USE OF VIRTUAL MANIPULATIVES IN DEVELOPING GRADE SIX STUDENTS' PROPORTIONAL REASONING SKILLS

By

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I dedicate this work to God, “In whom are hid all the treasures of wisdom and knowledge.” (Colossians 2:3)

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ABSTRACT

The lack of development of proportional reasoning skills has long been recognized in the literature as a mathematical topic needing specific attention. Also, several studies demonstrated the use of virtual manipulatives as an effective tool in the development of certain mathematical concepts; however, few researchers have pursued the application of virtual manipulatives to develop proportional reasoning skills. This study aimed to determine the impact of the use of virtual manipulatives on the development of proportional reasoning in grade six students, focusing upon the potential effect of gender, technology-input modality, and interaction effect between these two factors.

Two virtual manipulatives, Thinking Blocks and Number Pieces, were included as part of a mixed-method approach incorporating a convergent parallel design. During a ten-day period, a group of 56 participants completed a pre-test/post-test instrument measuring proportional reasoning, including a survey to determine touchscreen experience and preference of modality; in addition, five performance tasks involving proportional reasoning were completed at regular intervals. Finally, a group of six target students were observed during the data-gathering process as they worked during class time, being interviewed individually as they completed two additional tasks before and after instruction.

Findings from this research revealed participants who used Thinking Blocks and Number Pieces made greater gains on the post-test when compared to the participants

who did not; however, these gains did not demonstrate statistical significance. No statistical significance was found in the performance of participants with respect to gender or technology-input modality; also, the researcher noted no interaction effect between gender and technology-input modality.

Participants demonstrated growth as shown on the work generated from the first task to the fifth, but performance levels fluctuated from one task to another. Participants tended to revert to an additive approach when encountering a non-integral value in a proportional setting. All six target students used an additive approach when completing the pre-instruction interview task. However, the use of academic vocabulary and informal expressions of proportional reasoning concepts exhibited during the post-instruction interviews by the four target students who used Thinking Blocks and Number Pieces provided additional support for the premise of virtual manipulatives as effective tools to develop proportional reasoning skills.

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CHAPTER I: INTRODUCTION AND RATIONALE

Statement of the Problem

Innovations and improvements for the effective teaching of mathematics are touted on a routine basis. Displays at conferences, advertisements in magazines, visits by textbook representatives, and marketing emails inform educators about the latest products or programs that will meet the mathematical learning needs of students. Despite the claims, every approach or product may not be equally effective in achieving the goal of mathematical proficiency. Although there are several topics in mathematics, one specific area of critical need is the development of proportional reasoning skills.

Lamon (1999) reported that more than half of the adult population cannot be viewed as proportional thinkers; further, maturity and experience may not be sufficient to ensure the development of proportional reasoning skills. In her later work, Lamon (2012) estimated that the proportion of adults who cannot reason proportionally well exceeds 90%. Thus, a concerted effort is needed in mathematics classrooms in order to provide a learning setting that cultivates proportional reasoning skills. In light of the situation, it would behoove teachers to incorporate available resources to support the development.

The National Council of Teachers of Mathematics (NCTM) stated that "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (NCTM, 2000, p. 24). The rapid advance of numerous technological tools, including digital resources commonly called virtual manipulatives, has provided new resources for teachers to consider in their teaching mathematical content to students. A cursory inspection of the results one finds when performing an Internet search for virtual manipulatives yields applications in several mathematical topics, including fractions, place value, equations, and area. However, a review of the literature indicates research concerning the use of virtual manipulatives to promote the development of proportional reasoning skills has not been adequately pursued. This lack of application of virtual manipulatives to the topic of proportional reasoning suggests the recommendation of NCTM concerning the enhancement of student learning with the use of technology has not been sufficiently addressed.

Any assessment of students with respect to proportional reasoning should not be limited to an examination of just whole group data. Just as the calculation of a class average for a test masks the performance of subgroups and individuals, an examination of various subgroups is required in order to determine the performance of such subgroups with respect to an assessment of proportional reasoning skills. Delving deeper into the performance of the subgroups provides data that can, in turn, lead to decisions as to what impact the use of virtual manipulatives might have on subgroups; specifically, one can study the impact of the use of virtual manipulatives with respect to gender. Also, one can consider technology-input modality and the impact various modes of computer interaction presents while implementing virtual manipulatives in the development of proportional reasoning skills.

This dissertation considers the use of virtual manipulatives as a tool to assist grade six students in developing proportional reasoning skills. In this chapter, a rationale for this study is developed by examining the following: proportional reasoning; a background of manipulatives, both physical and virtual; gender differences; and technology-input modality. A summary of factors is provided, along with goals, research questions, definitions, and significance of this study. However, it is also appropriate to consider the theoretical framework upon which the rationale is grounded.

Conceptual Framework

In order to pursue the proposed research, it is helpful to declare the ideas that guide the research study. These ideas span the two major topics making up the background for the research study: proportional reasoning and virtual manipulatives. Although each idea stands independently, the synthesis of these ideas into a conglomerate concept provides the foundation upon which to build. With respect to proportional reasoning, it is not sufficient to say students either do or do not possess proportional reasoning skills; rather, students develop proportional reasoning skills. When considering virtual manipulatives, the growth of technology supports the introduction of virtual manipulatives into the classroom, but the perception of the role of virtual manipulatives in the mathematics classroom guides their use.

Levels of Proportional Reasoning

Karplus, Karplus, Formisano, and Paulsen (1977) identified levels of proportional reasoning in their research concerning adolescents in seven countries. There are four categories of proportional reasoning they employed in classifying the responses made on a proportional reasoning task: (a) Category I (Intuitive); (b) Category A (Additive); (c) Category Tr (Transitional) and; (d) Category R (Ratio). These categories are presented in the order of increased development; thus, Category I is the lowest level of proportional reasoning, while Category R is the highest level of proportional reasoning. A change in the label of the lowest category for proportional reasoning skills occurred in the literature as demonstrated by Karplus and other researchers in subsequent published works; instead of using the title Intuitive, the title Illogical appeared (Khoury, 2002).

A student in Category I gives no explanation or shows illogical computation concerning the proportional work at hand. Also, students who describe results as a guess belong in Category I. For Category A, the student's work is based solely upon an approach using addition or subtraction. Work that is classified in Category Tr has an additive component, but there is also an attempt to compare changes in a relative fashion. In Category R, the use of a constant ratio or conversion of units appears in responses (Karplus, Adi, & Lawson, 1980). As part of the current study, students completed proportional reasoning tasks and the researcher classified the results with regard to these four levels of proportional reasoning.

This leveled approach to determine proportional reasoning is not unique to Karplus. Langrall and Swafford (2000) also identified four different levels of strategies employed by students to complete proportional reasoning tasks: Levels 0-3. As summarized by Langrall and Swafford, these levels are described as follows: (a) Level 0 work shows no proportional reasoning; (b) Level 1 work involves informal reasoning concerning proportional situations with the assistance of manipulatives, pictures or other models; (c) Level 2 work demonstrates a more sophisticated strategy of quantitative reasoning without manipulatives or can link models with appropriate calculation; and (d) Level 3 work shows the ability of students to set up and solve proportions with full understanding of the structural relationships that exist. Since the current research study involved virtual manipulatives as an inherent component, all students participating in the treatment groups would start with Level 1 of Langrall and Swafford's classification. In light of this situation, the researcher deemed it better to incorporate the levels of proportional reasoning as advanced by Karplus.

Virtual Manipulatives as Cognitive Tools

Songer (2010) distinguished between digital resources and cognitive tools. Although virtual manipulatives are considered to be digital resources in that they are computer-based information sources, they also qualify as cognitive tools. A cognitive tool transcends the level of just providing information; it is a resource that is specifically designed to allow students to achieve particular learning goals on a topic of interest. For example, a teacher can generate a worksheet from a website so that students can practice certain mathematical skills. In this case, the teacher has used a digital resource. However, the worksheet provides only practice, not conceptual development. In the case of a cognitive tool, the website must allow for dynamic interaction and connectivity between changes that the student introduces and the results seen from such change. For the current study, one goal was to ascertain the impact that virtual manipulatives have when they are used in developing proportional reasoning skills. Since the websites intended for use in this research provided opportunities for students to build models and explore proportional relationships, the virtual manipulatives functioned as cognitive tools.

Proportional Reasoning

Proportional reasoning has been defined in various ways by several researchers throughout the years, including the following definitions:

- proportional reasoning involves recognizing an equivalence that exists between the comparison of two sets of two terms (Inhelder & Piaget, 1958);
- proportional reasoning involves mental assimilation and synthesis of the various complements of ratios in a proportion (Lesh, Post, & Behr, 1988);
- proportional reasoning has a critical component that involves a multiplicative relationship among the quantities that represent a situation (Cramer & Post, 1993);
- proportional reasoning involves mathematical relationships which are multiplicative in nature (Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998);
- proportional reasoning is the ability to think about and compare multiplicative relationships between quantities (Van de Walle & Lovin, 2006); and,
- proportional reasoning is the cognitive process involving comprehension of multiplicative relationships in proportions, the construction of appropriate proportional schemes, and the ability to model and solve a variety of ratio-proportion problems and tasks (Özgün-Koca & Altay, 2009).

A common theme in all of these definitions of proportional reasoning is the multiplicative relationship that exists in the nature of ratios. Since ratios are used to create proportions, the multiplicative concept is automatically embedded into the nature of proportions. As the understanding of proportional reasoning develops, the necessity for updating and refining its definition exists. According to Beckmann and Izsák (2014), “a robust understanding of proportional relationships includes understanding and using multiplicative relationships between two co-varying quantities and recognizing whether or not two co-varying quantities remain the same constant ratio”(p. 4). The refining of the definition of proportional reasoning from Inhelder and Piaget (1958) to the present suggests the understanding of proportional reasoning today requires more detail.

In spite of the definitions offered for proportional reasoning, Lamon (2012) insisted that “the term is ill-defined and researchers have been better at determining when a student or an adult does *not* reason proportionally rather than defining the characteristics of one who does” (pp. 2-3). The increased development of the concepts concerning proportional reasoning warrants further research into the methods that best lend themselves to the introduction and development of proportional reasoning topics in the mathematical classroom setting.

Teaching Proportional Reasoning

Since one purpose of this research was to investigate how well virtual manipulatives aid middle school students in learning proportional reasoning, it is helpful to know what is expected at the classroom level. According to Lobato, Orrill, Druken, and Jacobson (2011), research suggests that it is common for students to enter middle school without possessing proportional reasoning skills. Thus, one focus of the middle school mathematics curriculum should be the development of proportional reasoning skills. Based upon the Common Core State Standards in Mathematics (CCSSM), teachers of grade six students expect them to master skills concerning ratios and rates (Common Core State Standards Initiative, 2010a); there are also CCSSM concerning proportional reasoning skills in grade seven (Common Core State Standards Initiative, 2010b). CCSSM nomenclature labels the general category of skills addressing ratios and proportions with the letters RP. An examination of CCSSM shows the use of a combination of letters and numbers attached to the category of RP standards to distinguish between them. Table 1 lists the CCSSM in grade six concerning ratios and rates, while Table 2 gives the CCSSM in grade seven with a focus upon proportional reasoning.

Table 1

CCSSM in Grade Six

Label	Standard
6.RP	**Understand ratio concepts and use ratio reasoning to solve problems.**
6.RP.A.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
6.RP.A.2	Understand the concept of a unit rate *a*/*b* associated with a ratio *a:b* with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*
6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Table 1

CCSSM in Grade Six (continued)

Label	Standard
6.RP	**Understand ratio concepts and use ratio reasoning to solve problems.**
6.RP.A.3.A	Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
6.RP.A.3.B	Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
6.RP.A.3.C	Find the percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
6.RP.A.3.D	Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Table 2

CCSSM in Grade Seven

Label	Standard
7.RP	**Analyze proportional relationships and use them to solve real-world and mathematical problems.**
7.RP.A.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour*.
7.RP.A.2	Recognize and represent proportional relationships between quantities.

Table 2

CCSSM in Grade Seven (continued)

Label	Standard
7.RP	**Analyze proportional relationships and use them to solve real-world and mathematical problems.**
7.RP.A.2.A	Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
7.RP.A.2.B	Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
7.RP.A.2.C	Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and number of items can be expressed as t = pn.*
7.RP.A.2.D	Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate.
7.RP.A.3	Use proportional relationships to solve multistep ratio and percent problems.

A comparison of the expectations of students with respect to CCSSM as given in Tables 1 and 2 indicates a progression of competency within the area of ratios and proportions. In order for middle school teachers to help their students achieve mastery of proportional reasoning standards, it is not unreasonable to assume that teachers would want to implement available, effective resources. Manipulatives have the capacity to enable students to understand mathematical concepts and develop mathematical skill.

Manipulatives

Since the existence of organized classrooms and the utilization of materials with the intent of teaching mathematical concepts to students in these classrooms, questions concerning the effective use of these available resources abound. Montessori, Pestalozzi, Piaget, Froebel, and Bruner are some influential educational theorists who advocated for the use of materials and manipulatives to further mathematical instruction (Namukasa, Stanley, & Tuchie, 2009). Goldsby (2009) asserted that research on the use of manipulatives in the middle grades is not as extensive as research at the elementary grade level; additionally, the focus of research concerning manipulatives is upon special groups, such as students with disabilities. Dykema (2013) reported that the following results can be supported with the use of manipulatives:

- brain-based long-term memory connections are achieved by the use of color;
- deeper connections are made by the exposure to varied experiences;
- misunderstood concepts may be cleared up by seeing the concept expressed in a different way; and
- children think and reflect about mathematical ideas.

Manipulatives Defined

In order to help clarify what is meant by manipulatives, it is advisable to examine how such materials are defined. Several definitions for manipulatives are found in previous studies, including the following:

- learning aids, computers, adding machines, blocks, tools, models, and measuring devices (Davidson, 1968);

- common out-of-school tools, educational materials conceived for educational purposes, and games (Szendrei, 1996);
- concrete models incorporating mathematical concepts that appeal to several senses, which can be touched and moved around by students (Suydam, 1986); and,
- objects that the pupil can feel, handle, or move (Reys, 1971).

There are two important common qualities identified in these definitions: (a) manipulatives directly involve the various senses of the student, particularly visual and tactile senses; and (b) hands-on experiences are necessary in order for materials to be classified as a manipulative. Technology development has infused these two qualities into virtual manipulatives as well. Manches and O'Malley (2012) used the term *tangible technologies* to describe virtual manipulatives that require interaction via touchscreen instead of the use of a mouse; thus, there are distinctions that exist even among virtual manipulatives. Physical manipulatives and virtual manipulatives are distinct in that the former is represented by physical objects and the latter type is electronic in nature; it is conceivable to argue that virtual manipulatives are not considered concrete in that they are not able to be held or touched directly. However, the term *concrete* has a different meaning in the eyes of some researchers.

Clements and McMillen (1996) stated that virtual manipulatives should be considered just as concrete as physical manipulatives. The basis for this consideration is the degree of meaningfulness that the manipulatives, both physical and virtual, provide to students. So, the terms *physical* and *concrete* are not necessarily synonymous with each other with respect to manipulatives. Manches and O'Malley (2012) clarified Clements' meaning by presenting the root meaning of the term *concrete* as *grow together*; thus, both physical manipulatives and virtual manipulatives satisfy the condition of being concrete.

Virtual Manipulatives Defined

Some researchers define virtual manipulatives as digital representations of a physical manipulative (Mildenhall, Swan, Northcote, & Marshall, 2008), while other researchers have taken pains to differentiate between static representations and dynamic manipulatives (Moyer, Bolyard, & Spikell, 2002). Static visual representations are considered no more than pictures of physical models; as such, they lack interactive capability and cannot be classified as true virtual manipulatives. A true virtual manipulative must be interactive and dynamic, allowing opportunities for the user to construct mathematical knowledge (Moyer et al., 2002). Virtual manipulatives allow the same opportunities that physical manipulatives permit for students - namely, the ability to use materials to assist in the introduction and development of mathematical concepts.

For those studies that compare the use of physical manipulatives and virtual manipulatives, researchers have reached various conclusions. For instance, Brown (2007) concluded that students who used physical manipulatives performed better than those students who used virtual manipulatives, although both types of manipulatives enhanced the learning environment. Hunt, Nipper, and Nash (2011) suspected that different academic abilities of these two groups studied by Brown affected the results of her study; additionally, the two groups worked with manipulatives that focused upon differing mathematical content. Alternatively, Olkun (2003) reported that virtual manipulatives provided an effect concerning understanding of geometrical content equally as strong as physical manipulatives. Given these findings, the choice of the use of physical manipulatives or virtual manipulatives may depend upon the preference of the classroom teacher.

Examples of common manipulatives and their virtual manipulative counterparts appear in Figures 1-3. Although physical manipulatives and their corresponding virtual manipulatives share the same structure, virtual manipulatives allow the user to access and impose more features than the physical manipulatives, such as change of color, access to more pieces, and linkage to other modes of expression. Also, virtual manipulatives appear to create a natural bridge from the abstract to the concrete (Hunt et al., 2011).

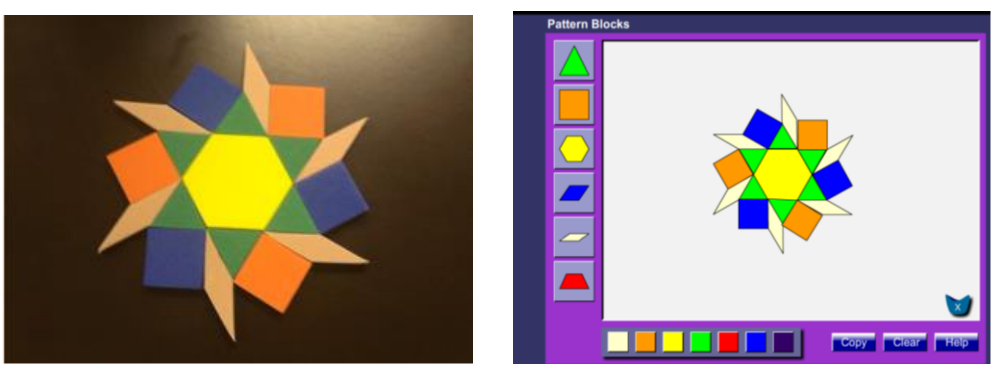


Figure 1. Pattern blocks - physical manipulatives and virtual manipulatives.

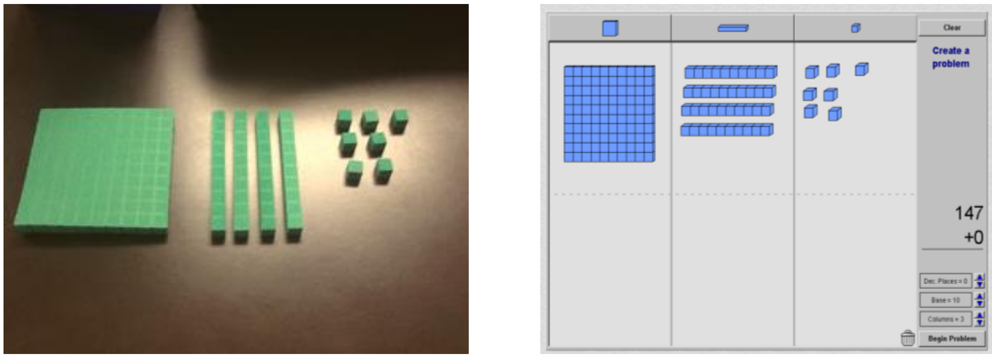


Figure 2. Base-ten blocks - physical manipulatives and virtual manipulatives.

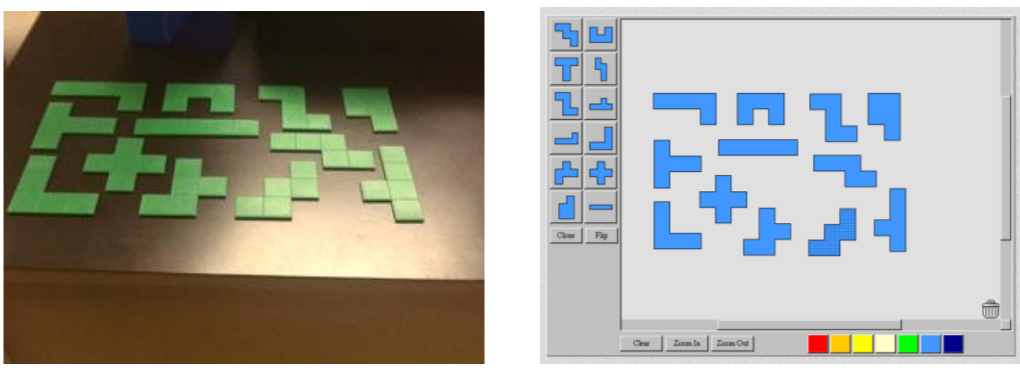


Figure 3. Pentominos - physical manipulatives and virtual manipulatives.

Research Concerning Virtual Manipulatives

With recent technological advances becoming available in classroom settings, virtual manipulatives are resources accessible to both students and teachers; however, differing opinions about their effectiveness abound. For example, some teachers consider virtual manipulatives to be entertaining, game-like diversions instead of a learning tool (Moyer, 2001). Weiss (2006) reported that some teachers consider manipulatives to be effective for younger learners but are not appropriate for older students. Weiss’ observation was based upon the premise that manipulatives are toys without any value with respect to higher-level mathematics. In addition, it is crucial that teachers not treat virtual manipulatives as some magical catalyst whose presence is supposed to ensure mastery of mathematical concepts. Speer (2009) cautioned that there are several variables to consider when determining how effective virtual manipulatives are in the classroom, including research design, sampling characteristics, and the type of manipulative used. Another area of concern is familiarity with the purpose of the manipulative materials used; such familiarity is required of both the teacher and students. According to Weiss (2006), "if the student does not easily identify the purpose of the manipulative, it is no longer a tool but a distraction" (p. 241). Therefore, it is crucial that instruction provided by mathematics teachers include the purpose of any manipulative that they choose to use.

Research concerning virtual manipulatives has involved various grade levels (Lee & Chen, 2010; Namukasa et al. 2009; Reimer & Moyer, 2005; Steen, Brooks, & Lyon, 2006; Zacharia & Constantinou, 2008) and specific topics (Mendiburo, 2010; Hwang, Su, Huang, & Dong, 2009; Suh & Moyer, 2007). Although research exists which focused upon students and teachers in the middle grades (Hunt et al., 2011; Moyer, 2001), the majority of research appears to focus upon topics for students at the elementary grade level (Mendiburo, 2010; Moyer-Packenham et al., 2013; Reimer & Moyer, 2005; Steen et al., 2006; Suh & Moyer, 2008). This is not to imply there are no virtual manipulatives intended for use by secondary students. In fact, two popular examples of virtual manipulatives that are available and intended for use by students at the secondary level are Geometer's Sketchpad and Geogebra.

Geometer's Sketchpad has been available since 1995 (Scher, 2000) and can be found at http://www.keycurriculum.com/; Geogebra started in 2001 as part of the master thesis work of Marcus Hohenwater (Carp, 2010). Both software packages allow students to investigate mathematical relationships between data and graphs; however, while Geogebra is a free resource, Geometer's Sketchpad is not.

Background Factors of Study

In order to determine any degree of effective use of virtual manipulatives, it is helpful to narrow choices of mathematical content to a particular focus of study. The conceptual framework presented previously in this chapter provides a focus upon proportional reasoning. Additionally, it is also helpful to consider other qualities or characteristics that might impact any effective use of virtual manipulatives, such as the availability of virtual manipulatives designed for proportional reasoning development, gender of students, and the gesturing aspect of technology-input modality. Any interactions between these characteristics should be considered as well.

Availability

Moyer-Packenham (2010) addressed the implementation of virtual manipulatives as an effective tool in the mathematics classroom for kindergarten through grade eight. As part of her text, sample lessons were included, covering topics in number and operation, algebra, geometry, data analysis, and probability. However, no virtual manipulative lesson addressing proportional reasoning was presented. An examination of various websites housing virtual manipulatives, such as the National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/vlibrary.html), Illuminations (http://illuminations.nctm.org/), and Shodor Interactivate (http://www.shodor.org/interactivate/), reveals many topics for which virtual manipulatives have been developed, including fractions, decimals, place value, algebraic expressions, and solving equations; again, few virtual manipulatives exist that are intended as tools for lessons concerning proportional reasoning skills. The apparent lack of virtual manipulatives that focus on the development of proportional reasoning skills could contribute to the lack of development of lessons incorporating the use of virtual manipulatives. One virtual manipulative website that focuses upon ratio and proportion is Thinking Blocks: Ratio and Proportions Practice, which is found at http://www.mathplayground.com/tb_ratios/thinking_blocks_ratios.html. Figures 4-10 show a sample problem completed step by step from the Thinking Blocks website.

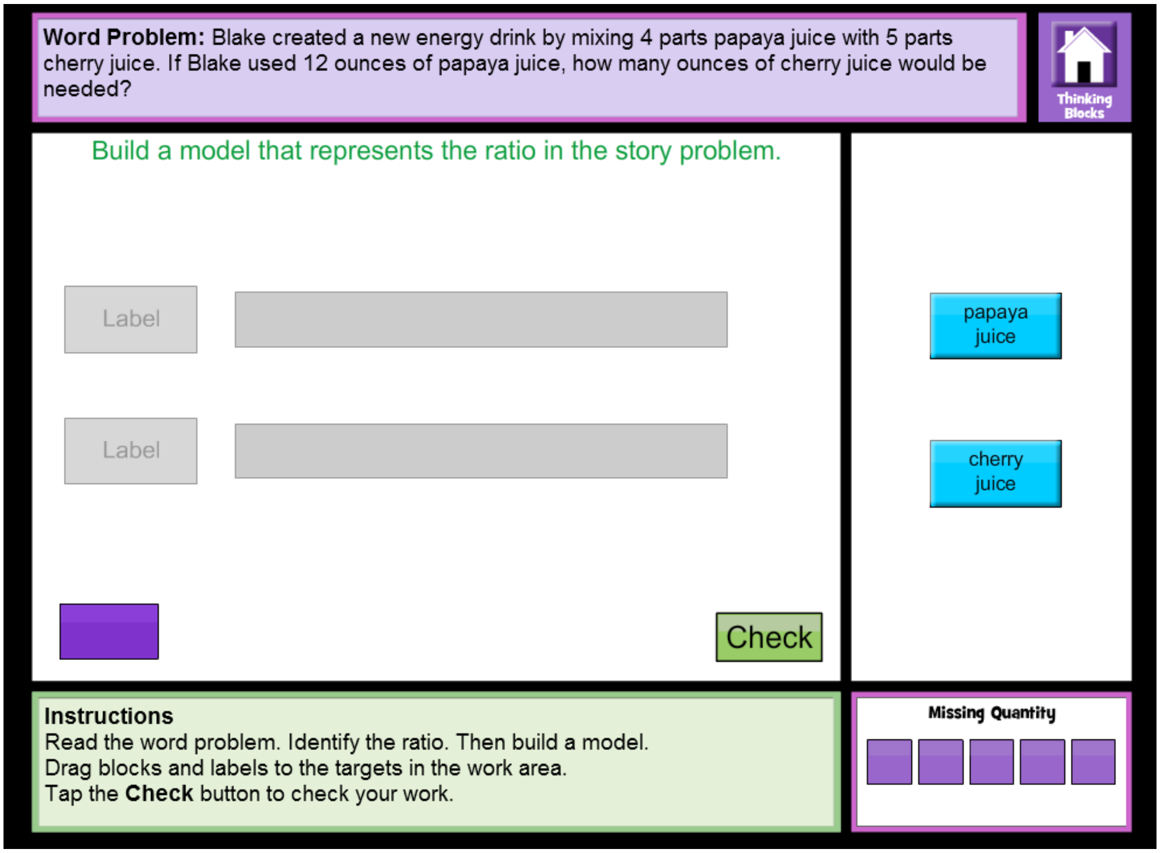


Figure 4. Thinking Blocks introduces the problem, blocks, and labels.

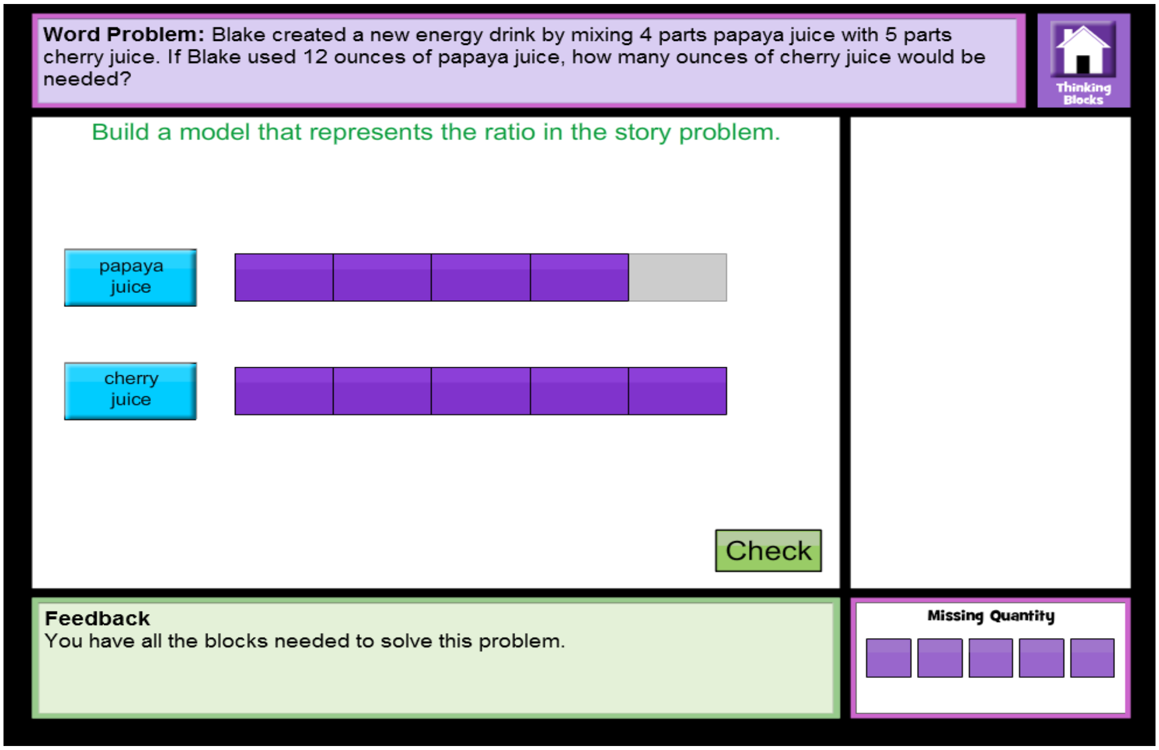


Figure 5. Block models are built and labels are applied.

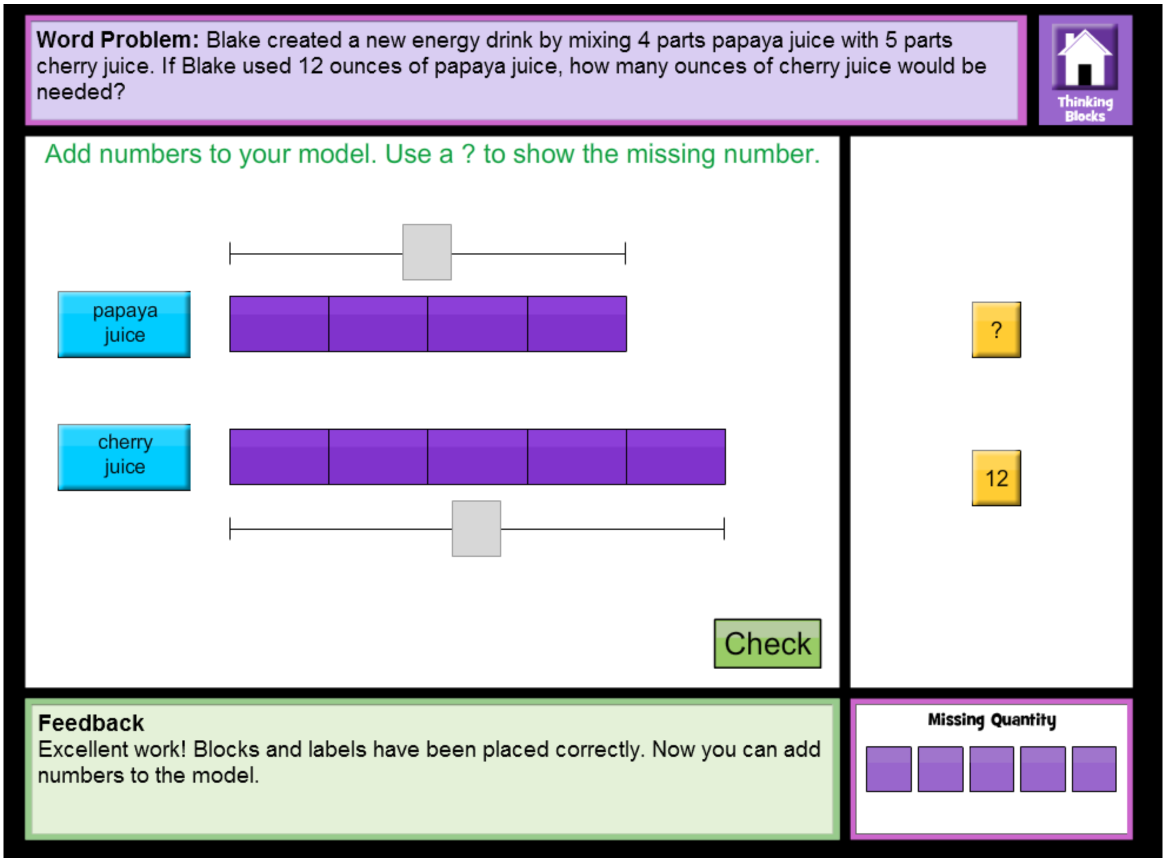


Figure 6. Work is verified and numbers are introduced.

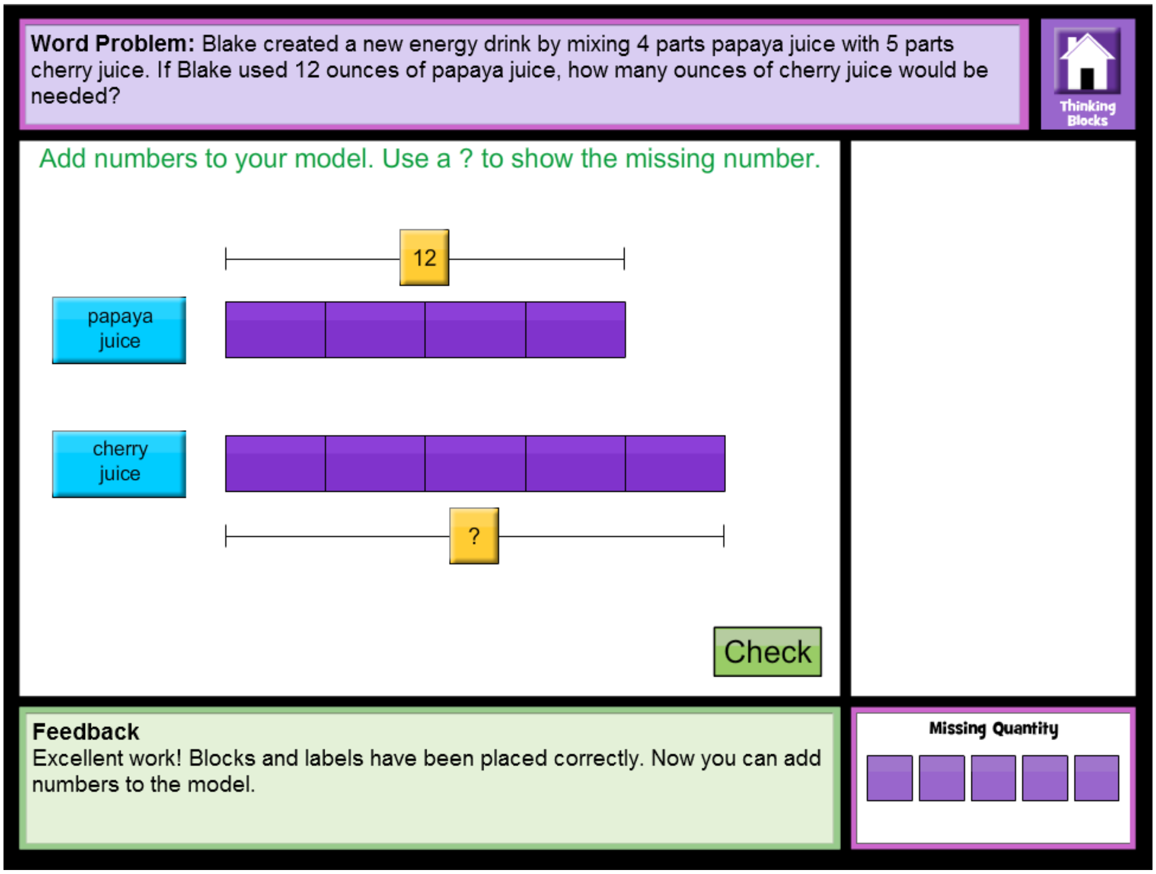


Figure 7. Numbers are applied to the block models.

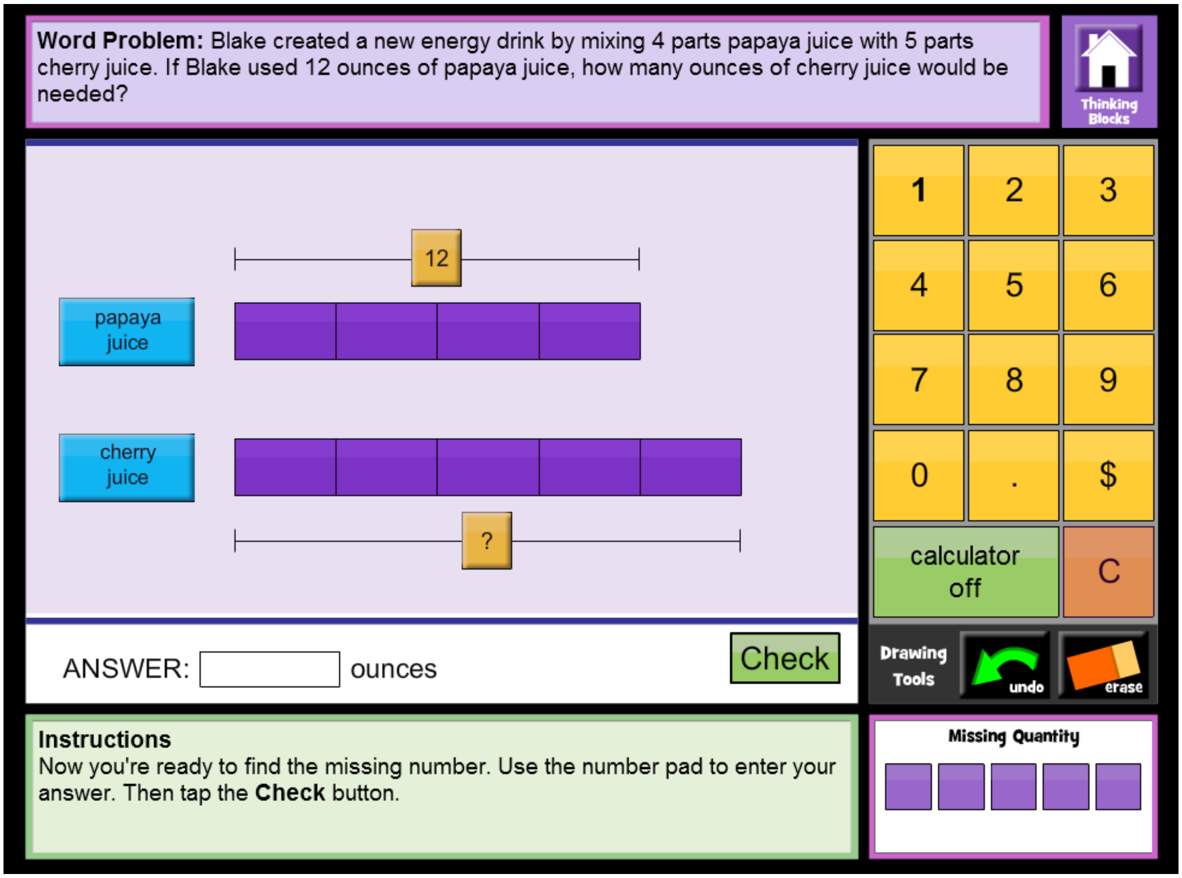


Figure 8. Numbers are verified and missing value can be calculated.

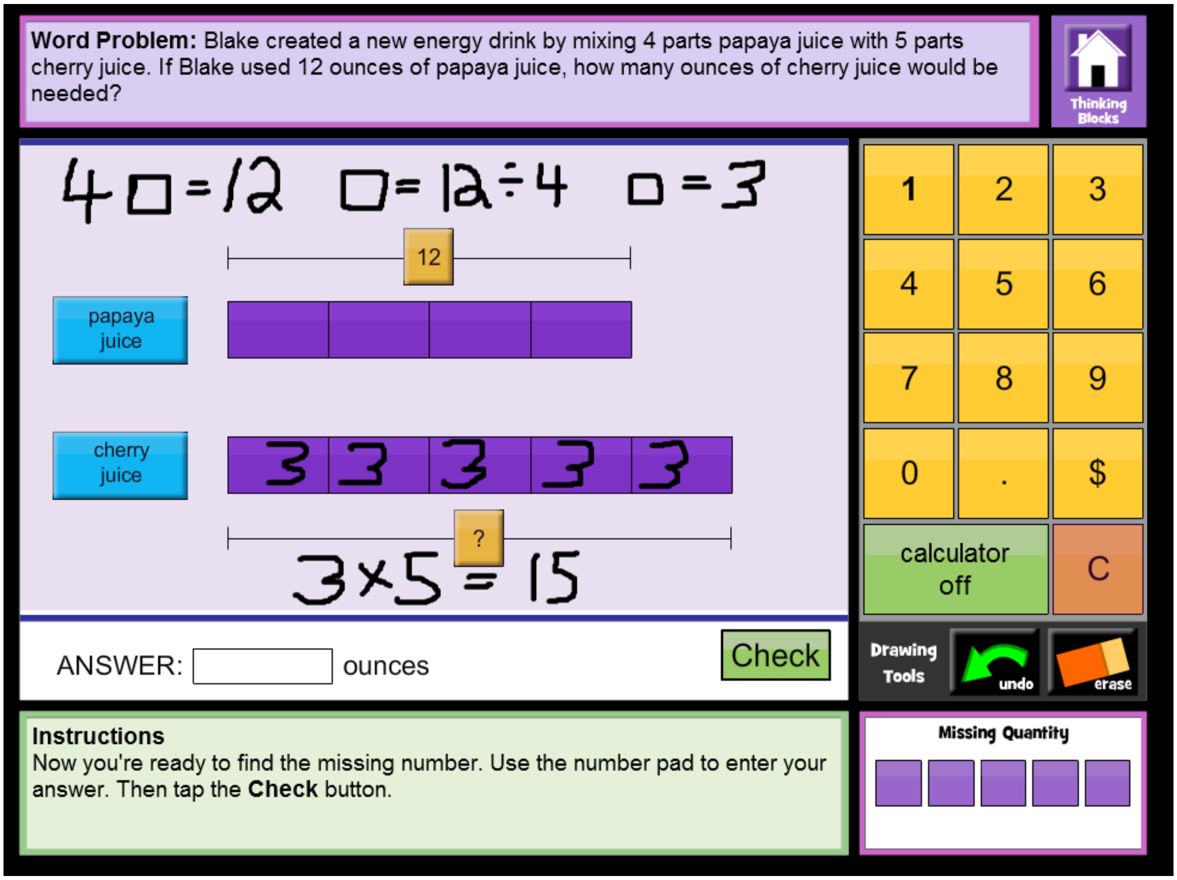


Figure 9. Calculations performed to find value of one block and missing value.

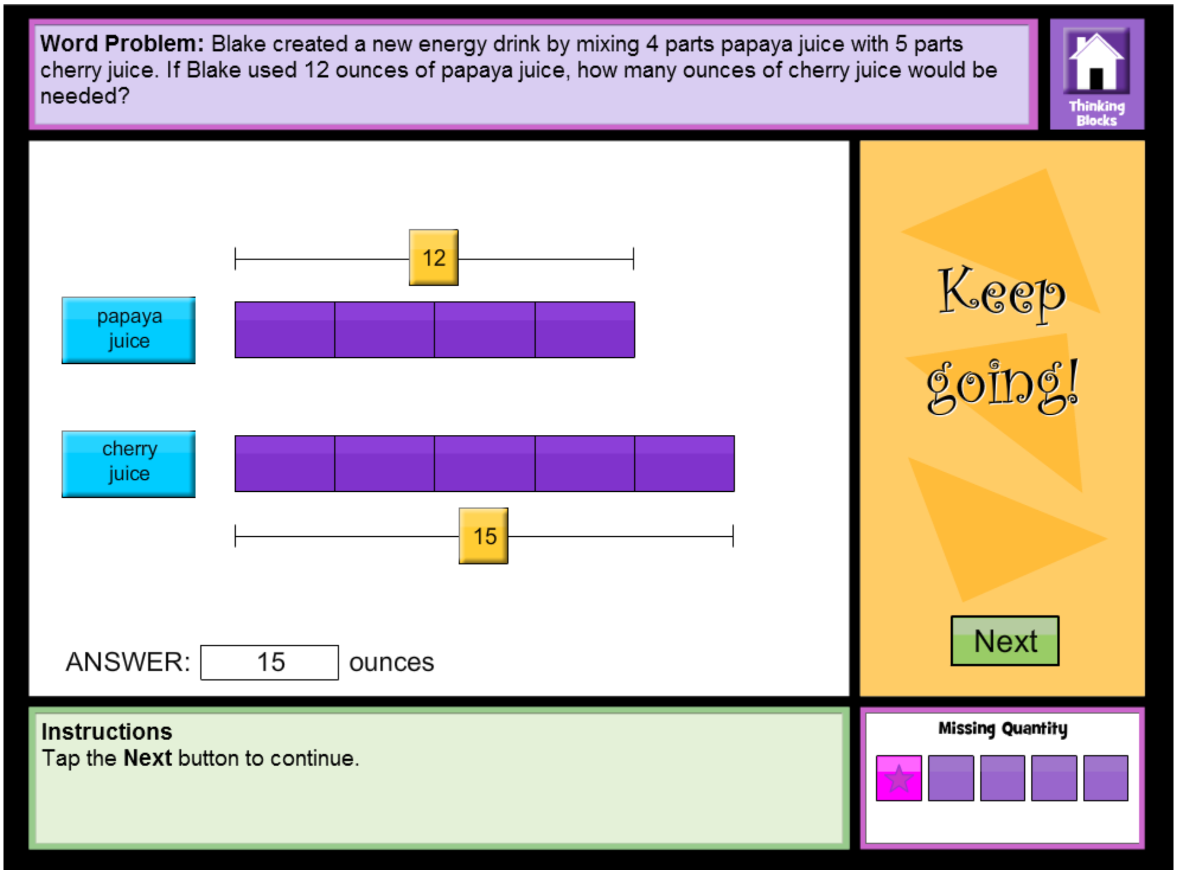


Figure 10. Work completed and question answered.

Another virtual manipulative that lends itself for modeling ratio and proportions problems is Number Pieces, which can be accessed at www.mathlearningcenter.org/apps. Originally, the intent of Number Pieces was for use in developing number sense concerning place value; however, the researcher found that Number Pieces also serves the purpose of modeling ratio and proportion problems as well. Unlike the Thinking Blocks virtual manipulative, Number Pieces does not provide support in a step-by-step fashion. A sample screen capture from Number Pieces appears in Figure 11.

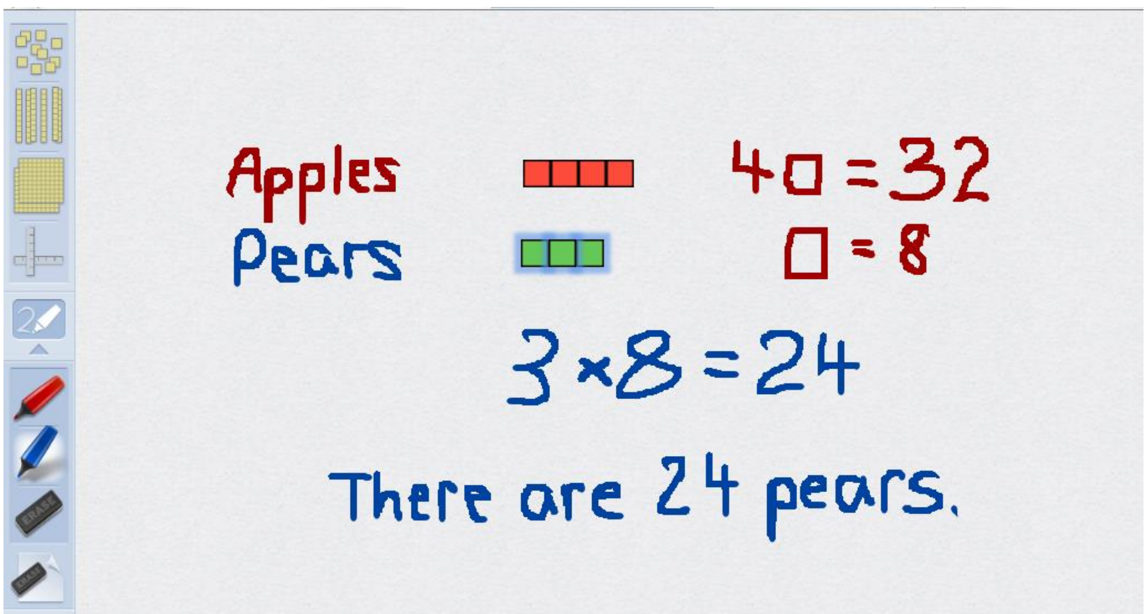


Figure 11. Number Pieces.

Gender Differences

The concept of gender differences in the teaching and learning of mathematics is not new, as evidenced by studies dating as far back as 1977. Peterson and Fennema (1985) concluded in a study of fourth grade students that boys and girls did not differ significantly in their mathematical achievement, but engagement in the classroom was influenced by the type of activities. One longitudinal study that focused upon gender differences in the elementary mathematics classroom noted that strong and consistent gender differences were found with respect to the strategies used to solve problems (Fennema, Carpenter, Jacobs, Franke, & Levi, 1998). As concluded by Fennema et al. (1998), boys have a tendency to use more abstract strategies, like symbolic manipulation, while girls tend to favor concrete strategies, such as modeling and counting. Klahr, Triona, and Williams (2007) reported in their study of middle school students that gender did not have a significant impact on the results when students were exposed to instruction based upon the use of manipulatives.

When technology is introduced into the mathematics classroom, an additional variable appears that can influence gender differences. Heemskerk, Ten Dam, Volman, and Admiraal (2009) stated there are indications that the use of technology affects girls differently than boys; in fact, this study concluded that girls especially experience benefits when educational tools are incorporated into the mathematics classroom. Goldstein and Puntambekar (2004) declared that data from their study suggested both girls and boys hold similar attitudes concerning computers and group work in a mathematical setting. Additionally, girls may actually participate more actively and persistently in a technology-rich collaborative environment. Considering the various findings from past research with gender differences, a study of any effect when virtual manipulatives are used in developing proportional reasoning may be able to contribute additional information to the current body of literature. For instance, the potential interaction between the use of technology and development of proportional reasoning skills exists, but the extent to which gender influences this interaction is unknown. Additionally, the influence of technology-input modality upon the development of proportional reasoning skills is also unknown.

Gesturing and Learning

Along with technological advances come the opportunities to implement new technologies into the classroom; however, not all hardware devices and software packages are compatible. For example, Jobs (2010) announced that Apple had adopted

HTML5, CSS, and JavaScript, which are software programs used in the design of computer applications. These software programs are designed to work in a touchscreen environment; for example, the tangible technologies described by Manches and O'Malley (2012) operate with touch instead of being mouse driven. Adobe Flash was specifically designed to work with personal computers (PCs) using mice, not touch screens using fingers (Jobs, 2010). With these choices of input modality, the opportunity presents itself to examine any advantages one input modality might have over the other. With respect to touchscreen input modality, the use of hands to interact with computer software may enhance learning, much like gesturing. Sinclair (2012) defined gesturing as hand motions that accompany speech.

Dewar (2013) summarized that gesturing enhances learning in several ways, including:

- freeing up working memory space in students, which reduces cognitive load;
- assisting students to retain what they have learned; and,
- enabling students to process mental visualization.

Not only does gesturing enhance communication (Alibali, Flevares, & Goldin-Meadow, 1997), gesturing while giving instructions assists children in learning tasks (Ping & Goldin-Meadow, 2010). Ehrlich, Levine, and Goldin-Meadow (2006) stated that gesturing improves performance of five-year old children when they attempt mental rotational tasks.

Gesturing occurs when the hands of an individual are unencumbered with objects or devices. Although the use of touch technology may not be classified as proper gesturing since the hands are engaged with an object, the use of hands in order to activate, operate, and manipulate touch technology may assist in the processing of the information that is generated in this input modality. At this time, little is known about how the use of a touch-screen interface influences or changes mathematics learning ("iPad Math Apps", 2013). By accepting the premise that touch technology can aid in learning, it is possible to extrapolate that the use of a mouse to interact with a virtual manipulative may hinder the processing of information; in fact, this question appears in findings reported by Manches and O'Malley (2012). By restraining a natural response to interact with the learning environment, mouse technology might erect a barrier to processing information.

Researcher's Background with Virtual Manipulatives

Since the fall semester 2008, the researcher has been an instructor for two different online classes for the state's Regents Online Degree Program (RODP), which can be found at http://www.rodp.org/. The two classes for pre-service elementary teachers are Number Concepts for Elementary Education and Geometry for Elementary Education. Both of these classes incorporate various websites with different types of virtual manipulatives into the assignments that constitute required coursework. There are several reasons as to why virtual manipulatives are used in this online learning environment:

- Availability of virtual manipulatives: The development of technology allows virtual manipulatives to be available to pre-service teachers. In fact, without virtual manipulatives, conceptual development would be more difficult to attain in the online classroom environment;

- Awareness of various types of virtual manipulatives: In order for pre-service teachers to implement virtual manipulatives into their future classrooms successfully, they must be aware of the existence of these tools; and,
- Experience using various types of virtual manipulatives: Experience with the use of virtual manipulatives enables the pre-service teachers to gain confidence in using these resources.

In the experience of the researcher, teaching pre-service elementary teachers in an online environment allows students to share their feedback concerning the websites used in the classes. Their feedback consistently includes an initial non-awareness of virtual manipulatives. However, a review of the literature indicates that acceptance and use of virtual manipulatives is not an automatic event; teachers must be trained on using manipulatives effectively. For example, a three-year study of 78 pre-service middle grades mathematics teachers who used both physical and virtual manipulatives concludes the use of manipulatives helps pre-service teachers build their own conceptual understanding, which provides them with sound pedagogical strategies for future use (Hunt et al., 2011). Concerning the learning of mathematics, an interview with Zoltan P. Dienes reported that the famous pedagogue believes that “what really matters is that actual learning can take place with the proper use of materials, games, stories and such and that should be our focus” (Sriraman & Lesh, 2007, p. 72). The availability of virtual manipulatives permits the study of their impact upon the development of mathematical concepts; particularly, the role of virtual manipulatives in the development of proportional reasoning skills should be explored.

Summary of Factors

As presented in this chapter, the topics of proportional reasoning, manipulative use in learning mathematics, gender differences in the mathematics classroom, and technology in the mathematics classroom lend themselves to research in and of themselves; however, these separate areas combine to form a specific area of interest for investigation. The circular diagram in Figure 12 models the relationships between the various areas of interest. Each topic is a region in which research may be performed separately; however, the overlap of the different topics provides its own area of research for consideration.

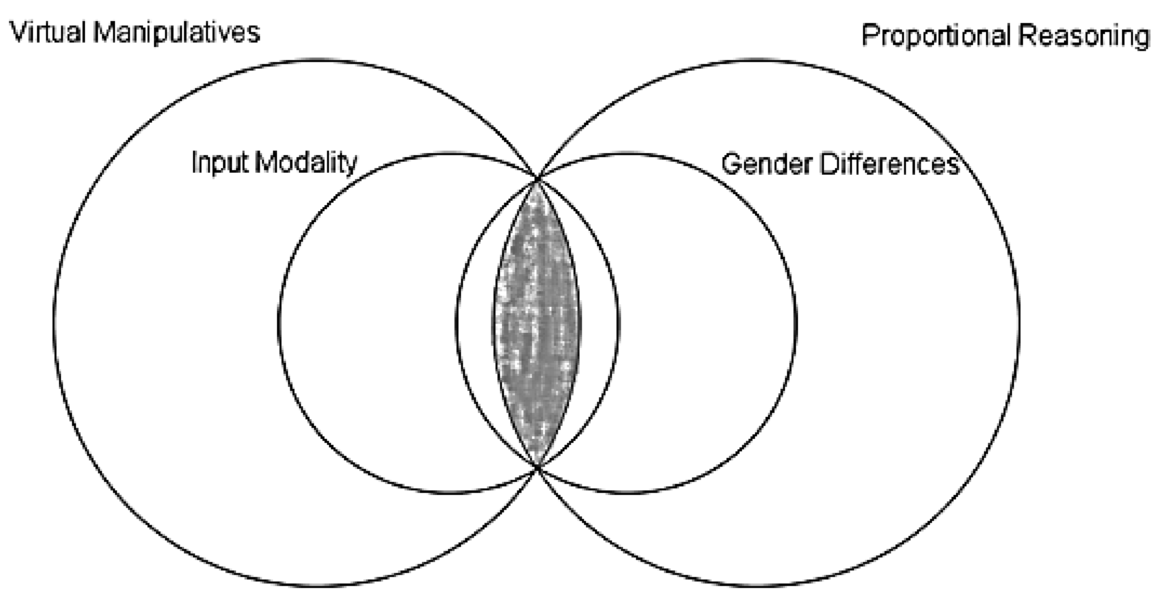


Figure 12. Summary of factors.

As shown in Figure 12, the shaded area of intersection involving the four circles represents the focus of this research study: the use of virtual manipulatives when teaching proportional reasoning skills to grade six students, with consideration of the possible impact of input modality with respect to gender. The circles representing virtual manipulatives and proportional reasoning are larger in comparison to the other circles to indicate the greater emphasis placed upon these two topics. Although each topic is worthy of research in its own right, it is the intersection of the topics that has drawn the attention of the researcher.

Purpose of Study and Research Questions

The primary goal of this research was to measure the impact of the use of virtual manipulatives on the development of grade six students' proportional reasoning skills. Although virtual manipulatives are required to help achieve this primary goal, it should be noted reliance upon virtual manipulatives in order to solve proportional reasoning problems should decrease, once students develop proficiency in working with problems in a proportional setting. If students must depend upon virtual manipulatives in order to work with ratios and proportions, then virtual manipulatives serve as a crutch instead of a scaffold. Secondary goals of this research involved the following: (a) to investigate whether gender of students exhibits any main effect when using virtual manipulatives to develop proportional reasoning skills; (b) to investigate whether technology-input modality (touchscreen or mouse) yields any differences when using virtual manipulatives to develop proportional reasoning skills; and (c) to investigate whether the factors of gender and technology-input modality interact when using virtual manipulatives to develop proportional reasoning skills.

In order to accomplish the research goals, the following questions were proposed. The primary question considered is listed first, followed by the secondary questions:

1. In general, what gains are made by grade six students when virtual manipulatives are used to teach certain aspects of proportional reasoning?
2. What differences exist with respect to gender when using virtual manipulatives to teach proportional reasoning to grade six students?
3. What differences exist between students who use touch technology and those who use mouse technology when studying proportional reasoning?
4. What interactions exist between gender and technology-input modality when students use virtual manipulatives when studying proportional reasoning?
5. How do grade six students who use virtual manipulatives differ from those grade six students who do not use virtual manipulatives when developing proportional reasoning skills?

Significance of Study

Conclusions from research in the areas of proportional reasoning and gender differences within mathematics have had an impact upon the field. However, the introduction of virtual manipulatives requires further research in order to ascertain whether they can be effectively utilized in the middle school mathematics classrooms as additional tools to help students acquire and strengthen proportional reasoning skills. Any influence that might be exerted with respect to gender should receive attention when virtual manipulatives are used to develop proportional reasoning skills. Also, any differences attributable to technology-input modality and the potential impact of these differences upon students as they learn are worthy of consideration. Finally, any discovery of interaction with respect to gender and technology-input modality when virtual manipulatives are used to develop proportional reasoning skills deserves investigation as well.

Chapter Summary

As seen in the body of literature, researchers recognized the development of proportional reasoning skills as a crucial topic requiring attention in the middle school mathematics classroom. This study endeavored to examine the potential impact of two virtual manipulatives, Thinking Blocks and Number Pieces, upon the development of proportional reasoning skills of grade six students. Additionally, any differences associated with gender or technology-input modality with respect to the development of proportional reasoning skills was considered. Also, the researcher considered any interaction between the factors of gender and technology-input modality in light of the development of proportional reasoning skills.

CHAPTER II: LITERATURE REVIEW

Introduction

The goals of this research involved the use of virtual manipulatives in attempting to develop proportional reasoning skills. The primary goal of this research was to measure the impact of the use of virtual manipulatives on the development of grade six students' proportional reasoning skills. Secondary goals of this research involved the following: (a) investigations of whether gender of students exhibits any main effect when using virtual manipulatives to develop proportional reasoning skills; (b) whether either technology-input modality exhibits any advantages over the other when using virtual manipulatives to develop proportional reasoning skills; and (c) whether gender and technology-input modality expresses any interaction with each other when using virtual manipulatives to develop proportional reasoning skills. A review of the existing literature provides the background of work already published. Additionally, the ideas that guide this research are based upon evidence found in the literature review, including the following: (a) the development of proportional reasoning skills is needed in order to build understanding of other algebraic topics, such as slope (Cheng, Star, & Chapin, 2013); (b) research has shown that manipulatives can help students in learning mathematics (Sowell, 1989); and, (c) virtual manipulatives have the potential to affect cognitive functions of students who work with them (Pea, 1987; Songer, 2010).

Conceptual Framework

As presented in Chapter I, the conceptual framework provided guidance to the researcher as he reviewed the existing literature. Specifically, the combined topics of proportional reasoning and virtual manipulatives drew the attention of the researcher to an area of need existing in the literature. With respect to proportional reasoning, it is not sufficient to say that students either possess proportional reasoning skills or they do not possess proportional reasoning skills; rather, proportional reasoning skills are developed. Similarly, the growth of technology supports the introduction of virtual manipulatives into the classroom, but the perception of the role of virtual manipulatives in the mathematics classroom guides their use.

Levels of Proportional Reasoning

In their research concerning adolescents in seven countries, Karplus et al. (1977) identified four levels of proportional reasoning; the lowest category of proportional reasoning changed through the years from Intuitive to Illogical, as seen in Khoury's work with Mr. Tall/Mr. Short (Khoury, 2002). Therefore, these four categories of proportional reasoning employed in classifying the responses made on a proportional reasoning task include: (a) Category I (Illogical); (b) Category A (Additive); (c) Category Tr (Transitional) and; (d) Category R (Ratio). These categories are presented in the order of increased development; thus, Category I is the lowest level of proportional reasoning, while Category R is the highest level of proportional reasoning. Other researchers modified these categories as part of their studies; for instance, Riehl and Steinthorsdottir (2014) elaborated upon distinctions within the Transitional category for their work.

For this study, a student in Category I gave no explanation or showed inaccurate computation concerning the proportional work at hand. Also, a student who described their result as a guess belongs in Category I. For Category A, the student's work was based solely upon an approach using addition or subtraction without any consideration of relative comparison. Work classified as Category Tr had an additive component, but there was also an attempt to compare changes in a relative fashion. Repeated addition of the same value, or partitioning, without introducing a multiplicative approach belonged to the transitional category. Also, any attempt to use ratios incorrectly or incorrect answers obtained using ratios were classified as part of Category Tr. In Category R, the use of a constant ratio or conversion of units appears in the response (Karplus et al., 1980). After students completed performance tasks in this study, an inspection of their work generated ratings at the four levels of proportional reasoning as presented by Karplus et al.

Clearly, the existing literature supports the idea of a leveled development of proportional reasoning skills. As stated previously, this leveled approach to determine proportional reasoning is not unique to Karplus and his colleagues. Langrall and Swafford (2000) also identified four different levels of strategies employed by students to complete proportional reasoning tasks: Levels 0-3. As summarized by Langrall and Swafford, these levels are described as follows: (a) Level 0 work shows no proportional reasoning; (b) Level 1 work involves informal reasoning concerning proportional situations with the assistance of manipulatives, pictures or other models; (c) Level 2 work demonstrates a more sophisticated strategy of quantitative reasoning without manipulatives or can link models with appropriate calculation; and (d) Level 3 work shows the ability of students to set up and solve proportions with full understanding of the structural relationships that exist. Since the researcher designed the study to incorporate Thinking Blocks and Number Pieces, any participant's work based upon the use of these virtual manipulatives automatically qualified for inclusion at Level 1.

Virtual Manipulatives as Cognitive Tools

Although the development of educational technology encouraged the use of such resources in the classroom, not all technological resources provided the same type of experiences. As stated earlier, Songer (2010) distinguished between digital resources and cognitive tools. A cognitive tool transcends the level of just providing information; it is a resource that is specifically designed to allow students to achieve particular learning goals on a topic of interest. Although virtual manipulatives are considered to be digital resources in that they are computer-based information sources, they also qualify as cognitive tools. A teacher can generate a worksheet or display a video from a website so that students can practice certain mathematical skills. In this case, the teacher used a digital resource. However, these resources provided practice rather than conceptual development.

In the case of a cognitive tool, the website must allow for dynamic interaction and connectivity between changes that the student introduces and the results seen from such change. For this research, one goal was to ascertain the impact that virtual manipulatives have when they are used in developing proportional reasoning skills. Since Thinking Blocks and Number Pieces provided opportunities for students to build models and explore proportional relationships, the virtual manipulatives function as cognitive tools.

Pea (1987) described cognitive technology tools as a means for users to act upon representations of mathematical objects, as well as sharing the cognitive load of the learner. Virtual manipulatives fulfill this description. As seen by the descriptions of Songer and Pea, the emphasis upon the cognitive aspect of virtual manipulatives by these two researchers cannot be overlooked.

Proportional Reasoning

McIntosh (2013) defined proportional reasoning to be "the deliberate use of multiplicative relationships to compare quantities and to predict the value of one quantity based on the values of another" (p. 7). Along with this definition, McIntosh also provided situations that were not considered to fulfill the requirement to be proportional reasoning; namely, setting up and solving a proportion by using cross multiplication to find a missing number. In order to develop proportional reasoning skills, it is necessary to understand the concept of ratio.

Ratios

A ratio must be understood as a relative comparison. Nikula (2010) considered that moving beyond absolute comparisons to relative comparisons is necessary for understanding ratios. The difference between absolute comparison and relative comparison is illustrated by the two phrases *how much more* and *how many times more*, respectively. Absolute comparison is additive, while relative comparison is multiplicative. As Lobato and Ellis (2010) suggested in their work with essential understandings concerning ratios and proportions, "a ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit" (p. 12).

Ratios and fractions are linked, but they are not identical. Students often believe that the terms *ratio* and *fraction* are interchangeable (Lobato & Ellis, 2010). However, Clark, Berenson, and Cavey (2003) illustrated the distinction between ratios and fractions with the golden ratio, written $\frac{\sqrt{5}+1}{2}$; the value of this ratio is an irrational number, whereas fractions are rational numbers. Another distinction between these two concepts was given by Lobato and Ellis (2010) concerning the type of comparisons that can be expressed. Ratios express part-to-part comparisons, while fractions express part-to-whole comparisons.

Concerns with Proportional Reasoning

Students can write and solve proportions while not having a developed sense of proportional reasoning. Ben-Chaim, Keret, and Ilany (2012) regarded the presence of a formal strategy as an indication of the existence of proportional reasoning and abstract thinking; that is, using the proportion formula $\frac{a}{b} = \frac{c}{d}, (a, b, c, d \neq 0)$ was a sign that the user possessed proportional reasoning skills. The context in which such a statement was made involved the use of algebraic expressions within the proportion. A search of the literature revealed that other researchers hold a different perspective concerning the use of cross-multiplication to solve a proportion. It is possible for a student to set up and solve a proportion using cross-multiplication while at the same time lack a developed set of proportional reasoning skills (Nikula, 2010). This situation was demonstrated by a student who set up and solved a proportion successfully, due to the mathematics problem being in a context that lent itself to such a procedure; however, the same student could not solve other tasks which required proportional reasoning (Lobato and Ellis, 2010). Lamon (2012) declared that students compensate for a lack of proportional reasoning by using rules learned in algebra and geometry.

Students often apply proportional relationships to situations that do not call for proportions due to superficial cues that are present in the problem (Lamon, 2012). The inappropriate use of proportions to solve a mathematical problem can indicate a weak sense of procedural fluency with respect to proportional reasoning. Procedural fluency has been defined as the knowledge of procedures and the knowledge of when and how these procedures are to be used (National Research Council, 2001). In the case of proportional reasoning, a misapplication of proportions in order to solve a mathematical problem often occurs when three numbers are present and a fourth value is sought. Lamon (2012) provided examples of problems in which students attempted to use proportions inappropriately in order to solve for a missing quantity:

- "If a football player weighs 225 lbs, how much will 3 football players weigh?" (p. 4)

In this first problem, the use of proportions assumes that all football players weigh the same. This assumption makes the term "football player" constant, much like a unit of measurement.

- "If Ed can paint his room by himself in 3 hours, and his friend Jake works at the same rate as Ed, how long will it take to paint the room if the boys work together?" (p. 4)

For the second problem, the situation involves an inverse relationship with the number of painters and the hours it takes to paint: as the number of painters increases, the time it takes to paint the room decreases.

- "Bob and Marty like to run laps together because they run at the same pace. Today, Marty started running before Bob came out of the locker room. Marty had run 7 laps by the time that Bob had run 3. How many laps had Marty run by the time that Bob had run 12 laps?" (p. 5)

On the last problem, the head start of running laps by the first person is an additive situation, not a multiplicative situation. When a student has a developed sense of proportional reasoning, they will know when and when not to use a proportion to solve a mathematical problem.

The development of proportional reasoning should be "one of the hallmarks of the middle grades mathematics program" (NCTM, 1989, p. 213). Langrall and Swafford (2000) found that students use proportional reasoning to consolidate their knowledge gained at the elementary level in order to build a foundation for future mathematical learning, especially with respect to algebraic reasoning. In her thesis, Korth (2010) wrote "students need to have a conceptual knowledge of proportions in order to apply the concepts to real life" (p. 3). However, several research studies have shown that proportional reasoning is challenging to develop successfully in students (Cramer & Post, 1993; Nabors, 2003; Norton, 2005; Singh, 2000; Yetkiner & Capraro, 2009). Larson (2013) reported that students must make significant shifts in their thinking in order to acquire proportional reasoning skills.

Karplus et al. (1977) identified a situation in which a connection between conceptual development and expression of the concepts in language existed. In their study, Karplus and his colleagues stated that students described proportional relationships in rudimentary terms. Munro (1989) affirms this situation is not unusual; rather, children construct new ideas and learn later the conventional language formats in order to express these ideas. Some researchers propose the idea of developing concepts and language simultaneously, stating "linking manipulation [mathematically] to the use of language is crucial in such learning experiences (Fisher & Blachowicz, 2013)." According to MacGregor (2002), no linguistic theory existed at that time which provided a satisfactory account of how comparison of quantities was conceptualized and expressed in words. So, attention to the use of language and vocabulary while students develop proportional reasoning skills is warranted.

Another issue to consider in the topic of proportional reasoning is the void created by the decreasing emphasis of research on the topic within the realm of mathematics education. According to Lamon (2007), major contributors to rational number research have deceased or retired. An examination of the dates for references in *Developing Essential Understanding of Ratios, Proportions, & Proportional Reasoning, Grades 6-8* (Lobato & Ellis, 2010) appears to support Lamon's assertion: only 10% of the references are dated after 2007. A similar inspection of the articles listed on the website for the Rational Number Project revealed that 98 articles or books were published in the years 1979-2013; however, only eight articles or books were published in the years 2007-2013 (Regents of University of Minnesota, 2013). It is possible to disagree with Lamon's assertion; however, this study is intended to contribute to filling any void in proportional reasoning research, especially the application of virtual manipulatives to the development of proportional reasoning skills.

Another possibility to consider when examining proportional reasoning research is the cyclical nature of trends with respect to such research. It is not unusual to see emphasis on certain mathematical topics in a given period of time, which in turn encourages researchers to pursue these topics. As the emphasis shifts to other mathematical topics, the research also shifts. At the NCTM Annual Meetings of 2013 and 2014, held in Denver and New Orleans respectively, numerous sessions focused upon proportional reasoning topics. So, although Lamon may have a point concerning a lull in research with a focus on proportional reasoning within a given period of time, more recent research may again focus upon proportional reasoning.

Background of Manipulatives

Teachers have tried to determine better ways for students to learn and master mathematical concepts and sharpen mathematical skills. Commenting upon this situation, Loucks and Gangloff (2006) declared that “once upon a time, the only way for students to practice math skills was using pencil and paper” (p. C3). The development of mathematical ability and the deepening of mathematical concepts in the minds of students do not solely depend upon menial drills. Manipulative materials have been introduced into the mathematics classroom for years. These materials have been accompanied by differing opinions as to their efficacy. However, there have been advocates for the use of manipulative materials with respect to education for centuries. Comenius, Allingham, Pestalozzi, Montessori, Froebel, and Goodrich are just a few educational scholars who recommended that education should not be restricted to just words and lecture alone;

rather, manipulatives should have an appropriate use in the mathematics classroom (Namukasa et al., 2009).

Effectiveness of Manipulatives

Research on the efficacy of manipulative materials has been conducted at various levels throughout the years (Namukasa et al., 2009; Suh & Moyer, 2007). Friedman (1978) reviewed the then-available four studies that focused upon the effectiveness of manipulative use in the mathematics classroom. He also examined 18 doctoral dissertations with a focus upon manipulative use in the classroom, produced between 1970 and 1978; of these studies, 14 of them demonstrated no significant differences or found mixed results. In light of these findings, Friedman determined that manipulative use produced effective results for students in the first grade only, arguing the studies involving students above the first grade did not produce any significant differences between groups using manipulatives and those groups that did not. In the same article, Friedman suggested manipulatives should be included as a tool in mathematics instruction and further research performed concerning effective use of manipulatives.

Meta-analyses have been performed concerning studies on the effects and influences of instructional aids and manipulative materials with respect to mathematics instruction and mathematical achievement. Sowell (1989) indicated that implementing manipulative materials into mathematics instructions in kindergarten through grade eight produced greater achievement as compared to mathematics instruction without such materials. Carbonneau, Marley, and Selig (2013) performed a meta-analysis on 55 different studies that compared instruction using manipulatives with instruction using only abstract mathematical symbols. Their findings indicated that the use of manipulatives in mathematics instruction produced a small- to medium-sized effect on student learning. Additionally, other instructional variables influenced the strength of this effect.

There are differing opinions with respect to the effectiveness of the use of physical manipulatives in the mathematics classroom. A study by Raphael and Wahlstrom (1989) concluded that teacher experience and extensive use of instructional aids in the geometry classroom contributed to a greater coverage of content; but, in general, teachers were not very enthusiastic about the efficacy of manipulatives. Fey's review of instructional methods research studies concluded that many of these studies had flaws, which cast dispersion upon the conclusions that were made by the researchers (Fey, 1980). McNeil and Jarvin (2007) reported on two possible theories which explained why manipulatives might hinder students in learning mathematical concepts instead of helping them. These hindrances were: (a) ineffective implementation of manipulatives by teachers; and (b) dual representation of manipulatives. With the former, the problem was in how teachers may choose to use the manipulatives, while the latter involved the objects themselves.

Active, hands-on learning can lead to conceptual development instead of just procedural mastery. Schoenfeld (2007) stated that "years of learning mathematics passively result in a population that tends to be mathematically passive" (p. 72). Virtual manipulatives provide an environment of activity and support, which in turn can engage the student in learning mathematics concepts, such as proportional reasoning. The overall concept of mathematical proficiency is an idea that is comprised of five different strands: (a) conceptual understanding – comprehension of mathematical concepts, operations, and relations; (b) procedural fluency – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately; (c) strategic competence – ability to formulate, represent, and solve mathematical problems; (d) adaptive reasoning – capacity for logical thought, reflection, explanation, and justification; and (e) productive disposition – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (National Research Council, 2001). Although virtual manipulatives may not address all five strands of mathematical proficiency equally, their use can contribute to conceptual understanding, strategic competence, and adaptive reasoning by actively involving students in the process of learning.

It is appropriate to consider how manipulatives influence and affect learning of mathematical concepts. The use of manipulatives in the classroom increased during the 1960s when rationales for their use were offered (Thompson, 1994), but some of the reasoning behind this increase was based more on availability than any particular scientific theory. In more recent years, Namukasa et al. (2009) cited the following rationales as being the most prevalent for manipulative use: psychological theories of concept development and children growth, theories of discovery and active learning, and different learning styles.

Effectiveness of Virtual Manipulatives

With respect to the conceptual framework for this research, several rationales support the idea that virtual manipulatives are cognitive tools. Dual coding (Clark & Pavio, 1991) and scaffolding (Suh & Moyer, 2008) are two cognitive theories suggesting virtual manipulatives provide support for students as they learn and master mathematical concepts. The linking of multiple modes of expression and step-by-step support as students work mathematical problems are features intrinsic to virtual manipulatives (Suh & Moyer, 2007); also, improved attitudes of students toward mathematics have been reported (Lee & Chen, 2010; Reimer & Moyer, 2005). In addition, factors of convenience, availability outside of the classroom, and increased time on task by students are reported (Mildenhall et al., 2008; Moyer et al., 2002).

One cognitive theory in particular that is associated with virtual manipulatives is Dual Coding Theory (DCT). DCT advances the idea that information for memory is processed and stored using two different sets of codes: visual codes and verbal codes. Virtual manipulatives can provide students access to both types of codes. According to Clark and Pavio (1991), these two different coding systems are interconnected. DCT suggests that information is more readily processed and retained through presentations that involve both images and text (Jacobs, 2005). Suh and Moyer-Packenham (2007) examined the use of both physical and virtual manipulatives with grade three students learning fraction and algebraic concepts, using both single-coded and dual-coded items. They concluded that dual-coded representations in virtual manipulative environments had the potential to be effective in teaching mathematical processes; additionally, the method of using dual-coded representations may aid in the learning of complex algorithmic processes. Thus, the research of Suh and Moyer-Packenham supports the premise that virtual manipulatives function as a cognitive tool.

Researchers report about the ability of manipulatives to bridge the gap between various levels of learning in mathematics. Manipulatives provide scaffolding for students as they internalize learning, which was a support mechanism proposed in Vygotsky's work (Namukasa et al., 2009). In their work with teaching fraction equivalence to a group of 19 grade four students, of which 10 were considered special needs students, Suh and Moyer (2008) concluded that unique features of virtual tools enabled special needs students to reduce the cognitive load associated with the task of maintaining both pictorial images and symbolic notations. This reduction of cognitive load allowed students to focus more upon the mathematical processes and relationships.

Virtual manipulatives appear to go beyond these listed rationales and achieve a level of efficacy based upon other factors. Suh and Moyer (2007) reported the explicit linking of multiple modes of expression in their research involving grade three students who worked with virtual and physical algebra balances. Virtual manipulatives afforded the opportunity to make meaning and see relationships that occurred when students engaged with technology (Moyer et al., 2002). Suh, Moyer, and Heo (2005) used virtual manipulative fraction models to allow grade five students the ability to use multiple representations, as well as the ability to translate between these representational models.

Step-by-step support with algorithmic processes is afforded in the mathematics classroom when students engage with virtual manipulatives. According to NCTM

(2000), children can use virtual manipulatives "to extend physical experience and to develop an initial understanding of sophisticated ideas like the use of algorithms" (pp. 26-27). In addition, immediate feedback with self-checking mechanisms is available to students when they use virtual manipulatives (Suh & Moyer, 2007).

Studies report an improved attitude toward mathematics with the introduction of virtual manipulatives into the classroom setting; Reimer and Moyer (2005) reported that virtual manipulatives enhanced students' enjoyment while learning about fractions. Lee and Chen (2010) studied the progress that grade nine students in Taiwan made while working with virtual manipulatives as a problem-solving tool. Using a Mathematics Attitude Scale (MAT), they concluded that students' attitudes toward mathematics improved as a result of working with virtual manipulatives. Additionally, students have demonstrated increased time on task as a result of using virtual manipulatives (Crawford & Brown, 2003).

In addition, specific factors that do not connect to learning theory per se are also necessary considerations when determining advantages that virtual manipulatives might offer. Technological advances have made access to virtual manipulatives feasible for students and teachers. With the increased access to the Internet in the classroom, virtual manipulatives are usually available as free resources (Reimer & Moyer, 2005). Teachers who are reluctant to send manipulatives home with students can offer access outside of the classroom via the Internet. Concerns about having enough materials for each student, as well as losing pieces of physical manipulatives, are overcome by the use of virtual manipulatives (Moyer et al., 2002). Mildenhall et al. (2008) commented on the improved use of time management and classroom management achieved by teachers using virtual manipulatives as compared to physical manipulatives.

Physical Manipulatives and Virtual Manipulative Counterparts

By comparing physical manipulatives with their virtual counterparts, it is possible to determine areas in which the physical models may have potential shortcomings for which virtual models may compensate. Physical manipulatives require storage space; in addition, the time it takes to distribute, collect, and inventory materials can reduce instructional time. Mildenhall et al. (2008) identified the following advantages for teachers, students, and parents with respect to the innate qualities of virtual manipulatives:

- it is possible to record and store the movements and actions of users;
- parents have the potential to assist students;
- there is potential for alteration;
- virtual manipulatives are available and accessible to diverse groups of students, such as students with special needs; and
- virtual manipulatives appeal to older students who believe that these forms are more sophisticated than manipulatives in concrete form.

There are advantages to implementing virtual manipulatives in the classroom; nevertheless, there are certain caveats as well. Since virtual manipulatives are technologically dependent, problems can arise when technology fails to function as planned. Websites that were once active might become inactive and unavailable at a moment's notice. Suh and Moyer (2007) cautioned that students do not automatically make connections between actions with manipulatives and actions with symbols; teachers must make a conscious effort to establish these connections. Although a lack of connection can occur with either type of manipulative, the danger of students perceiving the virtual manipulative models to be merely another type of game or form of entertainment must be considered and addressed to prevent any disconnection from the mathematical concept being studied.

Findings Concerning the Use of Virtual Manipulatives

The use of technology in the mathematics classroom surged as various hardware and software packages became available; virtual manipulatives are included in these advances made by technology. Other than availability, research shows that virtual manipulatives have been used effectively in the mathematics classroom setting with respect to teaching the concepts of equivalent fractions and adding fractions with unlike denominators (Moyer-Packenham et al., 2013; Moyer-Packenham & Suh, 2012). Li and Ma (2010) conducted a meta-analysis of the impact of computer technology on mathematics education in K-12 classrooms. They concluded a variety of studies have examined virtual manipulatives, and that these tools have had a positive impact on both student achievement and student attitude toward mathematics.

Another meta-analysis conducted by Moyer-Packenham and Westenskow (2011) compared the use of virtual manipulatives to other instructional treatments. Within 29 studies, 79 effect score cases emerged comparing virtual manipulatives with other instructional treatments. Based upon their findings, using virtual manipulatives was an effective instructional method for teaching mathematics when compared to other instructional treatments. Moyer-Packenham and Westenskow also found that combining virtual manipulatives with physical manipulatives as a treatment resulted in some of the largest effects produced in the meta-analysis.

Moyer-Packenham et al. (2013) designed a mixed method study, using a pre-test/post-test quantitative component and observation ethograms when comparing virtual manipulatives to other instructional treatments in a third- and fourth-grade classroom setting. They concluded that virtual manipulatives were just as effective as physical manipulatives in teaching students concerning equivalent fractions. While examining the use of virtual manipulatives with different achievement groups of fifth-grade students learning fraction equivalence and fraction addition with unlike denominators, Moyer-Packenham and Suh (2012) used a pre-test/post-test component and videotapes of classroom sessions to gather data from four different groups of students: one low achieving, two average achieving, and one high achieving. They found that although all groups made gains in their mean scores, the gains in the low achieving group were statistically significant, Pre-test mean score = 70.15, $SD = 21.44$, Post-test mean score = 81.31, $SD = 12.34$; $t(12) = -2.433$, $p = .032$.

Extent of Virtual Manipulatives Use

There seems to be differing opinions as to the extent of the research conducted on the efficacy of virtual manipulatives. Several articles report that the body of research in the area of virtual manipulatives is small but growing (Hunt et al., 2011; Martin & Lukong, 2005; Reimer & Moyer, 2005); being such, it is a challenge to find research that specifically addresses and documents work in classrooms with respect to the use of virtual manipulatives (Steen et al., 2006). Other reports indicate that the use of virtual manipulatives is a new trend with few studies investigating the impact of virtual manipulatives upon the attitudes of students toward mathematics (Lee & Chen, 2010). Alternately, some researchers report that research on manipulatives is abundant (Namukasa et al., 2009). In their work with virtual manipulatives that focus upon the concepts of heat and temperature, Zacharia, Olympiou, and Papaevripidou (2008) emphasized research with virtual manipulatives has existed for decades. One can attribute these contradictory opinions to the various definitions that exist for virtual manipulatives, the use of virtual manipulatives in different fields of study, and the different types of virtual manipulatives available.

Research on virtual manipulatives is neither limited to the mathematics classroom nor restricted to just one level or topic of study. Zacharia et al. (2008) studied the impact of virtual manipulatives on the understanding of the concepts of heat and temperature by undergraduate students in a physics classroom; moreover, the study was replicated (Zacharia & Constantinos, 2008). The researchers concluded that virtual manipulatives were an effective tool in the understanding of the physical concepts studied.

Several studies focus upon the use of virtual manipulatives in the elementary school. Steen et al. (2006) conducted research with grade one students and geometry instruction using virtual manipulatives. Grade three students learning fraction concepts with virtual manipulatives was the focus of the research by Reimer and Moyer (2005), while Suh and Moyer (2007) concentrated upon grade three students learning algebraic relationships with the aid of virtual manipulatives. Suh, Moyer, and Heo (2005) researched the development of fraction sense in grade five students using virtual manipulatives concept tutorials. A common finding emerging from these studies concerns the impact that the use of virtual manipulatives has when compared to the use of physical manipulatives: virtual manipulatives can be just as effective as physical manipulatives when used in the mathematics classroom.

Research focusing upon the use of virtual manipulatives in a middle school setting indicates that the appropriate implementation of virtual manipulatives can be effective. Lee and Chen (2010) studied the impact of virtual manipulatives' use on the attitudes of Taiwanese junior high school students toward mathematics; they found that the use of virtual manipulatives had a positive effect about learning mathematics. Moyer and Bolyard (2002) used virtual manipulatives while working with middle school students and geometric thinking. They concluded that virtual manipulatives assisted middle school students to transition from one van Hiele level of geometric thought to the next by engaging students with a variety of interactive geometric models.

Technology-Input Modality

The use of technology in the classroom is an issue which justifies investigation in its own right, because the continual improvements in technology impact the efficacy of virtual manipulatives. NCTM advocated the appropriate use of technology, stating that "students can learn more mathematics more deeply" (NCTM, 2000, p. 24). The advent of interactive whiteboards makes it possible for teachers to demonstrate virtual manipulatives, as well as allows students to present their findings to an entire class, modeling their processes for all to see (Hwang et at., 2009; Mildenhall et at., 2008).

However, there are teachers who believe that students become overly dependent upon technology in a mathematics classroom, which has an adverse effect upon mathematical development and computational skill. Teachers report watching students reach for a calculator to perform simple operations (Bing & Redish, 2008; Steen et al., 2006).

Clark (1983) cautioned that "media are mere vehicles that deliver instruction but do not influence student achievement any more than the truck that delivers our groceries causes changes in our nutrition" (p. 445). Based upon this premise, Clark might argue that the manipulative type needed for use in the classroom should be the least expensive, since the instructional medium is independent of the delivery medium. However, opponents of this view contend that, "media must be designed to give us powerful new methods, and our methods must take appropriate advantage of a medium's capabilities" (Kozma, 1994, p. 16). Kozma's view reflects the recommendation of NCTM concerning the role of technology in the mathematics classroom: "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (NCTM, 2000, p. 24). Hastings and Tracey (2005) voiced their opinion that improved computer capabilities and the Internet are not replaceable and that learning is affected by them. Given these two camps of thought concerning the impact of technology in the classroom, the research proposed in this dissertation can result in conclusions which support one of these views.

According to Martin, Svihia, and Smith (2012), evidence regarding physical action's effects on learning mathematics is inconclusive; different studies declare that actions help, hurt, or have no impact at all. With respect to technology-input modality, the availability of both touch screen technology and mouse technology implies that some differences in efficacy might exist. The availability of the same virtual manipulatives in different input modalities permits comparison between touch screen modality and mouse modality. Heather Kearney (personal communication, January 15, 2014) announced that Gizmos, mathematics and science simulations that help students conceptualize many important situations in a mathematical or science setting, are now available for iPad through a specifically-designed application. Other applications that are now available for iPad users include Geogebra and Geometer's Sketchpad. Moyer-Packenham et al. (2014) declared that research concerning the effectiveness of touch technology with respect to children's learning has not kept pace with the rapid development of virtual manipulatives in the touch technology input medium. Thus, additional research comparing the two different technology-input modalities in a similar setting is warranted.

Gesturing and Learning

Sinclair (2012) defined gesturing as hand motions that accompany speech. Studies that focused upon the importance of gesturing while learning in different content areas concluded that gesturing helps to reduce cognitive load (Goldin-Meadow and Wagner, 2005). Chu and Kita (2011) worked with a group of 132 students in which the students were instructed to rotate objects mentally, picturing how the objects would appear after the rotation had been applied. They concluded that the students using gesture outperformed the other students. In a study that focused upon gestures used during a mathematics lesson, Cook and Goldin-Meadow (2006) concluded that students who used gestures during the lesson did a better job remembering the correct strategy demonstrated in the mathematics lesson as opposed to those students who did not. Goldin-Meadow, Nusbaum, Kelly, and Wagner (2001) conducted an experiment in which they asked their subjects, both children and adults, to remember a list of items while explaining how they had solved a mathematics problem. The results indicated that subjects from both age groups remembered more items from the list if the subjects gestured during their explanations. Gesturing before a lesson or during the lesson improved later performance, which implied that gesturing is a factor that improved retention and recall (Cook, Mitchell, & Goldin-Meadow, 2008).

Gesturing associated with bodily movement while engaged in learning activities can impact learning. Gerofsky (2010) conducted research in which she considered the impact of gesturing on mathematics learning. One theoretical basis incorporated in her work was embodied learning theory. Embodied learning theory explores teaching methods that integrate body, sensory, and intellectual engagement on the part of the learner. Goldin-Meadow and Beilock (2010) suggested that experiences in the world affect how information is processed; specifically, these experiences involve a greater network of sensorimotor regions of the brain. Paek and Hoffman (2014) determined manipulating computer mice can be challenging, depending on the age of the child and their prior experience using mice; further, they stated "it is exciting to think about the potential of new input devices such as touchscreens and gesture detection to overcome these challenges (p. 175)." With respect to virtual manipulatives and the development of proportional reasoning skills, comparing work produced by students who use the two different technology-input modalities has the potential to reveal any differences attributable to each modality.

Gender Differences

Extensive research, especially in the 1980's and 1990's, addressed questions concerning the influence of gender on the learning of mathematics (Fennema et al., 1998; Peterson & Fennema, 1985; Steinthorsdottir & Sriraman, 2007). However, there is still disagreement as to how much gender differences exist with respect to learning mathematics. Some studies have found that no statistical significance occurs with respect to gender, even at various grade levels (Amit & Neria, 2002; Mayall, 2008; Steinthorsdottir & Sriraman, 2007). Other reports declare that gender differences exist, but they are decreasing (Heemskerk et al., 2009; Jackson, Brummel, Pollet, & Greer, 2013; NCES, 2003). Some studies focus upon gender differences with respect to the use of computer technology in the classroom (Christensen, Knezek, & Overall, 2005; Mims-Word, 2012; Plumm, 2008) and have found technology may contribute to gender differences instead of helping to remove them.

Lindberg, Hyde, Petersen, and Linn (2010) published a meta-analysis that analyzed gender differences in recent studies of mathematics performance. They concluded that males and females performed similarly in mathematics. However, this finding may not extend to the specific area of proportional reasoning. A study conducted by Karplus, Pulos, and Stage (1983) involving eleven year-olds and thirteen year-olds focused upon proportional reasoning. Although the researchers concluded that age did not have an effect upon performance, gender was not factored into the study. Additional studies in which gender was considered with respect to proportional reasoning resulted in different findings.

Meehan (1984) conducted a meta-analysis to determine if any gender differences existed with respect to formal operational thought, which included proportional reasoning. She concluded that males exhibit a better proportional reasoning performance than females, although the difference was small. In their study of the proportional reasoning skills of Turkish middle school students, Özgün-Koca and Altay (2009) concluded that no difference with respect to gender existed. Alternatively, Steinthorsdottir and Sriraman (2007) reported that a 2003 Programme for International Student Assessment (PISA) study indicated for 27 out of 41 participating countries, there were statistically significant gender differences in favor of boys. The one exception was Iceland, in which the PISA study indicated that gender difference in achievement favors girls. With respect to proportional reasoning, the study by Steinthorsdottir and Sriraman (2007) showed no gender differences in the overall success rate, although girls appeared to be more successful when working with symbolic problems while boys appeared to be more successful in part-part-whole problems. In light of these findings, any influence the use of virtual manipulatives provides in developing proportional reasoning skills warrants research, especially if any difference with respect to gender occurs.

Chapter Summary

Every development, refining, or advancement of technological resources provides both teachers and students the opportunity to enhance learning. However, availability is not a guarantee that such opportunities are equal or effective. A review of the literature indicated virtual manipulatives can be used effectively in the classroom, but there is a lack of research in which the implementation of virtual manipulatives for the purpose of developing proportional reasoning skills occurred. By conducting research at the classroom level, it may be possible to determine the following: (a) if the use of virtual manipulatives have an impact in developing proportional reasoning; (b) if gender has any impact when using virtual manipulatives in developing proportional reasoning; (c) if technology-input modality offers any advantages to students in developing proportional reasoning; and, (d) if there is any significant interaction between gender and input modality when using virtual manipulatives in developing proportional reasoning.

CHAPTER III: METHODOLOGY

Introduction

A review of the literature in Chapter II indicated a need for research with respect to the impact of virtual manipulatives on the development of proportional reasoning skills. This chapter describes a mixed method research design that allowed the researcher to gather both quantitative and qualitative data in a convergent parallel design. Though these data were gathered concurrently, they were analyzed separately. The research questions to be answered at the conclusion of this research are listed:

1. What gains are made by grade six students when virtual manipulatives are used to teach certain aspects of proportional reasoning?
2. What differences exist with respect to gender when using virtual manipulatives to teach proportional reasoning to grade six students?
3. What differences exist between students who use touch technology and those who use mouse technology when studying proportional reasoning?
4. What interactions exist between gender and technology-input modality when students use virtual manipulatives when studying proportional reasoning?
5. How do grade six students who use virtual manipulatives differ from those grade six students who do not use virtual manipulatives when developing proportional reasoning skills?

In December 2013, the researcher conducted a pilot study, which created an opportunity to work with the research design and the specific technology intended for use in the dissertation study. Chapter III contains the results and conclusions from the pilot study. The conclusions from the pilot study helped refine the research design of the dissertation study, which is discussed in length in this chapter as well.

Setting

The community from which the participants in this research come was located in the southeastern region of the United States. According to the census data, this community had a population of approximately 53,000 residents. Of these community residents, males constituted 49% of the population and females accounted for 51% of the population. The largest race group was White, which accounted for 92% of the population. The Black and Hispanic population was approximately equal, accounting for 3.5% and 3.8% of the population, respectively (United States Census Bureau, 2014). The school system that housed the participants in this study served approximately 4,500 students; there were six elementary schools, one middle school, one ninth grade academy, one high school, and one alternative school in the school district.

The school used in this study, called XYZ Middle School, was the sole middle school within the school system. XYZ Middle School was located in a rural setting that housed students in grades six - eight. The demographics for XYZ Middle School during the duration of the pilot study appear in Table 3.

Table 3

Student Demographics at XYZ Middle School during Pilot Study

Demographic	Grade Six	Grade Seven	Grade Eight	Percentage of Population
Male	141	150	157	47.0
Female	173	155	178	53.0
Total	314	305	335	100
Asian	0	0	2	0.2
Black	4	3	10	1.8
Hispanic	17	8	13	4.0
Indian	0	1	2	0.3
Multi-racial	4	3	2	0.9
Pacific Islander	0	0	1	0.1
White	289	290	305	92.7

As seen in Table 3, during the pilot study, there were slightly more female students than male students enrolled in XYZ Middle School. Also, the population of students at XYZ Middle School did not exhibit a great deal of ethnic diversity; the ethnicity with the largest population was White. Not only is this ethnic group considered to be the majority, it was also a plurality, given the difference between the two groups with the largest population is 88.7 percentage points, and thus greater than 50%.

With respect to XYZ Middle School's mathematics department, there were 10 teachers who served the general student population; three special education teachers who served special service students in either an inclusion setting or a pull-out setting; and one special education teacher who served students placed in a Comprehensive Developmental Classroom (CDC). One section of Algebra 1 was offered for those students in grade eight who qualified for enrollment, based upon standardized test scores from the previous school year.

Pilot Study

In order to establish an appropriate research design as well as anticipate and address potential problems with technology, the researcher conducted a pilot study during December 2013 at the XYZ middle school. The pilot study required the recruitment of a grade six host teacher, referred to as Ms. Yanth (a pseudonym). The pilot study began with 40 students from two grade six mathematics classrooms, both instructed by Ms. Yanth; due to absences, 37 students completed the work in the pilot study, with 18 students in the first class period (Group 1) and 19 students in the second (Group 2). All of the students participating in the pilot worked with iPad technology; as such, the pilot study did not consider technology-input modality as a factor.

In addition to providing an opportunity to address potential technological issues that could arise, the pilot study provided a setting in which to establish measures of reliability for the testing instruments intended for the dissertation research. From established resources, the researcher developed and administered two different forms of an assessment, identified as Forms A and B. Each form consisted of 28 multiple-choice questions: Form A served as the pre-test for half of the participants, while Form B served as the pre-test for the other half. Conversely, Forms A and B served as the post-test for each respective group. The use of this counterbalancing technique eliminated any serial effect that could have been introduced by using one version as the sole pre-test and the other version as the sole post-test. Appendix B contains the test Forms A and B.

The participants in the pilot study constituted a single group. Students were randomly assigned to the pre-test/post-test form in order to achieve a quasi-experimental design because the assignment of students to Ms. Yanth's class and to the specific period in which they received mathematical instruction required registration in advance. According to Kleinbaum, Kupper, Nizam, and Muller (2008), quasi-experimental studies required participant assignment to treatment conditions without complete randomization.

Background of Ms. Yanth

Ms. Yanth held a Professional Teacher License with an Elementary K-6 endorsement, PreK-4 endorsement, and a Middle School 4-8 endorsement. With these credentials, Ms. Yanth was licensed for a ten-year period and could teach any core academic subject in the K-8 grade band; in addition, Ms. Yanth was considered Highly Qualified in Mathematics K-8, which meant that the federal requirements for subject competency were fulfilled. At the time of the pilot study, Ms. Yanth had six years of experience teaching at XYZ Middle School in the area of mathematics.

Though comfortable with teaching ratio and proportion concepts to students, Ms. Yanth stated in an interview with the researcher that she had not had any specific training with this topic during her teacher preparation program of study. In addition, Ms. Yanth's work with manipulatives, both physical and virtual, in her mathematics classroom was minimal. With respect to the use of technology in the mathematics classroom, Ms. Yanth stated that she did not feel comfortable using technology in her classroom, although her students worked with calculators. Prior to the pilot study, Ms. Yanth did not use iPads in her classroom.

Demographics of Pilot Study Participants

The participants in the pilot study possessed the demographics displayed in Table 4. The information indicated the participants in the pilot study approximated the same demographics as the student population of the middle school, shown previously in Table 3. For instance, the percentages of male students and female students in XYZ Middle School were 47.0% and 53.0%, respectively, while the percentages of male students and female students for the pilot study participants in Ms. Yanth's classes were 51.4% and 48.6%, respectively.

Table 4

Demographics of Student Participants in Pilot Study

Demographic	Number	Percentage
Male	19	51.4
Female	18	48.6
Total	37	100
Black	0	0
Hispanic	3	8.1
Multiracial	1	2.7
White	33	89.2
Gifted	7	18.9

For the participants who completed all the components of the pilot study, 36 participants generated data from the previous year's state standardized summative assessments. Since Ms. Yanth taught three classes of students who qualified for enrollment in an accelerated mathematics class, the level of performance on the mathematics portion of the state standardized summative assessment for these participants was not representative of the entire grade six student population. All of the participants who took the state standardized summative assessment in 2013 as grade five students scored Advanced on the mathematics test; however, there was still variation within the Advanced performance level. Tables 5 and 6 show a breakdown of information concerning the performance of the participants on the mathematics portion of the state standardized summative assessment and the Reporting Category Performance Index (RCPI). The RCPI shows an estimate of student performance on a 100-point scale.

Table 5

Breakdown of Advanced Scores for Pilot Study Participants

Scale Score	Number of Student Participants
798	1
804	9
813	12
829	9
900	5

Table 6

RCPI Scores for Numbers and Operation Category for Pilot Study Participants

RCPI Score	Number of Student Participants
94	2
95	3
96	3
97	10
98	9
99	2
100	7

For Table 5, the state standardized summative assessment uses a range of 600-900 for scale scores. In order to be classified as Advanced, a student must score at least 795. As stated earlier, the RCPI Scores reported in Table 6 reflect the projected score that a student would be expected to make if there were 100 points available on the Numbers and Operation category of the mathematics portion of the state standardized summative assessment. The performance of the participants on grade five mathematics portion of the state standardized summative assessment indicated that students not only scored Advanced on the overall mathematics portion, they also scored Advanced on the Numbers and Operation category. On the grade six mathematics portion of the state mandated summative assessment, it is the Numbers and Operation category that contains the ratio and proportion standards.

Instruments

Four different sources supplied the questions used on the pre-test/post-test instruments in the pilot study: (a) the Trends in Mathematics and Science Study (TIMSS) item bank (NCES, 2013b) (b) the National Assessment of Educational Progress (NAEP) item bank (NCES, 2013a); (c) a ratio and proportions test posted online at https://grade8-math.wikispaces.com/; and (d) modified ratio and proportion items developed by the researcher similar to those items found in the TIMSS and NAEP item banks. All items were multiple-choice format with four choices for each question. Any question selected from the various sources that was originally offered with five choices had one distractor removed in order to fit the four-choice format. All of the questions from the four sources formed a 56-question item bank; the pre-test and post-test each contained 28 questions. The distribution of the questions in the item bank by source is displayed in Table 7. The researcher found more questions concerning proportional reasoning skills in a multiple-choice format in the TIMSS tests from various years of administration as compared to the NAEP tests, which accounts for more questions from the TIMSS source. The pilot study required questions from the online test, as well as some modification of questions from TIMSS and NAEP, in order to have a sufficient number of questions in the item bank.

Table 7

Distribution of Items by Source

Source	Number of Questions
TIMSS	22
NAEP	6
Online test	12
Modified items	16

Tasks

The researcher selected tasks for the pilot study from NCTM sources and created additional tasks that involved ratio and proportion skills. Specifically, the pilot study used the Mr. Tall/Mr. Short task (Khoury, 2002) and John's School from *Classroom Activities for Making Sense of Fractions, Ratios, and Proportions* (Magone, Moskal, & Lane, 2002). Tasks created by the researcher mirrored the same values used in the Mr. Tall/Mr. Short task or replicated the same type of problems that participants encountered on the virtual manipulative website, Thinking Blocks: Ratio and Proportion Practice. The tasks used in the pilot study appear in Appendix C.

Schedule

Conceived as a five-day event, a change in the pilot study schedule occurred due to inclement weather at the start of the pilot study window. Therefore, the pilot study occurred over four days instead of five. The events as they occurred are shown in Table 8.

Table 8

Pilot Study Schedule

Day	Events
1	Mr. Tall/Mr. Short Task Pre-test
2	Thinking Blocks: Ratio and Proportion Practice Researcher-developed Task 1
3	Thinking Blocks: Ratio and Proportion Practice Researcher-developed Task 2
4	John's School Task Post-test

The structure of the Thinking Blocks: Ratio and Proportion Practice website allowed participants to work at their own pace. Participants worked independently for most of the activities as opposed to working in groups. Ms. Yanth's classroom structure, which encouraged students to work independently, influenced this choice of work environment.

Each of the two classes lasted sixty-six minutes; within this time, normal classroom procedures occurred, such as distributing papers and taking attendance. Also, the distribution of iPads, calculators, and other materials required additional time. After considering the time required to complete these classroom procedures, there remained approximately fifty-five minutes per class period for participants to complete the pilot study work.

Pilot Study Data

Data gathered for the pilot study were of two types: quantitative data and qualitative data. Within each of these two categories of data, there were two sources of data: (a) pre-test scores; (b) post-test scores; (c) observations of participants as they worked with Thinking Blocks: Ratio and Proportion Practice website; and (d) tasks completed by the participants. The calculation of quantitative data statistics included descriptive statistics, measures of reliability, item statistics, item analysis, item test correlations, and distractor analysis. The researcher planned to observe participants working in the classroom setting in hopes of witnessing patterns while participants worked together; however, due to the nature of Ms. Yanth's approach to classroom management, minimal observation data emerged during the pilot study.

Quantitative Results

Mean scores, standard deviations, and standard error means for the results from the various tests and groupings are reported in Table 9. Both the pre-test and post-test used a 100-point scale. The researcher employed a paired samples *t*-test to check for statistical significance of test results. The pilot study relied upon SPSS, version 20 to calculate these descriptive statistics.

Table 9

Descriptive Statistics of Pilot Study Assessment

	Pre-Test			Post-Test		
Category	Mean	Standard Deviation	Standard Error	Mean	Standard Deviation	Standard Error
Whole Group (n = 37)	65.83	14.88	2.45	69.78	12.37	2.03
Male (n = 19)	68.61	14.84	3.40	74.63	11.86	2.72
Female (n = 18)	62.90	14.77	3.48	64.67	11.02	2.60
Group 1 (n = 18)	71.62	13.39	3.16	71.03	14.45	3.40
Group 2 (n = 19)	60.35	14.43	3.31	68.60	10.30	2.36

Using a paired samples t-test, the difference between the post-test mean score and the pre-test mean score for the category Whole Group was found not to be statistically significant. However, the results of an independent samples t-test showed that the difference in mean scores for Group 1 and Group 2 was found to be statistically significant, t (36) = 2.464, $p < .05$. The difference in mean scores for the gender groups was found not to be statistically significant. When considering the post-test, the results from the independent samples *t*-test indicated the difference in mean scores with respect to gender was statistically significant, $t(36) = 2.647$, $p < .05$, while the difference in mean scores with respect to Group 1 and Group 2 was found not to be statistically significant. Despite the Advanced standing of each participant on the grade five state standardized summative assessment in Mathematics, the minimum mean value score was 60.35 for the pre-test, while the maximum mean value score was 74.63 for the post-test. These results suggested one should not assume that students who apparently have mastered topics in the grade five mathematics curriculum already possess developed proportional reasoning skills.

The percent of participants who answered the questions correctly on the two different forms of the assessment instrument is given in Tables 10 and 11; in addition, the source of each question is identified in parenthesis. For those questions taken from previous TIMSS or NAEP assessments, the percentage of students in the United States who answered those specific questions correctly is provided.

Table 10

Item Statistics on Form A Test – Pilot Study

Question Number (Source)	Percent Correct	Percent US Correct
1 (A)	39	28
2 (A)	76	68
3 (B)	71	-----
4 (E)	32	-----
5 (B)	29	-----
6 (A)	74	55
7 (E)	61	-----
8 (A)	89	55
9 (E)	71	-----
10 (E)	87	-----
11 (E)	34	-----
12 (B)	63	-----
13 (E)	63	-----
14(A)	53	52

Note. (A) = TIMSS, (B) = TIMSS Modified, (C) = NAEP, (D) = NAEP Modified, (E) = Grade 8 test

Table 10 (continued)

Item Statistics on Form A Test – Pilot Study

Question Number (Source)	Percent Correct	Percent US Correct
15(A)	89	70
16(E)	66	-----
17(A)	21	11
18(A)	74	50
19(A)	89	63
20(A)	45	34
21(B)	92	-----
22(B)	79	-----
23(C)	92	75
24(C)	53	44
25(D)	58	-----
26(C)	26	39
27(D)	87	-----
28(D)	82	-----

Note. (A) = TIMSS, (B) = TIMSS Modified, (C) = NAEP, (D) = NAEP Modified, (E) = Grade 8 test

Table 11

Item Statistics on Form B Test – Pilot Study

Question Number (Source)	Percent Correct	Percent US Correct
1(E)	92	-----
2(A)	67	72
3(A)	69	59
4(A)	90	63
5(B)	100	-----
6(A)	69	48
7(A)	79	58
8(A)	82	55
9(B)	51	-----
10(E)	92	-----
11(A)	100	80
12(B)	56	-----
13(A)	95	78
14(B)	13	-----

Note. (A) = TIMSS, (B) = TIMSS Modified, (C) = NAEP, (D) = NAEP Modified, (E) = Grade 8 test

Table 11 (continued)

Item Statistics on Form B Test – Pilot Study

Question Number (Source)	Percent Correct	Percent US Correct
15(A)	38	40
16(B)	74	-----
17(A)	41	36
18(E)	56	-----
19(E)	49	-----
20(A)	90	73
21(A)	38	45
22(E)	85	-----
23(D)	85	------
24(D)	67	------
25(C)	77	72
26(D)	72	-----
27(C)	92	60
28(C)	72	57

Note. (A) = TIMSS, (B) = TIMSS Modified, (C) = NAEP, (D) = NAEP Modified, (E) = Grade 8 test

In comparing the two forms, an approximately equal distribution of questions from TIMSS and NAEP occurs on each test form. Form A contained 15 questions from TIMSS sources and six questions from NAEP sources, while Form B had 17 questions from TIMSS sources and six questions from NAEP sources. When comparing the performance of the participants in the pilot study with the national percentages, there are only four questions in which the national average percentages exceeded the student subject percentages (Form A: #26; Form B, # 2, 15, 21). The researcher removed one answer choice from any question with five answer choices as used from the TIMSS or NAEP item bank; thus, the higher scores seen when comparing the pilot study performance to national performance could be due to this reduction of answer choices.

Item Analyses. The researcher performed an item analysis, item test correlation, and distractor analysis for each test form. The information gathered from these analyses provided valuable feedback for the dissertation research test construction. Tables 12 and 13 contain the item analyses for Form A and Form B, respectively.

Table 12

Item Analyses for Form A – Pilot Study

Question	Percent Correct	Point Biserial	P-Value	A	B	C	D
1	39.47	0.44	0.39	17	15*	1	5
2	76.32	0.34	0.76	29*	7	1	1
3	71.05	0.35	0.71	27*	3	2	6
4	31.58	0.26	0.32	1	12*	24	1
5	28.95	0.02	0.29	20	11*	6	1
6	73.68	0.26	0.74	28*	2	8	0
7	60.53	0.42	0.61	12	23*	0	3
8	89.47	-0.02	0.89	0	0	4	34*
9	71.05	0.11	0.71	27*	5	4	2
10	86.84	0.27	0.87	4	1	33*	0
11	34.21	0.17	0.35	3	12	13*	9
12	63.16	0.29	0.63	6	1	24*	7
13	63.16	0.18	0.63	8	24*	3	3
14	52.63	0.03	0.53	20*	4	3	11
15	89.47	0.40	0.89	34*	2	0	2
16	65.79	0.44	0.66	25*	12	1	0
17	21.05	0.32	0.21	16	8	6	8*

Note. * denotes correct answer.

Table 12 (continued)

Item Analyses for Form A – Pilot Study

Question	Percent Correct	Point Biserial	P-Value	A	B	C	D
18	73.68	0.08	0.76	28*	2	1	6
19	89.47	0.21	0.92	2	34*	1	0
20	44.74	-0.03	0.46	4	7	9	17*
21	92.11	0.18	0.95	1	35*	1	0
22	78.95	0.12	0.81	6	1	30*	0
23	92.11	0.33	0.95	1	35*	1	0
24	52.63	0.47	0.56	6	20*	3	7
25	57.89	0.26	0.61	10	4	22*	0
26	26.32	0.20	0.29	12	1	11	10*
27	86.84	0.07	0.97	1	0	0	33*
28	81.58	0.00	0.91	0	0	31*	3

Note. * denotes correct answer.

Table 13

Item Analyses for Form B – Pilot Study

Question	Percent Correct	Point Biserial	P-Value	A	B	C	D
1	92.31	-0.06	0.92	1	36*	0	2
2	66.67	-0.03	0.67	2	26*	9	2
3	69.23	0.21	0.69	12	0	27*	0
4	89.74	0.03	0.9	35*	2	0	2
5	100.00	N/A	1	0	0	0	39*
6	69.23	0.23	0.69	3	0	9	27*
7	79.49	0.14	0.79	4	2	31*	2
8	82.05	0.16	0.82	3	32*	3	1
9	51.28	-0.24	0.51	2	3	14	20*
10	92.31	0.15	0.92	1	36*	2	0
11	100.00	N/A	1	0	39*	0	0
12	56.41	0.15	0.56	14	1	2	22*
13	94.87	0.18	0.95	1	0	37*	1
14	12.82	0.45	0.13	26	5*	6	2
15	38.46	-0.10	0.38	3	10	11	15*
16	74.36	-0.04	0.74	4	3	29*	3
17	41.03	0.15	0.41	8	16*	7	8

Note. * denotes correct answer.

Table 13 (continued)

Item Analyses for Form B – Pilot Study

Question	Percent Correct	Point Biserial	P-Value	A	B	C	D
18	56.41	0.19	0.56	2	9	6	22*
19	48.72	0.23	0.49	10	4	19*	6
20	89.74	0.07	0.92	0	3	35*	0
21	38.46	0.57	0.39	8	7	8	15*
22	84.62	0.27	0.87	4	33*	1	0
23	84.62	-0.01	0.87	1	1	3	33*
24	66.67	0.42	0.68	3	6	26*	3
25	76.92	0.32	0.81	7	30*	0	0
26	71.79	0.02	0.76	28*	2	4	3
27	92.31	0.22	0.97	36*	1	0	0
28	71.79	0.43	0.76	1	1	7	28*

Note. * denotes correct answer.

Based upon the results shown, 25% of the questions in Form A had more incorrect responses than correct responses (questions 1, 4, 5, 11, 17, 20, and 26). Similarly, 17.9% of the questions in Form B had more incorrect responses than correct responses (questions 14, 15, 17, 19, and 21). Also, there were two questions on Form A that had a negative point biserial value (questions 8 and 20), whereas six questions on Form B had a negative point biserial value (questions 1, 2, 9, 15, 16, and 23). In addition, two questions on Form B had no point biserial value in that all of the students answered those questions correctly (questions 5 and 11). The point biserial value is negative when students who performed poorer on the test answered those questions correctly more frequently than students who performed better on the test.

Reliability Measures. Since the development of national and international standardized examinations involved processes for item development, field testing, and item quality based upon test results, questions taken from TIMSS and NAEP have already passed lengthy development and quality checks that enable them to be used on the examinations in question; for instance, the TIMSS 2003 assessment development process spanned two and one-half years (Neidorf & Garden, 2004). However, by gleaning the questions from the examinations instead of using the entire existing examination structure, as created by test developers for TIMSS and NAEP, it was possible that reliability may have been compromised. It is helpful to recalculate reliability measures for the two test forms developed for the pilot study.

Considering each test form separately, the calculation of Cronbach's alpha yielded $\alpha \approx 0.73$ for Form A and $\alpha \approx 0.58$ for Form B. As a post-test, regardless of the form used by the student participants, the calculation of Cronbach's alpha yielded $\alpha \approx 0.63$. A scatterplot of the data from each subject's test results was used to determine if the two test forms were tau equivalent, which occurs when one test form is a linear function of the other test form. If the two test forms were tau equivalent, then one form could predict the results for the other form. Figure 13 shows this scatter plot.

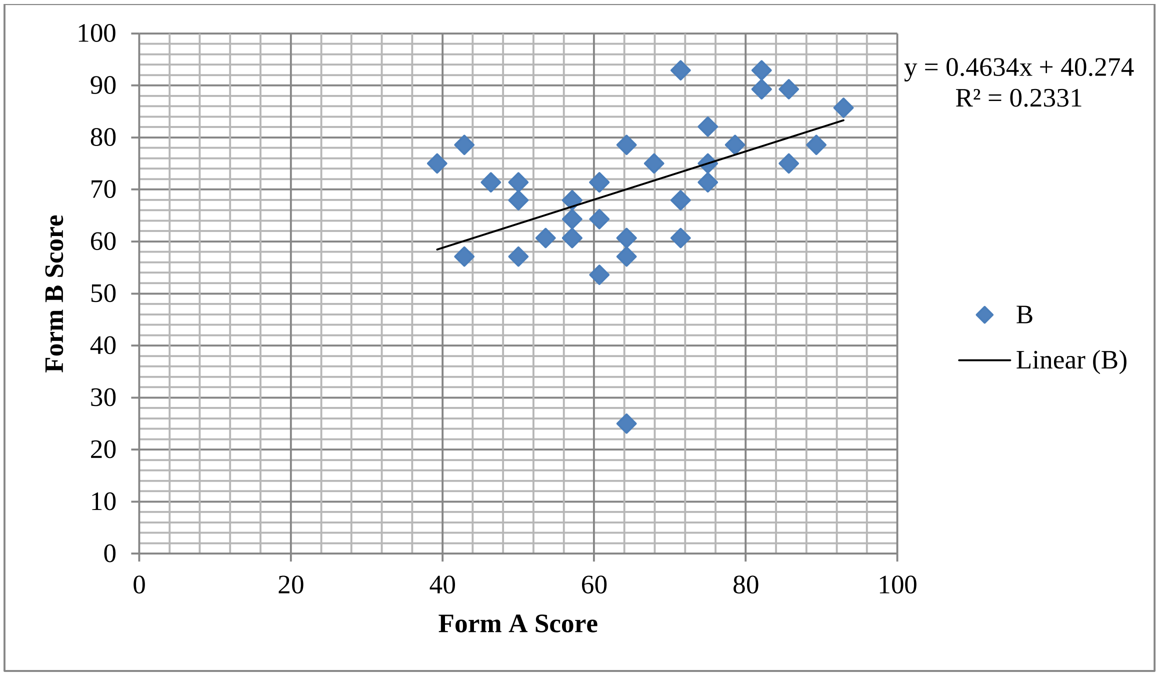


Figure 13. Scatter plot to determine tau equivalency.

Since the R^2 value from Figure 13 for the regression line is less than 0.5, Forms A and B were not considered tau equivalent; thus, these forms were not effectively interchangeable as pre-test and post-test. In light of the greater number of questions with negative point biserial values, lower test result validity, and lack of tau equivalency, the researcher rejected Form B from future consideration for use in this study.

Qualitative Results

The gathering of both quantitative and qualitative data occurred during the pilot study. Participants worked proportion problems on the Thinking Blocks: Ratio and Proportions Practice website; additionally, the researcher collected samples of participants' work with pencil-and-paper tasks, away from the website. Appendix C contains the tasks used during the pilot study. The qualitative data complemented, rather than replicated, the quantitative data gathered from the pre-test/post-test events.

Observations. In the course of conducting the pilot study, the intent of observation focused upon interaction among participants and interaction with the technology. Without exception, every participant worked with the technology and website without difficulty. No incident occurred in which a participant could not operate his or her iPad; similarly, participants worked independently and used the virtual manipulatives website's modeling approach to answer the various types of questions presented. Occasions in which participants asked questions of the researcher concerning procedure or understanding the models arose, but these occasions were few and did not hinder the participants from completing the modules of ratio and proportion problems found on the website.

The interaction between the participants with each other did not unfold as expected. Although Ms. Yanth arranged participants in groups in her classroom, the students did not work together in groups. Since the participants worked independently, there were no conversations to record any thought processes about any aspect of their work with the ratio and proportion problems. Ms. Yanth reinforced independent work habits in her classroom setting, despite the physical arrangement of participants in groups within the classroom.

Work Samples. Samples of work by each participant collected during the completion of the pilot study indicated a progression of organization and mathematical thought concerning the solving of problems involving ratios and proportions. The tasks used during the pilot study are found in Appendix C. One aspect of the conceptual framework for this research is the idea that proportional reasoning is developed. As presented by Karplus and his colleagues in their research with proportional reasoning skills (Karplus et al., 1977), there are four levels of work with respect to the development of proportional reasoning skills used in this pilot study: Level I (Illogical), Level A (Additive), Level Tr (Transitional), and Level R (Ratio). As students developed proportional reasoning skills, they advanced through these levels. Table 14 shows the number of participants who were assigned to the various levels, based upon the work they produced on questions contained in each task, along with the reasons they supplied to support the answers they produced. An inspection of the results shown in Table 14 indicated that the participants assigned to Level A decreased throughout the pilot study, while those participants assigned to Level R increased. Also, very few students exhibited work at Level Tr.

Table 14

Proportional Reasoning Levels by Task

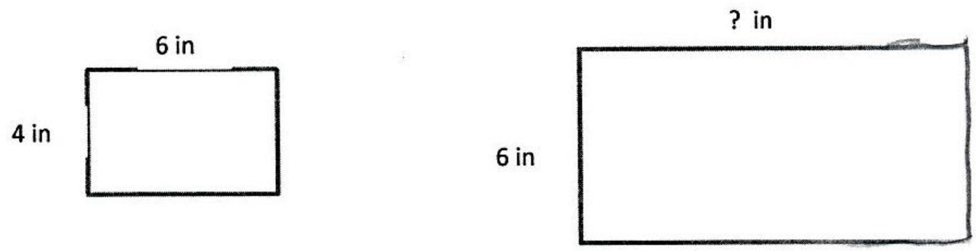
Task	Level I	Level A	Level TR	Level R
Mr. Tall/Mr. Short	1	31	1	6
Researcher-developed Task 1, Question 1	2	0	0	37
Researcher-developed Task 1, Question 2	16	3	0	20
Researcher-developed Task 2	4	28	0	7
John's School, Question 1	9	6	1	22
John's School, Question 2	11	1	0	26

Note. Level I = Illogical, Level A = Additive, Level TR = Transitional, Level R = Ratio

Although the questions and tasks used in the pilot study focused upon ratio and proportion skills, there is always the presence of other mathematical topics that can potentially interfere with the performance of the participants on these tasks. For instance, the question used for Researcher-developed Task 2 involved two similar rectangles (see Figure 14). The work produced by the participants indicated that the ideas of length and width of a rectangle confounded the results of the work obtained with ratios. Although the missing quantity for this problem was a horizontal side of one of the rectangles, some participants ignored their work and supplied an answer that was the measure of a vertical side of the rectangle.

Number 10412

These two rectangles are similar:



Find the missing width from the second rectangle by answering the following questions:

1) What is the ratio of length to width from the smaller rectangle?

6:4

2) If the length of the smaller rectangle were reduced to 1 inch, what would be the matching width?

1.5

3) Complete this table to show the relationship of length to width for rectangles of this shape:

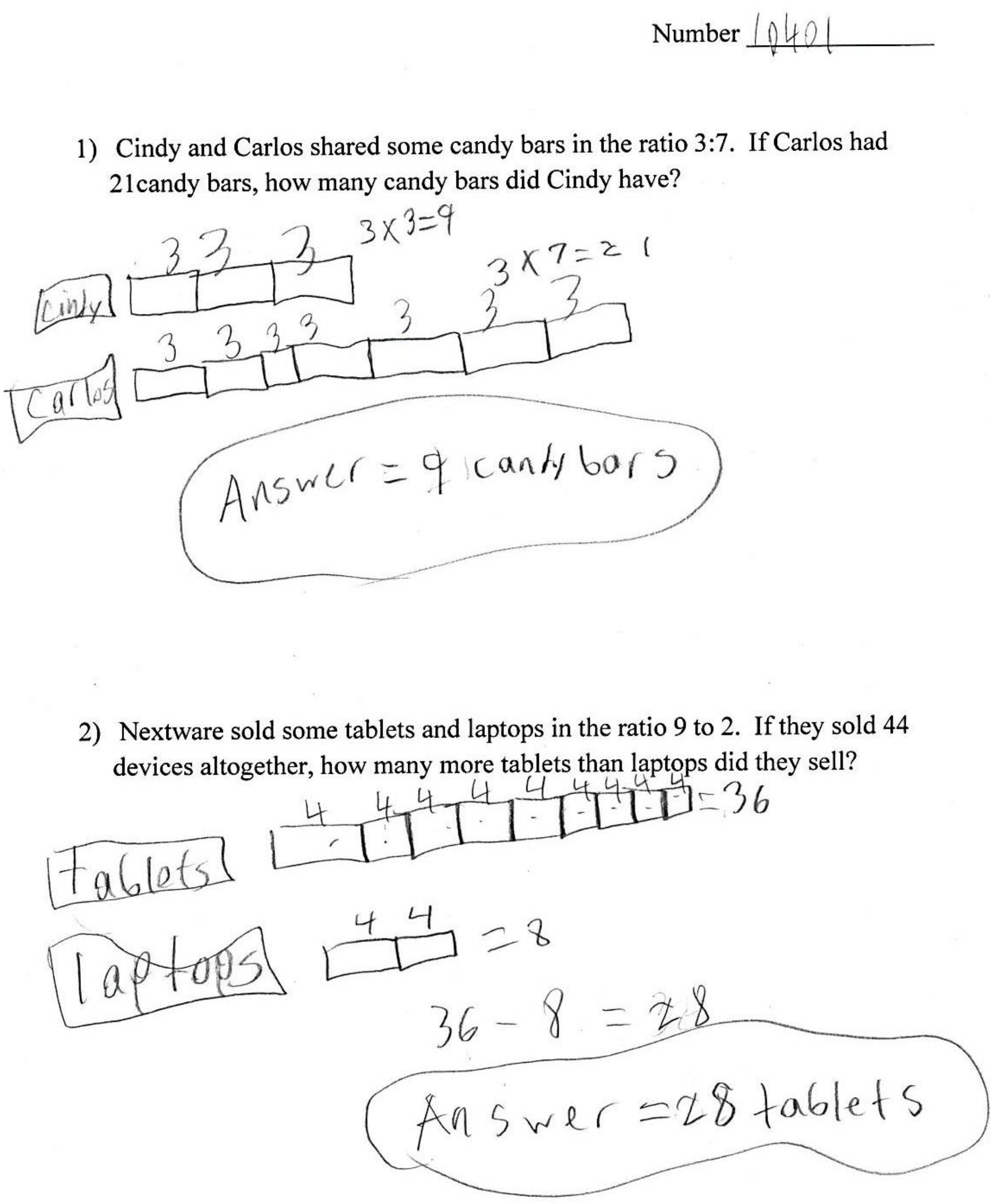
Length	Width
1 4	2
2 6	3
3 8	4
4 10	5
5 12	6
6 14	7

4) For a length of 6 inches, the width of the second rectangle is 6.

Figure 14. Example of vocabulary interference with proportional reasoning.

The issue concerning length and width can be seen in the sample response to the first question in that the order of comparison for the ratio does not match the order indicated in the directions. In fact, the participant appears to have correctly identified the scale factor for this problem, but did not use this scale factor in completing the table for question 3.

After working with the virtual manipulative selected for this pilot study, several participants began using the block model strategy to answer questions on the tasks provided away from the website. Incorporating a modeling strategy in order to solve proportional reasoning skills was a goal of this research. Figure 15 shows a sample of the block modeling strategy that participants encountered while working with the Thinking Blocks: Ratio and Proportions Practice website. The modeling block strategy showed that the participant recognized the need for a multiplicative approach to solve these problems instead of an additive approach.



Number 10401

1) Cindy and Carlos shared some candy bars in the ratio 3:7. If Carlos had 21candy bars, how many candy bars did Cindy have?

2) Nextware sold some tablets and laptops in the ratio 9 to 2. If they sold 44 devices altogether, how many more tablets than laptops did they sell?

Figure 15. Sample of block modeling strategy.

Pilot Study Implications for Dissertation Research Design

The intent of this dissertation research was to address the following research questions:

1. In general, what gains are made by grade six students when virtual manipulatives are used to teach certain aspects of proportional reasoning?
2. What differences exist with respect to gender when using virtual manipulatives to teach proportional reasoning to grade six students?
3. What differences exist between students who use touch technology and those who use mouse technology when studying proportional reasoning?
4. What interactions exist between gender and technology-input modality when students use virtual manipulatives when studying proportional reasoning?
5. How do grade six students who use virtual manipulatives differ from those grade six students who do not use virtual manipulatives when developing proportional reasoning skills?

One purpose of conducting a pilot study prior to the dissertation research gathering phase was to help address any issues that might have arisen during the pilot study concerning technology; in addition, any outcomes from the pilot study that could help refine the dissertation research design should be considered. With respect to technology, no issues occurred that would suggest students could not work with the hardware or software. No internal connectivity issues, such as lack of adequate bandwidth for Internet access to websites, manifested during the pilot study.

With regard to impacting the dissertation research design, the following ideas in the pilot study emerged as recommendations and were implemented accordingly:

- Instead of working with one virtual manipulatives website, another website was incorporated in order to support the participants in the development of the modeling block strategies;
- Pencil-and-paper tasks from the pilot study were revisited in the dissertation research, but additional tasks already featured in the literature were used as well; and
- Pre-test/post-test design was revisited with respect to reliability in order to select the more effective instrument.

People Associated With Dissertation Study

In order to perform research and gather data for this study, the research design involved grade six students as participants. Also, a grade six mathematics teacher willing to work with the researcher in the study process was needed. A description of the host teacher and the participants who agreed to partake in this study follows.

Host Teacher

As in the pilot study, students in the classes of one middle school mathematics teacher served as the participants. XYZ Middle School, which accommodated the pilot study, also accommodated the research study. The teacher who consented to host the research study, referred to as Ms. Xanth (a pseudonym), held a Professional Teacher License with an Elementary K-8 endorsement, which meant that she was licensed for a ten-year period and could teach any core academic subject in the K-8 grade band. In addition, Ms. Xanth was considered Highly Qualified in Mathematics K-8, which means that she has fulfilled the federal requirements for subject competency. Ms. Xanth had thirteen years of experience teaching at the middle school level in the areas of science and mathematics. Working with one teacher in this study avoided any teacher effect variations that could be introduced by differing teacher styles and strategies of different teachers.

Participants

In reporting the results of data analyses, a detailed description of the participants is first presented to reveal a more complete picture of their background. In this section, participants are classified by gender and by technology research group. Also, results from a state-mandated mathematics assessment are reported in order to determine competency of participants with respect to previous mathematical instruction. In addition, results from a survey administered to participants are presented in order to ascertain their experiences with touchscreen technology and their preferences for technology-input modality.

Gender and Technology. All of the students who participated in the dissertation research were enrolled in one of three grade six mathematics classes with Ms. Xanth as their designated classroom teacher. Originally, 68 students were invited to participate in the dissertation research activities; however, two students declined to participate and 10 students, whose absences prevented their inclusion, did not take part in the study. Therefore, a total of 56 students were included in the final data analyses. Table 15 displays the distribution of the participants among the three groups, distinguished by the type of technology used during the actual study: the control group used pencil and paper activities as the participants worked to develop proportional reasoning skills, while the two treatment groups used either iPads or laptop computers with a mouse to interact with the computer in an effort to develop proportional reasoning skills.

Table 15

Distribution of Participants

Group	Females	Males	Total
iPad	11	8	19
Mouse	7	12	19
Pencil-and-Paper	7	11	18
Total	25	31	56

An inspection of Table 15 shows an approximately equal number of participants in each group. Variations of gender distribution existed, with slightly more males than females in the total number of participants, while more females than males participated in the iPad group.

State-Mandated Mathematics Assessment. For the state in which the participants of this study reside, a state-mandated summative assessment in mathematics is administered each year to students enrolled in grades three through eight. Since the participants in this study were in the first semester of grade six, the most current mathematical summative assessment was administered in grade five. In order to further describe the participants in each group with respect to their mathematical background, the results from their performance on the grade five state-mandated summative assessment appear in Table 16. The performance levels on the summative assessment are listed: (a) Advanced; (b) Proficient; (c) Basic; and, (d) Below Basic. A student who scored Advanced demonstrated *complete mastery* of the mathematical standards; a student who scored Proficient demonstrated *mastery* of the mathematical standards; and, a student who scored Basic demonstrated *partial mastery* of the mathematical standards. Some participants did not have any scores on the grade five summative assessment in mathematics either because they were absence during the assessment or did not reside in the state during its administration.

Table 16

Performance Level on Grade Five Mathematics Summative Assessment

Group	Gender	Advanced	Proficient	Basic	Below Basic	No Score	Total
iPad	Male N = 8	2	3	1	0	2	8
	Female N = 11	3	4	4	0	0	11
Mouse	Male N = 12	4	5	2	1	0	12
	Female N = 7	5	0	0	1	1	7
Pencil and Paper	Male N = 11	1	6	2	2	0	11
	Female N = 7	0	5	0	0	2	7
Total	Male N = 31	7	14	5	3	2	31
	Female N = 25	8	9	4	1	3	25
	All N = 56	15	23	9	4	5	56

A comparison of the scores listed in Table 16 indicated that the three research groups were similar with respect to their performance on the grade five state-mandated assessment in mathematics; 14 participants in the Mouse group scored either Advanced or Proficient, while 12 participants in both the iPad group and the Pencil-and-Paper group scored either Advanced or Proficient. If one focused upon the Advanced category, a distinction between the groups emerged. While the Pencil-and-Paper group had only one student in that category, the Mouse group had nine students who scored Advanced. Further inspection indicates that one gender did not significantly outperform the other on the state-mandated assessment, either with respect to the whole pool of participants or with respect to the control group or treatment groups.

Demographics. The demographics of the students who participated in this research study are listed in Table 17, while the demographics for the entire XYZ Middle School appear in Table 18. When comparing the XYZ Middle School demographics from Table 18 with the student participant demographics in Table 17, it was concluded that the students participating in this research study closely approximated the demographic composition of the entire school.

Table 17

Demographics of Grade Six Participants

Demographic	Number	Percentage
Male	31	55.4
Female	25	44.6
Total	56	100
Gifted	3	5.4
Asian	1	1.8
Black	1	1.8
White	51	91.1

Table 18

Student Demographics of XYZ Middle School

Demographic	Grade Six	Grade Seven	Grade Eight	Percentage of Population
Male	145	134	156	46.4
Female	170	173	160	53.6
Total	315	307	316	100
Asian	2	0	0	0.2
Black	8	7	6	2.2
Hispanic	16	8	12	3.8
Indian	1	1	1	0.3
Multi-racial	0	4	3	0.7
White	288	287	294	92.6

Collaborative Lesson Design

In order to prepare Ms. Xanth to work with the technology and virtual manipulatives needed for this dissertation research, tutorial sessions with the researcher occurred. The tutorial component consisted of designing lessons around the tasks which incorporate proportional reasoning skills, as well as hands-on practice with using virtual manipulatives on both the PC and iPad technology. The tasks found in mathematics education texts, such as *Classroom Activities for Making Sense of Fractions, Ratios, and Proportions* (Magone et al., 2002) and *Teaching Fractions and Ratios for Understanding*

(Lamon, 2012), were selected to implement classroom lessons. Appendix D contains the tasks used in the dissertation study that were not included in the pilot study: Egg Carton (Lamon, 2012), Tree House (Lamon, 2012), Sticks and Rhombi (Lamon, 2012), Make a New Puzzle (Clement, 2002), and Cocoa (Billings, 2002).

The virtual manipulatives incorporated into the collaborative lesson design provided the opportunity to build models that support the development of proportional reasoning skills. Ms. Xanth studied with the researcher in order to develop proficiency in using virtual manipulatives when teaching proportional reasoning concepts. In addition to the Thinking Blocks: Ratio and Proportions Practice website used in the pilot study, it was necessary to incorporate a virtual manipulative website that allowed students to develop block models without the guided support found in Thinking Blocks; in particular, the virtual manipulative entitled Number Pieces (www.mathlearningcenter.org/apps), provided just such an environment. There are three main reasons why Number Pieces lends itself for inclusion in this dissertation research: (a) it is a free resource; (b) its structure is intended for place value applications, but the application can be used for the teaching of proportional reasoning by using block models; and (c) it is available for both PCs and iPads.

Research Design

Revisiting the first research question is appropriate at this time: In general, what gains are made by grade six students when virtual manipulatives are used to teach certain aspects of proportional reasoning? Gains can be measured both quantitatively and qualitatively; the presence of both types of research approaches was incorporated into a mixed method model for this study. A central premise of a mixed method approach is that the combination of quantitative and qualitative approaches provides a better understanding of research problems than either approach alone (Creswell & Plano Clark, 2011). The simultaneous gathering of quantitative and qualitative data occurred independently; given this approach to gathering the research data in this fashion, this type of mixed method research model is called a convergent parallel design. Although multiple reasons exist as to why a convergent parallel design is used, this research study appealed to the concept of complementarity. According to Hesse-Biber (2010), "complementarity allows the researcher to gain a fuller understanding of the research problem and/or to clarify a given research result" (p. 4). A diagram of the convergent parallel design for this research is shown in Figure 16.

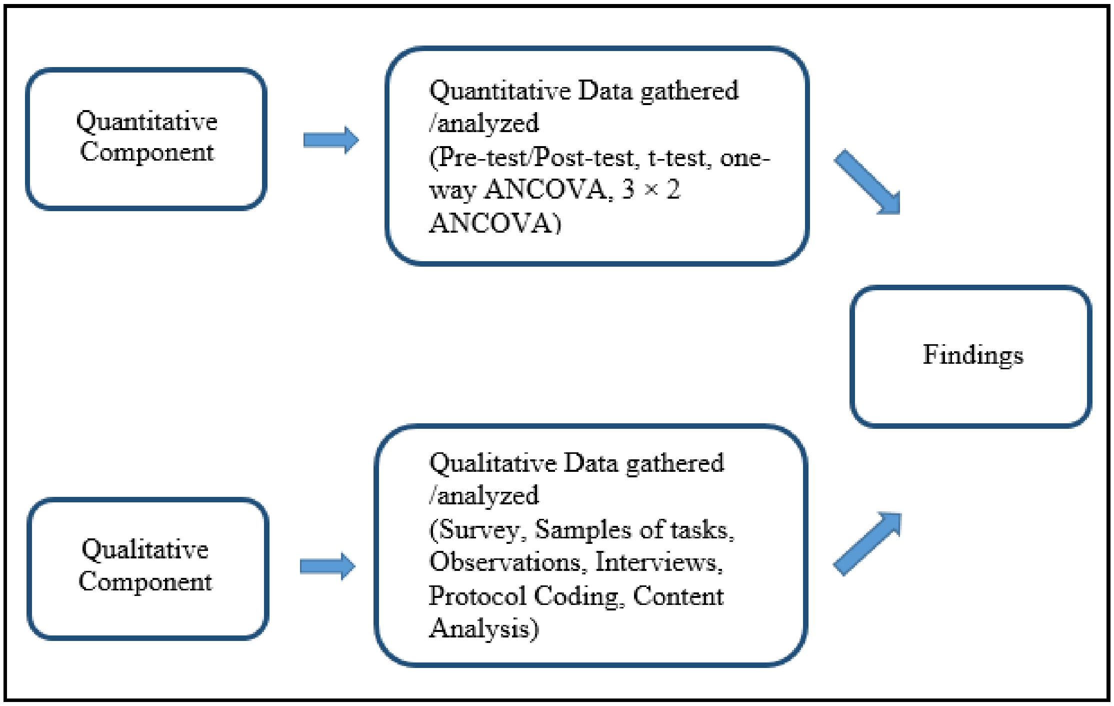


Figure 16. Convergent Parallel Design.

As seen in Figure 16, the quantitative and qualitative components of this research were kept separate throughout the data gathering portion of the study. The study was conducted over ten school days. By allowing ten days for this dissertation research, adequate time occurred for the components of this research design. After the data were collected and analyzed, findings from both branches of the model were summarized.

By gathering data using various methods, the researcher engaged in the process of triangulation. Gay, Mills, and Airasian (2009) defined triangulation as "the process of using multiple methods, data collection strategies, and data sources to obtain a more complete picture of what is being studied (p. 377)." Qualitative data sources included the field notes written by the researcher, samples of proportional reasoning tasks completed by participants, pre-instruction and post-instruction interviews with individual target students, and observations of target students during classroom instruction. Interviews and classroom observations were video graphed using an iPad.

On a different level of meaning, the convergent parallel design implemented for this study is also called a triangulation mixed methods design (Gay, Mills, & Airasian, 2009). By gathering both quantitative and qualitative data concurrently, the opportunity for a deeper, more complete depiction of the development of proportional reasoning skills emerged. Not only could growth be measured by comparing results from a pre-test to a post-test, but the various levels of proportional reasoning were ascertained and evaluated.

In order to avoid any effect that might be introduced by Ms. Xanth with respect to the development of proportional reasoning skills, data collection occurred at the time that she would normally teach such skills to her students; typically, the proportional reasoning topics are addressed during the fall semester. In addition, the students who participated in this research were randomly selected from three of the four classes taught by Ms. Xanth.

Dissertation Procedures

Pre-Study Process

After the successful defense of the dissertation proposal, the researcher obtained permission from the Internal Review Board (IRB) to conduct research pertinent to the dissertation study. See Appendix A for a copy of the IRB approval letter. As part of this approval, each parent/guardian signed a parental consent form allowing their child to participate in the study. Since the topic of proportional reasoning is part of the grade six mathematics curriculum, students could not choose to opt out of instruction; however, a student could choose not to participate in the study and no data were collected from any student who made this choice. In the end, 56 students agreed to participate in the study.

Control and Treatment Groups

This dissertation research involved three classes; one class of Ms. Xanth's students was designated as a control group. This control group completed the Mr. Tall/Mr. Short task, the Egg Carton task, the Tree House task, the Sticks and Rhombi task, and the John's School task to ascertain the development of proportional reasoning skills, implementing neither technology nor any virtual manipulatives. Additionally, the researcher reproduced word problems generated from the Thinking Blocks website for participants in the control group to complete while working only with pencil and paper. Appendix E contains the word problems the control group completed during the first four days of classroom instruction. The other two classes of Ms. Xanth's students comprised the treatment groups. The treatment group participants also completed the same tasks as the control group; however, these students worked with Thinking Blocks and Number Pieces in order to develop proportional reasoning skills. All participants implemented a block modeling strategy when answering proportional reasoning word problems, regardless of assignment to the control group or either treatment group.

For the treatment groups, classes were randomly assigned to work with a device using one of the two technology-input modalities: iPads with touch technology or PCs with mouse technology. This random assignment was made to the whole class in order to avoid potential difficulties of participants changing technologies without notice. The control group and the treatment groups took the same pre-test and post-test; additionally, participants in both groups worked with the same five tasks in the same order. The random assignment of classes as the control group or the treatment groups determined a quasi-experimental research design; participants were already enrolled in Ms. Xanth's classroom prior to the dissertation research.

Dissertation Study Schedule

Ten school days were allotted for the gathering of data for this study. Prior to the work of the participants in the classroom setting, Ms. Xanth administered the pre-test to all of the participants. Based upon the results of this pre-test, the researcher identified a group of potential target students, using the mean score from the pre-test to separate the participants into two groups: those participants scoring above the mean and those scoring below the mean. The researcher based the selection of target students upon a balance of gender and performance on the pre-test, creating an equal number of male and female target students, as well as Ms. Xanth's recommendation as to which students would offer a reasonable degree of conversation and explanation during the interviews.

Each of the class periods lasted sixty-six minutes; within this time, normal classroom procedures occurred, such as distributing papers and taking attendance. Also, the distribution of iPads, calculators, and other materials required additional time as well. After considering the time required to complete these classroom procedures, there remained approximately fifty-five minutes per class period for participants to complete the dissertation study work. Table 19 shows the research study schedule for each day.

Table 19

Daily Schedule of Study Activities

Day	Study Activities
1	P: Pre-test administered
2	T, R: Pre-instruction interview with Make a New Puzzle task
	P: Mr. Tall/Mr. Short task completed
	P: Introduction to Thinking Blocks tutorial videos
	P: Thinking Blocks modules completed independently
	R: Target students observed
3	P: Thinking Blocks modules completed independently
	R: Target students observed
4	P: Egg Carton task completed
	P: Thinking Blocks modules completed independently
	R: Target students observed
5	P: Thinking Blocks modules completed independently
	R: Target students observed
6	P: Tree House task completed
	P: Orientation to Number Pieces
	P: Number Pieces problems completed independently
	R: Target students observed
7	P: Number Pieces problems completed independently
	R: Target students observed
8	P: Sticks and Rhombi task completed
	P: Number Pieces problems completed independently
	R: Target students observed
9	P: Number Pieces problems completed independently
	R: Target students observed
10	P: John's School task completed
	P: Post-test administered
	T, R: Post-instruction interview with Cocoa task

Note. P = Participants, T = Target Students, R = Researcher

On a typical day of classroom instruction, Ms. Xanth displayed review questions for participants to answer at the start of class. Once the instruction concerning proportional reasoning commenced, participants used the block modeling strategy to answer proportional reasoning word problems. Participants in the two treatment groups used Thinking Blocks for the first four days of classroom instruction; Figures 4-10 presented a sample problem from one of the modules. Participants worked with Number Pieces, the second virtual manipulative in the study, during the last four days of classroom instruction. Participants in the control group used pencil and paper to draw the block models and answer the proportional reasoning word problems during all eight days of classroom instruction.

On the days when participants completed a performance task, Ms. Xanth distributed the tasks and instructed the students to work individually. After giving students time to complete the task, Ms. Xanth encouraged participants to share their results with the rest of the class. Next, participants participated in small group discussion concerning their results for tasks. Finally, Ms. Xanth led the class in discussion concerning the work generated by the participants on the tasks. Participants did not use virtual manipulatives while they completed the performance tasks.

Instruments and Data Sources

In order to collect data in this mixed method research design, the researcher utilized both quantitative and qualitative tools. For the quantitative component, a multiple-choice assessment instrument was used for the pre-test and post-test. Collection of completed tasks from all the participants constituted a portion of the qualitative component. In addition, the researcher observed and interviewed a set of six target students individually as they completed two additional performance tasks. A description of the researcher as an instrument of data collection is provided later in this section.

Quantitative Component

As part of the pilot study conducted prior to this study, the researcher developed two forms of a 28 multiple-choice question test. Based upon test results from the pilot study, one form continued to be used as the pre-test/post-test instrument for the dissertation study. Three mathematics education professionals agreed to examine the instrument in an effort to establish content validity. Calculations of Cronbach's alpha confirmed acceptable test reliability for the instrument.

Content Validity. As reported previously concerning the pilot study, the higher Cronbach alpha score and results from the item analyses supported the choice of Form A over Form B for the pre-test/post-test instrument used in the dissertation study. In an effort to establish content validity of the pre-test/ post-test instrument, the researcher asked Subject Matter Experts (SMEs) to examine Form A to determine if each question assessed proportional reasoning skills. Three SMEs responded to this examination request: a middle school mathematics teacher/numeracy coach, a university faculty member from a department of Mathematics with middle school mathematics experience, and a veteran mathematics education faculty member from a college of education at a different university. Percentages for interrater reliability were obtained by using ReCal 0.1 Alpha for 3+ Coders (Freelon, 2013). For this assessment, the three SMEs agreed that 92.9% of the questions involved proportional reasoning skills. The two university faculty members agreed that 100% of the questions addressed proportional reasoning skills, with the average pairwise percent agreement calculated as 95.2%. Appendix F contains the results of the examination of the multiple-choice questions by the SMEs.

Test Reliability. In Chapter III, the presentation of pilot study research introduced two multiple-choice, 28-question instruments that served as the pre-test and post-test for the study, labeled Form A and Form B. Appendix B contains these two forms of the test used in the pilot study. The test results from these two forms indicated that Form A had the higher reliability value as determined by Cronbach's alpha, $\alpha \approx 0.73$, compared to Form B, $\alpha \approx 0.58$. Furthermore, Form A was examined by Subject Matter Experts (SMEs) and was found to possess content validity; for these reasons, Form A served as the pre-test and post-test for the dissertation research. Although the same 28 multiple-choice questions appeared in the pre-test and post-test, the order of the questions changed from one assessment to the other. For the pre-test, using Cronbach's alpha, the results were found to be reliable, $\alpha \approx 0.72$. Further, the results for the post-test were also found to be reliable, $\alpha \approx 0.79$. Based upon the measures of reliability and validity obtained, Form A appeared to be an acceptable choice as the pre-test/post-test instrument for use in this dissertation study.

Qualitative Component

Although the primary research question for this study, which addressed gains made by grade six students, incorporated both quantitative and qualitative aspects, one of the research questions guiding this research study appealed to qualitative data in order to obtain answers: How do grade six students who use virtual manipulatives differ from those grade six students who do not use virtual manipulatives when developing proportional reasoning skills? In order to answer this question, a case study method with a holistic multiple-case design presented the best approach for gathering data from the selected target students. Also, participants answered a survey concerning experience with touchscreen technology and preference for technology type. Additional qualitative tools for this study included an observation protocol and an interview protocol.

Holistic Multiple-Case Design. According to Yin (2003), a multiple-case design provides evidence more compelling and robust as compared to a single-case design. For this study, there were three different classroom settings: the control group in a traditional instructional setting, a treatment group in which students used iPads to access virtual manipulatives, and a treatment group in which students used laptop computers with a mouse to access virtual manipulatives. Two students from each group comprised the case study members; thus, six individual case studies combined to create the multiple-case design. After completing the case study for each of the six target students, the researcher evaluated the multiple case in light of the development of proportional reasoning skills. Each individual represented one unit of analysis; thus, the design of the multiple case was classified as holistic rather than embedded. An embedded design requires multiple units of analysis.

The researcher was present in the classroom as instruction occurred in order to observe work habits of students as they developed proportional reasoning skills. In addition, samples of student work were collected and analyzed for any patterns that might have emerged. Additionally, interviews with the six target students occurred. These target students were selected after the pre-test was administered and the results separated into two groups: low scoring and high scoring.

As stated earlier in this chapter, tasks from the literature focusing upon proportional reasoning skills were used as part of the qualitative aspect of the dissertation research; particularly, tasks found in *Classroom Activities for Making Sense of Fractions, Ratios, and Proportions* (Magone et al., 2002) and *Teaching Fractions and Ratios for Understanding* (Lamon, 2012). The Mr. Tall/Mr. Short task and the John's School task, which were used in the pilot study conducted in December 2013, were incorporated into the dissertation research materials; research study participants completed these tasks as well.

Survey. On Day 1 of this study, participants completed a brief survey concerning their experiences with touchscreen technology and preferences for computer use; participants indicated their gender on the survey as well. The compiled results from this survey provided the researcher the opportunity to determine if inexperience with the use of touchscreen technology correlated with any difficulty in operating an iPad. Also, the results of the survey indicated any aversion to the use of touchscreen technology. Appendix G contains the survey instrument.

Observation protocol. As part of the observation protocol, the researcher focused upon the engagement of the six target students as they participated in class; for instance, did participants work with the block modeling strategy effectively? Also, did participants exhibit understanding of proportional reasoning concepts or did they exhibit confusion with respect to this topic? If so, what type of proportional reasoning questions appeared to confuse the participants? Additionally, a sketch of the room layout was included. Appendix G shows the organization of the observation protocol.

Interview protocol. As part of the work with the six target students, the researcher created an interview protocol used with the pre-instruction completion of the Make a New Puzzle task and the post-instruction completion of the Cocoa task. The interview protocol was semi-structured, allowing the target students to interact with the researcher as needed. Appendix G contains the initial questions used in each interview protocol.

Researcher as Instrument

One characteristic of qualitative data gathering is that the researcher serves as a key instrument (Creswell, 2007). Thus, the background and experiences of the researcher influenced aspects of the study; for instance, the organization of the design, the gathering of field notes, the structuring of interview questions, and the interpretation of the data reflected the professional work history of the researcher. The researcher for this study was a male with 30 years teaching experience in public education; within these 30 years of teaching experience, he taught students various mathematics subjects at the high school and undergraduate university level, including courses in an online setting. The researcher served as an administrator for 10 years as well, currently hired as Director of Middle School Instruction and Testing in a public school system.

The researcher earned three degrees from an accredited university: a Bachelor's of Science in Mathematics, a Master's of Science in Mathematics, and an Education Specialist in Curriculum and Instruction. He successfully completed qualitative research coursework as part of the doctoral program of study. In addition, the researcher analyzed and coded qualitative data as part of a previous study (Barlow, McCrory, & Blessing, 2013).

Data Analysis

After the data collection, the convergent parallel design incorporated for this study dictated that the quantitative data and qualitative data be kept separate and analyzed separately. Data analyses occurred in an appropriate manner and findings emerged by examining the results of the analyses. From the researcher's perspective, the findings provided a sense of complementarity; that is, the qualitative findings provided a more complete picture of the quantitative findings.

Quantitative Data Analysis

The statistical software package SPSS, version 20 provided descriptive statistics of mean score, standard deviation, and standard error for the pre-test and post-test results. Also, the use of a paired-samples *t*-test allowed the researcher to check for statistical significance with respect to the test results. An investigation of the main effects of gender and technology-input modality occurred; in addition, interaction effect between gender and technology-input modality was considered. The calculation of Cronbach's alpha provided levels of reliability for test results.

In order to consider the factor of technology use on the development of proportional reasoning skills as measured by the change of scores from the pre-test to the post-test, the researcher used a one-way ANCOVA with the pre-test serving as the covariate. As presented in Chapters I and II, gender and technology-input modality were factors considered in this research. Therefore, a 3 × 2 ANCOVA design was appropriate for this setting, which allowed for investigating any main effect with respect to gender on post-test scores, any main effect with respect to technology-input modality on post-test scores, and any interaction effect between gender and technology-input modality.

Qualitative Data Analysis

Since there are different sources of qualitative data in this dissertation study, each type of data required its own type of analysis. The tasks administered to all of the participants were analyzed with respect to the levels of development of proportional reasoning as advanced by Karplus et al. (1977) and described in the conceptual framework. The data obtained by observing and interviewing the six target students constituted a case study for each target student; thus, six individual case studies emerged. Data from each case generated descriptions of students at both low performance levels and high performance levels for each of the control and treatment groups.

The researcher employed content analysis to determine the various levels of proportional reasoning as demonstrated by all participants for all five classroom tasks. Weber (1990) defined content analysis as a research method using a set of procedures to make valid inferences from text. Content analysis permitted the qualitative data to be sorted and counted, which quantized the data. For this study, the researcher followed the example of Karplus et al. (1977) by classifying proportional reasoning into four categories. Table 20 gives a rubric illustrating the four categories and a description of each category.

While the four categories concerning the levels of proportional reasoning are distinct and separate, the possibility existed for a participant’s response to a task to lack clarity with respect to classifying it in one category or the next. For instance, a participant may have provided a correct response with inadequate or incorrect explanation. Whenever the researcher encountered a response of this nature, he made the choice to place the response in the lower category instead of the higher one. This selection permitted the researcher’s categorization to err on the side of conservancy and avoid any perceived inflation of results to show more growth in the higher categories of proportional reasoning development.

Table 20

Proportional Reasoning Rubric

Category	**Description** (Khoury, 2002)
Illogical (I)	No explanation is given. An illogical computation, a guess, or a general estimate is made on the basis of observation.
Additive (A)	The student focuses on the difference between the corresponding quantities, then assumes that the same difference exists between the other quantities.
Transitional (Tr)	The student uses an additive approach that focuses on the correspondence of the measures.*
Ratio I	The student uses a constant ratio relationship or makes a multiplicative comparison of the measures of both figures.

*The researcher added this description: The student implements a multiplicative approach unsuccessfully.

Using the labels as listed by Khoury (2002) to classify the levels of proportional reasoning for each of the five tasks, the researcher engaged in the process of protocol coding. As defined by Saldaña (2009), protocol coding is the collection and coding of qualitative data according to a pre-established system. Boyatzis (as cited in Saldaña, 2009) stated "the use of prior data and research as the basis for development of a code means that the researcher accepts another researcher's assumptions, projections, and biases (p. 130)." By using protocol coding, the researcher accepted the views advanced in previous studies concerning the levels of proportional reasoning.

While each case stands independently, a multiple case study situation exists. According to Stake (2006), "the single case is of interest because it belongs to a particular collection of cases (p. 4)". By examining specific characteristics and behaviors, commonalities emerged that described the group represented by the six target students; in this instance, the group of grade six students who were developing proportional reasoning skills. By considering the individual case studies as a multiple case, the potential recognition of themes and patterns concerning these grade six students developed.

Chapter Summary

The research goals for this study concerned the impact of the use of virtual manipulatives when developing proportional reasoning skills in grade six students, as well as determining if there was any main effect for gender on student gains or technology-input modality on student gains that occurred. The study investigated an interaction effect for gender and technology-input modality. In order to measure potential growth, quantitative data were collected in the form of a pre-test/post-test design. The gathering of qualitative data included collecting samples of completed performance tasks, observations of six target students and individually interviewing these same target students. This study applied a mixed method approach due to the dual nature of the data gathering. Because of the concurrent gathering of data, the selection of a convergent parallel design model allowed for the qualitative aspect to complement data from the quantitative aspect. In keeping with the nature of this convergent parallel design, these different types of data were kept separate until determination of findings.

CHAPTER IV: RESULTS OF DATA ANALYSES

Introduction

As stated in Chapter I, the research goals of this dissertation were: (a) to measure the impact of the use of virtual manipulatives on the development of grade six students' proportional reasoning skills; (b) to investigate whether gender of students exhibits any main effect when using virtual manipulatives to develop proportional reasoning skills; (c) to investigate whether touch technology or mouse technology-input modality exhibits any advantages over the other when using virtual manipulatives to develop proportional reasoning skills; and (d) to investigate whether gender and technology-input modality expresses any interaction with each other when using virtual manipulatives to develop proportional reasoning skills. In order to accomplish these goals, the following research questions were considered:

1. In general, what gains are made by grade six students when virtual manipulatives are used to teach certain aspects of proportional reasoning?
2. What differences exist with respect to gender when using virtual manipulatives to teach proportional reasoning to grade six students?
3. What differences exist between students who use touch technology and those who use mouse technology when studying proportional reasoning?
4. What interactions exist between gender and technological input modality when students use virtual manipulatives when studying proportional reasoning?

5. How do grade six students who use virtual manipulatives differ from those grade six students who do not use virtual manipulatives when developing proportional reasoning skills?

Either quantitative data or qualitative data provided information about the participants and the impact of virtual manipulatives on their performance with respect to the development of proportional reasoning skills; however, by considering both types of data in a mixed method approach, the potential for a synergistic situation existed. In light of the convergent parallel design implemented for this research, results from quantitative data analysis and qualitative data analysis were presented so that a fuller, more complete depiction of the development of the participants' proportional reasoning skills was revealed. In this chapter, background information concerning the technology preparation and preferences of the participants is presented; additionally, performance on past state-mandated mathematics assessments is shared. Moreover, quantitative data gathered during the study are presented. Finally, qualitative data from the entire group of classroom participants as well as the six target students are disclosed.

Survey Results

In order to determine participants' background in the use of touchscreen technology and any preference for technology-input modality, a survey was administered on the first day of the study, prior to the administration of the pre-test. The results from the survey concerning experience with touchscreen technology use are shown in Table 21, while the results indicating preference of technology-input modality are shown in Table 22. With respect to touchscreen technology, 83.9% of the participants indicated that they had at least one year of experience using a touchscreen device outside of the classroom. Also, 80.4% of participants indicated that they prefer using an iPad over other available options.

With respect to gender, the results from the survey were similar. Responses indicated that 87.1% of the males reported they had at least one year experience with touchscreen technology, while 80.0% of female participants had at least one year experience with touchscreen technology. The results from the survey pertaining to technology-input modality preference revealed similarities as well. For male participants, 80.6% indicated that they preferred to use an iPad over other options. As for female participants, 80.0% indicated that they preferred to use an iPad over other options.

Table 21

Survey Results Showing Experience with Touchscreen Technology

Group	Gender	Never	Less than 1 yr	Between 1 and 2 yrs	Between 2 and 3 yrs	More than 3 yrs
iPad	Male N = 8	1	1	2	2	2
	Female N = 11	0	2	3	3	3
Mouse	Male N = 12	0	0	3	1	8
	Female N = 7	0	3	0	1	3
Pencil and Paper	Male N = 11	0	2	3	1	5
	Female N = 7	0	0	1	3	3
Total	Male N = 31	1	3	8	4	15
	Female N = 25	0	5	4	7	9
	All N = 56	1	8	12	11	24

Table 22

Survey Results Showing Technology-Input Modality Preference

Group	Gender	iPad	Computer with mouse	No preference
iPad	Male N = 8	5	3	0
	Female N = 11	9	1	1
Mouse	Male N = 12	12	0	0
	Female N = 7	5	1	1
Pencil and Paper	Male N = 11	8	3	0
	Female N = 7	6	0	1
Total	Male N = 31	25	6	0
	Female N = 25	20	2	3
	All N = 56	45	8	3

The survey results reveal experiences of the participants with touchscreen technology and preference for use of such technology. However, considering the distribution of the survey results by group, it is presumed that no certain advantage or disadvantage occurred by the assigning a classroom to a particular treatment group. No group showed that its participants lacked experience in working with touchscreen technology; further, the majority of participants in all groups indicated a preference for working with an iPad. As such, either of the three grade six classes could have been assigned as the control group or a treatment group.

Quantitative Results

As part of the mixed method design implemented for this research study, a quantitative data component exists. This section includes descriptive statistics from a pre-test and post-test administered to the participants. A use of a paired-samples *t*-test allowed the researcher to check for statistical significance of the test results. In addition, a one-way ANCOVA on post-test results between two groups was employed, considering the factor of technology use with the pre-test serving as the covariate. Finally, a 3×2 ANCOVA was employed, considering the factors of technology-input modality and gender with the pre-test serving as the covariate. The results from these various ANCOVA models are reported following the upcoming discussion of how the students performed on the pre-test/post-test instruments.

Descriptive Statistics

In order to measure participants' growth attributable to proportional reasoning instruction, a pre-test and post-test were administered to all three grade six classes. The same test served as both the pre-test and post-test, but the order of questions was changed from the pre-test to the post-test. The testing instrument was examined by SMEs and found to exhibit content validity. Each test, comprised of 28 multiple-choice questions, was developed using question banks from NAEP, TIMSS, and an online assessment containing ratio and proportion questions. Chapter III contains details concerning the testing instrument. The statistical package SPSS, version 20 was used to calculate the descriptive statistics from the pre-test and post-test. The results from these assessments appear in Table 23.

Table 23

Descriptive Statistics from Pre-Test/Post-Test

	Pre-Test			Post-Test		
Category	Mean	Standard Deviation	Standard Error	Mean	Standard Deviation	Standard Error
Whole Group (n = 56)	38.97	16.02	2.14	45.54	18.39	2.46
iPad (n = 19)	38.72	15.26	3.50	50.00	18.48	4.24
Mouse (n = 19)	40.98	16.25	3.73	47.37	17.57	4.03
Pencil-and-Paper (n = 18)	37.10	17.20	4.05	38.89	18.25	4.30
Technology (n = 38)	39.85	15.59	2.53	48.68	17.84	2.89

In order to use a paired sample *t*-test, the normality assumption for the independent variables must be satisfied. Accordingly, the researcher performed a Shapiro Wilk test for each independent variable. The results of the Shapiro Wilk test indicated the independent variables were approximately normally distributed. For all participants tested, the mean score increased 6.57 points from the pre-test to the post-test. Using a paired sample t-test, this difference demonstrated statistical significance, $t(55) = 3.278$, $p < .005$, $R^2 \approx 0.394$. When considering those students who used either iPad or mouse technology during the research, the mean score increased 8.83 points from the pre-test to the post-test. Using a paired sample t-test, this difference also demonstrated statistical significance, $t(37) = 3.617$, $p < .001$, $R^2 \approx 0.362$. Similarly, when the different types of technology-input modality are considered separately, the mean score increased 11.28 points for the iPad group and 6.39 points for the Mouse group. Using a one-tailed *t*-test, both of these differences demonstrated statistical significance, $t(18) = 3.120$, $p < .01$, $R^2 \approx 0.334$ (iPad); $t(18) = 1.946$, $p < .05$, $R^2 \approx 0.415$ (Mouse). As for the Pencil-and-Paper group, the difference in mean score from the pre-test to the post-test increased 1.79 points; however, this difference did not demonstrate statistical significance. Based upon these test results, the use of virtual manipulatives supported the development of proportional reasoning skills as well as or better than the use of pencil and paper practice alone.

One-Way ANCOVA

In order to determine if the use of virtual manipulatives demonstrated significant influence while participants worked to develop proportional reasoning skills, the researcher employed a one-way ANCOVA on the post-test results between two groups using the pre-test score as a covariate. The results from the equal slope assumption test with respect to gender and pre-test revealed a non-significant result, $F(1, 55) = 1.106$, $p > .05$. Similarly, the results from the equal slope assumption test with respect to technology-input modality and pre-test also revealed a non-significant result, F (1, 55) = 0.883, $p > .05$. These results granted the use of an ANCOVA test. As previously seen in Table 23, the group of participants that used virtual manipulatives in developing proportional reasoning skills had a higher mean average on the post-test compared to those participants who used pencil and paper; however, the results from the one-way ANCOVA indicated the difference between the post-test mean scores of the two groups was not statistically significant. It was noted that the difference between the post-test mean scores for the virtual manipulative group and the control group approached statistical significance, $F\ (1, 53) = 3.785, p = .057, R^2 \approx 0.435$.

3 × 2 ANCOVA

To conduct comparison between the control group and the two treatment groups, a 3 × 2 ANCOVA model was employed to determine any effect technology-input modality and gender might have upon the post-test score; specifically, technology-input modality (iPad, Mouse, Pencil-and-Paper) and gender serve as the main effects with the pre-test acting as the covariate. As stated previously, the results of the equal slope assumption test justified the use of the ANCOVA test. Table 24 provides the results from the 3 × 2 ANCOVA process. When comparing the post-test mean score for the three groups with respect to gender, there is no consistent pattern of performance. For the iPad group, males (Mean = 59.82) had a higher mean score than females (Mean = 42.86); however, for the Pencil-and-Paper group and Mouse group, females (Mean = 42.35, Pencil-and-Paper; Mean = 51.53, Mouse) had a higher mean score than males (Mean =

36.69, Pencil-and-Paper; Mean = 44.94, Mouse). There was no significant interaction effect between gender and technology-input modality.

Focusing upon the post-test mean score for each group, both treatment groups had a higher average than the control group (iPad, mean score = 50.00; Mouse, mean score = 47.37; Pencil-and-Paper, mean score = 38.89). When applying a Bonferroni adjustment, the difference between the mean scores for the iPad group and Pencil-and-Paper group did not demonstrate statistical significance; similarly, the difference between the mean scores for the Mouse group and Pencil-and-Paper group also did not demonstrate statistical significance. Also, the difference between the mean scores for the treatment groups did not demonstrate statistical significance. Small sample size could be a contributing factor for the non-significance.

Table 24

Technology-Input Modality and Gender Post-Test Results

Group	Gender	Mean	Standard Deviation	Standard Error
iPad	Male N = 8	59.82	21.91	7.75
	Female N = 11	42.86	12.06	3.64
	Total N = 19	50.00	18.48	4.24
Mouse	Male N = 12	44.94	16.22	4.68
	Female N = 7	51.53	20.29	7.67
	Total N = 19	47.37	17.57	4.03
Pencil-and-Paper	Male N = 11	36.69	15.90	4.79
	Female N = 7	42.35	22.34	8.44
	Total N = 18	38.89	18.25	4.30
Total	Male N = 31	45.85	19.39	3.48
	Female N = 25	45.14	17.46	3.49
	All N = 56	45.54	18.39	2.46

Based upon the quantitative data presented, the following findings emerged:

- The use of Thinking Blocks and Number Pieces supported the development of proportional reasoning skills as well as or better than the use of pencil and paper alone;
- Neither gender nor technology-input modality use appeared as a factor of statistical significance; and,
- No interaction effect between gender and technology-input modality use appeared as a factor of statistical significance.

Qualitative Results

In an effort to provide complementarity (Greene, Caracelli, & Graham, 1989; Hesse-Biber, 2010; Onwuegbuzie & Leech, 2006) to the quantitative results of this research, a mixed method approach required a qualitative component. The convergent parallel design implemented for this research allowed the gathering of quantitative and qualitative data simultaneously; afterward, the examination of both types of data provided the potential for a more complete description of the development of proportional reasoning skills and the impact that the use of virtual manipulatives had on such development. In this section, the results of the grade six participants' work on five tasks are reported in levels of development with respect to proportional reasoning. Also, six individual case studies of target students who were tracked during the research are presented. Finally, a multiple-case analysis in which the six target students are considered in various groupings is considered.

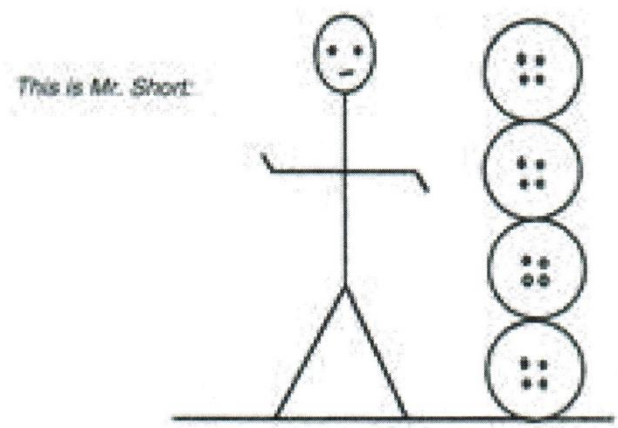
Proportional Reasoning Tasks

During the ten days of data collection for this study, five proportional reasoning tasks found in the literature were assigned to participants to complete during regular class time. Presented in the order listed, these tasks were titled: (1) Mr. Tall/Mr. Short; (2) Egg Carton; (3) Tree House; (4) Sticks and Rhombi; and (5) John's School. As presented in the conceptual framework for this study in Chapters I and II, proportional reasoning is considered to be developed in levels. The four categories reflecting such development are: (a) Illogical; (b) Additive; (c) Transitional; and (d) Ratio. Responses to the questions in each task were sorted into these four categories. Although there were 56 participants in this study, the number of responses for each task varied slightly from the participant count due to the absence of various participants throughout the ten days of data collection.

1) Mr. Tall/Mr. Short task. On Day 2 of the research schedule, participants completed the Mr. Tall/Mr. Short task. Participants read about the height of Mr. Short as measured in buttons and paper clips. Considering these measures, the task informed students about the height of Mr. Tall in buttons and asked them to determine the height of Mr. Tall in paper clips. Figure 17 shows a sample of a completed task. Using the response and explanation provided by each participant, the level of proportional reasoning development was determined. Results by group and gender are reported in Table 25.

Study Student Number 116

A



The length of Mr. Short is 4 large buttons.

The length of Mr. Tall (not drawn) is 6 large buttons.

When paper clips are used to measure Mr. Short and Mr. Tall, the length of Mr. Short is 6 paper clips. What is the length of Mr. Tall in paper clips? 8 paper clips

Please EXPLAIN how you arrived at your answer. I added 2 to 6 and that made 8

Figure 17. Sample of Mr. Tall/Mr. Short task.

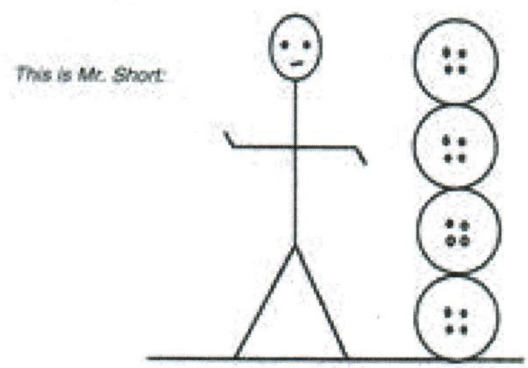
Table 25

Results from Mr. Tall/Mr. Short Task

Group	Gender	Illogical	Additive	Transitional	Ratio	Total
iPad	Male N = 8	1	6	0	1	8
	Female N = 11	1	8	1	0	10
Mouse	Male N = 12	0	10	1	0	11
	Female N = 7	1	5	0	0	6
Pencil and Paper	Male N = 11	2	5	1	0	8
	Female N = 7	2	5	0	0	7
Total	Male N = 31	3	21	2	1	27
	Female N = 25	4	18	1	0	23
	All N = 56	7	39	3	1	50

An examination of the results in Table 25 indicates that approximately 78% of the participants who completed the Mr. Tall/Mr. Short task exhibited an additive approach in completing the Mr. Tall/Mr. Short task. When considering results by gender, the percentage of participants exhibiting an additive approach is similar to each other and to all participants: 78.3% of females and 77.8% of males gave responses at the additive level of proportional reasoning development. Only one student of the 50 participants who completed the Mr. Tall/Mr. Short task gave a response that indicated they were working at the Ratio level of proportional reasoning. Figure 18 presents the work sample of Mr. Tall/Mr. Short at the Ratio level.

Study Student Number 106 R



The length of Mr. Short is 4 large buttons.

The length of Mr. Tall (not drawn) is 6 large buttons.

When paper clips are used to measure Mr. Short and Mr. Tall, the length of Mr. Short is 6 paper clips. What is the length of Mr. Tall in paper clips? 9 paper clips

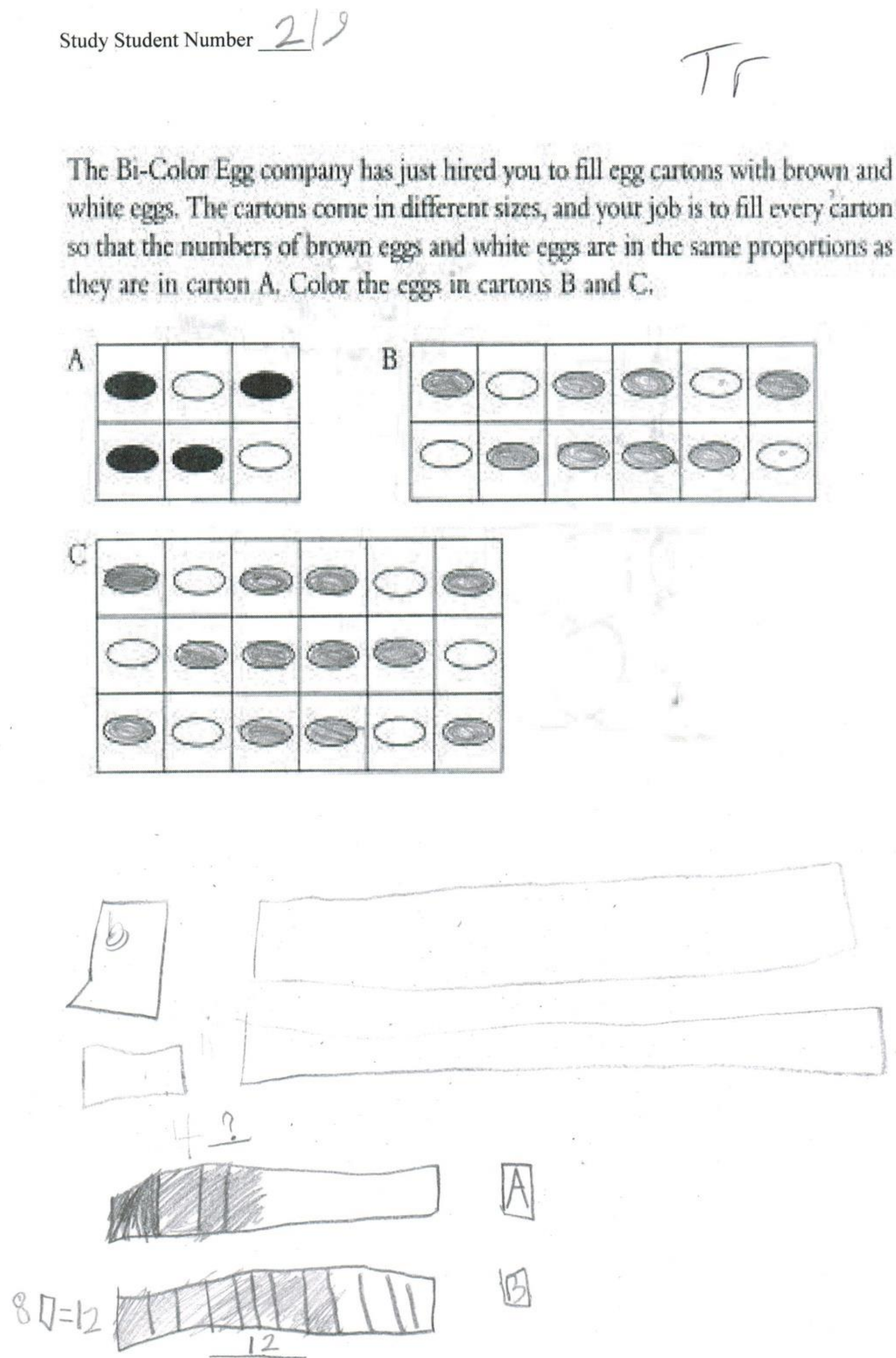
Please EXPLAIN how you arrived at your answer.

Mr. short had 2 buttons and for every 3 there were clips. so he had 6.

so Mr. tall had 6 buttons and 3 clips for 2 buttons – there were. so he had 9.

Figure 18. Mr. Tall/Mr. Short work sample at ratio level.

2) Egg Carton task. On Day 4 of the research schedule, participants completed the Egg Carton task. In this task, a picture of Carton A indicated this carton contained four brown eggs and two white eggs; participants had to color the eggs in two larger cartons, called Cartons B and C, in such a way that the proportion of brown to white eggs from Carton A remained. Carton B contained 12 eggs and Carton C contained 18 eggs. Figure 19 shows a sample completed Egg Carton task at the Transitional level. Based upon the responses from the 55 participants who completed the task, the levels of proportional reasoning demonstrated on the Egg Carton task are shown in Table 26.


Study Student Number 219

Tr

The Bi-Color Egg company has just hired you to fill egg cartons with brown and white eggs. The cartons come in different sizes, and your job is to fill every carton so that the numbers of brown eggs and white eggs are in the same proportions as they are in carton A. Color the eggs in cartons B and C.

Figure 19. Sample of Egg Carton task.

Table 26

Results from Egg Carton Task

Group	Gender	Illogical	Additive	Transitional	Ratio	Total
iPad	Male N = 8	1	0	2	5	8
	Female N = 11	5	0	6	0	11
Mouse	Male N = 12	3	0	3	6	12
	Female N = 7	1	0	1	5	7
Pencil and Paper	Male N = 11	1	0	4	5	10
	Female N = 7	4	0	2	1	7
Total	Male N = 31	5	0	9	16	30
	Female N = 25	10	0	9	6	25
	All N = 56	15	0	18	22	55

There is a distinct difference evident in the responses collected from the Mr. Tall/Mr. Short task and the Egg Carton task: no participant gave a response that exhibited an additive approach in completing the Egg Carton task. Upon reflection, the structure of the Egg Carton task did not lend itself to supporting an additive approach as readily as the Mr. Tall/Mr. Short task. However, the visual nature of the Egg Carton task revealed participant thinking concerning ratio and position of the eggs within the carton. Participants who attempted to preserve both count and arrangement of eggs within a carton revealed a transitional approach to completing the Egg Carton task.

The students worked with a block modeling approach for two days prior to completing the Egg Carton task; therefore, Ms. Xanth asked them to draw a block model to represent the problem for coloring the brown eggs in Carton B. At that point, another unique feature regarding the Egg Carton task manifested: the visual approach for the Egg Carton task and the visual approach for block modeling seemed to conflict with each other for several participants. Instead of drawing a block model to represent the ratio of brown eggs to white eggs, participants attempted to redraw the setting from Carton B. Figure 20 presents a sample of the Egg Carton task illustrating this visual conflict; this picture also provides a sample of the task at the Ratio level.

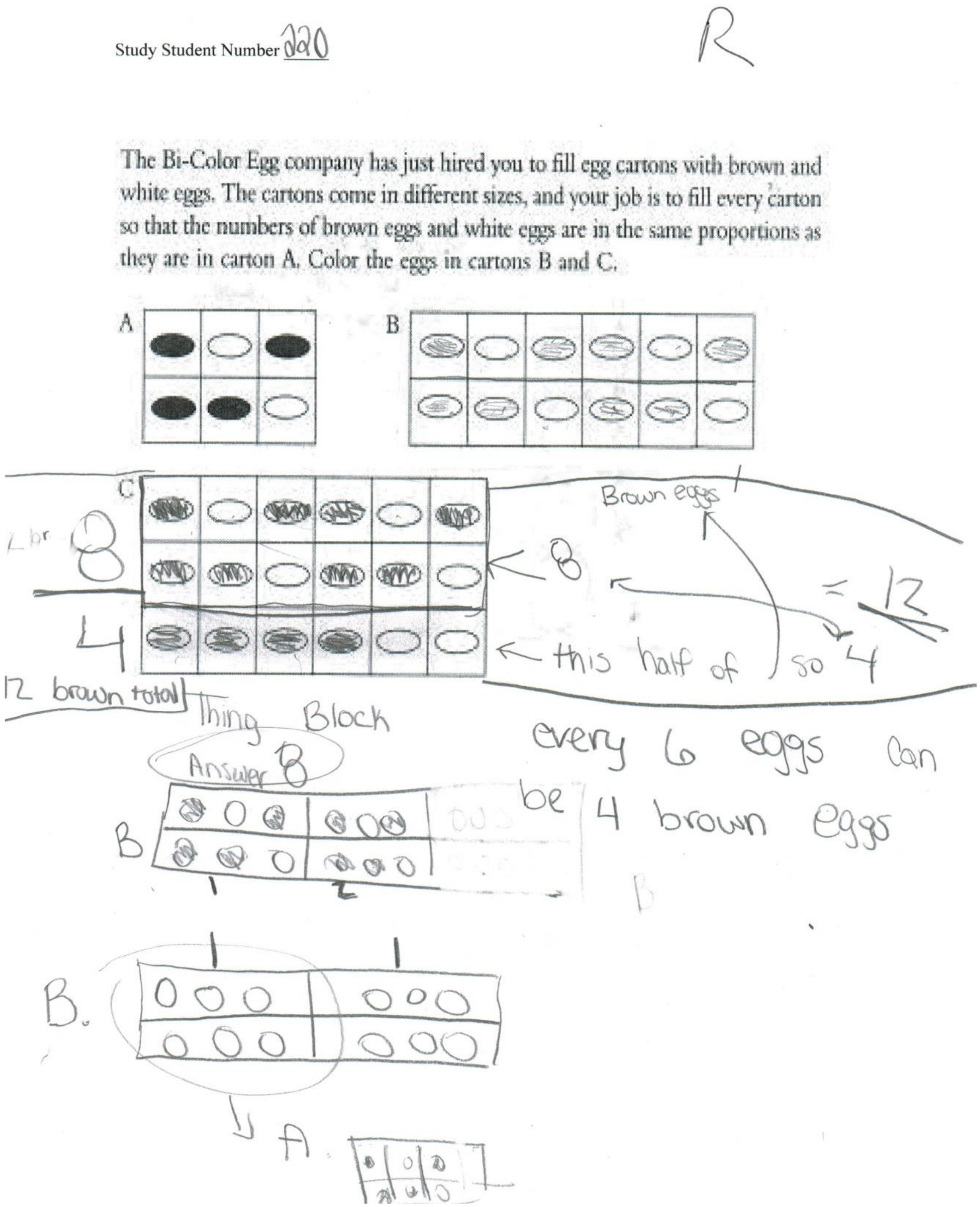
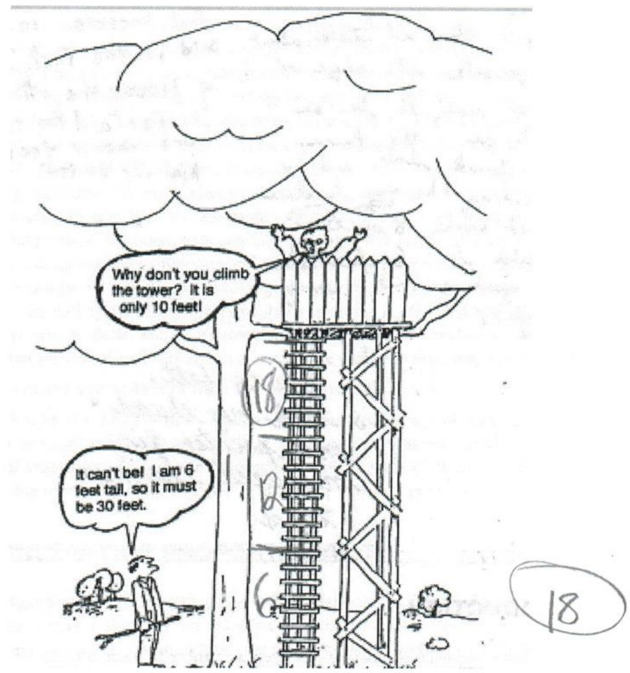


Figure 20. Visual conflict in Egg Carton task.

3) Tree House task. On Day 6 of the research schedule, participants completed the Tree House task. This task provided a drawing of a tree house, ladder, and two people. One person in the tree house invites the person standing on the ground to climb the tower ladder and declares that it is 10 feet tall; however, the person on the ground insists that the ladder must be taller than 10 feet, since the person on the ground is six feet tall. Participants calculated the height of the ladder based upon the information provided. Figure 21 contains a sample of a completed Tree House task at the Ratio level. For the 54 participants who completed this task, the levels of proportional reasoning are given in Table 27.

Study Student Number 217 R



Based upon the information provided in the drawing, how tall is the ladder? Show your work to support your answer.

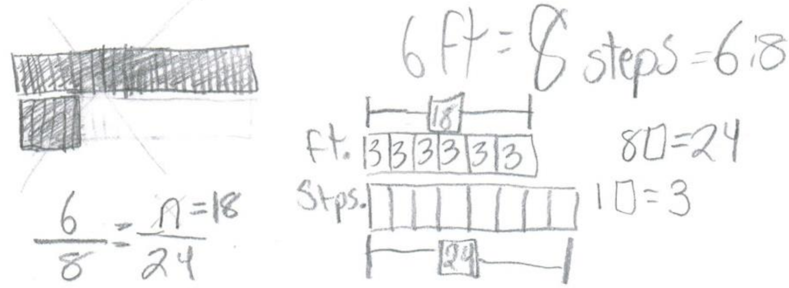


Figure 21. Sample of Tree House task.

Table 27

Results from Tree House Task

Group	Gender	Illogical	Additive	Transitional	Ratio	Total
iPad	Male N = 8	3	0	3	2	8
	Female N = 11	8	0	2	1	11
Mouse	Male N = 12	7	0	2	2	11
	Female N = 7	2	0	3	1	6
Pencil and Paper	Male N = 11	6	0	2	2	10
	Female N = 7	5	0	1	1	7
Total	Male N = 31	16	0	7	6	29
	Female N = 25	15	0	6	3	24
	All N = 56	31	0	13	9	53

More than half of the participants applied an illogical approach for the Tree House task; with respect to gender, 55.2% of males and 60% of females used an illogical approach to complete this task. In the control group, 60% of males and 71.4% of females used an illogical approach; 52.6% of males and 55.6% of females in the treatment groups completed the task with an illogical approach. The statement concerning the ladder being

10 feet tall seemed to interfere with how participants approached solving this task. Participants who simply took the statement as truth without any further calculation or consideration of the evidence provided in the task evidenced an illogical approach. Figure 22 shows an example of one student's work with this illogical approach to the Tree House task.

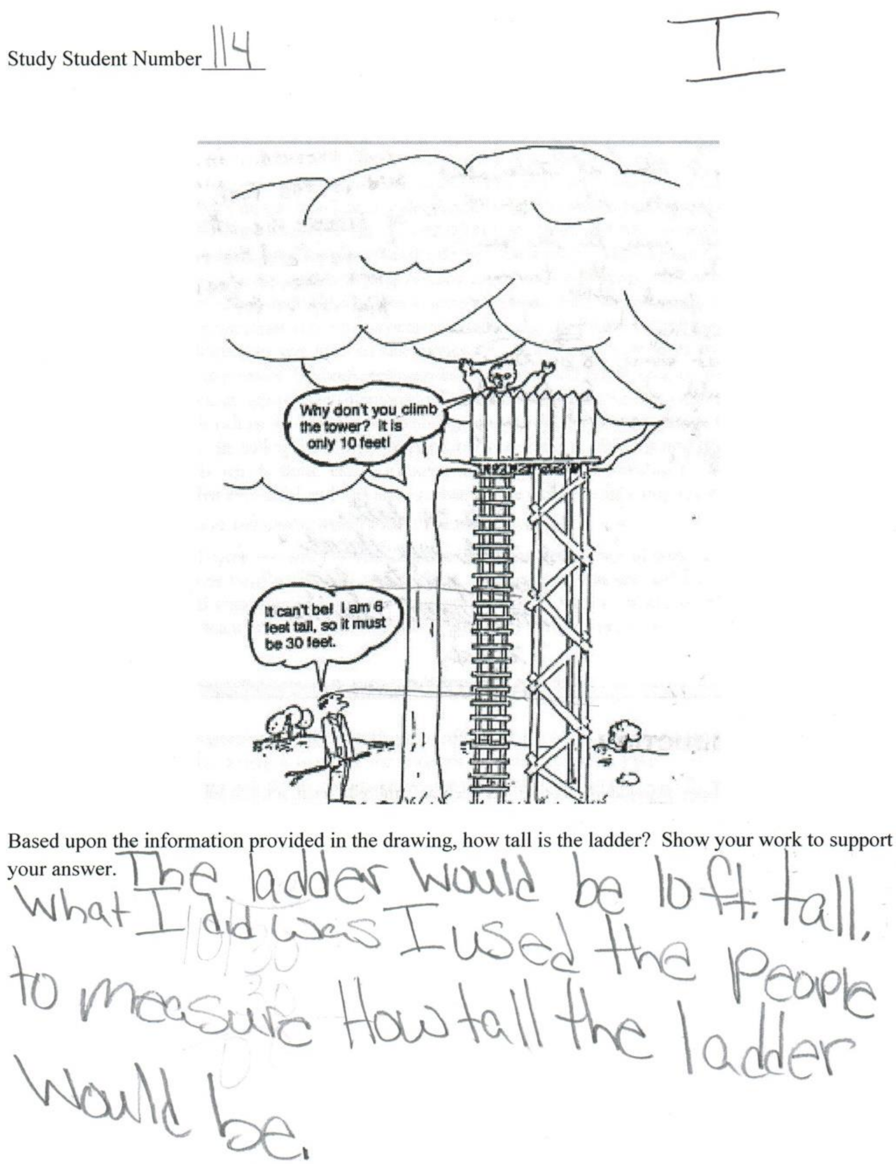
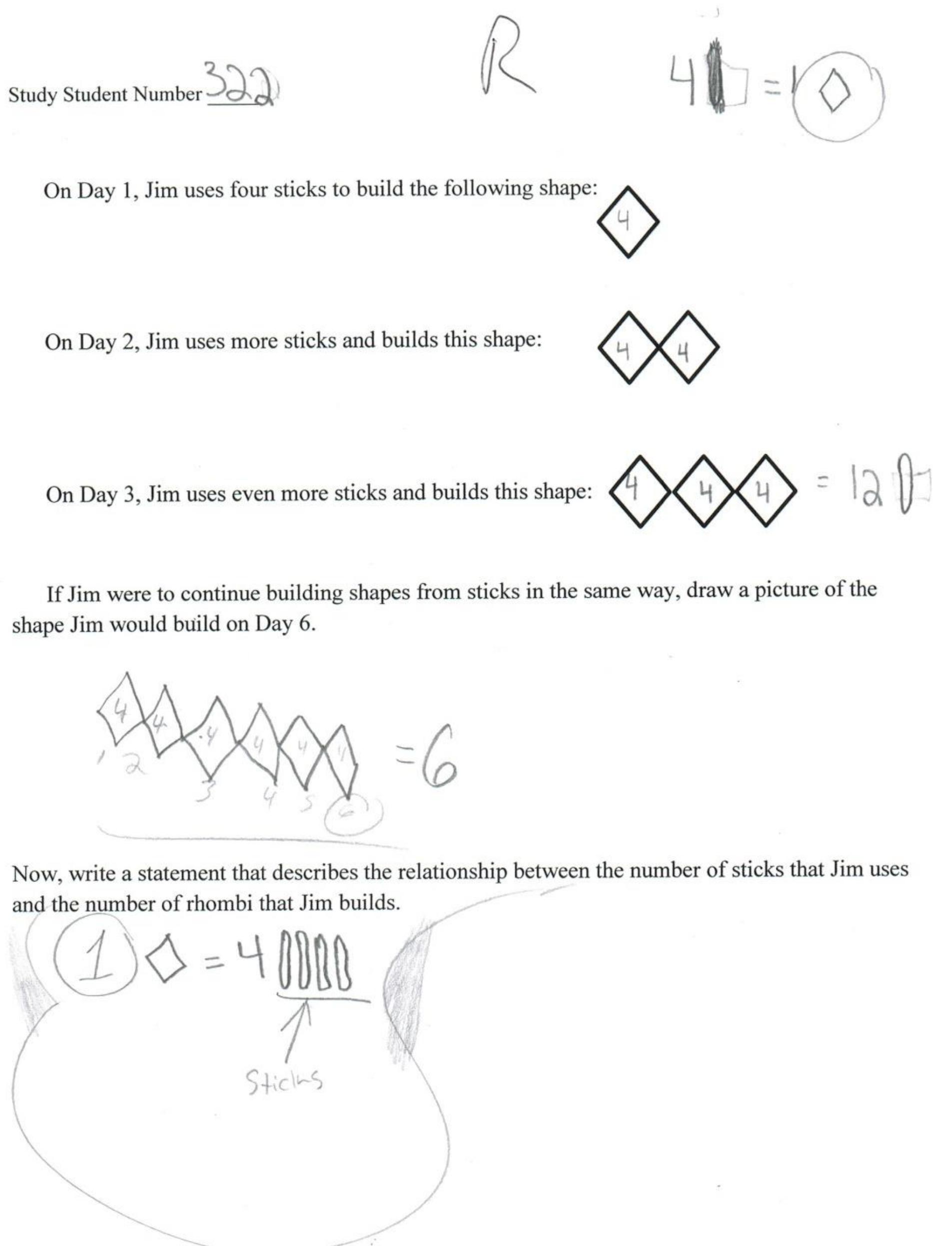


Figure 22. Illogical response to Tree House task.

4) Sticks and Rhombi task. On Day 8 of the research schedule, participants completed the Sticks and Rhombi task. In this task, a pattern concerning the number of sticks and number of rhombi built from the sticks is established. Participants extended the pattern and wrote a statement describing the relationship between the number of sticks and the number of rhombi. Figure 23 presents a sample of the Sticks and Rhombi task at the Ratio level. Due to absences, only 52 participants submitted work for this task. The results from the Sticks and Rhombi task are given in Table 28.


Study Student Number 322 R

On Day 1, Jim uses four sticks to build the following shape:

On Day 2, Jim uses more sticks and builds this shape:

On Day 3, Jim uses even more sticks and builds this shape: = 12

If Jim were to continue building shapes from sticks in the same way, draw a picture of the shape Jim would build on Day 6.

=6

Now, write a statement that describes the relationship between the number of sticks that Jim uses and the number of rhombi that Jim builds.

1 = 4

Sticks

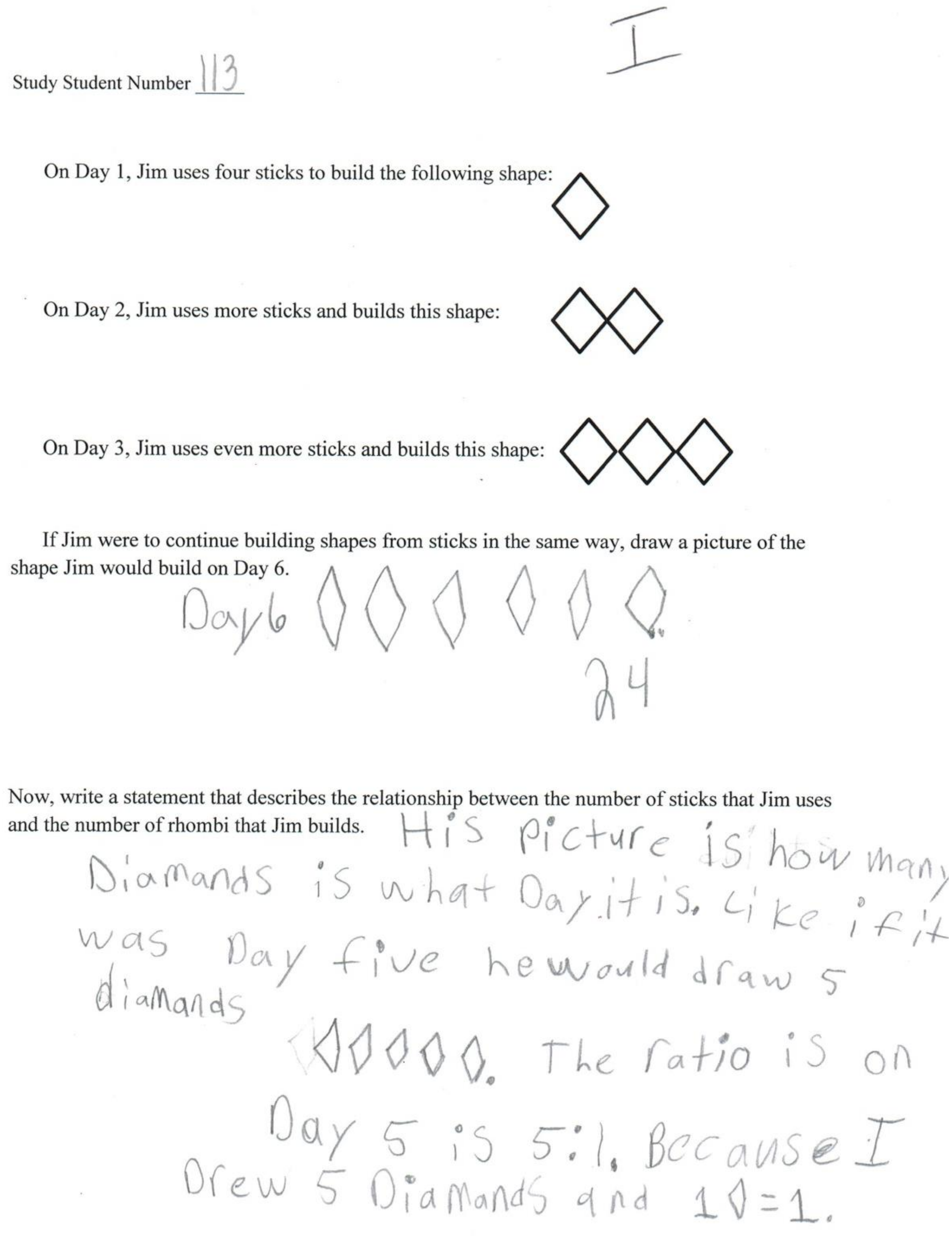
Figure 23. Sample of Sticks and Rhombi task.

Table 28

Results from Sticks and Rhombi Task

Group	Gender	Illogical	Additive	Transitional	Ratio	Total
iPad	Male N = 8	2	0	0	6	8
	Female N = 11	8	0	1	1	10
Mouse	Male N = 12	4	0	3	5	12
	Female N = 7	0	0	1	5	6
Pencil and Paper	Male N = 11	3	0	4	4	11
	Female N = 7	1	0	2	2	5
Total	Male N = 31	9	0	7	15	31
	Female N = 25	9	0	4	8	21
	All N = 56	18	0	11	23	52

The results from Table 28 indicated that participants did not employ an additive approach to complete the Sticks and Rhombi task; however, some participants demonstrated work at the higher levels of a transitional approach or a ratio approach. In fact, 65.4% of participants demonstrated work on the Sticks and Rhombi task at the transitional level or ratio level. Considering gender, 71.0% of males and 57.1% of females demonstrated work at the transitional level or ratio level on this task. Participants whose responses indicated an illogical approach appeared to be confused by the directions for the task. Instead of writing a statement that related the number of sticks to the number of rhombi, those participants related the number of days to the number of rhombi or the number of days to the number of sticks. A sample of the Sticks and Rhombi task demonstrating work at the Illogical level appears in Figure 24.



I

Study Student Number 113

On Day 1, Jim uses four sticks to build the following shape:

On Day 2, Jim uses more sticks and builds this shape:

On Day 3, Jim uses even more sticks and builds this shape:

If Jim were to continue building shapes from sticks in the same way, draw a picture of the shape Jim would build on Day 6.

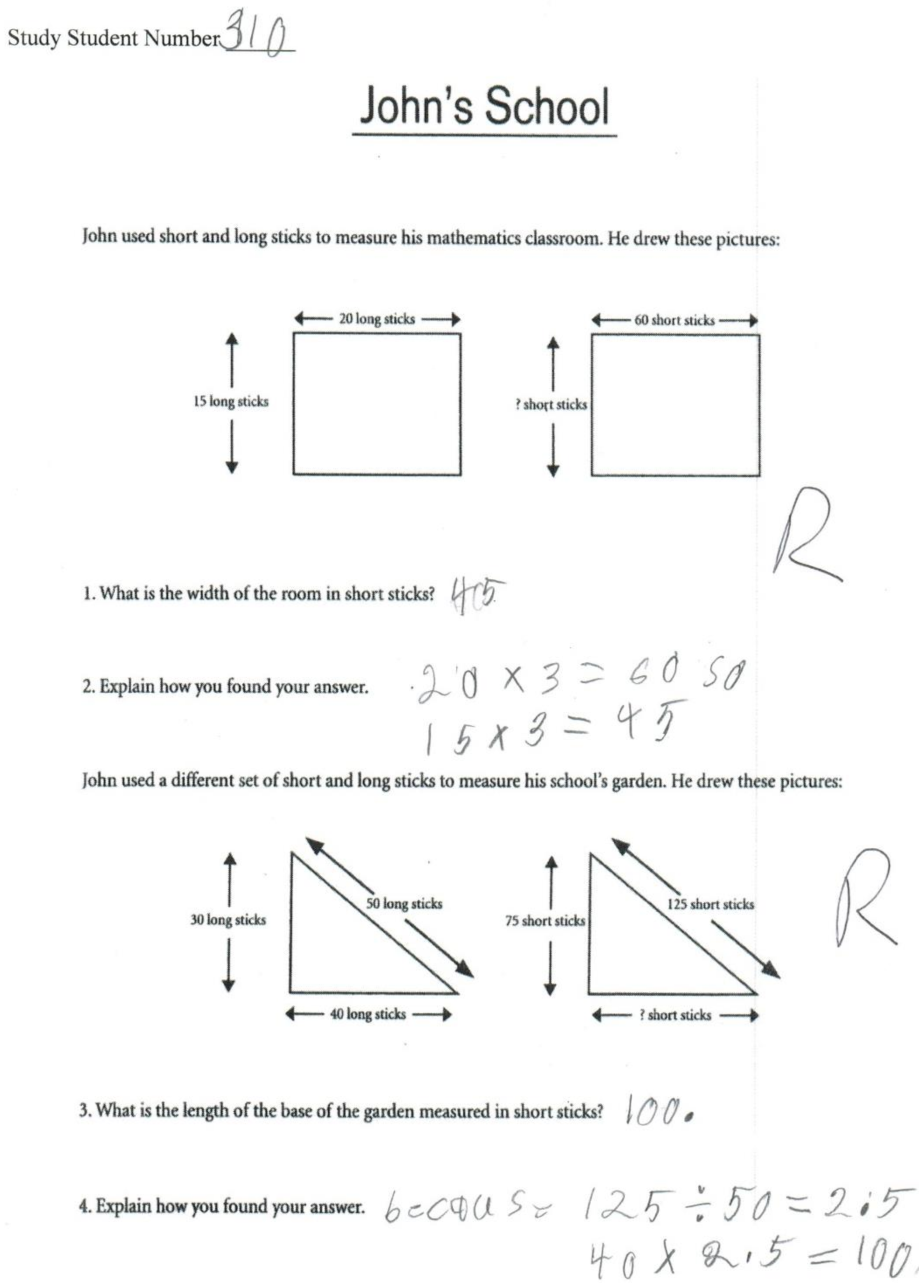
Day 6

24

Now, write a statement that describes the relationship between the number of sticks that Jim uses and the number of rhombi that Jim builds. His picture is how many Diamands is what Day it is. Like if it was Day five he would draw 5 diamands. The ratio is on Day 5 is 5:1. Because I Drew 5 Diamands and 1◊=1.

Figure 24. Sticks and Rhombi task at Illogical level.

5) John's School task. On Day 10 of the research schedule, participants completed the John's School task. This task had two parts, Questions 1 and 2, both involving measuring a classroom space or a garden space using short sticks and long sticks for the units of measure. Based upon the measurements provided, participants determined a missing quantity. Question 1 involved an integral-value ratio of short sticks to long sticks, whereas Question 2 involved a rational-value ratio of short sticks to long sticks. Figure 25 illustrates a completed sample of John's School task in which the responses to both questions demonstrated work at the Ratio level. Table 29 reports the results for Question 1, while Table 30 gives the results for Question 2.



Study Student Number 310

John's School

John used short and long sticks to measure his mathematics classroom. He drew these pictures:

R

1. What is the width of the room in short sticks? 45

2. Explain how you found your answer. 20 x 3 = 60 50
15 x 3 = 45

John used a different set of short and long sticks to measure his school's garden. He drew these pictures:

R

3. What is the length of the base of the garden measured in short sticks? 100.

4. Explain how you found your answer. because 125 ÷ 50 = 2.5
40 x 2.5 = 100.

Figure 25. Sample of John's School task.

Table 29

Results from John's School Task, Question 1.

Group	Gender	Illogical	Additive	Transitional	Ratio	Total
iPad	Male N = 8	2	2	0	4	8
	Female N = 11	1	3	2	5	11
Mouse	Male N = 12	5	3	1	3	12
	Female N = 7	2	1	0	4	7
Pencil and Paper	Male N = 11	2	5	0	4	11
	Female N = 7	1	0	4	2	7
Total	Male N = 31	9	10	1	11	31
	Female N = 25	4	4	6	11	25
	All N = 56	13	14	7	22	56

For the participants in the control group, 33.3% of the students produced work for Question 1 at the Ratio level, while 42.1% of participants in the treatment groups also demonstrated work at the Ratio level. Additionally, 36.4% of males and 28.6% of females in the control group exhibited work at the Ratio level whereas 35% of males and

50% of females in the treatment groups completed Question 1 at the Ratio level. So, no clear pattern of performance at the Ratio level with respect to gender appeared.

Table 30

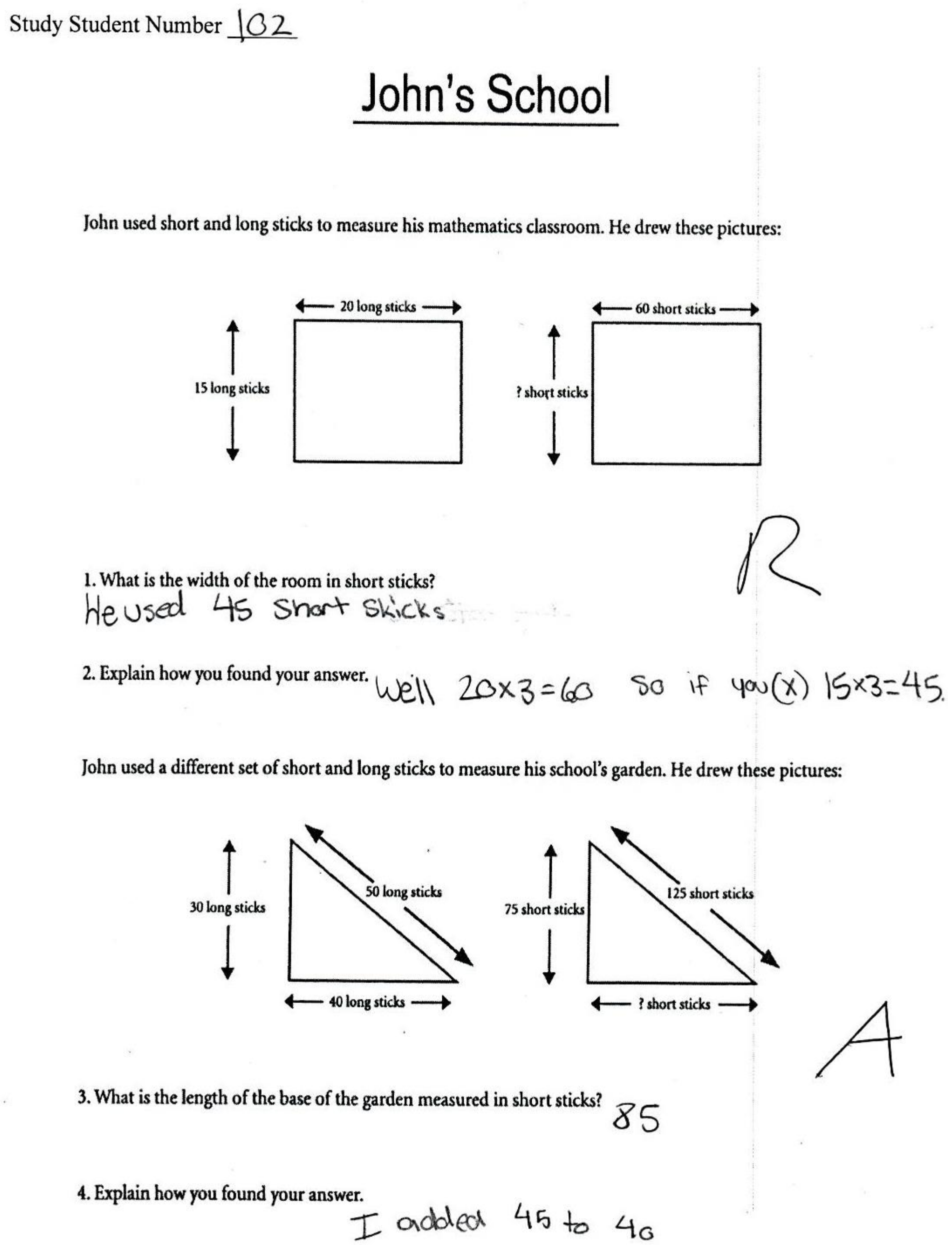
Results from John's School Task, Question 2.

Group	Gender	Illogical	Additive	Transitional	Ratio	Total
iPad	Male N = 8	1	4	1	2	8
	Female N = 11	2	5	2	2	11
Mouse	Male N = 12	6	3	3	0	12
	Female N = 7	1	3	1	2	7
Pencil and Paper	Male N = 11	2	6	0	3	11
	Female N = 7	2	2	0	3	7
Total	Male N = 31	9	13	4	5	31
	Female N = 25	5	10	3	7	25
	All N = 56	14	23	7	12	56

For the participants in the control group, 33.3% of the students produced work for Question 2 at the Ratio level, which is the same percentage of at the Ratio level for

Question 1 with this group. However, only 15.8% of participants in the treatment groups also demonstrated work at the Ratio level. Additionally, 27.3% of males and 42.9% of females in the control group exhibited work at the Ratio level whereas 10% of males and 22.2% of females in the treatment groups completed Question 2 at the Ratio level. So, a higher percentage of participants in the control group performed at the Ratio level as compared to the treatment groups; also, a higher percentage of females in both the control group and treatment groups demonstrated work at the Ratio level when compared to the male participants.

Comparing the results from Tables 29 and 30 reveals more participants appeared to employ an additive approach when using a rational-value ratio as opposed to using an integral-value ratio. This occurrence can be attributed to participants reverting to a familiar strategy of an additive approach when faced with more challenging situations in a proportional reasoning setting. Participants appear to recognize a proportional situation when using an integral-value ratio; however, the appearance of a rational-value ratio in a proportional reasoning situation seemed to confound participants who were expecting to use a multiple to find the missing value. Figure 26 gives an example of a student employing additive reasoning in completing John's School task, Question 2, yet the student used a ratio approach in completing Question 1.



Study Student Number 102

John's School

John used short and long sticks to measure his mathematics classroom. He drew these pictures:

20 long sticks
15 long sticks
60 short sticks
? short sticks

R

1. What is the width of the room in short sticks?
He used 45 Short sticks

2. Explain how you found your answer. Well 20x3=60 So if you(x) 15x3=45.

John used a different set of short and long sticks to measure his school's garden. He drew these pictures:

30 long sticks
50 long sticks
40 long sticks
75 short sticks
125 short sticks
? short sticks

A

3. What is the length of the base of the garden measured in short sticks? 85

4. Explain how you found your answer.
I added 45 to 40

Figure 26. Additive reasoning approach on John's School, Question 2.

Comparison of Groups within the Tasks

The results from the various tasks reported in Tables 25-30 provided data for all participants as a whole; it is appropriate to examine the results in relation to the control group (pencil and paper) and the two treatment groups (iPad group and mouse group). Additionally, performance by gender within each group provides another lens to look at data for each task. By considering the performance by task for each group and gender within each group, patterns by group and gender appeared. Although the actual numbers of participants at each level of proportional reasoning are reported in Tables 25-30, the percentages of participants at each level are presented in this section. Results for each task are reported by treatment group as well as gender within each treatment group.

Treatment Groups

Quantitative data gathered during this study supports the premise that all groups experienced growth as evidenced through mean scores on the pre-test and post-test. However, an inspection of the qualitative data as expressed through the performance tasks provides additional information that supplements the results obtained by testing. Tables 31-33 display the percentages of participants whose work suggested performance at each level of proportional reasoning by treatment group.

Table 31

iPad Group and Proportional Reasoning Levels by Task (percentages)

Group	Task	Illogical	Additive	Transitional	Ratio
iPad	Mr. Tall/Mr. Short	11.1	77.8	5.6	5.6
	Male	12.5	75	0	12.5
	Female	10	80	10	0
	Egg Carton	31.6	0	42.1	26.3
	Male	12.5	0	25	62.5
	Female	45.5	0	54.5	0
	Tree House	57.9	0	26.3	15.8
	Male	37.5	0	37.5	25
	Female	72.7	0	18.2	9.1
	Sticks/Rhombi	55.6	0	5.6	38.9
	Male	25	0	0	75
	Female	80	0	10	10
	John's School Q1	15.8	26.3	10.5	47.4
	Male	25	25	0	50
	Female	9.1	27.3	18.2	45.5
	John's School Q2	15.8	47.4	15.8	21.1
	Male	12.5	50	12.5	25
	Female	18.2	45.5	18.2	18.2

Table 32

Mouse Group and Proportional Reasoning Levels by Task (percentages)

Group	Task	Illogical	Additive	Transitional	Ratio
Mouse	Mr. Tall/Mr. Short	5.9	88.2	5.9	0
	Male	0	90.9	9.1	0
	Female	16.7	83.3	0	0
	Egg Carton	21.1	0	21.1	57.9
	Male	25	0	25	50
	Female	14.3	0	14.3	71.4
	Tree House	52.9	0	29.4	17.6
	Male	63.6	0	18.2	18.2
	Female	33.3	0	50	16.7
	Sticks/Rhombi	22.2	0	22.2	55.6
	Male	33.3	0	25	41.7
	Female	0	0	16.7	83.3
	John's School Q1	36.8	21.1	5.3	36.8
	Male	41.7	25	8.3	25
	Female	28.6	14.3	0	57.1
	John's School Q2	36.8	31.6	21.1	10.5
	Male	50	25	25	0
	Female	14.3	42.9	14.3	28.6

Table 33

No Technology Group and Proportional Reasoning Levels by Task (percentages)

Group	Task	Illogical	Additive	Transitional	Ratio
Pencil and Paper	Mr. Tall/Mr. Short	26.7	66.7	6.7	0
	Male	25	62.5	12.5	0
	Female	28.6	71.4	0	0
	Egg Carton	29.4	0	35.3	35.3
	Male	10	0	40	50
	Female	57.1	0	28.6	14.3
	Tree House	64.7	0	17.6	17.6
	Male	60	0	20	20
	Female	71.4	0	14.3	14.3
	Sticks/Rhombi	25	0	37.5	37.5
	Male	27.3	0	36.4	36.4
	Female	20	0	40	40
	John’s School Q1	16.7	27.8	22.2	33.3
	Male	18.2	45.5	0	36.4
	Female	14.3	0	57.1	28.6
	John’s School Q2	22.2	44.4	0	33.3
	Male	18.2	54.5	0	27.3
	Female	28.6	28.6	0	42.9

A close inspection of the data reveals some consistencies which could generate a pattern. With respect to the groups, the Mr. Tall/Mr. Short task was completed prior to instruction; so, no pattern from this task is attributable to growth due to development of proportional reasoning skills. Since the design inherent in the Egg Carton, Tree House, and Sticks and Rhombi tasks did not lend to using an additive approach, the low percentages of participants classified at the Additive level for these tasks should not be attributed to an increased development of proportional reasoning skills.

When focusing upon gender in the three groups, there were tasks in which males demonstrated a higher percentage using a ratio approach than females, such as within the iPad group. However, in the Mouse group, females demonstrated a higher percentage using a ratio approach than males in completing the tasks, with the exception of the Tree House task. In the Pencil and Paper group, males seemed to have a higher percentage using a ratio approach in the earlier tasks while the females appeared to have a higher percentage using a ratio approach in the later tasks.

Although the percentages vary from group to group, two consistent patterns emerged from this data:

- the percentage of participants demonstrating the use of a ratio approach increased from the first task to the last task; and,
- in light of the participants who implemented an additive approach on the last task, a lower percentage of participants used an additive approach on Question 1 of John’s School task as opposed to Question 2.

The first point is particularly relevant: the intent of developing proportional reasoning skills in grade six students influenced this study.

Even though participants demonstrated growth with respect to the use of a ratio approach across the five tasks, this growth was not always consistent from task to task; which is to say, growth in the ratio approach did not steadily increase from the first task to the last task. On the two questions from John's School task, the increased percentage of participants implementing an additive approach on Question 2 as compared to Question 1 supported the findings from other researchers in the literature concerning proportional reasoning problems involving non-integral ratios (Karplus, Karplus, & Wollman, 1974; Pulos, Karplus, & Stage, 1981; de la Cruz, 2013; Singh, 2000; Tjoe & de la Torre, 2014).

Individual Case Studies

As part of the mixed method approach used for this study, six target students comprised a focus group, with the intent of studying their work on two additional performance tasks. The mean score of the pre-test administered at the beginning of this study allowed the researcher to separate participants into two categories: participants who scored above the mean score and participants who scored below the mean score. In cooperation with Ms. Xanth, the researcher selected one student from each of the control and two treatment groups as per the two categories generated by the mean score. Hence, three male participants and three female participants were selected. During regular class instruction, video recording of the classroom with a focus on the six target students occurred. With the observation protocol in mind, video recordings focusing upon the six target students provided opportunity to study their work habits. To preserve confidentiality of the participants, the researcher supplied pseudonyms for each target student.

In addition to the five tasks completed by the 56 participants, the six target students completed two additional tasks in their pre-instruction and post-instruction interviews: Make a New Puzzle and Cocoa. Appendix D contains these two additional tasks. Students completed the Make a New Puzzle task prior to any work with the Thinking Blocks virtual manipulative or any block modeling strategy. The task involved increasing side lengths from a set of puzzle pieces, already cut out by the researcher and provided to each target student during the interview, in order to determine the side lengths of a larger puzzle with the same shape as the original. Figure 27 shows the completed work on Make a New Puzzle task by a target student.

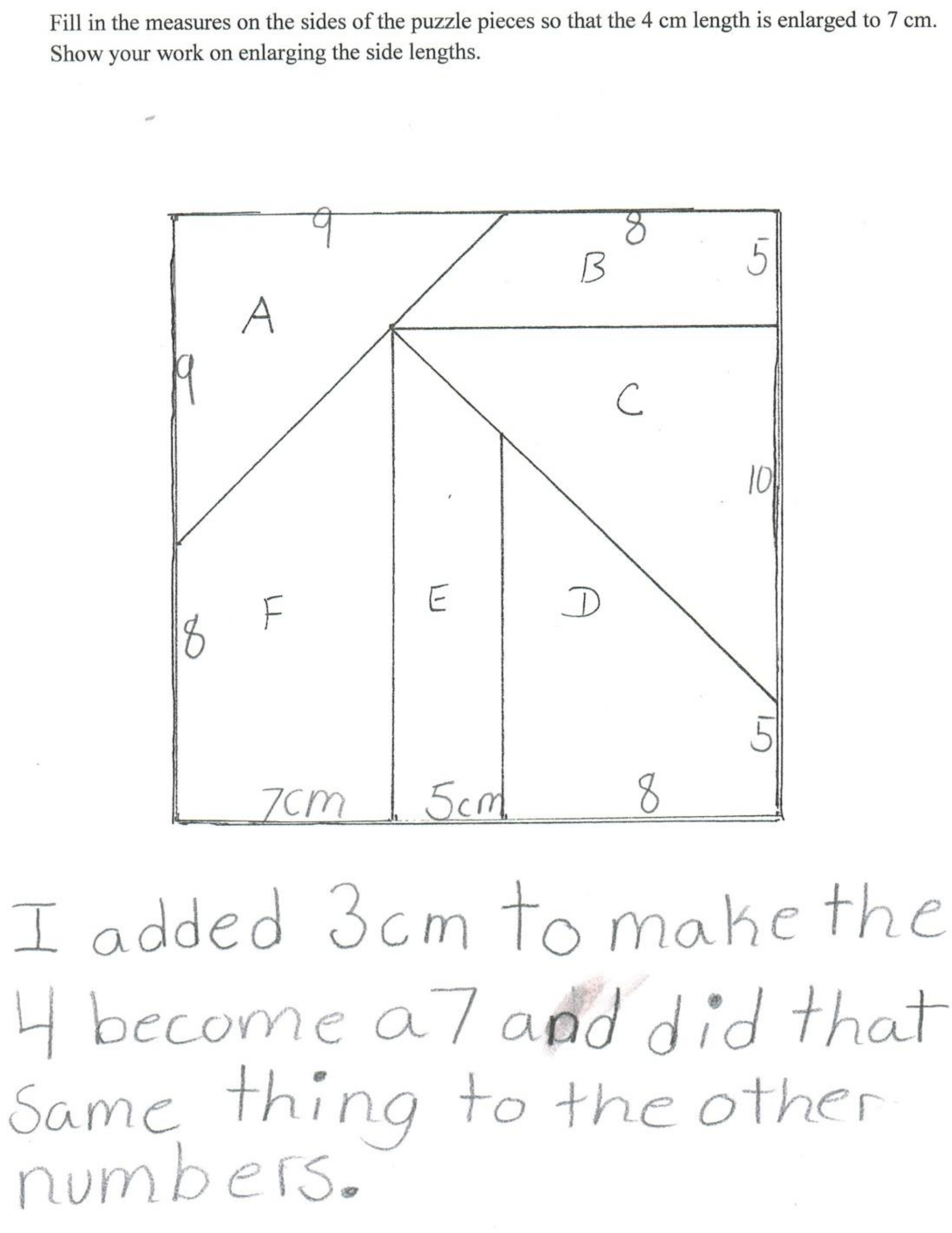
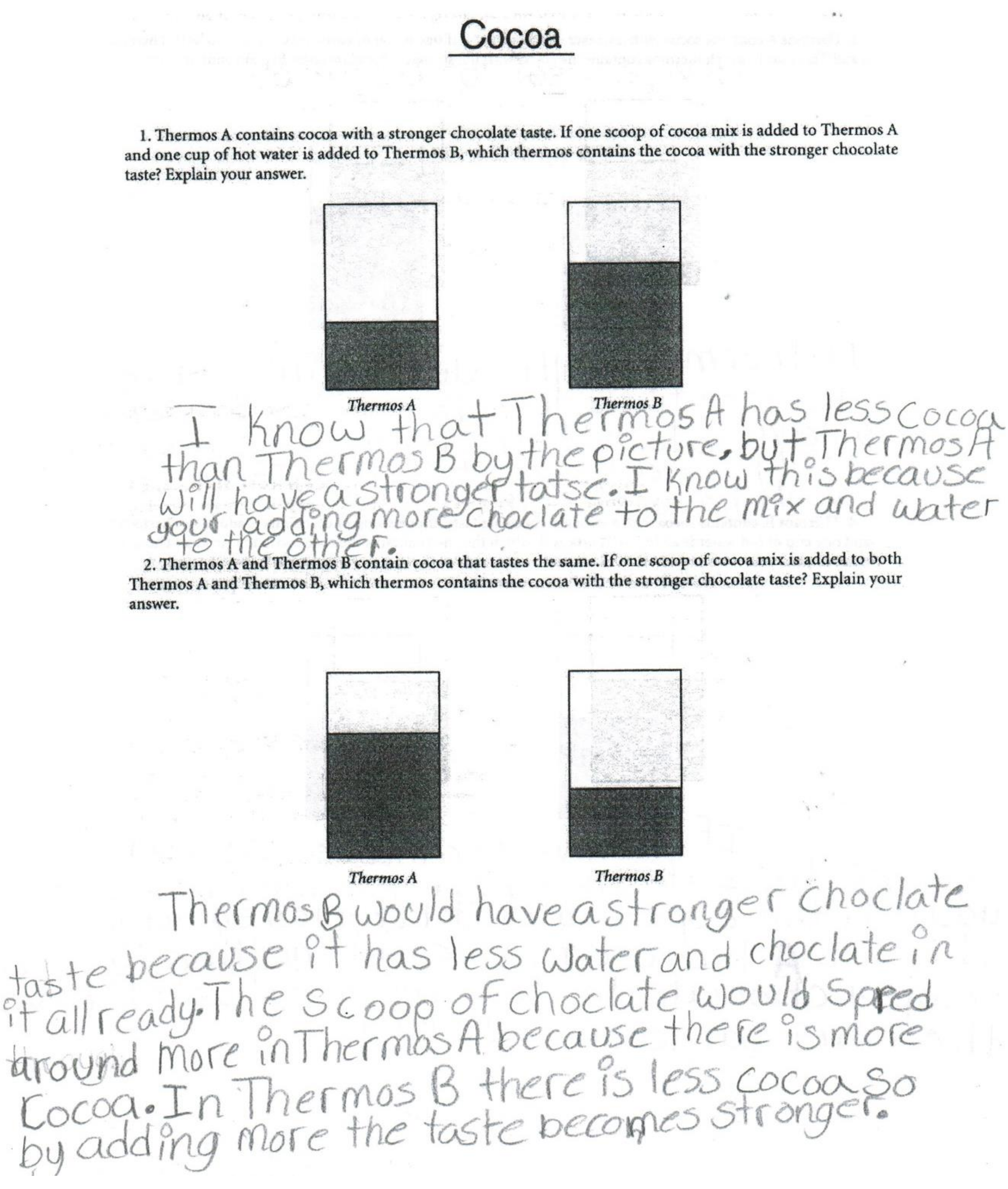


Figure 27. Sample of Make a New Puzzle task.

The target students completed the Cocoa task at the conclusion of the study, after working with both Thinking Blocks and Number Pieces virtual manipulatives for the two treatment groups and pencil-and-paper practice with the block modeling strategy for the control group. On the Cocoa task, Thermos A and Thermos B both contain cocoa. A description of the strength of the chocolate taste for the cocoa is given. For each of four problems in the task, either cocoa mix or hot water is added to one or both of the thermoses. The student must then decide which thermos has the stronger chocolate taste and explain their responses. Figure 28 presents a sample of a completed Cocoa task.



Cocoa

1. Thermos A contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Thermos A   Thermos B

I know that Thermos A has less Cocoa than Thermos B by the picture, but Thermos A will have a stronger tatse. I know this because your adding more choclate to the mix and water to the other.

2. Thermos A and Thermos B contain cocoa that tastes the same. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Thermos A   Thermos B

Thermos B would have a stronger choclate taste because it has less Water and choclate in it allready. The scoop of choclate would spred around more in Thermos A because there is more Cocoa. In Thermos B there is less cocoa so by adding more the taste becomes stronger.

Figure 28. Sample of Cocoa task.

The six target students were interviewed separately as they completed both tasks; as they performed the two tasks, the researcher videotaped the work of the target students and recorded the dialogue. Appendices H and I contain the transcripts of the interviews recorded while completing these two tasks. Students' actions, comments, and work were analyzed in order to determine any emerging patterns or trends. Descriptions follow of all six target students, whose pseudonyms are Alice, Alan, Candy, Carl, Betty, and Bob.

Alice

Background information. At the time of the study, Alice-an 11-year old female member of the iPad group- scored below the mean score on the pre-test. With respect to the state-mandated mathematics assessment, Alice scored Advanced as a grade five student. According to the responses to the technology use and preference survey, she had one-two years' experience working with touchscreen devices; in fact, Alice indicated that she prefers to work with an iPad.

Pre-instruction interview. For the Make a New Puzzle task, Alice was one of two target students who successfully assembled the original puzzle pieces into a square. Before doing so, Alice asked if she had to use all of the puzzle pieces; she was the only target student who asked this particular question. In calculating the measures of the new square puzzle, Alice used an additive approach to complete the task; additionally, she counted by three on her fingers to determine the change from 4 cm to 7 cm. When given the new set of puzzle pieces reflecting her calculated measures, she attempted to assemble the pieces into a new square. However, Alice exhibited hesitancy and confusion when she was unable to form the square. When asked about the situation, Alice replied "…the pieces don't make a square anymore, any way you put them." As she worked this task, Alice confirmed that the measures of the side lengths of the puzzle pieces were accurate; she didn't attempt to attribute her inability to assemble the puzzle pieces to any flaw in measurement of the new puzzle pieces. Figure 29 depicts Alice attempting to assemble the new puzzle pieces into a square, while Figure 30 gives a sample of Alice's work on the Make a New Puzzle task.

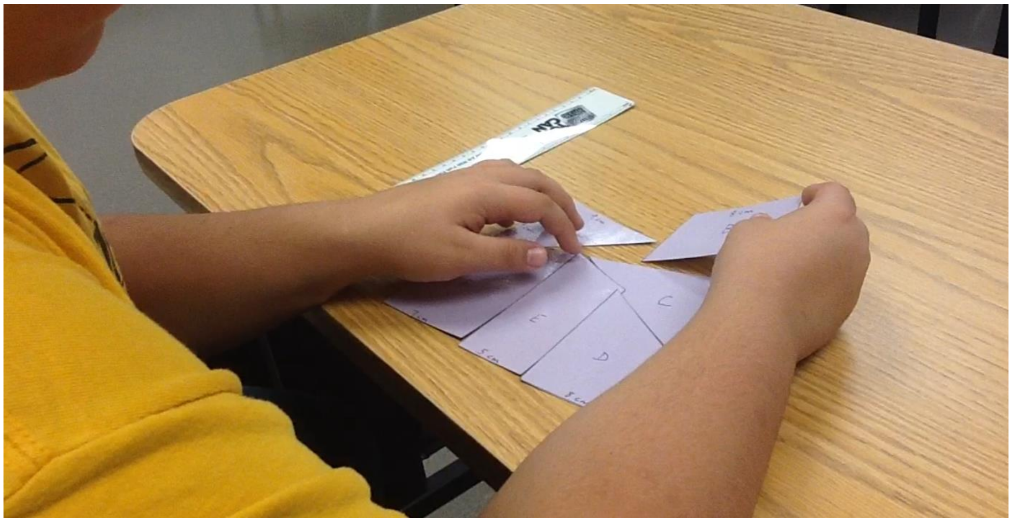


Figure 29. Alice assembling new puzzle pieces.

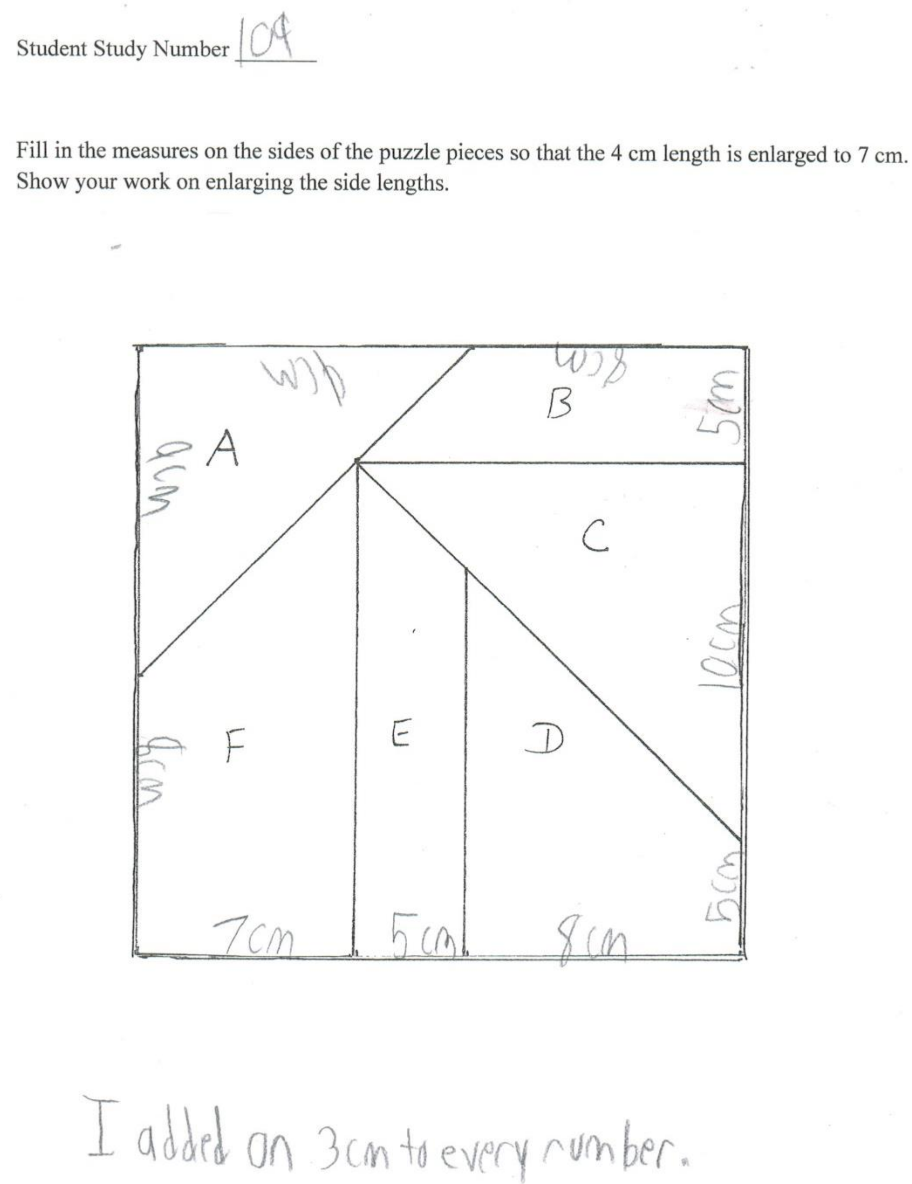
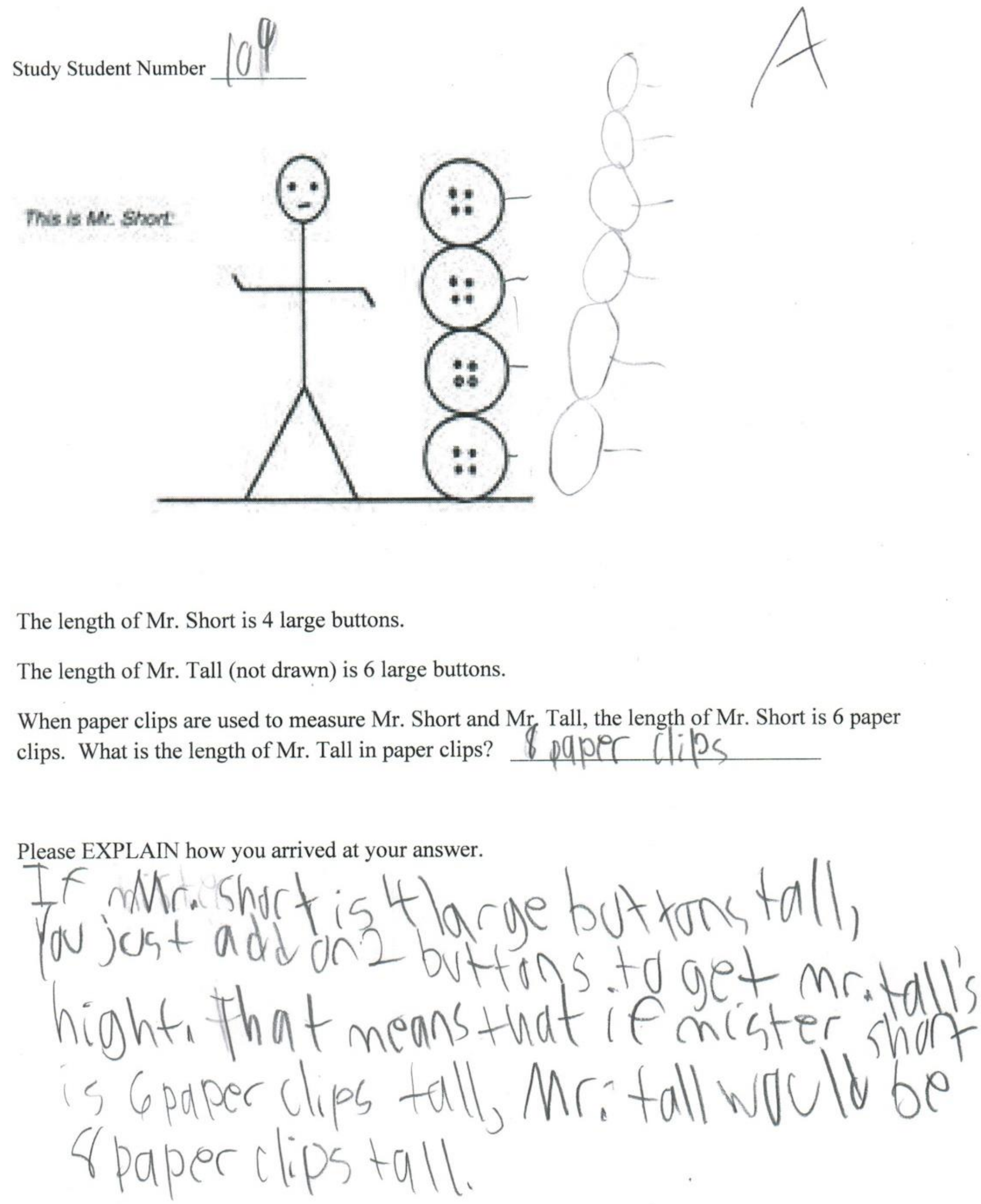


Figure 30. Alice's work on Make a New Puzzle task.

Classroom work with virtual manipulatives. During classroom instruction, Alice tended to work independently from other participants. When Ms. Xanth directed students to work in their table groups, Alice followed directions; however, she contributed little to group discussions. Alice did not experience difficulty in working with the virtual manipulatives during classroom instruction. She completed the modules on the Thinking Blocks website; with respect to the problems assigned for use with the Number Pieces website, she completed seven out of twenty problems.

Mr. Tall/Mr. Short task (1). Alice used an additive approach to complete the Mr. Tall/Mr. Short task. Her result of 8 paper clips for Mr. Tall's height and the explanation provided as to how she found her answer supported this classification. In her work, Alice first formulated a rule to find Mr. Tall's height based upon the information provided in the problem, then she applied the rule for specific values. On her paper, Alice sketched the height of Mr. Tall in buttons and used the visual relationship to establish her addition rule. Figure 31 shows Alice's work on the Mr. Tall/Mr. Short task.



Study Student Number 10[illegible] A

This is Mr. Short

The length of Mr. Short is 4 large buttons.

The length of Mr. Tall (not drawn) is 6 large buttons.

When paper clips are used to measure Mr. Short and Mr. Tall, the length of Mr. Short is 6 paper clips. What is the length of Mr. Tall in paper clips? 8 paper clips

Please EXPLAIN how you arrived at your answer.

If Mr. Short is 4 large buttons tall, you just add on 2 buttons to get mr. tall's hight. That means that if mister short is 6 paper clips tall, Mr. tall would be 8 paper clips tall.

Figure 31. Alice's work on Mr. Tall/Mr. Short task.

Egg Carton task (2). Alice correctly colored the number of brown eggs for each carton on the Egg Carton task; however, she attempted to maintain the physical arrangement of brown eggs and white eggs from Carton A as she completed coloring eggs in Cartons B and C. This effort to maintain the physical arrangement indicated Alice perceived a connection between the ratio of brown eggs to white eggs and their location in the cartons. Based upon her work, the researcher evaluated Alice's level of proportional reasoning to be at the transitional level. Figure 32 displays Alice's work on the Egg Carton task.

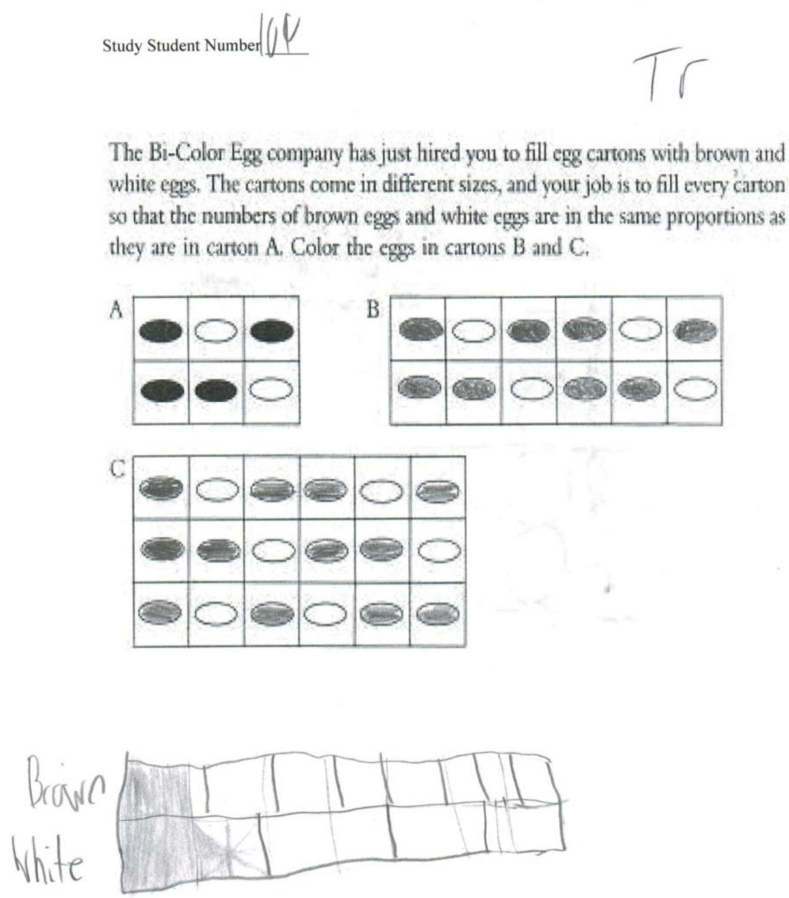


Figure 32. Alice's work on the Egg Carton task.

Tree House task (3). Alice did not find the correct height of the ladder on the Tree House task. Despite identifying a relationship between the man's height and the number of steps on the ladder (6 feet = 8 steps), she did not apply this information in calculating the height of the ladder. Also, Alice appeared to miscount the number of steps, which interfered with her calculations. As a result of her calculations, Alice exhibited work at the illogical level for this task. Figure 33 contains Alice's work on the Tree House task.

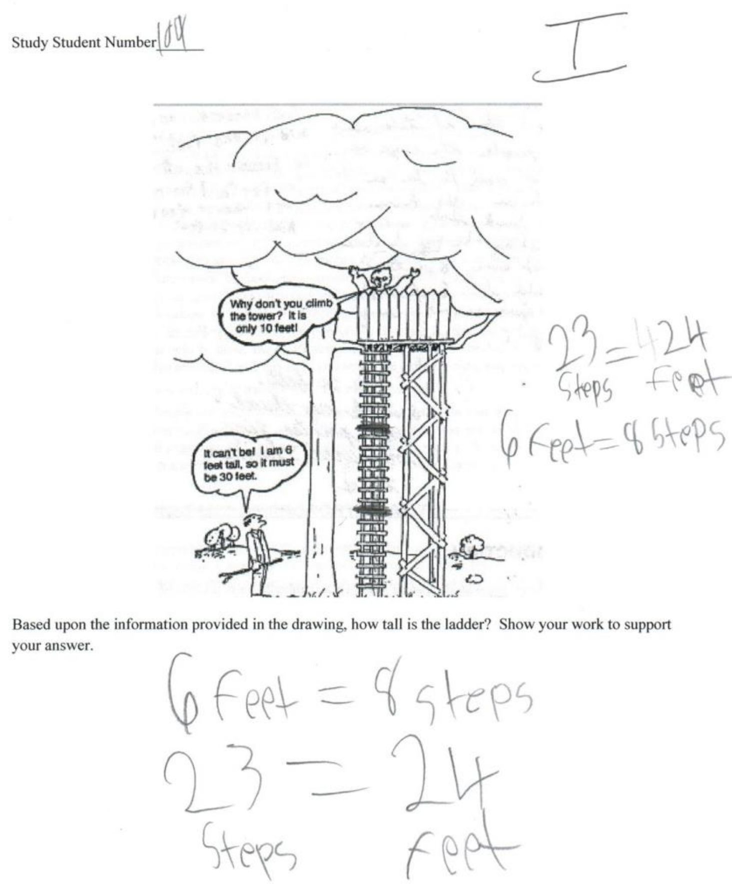
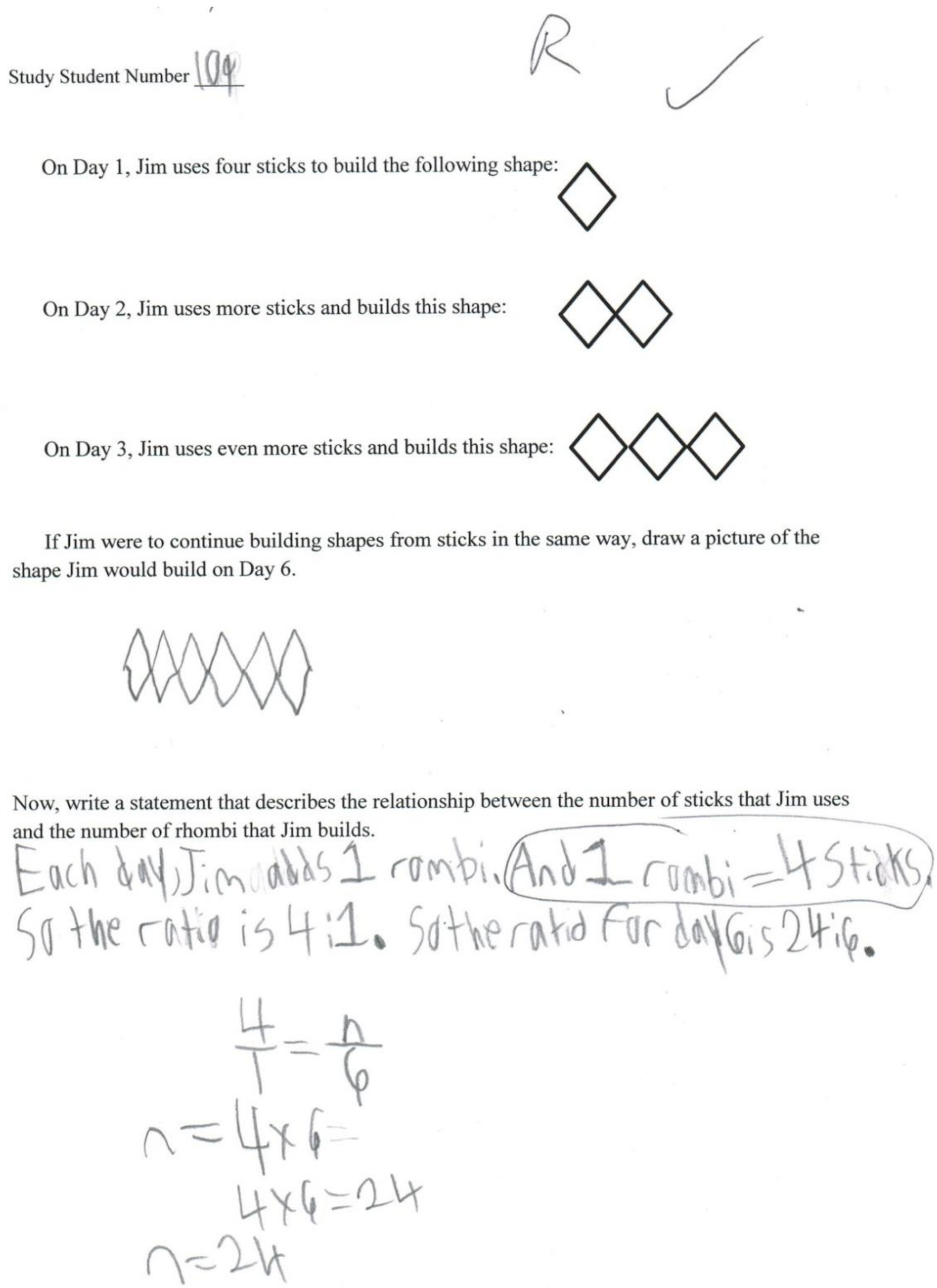


Figure 33. Alice's work on the Tree House task.

Sticks and Rhombi task (4). Alice correctly answered the questions on the Sticks and Rhombi task. Her sketch of the figure for Day 6 showed a set of connected rhombi. In addition, Alice stated the relationship between the number of rhombi and the number of sticks used to build the rhombi. When Alice used her ratio to determine the number of sticks used to build six rhombi, she changed the order of comparison; however, this change of order was not required to answer the question. Further, Alice wrote a proportion and solved the proportion correctly. Alice's work justified assigning the level of proportional reasoning at the ratio level. Figure 34 shows Alice's work on the Sticks and Rhombi task.



Study Student Number 104 R ✓

On Day 1, Jim uses four sticks to build the following shape:

On Day 2, Jim uses more sticks and builds this shape:

On Day 3, Jim uses even more sticks and builds this shape:

If Jim were to continue building shapes from sticks in the same way, draw a picture of the shape Jim would build on Day 6.

Now, write a statement that describes the relationship between the number of sticks that Jim uses and the number of rhombi that Jim builds.

Each day, Jim adds 1 rombi. And 1 rombi = 4 sticks. So the ratio is 4:1. So the ratio for day 6 is 24:6.

$\frac{4}{1} = \frac{n}{6}$

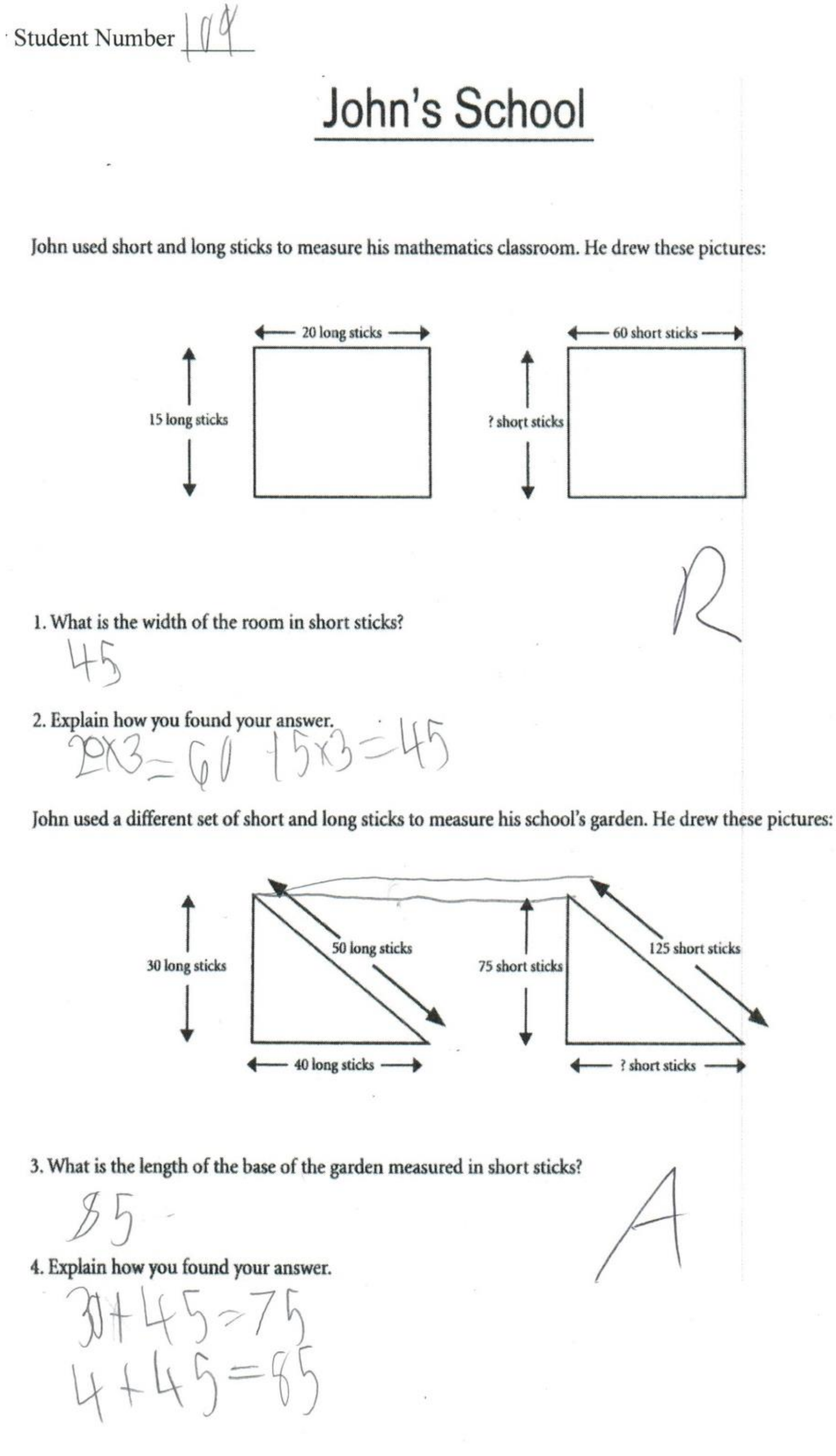
$n = 4 \times 6$

$4 \times 6 = 24$

$n = 24$

Figure 34. Alice’s work on the Sticks and Rhombi task.

John's School task (5). For this task, Alice correctly answered question 1, demonstrating work at the ratio level; however, her work on question 2 reverted back to an additive approach in order to find the missing side length. Alice did not provide any written explanation as to how she found the missing side lengths for each question. For question 1, her work indicated she identified the scale factor and used multiplication to find the missing side length. Instead of maintaining this multiplicative approach on question 2, Alice decided to implement an additive approach, writing an incorrect value in her addition equation. The presence of a non-integral value in question 2 seemed to affect Alice's choice of an additive approach for this situation. Alice's work on John's School task is found in Figure 35.



Student Number 109

John's School

John used short and long sticks to measure his mathematics classroom. He drew these pictures:

R

1. What is the width of the room in short sticks?

45

2. Explain how you found your answer.

20x3=60 15x3=45

John used a different set of short and long sticks to measure his school's garden. He drew these pictures:

3. What is the length of the base of the garden measured in short sticks?

85

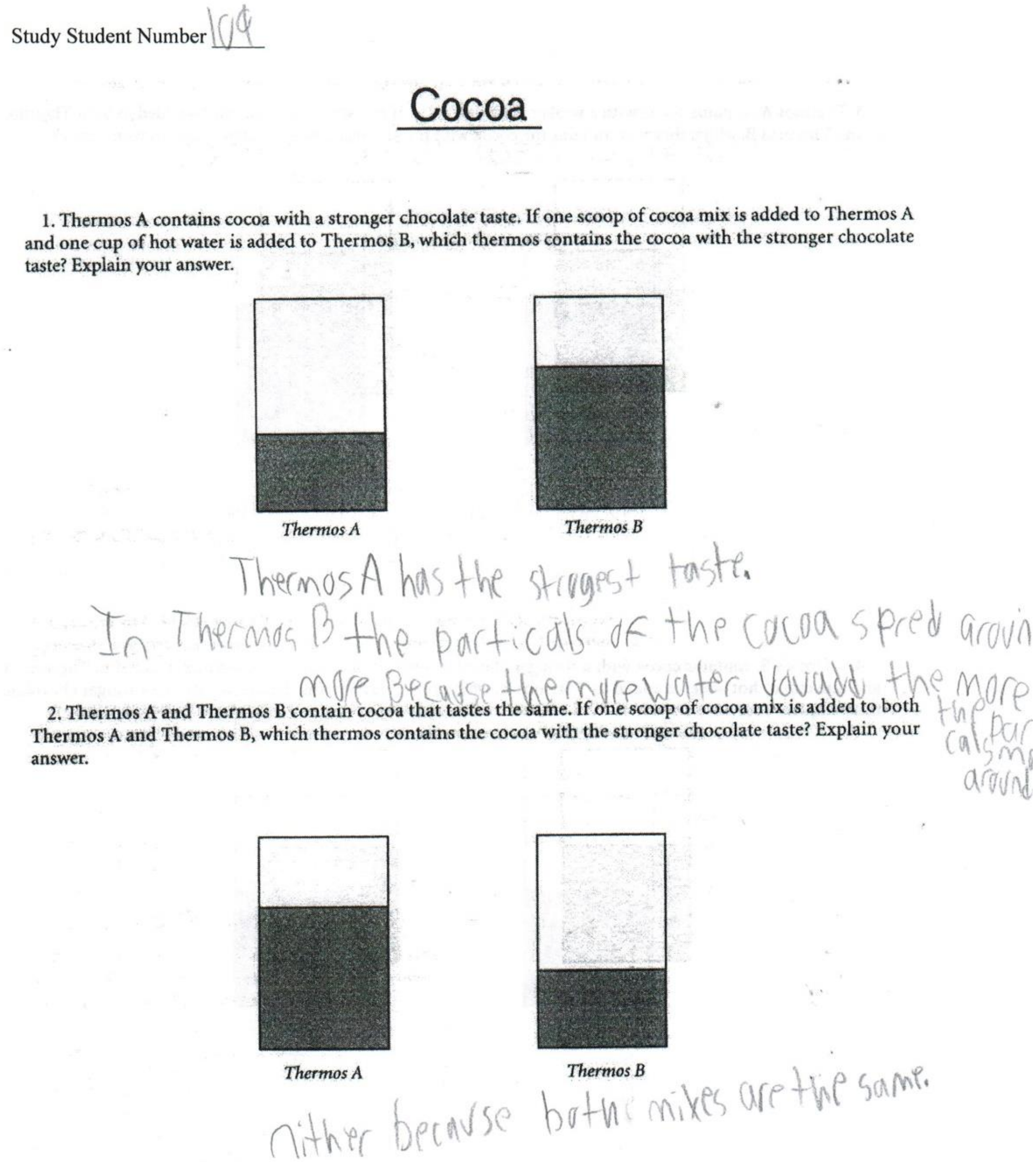
A

4. Explain how you found your answer.

30+45=75
4+45=85

Figure 35. Alice's work on John's School task.

Post-instruction interview. Completing the Cocoa task at the end of classroom instruction, Alice identified the correct thermos with the stronger chocolate taste twice out of the four problems presented. In question 1, she gave a response that indicated a sense of proportional reasoning when she said "the particles of the cocoa spread around more because the more water you add, the more the particles move around." Also, when asked if technology would have been of assistance to her in completing the Cocoa Task, Alice stated "No, not really, because it [the task] doesn't give you like a ratio." The correct use of academic vocabulary suggests an understanding of the term. Figures 36 and 37 provide Alice's work from the Cocoa task.



Study Student Number 109

Cocoa

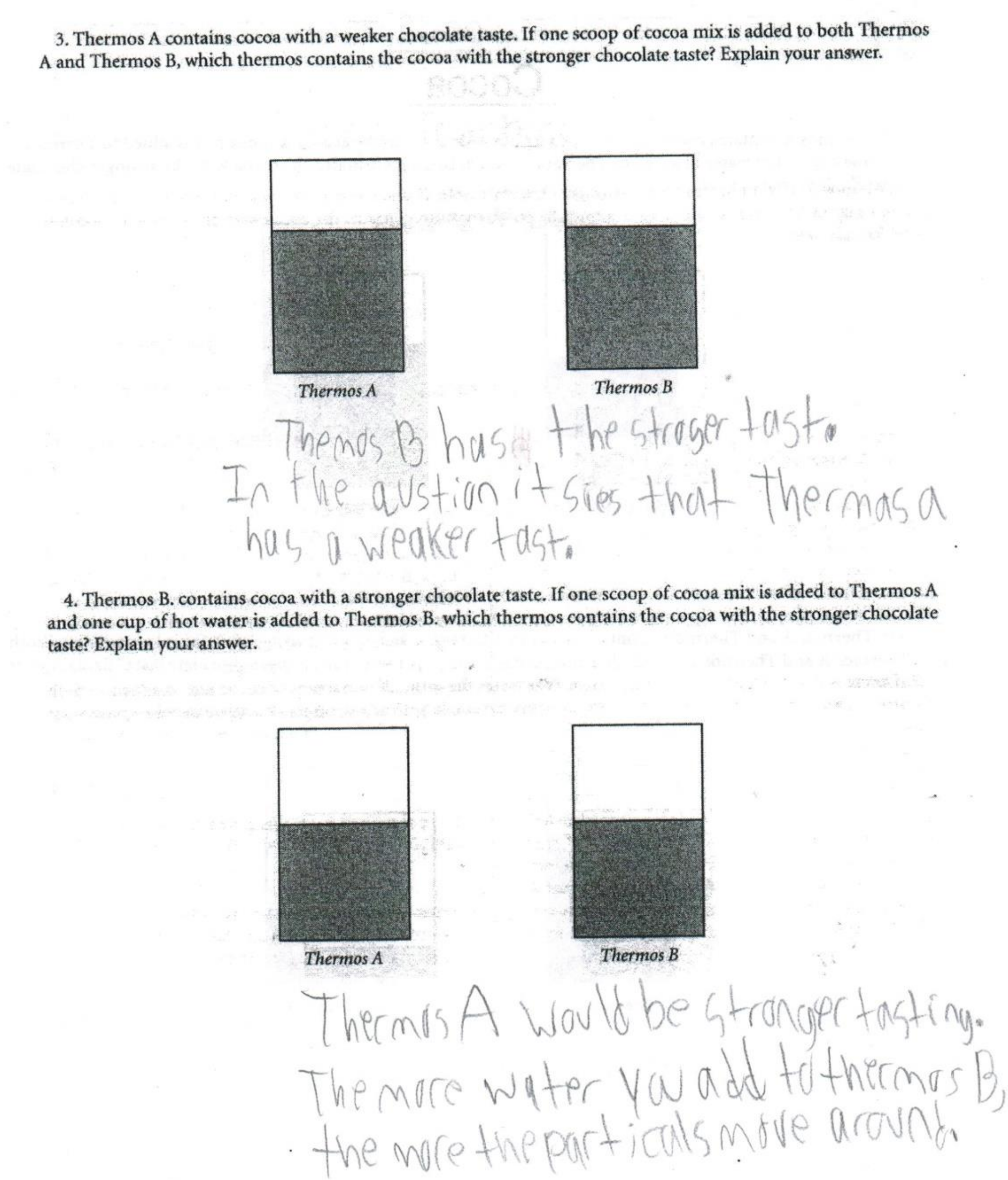
1. Thermos A contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Thermos A has the strogest taste.
In Thermos B the particals of the cocoa spred aroun more Because the more water you add the more the par cals m[ove] around

2. Thermos A and Thermos B contain cocoa that tastes the same. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

nither because both mixes are the same.

Figure 36. Alice’s work on Cocoa task, page 1.



3. Thermos A contains cocoa with a weaker chocolate taste. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Themos B has the stroger tast. In the qustion it sies that thermas a has a weaker tast.

4. Thermos B. contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B. which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Thermos A would be stronger tasting. The more water you add to thermos B, the more the particals move around.

Figure 37. Alice's work on Cocoa task, page 2.

Summary. Alice tended to work independently on class work during the instructional time. When given opportunity to collaborate with others, she seemed reluctant to speak out and contribute her thoughts or work to her table group. Alice did not indicate she encountered difficulty working with Thinking Blocks or Number Pieces. While completing the five tasks used in the research study, Alice's work was ranked at various levels of proportional reasoning; there was no perceived pattern of progression.

Alan

Background information. At the time of the study, Alan-an 11-year old male member of the iPad group- scored above the mean score on the pre-test. With respect to the state-mandated mathematics assessment, Alan scored Advanced as a grade five student. According to his responses on the technology use and preference survey, he had more than three years' experience working with touchscreen devices; just like Alice, Alan preferred to work with an iPad.

Pre-instruction interview. For the Make a New Puzzle task, Alan asked whether the square would have a hole in it of whether the pieces must connect; he was the only target student to ask this question concerning the puzzle. While measuring the puzzle pieces, Alan commented "It's not exactly, but its close." However, he acknowledged that the measures of the original puzzle pieces were accurate enough to form the square. To calculate the side lengths of the new puzzle pieces, Alan used an additive approach. When Alan attempted to assemble the new set of puzzle pieces into a square, he appeared hesitant and confused. When asked about this situation, Alan stated "…I might be putting them wrong, but I don't think they'll turn out to make a square." Although Alan experienced difficulty in trying to assemble the new puzzle pieces into a square, he did not attempt to blame his difficulty on any defect with the puzzle pieces. Figure 38 provides Alan's work on the Make a New Puzzle task.

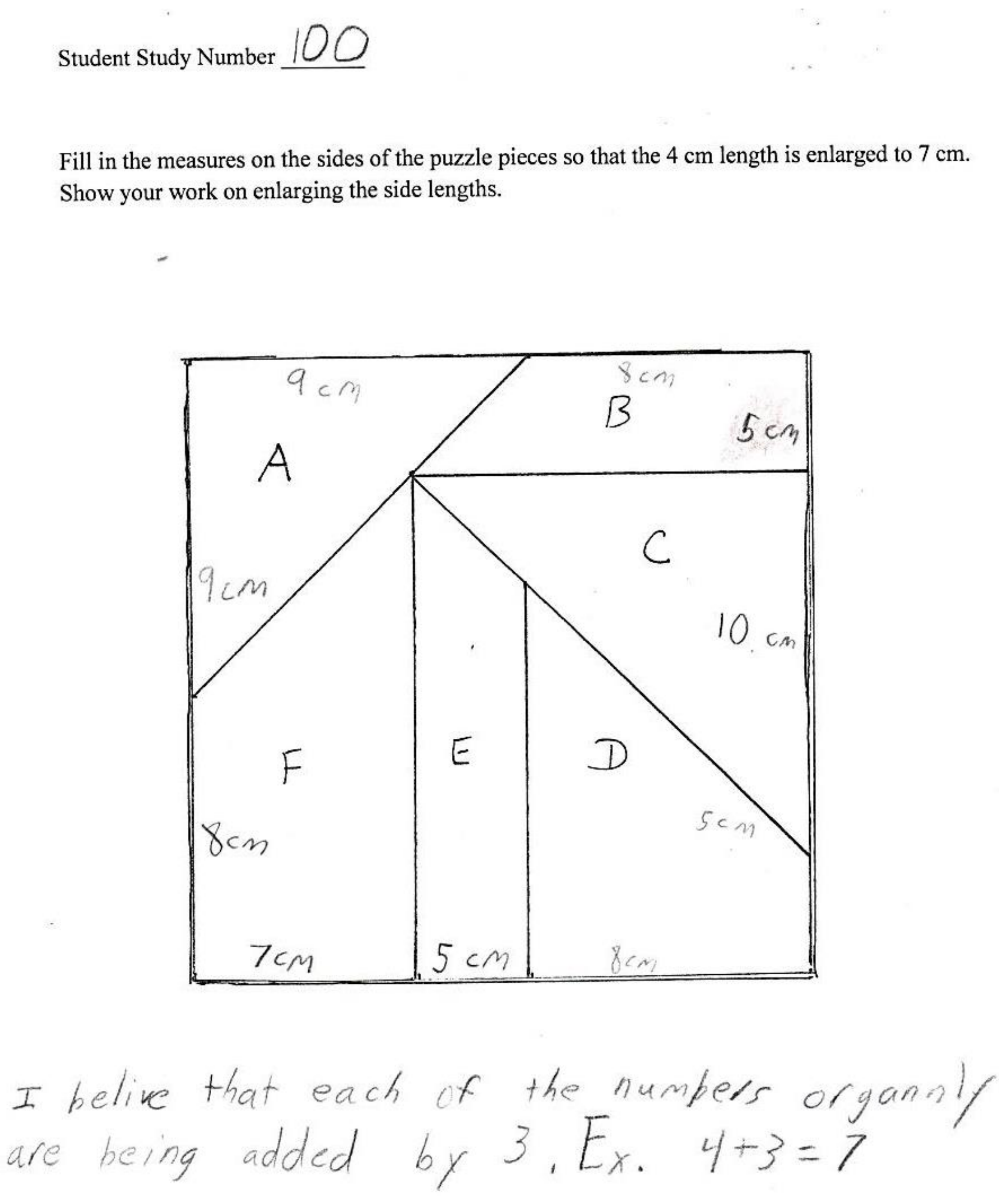
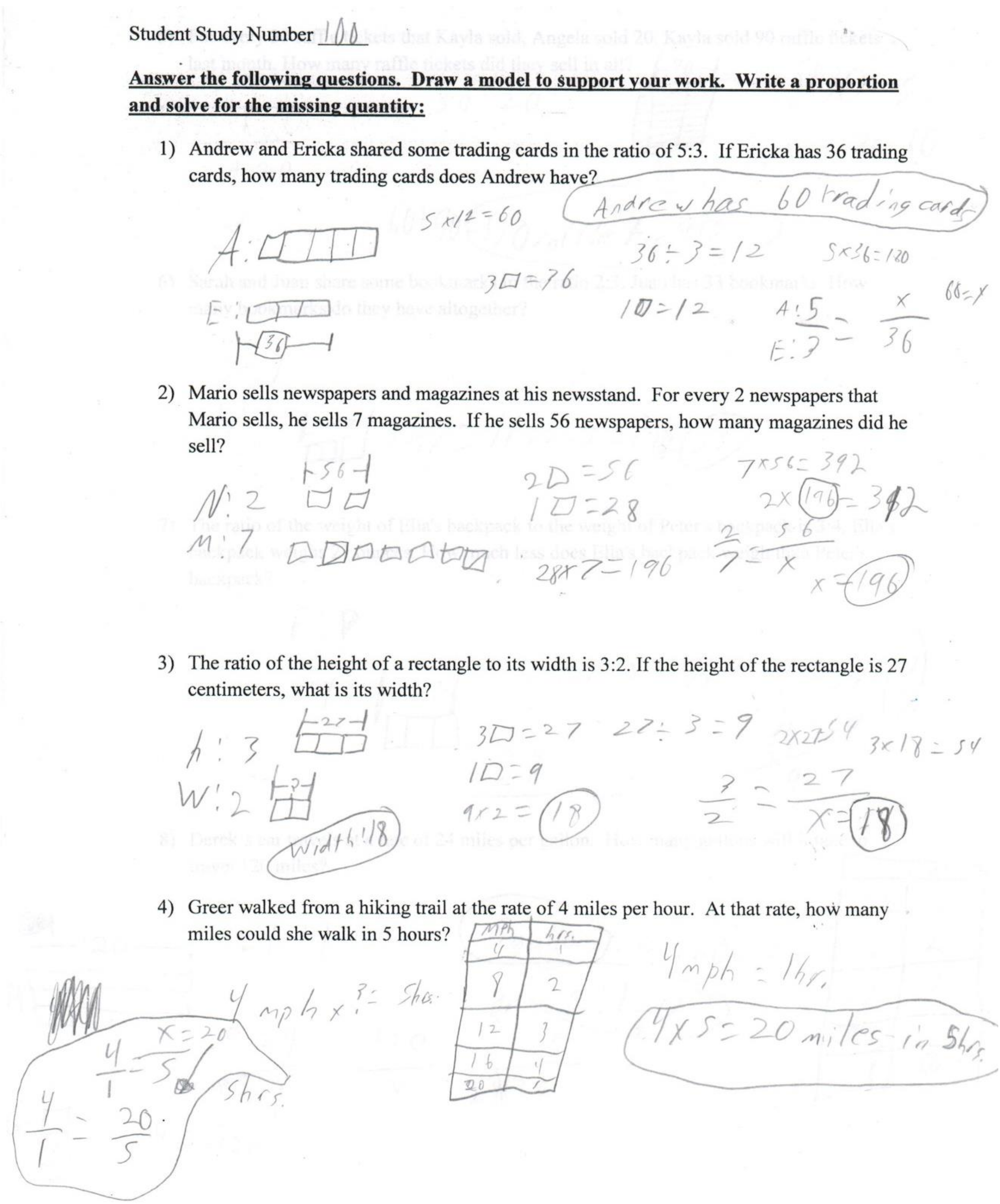


Figure 38. Alan's work on Make a New Puzzle task.

Classroom work with virtual manipulatives. At the onset of classroom instruction, Alan's work indicated he operated at the additive level of proportional reasoning. In fact, during classroom discussion on Day 2, Alan presented his explanation to the class as to why an additive reasoning approach should be used to find the height of Mr. Tall for the Mr. Tall/Mr. Short task. However, Alan started working with the Thinking Blocks virtual manipulative and completed the required modules ahead of most of the class; as a result, Alan began working on additional modules in the Thinking Blocks website.

When working with the Number Pieces website, Alan appeared to implement the block modeling strategy from the Thinking Blocks website as he completed the assigned problems. Figure 39 shows Alan's work from the first page of the assigned problems. Not only did Alan use the block modeling strategy, he also set up and solved proportions. In fact, on problem 4 in Figure 39, Alan used a table in answering the question. This use of different approaches when solving proportional reasoning problems indicated Alan developed proportional reasoning skills and operated at the ratio level.



Student Study Number 100

Answer the following questions. Draw a model to support your work. Write a proportion and solve for the missing quantity:

1) Andrew and Ericka shared some trading cards in the ratio of 5:3. If Ericka has 36 trading cards, how many trading cards does Andrew have?

2) Mario sells newspapers and magazines at his newsstand. For every 2 newspapers that Mario sells, he sells 7 magazines. If he sells 56 newspapers, how many magazines did he sell?

3) The ratio of the height of a rectangle to its width is 3:2. If the height of the rectangle is 27 centimeters, what is its width?

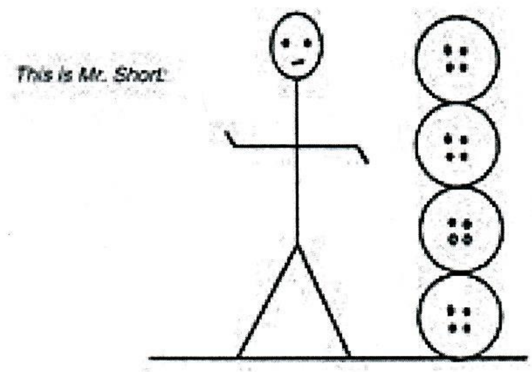
4) Greer walked from a hiking trail at the rate of 4 miles per hour. At that rate, how many miles could she walk in 5 hours?

Figure 39. Alan’s work from Number Pieces assignment.

Mr. Tall/Mr. Short task (1). As Alan completed the Mr. Tall/ Mr. Short task, he demonstrated work at the additive reasoning level. His response of 8 paper clips for the height of Mr. Tall, along with his explanation as to how he arrived at this answer, supported work at the additive level. Alan volunteered to share his explanation with the class concerning the height of Mr. Tall; however, after Ms. Xanth directed the participants to discuss their answers with their group, Alan changed his mind concerning the height of Mr. Tall. Alan's table partner during the task's discussion happened to be the one participant in the study who solved the Mr. Tall/Mr. Short task correctly. Apparently, this student convinced Alan that Mr. Tall was 9 paper clips tall. Also, Alan developed a table that expressed the unit rate between the buttons and paper clips used in the task; he was the only student during the study that implemented a table approach to find the height of Mr. Tall. Figures 40 and 41 show Alan's work on the Mr. Tall/Mr. Short task.

Study Student Number 100

A



The length of Mr. Short is 4 large buttons.

The length of Mr. Tall (not drawn) is 6 large buttons.

When paper clips are used to measure Mr. Short and Mr. Tall, the length of Mr. Short is 6 paper clips. What is the length of Mr. Tall in paper clips? 8 paper clips

Please EXPLAIN how you arrived at your answer.

I belive that when Mr. short's height goes from 4 buttons to 6 paperclips, you will add two to the original height.

Example:

Mr. short → regular: 4 buttons tall
New measurment: 6 paper clips tall

Mr. Tall → regular: 6 buttons tall
New measurment: 8 paper clips tall

Figure 40. Alan's work on Mr. Tall/Mr. Short task.

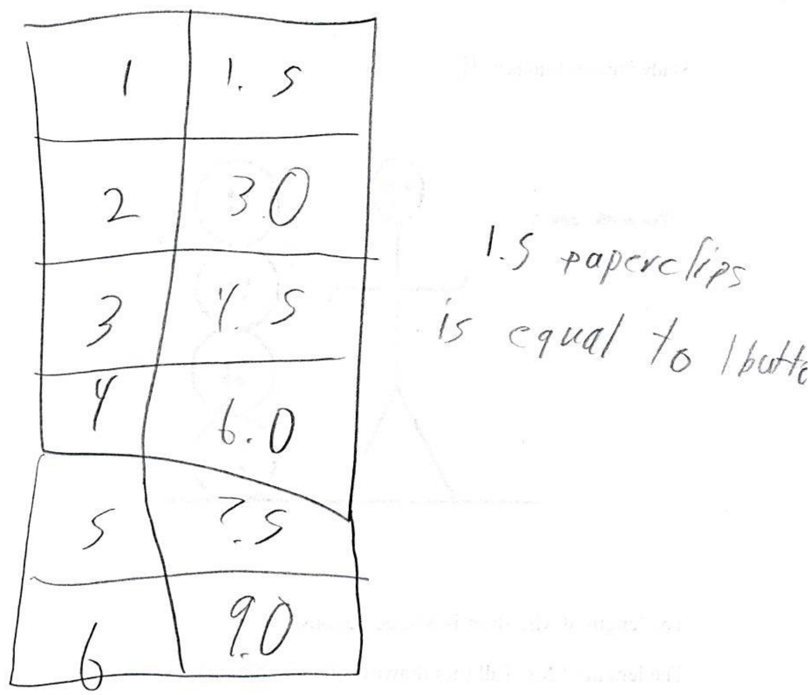
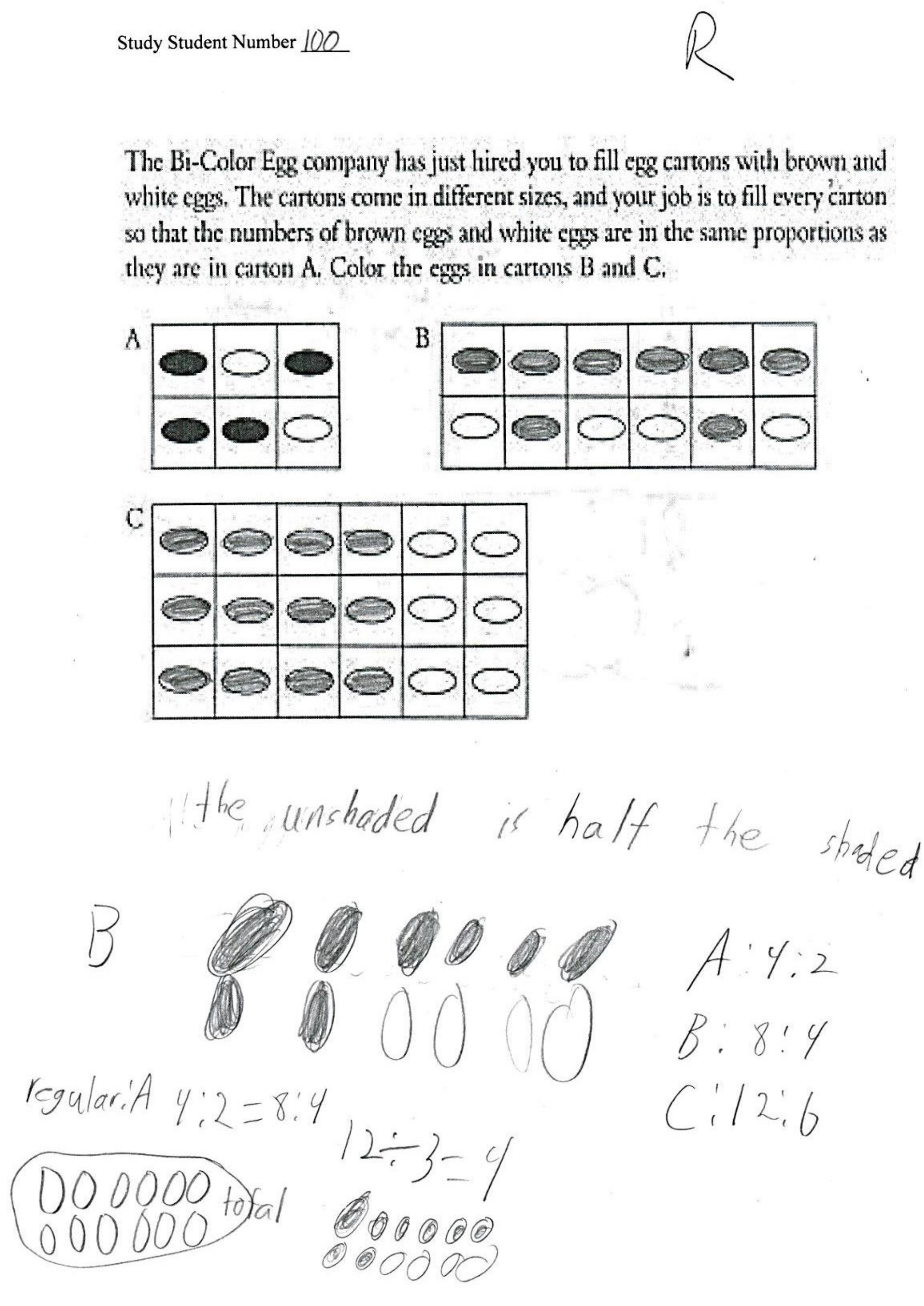


Figure 41. Alan's table after group discussion.

Egg Carton task (2). Alan completed the Egg Carton task correctly, coloring the brown eggs for Cartons B and C; however, he also replicated the contents of Carton B when trying to express a block model for the situation. As part of his work on this task, Alan expressed a ratio for each carton. In addition, he recognized the equivalence between the ratios for Cartons A and B. Finally, Alan also stated an overall comparison between the total number of brown eggs and the total number of white eggs. All of this work supported Alan's ratio approach for this task. Figure 42 displays Alan's work on the Egg Carton task.


Study Student Number 100

R

The Bi-Color Egg company has just hired you to fill egg cartons with brown and white eggs. The cartons come in different sizes, and your job is to fill every carton so that the numbers of brown eggs and white eggs are in the same proportions as they are in carton A. Color the eggs in cartons B and C.

A

B

C

the unshaded is half the shaded

B

A: 4:2

B: 8:4

C: 12:6

regular: A 4:2 = 8:4

12 ÷ 3 = 4

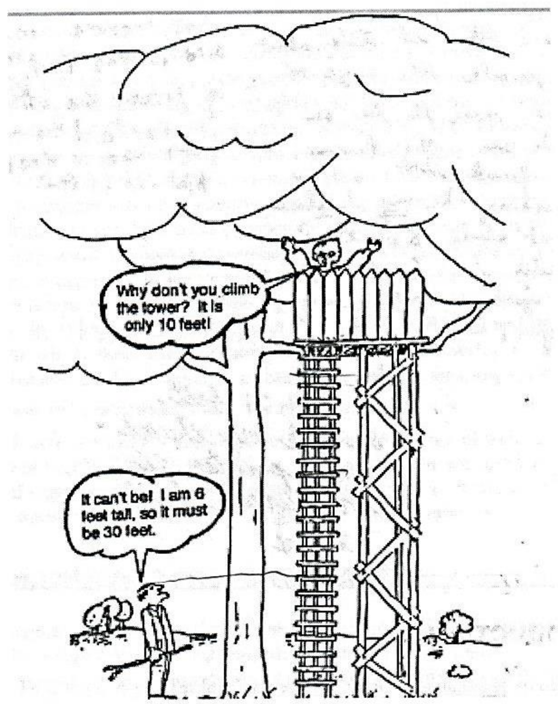
total

Figure 42. Alan’s work on Egg Carton task.

Tree House task (3). Alan completed the Tree House task and correctly determined the height of the ladder. His work indicated he disregarded information provided in the task that did not contribute pertinent data. Although he appeared to have a false start with counting the number of rungs, Alan's efforts demonstrated an understanding of the relationship between the number of rungs on the ladder and the height of the man on the ground. In light of the partitioning and building of values to find the height of the ladder, Alan appeared to be operating at the transitional level of proportional reasoning for this task. Figure 43 displays Alan's work on the Tree House task.

Study Student Number 100

Tr



Based upon the information provided in the drawing, how tall is the ladder? Show your work to support your answer.

18 feet

8:6
16:12
24:18

$\frac{6}{1} \times \frac{23}{6} = \frac{138}{6} = 23$

6 feet tall

Est: 30

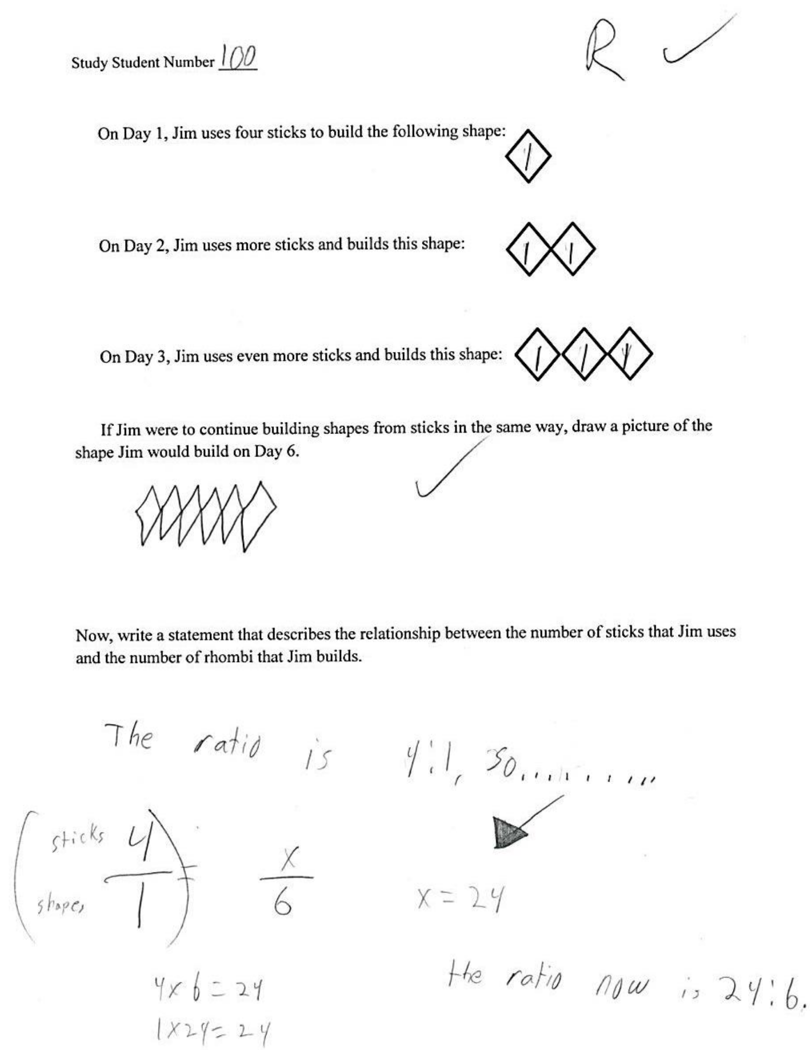
~~23 feet high~~

23 steps

Man is 6 feet tall

Figure 43. Alan’s work on Tree House task.

Sticks and Rhombi task (4). For this task, Alan drew the shape for Day 6 correctly; in addition, his statement relating the number of sticks to the number of rhombi accurately described the relationship. In his work, Alan expressed ratios in two different forms: using a colon and using a fraction. Further, Alan provided the specific ratio which represented the shape constructed on Day 6. Figure 44 shows Alan's work on the Sticks and Rhombi task.


Study Student Number 100

R ✓

On Day 1, Jim uses four sticks to build the following shape:

On Day 2, Jim uses more sticks and builds this shape:

On Day 3, Jim uses even more sticks and builds this shape:

If Jim were to continue building shapes from sticks in the same way, draw a picture of the shape Jim would build on Day 6.

Now, write a statement that describes the relationship between the number of sticks that Jim uses and the number of rhombi that Jim builds.

The ratio is 4:1, so.........

sticks 4 / shapes 1 = x / 6

x = 24

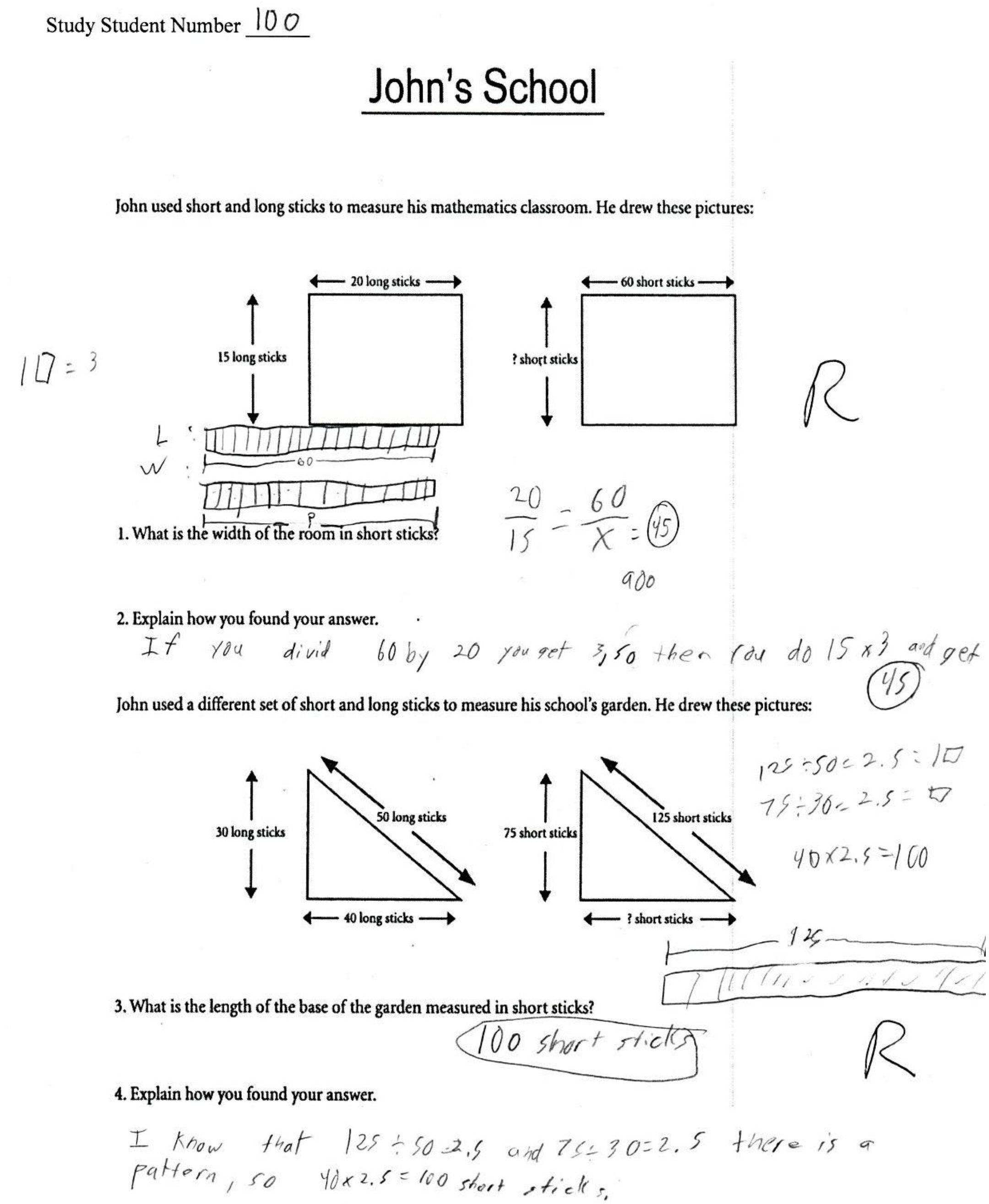
4x6=24

1x24=24

the ratio now is 24:6.

Figure 44. Alan's work on Sticks and Rhombi task.

John's School task (5). Prior to taking the post-test, Alan answered both questions for John's School task correctly. In finding the missing side lengths, Alan incorporated a visual approach with the block modeling strategy studied during classroom instruction. He identified the scale factor used to enlarge the values from one shape to another for each question by using division and labeled the unit value for a single block. For question 1, Alan used a proportion to solve for the missing side length, but he choose to express his work on question 2 by multiplying the unit value with the lengths of the sides measured in long sticks to find the missing length in short sticks. Unlike his work on the Tree House task, Alan did not employ a partitioning approach to find the missing side lengths. The answers provided for each question, along with the supporting work, indicated Alan functioned at the ratio level for this task. Figure 45 gives Alan's work on the John's School task.



Study Student Number 100

John's School

John used short and long sticks to measure his mathematics classroom. He drew these pictures:

1. What is the width of the room in short sticks?

$\frac{20}{15} = \frac{60}{X} = 45$

900

2. Explain how you found your answer.

If you divid 60 by 20 you get 3, 50 then you do 15 x 3 and get 45

John used a different set of short and long sticks to measure his school's garden. He drew these pictures:

125 ÷ 50 = 2.5

75 ÷ 30 = 2.5

40 x 2.5 = 100

3. What is the length of the base of the garden measured in short sticks?

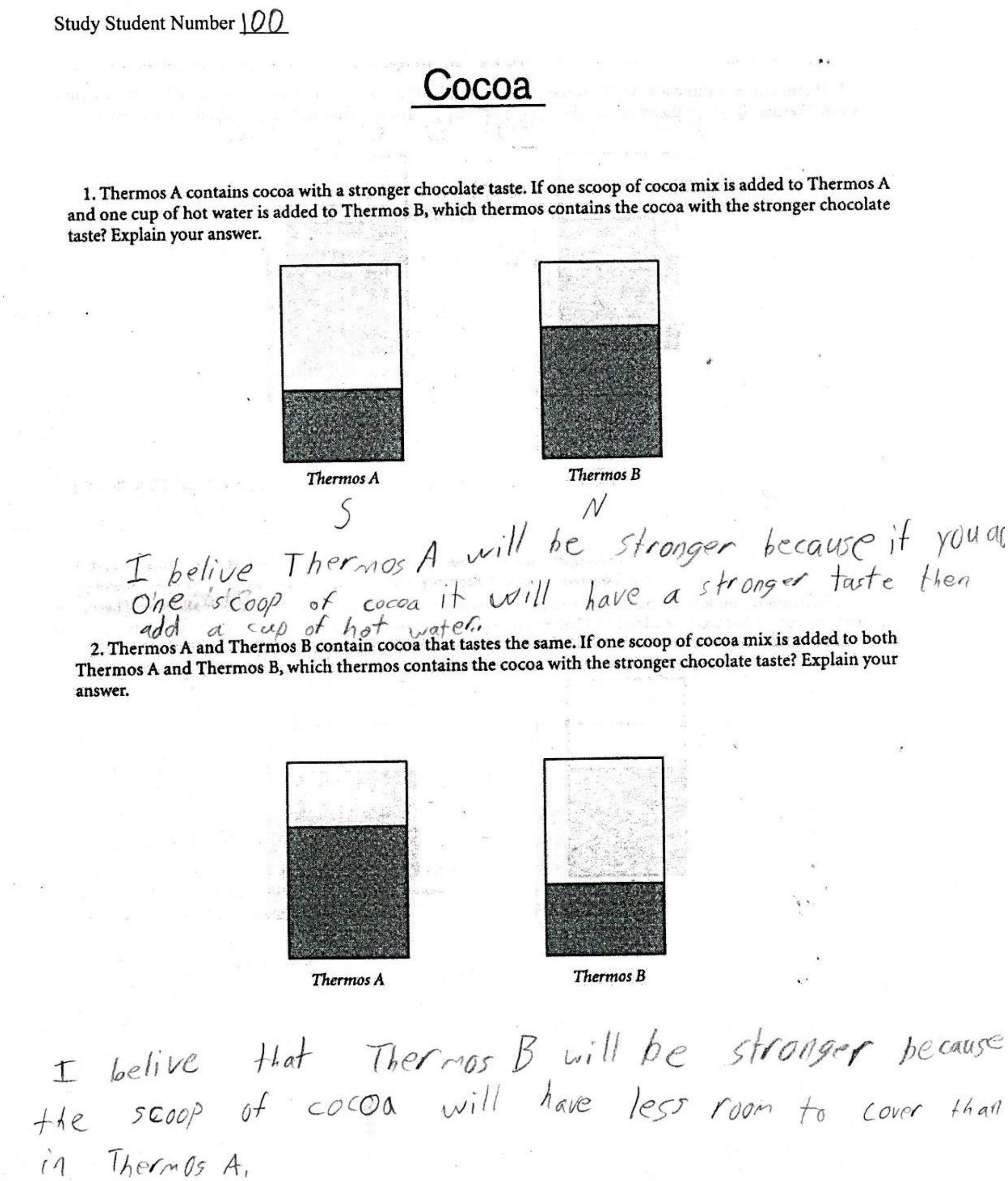
100 short sticks

4. Explain how you found your answer.

I know that 125 ÷ 50 = 2.5 and 75 ÷ 30 = 2.5 there is a pattern, so 40 x 2.5 = 100 short sticks.

Figure 45. Alan's work on John's School task.

Post-instruction interview. While completing the Cocoa task, Alan answered three out of the four questions correctly. For the fourth problem, he recognized something different occurred; Alan asked if only one thermos was the answer or could both thermoses be the answer. Although he recognized the difference, Alan did not answer question four correctly. During the task, Alan commented on the relative strength and weakness of the cocoa, which indicated an understanding of the ratio between the cocoa mix and the water. For instance, Alan stated "…if you add one scoop of cocoa, it will have a stronger taste because the chocolate mixed with the cocoa just means more chocolate. It will have more stronger taste than just adding a cup of hot water." On question 2, Alan commented "I believe that Thermos B will be stronger because the scoop of cocoa will have less room to cover than in Thermos A." When asked if technology could have helped him in completing the Cocoa task, Alan indicated technology could be helpful, but he didn't know how to answer these questions using technology since he couldn't draw a model or do a proportion with these questions. He further described these conditions as the "arithmetic of a problem." Figures 46 and 47 show Alan's work on the Cocoa task.



Study Student Number 100

Cocoa

1. Thermos A contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Thermos A Thermos B

S N

I belive Thermos A will be stronger because if you ad One scoop of cocoa it will have a stronger taste then add a cup of hot water.

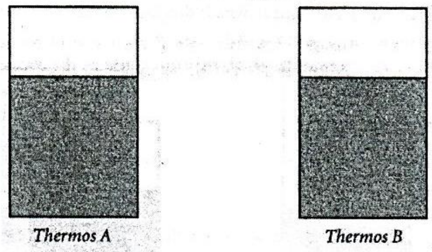
2. Thermos A and Thermos B contain cocoa that tastes the same. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Thermos A Thermos B

I belive that Thermos B will be stronger because the scoop of cocoa will have less room to cover than in Thermos A.

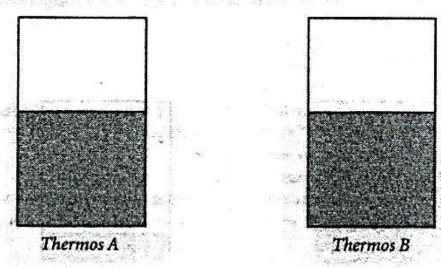
Figure 46. Alan’s work on Cocoa task, page 1.

3. Thermos A contains cocoa with a weaker chocolate taste. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.



I belive that Thermos B will be stronger because it tell us that Thermos A has a weaker chocolate taste, so if you add eve more cocoa to Thermas B it will be stronger.

4. Thermos B. contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B. which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.



I belive that both Thermos A and B will aproximitly the same taste because it tells us that Thermos B is stronger but than Thermos A, but when you put a cup of hot water in Thermos B and put a scoop of cocoa mix in Thermos A they will be about the same.

Figure 47. Alan’s work on Cocoa task, page 2.

Summary. Alan demonstrated that he could work both independently and collaboratively. He most often worked with a particular student at his table, but he was also willing to work with other students in his vicinity and other students in the classroom. When given the opportunity, Alan volunteered information and was willing to share his work with the class. After the class had discussed the Mr. Tall/Mr. Short task in small groups, Alan presented a solution in which he developed a table that related the number of buttons to the number of paper clips. He was the only student who implemented a table to show this particular relationship. During class instruction throughout the study, Alan demonstrated understanding of proportional reasoning concepts; for instance, he recognized 25 miles per gallon as a ratio when other participants did not. Also, Alan commented that equal proportions may be reduced to the same equivalent fraction. When completing the five classroom tasks, Alan progressed from an additive approach to a ratio approach.

Candy

Background information. At the time of the study, Candy - an 11-year old female member of the Mouse group - scored above the mean score on the pre-test. With respect to the state-mandated mathematics assessment, Candy scored Advanced as a grade five student. According to her responses on the technology use and preference survey, she had less than one year's experience working with touchscreen devices. Candy indicated that she prefers to work with an iPad.

Pre-instruction interview. When completing the Make a New Puzzle task, Candy assembled the original puzzle pieces into a square; she was one of only two target students who accomplished this assembly. Although she had no difficulty in measuring the side lengths of puzzle pieces to confirm the accuracy of the measurements, Candy expressed confusion as to what was expected in order to enlarge the original measurements to form the side lengths for the new set of puzzle pieces: "Oh, you're going to multiply it by, cause you multiplied by, you added three to four. So, you're going to make this [two] a five?" However, once she understood the directions, she completed the task by using an additive approach.

When invited to assemble the new puzzle pieces together to form a square, Candy seemed confused, looking back and forth from her model to the puzzle pieces. When asked about this situation, Candy said "It doesn't look like this one [puzzle piece] fits there." Figure 48 shows Candy attempting to assemble the new set of puzzle pieces into a square. During the interview, Candy asked more clarifying questions concerning the activity when compared to the other target students. For instance, Candy asked these questions during the part of the interview when she was to determine the side lengths of the new puzzle pieces:

- Can I add lengths together?
- You add three to it?
- You add it on to the seven?

Figure 49 shows Candy's finished work on the Make a New Puzzle task.

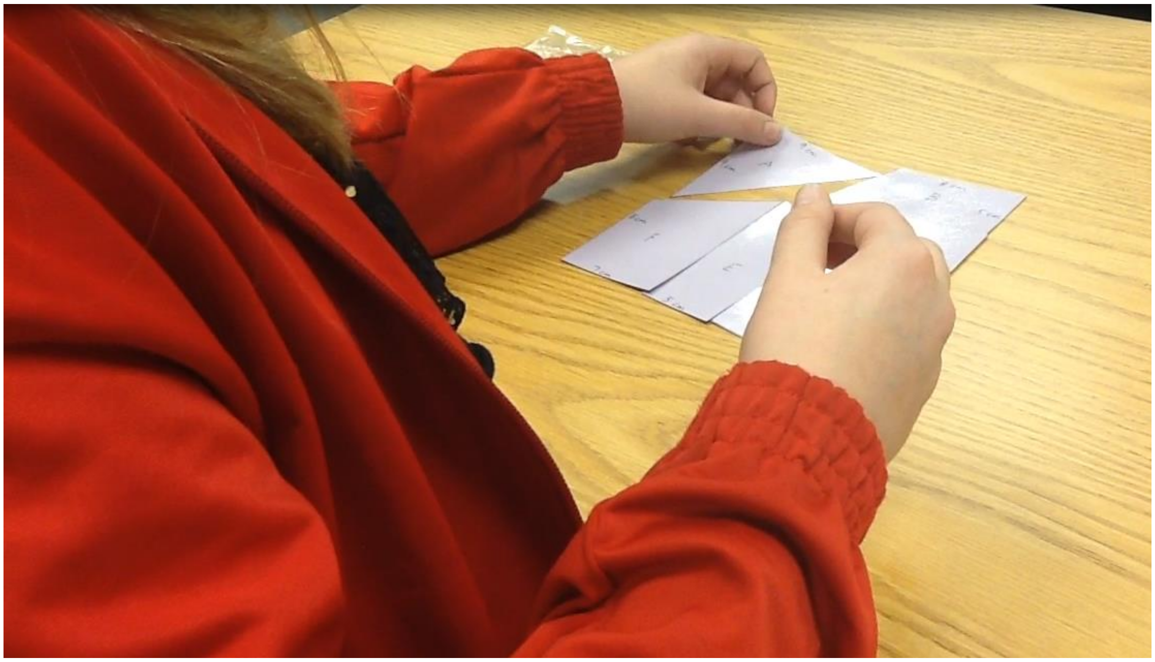
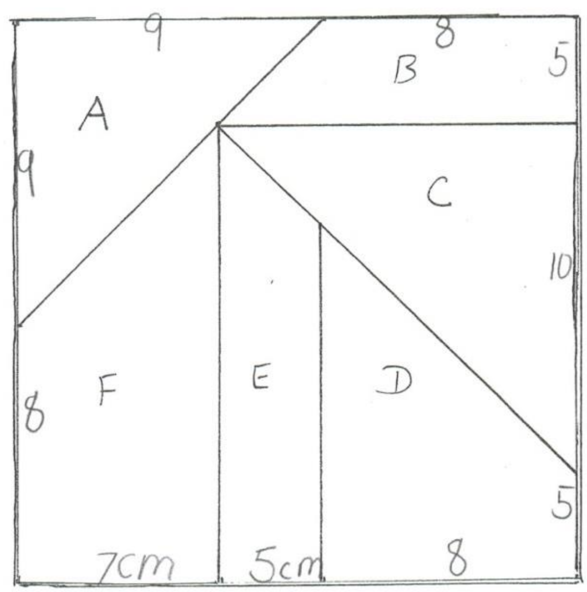


Figure 48. Candy assembling puzzle pieces.

Student Study Number 305

Fill in the measures on the sides of the puzzle pieces so that the 4 cm length is enlarged to 7 cm.
Show your work on enlarging the side lengths.



I added 3cm to make the 4 become a 7 and did that same thing to the other numbers.

Figure 49. Candy's Make a New Puzzle task.

Classroom work with virtual manipulatives. During the course of instruction while working with Thinking Blocks, Candy initially demonstrated a preference for the use of repeated addition when using the block modeling strategy; that is, as she worked with the word problems, Candy labeled her blocks with the unit value along with plus signs. This use of partitioning indicated that Candy functioned at a transitional level during this work; a recreation of Candy's work is found in Figure 50. As Candy continued her classroom work throughout the study, she started to implement a multiplicative approach when working with the block modeling strategy on the Number Pieces website. On Day 3 of this study, Candy's work consisted of repeated addition of the same value. By Day 7, Candy's work employed multiplication to solve the word problems. On Day 9, Candy used proportions to solve the two questions from John's School task.

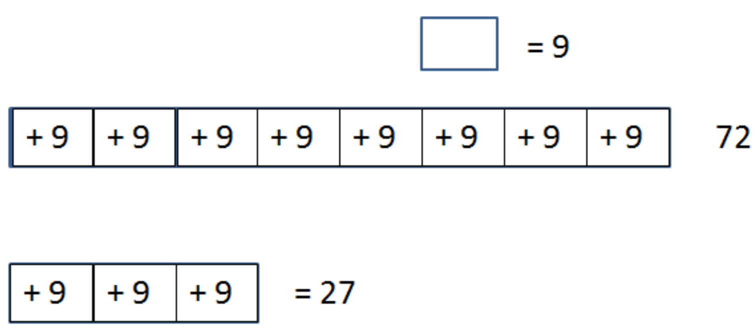
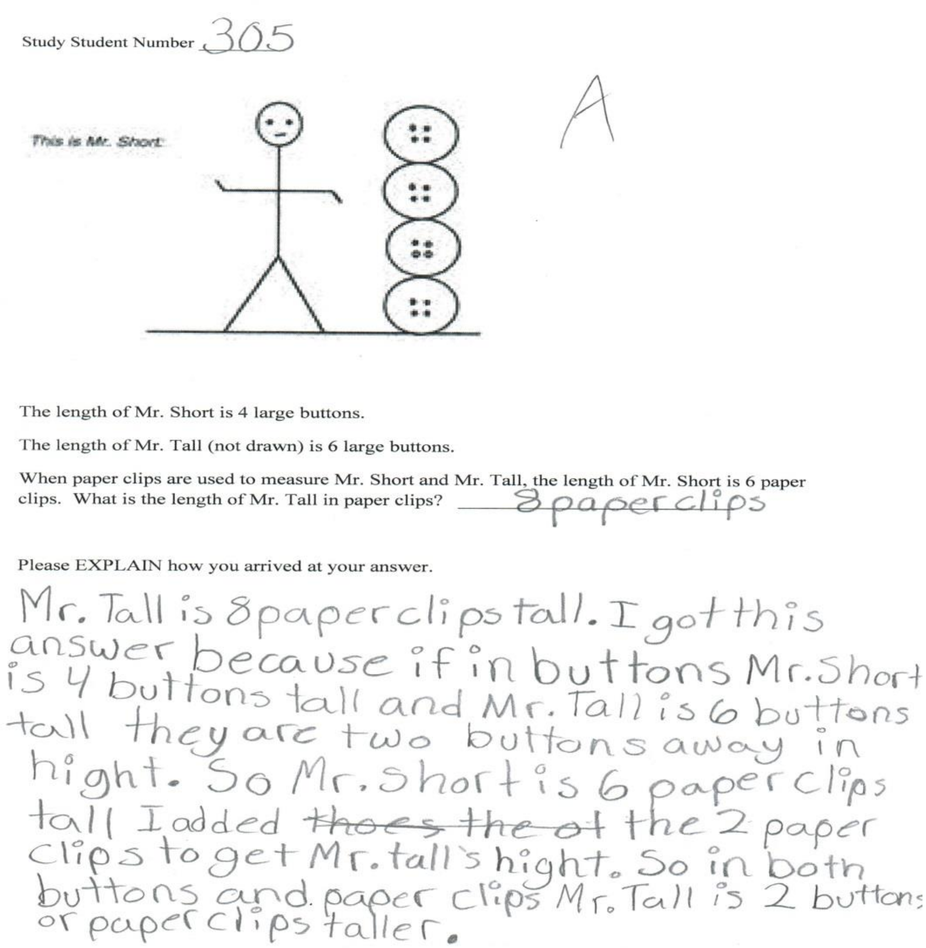


Figure 50. Candy's partitioning work sample.

Mr. Tall/Mr. Short task (1). While completing the Mr. Tall/Mr. Short task, Candy demonstrated work at the additive reasoning level. Candy responded 8 paper clips tall when asked the height of Mr. Tall in paper clips; this response supports the application of additive reasoning. Also, Candy's explanation as to how she determined the height of Mr. Tall indicated a use of additive reasoning when she wrote, "I added the 2 paper clips to get Mr. tall's hight." Candy's work on the Mr. Tall/Mr. Short task appears in Figure 51.


Study Student Number 305

A

This is Mr. Short

The length of Mr. Short is 4 large buttons.

The length of Mr. Tall (not drawn) is 6 large buttons.

When paper clips are used to measure Mr. Short and Mr. Tall, the length of Mr. Short is 6 paper clips. What is the length of Mr. Tall in paper clips? 8paperclips

Please EXPLAIN how you arrived at your answer.

Mr. Tall is 8paperclips tall. I got this answer because if in buttons Mr.Short is 4 buttons tall and Mr. Tall is 6 buttons tall they are two buttons away in hight. So Mr. Short is 6 paper clips tall I added ~~thoes the of~~ the 2 paper clips to get Mr. tall's hight. So in both buttons and paper clips Mr. Tall is 2 buttons or paper clips taller.

Figure 51. Candy's work on Mr. Tall/Mr. Short task.

Egg Carton task (2). As Candy completed the Egg Carton task, she used a visual approach to complete the task. The use of a visual approach suggested she had incorporated the block modeling strategy presented during class instruction. Candy ultimately shaded in correctly four brown eggs for Carton B (8:4 ratio) and she correctly shaded in 12 brown eggs for Carton C (12:6 ratio). Figure 52 shows Candy's work on the Egg Carton task.

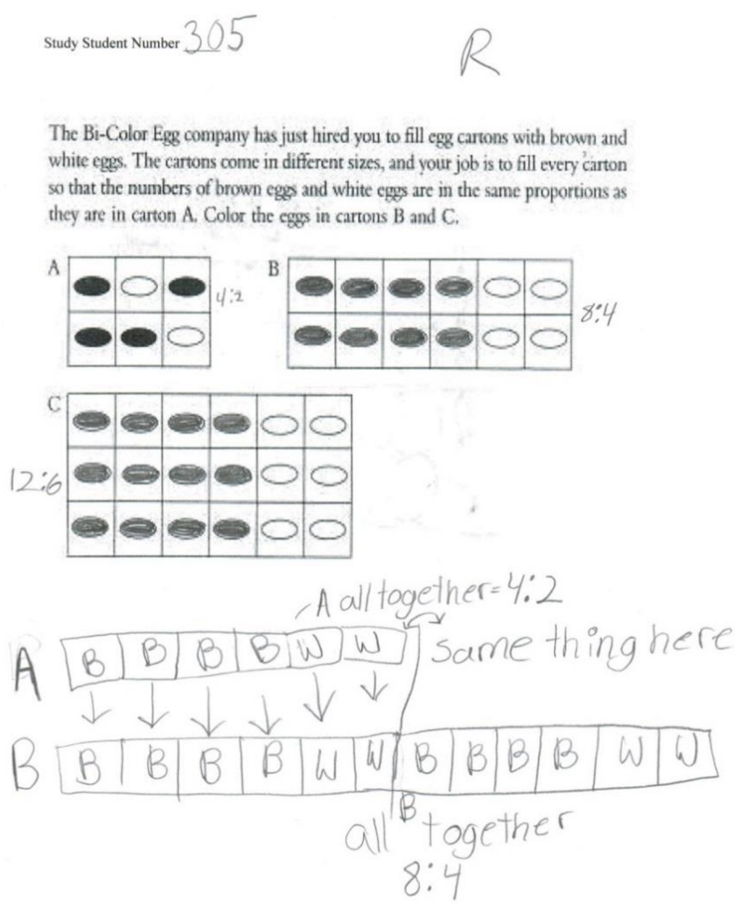


Figure 52. Candy's work on Egg Carton task.

Tree House task (3). Candy completed the Tree House task as directed by Ms. Xanth. During her work, Candy appeared to identify the information needed to complete the task, working quietly without assistance. An examination of Candy's work showed a transitional approach to completing the Tree House task. Although the use of addition may suggest the work to be appraised at the additive level, a closer look indicates that Candy partitioned the work into repeated additions. Since Candy added an additional 3 feet instead of adding the same amount each time implied she may have miscounted the number of rungs on the ladder and attempted to compensate. Figure 53 displays Candy's work on the Tree House task.

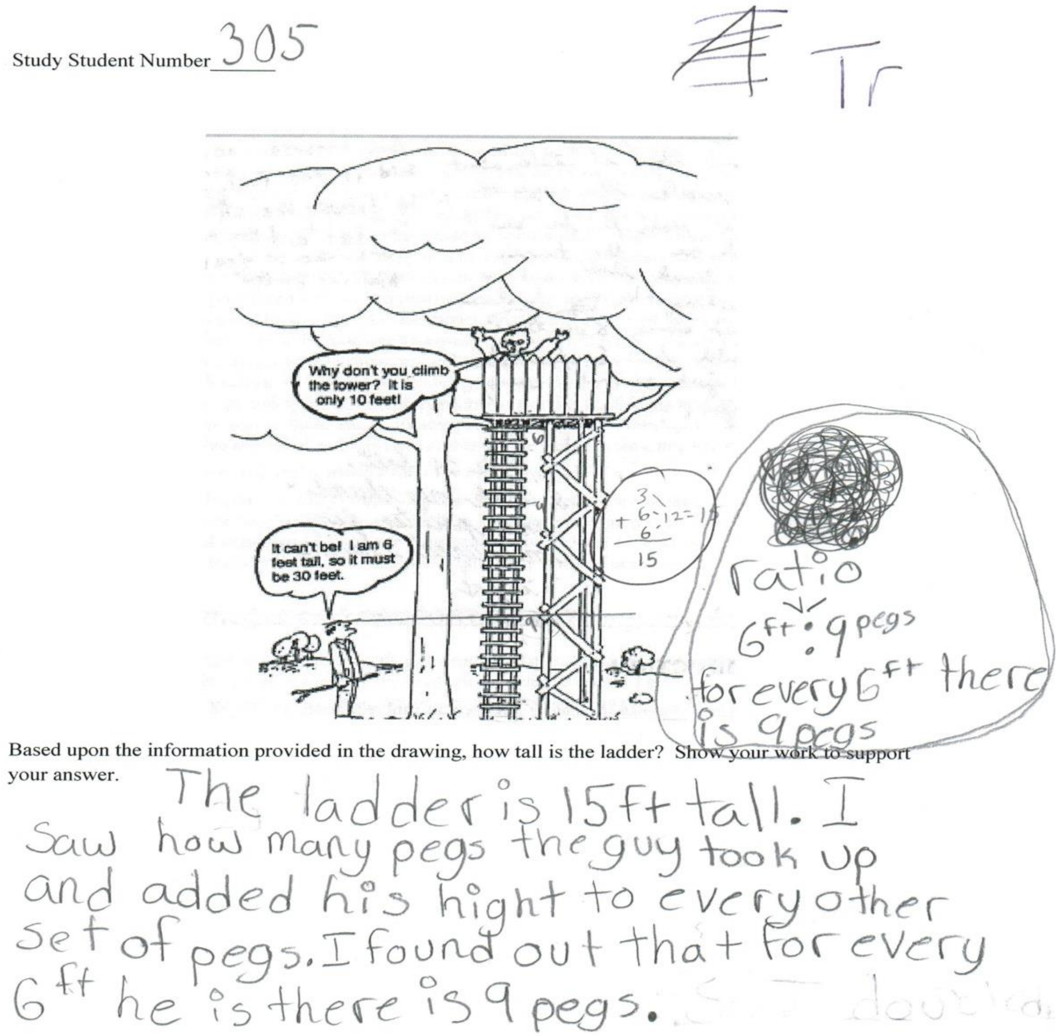
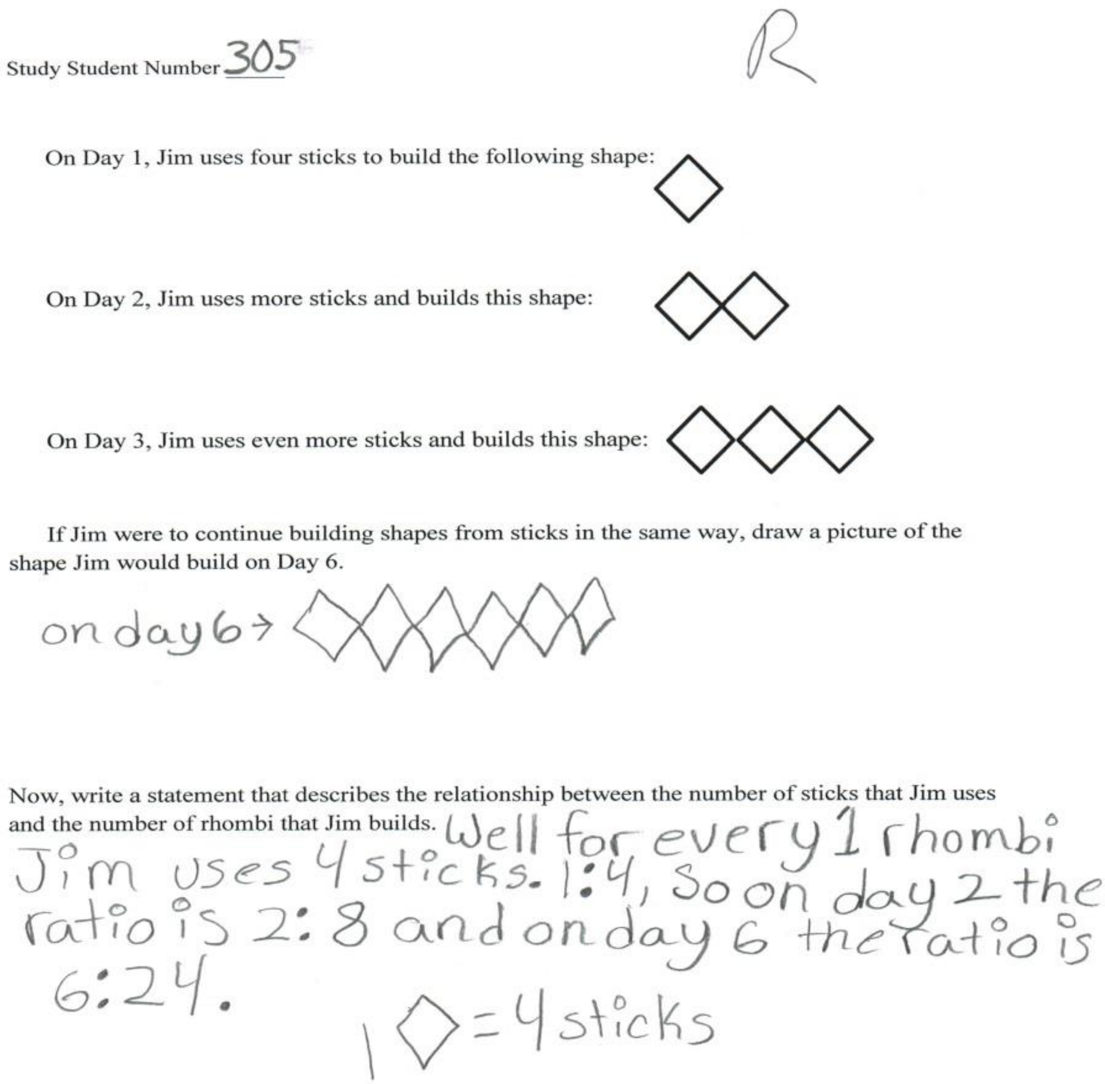


Figure 53. Candy's work on Tree House task.

Sticks and Rhombi task (4). Candy correctly completed the Sticks and Rhombi task without any indication that she encountered difficulty. The completed work for this task is shown in Figure 54. When Ms. Xanth facilitated discussion after participants completed the Sticks and Rhombi task, Candy stated, "For every one rhombus, there are four sticks." This use of ratio indicated Candy understood the comparative relationship present in the task.



Study Student Number 305

R

On Day 1, Jim uses four sticks to build the following shape:

On Day 2, Jim uses more sticks and builds this shape:

On Day 3, Jim uses even more sticks and builds this shape:

If Jim were to continue building shapes from sticks in the same way, draw a picture of the shape Jim would build on Day 6.

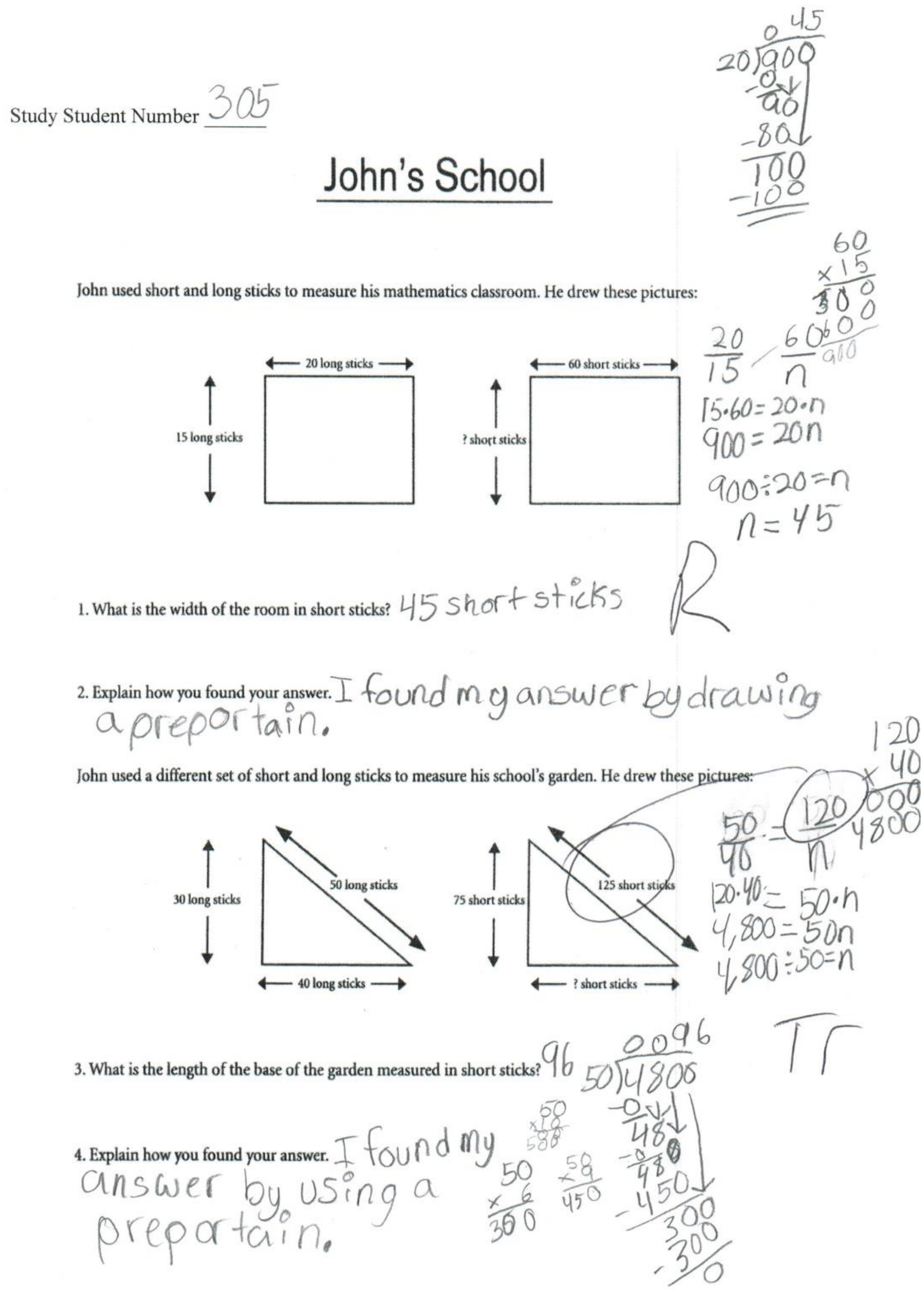
on day 6 →

Now, write a statement that describes the relationship between the number of sticks that Jim uses and the number of rhombi that Jim builds. Well for every 1 rhombi Jim uses 4 sticks. 1:4, So on day 2 the ratio is 2:8 and on day 6 the ratio is 6:24.

1 ◇ = 4 sticks

Figure 54. Candy's work on Sticks and Rhombi task.

John's School task (5). Participants completed John's School task prior to the administration of the post-test. Candy completed both questions from the task without any request for clarification or assistance. Candy applied the process of setting up and solving a proportion, which strategy Ms. Xanth introduced to her students after they worked four days with the Thinking Blocks virtual manipulative. For question 1, Candy constructed the proportion correctly and solved for the missing side length; however, for question 2, Candy miscopied the value from the drawing, using 120 instead of 125 as the numerator in the ratio, which affected her response for the missing side length. For question 1, Candy's work presented evidence of ratio level, while the work on question 2 was rated as transitional level. One could argue Candy's work for question 2 represented effort at the ratio level, especially since she employed a proportion and solved for the missing side length. However, since Candy's answer resulted in an incorrect value, her work appraised at the transitional level. Figure 55 displays Candy's work on John's School task.


Study Student Number 305

John's School

John used short and long sticks to measure his mathematics classroom. He drew these pictures:

1. What is the width of the room in short sticks? 45 short sticks

2. Explain how you found your answer. I found my answer by drawing a preportain.

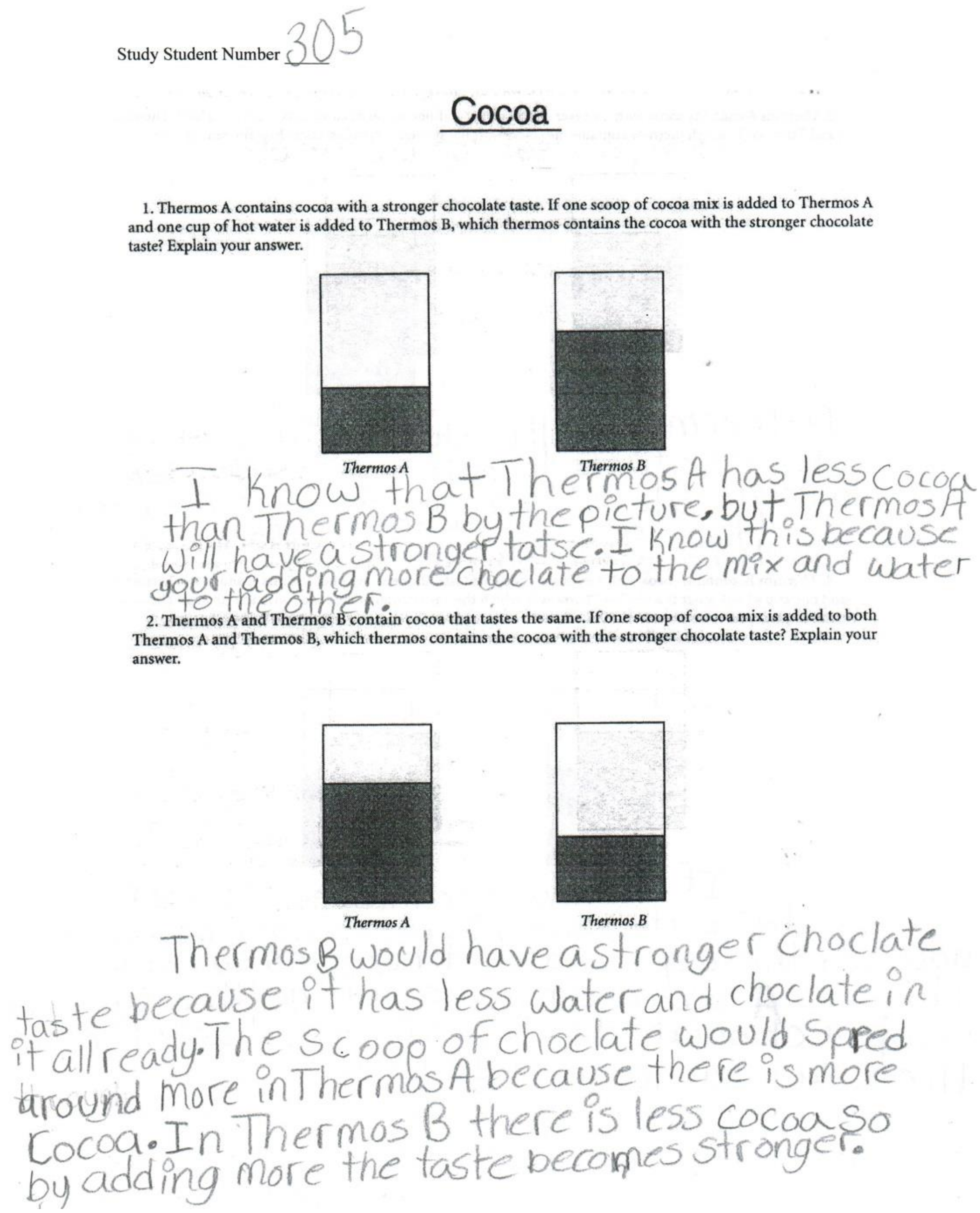
John used a different set of short and long sticks to measure his school's garden. He drew these pictures:

3. What is the length of the base of the garden measured in short sticks? 96

4. Explain how you found your answer. I found my answer by using a prepartain.

Figure 55. Candy's work on John's School task.

Post-instruction interview. On the Cocoa task, Candy answered three questions correctly out of the four questions presented; additionally, Candy's explanations demonstrated development of proportional reasoning skills, although she did not consistently employ academic vocabulary in her explanations. For instance, when describing why Thermos A had a stronger chocolate taste in question one, she said "I know how to solve it but I don't know how you do it involving ratios, though." During her explanation for her choice of thermos, Candy stated "…there is less water and stuff in there, and when you are adding just water to Thermos B, the chocolate taste is going away, is spreading out with the more water." On question four, Candy recognized that there was something different about the situation, but she assumed that the addition of cocoa mix to Thermos A and adding hot water to Thermos B generated the same chocolate taste in each thermos. When asked about the use of technology to answer the questions on the Cocoa task, Candy replied "It might have [helped] if I knew how to answer it, but I didn't know how to use Thinking Blocks or anything like that." Figures 56 and 57 show Candy's work on the Cocoa task.



Study Student Number 305

Cocoa

1. Thermos A contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Thermos A Thermos B

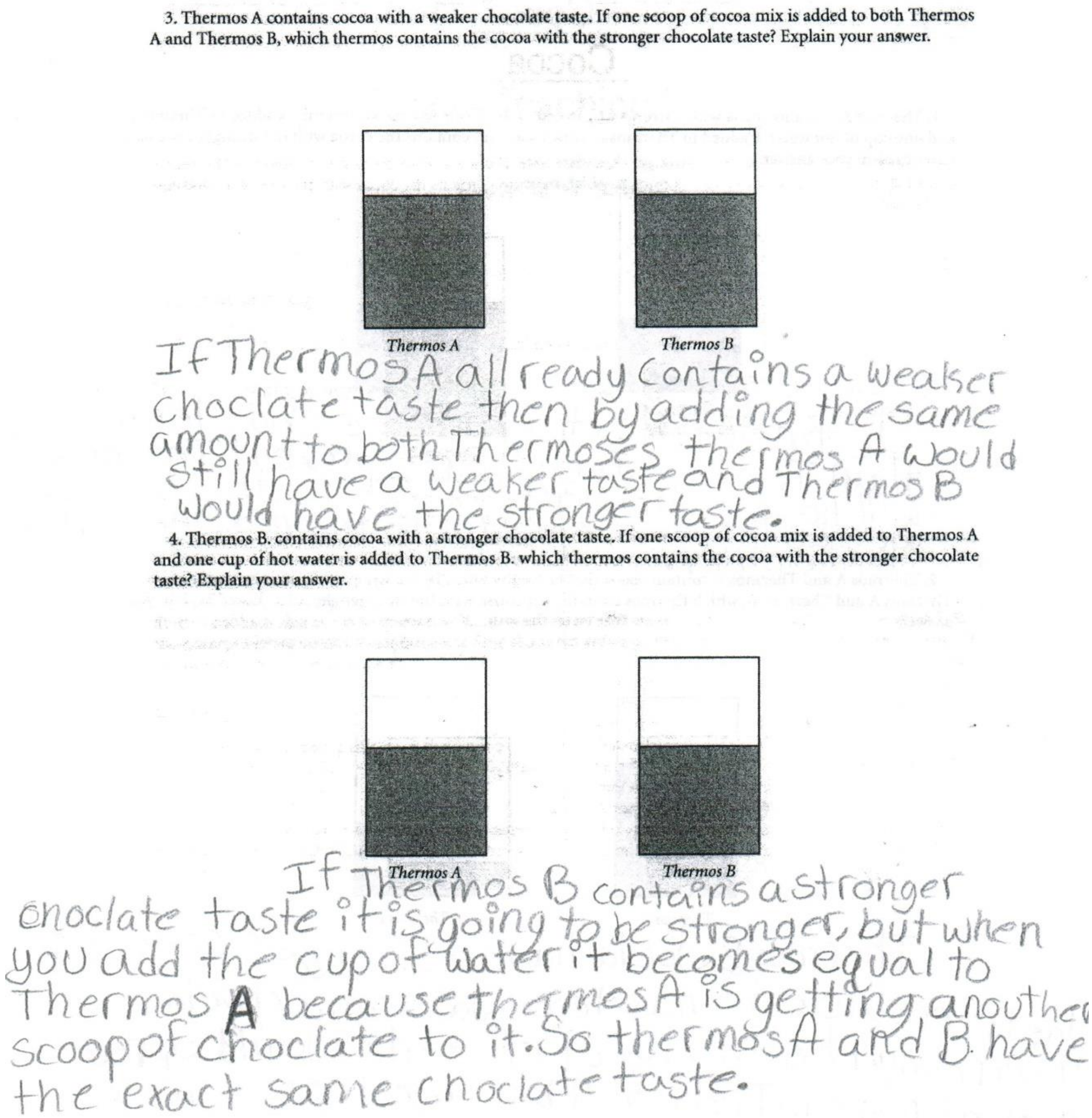
I know that Thermos A has less cocoa than Thermos B by the picture, but Thermos A will have a stronger tatse. I know this because your adding more choclate to the mix and water to the other.

2. Thermos A and Thermos B contain cocoa that tastes the same. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Thermos A Thermos B

Thermos B would have a stronger choclate taste because it has less water and choclate in it allready. The scoop of choclate would spred around more in Thermos A because there is more cocoa. In Thermos B there is less cocoa so by adding more the taste becomes stronger.

Figure 56. Candy's work on Cocoa task, page 1.



3. Thermos A contains cocoa with a weaker chocolate taste. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

If Thermos A all ready contains a weaker choclate taste then by adding the same amount to both Thermoses thermos A would still have a weaker taste and Thermos B would have the stronger taste.

4. Thermos B. contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B. which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

If Thermos B contains a stronger choclate taste it is going to be stronger, but when you add the cup of water it becomes equal to Thermos A because thermos A is getting anouther scoop of choclate to it. So thermos A and B have the exact same choclate taste.

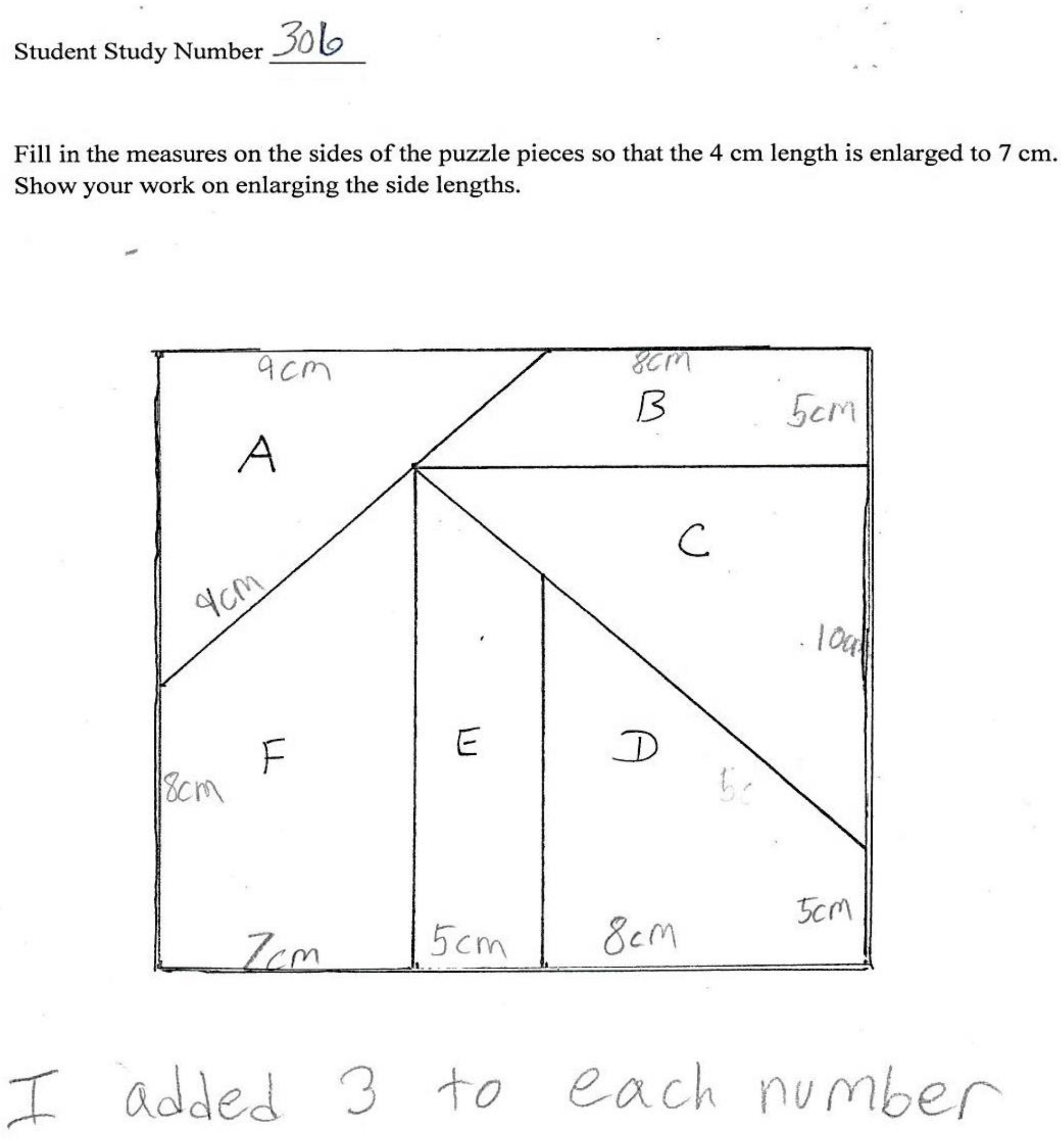
Figure 57. Candy's work on Cocoa task, page 2.

Summary. Candy demonstrated independent work during class instruction, asking relatively few questions of Ms. Xanth. With respect to the use of Thinking Blocks and Number Pieces, Candy appeared to use these virtual manipulatives successfully. Also, Candy evidenced development of proportional reasoning during the study. For instance, as students answered review questions for Ms. Xanth during class, Candy recognized 4 miles per hour as a ratio when other participants did not. Finally, Candy expressed a basic understanding of proportional reasoning in both mathematical symbol and language used to explain her work on several tasks. For instance, on the Egg Carton task, Candy wrote equivalent ratios of brown eggs to white eggs for each carton. On the Tree House task, Candy stated "for every 6 ft he is, there is 9 pegs." For the Sticks and Rhombi task, she wrote "for every 1 rhombi Jim uses 4 sticks." These expressions support the idea that Candy experienced development of proportional reasoning. The work samples generated by Candy as she completed the five tasks indicated a tendency to function at the transitional level of proportional reasoning development, although she demonstrated some work at the ratio level.

Carl

Background information. At the time of the study, Carl - an 11-year old male member of the Mouse group - scored below the mean score on the pre-test. With respect to the state-mandated mathematics assessment, Carl scored Proficient as a grade five student. According to his responses on the technology use and preference survey, he had more than three years' experience working with touchscreen devices; like the rest of the target students, Carl preferred to work with an iPad.

Pre-instruction interview. While working with the Make a New Puzzle task, Carl worked quietly as he completed the task. To calculate the side lengths of the new puzzle pieces, he used an additive approach. Prior to completing his calculations, Carl seemed confused concerning the completion of the task. While trying to assemble the new pieces into a square, Carl hesitated and glanced at the model for direction. When measuring the puzzle pieces, he indicated that most of the measures matched those written for the original side lengths, claiming "that one's a little bit off", referring to a segment five centimeters in length, and "…the numbers are almost all the same." Carl concluded that the new puzzle pieces would not form a square when assembled. Carl's work on the Make a New Puzzle task appears in Figure 58.


Student Study Number 306

Fill in the measures on the sides of the puzzle pieces so that the 4 cm length is enlarged to 7 cm.
Show your work on enlarging the side lengths.

I added 3 to each number

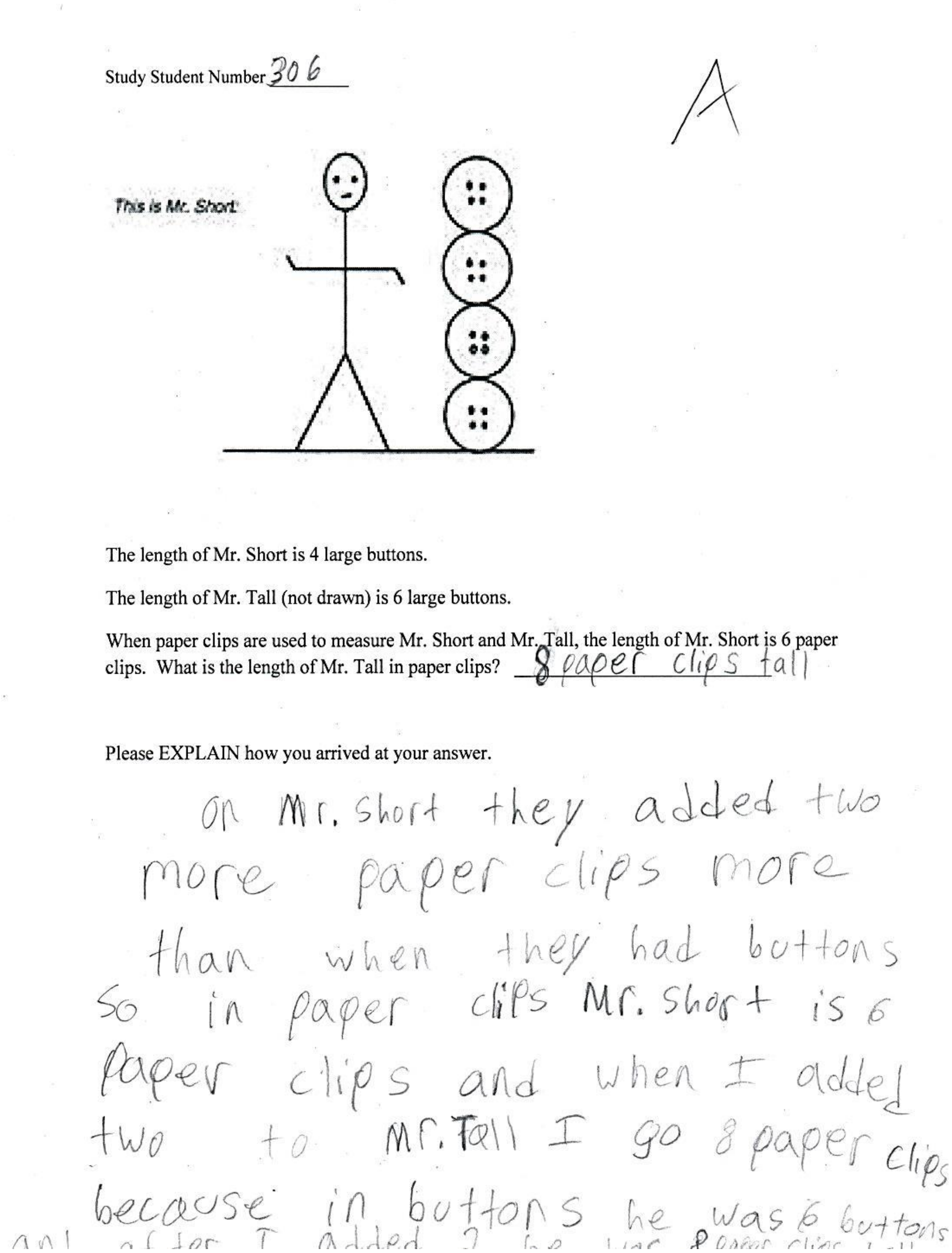
Figure 58. Carl’s work on Make a New Puzzle task.

Classroom work with virtual manipulatives. During the instructional phase of this study, Carl demonstrated a willingness to participate in class, either when he had answers to share or when he had a question for Ms. Xanth to consider. Carl completed more questions in the Thinking Blocks modules than Ms. Xanth requested, even to the point that he kept working when Ms. Xanth directed students to stop their work for the day. This willingness to work with Thinking Blocks suggested Carl enjoyed working with the virtual manipulative.

As Carl worked with Number Pieces, he encountered difficulty in constructing the block models; this difficulty presented itself in the work of other participants as well. One possibility explaining his difficulty is the lack of support in the Number Pieces virtual manipulative as compared to the Thinking Blocks website. As participants completed word problems in the Thinking Blocks setting, the virtual manipulative confirmed their work. The objective in confirming this work is the development of building and applying block models in a proportional reasoning setting. However, if participants worked through the problems in a perfunctory manner, the development of the block modeling strategy did not occur. Without the step-by-step confirmation in the Number Pieces website, participants who have not developed the use of the block modeling strategy may be at a loss. So, it is possible for students to complete the work in the Thinking Blocks environment and not carry over learning to the Number Pieces environment. Figure 59 shows Carl working with the Number Pieces virtual manipulative. As seen in the picture, Carl used gestures at times while working with the Number Pieces website.

Figure 59. Carl's work in Number Pieces.

Mr. Tall./Mr. Short task (1). Carl demonstrated work at the additive level as he completed the Mr. Tall/ Mr. Short task. His response of 8 paper clips for the height of Mr. Tall resulted from adding two to the height of Mr. Short, as evidenced in Carl's explanation as to how he found the height of Mr. Tall. Carl stated "On Mr. Short, they added two more paper clips more than when they had buttons." Figure 60 shows Carl's work on the Mr. Tall/ Mr. Short task.


Study Student Number 306

A

This is Mr. Short

The length of Mr. Short is 4 large buttons.

The length of Mr. Tall (not drawn) is 6 large buttons.

When paper clips are used to measure Mr. Short and Mr. Tall, the length of Mr. Short is 6 paper clips. What is the length of Mr. Tall in paper clips? 8 paper clips tall

Please EXPLAIN how you arrived at your answer.

On Mr. Short they added two more paper clips more than when they had buttons So in paper clips Mr. Short is 6 paper clips and when I addel two to Mr. Tall I go 8 paper clips because in buttons he was 6 buttons

Figure 60. Carl's work on Mr. Tall/Mr. Short task.

Egg Carton task (2). Carl completed the Egg Carton task without any indication that he encountered difficulty; however, Carl's thought concerning the relationship between the three cartons appeared incomplete. Although he colored in the correct number of brown eggs for each carton, he used an equal sign between the three cartons. The use of the equal sign indicated an understanding concerning the equivalent values of the ratios for each carton. During the class discussion, Carl stated he used the ratio 2:3 on the Egg Carton task; however, the ratio 2:3 did not appear in his work sample. Further, the statement that the cartons were equal disregarded the actual number of eggs found in each carton. Also, Carl added the number of brown eggs for each carton and found 24 brown eggs were present; he did the same type of addition for white eggs and obtained 12 white eggs. This action appeared irrelevant to the completion of the task. Figure 61 shows Carl's work on the Egg Carton task.

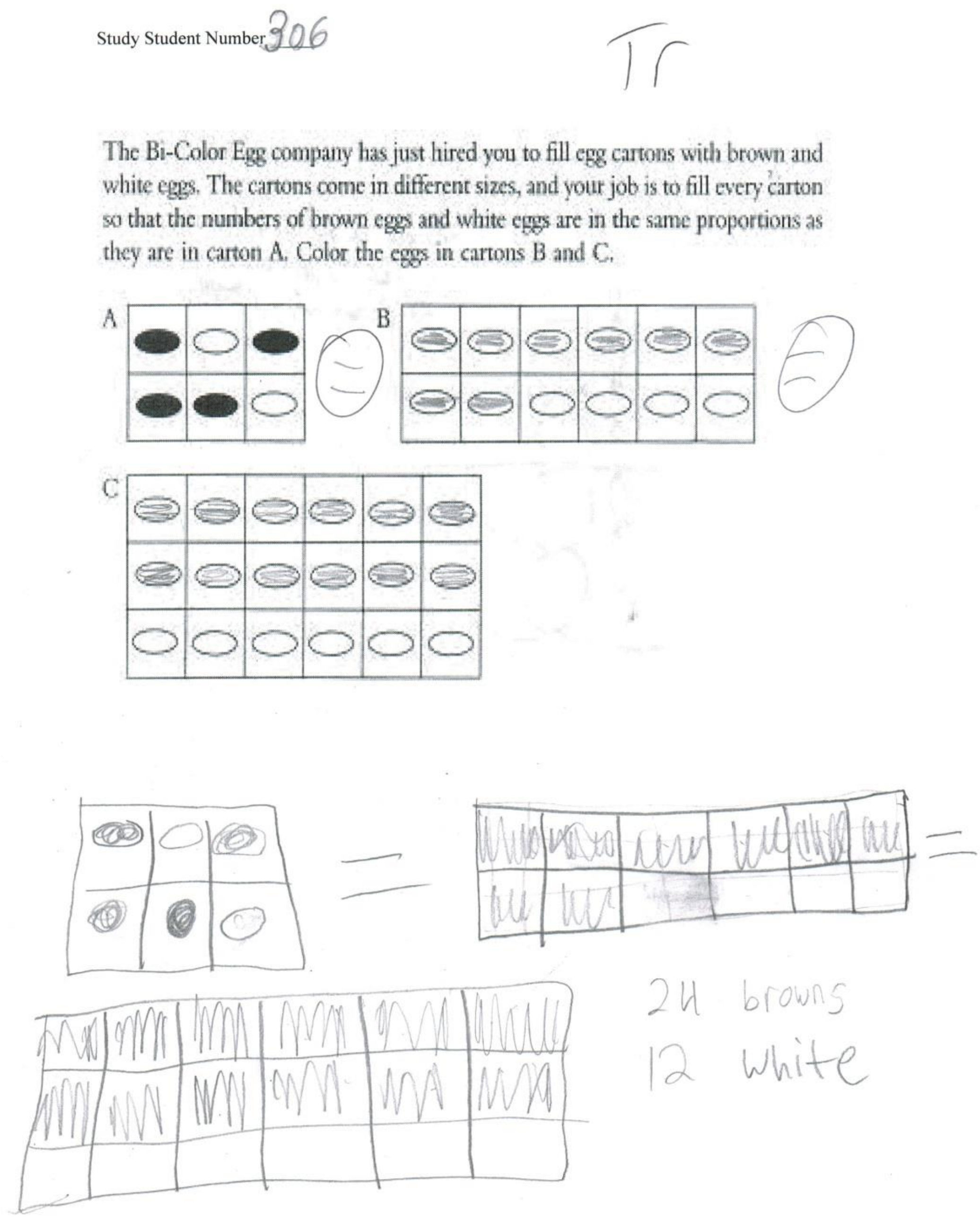


Figure 61. Carl’s work on Egg Carton task.

Tree House task (3). Carl did not complete the Tree House task. Ms. Xanth recognized that Carl encountered difficulty while attempting to complete the task. She hinted that some information presented in the problem may not be used in finding the height of the ladder. Although Carl acknowledged that 10 ft was not reasonable in light of the height of the man standing on the ground, he was not able to use the information provided in the task to determine the height of the ladder. Figure 62 shows what Carl produced on the Tree House task. Carl attempted to partition the ladder into equal sets of rungs, but he did not act upon this grouping.

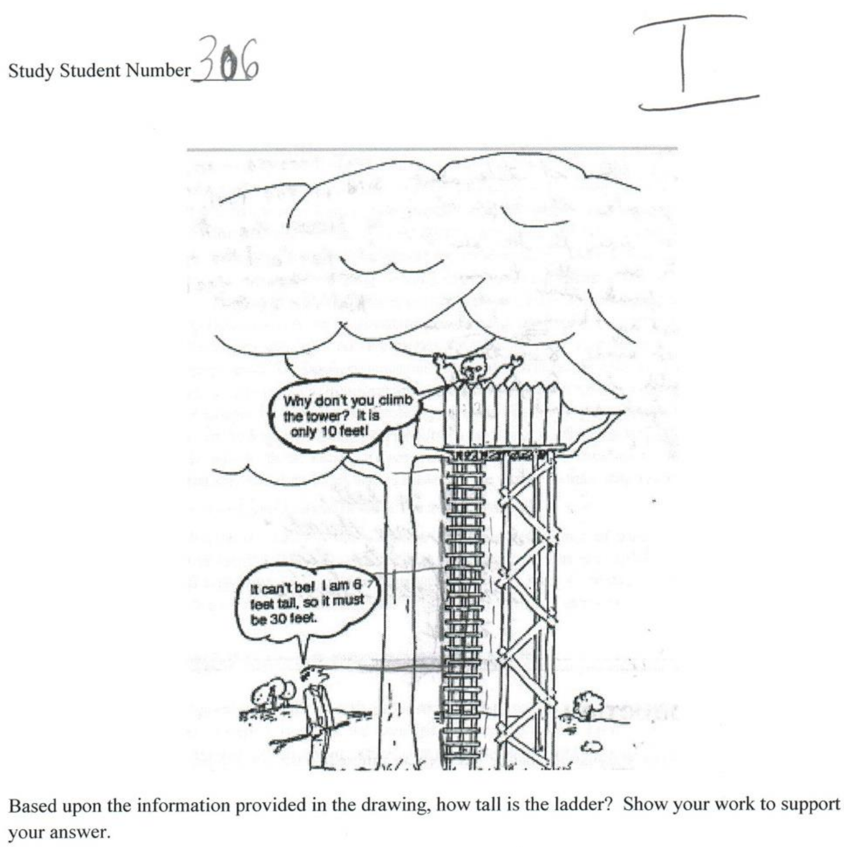


Figure 62. Carl's efforts on Tree House task.

Sticks and Rhombi task (4). As Carl completed the Sticks and Rhombi task, he drew the rhombi as several disconnected shapes as opposed to one continuous shape. Although his statement describing the relationship between the number of sticks and the number of rhombi appeared to ramble, he recognized the relationship. Figure 63 shows Carl's work on the Sticks and Rhombi task.

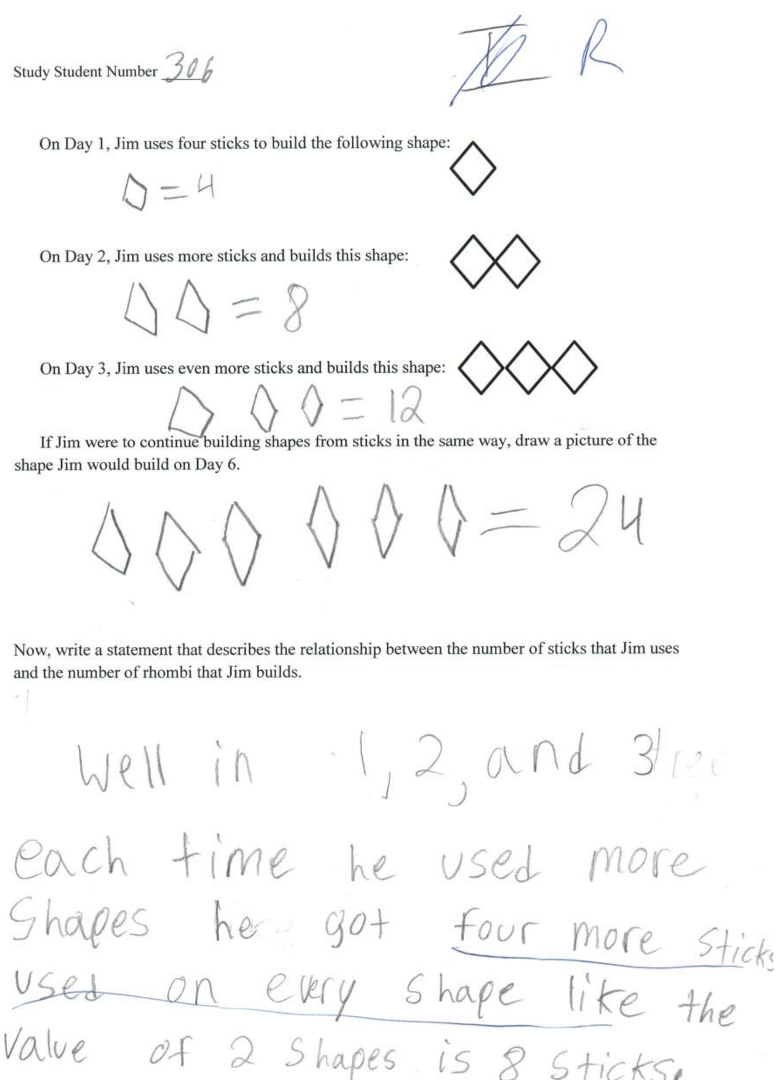
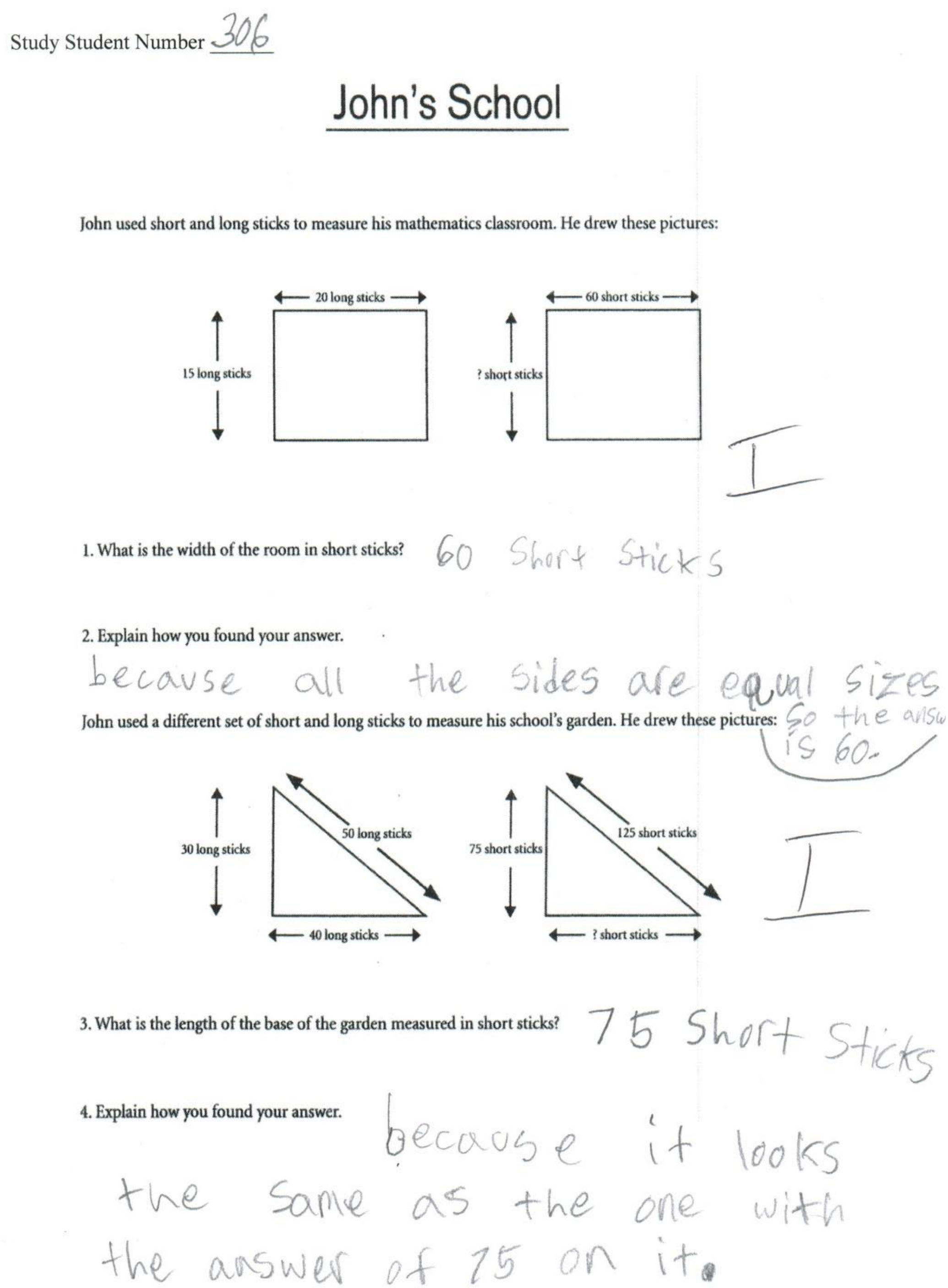


Figure 63. Carl's work on Sticks and Rhombi task.

John's School task (5). Carl completed John's School task prior to the post-test administration. Although he exhibited work at the transitional level and ratio level on prior tasks, Carl's work on John's School task indicated operation at the illogical level for both questions. On question 1, Carl interpreted the figures in the question as squares. The measurements given in question 1 presented a different length and width, so the assumption that the figures are squares was not supported. Also, Carl's assumption interfered with any calculation involving proportional reasoning concerning these figures. With respect to question 2, Carl based his response upon apparent congruence of segments from one side of the second triangle to another side of the same triangle. In both questions of this task, Carl ignored the measurements provided for the figures and based any work on visual inspection. Figure 64 presents Carl's work on John's School task.



Study Student Number 306

John's School

John used short and long sticks to measure his mathematics classroom. He drew these pictures:

1. What is the width of the room in short sticks? 60 Short Sticks

2. Explain how you found your answer.

because all the sides are equal sizes So the answ is 60.

John used a different set of short and long sticks to measure his school's garden. He drew these pictures:

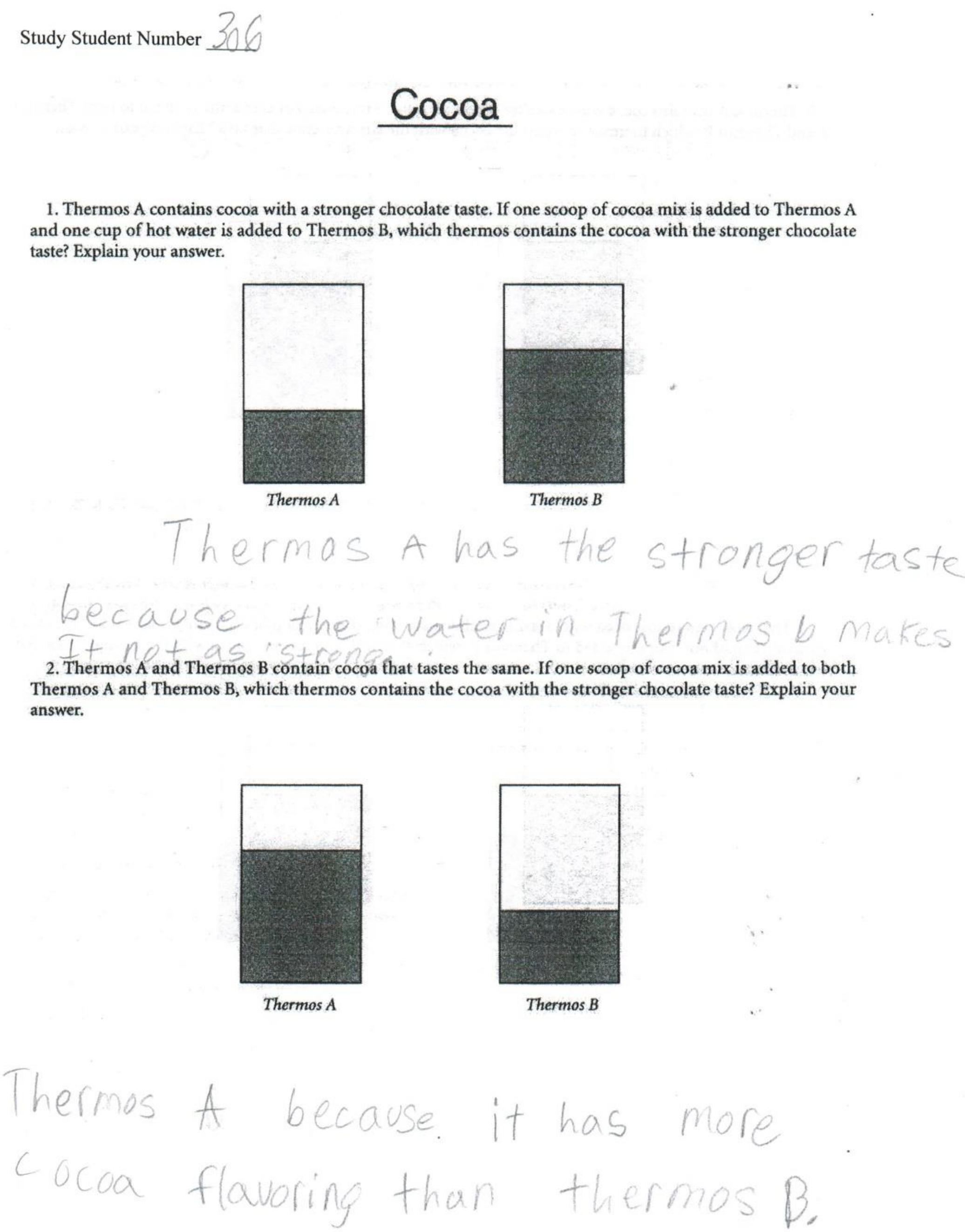
3. What is the length of the base of the garden measured in short sticks? 75 Short Sticks

4. Explain how you found your answer.

because it looks the same as the one with the answer of 75 on it.

Figure 64. Carl's work on John's School task.

Post-instruction interview. During the Cocoa task, Carl answered two questions correctly out of the four questions presented. Some of Carl's responses indicated a sense of proportional reasoning concerning the ingredients for the cocoa; for instance, on question one he stated "the water makes the chocolate kind of less strong." However, Carl seemed to relate volume to strength of chocolate taste when he remarked on question three "there has to be a lot more in A to be equal or more than B." When asked about how helpful it would be to use technology to answer the questions on the Cocoa task, he said "a little." In response to a request for further explanation, Carl replied that the technology "could have shown the blocks", referring to a Thinking Blocks model. Figures 65 and 66 provide Carl's work from the Cocoa task.


Study Student Number 306

Cocoa

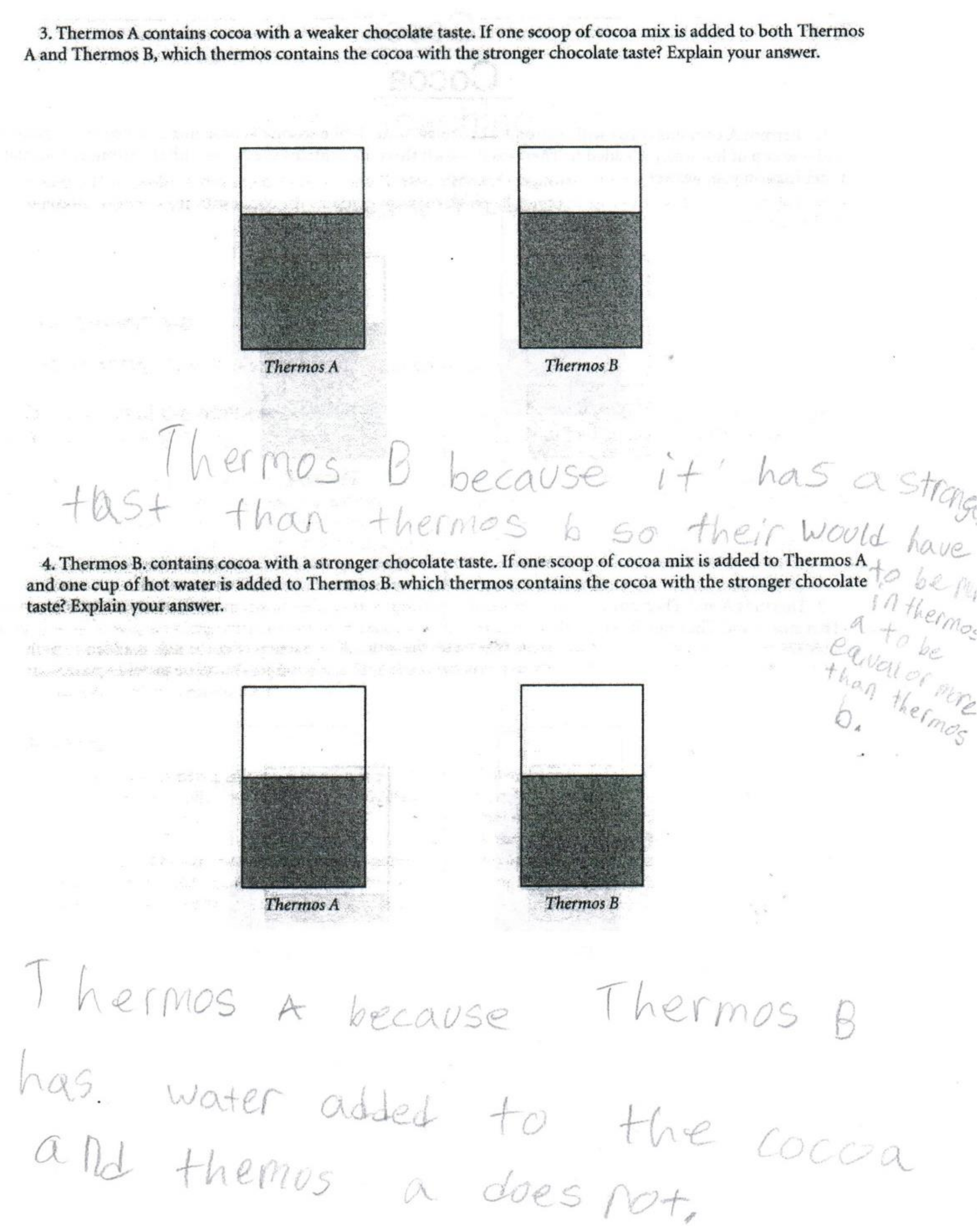
1. Thermos A contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Thermos A has the stronger taste because the water in Thermos b makes It not as strong.

2. Thermos A and Thermos B contain cocoa that tastes the same. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Thermos A because it has more cocoa flavoring than thermos B.

Figure 65. Carl's work on Cocoa task, page 1.



3. Thermos A contains cocoa with a weaker chocolate taste. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Thermos B because it has a strong tast than thermos b so their would have to be m in thermos a to be equal or more than thermos b.

4. Thermos B. contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B. which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Thermos A because Thermos B has water added to the cocoa and themos a does not,

Figure 66. Carl’s work on Cocoa task, page 2.

Summary. Carl showed a tendency to work independently from the other participants during the virtual manipulative website instruction. While working with the Thinking Blocks application, Carl showed indications that he enjoyed working with the virtual manipulative: he continued working with the website while other participants were off task, as well as reworking modules that he had already completed. On the Number Pieces website, Carl encountered difficulty in constructing his block models, which suggested he had not completely comprehended the work with the Thinking Blocks virtual manipulative.

With respect to completing the five tasks, Carl showed that he could work as part of a group; additionally, he volunteered to share information with the class. During the classroom instruction aspect of the study, Carl made statements or asked questions indicating a development of proportional reasoning skills. For instance, on Day 3, Carl asked about the order of comparison when writing a ratio from the Thinking Blocks website. On Day 6 of the study, Carl stated, "What I really would have done would be to simplify the fraction." Ms. Xanth acknowledged his approach, but also asked Carl what he would do if the fraction did not reduce; Carl did not offer any additional strategy. Similarly, Carl asked if a fraction was different from a ratio on Day 7. An analysis of Carl's work on the five tasks indicated no consistent pattern of proportional reasoning development.

Betty

Background information. At the time of the study, Betty - an 11-year old female member of the Pencil-and-Paper group - scored above the mean score on the pre-test. With respect to the state-mandated mathematics assessment, Betty did not take the assessment as a grade five student. Upon inquiry, the researcher learned Betty recovered from a concussion during the week of administration of the state-mandated mathematics assessment. According to her responses on the technology use and preference survey, she had more than three years' experience working with touchscreen devices; Betty prefers to work with an iPad.

Pre-instruction interview. As she completed the Make a New Puzzle task, Betty worked quietly; in fact, the researcher had to prompt Betty to speak aloud instead of just shaking her head during the interview. Betty used an additive approach to calculate the side lengths for the new puzzle pieces. As Betty tried to assemble the new puzzle pieces into a square, she hesitated and looked at the puzzle model for guidance. When asked about her hesitancy, Betty said "It doesn't seem like it [the new puzzle] goes together." When prompted to measure the side lengths of the puzzle pieces, Betty agreed that the measures of most of the side lengths agreed with her calculations. For one side length, Betty's measurement appeared to be 1/10 cm off from her calculation; she indicated that this amount should not keep the puzzle pieces from being assembled into a square. Figure 67 displays Betty's work on the Make a New Puzzle task.

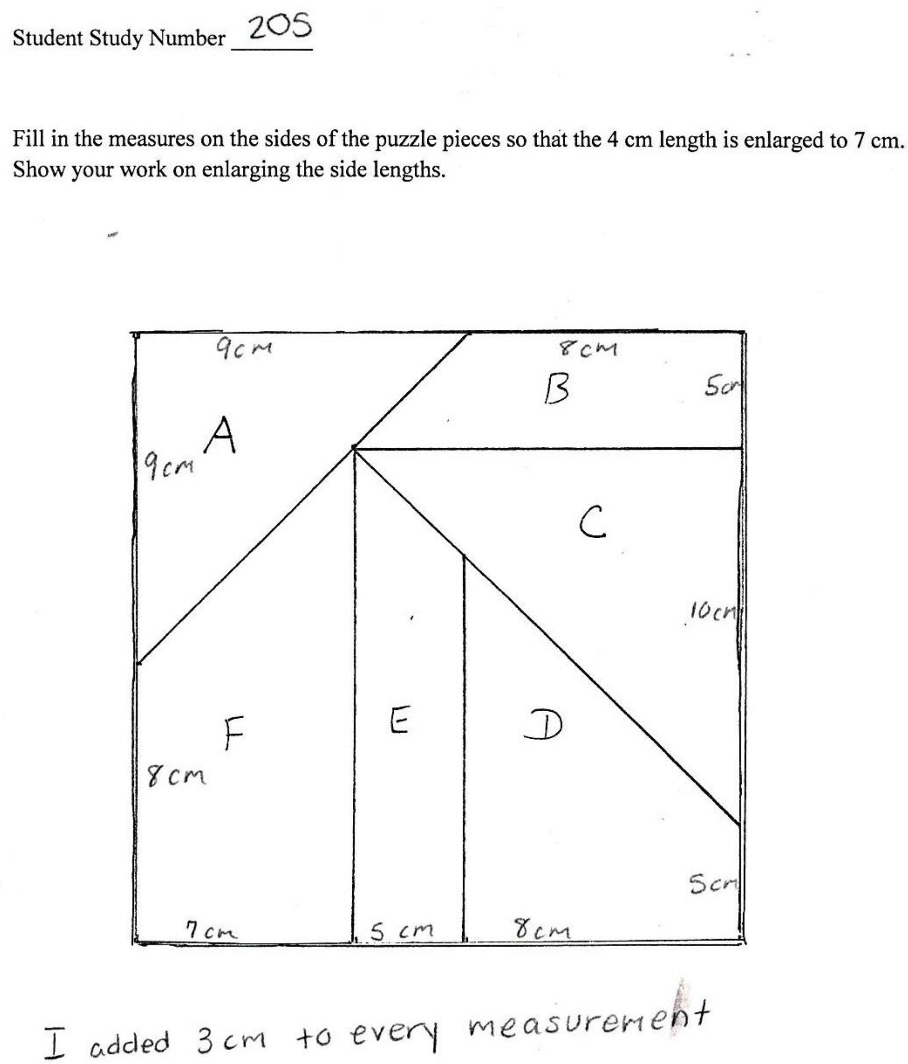
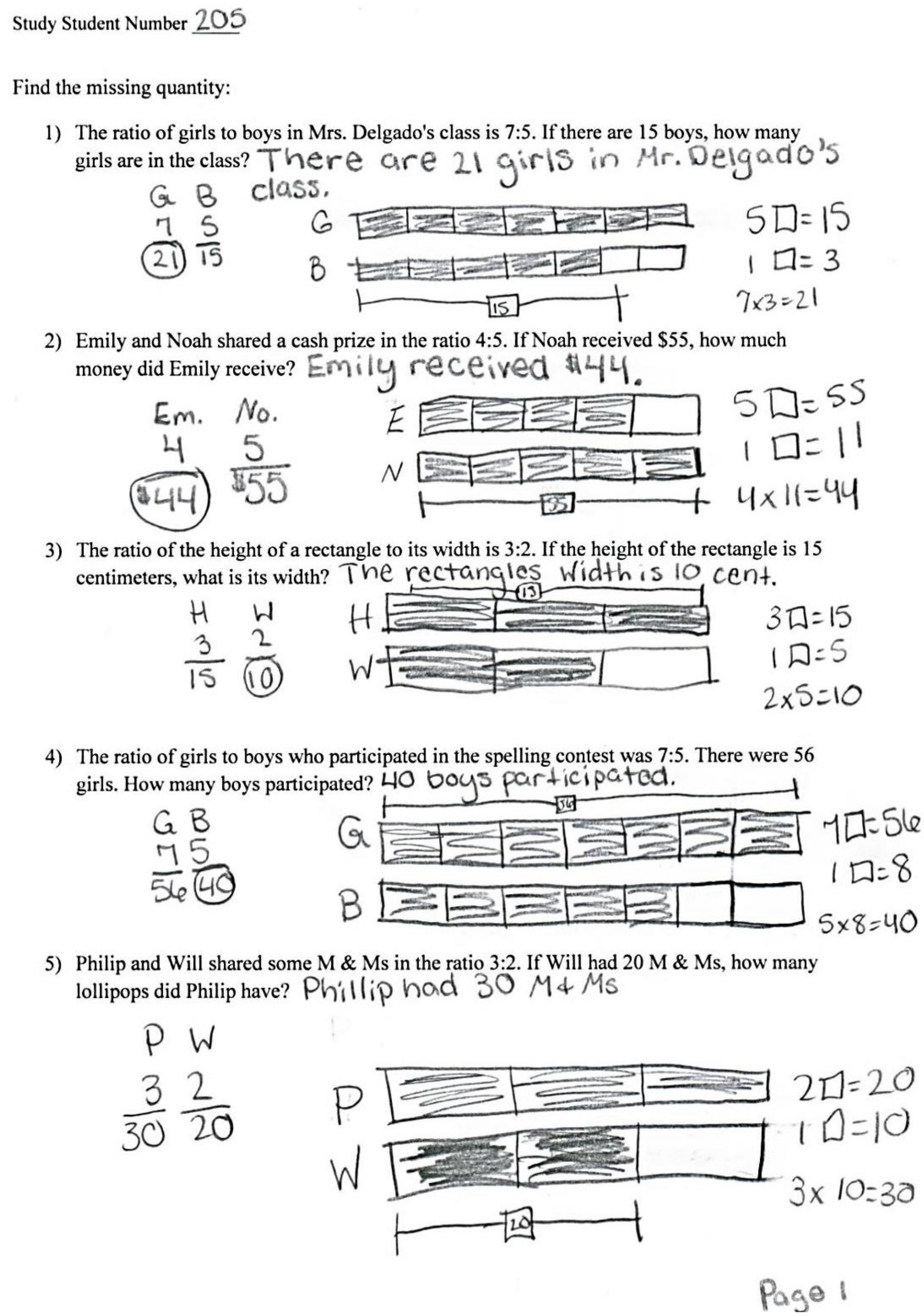


Figure 67. Betty's work on Make a New Puzzle task.

Classroom work with block modeling strategy. As part of the control group, Betty only used pencil and paper while working with the same block modeling strategy as presented with the Thinking Blocks virtual manipulative. Ms. Xanth used the tutorial videos found with the Thinking Blocks website to present the block modeling strategy to the participants in the control group; in addition, she modeled the strategy for the class in order for them to develop their understanding of the block modeling strategy.

Betty missed class on Day 3 of the study, which contributed to some confusion upon returning to class the next day. When presented with a word problem involving proportional reasoning and the application of the block modeling strategy, she stated "I know the answer, but how do you do the model?" Despite the declaration upon her return to class, Betty demonstrated mastery of the block modeling strategy on the word problems assigned during the classroom instruction. Not only did she show mastery of the block modeling strategy with respect to the assigned word problems, Betty also manifested preliminary work resembling proportions. Betty arranged information provided in each word problem into two categories and wrote two ratios; in fact, the only symbols lacking from her work with respect to a proportion are the equal sign and a variable for the missing quantity. Figure 68 provides a sample of Betty's work on the proportional reasoning problems assigned during the study.

Despite Betty's evidence of mastery in solving proportional reasoning problems, she insisted upon Ms. Xanth's attention to verify her work during the problem-solving process. The level of attention required from Ms. Xanth varied; at times, Betty raised her hand for Ms. Xanth only when the problem was complete. On other occasions, Betty requested Ms. Xanth to inspect her work at each step. This variance of attention seemed to depend upon how comfortable Betty and her table group felt about the work on each individual problem. Usually, Betty worked cooperatively with participants at her table.



Study Student Number 205

Find the missing quantity:

1) The ratio of girls to boys in Mrs. Delgado's class is 7:5. If there are 15 boys, how many girls are in the class? There are 21 girls in Mr. Delgado's class.

G B
7 5
(21) 15

G
B
15

5□=15
1 □=3
7x3=21

2) Emily and Noah shared a cash prize in the ratio 4:5. If Noah received $55, how much money did Emily receive? Emily received $44.

Em. No.
4 5
($44) $55

E
N
55

5□=55
1 □=11
4x11=44

3) The ratio of the height of a rectangle to its width is 3:2. If the height of the rectangle is 15 centimeters, what is its width? The rectangles width is 10 cent.

H W
3 2
15 (10)

15
H
W

3□=15
1□=5
2x5=10

4) The ratio of girls to boys who participated in the spelling contest was 7:5. There were 56 girls. How many boys participated? 40 boys participated.

G B
7 5
56 (40)

56
G
B

7□=56
1 □=8
5x8=40

5) Philip and Will shared some M & Ms in the ratio 3:2. If Will had 20 M & Ms, how many lollipops did Philip have? Phillip had 30 M+Ms

P W
3 2
30 20

P
W
20

2□=20
1□=10
3x 10=30

Page 1

Figure 68. Betty’s work with the block modeling strategy.

Mr. Tall/Mr. Short task (1). Betty did not complete the Mr. Tall/Mr. Short task. Betty attempted to identify the relationship between the numbers of buttons for Mr. Short's height to the number of buttons for Mr. Tall's height in a sketch, but she appeared to have changed her mind concerning this strategy. Further, she tried to perform some calculations, but Betty's work indicated she did not make any conclusion concerning the height of Mr. Tall in paper clips. Based upon the evidence provided in her work, Betty's attempt on the Mr. Tall/Mr. Short task rated at the illogical level of proportional reasoning. Figure 69 contains Betty's work on the Mr. Tall/Mr. Short task.

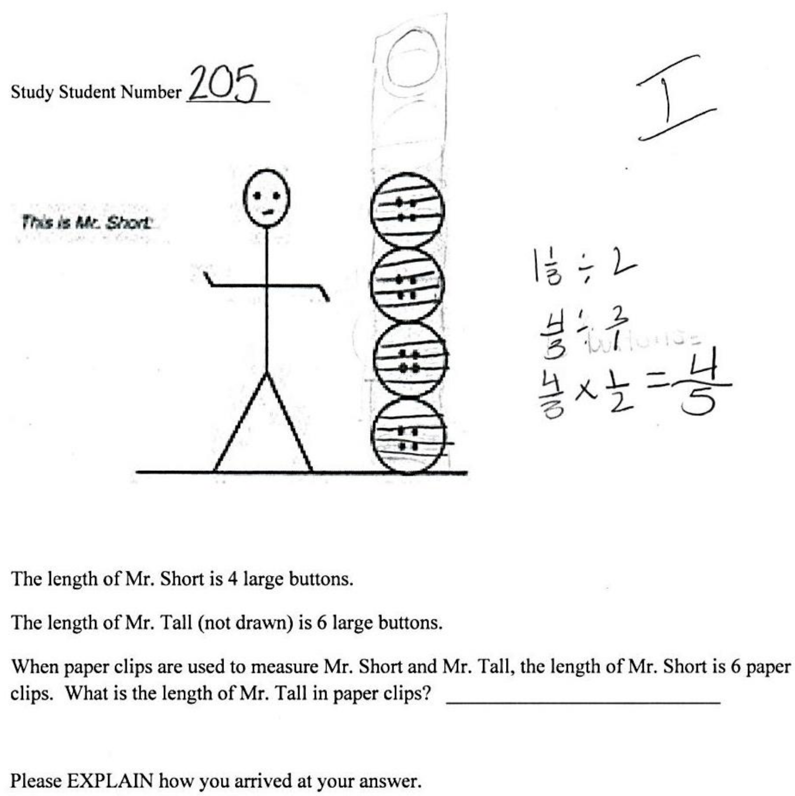


Figure 69. Betty's work on Mr.Tall/Mr. Short task.

Egg Carton task (2). Betty correctly colored the brown eggs in Cartons B and C for the Egg Carton task. She seemed to recognize the ratio of brown eggs to total eggs in each carton as opposed to the ratio of brown eggs to white eggs, as requested. By attempting to maintain the physical position of the brown eggs in Cartons B and C to reflect the position of brown eggs in Carton A, Betty evidenced work at the transitional level of proportional reasoning. Figure 70 gives Betty's work for the Egg Carton task.

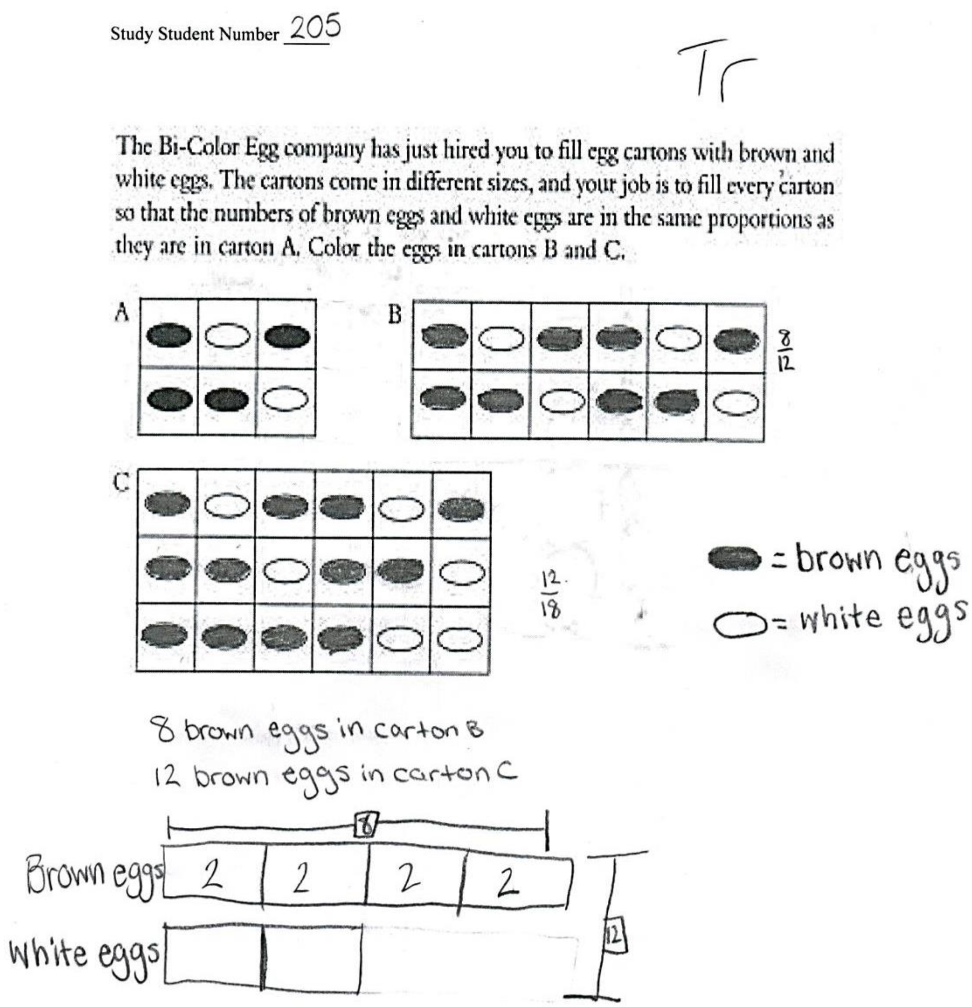


Figure 70. Betty's work on the Egg Carton task.

Tree House task (3). Betty's work on the Tree House task reflected stages of effort: her individual work and her group work. By marking out information provided in the problem, Betty recognized the statement concerning the height of the ladder being 10 feet was inaccurate. However, she appeared to relate the height of the man on the ground to an incorrect number of rungs on the ladder, stating that there were 7 ½ rungs for every 6 feet. Due to the use of an inaccurate ratio, Betty's work for this task rated at the transitional level of proportional reasoning.

After working with her table group, Betty concluded the ladder was 18 feet tall. Even after Ms. Xanth confirmed the number of rungs on the ladder in the class discussion concerning this task, Betty insisted a different number of rungs were to be found in the picture. Figure 71 shows Betty's work on the Tree House task.

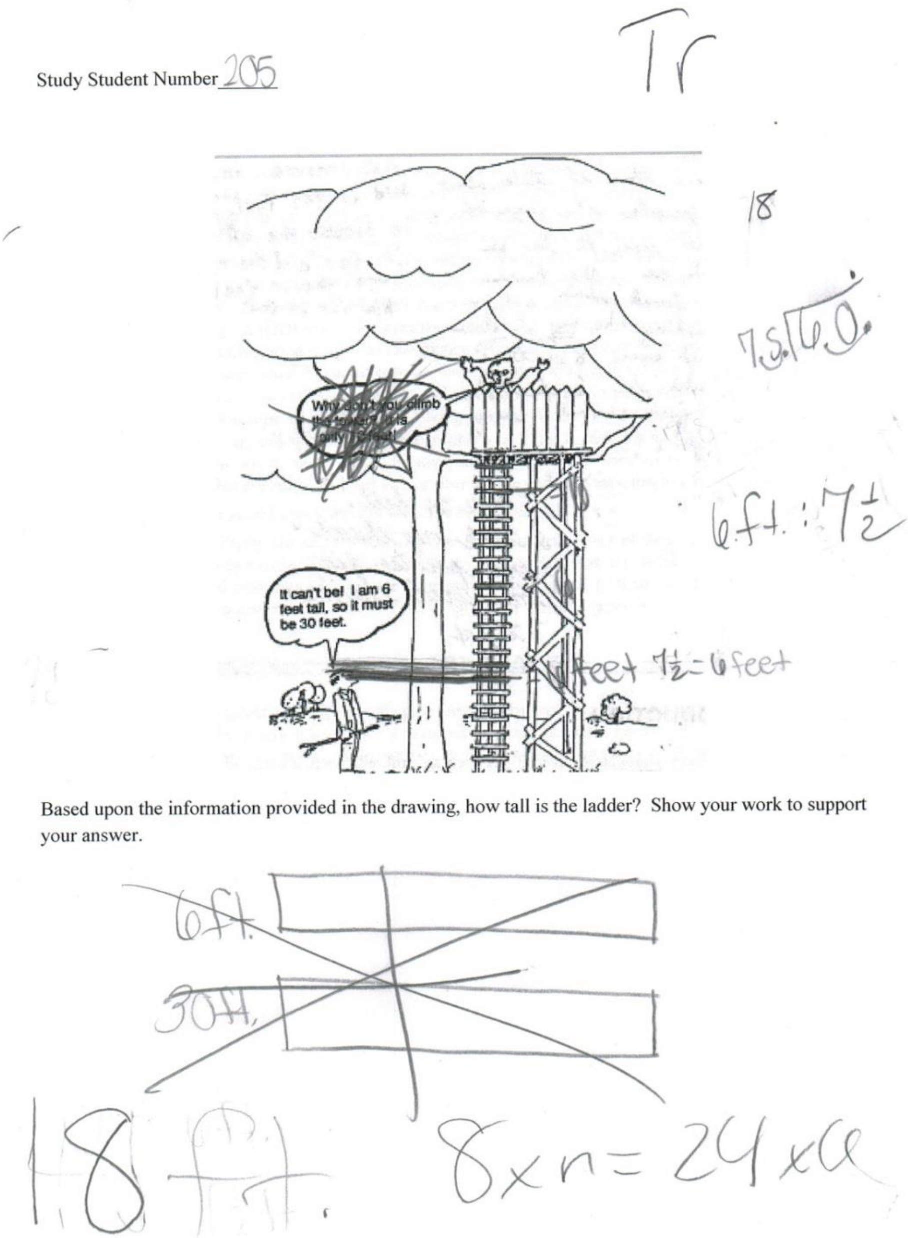
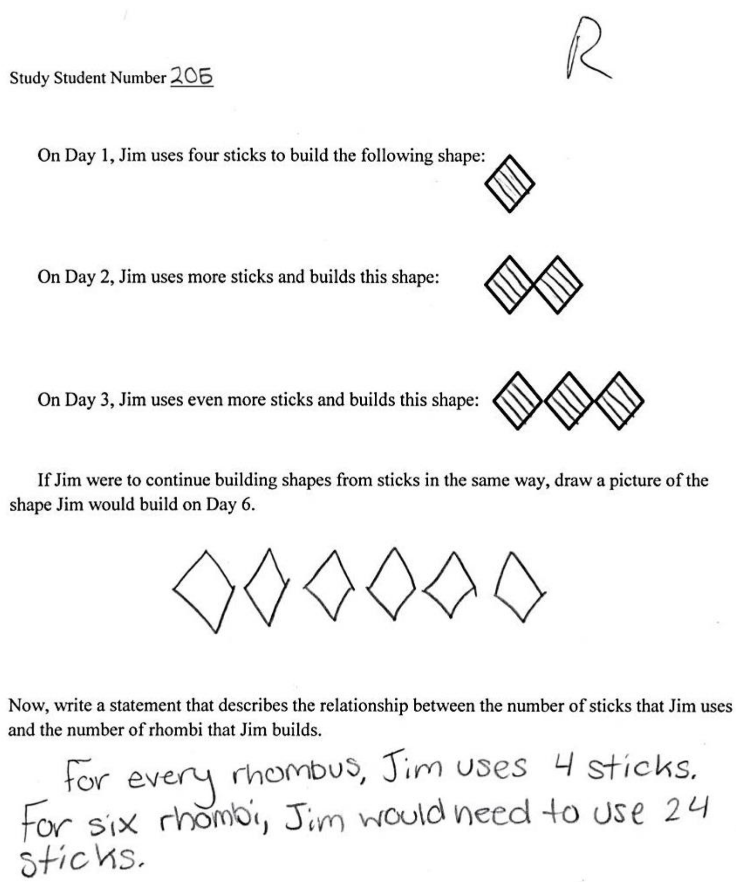


Figure 71. Betty's work on the Tree House task.

Sticks and Rhombi task (4). Betty correctly completed the Sticks and Rhombi task. In drawing the shape produced on Day 6, Betty's work showed six disconnected rhombi instead of one shape consisting of six connected rhombi. Not only did Betty write a correct statement concerning the relationship between the number of sticks and the number of rhombi, she also identified the number of sticks used on Day 6. The researcher rated Betty's work at the ratio level of proportional reasoning. Figure 72 provides Betty's work on the Sticks and Rhombi task.



Study Student Number 205

R

On Day 1, Jim uses four sticks to build the following shape:

On Day 2, Jim uses more sticks and builds this shape:

On Day 3, Jim uses even more sticks and builds this shape:

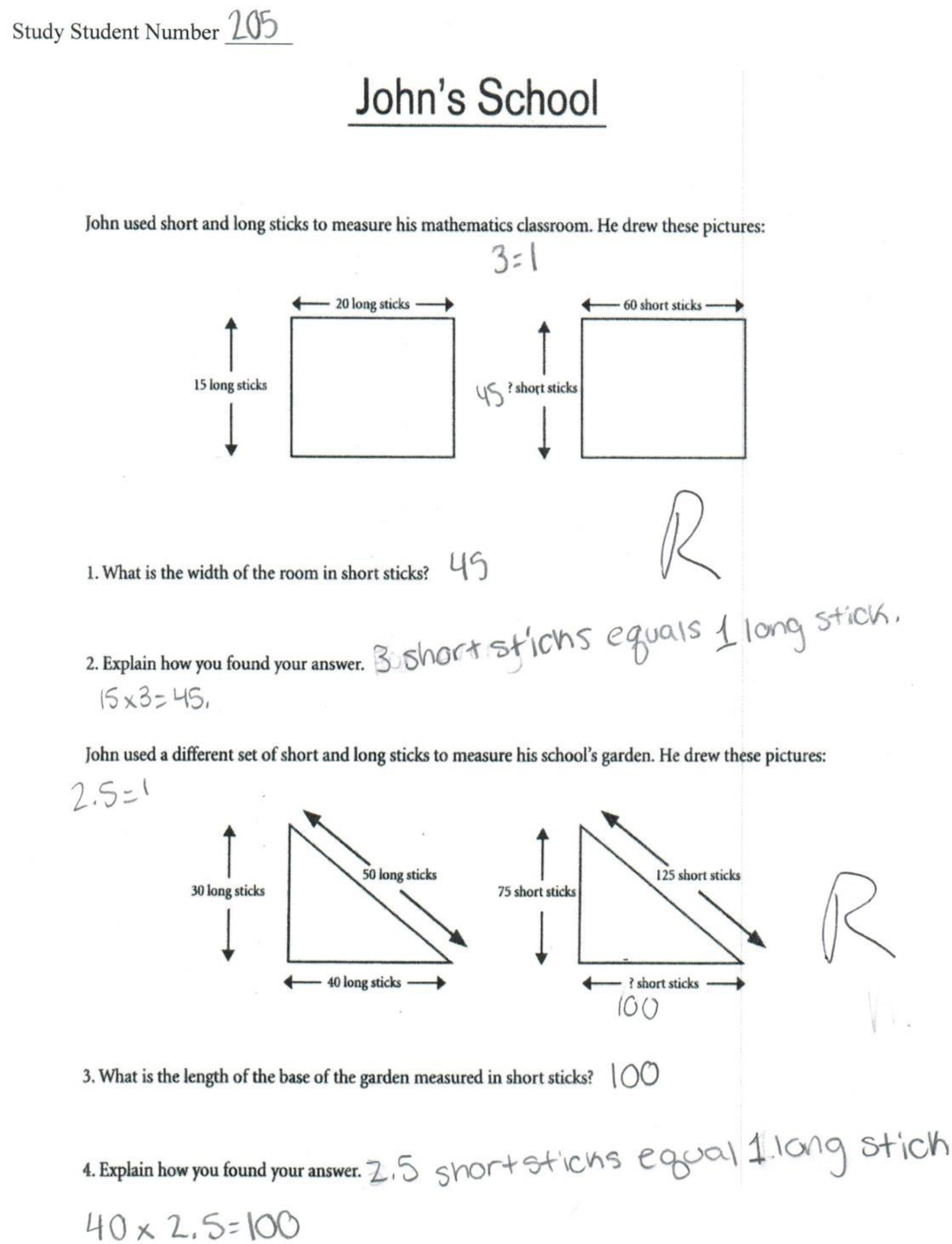
If Jim were to continue building shapes from sticks in the same way, draw a picture of the shape Jim would build on Day 6.

Now, write a statement that describes the relationship between the number of sticks that Jim uses and the number of rhombi that Jim builds.

For every rhombus, Jim uses 4 sticks. For six rhombi, Jim would need to use 24 sticks.

Figure 72. Betty's work on the Sticks and Rhombi task.

John's School task (5). For this task, Betty correctly answered both questions. As part of her work, Betty wrote an initial equation for each question which seemed incorrect: 3 = 1 and 2.5 = 1. However, Betty's explanation clarified the meaning of these equations. The relationship indicated by the apparently incorrect equations omitted the units of measure used in the task. In providing an explanation concerning the answer found for each question, Betty gave a statement relating the number of short sticks to the number of long sticks; also, she showed an equation in which she multiplied the unit rate with the corresponding side length. In her work, Betty indicated no difference in solving a problem involving an integral unit rate or a non-integral unit rate. Figure 73 gives Betty's work on John's School task.



Study Student Number 205

John's School

John used short and long sticks to measure his mathematics classroom. He drew these pictures:

3=1

1. What is the width of the room in short sticks? 45

R

2. Explain how you found your answer. 3 short sticks equals 1 long stick. 15x3=45.

John used a different set of short and long sticks to measure his school's garden. He drew these pictures:

2.5=1

R

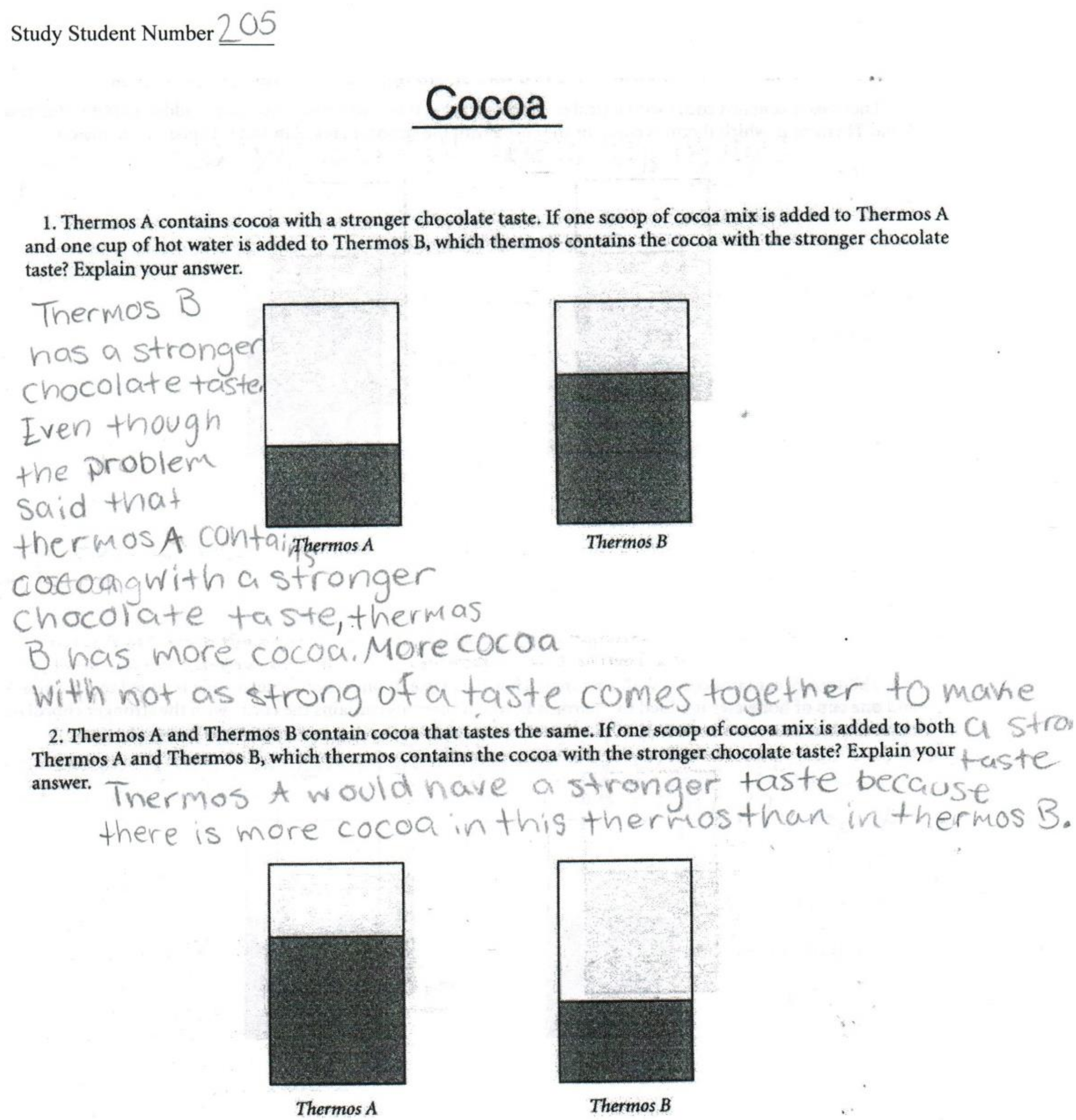
3. What is the length of the base of the garden measured in short sticks? 100

4. Explain how you found your answer. 2.5 short sticks equal 1 long stick

40 x 2.5=100

Figure 73. Betty's work on the John's School task.

Post-instruction interview. For the Cocoa task, Betty answered one question correctly out of the four questions. Based upon her responses, it appeared that Betty did not consider cocoa consisting of two components whose ratio influenced the chocolate taste; rather, Betty seemed to think of cocoa as a single quantity and more volume of liquid indicated more strength of chocolate taste. Betty stated, "… even though Thermos A says it has the cocoa with the stronger chocolate taste, there's more cocoa in Thermos B, so I thought that with all that cocoa when it comes together it will create a stronger taste in Thermos B." When asked if technology would have provided assistance with answering the questions from the Cocoa task, Betty responded, "Most likely not." When asked to explain her response, Betty said one would actually need to have the thermoses of cocoa available for tasting to see which one had the stronger chocolate taste. Betty's response indicates that she did not recognize the use of proportional reasoning in the Cocoa task. Figures 74 and 75 provide Betty's work on the Cocoa task.



Study Student Number 205

Cocoa

1. Thermos A contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Thermos B has a stronger chocolate taste. Even though the problem said that thermos A contains cocoa with a stronger chocolate taste, thermas B has more cocoa. More cocoa with not as strong of a taste comes together to make a strong taste

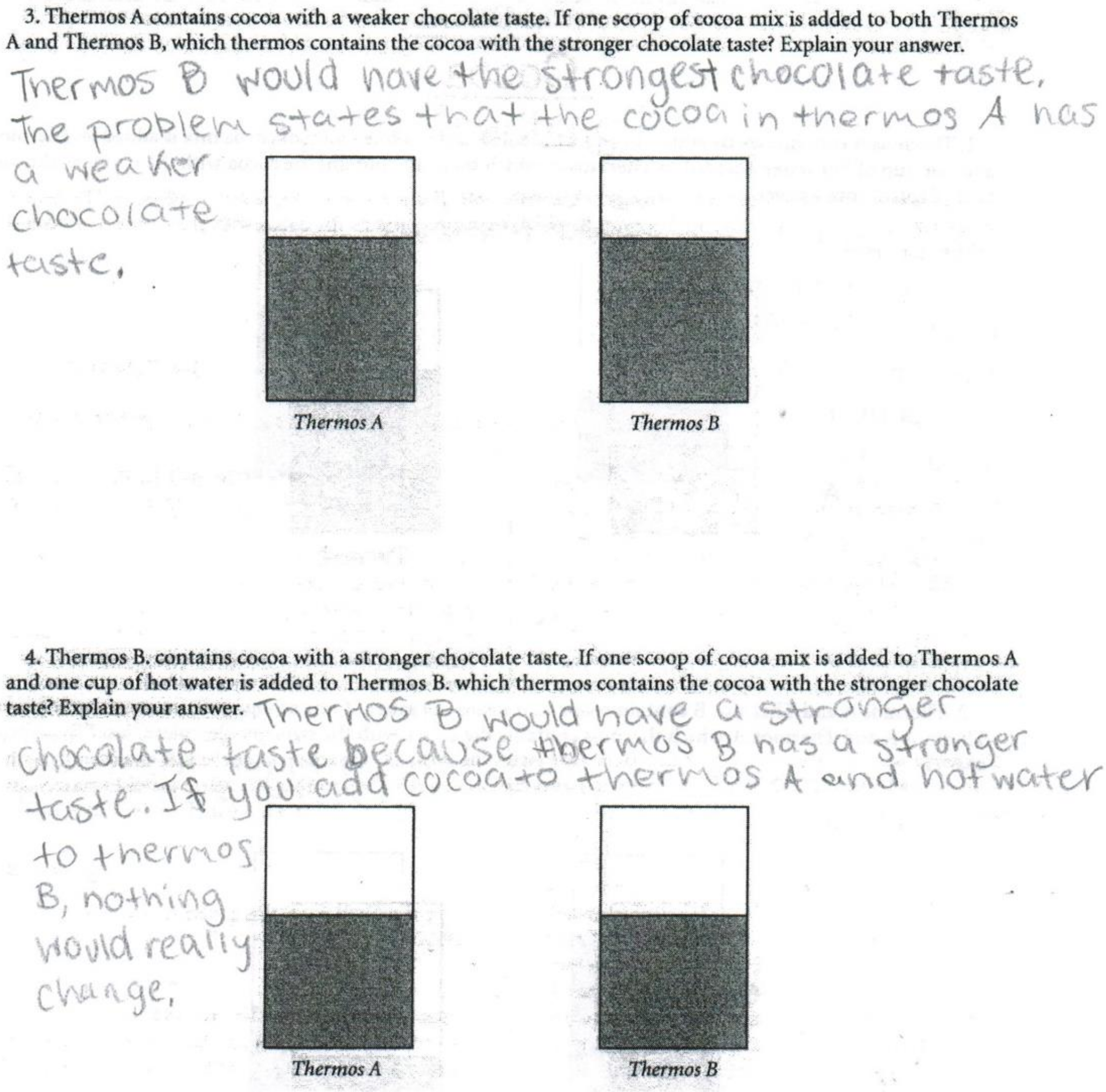
Thermos A Thermos B

2. Thermos A and Thermos B contain cocoa that tastes the same. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Thermos A would have a stronger taste because there is more cocoa in this thermos than in thermos B.

Thermos A Thermos B

Figure 74. Betty’s work on the Cocoa task, page 1.



3. Thermos A contains cocoa with a weaker chocolate taste. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Thermos B would have the strongest chocolate taste, The problem states that the cocoa in thermos A has a weaker chocolate taste.

Thermos A

Thermos B

4. Thermos B. contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B. which thermos contains the cocoa with the stronger chocolate taste? Explain your answer. Thermos B would have a stronger chocolate taste because thermos B has a stronger taste. If you add cocoa to thermos A and hot water to thermos B, nothing would really change.

Thermos A

Thermos B

Figure 75. Betty's work on the Cocoa task, page 2.

Summary. Betty worked both independently and collaboratively during this study. Being absent one day near the beginning of the study contributed to some confusion that Betty initially experienced concerning the block modeling strategy. In fact, Betty admitted some difficulty with the block modeling approach to solving proportional reasoning problems: she stated, “I know the answer, but how do you do the model?” In another instance, Betty indicated that she had difficulty reconciling the information used in drawing a block model and using the same information to write a proportion when she said, “This is like what I don’t understand.” Betty recognized 25 mph as a ratio, which indicated that she understood some proportional reasoning concepts. While completing assignments in class, she expressed confidence in her work. Betty’s confidence in her abilities also led to some disagreement with Ms. Xanth’s solution to the Tree House task; even when the class had agreed on the correct answer, Betty still insisted that the class was wrong. The source of disagreement was found to be the number of rungs on the ladder; Betty counted 23 rungs instead of 24 rungs. When completing the five tasks used in the research study, Betty demonstrated development of proportional reasoning from illogical to transitional to ratio.

Bob

Background information. At the time of the study, Bob - an 11-year old male member of the Pencil-and-Paper group - scored below the mean score on the pre-test. With respect to the state-mandated mathematics assessment, Bob scored Proficient as a grade five student. According to his responses on the technology use and preference survey, he had less than one year’s experience working with touchscreen devices; however, Bob prefers to work with an iPad.

Pre-instruction interview. Bob worked quietly with the Make a New Puzzle task, usually nodding his head to show his agreement. Whenever Bob calculated the new side lengths for the bigger puzzle pieces, he used his fingers to add three to each original side length. As Bob began to assemble the new puzzle, he hesitated and said, “It doesn’t work.” After confirming that the measures of the side lengths of the new puzzle pieces matched his calculations, Bob recognized that gaps still existed when trying to assemble the puzzle pieces into a square. In his efforts to assemble the puzzle, Bob neither attributed his lack of success in assembling the new pieces into a square on inaccuracy of measurement of puzzle pieces nor any lack of ability on his part. Figure 76 shows Bob’s work on the Make a New Puzzle task.

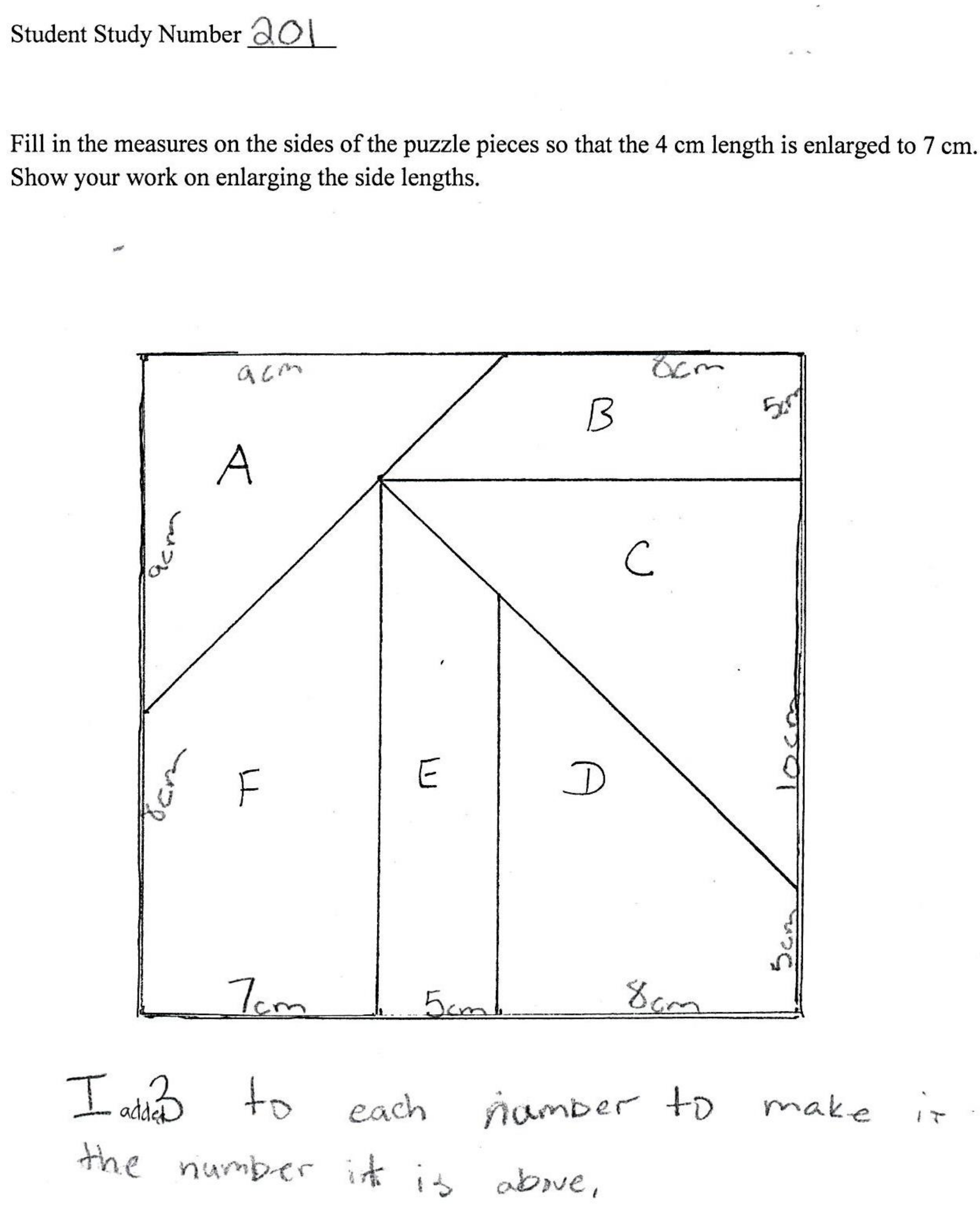
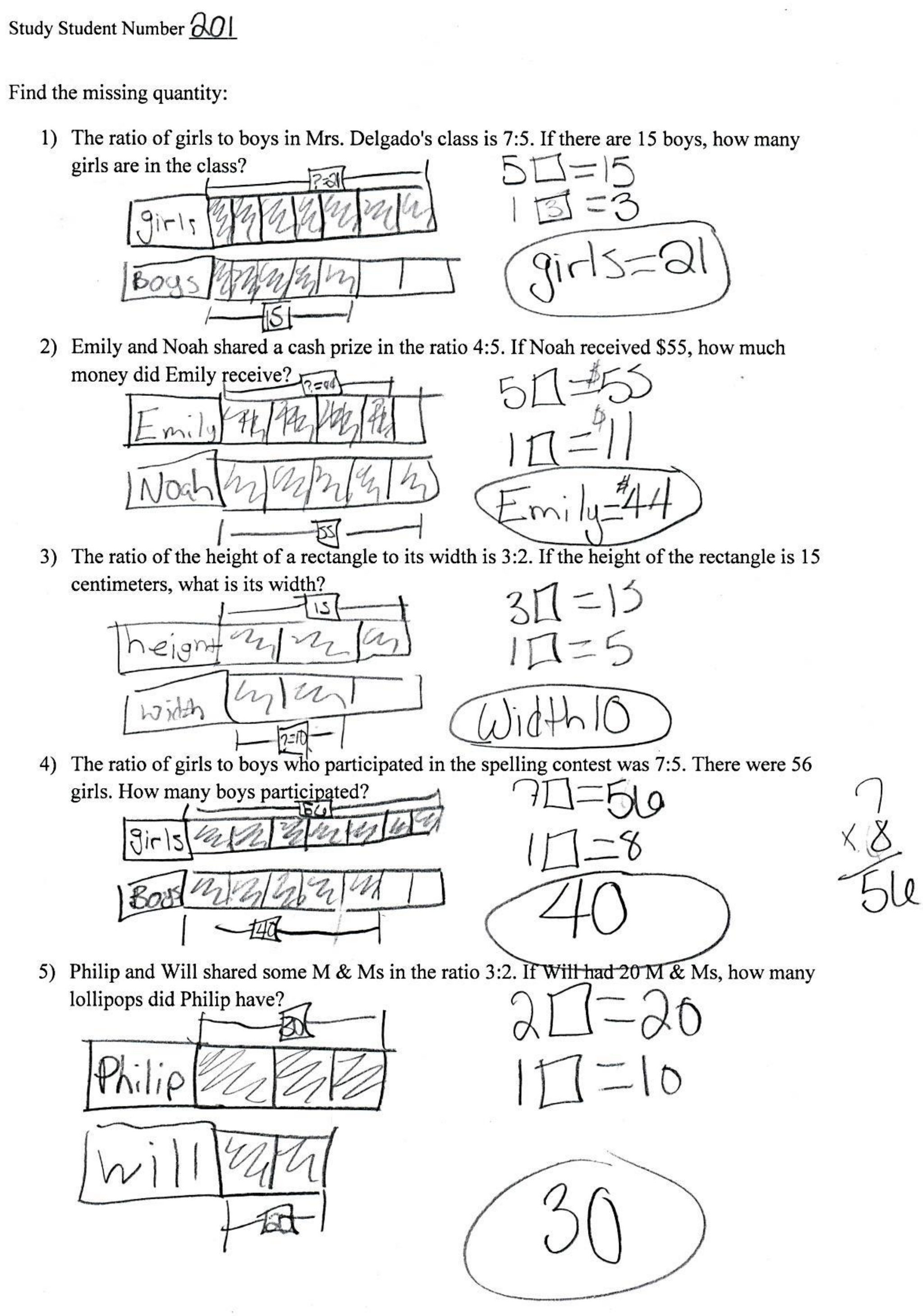


Figure 76. Bob's work on Make a New Puzzle task.

Classroom work with block modeling strategy. As with the other participants in the control group, Bob used pencil and paper as he worked with the block modeling strategy presented with the Thinking Blocks website. Bob's work showed consistency in following the examples presented with the block modeling tutorial videos and Ms. Xanth's step-by-step practice problems. Figure 77 shows Bob's work with the block modeling strategy.

After becoming accustomed to the block modeling strategy, Bob expressed his support of this approach in a conversation with another student in his table group. When Bob noticed a member of the table group answering questions without drawing block models, he stated using the block modeling strategy was easier. As Bob continued his work during classroom instruction, he appeared to have difficulty writing proportions as part of his work. On Day 7 of this study, Ms. Xanth encouraged participants to write proportions along with their block models. Bob asked, "What is a proportion?" In the same class period, he experienced difficulty in constructing proportions. As classroom instruction continued, Bob appeared to improve with respect to writing proportions.



Study Student Number 201

Find the missing quantity:

1) The ratio of girls to boys in Mrs. Delgado's class is 7:5. If there are 15 boys, how many girls are in the class?

5□=15
1 [3] =3
girls=21

2) Emily and Noah shared a cash prize in the ratio 4:5. If Noah received $55, how much money did Emily receive?

5□=$55
1□=$11
Emily=$44

3) The ratio of the height of a rectangle to its width is 3:2. If the height of the rectangle is 15 centimeters, what is its width?

3□=15
1□=5
Width 10

4) The ratio of girls to boys who participated in the spelling contest was 7:5. There were 56 girls. How many boys participated?

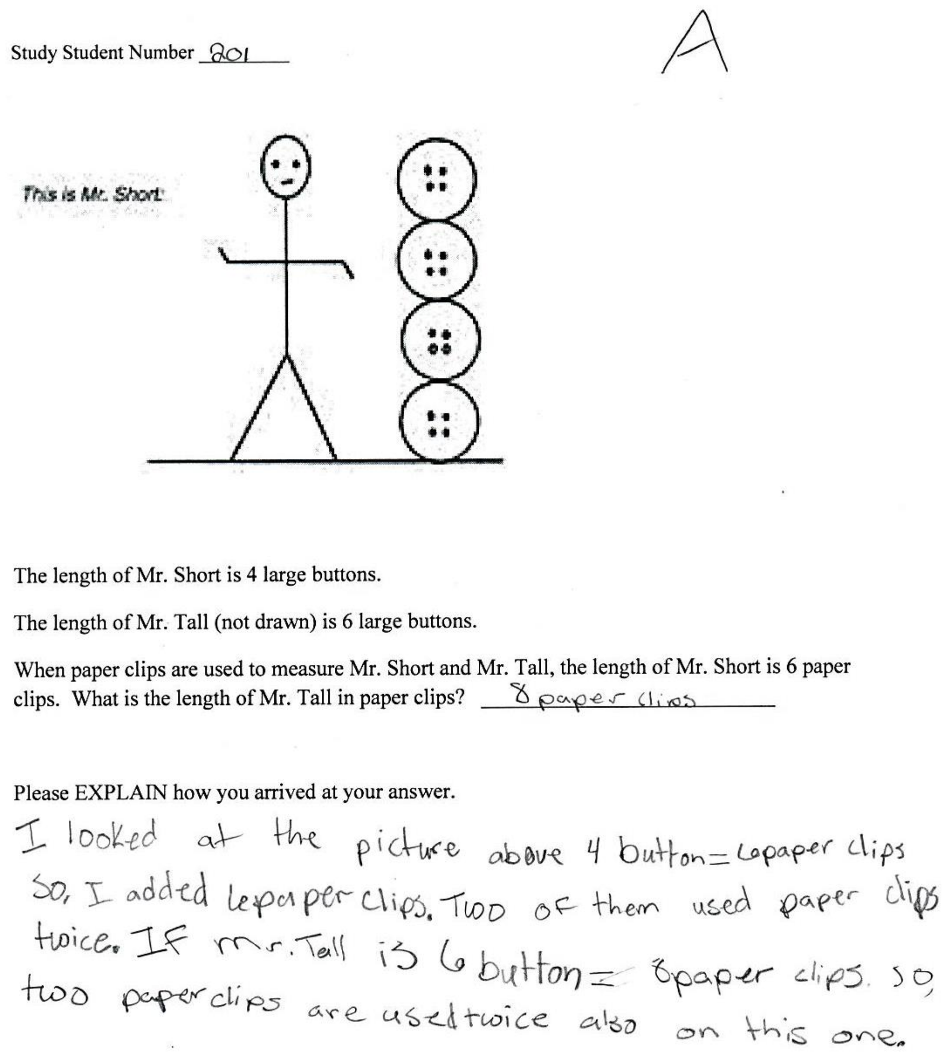
7□=56
1□=8
40

5) Philip and Will shared some M & Ms in the ratio 3:2. If Will had 20 M & Ms, how many lollipops did Philip have?

2□=20
1□=10
30

Figure 77. Bob's work with block modeling strategy.

Mr. Tall/Mr. Short task (1). On this task, Bob employed an additive approach to find the height of Mr. Tall in paper clips. His response of 8 paper clips and his explanation supported his work being evaluated at the additive level of proportional reasoning. In his explanation, Bob states a relationship between the buttons and the paper clips, but he did not pursue the multiplicative nature of the relationship. Figure 78 gives Bob's work on the Mr. Tall/Mr. Short task.



Study Student Number 201

A

This is Mr. Short

The length of Mr. Short is 4 large buttons.

The length of Mr. Tall (not drawn) is 6 large buttons.

When paper clips are used to measure Mr. Short and Mr. Tall, the length of Mr. Short is 6 paper clips. What is the length of Mr. Tall in paper clips? 8 paper clips

Please EXPLAIN how you arrived at your answer.

I looked at the picture above 4 button=6paper clips so, I added 6paper clips. Two of them used paper clips twice. If mr.Tall is 6 button= 8paper clips. so, two paperclips are used twice also on this one.

Figure 78. Bob's work on the Mr. Tall/Mr. Short task.

Egg Carton task (2). Bob correctly colored the number of brown eggs in Cartons B and C for this task. Although Bob wrote a correct ratio of brown eggs to white eggs for both Cartons B and C, he maintained the physical position of brown eggs in Carton B. So, Bob's work on the Egg Carton task evidenced effort at the transitional level of proportional reasoning. Figure 79 contains Bob's work on the Egg Carton task.

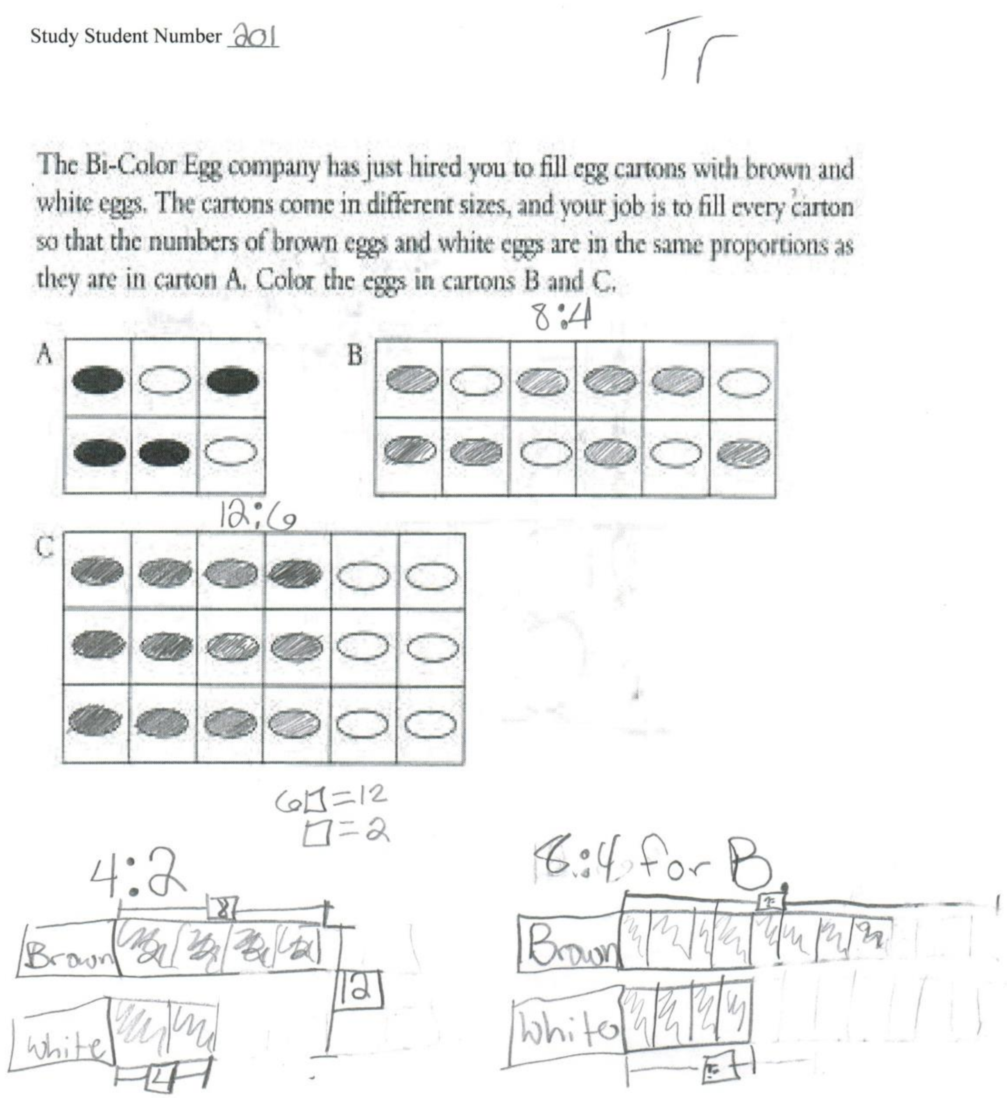


Figure 79. Bob's work on the Egg Carton task.

Tree House task (3). Bob correctly identified the height of the ladder in the Tree House task. He implemented the block modeling strategy initially; however, he also wrote and solved a proportion after discussing responses to the task with his table group. Based upon his work, Bob's response to the Tree House task rated at the ratio level of proportional reasoning. Figure 80 provides Bob's work on the Tree House task.

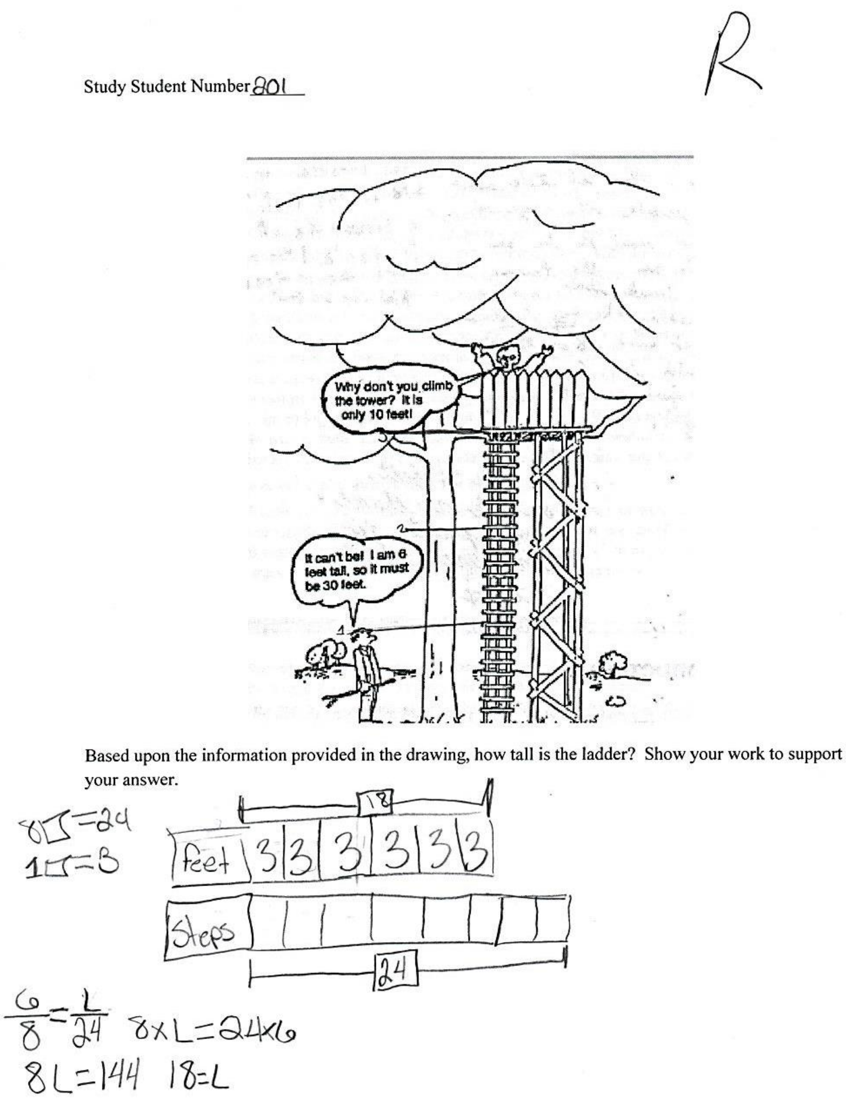
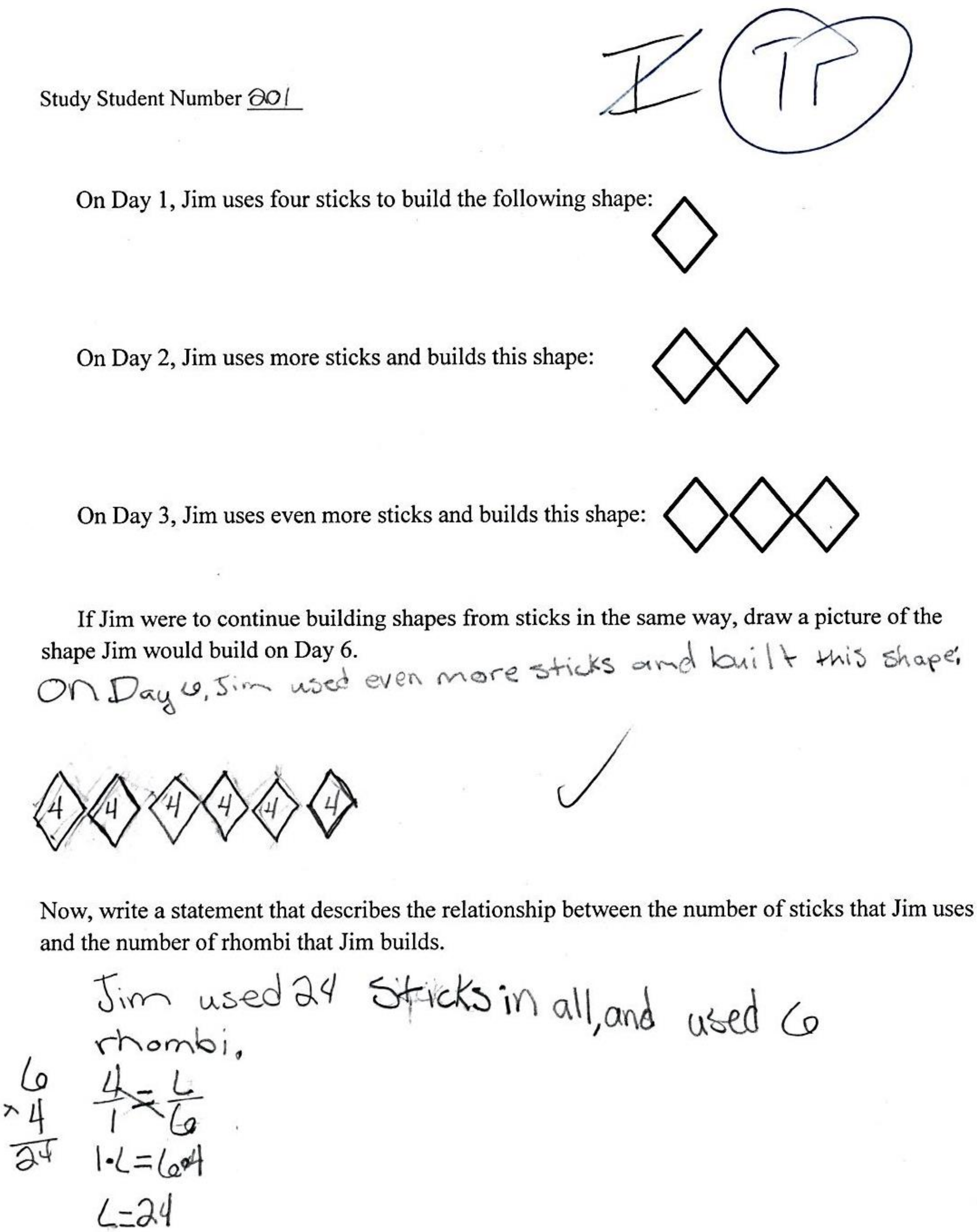


Figure 80. Bob's work on the Tree House task.

Sticks and Rhombi task (4). Bob used the correct number of Rhombi for the shape requested on the Sticks and Rhombi task. He drew some of the rhombi connected while others he did not connect. In writing his statement relating the number of sticks used and the number of rhombi built, Bob focused upon the shape that he drew instead of writing a statement reflecting the unit rate for this task. Additionally, Bob wrote and solved a proportion. In light of Bob's statement focusing solely on the shape for one particular day in the task, his work rated at the transitional level of proportional reasoning. Figure 81 shows Bob's work on the Sticks and Rhombi task.



I TP

Study Student Number 001

On Day 1, Jim uses four sticks to build the following shape:

On Day 2, Jim uses more sticks and builds this shape:

On Day 3, Jim uses even more sticks and builds this shape:

If Jim were to continue building shapes from sticks in the same way, draw a picture of the shape Jim would build on Day 6.

On Day 6, Jim used even more sticks and built this shape:

4 4 4 4 4 4

Now, write a statement that describes the relationship between the number of sticks that Jim uses and the number of rhombi that Jim builds.

Jim used 24 Sticks in all, and used 6 rhombi,

$6 \times 4 = 24$

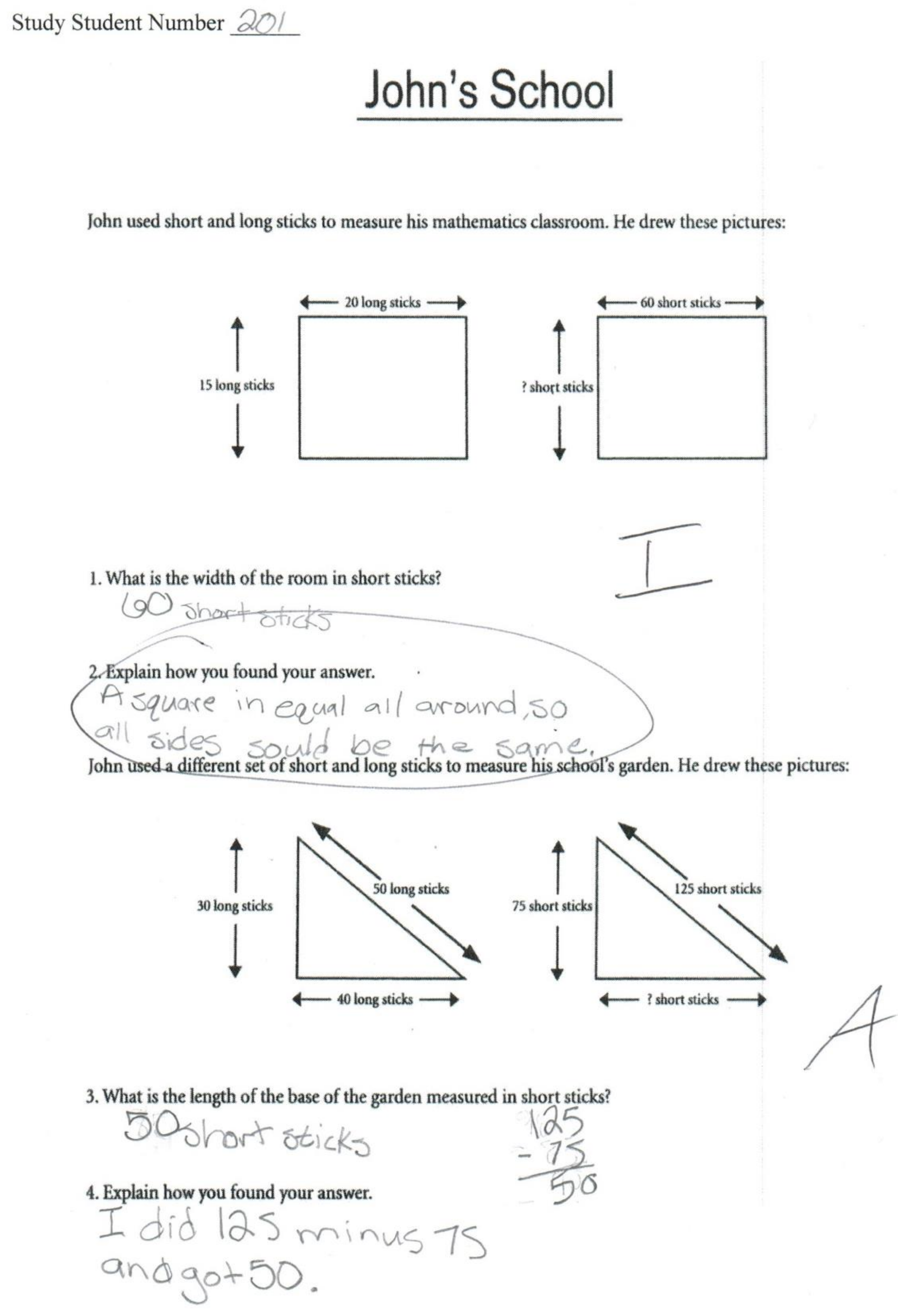
$\frac{4}{1} = \frac{L}{6}$

$1 \cdot L = 6 \cdot 4$

$L = 24$

Figure 81. Bob’s work on the Sticks and Rhombi task.

John's School task (5). Bob incorrectly answered both questions on John's School task. Although he worked at the transitional level and ratio level on previous tasks, Bob's work revealed a regression to an illogical level of proportional reasoning for question 1 and displayed work at an additive level of proportional reasoning for question 2. On question 1, Bob appeared to assume the shape was a square, despite the measurements provided in the drawing. For question 2, Bob attempted to use subtraction to find the missing side length; however, Bob used measures from the same unit called short sticks instead of establishing a relationship between short sticks and long sticks. Figure 82 gives Bob's work on John's School task.



Study Student Number 201

John's School

John used short and long sticks to measure his mathematics classroom. He drew these pictures:

I

1. What is the width of the room in short sticks?

60 short sticks

2. Explain how you found your answer.

A square in equal all around, so all sides sould be the same.

John used a different set of short and long sticks to measure his school's garden. He drew these pictures:

A

3. What is the length of the base of the garden measured in short sticks?

50 short sticks

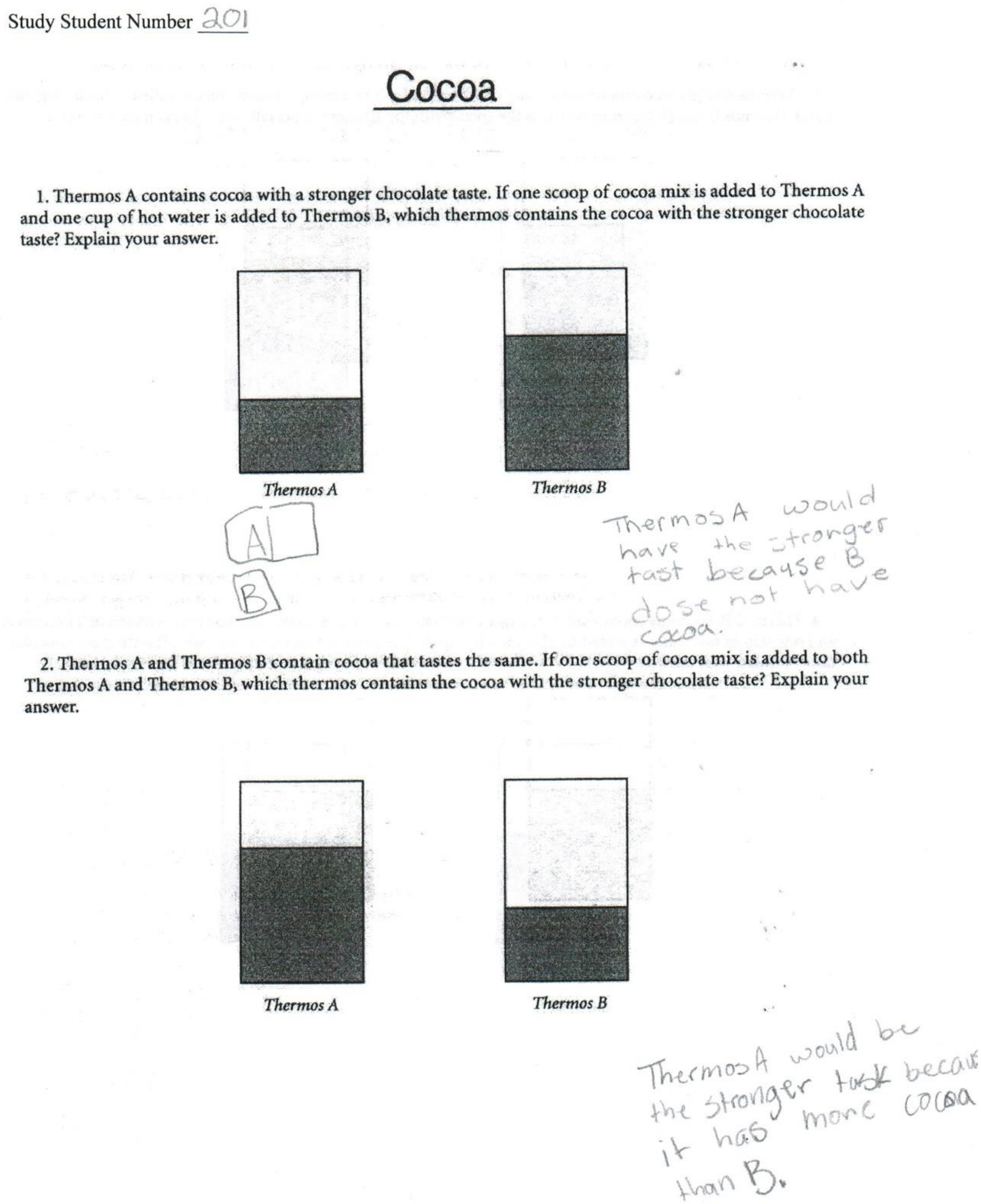
125
- 75
50

4. Explain how you found your answer.

I did 125 minus 75 and got 50.

Figure 82. Bob's work on John's School task.

Post-instruction interview. On the Cocoa task, Bob answered two questions correctly out of the four questions presented; however, some of the reasons that he provided on the correct responses indicated that he selected the correct thermos without having an adequate explanation to support his choice. For instance, Bob chose Thermos A as having the stronger chocolate taste for question one because he said Thermos B didn't have any cocoa in it, only water. Bob's work with the other questions showed an inconsistency in applying proportional reasoning to the situation. For instance, Bob responded as follows on question three: "Thermos B would be stronger because if Thermos A has a weaker taste, then it is just more stronger than Thermos B." This response demonstrated increasing the ratio of cocoa mix to water strengthens the chocolate taste of the cocoa in the thermos. On a different question, Bob's response showed a lack of understanding of proportional reasoning: "Thermos A has more cocoa in it than B; more cocoa would be stronger." This response implied more volume of cocoa meant an increase of chocolate taste. When asked about the use of technology to answer the questions from the Cocoa task, Bob indicated that it would be easier to work on paper, especially if there is a lack of understanding on how to work with the technology. Figures 83 and 84 display Bob's work on the Cocoa task.



Study Student Number 201

Cocoa

1. Thermos A contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

A

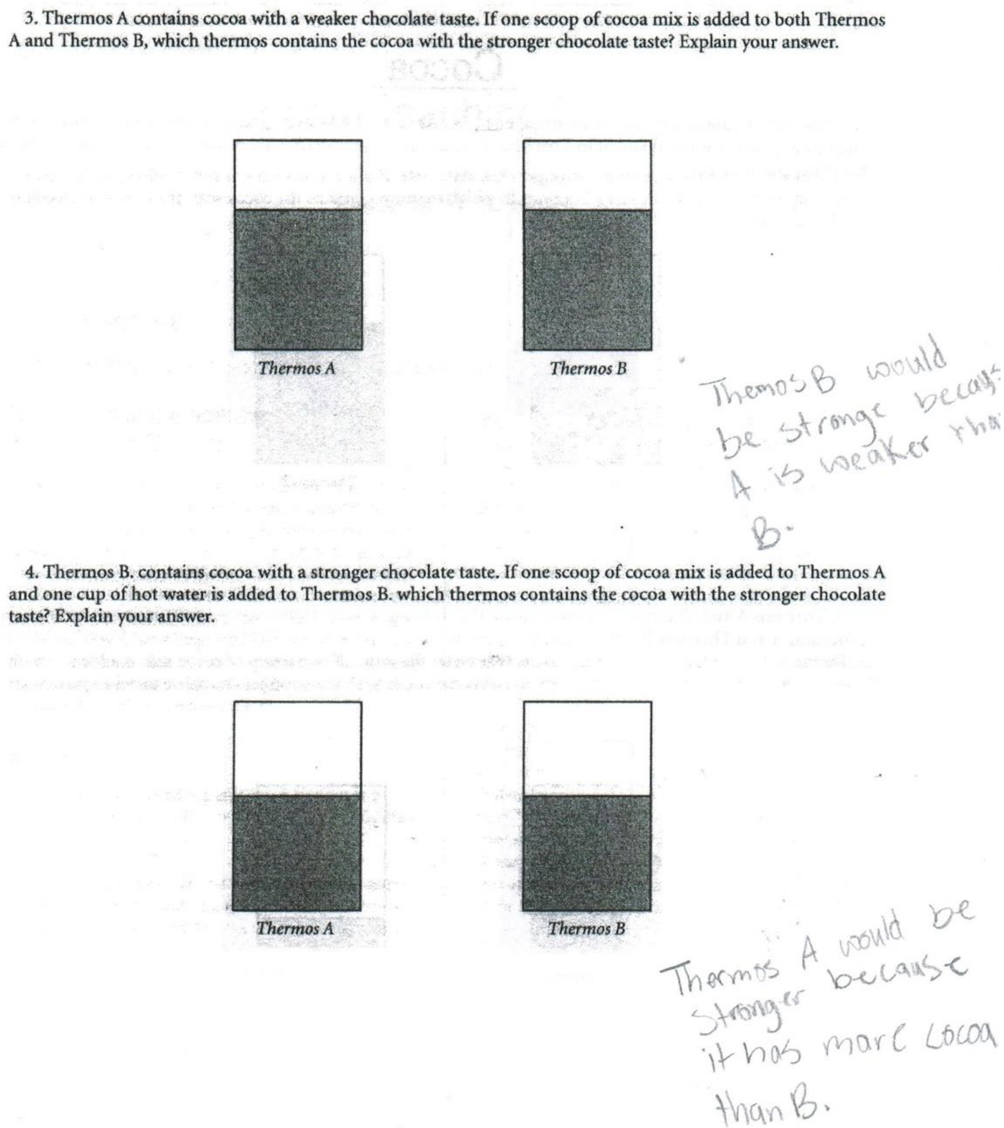
B

Thermos A would have the stronger tast because B dose not have cocoa.

2. Thermos A and Thermos B contain cocoa that tastes the same. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Thermos A would be the stronger tast because it has more cocoa than B.

Figure 83. Bob's work on the Cocoa task, page 1.



3. Thermos A contains cocoa with a weaker chocolate taste. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Themos B would be stronge becaus A is weaker tha B.

4. Thermos B. contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B. which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Thermos A would be stronger because it has more Cocoa than B.

Figure 84. Bob's work on the Cocoa task, page 2.

Summary. Bob demonstrated that he could work both independently and collaboratively during the study, but he also showed that he worked competitively. While working a problem using a block model strategy, Bob asked his table partner how they worked the problem without drawing a block model. The table partner indicated that he thought the problem was easy; Bob responded, “I think it’s easier using blocks.” Bob actively sought Ms. Xanth’s assistance when he had a question. Whenever Bob asked a question and received affirmation, he indicated his understanding with actions or statements. For instance, Bob stated “Now I get it; it’s kind of easy if you get it.” On another occasion, Bob indicated his success in answering a question by yelling out “Yes!” and pulling his elbows in to his side. During work with ratios, Bob recognized 7:4 and 4:7 represent different ratios, but he did not seem to recognize one may write either ratio with values from the same problem. Bob demonstrated inconsistent development of proportional reasoning skills while completing the five tasks assigned during this study.

Multiple Cases

Individual case studies permitted investigation of each target student; although such an investigation was informative, it limited the search for patterns or themes. By examining the six target students as a group or by forming various subgroups, the researcher considered potential themes and patterns associated with these groupings. In this section, summaries of the two treatment groups and the control group are presented, along with a summary of target student performance as a group.

Summary of the iPad Group Target Students. At the beginning of this study, both Alice and Alan generated work at the additive level of proportional reasoning. Both participants demonstrated that they worked with the virtual manipulatives without difficulty and were able to build block models to answer proportional reasoning questions using both Thinking Blocks and Number Pieces. Although Alice exhibited growth with respect to the development of proportional reasoning skills, her growth was not consistent; however, Alan's growth appeared to be more consistent in that he progressed and maintained work at the ratio level as evidenced by the classroom tasks. Both Alice and Alan indicated development of proportional reasoning skills on the Cocoa task by the use of academic vocabulary and rudimentary comparisons concerning the strength of chocolate flavoring in the respective thermoses.

Summary of the Mouse Group Target Students. As with the iPad group, both Candy and Carl generated work at the additive level of proportional reasoning at the beginning of this study. When using Thinking Blocks and Number Pieces, Candy demonstrated her ability to use these virtual manipulatives in order to answer proportional reasoning word problems. Although Carl completed the modules in Thinking Blocks, he encountered some difficulty in using the block modeling strategy in the Number Pieces setting. Candy and Carl both exhibited signs of development of proportional reasoning skills during this study. Samples of work from the various performance tasks and statements made during classwork instruction support this observation. While Candy showed consistent development and progress from the additive level to the transitional and ratio levels of proportional reasoning, Carl's work indicated a lack of consistency from task to task.

Summary of the Pencil and Paper Target Students. Just like the target students in the treatment groups, both Betty and Bob demonstrated work at the additive level at the beginning of this study. They both appeared to apply the block modeling strategy to proportional reasoning problems successfully. Betty's work indicated a readiness to apply proportions in solving word problems while Bob's work demonstrated a more gradual transition to the use of proportions. Although both participants exhibited growth with respect to proportional reasoning as determined by the five performance tasks, Betty's growth appeared to be more consistent than Bob. With respect to the Cocoa task, both Betty and Bob seemed to relate strength of chocolate taste to the volume of cocoa in the thermoses.

Summary of Target Student Performance

The six target students completed the pre-test, post-test, and performance tasks along with the other 50 participants; as such, their results were included in the overall data analyses presented in this chapter. However, inclusion of the performance data for the target students as a separate group provided the opportunity to identify potential themes and patterns associated with this group of target students. Table 34 summarizes all target students' performance on the pre-test and post-test, while Table 35 presents the level of proportional reasoning on the five performance tasks.

Table 34

Performance of Target Students on Pre-Test and Post-Test

Group	Student	Pre-test Score	Post-test Score	Difference
iPad	Alice	28.57	53.57	25.00
	Alan	67.86	75.00	7.14
Mouse	Candy	53.57	57.14	3.57
	Carl	32.14	39.29	7.15
Pencil-and-Paper	Betty	78.57	85.71	7.14
	Bob	32.14	53.57	21.43

Although all of the six target students improved their scores from the pre-test to the post-test, no additional patterns emerged with respect to control group or treatment group, gender, or technology-input modality. Alice, a female from the iPad group, made the largest gain; however, Bob, a male member of the control group, also made a large gain from pre-test to post-test. Candy and Carl from the Mouse group made minimal gains using technology as opposed to the other four target students. Based upon this data, performance increase as measured by the assessment instrument cannot be attributed to presence or lack thereof of technology use.

Table 35

Levels of Proportional Reasoning of Target Students on Performance Tasks

Student	Mr. Tall/Mr. Short	Egg Carton	Tree House	Sticks and Rhombi	John's School Q1	John's School Q2
Alice	A	Tr	I	R	R	A
Alan	A	R	Tr	R	R	R
Candy	A	R	Tr	R	R	Tr
Carl	A	Tr	I	R	I	I
Betty	I	Tr	Tr	R	R	R
Bob	A	Tr	R	Tr	I	A

Note. Level I = Illogical, Level A = Additive, Level TR = Transitional, Level R = Ratio

Regarding target students' proportional reasoning level on the classroom tasks as a group, no pattern emerged based upon unanimous results in a task. When variation from the group occurred on a task, usually a different student contributed the variation. For instance, Betty demonstrated work at the Illogical level on the Mr. Tall/Mr. Short task while Bob's work on the Sticks and Rhombi task was the only result rated at the Transitional level. No target student produced work consistently at the same level of proportional reasoning; in addition, the development of the proportional reasoning from task to task for each student did not follow any given pattern. Neither gender nor technology use generated any particular patterns.

Whole Group Patterns

Stake (2006) asserts "in multicase study research, the single case is of interest because it belongs to a particular collection of cases (p. 4)." The six target student cases developed for this study provide a sketch for each student individually, but any pattern for the group as whole emerged only when studying the target student data collectively. For this study, two major themes from the group of six target students were realized: (1) Additive reasoning is an initial approach to solving a proportional reasoning problem; and, (2) uncertainty (Zaslavsky, 2005) results from using an additive approach to solve a proportional reasoning problem.

Additive reasoning and proportional settings. Prior to the instructional phase of this research, six target students were selected based upon the results of a pre-test. All six target students were interviewed separately on the same day; the focus of the interview was the completion of the Make a New Puzzle task. As part of this task, each target student was required to enlarge a segment of length four centimeters to a new segment of length seven centimeters. Once the student decided how they would change from four centimeters to seven centimeters, they would apply this process to the remaining segment lengths from the original set of puzzle pieces.

All six target students applied the same additive reasoning to the problem at hand; by adding three centimeters, the segment of length four centimeters was enlarged to a new segment of length seven centimeters. Although this reasoning resulted in new side lengths for all of the new puzzle pieces, the additive approach did not preserve proportionality as seen in the original side lengths. Proportional reasoning requires the application of a multiplicative strategy in order to enlarge the side lengths in such a way as to maintain the ability for the puzzle pieces to be assembled into a square. Specifically, the target student should have multiplied the value of four centimeters by seven fourths in order to create a new segment of length seven centimeters. Multiplying seven fourths with the lengths of all the original segments generates the new side lengths of the lengthened segments.

Uncertainty in proportional settings. All six target students employed an additive approach when attempting to complete the Make a New Puzzle task. After these new side lengths were determined, the target students were directed to assemble the new puzzle pieces into a square, just like the first set of puzzle pieces. The target students attempted to assemble the puzzle pieces as requested, but each one discovered rather quickly that the puzzle pieces did not form a square. The hesitancy and confusion expressed by each target student indicated their expectations of forming the square were in place, but the realization produced by the difficulty encountered in assembling the pieces conflicted with these expectations. The target students attempted to rectify the situation by consulting the model to see if they had somehow misplaced one or more of the puzzle pieces; furthermore, in an attempt to make the puzzle pieces form a square, the target students abandoned the model and rearranged the pieces in different configurations. The result was always the same: the new puzzle pieces did not assemble into a square as the original puzzle pieces did. The hesitancy and confusion exhibited by the six target students is described in the literature as a type of uncertainty (Zaslavsky, 2005).

The foundation for the concept of uncertainty can be traced to Dewey and Piaget in their ideas of reflective thinking and disequilibrium (Zaslavsky, 2005). Festinger (1957) advanced a theory concerning cognitive dissonance and the resolution of such conflict, whereas Berlyne (1960) advocated conceptual conflict as a component in the acquisition of knowledge. However, the behaviors seen in the six target students can best be described as an unknown path or a questionable conclusion uncertainty. Zaslavsky (2005) described this type of uncertainty as being associated with inquiry, exploration tasks, and open-ended problems. With respect to the group, the uncertainty experienced by the six target students provided an opportunity to reconsider an additive approach for completing the Make a New Puzzle task.

Subgroup Pattern

Attention focused upon subgroups composed of members that represent points of interest for this study; namely, target students who worked with virtual manipulatives and target students who did not work. Alice, Alan, Candy, and Carl comprise the virtual manipulative subgroup, while Betty and Bob belong to the subgroup that did not implement virtual manipulatives while developing proportional reasoning skills. One major pattern that distinguishes the two subgroups is the use of proportional reasoning concepts in the working of the Cocoa task.

Betty and Bob, members of the subgroup that did not use virtual manipulatives during the study, both made statements during the completion of the Cocoa task relating volume to strength of chocolate task in the cocoa. On question two, Betty stated "…Thermos A had the stronger taste because there's more cocoa and it says that the cocoa tastes the same." For the same question, Bob replied "Thermos A would taste the strongest because it has more cocoa than Thermos B." In the same vein, Bob indicated "Thermos A has more cocoa in it than B; more cocoa would be stronger." It is possible that confusion arose concerning the difference between cocoa and cocoa mix.

Alice, Alan, Candy, and Carl all made statements that incorporated proportional reasoning relationships or academic vocabulary as they answered questions on the Cocoa task. While working with question three on the Cocoa task, Candy said "Thermos A contains the weaker, it's still going to be weaker, even with another scoop because both of them got the exact same scoop of cocoa added to them." Alice stated "the more water you add to Thermos B, the more the particles move around" when working with question four. In his respond to question one, Alan replied "…I believe Thermos A will be stronger because if you add one scoop of cocoa, it will have a stronger taste because the chocolate mixed with the cocoa just means more chocolate." On question four, Carl decided that Thermos A had the cocoa with the stronger chocolate taste "because it doesn't have any hot water to make it less strong." With respect to academic vocabulary, Candy referred to ratios and Thinking Blocks; Alice mentioned ratios; Alan described a process "like drawing a model or doing a proportion"; and, Carl stated using Thinking Blocks could have been helpful in answering questions on the Cocoa task.

Chapter Summary

In this chapter, survey results from the 56 participants disclosed information concerning experience with touchscreen technology and preferences for computer use. A background of the participants concerning performance on a state-mandated summative mathematics assessment revealed previous preparation before partaking in this study. Quantitative and qualitative data analyses provided information as planned in the mixed method approach designed for this study. Descriptive statistics, paired sample *t*-tests, results from a one-way ANCOVA, and results from a 3×2 ANCOVA provided instances in which the differences in mean scores were statistically significant. Levels of development of proportional reasoning on five tasks were determined and reported for all 56 participants. With respect to six target students, performance on a pre-instruction task and a post-instruction task was considered, both individually as six case studies and a multiple case study. Two patterns emerged with respect to the target student group, while a pattern contrasting the virtual manipulative subgroup and the subgroup that did not use virtual manipulatives was reported.

With respect to complementarity, the qualitative data provided elaboration concerning the results from the quantitative data. An increase in mean score values from the pre-test to the post-test suggested growth occurred with respect to the development of proportional reasoning for all groups identified in this study, but the disaggregation of development using levels of proportional reasoning in the qualitative data further explained this growth. Also, the use of performance tasks with the 56 participants as a whole as well as the six target students allowed opportunity to discover variations or inconsistent growth reflected in the answers and explanations provided by participants.

CHAPTER V: SUMMARY AND DISCUSSION

Introduction

The purpose of this study was to investigate any impact virtual manipulatives had on grade six students' development of proportional reasoning skills; in addition, this study aimed to determine any influence the use of Thinking Blocks and Number Pieces made with respect to gender and technology-input modality. In this final chapter, a review of the major findings of this study is presented along with a summary of the results. The research questions are addressed; implications from the study are considered as well. Connections to the literature and recommendations for potential future research are made.

Research Questions and Methodology Review

As stated in Chapter I, the research goals of this dissertation were: (a) to measure the impact of the use of virtual manipulatives on the development of grade six students' proportional reasoning skills; (b) to investigate whether gender of students exhibits any main effect when using virtual manipulatives to develop proportional reasoning skills; (c) to investigate whether technology-input modality (touchscreen or mouse) yields any differences when using virtual manipulatives to develop proportional reasoning skills; and (d) to investigate whether the factors of gender and technology-input modality interact when using virtual manipulatives to develop proportional reasoning skills. In order to accomplish these goals, the following research questions were considered:

1. In general, what gains are made by grade six students when virtual manipulatives are used to teach certain aspects of proportional reasoning?

2. What differences exist with respect to gender when using virtual manipulatives to teach proportional reasoning to grade six students?
3. What differences exist between students who use touch technology and those who use mouse technology when studying proportional reasoning?
4. What interactions exist between gender and technological input modality when students use virtual manipulatives when studying proportional reasoning?
5. How do grade six students who use virtual manipulatives differ from those grade six students who do not use virtual manipulatives when developing proportional reasoning skills?

Grade six students, taught by the same mathematics teacher at a local middle school, participated in this study. The structure of the study implemented a mixed method approach, while a convergent parallel design provided the structure for gathering quantitative and qualitative data at the same time. Participants took both a pre-test and post-test; additionally, as a part of regular classroom instruction, participants completed five tasks to assess their development of proportional reasoning skills. Prior to the instructional phase of this study, participants completed a survey in order to determine experience with touchscreen technology as well as to ascertain preference for the type of input for computer use. In addition, the researcher observed a focus group comprised of six target students during the study. As part of this observation, each target student took part in interviews before the start of the instructional phase of this study and at the conclusion of this study. Each target student interview focused upon the completion of a proportional reasoning task. The students completed the interviews individually; the researcher conducted no group interviews as part of this study.

Research Questions

To achieve the research goals set forth in this study, five research questions guided the implementation of the structure and design necessary to complete the study. Gathering and analyzing of both quantitative and qualitative data provided the information needed to answer the research questions. In this section, each question is revisited and answered.

Question 1

In general, what gains are made by grade six students when virtual manipulatives are used to teach certain aspects of proportional reasoning? Although the mean score increased 6.57 percentage points from the pre-test to the post-test for all 56 participants, the mean score for participants using Thinking Blocks and Number Pieces during the study increased 8.83 percentage points from the pre-test to the post-test. Those participants who did not use technology during the study showed an increase of only 1.79 percentage points from the pre-test to the post-test.

On the five tasks, participants demonstrated overall growth with respect to working at the various levels of proportional reasoning development. For the first task (Mr. Tall/Mr. Short), only one student produced work that indicated effort at the ratio level. However, on the last task (John’s School, Q1), 42.1% of participants who used Thinking Blocks and Number Pieces during the study produced work that indicated effort at the ratio level as compared to 33.3% of participants who did not use virtual manipulatives during the study produced work that indicated effort at the ratio level.

For the six target students, each one showed work at the additive level of proportional reasoning on the Make a New Puzzle task during their individual interviews before the instructional phase of the study. During the closing interviews, the four target students who worked with Thinking Blocks and Number Pieces made statements that indicated a degree of understanding concerning proportional reasoning concepts on the Cocoa task; some of these statements demonstrated correct use of academic vocabulary while others expressed the ideas in general terms.

Question 2

What differences exist with respect to gender when using virtual manipulatives to teach proportional reasoning to grade six students? Male participants who used Thinking Blocks and Number Pieces during this study had a mean score 14.2 percentage points higher on the post-test as compared to male participants who did not use virtual manipulatives, but this increase failed to demonstrate statistical significance. Female participants scored 3.88 percentage points higher on the post-test as compared to those females who did not use virtual manipulatives; again, this increase did not attain statistical significance. Also, the difference in mean scores on the post-test with respect to gender failed to achieve statistical significance.

When considering gender of the participants and the level of proportional reasoning exhibited while working with the five tasks, 77.8% of male participants produced work at the additive level on the Mr. Tall/Mr. Short Task while 78.3% of female participants produced work at the additive level on the same task. However, after using Thinking Blocks and Number Pieces during the study, 25% of males and 22.2% of females produced work at the additive level of proportional reasoning on the Johns' School Task, Q1. For the control group on the same task, 45.5% of males and 0% of the females produced work at the additive level of proportional reasoning. Therefore, although the percentage of participants producing work at the additive level decreased for both genders from the first task to the last task of the study, males in the treatment group and females in the control group demonstrated the greater decreases in percentages operating at the additive level.

Considering all participants who used an illogical approach from the first task to the last task, the percentage increased from 14% on the Mr. Tall/Mr. Short task to 23.2% on the John's School, Q1 task. However, when considering gender and the control group, males demonstrating work with an illogical approach showed a decrease from 25% on the first task to 18.2% on the last task (Q1) while females producing work with an illogical approach showed a decrease from 16.7% on the first task to 14.3% on the last task (Q1). For the treatment groups, males using an illogical approach showed an increase from 5.3% on the first task to 35% on the last task (Q1); but, females implementing an illogical approach demonstrated a slight decrease on the percentage of performance, going from 17.6% on the first task to 16.7% on the last task (Q1). So, the percentage of females, working at the illogical level in developing proportional reasoning skills and using virtual manipulatives, decreased in both the control group and treatment groups. Nonetheless, this percentage decrease was scant.

Question 3

What differences exist between students who use touch technology and those who use mouse technology when studying proportional reasoning? When considering the post-test average for each treatment group, although statistically not significant, participants in the iPad Group had a mean score of 50.00% while the participants in the Mouse Group had a mean score of 47.37%. Although both of these mean score averages were higher than the Pencil-and-Paper Group average of 38.89%, the difference between the mean score averages for the treatment groups and the control group failed to achieve statistical significance.

With respect to the five tasks used during the study, 47.4% of the participants in the iPad Group exhibited work at the ratio level of proportional reasoning on John's School Task Q1; for the Mouse Group, 36.8% of the participants exhibited work at the ratio level of proportional reasoning. For John's School Task Q2, 21.1% of participants in the iPad Group and 10.5% of participants in the Mouse Group exhibited work at the ratio level of proportional reasoning. This data suggests the touchscreen technology of the iPad group supported the development of proportional reasoning more than the technology of the Mouse group.

Question 4

What interactions exist between gender and technology-input modality when students use virtual manipulatives when studying proportional reasoning? Male participants in the iPad Group had a mean score of 59.82% on the post-test, compared to female participants in the iPad Group who had a mean score average of 42.86% on the post-test; however, females who used pencil and paper or a mouse during the study had a higher mean score average on the post-test than the males who used pencil and paper or a mouse. For the Pencil-and-Paper Group, females had a mean score average of 42.35% and male participants had a mean score average of 36.69%. In the Mouse Group, female participants had a mean score average of 51.53% while the male participants had a mean score average of 44.94%. Data analysis conducted with a 3 × 2 ANCOVA indicated that any interaction effect between gender and technology-input modality failed to achieve statistical significance.

Question 5

How do grade six students who use virtual manipulatives differ from those grade six students who do not use virtual manipulatives when developing proportional reasoning skills? All participants implemented a block modeling strategy during this study, either with Thinking Blocks, Number Pieces, or traditional practice with pencil and paper. Descriptive statistics, paired sample *t*-tests, results from a one-way ANCOVA, and results from a 3 × 2 ANCOVA provided opportunity to determine when the differences in mean scores were statistically significant. Although participants who used virtual manipulatives had a higher mean score on the post-test as opposed to those participants who did not, these differences were found overall not to be statistically significant.

Levels of development of proportional reasoning on five tasks were determined and reported for all 56 participants. Prior to classroom instruction, the majority of participants in all three groups generated work at the lower levels of proportional reasoning on the Mr. Tall/Mr. Short task: 88.9% of the participants in the iPad Group, 94.1% of the participants in the Mouse Group, and 93.4% of the participants in the Paper-and-Pencil Group produced work at the illogical or additive level. At the end of classroom instruction, the work generated for the two questions which comprised John's School task indicated some participants had advanced in their level of proportional reasoning. On question 1, 36.9% of participants in the iPad Group produced work at the transitional or ratio levels, 42.1 % of participants in the Mouse Group produced work at the transitional or ratio levels, and 55.5% of participants in the Paper-and-Pencil Group produced work at the transitional or ratio levels. For question 2, 36.9% of participants in the iPad Group produced work at the transitional or ratio levels, 31.6% of participants in the Mouse Group produced work at the transitional or ratio levels, and 33.3% of participants in the Paper-and-Pencil Group produced work at the transitional or ratio levels.

In their pre-study interviews, all of the six target students used the same additive approach when working with the Make a New Puzzle task at the beginning of the study. However, differences occurred as the target students completed the Cocoa task during the post-study interviews at the conclusion of the study. The four target students who worked with virtual manipulatives during the study responded with statements as they completed the Cocoa task indicating a rudimentary understanding of proportional reasoning concepts. The two target students who did not work with virtual manipulatives indicated volume of cocoa related to strength of chocolate taste in the cocoa.

Findings

The data obtained from this study were classified as either quantitative or qualitative; however, the nature of the convergent parallel design required one to consider both types of data to address the research questions. So, in order to view a more complete perspective, reference to both types of data is made in these findings. No preference is given to either type of data in this summary.

Based upon the data, the following points represent the findings in this study:

- Participants who used virtual manipulatives developed proportional reasoning skills as well as or better than those participants who worked with traditional pencil and paper;
- Gender of participants did not serve as a significant influence when using virtual manipulatives to develop proportional reasoning skills;
- Technology-input modality did not account for a significant influence when using virtual manipulatives to develop proportional reasoning skills;
- Interaction between gender of participants and type of technology-input modality did not aid as a significant influence when using virtual manipulatives to develop proportional reasoning skills;
- Participants initially seemed to resort to an additive approach when addressing a proportional reasoning task; and,
- Participants who encountered a proportional reasoning task in which they were unsure how to proceed resorted to an additive approach, which resulted in a state of uncertainty, especially if non-integral values were involved.

Discussion

The nature of research demands that results from a study are not maintained in a vacuum; connection and application from the study's results should link to the extant body of literature and potential use in appropriate settings. In this section, insights from the study data are presented, as well as the study's limitations. Additionally, findings from the current study are linked to prior research in the literature. Lastly, implications for practice are suggested, along with recommendations for further research.

Insights from the Study

Drawing block models versus modeling with virtual manipulatives. Although all three groups involved in the study were taught how to draw a block model when solving proportional word problems, the control group drew all of the block models by hand while the treatment groups interacted with Thinking Blocks to create block models. While Ms. Xanth taught all three classes, there were differences in place due to the use of technology or lack thereof. The greatest difference observed from the control group to the treatment groups involved the level of support accessible to participants.

For the control group, a typical day of classroom instruction involved drawing block models while answering proportional word problems. For each different type of word problem, Ms. Xanth played a tutorial video for the participants in order to introduce the block modeling approach for the particular problem type. Additionally, Ms. Xanth would complete a practice problem with the participants. Once participants completed the review for the day, they then would start working on the assigned proportional word problems. Ms. Xanth walked around the room as participants worked in order to be available whenever questions were asked.

Participants in the treatment groups viewed the tutorial videos from the Thinking Blocks website, just like the students in the control group. Also, Ms. Xanth started the treatment group classes with review problems, again like the control group. In addition to the support Ms. Xanth supplied, participants in the treatment groups accessed step-by-step reinforcement as they completed the modules of word problems in Thinking Blocks. When the treatment group participants started work on the Number Pieces website, students drew block models on the computer before sketching the models on paper. One could argue the treatment groups experienced additional practice with the block modeling strategy by working problems on the computer and paper, as well as accessing additional support on the Thinking Blocks website as they worked problems from the modules. Participants in all three groups completed word problems at their own rate and received support from Ms. Xanth as they worked, but students in the treatment groups did not have to wait for Ms. Xanth to receive step-by-step support from the Thinking Blocks website.

Virtual manipulatives as cognitive tools. As introduced in Chapter I within the discussion of the conceptual framework, virtual manipulatives function as cognitive tools. A cognitive tool transcends the level of just providing information; it is a resource that is specifically designed to allow students to achieve particular learning goals on a topic of interest. The results from this study's pre-test and post-test revealed that all groups demonstrated an increase in their mean scores; but, the two groups using virtual manipulatives showed higher gains in their mean scores from the pre-test to the post-test.

Additionally, the four target students who used virtual manipulatives demonstrated growth with respect to proportional reasoning by their descriptions of ratios in the Cocoa task and their use of proportional reasoning vocabulary. Both the gains realized by the treatment groups' participants and by the growth of the target students support the premise that virtual manipulatives serve as a cognitive tool.

One aspect of virtual manipulatives emerged during the study supporting the cognitive tool concept - namely, the step-by-step support provided to students as they worked with Thinking Blocks. Ms. Xanth made herself available to all participants who needed assistance as they worked; however, Ms. Xanth was a limited resource because she could not be available to all of the participants at the same time. For those participants working with the Thinking Blocks website, step-by-step support and feedback were available to them. Granted, participants could work mechanically in completing work each word problem on the Thinking Blocks website and attempt to finish the work only intending to satisfy Ms. Xanth's expectations. Nevertheless, feedback given to students through the website and repetition of effort on the various word problems provided the participants with the experience of solving word problems with a proportional reasoning emphasis.

Levels of proportional reasoning. As demonstrated in the literature, students develop their proportional reasoning skills in levels rather than in an "all or nothing" situation (Karplus et al, 1977; Khoury, 2002; Langrall & Swafford, 2000). The work generated on the various tasks used in this study illustrated this point: participants' work on tasks indicated a certain level of proportional reasoning. One observation gleaned from the work on the proportional reasoning tasks was the inconsistency demonstrated by participants with respect to the levels of proportional reasoning. Instead of increasing steadily from a lower level to a more advanced level of proportional reasoning, most participants generated work that would fluctuate; on one task, the level of proportional reasoning would be Transitional, while on the next task the level of proportional reasoning indicated would be Illogical. For example, the target student Bob's work on the five tasks during the classroom instruction did not develop consistently as instruction progressed; instead, no pattern of proportional reasoning emerged from his work. The type of task and the specific values involved in the tasks might have contributed to this fluctuation; in fact, the order of the tasks could have served as a factor in the fluctuation of these levels. One possibility for future research involves the selection of different proportional reasoning tasks or changing the order of the tasks used in this study.

Additive reasoning. A result found in previous studies emerging from this research was the tendency for participants to apply additive reasoning to a proportional reasoning setting. On the Mr. Tall/Mr. Short task, 78% of the participants used an additive approach to find the height of Mr. Tall in paper clips. Another example of an additive approach used in a proportional setting concerned the initial interviews of the six target students. The Make a New Puzzle task required students to enlarge a segment with length 4 cm to a new segment with length 7 cm. Based upon their experiences, the target students tended to recognize the additive relationship and applied it to the remaining segments to form the new segment lengths, which did not maintain the proportional relationship that existed with respect to the original square puzzle.

Of course, the additive relationship between the values of 4 and 7 is not the only relationship that exists between these two numbers; in fact, one can express an infinite number of relationships beginning with an input of 4 and ending with an output of 7 and can depict these relationships by the pencil of lines containing the point (4, 7). According to postulates of plane geometry, there are an infinite number of lines that contain a particular point; in this case, the point in question is (4, 7). Figure 85 illustrates a partial expression of the pencil of lines containing the point (4, 7).

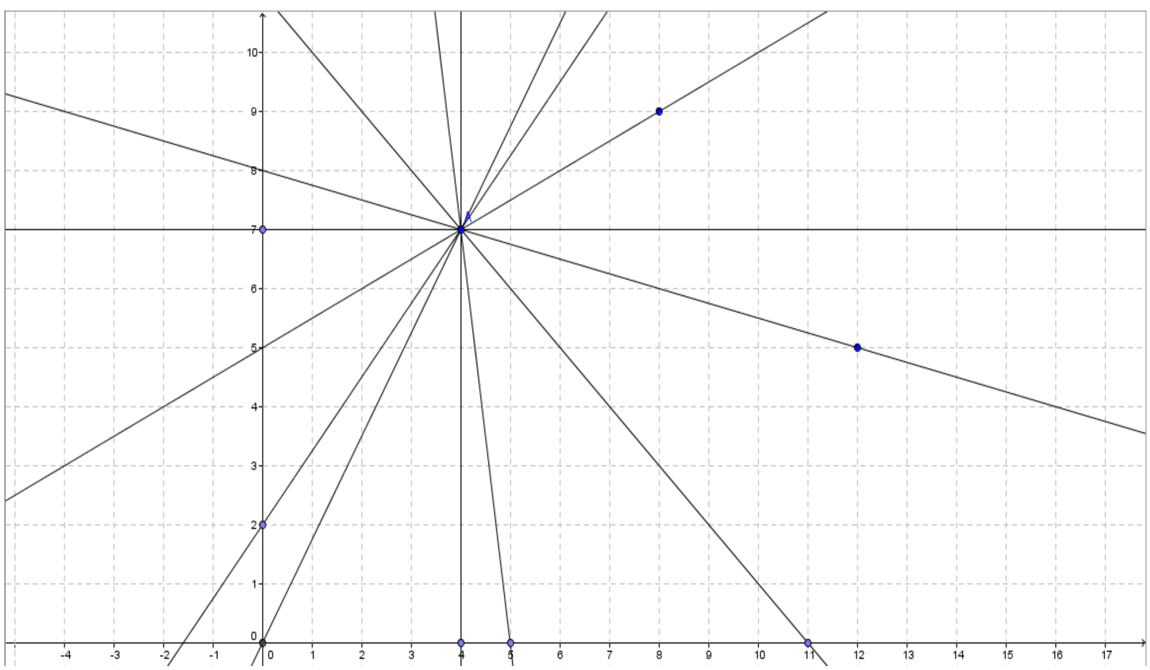


Figure 85. Lines from the pencil containing (4, 7).

When the target students were posed the question in the Make a New Puzzle task, these students were in essence asked to select a single line that contains the point (4, 7) in the Cartesian Graph Plane and also models a proportional relationship. Since the students only had a single point with which to work, it seemed natural for these students to rely upon their background and select the additive relationship. In this case, the additive relationship is modeled by the function $f(x) = x + 3$; the original side lengths from the task are values of the independent variable and the new side lengths are the corresponding values of the dependent variable. If the target students had been given two side lengths to enlarge, they most likely would not have selected the additive relationship. For instance, if the Make a New Puzzle task had asked the student to change 4 cm to 7 cm and to change 8 cm to 14 cm with the same rule, an additive relationship would not satisfy the requirement.

The structure of the Make a New Puzzle task used in this study permitted the six target students to pursue a relationship between the values of 4 cm and 7 cm. The selection of an additive relationship between these two numbers created a situation in which uncertainty emerged concerning the preservation of proportionality from the original puzzle pieces to the new set of puzzle pieces. This situation of uncertainty allowed the six target students an opportunity to consider relationships other than additive, which consideration was suggested by the researcher at the end of the pre-instruction individual interviews.

Connections to Previous Studies

The results from the data gathered during this research connect to past research in various ways; such connections may lead to affirmation of past research or present contradiction of results from past studies. In this section, a comparison is made to results from previous work in the development of proportional reasoning skills, specifically noting the various tasks used for this study. Also, results from studies using virtual manipulatives are revisited.

Development of proportional reasoning skills. According to de la Cruz (2013), teachers can assist students to develop a conceptual understanding of proportional relationships by postponing the introduction of the cross-multiplication algorithm; instead, engaging the student in well-designed problem situations in a proportional reasoning setting should be considered. One particular task found in the literature that relates to the development of proportional reasoning is the Mr. Tall/Mr. Short task (Khoury, 2002). Riehl and Steinthorsdottir (2014) report that the Mr. Tall/Mr. Short task appeared in the work of Robert Karplus and his colleagues in the late 1960's; Karplus, Karplus, and Wollman (1974) revisited and refined this task, because its structure did not require any understanding of physical principles that other Piagetian tasks possessed at the time.

When compared to results obtained from previous research with the Mr. Tall/Mr. Short task, more participants in this study produced work using an additive approach. Specifically, 78% of 50 participants utilized an additive approach and 2% demonstrated a use of a ratio approach when completing the Mr. Tall/Mr. Short task. Karplus, Karplus, and Wollman (1974) reported that 32% of 610 students in grades 4-9 implemented an additive approach and 37% used a ratio approach when working with the Mr. Tall/Mr. Short task; similarly, 44% of 412 students in grades 5-8 used an additive approach and 29% solved the Mr. Tall/Mr. Short task with a ratio approach in the study conducted by

Riehl and Steinthorsdottir (2014). It is noted that participants in this study were only in grade six, while students from grades 4-9 were included in the other studies. When narrowing the results from the Riehl and Steinthorsdottir (2014) study to students in grade six, 60% of the participants used an additive approach and 16 % generated a correct response using a ratio approach for the Mr. Tall/ Mr. Short task. The finding that grade six participants in this study seemed to apply an additive approach while trying to solve a proportional reasoning problem confirmed findings from previous studies.

An observation gathered from several studies in the literature is the tendency for students to revert to the use of an additive approach for solving proportional reasoning problems when non-integral ratio values are encountered (Karplus, Karplus, & Wollman, 1974; Pulos, Karplus, & Stage, 1981; de la Cruz, 2013; Singh, 2000). Tjoe and de la Torre (2014) proposed students who lack the mastery to think proportionally will fall back to additive reasoning when posed with a problem in which one part of the proportion is missing. With respect to the John's School Task used in this study, a similar effect was observed. Two questions were posed to participants in the John's School Task: Question 1 (Q1) involved an integral value whereas Question 2 (Q2) involved a rational value. For Q1, 25% of participants used an additive approach in answering the question and 39.3% of participants used a ratio approach. When compared to the results for Q2, in which a non-integral value was involved, an increase of participants using an additive approach occurred. For Q2, 41% of participants used an additive approach in answering the question and 21.4% of participants used a ratio approach. The availability of Thinking Blocks and Number Pieces for those participants who used virtual manipulatives during instruction did not appear to prevent students from reverting to the use of additive reasoning; both the control group and the two treatment groups showed an increase of participants using an additive approach when a rational value was involved. This same pattern of reverting to the use of an additive approach when a non-integral value was involved in the proportional setting was observed with respect to gender with one exception: 25% of male participants in the Mouse group used an additive approach in answering Q1 and Q2.

The Cocoa task used as part of the case studies for the six target students is considered non-numerical in that no numbers appear in the task and no arithmetical calculations are required to complete it. Billings (2001) states “one way that we can help students cultivate proportion sense is to strip problems of numbers, that is, provide nonnumeric proportion problems, which force students to examine the relationships between variables directly (p. 11).” In the literature, there are tasks similar in nature to the Cocoa task: a beverage composed of two parts is mixed in different proportions in two different containers. After flavoring or water is added to the containers, a decision must be made as to which container contains the beverage with the stronger flavor. Billings (2001) has a Coffee task with two carafes; she also created the Cocoa task (Billings, 2002). Noelting (1980) presented a study in which the Orange Juice task was the instrument used to study proportional reasoning in children of various ages. Regardless of the beverage, the idea is the same: the nature of the task is to ascertain whether or not the participant can conclude successfully which container holds the beverage with the stronger taste. In terms of ratios, the container holding the beverage with the stronger taste has a ratio of flavoring to water greater than the ratio of flavoring to water in the container with the beverage of weaker taste.

From the results of the Cocoa task, the four target students who worked with virtual manipulatives generated responses that indicated a rudimentary understanding of the importance of the ratio of flavoring to water. Although these students did not express their responses in formal mathematical terms, they demonstrated understanding of the relationship between cocoa mix and water in their responses. Karplus et al. (1977) findings also showed a lack of sophistication in expressing proportional reasoning and ratio relationships. In contrast, the two target students who were part of the control group generated responses that did not take the ratio concept into account; rather, their statements indicated that a greater volume of beverage was key to having a stronger taste.

Virtual manipulatives studies. Although a myriad of studies in which the development of proportional reasoning is investigated exists in the body of literature, there is a lack of research in terms of which particular types of virtual manipulatives are used in the development of proportional reasoning. In fact, this gap constituted a major reason why this study involved a Thinking Blocks and Number Pieces virtual manipulative component. Thus, a comparison to previous studies in which virtual manipulatives were used to develop other mathematical concepts is discussed herein.

Fleenor, Westbrook, and Rogers (1995) advocated the use of manipulatives over reading, lecturing, and drill in order to develop the mathematical reasoning of students at the middle school level. Also, Tourniaire and Pulos (1985) reported the use of lab activities provided students the opportunity to have a concrete experience with proportions. The use of a block modeling strategy and virtual manipulatives affords such an experience rather than initially implementing a cross-product algorithm approach. As reported previously, participants who used virtual manipulatives throughout this study demonstrated a greater gain from pre-test to post-test as opposed to those who did not; this gain adds support to the claim that virtual manipulatives contribute to the development of proportional reasoning skills.

Additional topics in which virtual manipulatives assisted in conceptual development are found in the literature, including place value (Jolicoeur, 2011), fractions (Moyer-Packenham & Suh, 2012; Moyer-Packenham et al, 2013; Reimer & Moyer, 2005), algebraic relationships (Suh & Moyer, 2007), and geometry (Steen, Brooks, & Lyon, 2006); in science topics, Zacharia, Olympiou, and Papaevripidou (2008) concluded the use of virtual manipulatives enhanced students' conceptual understanding when studying heat and temperature. Also, Klahr, Triona, and Williams (2007) determined in many cases virtual manipulatives were as effective as physical manipulatives in the design of mousetrap cars, an engineering project for middle school students. Findings in this current study support the premise that virtual manipulatives are effective in developing proportional reasoning skills showing: (a) gains demonstrated by participants in the treatment groups on the post-test as compared to the control group; (b) increased number of participants using a ratio approach on performance tasks at the conclusion of the study; and, (c) the language used to express rudimentary proportional reasoning concepts by the four target students who were members of the two treatment groups.

Implications for Practice

Although the use of technology holds a certain appeal for many teachers and students, the implementation of technology into the mathematics classroom just because the equipment is available does not lead to effective teaching practices. A purposeful, well-planned approach for the use of hardware and software must be kept in mind as a prerequisite for the successful application of technology with respect to the development of mathematical concepts. From the results of this study, two implications are evident: (a) virtual manipulatives afford the opportunity to develop proportional reasoning skills in methods different from the traditional methods espoused in many textbooks; and, (b) the type of technology used in the mathematics classroom can be relevant with respect to student preference.

Method for Developing Proportional Reasoning Skills

The literature is replete with articles and research in which mathematical instruction should focus on conceptual development and not merely procedural proficiency (Fleenor, Westbrook, & Rogers, 1995; NCTM, 2000; National Research Council, 2001). Yet, teachers tend to return to methods and materials with which they are familiar instead of overcoming the inertia of change, just as students often revert to the use of an additive approach when dealing with certain proportional situations (Karplus, Karplus, & Wollman, 1974). With respect to the development of proportional reasoning skills, the cross-product algorithm appears as the procedural approach to handling ratio and proportions situation; however, several researchers recommend postponing this procedure until students have engaged in problem solving situations requiring conceptual development (Billings, Coffey, Golden, & Wells, 2013; de la Cruz, 2013).

The block model strategy utilized in this study on the Thinking Blocks and Number Pieces websites permits students to build blocks that model the situation; specifically, the Thinking Blocks website supports the students at each step in solving the problem by providing feedback, redirecting the student when responses are not correct, and tracking progress as students complete problems successfully. The unit value associated with the problem is clearly identified as the value of one block in the model; in addition, the multiplicative relationship inherent in the proportional reasoning process is visible. Once the concept is developed, students can make a connection to a proportion and the cross-product algorithm. On Day 7 of the study, Ms. Xanth introduced the idea of writing and solving proportions after students had the opportunity to explore the basic concept of proportionality with block models.

Type of Technology

As part of this study, two distinct treatment groups were formed: one group used touchscreen iPad computers and the other group used laptop computers controlled with a mouse. Although both groups showed improvement on the mean score from the pre-test to the post-test, the iPad group gained more numerically as compared to the Mouse group. With respect to the technology-input modality, Manches and O'Malley (2012) stated:

> Interaction with objects involves actions, and it is possible that these actions generate motor schemas which provide metaphors from which children develop more symbolic concepts. . . Support for the embodiment of physical actions in numerical concepts has also come from studies looking at gesture

> use. . . Gesture studies continue to provide support for the notion that thinking may be grounded in motor actions and these studies have highlighted possible educational value, for example, in helping assess children's understanding as well as a means for teachers to support children's learning. If manipulating objects generate gestures, using manipulatives may not only provide a medium to support communication between learners and the teacher, but also provide a way to activate certain embodied processes (p. 413).

To an extent, the data gathered during this study supported Manches and O'Malley's view; participants in the iPad group demonstrated more improvement on the post-test than either the Mouse group or the Pencil-and-Paper group. However, this improvement did not demonstrate statistical significance when compared to the improvement on the post-test for the Mouse group. Future studies involving various technology-input modalities have the potential to investigate and reveal statistical significance in other settings or structures.

Another point to consider when distinguishing between touchscreen input and mouse input is the disposition of the students regarding the type of technology they prefer to use. According to the preference of technology survey results obtained from the participants at the onset of this study, 80.9% of the participants indicated that they preferred to work with an iPad. So, if the option of purchasing computers is available, the type of technology-input modality may be a deciding factor in the purchases that school systems and teachers make in the future. Finally, the following conditions influence choices with respect to the use of iPads in the mathematics classroom: (a) touchscreen technology continues to become more readily available; (b) software for touchscreen technology develops and improves; and, (c) students and teachers become more adept with touchscreen technology.

Limitations

As with all research, limitations exist that influence the data, which in turn influence the findings. For this dissertation study, the participants were not selected in a completely random fashion. Instead, the sample would best be described as a convenience sampling. Since the quantitative design is quasi-experimental instead of being truly experimental, the ability to generalize findings to all grade six students was hindered. Also, the timeframe of ten days devoted to this dissertation study may not have been sufficient to measure the development of proportional reasoning skills adequately.

The number of participants in this study was a limitation; small sample size limited the power of the statistical testing. Such limitation possibly contributed to the results leading to a non-significant outcome when considering the impact of gender or technology-input modality upon the difference from pre-test to post-test. The limitation of sample size also extended to any consideration of interaction between the factors of gender and technology-input modality.

When developing the research design, the various tasks selected served different purposes. The nature of each task provided a different aspect of consideration for proportional reasoning; for instance, the Egg Carton task was visually based, while the Cocoa task was non-numerical. The selection of the performance tasks used in this study potentially impacted the results; thus, choosing different tasks would also have impacted the progression of the development of proportional reasoning skills. Similarly, a different ordering of the same tasks may affect the development of proportional reasoning skills as well.

The researcher did not employ any measure of interrater reliability when determining the levels of proportional reasoning for the five classroom tasks completed by the participants during this study. If a second reviewer confirmed the results obtained by the researcher, such confirmation would have contributed an additional layer of credibility to the analysis of qualitative results. As such, the lack of application of interrater reliability resulted in an additional limitation for this study.

Overall, the status of the researcher as a novice with respect to qualitative methods should be viewed as a limitation. The researcher missed opportunities to probe further into target student responses during interviews; once the study concluded, it was not feasible to revisit the target students for additional interviews in order to clarify responses or probe deeper into their thinking about and work with proportional reasoning tasks.

Recommendations for Future Research

The two topics comprising the basis for this study were virtual manipulatives and proportional reasoning. Recommendations for future research could focus on either topic separately, but considerations of both topics in tandem can be considered as well. The following recommendations for future research are listed in no particular order of priority:

- As technology continues to develop and improve, one can examine virtual manipulatives aside from Thinking Blocks and Number Pieces to investigate what impact they have upon the development of proportional reasoning skills;

- One can reorder the tasks used in this study to see what effect such a change might have on the development of proportional reasoning skills;
- One might compare the use of block models, both physical and touchscreen, to determine which type, if any, supports the development of proportional reasoning skills; and,
- One might consider increasing the sample size and changing the design structure from quasi-experimental to experimental.

Summary

Advances in technology afford classroom teachers opportunities to incorporate innovative approaches for teaching students mathematical concepts, especially those topics considered challenging for middle school students to develop. Past research supports the premise that virtual manipulatives are effective tools in developing mathematical concepts in younger children at primary and elementary grade levels; place value, fractions, and algebraic equations are topics that have been the focus of such research. However, research in which virtual manipulatives were incorporated into the development of proportional reasoning skills is lacking.

As a result of this study, the data support virtual manipulatives as an effective tool in developing proportional reasoning skills in grade six students. Hence, instead of using the cross-product algorithm in solving proportions with a missing value, the block modeling strategy implemented in this study in a virtual manipulative context permitted students to visualize the relationship between quantities in a proportional situation, which in turn allowed students access to conceptual development of proportional reasoning.

Although this block modeling strategy exists separately from technology, virtual manipulatives makes this approach a more viable option for use in the classroom. Neither gender nor technology-input modality was revealed as a source of significant influence when using virtual manipulatives in developing proportional reasoning skills; this conclusion indicates researchers and teachers should consider other factors when incorporating virtual manipulatives in the development of proportional reasoning skills, such as students' preference for touchscreen technology over mouse technology.

It is imperative teachers make decisions that positively impact the development of mathematical concepts in students, including the materials used in the classroom. Not only are virtual manipulatives available for use in the mathematics classroom, this research demonstrated they are also an effective alternative for use in developing mathematical concepts. Given the completion of this study, the development of proportional reasoning skills joins the list of mathematical concepts in which virtual manipulatives assist students to learn.

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APPENDICES

APPENDIX A

IRB Approval Letter

11/2/2014

Investigator(s): Stephen W. Blessing, Dr. Michaele Chappell
Department: Mathematical Sciences
Investigator(s) Email: blessing@mtmail.mtsu.edu, Michaele.Chappell@mtsu.edu

Protocol Title: "Use of Virtual Manipulatives in Developing Grade Six Students' Proportional Reasoning Skills "

Protocol Number: 15-098

Dear Investigator(s),

The MTSU Institutional Review Board, or a representative of the IRB, has reviewed the research proposal identified above. The MTSU IRB or its representative has determined that the study poses minimal risk to participants and qualifies for an expedited review under 45 CFR 46.110 and 21 CFR 56.110, and you have satisfactorily addressed all of the points brought up during the review.

Approval is granted for one (1) year from the date of this letter for 70 participants.

Please note that any unanticipated harms to participants or adverse events must be reported to the Office of Compliance at (615) 494-8918. Any change to the protocol must be submitted to the IRB before implementing this change.

You will need to submit an end-of-project form to the Office of Compliance upon completion of your research located on the IRB website. Complete research means that you have finished collecting and analyzing data. **Should you not finish your research within the one (1) year period, you must submit a Progress Report and request a continuation prior to the expiration date.** Please allow time for review and requested revisions. Failure to submit a Progress Report and request for continuation will automatically result in cancellation of your research study. Therefore, you will not be able to use any data and/or collect any data. Your study expires **11/2/2015**.

According to MTSU Policy, a researcher is defined as anyone who works with data or has contact with participants. Anyone meeting this definition needs to be listed on the protocol and needs to complete the required training. **If you add researchers to an approved project, please forward an updated list of researchers to the Office of Compliance before they begin to work on the project.**

All research materials must be retained by the PI or faculty advisor (if the PI is a student) for at least three (3) years after study completion and then destroyed in a manner that maintains confidentiality and anonymity.

Sincerely,

Kellie Hilker
Institutional Review Board
Middle Tennessee State University

APPENDIX B

Pre-Test/Post-Test

Pilot Study

1. A runner ran 3000 m in exactly 8 minutes. What was his average speed in meters per second?

   A. 3.75
   B. 6.25
   C. 16.0
   D. 37.5

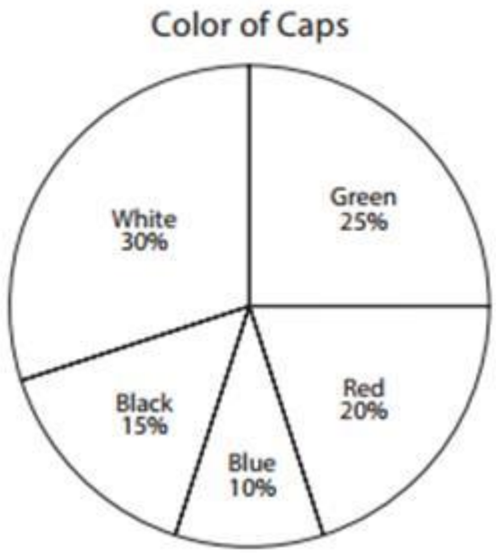
   TIMSS 1999, 39% ANSWERED CORRECTLY

2. If there are 300 calories in 100 g of a certain food, how many calories are there in a 30 g portion of this food?

   A. 90
   B. 100
   C. 900
   D. 9000

   TIMSS 1999, 68% ANSWERED CORRECTLY

3.



The pie chart shows the percentage of caps for sale at a sporting goods store. If there are 50 caps, what is the total number of caps that are either white or blue?

A. 20

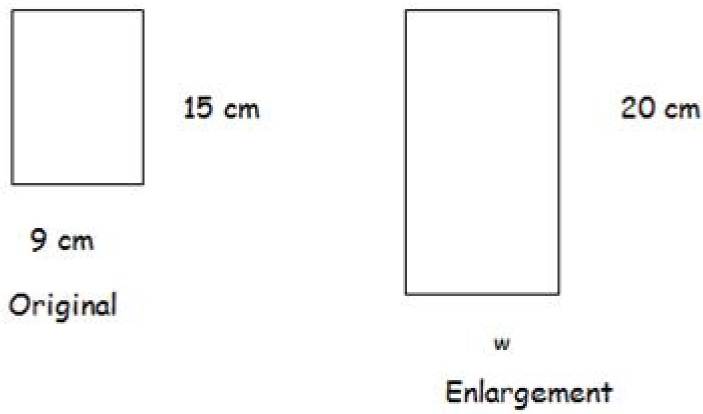
B. 25

C. 30

D. 40

TIMSS 2011 MODIFIED, 71% ANSWERED CORRECTLY

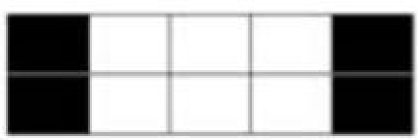
4. The drawing below represents a picture enlarged on a photocopier. Find the width, w, of the enlarged picture.

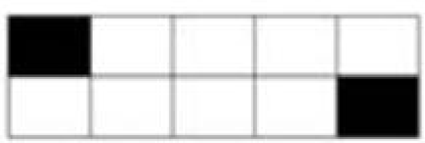


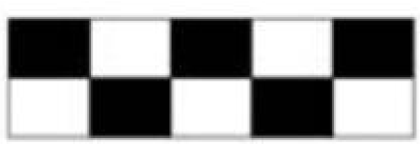
A. 10 cm
B. 12 cm
C. 14 cm
D. 18 cm

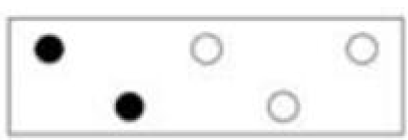
ONLINE TEST, 32% ANSWERED CORRECTLY

5. Which picture shows that $\frac{4}{5}$ is equivalent to $\frac{8}{10}$?

A.


B.


C.


D.


TIMSS 1999 MODIFIED, 29% ANSWERED CORRECTLY

6. The fractions $\frac{4}{14}$ and $\frac{x}{21}$ are equivalent. What is the value of *x?*

A. 6
B. 7
C. 11
D. 14

TIMSS 2011, 74% ANSWERED CORRECTLY

7. Which ratio is equivalent to 14:20?

A. 14.20:1
B. 0.7:1
C. 1:0.7
D. 1.4:1

ONLINE TEST, 61% ANSWERED CORRECTLY

8. In making a garden fertilizer, a gardener mixes 2 kg of a nitrate, 3 kg of a phosphate, and 6 kg of potash. What is the ratio of nitrate to the total amount of fertilizer?

A. $\frac{11}{9}$
B. $\frac{2}{3}$
C. $\frac{2}{9}$
D. $\frac{2}{11}$

TIMSS 1999, 89% ANSWERED CORRECTLY

9. Sarah bought bananas, apples, and pears. The ratio of bananas to apples was 3 to 5 and the ratio of apples to pears was 4 to 2. She bought 12 bananas. How many pears did she buy?

A. 10
B. 12
C. 20
D. 42

ONLINE TEST, 71% ANSWERED CORRECTLY

10. The ratio of girls to boys in a school is 2:3. The school has 405 boys. How many girls are there?

A. 135

B. 180

C. 270

D. 405

ONLINE TEST, 87% ANSWERED CORRECTLY

11. Which of the following teams has the best record?

A. The Predators: 17 wins in 26 games

B. The Ducks: 14 wins in 21 games

C. The Maple Leafs: 21 wins in 30 games

D. The Penguins: 15 wins in 22 games

ONLINE TEST, 34% ANSWERED CORRECTLY

12. From a batch of 3,000 light bulbs, 200 were selected at random and tested. If 10 of the light bulbs in the sample were found to be defective, about how many defective light bulbs would be expected in the entire batch?

A. 15

B. 60

C. 150

D. 300

TIMSS MODIFIED, 63% ANSWERED CORRECTLY

13. On a map, a 600 km distance is represented by a 7.5 cm segment. Using the same scale, a distance of 280 km would be represented by a segment of length

A. 2.1 cm

B. 3.5 cm

C. 16.1 cm

D. 80 cm

ONLINE TEST, 63% ANSWERED CORRECTLY

14. Two-thirds of the people present at the beginning of a meeting are men. Nobody leaves but 10 more men and 10 more women arrive at the meeting. Which of the following statements is true?

A. There would then be more men than women at the meeting.
B. There would then be the same number of men as there are women at the meeting.
C. There would then be more women than men at the meeting.
D. From the information given, you cannot tell whether there would be more women or men.

TIMSS 2003, 53% ANSWERED CORRECTLY

15. At a play, $\frac{3}{25}$ of the people in the audience were children. What percent of the audience was this?

A. 12%
B. 3%
C. 0.3%
D. 0.12%

TIMSS 2003, 89% ANSWERED CORRECTLY

16. Shawn bakes 2,000 bagels in an 8-hour work day. On average, how many bagels does he bake in a half-hour?

A. 125 bagels
B. 250 bagels
C. 8,000 bagels
D. 16,000 bagels

ONLINE TEST, 66% ANSWERED CORRECTLY

17. The rectangle below is twice as long as it is wide.

What is the ratio of the width of the rectangle to its perimeter?

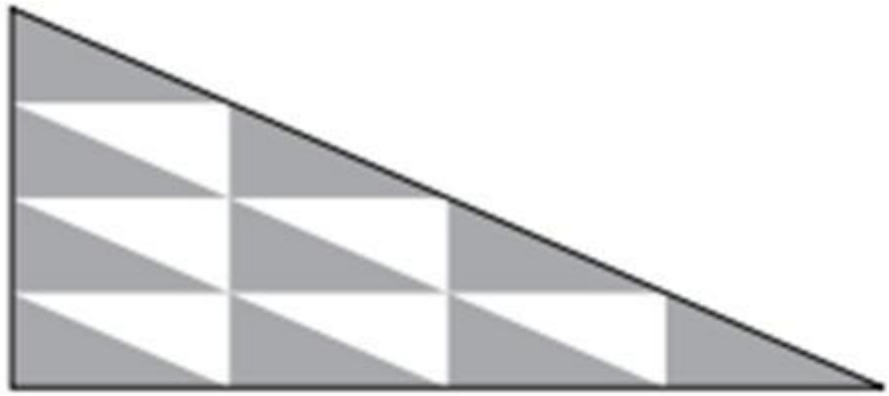
A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{1}{6}$

TIMSS 1999, 21% ANSWERED CORRECTLY

18. 

In the figure above, each of the smaller triangles has the same area. What is the ratio of the shaded area to the unshaded area?

A. 5:3
B. 8:5
C. 5:8
D. 3:5

TIMSS 2003, 74% ANSWERED CORRECTLY

19. Sound travels at approximately 330 meters per second. The sound of an explosion took 28 seconds to reach a person. Which of these is the closest estimate of how far away the person was from the explosion?

A. 12,000 m
B. 9,000 m
C. 8,000 m
D. 6,000 m

TIMSS 1999, 89% ANSWERED CORRECTLY

20. The table shows some values of *x* and *y*, where *x* is proportional to *y.*

x	4	8	*Q*
y	9	*P*	45

What are the values of *P* and *Q*?

A. P = 40 and Q = 13
B. P = 18 and Q = 17
C. P = 20 and Q = 18
D. P = 18 and Q = 20

TIMSS 1999, 45% ANSWERED CORRECTLY

21. A workman cut off $\frac{1}{4}$ of a pipe. The piece he cut off was 3 meters long. How many meters long was the original pipe?

A. 7 m
B. 12 m
C. 16 m
D. 18 m

TIMSS 2011 MODIFIED, 92% ANSWERED CORRECTLY

22. Three brothers, Bob, Dan, and Mark, receive a gift of 45,000 zeds from their father. The money is shared between the brothers in proportion to the number of children each one has. Bob has 2 children, Dan has 3 children, and Mark has 4 children.

How many zeds does Dan get?

A. 5,000
B. 10,000
C. 15,000
D. 20,000

TIMSS 2003 MODIFIED, 79% ANSWERED CORRECTLY

23. Stacie rides her bike 3 miles in 12 minutes. At this rate, how long will it take her to ride her bike 7 miles?

A. 22 minutes

B. 28 minutes

C. 36 minutes

D. 43 minutes

NAEP 2013, 92% ANSWERED CORRECTLY

24. The ratio of boys to girls to adults at a school party was 6:5:2. There were 78 people at the party. How many of them were adults?

A. 6

B. 12

C. 30

D. 36

NAEP 2013, 53% ANSWERED CORRECTLY

25. The school carnival committee sold a total of 400 tickets for the grand prize drawing. Sue bought enough tickets so that she had a 20 % chance of winning the grand prize. How many tickets did Sue buy?

A. 20

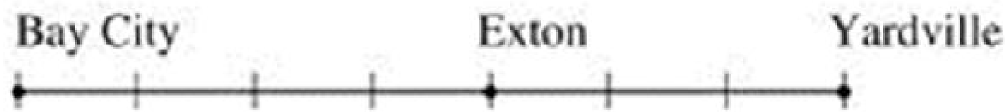
B. 40

C. 80

D. 800

NAEP 2009 MODIFIED, 58% ANSWERED CORRECTLY

26.



On the road shown above, the distance from Bay City to Exton is 60 miles. What is the distance from Bay City to Yardsville?

A. 45 miles
B. 75 miles
C. 90 miles
D. 105 miles

NAEP 2003, 26% ANSWERED CORRECTLY

27. Which of the following ratios is equivalent to the ratio 4 to 6?

A. 12 to 8
B. 8 to 6
C. 6 to 4
D. 2 to 3

NAEP 2003 MODIFIED, 87% ANSWERED CORRECTLY

28. In the model town that a class is building, a car 15 feet long is represented by a scale model 3 inches long. If the same scale is used, a house 25 feet high would be represented by a scale model how many inches high?

A. $\frac{45}{25}$
B. 3
C. 5
D. 7

NAEP 1990 MODIFIED, 82% ANSWERED CORRECTLY

Pilot Study

1. If a bricklayer lays 15 bricks per minute, how many bricks can he lay in a half-hour?

   A. 45
   B. 450
   C. 600
   D. 900

TIMSS 1999, 39% ANSWERERED CORRECTLY

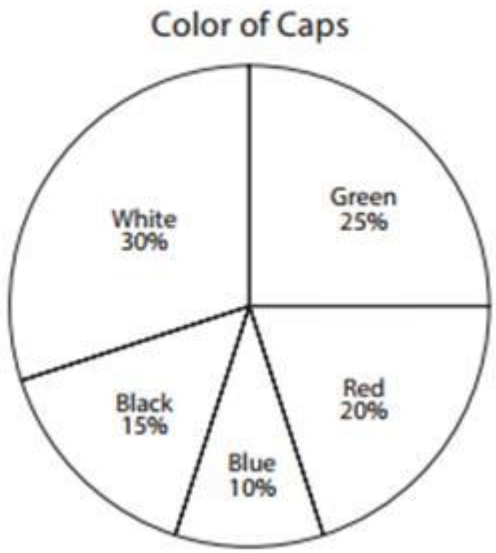
2. 

The car is 3.5 m long. About how long is the building?

   A. 18 m
   B. 14 m
   C. 10 m
   D. 4 m

TIMSS 1999, 76% ANSWERED CORRECTLY

3.



The pie chart shows the percentage of caps for sale at a sporting goods store. If there are 200 caps, what is the total number of caps that are either white or green?

A. 55

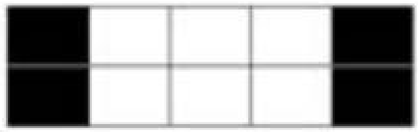
B. 100

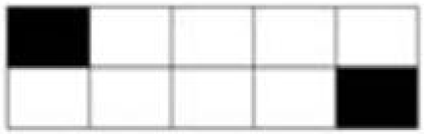
C. 110

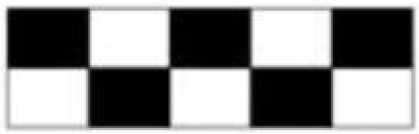
D. 145

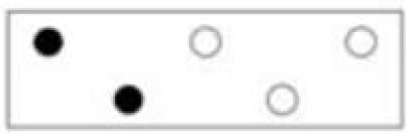
TIMSS 2011, 69% ANSWERED CORRECTLY

4. Which picture shows that $\frac{2}{5}$ is equivalent to $\frac{4}{10}$?

A. 

B. 

C. 

D. 

TIMSS 1999, 90% ANSWERED CORRECTLY

5. In making a garden fertilizer, a gardener mixes 2 kg of a nitrate, 3 kg of a phosphate, and 6 kg of potash. What is the ratio of potash to the total amount of fertilizer?

A. $\frac{11}{6}$

B. $\frac{2}{3}$

C. $\frac{3}{9}$

D. $\frac{6}{11}$

TIMSS 1999 MODIFIED, 100% ANSWERED CORRECTLY

6. Alice can run 4 laps around a track in the same time that Carol can run 3 laps. When Carol has run 12 laps, how many laps has Alice run?

   A. 9
   B. 11
   C. 13
   D. 16

TIMSS 2003, 69% ANSWERED CORRECTLY

7. From a batch of 3,000 light bulbs, 100 were selected at random and tested. If 5 of the light bulbs in the sample were found to be defective, about how many defective light bulbs would be expected in the entire batch?

   A. 15
   B. 60
   C. 150
   D. 300

TIMSS 1999, 79% ANSWERED CORRECTLY

8. A machine uses 2.4 liters of gasoline for every 30 hours of operation. How many liters of gasoline will the machine use in 100 hours?

   A. 7.2
   B. 8.0
   C. 8.4
   D. 9.6

TIMSS 2003, 82% ANSWERED CORRECTLY

9. Two-thirds of the people present at the beginning of a meeting are women. Nobody leaves but 10 more men and 5 more women arrive at the meeting. Which of the following statements is true?

A. There would then be more men than women at the meeting.
B. There would then be the same number of men as there are women at the meeting.
C. There would then be more women than men at the meeting.
D. From the information given, you cannot tell whether there would be more women or men.

TIMSS 2003 MODIFIED, 51% ANSWERED CORRECTLY

10. What percent represents a ratio of 8 to 50?

A. 8%
B. 16%
C. 42%
D. 58%

ONLINE TEST, 92% ANSWERED CORRECTLY

11. If $\frac{12}{n} = \frac{36}{21}$, then *n equals*

A. 3
B. 7
C. 36
D. 63

TIMSS 2003, 100% ANSWERED CORRECTLY

12. In the table below, x is proportional to y:

x	3	12	T
y	8	R	16

What are the values of R and T?

A. R = 17 and T = 11
B. R = 17 and T = 6
C. R = 32 and T = 11
D. R = 32 and T = 6

TIMSS 1999 MODIFIED, 56% ANSWERED CORRECTLY

13. If the ratio 7 to 13 is the same as the ratio *x* to 52, what is the value of *x* ?

A. 7
B. 13
C. 28
D. 364

TIMSS 1999, 95% ANSWERED CORRECTLY

14. The rectangle below is twice as long as it is wide.

What is the ratio of the length of the rectangle to its perimeter?

A. $\frac{1}{2}$

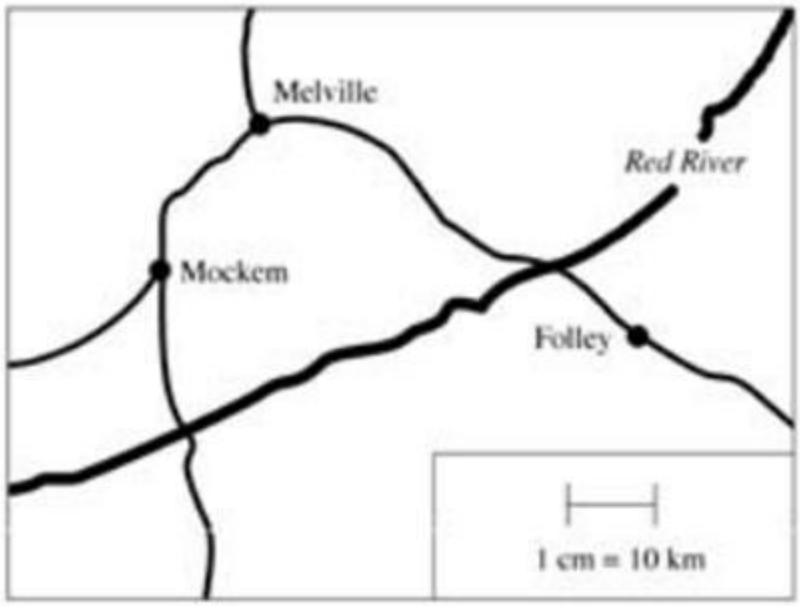
B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{1}{6}$

TIMSS 1999 MODIFIED, 13% ANSWERED CORRECTLY

15. On the map, 1 cm represents 10 km on the land.

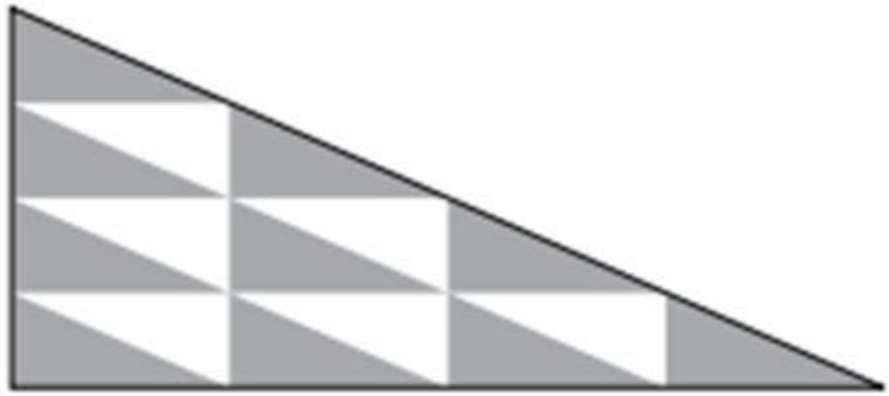


On the land, about how far apart are the towns of Melville and Folley?

A. 5 km
B. 30 km
C. 40 km
D. 50 km

TIMSS 1999, 38% ANSWERED CORRECTLY

16.



In the figure above, each of the smaller triangles has the same area. What is the ratio of the shaded area to the total area of the original figure?

A. 5:3

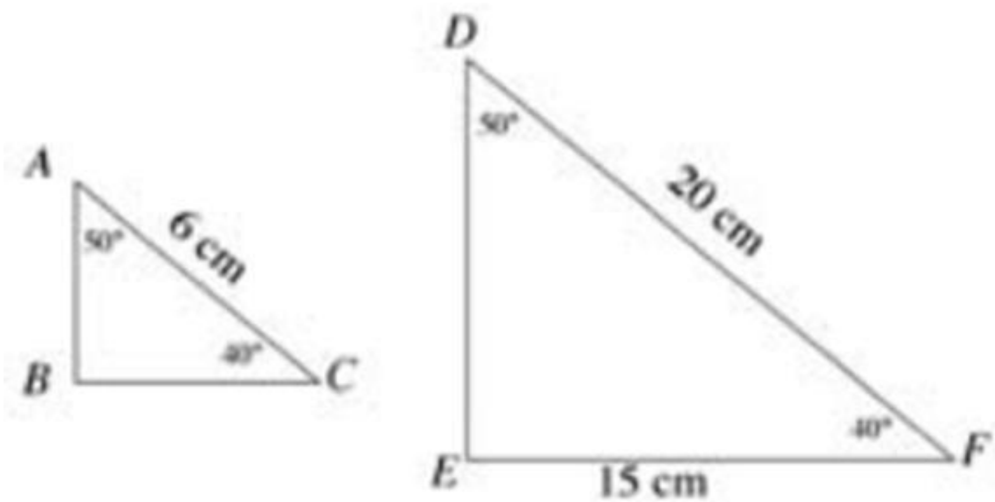
B. 8:5

C. 5:8

D. 3:5

TIMSS 2003 MODIFIED, 74% ANSWERED CORRECTLY

17. The figure represents two similar triangles. The triangles are not drawn to scale.



In the actual triangle ABC, what is the length of side BC?

A. 3.5 cm
B. 4.5 cm
C. 5 cm
D. 5.5 cm

TIMSS 1999, 41% ANSWERED CORRECTLY

18. If the Student Council has 36 members and the ratio of girls to boys on the council is 4:5, then the number of boys on the Student Council is

A. 5
B. 9
C. 16
D. 20

ONLINE TEST, 56% ANSWERED CORRECTLY

19. It takes one hour to drive 80 km. About how far can you drive in 20 minutes at that speed?

A. 20 km
B. 25 km
C. 27 km
D. 30 km

ONLINE TEST, 49% ANSWERED CORRECTLY

20. A workman cut off $\frac{1}{5}$ of a pipe. The piece he cut off was 3 meters long. How many meters long was the original pipe?

A. 8 m
B. 12 m
C. 15 m
D. 18 m

TIMSS 2011, 90% ANSWERED CORRECTLY

21. Three brothers, Bob, Dan, and Mark, receive a gift of 45,000 zeds from their father. The money is shared between the brothers in proportion to the number of children each one has. Bob has 2 children, Dan has 3 children, and Mark has 4 children.

How many zeds does Mark get?

A. 5,000
B. 10,000
C. 15,000
D. 20,000

TIMSS 2003, 38% ANSWERED CORRECTLY

22. The table shows some information about the grade 6 students in a school:

	Left-handed	Right-handed
Girls	12	36
Boys	12	60

What is the ratio of left-handed students to right-handed students?

A. 1:3
B. 1:4
C. 1:5
D. 1:10

ONLINE TEST, 85% ANSWERED CORRECTLY

23. Stacie rides her bike 3 miles in 12 minutes. At this rate, how long will it take her to ride her bike 10 miles?

A. 19 minutes
B. 28 minutes
C. 36 minutes
D. 40 minutes

NAEP 2013, 92% ANSWERED CORRECTLY

24. The ratio of boys to girls to adults at a school party was 6:5:2. There were 78 people at the party. How many of them were girls?

A. 6
B. 12
C. 30
D. 36

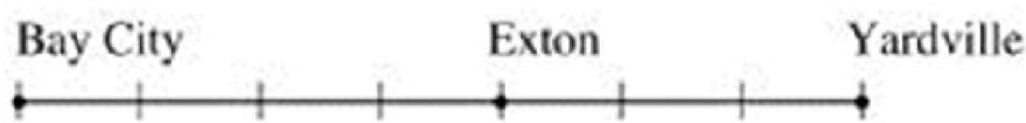
NAEP 2013, 53% ANSWERED CORRECTLY

25. The school carnival committee sold a total of 200 tickets for the grand prize drawing. Sue bought enough tickets so that she had a 20 % chance of winning the grand prize. How many tickets did Sue buy?

A. 20
B. 40
C. 160
D. 400

NAEP 2009, 77% ANSWERED CORRECTLY

26.



On the road shown above, the distance from Bay City to Exton is 60 miles. What is the distance from Exton to Yardsville?

A. 45 miles
B. 75 miles
C. 90 miles
D. 105 miles

NAEP 2003 MODIFIED, 72% ANSWERED CORRECTLY

27. Which of the following ratios is equivalent to the ratio 6 to 4?

A. 12 to 8
B. 8 to 6
C. 4 to 6
D. 2 to 3

NAEP 2003, 92% ANSWERED CORRECTLY

28. In the model town that a class is building, a car 15 feet long is represented by a scale model 3 inches long. If the same scale is used, a house 35 feet high would be represented by a scale model how many inches high?

A. $\frac{45}{35}$

B. 3

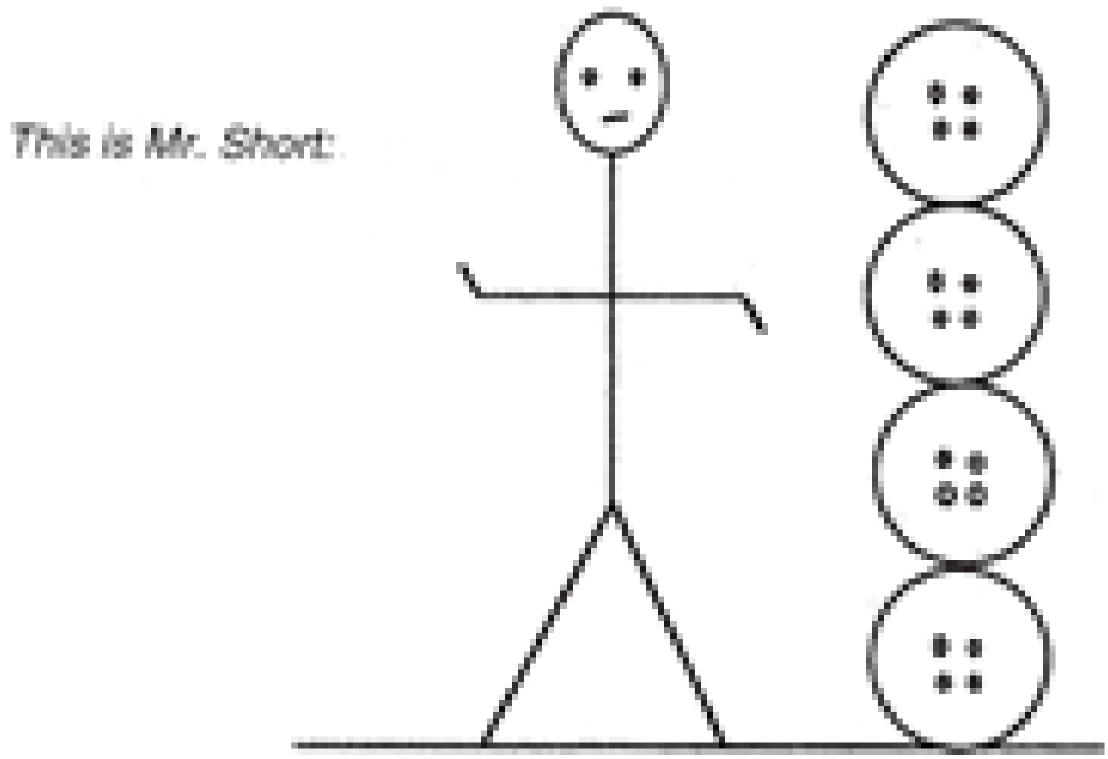
C. 5

D. 7

NAEP 1990, 72% ANSWERED CORRECTLY

APPENDIX C

Tasks from Pilot Study



The length of Mr. Short is 4 large buttons.

The length of Mr. Tall (not drawn) is 6 large buttons.

When paper clips are used to measure Mr. Short and Mr. Tall, the length of Mr. Short is 6 paper clips. What is the length of Mr. Tall in paper clips?

____

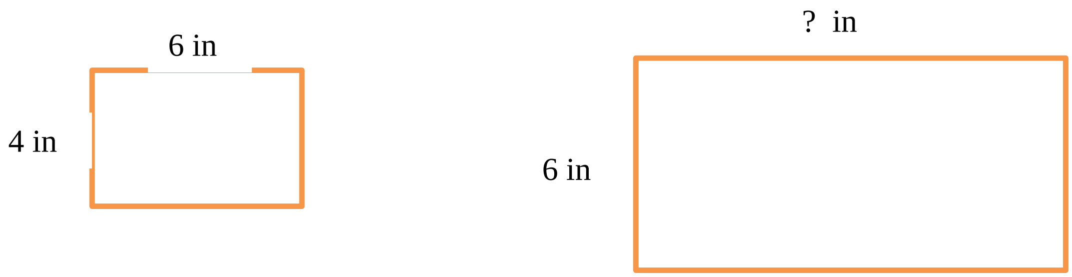
Please EXPLAIN how you arrived at your answer.

Number _______________

1) Cindy and Carlos shared some candy bars in the ratio 3:7. If Carlos had 21 candy bars, how many candy bars did Cindy have?

2) Nextware sold some tablets and laptops in the ratio 9 to 2. If they sold 44 devices altogether, how many more tablets than laptops did they sell?

These two rectangles are similar:



Find the missing width from the second rectangle by answering the following questions:

1) What is the ratio of length to width from the smaller rectangle?

2) If the length of the smaller rectangle were reduced to 1 inch, what would be the matching width?

3) Complete this table to show the relationship of length to width for rectangles of this shape:

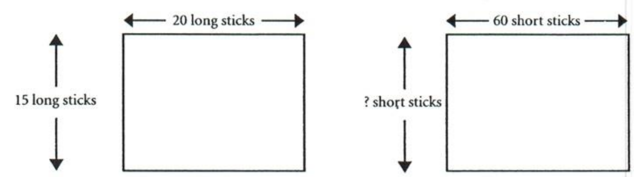
Length	Width
1	
2	
3	
4	
5	
6	

4) For a length of 6 inches, the width of the second rectangle is ____________.

Name ______________________ Date ______________________

John's School

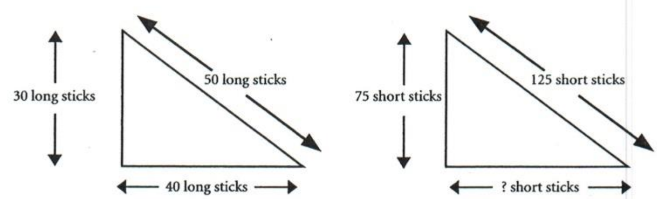
John used short and long sticks to measure his mathematics classroom. He drew these pictures:



1. What is the width of the room in short sticks?

2. Explain how you found your answer.

John used a different set of short and long sticks to measure his school's garden. He drew these pictures:



3. What is the length of the base of the garden measured in short sticks?

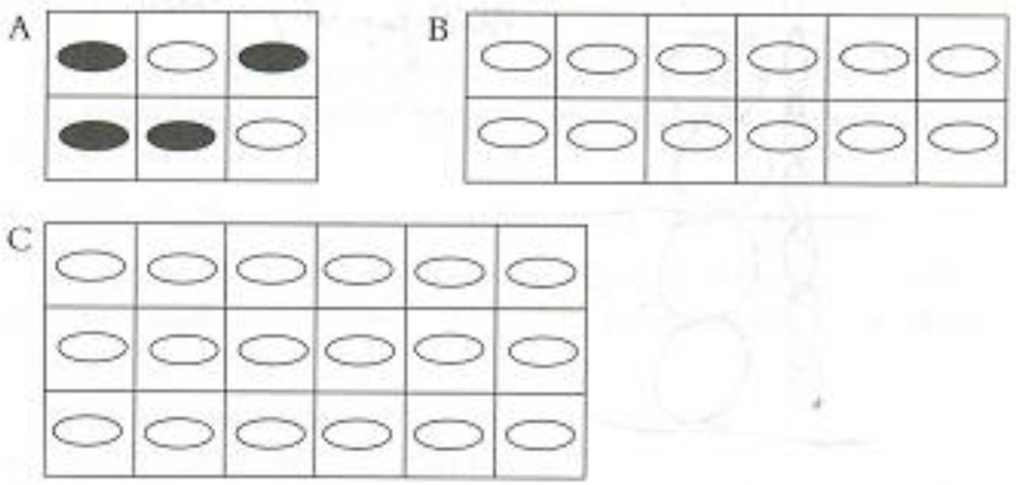
4. Explain how you found your answer.

APPENDIX D

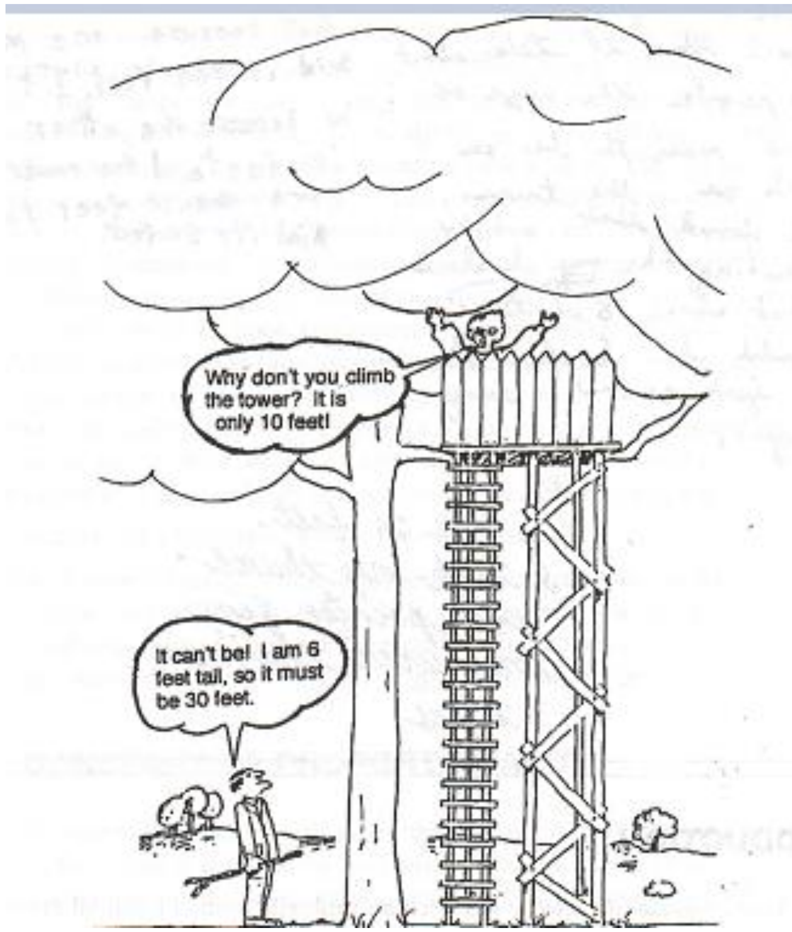
Additional Tasks in Dissertation Study

1. Egg Carton task

The Bi-Color Egg company has just hired you to fill egg cartons with brown and white eggs. The cartons come in different sizes, and your job is to fill every carton so that the numbers of brown eggs and white eggs are in the same proportions as they are in carton A. Color the eggs in cartons B and C.

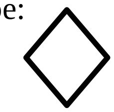


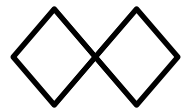
2. Tree House task

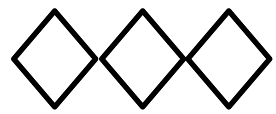


How tall is the ladder? Show your work to explain your answer.

3. Sticks and Rhombi Task

On Day 1, Jim uses four sticks to build the following shape: 

On Day 2, Jim uses more sticks and builds this shape: 

On Day 3, Jim uses even more sticks and builds this shape: 

If Jim were to continue building shapes from sticks in the same way, draw a picture of the shape Jim would build on Day 6.

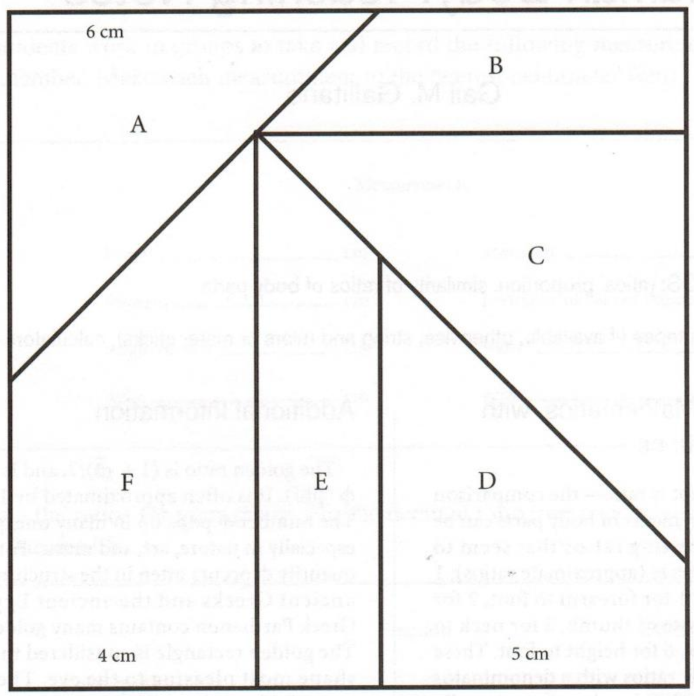
Now, write a statement that describes the relationship between the number of sticks that Jim uses and the number of rhombi that Jim builds.

4. Make a New Puzzle task

Name ______________________ Date ______________________

Make a New Puzzle

The pieces below may be cut out and then reassembled to make the square.



1. Make a new set of all six pieces so that Rule 1 is satisfied. Work in groups of three. Each group member selects two pieces to create and cut out. When each person is finished, you should be able to put the new pieces together to make a square.

   Rule 1: The segment that measures 4 cm on the original should measure 7 cm on the new version.

2. For each of Rules 2 and 3, make a sketch of all six pieces and indicate the measurements needed to get the new puzzle.

   Rule 2: The 5 cm segment on the original should measure 8 cm on the new version.

   Rule 3: The 6 cm segment on the original should measure 4 cm on the new version.

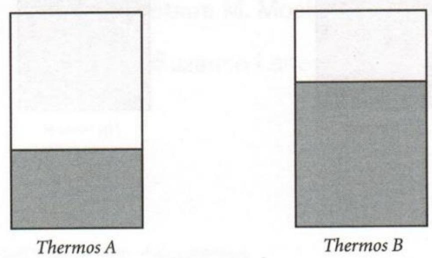
3. Write a set of instructions for enlarging and reducing the puzzle pieces. Describe any method that did not work, and explain why it did not work.

5. Cocoa task

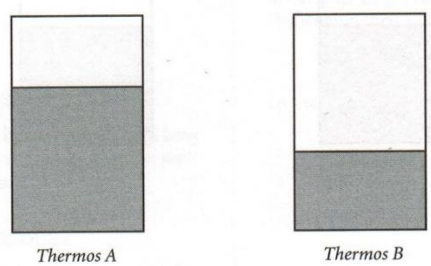
Name ______________________________ Date ______________________________

Cocoa

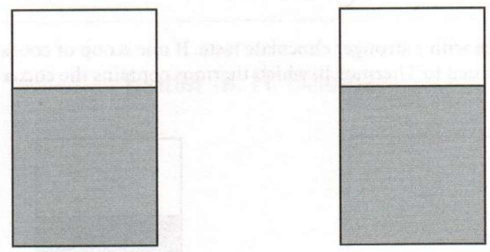
1. Thermos A contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.



2. Thermos A and Thermos B contain cocoa that tastes the same. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

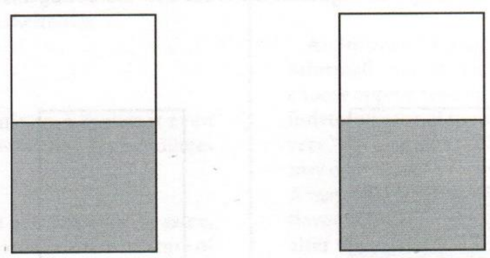


3. Thermos A contains cocoa with a weaker chocolate taste. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.



Thermos A *Thermos B*

4. Thermos B. contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B. which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.



Thermos A *Thermos B*

APPENDIX E

Proportional Reasoning Word Problems for Pencil-and-Paper Group

Find the missing quantity:

1) The ratio of girls to boys in Mrs. Delgado's class is 7:5. If there are 15 boys, how many girls are in the class?

2) Emily and Noah shared a cash prize in the ratio 4:5. If Noah received $55, how much money did Emily receive?

3) The ratio of the height of a rectangle to its width is 3:2. If the height of the rectangle is 15 centimeters, what is its width?

4) The ratio of girls to boys who participated in the spelling contest was 7:5. There were 56 girls. How many boys participated?

5) Philip and Will shared some M & Ms in the ratio 3:2. If Will had 20 M & Ms, how many M & Ms did Philip have?

Find the total or difference:

1) The ratio of girls to boys at the school play was 4:3. Doug counted 12 boys. How many children were at the school play altogether?

2) The Sport Court donated baseballs and helmets to the community center in the ratio 3:4. The store donated 28 helmets. How many more helmets were donated than baseballs?

3) The ratio of the number of video games in Monica's collection to the number of video games in Jada's collection is 7:3. Jada has 24 video games. How many video games do they have altogether?

4) For every 3 chin-ups Kayla does, Nancy does 2. If Kayla did 15 chin-ups, how many more chin-ups did Kayla do than Nancy?

5) Tommy and Corey share some CDs in the ratio 2:3. Corey has 27 CDs. How many CDs do they have altogether?

Use the total or difference to find a quantity:

1) The ratio of boys to girls in Mr. Walker's math class is 6:5. If there are 110 students altogether, how many boys are in the class?

2) Peter and Ashley shared a $132 cash prize in the ratio 3:8. How much money did Peter get?

3) The difference between two numbers is 3. The ratio of the bigger number to the smaller number is 4:3. What is the smaller number?

4) Last month, Jacob sold 4 calendars for every 3 that Abby sold. Overall, Jacob sold 12 more calendars than Abby. What was the total number of calendars sold last month?

5) The ratio of the weight of Rachel's sculpture to the weight of Griffin's sculpture is 2:3. Griffin's sculpture weighs 6 more ounces than Rachel's sculpture. How much does Griffin's sculpture weigh?

Work with Part:Total Ratios

1) Andy and Kendra worked together to deliver newspapers. Andy delivered 3 out of every 5 newspapers. When they finished the job, Kendra had delivered 8 fewer newspapers than Andy. How many newspapers did Andy deliver?

2) Philip built a gaming website. The website had 90 visitors on Friday. 3 out of every 10 visitors played Cosmic Blobs. The other visitors played Astro-Bots. How many visitors played Astro-Bots?

3) Mrs. Miller has 64 students in her class. 3 out of 8 students stayed after school yesterday for play practice. The other students stayed for soccer practice. How many students stayed for soccer practice?

4) Brady spends 4 out of every 7 dollars he earns on software. He uses the rest of the money to buy snacks. Last month, Brady spent 24 dollars on software. How many fewer dollars did he spend on snacks?

5) A group of 45 fourth grade students voted for their favorite sport. 2 out of 5 votes were for baseball. The other votes were for football. How many votes did football receive?

Work with Ratios that Compare Three Quantities:

1) Cole, Jake, and Griffin were the top three winners in the spelling contest. They shared a cash prize in the ratio 1:3:7. Griffin received $30 more than Cole. What was the amount of the cash prize?

2) The ratio of muffins to doughnuts to cupcakes left over after the bake sale was 1:2:7. There were 49 cupcakes. How many desserts were there altogether?

3) Blake, Karen, and Monica paid $54 for a gift for their friend. They shared the cost in the ratio 5:3:1. How much more money did Blake spend than Karen?

4) Three numbers A, B, and C, are in the ratio 7:2:1. The difference between the largest number and the smallest number is 36. What is the sum of the numbers?

5) Mr. Brown spent money on puzzles, chess sets, and word games in the ratio 6:4:1. The word games cost 25 dollars less than the puzzles. How much money did Mr. Brown spend in all?

APPENDIX F

Results from SMEs Examination of Pre-Test/Post-Test

Reviewer: SME 1

Please share your opinion as to whether the questions on the instrument provided to you assesses proportional reasoning. Indicate Yes or No. Provide any comments that you wish to share.

Question 1 Yes X No____ Comment:

Question 2 Yes X No____ Comment:

Question 3 Yes X No____ Comment: Given these numbers, there would be 7.5 Black caps and 12.5 Green caps, which does not make much sense realistically. Also, I wasn't immediately sure what was meant by "caps". I'm assuming you mean hats.

Question 4 Yes X No____ Comment: This picture does not seem like it is to scale. Should the directions mention that the shapes are similar?

Question 5 Yes X No____ Comment: Although the answer is B, this choice does not very clearly model the equivalence of 4/5 and 8/10. Was this purposeful? Is there a reason not to shade two smaller rectangles that are within the same column?

Question 6 Yes X No____ Comment:

Question 7 Yes X No____ Comment:

Question 8 Yes X No____ Comment:

Question 9 Yes X No____ Comment:

Question 10 Yes X No____ Comment:

Question 11 Yes X No____ Comment: The team with the most wins is also the team with the best record. Should there be a distractor (i.e., a team with a higher number of total wins than the Maple Leafs, but not having the best record)?

Question 12 Yes X No____ Comment:

Question 13 Yes X No____ Comment:

Question 14 Yes X No____ Comment:

Question 15 Yes X No____ Comment:

Question 16 Yes X No____ Comment:

Question 17 Yes X No____ Comment:

Question 18 Yes X No____ Comment:

Question 19 Yes X No____ Comment:

Question 20 Yes X No____ Comment:

Question 21 Yes X No____ Comment:

Question 22 Yes X No____ Comment: What are zeds? Could you use a more familiar monetary unit?

Question 23 Yes X No____ Comment:

Question 24 Yes X No____ Comment:

Question 25 Yes X No____ Comment:

Question 26 Yes X No____ Comment: Should this say that the drawing is "to scale"?

Question 27 Yes X No____ Comment: All other options (aside from the correct answer, D) have a larger number as the first entry in the "x to y". Should there be an additive reasoning distractor like "6 to 8" with the smaller number coming first?

Question 28 Yes X No____ Comment:

Reviewer: SME 2

Please share your opinion as to whether the questions on the instrument provided to you assesses proportional reasoning. Indicate Yes or No. Provide any comments that you wish to share.

Question 1 Yes X No____ Comment:

Question 2 Yes X No____ Comment:

Question 3 Yes X No____ Comment:

Question 4 Yes X No____ Comment:

Question 5 Yes X No____ Comment:

Question 6 Yes X No____ Comment:

Question 7 Yes X No____ Comment:

Question 8 Yes X No____ Comment:

Question 9 Yes X No____ Comment:

Question 10 Yes X No____ Comment:

Question 11 Yes____ No X Comment: Unless you ask the student to find the percentage, this is just a straight division problem.

Question 12 Yes X No____ Comment:

Question 13 Yes____ No X Comment: This is a great question for "reasoning", but not for proportional reasoning. A proportion would not be used to find the solution.

Question 14 Yes X No____ Comment:

Question 15 Yes X No____ Comment:

Question 16 Yes X No____ Comment:

Question 17 Yes X No____ Comment:

Question 18 Yes X No____ Comment:

Question 19 Yes X No____ Comment:

Question 20 Yes X No____ Comment:

Question 21 Yes X No X Comment: This question is a great algebraic thinking question. I would have set up an equation and solved 1/4x = 3. I do not think I would have thought to use a proportion.

Question 22 Yes X No____ Comment:

Question 23 Yes X No____ Comment:

Question 24 Yes X No____ Comment:

Question 25 Yes X No____ Comment:

Question 26 Yes X No____ Comment:

Question 27 Yes X No____ Comment:

Question 28 Yes X No____ Comment:

Reviewer: SME 3

Please share your opinion as to whether the questions on the instrument provided to you assesses proportional reasoning. Indicate Yes or No. Provide any comments that you wish to share.

Question 1 Yes X No____ Comment: Does the switch in units from minutes to seconds add to (or detract from) the proportional reasoning focus. E.g., would performance be better if the units remain the same? If so, is it really proportional reasoning that is the MAJOR focus?

Question 2 Yes X No____ Comment:

Question 3 Yes X No____ Comment: Certainly touches on pr, but the conditional "or" may, once again, cloak the student's understanding of the pr element of the item

Question 4 Yes X No____ Comment: Just a query- does a photocopier enlargement actually increase the two dimensions proportionally (or ROUGHLY proportionally)? Not sure

Question 5 Yes X No____ Comment: Does it help the item if these are identified as "fractions" as opposed to "ratios"? Or, perhaps, simply add, "Which picture shows that of the region is equivalent to of the region?"

Question 6 Yes X No____ Comment:

Question 7 Yes X No____ Comment:

Question 8 Yes X No____ Comment:

Question 9 Yes X No____ Comment:

Question 10 Yes X No____ Comment:

Question 11 Yes X No____ Comment: Determining "the best record" is subject to interpretation. Is the most wins better? Perhaps clarify that the item is after the highest winning percentage.

Question 12 Yes X No____ Comment:

Question 13 Yes X No____ Comment:

Question 14 Yes X No____ Comment:

Question 15 Yes X No____ Comment: Depends on the definition of pr being used. One was not provided- not necessary on MOST of the items included here but "fringe" elements of the "common" description of pr.

Question 16 Yes X No____ Comment: The assumption of a constant rate of baking throughout the work day is a bit unrealistic.

Question 17 Yes X No____ Comment:

Question 18 Yes X No____ Comment: While it is readily understood what is meant by "smaller" triangles, it is POTENTIALLY a bit confusing since there are many different sized triangles embedded in this image.

Question 19 Yes X No____ Comment:

Question 20 Yes X No____ Comment:

Question 21 Yes X No____ Comment: Does not REQUIRE proportional reasoning to solve (but, then, that is true of a few others as well).

Question 22 Yes X No____ Comment:

Question 23 Yes X No____ Comment:

Question 24 Yes X No____ Comment:

Question 25 Yes X No____ Comment: Should clarify that one ticket will be pulled to determine the grand prize winner.

Question 26 Yes X No____ Comment: Need to clarify that this linear map is marked off in equal units. This can't be assumed.

Question 27 Yes X No____ Comment:

Question 28 Yes X No____ Comment:

APPENDIX G

Survey, Observation and Interview Protocols

Please put a checkmark in the spaces provided to answer the following questions:

1) Gender _____Female _____Male

2) How long have you used an iPad or any other touch screen tablet device outside of school? Don't include cell phones when you answer this question.

   _____ I have never used an iPad or any other touch screen tablet device outside of school.

   _____ Less than a year

   _____ Between 1-2 years

   _____ Between 2-3 years

   _____ More than 3 years

3) Which of the following statements best describe your choice of computer use?

   _____ I prefer to use a computer with a mouse.

   _____ I prefer to use an iPad.

   _____ I have no preference working with a computer with a mouse or an iPad.

   _____ I would rather not work with computers of any kind.

Date:	Class Period:
What evidence suggests that participants are using virtual manipulatives effectively?	
What evidence suggests that participants are using the block modeling strategy effectively?	
What evidence suggests that participants are learning proportional reasoning concepts?	
What evidence suggests that participants are confused concerning proportional reasoning concepts?	
Sketch of the room:	

Pre-test Follow Up Questions

1) This is a task called Make a New Puzzle. Follow the directions and see if you can take the puzzle and make a bigger version of it.

2) Tell me about the work you just did.

Post-test Follow Up Questions

1) Here is a task similar to some you completed in class recently. Please answer the four questions from the Cocoa task.

2) Tell me about the work you just did.

APPENDIX H

Transcript of Student Interviews: Make a New Puzzle

Interviewer: Today, I'm going to have you work on a math task called Make a New Puzzle. What I'd like for you to do is take these pieces, they are puzzle pieces, out of the baggie and put them together and see if you can make a square.

Alice: (Alice takes pieces out of baggie and begins assembling the puzzle). Do I have to use all of the pieces?

Interviewer: You do, that's a good question. You have to use all of the pieces and I'll give you a hint: Do you see those numbers running along some of the edges? Those go on the perimeter of the square, or the outside.

Alice: (Alice continues to assemble the puzzle).

Interviewer: And so that you don't worry, the putting together of the pieces is not the main idea of this task, this is just to show you what we're working with and the task.

Alice: (Alice continues to assemble the puzzle).

Interviewer: You are doing great, keep on going.

Alice: (Alice assembles the puzzle into a square).

Interviewer: Excellent! Good job! You put the puzzle together, and here's a picture to show that the pieces go together the way that you put them together. Great! Now, what I'd like for you to do is to take this ruler on the centimeters and I want you to pick just a couple of the pieces and I want you to measure to see that the numbers that we have written here are true, that they are not mismeasured. So, you need to centimeters part of the ruler where it says metric and you can move the pieces from the puzzle; you can take it apart now and you can move it anyway you want to and just pick a couple. See the numbers that are written as if they measure out to be what's written on the puzzle.

Alice: (Alice selects pieces and begins to measure side lengths).

Interviewer: Okay, so did those work?

Alice: (Alice nods head and selects another piece to measure).

Interviewer: And that works?

Alice: (Alice nods head and continues to measure).

Interviewer: So, would you say that the puzzles are measured accurately? The numbers written on the pieces match the measures you are finding?

Alice: Yeah.

Interviewer: Great! Now, here's what you're going to do: you can slide those puzzle pieces away to your left if you like. Here's what we're going to do. First of all, if you would write your study number, if you remember that, at the top of the page.

Alice: (Alice writes number as directed).

Interviewer: Excellent. Now, here's what we're going to do. We're going to fill in the measures on the sides of these puzzle pieces so that the 4 cm length you head over there is going to become a 7 cm length. So, I know that its right here, if you go ahead and write a 7 cm right there, and if you would like to pick the puzzle piece that matches to see the numbers; for instance, that F piece that you have on your left-hand. You see the F? And you see the F here? That four became a seven. Now, what we want to do is the way that you make a four into a seven, I want you to fill in your new numbers here from the original puzzle pieces to the new sizes. If you have any questions, just let me know.

Alice: (Alice writes new measures on worksheet).

Interviewer: And you can move those pieces closer if you need to, whatever works for you.

Alice: (Alice continues to write measures on worksheet. Alice uses fingers to add three to the previous side lengths to get new side lengths).

Interviewer: And there's one more on the F side, there's one more number. Good! Now, if you'll put the blue puzzle pieces back in the baggie. That way it's all together, thank you. Now, what I'd like for you to do is, underneath the puzzle piece write me a sentence or statement that tells me what you just did. How did you make the four become a seven and how did you get all those new numbers? What did you do math wise?

Alice: (Alice writes sentence as requested).

Interviewer: Okay. So it says that I added on 3 cm to every number. Great! I just happen to have puzzle pieces with the numbers that you came up with. I'd like you to put those together for your new square that you just found measures for.

Alice: (Alice removes pieces from baggie and begins to assemble new puzzle. Alice hesitates while attempting to complete puzzle).

Interviewer: I noticed that you were glancing at your model and putting the pieces together.

Alice: (Alice continues attempt at assembling puzzle).

Interviewer: I see that you have an interesting approach in order to put it together; you are rearranging it differently than the original model. Do you think that will help?

Alice: No, not really, because the pieces don’t make a square anymore, any way you put them.

Interviewer: They don’t make a square anymore? You know, it could be that I didn’t really put the numbers right. Let’s measure, like we did with the blue pieces, we did some measuring. Pick a couple and see if the numbers are right. Measure them on the metric side. Pick a couple of pieces and see if the numbers match what you came up with on your paper model.

Alice: (Alice selects pieces and measures side lengths).

Interviewer: Which piece is that? Does D match the numbers you came up with?

Alice: (Alice nods head).

Interviewer: Okay. Let’s try the A piece that you have there at your hand; see if it matches.

Alice: (Alice measures puzzle piece).

Interviewer: Wow, so those numbers matched, too?

Alice: (Alice selects another piece to measure).

Interviewer: Okay. So, if those numbers are matching, it doesn’t seem to be that the numbers are off from what you found, they matched what you found, but you’re telling me they don’t make a square anymore.

Alice: (Alice makes inaudible response).

Interviewer: Well, I don’t want you to panic or worry about your calculations. What you described here and what you wrote here are correct numbers, but it turns out there’s more than one way to make 4 cm becomes 7 cm, and as you are working in math class this week and next week, you're going to find out about this way that will make these numbers change so that we keep everything in proportion, so that we do have a new square that does work together. Maybe you have some questions? Anything?

Alice: (Alice shakes head).

Interviewer: Okay. Thank you for taking the time to work with this puzzle today and I'm going to send you back to your first-period class.

Interviewer: What we're going to do this morning is work a math task called Make a New Puzzle. So, what I'd like for you to do is to take the six pieces out of this baggie and try to make a square out of them; they should form a square.

Alan: (Alan takes pieces and begins to assemble puzzle).

Interviewer: And if you'll turn them all face up, each one has a letter and some numbers. And I'll give you a hint: the numbers that you see along an edge are the edge of the square, so they will go on the perimeter or outside.

Alan: (Alan continues to assemble puzzle). Are there going to be holes in the middle or will it be all connected?

Interviewer: That's a good question. There should be no holes in the middle; it should be a square. You should see the perimeter and area on the interior, all been a square.

Alan: (Alan continues to assemble puzzle).

Interviewer: You know, the really important part of this task is not as much putting together the puzzle as it is the numbers, so let's see if you can now build a square. (Interviewer shows student 4 the paper model with the assembled square puzzle).

Alan: Oh! (Alan uses model to assemble puzzle, encounters difficulty with pieces staying together).

Interviewer: Those pieces want to flop around a little bit, but you've got them together approximately. So, they do go together to make a square. You'll notice numbers the numbers that are written along the edges, now that you see them on the outside or perimeter of the square. I want you to pick two or three pieces and measures those on the metric side of this ruler, to make sure that the numbers that are written on that puzzle are accurate. So, just pick a few pieces and measure them and see. You can move them around like you're doing; that's fine.

Alan: (Alan selects pieces and measures the side lengths). This one's not exactly, but it's close.

Interviewer: Okay.

Alan: This one's not exactly either, but it's close.

Interviewer: Okay.

Alan: (Alan measures another puzzle piece). It's not exactly, but it's close.

Interviewer: Okay, so you're saying those numbers are off; that might need you to error in chopping or trimming, but you agree that they do make a square?

Alan: Yes sir.

Interviewer: Okay. Now, here's what we're going to do: first of all, we're going to do some writing. If you will write your study number; do you remember your three digit number from Friday?

Alan: Yes sir. (Alan writes number as directed).

Interviewer: We're now going to build a new puzzle; we're going to make it bigger. I want you to fill in the measures on the sides of these puzzle pieces so that the 4 cm length is enlarged to 7 cm. So, let's find that; it's right here. If you notice, that's a four. Go ahead and write seven down there for the matching part. For all of these others, I want you to take all of these numbers you see and I want you to do the same thing to them that you do to make this four into a seven, and I want you to fill in those new measures on your puzzle pieces.

Alan: (Alan proceeds to find and write new measures).

Interviewer: Okay, good. Now, what I'd like for you to do is to write a sentence telling me what you did. How did you change the four into the seven or the two into the five? Tell me what you did, mathematically to get those new numbers.

Alan: (Alan writes sentence as directed). I can't spell, I'm sorry.

Interviewer: Oh, spelling doesn't count in this task. As long as I can tell the word, it's fine.

Alan: That's "number originally". (Alan reads statement). I believe that each of the numbers originally are being added by three. Example: 4+3 = 7.

Interviewer: Okay, thank you. So, your strategy that you used to change the four into a seven and all your other numbers is where you added 3 cm to the original to get the new.

Alan: Yes sir.

Interviewer: Okay, thank you. Would you would you mind putting those pieces back in the baggie?

Alan: (Alan puts puzzle pieces back into baggie).

Interviewer: Thank you. Now, I just happen to have pieces that are cut out to match the numbers that you came up with. Please take these and make them into a square.

Alan: (Alan proceeds to assemble new puzzle pieces). So, the measurements that I made are the measurements that are on here?

Interviewer: Check them. Compare them and see if they do say the same thing that you did.

Alan: (Alan compares measurements on puzzle pieces to measurements written on paper model). Yes.

Interviewer: Okay, so put them into a square. Again, you can use this as your model.

Alan: (Alan proceeds to assemble puzzle pieces into a square. Alan hesitates while attempting to assemble puzzle).

Interviewer: You are looking puzzled; maybe you should measure and see that I didn't mess up. Use the metric part of the ruler to measure those sides like you did with the previous puzzle pieces.

Alan: (Alan selects puzzle pieces to measure). That one's not nine, but it's really close.

Interviewer: You only have to measure the ones that have numbers beside them; there you go.

Alan: (Alan continues to measure puzzle pieces). Yeah, that one's estimated to eight, and that one's estimated to five.

Interviewer: So, you say the numbers and you that you came up with here are pretty close?

Alan: Yes sir.

Interviewer: Do you think you can make a square out of those?

Alan: I'll try one more time, but if I can't, then I know it's not possible.

Interviewer: Okay. (Alan attempts to assemble puzzle pieces into a square).

Alan: Well, I might be putting them wrong, but I don't think they'll turn out to make a square.

Interviewer: I don't think you're putting them together wrong. Here's, I think, what we need to recognize: you used a certain strategy to come up with your new numbers. It turns out to change a four into a seven, there's more than one way to do it. As you are working in your math class this week and next week, and I think you'll find this new strategy that will help you change, or in this case enlarge a four into a seven, there's more than one way to do it. When you use this new strategy, and you apply to all these pieces, you would get a square. So, as you are working this week keep in mind these new strategies that you will be learning and hopefully you will be able to be use in the future when you are working with enlarging or reducing, or working with things that are said to be proportional. Thank you for taking time to work with me today. I'm going to send you back to your second period.

Interviewer: I have in this baggie a puzzle. I'd like for you to put that puzzle together; it has six pieces and it should form a square. So, if you will try that for me, please.

Betty: (Betty takes baggie, opens it, removes pieces, and begins to assemble puzzle).

Interviewer: And don't panic, the putting together of the puzzle is not the important part; it's just to give you an idea of what we're working with today.

Betty: (Betty has difficulty assembling puzzle).

Interviewer: I'll also give you another hint. You see the measures that are written on the pieces? Those are the outside parts of the square.

Betty: Oh. (Betty still has difficulty assembling puzzle).

Interviewer: You are close with that. OK, what I want to do, just because this is not the important part of this math task, here's a guide to let you know how the puzzle pieces go together. So, if you would put that together and make sure that it does form a square.

Betty: (Betty uses guide to assemble puzzle).

Interviewer: Excellent! So it does work. We have a square, and you'll notice the measures that are written along the side. 0K, what I would like you to do is take a ruler, whichever one of these you'd like, and I want you to pick a couple of pieces and measure them to show that the number written down there are the numbers that they should be. Those are centimeters, so you'll need to use the centimeter side.

Betty: (Betty chooses ruler and measures selected pieces for accuracy).

Interviewer: So, do you agree that the measures there are accurate?

Betty: (Betty shakes her head yes).

Interviewer: 0K. And you can talk; it's no problem, because this iPad is recording both images and sound. Here's what I'd like for you to do: this task is called Make a New Puzzle, and we're taking those measures and we're enlarging it to form a new puzzle. If you want to put it back together as a square, you'll see which parts and which numbers are going where.

Betty: (Betty reassembles puzzle pieces to form a square).

Interviewer: And you have those numbers all the way around the square, and we're going to enlarge it, and here are the directions. We're going to fill in the measures on this page so that the puzzle pieces, so that this length 4 cm we want to make it 7 cm, and I want you to do that for all of the numbers. If four becomes seven, I want you to fill in those measures on this page, please.

Betty: (Betty starts calculating new measures).

Interviewer: 0K. And you're almost through there, yes. Now, if you don't mind, tell me what you did and write down here what you did to get those new numbers.

Betty: (Betty writes sentence on paper beneath the puzzle diagram: I added 3 cm to every measurement.).

Interviewer: Do you remember your student study number? If you will write that there.

Betty: And, could I head up to the cafeteria a little quick because I got to go to a meeting for FBLA?

Interviewer: We're just about through and we will get you there.

Betty: 0K.

Interviewer: I happen to have a puzzle with those measures, and I want you to put that together; it follows the same pattern.

Betty: (Betty takes new puzzle and begins to assemble pieces. Betty hesitates as she tries to assemble the pieces of the new puzzle).

Interviewer: You are hesitating a little bit.

Betty: It doesn't seem like it goes together.

Interviewer: Yeah. Do you have any ideas why you think it may not be going together?

Betty: (Betty continues to attempt to assemble the puzzle).

Interviewer: Check this: see if the measures are right. Take any one piece and measure to see if it matches what you wrote on your paper.

Betty: (Betty takes ruler and measures side length of puzzle piece). This is 7 1/10 cm.

Interviewer: 0K, what about the other part of that you had? (Betty selects another puzzle piece to measure). So, measure and see if you think that (Betty goes back to puzzle piece already measured). No, no; you've already measured that one. Let's try the other part, you see it's written with a number on it.

Betty: (Betty measures side length of another puzzle piece). That one is 9 cm.

Interviewer: Do you think that one piece being off by 1/10 of a centimeter would make it not come together as a square?

Betty: No, not really.

Interviewer: OK. Well, it turns out that the work that you going to be doing in your math class for the next few days will help you see why the strategy that you used in working this puzzle and enlarging it is not the only way to enlarge. And when you enlarge it with this new strategy, you'll find it will keep everything intact and it will make a square. Do you have any questions about the work that we did here?

Betty: (Betty shakes her head no).

Interviewer: I want to thank you for participating today and we will get you back to your class now.

Interviewer: Today, we're going to work a math task called Make a New Puzzle. I want you to take these puzzle pieces and I want you to put them together; they should form a square.

Bob: (Bob removes puzzle pieces from baggie and begins to assemble puzzle).

Interviewer: I'll give you a hint. Do you see those numbers that are written on certain sides? Those go on the perimeter of the square; they go around the square. See if that helps.

Bob: (Bob continues to attempt assembling the puzzle).

Interviewer: I don't want you to get frustrated, because putting together the puzzle is not the main idea of the task. Here's a model; let's see now if the pieces will go together now and form a square. See if you can put them together to match this.

Bob: (Bob uses model to assemble puzzle pieces).

Interviewer: Okay, so it's sort of comes together and makes a square. I know the pieces flop around a little bit, but I think we've got the idea. Excellent! Now, what I would like for you to do is take this ruler and I want you to use the metric part. I want you to pick just two or three pieces and measure where you see numbers written to see if what's written here matches on the ruler.

Bob: (Bob selects pieces and measures side lengths with the ruler).

Interviewer: Do they match?

Bob: (Bob nods head).

Interviewer: Okay. Pick a couple more, one or two more, and let's see if they match.

Bob: (Bob continues to measure additional pieces). That one's just a little off.

Interviewer: Okay.

Bob: That one seems all right.

Interviewer: So, even with what you think may not measure out, it still forms a square you believe?

Bob: (Bob nods head).

Interviewer: Well, here's what I want you to do. First of all, if you will write your study number at the top of that page.

Bob: (Bob writes number as directed).

Interviewer: We're going to make a new puzzle. Now, you see how the numbers are written here along the certain edges? What we're going to do, we're going to fill in the measures on the sides of this puzzle piece, so that the 4 cm is enlarged to 7 cm. So, go ahead and write 7 cm there on the F piece.

Bob: (Bob writes new measure as directed).

Interviewer: Now, what I want you to do is whatever you do to make the four into a seven, I want you to do the same thing to all these numbers and fill them in around the square. Do that, please.

Bob: (Bob writes new measures as directed). This one has two of them; do I just do one?

Interviewer: No, you do both. This one's the horizontal and this one's the vertical, so do both of them separately. (Bob uses fingers to calculate new measures). Okay, so you've gone around and filled in all those numbers. If you would write a sentence or statement telling me what you did mathematically to make the four into the seven and the two to a five; tell me what you did and write it down please underneath the puzzle.

Bob: (Bob writes sentence as directed).

Interviewer: Okay, if you will read that statement to me.

Bob: (Bob realizes he omitted a word from their sentence). It should have I added in it.

Interviewer: Just put in that other word in between; that's fine.

Bob: I added three to each number to make it the number it is above.

Interviewer: Excellent! So, you've gone through and you've enlarged those measures. If you put those puzzle pieces back into the baggie.

Bob: (Bob replaces puzzle pieces into baggie as directed).

Interviewer: Now, I just happen to have a baggie here that has the measures you came up with. Take these out of the baggie and see if you can put that together to form a bigger square, following this model.

Bob: (Bob begins to assemble new puzzle pieces. Bob hesitates.). It doesn't work.

Interviewer: No, it doesn't work. Hmm, let's measure. Pick a piece or two and see if they match the numbers you came up with.

Bob: (Bob takes ruler and begins to measure selected pieces). I can't pick that one.

Interviewer: Hmm?

Bob: Wait. Oh, duh. (Bob continues to measure puzzle pieces).

Interviewer: Okay, are you finding the numbers match what you came up with on paper?

Bob: (Bob nods head).

Interviewer: So, do you think that they'll come together and form a square?

Bob: No.

Interviewer: No? Okay.

Bob: (Bob continues to attempt and assemble puzzle). You would have to, like. (Bob points to gaps between puzzle pieces).

Interviewer: It seems to have some gaps that the other puzzle didn't have. Well, it turns out, I don't want you to stress over this, the strategy that you came up with to come up with these numbers is not the only way that you can make four become a seven. So, what you're going to learn in your math class this week and next week is this new strategy that will let you enlarge or reduce and keep things in proportion to each other. So, what you did here is an approach, but it didn't keep the square; everything seemed to get a little messed up. So, as you work this week, make sure as you are learning these new strategies that if you have questions, that you ask them so that we can figure out how to enlarge or reduce or make things in proportion. Thank you for working with me today; I'm going let you go back to your second period now.

Interviewer: This morning, I'm going to have you start by taking this baggie; it has six puzzle pieces. I'd like you to put them together; they form a square. So, if you would do that, please.

Candy: (Candy takes pieces and begins to assemble puzzle).

Interviewer: And I will give you a hint. The numbers you see along the edges, those go on the outside of the square.

Candy: (Candy continues to assemble puzzle).

Interviewer: And don't worry if you have difficulty, because the putting together of the square is not the most important part. I'm giving you an opportunity to see what we're talking about.

Candy: (Candy continues to assemble puzzle. Candy successfully assembles puzzle).

Interviewer: Good job! You put that square together. You figured it all out, that's wonderful. Here's a picture to see that the pieces do go together just like you put them together. If you would, go ahead and write your student study number at the top of that page, please.

Candy: (Candy writes number as directed).

Interviewer: And here's what we're going to do. I want you to take this ruler; just pick a couple of pieces and I want you to measure them on the centimeters to see that the numbers written down there are accurate. Just pick a couple of the blue pieces and measure where the numbers are.

Candy: (Candy begins to measure on handout instead of puzzle pieces).

Interviewer: No, on the blue pieces. There you go, just pick a couple. You can take the puzzle apart if you like, whatever works for you.

Candy: (After measuring sides of one piece): This one matches.

Interviewer: Okay.

Candy: This one matches, too.

Interviewer: Okay, so the pieces seem to be accurately made and the numbers fit what we see. Excellent! Here's what we're going to do, and this is the math task. Using this picture, which matches these pieces, and I want you to fill in the measures on the sides of the puzzle pieces so that the 4 cm length is enlarged and becomes 7 cm. You can do figuring, you can do writing, but I would like you to fill in those new numbers on this picture. If you would do that, please.

Candy: (Candy seems unsure how to proceed).

Interviewer: So, the first one you start with is that 4 cm and it's automatically going to be a seven; we want to make that 7 cm. You can write in the 7 cm, if you wish.

Candy: (Candy seems unsure how to proceed).

Interviewer: Can you see where that goes?

Candy: I'm confused.

Interviewer: Okay, what you're doing, do you see the F piece? Pick up the F piece. Do you see that there are two numbers on it? There are 4 cm on the bottom and 5 cm on the side. The 4 cm is going to become 7 cm, so write 7 cm. Now, I'm asking you will you to make that 4 cm become 7 cm, you have to do something, and I'd like you to do that same something to the other numbers and fill them in. So, on the E you're going to fill in here and on the D you're going to fill in two measures and on the C you're going to fill in these numbers using the same method or pattern that makes the 4 cm become 7 cm. So, take a few minutes and write those new numbers. Do you still have some questions, perhaps?

Candy: Yes. Can I add lengths together?

Interviewer: You do whatever you think makes the four become a seven. So, how do you make a four into a seven?

Candy: You add three to it?

Interviewer: Okay. So, what do you want to do then with that two?

Candy: You add it on to the seven? No, not the seven, to the four.

Interviewer: Well, that is for the F piece; this is the E piece. So, what number are you working with?

Candy: Two.

Interviewer: And what are you going to do now to that two, according to what you told me?

Candy: Oh, you're going to multiply it by, cause you multiplied by, you added three to four. So, you're going to make this a five?

Interviewer: Okay, so go to the next piece.

Candy: This one is five plus three. Eight. And two plus three is five. Then seven plus three is ten. And two plus three is five. Five plus three is eight.

Interviewer: And you have the other side of the F piece. Okay, you've taken and you decided to add three. I'd like you to write a statement on the paper, saying what you did, what pattern you followed to get those new numbers. Write a statement that describes what you did.

Candy: (Candy writes statement as directed).

Interviewer: Excellent! I just happen to have a new puzzle that has those numbers that you just came up with. I want you to move the blue ones back into the baggie and we're going to take this and make the new puzzle that you just came up with the new numbers.

Candy: (Candy takes new puzzle pieces and begins to assemble them. Candy looks confused and begins to hesitate).

Interviewer: You seem a little perplexed. I notice you're looking back to this model and you're looking at this puzzle. Is something wrong?

Candy: It doesn't look like this one fits there.

Interviewer: Oh, it doesn't, does it? Let's measure. Let's see if the numbers I have there are written correctly. Measure and see if that seven or the five; just pick a couple of pieces and measure and see if the numbers match what we have written here.

Candy: (Candy measures length of new puzzle pieces). That matches.

Interviewer: Pick another and let's see.

Candy: That one matches.

Interviewer: Okay, so it seems like the numbers are matching what we came up with this pattern to make the new puzzle, but it doesn't seem to want to go together. Well, it turns out there's more than one kind of strategy you can use to enlarge, to make in this case a new square puzzle. As you work this week and next week in your math class, you're going to learn about those new strategies so that when you enlarge those numbers that you’re going to keep everything in the correct proportion to make the new puzzle. Maybe you have some questions? Is there anything that you’ve written or saw that you have questions about?

Candy: (Candy shakes her head).

Interviewer: Okay, thank you for taking time to work this math task for me today. I’m going to send you back to your first-period class.

Interviewer: This morning, I want you to do a math task for me called Make a New Puzzle. Here's a baggie with some puzzle pieces; I'd like you to take those out and take those six pieces and try to make them into a square.

Carl: (Carl removes pieces from baggie and starts to assemble them).

Interviewer: I'll give you a hint; you see the numbers that are written on some of the edges? They go on the outside on the sides of the square, or the perimeter of the square.

Carl: (Carl continues to assemble puzzle pieces).

Interviewer: I don't want you to stress over this, because making the square isn't the main part of the task. Let's see; here's a model. See if now you can assemble those pieces into a square.

Carl: (Carl uses model to assemble puzzle pieces).

Interviewer: Remember, the numbered edge goes on the perimeter of the square.

Carl: (Carl continues to assemble puzzle pieces).

Interviewer: You have a piece named E that you need to see where it goes.

Carl: (Carl completes puzzle).

Interviewer: Okay. So we've got the square, good. What I'd like you to do is to take this ruler, and I want you to pick two or three pieces and measure the sides that have these numbers and see if there are accurate. So, use the metric part of the ruler. You can move the pieces; you don't have to keep them as a square. Move them however you like, and line them up to see if the numbers written there are accurate. You are on the inches, go to metric. There you go.

Carl: (Carl measures selected pieces. Carl begins to measure sides with no lengths indicated).

Interviewer: Okay, you don't have any measures on those. (Carl just pieces to measure side lengths with numbers indicated). There you go, the ones with numbers.

Carl: (Carl continues to measure).

Interviewer: Do the numbers seem to be matching what's written?

Carl: Yes ma'am, I mean Yes sir. Sorry, I'm used to talking to teachers who are girls cause I usually have all girl teachers.

Interviewer: That's okay. Here's what we're going to do now. First of all, I want you to write your study number right up there.

Carl: I think it's 301. (Carl writes number as directed).

Interviewer: And, I want you to follow these directions. We're going to fill in the measures such as on the sides of the puzzle pieces, so that 4 cm meters, which is down here, is going to be enlarged to 7 cm. So, go ahead and write a 7 cm for me down there. And if you will, you take your puzzle pieces and you can move them over here, and you are now going to fill in these new numbers to match where you are saying these old numbers. And I want you to do to those numbers that we did here to make the four into a seven.

Carl: (Carl seems confused).

Interviewer: You just fill in those numbers to do the same thing here that we have done to make a four into a seven.

Carl: (Carl still seems to be confused).

Interviewer: So, what you're going to do is just, again, you got this piece here and you see this. So, this is 7 cm and it's going to become a new number; so I want you to write that new number for me.

Carl: (Carl writes new number as directed).

Interviewer: Okay, and do that with all these pieces. This is C; now you can do the rest of F until they are all filled in.

Carl: (Carl feels in new measures as directed. Carl stops writing before finding all of the new measures).

Interviewer: Okay, and you see there is also, like on the B piece, there's a number here; if you want to fill in that number. There's one here for A, there's one here for D. Fill those in as well.

Carl: (Carl finds all of the new measures).

Interviewer: Good. Now, if you would write a sentence or statement here, telling me what you did mathematically to get these numbers.

Carl: (Carl writes sentence as directed).

Interviewer: Okay, thank you. Would you put the pieces back in the baggie?

Carl: (Carl replaces pieces back into baggie as directed).

Interviewer: I just happen to have a set that matches the numbers that you have written. I want you to take these out and try to make that new square with the numbers that you came up with. It should follow the same pattern.

Carl: (Carl begins to assemble new puzzle pieces. Carl hesitates).

Interviewer: I notice you glancing at the model as you're putting the pieces together. What do you notice?

Carl: Um, that the numbers are almost all the same.

Interviewer: The numbers are almost all the same?

Carl: They are all the same.

Interviewer: So, what's going on with your puzzle?

Carl: It's kind of hard to put these together.

Interviewer: Okay. Do you think they'll become a square?

Carl: No sir.

Interviewer: You don't? Okay, well let's measure to make sure. Take a couple of pieces and make sure that I didn't mess up and put wrong numbers. Measure the sides that have [side lengths].

Carl: (Carl begins to measure side lengths). That one's a little bit off.

Interviewer: You think that one's a little bit off? Okay, measure another piece.

Carl: (Carl continues to measure side lengths).

Interviewer: Do the numbers seem to be about right?

Carl: Yes sir.

Interviewer: Okay, well, I don't want you to get frustrated because they're not going to form a square. It turns out the strategy that you used to change the four into the seven is not the only strategy available to make a four into a seven. As you are working in math class this week and next week, you're going to learn about the strategies that will let you change numbers, enlarge them, reduce them, to keep things in proportion. So, as you are working this week and you're learning these strategies you should be able to recognize the different strategies available to you. Do you have any questions?

Carl: No sir.

Interviewer: Thank you.

APPENDIX I

Transcript of Student Interviews: Cocoa Task

Interviewer: Thank you for being part of this interview process and being one of the target students. Would you go ahead and write your study student number?

Alice: (Alice writes number as directed).

Interviewer: Let me explain this. This is called Cocoa, and there are four questions. They are going to ask you some things about the containers. You will look; for every question, you will see there is a different picture and what you see here is the level of the cocoa in the container. They could use a thermos, but they could just as well say glass, mug, or any sort of container. So, there’s nothing magical about the word thermos. What I would like for you to do is to read the question, look at the pictures, answer the question underneath, and give the reason for the answer, whatever you choose. So, if you will go ahead and do that for the first question.

Alice: (Alice begins working with problem one).

Interviewer: Of course, if you have questions, feel free to ask.

Alice: (Alice continues working with problem one).

Interviewer: Okay, and just for the recording: Thermos A contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and 1 cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Which one did you choose?

Alice: Thermos A.

Interviewer: Okay, and just tell me now basically what you've written; you can read it or say it.

Alice: Thermos A has the strongest taste. Thermos B; hold on. (Alice adds more writing to their statement). In Thermos B, the particles of the cocoa spread around more because the more water you add, the more the particles move around.

Interviewer: Okay, this look at the second question. Thermos A and Thermos B containing cocoa that tastes the same. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? You'll explain your answer, so consider the question and look at the picture and decide.

Alice: (Alice begins working with problem two).

Interviewer: Okay. So again, tell me what you selected and why.

Alice: I put neither because both mixes are the same.

Interviewer: Okay, thank you. If you'll turn the page over, we'll look at question three. Thermos A contains cocoa with a weaker chocolate taste. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Alice: (Alice begins working with problem three).

Interviewer: Okay, tell me which thermos you picked and why.

Alice: Umm, I put Thermos B has the stronger taste. In the question, it says that Thermos A has the weaker taste.

Interviewer: Okay, thank you. One more question. Thermos B contains cocoa with the stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Alice: (Alice begins working with problem four).

Interviewer: Okay. Which one did you pick and why?

Alice: I put Thermos A would be stronger tasting. The more water you add to Thermos B, the more the particles move around.

Interviewer: Okay, thank you for answering those questions; one more question that is not on this page. The laptop and the iPad were here for you to use. You chose not to use them. Do you think they would have been helpful to you in what you been working with in class, to use either of those on this task?

Alice: Umm, no not really, because it doesn't give you like a ratio.

Interviewer: Okay. Thank you so much, and I'm the let you go on now to your first-period class.

Interviewer: Thank you for being part of this interview this morning. If you will go ahead and write your study student number on the paper.

Alan: (Alan writes number as directed).

Interviewer: Now, I'll explain to you what is going on. This task is called Cocoa. There are four questions, and each one talks about two different thermoses: Thermos A and Thermos B. They give you information about it and ask you a question about which thermos has the cocoa with the stronger taste. Now, there's nothing magical about thermos; it could just as well have said cup or a container of any kind. What will do is read it, then you will answer it and we'll talk about your answer. Let's start with the first problem; of course, if you have any questions along the way, please ask.

Alan: (Alan reads number one). Thermos A contains cocoa with a stronger chocolate taste. (Alan starts to discuss problem).

Interviewer: Okay, if you would read the whole question first, then I'll let you do your work.

Alan: (Alan resumes reading number one). If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer. So, Thermos A contains cocoa with a stronger taste. One scoop of cocoa mix is added to Thermos A. So, at the beginning, this [Thermos A] was the strongest and this [Thermos B] was, I guess you would say, not so strong. So, if Thermos B, if one cup of hot water was added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Well, one cup of hot water, and that's not chocolate, but one scoop of cocoa mix, so if you add… (Alan pauses). So, if you were to add one chocolate to this [Thermos A], that would make it have a chocolate taste. If you were to add water to this [Thermos B], that would water it down. So, I don’t know how to do the math on this question, but...

Interviewer: It’s not asking for any math in terms of calculation, just which thermos you select and give your reason.

Alan: (Alan writes response to number one). Not chocolate, cocoa. I put I believe Thermos A will be stronger because if you add one scoop of cocoa, it will have a stronger taste because the chocolate mixed with the cocoa just means more chocolate. It will have more stronger taste than just adding a cup of hot water.

Interviewer: Okay, good. Let’s go on to the second one, if you want to read that for me.

Alan: Thermos A and Thermos B contain cocoa that tastes the same. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer. (Alan begins working with problem two). I put I believe that Thermos B will be stronger because the scoop of cocoa will have less room to cover than in Thermos A.

Interviewer: Okay, thank you. If you will turn the page over, let’s look at problem three.

Alan: Thermos A contains cocoa with a weaker chocolate taste. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer. Okay, Thermos A contains weaker chocolate. If one scoop is added to both Thermoses A and B, so if we know Thermos A is weaker than Thermos B, so if you added even more stuff to Thermos B, I believe it’s Thermos B. (Alan writes response to problem three).

Interviewer: Okay, so tell us about your choice.

Alan: I said I believe that Thermos B will be stronger because it tells us that Thermos A has a weaker chocolate taste. So, if you add even more cocoa to Thermos B, it will be stronger.

Interviewer: Okay. Let's look at the last question on the page.

Alan: Thermos B contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger taste? Now, on this question, is it one thermos that is the answer or could it be both?

Interviewer: Well, that's up to you to decide, based upon what you read and what you see.

Alan: Okay. Thermos B contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A, one cup of hot water… (Alan writes response to problem four).

Interviewer: Okay, tell us about your choice.

Alan: I put I believe that both Thermos A and B will be approximately the same taste because it tells us that Thermos B is stronger than Thermos A, but when you put a cup of hot water in Thermos B and put a scoop of cocoa mix in Thermos A, they will be about the same.

Interviewer: Okay, thank you. There's one more question; it's not on the paper, but it's about technology. There's a laptop and an iPad that you had available to you; did you think that using technology would help you answer these questions, based on what you've done in class these past few days and what you see in front of you?

Alan: I don't believe that the, umm, I bet that they could, but I wouldn't know how to answer these questions, like drawing a model or doing a proportion, like the arithmetic of a problem; I wouldn't know how to do that. I just tried to explain it the best that I could in my explanation.

Interviewer: Okay. Thank you very much for participating and I'll let you go on to your class, now.

Interviewer: Thank you for participating in today's interview. If you will go ahead and write your study student number at the top of the page.

Betty: (Betty writes number as directed).

Interviewer: Let me explain this task; it is called Cocoa. In each problem, there are two thermoses containing cocoa: Thermos A and Thermos B. You have a picture to show you how much is in each thermos, and you also have something that they are doing to the thermoses. After they perform these actions, they want you to decide which thermos has the stronger chocolate taste. You will write your answer and you'll explain your answer. If you have any questions during the task, please ask; no problem. Go ahead and let's start with problem one. We have Thermos A contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and 1 cup of hot water is added to Thermos B, which Thermos contains the cocoa with the stronger chocolate taste? You will write your answer and explain your choice.

Betty: (Betty begins working with problem one).

Interviewer: Okay, tell me about your choice and why you chose it.

Betty: I put that Thermos B contains the cocoa with the strongest chocolate, because even though Thermos A says it has the cocoa with the stronger chocolate taste, there's more cocoa in Thermos B. so I thought that with all that cocoa when it comes together it will create a stronger taste in Thermos B.

Interviewer: Okay. Let's move on to the second question. Again, we have Thermos A and Thermos B contains cocoa that tastes the same. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which Thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Betty: (Betty begins working with problem two).

Interviewer: Okay, tell me about your choice and why you chose it.

Betty: I thought Thermos A had the stronger taste because there's more cocoa and it says that the cocoa tastes the same.

Interviewer: Okay. Let's turn the page over, and you have two more questions. Let's look at numbers three. Thermos A contains cocoa with a weaker chocolate taste. A if one scoop of cocoa mix is added to both Thermos A and Thermos B, which Thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Betty: (Betty begins working with problem three).

Interviewer: Okay, tell me about your choice and why you chose it.

Betty: I think that Thermos B would have the strongest chocolate taste, because in the problem it says that Thermos A has a weaker chocolate taste already and if one scoop is added to both, then it wouldn't change anything. Like if it was a stronger chocolate scoop that they put in Thermos A, they would have to put a stronger scoop of cocoa in this one to so that it wouldn't really change that much.

Interviewer: Okay, look at number four. Thermos B contains cocoa with a stronger chocolate taste. It will come in if one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which Thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Betty: (Betty begins working with problem four).

Interviewer: Okay, tell me about your choice and why you chose it.

Betty: Okay, I thought that Thermos B would have the stronger taste still, because it says that Thermos B already has a stronger taste and it says that one scoop of cocoa mix, just like one normal scoop is added to Thermos A, it wouldn't really do anything to it, and if one cup of water is added to Thermos B, that wouldn't really do anything, either. So, I thought that Thermos B would still be the strongest taste.

Interviewer: Okay, thank you. I have one more question; it's not on the paper. There's a laptop and there's an iPad that was available to you, so you could have used technology to answer these. Do you think technology would have helped you to answer these questions?

Betty: Most likely not.

Interviewer: Okay. Because?

Betty: Because like, you couldn't tell, you couldn't really tell without trying to guess like on here, if which one was stronger unless you had the cocoa with you; you're actually tasting it and try to see which one had the stronger chocolate taste.

Interviewer: Okay. Thank you very much for participating, and I will send you back to class now.

Interviewer: Okay, thank you for being part of the interview this morning. If you will go ahead and write your study student number on the paper.

Bob: (Bob writes number as directed).

Interviewer: Let me explain what we're doing; this task is called Cocoa. There are different questions on these pages; there are four all together, and each one talks about two different thermoses: Thermos A and Thermos B. Both of them have cocoa in it, and the questions tell you about stronger taste or weaker taste and what they do then to each thermos, and then it will ask you to pick which thermos has the cocoa with the stronger chocolate taste. So, read it and I want you to make your selection and write your explanation, then tell me about it. So, will start with the first one; of course, any time you have questions feel free to ask, but let's start with number one. Thermos A contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Bob: (Bob begins working with problem one). Does it say that Thermos B has some hot chocolate added, too?

Interviewer: Well, let's read. It says that Thermos B has a cup of hot water added to it. So, Thermos A gets one scoop of cocoa mix is added to it and Thermos B has one cup of hot water added to it.

Bob: I would say that A has the strongest taste.

Interviewer: Okay, if you would just write down your reason, then.

Bob: (Bob writes reason for problem one).

Interviewer: Okay, so you chose Thermos A; tell me why.

Bob: Because Thermos B doesn't have any cocoa in it.

Interviewer: It doesn't have any cocoa in it?

Bob: Because it says it only has a scoop of water.

Interviewer: Okay. Remember, they both start with cocoa, so I'm not asking you to change your answer; I want you to remember what the pictures are telling you. The pictures are showing you the amount of cocoa that's in each thermos. So, you can stick with Thermos A, of course; but, please remember each thermos does have cocoa and you're doing something to it. So, that's what the pictures are telling you when you say the levels, that's telling you how much is in the thermos. Okay, let's go on to the next one. Thermos A and Thermos B contain cocoa that tastes the same. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Bob: (Bob begins working with problem two). So, this one [Thermos A] going to end up with one scoop and, so that one [Thermos A] will be the strongest.

Interviewer: Okay, so write down your answer and give me your reason why.

Bob: (Bob writes reason for problem two).

Interviewer: Okay, so tell me about your answer.

Bob: Thermos A would taste the strongest because it has more cocoa than Thermos B.

Interviewer: Okay, turn the page over and let's look at the third set. Thermos A contains cocoa with a weaker chocolate taste. If one scoop of cocoa mix is added to both Thermos

A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Bob: (Bob begins working with problem three). Thermos B would be because if that [Thermos A] has a weaker taste, then it just added more stronger to that one [Thermos B]. (Bob writes response to problem three).

Interviewer: Okay, let's look at the last question on the page. Thermos B contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Bob: (Bob begins working with problem four). I would say B because it has cocoa and water mixed into it. (Bob writes response to problem four).

Interviewer: Okay, so what did you pick and why?

Bob: Thermos A has more cocoa in it than B; more cocoa would be stronger.

Interviewer: Okay, thank you. I have one more question; it's not on the page. In front of you there is a laptop and an iPad, so you had technology available. You answered these questions, you didn't use that [technology]. Did you think that the technology would have been helpful to you in answering these questions, based on the work that you've done these past few days in math class?

Bob: I would think that paper would be a little easier, because if you don't know how to use one of the ease, it would be harder to do.

Interviewer: Okay, and that makes sense. Thank you very much for participating and I'll let you go on the class.

Interviewer: Thank you for participating this morning. Let's start by the study student number at the top of the page.

Candy: (Candy writes number as directed).

Interviewer: And let me explain to you this math task. It is called Cocoa. For each question, there are two thermoses, each containing cocoa or hot chocolate, whichever term you want to use: Thermos A and Thermos B. The picture gives you some information about how much is in each thermos, and then the question we are doing something to each thermos and then you have to decide, based upon what's in the picture and what's done in the problem, which thermos ends up with the stronger cocoa taste. So, you will write your answer and you'll explain why you chose what you chose. So, let's start with the first one. Of course, any time that you have questions on this, feel free to ask. (Interviewer checks iPad to make sure it is working). Let's look at problem one. Thermos Pay contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and 1 cup of hot water is added to Thermos B, which Thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

Candy: (Candy hesitates). I know how to solve it but I don't know how you do it involving ratios, though.

Interviewer: Okay. You come up with your answer based upon whatever way you can work that.

Candy: Okay. (Candy begins working with problem one).

Interviewer: Okay. Tell me about your choice and why you made that decision.

Candy: Well, Thermos A should have a stronger chocolate taste than Thermos B, because you are adding more to Thermos A. there is less water and stuff in there, and when you are adding just water to Thermos B, the chocolate taste is going away, is spreading out with the more water.

Interviewer: Okay, thank you. Let's look at question two. Thermos A and Thermos B containing cocoa that tastes the same. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer, please.

Candy: (Candy begins working with problem two).

Interviewer: Okay. Tell me about your choice and why you chose it.

Candy: Well, Thermos B has less liquid in it, so you're adding more, it can't, does not have as much liquid, it has to stay in that place and just mix around in that little bit, but with Thermos A, it spreads out everywhere in the whole thing.

Interviewer: Okay, thank you. Let's turn the page over and look at the next one. Thermos A contains cocoa with a weaker chocolate taste. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer, please.

Candy: (Candy begins working with problem three).

Interviewer: Okay. Tell me about your choice.

Candy: Well, in the first sentence it says that Thermos A contains cocoa with a weaker chocolate taste. So, if you're adding the same amount of cocoa mix to both thermoses, then, and it said that Thermos A contains the weaker, it's still going to be weaker, even with another scoop because both of them got the exact same scoop of cocoa added to them.

Interviewer: Okay. Let's look at number four. Thermos B contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and 1 cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer, please.

Candy: (Candy begins working with problem four).

Interviewer: Okay. Tell me about your choice for problem four.

Candy: Well, it says that Thermos B contains the stronger chocolate taste, but when you added the water to it, it made the chocolate taste spread out with the water. When you added the extra chocolate taste to Thermos A, which was weaker than Thermos B, it made it be the exact same taste as Thermos B.

Interviewer: Okay. I have one more question; it's not on this page. In front of you there was a laptop and an iPad, and you did not choose to use either of those when answering these questions. Do you think technology could have helped you answer these questions about the Cocoa task?

Candy: It might have if I knew how to answer it, but I didn't know how to use Thinking Blocks or anything like that.

Interviewer: Okay. Thank you so much for participating today and I will send you back to class now.

Interviewer: Thank you for being part of the interview this morning; if you will go ahead and write your study student number.

Carl: (Carl writes number as directed).

Interviewer: Let me explain the task to you. It is called Cocoa. There are four different questions about two different thermoses of cocoa, called Thermos A and Thermos B, and you are given information about the thermoses in a picture, and then you are given a description of what’s happening to each container; they want you to make a decision as to which thermos has the stronger chocolate taste. Now, there’s nothing magical about the word thermos; it could just as well say cup or a container of any kind. The information that is provided is in the picture and in the problem. Of course, if you have any questions during this, no problem. Please ask, and will try to address them. So, let’s start with the first problem. Thermos A contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? So, make your decision on that and explain your answer. Of course, you will write it then you’ll say it.

Carl: (Carl begins working with problem one). Thermos A has the stronger taste because the water makes the chocolate kind of less strong.

Interviewer: Okay, let’s look at the second. Thermos A and Thermos B contain cocoa that tastes the same. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer, please.

Carl: (Carl begins working with problem two). I think Thermos A because it has more flavoring in it.

Interviewer: Okay. Let's look at question three. Thermos A contains cocoa with a weaker chocolate taste. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer, please.

Carl: (Carl begins working with problem three).

Interviewer: Okay, tell me about your choice for number three.

Carl: I think it would be Thermos B because it has a stronger cocoa taste in it, and then there has to be a lot more in A to be equal or more than B.

Interviewer: Okay, let's look at the last thermos question. Thermos B contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer, please.

Carl: (Carl begins working with problem four). I think Thermos A because it doesn't have any hot water to make it less strong.

Interviewer: Okay, thank you. I have one more question; it's not on the pages. You have in front of you, there's a laptop and there's an iPad, so you had some technology available to you, but you chose not to use it to answer these questions. Do you think that technology could have helped you to answer these questions, based on the work that you've been doing in class these past few days?

Carl: A little.

Interviewer: You think it would have helped a little? Okay, how would it have been helpful to you, do you think?

Carl: [It would have] shown the blocks.

Interviewer: Shown the blocks? Okay.

Carl: I could have added them to equal up to this [Thermos B].

Interviewer: Okay. Thank you so much for participating today. I'll send you on to class now.