The Role of Mindset in a Mathematics Teacher's Interpretations and Enactments of Professional Development Activities

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This work is dedicated to my parents, C. W. and Elizabeth Willingham, and to my wife, Renee Crosswhite. Without your constant support and encouragement, nothing I have ever accomplished would have been possible.

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ABSTRACT

Although the motivational factors that underlie the process of mathematics teacher change have been under study for more than thirty years, the role of one of the factors in this process, a teacher's implicit theory regarding mathematical ability, has not been well examined. The implicit theories model posits that an individual's implicit assumptions about the nature of an ability lead directly to the type of goals he sets regarding that ability and the behaviors in which he engages to pursue these goals. Those that espouse an incremental theory tend to establish learning goals and focus on strategies that lead to the improvement of the ability in question. These individuals are said to have a growth mindset.

The purpose of this study was to explore the role of a teacher's mindset within the contexts of the teacher's professional development experiences and answer the research question: How do characteristics of the growth mindset influence a mathematics teacher's interpretations and enactments of professional development experiences, if at all? A holistic single-case study design was implemented to examine elements of an elementary mathematics teacher's change environment as she observed, adapted, and enacted a demonstration lesson from a professional development program into her own classroom. The study examined aspects of this environment including the teacher's beliefs and mindset regarding mathematics and the teaching and learning of mathematics, her classroom teaching practices, her perceptions of her past experiences in professional development, her areas of focus during the demonstration lesson, and her experiences and reflections as she implemented the demonstration lesson in her classroom.

The study produced results that were significant in at least four ways. First, the study presented evidence that tenets of self-regulation theory, including goal setting, goal operating, and goal monitoring, were utilized by the participant teacher to operationalize her mindset. These findings provide support for the use of self-regulation theory in examining mindset constructs and help extend the study of implicit theories to mathematics teacher professional development. Second, the results indicated that the teacher operated through goals at three distinct levels: long-term goals related to mathematical practices, mid-term goals related to her mathematics learning trajectory, and short-term goals related to mathematical content. These goal levels have potential applications for both classroom teachers and designers of professional development. Third, the study revealed the role of mindset, operationalized through self-regulation theory, as a mediator of the various domains of the teacher's change environment at each of her goal levels. The highly connected growth networks formed by these mediated pathways appeared to have been a factor in the sustained change in teaching practices and beliefs described by the study's participant. Finally, the case narrative produced in the study provided a deep, rich description of a teacher's interpretations and enactments of her professional development experiences that adds to our understanding of the variation present in these situations.

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CHAPTER I: INTRODUCTION

Introduction

This dissertation contains a description of an exploratory case study that examined the role of growth mindset characteristics in an elementary school teacher's participation in professional development activities focusing on a demonstration lesson. This chapter of the study contains an introduction to the research including a brief review of its background, a description of the nature of the problem addressed, and its potential significance. An introduction to the study's theoretical framework, its purpose, and definitions of its key terms will also be presented.

Background of the Study

Although the United States has experienced moderate gains in elementary and middle grades mathematics achievement over the past two decades (Mullis, Martin, Foy, & Arora, 2012; National Center for Education Statistics [NCES], 2013), there is much left to be achieved. This is particularly true in the secondary grades (NCES, 2013; Organisation for Economic Co-operation and Development [OECD], 2013a) and among traditionally underserved populations (Darling-Hammond, 2010). Gains in measures of mathematical problem-solving ability and self-efficacy, particularly among the strongest performers, are offset by deficiencies in overall mathematics achievement, especially among the weakest performers (OECD, 2013b). These deficiencies are left to be addressed by a growing body of mathematics education research and the nearly two million mathematics teachers that implement its results each day (National Council of Teachers of Mathematics [NCTM], 2014).

The Importance of Teacher Quality

Classroom teachers are potentially the best resource for continued improvement in mathematics understanding and achievement. An extensive research base indicates that the quality of the classroom teacher is the single most important school-based factor in student achievement (Baumert et al., 2010; McCaffrey, Lockwood, Koretz, & Hamilton, 2003; Rivkin, Hanushek, & Kain, 2005; Rowan, Correnti & Miller, 2002; Wright, Horn, & Sanders, 1997). In terms of the persistent achievement gaps that plague our schools, findings that the influence of effective teachers is robust, cumulative, and long-lasting are particularly important (Darling-Hammond, 2000; McCaffrey et al., 2003; Sanders & Rivers, 1996). Although a variety of factors influence mathematics teacher quality, including personal background, gender, culture (Blömeke, Suhl, & Kaiser, 2011), mathematical knowledge for teaching (Hill, Rowan, & Ball, 2005), general knowledge, teacher preparation, certification and licensure, teaching experience, and in-service training opportunities (Whitehurst, 2002), it is perhaps this final category that offers the most potential for long-term professional development and change in practice for classroom teachers (Desimone, Porter, Garet, Yoon, & Birman, 2002).

Professional Development and Teacher Change

In the 1980s, a series of empirical and theoretical works examining the processes of change in mathematics teachers' classroom practices expanded the focus on effective teaching from teachers' knowledge of mathematics to their conceptions of mathematics and its teaching (Ernest, 1989; Guskey, 1986; Thompson, 1984). This shift highlighted that constructs including teachers' beliefs, views, and attitudes about mathematics were essential components of their teaching practices. More recent research has examined how these dispositions impact professional development in at least four ways. First, increasingly sophisticated models emphasizing the interactions of these mental characteristics with teaching practices have been developed and empirically vetted (Clarke & Hollingsworth, 2002; Wilkins, 2008). Second, practical characteristics for professional development programs designed to support long-term changes in practice have been suggested (Garet, Porter, Desimone, Birman, & Yoon, 2001; Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003; Wilson & Berne, 1999). Third, stages through which teachers transition as their classroom practices evolve have been described (Andreasen, Swan, & Dixon, 2007; Farmer, Gerretson, & Lassak, 2003). Finally, the impact of other important factors on teaching practices, such as the school setting and types of learning activities implemented in the classroom, has been explored (Opfer & Pedder, 2011). Despite this emphasis on teacher conceptions of mathematics and mathematics teaching and learning, one potentially important influence on pedagogical practices that has not been well examined is the teacher's implicit theory, or mindset (Rattan, Good, & Dweck, 2012).

Implicit Theories

The basis of the implicit theories model is that an individual's implicit assumptions about the nature of an ability lead directly to the type of goals pursued regarding that ability (Dweck & Leggett, 1988). Based on the type of goals he establishes and the manner in which he pursues those goals, an individual is said to hold either a growth or a fixed mindset. Those who espouse the growth mindset tend to establish learning goals and focus on the improvement of the ability in question. Those with fixed mindsets adopt performance-oriented goals and either seek judgment for a well-developed ability or avoid judgement for a less-developed ability.

These mindsets and their associated goal pursuits thus create "a framework for interpreting and responding to events" (Dweck & Leggett, 1988, p. 260) that results in predictable behaviors. The theory was originally established by examining assumptions about intelligence, but it was later extended to support its generalizability to other traits and abilities. Additionally, instruments for measuring mindset constructs as they relate to specific abilities were constructed and validated (Dweck, Chiu, & Hong, 1995). The theory has proven robust, and although it has been widely considered as a general form of self-regulation (Burnette, Boyle, VanEpps, Pollack, & Finkel, 2013), as a factor in mathematics achievement (Blackwell, Trzesniewski, & Dweck, 2007; Dweck, 2008), and as a component of school and classroom culture (Boaler, 2013, 2016), little empirical research has considered it as a mediator of mathematics teaching practices (Rattan et al., 2012).

Theoretical Framework

This background literature suggested practical and theoretical elements to be considered in a study of this nature. As a theoretical framework provides the underlying structure for all research (Merriam, 2009) and shapes the research design process (Yin, 2014), two theoretical constructs guided this study. The first of these, the Interconnected Model of Teacher Professional Growth (IMTPG, Clarke & Hollingsworth, 2002), addressed teacher change as a series of interactions among internal traits (e.g., beliefs and attitudes), external factors (e.g., professional development opportunities), teaching experiences, and classroom outcomes. These domains helped define the context the study examined. Second, implicit theories (Dweck & Leggett, 1988), as previously introduced, represented the primary construct examined within the context of the IMTPG mentioned here. This theory was particularly useful as it explained the manner in which underlying personality traits produced motivational processes that impacted an individual's cognition, affect, and behavior. Together, these theories provided the lens used to examine the manner in which a teacher translated professional development experiences into classroom practices.

The Problem Statement

Coupled with a need to better understand the manner in which teachers' motivations and dispositions influence "the ways in which they approach and resolve certain kinds of issues about students . . . classroom structure and functioning, and the use of curriculum materials and resources" (Goldsmith & Shifter, 1997, p. 47), the background previously provided outlines the problem addressed in this dissertation. Specifically, motivational factors, which play a key role in the processes of teacher change, are not well understood (Guskey, 2002; Thoonen, Sleegers, Oort, Peetsma, & Geijsel, 2011). Empirical research is needed to examine the manner in which these factors influence teachers' daily practices (Goldsmith & Shifter, 1997), explore how they mediate other constructs which influence practice (Clarke & Hollingsworth, 2002), and articulate mechanisms for teacher change (Goldsmith, Doerr, & Lewis, 2014). Calls from more recent literature stress that studies of this nature should be situated in specific sets of activities, supports for learning, contexts, and characteristics of individual teachers (Opfer & Pedder, 2011). Additionally, these studies should focus on the interactions "between the individual teacher, the context of the professional development activity itself, and the teacher's work environment" (Wagner & French, 2010, p. 169).

Statement of Purpose

A complete model of mathematics teacher development must describe teachers' motivations and dispositions and their influence on factors such as the teacher's interpretations of professional development experiences, implementation of learning activities, and interactions with students and the classroom environment (Goldsmith & Shifter, 1997; Opfer & Pedder, 2011; Wagner & French, 2010). The purpose of this study was to explore one of these motivational factors, the teacher's mindset, within the contexts of the teacher's professional development experiences. These premises led to the primary research question of the study: How do characteristics of the growth mindset influence a mathematics teacher's interpretations and enactments of professional development experiences, if at all? To address this question, the influence of mindset as the teacher observed, interpreted, discussed, adapted, planned for, implemented, and reflected on a demonstration lesson was examined.

Significance of the Study

The study proved to be significant to the existing body of mathematics education research in at least five substantial ways. First, the study's focus on the influence of mindset on classroom practices helped to extend a well-examined theoretical construct into the realms of mathematics teacher professional development and practice, areas in which it had not been thoroughly examined (Rattan et al., 2012). Second, the design of the study allowed consideration of teacher change within a specific set of contexts, helping verify “our presumption of variation” (Opfer & Pedder, 2011, p. 394) regarding these processes (Opfer & Pedder, 2011; Van Driel, & Berry, 2012; Wagner & French, 2010). Third, the study contributed to an existing body of literature examining how the motivations and dispositions of mathematics teachers influenced their classroom practices and improved upon the sophistication of one of the current models of these processes (Clarke & Hollingsworth, 2002; Goldsmith & Shifter, 1997). Fourth, consideration of the teacher’s interactions with the demonstration lesson tested claims that this format allows teachers to reflect carefully on observed practices and helped distinguish the teacher’s areas of focus during the demonstration lesson (Clarke et al., 2013). Finally, the study looked closely into “the actions teachers *choose* to take and *do* take (and not just what they *claim* they will take or have taken)” (Clarke et al., 2013, p. 225) from a demonstration lesson. In line with calls from the research previously cited, this work extended the research base on teacher change and provided a detailed description of the manner in which a teacher utilized the concepts and practices observed in a demonstration lesson.

Definitions

Throughout this study key terms are referred to repeatedly. This section is intended to bring clarity to the meaning to these terms.

Attitude

The term attitude will generally be used to refer to attitudes towards mathematics. An attitude towards mathematics is an affective characteristic describing a “positive or negative response to mathematics that is relatively stable” (Hemmings, Grootenboer, & Kay, 2011). Examples include enjoyment and interest in mathematics, confidence in mathematics, and their opposites. This definition can also be extended to include attitudes towards teaching mathematics, which mirror the above descriptions in the context of teaching mathematics (Ernest, 1989).

Belief

The term belief will be used to describe a teacher’s system of beliefs, including his deeply held conceptions, values, and ideologies about a specific topic. The term will generally be used to refer to beliefs regarding either mathematics or the teaching and learning of mathematics (Ernest, 1989). Beliefs about mathematics fall into one of three broad categories: (a) instrumentalist views, which consider mathematics as a collection of disconnected rules, facts, and skills; (b) Platonist views, which see mathematics as a static, but unified body of discovered knowledge; and (c) problem-solving views, which perceive mathematics as a dynamic and expanding field of inquiry. Beliefs about teaching and learning mathematics will include conceptions of the teacher’s and students’ roles, including behaviors and mental activities, in the construction of mathematical understanding.

Demonstration Lesson

A demonstration lesson is a professional development approach in which teachers gather to observe the instructional practices of another teacher during a classroom lesson. The demonstration lesson utilized in this study was led by an expert teacher in a classroom hosted by one of the participants of the professional development program. Pre-lesson briefing, observation of the demonstration lesson, and post-lesson debriefing were utilized to promote participant reflection on the practices involved in mathematics teaching (Loucks-Horsley et al., 2003).

Instructional Practices

Instructional practices, also referred to as teaching practices or classroom practices, are the activities of the teacher that facilitate interactions between the teacher and students with the content being studied in a particular learning environment. These interactions include the connected work that occurs between the teacher and students over time to promote learning (Cohen, Raudenbush, & Ball, 2003). Eight Mathematics Teaching Practices are identified by the NCTM as essential components of mathematics instruction:

1. Establish mathematics goals to focus learning;
2. Implement tasks that promote reasoning and problem solving;
3. Use and connect mathematical representations;
4. Facilitate meaningful mathematical discourse;
5. Pose purposeful questions;
6. Build procedural fluency from conceptual understanding;

7. Support productive struggle in learning mathematics;
8. Elicit and use evidence of student thinking (NCTM, 2014, p. 10).

Mathematical Practices

Mathematical practices are the activities in which students engage in the learning of mathematics. They include the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections (NCTM, 2000) and the National Research Council's strands of mathematical proficiency related to adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (Kilpatrick, Swafford, & Findell, 2001). Eight Standards for Mathematical Practice are explicitly described in the Common Core State Standards for Mathematics (CCSSM):

1. Make sense of problems and persevere in solving them;
2. Reason abstractly and quantitatively;
3. Construct viable arguments and critique the reasoning of others;
4. Model with mathematics;
5. Use appropriate tools strategically;
6. Attend to precision;
7. Look for and make use of structure;
8. Look for and express regularity in repeated reasoning (Common Core State Standards Initiative [CCSSI], 2010, p. 6-8).

Mindset

Mindset will refer to the teacher's implicit theory regarding a specific attribute. The term growth mindset will refer to an incremental theory, in which the attribute is viewed as a malleable characteristic. This will generally be accompanied by establishing learning goals related to improving the attribute. The term fixed mindset will refer to an entity theory, in which the attribute is viewed as a rigid construct, with goals that tend toward performance and the measurement, or avoidance of measurement, of the attribute (Burnette et al., 2013; Dweck & Leggett, 1988).

Reform-oriented Instruction

Reform-oriented instruction is a broad set of instructional practices intended to shift student learning toward "conceptual understanding; the capacity for disciplined reasoning, analysis, argument, and critique; and the ability to communicate ideas and interact effectively with others" (Ball & Forzani, 2011). It is facilitated through the use of specific instructional practices (NCTM, 2014) intended to encourage mathematical practices in students such as those described in the CCSSM (CCSSI, 2010).

Teacher Change

Teacher change is a description of the process of teacher development, which consists of the reorganization of the teacher's conceptions of teaching and learning, from a position focused on the transmission of knowledge to one that respects students' current understanding and supports its growth. This progression is generally conceived of as a movement between stages, facilitated by psychological and sociocultural mechanisms, which act under the influence of motivational and dispositional factors (Goldsmith & Shifter, 1997). Teachers engaged in this process are said to be teachers in transition.

Chapter Summary

This chapter included a brief introduction to the case study presented in this dissertation. The study examined the manner in which a mathematics teacher's mindset influenced her participation in professional development activities related to a demonstration lesson. Chapter Two of this volume will provide a condensed summary of the literature, which guided the study. A detailed description of the methodology under which the study was conducted is included in Chapter Three. Chapter Four presents the complete results of the study, including a case description arising from thematic analysis and narrative samples of important observations. The volume concludes in Chapter Five, which presents an interpretive analysis and discussion of the study's results.

CHAPTER II: LITERATURE REVIEW

Introduction

Given the significant role that individual teachers play in advancing students' mathematical achievement (Baumert et al., 2010; Rivkin et al., 2005) and the importance of continued professional development in supporting effective teaching practices and school improvement (Darling-Hammond, 2010; Desimone, 2009), understanding the processes and motivations of teacher change are of paramount importance to mathematics education. The purpose of this study was to explore one of these motivational factors, the teacher's mindset, within the contexts of the teacher's professional development experiences. This chapter will begin with a brief review of a portion of the theoretical and empirical literature relevant to the processes of teacher change. This will be followed by an examination of literature regarding one potential motivating factor, the teacher's mindset. Based on these reviews, the theoretical framework of the study will be expounded, and the connections between this literature base and the current study will be discussed.

Under the recommendations of qualitative research theorists that the act of reviewing literature, particularly during the formative stages of a study, influences the ability of the researcher to engage in an authentic inductive analysis (Gay, Mills, & Airasian, 2011; Bogdan & Biklen, 1998), this literature review was not prepared as a comprehensive or exhaustive review of the extensive literature available on these topics. Rather, it was guided by three expert suggestions, which recommended a brief and focused literature review. First, the review presents only the underlying theoretical assumptions that are most central to this research study (Marshall & Rossman, 2014; Yin, 2014). Second, only empirical research that frames and supports the research question of the study was reviewed (Yin, 2014). Finally, and perhaps most importantly, the review focused only on works directly relevant to the design, justification, and theoretical framework of the study (Maxwell, 2006). Topics emerging during the course of the study that required further review are introduced throughout this volume, with only the most salient topics expanded in this chapter. Additionally, the conceptual framework of the study is not reported here, as it emerged during the study's interpretive analysis and requires evidence from the study's results to develop appropriately. This framework, which was justifiably allowed to evolve from the ongoing interaction among the theoretical concepts guiding the study and the research process (Camp, 2001; Maxwell, 2005), is instead reported in Chapter 5.

Teacher Change and Professional Development

Three decades ago, an important transition in the study of mathematics teacher change began when the largely ignored question of how teachers' conceptions of mathematics impacted their instructional practices was first widely considered (Thompson, 1984). Questions such as this expanded the focus on effective mathematics teaching from teachers' knowledge of mathematics to their conceptions of mathematics and its teaching (Ernest, 1989; Guskey, 1986; Philipp, 2007; Thompson, 1984). This shift highlighted that constructs including teachers' beliefs, views, and attitudes about mathematics were essential components of their teaching practices, that these practices slowly evolved in response to a myriad of other factors, and that teachers in transition operated in a dual reality between their espoused and enacted conceptions (Clark & Hollingsworth, 2008; Ernest, 1989; Guskey, 1986; Pajares, 1992; Philipp, 2007).

More recent research has considered how these dispositions impact professional development in four important ways. First, the models examining the influence of teachers' conceptions on their classroom practices have grown increasingly sophisticated and begun to account for the nonlinear relationships among the factors involved in these relationships (Clarke & Hollingsworth, 2002; Wilkins, 2008). Second, a variety of professional development programs have supported long-term changes in teachers' conceptions of teaching mathematics and their associated practices (Garet et al., 2001; Loucks-Horsley et al., 2003; Wilson & Berne, 1999). Third, researchers have presented a diverse range of models to describe the stages through which teachers transition as their conceptions and practices evolve (Andreasen et al., 2007; Farmer et al., 2003). Finally, research has explored the impact of a multitude of other factors, such as the school setting, the teacher's perspective concerning professional development activities, and the types of learning activities implemented in the classroom, on teachers' conceptions and implementation of teaching practices (Clarke et al., 2013; Opfer & Pedder, 2011). This section will elaborate the theoretical basis and empirical results of these works examining teacher change and professional development.

Models of Teacher Change

This section provides a description of three models of teacher change particularly relevant to the study. The third model presented here, the IMTPG, is a foundational element of the theoretical framework of the study.

An early description of teacher change. An early attempt to describe the association between teachers' conceptions and classroom practices was Guskey's (1986) model of the process of teacher change showing the relationship between four domains of teachers' experiences (see Figure 1). In this model, changes in teachers' beliefs and attitudes were believed to be contingent upon student outcomes that arose from changes in the teachers' practices (Guskey, 1986). These changes in classroom practices were

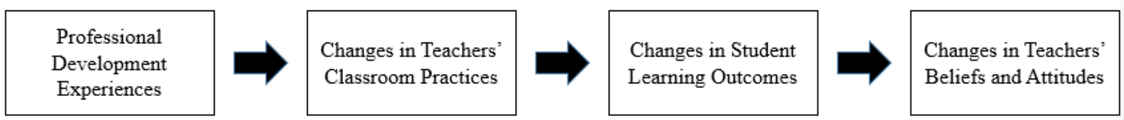


Figure 1. A model of the process of teacher change. Adapted from "Staff Development and the Process of Teacher Change," by T. R. Guskey, 1986, *Educational Researcher, 15*, p. 7.

thought to be directly related to the professional development activities in which the teacher engaged. Although other frameworks would dispute the linearity of this model (Clarke & Hollingsworth, 2002) and the sequencing of the events (Ernest, 1989), three important ideas regarding teacher change accompanied this model. First, teacher change was viewed as a gradual and difficult process that requires both time and effort from the teacher involved. Second, the model required teachers to regularly assess or otherwise receive feedback regarding their students' learning progression, with the idea that positive outcomes would reinforce the teaching practices that led to these outcomes while negative outcomes would result in the practices that led to them being extinguished. Finally, the model suggested that support for teacher change must be continued after the initial professional development experience in order for the teacher to develop confidence in the practices adopted.

The relationship between conceptions and practices. Ernest (1989) provided an alternate explanation of the association between teacher conceptions and practices. In this model, teachers' knowledge structures, beliefs, and attitudes regarding mathematics were explicitly defined, and the relationships between them were viewed in a slightly more interactive fashion (see Figure 2). Teachers' knowledge, beliefs, and attitudes were each described as directly influencing classroom practices, while knowledge structures regarding mathematics and its teaching were theorized to influence the teacher's attitudes regarding mathematics and its teaching, and vice-versa. Both knowledge and attitudes were believed to contribute to the teachers' beliefs about mathematics and its teaching.

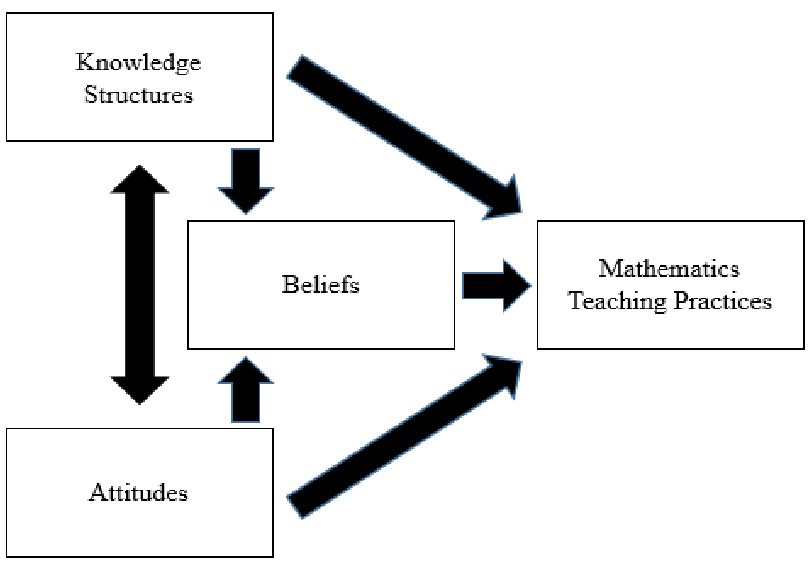


Figure 2. The relationship between attitudes, beliefs, knowledge, and practice. Developed from "The Knowledge, Beliefs and Attitudes of the Mathematics Teacher: A Model," by P. Ernest, 1989, *Journal of Education for Teaching, 15*, 13-33.

Ernest’s (1989) work also made distinctions between the theoretical characteristics of each conception held by a teacher and their more practical counterparts. The theoretical constructs were seen as those developed away from the practices of teaching and included knowledge of mathematics, teaching, and other content as well as beliefs and attitudes towards mathematics. The more practical conceptions were gained through the practices of teaching and related directly to the teaching and learning of mathematics. These elements were described as being linked, with the theoretical components believed to form the basis for their practical equivalents. The final significant aspect of this model was its description of teachers’ espoused conceptions versus those actually enacted in the classroom. The known disparity in these conceptions was accounted for by considering the depth and connectedness of the construct within a teacher’s other mental characteristics, the teacher’s awareness of and reflection on the construct, and the social contexts, particularly in the expectations of peer teachers and administrators, within which the teacher functions (Ernest, 1989).

An interconnected model. Tenets of both Ernest’s (1989) and Guskey’s (1986) models appeared in the IMTPG elaborated on by Clarke and Hollingsworth (2002). In this theoretical model (see Figure 3), the four domains identified by Guskey (1986) were expanded, and the relationships among them were described in terms of enactment and reflection similar to those posited by Ernest (1989). The domains of this model included the personal domain (knowledge, beliefs, and attitudes), the external domain (outside sources of information), the domain of practice (enacted classroom experiences), and the domain of consequence (salient outcomes involving student learning). Interactions among these domains were described to be mediated by two processes. The first of these, enactment, was viewed as the active process of operationalizing ideas from one domain into another. The second process, reflection, was described as a determined consideration of the experiences of one domain as they influenced another (Clarke & Hollingsworth, 2002).

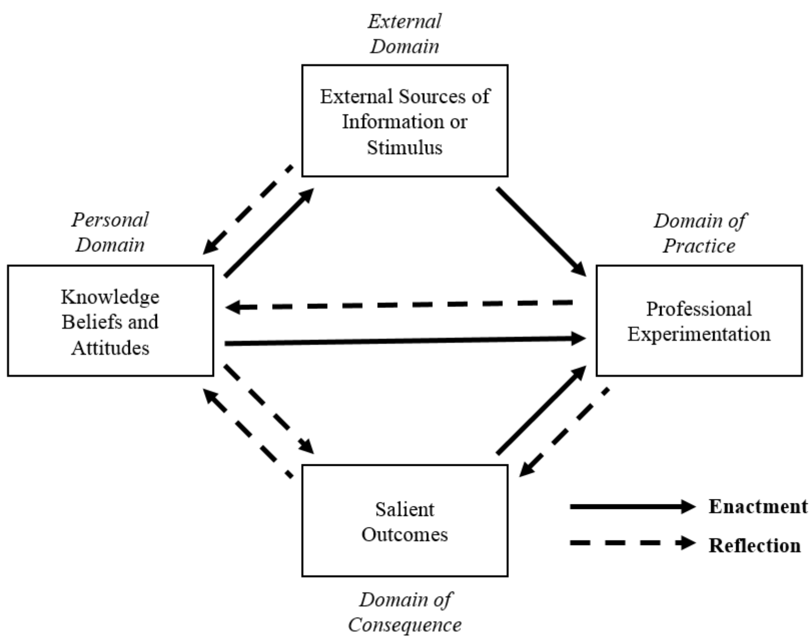


Figure 3. The Interconnected Model of Teacher Professional Growth. Adapted from "Elaborating a Model of Teacher Professional Growth," by D. Clarke and H. Hollingsworth, 2002, *Teaching and Teacher Education, 18*, p. 951.

Interactions within these domains were posited to occur within a specific change environment (Clarke & Hollingsworth, 2002). The change environment was described as consisting of a particular set of elements, unique to each teacher, that would facilitate or inhibit experiences within each of the model's domains. Facilitative examples of these factors might include being a member of a school community invited to participate in a professional development project, or having a network of supportive teachers and administrators with which to share the consequences of specific implementations. Inhibitory factors could include the perceived consequences of implementing a new practice based on an implicit belief about how teaching and learning should occur.

Within this change environment, two types of teacher shifts were theorized to arise as specific interactions occurred between the domains (Clarke & Hollingsworth, 2002). The first of these, termed a *change sequence*, occurred any time one domain exerted influence on another. These changes were often temporary and typically enacted in some form of professional experimentation that was quickly abandoned. However, change sequences occasionally lead to further interactions between the domains, resulting in a more permanent transition. These extended interactions were deemed part of a *growth network* and represented the product of teacher development.

Empirical Research on Professional Development

This section contains reviews of the methods and results of three empirical studies of professional development offering significant findings for the study. In these studies, the complexity of professional learning, the relationship between elementary teachers' mathematical conceptions and practices, and the area of focus of elementary teachers during demonstration lessons are examined.

Understanding the complexity of professional learning. Opfer and Pedder (2011) applied a complexity theory framework, in which reports on specific aspects of professional development programs were situated within a holistic view of professional learning, to review a wide base of literature concerning professional development and teacher learning. Their goal with this review was to consider how professional learning activities fit into the professional lives and real work environments of the teachers who participate in these programs. The review identified three systems deemed essential to impactful professional learning: the individual teacher, the classroom and school environment in which the teacher operates, and the professional learning activities in which the teacher is engaged.

The authors noted that conventional views of teacher professional development adopt a directional process-product view in which teachers engaging in professional development experiences are expected to translate these experiences directly into changes in practice (Opfer & Pedder, 2011). They argued that this is an oversimplified view of professional learning, in that substantial changes in practice do not occur in isolation but rather as the results of reciprocal and cyclic changes across all of the systems of influence that are active in the teacher's reality. Thus, in order to understand the processes of teacher professional development, we must realize that the change environment is unique for every teacher and that the features of any professional development program will operate differently on its participants depending on contexts unique to the participant. Therefore:

> We must expand our casual assumptions beyond the features of the learning process or activity to consider the reciprocal relationships that exist between the systems of activities in which teachers engage and the systems of influence that mediate and moderate these activities. (Opfer & Pedder, 2011, p. 386)

The relationship between conceptions and practices. In a large scale path analysis (*n* = 481), Wilkins (2008) considered the relationships among K-5 teachers' background characteristics, mathematical content knowledge, attitudes regarding mathematics, beliefs about reformed-based teaching practices, and self-reported utilizations of these practices. Using high reliability instruments (Cronbach's α = .80 to .88), he surveyed these factors for teachers initially entering a three-year professional development project. He then used a multiple regression procedure to complete a path analysis for these factors and reported a reduced model describing the significant correlations.

The findings of this study generally aligned with the model described by Ernest (1989) with one notable exception. Although all of the identified conceptions of mathematics directly impacted reported classroom practices, a greater level of mathematical content knowledge was found to be negatively related to a teacher's reform-oriented instructional beliefs and practices (Wilkins, 2008). More specifically, Wilkins found that although grades 3-5 teachers displayed significantly greater mathematical content knowledge and more positive attitudes toward mathematics than grades K-2 teachers, the grades K-2 teachers reported using reform-oriented practices significantly more often. This finding was in contrast to earlier studies (Ball, 1991; Fennema & Franke, 1992) that indicated a positive relationship between teachers' mathematics content knowledge and their reform-oriented instructional practices. Perhaps the most important finding of the study was that despite the disparities reported here, both groups of teachers showed similar beliefs in the effectiveness of reform-oriented instruction. This was significant as these beliefs were found to have the strongest relationship to classroom practices, suggesting that beliefs about the effectiveness of reform-oriented practices may serve as a mediator in the practices that are actually implemented in the classroom.

Areas of teacher focus during demonstration lessons. In a longitudinal professional development program, Clark et al. (2013) considered the areas of focus of elementary teachers observing demonstration lessons in mathematics and the types of changes in practice they expected to implement in their classrooms based on their observations. A random sample of 200 teachers participated in a survey, which asked teachers attending demonstration lessons in groups of around 20 to designate two areas of focus before the demonstration lesson: one related to teaching practices and one related to student learning. After the demonstration lesson, the teachers were asked what changes to their practices they might consider making based on their observations of the lesson.

The results of the survey revealed that teachers professed the most interest in teaching practices related to questioning, differentiation, structuring of the lesson, and the specific content, teaching practices, and academic language used by the expert teacher conducting the lesson (Clark et al., 2013). Their focuses on student learning were related to the students' specific understanding of the mathematical content being taught, the manner in which students communicated their ideas, their affective characteristics during the lesson, and the actions they engaged in during the lesson. Interestingly, more than 20% of the teachers did not provide meaningful responses to this survey item, indicating difficulty designating an area of student learning on which to focus during the demonstration lesson. After the lesson, many of the focus items designated prior to the observations were indicated as potential areas for changes in practice. These included methods to improve students' communication efforts, the types of questions to be asked, the manner in which lessons would be structured, and strategies for differentiating material for their students. However, the two areas that were most often reported for potential changes in practice (i.e., the materials and representations utilized in the lesson, and the pacing and wait time implemented by the expert teachers) were rarely considered points of focus prior to the lesson (Clark et al., 2013).

Significance of the Research

The research reviewed in this section supports the study in three substantial ways. First, it offers a description of the complexity of teacher change and the elements of the change environment in which the study should be grounded (Clarke & Hollingsworth, 2002; Guskey, 1986; Opfer & Pedder, 2011). Second, it frames the importance of the teacher's conceptions of mathematics in this change process and contains propositions regarding the manner in which these conceptions mediate classroom practices (Ernest, 1989; Wilkins, 2008). Finally, it delineates areas of potential impact during professional development activities such as demonstration lessons (Clarke et al., 2013) and includes suggestions for considering these in ways that are personally meaningful to the teacher involved (Opfer & Pedder, 2011).

Implicit Theories

Despite this emphasis on teacher conceptions of mathematics and mathematics teaching and learning, one potentially important influence on pedagogical practices that has not been well examined is the teacher's implicit theory, or mindset (Rattan et al., 2012). The basis of the implicit theories model is that an individual's implicit assumptions about the nature of an ability shapes his goal orientation related to that ability (Dweck & Leggett, 1988). These mindsets are measurable constructs (Dweck et al., 1995) and their associated goal orientations lead to observable behaviors that form the body of empirical research based in the theory. The theory has proven robust, and although it has been widely considered as a general form of self-regulation (Burnette et al., 2013), as a factor in mathematics achievement (Blackwell et al., 2007; Dweck, 2008), and as a component of school and classroom culture (Boaler, 2013, 2016), little empirical research has considered it as a mediator of mathematics teaching practices (Rattan et al., 2012).

Theoretical Basis of the Model of Implicit Theories

Situated in their prior research on goal orientation and behavior, Dweck and Leggett (1988) described the social-cognitive model of motivation and personality that has developed into the implicit theories framework. In this model, the authors posited that an individual's implicit assumptions about the nature of an ability lead directly to the type of goals he pursues regarding that ability and the behaviors he exhibits when faced with challenges to that ability (Dweck & Leggett, 1988; Dweck et al., 1995). These mindsets and their associated goal pursuits thus created "a framework for interpreting and responding to events" (Dweck & Leggett, 1988, p. 260) that promoted observable behavioral patterns when the ability under consideration is challenged. Two implicit theories, the *entity theory* and *incremental theory*, were elaborated on within the model.

Incremental theories. The model described individuals espousing an incremental theory as those who view attributes as malleable, with the potential for the related ability to grow over time. Subscribers to this *growth mindset* often establish learning goals that are focused on improvement of the ability in question (Dweck, 1986; Dweck & Leggett, 1988; Elliott & Dweck, 1988). When faced with challenging situations related to this ability, individuals with growth mindset characteristics display adaptive, mastery-oriented responses characterized by engagement with the challenges and persistence when faced with failure (Elliott & Dweck, 1988).

Entity theories. Individuals assuming an entity theory tended to view attributes as fixed, uncontrollable entities, for which ability was determined by factors over which the individual had no control. Those with these *fixed mindset* characteristics adopted performance-oriented goals to gain positive judgments for skills they had already mastered or to avoid negative judgments regarding talents they had yet to acquire (Dweck, 1986; Dweck & Leggett, 1988; Elliott & Dweck, 1988). When faced with challenges, these individuals displayed maladaptive, helpless responses characterized by lowered performance and avoidance of the imminent challenge (Elliott & Dweck, 1988).

Generalization of the model. Although the tenets of implicit theory were initially established through research regarding characterization of an individual's own intelligence (Dweck & Leggett, 1988), the model was soon generalized to other attributes and domains. The authors predicted that for any attribute of personal significance, "viewing it as a fixed trait will lead to a desire to document the adequacy of that trait, whereas viewing it as a malleable quality will foster a desire to develop that quality"

(Dweck & Leggett, 1988, p. 266). Applications of this prediction culminated in the validation of a simple instrument used to assess an individual's implicit theories for a variety of attributes (Dweck, Chiu, & Hong, 1995). Additional evidence supported the notion that the model holds for generalization to other traits, such as the character and attributes of other people (Erdley, & Dweck, 1993), or mathematical ability (Lischka, Barlow, Willingham, Hartland, & Stephens, 2015; Rattan et al., 2012; Willingham, Barlow, Stephens, Lischka & Hartland, 2016).

Empirical Research on Implicit Theories

This section contains reviews of the methods and results of three empirical studies of implicit theories offering significant findings for the study. In these studies, the roles of implicit theories in the mathematics classroom, the mediating factors of the incremental mindset on goal achievement, and the potential influence of a teacher's implicit theories on their pedagogical practices and feedback are examined.

Mediated pathways of the incremental theory. Burnette et al. (2013) conducted a large-scale (N = 28,217; k = 113) meta-analysis examining the relationship between implicit theories and self-regulation theory. In this study, three key processes of self-regulation theory (i.e., goal setting, goal operation, and goal monitoring) were operationalized through implicit theory constructs. Goal setting was aligned with performance versus learning goal orientation; goal operation was associated with helpless versus mastery responses; and goal monitoring was viewed through the dichotomy of negative emotional responses versus expectations of success. A sample of 236 citations broadly related to these constructs and published between Dweck and Leggett's seminal work of 1988 and October of 2010 was examined. This sample was reduced to 85 citations that contained bivariate effect size measurements relative to self-regulation characteristics and achievement outcomes, with these citations analyzed in the resulting meta-analysis.

Mean effect sizes and meta-regression characteristics were then reported for these citations (Burnette et al., 2013). The results of this analysis indicated a strong correspondence between the constructs of implicit theory and self-regulation theory. More specifically, the results described the relative strength of association for the mediators of incremental theory on goal achievement across a wide range of abilities, disciplines, and context (see Figure 4).

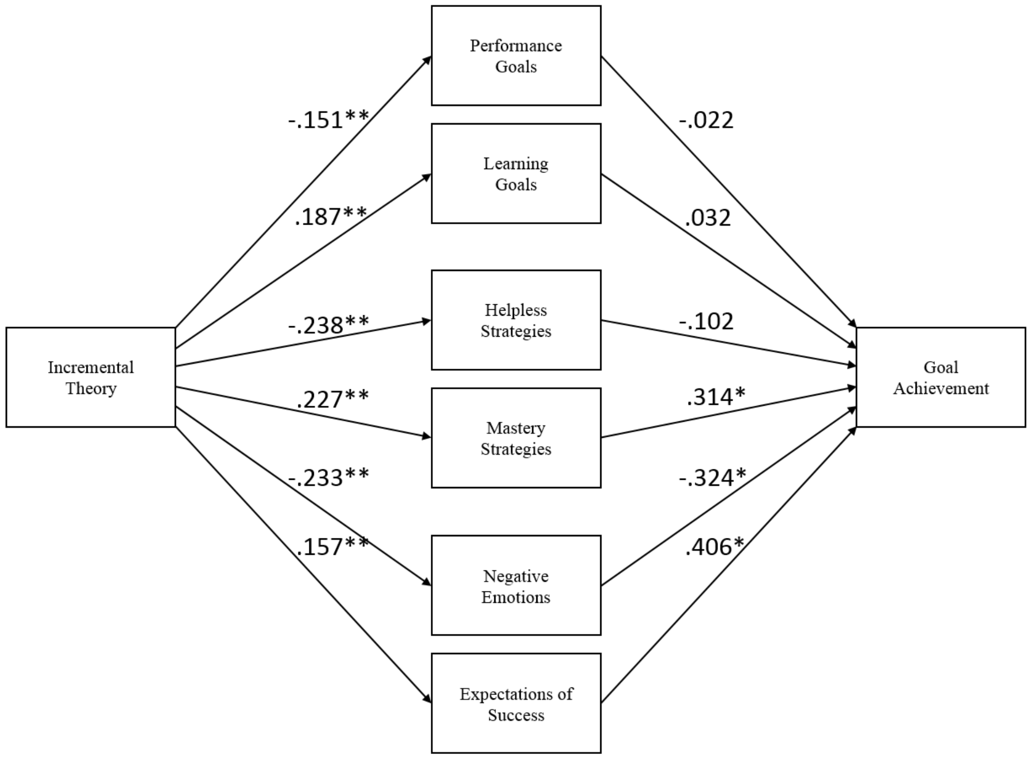


Figure 4. Mediated relationships between incremental theory and goal achievement. Adapted from "Mind-Sets Matter: A Meta-Analytic Review of Implicit Theories and Self-Regulation," by J. L. Burnette, E. H. O'Boyle, E. M. VanEpps, J. M. Pollack, and E. J. Finkel, 2013, *Psychological Bulletin, 139*, p. 106. *p<.01, **p<.001.

In alignment with prior research (e.g., Blackwell et al., 2007; Dweck & Leggett, 1988; Dweck et al., 1995), holding an incremental theory regarding an ability was associated with an affinity for learning goals, mastery strategies, and expectations of success regarding that ability, while being negatively associated with the pursuit of performance goals, helpless responses, and negative emotions regarding that ability (Burnette et al., 2013). However, this analysis also revealed significant findings regarding the positive strength of association between mastery-oriented responses and expectations of success with goal achievement and the negative relationship between negative emotions regarding an ability and goal achievement. The most significant findings resulted from considering the mediated paths between an incremental theory and goal achievement. For instance, examination of these pathways revealed that the incremental theory's avoidance of negative emotions is more strongly associated with goal achievement than its expectations of success.

Role of implicit theories in the mathematics classroom. Dweck (2008) presented the results of a sample of empirical literature examining the influence of mindset on mathematics and science in order to frame a specific set of recommendations regarding the role of mindset in these classrooms. Her review highlighted results indicating increased achievement in these subjects based on a learning-goal orientation (e.g., Blackwell et al., 2007; Grant & Dweck, 2003), the role of mindset in traditionally underrepresented populations in mathematics and science (e.g., Aronson, Fried, & Good, 2002; Good, Rattan, & Dweck, 2007), and interventions that impact mindsets (e.g. Blackwell et al., 2007; Aronson et al., 2002).

Based on this body of work, four specific recommendations were provided regarding the integration of mindset research into the classroom (Dweck, 2008). First, teachers should help establish growth mindset characteristics in their students by explicitly discussing brain development and its relationship to ability, placing value on effort and the role of mistakes in learning, and offering process-oriented praise and feedback. Second, teacher educators should mirror these implementations in teacher preparation, professional development programs, and curricular materials. Third, underachieving and underrepresented populations should be taught that traditional performance gaps are based on environmental factors that can be overcome through personal effort and educational support systems. Finally, high-stakes testing should be adapted to value growth mindset characteristics and allow teachers to focus on these premises.

The influence of implicit theories on pedagogical practices. Rattan et al. (2012) reported the results of a series of four experimental studies describing how incremental and entity theories influenced individuals' perceptions of student mathematical ability and responses to students based on this ability. They also examined how students perceived these responses in relation to entity and incremental mindsets and how the responses impacted student motivation. Participants in the studies (n = 41 to n = 95) were either undergraduate university students or graduate teaching assistants in mathematics-related areas.

In each study, the mindset characteristics of the subjects were measured in relation to mathematical ability and descriptions of their responses to imagined scenarios involving mathematics classrooms were collected (Rattan et al., 2012). Responses were classified in one of two ways: feedback or suggested pedagogical practices. Feedback responses were separated into either comfort-oriented or strategy-oriented feedback. Suggestions for pedagogical practices were separated into strategies that would be likely to cause disengagement from mathematics or practices that would motivate further engagement in mathematics.

The results of the four experiments indicated that individuals holding an entity theory with regard to mathematical ability were more likely to attribute poor performance in mathematics to a lack of mathematical ability rather than a lack of effort, while those holding an incremental theory reversed these associations. Additionally, entity theorists were significantly more likely to offer comfort-oriented feedback rather than strategy-oriented feedback and to suggest potential pedagogical practices that would result in students disengaging with mathematics (Rattan et al., 2012). These results were replicated across both undergraduate students imagining themselves in a teaching role and in the sample of graduate teaching assistants. Further findings revealed that students receiving comfort-oriented feedback perceived their teachers as having fixed views of mathematical ability, lower expectations, and less investment in their students than those receiving strategy-oriented feedback and felt less motivated to continue engaging with mathematics.

Significance of the Research

The research reviewed in this section is significant to the study for three primary reasons. First, the constructs of implicit theory, particularly when focused on perceptions of mathematical ability and the teaching of mathematics, represent an important conception of mathematics that has not been well examined with regards to pedagogical practices (Dweck & Leggett, 1988; Dweck et al., 1995; Rattan et al., 2012). Second, this literature supports the idea that tenets of implicit theory, specifically the growth mindset and its associated goal orientation and mastery responses, may act as mediators that directly influence classroom-teaching practices (Dweck, 2008; Elliott & Dweck, 1988;

Rattan et al., 2012). Finally, this review provides recommendations for operationalizing the constructs of the growth mindset through the tenets of self-regulation theory (Burnette et al., 2013) and descriptions of classrooms in which the constructs have been successfully enacted (e.g., Aronson et al., 2002; Blackwell et al., 2007; Dweck, 2008; Good et al., 2007).

Theoretical Framework

Two of the theoretical constructs identified in this review guided this study. The first of these was the IMTPG (Clarke & Hollingsworth, 2002). The personal, external, practice, and consequence domains of this theory delineated the teacher experiences examined in the study: mindset characteristics, professional development activities, classroom practices, and interactions with students. Data collection was structured to consider the interactions among these domains, and the resulting case narrative provided a rich account of the change environment within which the case-study teacher operated. The second theoretical pillar was the model of implicit theories, specifically the incremental theory (Dweck & Leggett, 1988). The growth mindset of the case-study teacher was the major construct under consideration throughout the study. The manner in which these characteristics were displayed acted as mediators within the IMTPG and directly addressed the study's primary research question. Additionally, the teacher's goal setting, goal operation, and goal monitoring in response to challenges were directly observed during data collection, with these behaviors used to provide insight into the significance of the growth mindset in the teacher's engagement in professional development. Together, these theories provided the framework used to examine the manner in which a teacher translated professional development experiences into classroom practices.

Connections of the Literature to the Current Study

In addition to providing the theoretical framework previously described, the reviewed literature supported the study in at least four areas: informing the context and design of the study, shaping its research question, suggesting appropriate data collection methods, and raising awareness of potentially confounding influences. This section will discuss relevant factors from the reviewed literature for each of these areas.

Context and Design

The literature reviewed in this chapter provided a deeper understanding of the context in which the study was set and offered suggestions that influenced its research methodology. The primary idea under examination in the study was the manner in which characteristics of the growth mindset influenced a teacher's engagement in professional development experiences (Burnette et al., 2013; Dweck & Leggett, 1988). As professional growth is a gradual and often difficult process that requires continued support and time (Guskey, 1986), the study focused on the interplay of several aspects of this learning over time in order to understand their significance (Opfer & Pedder, 2011). Reflective and enactive interactions among the personal, external, practice, and outcome domains (Clarke & Hollingsworth, 2002) were considered as the teacher observed, interacted with, interpreted, and implemented a demonstration lesson in her classroom (Clarke et al., 2013). A case-study methodology was used to explore the unique context of this teacher's change environment and to consider alignments and variations against established theories (Clarke & Hollingsworth, 2002; Opfer & Pedder, 2011; Wilkins, 2008).

Research Question Extensions

As the influence of growth mindset characteristics on the case-study participant's personal experiences with professional development were considered, the exploratory nature of the study could have allowed the researcher to examine many extensions of the research base presented in this chapter. However, four primary areas of focus emerged during the course of the study. First, the manner in which the teacher's conceptions regarding mathematics acted as mediators to her practices was explored (Burnette et al., 2013; Dweck, 2008; Rattan et al., 2012; Wilkins, 2008). Second, how principles and activities from the professional development environment were utilized by the teacher to continue professional learning outside this environment was considered (Clarke & Hollingsworth, 2002; Guskey, 1986). Third, the similarities and nuanced differences in the espoused and enacted conceptions of the teacher, and the factors that helped reconcile these, were examined (Clarke et al., 2013; Ernest, 1989; Wilkins, 2008). Finally, the impact of the teacher's interpretations of her professional development experiences on the reality of her classroom were studied (Clarke & Hollingsworth, 2002; Opfer & Pedder, 2011).

Data Sources

Directly related to these ideas, the literature reviewed provided broad guidelines for the collection of meaningful data. The data focused on the teacher's espoused, self-described characteristics in comparison to those observed in practice (Ernest, 1989;

Wilkins, 2008). Specific characteristics of the mindset in action, operationalized through the tenets of self-regulation theory (Burnette et al., 2013), were observed and questioned when possible. The influence of these characteristics on pedagogical decisions and practices was prioritized (Dweck, 2008; Rattan et al., 2012). The teacher's areas of focus during the demonstration lesson (Clarke et al., 2012) and processes of interpreting and implementing the premises of this lesson into her unique circumstances (Opfer & Pedder, 2011) were examined. Finally, the actions and relationships among multiple systems that were personally meaningful to the teacher were explored (Clarke & Hollingsworth, 2002; Opfer & Pedder, 2011).

Confounding Influences

Finally, the literature base suggested four factors that could have made analysis of the study's data difficult. First, the need for continued support, particularly in the absence of critical resources, and the time frames needed to see meaningful change (Guskey, 1986; Opfer & Pedder, 2011) could have been significant. Bounding the study within a single semester and examining the teacher's perceptions of past event and future plans helped lessen this concern. Second, without careful attention, the differences in the participant's espoused and enacted conceptions, or the true nature of these conceptions, could have been overlooked (Clarke et al., 2013; Ernest, 1989; Wilkins, 2008). The use of historical data regarding conceptions important to the study and multiple data sources, including interviews, journal entries, and classroom observations, collected over an extended period of time helped resolve this issue. Third, the complexity of the change environment could have been difficult, or impossible, to make meaningful sense of without substantial experience in this type of research (Clarke & Hollingsworth, 2008; Opfer & Pedder, 2011). The rich descriptions developed from the study's data along with its grounding in a strong theoretical framework assisted in making sense of these changes in context. Finally, careful attention was paid to differentiating the specific mechanisms through which the participant's mindset influenced her goal outcomes (Burnett et al., 2012). As the study's design was informed by this literature base and the researcher was aware of the potential issues arising in these areas, these obstacles were largely alleviated.

Chapter Summary

This chapter contained a review of theoretical and empirical literature supporting the current study, focusing primarily on the processes of teacher change and implicit theories. Although the review did not attempt to synthesize the extensive volume of literature existing in these areas, the literature that was examined was carefully selected to provide a strong base of support for the current study. To this end, the review elaborated further on the theoretical framework that grounded the study and considered how this research base informed the contexts of the research, the research question, and the methodological design of the study. The next chapter will provide details of this research methodology as informed by this literature review.

CHAPTER III: METHODOLOGY

Introduction

Although the United States has experienced some success in mathematics in recent years (Mullis et al., 2012; NCES, 2013), there is much work left to be done (Darling-Hammond, 2010; NCES, 2013; OECD, 2013a, 2013b). Many factors impact the quality of the classroom teachers who will do this work (Blömeke et al., 2011; Hill et al., 2005; Whitehurst, 2002), but perhaps none so much as the knowledge and dispositions of the individual teacher (Ernest, 1989; Goldsmith & Shifter, 1997; Pajares, 1992; Philipp, 2007; Wilkins, 2008). Although the impact of elements such as attitudes and beliefs about mathematics on teaching practices has been well examined (Wilkins, 2008; Clarke & Hollingsworth, 2002), other factors, such as the teacher's mindset, have not (Rattan et al., 2012). Additionally, although aspects of effective professional development programs for mathematics teachers have been described (Desimone et al., 2002; Garet et al., 2001; Loucks-Horsley et al., 2003), specific components, such as how teachers actually utilize demonstration lessons (Clarke et al., 2013) are still under consideration. Therefore, the purpose of this study was to explore one of these motivational factors, the teacher's mindset, within the contexts of the teacher's professional development experiences.

This chapter contains details of the research methodology utilized in the study. It begins with an overview of the design and describes the context within which the study was conducted. This is followed by a description of the study's participant selection, the types of data collected, and the instruments and procedures that were used to gather this data. Next, the chapter addresses measures taken to assure the study's trustworthiness along with its limitations and delimitations. Finally, the processes used to analyze the data in order to address the study's research question are presented.

Research Overview

The study utilized an exploratory, holistic single-case design (Yin, 2014) to consider how characteristics of the growth mindset influence a mathematics teacher's interpretations and enactments of professional development experiences. Four important features supported this choice of methodology (Yin, 2014). First, three attributes of the study's research question supported a case-study approach: the question focused on how mindset characteristics influenced a teacher's interpretations and practices, required observations of behavior in authentic environments, and addressed a problem of contemporary significance. The essence of this question made case-study methodology an appropriate choice and suggested a holistic approach in which the overall nature of the teacher's experiences was examined. Second, the exploratory nature of the study was supported through situating its purpose in prior research. This provided guidance as to what was to be explored and suggested criteria by which the study was considered successful. Third, the unit of analysis was defined, focusing on the critical case of a single teacher who displayed characteristics of the growth mindset and was transitioning to the use of reform-oriented teaching practices. Considering the teacher's interactions with a demonstration lesson helped to delineate boundaries for the case and further supported the single-case design. Finally, a variety of frameworks regarding influences on teaching practices were available in the literature, which laid out "key factors, constructs, or variables, and presumes relationships among them" (Miles & Huberman, 1994, p. 440). This variety was important as it provided scaffolding for the emergent data analysis and included alternate interpretations against which the study's significant findings could be compared.

Research Context

The elements of the research context presented here are described based on the selection of the case-study participant, which is detailed in the next section. Five elements of the study are described: the state, school district, and school in which the study took place; the professional development environment in which the participant was engaged during the study; and the participant's background. The study focused on an elementary grade mathematics teacher engaged in an ongoing professional development program in a predominantly rural school district within a southeastern state during Fall Semester of 2015. This section contains relevant details for each of these elements.

State

The study was conducted in a southeastern state, which has traditionally underperformed on the National Assessment of Educational Progress' (NAEP) 4th grade mathematics exam (U.S. Department of Education [USDE], 2014). After seven years of stagnant performance on this exam, the state adopted the Common Core State Standards for Mathematics (CCSSM) in July of 2010. This was followed by an incremental implementation of the CCSSM, alongside existing state standards during the 2011-2012 and 2012-2013 school years, with a full implementation of the CCSSM during the 2013-2014 school year. For the 2014-2015 school year, the CCSSM had been retained under a title specific to the state. On the 2013 NAEP 4th grade mathematics exam, the state saw its first statistically significant gains since 2003, with its overall results rising to the level of the national average (USDE, 2014). Although results for male and female students were not significantly different, significant achievement gaps remained for Black and Hispanic students and for students eligible for free or reduced school lunch, an indicator of low family income. These achievement gaps have not changed significantly in the last two decades (USDE, 2014). On the 2014-2015 statewide comprehensive assessment program's grade 3-8 mathematics achievement exams, 24.1% of students in the state earned an advanced score, 31.5% performed at a proficient level, 29.6% attained basic results, and 14.8% scored at a below basic level.

School District

The study was conducted in a rural school district centered approximately 65 miles from one of the largest cities in the state. During the 2013-2014 school year, the district serviced 4,588 students among nine schools through 311 teachers and 32 administrators. Enrollment demographics for students in the district included 90.9% White, 5.2% Hispanic or Latino, 3.0% Black or African American, 0.7% Asian, and 0.2% Native American or Alaskan. Economically disadvantaged students represented 59.1% of the population, and students with disabilities comprised 13.4%. On the 2014-2015 statewide comprehensive assessment program's grade 3-8 mathematics achievement exams, 24.3% of students in the district earned an advanced score, 34.0% performed at a proficient level, 29.6% attained basic results, and 12.1% scored at a below basic level. These results were similar to the performance for the entire state.

School

The study was conducted in a rural elementary school serving approximately 330 kindergarten through fifth grade students. During the time of the study the school employed 25 instructional faculty, 15 support personnel, and 2 administrators. Enrollment demographics for the school were similar to that of the district. On the 2014-2015 statewide comprehensive assessment program's grade 3-8 mathematics achievement exams, 14.0% of students in the school earned an advanced score, 33.3% performed at a proficient level, 38.0% attained basic results, and 14.7% scored at a below basic level. These results were slightly lower than that of the district and the state.

During the semester of the study, 3 of the school's 25 instructional faculty were involved in the professional development program described in the next section. This participation included 2 of the school's 3 second grade teachers, including the case-study participant. Based on the participant's description of the school's culture, these teachers planned their classes independently under a guided curriculum, but had opportunities to implement new instructional ideas and interact regarding their lesson designs and implementations on a regular basis. The study's participant also described taking advantage of these opportunities frequently through interactions with the other teachers engaged in the professional development project.

Professional Development Environment

The study was conducted within an ongoing professional development project, Project Influence, designed to support mathematics teachers from kindergarten through eighth grade in the development of effective mathematics teaching practices (NCTM,

2014) within a reform-oriented instructional environment. At the time of the study the project served approximately 120 in-service mathematics teachers across four grade bands: K-2, 3-4, 5-6, and 7-8. The project, in its third year of external funding at the time of the study, represented a partnership between a state university and four surrounding rural counties. The major activities of this year of the project included four academic year meetings, two rounds of demonstration lessons (spring and fall) comprised of four to six lessons targeting different grade bands, and a two-week summer institute designed to deepen mathematical content knowledge within the theme of composition and decomposition in mathematics.

Activities within the project were structured as immersion and practice-based experiences as recommended by Loucks-Horsley et al. (2003). Immersion activities offered participants the opportunity to directly engage in mathematical content and processes and were aligned to the grade-band content of the participants. Practice-based activities, including demonstration lessons, offered participants the opportunity to analyze student thinking directly through student work, segments of video of mathematics lessons, or real-time observations of mathematics teaching. In tandem, these activities were implemented to support development in the participants' mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008; Hill et al., 2005).

The Participant

Gale Martin, a Caucasian female in her mid-thirties, was selected as the critical case for the study. Ms. Martin was an elementary mathematics teacher in her second year of teaching second grade and her fifteenth year of teaching elementary school. She taught in a rural elementary school of approximately 330 students in a southeastern state. Ms. Martin's classroom hosted 17 students, 9 of which were female, and 8 of which were male. The majority of her class was Caucasian, with the exception of two African-American male students. Prior to teaching second grade, Ms. Martin had taught one year of kindergarten, two years of third grade, and 10 years of fourth grade, providing her some perspective in the mathematical content requirements of several elementary grades. During the course of this study, Ms. Martin was also engaged in her third year of the professional development project, Project Influence.

Participant Selection Process

Adhering to Yin's (2014) description of a critical case, Ms. Martin was selected for the study as she displayed critical elements of the theoretical constructs previously developed: a teacher engaged in the processes of change (Clarke & Hollingsworth, 2002) who displayed strong growth mindset characteristics (Dweck & Leggett, 1988). As multiple sources of evidence for these characteristics were drawn from the historical records of Project Influence, Ms. Martin was one of the participants given priority for the study, as she had a rich history within the project.

Ms. Martin was selected for participation in the study through the following participant selection process. An initial field of potential participants was identified based on conversations with project faculty regarding individuals who had displayed evidence of the critical characteristics of the study during previous professional development activities. Three sources of archived data were examined, as available, for each of these participants. First, changes in the participants' beliefs about mathematics, beliefs about knowing and learning mathematics, and beliefs about children's doing and learning mathematics were examined. These beliefs were considered based on results of the Integrating Mathematics and Pedagogy Web-Based Beliefs Survey (IMAP, Ambrose, Clement, Philipp, & Chauvot, 2004) completed by all participants twice per year of the project. Second, changes in the participants' classroom practices, based on prior observations using the Reformed Teaching Observation Protocol (RTOP, Sawada et al., 2002) were reviewed. As these observations were completed in a pre/post fashion for a limited sample of participants in each year of the project, they were reviewed as available. Finally, the mindset characteristics of each participant relative to intelligence, morality, the world, and mathematical ability were examined using a modified version of Dweck et al.'s (1995) mindset survey (Lischka et al., 2015; Willingham et al., 2016). Participants were then interviewed regarding the critical elements described here, with the participant who best represented these elements invited to participate in the case study.

Data Collection Sources and Instruments

The data collected throughout the study focused on how Ms. Martin's mindset characteristics influenced other experiences, beliefs, and practices. By focusing on this phenomenon throughout the study, and collecting multiple sources of data for each set of activities, the data was triangulated to strengthen the validity of the findings (Yin, 2014). Data sources for the study included semi-structured interviews, classroom observations, video samples from the demonstration lesson collected from Ms. Martin's point-of-view, observations of professional development activities, reflective journal entries, and artifacts from Ms. Martin's lessons. Each of these sources of data is described in this section. Additionally, as the researcher is considered a key instrument in qualitative data collection (Creswell, 2012), a description of the researcher's background is included.

Semi-structured Interviews

Semi-structured interviews were utilized to allow flexibility in the interviewing process due to items of significance that emerged during the study (Galletta, 2013). These interviews were completed six times during the course of the study, with different areas of focus for each interview. The first interview was conducted to facilitate selection of the case study participant (see Appendix A). The second interview gathered baseline information on Ms. Martin's background and mindset characteristics in relation to her perceived teaching practices (see Appendix B). The third interview, based on video from Ms. Martin's point-of-view in the demonstration lesson, considered her areas of focus and interpretations of the demonstration lesson and professional development activities, and asked how Ms. Martin expected to adapt the lesson for use in the classroom (see Appendix C). The fourth interview considered the actual adaptations made to the demonstration lesson for classroom use and discussed the unit planning process, individual lesson expectations, and goals of the instruction (see Appendix D). The fifth interview followed the unit's implementation and considered Ms. Martin's perceptions of the unit, changes to be made in the future, and specific practices based on video recorded during the unit (see Appendix E). The final interview included questions designed to have Ms. Martin reflect on the process of observing, adapting, and implementing the demonstration lesson and offer advice for future teachers considering this process (see Appendix F). All interviews were audio recorded so that significant exchanges could be transcribed to support analysis.

Classroom Observations

Classroom observations were used to examine the relationship between Ms. Martin's mindset characteristics and mathematical teaching practices. Sets of classroom observations were conducted at two times during the study, once prior to Ms. Martin's observation of the demonstration lesson and once while incorporating this lesson. An observation protocol (see Appendix G) was developed to focus observations on growth mindset characteristics enacted in the classroom (Burnette et al., 2013; Dweck, 2008; Dweck & Leggett, 1988; Rattan et al., 2012) and effective mathematics teaching practices (NCTM, 2014). The protocol guided general observations of the classroom with specific references to growth mindset characteristics and mathematics teaching practices cued from the included descriptions. The protocol was used to record observations for all classroom observations.

The lessons observed during these units were video recorded for three purposes. The first was to support corroboration between the observation protocol and the actual characteristics of the classroom. Second, the video was used to promote reflection and guide questioning during the fifth participant interview (see Appendix E). Finally, important portions of the video collected were transcribed to support analysis and provide the narrative descriptions of Ms. Martin's classroom practices included in the study's results.

Point-of-view Video

During Ms. Martin's observation of the demonstration lesson, she wore a pair of glasses with a small video camera attached. This camera supported an on-demand recording feature that captured the previous 30 seconds of Ms. Martin's observational focus onto video when she provided a signal. Ms. Martin was instructed on the use of the camera and allowed to record and view samples of the video generated during the baseline classroom observations in order to establish comfort with this technology. Using this camera, Ms. Martin recorded aspects of the pre-lesson briefing, the demonstration lesson, and the post-lesson debriefing believed to be most significant. These video samples were used to guide questioning during the third participant interview (see Appendix C).

Professional Development Observations

Ms. Martin's interaction in the professional development activities associated with the demonstration lesson were observed, with observations recorded utilizing a modified version of the classroom observation protocol described above (see Appendix G). During the professional development observations only the first page of the protocol was utilized in order to collect general notes about the day. Additionally, these activities were video recorded to support corroboration with the observations recorded through the protocol, allow transcription, and guide questioning during interview three (see Appendix C).

Reflective Journal Entries

A reflective journal was administered via email over the course of the study (see Appendix H). In general, the prompts in this journal were intended to elicit Ms. Martin's immediate interpretations of the activities in which she was involved and to inform the researcher of her perspective regarding upcoming events. These entries were used to support interview findings and allow some comparisons across time for questions that were asked in both formats.

Artifacts

Unit and lesson plans from the observed lessons, images of student artifacts from the observed lessons, and images of participant artifacts from professional development activities were collected to provide supporting documentation for observations. Additionally, these data sources were examined for evidence of the constructs under study as appropriate for their type.

The Researcher as an Instrument

The researcher served a significant role in the collection of data for the study (Creswell, 2012). Five sets of related experiences qualified the researcher to conduct the study. First, two years of doctoral coursework in mathematics and science education provided relevant knowledge regarding teaching and learning and educational research methodologies. Second, these classroom experiences were supplemented by involvement in a variety of qualitative research projects utilizing methodologies similar to the study. These projects were supervised by faculty with expertise in these methods and subjected to peer review through national-level presentations and publications. Third, two of the projects in which the researcher was most recently involved directly considered the role of implicit theories with in-service teachers. Fourth, these academic experiences were supplemented by 10 years of K-16 classroom teaching experience, with six years as a full-time secondary mathematics and science teacher. Finally, the researcher was actively involved in Project Influence as both a research and teaching assistant.

The researcher's involvement in Project Influence deserves further consideration. As a research assistant in this program over the course of its three-year history, the researcher established strong relationships within its community of practice with both the project's faculty and participants. More specifically, his relationship with Ms. Martin produced specific affordances and constraints to the study that warrant description. Due to Ms. Martin's strong commitment to Project Influence, the researcher was allowed unrestricted access to her participation in professional development activities, planning and implementation of classroom lessons, and other school-related actions. Additionally, this relationship likely facilitated Ms. Martin's willingness to participate in the study's activities, impacted the fullness of the answers she provided to questions asked by the researcher, and influenced her awareness of her classroom practices and interview responses. Additionally, the researcher's prior interactions with Ms. Martin combined with his observations of her classroom over an extended period of time likely influenced his interpretations of her words and actions. Readers should be aware of these potential biases as they in turn interpret the results of the study.

Data Collection Procedures

After receiving institutional review board approval (see Appendix I), the data sources described above were collected throughout the study in four distinct stages. The specific procedures utilized in each stage are described in this section.

Stage One: Participant Selection

Four primary activities were involved in the first stage of data collection, which was designed to select the case-study participant. First, the professional development project's faculty were informally interviewed to produce a list of potential candidates who exhibited the critical features required for the study. Once these candidates were identified, the second activity involved reviewing archived data from prior years of the project related to beliefs about mathematics, classroom teaching practices, and mindset characteristics. Third, after describing the purpose and procedures of the study and obtaining participant consent, each of the potential candidates was interviewed regarding the beliefs and mindset characteristics previously reviewed (see Appendix A). Finally, the candidate best exhibiting the desired characteristics, Ms. Martin, was invited to participate in the study, and the first set of reflective journal prompts (see Appendix H) was issued for responses. Additionally, Ms. Martin was surveyed regarding areas of mathematics content on which the study's demonstration lesson could be focused in order to facilitate its use in her classroom during the fall semester.

Stage Two: Baseline Observations

Once Ms. Martin was selected, the second stage was used to explore her background, consider her mindset characteristics in relation to her perceived teaching practices, and establish baseline observations of her actual teaching practices. This was facilitated through an audio-recorded interview focusing on these background characteristics (see Appendix B), a second set of reflective journal prompts (see Appendix H), and a series of classroom observations conducted during a unit of instruction in Ms. Martin's classroom. This unit of instruction encompassed an introduction to place value with two- and three-digit numbers within one week of instructional time. Baseline classroom procedures and mathematics practices were established and recorded during these observations, and behaviors which indicated the growth mindset in action were recorded. This was facilitated through the use of the observation protocol (see Appendix G) and documented through video recording in addition to the observation protocol. Additionally, Ms. Martin practiced recording sample video clips using the point-of-view camera during this phase in order to prepare for their implementation in stage three.

Stage Three: Professional Development Activities

The third stage of data collection was utilized to examine Ms. Martin's areas of focus, interpretations, and intentions for use of the demonstration lesson, and to observe her engagement in the professional development activities associated with this lesson. These activities were situated within a day of Project Influence on October 28, 2015. They consisted of a morning pre-lesson briefing, the demonstration lesson, a post-lesson debriefing, and an additional afternoon professional development activity unrelated to the study. During the morning activities, Ms. Martin wore the point-of-view camera with which she rehearsed in her classroom to record 30-second increments of video capturing important moments from the professional development activities and demonstration lesson. Additionally, Ms. Martin's interactions during the pre-lesson briefing and post-lesson debriefing were recorded and observed utilizing a modified, one-page version of the classroom observation protocol (see Appendix G). The afternoon activities were similarly recorded and observed, with Ms. Martin's interactions during these activities, focused recording segments, and initial intentions for the use of the demonstration lesson forming the basis of the follow-up interview (see Appendix C). Responses to reflective prompts were collected from the professional development activities and from the third section of the reflective journal (see Appendix H).

Stage Four: Demonstration Lesson Adaptation and Implementation

The final stage was used to consider the manner in which Ms. Martin adapted and implemented the demonstration lesson for classroom use. The classroom observations of the unit of instruction containing the adapted demonstration lesson occurred across three class days between November 12 and November 20, 2015. This unit of instruction contained a cohesive set of activities incorporating the mathematical content of the demonstration lesson and focused on the transition from subtraction with two-digit numbers to subtraction with three-digit numbers involving regrouping.

Approximately one week before these observations occurred, the fourth set of reflective prompts (see Appendix H) was issued for response. An interview focused on Ms. Martin's actual adaptations of the demonstration lesson, unit-planning processes, individual lesson expectations, and goals of instruction (see Appendix D) was conducted two days before the observations began. The classroom observations for the unit were video recorded for alignment with the classroom observation protocol described above (see Appendix G) and the reflective prompts accompanying each day of instruction (see Appendix H). Immediately after the unit's implementation, the final set of reflective journal prompts was issued (see Appendix H). Approximately four weeks later, a post-unit interview examining Ms. Martin's perceptions of the unit, ideas regarding improvement of the unit, and observations regarding video-recorded segments of the instruction (see Appendix E) was conducted. A final project interview, holistically examining Ms. Martin's processes of observing, adapting, and implementing the demonstration lesson (see Appendix F) followed on December 21, 2015.

Data Analysis

Based on Yin's (2014) recommendations, three general principles guided the data analysis for this case study. First, one aspect of the analysis was focused on constructs of teacher change and mindset as identified in the theoretical literature described previously. This grounded the study's results in prior literature and contributed to their overall significance. Second, a case description accompanied the data regarding Ms. Martin's observations, interpretations, adaptation, and implementation of the demonstration lesson. This provided clarity to calls in the literature regarding teachers' focus on and utilization of demonstration lessons (Clarke et al., 2013). Finally, the analysis considered a plausible rival explanation for the findings based on a conceptual framework developed from evidence in the study and related this framework back to the theoretical literature on which the study was founded.

More specifically, the data was initially compiled and reduced in the following manner. Ms. Martin's interviews were transcribed and, along with the completed observation protocols, were used to assist in identifying significant segments of the project's videos. These segments of video were transcribed and combined with the interviews, observation protocols, and collected reflective journal responses to constitute the major body of data to be analyzed. Artifacts of consequence, such as student work samples collected during the classroom observations, were similarly reduced.

This body of data was then organized in a chronological fashion, corresponding approximately with the data collection stages described above, and analyzed in the fashion of a simple time series (Yin, 2014). A holistic analysis of themes, "not for generalizing beyond the case, but for understanding the complexity of the case" (Creswell, 2012, p. 101), was performed for the first stage of data through open coding and reduction of these codes into themes consistent with the theoretical framework. The themes from the stage one analysis were then used to guide interpretation and coding of the stage two data, and the stage one codes were revisited for completion. This process was repeated through all four stages of data in order to produce a comprehensive set of themes to guide a written case description.

As an example of this coding process, consider the generation of the open codes and emergent themes for the stage one data (see Table 1). To generate the open codes in Table 1, the researcher read the transcribed copy of Ms. Martin's initial selection interview and her first set of reflective journal entries and coded Ms. Martin's responses in terms of distinct concepts and categories that originated in the text. A second reading

Table 1

Open Codes and Emergent Themes by Data Collection Stage

<table>
<tr><th colspan="2">Stage One Codes and Themes</th></tr>
<tr><th>Open Codes</th><th>Emergent Themes</th></tr>
<tr><td>AN – Avoidance of negative emotions
F – Description of fixed mindset
F/G – Differentiation of fixed and growth mindsets
G – Evidence of growth mindset
GM – Goal monitoring
GO – Goal operations
GS – Goal setting
HE – High expectations
LG – Learning goals
LTG – Long term goals
M – Awareness of mindset
MS – Mastery strategy
PG – Performance goals
PG – Purposeful goals</td><td>1) Evidence of Growth Mindset
a. Awareness of Mindset
b. Operationalization of Mindset</td></tr>
<tr><td>B→P – Conceptions influencing practices
B→E – Conceptions enacted externally
BM – Belief about mathematics
BT – Belief about teaching mathematics
C/A – Differences in thinking of children and adults
Con→Pro – Concept before procedure
Context – Importance of context
CO – Classroom outcomes
E→B – External influence on conceptions
IT – Student-centered mathematics
MCC – Mathematics as a connected web
O→B – Outcomes influencing beliefs
Trans – Evidence of transition
VSC – Value of structures and concepts</td><td>2) Evidence of Beliefs about Teaching and Learning Mathematics
a. Mathematics as a Connected System
b. Value of Mathematical Structures and Concepts
c. Value and Uniqueness of Students’ Thinking about Mathematics
d. Value of Students’ Communication about Mathematics</td></tr>
</table>

was then conducted to confirm that the initial codes were accurate and to axially code the initial list in a coherent manner based on the study’s theoretical framework. This process of organization led to the emergent themes recorded in Table 1. With these themes in mind, the researcher read the text from second stage of data collection and generated a new set of open codes for the stage two data. Newly generated codes appropriate to the stage one themes were incorporated into the list of open codes for stage one. The stage two texts were then submitted to the same axial coding and review process with each round of coding propagating forward into the next stage of data and used to complete the previous stage. When all four stages of data had been open coded, axially coded, and checked against the other stages, the final comprehensive list of codes and themes was generated. Table 1 is extended to provide this comprehensive list of codes and themes in Appendix J.

This first stage of data was then check against this comprehensive list of codes and themes, and a narrative for this stage of data was composed. This narrative was submitted to Ms. Martin for a member check. Once the first stage narrative passed this member check process, the same process was applied to the second stage, with the data confirmed against the comprehensive set of themes, and a narrative for the second stage developed and submitted for member checking. This cycle was repeated until all four stages of the data had been fully coded, developed into a narrative text, and vetted through the member check process. A final version of the full narrative for the study was then compiled and submitted for a final member check. This final narrative contained the full results of the study reported in Chapter 4 and was used for an interpretive analysis in which important constructs, processes, and relationships were considered (Miles & Huberman, 1994). The process resulted in the conceptual framework and other significant implications of the study presented in Chapter 5.

Boundaries of the Study

Boundaries arise in any research study due to design factors such as the study's context, the methodology selected to explore its research questions, and other similar characteristics that are both within and outside of the researcher's control. Additionally, high-quality qualitative research studies require that a certain degree of trustworthiness is established for their results to be considered valid. Their results should also be shown to reliably measure the constructs under consideration. This section first addresses these issues of trustworthiness before expanding on its specific limitations and delimitations.

Issues of Trustworthiness

The trustworthiness of the current study was established largely on the basis of its *credibility*, or its insurance that the full degree of complexity of the study's context was examined (Gay et al., 2011). This credibility was predicated on prolonged interactions with the case, persistent observations, and interviews, which resulted in an ample, varied body of data. These data collection methods and the data they produced were then used to support triangulation (Gay et al., 2011; Yin, 2014), which resulted in a more complete account of the situations under examination.

An example of this process is the manner in which Ms. Martin was first extensively interviewed about her perceptions of her classroom practices before these processes were directly observed over the course of a week during the baseline observations. Additionally, Ms. Martin responded to specific questions about each day's practices and outcomes in a reflective journal, which became the foundation of future interview questions and observations. Recordings of these interviews, videos of the classes observed, student artifacts from the classes, and the journal Ms. Martin kept were all retained as part of the case's data.

In an additional attempt to ensure the study's credibility, member checks were performed at five points throughout the study's data analysis. In each case, a portion of analysis was completed and a report of the findings was prepared and read by Ms. Martin for her verification and approval. These reports aligned with the major sections of Chapter 4, with the fifth member check reviewing the completed chapter. Feedback from Ms. Martin supported this analysis, and further analysis was delayed until she approved of the contents of each section. This process was essential in guaranteeing the credibility of the study and in ensuring that Ms. Martin's interpretations and experiences were accurately described.

Three additional factors, the study's transferability, dependability, and confirmability, helped ensure its trustworthiness. Its *transferability,* or descriptions within context (Gay et al., 2011), was derived from the rich descriptive data collected throughout the study and the situation of this data within the specific context of Ms. Martin's case. The study's *dependability*, or data stability (Gay et al., 2011), was supported by the overlapping methods of data collection described above. Additionally, all aspects of this data's generation were transparent due to the audit trail produced by this chapter's description of the instruments and processes used to collect, analyze, and interpret the data and the previously described artifacts generated in its production. Finally, the study's *confirmability*, or objectivity (Gay et al., 2011), was ensured through the process of triangulation and the researcher's awareness of the potential biases identified in the section describing the researcher's role as an instrument.

Limitations

The limitations of the study included issues that were largely out of the control of the researcher, either due to methodological, physical, or contextual reasons. Methodologically, the selection of a single-case study limits the study to analytic generalizations arising within or in response to the study's theoretical framework. Additionally, inherent researcher biases impacted factors such as the study's research question and interpretive analysis. Although the study's design was intended to mitigate concerns arising from these factors, their presence in research of this nature is largely unavoidable. Physical factors such as the timeframe in which the study was conducted and the volume of the report it produced also limit the study in specific ways. As the study's data was collected over a period of only three and one-half months, its results capture a brief and fleeting view of the processes under examination and rely on the perceptions of the case's teacher to situate this point-of-view. In the study of gradual processes such as teacher change, this factor is particularly limiting. Additionally, the volume of data collected during this timeframe also produced a thick description of the study's context whose length may limit the audience it reaches. Finally, the unique contexts in which the research was situated, particularly in its examination of a critical case, further bound the study. The availability of longitudinal, reform-oriented mathematics professional development projects, such as Project Influence, may limit the reproducibility of the study. This issue is compounded in trying to reproduce the degree of access provided to the researcher due to his relationship with the case teacher and the extreme nature of this teacher's mindset. Together these factors produced weaknesses in the study that were out of the control of the researcher.

Delimitations

Other characteristics that limit the scope of the study were under control of the researcher and helped define the study's boundaries. The initial problem that was selected, the study's research question, and the specific areas of focus examined within the research question were all guided by the researcher's choices. The parameters of the critical case under study, the participant selection process utilized, and the participant herself were all chosen directly by the researcher. The theoretical framework, which provided an interpretive lens for the research, and the study's methodology were also specifically selected due to their likely influence on the study. These factors were combined with others including the school system, professional development environment, and demonstration lesson to provide specific boundaries under which the study was conducted.

Chapter Summary

This chapter contained details of the methodology utilized in the holistic, single-case exploration of the role of growth mindset characteristics in an elementary mathematics teacher's interpretations, adaptations, and implementations of her professional development experiences. It provided a summary of the justification of the study and a brief research overview before developing the specific details of the methodology. The details included the study's research context, the sources of data and data collection techniques utilized in the study, and the methods of the data analysis performed. Additionally, the chapter presented issues of the study's trustworthiness and its limitations and delimitations. The next chapter, Chapter 4, presents the full results of the study arising from this methodology.

CHAPTER IV: RESULTS

Introduction

Many factors impact the effectiveness of mathematics teachers including constructs such as their knowledge structures, dispositions, attitudes, and beliefs (Ernest, 1989; Goldsmith & Shifter, 1997; Pajares, 1992; Philipp, 2007; Wilkins, 2008). The influence of some of these traits, such as teachers' attitudes and beliefs about mathematics, on their teaching practices has been well examined in the literature (Clarke & Hollingsworth, 2002; Wilkins, 2008). The influence of other factors, such as the teacher's mindset towards mathematics, has not (Rattan et al., 2012). Additionally, although many components of effective professional development programs for mathematics teachers have been described (Desimone et al., 2002; Garet et al., 2001; Loucks-Horsley et al., 2003), some aspects, such as how teachers utilize demonstration lessons in their classrooms (Clarke et al., 2013) are still under consideration. Therefore, the purpose of this study was to explore one of these motivational factors, the teacher's mindset, within the contexts of the teacher's professional development experiences, and answer the research question: How do characteristics of the growth mindset influence a mathematics teacher's interpretations and enactments of professional development experiences, if at all?

The results of this exploration are presented in this chapter in four parts. First, the teacher's perceptions of her own mindset and beliefs about the teaching and learning of mathematics are presented and compared to previously collected data regarding these constructs. Second, the teacher's descriptions of her classroom practices, activities, and outcomes and her observed mathematical teaching practices and activities are examined to establish a baseline for consideration of her enactment of the observed demonstration lesson. Third, the teacher's perceptions of her experiences during her recent professional development experiences and specific areas of focus during a demonstration lesson are examined, and her perceptions of the significance of these areas of focus recorded. Finally, the teacher's enactment of the demonstration lesson in her own classroom and her reflections regarding this enactment are considered.

Mindset and Beliefs Regarding the Teaching and Learning of Mathematics

Gale Martin, an elementary mathematics teacher in her second year of teaching second grade and her fifteenth year of teaching elementary school, was selected as the critical case for this study. Prior to teaching second grade, Ms. Martin had taught one year of kindergarten, two years of third grade, and 10 years of fourth grade, providing her some perspective in the mathematical content requirements of several grade levels and forming the basis of her horizon content knowledge.

> All my years, those 10 years of fourth grade have really been important [to] last year and this year [of teaching second grade] because teaching fourth grade all those years I saw what they needed. I also saw what they were missing. That has been an eye opener for me to say, okay, I've got an opportunity to fill in some gaps before they get to third and fourth grade. Those gaps that I saw in addition and subtraction and not knowing why to regroup, not knowing just the basic foundations of place value, that's why I have pushed it so heavily this last year and this year. (Background Interview, September 18, 2015)

This attention to students' understanding of foundational mathematical ideas and the desire to prepare students for long-term success in mathematics provided the foundation for many of Ms. Martin's beliefs regarding the teaching and learning of mathematics.

During the course of this study, Ms. Martin was also engaged in her third year of the professional development project which she referred to as Project Influence throughout these results. This section focuses on aspects of Ms. Martin's mindset and her transitioning beliefs regarding mathematics and the teaching and learning of mathematics which were relevant to this study.

Evidence of the Growth Mindset

Evidence of Ms. Martin's growth mindset towards mathematics and her awareness of the importance of this mindset was drawn from four sources: historical data collected through Project Influence, interviews with Ms. Martin, classroom observations of Ms. Martin's mathematics classes, and Ms. Martin's reflective journal. This section contains a report of these historical data on mindset as well as results from the current study.

Historical evidence of the growth mindset. Twice during Project Influence, in June of 2014 and again in April of 2015, Ms. Martin completed a mindset survey measuring her growth versus fixed mindset characteristics with regard to four attributes: intelligence, morality, one's ability to influence the outside world, and mathematical ability (see Appendix K). Scores on this survey ranged from 1 to 6, with averages of 3 and below indicating a fixed mindset for an attribute and scores of 4 and above indicating a growth mindset. At the time of the June 2014 survey, Ms. Martin's average results indicated a growth mindset for all of the attributes surveyed: intelligence (M = 5), morality (M = 5), worldview (M = 5.67), and mathematical ability (M = 6). Her average results at the time of the April 2015 survey corroborated these results: intelligence (M = 4), morality (M = 5.33), worldview (M = 5), and mathematical ability (M = 6). Regarding the attribute of interest to this study, mathematical ability, these results indicated Ms. Martin's mindset to be both growth-oriented and stable.

Evidence of the growth mindset from the current study. During an informal interview (September 1, 2015), the staff of Project Influence recommended Ms. Martin as a potential candidate for the study being reported. These recommendations were based on two characteristics: Ms. Martin's espoused mindset toward mathematics and the growth in her ideas related to the teaching and learning of mathematics she had displayed during the first two years of the project. The initial selection interview used to confirm Ms. Martin as a candidate for the study revealed her to have a strong understanding of mindset, to be aware of the characteristics of both her own and her students' mindsets, and to be able to describe ways in which she had operationalized mindset in her classroom. Each of these aspects related to mindset will be described in the following sections.

Understanding and awareness of the importance of mindset. During her initial selection interview, Ms. Martin established her understanding of mindset.

> Of course, you know, there is a fixed mindset and a growth mindset. A lot of people have a fixed mindset and believe that the things that they have, the abilities

> that they have, cannot be changed, but I guess I have, through the years, developed a growth mindset. (Selection Interview, September 9, 2015)

She further elaborated her views of her own growth mindset and mindset regarding teaching mathematics.

> When I first started teaching, I was one of those teachers that liked to do things the same. I didn't change my habits. I don't know if that was because I was new and I stuck to some type of structure or some familiarity kind of things, but over the years, education changes. Kids are different every year. I think you have to have a growth mindset because everything's always changing, especially in math. (Selection Interview, September 9, 2015)

With these thoughts, Ms. Martin established her awareness of the differences in the fixed and growth mindsets, described her transition into this mindset, and professed that she believed the growth mindset to be particularly important in the teaching of mathematics.

In addition to being aware of her own mindset, Ms. Martin often spoke of the importance of her students becoming aware of their mindsets regarding mathematics, and addressed conversations she had with her students regarding the importance of the growth mindset.

> I explain to my students the difference between the two mindsets so they understand our classroom expectations for having a growth mindset. Surprisingly, these young kids already have a fixed mindset in some ways, especially towards math. They already view math as "hard" and "they can't do it."

> We address that early so they will learn how to talk to themselves and each other positively to build that growth mindset. (Reflective Journal, September 17, 2015)

Seeing evidence of this fixed mindset towards mathematics in children so young was surprising to Ms. Martin, and she believed that her efforts to change their mindsets were both successful and important to the children and families with which she worked. She offered this anecdote from her previous year of teaching second grade.

> I thought they would just be, with these second graders, I thought "They won't have any kind of background that shuts them down yet." I think that kind of just builds over the years, but they did. I had this one little girl last year and her mom. I make the parents on open house write a wish for their child and we see, hopefully, that it comes true at the end of the year. I put it in their scrapbook. The mom's wish was, "I want my child to love math." So that mom had actually requested her kid be in here because she knew how I was. She told me at Christmas I could take the year off. I could take the rest of the year off, because her kid had totally changed their mindset. (Selection Interview, September 9, 2015)

In this quote, Ms. Martin indicated that she valued the mindset of the children in her classroom, believed that the growth mindset will help students to be successful in her class, and associated the growth mindset with positive affective experiences related to mathematics.

From these quotes, Ms. Martin's words suggested that she had a strong understanding of the tenets of mindset, recognized the significance of mindset in the teaching and learning of mathematics, and viewed both her own mindset and the mindsets of her students as powerful. Additionally, there was evidence that Ms. Martin believed that the results of her mindset influenced the success of her students, and that her growth as a teacher was a responsibility to her students.

> I have always prided myself in going to professional developments and doing things on my own, because I think if you stop learning, there's no real point to this. You've got to keep learning and changing the way you do things, because your kids need that. (Selection Interview, September 9, 2015)

This focus on the impact of her own mindset and professional growth on her students was evident throughout the study.

Ms. Martin's words in this section established that she understood the tenets of mindset, distinguished the characteristics of the growth and fixed mindset, and considered her own mindset towards mathematics and mathematics teaching to be growth-oriented. Additionally, Ms. Martin described attending to issues of mindset in her classroom and expressed a belief that students' mindsets influenced their success in mathematics. The next section contains evidence of the manner in which Ms. Martin described enacting the tenets of the growth mindset to support students in her classroom.

Descriptions of operationalization of the growth mindset. During her interviews, Ms. Martin described the means in which she managed her classroom in terms that indicated she had operationalized her growth mindset in a variety of ways. Among these, descriptions of the manner in which she set goals for herself and her students, the strategies she elicited to help achieve these goals, and the manner in which she managed the affective components of struggle emerged. This section includes Ms. Martin's descriptions of these practices in terms of goal setting, goal operation, and goal monitoring.

Goal setting. Ms. Martin discussed setting goals for her classroom that were purposeful and used to direct her lessons and student learning. She stressed the importance of this purpose in setting expectations for her students, knowing how she would respond to them, and setting the course of her instruction.

> Probably the biggest thing is just being purposeful with everything I do. I mean, planning is so important, and I need to know what to expect when I go into my lessons. I need to kind of already have in my mind maybe some things that they're going to say, what I am going to say back to those things. I don't write all of that out, I just know that in my head. I think being purposeful in everything that we do is just very important. We have to have a purpose. You have to know your end goal. You have to know where you want them to go before you can start. (Selection Interview, September 9, 2015)

She contrasted this with the actions of other teachers whose instruction is dictated by performance, and indicated that the goals that a teacher sets should be flexible and focused on her students.

> I think there are some teachers that get hung up on that, "Oh, I'm on day five. I have to do this on day five." Well, if my kids aren't there, you have to just back up and see where they are and look at what they've given you, and then plan your next day from that. (Selection Interview, September 9, 2015)

These examples demonstrated Ms. Martin's focus on learning goals over performance goals and preface the discussion in the next section of the strategies she described for achieving these goals.

Goal operation. Expanding on her previous comments concerning the manner in which some teachers focus on performance expectations, Ms. Martin noted the significance of rejecting a rote approach to teaching in favor of one that values productive strategies.

> I think you have to be willing to go against the norm. That's another thing. I don't like to use a basal. I don't want to be stuck to a basal, and there are people that still want to stick to a basal. I think you have to get away from that, "Oh there's one way to do it. This is my manual." I guess that's how I've approached it as well. There are several ways and several things that we can use and show as far as teaching strategies and curriculum that doesn't necessarily fit a basal. (Selection Interview, September 9, 2015)

Elaborating on one of these strategies, Ms. Martin addressed two of the primary goals she had set for her students: to have them speak about mathematics with purpose and understanding and to discuss their ideas with one another in meaningful ways. Here, she discussed this goal and described one of the classroom practices she used to help students move towards this goal.

> We do a lot of small groups, they share those kinds of things [their ideas about mathematics], especially at this grade level. I'm really working on getting them to be purposeful when they speak, so they're not just saying whatever. So we're

> really practicing on how to talk to each other, how to have those meaningful conversations. So when I'm going through, I will listen and I may ask a question so that I want them to maybe model the way I'm asking my question the next time I come through. (Selection Interview, September 9, 2015)

Together, these statements represented Ms. Martin as a teacher with goals for her students beyond the written curriculum, who had considered the approaches needed to be successful with these goals for both herself and her students.

However, Ms. Martin also reflected on what happens when her students are unable to meet these goals. When asked what she does when she tries something new and it does not go as well as expected, Ms. Martin explained:

> Oh, that happens, I think that's normal. We talk about backing up and punting, that's something I say in my [professional learning community] with the others. We just have to back up and punt and see what did the kids actually give you. Of course, they have to drive what you do. (Selection Interview, September 9, 2015)

Operationalizing this resistance to setbacks and focusing on the actual progress made by her students proved to be one of Ms. Martin's most effective strategies for continuing to progress toward the goals she had set. This idea is further examined in the following example.

Ms. Martin identified one day of class as particularly difficult when her students struggled to share their thinking regarding a task requiring them to group tens to make one hundred.

> I felt like today didn't go as well as the rest of the week has. I felt they were "off." I felt that the lesson got better toward the end. It just took them a while to get going. They surprised me with how well they answered the question. I was expecting some misconceptions about the number of 10s and hundreds so I was pleased with the discussion and outcome. (Reflective Journal, September 22, 2015)

On this day, Ms. Martin ended her class by praising the effort she had seen from her students, reinforced her expectations for understanding and communicating thinking, and provided feedback to encourage her students to build on their struggles.

> First of all, great job. I hear a lot of thinking, and different thinking, and that's great. Guys, let me remind you, when you're in your group, you need to hear what everyone is thinking, okay? And you also need to be able to defend your answer, so, "I got this answer because, here's why." You can't just say, "Because I got it, this is the answer." You have to be able to explain why you think the answer is the way it is. There may be somebody that says, and this is what we want, there may be somebody that says, "I don't see how this relates to this problem." There's a little bit of that going on, and we want that discussion . . . you've got to be able to explain to them so that they understand it. (Classroom Observation, September 23, 2015)

In addition to this feedback for her students, Ms. Martin recognized that her expectations and actions would impact the progress her students made. She later reflected on what her role would be in continuing to help her students progress towards her expectations.

> I must continue to foster a safe environment where they feel free to share their ideas. I want to introduce poster sessions as another way to encourage their teamwork and presenting in class. They are still learning how to formulate their ideas in their head and get it out. I want to do more modeling so they understand what it looks like and sounds like. I think the more we talk to each other the better they will become at sharing. (Reflective Journal, October 8, 2015)

In these quotations, Ms. Martin displayed a resilience to setbacks, continued focus on student thinking and development, and reinforcement of her high expectations in spite of struggle that assisted her in making progress toward achieving her classroom goals.

As seen in the two previous quotations, Ms. Martin provided feedback to her students that focused on the effort they made to describe their thinking to one another, and recognized that this is a learned ability rather than a naturally occurring trait. She expanded on this idea by referring to a mindset related story, *The Dot*, which she had discussed with her students at the beginning of the semester.

> *The Dot*, have you ever seen that story? . . . It's a book . . . about this little girl who said, “I can't draw, I'm no good at this,” and she just puts a dot on her paper. Then the art teacher's like, “Oh my gosh, that's the most beautiful thing I've ever seen,” so it totally changes this kid's frame of mind, totally changes that kid like “I can make a better dot than that.” That totally rebuilds that kid. Then what's great, at the end of the story is another kid says something about them not being able to do it, but that child is like, “Oh yes, you can, see this.” So she's changing. (Selection Interview, September 9, 2015)

This recognition of effort above ability was something Ms. Martin focused on throughout the semester and described valuing for her students, herself, and her family.

In this section, Ms. Martin indicated her desire to select instructional strategies that would facilitate her students' progress towards the long-term learning goals she had set for them and described the resilience of her responses when these strategies proved ineffective. This response included feedback focused on the efforts her students had made in alignment with her goals and reflection on her role in continuing to support her students' progress. Additionally, Ms. Martin discussed the role of effort in developing ability and described why this effort was personally meaningful to her. Ms. Martin's expectations in this situation, and her perspective on maintaining a positive outlook on the effort needed to improve one's mathematical ability, are discussed in the following section.

Goal monitoring. Monitoring one's goals requires being able to continuously evaluate both your current position relative to the goal and the rate at which you are making progress towards that goal. High expectations for mathematical understanding, reasoning, and communication formed the basis of the goal monitoring practices that Ms. Martin described for her classroom. These expectations allowed her to see the reasoning behind her students' answers and monitor their application of productive strategies beyond the solutions they provided. In the case of a student being able to see that the answer to 13 plus 7 was equal to 20, she described her expectations.

> You need to tell me, how do you know that's twenty? I want them to prove to me it's twenty, not just "I know it's twenty." That's where we would go in. I would

> want them to give me some sort of strategy that they use mentally, “I know that in thirteen there's a three and a ten and I know that seven needs a three to make a ten,” and so I would want them to explain their thinking. (Selection Interview, September 9, 2015)

The expectation of accountability for one’s mathematical understanding inherent in this statement was mirrored in other practices that Ms. Martin described to allow her access to her students’ thinking.

> Something that I try to do is, if I call on a kid and they said, “Oh, I was going to say what she said,” well, I want you to say it in a different way, tell me how you could have said that in another way. I don't just let them get out of not answering. (Selection Interview, September 9, 2015)

This combination of high expectations for mathematical understanding, communication, and accountability for their thinking provided one method Ms. Martin used to monitor students’ progress toward her classroom goals for them.

Additionally, one must feel that their goals are attainable in order to continue to make progress towards them. Ms. Martin advocated that her students use positive self-talk to mitigate negative perceptions of their abilities and continue to provide effort to improve these abilities.

> In the last several years, [the participants in Project Influence have] talked about your self-talk, the things that you say to yourself. If you believe you can't do something, well you're going to shut yourself down. You always want to speak

> differently to yourself. “I'm not good at this, but at least I can try.” (Selection Interview, September 9, 2015)

This quotation showed that Ms. Martin believed this positive self-talk provided her students a way to avoid helpless behaviors and to continue engagement with difficult subjects. She spoke of adherence to the growth mindset in a similar fashion, stating, “A growth mindset is so important because if you talk yourself out of it, if you talk yourself down, you're just dead in the water” (Selection Interview, September 9, 2015). Together, these ideas showed that Ms. Martin believed that self-talk and the growth mindset were important as they provided ways to avoid the negative emotions and helplessness that often accompany difficult experiences. Additionally, her descriptions of these constructs framed them as internal tools that allowed students to continue to make progress towards their own goals through effort rather than a reliance on ability.

Summary. Throughout this section, Ms. Martin’s understanding and awareness of mindset and the manner she professed to operationalize mindset in her classroom has been examined. Ms. Martin strongly advocated tenets of the growth mindset regarding mathematics and the teaching and learning of mathematics, which were confirmed through surveys of her mindset in previous years of Project Influence. Additionally, Ms. Martin’s mindset appeared to have been relatively stable over the course of her engagement in the project. In the next section her beliefs regarding mathematics and the teaching and learning of mathematics will be considered.

Evidence of Beliefs in Transition

The beliefs examined in this section describe Ms. Martin's deeply held conceptions, values, and ideologies about mathematics and the teaching and learning of mathematics. Her beliefs about the teaching and learning of mathematics included her conceptions of both the teacher's and students' roles in these processes and are demonstrated in the manner in which she described the behaviors and mental activities in which learners engaged as they constructed mathematical understanding. These beliefs were evidenced by three sources: historical results from the IMAP (Ambrose et al., 2004) administered during Project Influence, interviews with Ms. Martin, and her reflective journal. This section includes these historical results regarding her beliefs as well as results from the current study.

Historical evidence of beliefs. Over the course of Project Influence, Ms. Martin completed the IMAP at the beginning and end of each year of the project. The results of her initial survey, at the beginning of the first year of the project in June of 2013, and her most recent survey, at the end of the second year of the project in April of 2015, are reported in Table 2. For four of the seven beliefs measured by this instrument (Beliefs 1, 3, 6, and 7), there was greater evidence for the belief being held at the end of the second year of Project Influence than prior to the project, while the evidence for the remaining three beliefs (Beliefs 2, 4, and 5) remained consistent (note that scores of 1 and 2 for Belief 2 are both regarded as weak evidence for this item). Additionally, in the four cases of increased evidence for the belief, each evidentiary rating improved by at least two

Table 2

Ms. Martin's Beliefs as Measured by the IMAP

Belief	June 2013	April 2015
Belief About Mathematics		
Mathematics is a web of interrelated concepts and procedures (and school mathematics should be too). [Measured out of 3]	0	2
Beliefs About Learning or Knowing Mathematics		
One's knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts. [Measured out of 4]	2	1
Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures. [Measured out of 3]	1	3
If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts. [Measured out of 3]	3	3
Beliefs About Children's Learning and Doing Mathematics		
Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than adults expect. [Measured out of 4]	2	2
The ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children's initial thinking whereas symbols do not. [Measured out of 4]	0	2
During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible. [Measured out of 3]	0	3

Note. For each belief a score of 0 is equated with *No Evidence* for the belief, and the highest score, indicated for each item, is equated with *Strong Evidence* for the belief. Intermediate scores are equated with either *Weak Evidence* or *Evidence* for the belief.

levels, and for one of the beliefs that showed no change there was strong evidence that the belief was held prior to the project (Belief 4). These results indicated that substantial changes had occurred in these beliefs during the first two years of Ms. Martin's involvement with Project Influence, and allow some description of the profile of beliefs one could expect Ms. Martin to hold regarding mathematics, the learning and knowing of mathematics, and children's learning and doing of mathematics.

Data from the IMAP administered in April of 2015 indicated that Ms. Martin viewed mathematics as a web of interconnected concepts and procedures, in which the concepts were more powerful and generative than the procedures, and in which the concepts supported understanding of the processes they underlie. She expected students to think on their own to develop mathematical ideas and to build toward procedures from mathematical concepts. Additionally, there was some indication that Ms. Martin believed that being able to apply a procedure alone did not fully justify one's understanding of its underlying mathematical basis, and that her students understood mathematics in ways that were unique to them and fundamentally different than the approaches an adult might take. Further evidence for this profile is included in the next section which contains Ms. Martin's descriptions of her current beliefs regarding mathematics and the teaching and learning of mathematics.

Evidence of beliefs from the current study. Many of Ms. Martin's beliefs regarding mathematics and the teaching and learning of mathematics were situated in her early experiences with school mathematics. One of the core ideas she described was how students see mathematics differently from both other students and their teachers.

Although these ideas were only weakly evidenced in Ms. Martin's IMAP results, they appeared to have substantial personal meaning to her.

> I was always very good at math growing up, and I guess I saw things differently than maybe my teachers did. Sometimes I got in trouble for that, because you were supposed to only do it the teacher's way, and you weren't supposed to see things differently, and I would. I'd get in trouble for showing somebody a different way to do it, because that's not the teacher's way. I guess I've kind of taken my own childhood experiences and how that made me feel. (Selection Interview, September 9, 2015)

Her recognition of this need to "see things differently," make meaning in one's own way, and communicate these ideas to others was a strong theme throughout her descriptions. The remainder of this section considers this theme across her views of mathematics as a connected system, the value of understanding mathematical structures and concepts, the differences in children's and adults' views of mathematics, and the need for students to think and communicate about mathematics.

Mathematics as a connected system. Ms. Martin described her beliefs about the connectedness of mathematics in a straightforward manner with the statement, "I would say that mathematics is relationships within numbers" (Selection Interview, September 9, 2015). This simple statement appeared to underlie much of the organization and intention that guided her mathematics instruction both holistically and on a lesson-to-lesson basis. Additionally, the statement suggested that as much of Ms. Martin's focus in second grade was on the understanding of numbers and operations with numbers, when she spoke of mathematics she was generally referring to these specific aspects of mathematics. In describing the organization of the classroom time she utilized for mathematics instruction, she spoke explicitly of these holistic connections.

> All components of math in my classroom are related. Calendar time/morning meeting builds number sense around a number of the day, which leads to understanding in place value, which we are learning now, and other skills to come. My centers are used to reinforce the skills that are currently being taught, they are directly related to my instruction. This gives students a way to independently practice the current skills and be ready for the next day's instruction. (Reflective Journal, September 22, 2015)

These connections within Ms. Martin's daily curriculum were mirrored at a larger scale in her day-to-day instruction, which she described as building from her students' current understanding, and at a smaller scale in her daily lessons.

In discussing her typical day of mathematics instruction, Ms. Martin referred to her attempts to help students connect their representations of numbers and operations through the use of *number talks,* short conversations about concepts of number that provide ongoing opportunities to develop computational fluency (see Figure 5).

> I try to start my math lesson every day. . . . We start off math with a number talk like that. Yesterday, they had to tell me all the ways they could show fifteen. Today I asked them "What is seven plus three? How do you know what seven plus three is?" There were all different ways. Some of them used ten frames, tally marks, I count it on, there's tens blocks, I had seven and three and I put them

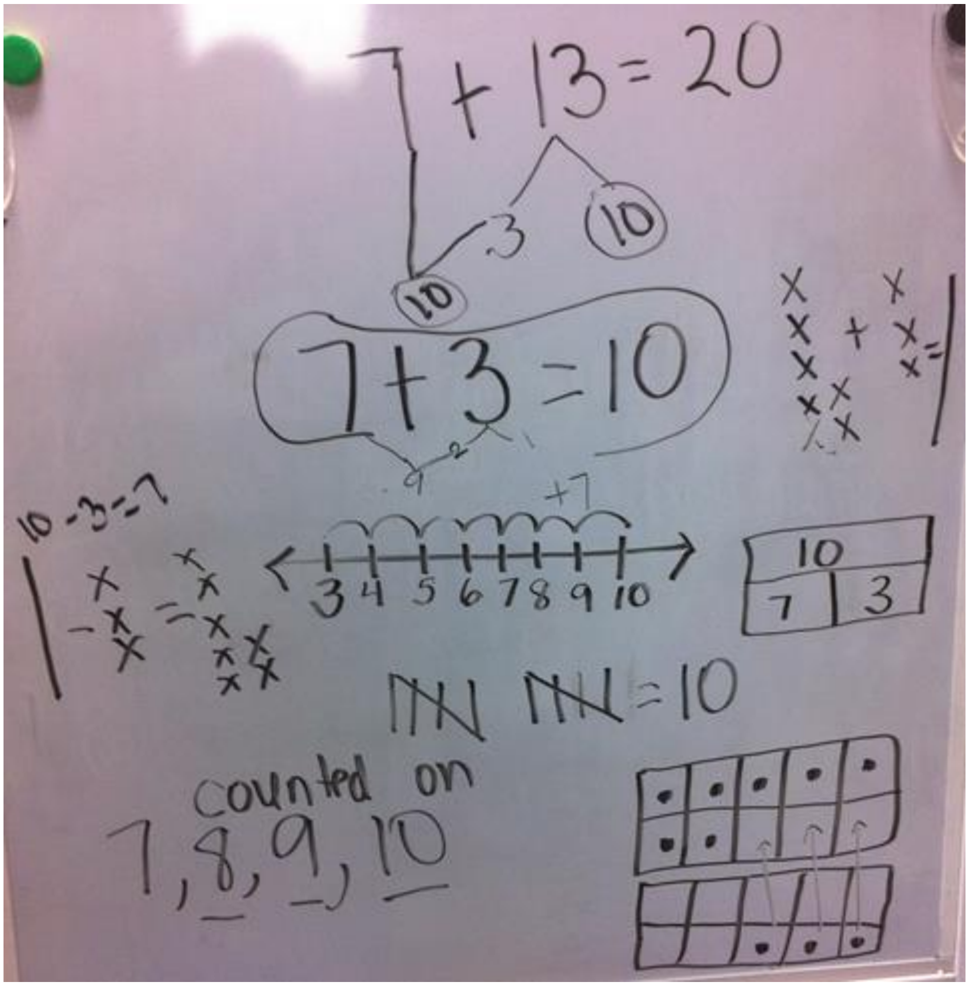


Figure 5. Student representations of 7 + 3 = 10 generated during a number talk and recorded by Ms. Martin on September 9, 2015.

> together, it could make a ten-rod. . . . I didn't say any of this, in a number talk, I don't give any examples, they have to give them all to me. (Selection Interview, September 9, 2015)

This quote showed that Ms. Martin viewed these talks as an opportunity for students to discuss their unique ways of understanding numbers and operations, as a chance to see how other students viewed these constructs, and as an opportunity to make connections among these different ways of visualizing the mathematics.

Ms. Martin also discussed the importance of encouraging her students to consider the relationship between their current mathematical understanding and the mathematical concepts they would encounter in the future.

> I think kids need to know that too. We are doing these things because this is what you're going to be doing later. Like numbers sense right now; I told them today, "If you know how these numbers are related to each other, when we get into these harder concepts, you're going to not think it's hard anymore, because you already have that understanding of all these little pieces." It's like a puzzle. That's what I tell them. It's all related. (Selection Interview, September 9, 2015)

Ms. Martin's description of number sense as a foundational concept and its supporting relationship with the more difficult ideas to follow gives further evidence of her views of the connectedness of mathematics. Additionally, her words conveyed a belief that understanding these relationships is a key to success in learning mathematics.

In this section, Ms. Martin's words provided evidence of her beliefs that both mathematics and mathematics teaching are connected systems. She described her focus on the relationships within numbers and the manner in which she structures her daily classroom activities, progression between lessons, and mathematics lessons to this effect. Additionally, she suggested that she regularly used number talks to allow students to see the connections among numbers, representations, and operations, and stressed the importance of understanding these relationships to her students. In the next section, evidence is presented that demonstrates the value that Ms. Martin placed on having students understand the concepts and structures that arose from these talks as they worked toward procedural fluency.

The value of mathematical structures and concepts. In the previous section, Ms. Martin's use of number talks was described as evidence of her views of mathematics as a connected, interrelated system of "relationships between numbers" (Selection Interview, September 9, 2015). Additionally, she described beliefs that this system was comprised of both conceptual understanding and procedural fluency and the need for students to master both of these components to fully understand mathematics.

> I feel that learning math requires thinking and training, just like learning anything else. Learning math is a combination of conceptual understanding and procedural fluency. They need both to understand mathematics. (Reflective Journal, September 17, 2015)

It was also her belief that the ideas being supplied by her students in these talks were highly conceptual, based on what they already understood about mathematics, and that their underlying structures would be useful to them when they started to examine other processes and procedures.

> To me, every bit of that was conceptual, because I did not teach a procedure. I just wanted them to add seven plus three. I guess you would say, if I said, "We always take this number and add it to this number," I taught them an actual procedure, how to do something, but that to me it is all conceptual. It is what they

> already know. The understanding that they already have and how they're going to apply that to other procedures later. (Selection Interview, September 9, 2015)

This description of the number talk she referenced suggested that Ms. Martin valued both building from the ideas that her students generated and developing a strong understanding of the concepts and relationships that underlie numbers and operations before beginning classroom discussions of procedures.

She further discussed the value of this type of conceptual understanding as her students transferred their thinking to a more difficult problem, and explicitly stated her belief that conceptual understanding must be developed before procedures can have meaning.

> So I put a string problem right above it, seven plus thirteen equals, I didn't put the equals, but they told me it was twenty. Seven plus thirteen, “Okay, you guys just did a number bond using three, two, and one. How would I use a number bond with thirteen? That's a two-digit number?” A procedure would be “Let's line them up vertically,” you're not teaching them the reason why. They've got to have that mental conceptual understanding before you can talk about a procedure. (Selection Interview, September 9, 2015)

Here again, Ms. Martin acknowledged the belief that these connected concepts build on one another, and that mastery of an idea provided access to increasingly sophisticated understanding of mathematics. She concluded her thoughts regarding this day’s number talk by reflecting and setting new goals for the next day’s instruction which she hoped would continue to build on these fundamental understandings.

> That's something I try to do a lot too, so tomorrow what I may do is do a whole string with seven plus three, seven plus thirteen, seven plus twenty-three and show them how it will relate as they continue to get into higher numbers. You don't have to be scared of those higher numbers, because you can use the relationships you've already seen within your basic facts. My kids really ran with that last year. Their fluency was amazing. (Selection Interview, September 9, 2015)

In this quote, Ms. Martin described the teaching and learning of mathematics as leveraging the underlying structures of mathematics which students already understand in order to help them access new conceptual ideas and referenced fluency as the ability to apply these relationships in new settings.

These quotations from Ms. Martin provided evidence of her view of the relationships between mathematical concepts and the procedures they support and described the need for mastery of both in order to understand mathematics. She elaborated on the manner in which concepts arise from what students already know about mathematics and discussed the way that mathematical procedures are supported by these understandings, including the ability to transfer understanding of an important concept to novel processes. Additionally, she described this progression between concepts and procedures as being useful for designing instruction. The next section contains descriptions of Ms. Martin’s beliefs regarding the ways that students’ understandings of mathematics differ from adult understanding.

The differences in children's and adults' views of mathematics. To this point, evidence has been examined which described views held by Ms. Martin regarding the connectedness of mathematics and the relationships between concepts and procedures that were also strongly evidenced in her most recent IMAP results. However, in her descriptions of her views on learning mathematics, she also focused on the idea that adults and children see mathematics in different ways. This idea was only weakly evidenced in the historical data, but was a focal point of her discussion of how children learn mathematics. In the following quote, she presented this view and offered some insight into why she believed children and adults view mathematics differently.

> I think we're so pre-programmed, the things that we learned as kids still stick with most people. . . . I think [the children] aren't so pre-programmed as much. I think that they're able, like a little sponge, they're able to see things maybe in different ways than adults have seen, because they just didn't think about numbers that way, or they didn't learn math that way. That's why I tell them it's so important to go home and explain to their parents how to do this. (Selection Interview, September 9, 2015)

Referencing the number talk discussion, she elaborated on this idea, and indicated that the way she teaches mathematics and expects her students to learn mathematics was also fundamentally different than the way her students' parents likely learned mathematics.

> Parents, they just know seven plus three is ten, the number talk today that we did is on the board over there. . . . All the different ways that they showed me how they can add seven plus three to get ten, I want parents to see that. We're pre-

> programmed as adults. I told the kids that today. Your parents know seven plus three is ten because they just learned a basic fact, but did they really learn relationships between numbers? Or did we just learn algorithms and facts and things like that? Did they understand? (Selection Interview, September 9, 2015)

Ms. Martin acknowledged a lack of deep understanding among many adults, and further stated, "There are so many things that [the children] can use and see that I just don't think we saw as adults, because we weren't exposed to it" (Selection Interview, September 9, 2015).

In this section, data was presented in which Ms. Martin elaborated on her views of the differences in the ways children and adults understand mathematics, and attributed these differences to the manner in which mathematics was learned. Additionally, she questioned the depth of understanding of mathematics that many adults have developed and attributed this shallow understanding to a lack of exposure to the subject. The importance of correcting this lack of exposure is inherent in the following section, which examines Ms. Martin's ideas about the role of students' thinking and communicating in learning mathematics.

The role of students' thinking and communicating in learning mathematics. As referenced previously, Ms. Martin remembered her childhood experiences in mathematics classrooms as limiting her thinking and discussion about mathematics to ideas generated by the teacher. However, according to her initial IMAP data, there was little evidence that she had operationalized these feelings in an actionable way as she showed no evidence of recognizing the need for students to do as much of the thinking as possible in their learning of mathematics. She recognized the role that Project Influence had served in reestablishing her belief that learning mathematics is intensely grounded in students' thinking and communicating about mathematics.

> When I first started [Project] Influence, I was like, "Yeah, I remember having my own way to do something, and it wasn't the teacher's way," so I really embraced that because I remembered how that felt as a kid. I want my kids to feel that way too, that all of their opinions matter and how they see something matters and it gives those other kids an opportunity to say, "Yeah, I didn't think about that, and I might want to use that another time because that might be easier for me." (Selection Interview, September 9, 2015)

She professed to believe that a focus on student thinking and ownership of their mathematical ideas is one of the most essential elements of her classroom environment that allows students to learn mathematics effectively. She described her thoughts on the matter, referring to how a student from the previous year's class was able to ultimately be successful in learning mathematics.

> She was able to share her ideas, because she heard somebody else share their ideas. So just building that community of "I am my own person in here," and that's okay. "I can show you how I know something. It's just not Ms. Martin's way. I can have my way. So and so can have their way. Yeah, Ms. Martin shows us things, but if I don't see it the way Ms. Martin shows us, then I can still use my way. I'm able to do that." (Selection Interview, September 9, 2015)

Ms. Martin credited this student's willingness to value and share her own ideas to the fact that she first heard another student share their thinking and the norms of the community she had built in her classroom.

This relationship between students' willingness to communicate their ideas and their increasing level of understanding of mathematics appeared to be one of the motivating beliefs behind much of Ms. Martin's thoughts regarding teaching and learning mathematics. In the following quote, she spoke directly to the notion that understanding mathematics and communicating about mathematical ideas were inexorably linked.

> That's why I always tell them, "If you can explain, you can go home and teach mom or teach brother or come to me and show me and explain it in your own words, I think that's how you understand it." That's why I like for them to do a lot of talking, obviously, because I want them to share their ideas with each other and understand it, especially in kid terms. Because there are times when I have said something and a kid will say it differently, and I feel like we've said it the same way. If a kid says it, [other] kids are like, "Oh yeah, that makes total sense." (Selection Interview, September 9, 2015)

It appeared that Ms. Martin believed that this focus on student thinking and communication was essential to student learning and would also help to reconcile the differences between children's and adults' thinking about mathematics.

Summary. Although historical data from Project Influence showed Ms. Martin's mindset regarding mathematical ability to be stable, her beliefs about mathematics and the teaching and learning of mathematics appeared to have evolved over the course of the previous two years. In describing her beliefs about mathematics and the teaching and learning of mathematics, Ms. Martin validated that she held many of the beliefs predicted by her most recent IMAP results. As expected, her descriptions supported views of mathematics as a connected system of concepts and representations that provided a foundation of understanding for procedures. Her descriptions of the teaching and learning of mathematics reflected this structure and required student engagement, exploration, thinking, and communication of their mathematical ideas. However, another major area of her accounts focused on the differences in thinking between children and adults and the variety of ways children organize and understand mathematics. These findings were only weakly evidenced in Ms. Martin's IMAP results and may have indicated continuing transition in these belief structures.

Summary of Ms. Martin's Mindset and Beliefs

The data presented in this section provided evidence of Ms. Martin's awareness of her mindset and the ways in which this mindset was potentially operationalized in the classroom and demonstrated many of Ms. Martin's current beliefs about mathematics and the teaching and learning of mathematics. Additionally, these constructs were contrasted with historical evidence of Ms. Martin's mindset and beliefs regarding the teaching and learning of mathematics in order to understand their recent evolution. Together, the first portion of these findings made visible the hidden, internal constructs that motivated Ms. Martin's classroom practices and her interpretations and enactments of her professional development experiences. The second portion of the results will focus directly on Ms. Martin's perceived and actual classroom practices and activities.

Described and Observed Baseline Teaching Practices, Activities, and Outcomes

Evidence of Ms. Martin's teaching practices and classroom activities was gathered in order to establish a baseline understanding of her classroom environment and to allow consideration of the manner in which her mindset, beliefs, and professional development experiences impacted this environment. Additionally, historical evidence of her teaching practices was examined to be aware of the changes in these practices that had occurred during her involvement with Project Influence. This section is focused on the aspects of these practices and activities most relevant to this study, with evidence of these practices and activities drawn from four sources: historical RTOP (Sawada et al., 2002) results from classroom observations occurring during the first two years of Project Influence, more recent classroom observations conducted during fall of 2015, interviews with Ms. Martin, and her reflective journal. These results are organized in sections encompassing the historical observation results, Ms. Martin's descriptions of her teaching practices and classroom activities, and recent observations of Ms. Martin's actual classroom practices and activities.

Historical Evidence of Classroom Practices

Over the course of Project Influence, project staff observed Ms. Martin's teaching practices three times utilizing the RTOP as the observation instrument: once in February of 2013, again in August of 2013, and finally in April of 2014. The results of these observations, which correspond approximately with the beginning, middle, and end of Ms. Martin's first year of involvement with Project Influence, are reported in Table 3. These results indicated an increase in the alignment of Ms. Martin's teaching practices with the tenets of reform teaching as defined by the RTOP, which parallel the description of reform-oriented instruction utilized in this study. Over the course of her first year of involvement with Project Influence, Ms. Martin showed growth of at least two levels in 16 of the items measured by the RTOP, growth of at least one level in 22 of the items, and did not show a decline in any of the items.

Table 3

Ms. Martin's Teaching Practices as Measured by the RTOP

Item	February 2013	August 2013	April 2014
Lesson Design and Implementation (Range 0 to 4)			
The instructional strategies and activities respected students' prior knowledge and the preconceptions inherent therein.	1	1	4
The lesson was designed to engage students as members of a learning community.	1	1	3
In this lesson, student exploration preceded formal presentation.	0	1	3
This lesson encouraged students to seek and value alternative modes of investigation or of problem solving.	1	1	2
The focus and direction of the lesson was often determined by ideas originating with students.	1	0	2
Content: Propositional Knowledge (Range 0 to 4)			
The lesson involved fundamental concepts of the subject.	1	2	3
The lesson promoted strongly coherent conceptual understanding.	0	1	3
The teacher had a solid grasp of the subject matter content inherent in the lesson.	0	2	3
Elements of abstraction (i.e., symbolic representations, theory building) were encouraged when it was important to do so.	2	1	3
Connections with other content disciplines and/or real world phenomena were explored and valued.	0	0	3
Content: Procedural Knowledge (Range 0 to 4)			
Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent phenomena.	1	1	3

(continued)

Table 3 continued

Item	February 2013	August 2013	April 2014
Students were actively engaged in thought-provoking activity that often involved the critical assessment of procedures.	2	0	2
Students made predictions, estimations and/or hypotheses and devised means for testing them.	0	0	0
Students were reflective about their learning.	0	0	0
Intellectual rigor, constructive criticism, and the challenging of ideas were valued.	1	1	3
Classroom Culture: Communicative Interactions (Range 0 to 4)			
Students were involved in the communication of their ideas to others using a variety of means and media.	1	2	2
The teacher's questions triggered divergent modes of thinking.	0	1	3
There was a high proportion of student talk and a significant amount of it occurred between and among students.	1	1	4
Student questions and comments often determined the focus and direction of classroom discourse.	0	0	3
There was a climate of respect for what others had to say.	1	1	3
Classroom Culture: Student/Teacher Relationships (Range 0 to 4)			
Active participation of students was encouraged and valued.	1	1	2
Students were encouraged to generate conjectures, alternative solution strategies, and ways of interpreting evidence.	0	1	1
In general the teacher was patient with students.	1	1	3
The teacher acted as a resource person, working to support and enhance student investigations.	0	0	3
The metaphor "teacher as listener" was very characteristic of this classroom.	0	0	3

These results indicated that substantial changes had occurred in Ms. Martin’s teaching practices during her involvement with Project Influence and suggested that she would be receptive to project activities, such as demonstration lessons, that were based in reform-oriented instructional approaches. More specifically, from these results it would be reasonable to expect Ms. Martin to be receptive to reform-oriented teaching practices for which she had shown strong evidence of past engagement. Examples of these practices evidenced in Ms. Martin’s most recent RTOP data included promoting a learning community which builds on students’ prior knowledge and mathematical explorations, teaching lessons that involve fundamental mathematical concepts represented in multiple ways, and emphasizing student-to-student interaction and strong questioning practices. The following section corroborates these findings with Ms. Martin’s descriptions of her own classroom practices.

Ms. Martin’s Descriptions of Her Practices, Classroom Activities, and Outcomes

Although the RTOP results presented above provided a relevant perspective on Ms. Martin’s past teaching practices, that of an outside observer to her classroom, Ms. Martin’s perceptions of her own teaching practices were equally pertinent to the study. This section consists of Ms. Martin’s perceptions of her classroom practices and their resulting outcomes as evidenced through her descriptions of these practices and outcomes. These results were drawn from interviews with Ms. Martin and entries in her reflective journal, and are presented in two parts: Ms. Martin’s descriptions of her classroom practices and her reflections on her students’ outcomes.

Ms. Martin's descriptions of her classroom practices and activities. Ms. Martin directly addressed the importance of having her students engage in mathematical practices each day. During an interview at the end of a school day early in the academic year, she reflected back to her mathematics instruction for the day and summarized what she thought to be the most important mathematical practices her students had engaged in that day.

> Oh gosh, just making sense of the problem. For example, like today, we did several mathematical practices today. We made sense of the problem, we looked for patterns, used tools, or their models, several modeled certain things, they used those ten frames. I definitely think that just making sense right now, understanding what I'm asked to do and then taking a tool or taking a model and applying that is really important right now, because I think that has to be really enforced before they pick up on that. You've got to do it a lot before they understand what they're doing. I think that's very important. (Background Interview, September 18, 2015)

In this reflection, Ms. Martin emphasized the importance of having her students make sense of the problems they were solving and applying appropriate mathematical tools and models to solve those problems. Additionally, she alluded to the need for repeated exposure to these mathematical practices in order for students to understand their value. In her elaborations on the part she played in engaging students in these mathematical practices, five distinct roles emerged: establishing a supportive learning community, engaging students in thinking about and discussing mathematics, facilitating mathematical discussions in a productive manner, holding students accountable for their thinking, and ensuring the success of students at all levels of ability. Ms. Martin's perceptions of each of these five roles will be described in the sections that follow.

Establishing a supportive learning community. Ms. Martin emphasized the importance of establishing classroom norms that focused on her high expectations for her students' thinking and communicating about mathematics. During the previous school year, her classroom had been utilized for a demonstration lesson for Project Influence, and she believed that her students had been extremely successful in this lesson. When asked about the reasons for their success, she credited the strong norms she had in place and elaborated on her expectations.

> Just our norms. They know that their names are going to be [randomly] pulled. That's another thing, I use their name cards. They know they have to be ready at all times to speak or give their thoughts. That was already in place. They knew they were going to have to discuss things with each other and share their ideas and work together. They had already had poster sessions [public presentations]. So whether or not they were going to get a poster, I didn't know, but we had already done several poster sessions with them, and they ran with that also. They enjoyed that. They would tell me, "No, this is not how we did math before." They loved it. They would present to everybody and come up, those little babies, at the beginning of year, they're like first graders still. (Selection Interview, September 9, 2015)

In addition to attributing her students' success to her classroom norms and describing her expectations for her students, this quote also showed Ms. Martin to believe that this classroom environment impacted her students' dispositions towards mathematics.

Ms. Martin also credited this environment for the success of individual students in her classroom. When asked about the reasons for the improvement one of her students had made in mathematics over the course of the last school year she referenced strategies for establishing a productive learning environment she had encountered during Project Influence.

> All those Project Influence strategies where we, first of all, you have to have a community, the whole [set of] norms. I've always been big on that anyway, before Project Influence. That has really helped me kind of validate what I was already doing. Management has always been one of my big pluses. Just letting [my student] know, first of all, if she messes up, she's going to be okay. She's safe. Nobody's going to judge her because she messed up. (Selection Interview, September 9, 2015)

With these words, Ms. Martin suggested that Project Influence had helped her build on her strength in classroom management and establish an environment in which her students felt safe and willing to take mathematical risks.

After the baseline classroom observations, Ms. Martin also noted that many of her students' interactions were in a formative stage and expressed her perceptions of what would be required for her students to mature. Her first concern was establishing a safe, comfortable environment in which her students could practice these interactions.

> I want them first to be comfortable presenting and sharing, and just like we talked about before, just kind of getting those words, developing that communication skill from the inside of their head for it to come out. I think some of them are still working on that, so I think once that starts moving smoothly in here, then we'll start allowing for questions or allowing for comments or things like that. I just want to build the comfort level first, I think, with this age group. (Point of View Interview, November 4, 2015)

Once this environment had been established, Ms. Martin believed her students would begin to engage in more mature interactions on their own.

> I think things like that happen once they become safe with each other, like they're doing right now. They're still learning how to talk to each other politely and critiquing each other, because that's hard for this group. I mean, you got some that are still, "I'm right, you're wrong." So they have to be taught how to give constructive criticism and feedback, and they don't just automatically know how to do that. So once they can, I think that will facilitate itself, I really do. Once they get that idea of, "I'm safe in here, I know that I can share my answer without anyone making me feel like I've done something wrong." (Point of View Interview, November 4, 2015)

As evidenced by the various classroom observations presented in this chapter, establishing this environment was one of the guiding principles under which Ms. Martin operated throughout the semester.

Evidence for Ms. Martin's role in establishing a supportive learning environment was further substantiated when she described her experiences as a departmentalized fourth and fifth grade mathematics teacher, and the reason she was more successful as a generalist.

> Don't get me wrong, if I could teach all that math and be in here with these kids all day and just have a math class, I'd be fine, but I've never felt like I could build the relationships that I needed to build teaching a different group of kids every 45 minutes. Relationships are important to me and I think that's part of the bargain with kids. If you build a relationship with them, then you're going to get so much more out of them. (Background Interview, September 18, 2015)

Ms. Martin credited this emphasis on relationships, a key part of a supportive learning community, for much of her students' success.

In this section, evidence has been presented that highlighted Ms. Martin's perception of the importance of her role in establishing a supportive learning community. This role included reinforcing classroom norms related to thinking and communicating about mathematics, creating a learning environment in which students felt safe and willing to take mathematical risks, and forming relationships with her students that would allow her to support them in their learning. The next section contains descriptions of the ways this environment is utilized for engaging students in thinking and discussing mathematics.

Engaging students in thinking about and discussing mathematics. One of Ms. Martin's ultimate goals in establishing the learning community described above was to ensure that much of the thinking and conversation about mathematics occurred among her students. She described facilitating these interactions in small groups in order to help her students purposefully explain their thinking to one another.

> We do a lot of small groups. They share those kinds of things, especially at this grade level. I'm really working on getting them to be purposeful when they speak, they're not just saying whatever. So we're really practicing on how to talk to each other. How to have those meaningful conversations, so when I'm going through, I will listen and I may ask a question so that I want them to maybe model the way I'm asking my question the next time I come through. (Selection Interview, September 9, 2015)

In this quote, Ms. Martin expressed the value in having her students interact meaningfully about mathematics, but also acknowledged that this was something they must be taught to do. She described modeling these interactions with small groups to help her students develop this ability and elaborated on why it was important for the conversations to occur from student to student.

> That's why I like for them to do a lot of talking, obviously, because I want them to share their ideas with each other, because if they understand it, especially in kid terms, because there are times when I have said something and a kid will say it differently, I feel like we've said it the same way. If a kid says it, kids are like "Oh yeah, that makes total sense." (Selection Interview, September 9, 2015)

This combination of student-to-student interaction and purposeful discussion of mathematics formed the core of many of the classroom practices and activities that Ms. Martin described.

Ms. Martin also recounted having her students evaluate each other's mathematical strategies as another important aspect of sharing their mathematical thinking with one another. She described her actions as having the goal of students communicating in mathematical terms and recognizing when another student had contributed a useful idea to the conversation.

> I want them to be able to hear it and see it. Just being immersed in that language. . . . I want them to hear their strategies and know that, "Hey, this might be something that I can use." (Background Interview, September 18, 2015)

To encourage this type of interaction, Ms. Martin described having students present their own ideas as well as the ideas of others.

> They would come up and present in the classroom with their posters, and I would make them present each other's work. Not just presenting their own, but, "You have to present so and so's." I think that's powerful too, and that's something I would have never thought about doing before Project Influence, is presenting somebody else's work. "Tell me what you think she did?" (Selection Interview, September 9, 2015)

These types of instructional practices and questioning were common in Ms. Martin's descriptions of her mathematics classroom.

Together, the quotes in this section emphasized structures and practices Ms. Martin utilized to engage her students in thinking about and discussing mathematics in her classroom. The next section contains further descriptions of Ms. Martin's views of her role in facilitating these mathematical discussions.

Facilitating mathematical discussions in a productive manner. Ms. Martin stressed the importance of her role as a facilitator of students' mathematical discussions rather than as a central mathematical authority from which they were to receive answers. She recognized this as the area in which she had grown the most as a teacher and credited Project Influence for much of the progress she had made.

> I think Project Influence has really helped me just be purposeful with what I'm doing and also my teaching practices. Me getting away from the front of the room and not being the leader of the room; walking around, making sure the kids look at each other when they talk. That's just something I guess maybe you don't think about until you see it in practice. They would always look to me and tell me the answer. "I want you to tell them. I already know. I want you to tell them." That is something I've tried to implement also. (Selection Interview, September 9, 2015)

In describing the value of this practice, Ms. Martin acknowledged that her teaching practices had changed due to seeing others enact these techniques. Ms. Martin explained that this recognition of her role in ensuring that conversations occurred between her students was transformative in her day-to-day teaching.

> I feel that I have grown more as a facilitator in the last 5 years. When I first started teaching, my classroom was very teacher-led and had very little group discussion. Now, my classroom operates this way every day. I've learned how important it is for students to lead class discussions and share their own ways of thinking. (Reflective Journal, September 17, 2015)

In these quotes Ms. Martin recognized the importance of distributing mathematical authority to her students and alluded to the significance and genesis of this idea.

Ms. Martin continued to emphasize the importance of this role and the changes she had made with regards to these practices throughout the study. She elaborated on the quality of the changes she had seen in herself, and again credited much of this change to watching others teach.

> I can tell you that when I first started, I wanted to be the one in front of everybody. I wanted to be the one talking. “I'll show you what to do.” I have really grown . . . in developing my role as a facilitator. I guess that's just exposing myself to different things, watching other people teach. . . . I don't want to be the one that holds all the information. I want them to share what they know. I feel like I will guide them based on what they give me. (Selection Interview, September 9, 2015)

In describing what had precipitated these changes she acknowledged her previous teacher-centered perspective as being detrimental to her students and again credited the type of learning environment she had experienced in Project Influence as being a key element in her changes in instructional practice.

> I've given up a lot of that for their benefit, because I was hindering them being so teacher-led, and, “Here let me give it to you,” and, “This is the way we're going to do it.” That was how we learned as students growing up so that was all you knew. Now here you are being immersed in this different teaching environment, this different learning environment [Project Influence], and you say, "Oh, this can be different and beneficial." (Background Interview, September 18, 2015)

In this quotation, Ms. Martin acknowledged what she perceived as flaws in the teacher-centered classroom and suggested that this approach is typical due to limited exposure to other learning environments. She then implied that immersion in a different style of learning environment was a precipitating factor in the changes she had implemented in her instructional practices.

Throughout this section, Ms. Martin’s words provided examples, which served as evidence of her current emphasis on her role as a facilitator of student discussion rather than as a central mathematical authority and described how this change occurred. In this description, she referenced immersion in a type of learning environment that differed from the traditional, teacher-centered classroom and the opportunity to view the teaching of others as significant elements of this change. In the next section, Ms. Martin’s descriptions offer further evidence of the instructional practices that she utilized to hold her students accountable in this environment.

Holding students accountable for their thinking. In describing the manner in which she orchestrated mathematical discourse in her classroom, Ms. Martin discussed three techniques she used to ensure that all of the students in her classroom were accountable to the ongoing conversation: randomly selecting students for questioning and sharing their thoughts, purposefully selecting students for questioning and sharing their thoughts, and having students re-voice the ideas of others. As an example of randomly selecting students, she explained her system for using randomly drawn name cards to have students answer questions.

> They know that their names are going to be pulled. That's another thing, I use their name cards. They know they have to be ready at all times to speak or give their thoughts. (Selection Interview, September 9, 2015)

However, Ms. Martin also implied that there were times when she wanted to know what a specific student was thinking, but elected to maintain the appearance of randomly calling a student.

> You can say you pulled their card but you maybe didn't pull their card. I might have said, “Oh yes, I pulled so and so's,” but I didn't. Because I wanted to hear, after I read the problem, I wanted to hear them, “But what did you hear in the problem? What do we know?” I want to hear from certain kids. (Background Interview, September 18, 2015)

As an example of this technique, she described the setup of a task implemented earlier in the day in which she had identified a specific student that she knew to be likely to become lost in the context of the problem.

> Immediately after [choral reading of the problem] I was hoping that she was paying attention as we were reading, so I didn't want her to get lost in the context of the problem. While it was fresh on her mind, I wanted her to give me

> something from the problem to see if she picked up understanding. (Background Interview, September 18, 2015)

Ms. Martin suggested that this combination of randomly and purposefully selecting students for questioning and sharing allowed her to keep all of her students prepared to speak or give their thoughts at any time while still assessing or advancing the understanding of specific students. Additionally, she described how she addressed a common manner in which students might try to avoid answering questions by appropriating another student's response.

> Something that I try to do is, if I call on a kid and they said, "Oh, I was going to say what she said," well, I want you to say it in a different way, tell me how you could have said that in another way. I don't just let them get out of not answering. (Selection Interview, September 9, 2015)

Together these examples illustrated specific strategies Ms. Martin utilized to hold her students accountable to ongoing mathematical discussions.

More broadly, Ms. Martin described her use of questioning as both a strength and an area in which she hoped to improve.

> I think that's the challenge. It's something that I'm still working on is my questioning. When I have my observations they always tell me my questioning is good but for whatever reason I still think I can get better. (Background Interview, September 18, 2015)

She explained that "with the right questions, yeah, for all of [the students] really on their different levels," (Background Interview, September 18, 2015) she believed that she could serve all of her students effectively. Although she felt that her current questioning practices were strong, Ms. Martin credited Project Influence for the changes she had made and her desire to continue improving.

> As far as teacher moves and things, I think I'm already doing those types of things and the questioning. I feel like I'm already doing that. Of course I know I can improve at my questioning but I do feel like I'm asking a lot of questions all the time, which is better than what I used to do. Instead of giving a lot of information, I'm asking for them to give me that information. So I still want to continue that shift in my practices, and I see that going on daily no matter what we're doing whether it's math, language, arts, whatever. (Professional Development Interview, November 3, 2015)

This pattern of recognizing an opportunity for improvement, refining the practice through experimentation in the classroom, and continuing to seek occasions to improve defined many of Ms. Martin's descriptions of change in her professional practice. In describing one way she would like to see her questioning improved, Ms. Martin referred to the actions of the expert teacher during a demonstration lesson.

> I think she's still doing a lot of what we've put in place, the going around to each table and having them explain what they're doing and her advancing questions, she would try to leave them with something and then walk off and leave. That's something that I'm still trying to do in here that I feel like I'm getting better at, but I want to continue to improve that, is asking them a question and getting their thinking maybe to change direction and then of course leaving them. I still

> always kind of listen behind as I'm walking away. (Professional Development Interview, November 3, 2015)

In this passage, Ms. Martin defined a specific goal for improving her mathematical questioning and acknowledged the significance of allowing students to engage with her questions independent of her presence. When asked how she would monitor the progress she had made towards the goal of improving her questioning, Ms. Martin described the desire to encourage divergent thinking in her students.

> I guess to see did the kids change their thinking? Did they, were they at a standstill when I ask them that question, and did they change their thinking, did they come up with something different? And [when] I pull a card or something and get those kids, that group to talk, are they giving me something different than what I heard from before? So that helps me know that I asked a good question or if they didn't move, I need to change my questioning. (Professional Development Interview, November 3, 2015)

In these quotes, Ms. Martin recognized the importance of the questions she asked her students and suggested that although she is confident in her questioning, it is an area in which she hoped to continue improving. Additionally, she described specific goals for improving her questioning that would allow her students to more fully engage with her questions and think about mathematics in different ways.

This section addressed Ms. Martin's use of questioning and the instructional practices she used to hold her students accountable to their mathematical discussions. Additionally, it evidenced the value Ms. Martin placed on these instructional practices, her desire to continue improving in this area, and the role that Project Influence played in facilitating these changes. The next section contains further evidence of the manner in which she used questioning and other instructional practices to help ensure the success of all of her students.

Ensuring the success of students at all levels of ability. Unlike the instructional practices examined in the four sections above, the descriptions of which arose naturally during the course of interviews and reflections regarding Ms. Martin's classroom and instruction, the responses Ms. Martin provided in this section were derived from specific questions about her role with students of differing mathematical abilities. The categories of student ability discussed below were inherent to the questions asked during a single interview, and outside of her responses to these questions Ms. Martin rarely, if ever, distinguished among her students by ability level. Additionally, throughout this segment of the interview, Ms. Martin stressed that many of the classroom practices she was describing were appropriate for all of her students.

When asked what her role with her best mathematics students was, Ms. Martin described her desire to continually challenge these students within the activities she was already implementing in her classroom.

> Probably challenging my higher ones. Like I say, I want to give them leadership roles because they are the higher ones and they do see things differently than most of those kids but how can I continue to challenge them? I know these [mathematical tasks] that we're doing already kind of differentiate themselves because of the way they're able to solve problems, and I don't have to plan 16

> different lessons, but yes, how can I challenge them a little more even though we're using the same problem? How can I challenge them? (Background Interview, September 18, 2015)

In this response, Ms. Martin acknowledged that her higher functioning students are likely to see mathematics in ways that are different from other students and that they should be asked to share this thinking with other students. However, she also stressed that finding ways to challenge them and extend their thinking within the tasks being used was essential.

These expectations for communicating ideas about problems and challenging one's way of thinking were extended to Ms. Martin's description of her role with students of average and lower than average ability. In describing her role with average students, she stressed the importance of helping them see that there are multiple ways to think about and represent mathematics.

> I still think the conversations and still working together is a big part for them because they need to, a lot of these will just have that one way to show something, but I think they need to grow out of that and see that there are other ways, and, "Can you show me this in a different way?" So just pushing them to show multiple representations and not just sticking to one way to show something. I think that can help them. (Background Interview, September 18, 2015)

When addressing her role with students of lower than average ability she continued to stress the importance of having them work with other students and share mathematical ideas, but suggested that the types of questions she would ask would be more focused on advancing their understanding of specific ideas.

> Right now I feel like I'm still having to provide maybe more support, like personal teacher support for them. They're still in those cooperative groups, and they're still working together and hearing each other share their ideas but sometimes I feel like I have to give more advancing questions for them so that they will, like, I know where they're at, so I have to kind of question, strategically question, them to get them to move on to that next little thing. I mean, I think that's important for all of them but those ones that are the lowest, I think I have to pull a little more and be strategic in questioning. (Background Interview, September 18, 2015)

Again in this example, Ms. Martin emphasized that many of the practices she had described for use with other students are equally valid for use with this group, and acknowledged that even the specific strategy she described for use with this group was broadly useful for all students.

From her descriptions of her roles with different students it was evident that many of Ms. Martin’s core instructional practices are used with all of her students regardless of their perceived ability level. Although she described specific instructional practices for each ability level of students when directly questioned, she also displayed a willingness to utilize any of the practices she described with students at any level. Additionally, she rarely referenced ability outside of this specific line of questioning and tended toward discussions and descriptions of her practices in response to the needs of specific students, as evidenced in the previous four sections.

Summary. This section contained Ms. Martin's descriptions of the instructional practices and activities she utilized on a daily basis, which aligned with five interrelated themes. The first of these themes addressed Ms. Martin's classroom norms and expectations in service of building a supportive learning community. The second theme detailed the manner in which Ms. Martin emphasized student thinking and student-to-student discussion of mathematics within this learning community. The third theme described Ms. Martin's growth as a facilitator of these discussions. The fourth theme examined the means by which Ms. Martin held her students accountable for their thinking and speaking in the classroom. The final theme presented Ms. Martin's descriptions of attending to the needs of students of all ability levels in her classroom. The next section contains Ms. Martin's observations regarding the outcomes of these practices.

Ms. Martin's descriptions of relevant outcomes. In her interviews and reflective journal, Ms. Martin described three types of outcomes relevant to the current study. First, she talked about specific outcomes in her students' abilities as problem solvers and with mathematical concepts. Second, she described her students' progression in connecting their mathematical strategies to one another. Finally, she discussed outcomes in achievement and affect across individual students and her classes as a whole. Ms. Martin's descriptions of these outcomes comprise this section.

Outcomes related to problem solving and specific mathematical concepts. Ms. Martin described the biggest change that had occurred in her classroom in recent years as involving a shift from problem performing to problem solving (Rigelman, 2007). She elaborated on how her changes in instruction had impacted her students' learning and described the importance of problem solving for her students.

> It is a big shift. It's probably the biggest shift that I have just seen, the impact that it has on their learning. They are learning, like the whole of problem solving is so important. . . . for them to be able to have their own ideas and me not say, "Oh, this is how we're going to do it," or, me give them a thought or an idea and let them cling to that because they will. If they think that this is Ms. Martin's way, they want to do what Ms. Martin is doing. (Background Interview, September 18, 2015)

In describing this change in learning, Ms. Martin emphasized the independent problem-solving ability that had developed in her students. She contrasted this independent thinking with that of problem performers, who she described as focused on the solution to a problem rather than understanding the process of solving it.

> If you're just performing you're just giving an answer. Maybe you really don't know why you got that answer or how you came to that answer, you're just giving the answer because that's the final result. Problem solving involves so many more life skills that these kids are going to need, not just finding a solution, but there are just many different ways that problems can be solved and there's not just one right path and one right answer. (Background Interview, September 18, 2015)

The combination of independent thinking, a focus on deep understanding, and the ability to recognize that problems could be solved in more than one way formed the basis of the life skills Ms. Martin associated with her students' transition to problem solving over problem performing.

Later in the school year, Ms. Martin referenced how these changes in problem-solving approaches had caused other changes in her students' abilities to see connections between their thinking. She also described the reasons for this success.

> Well, for example today in math, I had them sharing out their examples or their solutions strategies to the test that we did at the end of class. And you would have certain kids say, "Well, my strategy looks a lot like Sarah's strategy." So they're mirroring or they're recognizing that their strategies looks similar to another strategy in the room. And here it is November, so we're moving along on that trajectory. And as far as continuing that, I think you just still tell them, pulling those cards and making everybody responsible for an answer. And they know that they're going to have to provide some sort of answer and some sort of discussion. (Point of View Interview, November 4, 2015)

In this quotation, Ms. Martin recognized that her students are progressing toward the goal of recognizing and connecting different mathematical ideas and attributed this success to her classroom practices related to accountability. She also expressed a desire to continue utilizing these practices in order to encourage and support these changes.

Ms. Martin also referenced specific concepts of number and operation and their transfer to problem solving that her students had developed as a result of her use of number talks. One instance of this is illustrated in the discussion of Ms. Martin's belief in mathematics as a connected system earlier in this chapter. In describing the number talk from one day of class, she referenced the variety of representations her students had generated to explain their reasoning for single-digit addition (i.e., 7 + 3).

She further explained that because of their ability to represent this single-addition problem in a way that was meaningful, they had been able to resolve the addition of a single-digit number with a two-digit number (7 + 13) by transferring their understanding.

> Yes. So they broke up, they knew that because we took seven plus three, the first one we did was seven plus three equals ten, so in that thirteen there's that three that I can put with seven to make my ten and here's my other ten and that gets me twenty. They saw that immediately after we talked about the number bond just within three. (Selection Interview, September 9, 2015)

Ms. Martin also provided specific references to students utilizing concepts and strategies from her number talks and tasks to help perform operations on numbers and encounter opportunities to become comfortable with new approaches.

> It definitely lets them use their strategies that they feel most comfortable with. For example, today we've looked at, some of them were counting backwards on a number line for the subtraction. Some were just using those place value, “This is 53, if I'm 53 minus 20, I'm breaking apart my 50, and my 3, and I'm subtracting my 10s,” so there are still so many strategies. (Professional Development Interview, November 3, 2015)

Throughout her instruction and interviews she emphasized the importance of students being exposed to a variety of strategies and approaches to dealing with concepts of number in order to help them see the relationships among these ideas.

Additionally, Ms. Martin described a further transfer of this understanding from the context of her number talks to specific instances of problem solving. As an example, when evaluating how a problem-solving task about packing crayons in boxes of ten and singles had gone, Ms. Martin referenced the students' use of strategies from their morning number talk regarding the meaning of the number 32.

> I felt the lesson went well. I was glad to see how using the number talk was useful for students to strategize in solving the task. I still have some who want to use all strategies instead of context specific strategies, but that will come. (Reflective Journal, September 20, 2015)

This ability to transfer understanding both within and across numerical contexts was a key outcome of the student learning described by Ms. Martin.

This section contained Ms. Martin's descriptions of her students' outcomes in two related areas of mathematics: problem solving and operations with numbers. These results, along with broad improvements in her students' abilities to think and communicate about mathematics, constituted the most relevant learning outcomes Ms. Martin described for her students. The next section contains Ms. Martin's descriptions of how these outcomes impacted the achievement and affective characteristics of students in her classroom.

Outcomes related to achievement and affective characteristics. Ms. Martin described changes in both her students' attitudes towards mathematics and their mathematics achievement based on their exposure to the learning environment she had crafted. She described the impact of this environment and the changes it initiated in a student who had started the previous school year with a negative disposition towards mathematics.

> She is so bright, so intelligent. She could be gifted I even think, but it was like she saw things differently. I'm not saying the teacher she had before really turned her off, I don't know what it was, I don't know if she was turned off by something, but she never felt the opportunity to express herself with what she saw. So she just blossomed in here and was able to totally change the way she thought about math. She loves it. (Selection Interview, September 9, 2015)

Ms. Martin explained how this transformation had been mirrored across her entire classroom and discussed changes in her students' levels of engagement and excitement about their mathematics lessons related to problem solving. She highlighted both her own excitement and the excitement and engagement of her students when they hosted a demonstration lesson for Project Influence.

> That also excited me because they didn't go in there and they weren't little crickets just sitting there like "Oh, we're going to do something fun. This is going to be good." They were on it. They were talkative, but they were talkative about what was going on. I was really pleased with how they handled it. They were excited, they were excited to do that. (Selection Interview, September 9, 2015)

From Ms. Martin's accounts, these affective changes accompanied better results in student achievement. As evidence, she described how the sum of these changes impacted her students' achievement on state exams across multiple years.

> When I first started Project Influence and the things we did in fourth grade and the kids would go on to fifth grade, their scores would be better even in the fifth grade. Fifth grade teachers would get all the applause for that, but I had several people to come to Jessica [another teacher involved with Project Influence] and I and say, “We know it wasn't all them. We know a lot of it was because of the strategies you brought to the table to begin with.” (Selection Interview, September 9, 2015)

This quotation illustrated that Ms. Martin both received recognition for the work she had done in establishing this learning environment and believed the changes she had initiated in her students to be long lasting.

Summary. The outcomes examined in this section illustrated areas which were both perceived to be important by Ms. Martin and relevant to the current study. They included descriptions of the development in students’ problem-solving skills and mathematical understandings as well as shifts towards positive affective characteristics, higher levels of excitement and engagement with mathematics, and improved mathematics achievement. Ms. Martin provided examples of these outcomes for both individuals and classes as a whole, and referenced the long-term implications of these results. The next section substantiates Ms. Martin’s descriptions of her classroom practices and outcomes through evidence collected during classroom observations of her mathematics instruction.

Observations of Ms. Martin's Classroom Activities

As described previously, Ms. Martin credited a desire to have her students share their own thinking about mathematics and to begin to value the thinking of others as the motivation for much of her mathematics instruction.

> I want my kids to feel that way too, that all of their opinions matter and how they see something matters and it gives those other kids an opportunity to say "Yeah, I didn't think about that, and I might want to use that another time because that might be easier for me to use." (Selection Interview, September 9, 2015)

This motivation, other facets of Ms. Martin's beliefs about the teaching and learning of mathematics, and many of the instructional practices Ms. Martin reported using on a regular basis were evidenced during one week of baseline classroom observations conducted during the first half of the Fall 2015 semester. The sections that follow contain descriptions and narrative reports of the primary and secondary learning activities observed during this period. Ms. Martin described this week of instruction as an introduction to place value with two- and three-digit numbers and referenced the CCSSM for Understanding Place Value in the standards for Number and Operations in Base Ten in her lesson plans for the week (Lesson Plans, September 14 – 25, 2015). The standards for Understanding Place Value are shown in Table 4 and appear to be well aligned with the instruction reported in this section.

Table 4

CCSSM Grade 2 Standards for Number & Operations in Base Ten – Understanding Place Value

Standard	Description of the Standard
CCSS.Math.Content.2.NBT.A.1	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
CCSS.Math.Content.2.NBT.A.1.a	100 can be thought of as a bundle of ten tens — called a "hundred."
CCSS.Math.Content.2.NBT.A.1.b	The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
CCSS.Math.Content.2.NBT.A.2	Count within 1000; skip-count by 5s, 10s, and 100s.
CCSS.Math.Content.2.NBT.A.3	Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
CCSS.Math.Content.2.NBT.A.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

During each of five days of classroom observation, 95 minutes of mathematics instruction was scheduled, with 50 minutes per day allotted to whole-group and small-group instruction and activities and 45 minutes per day reserved for mathematics centers, for a total of 475 minutes of mathematics instruction scheduled during these observations. Over the course of the five days of observation, approximately 442 minutes of this time (93.05%) were utilized for actual mathematics instruction with the remaining time used for classroom transitions and other logistical needs. Of this instructional time, 277 minutes (62.67%) were utilized for whole-group and small-group instruction and activities, referred to as *morning instruction* for the remainder of this section, and 165 minutes (37.33%) were used for afternoon mathematics centers.

Throughout these observations, three primary types of classroom activities comprised the majority of Ms. Martin's mathematics instruction. The first two of these activities, the use of mathematical tasks and number talks, occurred predominantly during morning instruction and took place on four of the five days of instruction observed. The use of mathematical tasks consisted of students engaging in problem solving with a mathematical task through small-group and whole-group interactions. Number talks, as previously described, consisted of short discussions about numbers and operations that were facilitated by the teacher. The third type of activity, mathematics centers, occurred during the afternoon of each day of observation and consisted of small groups of students engaging with a variety of mathematical activities either independently or with the assistance of Ms. Martin or a teaching assistant. In addition to these primary mathematics activities, three types of secondary activities were observed to occur during morning instruction and appeared to support the goals of the primary mathematics instruction. These secondary activities included multimedia-supported skills practice, formal assessment, and a mindset discussion. The remainder of this section consists of representative examples of each of these six activities as they were implemented in Ms. Martin's classroom.

Primary mathematics learning activities. The activities reported in this section comprised the majority of the mathematics instruction occurring in Ms. Martin's classroom and were each observed to occur on multiple occasions. They included the use of mathematical tasks, number talks, and mathematics centers and accounted for approximately 65.70% of Ms. Martin's morning instruction and 78.51% of the overall instruction observed. The primary focus of these activities appeared to include learning new mathematics, transferring current understanding into new situations, reviewing previously encountered ideas, and sharing one's thoughts about mathematics.

The use of mathematical tasks. Ms. Martin utilized mathematical tasks throughout her instruction as a way of providing context for operations and procedures and reviewing content with her students. In both cases, tasks were presented through a problem-solving approach in which students engaged with a small number of mathematics problems, typically one or two, presented within a real-world or representational context. Typically, students briefly considered the problems individually before attempting to solve them with pairs or small groups, reported their approaches and results to the whole group via discussions facilitated by Ms. Martin, and shared ideas that either they or Ms. Martin deemed important in small- and whole-group formats. In this fashion, the use of mathematical tasks comprised the dominant learning activity during morning instruction, comprising 50.18% of instruction observed during this period and 31.45% of all instruction observed. The problems used during each day of observation are recorded in Table 5.

Table 5

Problems Used for Mathematical Tasks

Observation Day	Mathematical Task Problem(s)
Day 1	Problem #1: Sara has 36 crayons. She can pack them in boxes of 10 crayons or as single crayons. What are all of the ways Sara can pack the crayons?
Day 2	No problems utilized due to formal assessment.
Day 3	Problem #1: How can I show 70 ones with base-ten blocks? Problem #2: What are different ways to represent 100 with base-ten blocks?
Day 4	Problem #1: Create a quick drawing of 249 using base-ten blocks. Problem #2: Pencils are sold in boxes of 10. Mr. Lee needs 100 pencils. He already has 40 pencils. How many boxes of pencils should he buy?
Day 5	Problem # 1: Kendra has 130 stickers. It takes 10 stickers to fill a page. How many pages can she fill?

Task setup. Each implementation of a mathematical task was preceded by another mathematical activity, such as a number talk or skills practice, and was thus prefaced by a set of instructions used to transition to the task. The following setup, observed prior to the Day 1 task and used to review concepts related to the composition and decomposition of two-digit numbers, was typical of the instructions provided to students prior to their engagement with a mathematical task.

Ms. Martin: Today in our lesson, when we go back to our tables, I'm going to give you a word problem. When we look at our word problem, remember that we're going to first understand the problem, because if we don't understand the problem, what's going to happen?

Kyle: We don't know what to do.

Ms. Martin: We don't know what to do. So we have to understand our problem, so we're going to read it carefully, and pick out all of the pieces that we need, so that we can understand our problem. Then I'm going to give you a little time to work on your own, to think about all the different ways that you could answer that problem, because there will be more than one, then we'll have time to talk in our groups, then I'll want you to share out with me what you've come up with. Ok? (Classroom Observation, September 18, 2015)

These instructions referenced at least four ideas shown to be significant to Ms. Martin in this chapter. These included her desire for students to: make sense of mathematical situations; understand mathematical ideas in their own ways; consider multiple representations and approaches to solving problems; and communicate their thinking about mathematics to one another. Additionally, in prefacing her mathematical tasks in this fashion, Ms. Martin explicitly made her expectations with regards to these ideas transparent to her students and provided opportunities for them to engage in practices that support growth in these areas.

Task implementation. The mathematical tasks utilized by Ms. Martin involved real-world contexts during three of the four days they were observed, the use of manipulatives (base-ten blocks) to represent quantities on a single day, and a variety of student models and representations of quantities each day. In order to fully describe Ms. Martin's implementation of these tasks, this section consists of a synopsis, in narrative form, which summarizes the key characteristics of her implementation of a task involving a real-world context and student representations during the first day of observation (Classroom Observation, September 18, 2015). This narrative presents the words and actions, predominantly of Ms. Martin but also of her students, as they considered the following problem: Sara has 36 crayons. She can pack them in boxes of 10 crayons or as single crayons. What are all of the ways Sara can pack the crayons?

After receiving the instructions described in the previous section, Ms. Martin's 16 students returned to their table groups consisting of four students per table and were asked to read along as the problem was displayed on a whiteboard at the front of the classroom. The students were asked to read the question to themselves a second time and Ms. Martin posed the following question.

> I want you just to think about it, in your head, for just a minute. You may even want to read it again yourself, silently, and just think about what do I know in this word problem. What do I know?

Approximately thirty seconds later Ms. Martin randomly selected a student by drawing her name from a set of index cards containing all of the students' names and asked her to respond to the question. When the response was provided, Ms. Martin rephrased it to the group and recorded it on the whiteboard, then asked for volunteers to explain whether or not they agreed this idea was important. The process was repeated three more times to produce the following list of ideas from the students:

- The question is all the ways that Sara can pack her crayons;
- Sara has 36 crayons;
- If you cut the crayons in half you would have more crayons; and
- Sara can pack the crayons in boxes of tens or as ones.

Students were then provided a sheet of lined paper and asked by Ms. Martin to "come up with as many ways as you can to pack Sara's crayons."

Although most students began working on the problem independently right away, others were more hesitant, including Samuel, who raised his hand and asked Ms. Martin directly, "How are we supposed to do this?" Ms. Martin replied "I don't know how you're going to do it, you've got to tell me!" and then turned to the whole class.

> We know what we're looking for, so now we have to come up with a solution. How's she going to pack these crayons? There are lots of ways. Think about everything that we've been working on this week, think about all the different ways we can show numbers, and let that help you solve this problem.

Students were then allowed to work alone quietly for four minutes as Ms. Martin circulated, examined their work, and monitored their progress. As the first few students began to find solutions, Ms. Martin encouraged them to look for alternatives by stating, "If you find one solution, see if you can find another solution. Is that the only way she can pack them? Think about finding another solution once you've come up with one."

After another minute, she stopped the students and asked them to begin to share what they were finding with their groups.

> Let's stop what you're doing, you may not be completely finished with your answers, but that's ok, let's take a minute and stop. Now, I want you to remember what we're doing when we discuss in our groups. I want you to share your strategies. And guys, your task, when you're working with each other in your groups, you share your strategy, but then as a group, you're deciding, "Is that going to help us solve this problem?" Because we have a lot of strategies, don't we, to solve problems, and we can come up with so many ways to show numbers, but we have to make sure it fits this situation. So as I was walking around I saw lots of ways to show 36, but does it fit this situation? We're talking about boxes of crayons, and she can pack them in boxes of 10, or, as single crayons, so remember that as you're talking to each other at your tables. Do your strategies help solve this problem?"

Over the next nine minutes, as students began sharing their thinking with one another, Ms. Martin circulated to each group, observed the conversations that were taking place, and asked questions to advance the discussion and encourage students to continue talking with one another. During this time, the majority of Ms. Martin's questions related directly to the representations and strategies students were using to solve the problem, with other questions used to allow students to elaborate on their thinking or to consider mathematical ideas that arose from their discussions. A comprehensive list of the questions asked by Ms. Martin during this time is presented in Table 6.

Table 6

Questions Asked by Ms. Martin during Day 1 Task Implementation

Questions about Representations and Strategies
So how is this going to show us how we can pack the crayons?
I see a lot of different representations of thirty-six, but are they all going to help me solve this problem?
How does a ten frame help you in this situation?
How's this going to help me with the problem?
How is writing thirty-six in word form going to help you solve this problem?
How is this going to help you know what to put in each box?
Do you think this is a useful strategy if you're talking about boxing crayons?
How are your ten frames like the boxes?
How is that a useful strategy?
So, since I left, you had a discussion on picking and choosing strategies that would help us solve this. So what have you guys come up with?
Other Questions about Mathematics
What are all the ways Sara can pack the crayons?
Why are you showing four boxes?
How do you know that's the most that can fit into a box?
Questions about Student Thinking
Since I left, what have you guys come up with, anything different?
Help me understand the problem, what are we doing?
What are you guys coming up with here?
This is different here, so now tell me what you're doing?
What's my question?
Will you guys share that out for me when we get ready to share out our strategies?

Once she had visited each group at least twice and provided students time to discuss the last question asked at each group, Ms. Martin drew the students' attention to the front of the room in order to lead a group discussion of their thinking.

> I saw a lot of thinking, but we want to make sure our thinking is on what? On track with this problem, ok? So we've got several things that we want to share, several groups that I want to come up and show what they were thinking, ok? So Ally, why don't you bring your paper up?

As Ally moved to the front of the room, she and Ms. Martin placed her work (see Figure 6) on a document camera projected onto the whiteboard, and Ms. Martin concluded her instructions and turned the discussion over to Ally.

Figure 6. Work presented by Ally during the Day 1 mathematical task. The ten frames she referenced, which do not align exactly with her description, have been outlined.

Ms. Martin: So everybody is going to give Ally their attention, all right? Everybody looking at Ally, Ally speak out loud to your classmates so they can hear your thinking.

Ally: I was thinking that the tens frames could be the boxes, cause she said that, we could, um, that she put ten crayons in the boxes, so I was thinking that was the boxes, and I put three full ten frames . . . and a ten frame with just six. Because, um, the three full ten frames stand for 30, and the six just stand for six. So that is 36.

Ms. Martin: Boys and girls, remember when a speaker is speaking you're not writing, you're not talking, you're listening to this strategy. Because, why do we want to hear our strategies?

Students: So we can learn from each other.

Ms. Martin: So we can learn from each other, that's right. We want to hear these different ways of thinking, because you might not have thought this way. So, what were we trying to solve in this problem? Janet, restate what we were saying in in the problem, what are we trying to solve?

Janet: We were trying to solve how many ways Sara can put [the crayons in] boxes.

Ms. Martin: So how many different ways she can put them in boxes. So what's one way Sara can arrange them in boxes, according to Ally's work here? [Randomly drawing a student's name] Lisa, how are we going to arrange these crayons according to Ally's work?

When Lisa was hesitant to respond, Ms. Martin directed the question to the small groups and moved to Lisa's group to repeat the question to her.

Ms. Martin: [To the whole group] Everybody, turn and talk to your table. What's the arrangement of crayons using Ally's ten frames? How many boxes of ten and how many singles, using this drawing?

Ms. Martin: [To Lisa] If we can arrange them in boxes of tens and singles, how's Ally's drawing going to help me show how many boxes of tens she has versus how many singles?

One minute later, after giving the groups a chance to discuss the question and allowing Lisa the opportunity to share her thinking one-on-one, Ms. Martin called the class back together.

Ms. Martin: All right, back up here, so Lisa's going to tell us how this model is one solution to the problem.

Lisa: Ally has 10 boxes of crayons . . . I mean three boxes of 10 crayons and she has six singles.

Ms. Martin: Kyle, tell me what Lisa just said.

Kyle: She said that now we have three boxes of 10 crayons . . . and six singles.

Ms. Martin: So that equals what?

Kyle: That equals 36.

As Lisa spoke, Ms. Martin recorded “3 boxes of 10” and “6 singles” on the whiteboard, adding “= 36” as Kyle concluded the exchange. She then asked the class as a whole “Three boxes of 10 is how many?” and “How many singles?” and proceeded to record “30 + 6” underneath 36 as the class replied in chorus (see Figure 7).

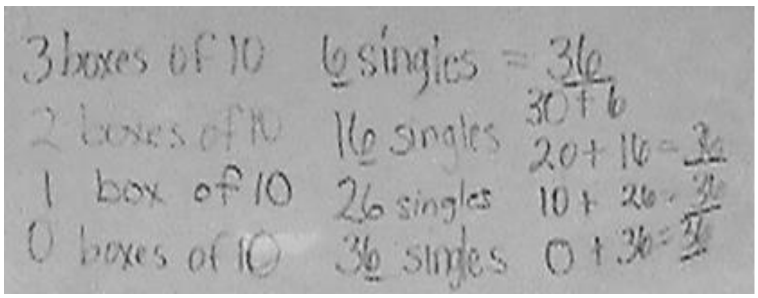


Figure 7. Ms. Martin’s recording of the compositions of crayons suggested by student responses during the Day 1 task. The first and last lines were filled in directly from work presented by students while the second and third lines were generated from students’ descriptions based on their observation of a pattern.

With this idea recorded, the class applauded Ally’s work and Ms. Martin asked the group to once again return to the original problem and consider compositions of 36 crayons.

> Before [the next student presents] I want you to think . . . I want to give you just a minute to talk to your tables. What’s another way I could show these 36 crayons in boxes of tens and single ones?

Students were allowed to discuss the problem for three more minutes as Ms. Martin circulated among groups asking questions and conversing with students about their different approaches and possible compositions. She then brought the group back together and congratulated them on their thinking as she introduced the next presenter.

Ms. Martin: All right guys, I like the way our change has gone. We just needed to see a little something [Ally's work], and we've changed our thinking just a little bit. And Olive, forgive me, but you had something I wanted you to show. . . .Let's turn our attention to the front, and let Olive show one way that she said we could arrange.

Olive: [Olive moved to the front of the room and displayed her work on the document camera (see Figure 8)] I thought about tens and ones, and I thought about having no tens, and then 36 ones.

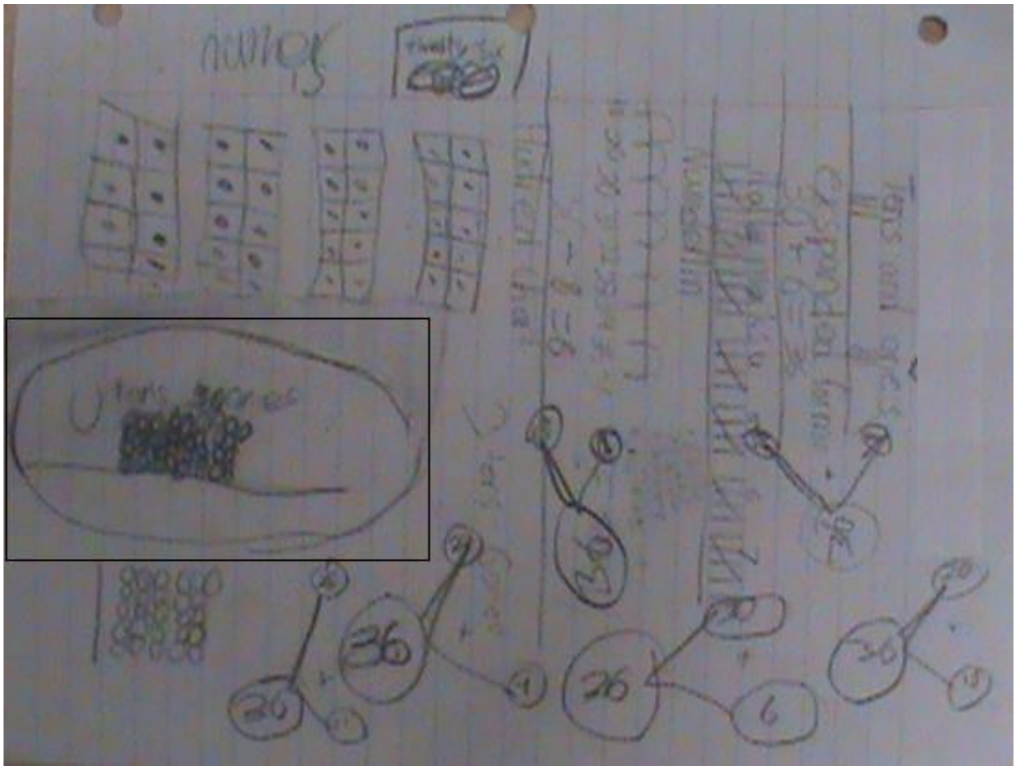


Figure 8. Work presented by Olive during the Day 1 mathematical task. The collection of ones she described has been outlined.

Ms. Martin: There were some others of you thinking that way, who else was thinking you could have no boxes of tens and 36 singles?

As several students raised their hands and described thinking of single units to their groups, Ms. Martin asked Sarah to share something she had been thinking about, to which Sarah replied "Money." When Ms. Martin asked her to elaborate Sarah went on to parallel 36 pennies with 36 crayons and Ms. Martin pointed out that counting each of these things worked in a similar fashion but reminded students, "We still need to remember that we are talking about crayons, so don't forget the context of the problem." From these suggestions, Ms. Martin returned to the white board (see Figure 7) and recorded “0 boxes of 10” and “36 singles” and asked Olive how she could record the total, to which Olive replied, “Zero plus thirty-six is thirty-six” from which Ms. Martin recorded “0 + 36 = 36.”

With this information recorded, Ms. Martin returned Olive to her group and posed her next question.

> I want you to notice what we already have. Three boxes of 10, six singles. Zero boxes of ten, 36 singles. Does anybody see any kind of pattern yet? Don’t say it out loud if you do, just notice, do we see any kind of pattern starting here? Write something down on your paper that you think could go into that pattern.

Students spent the next two minutes recording their observations as Ms. Martin circulated among the groups before asking “Who thinks they see something that might fit in this pattern? Brad?”

Brad: Um, 20 in tens and 16 ones.

Ms. Martin: Twenty in tens and 16 ones. How’s that going to fit? How does that fit with what you see?

Brad: Because, um, there’s zero boxes of tens and 36 ones and you could, those tens we are using are the same thing, except I had tens but I also had ones.

Ms. Martin: So you’re showing them in another way, so we don’t have to use three tens every time, do we? We can show, Linda, tell us what you saw?

Linda: [Linda moved to the front to display her work (see Figure 9), where Ms. Martin met her and covered the bottom of the page] We can show two boxes, and sixteen ones.

From these exchanges Ms. Martin and the class filled in the next line on the whiteboard (see Figure 7) with “2 boxes of 10” and “16 singles” which the class translated to “20 + 16 = 36” and Ms. Martin recorded. Ms. Martin then asked the class to consider the pattern once again, “Now do you see a pattern, and what is going to fit in that missing row? Tell me on your paper what’s going to fit in that missing row? What’s missing?” Ms. Martin circulated among groups checking their responses and making sure that all students replied for two minutes. At the end of this time, she once again pulled a card and asked a random student, Ty, to answer. From his response she completed the recordings in Figure 7 with “1 box of ten” and “26 singles,” which the class translated to “10 + 26 = 36”.

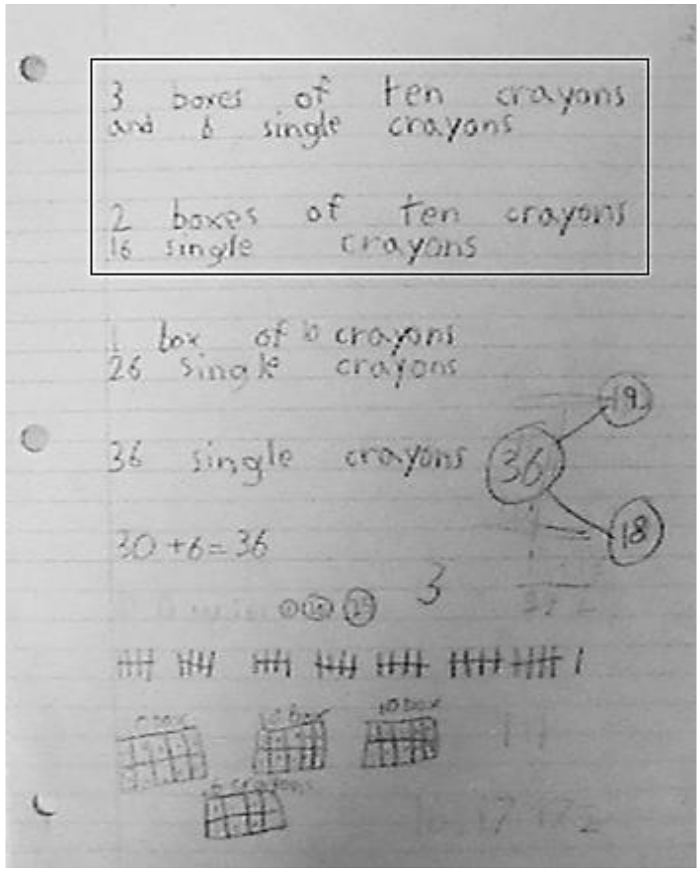


Figure 9. Work presented by Linda during the Day 1 mathematical task. The portion of the work initially shown to the class has been outlined.

After this exchange Ms. Martin asked the students a final question to consider as they lined up for lunch.

> Now, here's my final question before we line up for lunch. How did this pattern help us solve our problem? How were we able to look at all of our tens and ones, and how did that help us solve our problem? I just want you to think about it, and I'm going to ask you after you come back from gym. So, I'm going to give you just a minute or two to think about it, and look at all of these on the board.

Upon the class's return from lunch, approximately 30 minutes later, Ms. Martin concluded the task by asking students for their reflections.

> So I asked you to look at the pattern and think about how are we able to put this pattern together based on what we know about numbers so far, all the different ways that we can make these numbers, how were we able to put that pattern together? Who has come up with something they want to share about how we are able to put this pattern together?

She then pulled Olive's name randomly and had her start the discussion.

Olive: We know we're trying to make 36, we're following the numbers, but we're going backwards.

Ms. Martin: Ok, tell me a little more about that. I see what you're saying, but explain that a little bit more, what do you mean by going backwards?

Olive: Umm, going in order [pauses briefly] backwards [pauses for several seconds]

Ms. Martin: Who sees what she's saying? How are we going in order backwards? What does she mean? Becky, what does she mean, going in order backwards?

Becky: Well, there's three, two, one [pointing out the pattern in the tens].

Ms. Martin: Ok, you see the three, the two, then one [pointing to the pattern on board]. But what are we showing here?

Becky: Tens?

Ms. Martin: Yeah, groups of 10! [Nodding in agreement and pointing out numbers on the board as she speaks] Three groups of 10, six singles. Two groups of 10, 16 singles. These singles are also called what?

Class: Ones.

Ms. Martin: One box of 10, 26 ones. Zero boxes of 10, 36 ones. So what helps us put that together? Janet?

Janet: The [pauses] I can see the zero, one, two, three [pointing].

Ms. Martin: Ok, so I'm using different numbers of?

Class: Tens.

Ms. Martin: Different numbers of tens, and grouping them with different groups of ones, all to make what?

Class: Thirty-six.

Ms. Martin: [Underlining 36] Every single time. So we were able to use what we already know, we know 36 is not just three tens and six ones, we know it is represented in many different ways. We were able to use our knowledge of tens and ones to show multiple ways to package those 36 crayons, and we were able to put them into a pattern.

At this point, Ms. Martin ended the discussion of the task and transitioned into a conversation regarding mindset, which is described later in the chapter, in the section on Ms. Martin's secondary teaching practices. During the implementation of this task many, if not all, of the beliefs and practices espoused by Ms. Martin appeared to be evidenced to some degree. The following section contains a summary of this evidence supported by further comments from Ms. Martin regarding her use of mathematical tasks.

Evidence of beliefs and practices. The core activities of the task reported above, including sense making around a mathematical problem, generating multiple representations of situations involving number and applying them strategically, and communicating one's thinking about mathematics with others, were observed during each of Ms. Martin's tasks. Additionally, when asked to reflect about her implementation of this task, Ms. Martin suggested that she intentionally looked for connections among her students' representations that would support their conceptual understanding.

> I chose my students based on the strategies they used to solve the problem. The first one that presented had used ten frames to relate to boxes of crayons in the problem (3 tens 6 ones). She used an applicable strategy from the number talk. I also chose a student who showed the number 36 using all ones, which was a different way to represent 36 and also applied to the problem. The last student was chosen because she was able to show all the possible ways to arrange 36 crayons in the boxes. I felt as if this progression is what I want my students to be able to do when they solve these multiple solution tasks, to be able to show more than one solution. This way I know they see those numbers in different ways, which tells me they conceptually understand the material. (Reflective Journal, September 20, 2015)

By selecting student work to present in this order, Ms. Martin facilitated a productive discussion of the place-value ideas involved in the decomposition of a two-digit number and encouraged students to participate in a discussion about the underlying structure of this system. Additionally, as evidenced by her comments regarding a later task involving three-digit numbers, she believed that these ideas were well understood by students and were able to be transferred to new situations.

> I feel they did very well. Today was a hands on, exploratory lesson that I felt we needed after the assessment and to get them thinking in terms of one hundreds. I think the connections they were making with what we learned with ones and tens now to tens and hundreds were important. Them using the blocks to actually prove that all the representations they were making were all the same number is key. (Reflective Journal, September 22, 2015)

In combination, this sequence illustrated how Ms. Martin attributed her students' success with novel tasks and problems to an understanding of mathematical concepts that was generated by engagement with tasks through the core practices described above.

In addition to the specific conceptual understandings generated by Ms. Martin's use of mathematical tasks, she believed that her overarching goals of encouraging students to think independently and strategically about mathematics and communicate these ideas with others were well served by her task implementation.

> I feel that a lot of strategies were being mastered this week by the way they are able to apply them to different situations. They are learning to share ideas more

> freely and I can tell they are using strategies they've heard in class. (Reflective Journal, October 8, 2015)

This sharing of ideas, predominantly in small group interactions but also in whole group presentations and discussions, proved to be a core practice in all of the primary learning activities observed in Ms. Martin’s classroom. Additionally, Ms. Martin used these tasks to promote equity among her students by allowing and encouraging all students to speak and share their ideas while also holding them accountable to explaining their thinking and understanding.

> You’ve got to be able to prove your answer. Give me five. Give me five. You’ve got to be able to prove your answer. If I come to you, and you give me an answer, and you don’t know how you got it, do you really know what you’re doing? You have to be able to prove how you got your answer. (Classroom Observation, September 24, 2015).

These observations further evidenced Ms. Martin’s commitment to valuing her students’ thinking and the use of classroom practices which encourage understanding, communicating, equity, and accountability.

Finally, tenets of Ms. Martin’s mindset were observed through her practices in both implicit and explicit manners. Implicitly, her consistent focus on learning goals and understanding rather than performance characteristics was well aligned with the goal structures expected from the growth mindset. Additionally, her emphasis on the strategic use of representations and practices which made student thinking visible provided further evidence of the goal operating and goal monitoring practices discussed previously.

Explicitly, Ms. Martin both set aside time to discuss tenets of mindset with her students, as reported in the section on secondary activities, and referenced the ideas as students engaged with mathematical tasks.

> Give me five [holding hand in the air to focus the class's attention] for just a minute. I've seen lots of struggle, but I love it, because we know that productive struggle is what we want to see, because our brains are growing, we're learning, and that's what we want! Some of you are getting closer, and closer and closer, I can see it! But it's just not quite there yet, and that's ok, because that's what we want. (Classroom Observation, September 22, 2015).

Overall, the combination of Ms. Martin's actions and words during her task implementation provided further evidence of her operationalization of growth mindset tendencies during task implementation.

This section examined the dominant mathematics learning activity utilized during Ms. Martin's morning instruction, the use of mathematical tasks. A narrative example of her implementation of one of these tasks, including its full setup and implementation, was provided in order to illustrate the general manner in which Ms. Martin utilized mathematical tasks. The following section examines the second most utilized mathematical activity from Ms. Martin's morning instruction, number talks.

Number talks. Continuing the account of Ms. Martin's primary mathematics learning activities, her second most often utilized activity during morning instruction was the number talk. Although Ms. Martin typically used number talks to set up her other morning activities, this activity is reported on after her use of mathematical tasks due to the relative amount of time she devoted to the use of each activity. Ms. Martin used number talks on four of the five days observed to initiate her mathematics instruction. The talks, which consisted of students giving their ideas about a prompt (see Table 7) provided by Ms. Martin at the start of the talk, lasted between 10 and 12 minutes each and were aligned with the mathematics instruction which followed. During the talks, students sat in rows on a carpet in front of a small whiteboard where Ms. Martin called on students to share their ideas and she recorded their thoughts. Number talks were a relatively new activity in Ms. Martin's classroom, which she had adopted from her most recent experiences in Project Influence.

> I had never seen a number talk and never used one before [the third year] of Project Influence. I decided to use them because I like the way it brought us all together to share our ideas and strategies. It also is a good way to pre-teach an idea that we will discuss in our lesson following the number talk. (G. Martin, personal communication, February 27, 2016)

Although they represented a new classroom activity, Ms. Martin invested a substantial amount of time in their implementation. Number talks comprised 15.52% of the morning instruction observed and 9.73% of overall instruction.

Table 7

Prompts Used for Number Talks and the Problems Used for Tasks that Followed

Observation Day	Prompt for Number Talk	Mathematical Task Problems
Day 1	"All the different ways to make 32."	Problem #1: Sara has 36 crayons. She can pack them in boxes of 10 crayons or as single crayons. What are all of the ways Sara can pack the crayons?
Day 2	"All of the different ways I can represent 25."	No problems utilized due to formal assessment.
Day 3	"Counting on from 300 by tens."	Problem #1: How can I show 70 ones with base-ten blocks? Problem #2: What are different ways to represent 100 with base-ten blocks?
Day 4	"How you could represent 135."	Problem #1: Create a quick drawing of 249 using base-ten blocks. Problem #2: Pencils are sold in boxes of 10. Mr. Lee needs 100 pencils. He already has 40 pencils. How many boxes of pencils should he buy?
Day 5	No number talk used on this day due to skills practice.	Problem # 1: Kendra has 130 stickers. It takes 10 stickers to fill a page. How many pages can she fill?

The remainder of this section consists of a narrative synopsis of a number talk that was representative of those that occurred in Ms. Martin's classroom. The narrative is drawn from the number talk which took place on the first day of the baseline observations as students reviewed the different ways they knew to represent the number 32. The talk preceded the mathematical task described in the previous section of this report and along with it comprised a full period of morning mathematics instruction.

Ms. Martin's 16 students were seated in rows of four on a carpet facing a small whiteboard. Ms. Martin sat to the right of the whiteboard, facing the students. As she brought together the group's attention, she provided a brief description of the activity in which they were about to engage and reminded students of the signals they would be using to communicate with her.

> We're going to start our math lesson with a math talk, which we've done several times, and I want you to remember your signals that you're going to show me. Remember? We're not going to raise our hand, because other boys and girls may get distracted because they're still thinking, and if you're waving your hand in the air, they may not remember their thinking. So you're going to give me your little thumbs up right here in front of you [holding hand with thumb pointed up against chest], it's a private response, nobody's going to see it. If you're ready to share, thumbs up, if you're still thinking, that's ok, then you're going to keep your thumb down. All right? So thumbs up if you're ready to share, thumbs down if you're still thinking.

After providing these instructions Ms. Martin went on to reveal the prompt for the talk and wrote and underlined the number 32 in the upper-middle portion of the whiteboard.

As students signaled that they were prepared to respond to the prompt, Ms. Martin called on students to share their thinking, re-voiced their ideas, recorded their thoughts on the whiteboard (see Figure 10), and asked questions to clarify her recording and further the discussion.

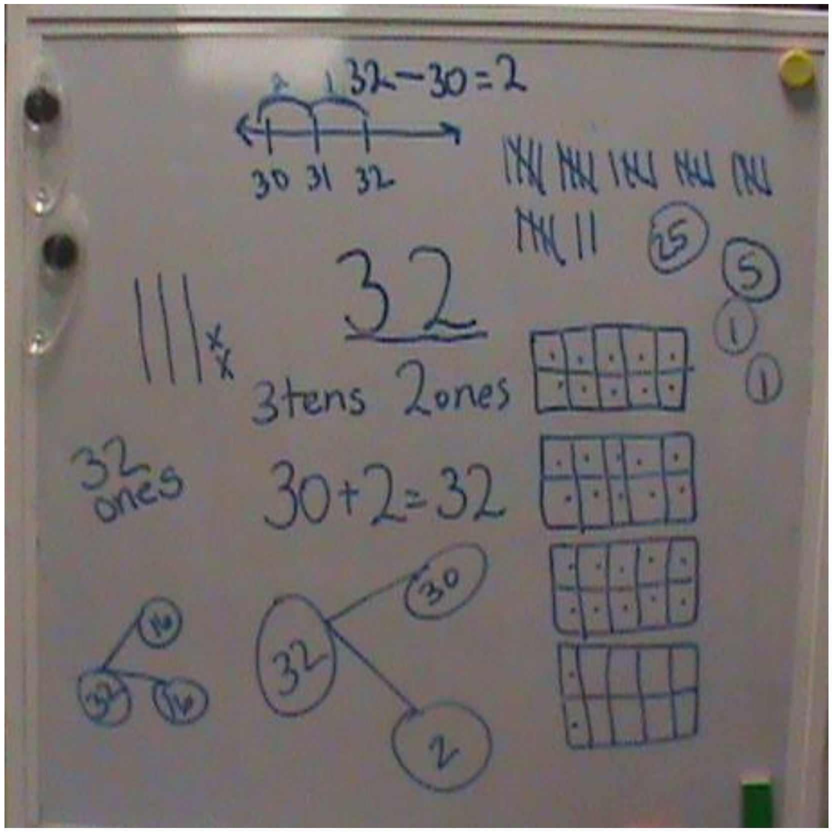


Figure 10. Ms. Martin's recording of the ideas presented by her students during a number talk.

Ms. Martin: I want us to review some things we've been working on. We've been working on counting by ones, fives, and tens, and we've been working on showing numbers in different ways. So today, I just want you to think about how you can show me all the different ways to make thirty-two. [Two students raise their hands] Remember your signals. I'll give you some thinking time and when you're ready to share, just give me your thumbs up. [Ten seconds pass] Kyle, we'll start with you.

Kyle: We can put it in ten frames.

Ms. Martin: Ok, I can show thirty-two in ten frames [beginning to draw the first ten-frame]. How many ten frames do I need?

Kyle: [Thinking briefly] Four.

Ms. Martin: Kyle, why four [continues drawing ten frames]?

Kyle: Because you can't fit thirty-two in three ten frames, because ten frames can only hold ten.

Ms. Martin: So ten frames can only hold ten? How many tens do I have here?

Kyle: Three tens.

Ms. Martin: [Filling in ten frames] So I have three full ten frames. And how many ones in this one [indicating the last ten frame].

Class: Two.

Ms. Martin: Very good. Does that show thirty-two?

Kyle: Yes ma'am.

Ms. Martin: Yes, good job. Ally, what would you like to share?

Ally: You could do it in tally marks.

Ms. Martin: I could do it in tally marks. How many groups of five will I make using thirty-two?

Ally: Six.

Ms. Martin: Six [drawing six groups of five tally marks]. So there's six groups of five, which is how many?

Ally: Thirty.

Ms. Martin: And how many more do I need?

Ally: Two.

Ms. Martin: Two more, very good, so that's another way to represent thirty-two. What's another way, Linda?

Linda: Thirty plus two?

Ms. Martin: [Repeating as she writes] Thirty plus two. What form do we call that?

Class: Expanded form.

Ms. Martin Expanded form, very good. Now, that also shows my place value doesn't it? It shows the value of my three. When I look at this thirty-two, it's not just three, it shows the value, that that [indicating the three in thirty-two] is actually thirty [indicating the thirty in expanded form] plus two [indicating the two in expanded form] equals thirty-two. Nice job. Karen?

Karen: A number tree.

Ms. Martin: So Karen says I can show [Begins to draw a small rectangle, and erases it, replacing with a two-pronged number bond] are you thinking of this Karen? A number bond?

Karen: Yes.

Ms. Martin: Ok, so what's one way I could show thirty-two? There are lots of ways aren't there? What's one way that you think I could show it?

Karen: Thirty and two.

Ms. Martin: Thirty and two [repeating as she fills in the number bond]. Who can think of a different way to show, not just thirty plus two, but what's another way that I could show that? Linda?

Linda: You could use sixteen and sixteen.

Ms. Martin: So, sixteen and sixteen [drawing a new number bond]. What did she use?

Kyle: Half. Doubles.

Ms. Martin: She did, so half of thirty-two is sixteen, and we also know those as doubles, that's exactly right. All right, Leia? What's something else you could share with us?

Leia: All ones.

Ms. Martin: [Nodding] We can just show all ones? How many ones are in thirty-two?

Olive: Two.

Ms. Martin: Well, there's two here [indicating the two in thirty-two], but remember you've got a three over here. What does that three represent?

Leia: [Pauses briefly] Thirty.

Ms. Martin: Thirty. So how many ones are there altogether?

Leia: Thirty-two.

Ms. Martin: Thirty-two ones [writing "32 ones"]. Very good.

Leia: Why don't you draw all of them?

Ms. Martin: No, forgive me for not drawing all of them, but you're exactly right, I can show that as thirty-two ones. Dale?

Dale: Um, you can do it in tens sticks.

Ms. Martin: I could show it using, like a quick drawing?

Dale: Yes [nodding].

Ms. Martin: How many tens would I draw?

Dale: Three.

Ms. Martin: And how many ones?

Class: Two.

Ms. Martin: Very good. Could I have done that with these thirty-two ones [indicating "32 ones"], drawn all of my thirty-two ones out?

Class: Yes.

Ms. Martin: Yes, I could. And yes, I would want you to do that, but for time's sake I'm going to pass on that. Ty?

Ty: [With thumb held high in the air] You could, um, [pauses for eight seconds seconds] I forgot.

Ms. Martin: Ok, I'll come back to you, try to get it back. Becky?

Becky: You could write three tens and two ones.

Ms. Martin: Ok, so I could say that there are three tens and two ones [records "3 tens and 2 ones"]. Something different. Olive?

Olive: A number line?

Ms. Martin: How would I show thirty-two on a number line? [Draws a blank number line]

Olive: Um, start at thirty and [pauses].

Ms. Martin: Start at what?

Olive: Thirty.

Ms. Martin: Thirty, ok [labels 30, continues labeling 31 and 32 as Olive counts].

Olive: Thirty-one, thirty-two. Then at the top, um, connect them.

Ms. Martin: What would I connect? Remember, I'm trying to represent thirty-two. How would that number line be useful?

Olive: You can count backwards.

Ms. Martin: Ok, so if I want to count backwards from thirty-two, I could say thirty-two minus [records "32 –" above the number line]?

Olive: [Quietly] Thirty.

Ms. Martin: What?

Olive: Thirty.

Ms. Martin: Ok [records "32 – 30"] equals?

Olive: Two.

Ms. Martin: [Records "32 – 30 = 2"] So if I start here [indicating 32] and I go [connects backward to 31, then 30], one, two. There's one, and there's two [labels the connections 1 and 2, counting backwards]. So I've got two [marks the 1, then erases it], oops, sorry about that, so here's one jump, and here's two. So [indicating numbers in "32 – 30 = 2" as she speaks], from thirty-two, going backwards to thirty, is two. Very good. One more, one more. Ty, did you remember yours?

Ty: [Nodding] Money.

Ms. Martin: I can show it using money. Ty, tell me how I can show that?

Ty: [Hesitant] It would be, um, thirty pennies.

Ms. Martin: I can do thirty pennies [draws a circle with "1" in it and writes 30 beside"].

Ty: And two nickels.

Ms. Martin: Two nickels. How much is a nickel worth?

Ty: Five cents.

Ms. Martin: [Nodding] Five cents. So if I did two nickels that would give me ten more cents. So think how I could show that in another way? [Other students raising hands] I'm still working with Ty, let him think.

Ty: A quarter.

Ms. Martin: You want me to change it? [Ty nods] Ok [erasing]. Now you're saying a?

Ty: A quarter.

Ms. Martin: A quarter [draws a circle with 25 inside]. How much is a quarter?

Ty: Twenty-five cents.

Ms. Martin: Twenty-five cents. Ok. [Some students beginning to fidget] Everybody watching, because you might need this another day. [To Ty] What else? [Pauses for five seconds] You're trying to make thirty-two, so what do you need to add to that twenty-five that will give you thirty-two? [Pauses for fifteen seconds] Let's think, what's the next ten I want to make if I'm at twenty-five? I go up to twenty-five and then, what's the next ten?

Ty: Twenty?

Ms. Martin: Not twenty. [Pauses four seconds] Twenty, twenty-five?

Ty: Thirty.

Ms. Martin: Thirty is my next ten. [Drawing a blank circle beside the quarter] So what can I put with that twenty-five cents to make thirty cents?

Ty: Five more.

Ms. Martin: What's that piece of money called? [Pauses twelve seconds] What's a five cent piece called?

Ty: A nickel.

Ms. Martin: A nickel [fills in the blank circle with a 5], very good. So [indicating the circles representing coins] so twenty-five plus five is now?

Ty: Thirty.

Ms. Martin: Not thirty-two, so thirty, and now how much more do I need to make thirty-two?

Ty: Two pennies.

Ms. Martin: Two pennies [drawing two circles with 1 inside], very good, Ty. All right guys, you did a great job with this little number talk, there are lots of ways that I could show thirty-two.

One noticeable difference in Ms. Martin's implementation of number talks in comparison to her use of mathematical tasks has to do with the type of communication about mathematics each activity produced. Throughout the narrative of the mathematical task, Ms. Martin provided students with opportunities to organize their mathematical thinking and communicate this thinking to their peers. Although these opportunities were not fully realized, most likely due to the example being taken from early in the school year, the structure of the activity allowed for a high proportion of student-to-student interactions in both small group and whole group formats. In contrast, the majority of the interactions in Ms. Martin's number talks occurred directly between the teacher and a single student. Additionally, much of the organization of the mathematical ideas shared by the students was completed by Ms. Martin during the number talks as she transcribed student descriptions on the white board. As noted previously in this section, number talks were a relatively new activity in Ms. Martin's classroom, having only been introduced during the semester of the current study.

The use of mathematical tasks and number talks constituted the primary mathematics learning activities utilized by Ms. Martin during her morning instruction. Although the styles of their implementation were quite different, these activities were often utilized in tandem, with a relevant number talk introducing a mathematical task.

Ms. Martin's other primary mathematics learning activity was utilized exclusively during afternoon instruction and is described in the next section.

Mathematics centers. Ms. Martin's final primary mathematics learning activity was the use of mathematical centers, which are common in elementary classrooms. In addition to approximately 50 minutes per day of morning instruction, each afternoon students engaged in 45 minutes of individual and small-group work in one of the five mathematics centers described below. Groups rotated through these activities spending one 45-minute period working in each center per week. The centers were designed for students to engage in these activities either individually, in groups of three to five students working together independently, or in groups of three to five students under the guidance of the teacher or a teacher's assistant. These mathematics centers comprised 37.33% of all of the mathematics instruction observed during the week.

Teacher assistance. In this center, students worked directly with Ms. Martin in a small group focused on activities from morning instruction with which they needed more exposure. During the week of observation, these activities included a worksheet focused on modeling and drawing numbers up to 500 using base-ten blocks and practice with skip counting by twos, fives, and tens up to 120. During this time, Ms. Martin provided direct instruction on these topics to the students, asked questions to individuals and the group as a whole, and encouraged discussion among the group of the answers the students provided to these exercises.

Place value. Students in this center worked as an independent group, with the help of a teaching assistant, or with occasional guidance from Ms. Martin to match sets of cards featuring a number and its representation in expanded form, word form, and base-ten blocks (see Figure 11). As students generated matched sets for two-digit numbers they individually recorded each representation on an accompanying worksheet.

Figure 11. Sets of matched representations from Ms. Martin's place value center.

Coin counting. This center consisted of students randomly selecting a card with images of pennies, nickels, and dimes (see Figure 12) and working as a group to determine and record the total value of the coins shown on the card. Students at this center worked as an independent group, with the help of a teaching assistant or with occasional guidance from Ms. Martin.

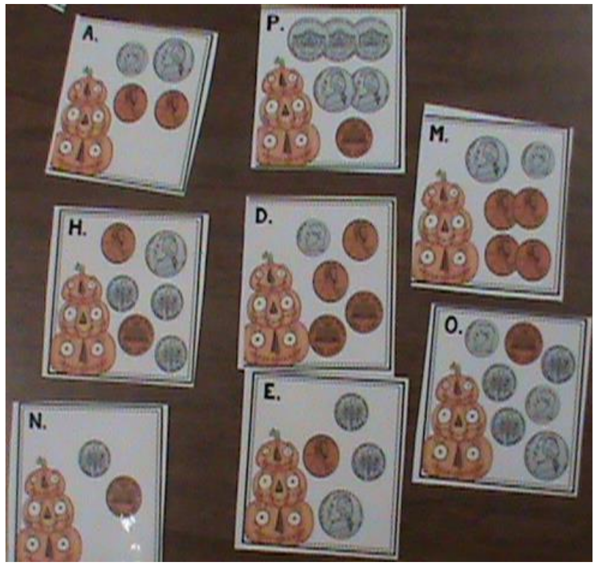


Figure 12. Sets of representations from Ms. Martin's coin counting center.

Skip counting. In this center students worked as either an independent group or with assistance from an adult to select a skip counting card (see Figure 13), determine the number by which the card was being counted, and record the entire skip counting sequence on an accompanying worksheet. This activity was also used with the whole group as part of the skills practice observed during morning instruction of Day 5 of observation.

Figure 13. Sets of prompts from Ms. Martin's skip counting center.

Computer-guided study. Students in this group worked independently at one of the classroom's four computers to review topics including place value, the representation of numbers via base-ten blocks, and number lines. The selected software, *Go Math!*, provided guided reading regarding these topics along with interspersed multiple-choice and fill-in-the-blank questions that provided feedback to students based on their responses. The next section addresses the secondary activities Ms. Martin used to support these primary learning activities.

Secondary activities. The activities reported in this section were each observed on a single occasion and appeared to be used to support primary learning activities in some fashion. These activities included formal assessment, skills practice, and a discussion of mindset. Together they accounted for approximately 34.30% of Ms. Martin's morning instruction and 21.49% of the overall instruction observed. Each of these activities are briefly described in this section in order to provide a full account of the manner in which instructional time was utilized in Ms. Martin's classroom.

Formal assessment. Day 2 of the baseline observations included the students' first formal assessment of the semester in the form of an exam. This activity occupied 18.77% of the morning instruction observed and 11.76% of all instruction. After completing a brief number talk in which ideas of place value, composition and decomposition of number, and representation of a two-digit number were discussed, students returned to their tables and placed three-ring binders as partitions between their workspaces. An 11-question exam, printed from the chapter resource materials that accompanied the class's textbook, was then distributed to the students and displayed on the whiteboard. As the class read the exam questions together, Ms. Martin pointed out key features of each question and provided explicit instructions regarding her expectations for the students' responses. Three sample items from the exam, along with the instructions provided by Ms. Martin for each item and a sample of full-credit student work for each, are included in this section. These items were selected as they illustrated the general nature of the content of the exam, highlighted the fashion in which Ms. Martin interacted with her students regarding the questions, and demonstrated the depth of reasoning Ms. Martin required for a question to receive full credit.

Exam question #2. Ms. Martin read the question (see Figure 14) aloud, called for questions, and then elaborated on the type of picture that was to be included in the response.

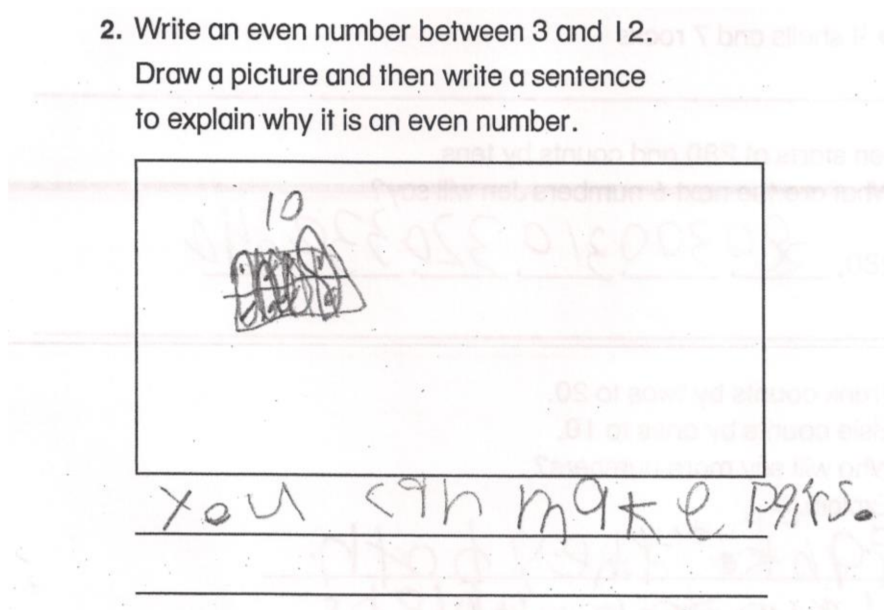


Figure 14. Exam question #2 with one of Ms. Martin's students' responses that received full credit.

Ms. Martin: Number two. Write an even number between three and twelve, you get to choose, ok? You get to choose an even number between three and twelve, draw a picture, and then write a sentence to explain why it is an even number. So I'm going to see some writing, and I'm going to see a picture, ok? After you choose your number, draw a picture to show it, and tell me why it is even. Questions on that? You see the lines for the writing and the box for the picture. Questions? Please ask them. Ally?

Ally: Does it mean, like draw a number in the box?

Ms. Martin: You can write a number in the box, but then draw a picture that shows that number. Ok? How would you represent, just like we've been doing on our math talks, how would you represent that number in a picture? Ok? Anything that represents that number.

Exam question #6. Ms. Martin read the question (see Figure 15) aloud and emphasized that an explanation as to why the answer was correct was required.

> All right, let's read number six everybody. Frank counts by twos to twenty. Elsie counts by ones to ten. Who will say more numbers? Explain. Ok? Again, explain. There are lines for you to write, you will tell me who counts more numbers, but then you're going to tell me why.

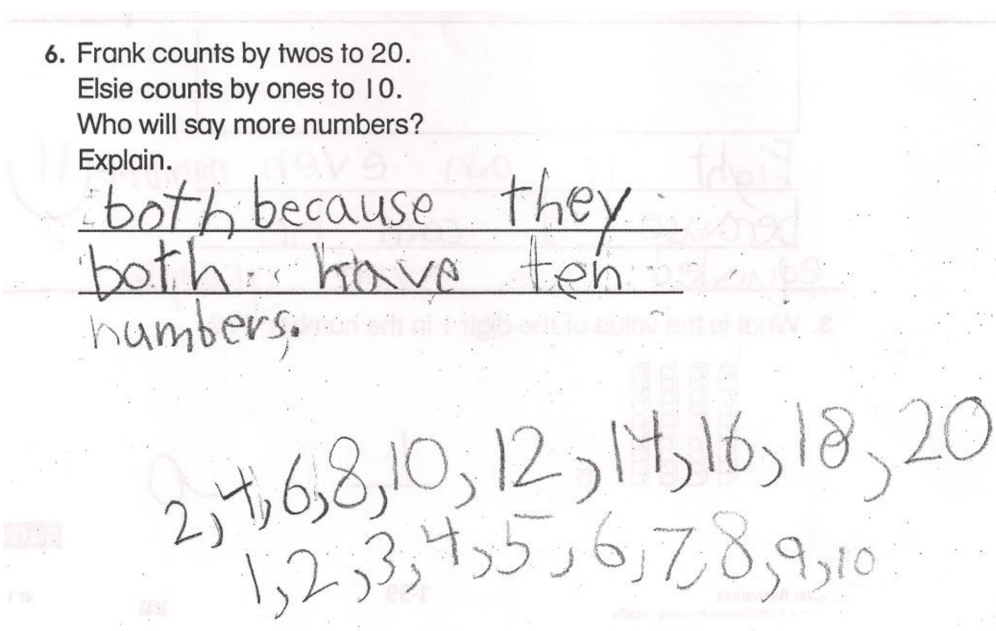
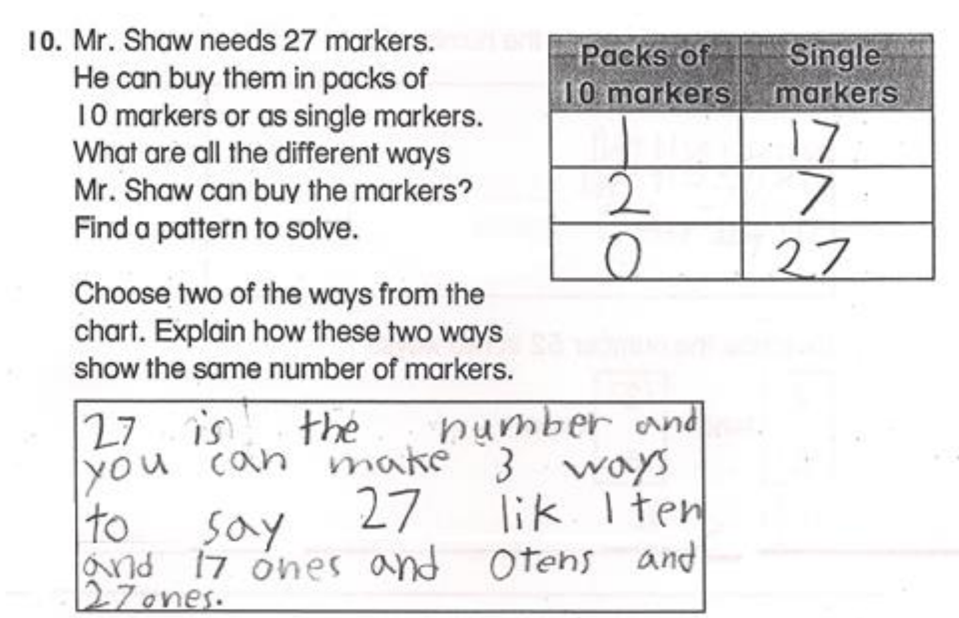


Figure 15. Exam question #6 with one of Ms. Martin's students' responses that received full credit.

Exam question #10. Ms. Martin reminded students of the task they worked on during the previous day and read the question (see Figure 16) aloud. She then emphasized the need for an explanation in order to receive credit for the question.

> Number ten looks a lot like our task from Friday. Mr. Shaw – can y'all read that with me? Mr. Shaw needs 27 markers. He can buy them in packs of ten markers or as single markers. What are all the different ways Mr. Shaw can buy the markers? Find a pattern to solve. Ok? So it gives you the box over here to be able to find the pattern, so use that box. But then look at this box right here. "Choose two of the ways from the chart." So once you fill this chart in, pick two ways. "Explain how these two ways show the same number of markers." So pick two ways and show how they are the same. Explain how those two ways are the same number.



10. Mr. Shaw needs 27 markers. He can buy them in packs of 10 markers or as single markers. What are all the different ways Mr. Shaw can buy the markers? Find a pattern to solve.

Packs of 10 markers	Single markers
1	17
2	7
0	27

Choose two of the ways from the chart. Explain how these two ways show the same number of markers.

27 is the number and you can make 3 ways to say 27 lik 1 ten and 17 ones and 0tens and 27ones.

Figure 16. Exam question #10 with one of Ms. Martin's students' responses that received full credit.

Ms. Martin's actions during and reflections on the exam. Throughout the exam Ms. Martin circulated among students, monitored the work that was being done, and helped students understand directions and read questions. Occasionally she called the group together to continue to stress the need for explanations of thinking as a justification of how the students knew their answers to be true.

> Guys, remember an explanation is telling me why. Explain means why you think the answer is what you choose, or why it is what it's asking. You've got to explain why. How do you know?

Additionally, she reminded students that the exam was being used to determine what ideas on which the class needed to continue to work and stressed that the exam was to be used in helping her assess the students' understandings and monitor their progress to this point.

> Guys, this is for me to understand, what you know, and what we still need work with. I can't give you any answers, you guys have to do this on your own. This is for me to see where we are at.

As the exam progressed, Ms. Martin continued to circulate as before, but also offered students encouragement, suggested test-taking strategies such as considering questions individually, and offered guiding questions to help align student responses to the exam questions.

After the exam, Ms. Martin reflected on why she had chosen this format and described its alignment to her goals of exposing students to different ways of thinking about mathematics. Additionally, she stressed the importance of students developing the ability to understand the questions they were being asked and applying their knowledge of mathematics.

> This assessment was chosen based on the types of questioning it uses. This aligns with my goals in the way that I am preparing the students to be exposed to many different kinds of questioning. Next year, they will be responsible for these types of questions. These questions are not just picking out an answer. Students must analyze the question and apply their knowledge. (Reflective Journal, September 21, 2015)

She also emphasized that "each question is standards-based" and that she would "determine individual mastery and non-mastery and base my next instructional strategies on the outcome" (Reflective Journal, September 21, 2015).

Ms. Martin continued to focus on these ideas when she marked the exams, where rather than providing a grade based on correct and incorrect answers she offered feedback to her students and circled problems on which they should continue to work. She also revisited material with which multiple students had struggled in both whole group and individual settings. An example of this was with problem number six above, which few students answered appropriately initially. In response she emphasized practice with skip counting during her morning skills practice on Day 5 of these observations (Classroom Observation, September 24, 2015) and worked with students in small groups throughout the week in the skip counting center (Classroom Observations, September 22-24, 2015). After approximately one week in which students were allowed to ask questions and make corrections to their exams, Ms. Martin re-marked the exams and provided a grade to her students.

This section presented the use of a formal assessment from Ms. Martin's classroom along with descriptions of the fashion in which it was implemented and Ms. Martin's descriptions of her rationales in its use. Additionally, sample items from the assessment were presented in order to provide the reader with a sense of the content of the exam and Ms. Martin's expectations for her students' responses. The next section contains a description of a secondary mathematics learning activity Ms. Martin referred to as skills practice.

Skills practice. During the morning of Day 5 of the baseline observation, Ms. Martin facilitated skills practice with her students in three forms: direct instruction and practice with skip counting, multimedia-assisted work with place value, and whole group completion of a worksheet on base-ten representations. These activities, which are briefly described in this section, comprised 11.19% of morning instruction and 7.01% of all instruction observed.

Skip counting. During this initial activity Ms. Martin displayed skip counting cards from the mathematics center activity (see Figure 13) on the whiteboard via document camera and assisted students in determining the number which was used for skip counting and completion of the skip counting sequence. Eight sequences of the form [66, X, 70, 72, X, 76, X, X], [80, X, 90, X, 100, 105, X], [X, X, 29, 39, 49, X, 69, X] were displayed as Ms. Martin took suggestions for the number used for counting, led attempts to count by the suggested number, elicited observations of patterns from students, offered strategies for determining the number a sequence was being counted by, led choral counting of the sequences, and fielded questions from students.

Place value. A short video, provided as a supplementary resource by the textbook publisher, was viewed which walked students step-by-step through a variety of questions involving place value with three-digit numbers. Ms. Martin supplemented the video by pausing it prior to each step and allowing the class, either as a whole group or with random selection, to suggest responses to the questions being posed and referring to representations and methods of counting that had been used during the previous week. For example, when a field of 150 mushrooms, grouped in bundles of 10 was shown, the video was paused to allow students the opportunity to discuss how the mushrooms could be counted before the video's narrator went on to explain: "I counted the mushrooms in groups of 10. This is one group [group of 10 mushroom shown]. There are 15 groups of mushrooms." A follow up question of "How can I write a number to show how many mushrooms there are?" was then addressed in whole group discussion, with reference to past work using base-ten and quick draw representations before the solution was allowed to play.

Base-ten representations. After viewing the video, the class completed a worksheet in which 300 units, represented as three sets of 10 base-ten rods, were circled to show groups of 100 and then counted by hundreds and units. A second set of problems replicated this task with sets of 120, 130, and 140 and asked students to circle groups of 100 and count in sets of tens, one hundreds and tens, and as units.

These activities were largely an extension of the skills rehearsed during afternoon mathematics centers implemented in a whole-group format. Many of the answers to the questions asked during this time were answered in a choral style, with students occasionally being called on to answer the questions individually. Additionally, during the place value video, Ms. Martin paused the video at one point and implemented one of its questions, “Kendra has 130 stickers. It takes 10 stickers to fill a page. How many pages can she fill?” in the form of a mathematical task. Although this is the final activity reported that directly addressed mathematics content, the mindset discussion described in the next section appeared to have an important relationship to Ms. Martin’s students’ thinking about the nature of mathematics, and is thus reported as a secondary mathematics learning activity.

Mindset discussion. At the end of morning instruction on Day 1 of the baseline observations, Ms. Martin and her students engaged in a conversation related to mindset supported by their experiences reading *The Dot* by Peter Reynolds (2003). This conversation occupied 4.33% of the morning instruction observed and 2.71% of the overall instruction. A portion of this conversation is included here because it provides a complete account of the instruction witnessed during the baseline observation and is immediately relevant to the study being reported. During this discussion, Ms. Martin and her students were seated in a large circle on the carpet in the front of her room as they shared the things they had made their “dot” for the week.

Ms. Martin: This week, our mission for the week was to make something your dot. So, we needed to all focus on something, like the little girl’s dot, Vashti's dot. In the story, she just felt like she could not draw, right? But she put a dot on a piece of paper, and her teacher saw all these different things, didn't she? So we wanted to charge you, this week, with making something your dot. So when we go to the carpet in just a minute, we're going to share those things that you made your dot for the week. What is something that you really wanted to change your mindset about, because you just felt like it just wasn't going well, something just wasn't working out for you, and you wanted to change your mindset about it. So I want to hear how you did that, and what your dot was for the week. [Ms. Martin and her students moved into a circle on the carpet at the front of the room.] So, let's hear from some of you about your dot. [Many volunteers raised their hands] Janet.

Janet: I wasn't as good at math [laughing]. I just kind of stepped it up a little bit, and now I'm doing good at it.

Ms. Martin: Ok, so can you tell me why you feel like you're getting better? What are some things that have changed in your mind to help you get better at math?

Janet: Well, I always, I thought that if I kind of step up, and make math my dot, I would kind of be, kind of a little better on track.

Ms. Martin: So maybe you just put a little more focus on it this week? Ok.

Janet: [Nodding] And now that I did that, I feel really good.

Ms. Martin: Very good Janet. Becky?

Becky: My dot was better handwriting.

Ms. Martin: Oh, better handwriting. Ok, tell me a little about that. Or tell your classmates a little bit about that.

Becky: At first, when I started out, I was very bad at handwriting. But now, I'm doing a little better at it.

Ms. Martin: Ok, so did you have to really focus on that this week since you felt like that was something that you weren't real good at, did you put a lot of focus into that? [Janet nodding] And how do you feel about your handwriting this week?

Janet: Better about it.

Ms. Martin: You feel better about it? Ok. All right, Leia, tell me about your dot for the week.

Leia: It's crazy! But, getting up in the morning and getting dressed.

Ms. Martin: Ok. So, when we talked, we could change our mindset about anything. It doesn't have to be something at school, it could be something at home, so, is that something that's a struggle for you? [Leia nodding] Ok, so Leia, tell me how you approached that. How did you, why did you make it your dot, and how did it change for you this week?

Leia: Well, my mom started getting me up in the morning to watch her . . . and I got up and I took a shower with her, I got up really early and took a shower with her, and then I would get dressed, and I would get my clothes on, and she would get all her clothes on, and she wears like five gallons.

Ms. Martin: [Ms. Martin allowed Leia to finish and redirected the conversation] So was it a struggle though this week, or was it something that was a little bit better because you were focusing on it and you had a different mindset?

Leia: Yeah.

Ms. Martin: It was better?

Leia: It was much better [indistinguishable].

Ms. Martin: But you saw a change though, because you were making it something on your mind, you saw a change in that?

Leia: I was getting up earlier in the morning, but then I was late for the shower.

Ms. Martin: So, yeah, well that happens. All right, Ally?

Ally: I got better at my morning work, when we would [take] half.

Ms. Martin: Oh yeah?

Ally: I got better at that because, how it rotates every morning, it rotates every day, like sixteen, then sixteen and a half, and all that.

Ms. Martin: So you saw like a pattern? [Ally nodding] Ok, so you were able to focus on that just a little bit and you were able to see a pattern, and now it’s easier for you? [Ally nodding] Good Ally, thank you for sharing that. Brad?

Brad: Now that I think of it, I’ve actually now noticed that I’ve made swimming my dot, and I never even noticed it.

Ms. Martin: Ok?

Brad: Because, I’ve been doing really, really, really, I don’t know how many reallys, work on breaststroke, and I just put my mind to it, and I was going a little faster day by day.

Ms. Martin: That’s great! And Brad, that’s what we want, isn’t it? We just want to just change our mindset, just a little, from, “I can’t do it” or, “I’m not good at it” to, “Hey, if I practice I can [pauses] what?”

Brad: Do it!

Ms. Martin: Get better, and I can do it, that’s right.

Brad: I worked so hard. I actually, in one week, I actually go to swimming practice twice a week, and I put my mind so much to it, that I got great at it by Thursday.

Ms. Martin: That’s very good. Olive?

Olive: Um, I made running my dot.

Ms. Martin: Ok?

Olive: At first, that couple of days at second grade, Samuel was starting to get faster as me, but when I went, and I was practicing around my neighborhood, I started getting better.

Ms. Martin: Ok, so you wanted just to improve your speed on your running? Ok, and I want you to think, all of these different examples [pointing at students around the circle], what did it take to get better?

Class: Practice!

Ms. Martin: Practice. Practice makes perfect, doesn't it?

Brad: But nobody is perfect.

Ms. Martin: No, we know that, we know that.

Leia: If we want to be perfect we'd have to practice a longgggg time.

Ms. Martin: [Several students talking at once] Listen. Listen. We know . . . we know that if we practice something, no matter how hard it seems, no matter how impossible it seems, if we practice, and we change our frame of mind, and our mindset, we can get better. Right? Just like with learning, getting up early in the morning, swimming, handwriting, whatever it is, if we just change our mindset, and we practice, we will get better.

After some other students share, Leia raised her hand with a question for Ms. Martin.

Leia: [To Ms. Martin] What's your dot?

Ms. Martin: I did have a dot this week. [Several students lean into the circle] My dot, was to get better [pauses] at housework! [Students laughing and clapping] Because, I first of all, don't like it, and I don't want to do it. But, is that going to accomplish anything?

Class: Nooooo.

Ms. Martin: What's going to happen to my house?

Class: [Students speaking at once] It's going to get messy! You'll have a pigpen.

Ms. Martin: If I don't like it, and I don't want to do it, is that a bad mindset?

Class: Yessss.

Ms. Martin: So even things like that, even adults, we have things we have to work on too, all right? There's lots of things we can make our dot, but I had to train myself this week, [listing on fingers] all right, this is what I've got to do, and this is what I've got to do, and this is what I've got to do. So before I go to bed, I had to make sure I got all my things done. Did I want to do it?

Class: Nooooo.

Ms. Martin: Did I probably have a grumpy face on?

Class: Yesssss.

Ms. Martin: But did I do it? Yes I did, and guess what? At the end of the week, how do I feel? [Multiple students responding positively] I do, I feel good, even though I was, "I don't want to do this, this is

> no fun, this is too hard, I just want to go to bed." I still did it, because I knew that if I did that, every day, that at the end of the week I would have something to show for it, and I would be proud of that. And guys, that's what I want you to continue to do, no matter what it is, your reading, your writing, your math, tying your shoes, whatever it is, whatever. If we get better, and we change the way we think about it, we're all going to be successful.

After reading the story of *The Dot* and participating in activities related to the story for one week, students were asked to select anything they found to be particularly difficult to make their personal dot over the course of the semester. Although the explicit discussion of mindset features included in this discussion of *The Dot* is directly relevant to the current study, the inclusion of this activity as a secondary mathematics learning activity came after considering the topics Ms. Martin's students selected as their personal dots for the semester. Of the 16 students participating in this semester-long activity, 10 selected mathematics as their personal dot, indicating that it was the single subject they found to be the most difficult and in which they most wanted to improve. This discussion provided a frame of reference for this semester-long activity which occurred in the background of the current study.

Summary. This section held an examination of Ms. Martin's secondary learning activities, consisting of a full description of her assessment practices and skills rehearsals, and a partial narrative of a discussion of student mindsets which took place in her classroom. These activities, which each occurred on a single occasion, comprised

21.49% of the instruction observed during the five days of baseline observations and were used to support other learning activities. Descriptions of these activities completes a full accounting of the activities and practices observed during these baseline observations.

Summary of Ms. Martin's Teaching Practices and Outcomes

As a whole, this section contained an examination of Ms. Martin's perceptions of her teaching practices and outcomes as well as evidence of these practices generated by direct observation. Evidence of these constructs was presented through a selection of quotations collected during interviews and through a reflective journal as well as from classroom observations occurring over a period of five days early in the fall semester of 2015. With regard to her practices, Ms. Martin chronicled accounts which were organized into five related themes: establishing a supportive learning community, engaging students in thinking about and discussing mathematics, facilitating productive classroom discourse, holding students accountable for their thinking and speaking, and supporting all students in their mathematics learning. Through the application of these practices Ms. Martin recounted areas of significant student outcomes including improvements in problem-solving skills, mathematical understanding, affective characteristics, and achievement. These espoused practices and outcomes appeared to be well aligned with the primary and secondary learning activities observed in Ms. Martin's classroom. The next section contains an account of Ms. Martin's areas of focus as she observed another teacher conducting a lesson with second grade students during a demonstration lesson.

Perceptions of Experiences During Project Influence

As evidenced to this point in the chapter, Ms. Martin spoke of her beliefs about the teaching and learning of mathematics and her classroom practices as having undergone substantial changes in recent years, and direct observations of her teaching practices confirmed the current state of many of these espoused changes. She attributed much of this change to her involvement with Project Influence and explicitly spoke about how the combination of her own mindset and the opportunities afforded to her by Project Influence helped promote these changes.

> I guess it was already happening before Project Influence, but when I first got involved with Project Influence, oh I totally ran with it, because I guess I was already in that mindset of, "If things are changing, I've got to change." (Selection Interview, September 9, 2015)

Her claim that "I think I've changed a lot of things because of what I've done in Project Influence" (Selection Interview, September 9, 2015) was supported by her descriptions of her current ideas and practices in a variety of areas that initiated from experiences in Project Influence. These included her beliefs about the value of student thinking and the shift to a student-centered classroom, the norms she used to support her classroom's learning community, the long-term goals she set for her students and the interactions she encouraged among them, the methods she used to hold students accountable for their thinking, and her questioning practices. In addition to ascribing these changes directly to Project Influence, Ms. Martin also praised the program and suggested that changes of this nature were attainable by other teachers who were willing to commit to the project.

> I told you that in the last interview that this is the best professional development by far and if I could just convince everyone else to be a part of it I think their teaching styles and their lives would change, but you can't, you just can't make everybody jump on board . . . Project Influence by far blows everything else out of the water. (Background Interview, September 18, 2015)

The remainder of this section consists of an account of Ms. Martin's experiences with Project Influence which made her a strong proponent of the project.

This account contains two parts. The first portion will examine Ms. Martin's perceptions of her past experiences engaging with the immersion and practice-based activities of Project Influence. This section will present the factors she identified as having the most influence during her immersion experiences and explore her perceptions of the demonstration lessons she has attended in the past. The second portion will depict Ms. Martin's areas of focus during a demonstration lesson occurring during the current study and consider her interpretations of the significance of her observations.

Ms. Martin's Past Experiences with Project Influence

Ms. Martin's reflections regarding Project Influence focused on two dominant types of professional development experiences: immersion activities and practice-based activities. Her experiences with immersion activities included contributions to problem-solving exercises and pedagogical discussions during three two-week summer institutes and six Saturday meetings between February of 2013 and August of 2015. Her involvement with practice-based activities included summer institute activities focused on assessing students' mathematical work and participation in six demonstration lessons taking place between February of 2013 and March of 2015.

Influences during summer institute activities. Ms. Martin directly cited her involvement with immersion activities as one of the most impactful experiences of Project Influence and attributed much of the change she had implemented to these opportunities to experience effective teaching and learning practices first hand.

> I've told people this, and I'll continue to tell people this. Project Influence is the best professional development I've ever had, because it's so useful and it's so purposeful. It helps me be a better teacher, because I see it in action, I'm immersed in it. So it's not somebody standing in the room telling me all of these things I need to do, I'm in the middle of those practices. Us being the student with the teacher, helps us come back to our classroom and know how we need to do that with our kids. That's just been the most meaningful thing. (Selection Interview, September 9, 2015)

This description suggested that in addition to highlighting effective teaching practices these experiences provided opportunities to return to the classroom and experiment with the methods of instruction that had been encountered and to evaluate their success with elementary students. In addition to the practices she encountered, Ms. Martin also cited the influence of the project faculty who modeled these instructional techniques.

> [The Project Influence facilitator], just her enthusiasm and the way she ran the classroom, I really thought, "Hey, this is definitely something that I can do, I'm

> already doing a lot of this." So I guess it just fed into what I was kind of already doing. (Selection Interview, September 9, 2015)

This enthusiasm and affirmation of her teaching practices, both those that were effective and in place prior to involvement with the project and those adopted from the project itself, appeared to have provided continued motivation for change for Ms. Martin.

When asked to identify specific teaching practices that had been modeled during these activities, Ms. Martin described the facilitator allowing learners to have their own ideas and guiding conversations about mathematics from those ideas rather than toward solutions.

> The way she facilitated the classroom, the way she let us have our own ideas and never shot anybody down. That's another thing that I really like, we don't talk about answers. That was another thing that I had to change, because yeah, they want to know the answer, I want to know the answer, that's just something that you've always done. I've changed that also. (Selection Interview, September 9, 2015)

In this quote, Ms. Martin once again attributed specific changes in her classroom practices to her experiences during Project Influence and alluded to a belief about mathematics teaching, allowing learners to do the thinking about mathematics, which appeared to have evolved during her experiences with the project. Additionally, she reinforced this valuing of student thinking as she described observing students interacting with mathematics content she found to be particularly difficult.

> It may just have been content because I think we did geometry that first year and that is not my strong suit, so I was thinking, "What have I got myself into. We're already taking a test." But no, I loved the videos and watching the teaching situations because that was very eye opening. Seeing all the different ways like the kids would solve a problem and maybe decide how would you address that if a kid answered a problem that way or what do you know about that child's learning because they answered that? (Background Interview, September 18, 2015)

The second half of this quotation illustrated the revelatory nature of Ms. Martin's experiences observing students engaged in the exploration of difficult mathematical ideas and suggested that this approach to assessment and instruction was both novel and valuable to her. Additionally, this emphasis on the modeling of teaching practices and focus on student thinking foreshadows Ms. Martin's experiences with demonstration lessons as described in the next section.

Perceptions of past demonstration lessons. As with her experiences engaging in summer institute activities, Ms. Martin described her focus during past demonstration lessons as centering on the teaching strategies used during the lesson and the manner in which they supported and reinforced her own classroom practices. However, she also emphasized the role that demonstration lessons served in translating the practices observed in summer institute activities to an actual classroom environment.

> When I see a demonstration lesson I think that, first of all, I think it's important that we see how the professors [teach] with the kids, because with them

> presenting to us and working with us, we're different. I think it's important that we see [the professors] in action with the kids and maybe we can pick up on some strategies that they're using with the kids. It's different when you're with adults. (Selection Interview, September 9, 2015)

Ms. Martin did not acknowledge these differences when discussing her participation in the immersion activities, but recognized in this quote that these differences in children and adults existed and stressed the importance of seeing the same teaching practices modeled with elementary students in order to validate their effectiveness. Additionally, she pointed out that the lessons in a demonstration environment would not always go perfectly according to plan and discussed the value in observing how the project's faculty handled this situation.

> The demo lessons are very helpful because we get to see the strategies we learned in the summer put into play. It helps us see the structure of a lesson and what to do when kids are present and what to do when a lesson doesn't go as expected. (Reflective Journal, October 25, 2015)

Together, these quotations asserted the importance perceived by Ms. Martin of the demonstration lessons in bridging the practices and values modeled throughout other aspects of the professional development environment with the reality of the elementary classroom.

This theme continued in Ms. Martin's reflections on her own students' engagement in a previous demonstration lesson. She described how her focus on establishing norms to support the learning environment put her students at ease and allowed them to engage productively during the demonstration lesson.

> My own personal kids, I thoroughly enjoyed that because, yes, I already had a lot of those practices in place, so they were so enthusiastic, it cracked me up because they just loved it and you could tell they were just embracing everything that she did. That also was exciting. (Selection Interview, September 9, 2015)

Additionally, she recalled how well her struggling students had performed in the lesson and considered the role that using manipulatives had played in allowing these students access to the mathematics of the lesson.

> I can think back to my class's demo lesson where we made pattern block fish. It amazed me that my struggling math students did so well. My thoughts were since they were able to use manipulatives, this made the math easier for them. (Reflective Journal, October 25, 2015)

Ms. Martin later provided evidence to reinforce the importance of the ideas of access and equity that were visible in the demonstration lessons by referencing a discussion that took place after one such lesson.

> I think the big discussion about equity was really important. I really enjoyed that . . . because I think a lot of the things that you're doing with Project Influence give your kids or it shows the equity in the classroom that all of the kids have access to the material, can solve these problems in any way they choose. (Professional Development Interview, November 3, 2015)

This recognition of the combination of the importance of the learning environment and the ability of the teacher to find ways to allow access to mathematical ideas for all students was evidenced in both Ms. Martin's descriptions of her current teaching practices and the baseline lessons which were observed.

Together, the perceptions described in this section suggested that Ms. Martin found that not only did the practices and norms emphasized throughout Project Influence translate to her own classroom, but also that these constructs prepared students to engage more completely in novel situations involving mathematical thinking regardless of the specific teacher delivering the instruction. This thinking influenced many of the long-term learning goals Ms. Martin described establishing for her students and influenced her desire to share these practices and norms with other teachers, no matter how difficult they were to reach.

> That's really hard when [other teachers] have fixed mindsets and, "I've taught for twenty years. This is the way I've always done it. This is the way I'm going to continue to do it. I'm sticking to that basal." That's something that we struggle with, is trying to get other people on board. . . . That's all you can really do. You can try to share your ideas and your strategies and invite people to come and watch. (Selection Interview, September 9, 2015)

This invitation to "come and watch" and share ideas and strategies related to what was observed is one of the core principles underlying the demonstration lessons in which Ms. Martin participated. The next section addresses exactly what Ms. Martin focused on as she observed a specific demonstration lesson and why she felt that these ideas were important to share.

Ms. Martin's Areas of Focus and Interpretations of a Demonstration Lesson

Early in the participant selection process, Ms. Martin responded to a question regarding the mathematical content focus she would most like to see included in a demonstration lesson that would take place during the semester of this study. Considering both the timing of the demonstration lesson and the content with which she felt her students were most likely to struggle, she recommended the subtraction of numbers within 1000.

> I remember last year at that time we were doing adding and subtracting like within a thousand, numbers up to 999. Not specifically adding, but the subtracting. I think at this age the re-grouping is something that's hard initially, but as much as I work, like we're working on expanded form and different forms of numbers, word form, standard form, expanded form, when they know what seven really means in 76 or when they know what nine in 959 means, I think that helps. (Selection Interview, September 9, 2015)

As this topic was well aligned with both the learning trajectory of Ms. Martin's classroom and the curriculum and pacing guides for the counties involved in Project Influence, the project staff designed and implemented a demonstration lesson for kindergarten to second grade teachers around this topic. This section surveys Ms. Martin's participation in that demonstration lesson by summarizing the lesson she observed, recording her general observations related to the lesson, examining the specific segments of the lesson she deemed most important, and considering the reasons she suggested for the importance of her observations.

Description of the observed demonstration lesson. This section contains a summary description of the demonstration lesson observed by Ms. Martin on October 28, 2015. The lesson was planned by Project Influence faculty (see Appendix L) and occurred with a second grade class of 24 students in an elementary school of approximately 650 students in a county adjacent to the one in which Ms. Martin taught. Demographically, the county, school, and classroom were similar to those of Ms. Martin. Approximately 30 kindergarten, first grade, and second grade teachers observed the demonstration lesson, which was facilitated by members of the Project Influence faculty. The expert teacher, Dr. Monroe, who delivered the instruction observed by the teachers, was a member of the project faculty with 23 years of experience in mathematics education including 13 years participating in the design and delivery of professional development for K-12 teachers. The lesson took place in the school's library, with students seated at round tables facing a whiteboard, an easel which held poster paper, and a document camera.

The general format of the lesson involved students engaging with a problem-solving task related to the subtraction of three-digit numbers with regrouping, and it featured extensive talk between students in pairs, small groups, and whole group presentations over the course of 55 minutes. As the students entered the library they were seated in groups of four with two half sheets of poster paper, a set of markers, and a bag of base-ten blocks per group. After the groups were seated, Dr. Monroe briefly introduced the teachers observing the lesson and began to teach.

Dr. Monroe initiated the lesson by introducing the students to the base-ten blocks at their tables and asking them to discuss in pairs what they knew about these blocks. After approximately one minute, she randomly selected students to respond and recorded their responses on a full sheet of poster paper at the front of the room. The students offered a variety of responses including: the naming of the blocks as flats, longs, and units; the association of these respective names with quantities of 100, 10, and 1; the fact that the blocks were used in mathematics classes; and that the blocks could be used to generate larger numbers. After this activating exercise Dr. Monroe asked the students, “How would you use these blocks to represent 127?"

Students discussed this question for about two minutes before a random student was selected to present his representation. As he described using one flat, two longs, and seven units to represent 127, Dr. Monroe placed these blocks under the document camera for the class to view. After asking if the class agreed or disagreed with this representation and receiving universal agreement, Dr. Monroe asked the students if they were ready for a real challenge. She then proceeded to ask the students, “What is another way to represent 127?”

After one minute, three students were randomly selected to provide their ideas. The first student suggested using the same blocks but placing them in a new orientation, with the longs placed on top of the flat. The second recommended using the same blocks, but verbally described adding the units, longs, and flat. The final student proposed that some blocks could be traded for an equal quantity in another form, adding that a flat could be taken away and replaced by ten longs. With these suggestions verbalized, Dr. Monroe introduced the students to the following problem by reading it aloud, asking students to consider it individually for ten seconds, and then working with their partners to produce an answer on the chart paper provided.

> On Thursday, Tara was at home representing numbers with base-ten blocks. The value of her blocks was 304. When she wasn't looking, her little brother grabbed 2 longs and a flat. What is the value of Tara's remaining blocks? Use pictures, words, and/or symbols to describe how you solved the problem.

As students explored the problem, Dr. Monroe circulated among groups, asking questions and encouraging pairs to continue thinking and recording their ideas.

After seven minutes Dr. Monroe asked the students to stop working and represented 304 using a flat and four units via the document camera. She removed a flat and asked the students to consider how to take away two longs. Students immediately volunteered that it did not make sense to do so because there were no longs available. Dr. Monroe then demonstrated an approach she had seen used among the pairs of students, by first adding two longs to the remaining blocks and then taking those two longs away. After a brief discussion of how this changed the value of the blocks present and would not be allowed, Dr. Monroe asked the students, "What's something else that we can do to those blocks so we can take away the longs?" She then allowed volunteers to respond, with the first two students suggesting to just take away another flat, leaving 104 blocks and that it could not be done. The third student then asked, in a questioning manner, “Can you regroup?” Dr. Monroe asked her to elaborate on this response and the student described trading a flat for ten longs. With this information the students were directed to return to the task with their partners as Dr. Monroe continued to circulate, question, and observe for six minutes.

At the end of this time, two groups of students were selected to present their work with the problem at the front of the room. In the first group, there was disagreement on the solution of the problem as one student suggested through drawings that one flat could be removed and a second could be partitioned into ten equal segments representing longs, so that two of those partitions could then be removed, leaving 184. However, her partner believed that there were no longs to be taken away from the physical flats and that the answer was 204. The second group of presenting students modeled a solution using the base-ten blocks in which one flat was removed and a second was traded for ten longs allowing two longs to be removed and matching the solution of 184 presented by the first student. However, as another student attempted to re-voice this solution, it became evident that there was still confusion with this idea among many of the students.

Dr. Monroe then asked the students to consider if three flats and four units was the only way that 304 could be represented and allowed the groups two minutes to consider alternative representations. A third group of students was then asked to come to the front of the room to model an alternate representation with the base-ten blocks. They initially placed three flats and four units under the document camera and then removed a flat and replaced it with ten longs. After a brief discussion of how the trading of blocks helped with the problem, Dr. Monroe presented an exit ticket for students to complete individually by answering the question "How would you represent 407 so that three longs could be taken away? Write a sentence explaining how you know you are correct."

Students were allowed six minutes to complete these exit tickets and Dr. Monroe posed a final question, "What is it that you think you learned by solving this problem?" After 45 seconds, index cards were used to select random students to answer the question. Of the four students asked to respond, two were unable to provide a response, one suggested that when you needed to take away blocks that were not present that you could trade out blocks to make it possible, and the final described how numbers could be represented in a variety of ways. Then, Dr. Monroe thanked the students, asked the observing teachers to give them a round of applause, and ended the lesson.

This section presented the demonstration lesson observed by Ms. Martin in narrative form. Throughout this lesson, Ms. Martin observed from a vantage point near the front of the classroom with a clear view of four pairs of students seated at two nearby tables. As she observed she used a small camera attached to a pair of glasses to record 17 30-second increments of video of the lesson which she deemed to be its most critical points. She later participated in a debriefing interview related to the demonstration lesson in which she described her general impressions of the demonstration lesson and offered explanations as to the importance of the 17 short video segments she filmed. The next section reports Ms. Martin's general impressions of the lesson and leads into a discussion of the specific observations she made regarding the video segments she filmed.

Ms. Martin's general impressions of the demonstration lesson. During the demonstration lesson debriefing interview, Ms. Martin immediately saw the relevance of the lesson's mathematical content to her classroom and believed that the task in which the students had engaged would be useful in her learning trajectory. She also started to consider how her students would apply the ideas and strategies they had been working on to this point in the semester to this task, focusing on the representations students used to model quantities.

> Well, definitely it was significant because that's what we are covering in here and I saw that fit perfectly into the things that we're working on. Immediately, as soon as the warm up, the kids were representing whatever number it was, and then she asked could you represent that in a different way, but I was like, "Oh gosh we're spending so much time doing that, how great, I hope that when they see this task, they will apply those strategies as well." So I really enjoyed seeing that math task because I knew that was something that I can use immediately with my kids. (Professional Development Interview, November 3, 2015)

Although she spoke of using the task as it was implemented immediately, Ms. Martin actually implemented the task in her classroom nearly three weeks later, in line with her planned curriculum, after completing a unit on models and representations for the addition and subtraction of two-digit numbers.

In addition to her observations regarding the relevance of the content and usefulness of the task, Ms. Martin noted the instructional practices of Dr. Monroe, particularly when it came to her questioning practices.

> I think she's still doing a lot of what we've put in place, the going around to each table and having them explain what they're doing and her advancing questions,

> she would try to leave them with something and then walk off and leave. That's something that I'm still trying to do in here that I feel like I'm getting better at, but I want to continue to improve that, is asking them a question and getting their thinking maybe to change direction and then of course leaving them. (Professional Development Interview, November 3, 2015)

These two areas of focus, the student representations of quantity described in the previous paragraph and Dr. Monroe’s questioning practices, were noted as highly significant due to Ms. Martin’s descriptions of them and the fact that they comprised the majority of the focus video she recorded.

In addition to these two areas of emphasis, Ms. Martin stressed the significance of the lesson’s focus on student understanding, particularly due to the ad hoc changes Dr. Monroe made to the lesson as a result of some of the students’ confusion with the idea of regrouping. She compared the enacted lesson to the planned lesson (see Appendix L) and noted the students’ struggles with using alternate representations of numbers.

> I was trying to compare what Dr. Monroe was actually doing to the actual written plan and I know at one point, because there were some points where the kids just weren't moving forward I think like she wanted, so she really had to change the way the lesson was going. I think there, I was sitting in one part, so I couldn't see everybody, but the majority of the kids I think were kind of at a standstill with representing those numbers in different ways. (Professional Development Interview, November 3, 2015)

Ms. Martin perceived these changes to the lesson, which included the discussion of how the value of a base-ten representation changes when additional blocks are added, the alternative representation of 304 with base-ten blocks, and the reflective question on what was learned from engaging with the problem, as providing authenticity to the lesson and making it more relatable and student focused. “I like the way she modified the lesson when things weren't working as planned. I feel like I already do this. My kids lead my instruction, not my lesson plans” (Reflective Journal, November 11, 2015).

Additionally, Ms. Martin described how Dr. Monroe elicited the students’ thinking and allowed it to guide the course of the lesson.

> So I think she had to maybe kind of stop what she was doing and pull a kid or two up there to explain their thinking, which is what I really try to do if we're at a place where not a whole lot's going on. I really try to do that, so I was glad to see that, just because you think a lesson's going to go one way, we all know that it may not, and it's okay to let your kids lead the instruction because they're the ones that need whatever at that moment, so I don't feel like just because it's on that lesson plan, you have to follow it to a T. So I was glad to see even her, who we all look up to because that's Dr. Monroe, we saw that she maybe changed it up a little bit. (Professional Development Interview, November 3, 2015)

In this quotation, Ms. Martin described how Dr. Monroe’s modeling provided reinforcement for practices in which she already engaged. Additionally, this quote evidenced how the demonstration lesson supported her emerging ideas about understanding and evaluating student thinking and adapting instruction to fit her students and her long-term learning goals.

Finally, Ms. Martin noted two reasons she believed students found these particular mathematical ideas to be so difficult. First, she described students' difficulties with regrouping and translating their concrete representations into pictures and symbols.

> Well, because you've got a zero in the tens place. You've got a three-digit number, you've got a zero right in the middle, and they're subtracting. They don't really know they're subtracting, but you had this one kid actually write the subtraction problem and we didn't even discuss this in the demo lesson or after the debriefing, that she was ready to move from that concrete example to that pictorial representation. She was almost ready to do the abstract algorithm, she had that written out. But I think that's hard for them because you can't take something from zero, so they see that zero and they're almost stuck. (Professional Development Interview, November 3, 2015)

After noting this difficulty, she suggested that the key point to this transition is in the use of an appropriate representation for the given mathematical context and hoped that the time she had invested working on these ideas with her students would be beneficial.

> I hope when I do this with mine, they'll remember, "Oh well we can show this in another way, it doesn't always have to be three hundreds, zero tens, and four ones." That's what you hope anyway, that they will remember, "Oh I can show this with two hundreds, ten tens, and four ones, if you regroup it that way," but I think that's just hard. (Professional Development Interview, November 3, 2015)

However, this struggle to link mathematical contexts to real-world situations led directly to her second observation of the reason students struggle with these ideas: an artificial separation of the models used to represent numbers from the situations in which the numbers arise.

> So I think that was the challenge. I think they do really well with the representations of numbers and using those different place-value blocks to represent those numbers, but I think sometimes when you put it in context, I think maybe, do they really think back and remember, "Oh I remember I can show that number in so many different ways." Do they really think, "I can apply that here?" I think they've got to make a connection that just, you know, "I'm not just using this with random numbers, I can apply those strategies also into these word problems." (Professional Development Interview, November 3, 2015)

To a large extent, the difficulties Ms. Martin described students encountering with this task were the very ones that dictated her goals of having students communicate their thinking with one another in order to allow them to encounter and understand a variety of representations and strategies that could then apply in a meaningful way.

This section included evidence of Ms. Martin's general observations of the lesson she observed, including the importance of students' representations of numbers to support their thinking about operations, the practices Dr. Monroe utilized to elicit student thinking and modify the lesson to support their understanding, and the difficulties students encounter with these ideas. The next section will present data detailing the specific areas of the lesson on which Ms. Martin focused through a description of the video segments she recorded and her perceptions of their significance.

Ms. Martin's areas of focus during the demonstration lesson. Three days before the demonstration lesson, Ms. Martin reflected on the areas she would be observing most closely during the lesson. Based on her participation in six demonstration lessons during the first two years of Project Influence, she described a strong focus on the instructional practices of Dr. Monroe, particularly as they related to the logistical and epistemological facilitation of the lesson's task. Specifically she stated, "I will be looking for how Dr. Monroe sets up the lesson, what tools does she use to help them with the problem, and how she moves the task along [through] her questioning" (Reflective Journal, October 25, 2015). These areas of focus were prominent in the video segments Ms. Martin recorded, accounting for approximately one-half of all of the video recorded. The remaining portions of video were heavily focused on the actions and words of students as they engaged with the lesson.

Immediately prior to the demonstration lesson Ms. Martin was fitted with the small video camera that she had previously practiced using in her classroom and asked to record any moments during the demonstration that she deemed important. She verbalized understanding the functionality of the camera and these directions and verified that there was no limit to the amount of footage that she could record. Over the course of the lesson she recorded 17 video segments lasting approximately 30 seconds each and accounting for 15.45% of the total lesson. Approximately one week after the demonstration lesson, Ms. Martin reviewed these video segments with the researcher and was asked to comment on any of the segments she found significant. Ms. Martin elected to provide comments on 11 of the 17 video segments she had previously recorded, with the six segments she did not comment on deemed trivial or made in error. The remainder of this section provides a description of each video segment Ms. Martin recorded and discusses the comments she made regarding their significance. The segments are presented in chronological order.

Video segment 1, 2:00 into lesson. A student named the base-ten blocks displayed under the document camera as a flat, a long, and a unit as Dr. Monroe recorded these images and names on chart paper in the front of the room. Ms. Martin described the segment as important as it activated students' prior knowledge and prepared them to engage in the task.

> I felt like that was important because, well, first of all, she's setting up for the task. But she's also pulling out their prior knowledge. And I liked that they were very familiar with those Base-Ten Blocks and the vocabulary that was tied to that. So I felt like that was important and also going to help them solve this task. (Point of View Interview, November 4, 2015)

Video segment 2, 5:50 into lesson. The pair of students nearest Ms. Martin represented 127 with one flat, two longs, and seven units. No comments were provided for this segment.

Video segment 3, 7:45 into lesson. After a group displayed 127 as one flat, two longs, and seven units, Dr. Monroe asked, "Are y'all ready for a challenge? What is another way to use those blocks to represent 127?" This question was then discussed between partners. Ms. Martin viewed this as an extension of the setup for the task and recognized that the use of an alternate representation would be essential to the main task.

> Okay, I remember this was at the beginning where she had them represent a number and then she asked to do it in another way. So again, that sets up for the actual task because she's getting them all ready to think about, “Well this is one way to represent this number with Base-Ten Blocks. Is there another way?” Which is what you want them to do in the task without having to say that. . . . They did, and then she's got them going, or to show it in another way as well, “Can you represent this number in another way?” (Point of View Interview, November 4, 2015)

Video segment 4, 9:05 into lesson. The boy in the pair nearest Ms. Martin offered a physically different representation of 127 to the whole group, “You can get two of the tens and set them on top of the one hundred.” Dr. Monroe re-voiced this as, “Okay, so you’ve used the same blocks, but moved this one over here [modeling on document camera].” Ms. Martin noted this as evidence that this student initially did not realize that there were alternate representations of 127 using different combinations of base-ten blocks and acknowledged his understanding of the word different as a misconception.

> That really struck me because that child, he thinks different, or here's what I thought he meant, he thinks different is just, “How can you arrange those same blocks differently?” So he just puts them on top of them, so his idea of representing numbers differently was not there. He was thinking of, "Oh, I can

> arrange these blocks in a different way." Not necessarily making that number in a different way. So that I think is a misconception for him as far as how to show that number in a different way. (Point of View Interview, November 4, 2015)

However, she also noted that when a student who presented soon after him offered the idea of trading a flat for ten longs that he may have acknowledged this difference in meaning.

> [Speaking from the students voice] “I see these other ways now.” We've got these boys and girls over here showing, "Oh, well, you have still got the same number shown, but it's in a different way using different blocks." So, I think once you put those two tens and those ten, ten rods up there . . . they see it. There's still 300, but it's in a different way. He may have not realized that that's what it meant by showing it differently. (Point of View Interview, November 4, 2015)

When asked if she believed this group would continue to use these alternate representations the next day, Ms. Martin responded affirmatively and suggested that this was one of the potential difficult points for the lesson she had considered beforehand.

> I think they would probably. I would hope, or I don't know if I would hope, but I would think that they would probably go ahead and put some of those in tens because they saw that you could do that. And maybe they were stuck on that idea, which is one of the things I really thought before we started this lesson.

Video segment 5, 10:10 into lesson. Dr. Monroe asked for other ways to represent 127 and a student suggested, “You can like, take the hundred away and add ten more tens, so that you like, ten tens equal 100, so then you can just take the two tens and seven ones." Ms. Martin, having referred to this idea with the previous segment, offered no additional comments.

Video segment 6, 14:43 into lesson. The pair of students nearest Ms. Martin represented 304 with three flats and four units. The students then removed one flat, added two longs to the representation, and removed those two longs, leaving 204 blocks. They then put their base-ten blocks away and began to transfer this idea to poster paper. Ms. Martin offered no comments on this video segment, but referred to the actions in her description of video segment 10.

Video segment 7, 15:19 into lesson. The pair of students nearest Ms. Martin continued to record their idea while Dr. Monroe prompted a nearby group of students to begin recording their work with the base-ten blocks as numbers or pictures on their poster paper. Ms. Martin provided no additional commentary for this video segment.

Video segment 8, 17:40 into lesson. Dr. Monroe brought the students together and noted an observation she had made of a pair of students' work. The students' suggestion was to take away one flat and two units (rather than two longs) from a representation of 304 using three flats and four units, leaving 202 blocks. She then asked the class "Does that make sense? When we take away two from that four [indicating the units] are we taking away two longs from that four, or two units from that four?" Students were directed to talk to their partners about this idea. Ms. Martin viewed this as a question to help clarify a misconception and promote students' understanding of the problem.

> Okay, there again, I liked the way she's clarifying some misunderstandings there. I think she brought in, if I can remember right, I think she showed an example of what someone had done, and so she was getting them to look at that and asked, "Does that make sense?" So she was clarifying maybe some misunderstandings to hopefully clear that up so that they could move on. And that focus on place value, too, "Where are we actually subtracting from? Will it make sense if we take something away from this? Are we in the same place value?" So I think she was really focused on that whole idea of place value. (Point of View Interview, November 4, 2015)

Video segment 9, 20:07 into lesson. As a pair of students at the table to Ms. Martin's left suggested that you cannot take away tens from a flat, Dr. Monroe asked the small group, "Why do you have so many tens there, I mean, so many longs there? Why does she have so many tens and y'all are saying there are no tens?" Ms. Martin suggested that this line of questioning was intended to have both sets of students justify their reasoning and to help rectify the two different representations being used.

> I guess I was focused on her conversation there, getting them to justify their reasoning. And I think two different things was going on at that table, so she was getting them to clarify what was going on [with] her questioning. . . . I think that's why I was really looking at why you have so many longs there. Because I think that's where she had regrouped that 100 into those longs. So, I think that is what she really wanted, that child to clarify her thinking, that's what you wanted the kids to do was just, or to show that number in a different representation. . . . Yes,

> "Why do you have so many longs? We had none on the original amount so why do you have so many?" I think is what she was doing there. (Point of View Interview, November 4, 2015)

Video segment 10, 21:30 into lesson. In a whole-group discussion a student from the table at Ms. Martin's left stated, "It doesn't make sense to take two longs away when there isn't any." As students began to talk to their groups, Dr. Monroe brought them together and relaunched the problem, "So, the problem we've got to figure out, this is where we have to scratch our heads to figure this out, is, looking at this picture [three flats and four units on the document camera], I've taken the flat away [removed a flat]. I can't just stick two longs up here [places two longs under the camera], that's what some of you are doing. . . . When I stick two longs up there like this, what's the value of this?" After many students in the class responded with 324, Dr. Monroe continued, "What's something else that we can do to those blocks so that we can take away some longs?"

Ms. Martin alluded to the fact that the group of students directly in front of her had attempted this approach and that this question was intended to help resolve this misunderstanding.

> Yeah, and I kind of even chuckled in that video, because I know why I chose that clip. Because she was again trying to clear up the misconception. Because you had some kids [including those she was directly observing], "Oh, well, since I need to take two away, I'll just add two to the original number and then take them away." So she was just clearing that misconception up. You can only start with a

> certain number, you can't add to it and then subtract. You're only allowed to start with that certain number, yeah. (Point of View Interview, November 4, 2015)

This segment also induced Ms. Martin to consider what the root issue for this misconception might be and to consider if any of her students might have a similar response.

> Right, “I'm going to make my blocks fit this situation.” Or, “I'm just going to add blocks to make this situation fit, I'm not really thinking about the numbers.” So it makes you wonder is there, if there's number sense missing there because they're not making that connection between their numbers and the base-ten blocks? . . . And so I was actually sitting there, I can remember sitting there thinking, "Would I have kids that actually do that?" And right off the top of my head, I can't imagine any kids, at this point where we are and [after] a lot of the discussions that we had, actually doing that. They might surprise me and do that. (Point of View Interview, November 4, 2015)

Video segment 11, 23:25 into lesson. Dr. Monroe asked the whole group how two longs can be taken from one flat. The young man at the table to Ms. Martin’s left continued to insist, “You can’t since there’s no tens for you to take away.” However, his partner then suggested to him privately, “You can take away the tens from the one hundred.” Across the room another student quietly made the same suggestion to the whole group, “Can you regroup?” Dr. Monroe prompted her to speak more loudly, “Say it out loud?” to which the student more confidently said, “Can you regroup?” Dr. Monroe repeated the student’s question and probed for more information, “Can you regroup? What is that?" The student responded, "Like, you trade the tens out for one hundred?" Dr. Monroe then turned the idea back to the whole group, "There's an idea, did y'all hear that?"

Ms. Martin noted this exchange as being extremely important, both due to the fact that the first student to speak had not yet grasped the idea of regrouping and that the suggestion for the idea came directly from another student via Dr. Monroe's facilitation.

> Yeah, so that really stuck out to me that, first of all he's saying, "Oh, there's no tens, there's a zero." So again, I don't think he's making that connection [that] he can go to the hundreds and have some tens. Just because there's a zero there, in his mind there are no tens at all. So he's not making that connection, "I can go over here and I can regroup those," and that's why I love that child spoke up and it really surprised me that she said the words, 'regroup' already. And I've got some that know that but that really surprised me. And then she's saying, "Well, can't we take a 100 and break that into tens?" So, I was excited to see that. (Point of View Interview, November 4, 2015)

Video segment 12, 29:25 into lesson. Dr. Monroe, circulating around the room as small groups discussed the ideas from the previous segment, observed the group of students to Ms. Martin's right. She then interjected, "That's where the 204 came from [taking one flat from 304], by removing a flat. I've got to take more away, so is my answer going to be more or less than 204?" After seven seconds of wait time, one of the students replied, "less than." Nodding, Dr. Monroe continued, "It's going to be less than

204, so y'all have to figure out what it is, and now you know it's less than 204." Dr. Monroe then redirected the students to talk to one another as she left the group.

Ms. Martin noted the significance of this interaction as it provided the students with some support to continue considering the question that had been asked without taking over the students' thinking.

> Because that group was having a hard time, she didn't just leave them hanging. She's asking them questions so she can kind of, hopefully move them along, because I think they were at a standstill. So, I think she got them thinking about the idea of subtraction, "Are you going to have more? Are you going to have less?" And hopefully that could move them in the direction of what base-ten blocks they would have left. So I noticed that her questioning was trying to get them to move along. (Point of View Interview, November 4, 2015)

Video segment 13, 30:27 into lesson. As some students began to become restless with their conversations, Dr. Monroe called them together to discuss their solutions to the main task as a large group, stating, "Ok, let me ask y'all to stop for just a second. I think that we are ready now to talk a little about some of our solutions that we found, or some of our answers that we found." Ms. Martin offered no additional comments for this segment.

Video segment 14, 33:08 into lesson. After a student quickly offered her approach to the problem to the whole class, Dr. Monroe asked, "Who feels comfortable repeating what Tina says to do?" After five seconds with no volunteers Dr. Monroe turned back to Tina, "Okay, Tina, you've got to say it again. Say it out loud so that everyone can hear." Tina then repeated her strategy, "I think that since there's no tens over here, that you should take away two tens away from the hundreds [indicating a drawing of a flat partitioned into ten longs]." This segment stood out to Ms. Martin due to Tina's degree of understanding and the combination of concrete, pictorial, and symbolic representations she used to explain her thinking.

> That child really stood out to me because we actually had a conversation afterwards about where was she in the math spectrum? Was she working concretely? Was she working pictorially? You want to move the kids to an abstract thinking, so she had, she was able to put her base-ten blocks on paper, which is what you want. You want them to get away from actually having to manipulate it and put those blocks pictorially. So she was able to do that, she marked those two tens out. She said, "I have no tens over here. I can go to this hundred. I can take two away." So she was able to do that pictorially and she even had a number sentence to match that picture. So she really stood out to me because I think she is further along than a lot of those kids are. (Point of View Interview, November 4, 2015)

Additionally, she noted that this advanced understanding may have been the reason that none of the other students were prepared to re-voice her approach.

> Yes, and I think no one can repeat what she's trying to say. Because I think her thinking, again, these kids were so involved with these base-ten blocks. She's got a picture and she's taking two out of that one, that flat, and I think a lot of them were like, "What?" They're still stuck on using those manipulatives and being

> able to exchange that flat for those tens. They were still hung up on the exchange in place values. (Point of View Interview, November 4, 2015)

Video segment 15, 43:30 into lesson. The group which had been seated nearest Ms. Martin presented their work on the problem using base-ten blocks under the document camera. They placed an initial representation of 304 using three flats and four units under the camera, replaced one of the flats with ten longs, and removed one flat and two longs to leave a total of 184 blocks. Dr. Monroe then re-voiced their approach, “Okay, do y’all see what they did? They traded one of those one hundreds, they took it and they traded it and they put how many longs up here?” When the class replied with 10, Dr. Monroe asked, “Now is it possible for the little brother to grab two longs?”

This segment was significant to Ms. Martin as it provided evidence of change in student thinking that occurred throughout the lesson. This group of students was Ms. Martin’s primary area of focus outside of Dr. Monroe and she described a substantial change in thinking that she had observed occurring due to Dr. Monroe’s instruction.

> Okay, that was the group, that was the little boy that was stacking his blocks or, yeah, his base-ten blocks on top of each other. So now here they've gone, and I'm sure the little girl, they've got some interaction going on together, so it might not have been just him but he's in that little group. They've gone from that, to being able to make that regrouping happen. They've taken that flat and they've traded it in for those ten, ten rods. (Point of View Interview, November 4, 2015)

Ms. Martin also pointed out that Dr. Monroe's question about the little brother's ability to now take away two longs helped students to consider the problem in context and address what she saw as one of the primary difficulties of the lesson.

> I liked the way [Dr. Monroe] asked . . . "So now is it possible for the brother to have taken two longs?" Because they were also hung up on that, too. The contextual part I don't think they were really paying attention to. Because can you just take away two tens if there are not any there? So that's why they were still, a lot of those kids were still stuck on, "I have to have three flats." And you can't just take, you can't pull two away from that flat, so how am I going to have him to take two away? (Point of View Interview, November 4, 2015)

Finally, when asked if she believed this understanding would be resilient for this student she again alluded to his progression in thinking and suggested the change would be substantial. "Oh, absolutely. Because it's hopefully going to move him from this way of thinking to seeing other ways of representing those numbers with those blocks. (Point of View Interview, November 4, 2015)

Video segment 16, 51:39 into lesson. Students completed exit tickets individually as Dr. Monroe circulated among the groups observing their responses. Ms. Martin offered no additional comments for this segment.

Video segment 17, 54:49 into lesson. A student named Abby responded to Dr. Monroe's final question regarding what she and her partner had learned from today's problem. "Well, we were saying that, if you have to take away some tens and there are none, that you have to switch them around with another one." Dr. Monroe re-voiced this comment to the whole group, “So Abby is saying that one thing that she learned, that her and her partner were talking about, is that if you were trying to take away some longs, and they’re not there, you have to switch them around so that you can take them away. Thank you, Abby.”

Ms. Martin described this segment as significant due to the student’s ability to verbalize one of the main ideas related to the lesson’s goal and as further evidence of student understanding arising from the day’s instruction.

> Yeah, again at the end of the lesson, she's going back to say, "What is something that you've learned today?" So she's pulling those concepts out that they took away, and that was good that that child was able to say that, because that's what you wanted. You wanted the different representations of the number, and that child knew, well sometimes if there are zero tens, she said, "You've got to change things up a little bit." So she made that connection, I have no tens, I've got to do something different with my hundreds. So that was good. (Point of View Interview, November 4, 2015)

Although this example showed a single student’s understanding, Ms. Martin’s description appeared to be more focused on Dr. Monroe’s final question and the way in which she interacted about the question with this student to close the lesson.

These video segments and their supporting comments provided substantial indication of the factors of the demonstration lesson Ms. Martin deemed to be the most significant. Although Ms. Martin identified a strong instructional focus prior to the lesson, her actual observations were equally divided between Dr. Monroe’s instructional practices and the thinking, actions, and interactions of the students involved in the lesson. A summary of the description of the video clips recorded by Ms. Martin and her area of focus for each, derived from both the videos and her descriptions of their significance, is presented in Table 8. Approximately 60% of the segments, including numbers 1, 3, 7, 8, 9, 10, 12, 13, 16, and 17 were well aligned with the instructional practices Ms. Martin identified as her areas of focus before the demonstration took place. However, almost 60% of the segments, including numbers 1, 2, 4, 5, 6, 7, 11, 14, 15, and 16, were focused on the words, actions, representations, and thinking of students. These approximations do not add up to 100% as three of the video clips (numbers 1, 7, and 16) represented a focus that was shared between the students' thinking and Dr. Monroe's practices.

Table 8

Summary of Ms. Martin's Recorded Video Segments and Their Areas of Focus

Video Number	Time	Description	Area of Focus
1	2:00	Student named base-ten blocks as flat, long, and unit to whole group; Dr. Monroe recorded.	Combined
2	5:50	Students in small group represented 127 as one flat, two longs, and seven units.	Student Oriented
3	7:45	Dr. Monroe asked whole group to represent 127 in a different way.	Instructional Practice
4	9:05	Student offered an alternative representation of 127 to the whole group by stacking base-ten blocks.	Student Oriented
5	10:10	Student offered an alternative representation of 127 to the whole group by exchanging the flat for 10 longs.	Student Oriented
6	14:43	Students in small group added two longs to 304 and then removed these two longs and a flat.	Student Oriented
7	15:19	In small groups, one group of students recorded ideas; Dr. Monroe asked another group to begin recording.	Combined
8	17:40	Dr. Monroe asked the whole group if it made sense to take away two units rather than two longs.	Instructional Practice
9	20:07	Dr. Monroe asked a small group of students to explain the differences in their representations.	Instructional Practice
10	21:30	Student suggested to the whole group that there are no longs to take away; Dr. Monroe relaunched the task.	Instructional Practice
11	23:25	A student suggested regrouping a flat to ten longs to the whole group.	Student Oriented
12	29:25	Dr. Monroe asked a small group if the final answer would be greater than or less than 204.	Instructional Practice
13	30:27	Dr. Monroe called the whole group together to examine student work.	Instructional Practice

(continued)

Table 8 continued

14	33:08	Dr. Monroe asked a student to repeat her hurried explanation to the whole group.	Student Oriented
15	43:30	Student pair offered a solution of 184 based on regrouping one flat to ten longs; Dr. Monroe re-voiced.	Student Oriented
16	51:39	Students complete exit tickets individually; Dr. Monroe circulated and examined responses.	Combined
17	54:49	Dr. Monroe re-voiced a student response regarding what she had learned to the whole group.	Instructional Practice

This section has presented Ms. Martin's specific areas of focus during the demonstration lesson she observed on October 28, 2015, as evidenced by the video segments she recorded as the most significant events during the lesson. In addition, it included Ms. Martin's explanations of the significance of each event as well as a summary of her areas of focus during the lesson. The next section presents a general summary of the importance of the demonstration lesson as described by Ms. Martin.

Ms. Martin's perception of the importance of the demonstration lesson. Ms. Martin placed a great deal of emphasis on the role that communication about mathematics had played in the demonstration lesson, both in her descriptions of Dr. Monroe's facilitation of the lesson and in her comments regarding the students' ability to communicate their ideas with one another. She described this as a shift from the current milieu of elementary mathematics education and stressed the emphasis of Project Influence on creating problem solvers rather than problem performers.

> I think just our whole world is centered on communication. So, they have to be able to communicate their thoughts, their way of thinking. It's just so much effort

> or emphasis has been put on a test and little test takers, and I think we've got to get away from that. They've got to be able to communicate, they've got to be able to share their ideas, share their thoughts, and not just take a test. Like we've always said in Project Influence, problem solvers and not problem performers. That's a huge deal. (Professional Development Interview, November 3, 2015)

This focus on the development of communication skills, both in the project at large and in the demonstration lesson in particular, provided an opportunity for Ms. Martin to continue her reflection on the importance of shifting the responsibility for mathematical thinking to her students. The significance of these thought processes, even in the absence of a student’s ability to write down or speak comprehensively about their ideas, was obvious in her reflections.

> It's still hard for them to get their thinking out. It's like they know it but they have a hard time getting it out. They can put it on paper, but being able to speak it is hard for a lot of these at this age. I can see they're thinking, I know exactly what they've done, but, even in mid-explanation, someone will stop because I mean they're still seven and eight and they're still developing those communication skills. So that's why I think it's so important in all we do to stop doing all the talking and for them to do all the talking. (Professional Development Interview, November 3, 2015)

As evidenced by her observations and comments, the demonstration lesson provided ample opportunities for Ms. Martin to view high expectations for students’ thinking and communicating about mathematics, instructional practices that shifted these responsibilities to students, and salient outcomes that depended on these expectations and practices.

Ms. Martin also extended these ideas regarding the importance of communication to describe why this type of lesson was so important to the early elementary grades. She stressed that ideas from teachers could become embedded in students without their full understanding and that practice in communicating about mathematics now was necessary for later grades.

> Well, because I think kids need to be able to share their thinking and other kids need to hear that thinking. If they're always listening to just my ideas and my thoughts and my examples, they get that stuck in their head, so it's so important for them to always share their thinking and especially at this age. I came from fourth grade and I'm coming down to second, so I saw the struggle it was to get some things out of fourth graders, so I think if I do those things now with these, that's going to be so easy for them down the road to explain their thinking. (Professional Development Interview, November 3, 2015)

Many of the practices Ms. Martin focused on in this lesson and described taking back to her classroom promoted this student-to-student discussion and encouraged students to be prepared for greater expectations and responsibilities in their future mathematics classrooms. She also attributed future engagement with mathematics to a student's ability to converse about their mathematical ideas and suggested that many students withdraw due to a lack of exposure to thinking and talking about mathematics.

> I think because I guess, again, I see where they're going and what their expectations are going to be down the road, and I think those things right now are very important because they have to be trained to do those things. I think it's almost kind of too late if you wait in fourth and fifth grade to get them to start discussing and sharing their thoughts. I think they're at a point then where a lot of them will shut down because if they haven't already been exposed to a lot of that. (Professional Development Interview, November 3, 2015)

These ideas of providing a strong foundation for students to build from throughout their lives were well aligned with Ms. Martin’s espoused beliefs and practices, observations of her classroom, and her focus areas during the demonstration lesson she observed.

Summary of Ms. Martin’s Professional Development Experiences and Foci

This section examined Ms. Martin’s perceptions of her experiences throughout Project Influence, including a discussion of her involvement in immersion and practice-based professional development activities. It contained an account of her experiences engaging in mathematical problem solving throughout the project and described the role that demonstrations lessons served in bridging these experiences to her classroom practice. Additionally, it described a demonstration lesson observed by Ms. Martin and examined the specific areas on which she focused during the lesson and her perceptions of their importance and the general importance of lessons of this nature. The next section will present an account of Ms. Martin’s adaptation and implementation of this demonstration lesson in her own classroom.

The Participant's Enacted Lesson and Reflections

On November 13, 2015, two and one-half weeks after observing the demonstration lesson, Ms. Martin enacted her version of the lesson in her classroom. Although many of the surface features of her enacted lesson were identical to the lesson she observed, her descriptions of the rationale for her timing and implementation of the lesson and the instructional decisions she made during the lesson indicated a substantial degree of thought went into preparing for its use. Additionally, Ms. Martin's students' growth in areas such as their ability to communicate with one another about mathematics and to justify their reasoning in small groups, large groups, and one-to-one discussions was evidenced during the lesson and highlighted by Ms. Martin's instructional practices.

This section reports the results of Ms. Martin's enactment of the demonstration lesson in three parts. The first portion presents Ms. Martin's initial ideas regarding the manner in which she planned to utilize the demonstration lesson and her descriptions of her planning for this enactment. The second part contains a narrative description of Ms. Martin's implementation of the lesson interspersed with comments she made as she viewed video of her version of the lesson. The final segment comprises Ms. Martin's reflections on the enacted lesson and the unit in which it was contained.

Ms. Martin's Planning for the Enacted Demonstration Lesson

As Ms. Martin prepared to implement the demonstration lesson in her classroom, three areas of focus emerged in her interviews and reflective journal responses. The first area of focus was the sequencing of the lesson and a consideration of how it fit into her overall learning trajectory for the semester. The second focal area involved the learning goals Ms. Martin established for her implementation of the demonstration lesson. The final area of focus was how well prepared for the lesson she thought her students would be and the ideas with which she most expected them to struggle. This section presents evidence of Ms. Martin's focus on each of these areas.

Ms. Martin's sequencing for the enacted lesson. Ms. Martin's initial thoughts on using the demonstration lesson in her classroom focused on the manner in which the lesson would build upon the place-value understanding she had emphasized in her classroom to this point in the semester and the ease with which she could adapt the lesson for her own use. She believed the lesson would be useful to her classroom soon after she observed the lesson, stating, "We are currently using our place value strategies to add and subtract numbers to 100, so this task will be something I can use soon" (Reflective Journal, November 11, 2015). She described her current classroom focus on strategies for adding and subtracting two-digit numbers as providing a strong foundation for operations with larger numbers.

> I think my emphasis is still on just these two-digit adding and subtracting strategies, for them to get really comfortable with two digits before we move to the 100s, because that's when you really start to have more regrouping and looking at those different ways to represent those numbers. So, I think we really build a good foundation with these two-digit numbers and then we move towards those three-digit numbers. (Professional Development Interview, November 3, 2015)

This emphasis on the use of strategies that would transfer between mathematical contexts was consistently present in Ms. Martin's classroom activities and predicted by her beliefs about the teaching and learning of mathematics as described in this chapter.

Additionally, Ms. Martin believed that this emphasis on mathematical strategies was a key feature of the demonstration lesson and offered a path through which to connect the demonstration lesson to her classroom.

> I really liked the demo lesson, first of all, because it tied to the strategies that I was already teaching with place value and I saw it fit perfectly with where we were going with adding and subtracting. . . . We are [now] adding and subtracting using just place value strategies to add and subtract. So we have moved from place value and . . . [are] now applying those [ideas] to adding and subtracting. (Professional Development Interview, November 3, 2015)

These quotations, in combination with the mathematical goals and lessons described as part of the baseline observations of Ms. Martin's classroom, helped to clarify the mathematics learning trajectory Ms. Martin envisioned for the semester. Her early focus on students' representations of numbers led into lessons involving the composition and decomposition of numbers with an emphasis on place-value concepts. These concepts were then leveraged to present addition and subtraction strategies for two-digit numbers, including regrouping, which could be transferred to larger numbers.

This learning trajectory, along with the mathematics emphasized in the demonstration lesson, allowed Ms. Martin to utilize the ideas and task from the demonstration lesson with little need for adaptation. However, this implementation did not come without consideration of how to adapt her instruction prior to the lesson.

> I definitely want to use that task like she has it presented. I would start the same way that she did where she asked them to get their base-ten blocks and represent a number, and I would also ask them the same thing, you know, "Is this the only way you could represent that number?" And I might even [pauses to think], you know, would you give them a number with a 0 before you give them 304? You know, I was kind of thinking that, that day, if you're scaffolding and you want, or is that too, is that giving away too much? [Or] would you give them a number with a zero in the 10s place. So that's something that I would maybe think about. . . . I'd almost like to see what would they do before I give them the task. (Professional Development Interview, November 3, 2015)

Although her meaning in the second half of this quotation was not entirely clear, Ms. Martin later revealed that she typically introduced new mathematical ideas without a real-world context, and this quotation indicated that she was considering this approach for the demonstration lesson's task. This initial reflection on the manner in which she would introduce the ideas from the demonstration lesson illuminated one of the key questions Ms. Martin considered in her adaptation of the lesson. Ultimately, Ms. Martin decided to utilize a task that supported her students in thinking about regrouping with two-digit numbers on the day before she implemented the demonstration lesson task. She first asked her students to consider the question, "Mrs. Smith had 60 pencils and gave Ms. Martin 30 of them. Does she have enough to give Mrs. Moss 23 pencils?" Once the class had determined that there were indeed enough pencils to share with Mrs. Moss, she followed up with the question of how many pencils Mrs. Smith would have left.

Ms. Martin also defined specific criteria she would utilize to determine if her students were prepared to confront the mathematical concepts inherent in the demonstration lesson.

> I think automatically exchanging those ones, or that 10 for those ones, and doing it without a struggle. I hope that even though I add a place value to it, they will still be able to exchange and carry that over into those three-digit numbers. My goal for them, I would like to see all of them do that with ease. When you don't have enough ones, you're regrouping that 10 to get 10 ones. That's what I would like to see them be able to do on their own on Thursday so when I do give them the three-digit numbers on Friday, hopefully it won't be that much of a struggle. (Planning Interview, November 10, 2015)

On the Thursday in question, Ms. Martin's students solved the problem of the pencil exchanges in a variety of ways, including direct modeling with base-ten blocks. For example, one student who presented to her classmates represented the 30 pencils that remained after the initial transaction using three longs. She then suggested trading one of these longs for 10 units in order to take away 23, in the form of two longs and three units, resulting in a final solution of seven pencils represented by seven unit cubes.

Her students' success with this task supported Ms. Martin's decision to follow up with an introduction to subtraction with regrouping in three-digit numbers that mirrored the demonstration lesson she had observed.

> I'm not planning on changing the task. I feel like some of the things that I'm doing now and leading up to the task will hopefully help. I really want to see what they'll do with the three digits and with the zero. Right now, we're still focused on the two digits, mental strategies, and those kinds of things. We've not gotten into any kind of algorithm at all, which I'm not planning on getting into an algorithm just yet. We're still building those mental strategies and using the base-ten system to help us add and subtract. (Planning Interview, November 10, 2015)

From this quotation, Ms. Martin's decision to use the demonstration lesson as it was observed appeared to serve three functions. First, the lesson provided a fluid transition from strategies and concepts of two-digit operations to three-digit operations. Second, the lesson allowed the introduction of a new mathematical context, regrouping for subtraction with three-digit numbers including a zero, which Ms. Martin's students had not previously encountered. Third, the lesson prefaced an eventual transition to symbolic representations of addition and subtraction with concepts needed to make sense of these representations. These three functions aligned with both the learning trajectory described above and the beliefs about the teaching and learning of mathematics espoused by Ms. Martin throughout the study.

The description presented in this section provided evidence of the rationale for the manner in which Ms. Martin utilized the task from the demonstration lesson in her adapted lesson. Rather than moving from concepts of place-value and two-digit addition and subtraction to a standard algorithm for these operations, Ms. Martin used the demonstration lesson's task to introduce subtraction with regrouping across a zero in three-digit numbers based on her students' current understanding of these concepts with two-digit numbers and the learning trajectory she had emphasized throughout the semester. The next section provides evidence that this use of a task to introduce a new mathematical idea was innovative for Ms. Martin and describes her goals for using the task in this manner.

Ms. Martin's goals for the enacted lesson. The idea of using a mathematical task to introduce new material was novel to Ms. Martin. When asked how often she utilized tasks in this manner, Ms. Martin revealed that most of her tasks were implemented after a strategy had been reviewed in a decontextualized manner.

> Probably not as often as I should. We do a lot of, I do give them some word problem type task but right now, but I feel like we're doing a lot of the basic strategies just decontextualized. So, I feel like when they've got a good idea or a good grasp on those strategies is when I can give them a task, and that might not be how we should think about it. (Professional Development Interview, November 3, 2015)

Although Ms. Martin had been using tasks as review, application, and practice for the mathematical concepts and skills her students had previously encountered, this quote indicated that she was reconsidering the possibility of using a task to introduce a new mathematical idea. When asked to elaborate on her thoughts about using a task in this fashion, Ms. Martin described the events that would lead to this type of task implementation.

> No, that's a really good question, and I'm still kind of thinking about that because I almost would like to give it as an introductory point to three digits just to kind of see where we are. We've spent a lot of time today subtracting. Yesterday we spent a lot of time adding and we've tried to apply the same strategies to subtracting today and they've been hung up a little bit. They still have a hard time switching gears from adding to subtracting. So, I know we're still spending a lot of time on that but I almost think by that point, that's still several days out . . . eight or so school days. I almost can see doing [the task] as an introductory lesson, especially to subtraction because that's what that task wants to pull out is that subtraction with the regrouping. (Professional Development Interview, November 3, 2015)

This quotation illustrated that although it was Ms. Martin's initial plan to use the task to introduce a new concept, she believed that her students needed to be well prepared to engage with this new idea. The previous section's description of her lesson sequencing highlighted how she engaged in this preparation.

Ms. Martin also spoke explicitly about her learning goals for her implementation of the demonstration lesson and how these goals related to her students' past and future study of mathematics. In describing these learning goals she referred directly to her students' current understanding of strategies for operating with two-digit numbers.

> I would think that my students need to be able to represent the two-digit number in various ways to subtract. . . . I feel like the goals are kind of the same, we're just kind of changing, we're moving from two-digit numbers to three-digit

> numbers. I feel like our goal is the same, can they look at these numbers and understand that I need to maybe represent the number in a different way in order for me to subtract. I guess that really works for this whole unit. (Planning Interview, November 10, 2015)

In elaborating on the value of this ability to represent numbers in a variety of ways, Ms. Martin described how this principle generalized to regrouping in different mathematical contexts.

> What we're leading them up to is to be able to regroup, if they're able to represent these numbers in different ways depending on the situation. Sometimes they won't have to regroup, but if they see, "Oh I've got 9 ones, I only have 8 ones, and I need to subtract 9 ones," I want them to be able to see, no matter what situation they're in, whether it's just us working with numbers now, or working with numbers in context, for them to understand that sometimes I may have to regroup to get what I need or to subtract what I need. (Planning Interview, November 10, 2015)

In these two quotations Ms. Martin established her specific mathematical goals for this lesson as extending students' abilities to represent numbers in a variety of ways into using these representations with purpose in the form of regrouping for subtraction.

Additionally, she described how this goal connected to her students' recent areas of study and her future goals for the semester.

> To me, it's definitely understanding that not only can we represent numbers in different ways, but we can do that when we are subtracting as well. I see where

> we were several weeks ago where we were really focused on the place value and representing those numbers in different ways. I really hope that they can see the tie-in with that to the subtraction. I really think if they make that big connection, then once we get to the algorithm of regrouping this will be no issue. (Planning Interview, November 10, 2015)

With these words Ms. Martin confirmed the goals described in this section and explained the connections among these goals, her emphasis early in the semester on representations of number and place value, and the future objective of having her students symbolically represent operations and utilize algorithms.

This section examined Ms. Martin's goals for her enactment of the demonstration lesson for both herself and her students. In considering how to utilize the demonstration lesson in her classroom Ms. Martin recognized a new use for mathematical tasks and considered the measures needed to prepare her students to successfully engage with a task used to introduce new mathematical ideas. Additionally, she explicitly described learning goals for her students for the enacted demonstration lesson that included extending their ideas of place value and numeric representations to support regrouping in three-digit subtraction. The next section examines the struggles Ms. Martin anticipated her students to experience during the lesson enactment.

Ms. Martin's anticipated student struggles for the enacted lesson. In the days leading up to her enactment of the demonstration lesson, Ms. Martin reflected on how prepared her students were to engage with the new ideas to be presented in the lesson.

Three days before the lesson, she explained that although the task would be a challenge, she hoped the ideas with which her students were currently working would transfer.

> I think for some of them it will be a challenge, but I'm hoping that they will bridge what we've been doing with these two-digit numbers and it will carry over to the three-digit numbers. (Planning Interview, November 10, 2015)

In considering the specific factors she hoped to see the students bridge between two-digit and three-digit operations, Ms. Martin highlighted the need for students to recognize that different representations of numbers would allow them to regroup when needed and suggested that some students were more prepared than others to engage with this idea.

> I already, kind of, have in my mind the ones that I feel like will do well with it. I guess what I'll be looking for is the same kind of things that I am looking for now with using the two-digit numbers. Can they see a different representation or is it always going to be using the smallest number of blocks possible? Can they already go ahead and see an issue, “Oh I can use a different representation here because I've got this zero?” Where I was doing, you know, 68 minus 19 today, there are not zeros, but it's still the same concept. . . . I just, you know, wonder. I hope that they will make that connection regardless of what number it is, “I don't have enough ones. I need to do some regrouping.” (Planning Interview, November 10, 2015)

Ms. Martin described the ideas inherent in the problem of 68 minus 19 as mirroring those in the upcoming enactment of the demonstration lesson and questioned how well the concepts would transfer between the tasks. Most of Ms. Martin’s concerns for her students' learning from the demonstration lesson's task centered on this transfer and their ability to recognize the usefulness of the different representations of numbers they had been working with throughout the semester.

Ms. Martin also noted the struggles that many of her students had encountered with regrouping for subtraction with two-digit numbers three days before the enacted demonstration lesson.

> You could definitely see the yield signs, like, "What's happening I don't have enough" . . . . You could see, "I need to take one," some of them just weren't sure, "I need to take one from this rod," then several of them are like, "Oh, I know what we can do, we can just trade that rod out for this other one." They're still struggling with it, which is good. (Planning Interview, November 10, 2015)

Ms. Martin acknowledged the value of these struggles and suggested that students' initial questions and interactions with the idea of regrouping had led to some revelations about the value of representing numbers in different ways to facilitate operations with numbers.

In her reflections regarding her students' performance with the pencil sharing task she implemented the day before the enacted demonstration lesson (described previously on page 208), she suggested that the context of this problem was a useful feature that was shared with the demonstration lesson and had helped her students succeed with these ideas. "They did well [with the pencil sharing task]. I enjoyed seeing their ways of thinking about the context of the problem that will be really important when we do the [demonstration lesson] task next" (Reflective Journal, November 12, 2015). Additionally, she acknowledged that this context and their practice with the ideas had helped her students think about the significance of the numeric representations with which they had been working. “They were able to think about representing numbers in different ways which will tie in to the [demonstration lesson] task” (Reflective Journal, November 12, 2015).

Ms. Martin also addressed her students’ progress with her larger goals for them to think and communicate about mathematics and to work together in groups to share their thinking. With regards to their progress in communicating their ideas, Ms. Martin acknowledged that although some students continued to have difficulty putting their thoughts into words, she continued to maintain high expectations for these practices and to offer students the opportunity to practice their communication skills in a variety of settings.

> I do feel like yes, we're making some progress. I still have some that struggle verbalizing what they want to say when I pull their card. Even though they know to be ready at any time, I still think some of them are having a hard time with that. . . . [I] try to give them as much wait time as I can without taking up a whole lot of time. What I try to do with that is, if there is a really long pause, I'll say let's take a minute and let's get back in our groups and let’s kind of rethink about what we had to say about this or whatever it was we were doing. Then, hopefully call on that person again so that they do have something to say. Hopefully by doing that I'm training them to give it a little time and think about what you need to say, talk to a partner. Maybe that will help you form your words. They're still really

> young and it's hard for some of them to think about what they want to say and how they want to say it. (Planning Interview, November 10, 2015)

In this quote, Ms. Martin attributed the growth her students had experienced in their ability to communicate to both her positive expectations for their use of these abilities and the specific strategies she used to support her students in practicing these skills. She also described the progress her students had made in working together in small groups, but noted that some students continued to struggle in this area.

> I think they're all progressing really well. They are working well in groups, with the exception of a couple. Some of them still want to kind of lean back and let everybody else do the work. I think that's natural. I'm trying to get some of them out of that. You have a part to contribute as well, it's not just so-and-so doing all the work or doing all the talking. (Planning Interview, November 10, 2015)

In both of these instances Ms. Martin noted the progress her students had made in these areas, acknowledged that they continued to struggle and grow in these areas, and discussed her actions and thoughts regarding the role she played in this development. Additionally, she recognized these as ongoing goals for the year and described their success in terms of the progress they had made this far into the semester. “I think it's just a training process and it is November, but I do see that they are progressing like I'd like them to” (Planning Interview, November 10, 2015).

Finally, Ms. Martin also considered how the practices used by Dr. Monroe could help scaffold her own students’ understanding of the mathematics embedded in the demonstration lesson’s task. She cited specific instances from the demonstration lesson she observed that had caused her to reflect on how her own students would respond and how she could adapt her teaching practices to better facilitate their learning. As an example, she described how the introductory activity in the demonstration lesson was designed to encourage students to think directly about representing numbers in different ways and recognized that this explicit questioning could help her students be more successful with regrouping concepts.

> Before the task ever starts she's setting up the lesson and having them show a number and then she asks them can you show this number in a different way? I know that's still partly related to the math, but I definitely want to use that questioning. I think that gets them going. Where I'm not doing that now, you know I'm not asking the question, “Well, can you show these numbers in different ways,” I'm kind of letting them grapple with it on their own. Then maybe in that task I will follow that lesson plan like Dr. Monroe did, where I actually have them represent a three-digit number but then ask them is there another way you can show this number. (Planning Interview, November 10, 2015)

Ms. Martin believed that this explicit questioning would support her students in recognizing that different representations of a number were useful in different situations and lead to success when they struggled with regrouping across the zero during the demonstration lesson task. Ms. Martin also considered how she might react differently than Dr. Monroe had during the lesson based on the outcomes she had witnessed. Referring to the struggles many students had experienced with trading out their flats to be able to remove longs she pointed out how Dr. Monroe had stopped to assess the students' progress and asked them to return to the idea of using their blocks differently.

> I really like the way that Dr. Monroe kind of stopped the lesson and she kind of assessed where they were. Maybe they just get their place-value blocks out again and go back to where they were in the beginning and say all right, well is there another way that you can show me the number in a different way? Can you use different blocks? You have to be careful with your wording. (Planning Interview, November 10, 2015)

However, she also considered the potential of using a simpler question based on two-digit numbers to help scaffold students into the three-digit scenario.

> I know Dr. Monroe kind of felt like she was stuck like that at a certain point. You just kind of have to regroup I think. There could be a time where if I really feel like I need to really back up to the front, I would give them the same type of question with just two digits. Change the task question to two two-digit numbers instead of two three-digit numbers. Then there is part of me that doesn't want to, I almost say dumb it down, but don't want to change those numbers because I still want them to grapple with those three-digit numbers. (Planning Interview, November 10, 2015)

These examples demonstrated the manner in which Ms. Martin anticipated her own students' potential struggles with the demonstration lesson, evaluated the teaching practices she had witnessed Dr. Monroe utilizing during the demonstration lesson, and made decisions to either retain or adapt these practices. The statements also provided evidence of Ms. Martin basing these decisions on a combination of the outcomes she had witnessed in the demonstration lesson and the fashion in which her own students would respond to the lesson.

Summary. This section recounted Ms. Martin's preparations for enacting the demonstration lesson she had observed in her own classroom through four vantage points: her sequencing of the lesson based on the learning trajectory she had developed in her classroom, the learning goals she established for the enacted demonstration lesson, the areas in which she most expected her students to struggle with the lesson, and the adaptations she considered making to support them in this struggle. The next section describes her enactment of the demonstration lesson based upon these considerations.

Ms. Martin's Enactment of the Demonstration Lesson

Ms. Martin enacted her version of the demonstration lesson in her classroom on November 13, 2015. At the end of the semester, just over a month later, she viewed a video of her enactment of the lesson and shared her thoughts regarding the most significant occurrences from the lesson. This section contains a narrative account of this lesson's enactment interspersed with the comments she made as she viewed her version of the lesson.

Lesson setup: Modeling 127. As the lesson began, Ms. Martin's 17 students were seated in groups of four or five talking quietly. Ms. Martin called the class together by reminding them of the classroom norms for working with manipulatives, praising the work the students had done during their most recent class, and asking the class to begin by representing the number 127 using base-ten blocks.

Ms. Martin: All right, everybody should be seated, all books put away. Thank you to those of you who are following directions quickly. All right, today we are going to be using our base-ten blocks. Here's the thing, since we don't have a whole lot, we are going to have to do some sharing at the tables, okay? So I'm going to be looking for boys and girls who are sharing, who are speaking nicely to each other, in order for us to accomplish our task. Now guys, I haven't said anything about getting the bags. What should you be doing right now?

Student: Listening.

Student: Leave them still.

Ms. Martin: That's right, just leave them still, and follow directions. All right, here we go. We have been working on several different things. First of all, we finished up a task yesterday. . . . We almost finished, but I'll put a little extension on it that we weren't finished just yet. So I got to thinking about that last night and I really liked the way we were representing numbers, not just in a way that we're using the least amount of blocks, but some of us were representing those numbers in many different ways. We're going to continue representing numbers today, but we're going to use three-digit numbers, okay? So here's what I would like for you to do, here's what I would like for you to do. I'd like for you to get your place-value blocks, your base-ten blocks, and I'd like for you to show me 127.

The students then took out their base-ten blocks and began to generate representations within their small groups with noticeable more discussion beginning immediately after the task was introduced. Ms. Martin circulated among the groups and monitored their behavior, repeated the question, and observed her students' work. Many of the individuals, and eventually all of the groups, used one flat, two longs, and seven units to represent 127. After approximately 90 seconds, Ms. Martin called the class together to view one of these representations presented by Nathan.

Ms. Martin: All right, let's pick a card. Nathan, can you go up and use my blocks, and show us how to represent 127? Everyone should have their hands free, eyes are up here watching Nathan.

Nathan: [Placing a flat under the document camera] One hundred, [placing two longs under the document camera] the two tens represent 20, [counting out seven units] and seven.

Ms. Martin: All right, thumbs up if you agree that shows 127. [After most of the class hold thumbs up] Nathan, I really like the way you said these represent a certain value. As you were putting them up there I heard you say, "These two tens represent 20." That was very nice. I like the way you did that. Thank you, Nathan, you can have a seat. I'm just going to leave that up there. Now here is your next task. Can you show 127 in another way?

Warm-up exercise: Alternative representations of 127. Several students reacted visibly to this request, with one in the background quietly saying, "Oh no." However, the students quickly began to work on Ms. Martin's request with animated discussions occurring in their small groups. After about 20 seconds, Ms. Martin addressed the whole group, "I already heard Dale say, 'We're going to have to share.' I wonder why he would have said that? I'm curious why he said we're going to have to share?" As students continued to discuss the question and manipulate their base-ten blocks, Ms. Martin circulated, observing and asking questions. As many students replaced their flat with ten longs, Ms. Martin referred to this representation and asked a student in a small group, "How do you know this is 100? Show me." As the student counted the longs and found there to be less than ten, Ms. Martin repeated the question and left the group. After about two minutes, she again addressed the whole group.

Ms. Martin: All right, guys [raising her hand to get attention], give me five for just a minute. Give me five for just a minute, let me draw your attention to something. I overheard one of you say, "Oh we can't do this, we don't have enough blocks." Yes, you do, you're just going to have to share. Remember, you're just going to have one bag per person, but you're able to share and work together at your tables. So, I want to hear that conversation, I want to see what you can use to represent this in a different way.

During her review of the lesson, Ms. Martin paused the video here to offer her first comments. She began by explaining the purpose of the warm-up exercise and its relevance to the task to come.

> They're representing the different number. We're leading into that task. This is hopefully going to get them thinking, because when they get to the task, they're going to have to represent the number in a different way. Not just using the minimal number of base-10 blocks, they're going to have to show other ways. By doing that, you hope that they're thinking, "Okay, I can show these numbers in a different way. Oh, let's do that. We just did that, so let's do that here in this task," hopefully. (Reflection Interview, December 21, 2015)

She further described how she remembered students interacting with the warm-up exercise, "I do remember. Some of them were changing out their hundred, which I think you showed. Some of them were also just changing out maybe a 10 [with] 10 ones, so they're not necessarily changing the hundred" (Reflection Interview, December 21, 2015). Ms. Martin also indicated that she was attending to her questioning and other interactions with the students while reviewing the video and that she was content with these interactions thus far. "I'm trying to listen to my questioning and clarification. I think I'm clarifying things and answering questions okay. I don't see anything so far that I would necessarily change" (Reflection Interview, December 21, 2015).

Student representations: Linda's model. As students continued working with the warm-up exercise, Ms. Martin circulated the room listening to student discussions and asking questions to advance these conversations. After an additional two minutes she brought the group together to consider the work of their peers.

Ms. Martin: All right, give me five. I've asked a couple of people to come up and give us representations up on the board. You don't have to bring your blocks, you can use mine up here, okay? So use as many as you need to show. I've asked Linda to come first, so if everybody will leave their blocks still, remember, when someone is speaking you are showing your best listening. So all eyes should be up on Linda.

Linda: [Displayed a flat and counted out 27 units] A one-hundred twenty sev- [restarts] I have 127 ones, I mean 127, and then I took away two tens and put twenty ones.

Ms. Martin: Ok, so does she still have 127 represented here?

Class: Yes.

While reviewing the video of the enacted lesson, Ms. Martin noted that many of her students had used a similar representation to Linda's model and described this as part of the reason she had selected Linda to present her work first.

> She changed all of her tens. She was one of the ones, I think, that's one of the reasons why I picked her first. She had kept her flat, her 100 together, and she decomposed her tens. Whereas we have Dale, I think, here in a minute, I think it's him, or one of them, is going to present that they changed their hundred, I

think is what happens here in a minute. (Reflection Interview, December 21, 2015)

Although Dale's model was not actually presented, Ms. Martin elected to display Linda's model first as she believed it would be familiar to most of her students and prepare them to discuss other representations that involved decomposing quantities into units.

Student representations: Janet's model. As the lesson continued, Ms. Martin thanked Linda and invited Janet to present.

Ms. Martin: Yes. Linda, very good and good explanation. I liked the way you explained that, thank you very much. Janet, I've asked Janet to go next, so everybody will continue to show your best speaking and listening to our presenter. Janet's going to get her model ready and she's going to tell us about her model.

Janet: [Laid out a flat, a long, and ten units side-by-side with seven units below them] I took away one ten and I saved it for ten ones and I still kept the hundreds place the same and the ones place the same.

Ms. Martin: Well, is your ones place a little different now that you've exchanged that ten rod? [Janet nodding yes] Yeah, so you've got maybe more ones now than you did before, so let's look at her model. Does she still have 127?

Class: Yes.

Ms. Martin: Yes, she does. [Moving to the board and pointing to the blocks as she spoke] She's got a one-hundred flat. She's got a ten here and

> ten ones, together that makes? [Class answers 20] Twenty, and then she has [indicating seven ones to which the class replies 7]. So she's got 100 plus 10 plus 17, so that gives her 127. Thank you, very nice, Janet, and I've asked Karen to go last. So one more representation.

In her review of the lesson, Ms. Martin noted that the precision with which Janet had placed her blocks under the document camera for her classmates to view prompted Ms. Martin to discuss the practices that she and her students had adopted for modeling place value.

> We also talk about precision in our models, and not just having a big pile of units. To actually go ahead and model them out to where they actually look like a ten rod, but you can see the separate pieces. I try to do that also abstractly when they're doing that on their paper models, "Pretend that that's a ten frame, and you've got one, two, three, four, five, six, seven, eight, nine, ten." You've got those, so when you get ready to compose them, they're already there in a bundle. You bundle them up and move them over to the tens place. We really try to talk about, "If you just throw up a bunch of single units, am I going to be able to see that right off how many you have?" I really try to make them be more accurate and use precision. (Reflection Interview, December 21, 2015)

This precision was evident throughout the enacted demonstration lesson, particularly in the models that were presented by individual students to the whole class.

Student representations: Karen's model. As Janet returned to her seat Karen moved to the front of the class. Examining Janet's model, Karen removed the flat and looked back at Ms. Martin:

Karen: [Removing the flat from Janet's model] I traded the hundreds one for ten ones [looking back at Ms. Martin].

Ms. Martin: Okay, so pull out whatever you need right there. So listen and watch Karen.

Karen proceeded to count out ten longs, looking back to the model her group had created at her table, and replaced nine of the units from Janet's model with a long, leaving twelve longs and eight units.

Ms. Martin: All right, so double count, double check yourself to make sure you've got what you want to represent 127.

Karen checked her representation, removing the extra one.

Ms. Martin: All right, let's take a look. Let's see what she's got. I want you to be counting up those tens and counting up those ones. Did she do it? Thumbs up if you think that's 127. All right, so Karen, one more time, tell us what you did. I know you told us when your back was to us, but tell us now that you're facing out to the front.

Karen: I traded, um, a one hundred block for ten ones, [correcting herself] tens, and then I added two tens for the twenty and added seven ones.

At this point in her review of the lesson, Ms. Martin paused the recording and described why she had selected these three students to present, focusing on Karen's method as an entry point for the regrouping needed in the lesson's primary task.

> I think, strategically, I picked out these three because, first of all, there was variation in all their models. I think why I chose her to go last is because, when we get ready to regroup here in just a moment in that subtraction, you're going to have to break up [interrupted as the interviewer clarified her rationales for the first two students to present] . . . . That regrouping is going to have to carry over into the demonstration lesson. Hopefully they'll see that. (Reflection Interview, December 21, 2015)

Ms. Martin thus selected three models to present to the class. The first was selected based on being a common method used by several of her students, which related to the class's previous work with two-digit numbers. The second was chosen due to its relationship to the first and the precision with which the student presented the model. The final was selected directly for its relevance to the upcoming task.

Primary task: Setting up the problem. After these students had presented their models of alternative representations of 127, Ms. Martin encouraged the groups to continue to work together as she presented them with the day's primary task.

> Ms. Martin: All right, do you guys agree that that's 127? [Members of the class nod and confirm their agreement] All right, nice job, Karen, thank you. Thank you to all three of our presenters who came up and showed their representations. All right, we're ready for our task

[clearing base-ten blocks from camera]. Here's what I want you to do. Remember, you're going to have to work, you'll see why in a minute, you're going to have to use these blocks as a table. So, I'm going to be coming over to each table, and I don't want to see just one person working, I want to see the table working together. Okay? And I've already seen a lot of that already today, lots of good discussion already, and it's going to help even more when we get ready to look at this task. All right, here we go. Give it just a second [as the task in Figure 17 was focused on the document camera].

On Thursday, Tara was at home representing numbers with base-ten blocks.

The value of her blocks was 304.

When she wasn't looking, her little brother grabbed 2 longs and a flat.

What is the value of Tara's remaining blocks?

Use pictures, words, and/or symbols to describe how you solved the problem.

Figure 17. Primary mathematical task presented to students during Ms. Martin's enactment of the demonstration lesson.

Ms. Martin: All right, I'm going to give you a minute to look at it quietly and then we'll go over it together. Hey, we're not working yet, we're

> not working yet. Eyes on the board reading the problem to yourself.

As her students read quietly for approximately one minute, Ms. Martin interacted with individuals with questions and statements such as, “We’re not working yet, we’re just reading,” “Have you read this? Can you explain that problem to me?,” and “Can you explain it to me when I ask you about it?”

During the interview concerning the enacted lesson, Ms. Martin described this private reading time as being significant to her for multiple reasons involving both literacy and mathematical reasoning. She first explained her general rationale for introducing a problem in this manner.

> I always try to do that. I don’t always start out right away reading [the problem] with them. I want them to read it first on their own and process it. I think Dr. Monroe does that as well. She makes them read it on their own, and then we go over it together so we can understand what’s going on. (Reflection Interview, December 21, 2015)

Although she later elaborated on the role that this processing played in mathematical understanding, Ms. Martin first explained the importance of this private reading time for both her struggling and stronger readers.

> I’ve still got some struggling readers in my room. Then, I’ve got kids that can process anything I can put in front of them. So, I think by going ahead and, I guess for two reasons, both sets of kids. The one group that can process anything, I think they’re going to go ahead, read it on their own, and try to understand it.

> Then the same thing with the ones who can't read it yet. They still have to struggle with trying to understand it and make sense of it without me, because eventually they've got to do this on their own without me, having it read to them, and without them hearing it out loud. I think it's important that they read it on their own and try to make sense of it on their own. (Reflection Interview, December 21, 2015)

This idea of struggling to improve sense making and literacy skills that support independent thinking mirrored those Ms. Martin advocated for mathematics thinking and reasoning throughout the semester. Additionally, Ms. Martin spoke to the importance of these skills in allowing students access to the mathematical context of a problem.

> I guess what I want them to do is to figure out the context of the problem, and see that understanding, and start thinking about what mathematical ideas will go along with that context. The kid coming to grab them, what's happening when somebody grabs something? Putting a mathematical idea with the story, what's happening in the problem? Not just looking at the numbers and say, "Oh, these two numbers. Let's just add them, or let's just subtract them." Really turn ideas to the actual context of the problem. (Reflection Interview, December 21, 2015)

Together, this combination of literacy skills that allow students to decipher the story being told by a problem and mathematical understanding that allows them to begin to make sense of the problem's context constituted the processing Ms. Martin expected her students to engage in when first encountering a mathematical problem.

After allowing about a minute for this processing to occur, Ms. Martin and the class read the problem out loud. Ms. Martin then described her expectations for the problem and Olive posed a question.

Ms. Martin: All right, let's read it together [class reads the problem out loud]. So you have tools, oh gosh, I'm talking and I hear people talking. I will let you know when it's time to start working. I'm excited that you're thinking and that you're ready to get going, but we have to make sure that we're ready before we start, and that we understand the problem. You have tools to use as a table. I have to also have something written, so I want to see some pictures, some words, something on your paper after you have modeled your solution, okay? All right, any questions? Does everybody understand the task? Olive?

Olive: Um, how could she take, um, how could her brother take two longs when there's no longs?

Ms. Martin: That's what you're about to discover, okay? All right, everybody get going.

During the interview, Ms. Martin commented on this question, noting with interest that Olive had struggled with regrouping in previous lessons and spotted the lack of longs right away.

> She says, "How can the brother take two longs away when there aren't any longs to be taken away?" My response was, "That's what you're about discover."

> Then, of course, I'm going to let her, in their groups, figure that out. She's already made the connection of zero, there's nothing. She's one of the ones that struggles with that whole regrouping idea. Yeah, that's very interesting that she sees that right away. . . . [She] was confused how she's going to take two away from zero. (Reflection Interview, December 21, 2015)

This question, and its apparent resolution, are reexamined later in the narrative when students present their work on the task to the whole class.

Primary task: Small group work. Over the next 12 minutes the small groups worked independently with a high degree of active student-to-student discussion. As the groups began discussing the problem, Ms. Martin circulated, clarified the question to groups, observed student discussions, and asked students to explain what the problem was asking. In her interactions with the small groups, she encouraged students to look to the problem for details they might have missed, prompted groups to model the problem with their base-ten blocks, redirected table discussions so they occurred from student-to-student rather than from student-to-teacher, and asked groups to explain their reasoning to her. Three examples of these small group interactions are described in this section.

Small group interactions: Ally and Nick's group. Ms. Martin approached a group consisting of Ally, Nick, and two other students and observed their conversation briefly. She then asked the group to explain their thinking from the beginning.

Ms. Martin: Show me what you started with, okay?

Ally: [Placed three flats and four units between herself and Nick] Okay, we started with this and took one flat away, so we'll take that out [removing flat], and we took one flat away [removing a second flat] and exchanged it with this [ten longs].

Ms. Martin: Why did you do that, because it didn't say that the little brother grabbed two flats, it said he grabbed two longs?

Nick: Because we've got to exchange.

Ally: We exchanged this flat for ten of these [longs].

Ms. Martin: Okay, so you did that because why?

Ally: Because then we could take two away [removing two longs].

Ms. Martin: [To two other students in the group] Do you guys agree with this?

Students: Yes!

Ms. Martin: Okay, now show me something on your paper. . . . See if you can find another solution [leaving].

As she reviewed this exchange via video, Ms. Martin commented on her use of questioning during this interaction.

> There's a lot of good dialogue and questioning and answering right there. . . . That's something that I've really had to work on over the years is my questioning. Getting my ideas out of the equation, and my thoughts about it, and really opening it up to them and hearing what they have to say. We've talked about that, too. Once I insert ideas, then they cling to that, "Oh, this is Miss Martin's thought." . . . I feel like I've gotten a whole lot better with that over the years. I know Project Influence has helped with that big time, because they give you no thoughts and no

> ideas, and they just leave you hanging. (Reflection Interview, December 21, 2015)

Questioning of this nature was common during Ms. Martin's interactions with her students, and this comment alluded to part of the rationale behind its use. Additionally, Ms. Martin noted the importance of the specific directions with which she left this group working.

> Oh, that's good, they found one way to do it, and so I asked them, which would be more an advancing type question, if you can find another solution, or another way that they could have done it. . . . Because you try not to let them just sit and simmer on that until everyone else [is done], so you try to get them to come up with another answer to keep the thinking going, and not waiting on everyone else to finish. Something else I try to work on and get better at. (Reflection Interview, December 21, 2015)

This specific type of request, which Ms. Martin referred to as *advancing*, was unique to this exchange due to this group's immediate success with the problem. Her next interaction was with a small group struggling to make sense of the problem and required a different type of discourse.

Small group interactions: Dale, Leia, Karen, and Samuel's group. As Ms. Martin approached this group the students sat relatively quietly with one flat and four units in the center of the table.

> Ms. Martin: All right guys, let me see what you've got here. Is this [one flat and four units] what you've ended with?

Karen: Yes.

Ms. Martin: So, will you model for me how you got this solution? Let's go back to the original problem. Will you model it for me, will you actually show me?

As Karen reset the blocks by laying out three flats and four units, Samuel attracted Ms. Martin's attention and asked, "Is this right?" while pointing at his answer.

Ms. Martin: I don't know, that's what we're getting ready to discover here. I want to actually see your process. [As Karen stacked the flats and longs, Ms. Martin separated them] Okay, can I do this, so that we can see each piece here? Okay, so is this what she started with, it says 304?

Karen: Yes. Then we take away two tens [removing two flats and correcting language], two one hundreds, because it says two longs and a one hundred.

At the same time, Leia picked up two units and looked questioningly at Karen as Dale spoke.

Dale: [Interjecting] Two flats.

Ms. Martin: Okay, does it, let's read that part again. Her little brother grabbed two longs and a flat.

Karen: Oh, and a flat [emphasizing "a"], that means one.

Ms. Martin: That means one flat, okay. So show me how he would grab that.

Karen replaced one flat and sets another flat to the side.

Ms. Martin: Now it says he grabbed two longs.

Dale: Woah.

Ms. Martin: That's what you're still figuring? All right, well I'm going to let you keep thinking about that if you're still figuring.

Karen: Ahhh, I've got it!

Ms. Martin: I'll come back in a minute, I want to see what you've got.

As Ms. Martin left the group, Karen was again resetting the model and directing the others students' attention to it.

In her reflections regarding this interaction, Ms. Martin first assessed the group's progress with the task thus far. "How can you take away two longs when you don't have any tens? They're still, even at this point, grappling with that, where everyone else has made that connection" (Reflection Interview, December 21, 2015). She then elaborated on her approach to this interaction, which focused on having the students recognize their logical inconsistency as they tried to justify their solution.

> You're not giving them an answer, and not letting them believe they're right or they're wrong. They're proving to me again how they got this answer. So I actually go through, we stopped, and we did some more clarification. They're still struggling with it. But there was a lot of conversation there. Just some clarification, redirecting them back to the problem, making sure they understand what's happening. That's good discourse. (Reflection Interview, December 21, 2015)

This type of interaction, which Ms. Martin described as *redirection*, appeared to be at least moderately successful for this group. Ms. Martin pointed out Dale's behavior as she left the group as an indicator of this success.

> Dale is saying right there, he's telling how many, they're going back, yes. Listen, they're going back in within the problem. He's over here already regrouping his. He's got a bundle of 10 tens in his hands, so he's already making that connection of, "We've got to regroup." They're all on task, which is of course the goal. (Reflection Interview, December 21, 2015)

This nonjudgmental approach was used throughout the lesson to encourage students to continue their mathematical discourse after recognition of discrepancies in their reasoning.

As Ms. Martin further considered this interaction, she noted two potential reasons this group might have struggled and considered how she could have interacted with them differently. First, she recognized that this group may have been transferring invented practices from their work with two-digit numbers.

> They were so focused on those two-digit practices, and then when they were moved to a three digit, they were only looking at that outside number. The tens used to be just that last digit, now you've added another digit. . . . They're thinking of, I guess, just that last place value, "I've got two here, so let me just take those two," to where they really have to think about, like we said, the context of the problem. We're taking longs away and not flats. There was a lot of questioning going on there to hopefully redirect them back. I think they did make

> some progress on that, but you can see that happen there. (Reflection Interview, December 21, 2015)

This proposed explanation for the group's thinking occurred after Ms. Martin had a chance to review this interaction on video and thus did not impact her instruction directly. Similarly, viewing the video allowed Ms. Martin to consider how the students continued to interact after she left the group and caused her to reconsider the redirection she had used during the lesson.

> See they're so caught up in what they're taking away, and not really focused on right now first representing the number to see what they can take away. They just want to start taking away stuff. They never really looked at the problem to see what they started with so that the little kid could have taken away something. . . . It's great that they understand that they're taking something away, because they grab them, but they're not really focused on what they started with. . . . I probably would have said, "Go back into the problem and read about what's happening, from what you start with, what's been taken away." Just maybe redirected them to read the problem again. (Reflection Interview, December 21, 2015)

In both of these quotations, Ms. Martin's assessment of the situation in the classroom changed due to her review of the lesson on video.

These examples of considering an instructional route different from the one implemented during a lesson and observing students' behavior without the teacher present provided evidence of situations that paralleled the benefits Ms. Martin noted during the demonstration lesson. In these instances, the opportunity to observe one's own teaching or hidden student outcomes substituted for observing another teacher's practices and outcomes during a demonstration lesson. In both cases, this video review provided opportunities for Ms. Martin to reflect on her instructional practices and student thinking in a manner that would not otherwise have been possible.

Small group interactions: Jason, Kyle, Nathan, and Olive's group. On Ms. Martin's first visit to this group, they were engaged in a discussion of how to represent the initial 304 blocks called for in the problem. The group had decided to count out all of their unit cubes to replace one of the flats, but they did not have enough blocks to complete this plan. Ms. Martin initially left the group with the suggestion, "If this solution didn't work, see if you can find another solution that does." As she walked away, Kyle suggested to the group that they use the longs to replace some of the units that they needed. When Ms. Martin returned to the group she followed up on this line of thinking.

Ms. Martin: All right, did you guys change a little of what you were doing?

Kyle: Yes, but he [Nathan] says it's different, we were arguing.

Ms. Martin: Well, let's see what you were doing.

Kyle and Samuel placed a stack of blocks at the center of the table as two other group members watched actively. However, Olive was withdrawn from the group as the discussion continued.

Kyle: We start with three of them [picking up three flats], then we change a one hundred flat, we change a hundred, we change a hundred flat for one, two, three, four, five [finished counting out ten longs silently into one hand, but unknowingly dropped one of these and he placed nine longs with two flats in the center of the table] these tens.

Ms. Martin: Olive, come up here, honey, I want you to see this.

Nathan: I said we couldn't, I said we couldn't do this.

Ms. Martin: Okay, well let's just see what's going on here.

Kyle: Then what we done was add the four ones and what it said was, "her brother took two longs" [Nathan removed two longs] "and a flat" [Kyle removed a flat], which what we had left was one hundred [pointing to the flat and interrupted as he began to count longs].

Nathan: Wait! [Begins counting at the flat and proceeded to count longs] One hundred seventy-seven, [turning to Ms. Martin] one hundred seventy-seven.

Kyle: [Continued to count the units after the longs] One hundred seventy-four.

Ms. Martin: Okay, can I ask you a question? How many of these [indicated longs] did you exchange for this [indicated flat]?

Kyle: Ten.

Ms. Martin: Ok, double check yourself. If there were really ten there.

Jason: One, two, three, four, five, six, seven, eight [as Samuel counted, Kyle touched each long starting with the first as Samuel said, "two."]

Nathan: And we took two away [holding up the two longs which had previously been removed].

Ms. Martin: Well, there's [stopped briefly] how many are here [pointing to the seven longs]?

Nathan: Eight!

Ms. Martin: [Touching each long as she counted] One, two.

Students: Three, Four, Five, Six, Seven.

Kyle: [Adding a long] Eight!

Ms. Martin: Why did I know that there has to be eight here?

Nathan: Because eight plus two is ten.

Kyle: See [stacked the eight longs on top of a flat and added two additional longs to cover the flat].

Ms. Martin: Okay, look at me, [to Kyle as he organizes blocks] look at me. You've got to be very careful that you are doing this accurately and not get in a hurry. I love that you're so excited and you're all kind of rushing around, that's great, but you've got to be accurate and check yourself. Keep working on this idea here.

As Ms. Martin left the group she once again leaned over to Olive and asked her to "come up here with the group."

While viewing the video of the lesson, Ms. Martin noted both her communication with the group and Olive's withdrawal from the group as significant events during this interaction. With regards to the group as a whole she applauded herself as she believed the interaction was positive and had become automatic for her.

> Fabulous question, Miss Martin [referring to the question "Why did I know that there has to be eight here?"]. . . . That's such a good question and a good connector, because we're trying so hard to focus on that idea of making 10, which is so crucial when they're adding and subtracting. I've really tried to push that. That was a good, I'm proud of myself, because that was automatic. (Reflection Interview, December 21, 2015)

This line of questioning and Ms. Martin's accompanying actions appeared to have been used to prompt multiple members of the group to engage in a discussion of their work in a form of *facilitation.* However, this facilitation proved to be ineffective for Olive during this exchange, and Ms. Martin noted her withdrawal and speculated on the reason for it.

> Remember, at the beginning, she was like, "How can you take two if there's a zero?" She's pulled herself out of the conversation here. Earlier, they were going back and forth. Again, that's that maturity thing. She's just shutting down, because she's not understanding, and he's not seeing her point of view. . . . She's not staying in actively, and participating, and trying to hear what's going on. She's shutting down mentally. She's going to have to get to the point where she's, "Okay. I understand I may not know, or I may not understand right now, but let me hear what they have to say, and it might make some sense. But I've got

> to persevere and stay in it." She's having problems with that right there. (Reflection Interview, December 21, 2015)

Ms. Martin also observed that this behavior was not unusual for Olive, described the success Olive had experienced as she began to overcome these issues of withdrawal, and spoke to the practices that supported this success.

> We've already made some gains, because when she gets it, her face just lights up. I mean, you can totally see the light bulb go off. I try to pinpoint those ideas. "See, you didn't give up. You stayed with it, and you got it." That would be where I stay on her and remind her that, "You may not get the success right away, but it will come. You just have to keep working with it." Because when she does get it, she is so proud of herself. (Reflection Interview, December 21, 2015)

This quotation offered further evidence of Ms. Martin's emphasis on effort-focused feedback in a wide variety of classroom settings.

Primary task: Individual recording. As the small-group discussions started to wane, Ms. Martin asked her students to take a few minutes to reflect on their ideas and strategies thus far and to record these on paper individually.

> All right, give me five. Give me five. Give me five. Here's what I want you to do. We're going to have five minutes of quiet, just think time. I hear a lot of fabulous discussion and you've had a lot of time to work with your models and move things around. Here's what I want you to do. I want you to take five minutes and I want you to get your ideas and your strategies on your own paper. Okay? I already think I have in mind who I want to present for us, but I'm still

> looking, so I want you to get your ideas down on your own paper. [Paused for 15 seconds] I want to add one more thing. If you disagree with what's going on in your group, then you need to be able to prove something different on you own paper. Okay?

As students recorded their individual responses, Ms. Martin circulated among the groups examining their work.

During the interview, Ms. Martin identified this activity as an opportunity for students to present their individual thoughts outside of their groups and for her to assess the thinking of individual students.

> I guess something for me to think about is that, at this point, I wanted their individual thoughts. I still think why I did that is because there's so much floating around in that room, and I want to see who can do what. I don't really want group things to take over. If you and I are in a group, "I'm just going to let him put his ideas down." I guess, at that point, I wanted to see what each one of them could do for me. (Reflection Interview, December 21, 2015)

She also viewed this individual think time as an opportunity to reinforce the big ideas with which students were working, namely the idea of regrouping for subtraction.

> That's just more opportunities like this one, to reinforce that strategy that I want them to come out with. That's that whole idea, "I have a zero in the middle of this number. How am I going to get anything from that?" which a lot of them were really struggling with that. Now that we've moved forward, we haven't worked on zeros as much, but they understand, "If I don't have enough, I've got

> something that I can go regroup and get more from." They are making that transition. (Reflection Interview, December 21, 2015)

The assessment practices described in these quotations were well aligned with Ms. Martin's goal monitoring activities throughout the semester.

At the end of the lesson, Ms. Martin collected this student work, which continued to evolve throughout the lesson, and used it to assess the changes in thinking that had occurred during the lesson. The individual work generated by students at this point in the lesson also provided a method to present their ideas for peer review and a platform to record changes in their thinking that occurred as they interacted with their peers. This is evidenced throughout the whole group discussion presented in the next section.

Primary task: Student presentations. After allowing approximately three minutes for students to record their individual thoughts, Ms. Martin called the class back together to examine work from their peers and engage in a whole-group discussion.

> Ms. Martin: All right, I've asked a couple of people to come up and show some ideas, so here's what I want you to do. Remember, when someone's presenting, you're listening, and after each person presents I want to have some discussion on their findings. So Brad, I'm going to let you start, and let's just discuss what Brad's got here. . . . So stop what you are doing and let's just study his model for a moment.

Brad moved to the front of the room and placed his written work under the document camera. His representation, shown in Figure 18 in its final form, consisted of three squares representing flats and four x's representing units. In the work he presented to the class one of the flats was darkened to show its replacement by 10 tally marks, representing longs, which were partitioned into a set with a value of 80 and a set with a value of 20. He then described his thinking to his classmates.

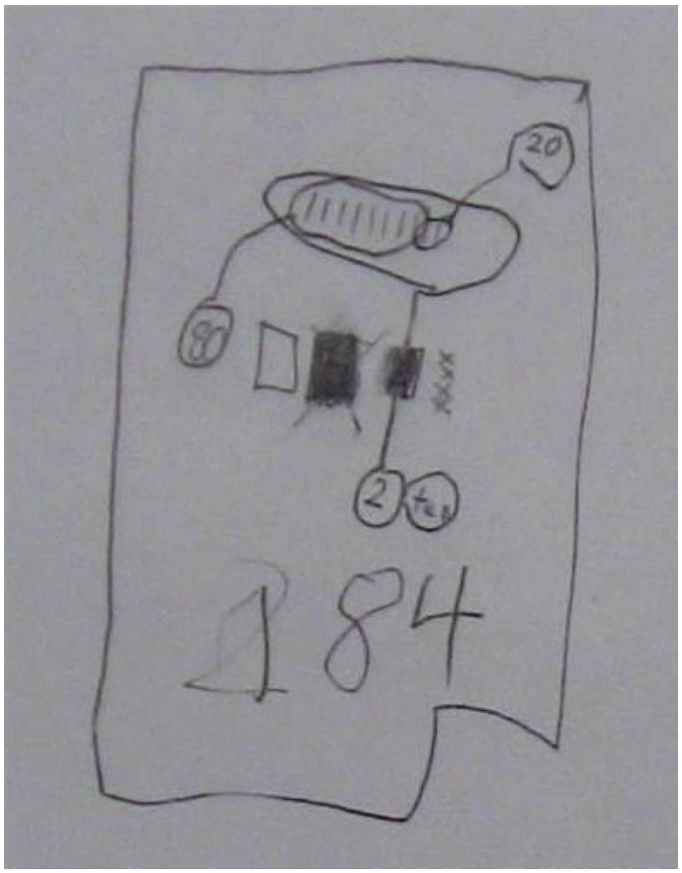


Figure 18. Brad's work after corrections stemming from his whole-group presentation. In the work as originally presented the middle square (representing a flat) was not darkened and the solution was shown as 284.

Brad: I thought, um, that I could take away that one 10 and, um, split it into 10 ones cause that's what um, that's what um, 10 ones, [correcting himself] 10 tens, you can exchange those 10 tens for one 100. And, um, I thought, um, and I thought if I could take away 20 and, um, I had 80 left, so I thought, um, I thought, I thought that it was 284.

Ms. Martin: Okay. Thoughts or questions about Brad's? Anybody wondering or thinking anything when they see this? [Nick raised his hand] Nick?

Nick: Ugh, I wondered if, I wondered if he messed up, by accident, on that.

Ms. Martin: Okay, well what would you think should be done differently?

Nick: Um I thought, I think he should go over it one more time, 'cause I didn't really understand it.

Ms. Martin: Well let's look what he's done. Does it fit the problem? It says the brother grabbed a flat and two what?

Class: Longs.

Olive: But he couldn't have done what Brad has done.

Ms. Martin: But look what Brad has done.

Brad: I should have took away one more 10 [indicating a flat], that's what it was.

Ms. Martin: Okay, did you hear what Brad just said?

Nick: That he should have taken away the 100, that's what I was talking about.

Ms. Martin: So Brad just says, "Oh maybe I should have taken away another 100." All right, thank you for sharing, Brad. Becky, no hang on, I think I'm going to go right to Nathan, so Nathan.

As Brad returned to his seat, Nathan moved to the front of the room.

While reviewing the lesson, Ms. Martin noted Brad's self-corrections and the fashion in which Nick pointed out the error in Brad's work as significant events in this exchange. She viewed Brad's self-corrections as a positive aspect of his presentation.

> He said ten instead of, he self-corrected himself there. He really was talking about ones and tens, but he self-corrected and said tens and hundreds, which is what you want them to do. (Reflection Interview, December 21, 2015)

Ms. Martin also appreciated the manner in which Nick engaged in the discussion and the respectful language he used to point out Brad's error.

> He's trying to be very respectful, "I think he should go over it one more time." Not just saying, "He's wrong, he needs to change this." That cracks me up to hear them talk like that, "I think he needs to go over it one more time. I think, maybe by accident, he overlooked something." (Reflection Interview, December 21, 2015)

Ms. Martin viewed this exchange as particularly important for Nick, who had started the year in third grade and had been moved into Ms. Martin's second grade class after his family relocated to the area. Nick had struggled with the content and interactions of third grade, and Ms. Martin saw this exchange as indicative of his growth in these areas.

> I think the big idea is just that he can communicate his ideas with others. He can work better in a group, whereas I don't know that he did a whole lot of group work beforehand. There was a lot of struggle at the beginning just learning how to be in a group. Just hearing him talk right there almost gives me tears, because he's looking at that piece, and he recognizes there's an error. He's trying to critique that respectfully. I think that's a big deal for these kids. He should already be a third grader. Just being able to communicate, and understand that everybody's got their own ideas and their own thoughts, and we can critique those, and we can judge each other respectfully. (Reflection Interview, December 21, 2015)

This combination of self-correction and reexamination of his work based on the suggestion of a peer allowed Brad to produce the final representation shown in Figure 18.

As Nathan reached the front of the room he placed his work under the document camera for his peers to review (see Figure 19). His representation consisted of two large squares, representing flats, 10 smaller rectangles beside them representing longs, and four x's representing units. One of the flats and two of the longs had then been crossed through. He turned toward the board and explained his work to his classmates.

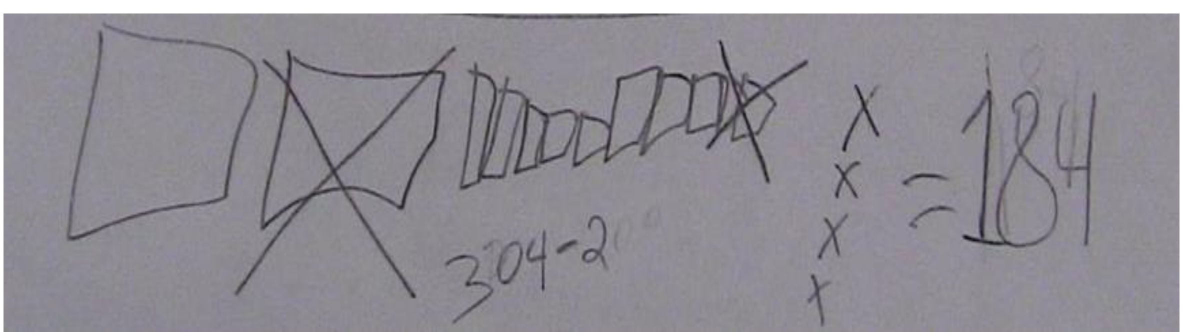


Figure 19. Nathan's work as presented to his classmates.

Nathan: [Facing the board and pointing as he spoke] Okay, she said she had three, three, 304, so I took, [corrected] so her little brother took away one 10 and two flats, so I x-ed those out and then I knew, then I counted the 10s and they were eight, so it was 184.

Ms. Martin: Okay, turn this way and tell us again. You said the brother grabbed what?

Nathan: One 100 and two longs [correcting previous statement of two flats].

Ms. Martin: Okay, so a flat and [interrupted]

Nathan: [Nodded] two longs. So it was 184.

Ms. Martin: Okay, so questions or comments for Nathan? [Olive quickly raised her hand] Olive?

Olive: There was three 100s, [turning to Ms. Martin] because our number was 304.

Ms. Martin: Turn and talk to Nathan.

Olive: [Turning to Nathan and speaking quietly] There was three 100s, but you only have two, although our number was 304.

Ms. Martin: Okay, so can you tell us? She, what Olive is saying if you can't hear, Olive is saying, "We have three 100s in the problem, but he only has two 100s." So Nathan, can you answer that, or talk about that for just a minute? Everybody should be having their pencils resting, there is a lot of good stuff going on. Stop what you're doing and listen.

Nathan: I exchanged the one hundred for 10, um, 10s, and ugh, so her little brother took out two of those, so two long ones and one big flat one, so then I counted them and there was 184. [Turning to Olive] That's how I know what it is.

Olive: If you exchange one from 300 that still is left with two 100s, or are you exchanging two 100s?

Nathan: Two 10s, I mean, I'm exchanging the 10s for that 10, [correcting himself] ten 10s for a 100.

Ms. Martin: So Nathan, can I ask you a question? Olive, what he says, he took these 10s [pointed at the 10 longs drawn on Nathan's work] and exchanged it for a flat, so does he have 300 represented? [Olive nods and another student said, "Yes."] Well that's what we're trying to decide. Does he have 300?

Class: Yes.

Ms. Martin: Do you always have to use flats for 100?

Class: No [Olive shaking her head no].

Nathan: You can exchange it for 10s to be 100.

Ms. Martin: Okay, thank you Nathan, very nice.

At the end of this exchange, Ms. Martin moved to the front of the room and prepared the class for their final activity of the lesson.

While viewing the video of this exchange, Ms. Martin allowed the video to continue to play as she offered comments regarding the discussion. She tracked the discussion between Nathan and Olive and noted that there was a combination of self-correction from Nathan and a resolution to Olive's question, which had lingered throughout the lesson, during this exchange.

> Now Olive's talking. . . . He's fixing himself. . . . That's lovely when all of that comes out. They're fixing their own problems. . . . This is a big point right here. . . . There's 300, but you only have two. Where he's got that other one represented, but she's still seeing those two hundreds. . . . They're having a good conversation back and forth. (Reflection Interview, December 21, 2015)

Although Olive appeared to have resolved her question regarding the missing longs during this exchange, Ms. Martin noted that the individual work generated during the lesson indicated that approximately one-third of the class continued to struggle with the idea.

> They really struggled with that. You can tell that they're still, just a lot of those ideas, there were some that were still able to get it, because they made that connection to the first problem. But you can tell there's still a lot of struggle with that. Just that concept in general. They're so caught up with there's nothing there. (Reflection Interview, December 21, 2015)

This struggle continued during the conclusion of the lesson, when students were asked to reflect on and make corrections to their individual work, and extend this idea to a parallel problem.

Lesson conclusion: Reflection and extension. As the final presenter returned to his seat, Ms. Martin thanked him and the class for their participation and asked the class to consider if what they had heard from their peers had influenced their thinking.

Ms. Martin: All right, so here's what I want you to do. I want you to kind of go back and look at yours, see if there's something that maybe you've changed your mind about before we ask our last question. We've heard some presentations, I want you to go back and look and see if there's anything you might want to do differently.

While students made corrections on the individual responses they had generated earlier in the lesson, Ms. Martin circulated among them and observed their work. After a brief exchange with Dale she addressed the class again.

Ms. Martin: That's what I want to do. Dale told me he's got something different to write down, that's what I want you to do if you've changed your mind or ideas about something. If you're still okay with yours then you don't have anything to change.

After approximately two minutes of allowing students to work quietly on their own, Ms. Martin extended her request.

Ms. Martin: Something else I'm wondering. Can you write an equation that shows your answer? Could you write an equation? I see fabulous models and drawings, but could you write an equation? Here's the problem, look back at the problem, could you write an equation that matches this?

In total, this private think time lasted about four minutes from the end of the student presentations. Ms. Martin circulated among her students during this time, observing their work and answering questions quietly. When the students had completed their corrections, Ms. Martin placed a final question on the board, which she read aloud with the class.

Ms. Martin: All right, here is your last question. Let's read it together. Guys, listen, we've got five minutes, and this is our last thing. Ready? [Read aloud and recorded on the whiteboard] How could you represent 407 so that 3 longs could be taken away? Write a sentence explaining how you know.

Ms. Martin: So you may turn your paper over and I would like for you to answer this question. You have to write a sentence, but I don't mind if you use a drawing to show it as well. But I do want a sentence. This is quiet on your own.

After allowing three minutes of individual work on this question, Ms. Martin brought the lesson to a close by asking the students to stop working and thanking them for their effort with the task.

In her reflection interview after the lesson, Ms. Martin's overall assessment of the lesson was that it had been successful and that the majority of students had developed further understanding of the need to represent numbers in different ways in order to support operations such as subtraction.

> I'd say three-fourths of the classroom. . . . I think they were [able to address the exit ticket]. I could pick out maybe five or six that were not. I think, for the most part, they were able to. . . . They understood that you had to represent a number a different way in order for them to subtract, which leads to the whole idea of the algorithm and regrouping. I feel like the trajectory is on target and moving towards the overall goal. (Reflection Interview, December 21, 2015)

The final section of this chapter contains an account of Ms. Martin's more general reflections regarding her use of the demonstration lesson and the unit in which it was contained.

Ms. Martin's Reflections on the Demonstration Lesson and Unit

In addition to the specific comments Ms. Martin made regarding her enactment of the demonstration lesson, she also offered more general reflections regarding her use of the demonstration lesson and the unit in which it was contained. This section presents those reflections in four parts. The first part addresses her use of the demonstration lesson task and her students' engagement with the task. The second piece examines her plans for the lessons immediately following the enacted demonstration lesson. The third portion considers her long-term mathematical goals for the remainder of the semester and school year. Finally, the fourth segment contains her reflections regarding her students' progress toward her larger goals for them for the semester.

Reflections on the enacted lesson. Ms. Martin's immediate reflections on the enacted demonstration lesson indicated that she believed the lesson to have been successful. She felt as though her students had effectively communicated their ideas and that the majority had gained an understanding of the value of representing numbers in different ways.

> I felt like they did really well! I love hearing the conversations and explanations that came out of the lesson. Their use of vocabulary pleased me also. Most of the students were able to make the connection to showing numbers in different ways. (Reflective Journal, November 13, 2015)

She was also pleased with the “connections they were able to make with their manipulatives and regrouping” (Reflective Journal, November 26, 2015) and the associations the students developed between using different representations of numbers and subtraction.

> They did well. There are some still struggling with the regrouping idea, but I can see it improving. The most important things that happened were their different representations of numbers and understanding that you show numbers in different ways in order to subtract. (Reflective Journal, November 28, 2015)

These comments, collected at different times during the two weeks following her enactment of the demonstration lesson, converged on the central learning objective accomplished by the lesson: students began to recognize the value of the variety of ways they had learned to represent numbers during the beginning of the semester and to transfer these concepts to numeric operations.

In addition to the lesson’s benefits to her students, Ms. Martin pointed out two ways in which enacting the demonstration lesson had been useful to her. First, she alluded to the value of returning to the lesson via video as it allowed her to consider aspects she had not previously inspected and spurred self-reflection on her implementation of the lesson.

> It feels so funny to watch. . . . It's all this stuff that I don't see. . . . Yes, and it's very eye-opening. . . . This is funny how it all comes back to you. . . . It is a good thing to self-reflect. (Reflection Interview, December 21, 2015)

Many of these reflections were recorded with the narrative of the lesson's enactment and led directly to considerations of future changes in practice.

The second benefit Ms. Martin noted was in using the lesson as an introduction to a new mathematical concept.

> I did like to use it as an introduction, because it's almost like a bridge from their understanding with two digits to adding that third digit. My thinking on that was, if they got the foundations, and they understand what's happening between ones and tens, then that should carry over from tens to hundreds. I do feel like it was placed well. . . . I loved using that zero, because that really threw them off and made them think, which is what we want them to do. (Reflection Interview, December 21, 2015)

This use of a task to introduce a new mathematical idea was a novel approach for Ms. Martin, and its success was likely imperative in considering the use of a task in this fashion in future lessons.

Considerations regarding future lessons. As Ms. Martin considered her immediate goals for following up with this lesson, she planned to have her students "work on more subtraction problems using their base-ten blocks and drawing models"

(Reflective Journal, November 13, 2015). Additionally, as she planned to have a substitute in class early in the week following the demonstration lesson, she "spoke with her about making sure she knows the students aren't ready to see the vertical representation of the problems and to let them use their blocks and drawings" (Reflective Journal, November 13, 2015). Depending on her students' degree of success with their continued exploration of these ideas, Ms. Martin planned to introduce the term regrouping and to align the procedures of the standard algorithm with the models they were generating.

> My plan right now is to see how they do with the subtraction I left for them. We may then move to using the term regrouping with 3 digits, still using base-ten blocks along with the algorithm. (Reflective Journal, November 13, 2015)

Once this idea had been fully developed and an algorithm had been introduced, Ms. Martin's plan was to bridge the concepts of addition and subtraction into measurement concepts late in the semester.

> We're going to continue. We should be able to wrap up adding and subtracting within a thousand. . . . We will move into measurement. That's what I really thought, but I didn't want to misquote myself. We'll take what we've learned from the composing and the decomposing, and adding and subtracting, to measurement. (Reflection Interview, December 21, 2015)

This planned instructional sequence complemented the learning trajectory presented for the first half of the semester to reveal Ms. Martin's overall mathematics learning trajectory for the semester.

In this trajectory, an early focus on students' representation of numbers led into lessons involving the composition and decomposition of numbers with an emphasis on place-value concepts. These concepts were then leveraged to present addition and subtraction strategies for two-digit numbers, including regrouping, which could be transferred to larger numbers. Substantial work with direct modeling of these strategies then led to a discussion of the formal language involved in numeric operations along with an introduction of algorithms and symbolic representations for these operations. The next section includes descriptions of Ms. Martin's perceptions of her students' progress along this trajectory over the course of the semester.

Reflections on students' mathematical progress. Near the end of the unit in which the demonstration lesson was enacted, Ms. Martin described her focus for the remainder of the semester, and at the end of the semester she looked back at how successful her students had been with the goals she had set. Her initial plan for the end of the semester was to begin the transition from concrete representations of addition and subtraction into more abstract representations.

> My focus will be adding and subtracting with regrouping using place value strategies, moving from horizontal to vertical notation. The goal is to move them from concrete to pictorial to abstract calculations. (Reflective Journal, November 28, 2015)

In elaborating on how she knew her students were prepared for this transition, she referred to their success in the demonstration lesson's unit. "I can see they accomplished the goals in their work with manipulatives. Now, we will move to pictorial and abstract notations" (Reflective Journal, November 28, 2015).

At the end of the semester she described her students' current mathematical activities and explained what this transition had consisted of in her classroom.

> Still looking at those abstract models. Making the models progressive by moving away from the actual [base-ten blocks] and those drawings to just the place value chart with the dots and the tens, or the one's, ten's, and hundred's place. We've been moving progressively away from the model so that they would understand what's going on in the algorithm. We also moved to a vertical representation as well, so their model looked a lot like their algorithm. (Reflection Interview, December 21, 2015)

This view of Ms. Martin's goals and the reality of what was accomplished in her classroom established additional context for the manner in which the enacted demonstration lesson was used.

Ms. Martin elaborated on how her students had progressed with these ideas since the demonstration lesson. With regards to their initial encounters with the concepts of regrouping experienced during the demonstration lesson, Ms. Martin indicated that the models and representations they used during the lesson continued to be of use.

> Yeah. I feel like [the struggles they experienced] in the demonstration lesson we were able maybe to overcome once the trajectory continued. I didn't totally get away from these ideas, those ideas just built, still using models. That's why I'm saying with Olive, she still clings to her model, because it just hasn't clicked with

> her. She still understands, looking at that model, how I can represent, not using those blocks, but in my model, I'm still able to show why I broke this 10 apart and then moved it. (Reflection Interview, December 21, 2015)

Although the concrete representations used for regrouping during the demonstration lesson continued to be directly useful for some students, the concepts that were represented within these models appeared to be valuable to all of Ms. Martin's students.

When asked how well the class as a whole had transitioned to the more abstract representations that had been introduced, Ms. Martin indicated that the transition had been successful, but that her students were still dispersed along a wide continuum of concrete to abstract representations.

> I've got, I'd say, 75% of the class not using models anymore. They're just ready to do it. However, there are times where I want them to give me a model, because I want them to be able to explain, "Why did you want to do that?" I still want them to give me a representation. But there are some still clinging to that model, there are some that will still want to draw it out every time. What I love, though, is if they make an error, they can have something to refer back to. I tried to explain that to them too. If you make an error in your algorithm, and you didn't use a model, use a model and see what happened. It's so visual. Some of them have just really clung to that, and some of them are like, "Oh, I know what I'm doing, and I'm moving on." It is neat to see the different levels of ability and where they're all at. (Reflection Interview, December 21, 2015)

This variety of approaches and the students' ability to refer back to more concrete representations in order to make sense of complex situations supports the power of the modeling approach used throughout the demonstration lesson's unit. Ms. Martin also commented on why she believed her students had been so successful in their transitions to abstract representations.

> I feel like overall, with their regrouping, they've done really well. Especially now that I see we've moved to subtracting vertically in the algorithm. They can see the why picture. This might be a little tidbit, but some of them said, "We tried to do this last year in first grade." My question was to them, "Well, did you know why you were doing this?" They said no. "Now do you understand why you're marking this number out?" What they'll tell you, "We're just marking this number out and making it a four, or whatever." Now they understand why that number in the tens place or the hundreds place is changing, because they need to have something to subtract from. (Reflection Interview, December 21, 2015)

This combination of making sense of abstract mathematical ideas and communicating their thinking about these concepts were well aligned with Ms. Martin's larger mathematical goals for her students. The next section presents Ms. Martin's views of other aspects of these larger goals that were achieved throughout the semester.

Reflections on progress towards long-term goals. Early in the semester, Ms. Martin established three long-term goals for her students: to make sense of mathematics, to think independently and value one's own thoughts about mathematics, and to clearly communicate one's thinking about mathematics with others. While the previous section addressed the semester's outcomes for the goal of making sense of mathematics, this section addresses Ms. Martin's perceptions of her students' progress toward the remaining goals.

When broadly assessing their progress at the end of the enacted demonstration lesson's unit, Ms. Martin complimented her students' understanding and communication skills. "I do think my students are progressing well. Their understanding at this point is where I want them. They are able to communicate more effectively than before, which is a huge goal" (Reflective Journal, November 26, 2015). Elaborating on the meaning of this success at the end of the semester, she spoke of her students' increasing maturity.

> I guess, if this is even a thing, it's their classroom maturity, just able to handle a challenge and able to just deal with the classroom. I'll say, there are still some issues, but some of them have grown so much in how they present themselves, and how they talk to each other. . . . Do you know what I mean? If you really stop and think about it, some of these kids in this room are still seven years old. That is little. They're doing some really big kid things here. It's just neat to see. (Reflection Interview, December 21, 2015)

Earlier in the year, Ms. Martin had described the features of the classroom she believed necessary in order for students to develop this classroom maturity. The key feature of her description was a level of comfort with other students that would allow them to interact in an authentic way. Ms. Martin's perceptions of the environment needed to support the development of this classroom maturity and the evidence of its existence in her current students comprise the remainder of this section.

The primary components of the maturity of which Ms. Martin spoke in her students included their independence in thinking and communication, their inclination to share their ideas and to credit ideas to others, and a willingness to question others. With regards to their independence, Ms. Martin referred to her students' eagerness to engage one another in meaningful mathematical discussions.

> I will say, as a whole, they are able to handle a situation on their own, and be independent, and talk with each other. Here's how I know that. If we're in larger group, a lot of times, before I can even get out what I want to say, they're ready to turn and talk, and get ready to work. That makes me excited, because they don't need me. That's what we want, you know? (Reflection Interview, December 21, 2015)

This independence extended beyond their one-on-one and small group discussions to their interactions with her and the whole class. Ms. Martin described a sign of her students paying attention to one another's thoughts as they began to refer to each other's ideas to support their reasoning.

> A lot of times, they all want to volunteer, which they know I'm going to pull cards. Sometimes, I will still take volunteers, just because I don't want that to go away. They want to share, they're willing to share. Something else that I've heard is them say, "I saw so-and-so do this." That's a big adjustment as well, and that's what we want them to do also. Looking at the reasoning of others, and being able to say, "I saw so-and-so do this, and I really liked that," or, "I wasn't sure why so-and-so did this." (Reflection Interview, December 21, 2015)

In addition to this constructive use of other's thinking, Ms. Martin cited instances in which her students had begun to question each other's thoughts and ideas.

> For example, if they're presenting their findings to the whole group, because we'll ask, "Do you have any questions or comments?" Some of them will say, "I just don't understand what's going on right here." Which is good, because then the presenter has another opportunity to maybe say it a different way. You already know this, but this whole process, it's amazing to see these little kids do this. I'm doing nothing, just providing opportunities. (Reflection Interview, December 21, 2015)

These three areas represented substantial changes in her students' classroom interactions that Ms. Martin believed show substantial growth toward her long-term goals for her students.

Summary. This section presented Ms. Martin's reflections across four key areas related to the demonstration lesson she enacted in her classroom. The first of these areas was the enacted demonstration lesson itself. This was followed by Ms. Martin's reflections on the lessons and learning trajectory that occurred immediately after the enacted lesson. The third area was Ms. Martin's considerations of her students' mathematical progression after the enacted demonstration lesson. Finally, Ms. Martin's perceptions of the growth of her students' classroom maturity over the course of the semester were examined. Together these reflections comprise a rich view of the outcomes of Ms. Martin's enactment of the demonstration lesson in her classroom.

Summary of Ms. Martin's Enacted Demonstration Lesson

The data presented in this section provided a full account of Ms. Martin's enacted version of the demonstration lesson she witnessed through Project Influence during the fall of 2015. This account was presented in three parts examining her initial planning for the demonstration lesson, her enactment of the demonstration in her classroom and the specific comments she offered as she reviewed her teaching of the lesson, and her general reflections regarding the enacted demonstration lesson and its effects on her students.

Chapter Summary

This chapter presented the results of an exploration of the question of how the characteristics of the growth mindset influenced an elementary mathematics teacher's interpretations and enactments of her professional development experiences. These results were presented in four parts. First, the teacher's perceptions of her own mindset and beliefs about the teaching and learning of mathematics were presented and compared to previously collected data regarding these constructs. Second, the teacher's descriptions of her classroom practices, activities, and outcomes and her observed mathematical teaching practices and activities were examined to establish a baseline for consideration of her enactment of the observed demonstration lesson. Third, the teacher's perceptions of her experiences during her recent professional development experiences and her specific areas of focus during a demonstration lesson were examined, and her perceptions of the significance of these areas of focus was recorded. Finally, the teacher's enactment of the demonstration lesson in her own classroom and her reflections regarding this enactment were considered. Chapter 5 will contain a summary of these results, an interpretive analysis of the findings, and a discussion of their importance.

CHAPTER V: SUMMARY AND DISCUSSION

Introduction

Despite moderate gains in elementary and middle grades mathematics achievement over the past two decades, there is much left to be accomplished (Mullis et al., 2012; NCES, 2013). Extensive research bases suggest that the quality of the classroom teacher is a major factor in this improvement in achievement (Baumert et al., 2010; McCaffrey et al., 2003; Rivkin et al., 2005; Rowan et al., 2002; Wright et al., 1997), that the teacher's conceptions of mathematics and the teaching and learning of mathematics share a complex relationship with the quality of their classroom instruction (Clarke & Hollingsworth, 2002; Handal, 2003; Philipp, 2007), and that effective professional development experiences offer a route to shaping these conceptions and practices (Desimone et al., 2002, Garet et al., 2001; Loucks-Horsley et al., 2003). One potentially impactful set of conceptions, implicit theories (or mindsets), has received little empirical investigation into its role as a mediator of mathematics teaching practices (Rattan et al., 2012). The implicit theories model posits that an individual's implicit assumptions about the nature of an ability, such as mathematical ability, impact the nature of the goals he sets for this ability and dictate a mindset for how he interprets and responds to events related to these goals (Burnette et al., 2013; Dweck & Leggett, 1988; Lischka et al. 2015).

The purpose of this study was to explore one of these motivational factors, the teacher's mindset, within the contexts of the teacher's professional development experiences, and answer the research question: How do characteristics of the growth mindset influence a mathematics teacher's interpretations and enactments of professional development experiences, if at all? As an aid to the reader the final chapter will contain a restatement of the research problem, a review of the methodology utilized in the study, and a summary of the results of the study. This review will be followed by a discussion of the results of the study, which will include its connections to prior research, theoretical and practical implications, and recommendations for future research.

The Research Problem

Coupled with a need to better understand the manner in which teachers' motivations and dispositions influence their interactions with students, curriculum, and their classrooms (Goldsmith & Shifter, 1997), the background provided in the introduction offers an outline for the key problem addressed in this study. Specifically, motivational factors, which play an important role in the process of teacher change, are not well understood (Guskey, 2002; Thoonen et al., 2011). Empirical research is needed to examine the manner in which these factors influence a teacher's daily practices (Goldsmith & Shifter, 1997), explore how they mediate other constructs which influence practice (Clarke & Hollingsworth, 2002), and articulate mechanisms for teacher change (Goldsmith et al., 2014). Calls from more recent literature stress that studies of this nature should be situated in specific sets of activities, supports for learning, context, and characteristics of individual teachers (Opfer & Pedder, 2011), and that these studies should focus on the interactions "between the individual teacher, the context of the professional development activity itself, and the teacher's work environment" (Wagner &

French, 2010, p. 169). A case study design, reviewed in the next section, was developed to address these concerns.

Review of Methodology

An exploratory, holistic single-case design (Yin, 2014) was utilized to consider how characteristics of the growth mindset influenced a mathematics teacher's interpretations and enactments of her professional development experiences. A single participant, Ms. Martin, representing the critical case of a teacher displaying strong growth mindset characteristics and engaged in the processes of teaching change, was selected as the focus of the study. Multiple sources of data, including historical records, interviews, classroom observations, video self-analysis, and a reflective journal, were used to develop a rich description of Ms. Martin's unique change environment. The domains of this environment that were examined included Ms. Martin's beliefs and mindset regarding the teaching and learning of mathematics, her described and observed mathematics teaching practices, her broad perceptions of her experiences during an ongoing professional development project (Project Influence), her specific areas of focus during a demonstration lesson, her enactment of this demonstration lesson in her classroom, and the classroom outcomes arising from these domains. The data generated in these domains was reduced and organized in a chronological fashion, with a series of holistic themes generated through a simple time series analysis. A narrative description of these themes comprised the full results of the study, which are summarized in the next section.

Summary of Results

The results in Chapter 4 of this dissertation presented four related aspects of Ms. Martin's professional change environment. First, aspects of Ms. Martin's mindset and beliefs related to mathematics and the teaching and learning of mathematics were examined in order to establish a context through which to examine her teaching practices, experiences in professional development, and enactment of these experiences in her classroom. With this perspective as a lens, Ms. Martin's perceptions of her teaching practices and the researcher's observations of these practices were presented with the intention of establishing a baseline understanding of the daily activities occurring in Ms. Martin's mathematics instruction. Next, the impact of external constructs on this teaching environment were examined, first by considering Ms. Martin's perceptions of her past experiences in an ongoing mathematics teaching professional development project and then by looking into her specific areas of focus during a demonstration lesson that was part of this project. Finally, Ms. Martin's enactment of this demonstration lesson in her own classroom and her reflections regarding this enactment and its outcomes were reported. Brief summaries of the results from each of these areas constitute the remainder of this section.

Mindset and Beliefs Regarding the Teaching and Learning of Mathematics

Ms. Martin was found to exhibit strong characteristics of a growth-oriented mindset regarding mathematical ability and to espouse beliefs about the teaching and learning of mathematics, which were well aligned with the definition of reform-oriented instruction presented in this study. Historical survey data supported the claim of Ms.

Martin's growth-oriented mindset, and evidence was presented that supported both Ms. Martin's understanding and awareness of the importance of mindset in mathematics teaching and learning and her operationalization of this mindset through the tenets of goal setting, goal operating, and goal monitoring. Historical evidence of Ms. Martin's beliefs indicated shifts during the last two years towards an understanding of: the nature of mathematics as a web of interrelated concepts and procedures, the power and generative nature of mathematical concepts, the uniqueness and value of children's ways of thinking about mathematics, and the role of children's thinking about mathematics in their learning of mathematics. Additionally, Ms. Martin was found to espouse current beliefs about mathematics as a connected system, the value of mathematical structures and concepts in understanding mathematics, the differences in children's and adults' views of mathematics, and the importance of students' thinking and communicating in learning mathematics.

Described and Observed Baseline Teaching Practices, Activities, and Outcomes

Ms. Martin described her teaching practices in a manner consistent with the tenets of reform-oriented instruction, and baseline observations of her classroom largely corroborated these descriptions. Additionally, historical records of observations of Ms. Martin's classroom during the first two years of Project Influence indicated a shift towards more reform-oriented teaching practices during this time. Ms. Martin's described and observed teaching practices were found to serve five distinct roles: establishing a supportive learning community, engaging students in thinking about and discussing mathematics, facilitating mathematical discussions in a productive manner, holding students accountable for their thinking, and ensuring the success of students at all levels of ability. Ms. Martin credited a high proportion of her use of these practices and her specific classroom activities (i.e., the use of mathematical tasks, number talks, and mindset discussions) to her experiences in Project Influence. Based on these activities and practices, Ms. Martin described seeing positive outcomes in her students' mathematics achievement, their understanding of specific mathematical concepts, their abilities as problem solvers, their abilities to relate mathematical strategies to one another and transfer conceptual ideas to new contexts, and their affective associations with mathematics.

Perceptions of Experiences During Project Influence

In describing her most significant experiences during Project Influence, Ms. Martin cited her involvement in both immersion and practice-based activities as influencing specific aspects of her beliefs and teaching practices. She described her experiences being immersed in mathematical problem solving as valuable due to the exposure she received to new teaching practices, the fashion in which her instructors modeled these practices in use, and the opportunity they provided for her to engage in learning mathematics in a student-centered environment. In referencing her practice-based experiences, she spoke explicitly about the role of the project's demonstration lessons as a bridge between the practices modeled by the project's faculty and the authentic elementary classroom, about the importance of classroom norms that emphasize student thinking and communication, and about the value of teachers observing one another's teaching practices. Ms. Martin credited the combination of these experiences during Project Influence with facilitating at least seven significant changes in her beliefs and teaching practices related to mathematics. These changes centered on the mathematical goals she set for her classroom, the manner in which she assessed and utilized student thinking about mathematics, her role in facilitating students' communication, her questioning practices, and the norms she established for her mathematics classroom.

In addition to describing her past experiences with Project Influence, Ms. Martin participated in a demonstration lesson during the course of the study and discussed her planned and actual areas of focus during the lesson. Prior to the lesson, Ms. Martin indicated that she would be focusing exclusively on the instructional practices of the expert teacher, particularly as they related to the logistical and epistemological facilitation of the lesson's task including the lesson's setup, the supports and scaffolding provided for students, and the questioning practices of the expert teacher. Although approximately one-half of Ms. Martin's recorded observations focused on these areas, the remaining one-half centered on the actions, representations, and words of the students as they engaged with the lesson. In general, Ms. Martin focused on the role communication about mathematics had played in the lesson, both through the expert teacher's facilitation and as students shared their ideas with one another. She also discussed the importance of the expert teacher's high expectations for students' thinking and communicating about mathematics, the instructional practices that shifted these responsibilities to students, and the salient outcomes that depended on these expectations and practices.

The Participant's Enacted Lesson and Reflections

After participating in the demonstration lesson through Project Influence, Ms. Martin planned for, enacted, and reflected on a version of the demonstration lesson in her classroom. In her planning, she focused on setting learning goals for her students which would arise from the lesson, sequencing the lesson in a way that made sense in her mathematical learning trajectory, and anticipating and preparing for the difficulties her students would experience during the lesson. She also elected to utilize the task presented in the lesson as an introduction to new mathematical content, an approach she described as novel for her classroom. Ms. Martin's enactment of the demonstration lesson shared many features with the lesson she observed, and featured an increased proportion of student-to-student discussion in small groups and student-to-student interactions during whole-group discussions over her baseline lessons. She devoted the majority of her instructional efforts to coordinating logistical aspects of the lesson while directing epistemological authority to students through questioning and interactions she described as advancing, redirecting, and facilitating. In her reflections on the lesson and the unit in which it was contained, Ms. Martin spoke of her students' success at three levels: with content-specific learning goals, as progress along her envisioned mathematical learning trajectory, and through increasing students' classroom maturity marked by independence in mathematical thinking and communication.

Discussion of Results

The results of the current study are significant in four primary ways. First, they connect to prior research by providing evidence that further supports the models of professional change (Clarke & Hollingsworth, 2002) and self-regulation (Burnette et al., 2013) from which the theoretical framework of this study was derived. Similarly, they provide a rich description of the role of a key motivational factor, mindset, on teacher change situated in a specific set of professional development activities and context as called for by recent literature (Opfer & Pedder, 2011; Wagner & French, 2010). Second, the results offer theoretical implications as they extend the implicit theories model (Dweck & Leggett, 1988) into mathematics teacher professional development and classroom practice, offer evidence of the growth mindset as a mediator of domains in the IMTPG (Clarke & Hollingsworth, 2002), and build upon current evidence regarding teachers' focus during demonstration lessons (Bruce, Ross, Flynn, & McPherson, 2009; Clarke et al., 2012; Goldsmith, Doerr, & Lewis, 2009). Third, the results offer suggestions for practice for designers of mathematics teacher professional development and classroom mathematics teachers regarding layered goal structures and the cultural processes involved in pursuing these goals. Finally, they recommend questions and considerations for future research to extend the findings of the current study. The remainder of the chapter presents a discussion of each of these important factors.

Connections to Prior Research

The results of this study connect to prior research in three important ways. First, they provide further evidence to support the validity of the IMTPG proposed by Clarke and Hollingsworth (2002). Second, they reinforce the associations between the tenets of self-regulation theory and implicit theories advocated for by Burnette and her colleagues (2013). Finally, they offer a rich description of teacher change motivated by the growth mindset within a specific set of professional development activities and classroom context as called for in recent literature examining teacher change processes (Opfer & Pedder, 2011; Wagner & French, 2010). This section explores each of these connections in detail.

Support for the Interconnected Model of Teacher Professional Growth. The description of Ms. Martin presented in Chapter 4 suggested a reflective practitioner deeply invested in the process of transitioning to reform-oriented instruction. However, the multitude of connections between Ms. Martin's conceptions of teaching and learning mathematics, mathematics teaching practices, experiences in Project Influence, and insights into her students' mathematical development offer substantial evidence that these change processes do not occur in isolation and require an extensive support network to initiate and maintain. Based on the evidence presented regarding these factors and their relationships, a generalized growth network for Ms. Martin's change environment is presented in Figure 20.

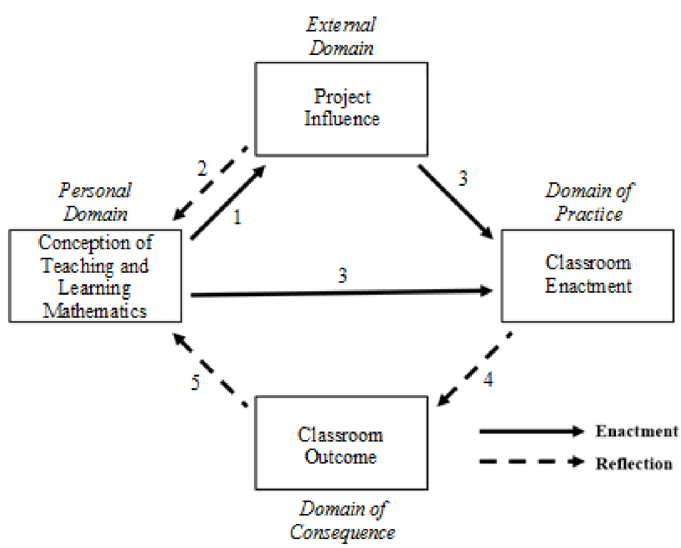


Figure 20. The generalized growth network for Ms. Martin's change environment.

This growth network presents a model of the incremental changes Ms. Martin described experiencing throughout her time in Project Influence combined with the processes observed as she adapted a demonstration lesson from Project Influence for use in her classroom. In this generalized network, some conception of the teaching and learning of mathematics, such as Ms. Martin's valuation of students' thinking and communicating about mathematics or her ideas regarding the manner in which a specific piece of mathematics content should be taught, influenced her interpretations of and interactions during an activity from Project Influence (Arrow 1). Examples of these activities included immersion in a problem-solving task or the observation and debriefing of a demonstration lesson. Ms. Martin's reflections on this experience (Arrow 2) then served to either confirm or incrementally reshape the conception in question. Under the recent influence of this conception, Ms. Martin then adapted some aspect of the Project Influence experience, such as a new classroom norm, questioning practice, or mathematical task, for enactment in her classroom (Arrow 3). Reflecting on this enactment's outcomes, such as the success of a lesson or changes in her students' affect or mathematical understanding (Arrow 4), served to further reinforce or extinguish the conception (Arrow 5) and reinitiate the cycle through external interactions or further classroom experimentation. Although specific examples of this process are provided later in this discussion, the current example serves to explain the general processes in play as Ms. Martin's beliefs and classroom practices changed.

Although this proposed growth network strongly supports the IMTPG, some aspects of the results of this study are difficult to describe in the current model. For example, Ms. Martin cited specific instances in which the student outcomes during an observed demonstration lesson acted much like an aspect of the domain of consequence, directly influencing her planned implementation of the demonstration lesson. Additionally, she cited examples of outcomes with her own students, either as they participated in past demonstration lessons or as they influenced her interactions within Project Influence, which would appear to necessitate a reflective pathway between the domain of consequence and the external domain. Instances such as these bring into question the isolation of the external domain from the "individual teacher's professional world of practice" (Clarke & Hollingsworth, 2002, p. 951), and prompt questions about how well certain external influences can integrate into this professional world.

Support for operationalizing implicit theory through self-regulation theory. The operationalization of implicit theory constructs through the self-regulation theory tenets of goal setting, goal operation, and goal monitoring (Burnette et al., 2013) proved to be extremely robust throughout this study. Ms. Martin's extensive use of learning goals over performance goals (i.e., goal setting) in areas such as her own improvement in mathematical teaching practices, her overall goals for personal development for her students, her mathematical learning trajectory, and her specific mathematical content goals proved to be one of the defining features of her growth mindset. Her focus on mastery responses (i.e., goal operation) for both herself and her students was evident in the specific strategies she developed to help students progress towards her mathematical goals for them (i.e., advancing, redirecting, and facilitating interactions in small groups and mastery-focused assessment practices) and the areas in which she focused her questions and interactions with her students (i.e., strategies, representations, and transferable concepts). Finally, her goal monitoring practices were widely evident and included encouraging students to describe their thinking in order for her to assess their progress, utilizing evidence of student thinking to shape her planning for future lessons and to modify daily instruction as needed, directing specific questions to individuals and groups based on their progress towards lesson goals, and tracking the progress of students along her envisioned mathematics learning trajectory. These examples illustrate a high degree of alignment between the two theories and suggest practical recommendations for operationalizing mindset through interventions based on self-regulation.

Teacher change in context. Analyzed together, the generalized growth network and operationalization of the growth mindset through self-regulation practices presented here provide a broad overview of the motivations and processes behind Ms. Martin's interpretations and enactments of her professional development experiences. Combined with the thick description of these motivations and processes presented in Chapter 4, these models offer the rich account of teacher change in context called for by recent professional development literature (Opfer & Pedder, 2011; Wagner & French, 2010). Additionally, they suggest an alternative framework for interpreting the specific aspects of professional development activities teachers take to their classrooms.

As an example of this type of interpretation, consider Ms. Martin's enactment of the demonstration lesson she observed in her classroom. Many, if not all, of the surface features of the enacted lesson were appropriated directly from the observed lesson. In an interpretive framework such as that proposed by Farmer and his colleagues (2003), these direct appropriations could make it difficult to determine if the teacher was acting from a practitioner, professional, or inquiry stance. In this framework, practitioners are viewed as those who appropriate concrete activities and content directly from a professional development experience. Professionals, in addition to these appropriations, begin to integrate knowledge regarding the activities' underlying principles into their practices. At the highest tier, in addition to adapting concrete elements of the activity and integrating its principles, teachers adopt an inquiry stance in which they learn from the process of teaching.

Noting features of Ms. Martin's use of the lesson such as the specific content goals she set, the goal operating practices she utilized to interact with small groups, and the goal monitoring fashion in which the lesson was matched to her ongoing learning trajectory provides additional insight into her motivations and utilization of the lesson. Combined with the incremental changes in beliefs and practices suggested by her observations on the importance of the lessons' context in supporting her students' learning goals, her novel use of the task to introduce new mathematical ideas to her students, and her recognition of the importance of structured reflection on her implementation of the lesson, a strong case for Ms. Martin as one who "also see[s herself] as learning from (or, perhaps more appropriately, in) the process of teaching" (Farmer et al., 2003, p. 342) can be built. These observations place Ms. Martin firmly in the inquiry stance for this particular lesson adaptation.

This section has provided examples of the manner in which this study supported, questioned, and offered alternate interpretations to prior research. In addition to these important roles it has also previewed the potential of growth mindset practices for mathematics teachers, suggested some overlap between the IMTPD and self-regulation theory, and started to examine how a teacher's focus during professional development impacts her classroom practices. These theoretical implications will be addressed explicitly in the next section.

Theoretical Implications

The results of this study present at least three important theoretical implications. First, the results provide direct, explicit evidence of the operation of growth mindset tendencies in the daily activities of an elementary mathematics teacher. Second, they offer evidence of the role of the growth mindset as a mediator between the domains of the IMTPD. Finally, they add to the findings of an emerging body of research into teachers' areas of focus during participation in demonstration lessons. This section contains discussions of each of these theoretical implications.

Extending the ideas of implicit theory into teacher practice. As described previously, utilizing the self-regulation constructs of goal setting, goal operating, and goal monitoring to observe Ms. Martin's operationalization of her mindset in the classroom proved to be extremely effective. Perhaps one of the most important results of this framework was the revelation of three distinct layers of goals under which Ms. Martin operated throughout the semester (see Figure 21). Although hierarchical language

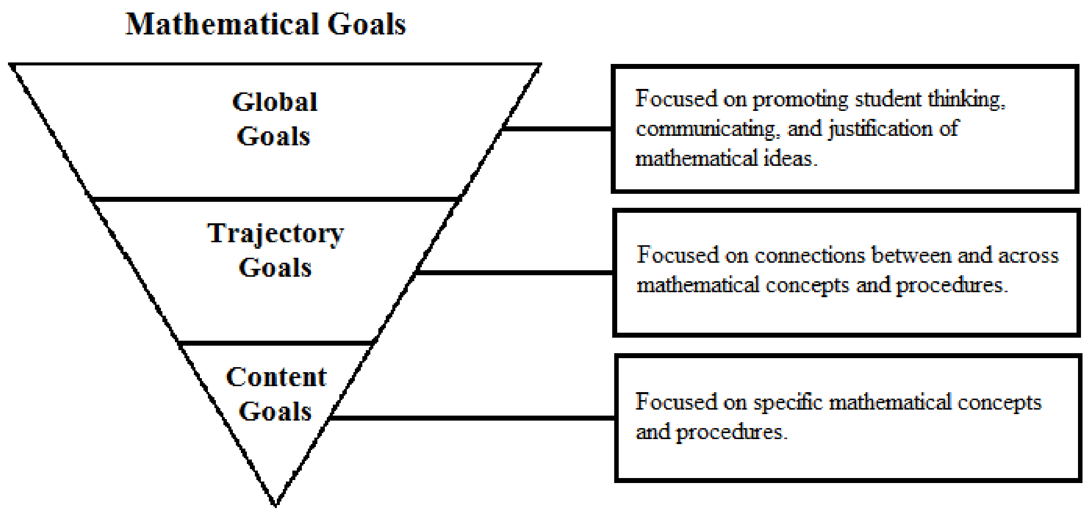


Figure 21. Ms. Martin's layered mathematical goals.

is used to describe these goal structures in the following paragraph, this language relates only to the relationships among the goals themselves and not the value which Ms. Martin ascribed to these goals as they were all spoken of with equal importance.

At the highest layer, Ms. Martin established *global goals* which spanned the length of the school year. These goals tended to focus on widely applicable student mathematical practices such as thinking independently about mathematical ideas, communicating these ideas to others, justifying this thinking, and critiquing the reasoning of others. In an intermediate tier, Ms. Martin described *trajectory goals* that involved assessing and moving students along an evolving mathematical trajectory by helping them connect various mathematical concepts and representations throughout the semester. These goals were spoken about at the level of sequences of lessons and classroom activities, units of instruction, and conceptually related mathematical topics. These goals appeared to be more fluid than the global goals and evolved as the semester progressed based on her students' current understanding. At the lowest level were Ms. Martin's *content goals*, which aligned roughly with her learning goals within a lesson or brief sequence of lessons. Each layer of goals offered distinct opportunities for teacher change during the semester, which are described in the next section.

These layered goals also offered a variety of opportunities to observe Ms. Martin engaging in goal operating and goal monitoring practices on a daily basis (see Table 9). In general, her goal operating practices aligned with utilizing specific instructional strategies to advance individual students, small groups, or the whole class towards her goals for them at different layers. Additionally, she focused heavily on her students' use of mastery strategies throughout her interactions with them. Ms. Martin's general goal-monitoring strategies were focused on making students' mathematical thinking visible to her and the students' peers. This thinking could then be used for assessment purposes as Ms. Martin compared students' progress to her learning goals, for redirection of small groups or facilitation of whole-group discussions, or for discussion and critique from the students' peers.

Table 9

Teaching Practices Indicative of Ms. Martin's Growth Mindset

Goal Operating Practices
Explicit discussions of mindset during mathematical tasks and through *The Dot*
Encouraged effort and engagement for all students through selection of accessible tasks, questioning practices, and accountability practices
Publically offered appreciation for struggle and provided scaffolding within and across lessons to support productive struggle
Modeled accountable talk and questioning practices to promote student interactions
Offered praise, feedback, and questioning of students' numeric representations, mathematical processes and strategies
Utilized strategies for advancing and redirecting student thinking within lesson goals
Respected and examined mathematical mistakes publically and without judgement
Goal Monitoring Practices
High expectations for student thinking and communicating about mathematics for both self/peer monitoring and informal assessment purposes
Suggested positive self-talk to students to help avoid negative affective associations
Encouraged students to monitor their own and peer progress during mathematical tasks
Utilized student thinking through informal assessments to inform and adapt daily instruction and planning for future lessons
Held students accountable for engagement and thinking during lessons
Aligned formal assessments with goals at all levels and assessed using a mastery/retesting format

Extension of the models of the theoretical framework. Perhaps the most significant implications of the study's results, and the most direct answers to its research question, involve two sets of overlaps within its theoretical framework. The first of these overlaps exists among the connected growth networks of Ms. Martin's change environment and is evidenced by considering specific examples of the generalized growth network proposed in this chapter across Ms. Martin's three goal layers. The second overlap, between the theoretical propositions of the IMTPG and implicit theories model, becomes apparent when considering the role of mindset as a mediator between the domains of the growth networks in these specific examples. It is from this second case that a new conceptual framework merging these theories arises.

Overlaps within Ms. Martin's generalized growth network. As suggested at the onset of this discussion, the generalized growth network for Ms. Martin's change environment was derived from specific descriptions of her past experiences in Project Influence combined with evidence collected as she adapted the demonstration lesson she observed as part of the study for use in her classroom. Three examples of the specific evidence for this growth network, one related to each of Ms. Martin's goal levels, are provided here in order to illustrate their impact on the process of teacher change (see Figures 22, 23, and 24). These models each share a common pathway (Arrow 1 in each diagram) representing the influence of Ms. Martin's personal domain characteristics on her interpretations of her professional development experiences. Additionally, as the examples provide evidence of the same growth network pathways across different domain characteristics depending on the goal level involved in the network, the diagrams share common labeling of these pathways tailored to each diagram (e.g., 2a, 2b, 2c represent the reflective pathway between the external domain and the personal domain at three different goal levels).

The first example, situated at the level of Ms. Martin's global goals, involves changes in Ms. Martin's beliefs and teaching practices regarding the value of students' thinking and communicating about mathematics (see Figure 22). Although the primary evidence for this growth network was derived from Ms. Martin's descriptions of her changes in practices based on her earliest experiences with Project Influence, the network is paralleled in the reinforcement of these beliefs and practices during the most recent

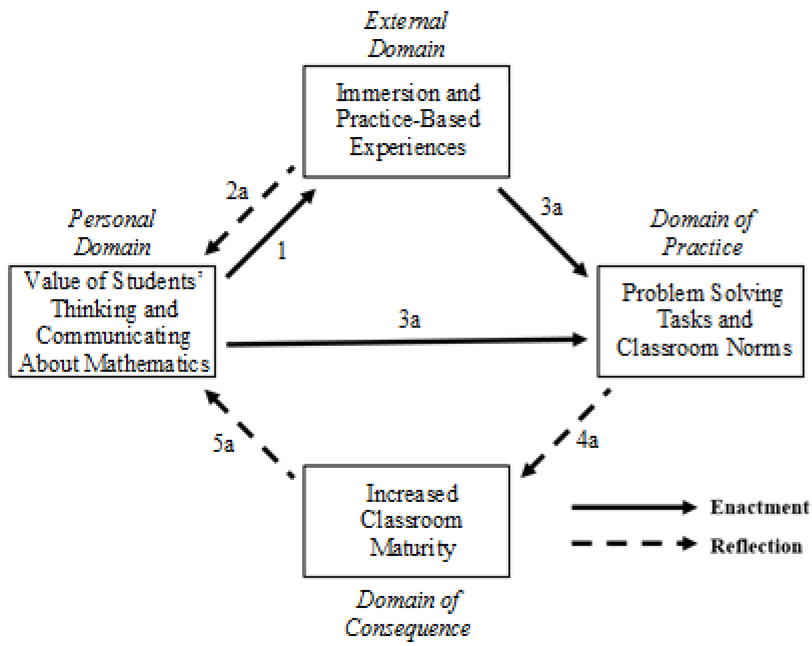


Figure 22. The growth network for Ms. Martin's beliefs and practices regarding the value of students' thinking and communicating about mathematics.

demonstration lesson. In this growth network, Ms. Martin became involved with Project Influence due to her desire to continue improving her abilities as a mathematics teacher, which she attributed to her own growth mindset and love of mathematics (Arrow 1). Note that in each example provided, the specific personal domain focus of the growth network was not established until after Ms. Martin's experience in Project Influence. During her first experiences with Project Influence, Ms. Martin became aware of reform-oriented teaching practices related to the value of students' thinking and communicating about mathematics due to the modeling of Project Influence's faculty and witnessing these practices in use in a demonstration lesson. Based on these experiences Ms. Martin developed goals for her classroom aligned with these practices (Arrow 2a). Operationalizing these goals, Ms. Martin described adopting the norms and problem-solving approaches she had experienced in Project Influence to her own classroom (Arrow 3a), and noted the influence that these practices had on her students' abilities in this area, which she later described as students' classroom maturity (Arrow 4a). Ms. Martin described these changes as transformative to both her way of thinking about (Arrow 5a) and teaching mathematics (Arrow 3a) and elected to continue her involvement with Project Influence when given the chance (Arrow 1), initiating a cyclic process.

In other iterations of this growth network, Ms. Martin's goals at different levels were involved. As an example, during her activities involving the demonstration lesson of this study, she described considering how the lesson matched with her envisioned mathematics learning trajectory (see Figure 23). Her reflections on aligning this lesson involved finding an appropriate place in the trajectory for its use (Arrow 2b) and her planning for the lesson involved preparing her students to engage with its content by transferring concepts from earlier in the year and considering the types of interactions she would use to address struggles she had witnessed from students in the observed demonstration lesson (Arrow 3b). Once the lesson was enacted, Ms. Martin reflected on its specific outcomes (Arrow 4b), and considered these outcomes in terms of the sequence of lessons and unit in which it was situated (Arrow 5b).

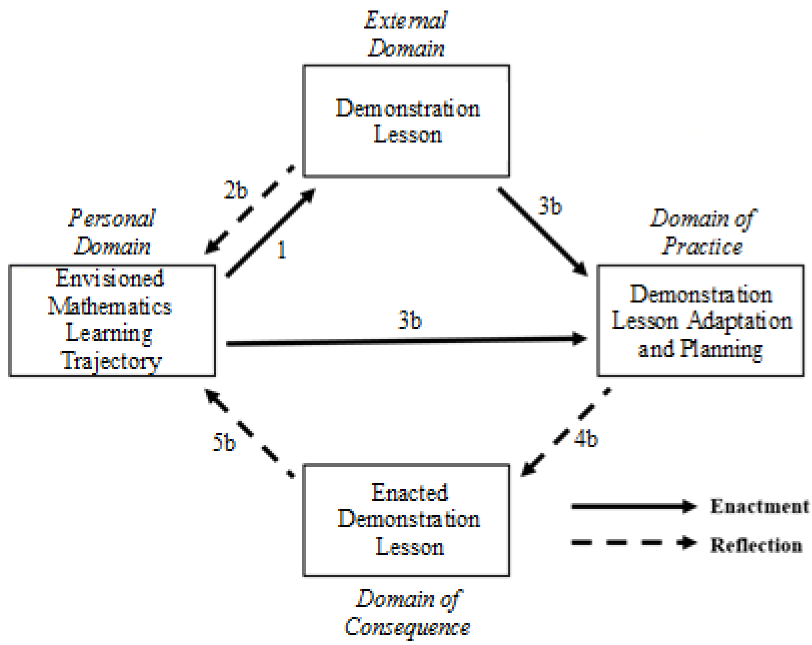


Figure 23. The growth network for Ms. Martin's envisioned learning trajectory.

The same set of contexts can be reexamined to look at changes based on Ms. Martin's specific content goals for her enacted demonstration lesson (see Figure 24). In this case, her reflections from the demonstration lesson focused on the manner in which she would prepare her students to examine how different representations of numbers could be used to support thinking about numeric operations such as subtraction (Arrow 2c). These reflections, along with the core activity from the demonstration lesson, were then used to prepare her students to work with this idea during the enacted demonstration lesson by scaffolding the idea in the lesson immediately preceding it (Arrow 3c). Immediately following this lesson Ms. Martin described monitoring her students' progress with the content goals of the enacted lesson (Arrow 4c) in order to determine the course of the remaining lessons in the sequence (Arrow 5c).

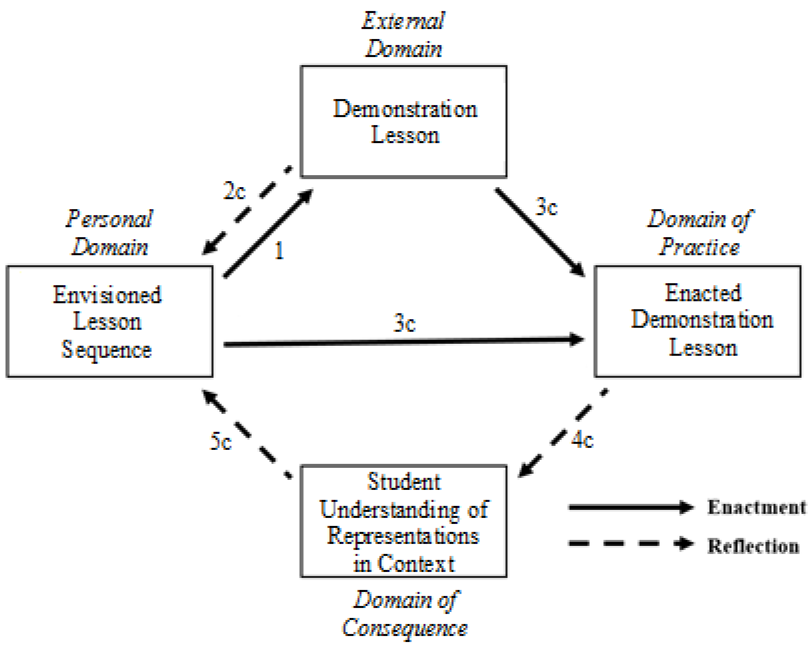


Figure 24. The growth network for Ms. Martin's lesson goals for her enacted demonstration lesson.

Recall that Ms. Martin's descriptions supported interpreting the first example given here as reinforcing her current beliefs and practices due to her experiences with the demonstration lesson in this study. In this case, all three of the examples shown share the same form of initial change sequence as their first enactment pathways (Arrow 1 in Figures 22, 23, and 24). In these shared pathways, a goal-dependent conception of the teaching and learning of mathematics, each at a different goal level, influenced Ms. Martin's interpretations of the demonstration lesson. These layered change sequences then lead to unique connected growth networks in which the mediating pathways are identical while the specific domain foci are dependent on the goal being operationalized. This highly connected, multidimensional growth network offers a potential explanation for why Ms. Martin's experiences in Project Influence have been so influential on her beliefs and practices.

Overlap between the IMTPG and implicit theory. Examining the growth networks presented in this chapter through the lens of implicit theories reveals an abundance of connections between the IMTPG and the implicit theories model. In general, Ms. Martin interacted in a professional development experience, established a set of goals based on the experience, adapted some feature of her classroom based on these goals and experiences, and monitored the results of these adaptations. More specifically, the tenets of goal setting, goal operating, and goal monitoring are shown to act as mediators of the domains of these growth networks, serving as both enactive and reflective pathways. Pathways 2a, 2b, and 2c (see Figures 22, 23, and 24) illustrate goal setting as a reflective process based on Ms. Martin's interpretations of her experiences in Project Influence. Pathways 3a, 3b, and 3c demonstrate Ms. Martin operating on these goals in an enactive fashion as she adopts, plans for, and implements features of her professional development experiences based on the goals she established. During and after these enactments, Ms. Martin monitored either her own or her students' progress towards the goals she set in a reflective manner via pathways 4a, 4b, and 4c and evaluated the goal itself, and perhaps its underlying beliefs, in relation to this progress in pathways 5a, 5b, and 5c. These examples offer consistent evidence of aspects of Ms.

Martin's mindset influencing her interpretations and enactments of her professional development experiences and provide an explicit conceptual framework to address the study's research question.

A new conceptual framework arising from these overlaps. Together, the examples in the previous two sections provided converging evidence of the tenets of self-regulation theory most closely associated with the growth mindset mediating changes within Ms. Martin's growth networks at different goal levels. Combined with the initial theoretical framework of the study, this empirical support provides the foundation for an analytic generalization of the manner in which these processes serve as mediators of the change environment (see Figure 25). This goal-mediated model establishes a new conceptual framework emerging from the ongoing interactions between the theoretical concepts guiding the study and the empirical evidence generated during the research process.

More specifically, for the growth mindset, an individual recognizing that he is capable of changing an ability interprets his experiences in the external domain in a manner that supports this change. Through reflection on these experiences, he establishes goals that incorporate specific aspects of the experience into the personal domain. As evidenced by Ms. Martin's case, multiple goals at different levels can arise from a single experience and produce a highly connected, multidimensional growth network that supports sustained changes at each level. These changes occur through the enactive process of goal operation and the reflective process of goal monitoring.

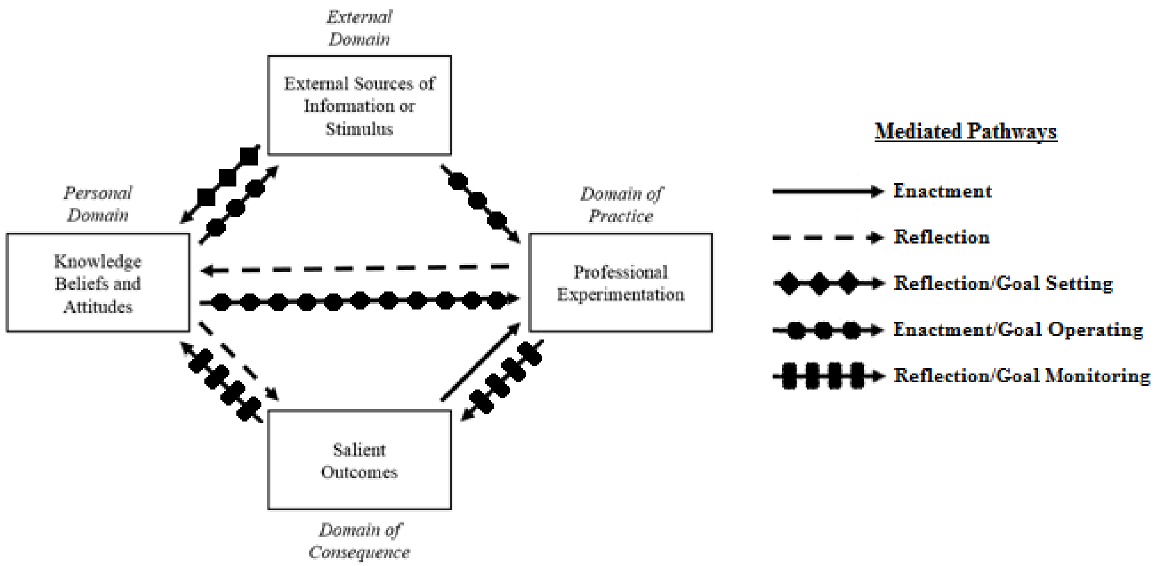


Figure 25. An adapted model of teacher professional growth incorporating tenets of self-regulation theory as mediators of the change environment. Aspects of the growth mindset are incorporated into this model through the self-regulation processes of goal setting, goal operating, and goal monitoring. Adapted from "Elaborating a Model of Teacher Professional Growth," by D. Clarke and H. Hollingsworth, 2002, *Teaching and Teacher Education, 18*, p. 951.

Goal operating occurs as the individual adapts his experiences in the external domain, in a mastery fashion, through the lens of his newly established goals to produce a change in the domain of practice. Depending on the goal level involved, these changes can include practices such as the establishment of new classroom norms, the planning and enactment of a lesson sequence, or the implementation of a specific task or activity encountered in the external domain. Based on outcomes occurring during and after this experimentation, the individual engages in goal monitoring to track progress toward the goal, tune his expectations based on the reality of the implementation's outcomes, and adjust his practices to continue progressing toward the goal.

This adjustment of practices is particularly important as it represents an enactive interaction between the domain of consequence and the domain of practice that is not labeled in the model in Figure 25. The pathways labeled in this model represent cases with sources of evidence from each of Ms. Martin's goal layers, and the adjustment of practices pathway was only observed directly as she monitored students' progress toward her content goals during the enacted demonstration lesson and adjusted her instructional practices accordingly. Similarly, although it can be inferred that Ms. Martin's classroom experimentations influenced her personal domain characteristics and that these characteristics influenced her monitoring of relevant outcomes, these pathways were not included in the model as they were not directly observed for multiple goal levels.

Teacher's focus during demonstration lessons. One area of the study that yielded unexpected results involved Ms. Martin's areas of focus during the demonstration lesson she observed. The areas she proposed to focus on prior to the demonstration lesson included the instructional practices of the expert teacher, particularly as they related to questioning and supporting struggling students, and the structural features of the lesson. These focus areas were well aligned with previous literature on the topic, as was the fact that Ms. Martin failed to identify a student-related observational focus (Clarke et al., 2013). However, in her descriptions of her areas of focus after the demonstration lesson, it was revealed that nearly 60% of her observational focus had indeed centered on students, particularly in their communication, understandings and representations of the lesson's content, and actions during the lesson. Additionally, half of these student-focused observations related to collaborative interactions among students, which is a rarely identified area of focus for teachers observing demonstration lessons, with Clarke and colleagues (2013) identifying only 2.5% of 200 teachers reporting this as an area of focus. Finally, Ms. Martin offered a unique insight into the role of the demonstration lesson as a bridge between her other experiences during professional development and the realities of the elementary classroom.

Although some studies report that a student-related observational focus may develop over time (Goldsmith et al., 2009), Ms. Martin's lack of an identified student focus combined with the high proportion of her actual observations in these areas and the nature of the observations she made suggests her case may be unique. What factors of her case prompted this observational focus and further investigation into how these focus areas develop through participation in multiple demonstration lessons is likely warranted. Additionally, these findings suggest the need for an explicit call for both instructional and student-focused observational foci during programs featuring demonstration lessons that wish to highlight these features.

This section focused on the theoretical implications of the current study and the extensions it offers to earlier theoretical models and other empirical research. The most significant implications of the study were found in two theoretical extensions. The first of these related to the description of a layered, multidimensional version of the growth network for Ms. Martin's case arising from the IMTPG. The second proposed an explanation of the manner in which the implicit theories model, described in terms of self-regulation theory, acted as a mediator for the domains of the IMTPG in both enactive and reflective capacities. Additionally, the section contained a discussion of new insights into the focus areas of teachers during demonstration lessons that offer some contrast to previous literature. The practical implications of these findings are described in the next section.

Suggestions for Practice

The results of the study offer several practical suggestions to inform both developers of mathematics teacher professional development programs and classroom mathematics teachers. In both cases, the suggestions revolve around leveraging the layered goal system and growth mindset characteristics exhibited by Ms. Martin to different effects. For designers of professional development, examining the growth networks presented in this chapter provides an opportunity to consider processes of incremental cultural change in the mathematics classroom. For practitioners, Ms. Martin's self-regulation characteristics are particularly informative. This section will discuss the application of the study's results in both of these settings.

Considerations for designers of professional development. The idea of teaching as a cultural activity is well established in the world of mathematics education (Lerman, 2000; Stigler & Hiebert, 1999). The study's results offer two significant insights into the incremental nature of cultural change in the classroom, both arising from the growth networks examined throughout this chapter. The first of these relates to the cyclic nature of the growth networks exhibited by Ms. Martin. The second attends to the fact that a single professional development activity can offer opportunities for development along multiple concurrent routes. In both cases, Ms. Martin's familiarity and comfort within the professional development environment suggested it was "organized in ways that closely align to teachers' professional practice, including opportunities to enact certain (innovative) instructional strategies and materials and to reflect, individually and collectively, on their experiences" (Van Driel & Berry, 2012, p. 27).

Ms. Martin's experiences suggested that long-term professional development programs should be designed around sets of core principles that serve a variety of goal levels with repeated opportunities to engage and reflect on these principles over time. The recommendation for repeated exposure is supported by Ms. Martin's success in enacting the observed demonstration lesson in her classroom. Her ultimate success in this enactment appeared to be dependent on two key factors: her classroom norms related to problem solving and communicating one's thinking about mathematics and her ability to effectively monitor her students' progress along her envisioned learning trajectory in order to determine an appropriate time to implement the lesson. Without these two elements in place, her implementation would likely have been much less successful. Critically, both of these elements were seen to have evolved via the cyclic growth networks arising from her ongoing engagement with Project Influence. The incremental adaptations she had made through adopting key features of her prior experiences in Project Influence effectively primed her classroom for the learning goals and operational strategies she utilized to enact the demonstration lesson.

Further evidence for the recommendation of repeated exposure to core principles derives from a comparison of Ms. Martin's use of mathematical tasks to her implementations of number talks. In her use of mathematical tasks throughout the semester, key lesson structures were in place to allow students multiple opportunities to organize their thinking and interact with one another regarding the mathematics under study. As an example, her use of the mathematical task in the demonstration lesson built on key features of her daily task implementations, which in turn built upon her prior exposure to previous demonstration lessons. In contrast, during her number talks, which she had only started implementing during the semester of the study, the vast majority of communication occurred directly between the teacher and one student at a time, and much of the organization of the mathematics described was directed by the teacher. This contrasting style of implementation illustrates the value of repeated exposure as activities with which the teacher is more familiar are implemented more effectively.

This example also illustrates the importance of designating core principles to serve a variety of goal levels, as it is likely that the norms, mathematics learning trajectory, and mathematical teaching practices Ms. Martin had in place in her classroom will ultimately allow her experimentation with number talks to be successful. In the brief time she had been utilizing this activity, she had already observed its value in helping her students see connections across mathematical representations and concepts, facilitating her goals of moving students along a trajectory of connected mathematical ideas. It is not difficult to imagine that as her students become more comfortable in the norms of the classroom they will begin to interact with one another in this setting just as they do in their other activities. Activities such as the use of mathematical tasks and number talks are unlikely to succeed without first establishing productive norms, mathematical learning trajectories, and content goals in which to embed them. Ms. Martin's case suggests that professional development programs that recognize the value of establishing core principles at all of these levels offer opportunities for growth at all of these levels and promote sustained teacher development.

Considerations for classroom teachers. The most directly useful aspect of Ms. Martin's case for the classroom teacher is likely the fashion in which her mindset was operationalized through her goal setting, operating, and monitoring practices. Although all classroom teachers may not operate under the tenets of a growth mindset on a daily basis, many of these goal-related practices can be easily adapted to any classroom. Setting goals that support student interactions about mathematics and that focus on mathematical concepts and strategies that transfer are broadly useful. Operating toward these goals by interacting with students via advancing, redirecting, and facilitating strategies appears to require little adaptation to the questioning approaches many teachers already use. Goal-monitoring practices such as focusing on student thinking and evaluating student progress against a mathematical learning trajectory align well with globally accepted assessment practices. However, utilizing these approaches in isolation, without acknowledgement of the classroom culture that underlies their success in this study will likely produce little lasting change.

To that end, identifying and reflecting on the most foundational classroom norms from Ms. Martin's case would help establish the environment within which her goal-related practices were enacted. Although it is difficult to distill such a complex environment into a brief list, the pillars of Ms. Martin's classroom environment appeared to be her high expectations for her students to think about and engage with mathematics, her requirements for her students to communicate and justify their mathematical ideas, the degree of accessibility provided by the mathematics activities she utilized, and the degree of accountability to which she held all students to these norms. These norms appeared to facilitate all of the other interactions in Ms. Martin's classroom and would thus provide an excellent starting point for implementing the practices described above. In combination, these norms and practices are likely to promote a classroom culture leading to the classroom maturity and mathematical success Ms. Martin's students experienced.

This section has elaborated on the practical implications of the study, culminating in specific recommendations for both designers of mathematics teacher professional development and classroom mathematics teachers. The primary recommendation for professional development programs is to establish a set of core principles at multiple goal levels which align with the program's values and provide multiple opportunities for participants to interact with and reflect on these principles over time. For classroom teachers it is suggested to look at not only the specific goal-related classroom practices utilized by Ms. Martin, but also at the culture of the classroom in which they were implemented. The final section of this chapter offers recommendations for future research based in the findings of this study.

Recommendations for Future Research

This study explored how characteristics of the growth mindset influenced a mathematics teacher's interpretations and enactments of her professional development experiences and provided a rich description of one teacher's experiences with this topic. From this exploratory case, an evidence-based conceptual framework was offered that suggested the manner in which aspects of the growth mindset, operationalized through self-regulation theory, served as enactive and reflective mediators of the domains of a teacher's change environment. Although this initial framework provided a tentative answer to the study's research question, it also leaves many questions to be addressed.

The most important questions focus on the robustness and durability of the proposed framework. In order to establish its validity as anything more than a theoretical construct, the framework must be examined in a variety of environments and with a wide range of participants. Assuming that it withstands these initial examinations, it needs to be tested with larger samples in order to examine its generalizability. Additionally, while multiple layers of Ms. Martin's connected growth network were presented, questions regarding the effective order of development of these layers remain. Based on her descriptions of her earliest experiences in Project Influence, Ms. Martin's case suggested that the global layer likely developed first, but this is highly dependent on her personal experiences in the project. It is likely that many teachers establish these layered goals for their classrooms, and it would be interesting to consider how professional development experiences and goal networks influence one another.

Based on prior research commentary that suggested that many teachers' "main focus during demonstration lessons was on the actions of the teacher and that adopting a student focus was difficult for them" (Clarke et al., 2013, p. 221), Ms. Martin's strong emphasis on students' mathematical thinking and classroom interactions may also be atypical. Although this focus appeared to have been influence by her mindset, this is certainly not the only factor involved. Research indicating increased comfort with a student focus during demonstration lessons based on repeated exposures to the format (Goldsmith et al., 2009) suggests an alternate influence and supports the recommendations for repeated exposure offered in this chapter. The circumstances leading to this focus, particularly with its prominence in Ms. Martin's descriptions of the study's demonstration lesson, present an important consideration for professional development programs that wish to promote a student-oriented focus. Additionally, Ms. Martin's reflections on the study's demonstration lesson evidenced that multiple goals and classroom strategies arise from an experience such as this. How teachers of different mindsets elect which goals to pursue and what strategies they use to pursue them would likely provide further insight into this study's results.

Finally, if the study's proposed framework proves to be valid, a variety of interventions for both teachers and students could easily be developed based on the tenets of goal setting, goal operating, and goal monitoring. Grounding these interventions in strong theoretical constructs and empirically testing their effectiveness in mathematics professional development and mathematics classrooms against proven metacognitive strategies would further validate the findings of this study.

Chapter Summary

Teacher conceptions of mathematics and the teaching and learning of mathematics share a complex relationship with the quality of their classroom instruction. One set of these conceptions, implicit theories, has not been well examined within mathematics teacher professional development. Thus, this study examined how characteristics of the growth mindset influenced a mathematics teacher's interpretations and enactments of her professional development experiences via an exploratory case study considering aspects of the teacher's beliefs, classroom practices, perceptions of her professional development experiences, and enactments of these experiences in her classroom. A rich description of these aspects was developed, which showed a complex relationship among the teacher's beliefs, goals, mindset, experiences, and classroom practices.

This chapter discussed these complexities by examining how they connected to prior research, interacted with the study's theoretical framework to propose a new conceptual framework addressing the research question, recommended specific practices for classroom teachers and designers of professional development, and suggested further avenues of research related to the study. Overall, the study's results were found to support the canon of its theoretical framework. In addition, a new conceptual model in which tenets of the implicit theory were shown to serve as enactive and reflective mediators between the domains of a teacher's change environment was introduced. Additionally, the study's results provided insights into a layered goal structure under which its case teacher operated. The results suggested that designers of professional development take advantage of these findings by offering participants repeated opportunities to interact with and reflect on core principles at multiple goal levels. Similarly, classroom teachers should be aware of how setting goals, operating toward them, and monitoring their progress can impact their classrooms if implemented in an appropriate classroom culture. Recommendations for future research included testing the robustness and validity of the study's conceptual framework and examining the details of how goal-related practices develop.

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APPENDICES

APPENDIX A: PARTICPANT SELECTION INTERVIEW PROTOCOL

S1. During the last year of our professional development program, we have often referenced the idea of mindset. What does mindset mean to you?

S2. Would you describe your mindset as fixed, or growth? Why?

S3. What are some areas you have the most growth-oriented mindset toward? What evidence do you have for this?

S4. What are some areas you have the most fixed-oriented mindset toward? What evidence do you have for this?

- If mathematical ability and mathematics teaching are not mentioned in S3 or S4, ask about them now.

S5. How would you describe the subject of mathematics? What are its most important features?

S6. What does it mean for a student to learn mathematics?

S7. What is the relationship between mathematical concepts and mathematical procedures?

S8. Do students think about mathematics the same way as adults? How is their thinking alike or different?

S9. What is the teacher's role in teaching mathematics? The student's role in learning?

S10. What has been your overall experience with our professional development project?

S11. Do you see changes in yourself or your teaching practices based on the project? Can you describe those changes?

S12. How do you typically use the demonstration lessons you observe in our professional development?

APPENDIX B: BACKGROUND AND MINDSET INTERVIEW PROTOCOL

B1. Describe your teaching background. How long have you taught, what grades and subjects, what type of teacher preparation, professional development, etc.

B2. How did you become involved in our professional development program?

B3. What are the most important experiences and things you have learned from our professional development program?

B4. What do you hope to get out of the program in the future?

B5. Describe some of your best math students. What do they have in common? As a math teacher, what is your role with these students?

B6. Describe some of your students who struggle the most with math. What do they have in common? As a math teacher, what's your role with these students?

B7. Why do you think some students struggle more with math than others?

B8. What do think you can do to improve your teaching for all of the students in your math classes?

B9. How does a student's mindset regarding their mathematical ability impact their mathematics learning?

B10. Do you believe that your mindset impacts your students' learning? How so?

B11. What type of goals have you set for yourself and your students this year?

B12. How did you develop these goals?

B13. Do you have specific plans to help achieve them? How do you develop your plans to help achieve goals such as these?

B14. Are you implementing any new classroom or teaching practices to help accomplish these goals? Can you describe these?

B15. Are there specific resources of you will use to reach you goals for the year? Are there any resources you need, but don't have access to?

APPENDIX C: PROFESSIONAL DEVELOPMENT INTERVIEW PROTOCOL

D1. Thinking back to the demonstration lesson you observed, what were your overall impressions of it?

D2. What portions of the demonstration lesson did you think were most important?

D3. Did any particular teaching practices you observed during the lesson stand out to you? Can you describe these?

D4. What were the most important mathematical ideas in the lesson?

D5. During the lesson, you recorded this video segment [play participant recorded video segment]. Why did you think this moment was particularly important?

- This question will be repeated approximately five times regarding different segments of participant recorded video.

D6. What aspects of the demonstration lesson will you be able to best use in your classroom?

D7. Did you see anything in the lesson that you could use on a daily basis? Why are these (ideas/techniques/practices) so important to you?

D8. What are your initial thoughts about incorporating the demonstration lesson's content focus into your lesson plans for this semester?

D9. How will this approach support your instructional goals?

D10. How would a lesson like this fit into your overall unit design?

D11. During the (pre-lesson briefing/post-lesson debriefing/afternoon professional development activity) you (observed/stated/participated in) ____________________. Can you tell me a little more about (what you were thinking/why this was important to you/how you will use this)?

- This question format will be repeated three to five times, depending upon observations of the participants interactions during the professional development activities.

D12. Is there anything else about the day that you would like to note?

APPENDIX D: PLANNING AND GOALS INTERVIEW PROTOCOL

G1. When we last met, you described using the demonstration lesson you observed ________________. Has that idea changed as you've planned for the upcoming unit of instruction?

G2. What factors influenced the changes that you've made?

G3. Were there specific people or resources you used in your planning? How did they help shape your instructional plans?

G4. Are your students prepared for the upcoming unit? How do you know?

G5. Can you describe the upcoming unit to me? How does the content from the demonstration lesson fit into the plan?

G6. Are there non-content aspects of the demonstration lesson you hope to utilize in this unit of instruction?

G7. What are your overall goals for this unit of instruction?

G8. Can you tell me more about the instructional goals of each lesson?

G9. What are the key points of each lesson that would best support your students' success with the lesson and unit?

G10. What do you expect the most difficult points of each lesson to be?

G11. What will you do to support your students at these difficult points?

G12. Can you describe your classroom and students for me? What types of things should I know to help me better understand how you will be teaching?

G13. How would you describe your school? Are there things I should know to help me understand your students and teaching?

G14. What aspects of the local community set it apart from others? How do these impact your school and teaching experiences?

G15. Is there anything else that you can tell me that will help me better understand your teaching?

APPENDIX E: EVALUATION AND REFLECTION INTERVIEW PROTOCOL

E1. What were your overall impressions of the unit? What went best and where were the struggles?

E2. How did your use of the demonstration lesson work with this unit? Would you change the manner in which you used it? How so?

- If both content and practices are not described, ask about them here.

E3. Are there any particular (activities/discussions/moments) from the unit that stood out to you? Why were these important?

E4. [Play video from the description in E3, or response in the reflective journal]. What stands out to you about this video segment? Did you notice anything new? Why was this event (so successful/such a struggle/etc.)?

E5. How would you (use/change) what you were doing in the video segment in the future? With this (use/change) as a goal, what can you do to make sure you reach it?

- Questions E4 and E5 will be repeated approximately five times regarding different segments of participant recommended video. The researcher will prepare additional segments, based on observations, to supplement this total.

E6. Overall, how successful do you think your students were with this unit? What evidence do you have for their success?

E7. Looking back at the key points for the lessons you described (G9), do you still think these are the most important points? If not, what has changed?

E8. Looking back at the points you expected to be the most difficult in each lesson (G10), have your perceptions changed? How so?

E9. Are there specific things you would do differently to help students at these key points?

E10. Were your goals for the unit, its lessons, and your students appropriate? Would you change these?

E11. How would you modify the unit in the future?

E12. What types of changes in your teaching practices could help you better support you students in this unit in the future?

E13. What resources do you have available to help you plan and implement changes like these? Are there things that you need but do not have access to?

E14. What is the most difficult aspect of making the changes you have described today? How are you able to overcome obstacles such as these?

E15. Is there anything else about the unit, or your future plans that you would like to share?

APPENDIX F: FINAL INTERVIEW PROTOCOL

F1. Looking back across the process of observing, adapting, and implementing the demonstration lesson in this unit, what were the most important things you learned?

F2. What are the most difficult parts of adapting a resource like this for use in your classroom?

F3. How did you overcome those difficulties?

F4. What aspects of the demonstration lesson and this unit did you find most useful?

F5. Can you describe any aspects of the demonstration lesson, or your implementation of it, that you will be able to use regularly in your classroom?

F6. Would it be possible to extend the process you used to adapt the demonstration lesson to other resources? Why or why not?

F7. How did your work in our professional development program over the last two/three years help you in adapting and implementing this lesson?

F8. How did feedback from your students impact the way you delivered the unit?

F9. How supportive are your peers and administrators in implementing this type of instruction?

F10. How similar was this unit to a typical unit of instruction in your classroom? What were the major similarities and differences?

F11. What are the advantages of this type of unit over (your/a) typical unit of instruction? What are the disadvantages?

F12. What kind of factors would prevent (you/other teachers) from teaching in this fashion daily?

F13. How would you go about overcoming (each of the obstacles described in F12)?

F14. What advice would you offer to other teachers preparing to adapt a demonstration for use in their classroom?

F15. Is there anything else you would like to tell me? Any final thoughts about the process that are on your mind?

APPENDIX G: CLASSROOM OBSERVATION PROTOCOL

Time	Persons Involved	Observation and Reference to Mindset Characteristic or Mathematics Teaching Practice Involved

Provide a timestamp and description of the mindset characteristic.	
Goals Focused on Learning	
Strategies for Dealing with Setbacks	
Expectations of Success	
Attributions of Success	
Feedback	
Effort and Challenge	

Provide a timestamp and description of the mathematics teaching practice.	
Establish mathematics goals to focus learning	
Implement tasks that promote reasoning and problem solving	
Use and connect mathematical representations	
Facilitate meaningful mathematical discourse	
Pose purposeful questions.	
Build procedural fluency from conceptual understanding	
Support productive struggle in learning mathematics	
Elicit and use evidence of student thinking.	

Growth Mindset Characteristic Cues

Possible cues to the presence of growth mindset characteristics	
Goals Focused on Learning	• Evaluates situation to establish learning goals • Monitors progress towards goals • Values learning goals over performance goals
Strategies for Dealing with Setbacks	• Perseverance in face of challenge • Adjustments based on evaluation • Develops new goals and strategies to overcome obstacles
Expectations of Success	• Communicates high expectations for all students • Maintains expectations in face of poor performance • Provides resources to support success
Attributions of Success	• Attributes success to effort rather than ability • Values and rewards productive effort • Celebrates success and growth in students
Feedback	• Offers constructive criticism • Focused on effort as opposed to current ability • Focused on strategy verses comfort
Effort and Challenge	• Models effort and embrace of challenge • Provides encouragement and support for students • Displays increased effort in face of challenge

Developed from the work of:

- Burnette, J. L., O'Boyle, E. H., VanEpps, E. M., Pollack, J. M., & Finkel, E. J. (2013). Mind-sets matter: A meta-analytic review of implicit theories and self-regulation. *Psychological Bulletin*, *139*, 655-701.
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- Dweck, C. S., & Leggett, E. L. (1988). A social-cognitive approach to motivation and personality. *Psychological Review*, *95*, 256-273.
- Rattan, A., Good, C., & Dweck, C. S. (2012). “It's ok—Not everyone can be good at math”: Instructors with an entity theory comfort (and demotivate) students. *Journal of Experimental Social Psychology*, *48*, 731-737.

Descriptions of Mathematics Teaching Practices

Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense-making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Source: National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: Author.

APPENDIX H: REFLECTIVE JOURNAL PROMPTS

Period 1: Prior to Baseline Classroom Observations

R1. How do aspects of the growth and fixed mindset impact your classroom?

R2. What does it mean for a student to learn mathematics?

R3. How have you changed the most as a teacher in the last five years?

Period 2: Baseline Classroom Observations

R4. How do you feel today's lesson went? What were the most important things that happened?

R5. Based on today's lesson, how will your plans for tomorrow change?

R6. [A prompt based on each day's lesson observation will be included here].

- Prompts R4, R5, and R6 will be provided for each day of observation.

Period 3: Demonstration Lesson Follow-up

R7. What were the most important mathematical goals of the lesson you observed?

R8. Did you observe any teaching practices that you thought were particularly useful?

R9. What are your initial thoughts on how you will use ideas from the demonstration lesson in your classroom?

- The participant's responses to any reflective prompts utilized during the professional development activities will be recorded here.

Period 4: Implementation of the Demonstration Lesson Unit

R10. How did you adapt the demonstration lesson to the upcoming unit of instruction?

R11. What are your most important goals for this unit?

R12. How do you feel today's lesson went? What were the most important things that happened?

R13. Based on today's lesson, how will your plans for tomorrow change?

R14. [A prompt based on each day's lesson observation will be included here].

- Prompts R12, R13, and R14 will be provided for each day of observation.

Period 5: Post Demonstration Lesson Unit

R15. Overall, how do you feel the unit went? What were the most important things that happened?

R16. Did you accomplish your goals for the unit?

R17. How do you know your students learned what you intended for them to learn?

R18. What will you change the next time you teach the unit?

APPENDIX I: INSTITUTIONAL REVIEW BOARD APPROVAL

MIDDLE TENNESSEE STATE UNIVERSITY

8/28/2015

Investigator(s): Alyson Lischka
Department: Mathematics
Protocol Title: Implementing Mathematical Practices and Content into Teaching 3
Protocol Number: #15-162

Dear Investigator(s):

We have reviewed your research proposal identified above and your requested changes. The following changes have been approved:

Addendum request dated 08/19/2015 has been approved

Please note that any unanticipated harms to participants or adverse events must be reported to the Office of Compliance at (615)-494-8918 or compliance@mtsu.edu. Any change to the protocol must be submitted to the IRB before implementing this change.

You will need to submit an end-of-project report to the Office of Compliance upon completion of your research. Complete research means that you have finished collecting data and are ready to submit your thesis and/or publish your findings. Should you not finish your research within the one (1) year period, you must submit a Progress Report and request a continuation ***prior*** to the expiration date on your approval letter. Please allow time for review and request revisions.

According to MTSU Policy, a researcher is defined as anyone who works with data or has contact with participants. Anyone meeting this definition needs to be listed on the protocol and needs to complete the online training. If you add researchers to an approved project, please forward an updated list of researchers to the Office of Compliance ***before*** they begin to work on the project.

Office of Compliance
010A Sam Ingram Bldg.
Middle Tennessee State University
1301 E. Main St. Murfreesboro, TN 37129

MIDDLE TENNESSEE STATE UNIVERSITY

Please note: **all research materials must be retained by the PI or a faculty advisor** (if the PI is a student) for at least **three (3) years after study completion**. Should you have any questions or need additional information, please do not hesitate to contact our office.

Sincerely,

Institutional Review Board Member
Middle Tennessee State University

Office of Compliance
010A Sam Ingram Bldg.
Middle Tennessee State University
1301 E. Main St. Murfreesboro, TN 37129

APPENDIX J: CODES AND THEMES

Table 1

Open Codes and Emergent Themes by Data Collection Stage

Stage One Codes and Themes	
Open Codes	Emergent Themes
AN – Avoidance of negative emotions F – Description of fixed mindset F/G – Differentiation of fixed and growth mindsets G – Evidence of growth mindset GM – Goal monitoring GO – Goal operations GS – Goal setting HE – High expectations LG – Learning goals LTG – Long term goals M – Awareness of mindset MS – Mastery strategy PG – Performance goals PG – Purposeful goals	1) Evidence of Growth Mindset a. Awareness of Mindset b. Operationalization of Mindset
B→P – Conceptions influencing practices B→E – Conceptions enacted externally BM – Belief about mathematics BT – Belief about teaching mathematics C/A – Differences in thinking of children and adults Con→Pro – Concept before procedure Context – Importance of context CO – Classroom outcomes E→B – External influence on conceptions IT – Student-centered mathematics MCC – Mathematics as a connected web O→B – Outcomes influencing beliefs Trans – Evidence of transition VSC – Value of structures and concepts	2) Evidence of Beliefs about Teaching and Learning Mathematics a. Mathematics as a Connected System b. Value of Mathematical Structures and Concepts c. Value and Uniqueness of Students' Thinking about Mathematics d. Value of Students' Communication about Mathematics

(continued)

Table 1 continued

Stage Two Codes and Themes		
Open Codes		Emergent Themes
B→P – Influence of conceptions on practice B→O – Beliefs influencing interpretations of outcomes CS – Connected strategies ES – Eliciting strategies Mod – Modeling of practices O – Relevant outcomes O→P – Outcomes influencing practices	P – Classroom practices P→O – Impact of practices on outcomes Praise – Praise for effort Resp – Positive response to challenge SC – Students' communications SCR – Students' critiques of reasoning ST – Students' thinking	1) Participant's Perceptions of Classroom Features a. Descriptions of Teaching Practices b. Descriptions of Classroom Activities c. Descriptions of Outcomes
Abs – Abstraction Acc – Accountability practices Ach – Achievement related outcomes AI – Affective outcomes C – Mathematics centers C/D – Focus on composition or decomposition of number CM – Communicating about mathematics ConO – Content outcomes CO – Classroom outcomes CR – Focus on connections between representations E→P – Impact of external influence on practices EM – Examining mistakes Eq – Equity practices FA – Assessment supports GM – Goal monitoring GO – Goal operations IA – Informal assessment during lesson IPS – Improved problem-solving ability LC – Focus on learning community MD – Mindset discussion MMR – Multiple methods and representations MSM – Making sense of mathematics MT – Use of mathematical task	Norms – References to classroom norms NR – Focus on representation of numbers NT – Number talk O→P – Outcomes influencing practices P→O – Impact of practices on outcomes PK – Building on prior knowledge Praise – Praise for effort PV – Focus on place value Q – Teacher questioning Resp – Positive response to challenge RM – Respect for mistakes S/S – Student to student communications S/T – Student to teacher communications S/WG – Student to whole group communications Skills – Skills practice SMM – Students' using models or manipulatives SR – Student reflection T/S – Teacher to student communications T/SG – Teacher communication to small group T/WG – Teacher communication to whole group	2) Observations of Classroom Features a. Observations of Teaching Practices b. Observations of Classroom Activities

(continued)

Table 1 continued

Stage Three Codes and Themes		
Open Codes		Emergent Themes
B→E – Conceptions enacted externally DS – Descriptions of her students during past demonstration lesson EX – External influence Bridge – Demonstration lesson as a bridge between professional development and classroom E→B – External influence on conceptions E→P – Impact of external influence on practices IA – Experience with immersion activity PB – Experience with practice-based activity	PD – Description of professional development PPD – Perception of professional development MD – Description of modeling of teaching practices VE – Value of equitable practices VO- Value of observing other's teaching practices VS – Value of productive struggle VT – Value of learner's thinking about mathematics	1) Perceptions of Experiences in Professional Development a. Influences and Perceptions of Summer Institutes b. Influences and Perceptions of Demonstration Lessons
FEST – Focus on teacher's eliciting/assessment of students' thinking FLC – Focus on lesson's content FLS – Focus on lesson's structure FSC – Focus on students' collaborative interactions FSC – Focus on students' communications FSTM – Focus on students' thinking about mathematics FTIP – Focus on teacher's instructional practices FTQ – Focus on teacher's questioning	FTS – Focus on scaffolding and supports provided by teacher FTSG – Focus on teacher's interactions with small groups S/S – Student to student communications S/T – Student to teacher communications S/WG – Student to whole group communications T/S – Teacher to student communications T/SG – Teacher communication to small group T/WG – Teacher communication to whole group	2) Areas of Focus During Demonstration Lesson
CM – Role of communication about mathematics HE – High expectations from teacher LTG – Importance of long-term goals	SC – Role of student's communication about mathematics SS – Role of student to student communication ST – Role of student thinking	3) Perceptions of the Importance of the Demonstration Lesson

(continued)

Table 1 continued

Stage Four Codes and Themes		
Open Codes		Emergent Themes
Adapt – Possible adaptations AIS – Adaptation of lesson sequence APL – Assessment in preparing for lesson LS – Descriptions of lesson sequence LT – Descriptions of learning trajectory OT – Off task behaviors P – Planning PV – Focus on place value	Quest – Possible questions R – Reflection RO – Representations to support operations Scaff – Considerations for scaffolding SR – Struggle with regrouping SSQ – Descriptions of sequencing TIC – Task to introduce content ideas	1) Planning for the Enacted Demonstration Lesson a. Lesson Sequencing b. Lesson Goals c. Anticipated Struggles
A – Advancing interaction Abs – Abstraction Acc – Accountability practices Ach – Achievement related outcomes AI – Affective outcomes C – Mathematics centers C/D – Focus on composition or decomposition of number CM – Communicating about mathematics CO – Content outcomes CR – Focus on connections between representations EM – Examining mistakes Eq – Equity practices E→P – Impact of external influence on practices F – Facilitating interaction FA – Assessment supports GM – Goal monitoring GO – Goal operations IA – Informal assessment during lesson IPS – Improved problem-solving ability LC – Focus on learning community MD – Mindset discussion MMR – Multiple methods and representations	MSM – Making sense of mathematics MT – Use of mathematical task Norms – References to classroom norms NR – Focus on representation of numbers NT – Number talk O→P – Outcomes influencing practices P→O – Impact of practices on outcomes PK – Building on prior knowledge PV – Focus on place value Q – Teacher questioning R – Redirecting interaction Resp – Positive response to challenge RM – Respect for mistakes S/S – Student to student communications S/T – Student to teacher communications S/WG – Student to whole group communications Skills – Skills practice SMM – Students' using models or manipulatives SR – Student reflection T/S – Teacher to student communications T/SG – Teacher communication to small group T/WG – Teacher communication to whole group	2) Lesson Enactment a. Lesson Setup b. Lesson Warm-up c. Primary Task
LG – Lesson goals IC – Importance of context CC – Connections across concepts BPI – Building on prior instruction CAI – Building connections to abstract representations B→O – Beliefs influencing interpretations B→P – Influence of conceptions on practice CLT – Change in learning trajectory ESS – Evidence of student struggle HI – Insight gained from reviewing video of lesson ICM – Increased classroom maturity	ICS – Increased connections between strategies IIT – Increase in independent thinking IWC – Increased willingness to communicate thinking MAS – Mathematical models to support thinking O→P – Outcomes influencing practices P→O – Impact of practices on outcomes PVA – Connecting place value to abstract representations R – Reflection SUR – Student understanding of regrouping VTI – Value of mathematical task to introduce content	3) Reflections and Considerations a. General Reflections Regarding the Lesson b. Reflections on Students Content Progress c. Considerations of Lesson's Impact on Lesson Sequence d. Reflections on Students' Progress to Global Goals

APPENDIX K: MINDSET SURVEY

Name:__________________________________ Date:________________ Year: _1 or 2_

For each of the following statements, rate how strongly you agree or disagree with the statement.	**Strongly Agree**	**Agree**	**Somewhat Agree**	**Somewhat Disagree**	**Disagree**	**Strongly Disagree**
You have a certain amount of intelligence and you really can't do much to change it.	**1**	**2**	**3**	**4**	**5**	**6**
Your intelligence is something about you that you can't change very much.	**1**	**2**	**3**	**4**	**5**	**6**
You can learn new things, but you can't really change you basic intelligence.	**1**	**2**	**3**	**4**	**5**	**6**
A person's moral character is something is something very basic about them and it can't be changed much.	**1**	**2**	**3**	**4**	**5**	**6**
Whether a person is responsible and sincere or not is deeply ingrained in their personality. It cannot be changed very much.	**1**	**2**	**3**	**4**	**5**	**6**
There is not much that can be done to change a person's moral traits (e.g. conscientiousness, uprightness, and honesty).	**1**	**2**	**3**	**4**	**5**	**6**
Though we can change some phenomena, it is unlikely that we can alter the core dispositions of our world.	**1**	**2**	**3**	**4**	**5**	**6**
Our world has its basic and ingrained dispositions, and you really can't do much to change them.	**1**	**2**	**3**	**4**	**5**	**6**
Some societal trends may dominate for a while, but the fundamental nature of our world is something that cannot be changed much.	**1**	**2**	**3**	**4**	**5**	**6**
A person has a certain amount of mathematical ability and they really can't do much to change it.	**1**	**2**	**3**	**4**	**5**	**6**
A person's mathematical ability is something about them that they can't change very much.	**1**	**2**	**3**	**4**	**5**	**6**
A person can learn new things about mathematics, but they can't really change their basic mathematical ability.	**1**	**2**	**3**	**4**	**5**	**6**

APPENDIX L: DEMONSTRATION LESSON PLAN

Handouts:	**Materials:**
• Block Problem sheet	• Elmo & projector • Chart paper & Markers • Poster norms poster • Notebook paper & pencils • Base-10 blocks

Lesson Goal: The goal of this lesson is to engage students in thinking about subtraction with regrouping, potentially representing the process symbolically.

Prior to beginning the lesson, place on students' desks a half-sheet of chart paper, a marker, and base-10 blocks for each pair of students.

Warm-up (15 minutes)
Display a unit block, a ten block, and a hundred block.

Think-pair-share: What are some things you know about these blocks? Record these on chart paper.

Ask: How would you use the blocks to represent 127?

The Block Problem (15 minutes)
Display the problem sheet. Read the problem to the students.

Think-pair-share: What are some things you are thinking about this problem?

Tell students that they are to work with their partners to represent and solve this problem on their chart paper. Encourage them to use pictures, words, and symbols (refer to poster norms).

Approximately 2-3 minutes into the work time, it may be necessary to stop the group and re-direct (make sure everyone understands the goal of their work) or to share some work done thus far (to provide inspiration to those who are slow to engage).

Monitor students' representations and select 2 or 3 different representations to be discussed.

Debrief (20 minutes)
Call time. Acknowledge to students that not everyone may have finished and that's ok. Explain that there were a lot of good representations being used but that you have asked three pairs to share.

Review presentation expectations (eyes on speakers, listen, etc.) and instruct students to think about how the representations are alike and different.

Ask the three groups to present. Tape each poster at the board while the pair presents. Encourage pairs to focus on **their solution strategies** rather than their computations.

After each poster, use think-pair-share to facilitate students' thinking about the solution method of the poster. Poster questions will be based on student representations.

After all presentations, facilitate a class discussion of the work. Possible discussion questions include:

- How are these representations alike? Different?
- Write the problem (304 – 120) vertically at the elmo. How can we represent symbolically what _______ did with his/her blocks?

Summary (10 minutes)
Distribute the notebook paper and pencils. Ask students to respond to the question prompt individually in writing. Share out if time permits.

Possible question prompts:
- Even though we might say there are no tens in 304, how was the little brother able to grab 2 longs?
- How could you represent 407 so that 3 longs could be taken away? Write a sentence explaining how you know you are correct.

The Block Problem
On Thursday, Tara was at home representing numbers with base-10 blocks.

The value of her blocks was 304.

When she wasn't looking, her little brother grabbed 2 longs and a flat.

What is the value of Tara's remaining blocks?

Use pictures, words, and/or symbols to describe how you solved the problem.