissions is employed to the count growth rate of drugs seized of the various drug types predominately trafficked by Mexican TCOs, aiming to account for potential unobserved heterogeneity in drug trafficking patterns. A growth rate transformation is applied to the dataset to aid model stability and convergence. The application is done separately for each drug type, acknowledging that each drug may have distinct trafficking patterns driven by different factors like demand characteristics.

The hidden state is represented by x_t be at time t, which follows a Markov process and takes values from the state space {c,e}. Each state corresponds to a different regime of the drug growth rate, contractive (c), and expansive (e); the classification is based on the means associated with each state. The transition probabilities between the states are described by the 2×2 transition matrix $A = [a_{ij}]$, where $a_{ij} = P(x_{t+1} = j|x_t = i)$ for $i, j \in \{c, e\}$.

The initial state probabilities $\pi = (\pi^c, \pi^e)$ are estimated during the model fitting process by initializing them to be uniform, which operates under the assumption that each hidden state is equally likely at the start of the process. The observed drug count growth rate, denoted by y_t, for each drug type at time t, is assumed to be Gaussian-distributed: $y_t|S_t = i \sim N(\mu_i, \sigma_i^2)$, where μ_i and σ_i^2 are inferred from our data during the model fitting process.

The equations for the baseline model (2) and the covariate model (3) are represented below:

$$y_t|\text{state} = i \sim N(\mu_i, \sigma_i^2) \tag{2}$$

$$y_t|\text{state} = i \sim N(\mu_i + \beta_i \times \text{Legalization}_t, \sigma_i^2) \tag{3}$$

In the covariate model, β captures the impact of the preceding period's marijuana legalization index and is interpreted as the change in the growth rate for a one-unit increase in the legalization index. The Expectation-Maximization (EM) algorithm, combined with bootstrapping, fits the model on our dataset comprising 295 observations over 58 months of monthly data (resulting in the loss of one observation during data transformation).

3.2 Results

The HMM trains on the growth rate data. Table 2 shows the estimates of μ_i, σ_i^2, and standard errors associated with our emission probabilities for our data set, which determine the shape of the distribution for each state:

Table 2: Baseline Model Monthly Parameter Estimates

Drug Type	μ_c	σ_c^2	μ_e	σ_e^2
Cocaine	-0.140	0.028	0.376	0.094
	(0.072)	(0.045)	(0.235)	(0.058)
Fentanyl	-0.122	0.066	1.008	0.674
	(0.075)	(0.218)	(0.523)	(0.293)
Heroin	-0.123	0.129	1.729	2.754
	(0.129)	(0.796)	(2.089)	(1.306)
Marijuana	-0.045	0.028	0.053	0.006
	(0.030)	(0.009)	(0.096)	(0.010)
Methamphetamine	-0.060	0.022	0.302	0.010
	(0.069)	(0.011)	(0.108)	(0.013)

One notable observation from Table 1 is that the markets for fentanyl and heroin display the most volatility, likely due to their intertwinement, whereas methamphetamine shows specific periods of high uncertainty. All contractive states across all drug types in this first model are negative. Figure 2 below presents the posterior probability results and transition matrices:

Table 3 below shows the parameter results of the covariate model:

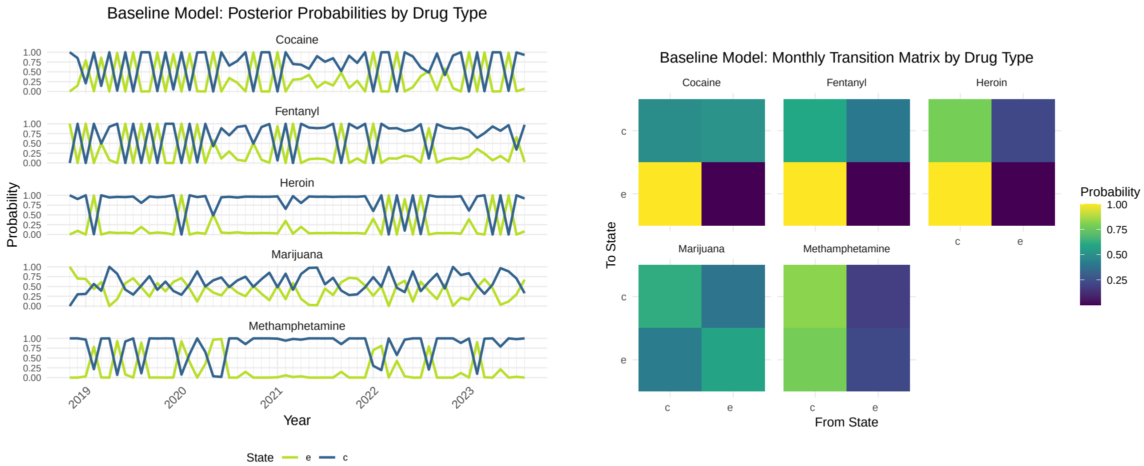

Figure 2: Baseline Posterior Probabilities and Transition Matrices)

Table 3: Covariate Model Monthly Parameter Estimates

Drug Type	**State** μ_c	σ_c^2	μ_e	σ_e^2
Cocaine	0.012	0.156	0.070	0.084
	(0.079)	(0.050)	(0.270)	(0.063)
Legalization$_{t,\text{Cocaine}}$	0.245	0.001	0.396	0.002
	(0.026)	(0.002)	(0.048)	(0.003)
Fentanyl	-0.087	0.079	1.263	0.617
	(0.066)	(0.279)	(0.631)	(0.371)
Legalization$_{t,\text{Fentanyl}}$	0.309	0.007	0.253	0.004
	(0.025)	(0.002)	(0.026)	(0.002)
Heroin	-0.121	0.127	1.777	2.775
	(0.119)	(0.949)	(2.092)	(1.629)
Legalization$_{t,\text{Heroin}}$	0.287	0.006	0.343	0.008
	(0.022)	(0.002)	(0.050)	(0.003)
Marijuana	-0.012	0.027	0.026	0.014
	(0.027)	(0.009)	(0.105)	(0.010)
Legalization$_{t,\text{Marijuana}}$	0.245	0.001	0.396	0.002
	(0.031)	(0.002)	(0.047)	(0.003)
Methamphetamine	-0.012	0.029	0.029	0.052
	(0.070)	(0.012)	(0.111)	(0.016)
Legalization$_{t,\text{Methamphetamine}}$	0.358	0.003	0.222	0.001
	(0.041)	(0.002)	(0.048)	(0.003)

To summarize the results of Table 3, a stable market presence is observed for cocaine, where the contractive state has now become positive, perhaps suggesting a stable strategy in cocaine trafficking. Heroin and fentanyl still demonstrate high variability and significantly larger expansive states compared to the baseline model and the other drug types. The expansive state of marijuana has dropped significantly from the baseline model, reflecting the steady decline of the drug, especially in an operational landscape heavily impacted by domestic production. Figure 3 depicts the posterior probabilities and transition matrices for the covariate model:

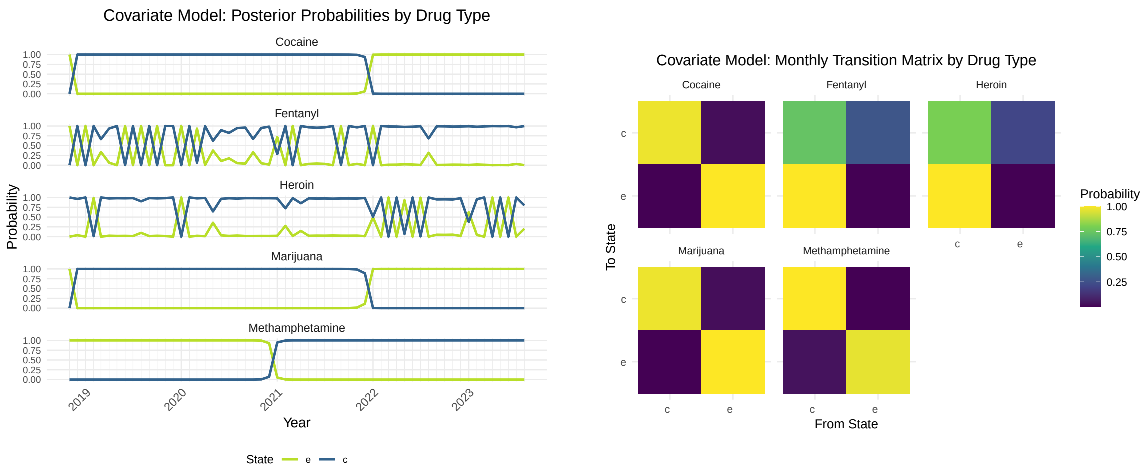

Figure 3: Covariate Posterior Probabilities and Transition Matrices)

Acknowledging the notably high standard errors, which could stem from the inherent variability and uncertainty of drug trafficking data and potentially limit the model's accuracy, the comparative analysis of both models for each drug type is presented below:

Cocaine: Upon including the covariate, the contractive state is observed to shift from a negative to a positive value and a reduced pace of increase in the expansive state. This could suggest that as more states legalize, there is more stability in the cocaine trade. There are also more deterministic shifts in the transition matrices and posterior probabilities compared to the baseline model.

Notably, out of all the drugs under consideration, Mexican TCOs do not have production control over cocaine. They are in a strategic position to traffic the drug as to their North is a market with a significant demand for cocaine, and to their South is the largest supply of cocaine. Since they largely depend on intermediaries in the cocaine supply chain, there could be a host of factors that can influence their operational strategy, as well as other economic factors.

Heroin: Comparing the models, there is very little change in the parameters. Out of all the drug types, heroin demonstrates the most variability and unpredictability, which could imply more sporadic activities or shifts in supply and demand. In the evolving landscape, heroin overdoses have declined substantially, while uncertainty around drug purity is increasing.

Marijuana: With the addition of the covariate, there is a decrease in the expansive state, further accelerating the decline when accounting for increasing legalization. There are more deterministic shifts between the states in the covariate model versus the baseline, as depicted by the posterior probabilities, which lean primarily toward a contractive trend.

Methamphetamine: The incorporation of the covariate slows the rate of decline in the growth rate of seizures for methamphetamine and demonstrates more deterministic shifts between states as observed by the posterior probabilities and transition matrices.

Fentanyl: Similar to heroin, even with the addition of the covariate, fentanyl demonstrates a very volatile trend. Perhaps pointing to the intertwinement of these markets and the fentanyl epidemic that has unfolded in the United States in recent years. The contractive and expansive states decrease and increase, respectively, with the addition of the covariate, pointing to an increase in fentanyl operations.

In conclusion, with the Marijuana Legalization Index as a covariate, there are notable shifts in the count growth rate of trafficking, suggesting adaptive modifications in Cartel operations. Overall, the model achieved an enhanced fit, as denoted by the BIC. Incorporating the covariate led to more deterministic transitions across drug types like cocaine, marijuana, and methamphetamine, whereas heroin and fentanyl showed higher variability. However, the model, limited by large standard errors due to drug trafficking's inherent variability, calls for further analysis. Certainly, further study is needed to address changes in law enforcement strategies, technological advancements, and political factors that impact drug trafficking.

4 Theoretical Model

The goal of Mexican TCOs is to capitalize on the illicit drug market by catering to a global demand. What stands in their way is law enforcement, whose objective is to disrupt the flow of these illegal substances. A risk-neutral Drug Cartel trafficks two types of drugs, q_1 and q_2, and is subjected to the threat of drug seizures s_1 and s_2, where $q_i > 0$ and $s_i \geq 0$ for $i = 1, 2$. The following represents the Drug Cartel's expected profits:

$$\Pi = \theta(s_1)[p_1(\phi_1)q_1] + [1-\theta(s_1)][-\varphi(q_1)] - (1-\phi_1)c(q_1) + \theta(s_2)[p_2q_2] + [1-\theta(s_2)][-\varphi(q_2)] - c(q_2) \quad (1)$$

The term $\theta(s_i)$, for $i = 1, 2$, denotes the probability of successfully trafficking drug i without seizure, influencing revenue potential. Where $\theta(s_i)$ is a decreasing function of s_i and $0 \leq \theta(s_i) \leq 1$. Each drug's revenue is modulated by $p_i(\phi_i)q_i$, where $p_i(\phi_i)$ is the market price function for drug i, dependent on its adulteration level ϕ_i, and $\frac{dp_i}{d\phi_i} < 0$ reflects the reduced market value of more adulterated drugs. The model assumes that adulteration can only be applied to drug 1, while drug 2 cannot be. Drug adulteration serves as a mitigation strategy that reduces the loss resulting from increasing drug seizures, where the Cartel can essentially sustain the quantity of drug 1 and maintain market potency.

The quantity q_i signifies the amount of drug i trafficked, with larger quantities posing higher risks and potential penalties, captured by the penalty function $\varphi(q_i)$. The production cost for drug i is $c(q_i)$, adjusted for drug 1 by $(1-\varphi_1)$ to account for reduced costs due to adulteration. The model portrays a nuanced operational landscape where the Cartel's profit maximization strategy is contingent on balancing the risks of seizure, the effects of adulteration, and the interplay between production costs and market pricing.

5 Comparative Statics

This section will examine how a Drug Cartel's expected profit, Π, responds to changes in model parameters. Comparative statics are employed to deduce the Cartel's strategies and the potential impact of external factors on its profitability.

The first-order conditions for Π illustrate the trade-offs inherent in varying the quantities of drugs trafficked (q_1, q_2) and the level of adulteration (ϕ_1). The partial derivative of Π with respect to q_1 is:

$$q_1 : \theta(s_1)p_1(\phi_1) - [1-\theta(s_1)]\frac{d\varphi(q_1)}{dq_1} - (1-\phi_1)\frac{dc(q_1)}{dq_1} = 0 \quad (2)$$

For equation (2), increasing q_1 potentially increases revenue due to the term $p_1(\varphi_1)q_1$, which captures the price-revenue relationship. However, a higher q_1 elevates the risk of incurring penalties, as indicated by $\frac{d\phi(q_1)}{dq_1}$, which assumes $\frac{d\phi}{dq_1} > 0$. The term $(1-\varphi_1)\frac{dc(q_1)}{dq_1}$ signifies that while production costs increase with q_1, they are partially offset by adulteration, given $(1 - \varphi_1) < 1$. Adulteration has the dual effect of diminishing the unit price of drug 1 while simultaneously reducing production costs. This strategy reduces costs, which is pivotal for the Cartel to maintain drug volume and potency in the market, as it compensates the revenue loss from a lower price point without compromising the supply.

The partial derivative of Π with respect to q_2 is:

$$q_2 : \theta(s_2)p_2 - [1 - \theta(s_2)]\frac{d\varphi(q_2)}{dq_2} - \frac{dc(q_2)}{dq_2} = 0 \tag{3}$$

In this equation (3), the revenue generated from drug 2 is directly relates to the product of its market price p_2 and the quantity q_2, without any moderating effect from adulteration. This relationship suggests that the Cartel has less flexibility in adjusting the effective purity and cost of drug 2, thereby having a more inelastic supply response to market and enforcement pressures. The term $\frac{dc(q_2)}{dq_2} > 0$ indicates rising marginal costs with increased production of q_2. Moreover, increased interdiction efforts (s_2) could restrict the market supply of drug 2, potentially exerting upward pressure on the price p_2. This dynamic could result in higher prices passed on to users as the drug network attempts to compensate for the increased risk of seizures and loss of product.

The partial derivative of Π with respect to ϕ_1 is:

$$\phi_1 : \theta(s_1)\frac{dp_1(\phi_1)}{d\phi_1}q_1 + c(q_1) = 0 \tag{4}$$

Equation (4) addresses the optimization of the adulteration level φ_1 for drug 1. The condition implies that the cost savings in production offset the marginal revenue loss from price reduction due to additional adulteration. Revealing a nuanced trade-off: the Cartel can utilize adulteration to lower costs, but an excess of adulteration could significantly decrease the drug's value and revenue. Intuitively, over-adulteration would be lethal and approach a natural limit.

5.1 Analysis of s_1

The impact of law enforcement intensity, represented by s_1, on drug cartel operations concerning drug 1 is examined through the derivatives of the quantity trafficked, q_1, and the adulteration level, ϕ_1. Our assumptions include the convexity of the penalty function, $\frac{d^2\varphi(q_1)}{dq_1^2} > 0$, and the diminishing marginal impact of adulteration on price, $\frac{d^2p_1(\phi_1)}{d\phi_1^2} > 0$.

The equations representing the derivatives with respect to s_1 are shown below:

$$\begin{aligned} \text{q1:} \quad & \left[p_1(\varphi_1) + \frac{d\phi(q_1)}{dq_1} \frac{d\theta(s_1)}{ds_1} \right] \\ & + [1 - \theta(s_1)] \frac{d^2\phi(q_1)}{dq_1^2} + [1 - \varphi_1] \frac{d^2 c(q_1)}{dq_1^2} \frac{dq_1}{ds_1} \\ & + \left[\theta(s_1) \frac{dp_1(\varphi_1)}{d\varphi_1} + \frac{dc(q_1)}{dq_1} \frac{dq_1}{ds_1} \right] = 0 \end{aligned}$$

$$\text{q2:} \quad 0$$

$$\begin{aligned} \phi_1: \quad & \left[\frac{dp_1(\varphi_1)}{d\varphi_1} q_1 \frac{d\theta(s_1)}{ds_1} \right] + \theta(s_1) \left[\frac{dp_1(\varphi_1)}{d\varphi_1} + \frac{dc(q_1)}{dq_1} \frac{dq_1}{ds_1} \right] \\ & + \left[\frac{d^2 p_1(\varphi_1)}{d\varphi_1^2} q_1 \right] \frac{d\varphi_1}{ds_1} + c(q_1) \frac{d\varphi_1}{ds_1} = 0 \end{aligned}$$

Solving yields equations (5) and (6):

$$\begin{aligned} \frac{dq_1}{ds_1} = & \frac{\frac{d\theta(s_1)}{ds_1}}{\text{det}} \\ & \left\{ \left[-\frac{d^2 p_1(\phi_1)}{d\phi_1^2} + \frac{dp_1(\phi_1)}{d\phi_1} \right] \left[p_1(\phi_1) + \frac{d\varphi(q_1)}{dq_1} \right] \right. \\ & \left. + \left[-\frac{c(q_1)}{q_1} + \frac{dc(q_1)}{dq_1} \right] \left[\frac{dp_1(\phi_1)}{d\phi_1} q_1 \right] \right\} \end{aligned} \tag{5}$$

Equation (5) demonstrates the Drug Cartel's response to changes in law enforcement efforts. Under the assumptions of convexity for the penalty function $\phi(q_1)$, $d^2\phi(q_1)/dq_1^2 > 0$ and the price function $p_1(\varphi_1)$, $d^2 p_1(\varphi_1)/d\varphi_1^2 > 0$, the determinant is found to be negative. This outcome aligns with the economic reasoning that increased quantities of trafficking would lead to diminishing returns to adulteration in price terms. The strategy of adulteration, while effective to some extent in expanding supply, faces limitations, particularly when considering the potential harm of excessive adulteration in potent substances.

The term $\frac{d\theta(s_1)}{ds_1} < 0$ reflects a decrease in the probability of successful trafficking without seizure as law enforcement intensifies. The marginal cost of production represented by $-c(q_1)/q_1 + dc(q_1)/dq_1$ is greater than the average cost, suggesting an increasing cost structure as the level of production rises.

Furthermore, the negative gradient of the price function with respect to adulteration, given by $\frac{dp_1(\phi_1)}{d\phi_1} < 0$ confirms that higher levels of adulteration reduce the drug's market value. The convex nature of the price's response to adulteration is further illustrated by the term $d^2p_1(\varphi_1)/d\varphi_1^2 - dp_1(\varphi_1)/d\varphi_1$ which is positive, indicating that the marginal benefit of increasing adulteration diminishes.

In economic terms, the derivative $\frac{dq_1}{ds_1} < 0$ implies that an increase in law enforcement efforts results in a decrease in the quantity of drug 1 trafficked. This decrease is driven by heightened seizure risks, rising production costs, and the reduced efficacy of adulteration as a cost-minimization strategy under a volatile operational landscape.

Equation (6) demonstrates the relationship between law enforcement efforts and the Cartel's adulteration strategy for drug 1. The formulation of the equation is as follows:

$$\frac{d\phi_1}{ds_1} = \frac{\frac{d\theta(s_1)}{ds_1}}{\det}\left\{\left[-\frac{c(q_1)}{q_1} + \frac{dc(q_1)}{dq_1}\right]\left[p_1(\phi_1) + \frac{d\varphi(q_1)}{dq_1}\right]\right\} + [1-\theta(s_1)]\frac{d^2\varphi(q_1)}{dq_1^2} + (1-\phi_1)\frac{d^2c(q_1)}{dq_1^2}\left[\frac{dp_1(\phi_1)}{d\phi_1}q_1\right] \tag{6}$$

Interpreting the terms similar to (5), the derivative $\frac{d\phi_1}{ds_1} > 0$ economically suggests that an increase in seizure risk incentivizes the Cartel to enhance the adulteration level (ϕ_1). This strategy is employed to counterbalance the rising costs and penalties, attempting to maintain profitability under heightened enforcement pressures. The equation captures the Cartel's adaptive measures in response to external law enforcement actions, highlighting the nuanced interplay between risk, cost control, and market pricing strategies.

5.2 Analysis of s_2

The analysis now focuses on the impact of law enforcement intensity s_2 on the quantity of drug 2 trafficked, q_2, where drug 2 is not subject to adulteration strategies:

$$\text{q1:} \quad 0$$

$$\text{q2:} \quad \frac{d\theta(s_2)}{ds_1}\left[p_2 + \frac{d\phi(q_2)}{dq_2}\right] - [1-\theta(s_2)]\frac{d^2\phi(q_2)}{dq_2^2} + \frac{d^2c(q_2)}{dq_2^2}\frac{dq_2}{ds_2} = 0$$

$$\phi_1: \quad 0$$

Solving, yields equation (7):

$$\frac{dq_2}{ds_2} = \frac{\frac{d\theta(s_2)}{ds_1}\left[p_2 + \frac{d\varphi(q_2)}{dq_2}\right]}{[1-\theta(s_2)]\,\frac{d^2\varphi(q_2)}{dq_2^2} + \frac{d^2c(q_2)}{dq_2^2}} < 0 \tag{7}$$

The numerator of (7), $\frac{d\theta(s_2)}{ds_2}\left[p_2 + \frac{d\varphi(q_2)}{dq_2}\right]$, captures the dynamics between the market price of drug 2 and the marginal penalty for trafficking. The term $\frac{d\theta(s_2)}{ds_2} < 0$ reflects the decreasing likelihood of trafficking success under intensified law enforcement. The combined effect of market price p_2 and the marginal penalty $\frac{d\varphi(q_2)}{dq_2}$ demonstrates the balance between the potential revenue from trafficking and the increasing marginal cost of penalties as trafficking quantity increases.

In the denominator, $[1-\theta(s_2)]\,\frac{d^2\varphi(q_2)}{dq_2^2} + \frac{d^2c(q_2)}{dq_2^2}$, the combined effect of the probability of seizure and the convex nature of the penalty and cost functions is inferred. The term $[1-\theta(s_2)]\,\frac{d^2\varphi(q_2)}{dq_2^2}$ highlights how the risk of penalties scales with increasing quantities, especially under the pressure of higher law enforcement scrutiny. The positive term $\frac{d^2c(q_2)}{dq_2^2}$ indicates increasing marginal costs of production, a factor that becomes increasingly significant as the quantity trafficked rises. The result of $\frac{dq_2}{ds_2} < 0$ implies that an intensification of law enforcement efforts, represented by an increase in s_2, leads to a reduction in the quantity of drug 2 trafficked. This direct response showcases a more inelastic approach by the Cartel to law enforcement pressures for drug 2. This inelasticity stems from the Cartel's limited ability to adjust drug 2's purity or production costs in response to increased risks, underscoring the heightened effectiveness of law enforcement strategies against drugs that cannot be easily adulterated.

5.3 Discussion

Our theoretical model presents distinct strategic margins for two different types of drugs trafficked by Drug Cartels. Drug 1 has dual margins in quantity (q_1) and adulteration levels (ϕ_1), whereas Drug 2 operates on a single margin. This distinction carries several implications for the Cartel's operational and strategic decision-making.

The dual-margin strategy for Drug 1 allows for greater flexibility, enabling Cartel to adjust both the quantity and the level of adulteration in response to the volatile operational landscape. The adulteration level (ϕ_1) serves as a method to control production costs and manage risk, especially when subjected to substantial law enforcement pressure (s_1). Meanwhile, the single-margin strategy for Drug 2 limits Cartel's responses to changes in law enforcement intensity (s_2). Without the option to adulterate, the Cartel's strategy for Drug 2 is less adaptable, primarily relying on quantity adjustments to navigate market and enforcement dynamics. This would allow law enforcement to have a greater influence on prices through disrupting supply.

The theoretical implications align with observed real-world dynamics in drug markets, particularly in the context of adulteration strategies. A notable example is the growing intertwinement of the fentanyl and heroin markets. The economic incentive to adulterate heroin with fentanyl mirrors the strategy to adjust ϕ_1 in Drug 1, allowing operatives in the drug network to enhance potency and reduce costs. However, a nuance here would be perhaps the pharmacological incompatibility of mixing a depressant like fentanyl with a stimulant such as cocaine, reflecting the limitations faced by Drug 2 in our model, where adulteration is not a viable option. While economically advantageous from a cost and potency perspective, the adulteration strategy also changes the risk profile of users and responses to legal scrutiny on the drug trade, thus impacting market dynamics and law enforcement approaches.

In summary, the model demonstrates the counterintuitive strategies that the Drug Cartel may employ in their seemingly insatiable desire to maximize profits, specifically emphasizing the role of drug adulteration in response to heightened drug seizure levels across drug markets. The analysis suggests that as law enforcement activities intensify, Drug Cartels may strategically increase drug adulteration to hedge against risk and to sustain market potency, albeit up to a certain extent. However, this approach encounters a natural limit, as excessive adulteration can compromise the health of users and, ultimately, the Cartel's market position. The dynamics are embedded in a complex interplay of variables not fully captured by the model, including market demand changes, Cartel competition, and evolving law enforcement tactics. As such, our model contributes to a nuanced understanding of the evolving dynamics of the illicit drug trade, and more comprehensive empirical scrutiny is needed to validate its theoretical implications.

6 Conclusion

This research touches on a small facet of the complex dynamics regarding the illicit drug trade, specifically the diversification strategies employed by Mexican Drug Cartels in a volatile operational landscape. A multifaceted approach is employed, integrating empirical and theoretical constructs. The Hidden Markov Model analysis provides insights into the fluctuations in drug trafficking over time, particularly in response to regulatory changes.

Simultaneously, our theoretical model offers a perspective on how drug adulteration serves as a mitigation strategy in response to drug seizures. However, our model shows that adulteration is not a panacea but only beneficial to an extent. Beyond its natural limit, it jeopardizes user health and can undermine the Cartel's market position. These findings contribute to a deeper understanding of the operations of drug syndicates, expounding on the precarious balance between profit maximization, product quality, market position, and regulatory shifts.

With Drug Cartels demonstrating their adaptability and resilience, the effect of drug seizures on the street prices of drugs can be less significant. Though essential, traditional strategies aimed at disrupting supply may inadvertently bolster the drug network by incentivizing practices such as drug adulteration. Their strategic evolutions call for policy responses beyond a supply-focused approach, which appears to be the policy stance endorsed by recent administrations, as evidenced by increased funding in prevention and treatment programs. Law enforcement and policy must adapt to Mexican TCOs to combat the illicit drug trade and curb the profound human toll and pervasive rise in drug-induced deaths.

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