THE STUDY OF POWER CONSUMPTION DURING RADIAL AND AXIAL SEGREGATION IN HORIZONTAL ROTATING CYLINDERS

by

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ABSTRACT

When binary granules of different material properties are rotated in a horizontal rotating drum they often segregate. Within the first few seconds, the smaller particles move towards the radial core in what is known as radial segregation. Depending on granular properties, drum speeds, and drum dimensions, the granules can segregate into "bands" known as axial segregation. Prior experiments by Gebrehiwot show that while rotating black beans and rice at a mixed state axial segregation as well as power dissipation were observed. Similar experiments were performed using barley, black beans, lentils, pintos, rice, and safflower. In the experiments that axial segregation occurred there was a substantial decrease in torque and power. Density, particle size by volume, and static angle of repose calculated by the fixed funnel method were measured for each granule. Axial segregation only occurred when the density ratio of the materials was between 1-1.2 (similar density's), and the size ratio was greater than 4 (large size difference). In relation to static angle of repose, axial segregation was observed when the angle ratio of the mixed granules was 0-1 (similar) and 1.1-3 (different). A separate experiment using nylon hose shows that radial segregation does occur in the absence of the avalanche effect where the larger granule concentrates to the core.

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1 INTRODUCTION

Granular segregation is a phenomenon that has been studied extensively in the last century. Segregation of granules happens both in nature and real world applications such as pharmaceuticals, agricultural, mineral extraction, construction industry, and food production. In most real world applications segregation is not desirable and in some cases be very expensive to avoid. Oyama first observed axial segregation in 1939 [2] and even after many years of both experimental and theoretical approaches by researchers, it is still a mystery why segregation occurs. In July 2005, the 125th anniversary issue of Science named the flow of granular material as one of the 125 big questions in science [3]. Many different approaches have been taken to better understand this phenomenon. Some of the different equipment used in experiments include rotating cylinders, Magnetic Resonance Imaging (MRI) devices [4], biaxial rotary mixer [5], tilted rotating drum [6], chutes [7,8], and a vertically vibrated container [9]. Researchers have studied the effects of many variables and their relationship on segregation such as dynamic angle of repose [8,10,11,12,], density [13,14,15,16], friction [11,15], as well as particle size [12]. All experiments presented in this paper deal with granules with different size, density and repose angle, being subjected to rotation in a horizontal rotating cylinder.

1.1 Purpose and significance of study

Power consumption during granular segregation, has been studied very little by researchers. Studies by Gebrehiwot, determined that when an equal mass of mixed binary granules, rice and black beans were rotated in a horizontal cylinder radial segregation occurred followed by axial segregation [1]. To determine the effect power has on the experiment Gebrehiwot observed parameters such as torque, angular speed and the dynamic repose angle. Gebrehiwot recorded energy dissipation during the segregation process, in which the torque decreased within the first hour of the experiment. Three similar experiments were conducted using only rice, only black beans, and unmixed rice / black beans in which energy did not dissipate. This research has built onto Gebrehiwot's work by performing similar experiments on not only rice, black beans but also pinto beans, lentils, barley, and safflowers. The main focus of all the experiments was to observe any changes in power consumption or if any segregation occurred. Experimental parameters of torque and angular speed were measured during the duration of each experiment. Material properties such as static repose angle, density, and size were also measured and recorded. Another experiment that was performed was rotating rice and black beans in women's leg hosing to examine the effect of friction on radial and axial segregation. It also, examines how particle characteristics effect both segregation and power consumption. The purpose of this research is to examine how power consumption varies from experiments, which rotate a single granule, binary granules that are unmixed, and finally binary granules that are mixed.

1.2 Outline of Thesis

Chapter 1 introduces the segregation phenomenon of granular materials. Chapter 2 is literature review on segregation types, friction types, avalanche, and the coefficient of restitution. Chapter 3 deals with the experimental methods and procedure. It includes measuring torque and angular speed for experiments to determine power consumption. It also deals with determining granular properties such as static angle of repose, size, and density. Separate experiments were also conducted using women's nylon leg hosing to study the effects of friction. Finally another experiment was performed to examine how different granules react when one type of granule is free fallen onto another. Chapter 4 interprets the results of power consumption in the experimental investigation of segregation. It compares power consumption for all experiments focusing on the relationship of power to axial segregation. It also examines the relationship if any of certain particle characteristics to both segregation and power consumption. Chapter 5 provides suggestions for future works. Appendix A shows incremental pictures for each experiment.

2 LITERATURE REVIEW

2.1 Granular segregation

Granular segregation occurs when two or more granules are subjection to a vibration that is horizontal, vertical, or in some cases both. One of the most well-known and simplest forms of granular segregation is known as the Brazilian nut separation, which granular particles with different sizes are separated by vertical vibrations [9]. There are three types of granular segregations. S-systems are where granules have different particle size, D-systems are where particles have different density and SD-systems where particles have both different size and density [17]. Depending on size and density ratios the type of flow can vary from a free surface flow, rolling regime, or an avalanching regime [17]. Extensive research has been conducted on all three types of granular segregation when subjected to rotation in a horizontal rotating cylinder [12,15,18,19]. Figure 2-1 shows that changing the size of the granules; cylinder diameter, or rotation speed can determine if axial segregation of the granules will occur [18]. Granular size ratio is not only important to axial segregation but it can also in some cases determine if de-mixing will occur [12]. De-mixing is when binary granules that are in a segregated state can be rotated to go back to a mixed state; this is usually achieved by simply changing the speed. As for density most research suggests that for axial segregation to occur granules must be of the same density. Sanfratello and Fukushima experimentally investigated axial segregation due to density ratios and found that axial segregation did not occur for ratios as high as 4.9 [14]. Dury and Ristow found that the density ratio may not have a direct relationship to axial segregation but it does on radial segregation. They found that increasing the density ratio enhances radial segregation and if the density ratio is decreased enough then the larger particles will move to the radial core [13].

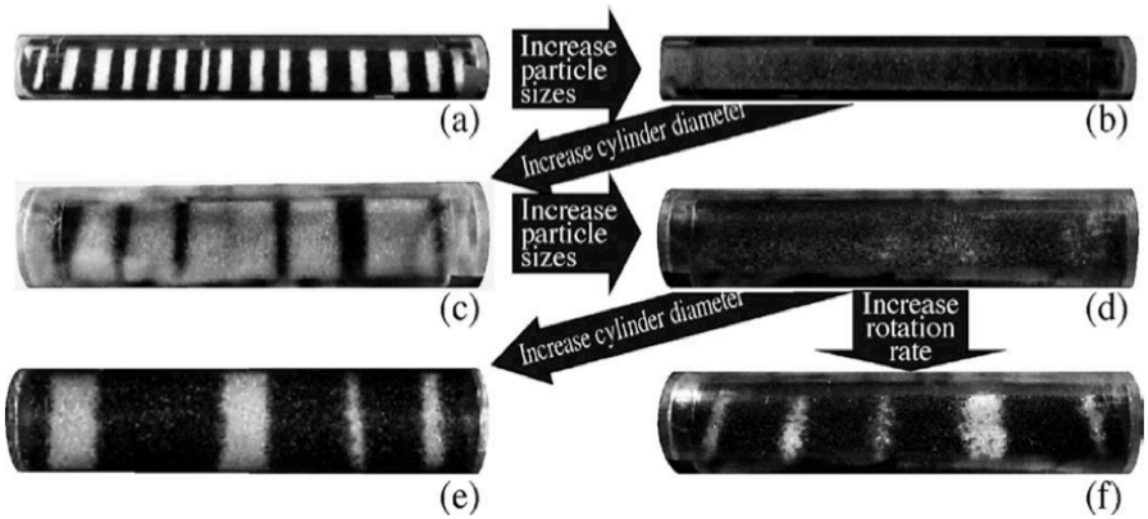


Figure 2–1 An illustration of particle size, cylinder diameter, and speed size relationship to axial segregation [18]

2.1.1 Avalanche

As the cylinder rotates, the granular material avalanche effect takes place. The material moves to an area called the marginal stability (θ_m), which is the maximum point in the avalanche, as this occurs the material becomes unstable and detaches from other granules. After the material has detached it free falls at an angle known as the dynamic angle of repose (θ_r). The avalanche process is shown in Figure 2-2. In the procedural set up for our experiments the avalanche is best seen at moderate speeds (10-12 rpm) and can easily be viewed throughout the experiment. Froude's number, which is the ratio of the inertia forces to the gravitational forces, is also very important to the avalanche effect. Depending on the relative degree of filling of granular material, Froude number of the drum as well as rotational speed flow regimes of the avalanche can be sliding, slumping, rolling, cascading, contracting, or centrifuging [20]. During the avalanche particles collide then disperse based on particle size, the larger particles travel a longer axial distance because of their higher momentum where the smaller particles are trapped in charge voids [16].

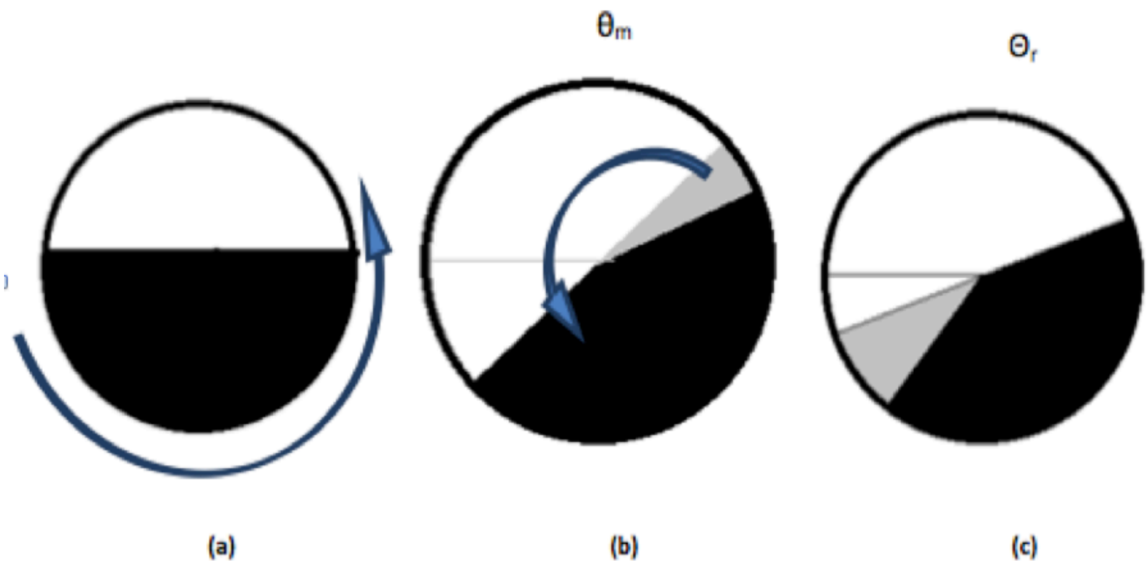


Figure 2–2 Avalanche in a rotating cylinder (a) Initial condition (b) Marginal or critical point before an avalanche (c) After an avalanche [21]

The angle of repose has been studied by many researchers [10,11,12,14] and although they all agree that having a difference in dynamic angle of repose is necessary for axial segregation, it is not the only determining factor. Rotational speeds change flow regimes of the avalanche effect that can change the dynamic repose angle. Khan states that angle of repose also has a direct relationship to weather de-mixing or reversible axial segregation occurs when an increase of speed increases the repose angle [10].

2.1.2 Radial segregation

The first segregation that is viewed during mixed binary granule experimentation is radial segregation. During the last phase of the avalanche, the particles free surface where typically the smaller particles concentrate to the core of the cylinder and the larger particles periphery of the cylinder. Radial segregation is often observed after only a few revolutions. Figure 2-3 illustrates radial segregation where the smaller particles can be seen at the core and the larger particles at the outside after just 60 revolutions.

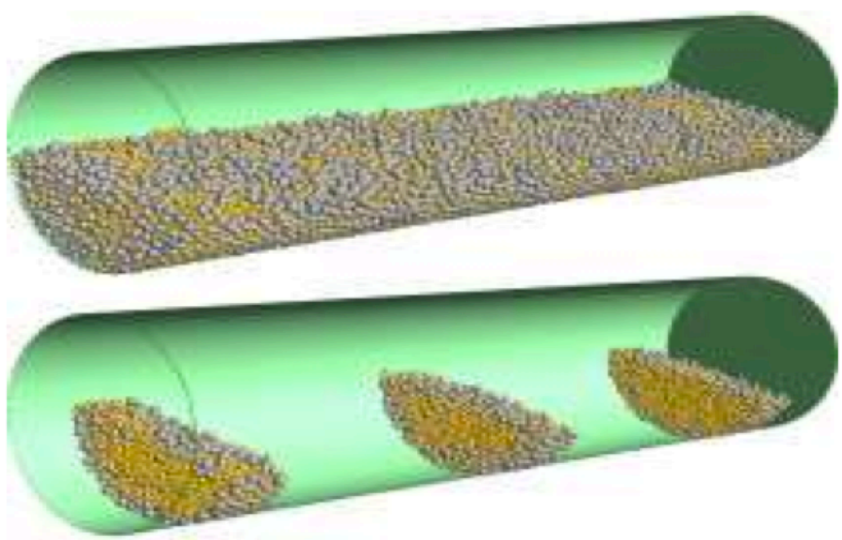


Figure 2–3 Smaller particles seen at the core after 60 revolutions [15]

Radial segregation of this nature occurs at lower rotational speeds where the avalanche effect can take place. It is often hard to view the inner core of a rotating cylinder during an experiment. Recently experiments have been performed using MRI technology that can view the radial core better and can better estimate the radial core as well as the avalanche in an experiment [4]. In recent studies by Chand et al, tests showed that radial segregation is directly dependent on drum length, and the friction on the non-rotating end plates [22]. They concluded that in longer drums the friction on the end plates show little effect whereas the shorter drums show a higher segregation ratio as the end plates friction is decreased. Finally when they roughened up the end plates they observed an increase in granular temperature but no segregation was seen.

2.1.3 Axial segregation

When two binary granules of a set size and density are rotated axial segregation can occur, where the granules segregate from one another into bands as shown in Figure 2-4. Although all the variables associated with the axial band formation are not totally understood, axial segregation for given binary granules will only occur in certain setups. Depending on granular materials axial segregation will only occur at specific speeds, cylinder lengths, and cylinder diameters. Band formation is usually visible within minutes and narrow bands merge into larger bands as time elapses known as coarsening [23]. Rapaport found that even if performing the same experiment with the same setup and particles, it is unlikely that the same band patters and band sizes will occur [23]. De-mixing or also known as reversible axial segregation is possible in certain experiments. Hill and Kakalios performed many tests studying de-mixing and found in certain tests simply changing the rotation speed granules that form segregation bands will return to a homogenous mixed state [12].

Figure 2–4 Axial segregation of rice and black beans shown in Gebrehiwot's experiment [1]

2.1.4 Energy dissipation in granular segregation

Although there has been a lot of research on axial segregation over the last 70 years, there has been little research on the relationship between energy and axial segregation. In the experiments presented in this paper all energy comes directly from the wall of the rotating cylinder transferred from mechanical energy. This energy rotates the cylinder and is inputted to the granular particles. Before the granules free fall is when they have the maximum potential energy and rotational kinetic energy. To better understand the relationship of power and segregation, Gebrehiwot performed experiments that measured torque, rotational speed, and dynamic angle of repose for rice and black beans while being tumbled in a rotating cylinder. Experiments were performed that measured these parameters while rotating each of the granules in a half filled cylinder. Then other experiments were conducted with equal amounts by mass of each granule in an unmixed state as well as a mixed state. Gebrehiwot concluded that energy dissipated only in the mixed experiment in which axial segregation occurred [1]. The same setup and materials are used in this research as that of Gebrehiwots. Overall power consumption is calculated from motor speed and torque since:

$$P = \tau \times S \times \frac{2\pi}{60} \qquad (1)$$

Where P = power (watt), τ = Torque (Nm), and S = rotational speed (RPM)

2.2 Frictions (particle to particle, particle to tube)

During experimentation, there are frictional losses that occur due to particle-to-particle interactions as well as particle to tube. Although experiments have been performed to understand the effect of particle-to-particle friction on segregation, no clear-cut results were found [15]. There have also been many experimental studies on particle to tube friction. It was discovered that when the friction coefficient of the large particle and cylinder wall was greater than the small particle and cylinder wall, axial segregation occurred regardless of the ratio of the large and small inter-particle friction [15]. It was also observed that if the friction coefficients were equal for the large and small grain particle-wall, axial segregation occurs if the large grains have a higher inter particle friction coefficient. The same results were found in experiments performed by Ram Chand et al [22]. End-bands are also due to the face that the tube caps have a different frictional property then of the grains [10]. To test the effect that wall friction has on segregation A. Santomaso et al mixed calcium carbonate and granulated cellulose in a horizontal cylinder with different wall materials of smooth PVC, rough rubber, and rubber spiral [24]. The materials were rotated for 90 minutes. Figure 2-5 shows that the wall material has a significant effect to banding patterns.

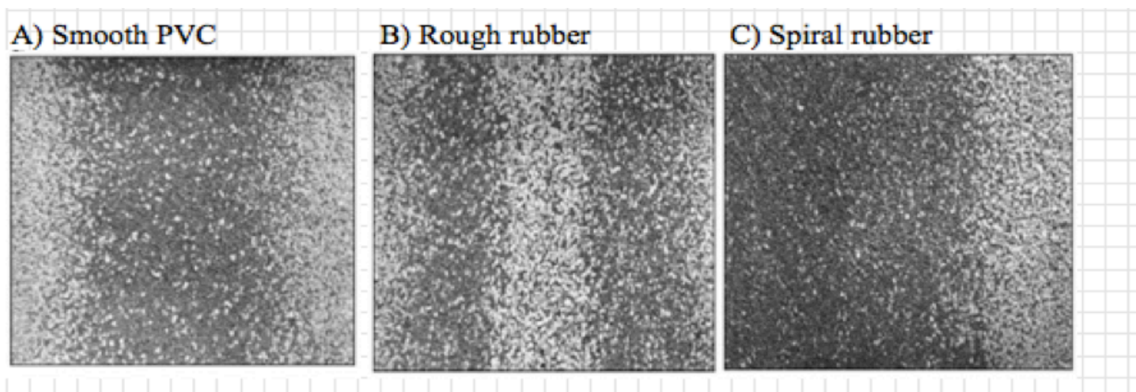


Figure 2–5 Band formations due to mixing calcium carbonate and granulated cellulose for 90 minutes with different end wall materials [24].

2.3 Coefficient of restitution (C.O.R.)

The C.O.R. is the decimal fractional value that represents the ratio of velocities before and after of two colliding entities [25]. Understanding how a granule acts after it has collided with a similar or different granule could be of great importance in later research. After avalanching in the rotating cylinder granules will act differently depending on which type of granule it comes into contact with first. Figure 2-6 is a graph by Mangwandi et al that relates the C.O.R. to the impact velocity for binderless granules. As the C.O.R. decreases for binderless granules the impact velocity also decreases [28]. The formula for the Coefficient of restitution is given by:

$$C_R = \frac{V_b - V_a}{U_a - U_b} \qquad (2)$$

Where: C_R = coefficient of restitution, V_a = final velocity of the first object after impact, V_b = final velocity of the second object after impact, U_a = initial velocity of the first object before impact, U_b = initial velocity of the second object before impact.

The effect of the coefficient of restitution has on granular segregation has not been researched much and this paper briefly touches upon it. Understanding the C.O.R. values for the granules may help determine flow regimes and displacement.

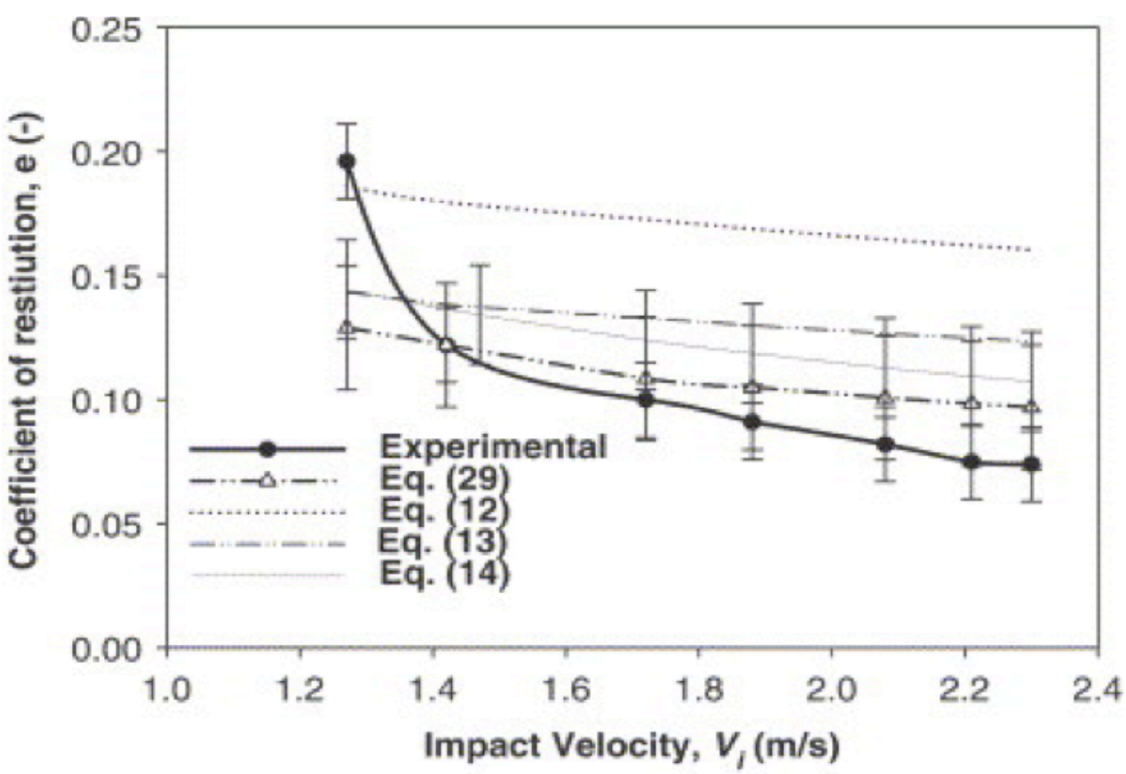


Figure 2–6 Relationship for C.O.R. to impact velocity for binderless granules [26]

Although the C.O.R. is intended for direct hits and our experiment deals with glancing hits it may still be of interest in later research. During rotation of the cylinder and after a particle has completed an avalanche it does not completely come to a stop. In the rice and black beans experiment the rice particles tend to cause more “chaos” as they move more radically then the larger black beans. When the larger black beans complete an avalanche cycle they seem to fall to a stop almost like a golf ball in a sand trap. As for the rice particles they often bounce around even after they complete their avalanche cycle almost like when a golf ball hits a larger rock. Although how particles react when they come into contact with just glancing hits is not directly related to C.O.R., future research on how particles react after they complete their avalanche cycle should be investigated.

3 EXPERIMENTAL METHODS AND PROCEDURES

This chapter describes the experimental methods used in this research and how granule properties such as density, repose angle, and size were determined. The granules used were rice, black beans, barley, safflower seeds, lentil, and pinto beans. Power consumption was monitored throughout the experiments by measuring torque and speed applied to the rotating cylinder. Pictures were taken throughout each experiment to analyze if energy dissipation occurred to specific visual changes such as radial segregation or axial segregation. Each granule was subjected to rotation in the horizontal cylinder separately as well as with other granules. When two granules were subjected to rotation two separate tests were done, one with the granules mixed and one with them unmixed. For the binary granule experiments a weight ratio of 50:50 was used, in most cases making the cylinder half full. A separate experiment was conducted to study the "bounce" of granules after they have come in contact with the same type of granule and a different type of granule. Finally another separate test was performed to study how friction effects radial and axial banding. In this test rice and black beans were mixed and placed in women's nylon leg hosing where the particles were not able to perform the avalanche effect freely.

3.1 Experiment equipment and set up

A clear acrylic rotating cylinder shown in Figure 3-1 is used for the experiments, which is 4ft (1.22m) in length and has an inside diameter of 11.38" (0.29m). Both ends of the cylinder have acrylic disks with one side that is clamped and fastened by screws.

This is referred to as the “right” side throughout this paper and this side is also where granules are fed and discharged. The other side referred to as the “left” side is fully closed and connected to a torque sensor. Before each experiment a bubble level was used to make sure the cylinder was level.

Figure 3–1 Acrylic-rotating cylinder used for all experiments

Torque is measured by an Omega TQ 513 torque sensor, which has a 200in-lb (23Nm) maximum reading. A 12V DC excitation is used that has a corresponding nominal reading of 24V. The sensor has a full-scale output reading of 2.0 mV/V nominal. Whenever the reading is below 40mV, it is linearly related to the torque. Therefore when the sensor reads 13 mV DC it will equal 110.5 lb-in (12.48 Nm). The torque sensor has a four-pin connector for both input and output readings. Impulses are sent to an Omega 2401 full speed thermocouple/voltage input data acquisition module. The Omega 2401 converts the impulses to mV and captures the pulses every second to the DAQ Central software. The software then converts the mV values to Nm. Standard errors were calculated for all experiments. The maximum error calculated was that of rice which had a ± 0.03 Nm.

A 1/8 HP Bison 336 series DC Parallel Shaft gear motor was used as a driver, which is connected to the same shaft as the torque sensor. An AC/DC inverter is used to control the speed of the horizontal cylinder. A shaft encoder is connected to the gear motor that sends frequencies to a multimeter. The frequency could be converted to RPM as shown in the equation below.

$$RPM = \frac{f * 60\frac{sec}{min}}{P * G_r} \qquad (3)$$

where f = frequency in Hz, P = pulses (100 for current encoder), G_r = Gear ratio 81.8

Using equation 3 a frequency reading of 1,500 Hz or 1.5 KHz would convert to 11.0 R.P.M. For the purpose of this paper the speed is used in the units of KHz. Speed varied very little throughout all experiments as it ranged from 1.46 KHz (10.7 RPM) and 1.52 KHz (11.1 RPM). An insignificant 0-1% increase in speed was observed in all experiments

3.2 Experimental methods

Gebrehiwot noted nearly a 10% power consumption decrease, when mixed rice and black beans were rotated in a horizontal cylinder for 5000 seconds. There was a minimal increase in speed but a substantial decrease in torque [15]. Radial segregation followed by axial segregation was also noted. This paper looks to investigate not only the mixtures of rice and black beans but also the binary mixtures of lentil, pinto beans, barley, and safflower. Table 3-1 gives all the material properties that include granule density, shape, and volume. The static angle of repose of each material was determined using the fixed funnel method.

Table 3–1 Granular properties and results for single granule experiments

Granular Properties and Results of Single Granule Experiments						
Granule Type	**Density Kg/m^3**	**Shape**	**Volume mm^3**	**Average Torque (Nm)**	**Average Speed (RPM)**	**Average Power (watt)**
Black Beans	1,210 ± 30	Kidney shape	132 ± 8	13.98 ± 0.02	10.91	152.5
Barley	1,340 ± 30	Ellipsoid	24 ± 3	14.09 ± 0.02	10.96	154.4
Pinto	1,200 ± 25	Kidney shape	204 ± 10	13.06 ± 0.02	10.94	142.9
Rice	1,270 ± 30	Skinny oval	12 ± 2	15.46 ± 0.03	10.93	169.0
Lentil	1,230 ± 30	Flat circular	25 ± 3	13.47 ± 0.02	11.00	148.2
Safflower	805 ± 20	Oval	42 ± 5	8.05 ± 0.01	10.86	87.4

Standard errors were calculated from 5-10 tests for values of density, angle of repose, and volume using:

$$SE_x = \frac{s}{\sqrt{n}} \qquad (4)$$

Where SE_X = Standard error, s = sample standard deviation, n = sample size

Although the fixed funnel method was used for calculating the angle of repose, future experiments should also calculate it by measuring the angle as avalanche occurs before axial segregation. This can be done by placing cameras at the ends of the cylinder and calculating the angle which sliding first occurs. The fixed funnel method will measure a static process and the angle as it rotates will measure a steady flow process. Doing so is significant because the angle of repose will change as speed of the rotating drum changes as explained by K. Hill [12]. The angle of repose is the same as the angle of friction and whenever the slope angle is greater then that of the angle of repose sliding occurs as in the avalanche process [27].

First experiments to find power consumption for each granule by itself were performed. A total of 78.4lb (35.56kg) of each material were placed into the rotating cylinder and tests were done. Similar experiments were performed which consisted of placing an equal mass 39.2lb (17.78kg) of two granules to the cylinder at an unmixed state. Finally an equal mass of 39.2lb (17.78kg) of two granules that were mixed was placed in the cylinder. The cylinder was then rotated at speeds between 10.5-11.5 RPM, controlled by an ac/dc invertor. Torque and speed were measured throughout the duration of the experiment. Pictures were taken at different time intervals to monitor the change of axial and radial segregation over the experiments. A total of 23 experiments were performed in the rotating cylinder, 6 single granule experiments, 9 unmixed binary granule experiments, 8 mixed binary granule experiments, and one experiment in which a band was manually created in the center of the cylinder initially. Table 3-2 shows the different binary granule experiments performed that investigated the effect of granule size, density, and repose angle. Although all different possible combinations weren't tested due to complexity and time consumption for each test, a good variety of each parameter was tested. Finding granules that meet the desired properties can be challenging. All the performed tests of mixed binary granules differed in either density, size, or in the angle of repose difference. More work should be done on granules that have a size ratio between 2-4. Using the fixed funnel method angle of repose values were calculated and averaged on 5 tests. The static repose angles were found to be 23.9° ±0.4° for lentil, 28.0° ±0.3° for pinto, 28.7° ±0.3° for barley, 31.1° ±0.5° for black beans, 31.4° ±0.6° for rice, and 32.4° ±0.5° for safflower.

Table 3–2 Mixed binary granule experiments

Mixed Binary Granule Experiments				
Density ratio	**Size Ratio**	**Angle of repose difference**		
		Small (0-1)	Medium (1-3)	Large (3-6)
Small (Ratio 1.0–1.2)	Small (1-2)			*Black Beans / Pinto*
	Medium (2-4)			
	Large (4-12)	*Rice / Black Beans*	*Black Beans / Barley*	*Pinto / Lentil*
Big (Ratio 1.2-2.0)	Small (1-2)			
	Medium (2-4)		*Safflower / Black Beans*	
	Large (4-12)			*Safflower / Pinto*

4 Results

4.1 Statistical approach

Standard errors of power were calculated for each of the 23 experiments. The standard error values were expected to be very small for the single granule experiments and that was the case. The highest error of power for the single granule materials was of lentil with ±0.03 W using equation 4 in chapter 3. Table 4-1 shows all the standard error values of power for each experiment. Graphs of moving average lines were used to plot torque vs time and speed vs time for all experiments. Moving average lines are important makes it easier to see at gradual and severe changes with respect to time. Graphs representing power vs time are shown using linear trendlines because the amount of change during the experiment is most important. Figures 4-1, 4-2, 4-4, 4-7, and 4-10 represent moving average line graphs of torque and speed. Linear trendline graphs are represented in Figures 4-3, 4-8, and 4-13. When there were high changes in power R^2 were also labeled using excel. Using the five step hypothesis testing it was observed that the correlation was always significant. A 0.05 level of significance was used and r critical values were looked up using a one tailed test in Pearson's correlation coefficient chart shown in Appendix B. Since sample sizes were always greater then 500 *df* values of infinity used as suggested by Pearson's chart. R^2 values were calculated using Excel spreadsheet.

Table 4–1 Standard Error for Power Measurements

Standard Error for Power Measurements					
Single Granule	Standard error ± (W)	Binary Unmixed	Standard error ± (W)	Binary Mixed	Standard error ± (W)
R	0.02	***BB / R***	0.12	***BB / R Day 1***	0.01
BB	0.01	***R / BB***	0.16	***BB / R Day 2***	0.01
B	0.02	***R / BB Reverse***	0.16	***P / L***	0.01
P	0.02	***S / BB***	0.11	***BB / P***	0.01
L	0.03	***P / L***	0.10	***B / BB***	0.01
S	0.02	***L / P***	0.11	***R / BB Band***	0.01
		B / BB	0.12	***P / S***	0.01
		S / P	0.10	***BB / S***	0.01
		P / BB	0.11		

Where R = rice, BB = black beans, B = Barley, P = Pinto, L= Lentil, S = Safflower

4.2 Single granule experiments

Six single granule tests were conducted; one for each of the granules. The granules were subjected to rotation for about 3000 seconds. Torque and speed measurements were simultaneously recorded to calculate power consumption. As expected there was only minimal torque and speed changes shown in Figures 4.1 and 4.2. The average torques were found to be, rice 15.45 Nm ± 0.03 Nm, black beans 13.98 Nm ± 0.02 Nm, barley 14.09 Nm ± 0.02 Nm, Safflower 8.05 Nm ± 0.01 Nm, lentil 13.47 Nm ± 0.02 Nm, and pinto 13.07 Nm ± 0.02 Nm respectively. Speed increased by less then 0.1 RPM for all experiments which is most likely due to the reshaping of granules as they tumble in the cylinder. Overall power consumption had no significant change during the single granule experiments. Figure 4-3 represents power consumption calculated from equation 1 using the speed and torque readings.

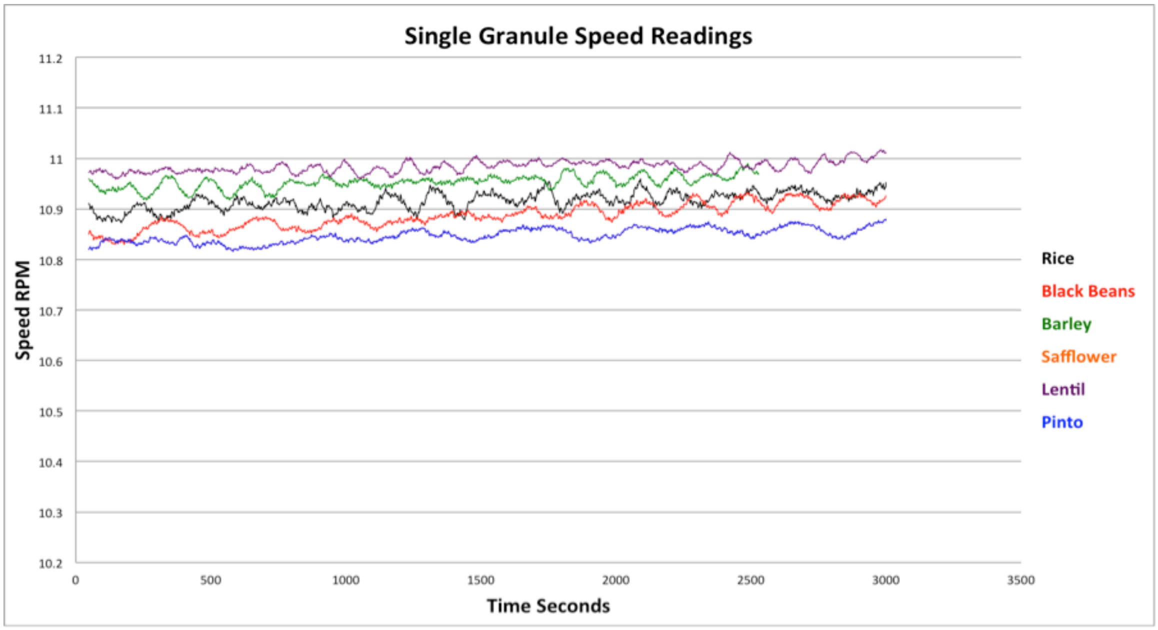


Figure 4–1 Speed recordings (RPM) for single granule experiments

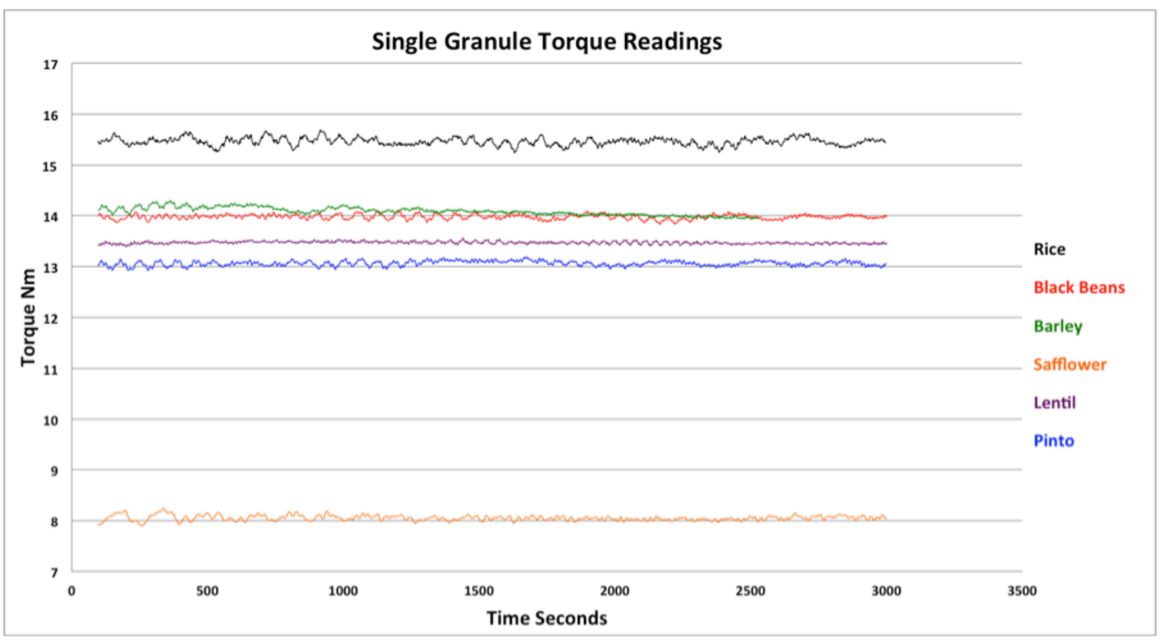


Figure 4–2 Torque (Nm) readings for single granule experiments

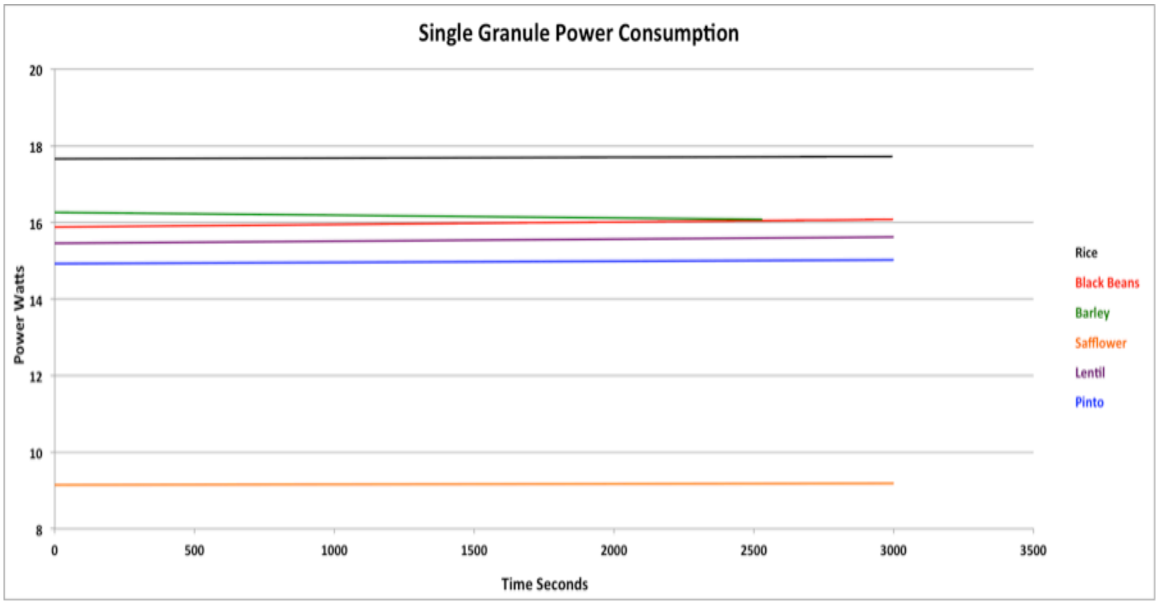


Figure 4–3 Power readings for single granule experiments

4.3 Unmixed binary granule experiments

Experiments were also conducted in which two equal quantities of granules by weight were placed into the cylinder at an unmixed state. A total of nine experiments of unmixed binary granule were performed. Figure 4-4 shows the speed for all nine experiments. When referring to an unmixed binary setup the first mentioned granule is always on the left and the second mentioned is on the right. Again speed increases but it is insignificant. It increases at about the same rate as the single granule experiments at about 0.1 RPM for every 3000 seconds.

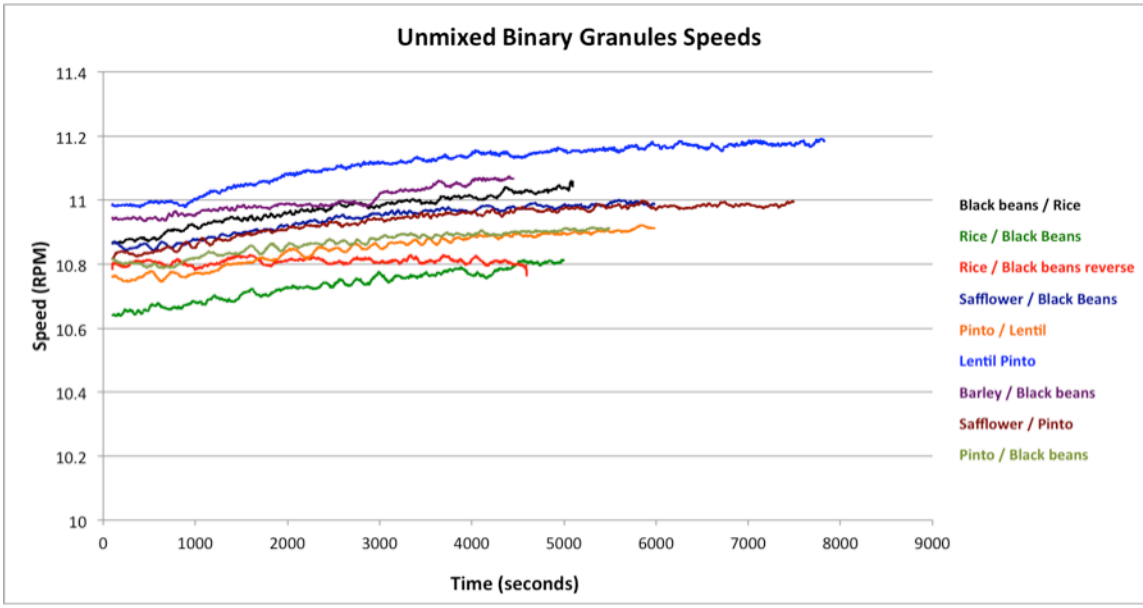
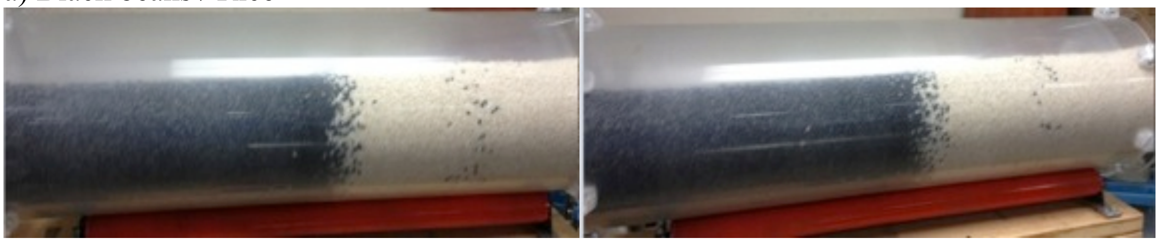


Figure 4–4 Speed recordings (RPM) for unmixed binary granules

Three of the experiments consisted of using rice and black beans. The first experiment was rice on the right / black beans on the left, second experiment was rice on the left / black beans on the right and finally the third was rice on the left / black beans on the right but reverse rotation. The purpose of these tests was to examine power change if any for each experiment, primarily torque. Secondly, it was to examine if the side in which a granule started out at or if rotation order effected segregation / power. All three experiments ran for about 5000 seconds and as seen below no mixing occurred.

a) Black beans / Rice



b) Rice / Black beans

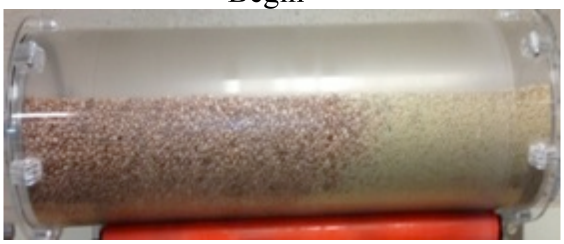
c) Rice / Black beans reversed

Figure 4–5 Begin (on left) and end of experiments (on right)

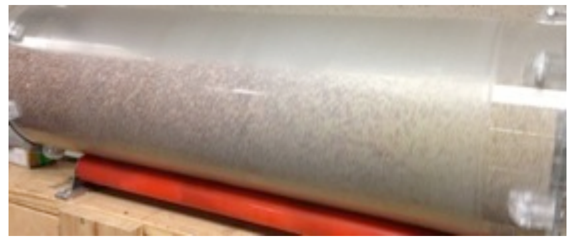
Just as there was no mixing of the two granules there was no significant change in torque as shown on Figure 4-7. The next unmixed binary granules experimented on were lintel and pinto. Two separate tests were done, one with pinto / lentil and the other lentil / pinto. In the pinto / lentil experiment there was no mixing and no changes in torque throughout the experiment. Contrary to that in the lentil / pinto experiment the pinto beans shifted to the left almost as a group as shown below. There was also a small decrease in torque in about 3% during this experiment as shown on Figure 4-7. Multiple tests were done with double-checking of the leveling bubble and the same results were observed.

Begin (left) and end (right) pictures of pinto / lentil experiment. No changes observed

Begin

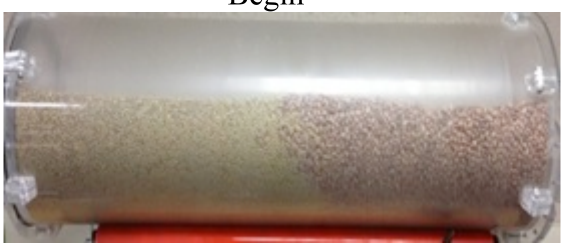


End



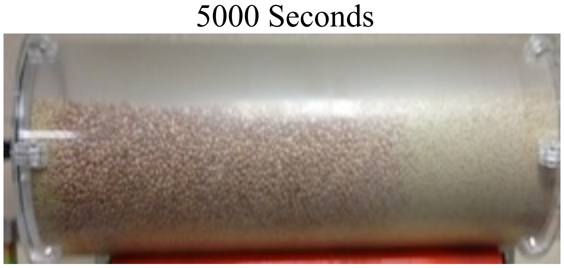
Lentil / pinto experiment, pictures taken at begin, 4500 seconds, 5000 seconds, and end. Shift of pinto beans observed.

Begin



4500 Seconds

5000 Seconds



End

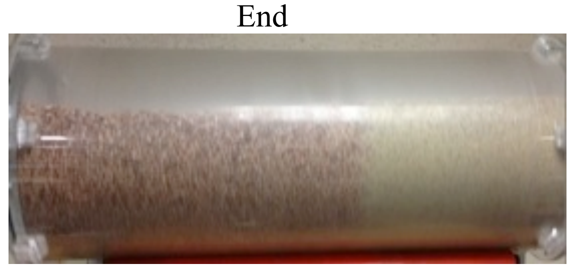
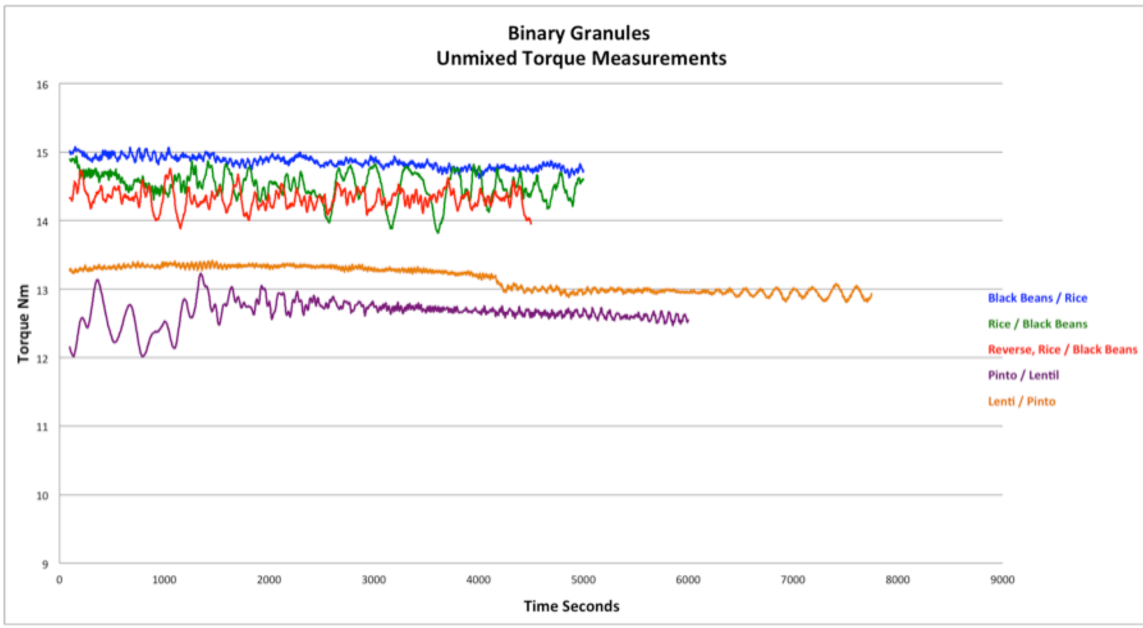


Figure 4–6 Comparison of pinto / lentil and lentil / pinto experiments. Granule shift can be seen for lentil / pinto experiment

The pinto beans shift to the left on the outside while the lentil move shift to the right through the center of the cylinder. The torque decrease is most apparent at about 4000 seconds at which the pinto beans are at the center of the cylinder and the lentils are at the outside edges. The shift in pinto beans starts at about 300 seconds but a decrease of torque isn't seen until about 3000 seconds in. In the pinto / lentil experiment high fluctuation occurs initially for about 2000 seconds but then levels off. For most of the experiments torque stayed constant but when there was some shift between granules some torque decrease was evident.



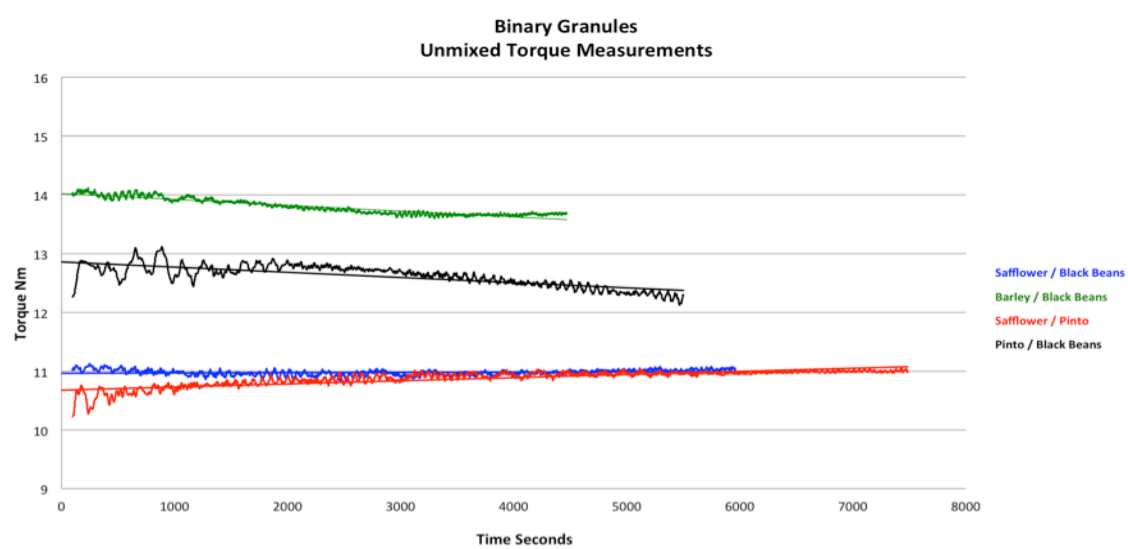


Figure 4–7 Torque readings (Nm) for all nine unmixed binary granule experiments

This shift seems to be related in the decrease of torque seen on Figure 4-7. Finally, the other four unmixed binary granule experiments were safflower / black beans, barley / black beans, safflower / pinto, and pinto / black beans. Appendix A shows images for each during the duration of the experiments. About a 3% decrease in torque is observed for the pinto / black beans experiment during which black beans form end bands. The smaller black beans move to the radial core of the cylinder and then to the ends. During the safflower / pinto experiment a 4.5% increase in torque is observed with no apparent movement or shift of granules. This experiment was performed multiple times and it isn't understood why there is such an increase in torque.

Figure 4-8 shows a summarization of power consumption for each of the unmixed binary granule experiments. A decrease of power is most evident in the experiments where most movement occurs. The safflower / pinto experiment is the first time a significant increase of power has been observed as seen from the R^2 values in Figure 4-8. Although the R^2 value (0.086) is very close 0 it still does have a significant relationship to time according to Pearson's r critical chart values shown in Appendix B. In Pearson's chart df represents the degree of freedom which is subjects minus 2. In our experiments we use infinity since our tests all have over 500 subjects. For the safflower / pinto experiment he r value of the R^2 is 0.293 and using a level of significance of 0.05 for a one tailed test with a *df* of infinity Pearson's chart shows a critical r value of 0.073. Our experiment has a r value higher then that and therefore there is a significant relationship. To summarize, the pinto and lentil experiments show that the in an unmixed experiment the side that the granules start on does impact what happens as evident in the images of Figure 4-6 and the graph in Figure 4-8. Also, the safflower and pinto experiment yields about a 5% ± 1% increase of power as shown in Figure 4-8. Both experiments were done multiple times and the same results happened. It is unclear why each happens but the shift in the lentil / pinto experiment and not in the pinto / lentil experiment suggests that there might be a set-up problem.

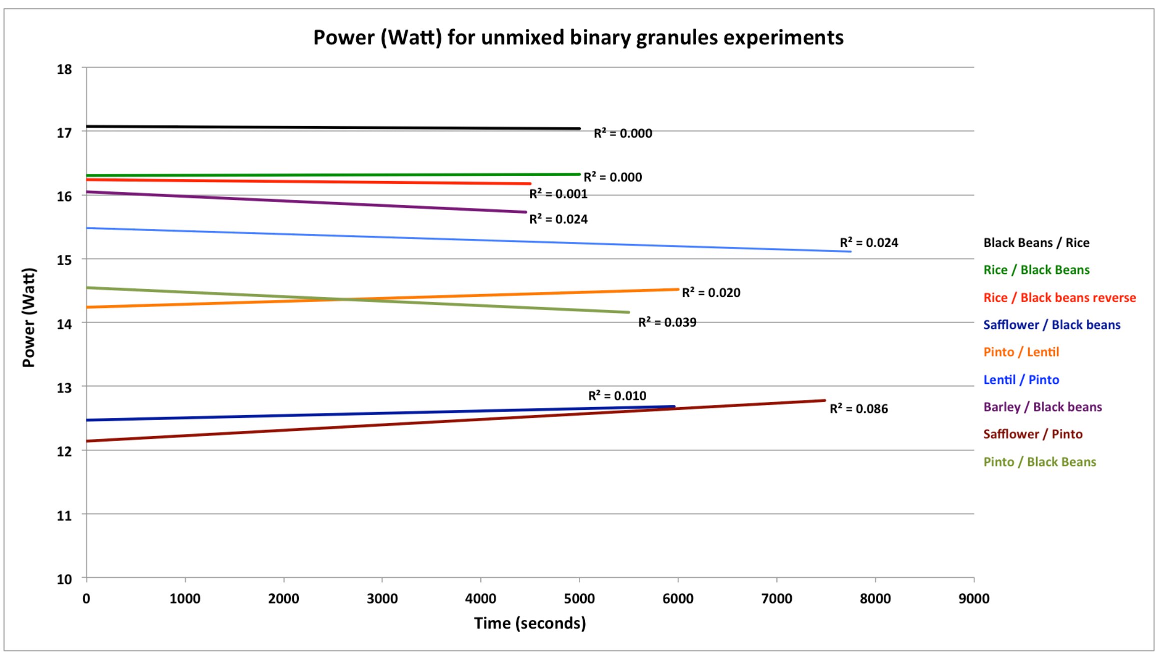


Figure 4–8 Power (Watt) for all unmixed binary granule experiments

4.4 Mixed binary granule experiments

Eight experiments were done with mixed binary granules. Tests were performed on binary mixtures of pinto / safflower, black beans / safflower, black beans / barley, pinto / lentil, black beans / pinto and two separate types of tests on black beans and rice. The two types of tests on black beans and rice were a two-day test as well as an experiment in which a band was manually created prior to rotation. Within the first few rotations radial segregation was seen in all experiments. The smaller granule concentrated to the core of the cylinder whereas the larger granules surround the smaller particles.

(a) (b)

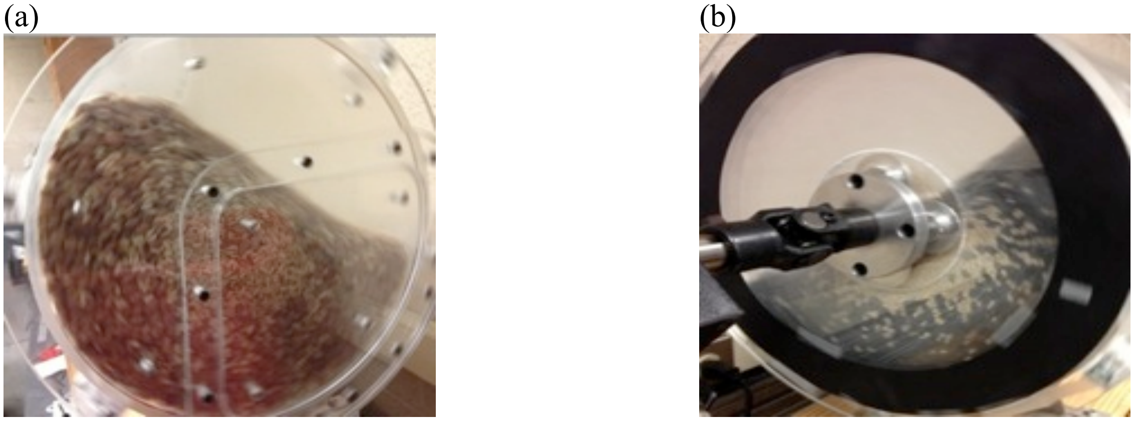
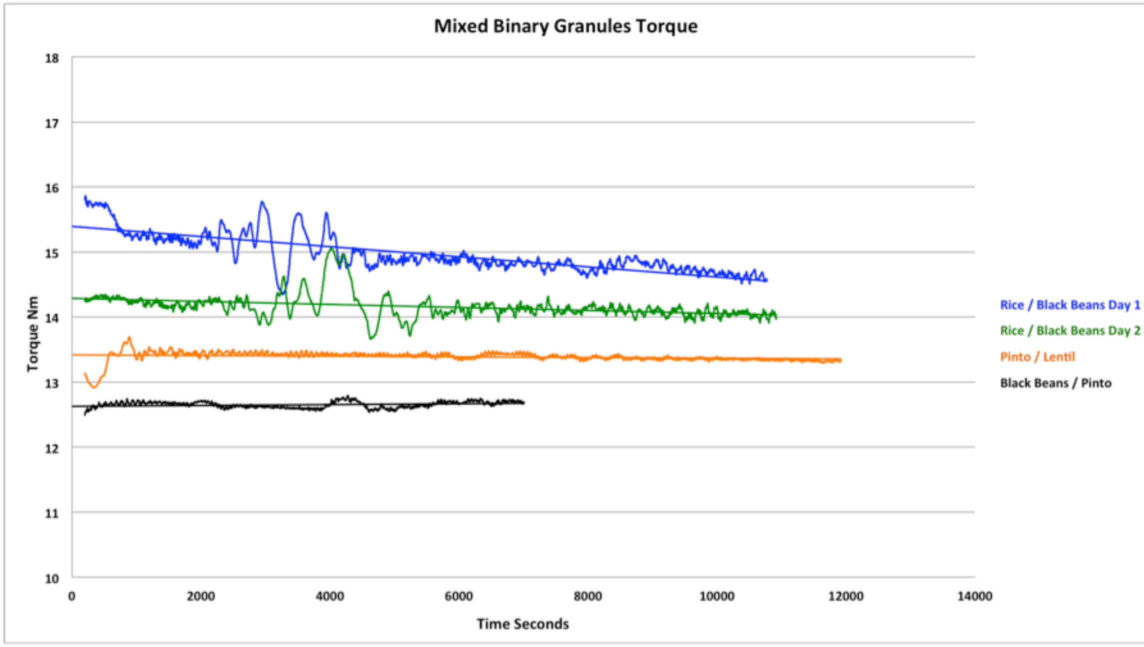


Figure 4–9 Radial segregation can be seen for a) Mixed pinto and safflower b) Mixed barley and black beans

Another thing that was the same in all experiments was that there was a very small increase in speed ≈ 1% or 0.2 RPM. This was the case not for just the mixed experiments but all experiments. The best reason for this is that the particles reshape because of particle-to-particle impact as well as particle to tube impact over time. This causes the particles to become more of a round shape and they avalanche faster. However one thing that did vary in the mixed experiments was torque. The torque changed very little (0-2%) in the black beans / safflower, pinto / lentil, pinto / safflower, black beans / pinto, and day 2 of rice and black beans. Contrary to that there was a large decrease (8-10%) in torque in the black beans / barley, rice / black beans day 1, and rice / black beans with manual formed band experiments as seen in the graphs of Figure 4-10.



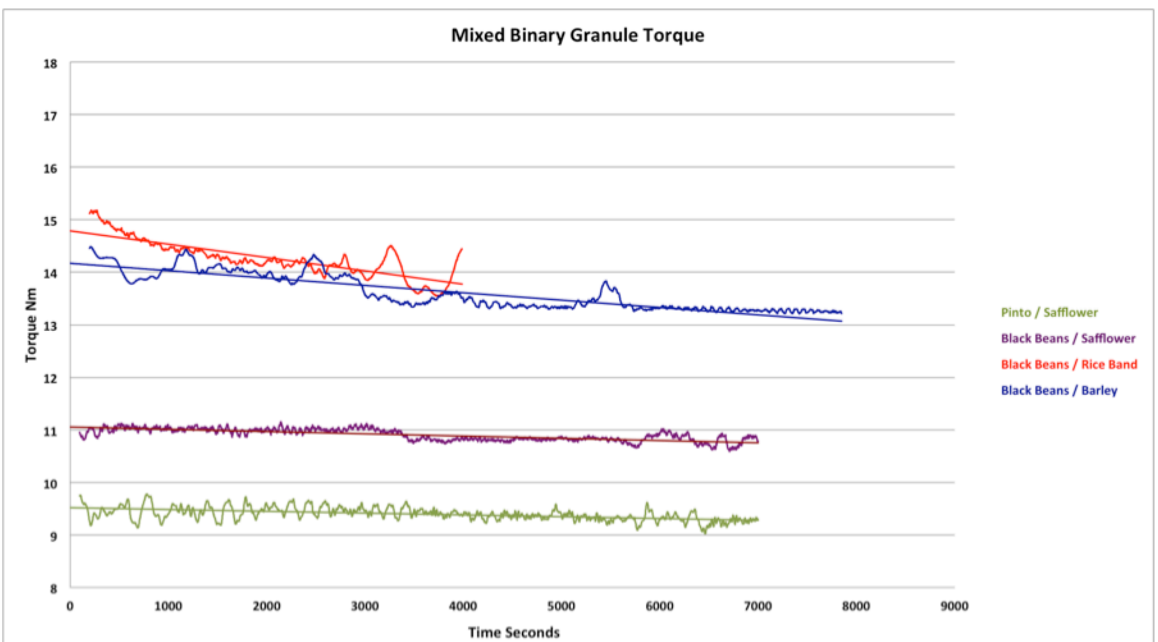


Figure 4–10 Torque (Nm) readings for mixed binary granule experiments. Power dissipation up to 10% can be seen in some experiments

So what is the difference between the experiments that have a big decrease in torque to the ones that have little or no change in torque? The difference is that when there is a high decrease in torque axial banding is clearly visible as to when there is little to no change in torque axial banding is not visible. In the pinto / lentil experiment end band formation is clearly visible but there is not axial segregation. After radial segregation where the smaller particle lentil concentrates to the core of the cylinder they then create bands at each end of the cylinder. The reasoning given for end band formation is because of frictional properties of the end of the cylinders and the friction of particles [11]. In every experiment that black beans were used, the black beans formed on the ends. Interestingly about the same decrease in torque is recorded in all three of the experiments when axial banding is visible. As shown in Appendix A the black beans / rice start out mixed and after about 1000 seconds

a) Band formation of rice and black bean 1

b) Band formation of barley and black beans

c) Band formation of rice and black beans with band

Figure 4–11 Axial bands

of rotation 2 rice bands form 6" from away each end of the cylinder. Shortly after three more rice bands form in between the two existing bands. So at maximum there are 5 rice and 6 black bean bands 4000 seconds in as shown in Figure 4-11. The bands start to merge and at the end of first day of rotation (or 10000 seconds) there are 4 rice and 5 black bean bands. The first day accounts for about 8% of the decrease in torque or ≈ 1.2 Nm. The second day of the same experiment also 10000 seconds didn't have the same changes. Although the bands merged into 3 rice and 4 black beans (that started to form at the end of the first day) there was only about a 2% decrease of torque or ≈ 0.3 Nm. This suggests that the decrease of torque is directly related to the band formations and mergers. In the mixed black bean / barley experiment there is also an 8% decrease in torque but the band formations happens differently. Two barley bands form about 4" from the ends of the cylinder and increase throughout the experiment. The experiment ends as shown in 10b with two bands of barley, two thin bands of black beans on the outside and a large band of black beans in the center of the cylinder. Lastly, an experiment was done that manually had a rice and black bean band formed as shown in Figure 4-12. In the first minute the bands started to mix and shortly

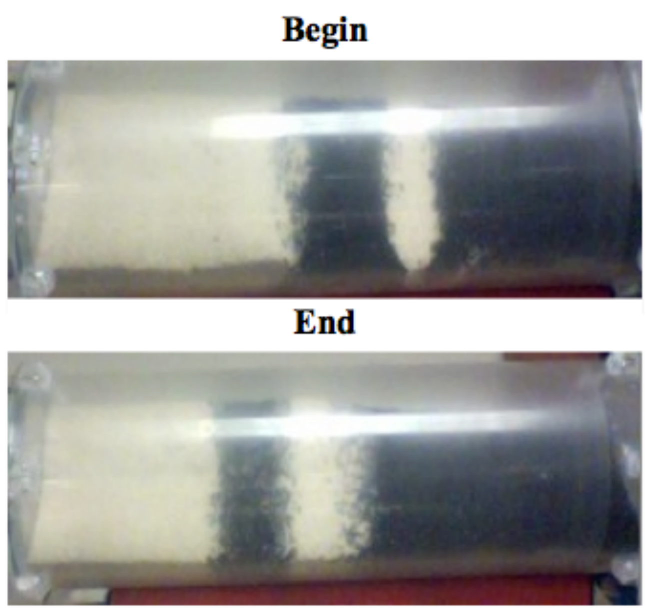


Figure 4–12 Band shift

after they segregated again forming a similar band it started out with but shifted to the left about a foot. Throughout the 4000-second experiment an 8% torque decrease was recorded. In summary when mixed binary granules are subjected to rotation and axial bands are formed, there is a significant decrease in torque and power. Figures 4-13 and 4-14 show power during the mixed experiments. From Pearson's r critical value chart in Appendix B significant relationships are calculated as explained in section 4.3.

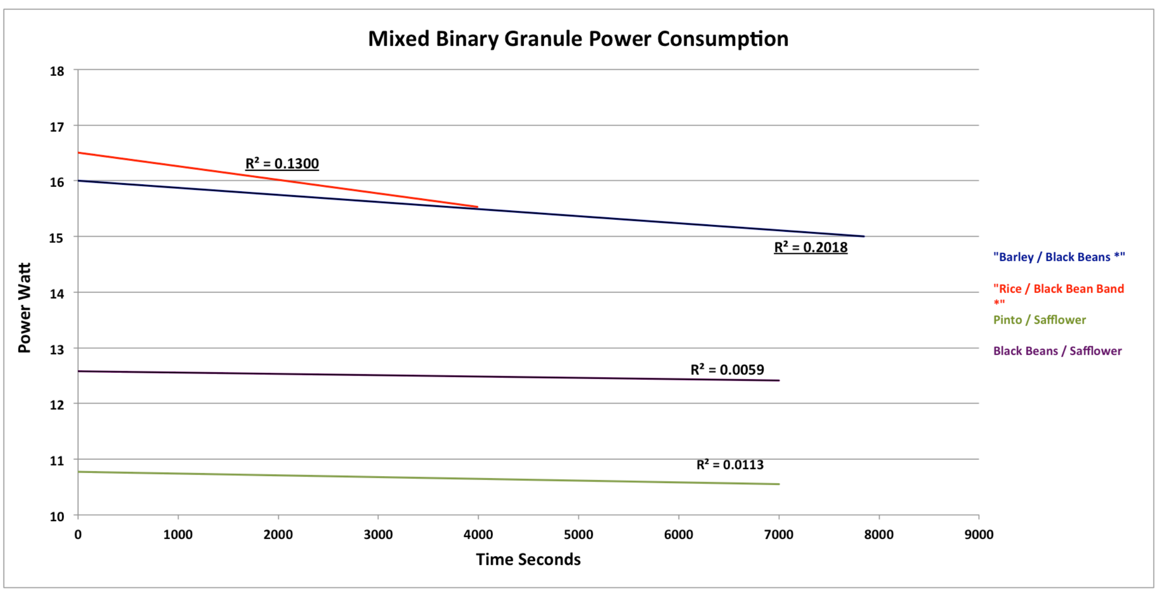


Figure 4–13 Power consumption (Watt) in mixed binary granule experiments

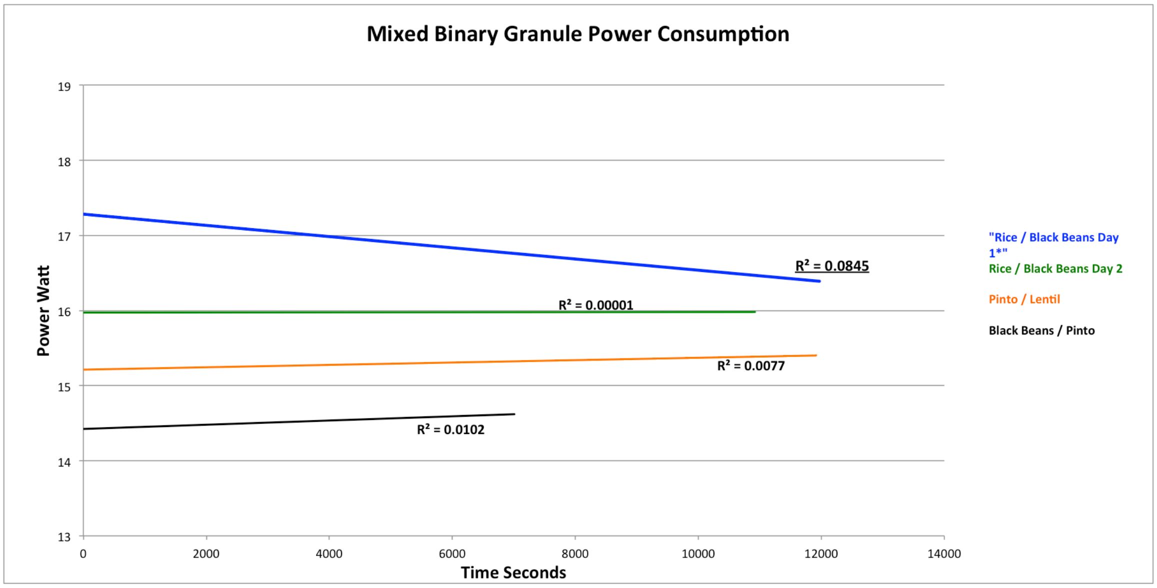
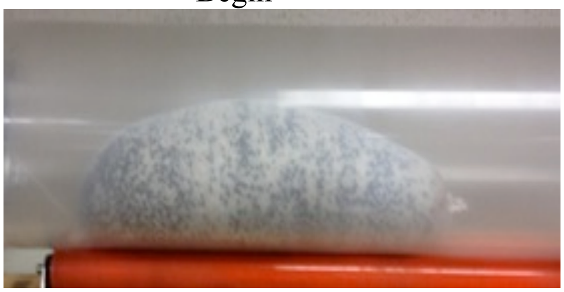


Figure 4–14 Power consumption (Watt) in mixed binary granule experiments continued

4.5 Friction effect on radial and axial segregation

Particle to particle friction as well as particle to tube frictions occurs during the rotation of the horizontal cylinder. Friction is important to segregation since it is directly related to the angle of repose [20]. Friction between particles also leads to some heat loss and can be a key contributor into energy dissipation. An experiment was done in which mixed granules were placed in a nylon hose and subjected to rotation at a speed of about 11 RPM. The purpose of this experiment was to observe the absence of an avalanche free surface and the effect it would have on radial and axial segregation. From Figure 4-15 we can see that the majority of the black beans have moved to the radial core of the nylon hose and also end-band formation has occurred on the left side. Radial segregation does occur but is very different in any of the other experiments because the larger of the two granules has moved to the core.

Begin



End (4500 seconds)

Figure 4–15 Nylon hose experiment (absence of an avalanche free surface)

4.6 Bounce effect

A separate test was done to measure the distance a grain would travel if it were dropped into a container of different grains. A 12” x 12” container was filled with one granule type and a different grain was dropped one at a time from 8 inches. When a larger granule was dropped into the container with the smaller granule, the larger granule would just fall into its place and move very little (1-2 inches). On the other hand when a smaller granule was dropped into the container with a bigger granule, the smaller granule would bounce around a couple of times and move erratically (8-10 inches). Another test was done where a grain was dropped into a container with the same type of grain, the grain would move very little (1-3 inches). Although smaller particles will not always be more mobile to larger ones based on their material properties, all the granules in this experiment suggests that the smaller particles are more mobile. The free fall of a granule on a straight path doesn't directly apply to the avalanche effect so further experimentation on the movement of granules after they complete the avalanche should be studied.

5 SUMMARY AND FUTURE WORKS

5.1 Summary

The mixed binary granule experiments agree with prior experiments that radial segregation can occur if two granules have significant density differences but axial segregation will not occur [2,7]. They also demonstrate that energy dissipation is directly related to axial segregation. Only when there was a decrease in torque did axial bands form. The amount of bands formed or the pattern of axial bands does not seem to affect the amount of energy dissipated. Also, in the unmixed experiments with pinto / lentil and lentil / pinto the pinto beans always ended on the left. It is unclear why this occurs but a decrease in torque was only seen when there was movement and the pinto beans shifted to the left. Based on the results of the current experiments, I believe that energy dissipates as a result of axial segregation. The results for power change in the mixed rice / black beans, mixed barley / black beans and the rice / black beans band experiments lead me to this assumption. In all three of those experiments torque and speed all decrease about the same value but the band patterns and sizes are very different. If axial segregation was a result of energy dissipation then the values of torque decrease would vary based on the amount of bands formed and their sizes. I believe that axial segregation is due to the "right" combination of repose angle, friction (granule-granule, granule-cylinder), size and density ratio. The friction angle is significant since it determines which granule will form bands at the end sections. For every experiment that had black beans as one of the binary granules, the black beans formed at the ends of the cylinder. Videos of each experiment were recorded and filed at M.T.S.U.

KEY POINTS

- Significant power dissipation is recorded when axial segregation occurs as shown in Figures 4-13 and 4-14
- Density ratio of granules must be between 1 and 1.2 for axial segregation to occur from Table 4-1.
- Axial segregation only occurred when size ratio was greater than 4 as shown in Table 4-1
- In the absence of the avalanche effect radial segregation did occur (large granule to the core) but axial segregation did not, shown in Figure 4-15 (nylon hose experiment)
- Axial segregation occurred both with the similar and different static angle of repose as shown in Table 4-1

5.2 Recommendations for future works

It is evident that axial segregation causes a decrease in power consumption. To better investigate the overall effects of density, size, and static repose angle more experiments should be done to fill in void areas of Table 3-2 as it shows that only 6 out of the possible 18 combinations of experiments were performed. However, this research agrees with prior studies that there must be a size ratio greater then 2 and the granules must have similar density for segregation to occur. More experiments must be performed to understand what the effects of static and dynamic angle of repose have on segregation. I believe that power dissipation occurs because of axial segregation that depends on specific granular and set up parameters. From the mixed rice and black bean with band experiment it is observed that the band mixes and shifts over to the left where it requires less torque to rotate. The shift in the axial band caused a power dissipation and this was the same case in the unmixed lentil / pinto experiment where power dissipation was observed as the pinto beans shifted to the left.

It is not one particular granular property such as density, size, repose angle or one experimental set up parameter such as length of cylinder, diameter of tube or speed that determines axial segregation but it is a "right" combination it needs to occur. Future experiments using different parameter combinations will help understand why segregation occurs. Would desegregation cause an increase in power consumption? After bands have formed they should then be desegregated by simply increasing or decreasing speed [4]. Future work should also inspect the effect how granules "react" when colliding during the avalanche effect. Another experiment that will help determine if in fact torque causes segregation would be to create a setup that doesn't change torque to the system but instead automatically adjusts speed. If segregation still occurs and there is no significant change in power then it is solely material properties that cause the segregation. On the other hand if this setup causes a significant decrease (6-10%) in speed, which would cause a significant decrease in power, then power has a big role in segregation. Finally, if no segregation occurs with any significant decrease in torque and speed then torque decrease is significant to segregation. Any future work should investigate the effects of static repose angle and the repose angle during rotation.

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APPENDIX A

Unmixed Black beans / Rice Experiment

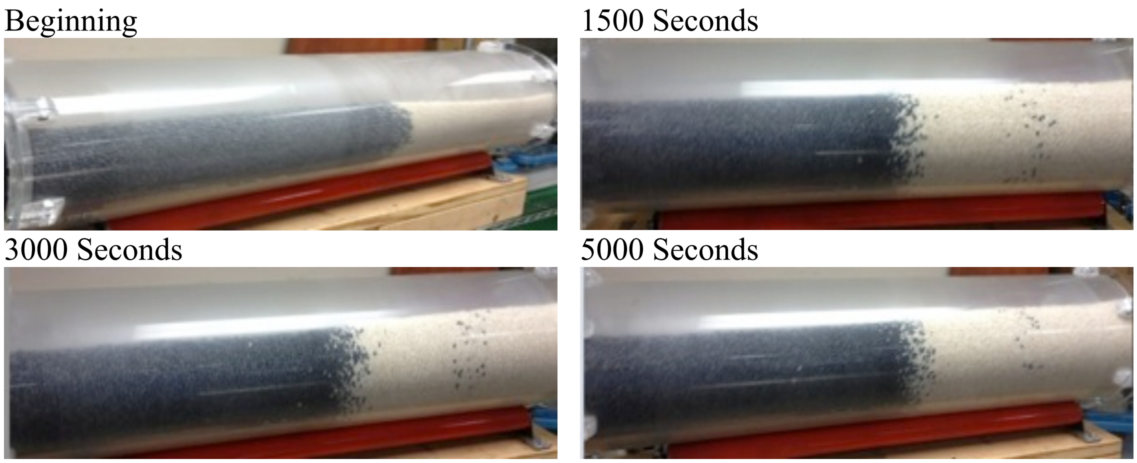


Figure A- 1 Unmixed black bean / rice experiment no changes observed

Unmixed Rice / Black beans Experiment

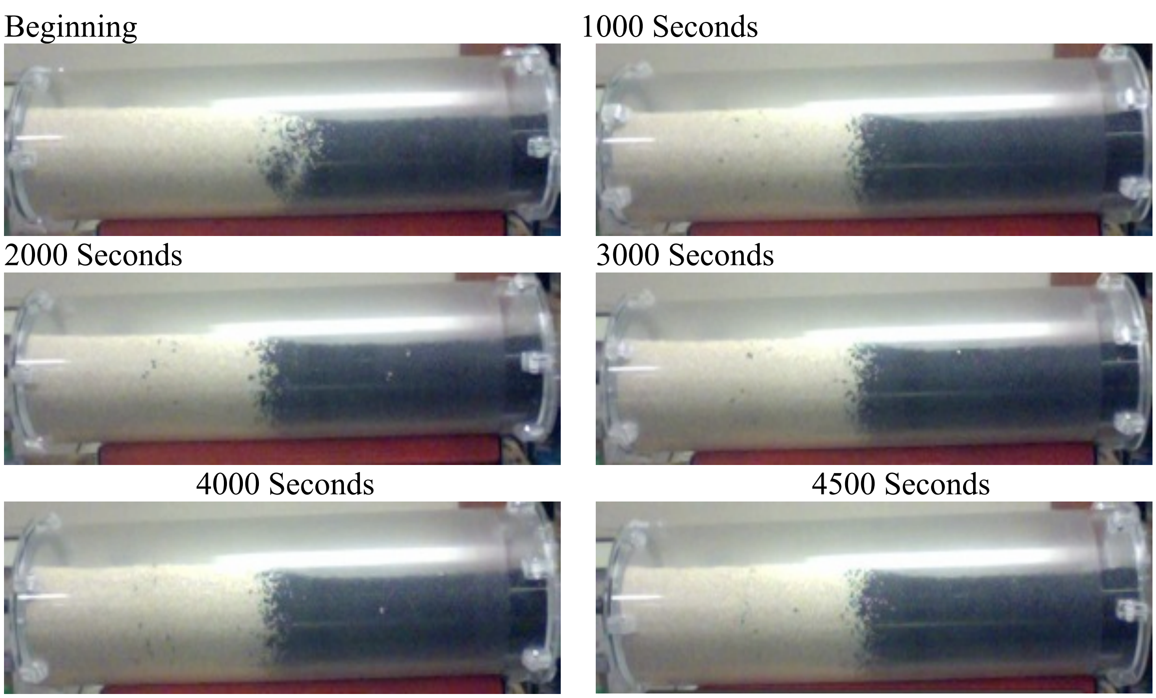


Figure A- 2 Unmixed rice / black beans experiment, no changes observed

Unmixed Rice and Black Beans **Rotating Reversed** Experiment

Beginning

1000 Seconds

2000 Seconds

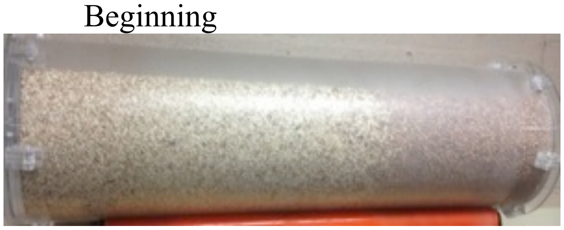
3000 Seconds

3700 Seconds

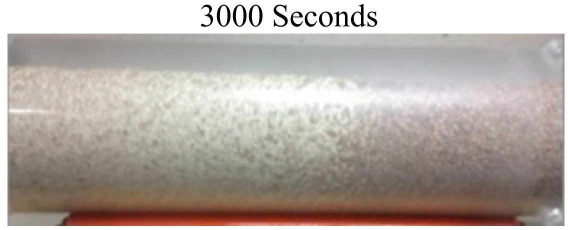
Figure A- 3 Unmixed rice / black bean experiment with reverse rotation, no changes observed

Unmixed Pinto and Safflower Experiment

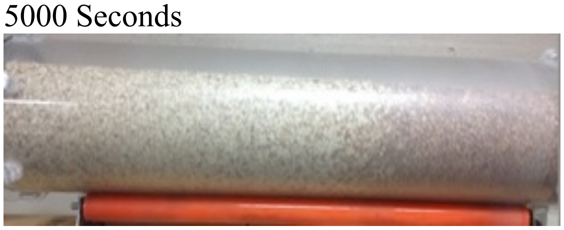
Beginning



3000 Seconds



5000 Seconds



7500 Seconds

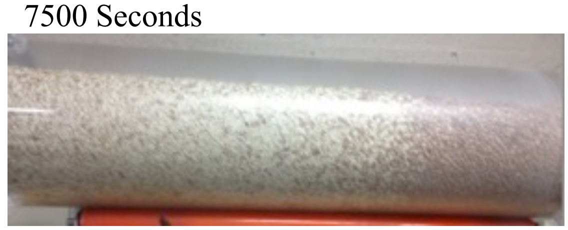


Figure A- 4 Safflower / pinto experiment, no changes observed

Unmixed Lentil and Pinto Experiment

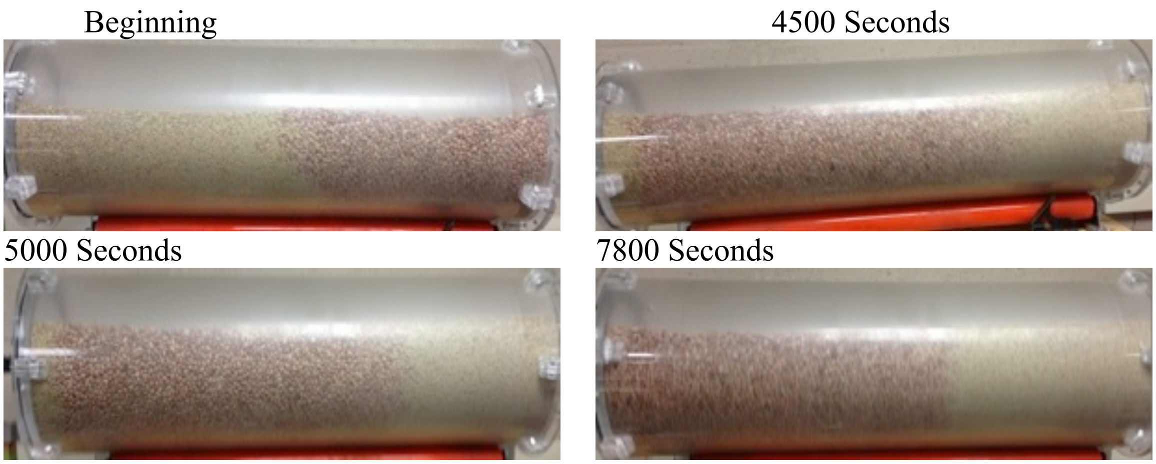


Figure A- 5 Unmixed lentil and pinto experiment, granules swap sides after 7800 seconds

Unmixed Pinto and Lentil Experiment

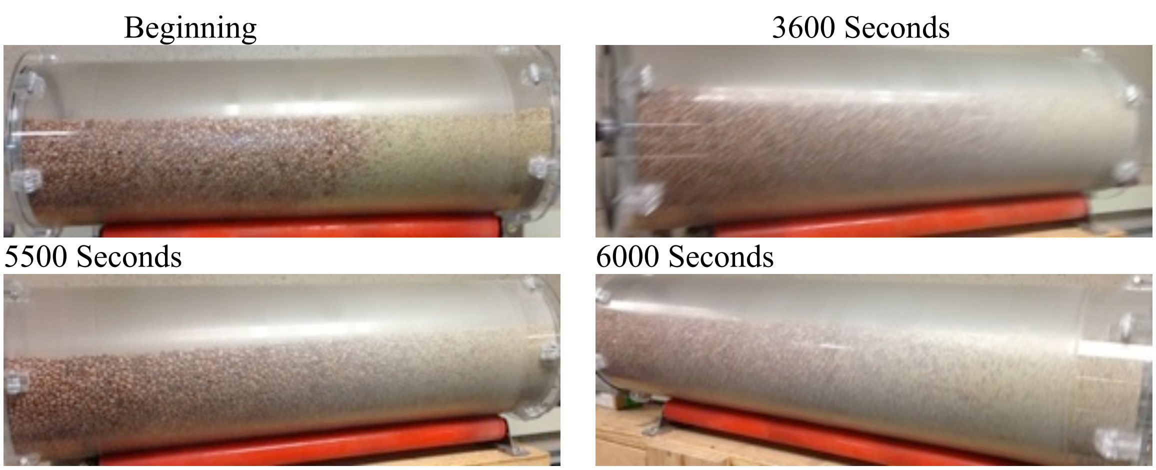


Figure A- 6 Unmixed pinto and lentil experiment, no changes observed

Unmixed Rice and Barley Experiment

Beginning

2700 Seconds

4500 Seconds

Figure A- 7 Unmixed rice and barley experiment, black beans seen starting to shift left

Unmixed Safflower and Black Beans Experiment

Beginning

1000 Seconds

2800 Seconds

6000 Seconds

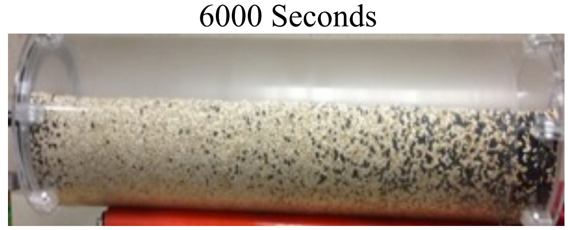


Figure A- 8 Unmixed safflower and black beans experiment, black beans seen starting to form end bands

Unmixed Pinto and Black Beans Experiment

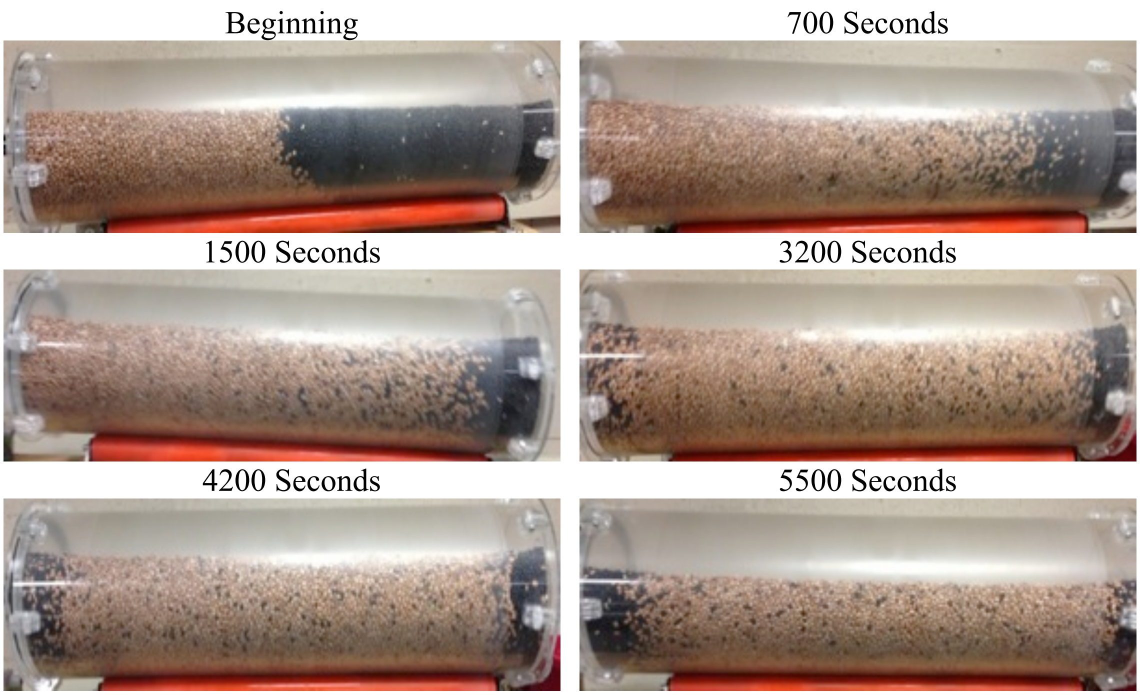


Figure A- 9 Unmixed pinto and black beans experiment, black beans seen forming end bands

Mixed Rice and Black Beans Day 1 Experiment

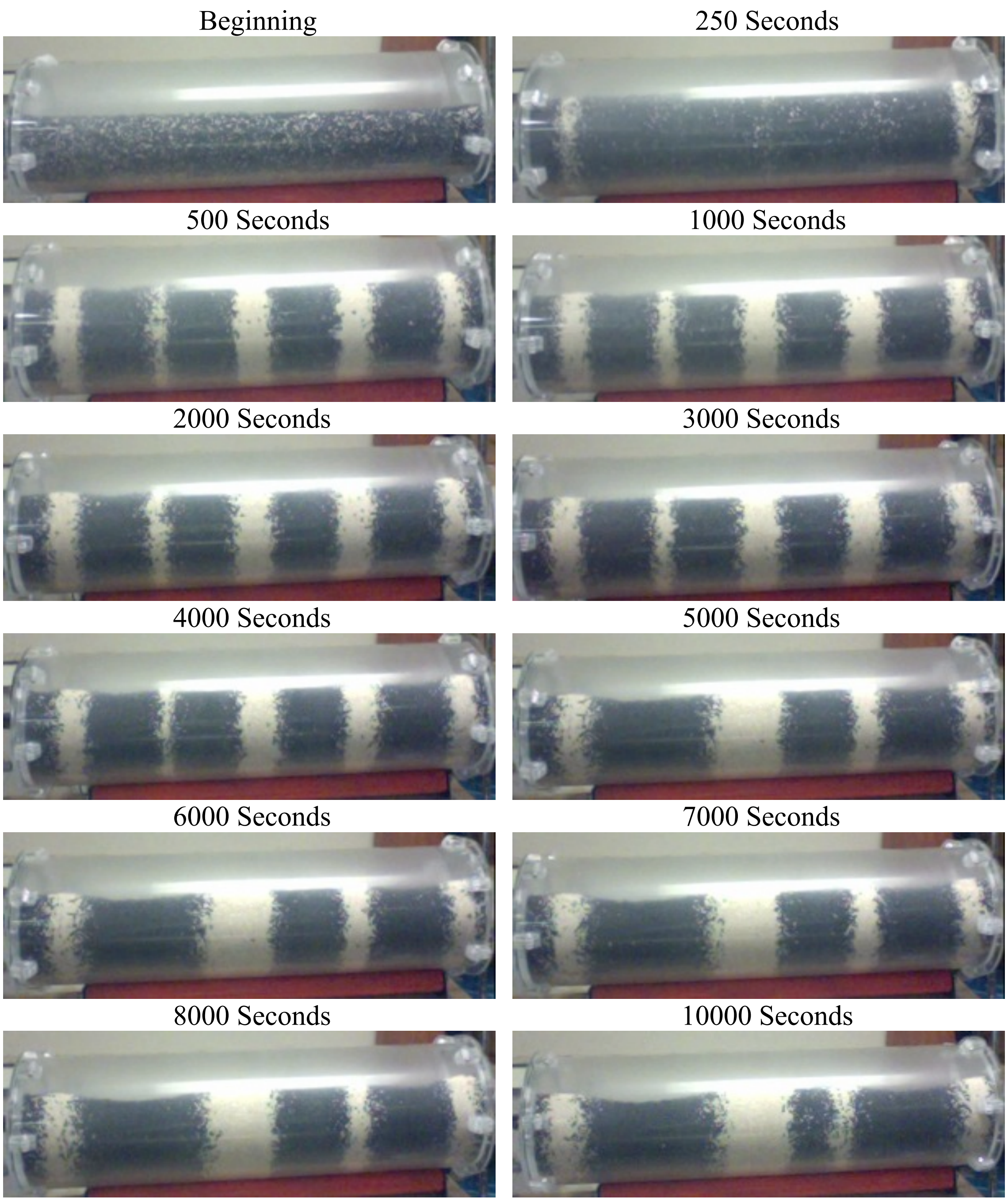


Figure A- 10 Mixed rice and black beans Day 1 experiment, axial bands are formed within minutes then they merge after 6000 seconds

Mixed Rice and Black Beans Day 2 Experiment

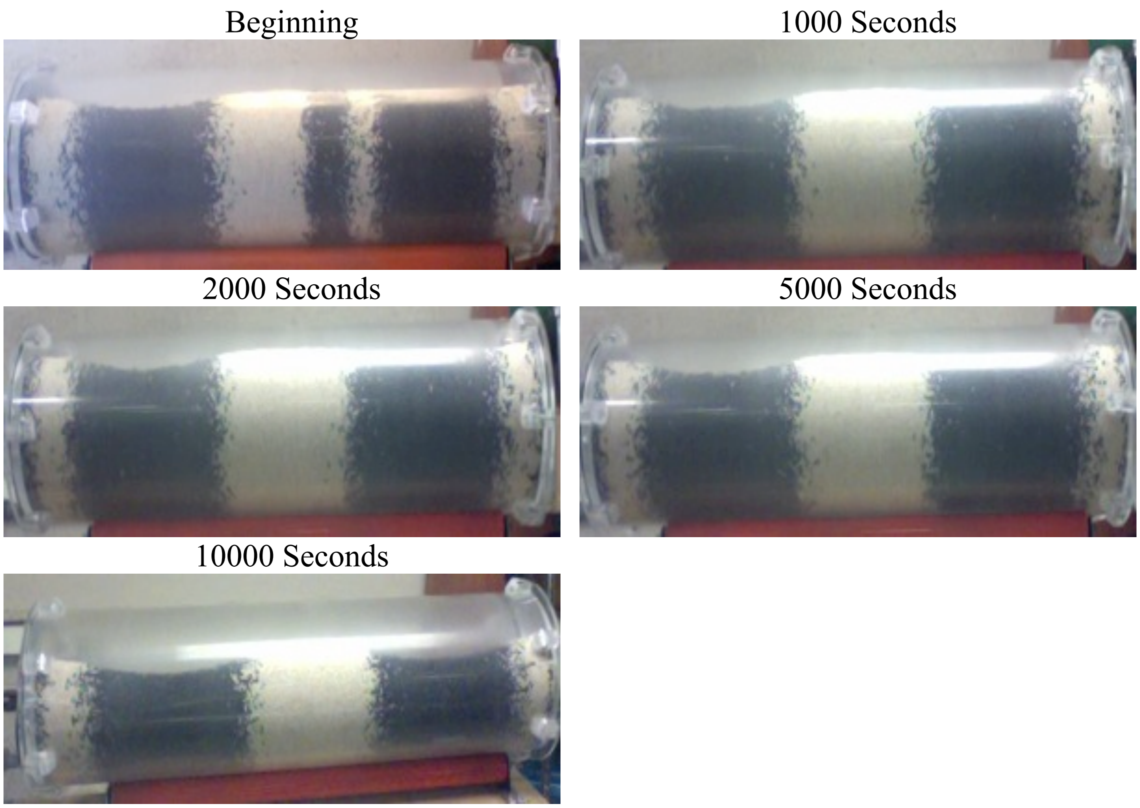


Figure A- 11 Mixed rice and black beans day 2 experiment, band merging continues for about 2000 seconds then bands stay constant for remaining 8000 seconds

Mixed Pinto and Lentil Experiment Experiment

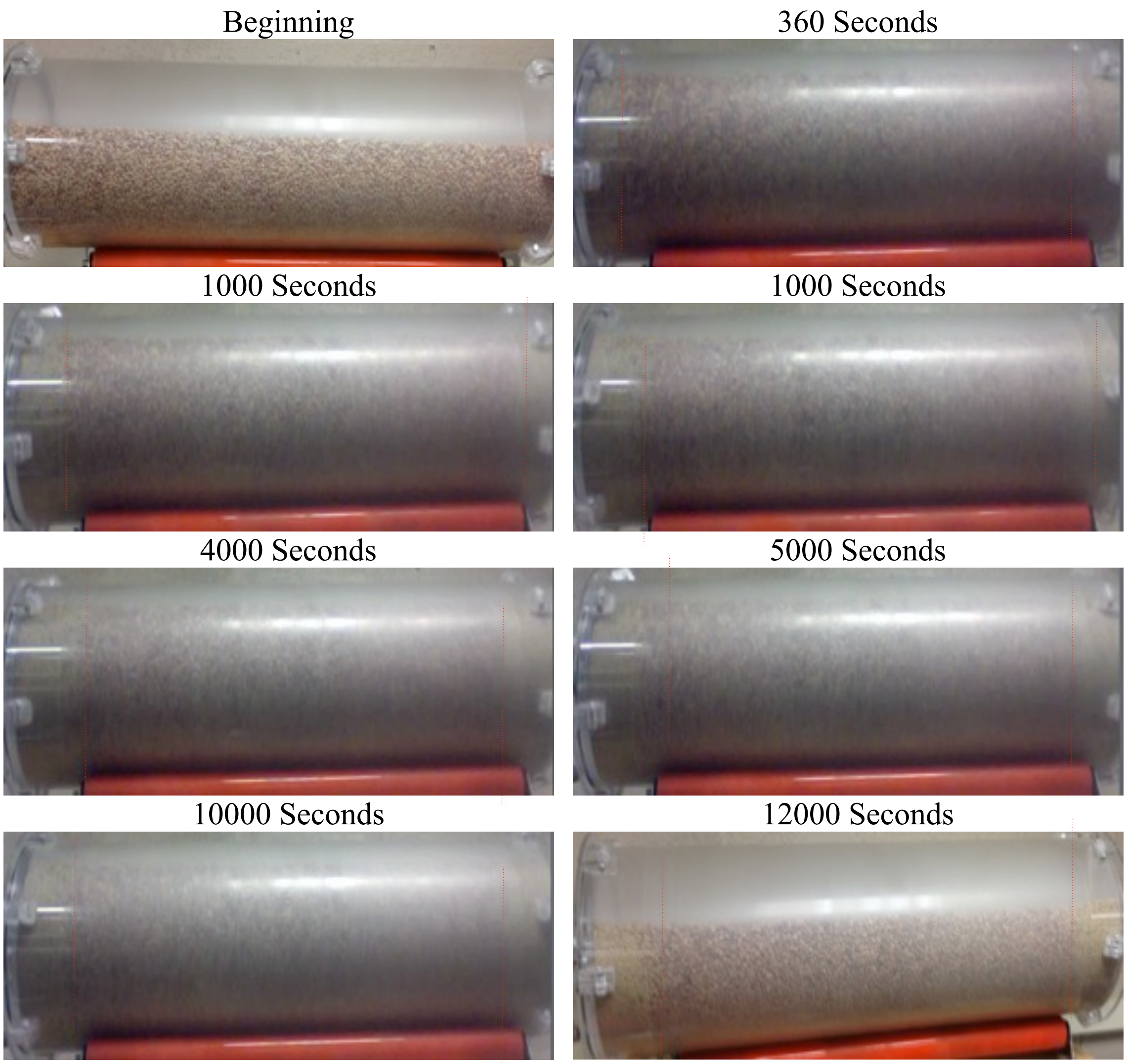


Figure A- 12 Mixed pinto and lentil experiment, lentil seen forming end bands

Mixed Pinto and Safflower Experiment

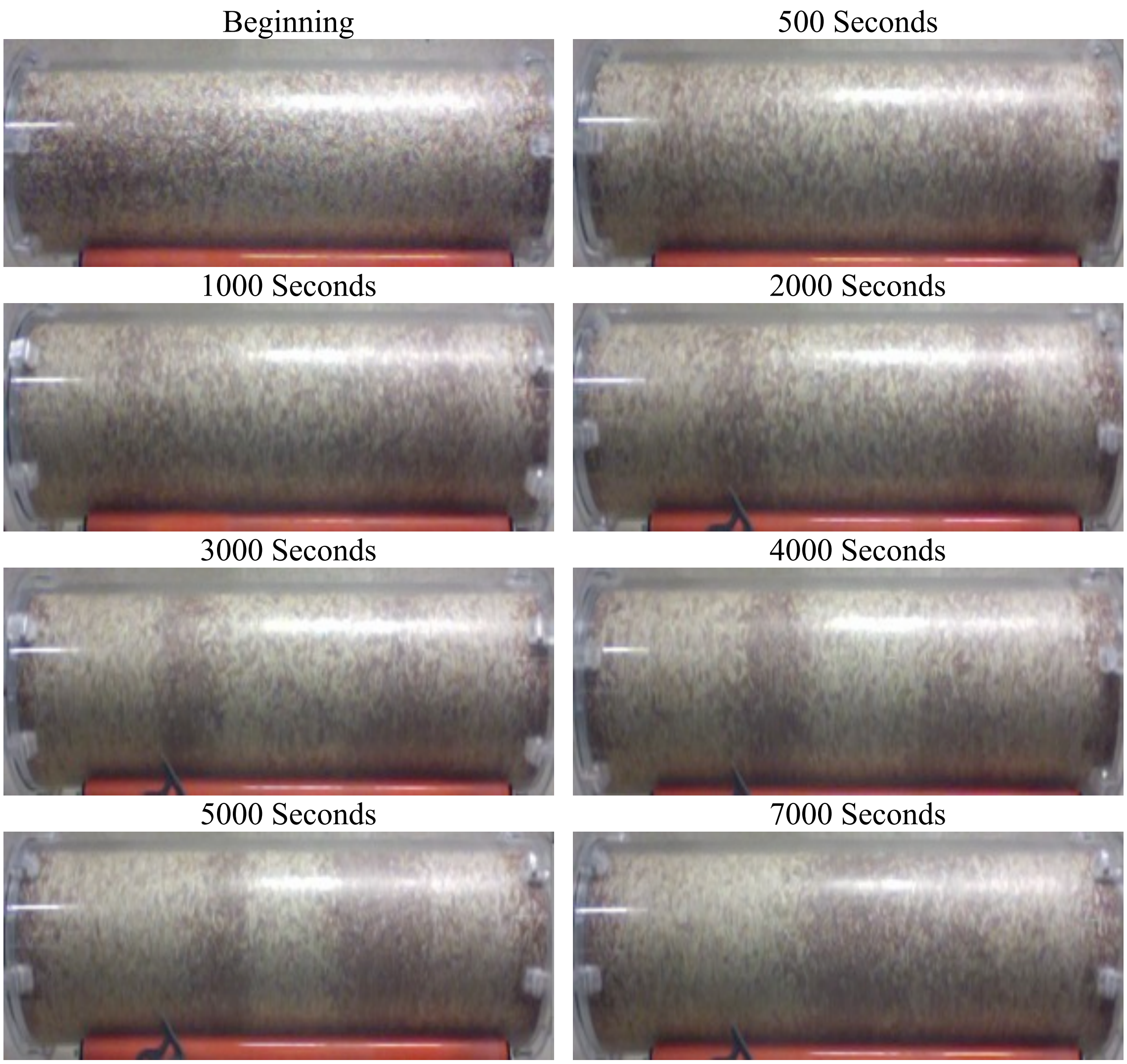


Figure A- 13 Mixed pinto and safflower experiment, traces of banding seem to appear but no clear banding observed

Mixed Safflower and Black Beans Experiment

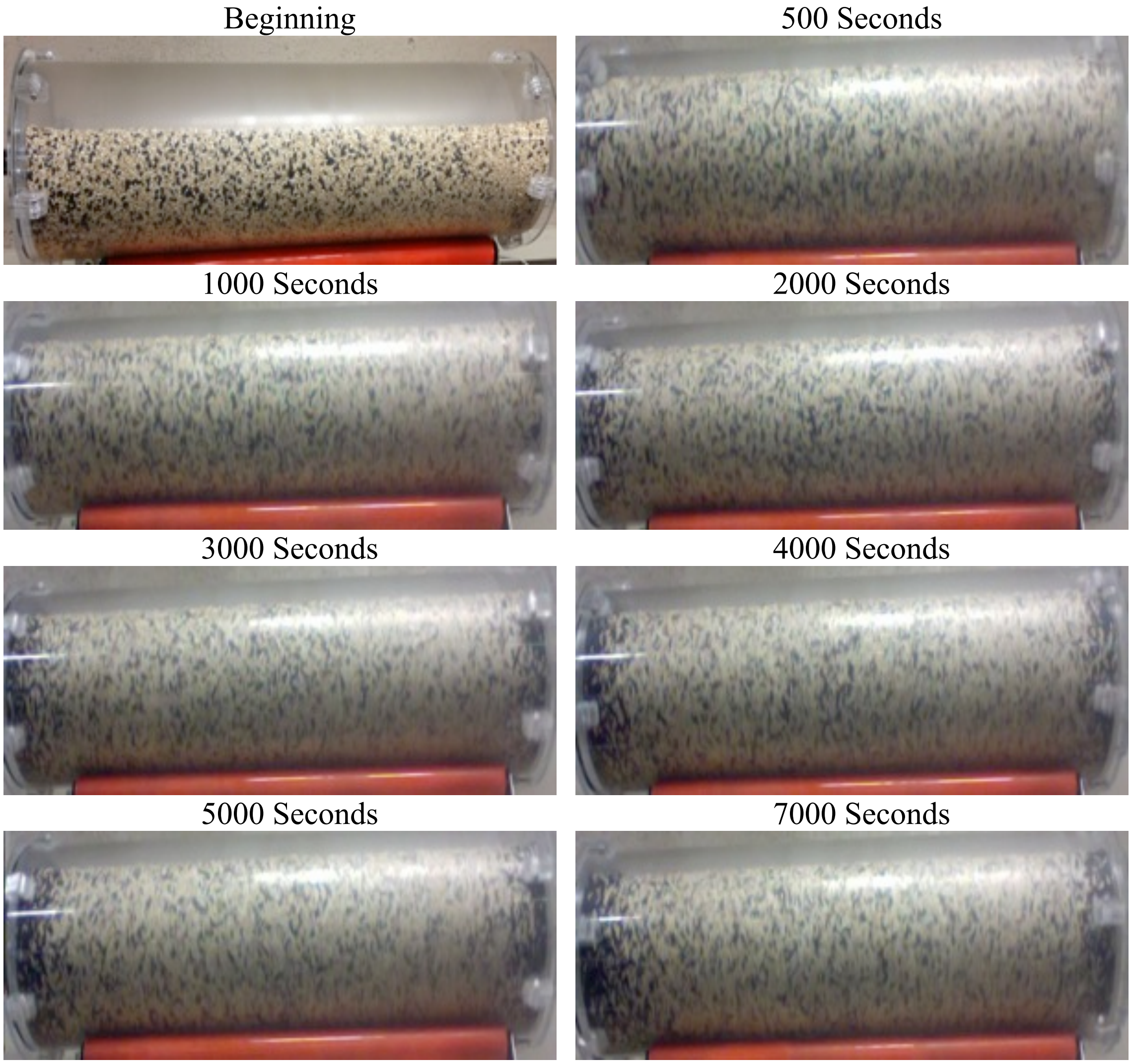


Figure A- 14 Mixed safflower and black beans experiment, black beans form end bands

Mixed Barley and Black Beans Experiment

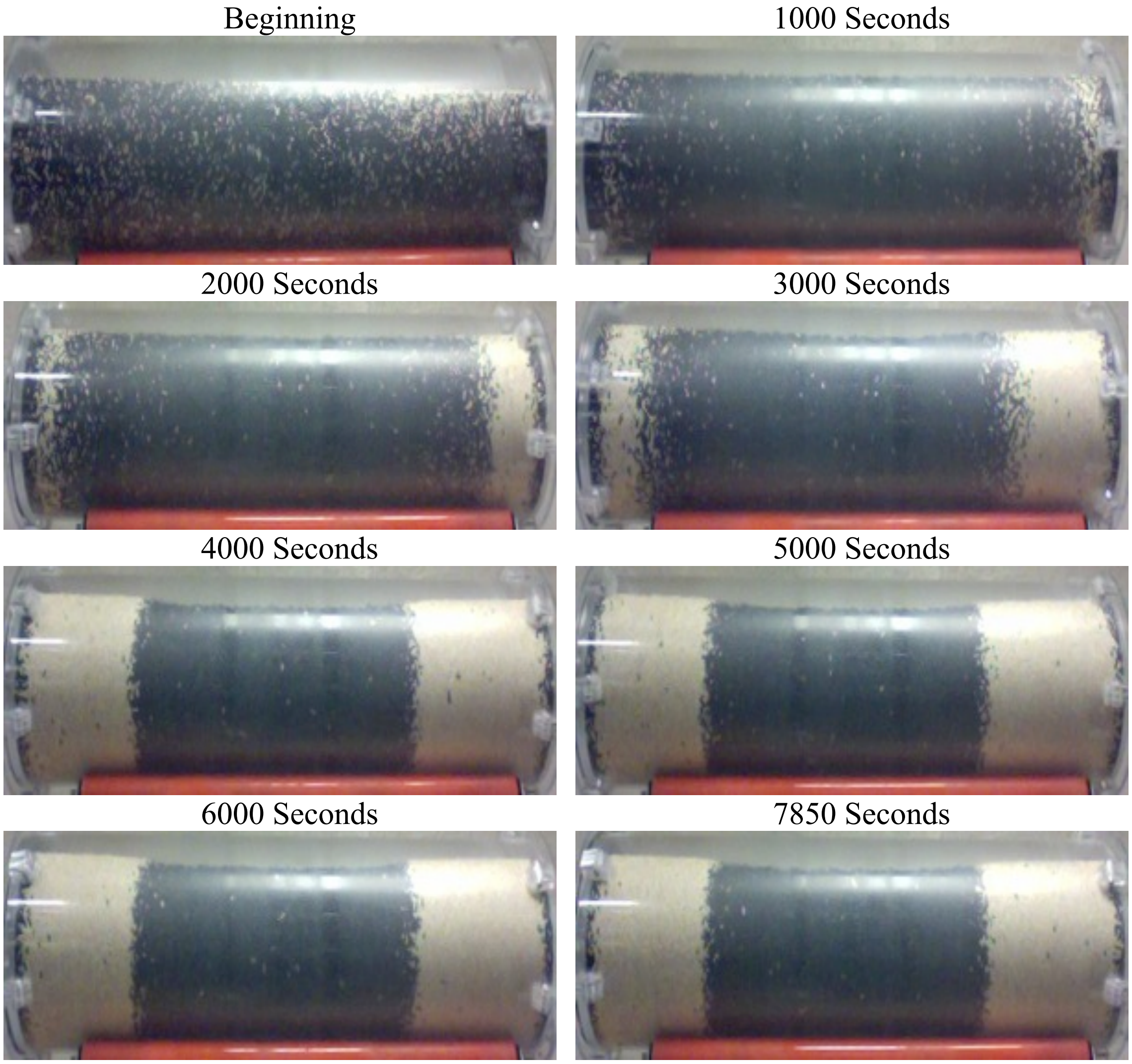
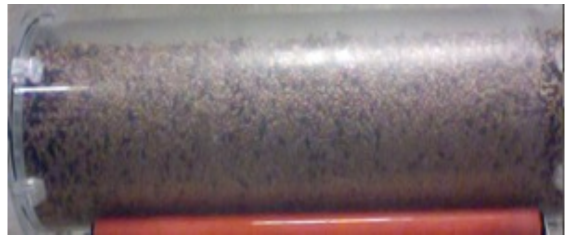


Figure A- 15 Mixed barley and black beans experiment, axial banding seen with thin line of black beans at ends

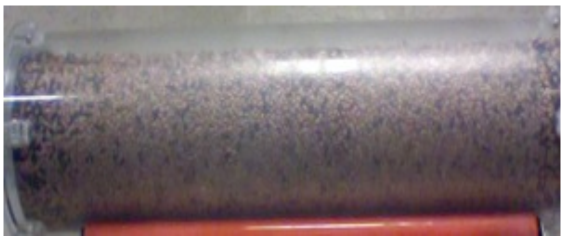
Mixed Pinto and Black Beans Experiment

Beginning

500 Seconds



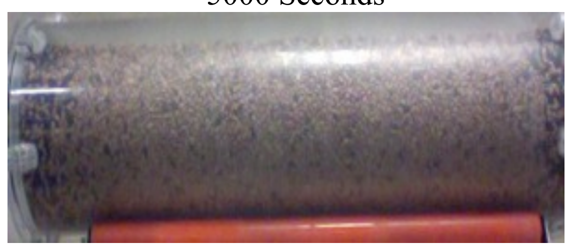
1000 Seconds



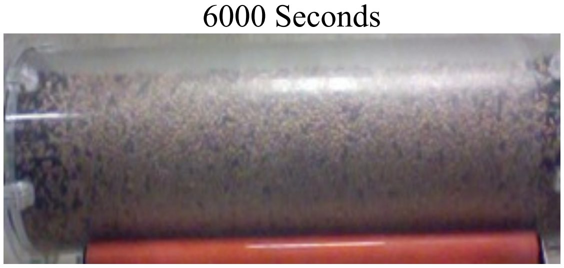
2000 Seconds

3500 Seconds

5000 Seconds



6000 Seconds



7000 Seconds

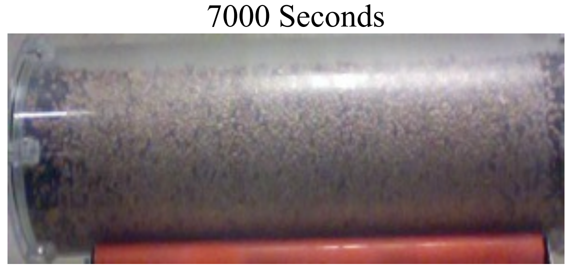


Figure A- 16 Mixed pinto and black beans experiment, black beans form end bands

Mixed Black Beans and Rice with Band Experiment

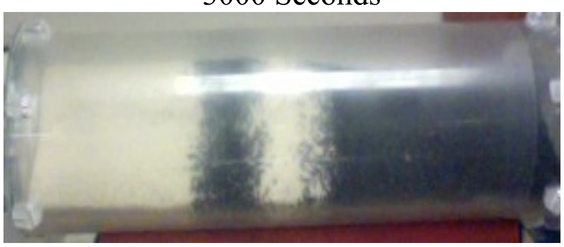
Beginning

100 Seconds

1000 Seconds

2000 Seconds

3000 Seconds



4000 Seconds

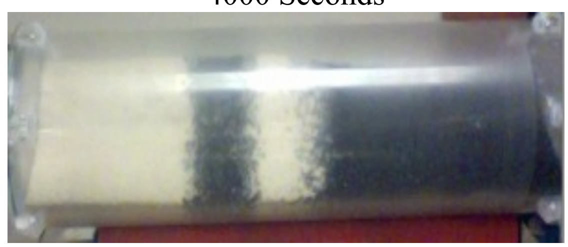


Figure A- 17 Rice and black beans with band at the middle, band mixes in the first 100 seconds but then axial band forms again but shifted over to the left

APPENDIX B

Table of Critical Values for Pearson's *r*

	Level of Significance for a One-Tailed Test					
	.10	.05	.025	.01	.005	.0005
	Level of Significance for a Two-Tailed Test					
df	.20	.10	.05	.02	.01	.001
1	0.951	0.988	0.997	0.9995	0.9999	0.99999
2	0.800	0.900	0.950	0.980	0.990	0.999
3	0.687	0.805	0.878	0.934	0.959	0.991
4	0.608	0.729	0.811	0.882	0.917	0.974
5	0.551	0.669	0.755	0.833	0.875	0.951
6	0.507	0.621	0.707	0.789	0.834	0.925
7	0.472	0.582	0.666	0.750	0.798	0.898
8	0.443	0.549	0.632	0.715	0.765	0.872
9	0.419	0.521	0.602	0.685	0.735	0.847
10	0.398	0.497	0.576	0.658	0.708	0.823
11	0.380	0.476	0.553	0.634	0.684	0.801
12	0.365	0.457	0.532	0.612	0.661	0.780
13	0.351	0.441	0.514	0.592	0.641	0.760
14	0.338	0.426	0.497	0.574	0.623	0.742
15	0.327	0.412	0.482	0.558	0.606	0.725
16	0.317	0.400	0.468	0.542	0.590	0.708
17	0.308	0.389	0.456	0.529	0.575	0.693
18	0.299	0.378	0.444	0.515	0.561	0.679
19	0.291	0.369	0.433	0.503	0.549	0.665
20	0.284	0.360	0.423	0.492	0.537	0.652
21	0.277	0.352	0.413	0.482	0.526	0.640
22	0.271	0.344	0.404	0.472	0.515	0.629
23	0.265	0.337	0.396	0.462	0.505	0.618
24	0.260	0.330	0.388	0.453	0.496	0.607
25	0.255	0.323	0.381	0.445	0.487	0.597
26	0.250	0.317	0.374	0.437	0.479	0.588
27	0.245	0.311	0.367	0.430	0.471	0.579
28	0.241	0.306	0.361	0.423	0.463	0.570
29	0.237	0.301	0.355	0.416	0.456	0.562
30	0.233	0.296	0.349	0.409	0.449	0.554
40	0.202	0.257	0.304	0.358	0.393	0.490
60	0.165	0.211	0.250	0.295	0.325	0.408
120	0.117	0.150	0.178	0.210	0.232	0.294
∞	0.057	0.073	0.087	0.103	0.114	0.146

Adapted from Appendix 2 (Critical Values of *t*) using the square root of $[t^2/(t^2 + df)]$
Note: Critical values for Infinite *df* actually calculated for *df*= 500.

Figure B- 1 Table of critical values for Pearson's r, values for infinite used for *df* since sample size was always over 500 [28].