The Role of Mathematics Vocabulary in College Algebra Performance: Examining Reading Comprehension, Task Type, and Vocabulary Alignment

By

William E. Puckett

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Literacy Studies

Middle Tennessee State University
December, 2025

Dissertation Committee:

Dr. Eric Oslund, Chair

Dr. Amy Elleman

Dr. Sungyoon Lee

DEDICATION

To my wife Kelly, whose steady support and encouragement have carried me through this work.

Thank you for believing in me even on the days I didn’t believe in myself.

ACKNOWLEDGEMENTS

I am especially grateful to our program director, Dr. Amy Elleman, whose belief in the value of both this work and in me came at exactly the moments I needed it. Thank you for encouraging me to keep moving forward when the path felt overwhelming, and for reminding me that progress is built through persistence. Your guidance helped me rebuild confidence in my abilities during times when I struggled to see them myself. Your support and your gentle refusal to let me discard my data and walk away made this dissertation possible.

I also extend sincere thanks to Dr. Eric Oslund and Dr. Sungyoon Lee for your insight, expertise, and steady encouragement. Your perspectives strengthened the clarity and rigor of this study, as well as my instructional practice and professional growth.

To my colleagues Gina Johnson and Michelle Pheneger in the University Studies Department, thank you for welcoming this project into your classrooms and schedules. Your collaboration, flexibility, and trust allowed me to collect data that genuinely reflects the work happening in our instructional spaces. I am grateful for the professional community we share.

To my friends and cohort in the Literacy Studies program, your kindness, encouragement, and willingness to listen to my stress, doubts, and overthinking transformed this experience. Thank you for making the work feel not only manageable, but meaningful and joyful. This journey was never solitary because of you.

Finally, to my children, Elijah and Holly, thank you for your laughter and the joy you bring to my life each day. You reminded me what balance looks like and why perseverance matters.

ABSTRACT

This study examined how mathematics vocabulary knowledge predicts college algebra performance while accounting for reading comprehension and testing potential moderation by task type and vocabulary alignment. Participants were 107 undergraduates enrolled in prescribed College Algebra courses at a large public university. Students completed a researcher-developed mathematics vocabulary assessment, a mathematics performance measure consisting of computation and word problem items from the KeyMath-3 Diagnostic Assessment, and the Gates-MacGinitie Reading Test (GMRT) Reading Comprehension subtest. Data were analyzed using multiple and repeated-measures regression models.

Mathematics vocabulary knowledge was observed to be a statistically significant positive predictor of mathematics performance even after controlling for reading comprehension ($\beta = .26, p = .012$). Reading comprehension contributed minimal variance and did not statistically significantly moderate this relationship, indicating that vocabulary supported performance similarly across reading levels. Task type moderated this relationship with vocabulary more strongly predicting computation performance than word problem performance. Finally, general mathematics vocabulary emerged as a statistically significant predictor of performance while vocabulary directly aligned with performance items did not, suggesting that students' vocabulary knowledge supported problem solving even when known terms did not appear on the assessment.

These findings highlight mathematics vocabulary as a central cognitive linguistic resource for accessing algebraic content, particularly for academically at-risk college students. Mathematics vocabulary is positioned not only as a target for improving problem-solving performance, but also as a diagnostic marker of students' underlying conceptual grasp of algebraic ideas. Results support disciplinary literacy perspectives by showing that mathematics-specific vocabulary contributes uniquely to algebra performance beyond general comprehension. Instructionally, results provide evidence for the importance of embedding explicit mathematics vocabulary instruction within college algebra curricula. Future research should investigate cognitive moderators and receptive versus expressive vocabulary to further clarify vocabulary's role in postsecondary mathematics learning.

Keywords: mathematics, mathematics vocabulary, disciplinary literacy, comprehension, cognitive skills, algebra performance

TABLE OF CONTENTS

Chapter 1

Introduction

Current descriptions of mathematical proficiency include understanding concepts, operations, and relations, procedural fluency, and adaptive reasoning (Kilpatrick et al., 2001). Through modalities such as words, images, equations, symbols, sounds, gestures, graphs, and artifacts (Gee 1996, 1999), students and teachers can show mathematical proficiency. However, deficits often arise when individuals must independently read, express, and leverage their understandings using academic language that is either not known or not clear to them. The National Council of Teachers of Mathematics (2014) even states the importance of communicating mathematically and follows up with requirements for students to both solve and support correct solutions using the academically accepted mathematical language.

Academic language includes the vocabulary, grammar, and linguistic functions students use to acquire knowledge and complete tasks in various academic disciplines, such as mathematics (Cummins, 2000). The discipline-specific ways of using language are essential for helping students understand how experts in the discipline organize knowledge (Fang, 2012), making it crucial for them to master this academic language to succeed academically (Kleemans et al., 2018; Townsend et al., 2012). Numerous studies have shown that academic language is not only linked to academic performance but also serves as a predictor of future academic achievement (e.g., Purpura, Logan, et al., 2017; Purpura, Napoli, et al., 2017; Townsend et al., 2012).

Mathematics, due to its specialized language, is particularly difficult for many students (Berch & Mazzocco, 2007). One essential component of mathematical language is mathematics vocabulary (Moschkovich, 2015; Simpson & Cole, 2015), which includes understanding specific terms, phrases, abbreviations, and symbols commonly used in math textbooks, teaching, and assessments (Monroe & Orme, 2002; Moschkovich, 2015). Mathematics vocabulary first appears in the form of number words, terms for quantities, and spatial relation words, becoming increasingly complex as students progress in content (Lin et al., 2021). By middle school, students must master hundreds of mathematical terms (Hughes et al., 2018; Powell et al., 2017). In parallel, the impact of vocabulary knowledge on reading comprehension grows as students progress in their education, eventually accounting for more variation in comprehension than decoding skills or background knowledge by high school (Cromley and Azevedo, 2007). The relation between mathematics vocabulary and mathematics performance may also increase with age because of the complexity of required higher-order mathematics tasks (Common Core State Standards Initiative, 2010)

A solid foundation in mathematics vocabulary often supports later mathematical understanding and skill development (Han, 2020; Kotsopoulos, 2007; Harmon, 2005). An iterative approach suggests that mastery of mathematics vocabulary can lead to improvements in mathematics performance, with reciprocal effects over time (Rittle-Johnson, 2017; Rittle-Johnson et al., 2001). Specifically, mathematics vocabulary equips students to construct, select, and apply problem-solving procedures effectively, which can enhance their mathematical performance. Conversely, active engagement in mathematical procedures can deepen students' understanding of mathematics vocabulary, particularly when practice activities are structured to highlight foundational concepts. This relationship means that as students learn and apply mathematical concepts, they strengthen their procedural and problem-solving skills through a deeper vocabulary comprehension. For instance, understanding the term "inverse property" can be enhanced through practice equations that illustrate the term in action, solidifying both vocabulary and comprehension through contextual application (Rittle-Johnson, 2017; Rittle-Johnson et al., 2001).

Mathematics Vocabulary and Problem Solving

Knowledge base comprises different aspects of real-world and academic memory contents including intuition and informal understandings (Carpenter et al., 1989; Zhang et al., 2021). This degree of knowledge of facts influences procedures taken within the problem-solving process since individuals with deeper understanding of content are more likely to implement correct procedures with mathematics vocabulary corresponding to mathematics conceptual knowledge that is stored in long-term memory (Schleppegrell, 2007). On the other hand, mathematics vocabulary might be less crucial for foundational math tasks that primarily focus on procedural accuracy and fluency of arithmetic operations (Fuchs et al., 2015; Lin, 2020; Peng & Lin, 2019). Mathematics vocabulary can be theorized to play a crucial role in one's conceptual understanding of mathematics stored in long-term memory; however, research suggests mathematics vocabulary may also serve as a proxy for comprehension and cognitive skills.

Comprehension skills (e.g. reading and listening comprehension and general vocabulary) are foundational for mastering mathematics vocabulary and learning mathematics with understanding, as they support the ability to process and integrate terms' meanings (e.g., Peng & Lin, 2019; Hornburg et al., 2018). Cognitive skills, including working memory and nonverbal reasoning, also contribute significantly to the relationship between mathematics vocabulary and mathematics performance by enabling students to maintain and process mathematics vocabulary in real time while reasoning through mathematical concepts. Mathematics vocabulary can serve as a mediator of cognitive demands among higher-order mathematics tasks given those tasks are innately more cognitively demanding (Bloom, 1956; Lai, 2011; Lin et al. 2021).

Heuristics references problem-solving strategies within mathematics (Schoenfeld, 1992). Individuals with automaticity built from mathematics vocabulary should be more likely to spend more cognitive resources on non-conceptual related processes during higher-order mathematics performance as opposed to spending those cognitive resources to parse out what problem is being presented, what conceptual knowledge is needed, and procedural steps, thus decreasing performance (Fuchs et al., 2015; Lin, 2020; Peng & Lin, 2019; Peng et al., 2020).

Self-regulation falls under metacognition and can influence reading and appropriate use of content knowledge (Schoenfeld, 1992). When solving mathematical problems, students often encounter uncertainty about next steps or previous accuracy, prompting them to backtrack, reassess the problem's requirements, and evaluate their solution's coherence—a process aligned with self-regulation, which involves planning, outlining, exploring options, and revising as needed to construct meaning (Montague, 1992; Zimmerman, 2002; Schoenfeld, 1987). Self-regulation also can support vocabulary acquisition by enabling learners to set goals, monitor comprehension, and apply strategies that enhance word knowledge (Li and Gan. 2022; Roberts et al., 2020). Duke and Cartwright's (2021) Active View of Reading model identifies self-regulation as essential for vocabulary development, emphasizing its role in sustaining motivation, engagement, and strategic word learning.

Receptive and Expressive Vocabulary

Receptive vocabulary refers to the ability to comprehend words and concepts when they are encountered in spoken or written form (Snow & Uccelli, 2009). For mathematics students, this includes contextualizing and understanding discipline specific meaning of terminology embedded in word problems, such as "factorize," "simplify," or "distribute." When a student encounters a complex problem, they rely on receptive vocabulary to interpret the language of the problem correctly and connect it to their existing mathematical knowledge. For example, understanding the phrase "determine the roots of the equation" requires the student to recognize "roots" as the solution to a quadratic equation. Receptive language skills are crucial during problem-solving, as solving word problems requires students to interpret and translate academic language into appropriate operations (Snow, 2010). By employing metacognitive strategies such as self-monitoring, planning, and evaluating, students can better navigate complex problems and enhance their mathematical performance (Schoenfeld, 1992; Zimmerman, 2002). This aligns with findings by Powell (2022) and Lin et al. (2021), which highlight the importance of precise word comprehension in navigating mathematical tasks and ensuring problem-solving success... In contrast, expressive vocabulary involves the ability to produce and convey information, either orally or in written form (Bailey et al., 2020). In mathematics, this could manifest as writing equations, labeling diagrams, or providing a step-by-step justification of problem-solving processes. Expressive vocabulary enables students to communicate their reasoning clearly and justify their steps, which is essential for demonstrating understanding and addressing errors. When students "show their work" and articulate why they chose specific methods, they engage their expressive vocabulary. Expressive vocabulary is thus integral to evaluating their steps and defending their decisions, making it another tool for self-regulation.

Receptive and expressive vocabulary are interdependent in mathematical problem-solving. A student with strong receptive vocabulary may understand a problem but struggle to articulate their reasoning, while a student with strong expressive vocabulary may effectively communicate incomplete or incorrect understandings. Research suggests that receptive vocabulary is typically larger than expressive vocabulary, as students often comprehend more words than they can produce (Fuchs et al., 2015). This discrepancy underscores the importance of instructional strategies that develop both types of vocabulary. For example, explicit teaching of mathematics-specific vocabulary, combined with opportunities for students to practice articulating their reasoning, can bridge the gap between understanding and communication (Sidney et al., 2015; Harmon et al., 2005).

Literacy's Role in the Mathematics Classroom

Waller and Flood (2016) reviewed case studies of mathematics classroom interactions in different multilingual groups and discovered three emergent themes. Mathematics provided multilingual students in a Brazilian classroom tools of reasoning and logic which showed several instances of personal empowerment among students approaching difficult problems. Using the vocabulary and concepts provided by the teachers, the students were able to corroborate their own experiences and language to learn and implement mathematical concepts. The case studies also revealed "mutual intersection" between the students in classrooms and provided support for the argument that mathematics is universal as fundamental concepts held true for all students regardless of their language or culture. One stated example showed how the students all shared the same number sense when explicitly working with manipulatives and a number line.

Using language as a resource in the mathematics classroom may allow students to understand content better while allowing students to position their own ideas about mathematics to construct their own mathematical identities (Herbel-Eisenmann et al., 2013; Horn, 2012). Barwell (2018) saw students who were both English and French speakers make meaning of mathematical terms and processes by using each language in tandem to relate aspects of each to come to correct solutions. The problems were read in French in which the students determined meaning by relating words in French and English to understand the context and what was being asked. Calculations were observed to be discussed primarily in English and then solutions presented in French. Though some words were not found in each language, the teacher was also able to provide written representations of French exclusive terms such as “demi-litre” which was presented to the students prior to working on the problems. This meaning was made using a diagram drawn on the blackboard along with other preferred mathematical vocabulary and used by the students when translating the problems into English providing multiple sources of language to make meaning.

Some researchers advocate the universal use of language in mathematics as expert mathematicians follow a similar format that Jamison (2000) discusses as being consistent and historically used for mathematical exposition. This definition-theorem-proof format may be widely used by expert mathematicians even today, but fragmented as it makes it way to the classroom and even further when it comes down to daily use. Teachers must not only defend mathematics usefulness through systematic argumentation and discourse but bridge the gap between the expected mathematical expression and common language usage. Gibbons (2003) discusses the use of language in every classroom for a range of purposes that students must learn, and teachers must mediate. Teachers must navigate the shift from how students use language in their everyday expressions to content area specific expressions (Gibbons, 2006). The need to read and write mathematical expressions in a specific manner, especially in an educational setting, provides a seeming paradox of meaning making. While documented and near universal, mathematical literacy requires individuals to adopt a potentially new way of communication that might be different than conversational or literature usage of vocabulary. Some of the language features in mathematics may be different with terms having different meanings in a mathematical context compared to day-to-day language usage.

Cartwright et al. (2022) used the term "lexical ambiguity" to describe the differences between word meanings in everyday conversation compared to the same word with a different meaning in a mathematics context. For example, the word "difference" can refer to objects being dissimilar in common language or the result of a subtraction problem in mathematics. Similarly, "volume" can describe the space inside a three-dimensional object in geometry or the loudness of a sound. Such lexical ambiguity can confuse students, especially when they rely on everyday meanings without considering content-specific vocabulary (Cartwright et al., 2022; Durkin & Shire, 1991). Cartwright et al. (2022) found that word problem-solving proficiency had a statistically significant positive correlation with knowledge of lexically ambiguous math words. Their findings also revealed a significant main effect for grade level, indicating that students' knowledge of these words increased as they progressed through school. Interestingly, no interactions were found with language group, suggesting that lexical ambiguity affected both

English monolingual and emergent bilingual students similarly. These results support the Peng et al. (2020) who propose that language serves as both a medium and a tool for higher order thinking in mathematical reasoning.

Pierce and Fontaine (2009) emphasize that high-stakes mathematics assessments, which primarily feature word problems, exacerbate the difficulty of mathematics vocabulary. A review of the Partnership for Assessment of Readiness for College and Careers (PARCC, 2015) Grade 5 Math items revealed that only 8 out of 35 word problems were written in simple, everyday language. As a result, despite the importance of mathematics vocabulary in mathematics, mastering it remains a challenge for many students (Rubenstein & Thompson, 2002).

Mathematics Disciplinary Literacy

Fulda (2009) demonstrates the difference between approaching math text through a content lens vs a disciplinary lens. For example, “$x < y$" includes letters that are seen in general literature; however, approaching this statement from a mathematician’s standpoint, they represent unknown values with a specific relationship. To master the content, the relationship between “x” and “y” as representations of unknown values with a specific relationship must be understood. Discourse theory in mathematics education explores how communication, including language, symbols, and interactions, facilitates the construction of mathematical knowledge (Moschkovich, 2002; Radford, 2008; Sfard, 2008; Walshaw & Anthony, 2008). Sfard (2008) introduced "commognition," combining communication and cognition, to emphasize that learning mathematics involves mastering its unique discourse, such as vocabulary, visual mediators, and routines. Similarly, Radford (2008) highlighted the semiotic and cultural dimensions of mathematical discourse, emphasizing the collaborative role of gestures, speech, and symbols in constructing meaning. Together, these perspectives hypothesize that mathematical understanding is developed through active participation in discourse.

An issue with discourse theory according to Shannahan (2011) is that it focuses on students talking about mathematics concepts but does not recreate a normative standard to use. Student inability to generalize concepts disallows pattern recognition between problems or concepts, not allowing them to transfer their knowledge or often even replicate results. Students need to learn the content of the subject, how content is created, how mathematicians think or approach problems, and how experts in the field read and write to express and understand concepts. The role of literacy in mathematics includes recording methods and results, making methods replicable, interpreting results appropriately, determining what is known and unknown, making sense and explaining phenomena. Understanding mathematical vocabulary leads to automaticity of concept recognition. If a student can approach mathematical statements in the wider relationship between concepts, they can identify patterns and transfer across multiple problems. This involves a deep understanding of the mathematics vocabulary to different levels from words (quantities, how the inequality is written in words), narratives (definitions, proofs, theorems), mediators (symbols, formulas, diagrams, graphs), to routines (patterns and properties) (Sfard, 2007). Individuals that can show knowledge of a discipline using the technical vocabulary of the discipline can align with the core principles, understandings, and practices in that field allowing practical use of expert patterns, interpretations, normative standards, skills, and social discourse of the discipline (Shanahan & Shanahan, 2020).

Standardized Measures of Mathematics Vocabulary

Standardized measures that look to assess mathematics vocabulary often focus on terms critical for understanding math concepts. For example, the Smarter Balanced Mathematics Assessment emphasizes "construct-relevant vocabulary," including terms related to operations, measurement, geometry, and algebra, to ensure students can comprehend and apply mathematical concepts across grade levels (Smarter Balanced Assessment Consortium, 2023). Similarly, the Woodcock-Johnson III Tests of Achievement (WJ-III; Woodcock et al., 2001) include the *Quantitative Concepts* subtest, which assesses knowledge of math vocabulary, symbols, and relationships, as well as the *Applied Problems* and *Math Fluency* subtests that contribute to a broader understanding of math competence. Though vocabulary is not isolated as its own domain, it is interwoven within these conceptual assessments.

The Test of Mathematical Abilities – Third Edition (TOMA-3; Bryant et al., 2012) provides a more targeted approach, with a dedicated *Mathematical Symbols and Concepts* subtest assessing students' understanding of math-specific terms, symbols, and language. The TOMA-3 is norm-referenced and includes several complementary subtests (e.g., *Computation*, *Story Problems*) to provide a comprehensive profile of mathematical ability. It has demonstrated strong reliability and validity, supporting its use in identifying areas of strength and need in mathematical development.

Among these tools, the KeyMath-3 Diagnostic Assessment (Connolly, 2007) offers one of the most comprehensive, standardized approaches for assessing both mathematical performance and vocabulary knowledge. Although vocabulary is not isolated in a single subtest in the 2007 edition, the assessment includes numerous items across the Numeration, Algebra, and Geometry subtests that require students to understand and apply key mathematical terms. Additionally, its Foundations of Problem Solving and Applied Problem Solving subtests assess reasoning in linguistic contexts, offering opportunities to examine the role of math vocabulary in problem interpretation. The KeyMath-3 is a norm-referenced, research-supported instrument that provides a detailed breakdown of students' skills, making it a valuable tool for evaluating student understanding mathematical vocabulary and performance for all ages.

Vocabulary, Reading Comprehension, and Mathematics Skills

Theories of reading comprehension consistently recognize vocabulary knowledge as a core element (e.g., Gernsbacher, 1990; Kintsch, 1988; Tunmer & Chapman, 2012). Vocabulary knowledge as fundamental to reading comprehension has been explored through numerous studies (e.g., Cromley & Azevedo, 2007; Kendeou et al., 2009; Stahl & Nagy, 2005; Stanovich, 1986; Tannenbaum et al., 2009) repeatedly showing that vocabulary knowledge influences comprehension (Elleman et al., 2009). Vocabulary knowledge alone does not guarantee reading comprehension, but limited vocabulary knowledge constrains comprehension processes (Perfetti, 2007; Perfetti & Stafura, 2014).

Core mathematics skills (e.g. understanding numbers, combining them, and performing basic operations) are foundational for tackling advanced mathematical problems (Powell et al., 2013; Sayeski & Paulsen, 2010). Higher-level skills, like solving word problems, working with fractions, and understanding algebra, require following complex procedures and integrating several concepts. These advanced tasks often demand greater working memory and involve nonverbal reasoning to manage the multiple steps and concepts involved (Peng et al., 2020). The Program for International Student Assessment (PISA) describes proficiency across levels, from basic to advanced including references to vocabulary. At lower levels, students rely on vocabulary to decode instructions and solve simple tasks. As proficiency increases, students engage with complex representations and mathematical contexts, requiring precise language to interpret problems and link abstract concepts to real-world applications. At the highest levels, advanced vocabulary supports creativity and critical thinking, enabling students to articulate reasoning and justify solutions effectively (Organization for Economic Co-operation and Development, 2023).

Sharma's (2013) "Levels of Knowing Mathematics" provides another framework emphasizing progression from concrete to abstract understanding using appropriate vocabulary. At the initial stages, vocabulary helps students describe and label tangible phenomena. Intermediate levels use mathematical terms to connect visual representations with abstract ideas, such as equations or functions. At advanced stages, vocabulary becomes essential for generalizing concepts and communicating reasoning in real-world and academic contexts. This structured progression highlights vocabulary as a tool for transitioning between intuitive understanding and sophisticated problem-solving.

Across these frameworks, vocabulary is central to mathematical literacy. It underpins the ability to decode problems, articulate reasoning, and generalize knowledge across contexts. with advanced proficiency involving not only solving problems but also using precise language to explain solutions and apply mathematical concepts creatively (Riccomini et al., 2015). Educators must prioritize the explicit teaching of math-specific vocabulary to enhance students' conceptual understanding and performance at all levels of proficiency (Monroe and Orme, 2002).

Chapter 2

Literature Review

According to the National Mathematics Advisory Panel (2008), algebra serves as a foundation for success in mathematics and STEM careers. This can be due to how algebra prepares students for advanced mathematical thinking and problem-solving (Kilpatrick, Swafford, & Findell, 2001), its role in fostering logical reasoning (The National Research Council, 2013), or its importance for academic equity and career access (Moses & Cobb, 2001). The National Research Council (2001) describes procedural fluency and conceptual understanding as interconnected components essential for mathematical proficiency. Research has shown conceptual understanding to enhance procedural adaptability, particularly in algebraic problem-solving (Star & Rittle-Johnson, 2008) with effective teaching building procedural fluency on a foundation of conceptual knowledge (National Council of Teachers of Mathematics, 2014). Arithmetic skills, such as fraction proficiency and proportional reasoning, have also been identified as significant predictors of algebra success (Brown & Quinn, 2007; Rittle-Johnson & Alibali, 1999). Additionally, cognitive skills, including working memory, spatial visualization, and number sense, are essential for both arithmetic and algebra performance (Lee et al., 2004; Tolar et al., 2009). However, a more comprehensive understanding of how these factors interact to influence algebra preparedness is needed.

Despite its theoretical importance, the relationship between mathematics vocabulary and mathematics performance remains unclear. Some studies have found a weak or non-significant correlation (e.g., Bradley, 1987; Pina et al., 2015), while others report a moderate correlation (e.g. Brenner, 1981; Fuchs et al., 2018; Peng & Lin, 2019; Vukovic, 2006), and some indicate a strong correlation (e.g., Bae, 2013; McEntire, 1981). Cirino et al. (2013) found that vocabulary consistently predicted algebraic performance, even when controlling for arithmetic factors. Conceptual understanding, particularly of fractions, and arithmetic proficiency were also significant predictors of algebra success.

While vocabulary is crucial in early learning stages for building foundational literacy and comprehension skills, its predictive power diminishes with age as other factors such as cognitive skills (e.g., working memory, reasoning), background knowledge, and problem-solving strategies become more influential (Suggate et al., 2014; Torgesen et al., 2001). This shift suggests that although a strong vocabulary base supports initial mathematical understanding, higher-order algebraic tasks rely increasingly on integrative skills, including conceptual reasoning and procedural fluency (National Research Council, 2001). Additionally, as algebra introduces specialized language and symbols, the interaction between mathematical vocabulary and cognitive demands becomes more complex (Fuchs et al., 2018; Peng & Lin, 2019). This dynamic highlights the importance of understanding the evolving interplay between linguistic and cognitive factors in shaping algebra performance over time.

Measures of mathematics vocabulary typically focus on assessing the understanding of math-specific terms, concepts, and their application. Construct validity for these measures is often ensured through various approaches, such as expert reviews and factor analysis, confirming that the items accurately reflect the underlying mathematical constructs (Powell et al., 2017; Baker, 2016). Reliability is frequently evaluated through internal consistency (e.g., Cronbach’s alpha) or inter-rater reliability when subjective scoring is involved. Cronbach's alpha is commonly used to ensure that all items within a test consistently measure the same underlying construct (Powell, 2015; Powell et al., 2016).

Research on mathematics vocabulary tests distinguishes between receptive (understanding terms) and expressive (using terms in context) vocabulary knowledge, often using factor analysis to validate these dimensions. Factor analysis confirms the validity of items by identifying those with strong correlations to the intended constructs and removing those with low factor loadings, ensuring that both receptive and expressive vocabulary are adequately assessed (Powell et al., 2017; Güler & Gelbal, 2022). Statistical analyses, combined with expert judgment, further refine item selection, retaining items with high discriminatory power to accurately measure students' vocabulary mastery.

Prior Reviews

Aiken (1972) performed a literature review investigating the critical role of language in mathematics learning with results highlighting strong correlations between verbal abilities and mathematics achievement, particularly in word problem-solving ($r = 0.40$ to 0.86). The review emphasized the unique linguistic structure of mathematics and the importance of addressing mathematical vocabulary and syntax in instruction. Effective strategies included integrating explicit language instruction, improving the readability of materials, and using linguistic analogies, while nonverbal approaches may benefit students with weaker verbal skills or language barriers. The findings suggest that verbalizing mathematical concepts enhances understanding and retention, though early verbalization might hinder immediate problem-solving for some. The researcher recommended further studies into the impact of teacher verbal behaviors, the long-term effects of verbalization, readability interventions to refine instructional practices, and the role of early language experiences in mathematical development.

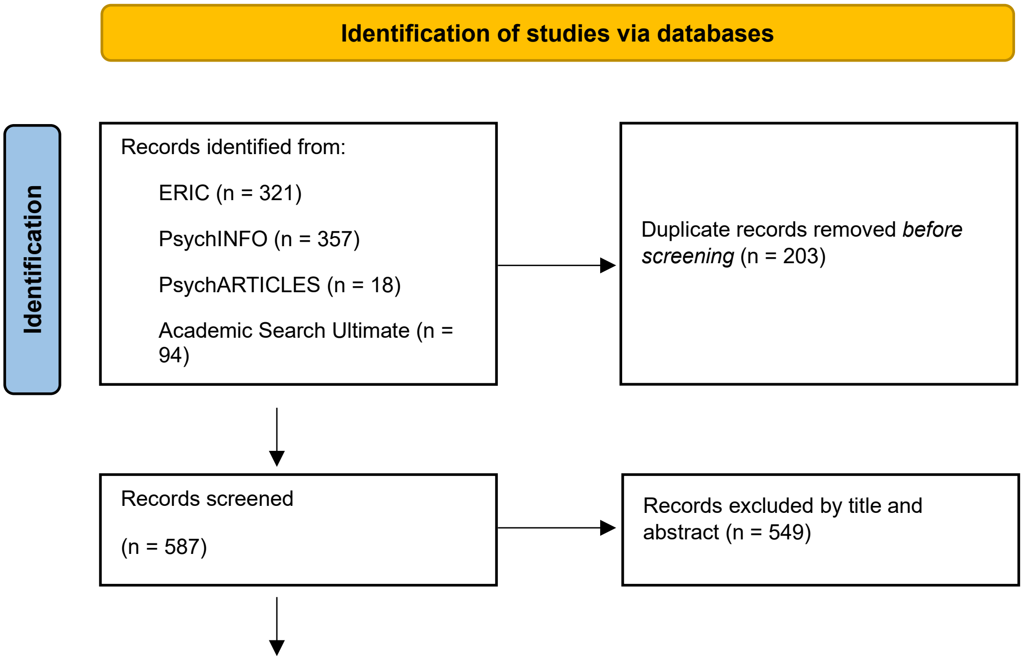
Lin et al. (2021) performed a meta-analysis that examined the link between mathematics vocabulary and mathematics performance across 40 studies. Findings showed a moderate correlation between mathematics vocabulary and mathematics tasks, with higher-order tasks (e.g., algebra, problem-solving) exhibiting stronger associations compared to foundational tasks like computation. Mathematics vocabulary was reported to facilitate cognitive reasoning during complex tasks through automaticity enabling efficient cognitive resource allocation. Controlling for comprehension and cognitive skills revealed that mathematics vocabulary had unique contributions, emphasizing its role beyond simple retrieval of knowledge. The researchers suggested future research investigating how mathematics vocabulary development interacts with age and task complexity, particularly regarding interventions targeting cognitive reasoning skills.

Lin and Powell (2022) performed a meta-analytic structural equation modeling study investigated the longitudinal contributions of initial mathematics, reading, and cognitive skills to later mathematics outcomes using data from over 580,000 participants in 250 studies. Results indicated that working memory and reading comprehension were among the strongest predictors of subsequent math performance, especially for older students. Age moderated these relationships, with the effects of comprehensive mathematics skills and working memory increased with age and the impacts of attention and self-regulation declined. The researchers recommended integrated interventions that bolster cognitive and linguistic capacities to support math learning, particularly among struggling students.

A review of the literature will be performed to synthesize the current body of work pertaining to mathematics vocabulary, how it relates to mathematics achievement, and what other potential covariates moderate the relationship. This review will detail established measures of mathematics vocabulary and achievement used in prior research. Outcomes will also include sample characteristics, study characteristics, and moderators that researchers have explored with measures for each. Lastly, final synthesis will provide conclusions shaping the study purpose, research questions, and current practices of data analysis to shape methodology.

Search Procedures

The databases of ERIC, APA PsychInfo, APA PsychArticles, and Academic Search Ultimate were searched using the terms “Vocabulary AND Math* Achievement” resulting in 790 identified articles. After duplicate removal, 587 unique articles were screened. A title and abstract screening was performed resulting in 38 articles selected for full-text review. The final number of studies identified for full inclusion was 10. The details of screening can be found in the PRISMA diagram (Moher, Liberati, Tetzlaff, & Altman, 2009) found in figure 1.



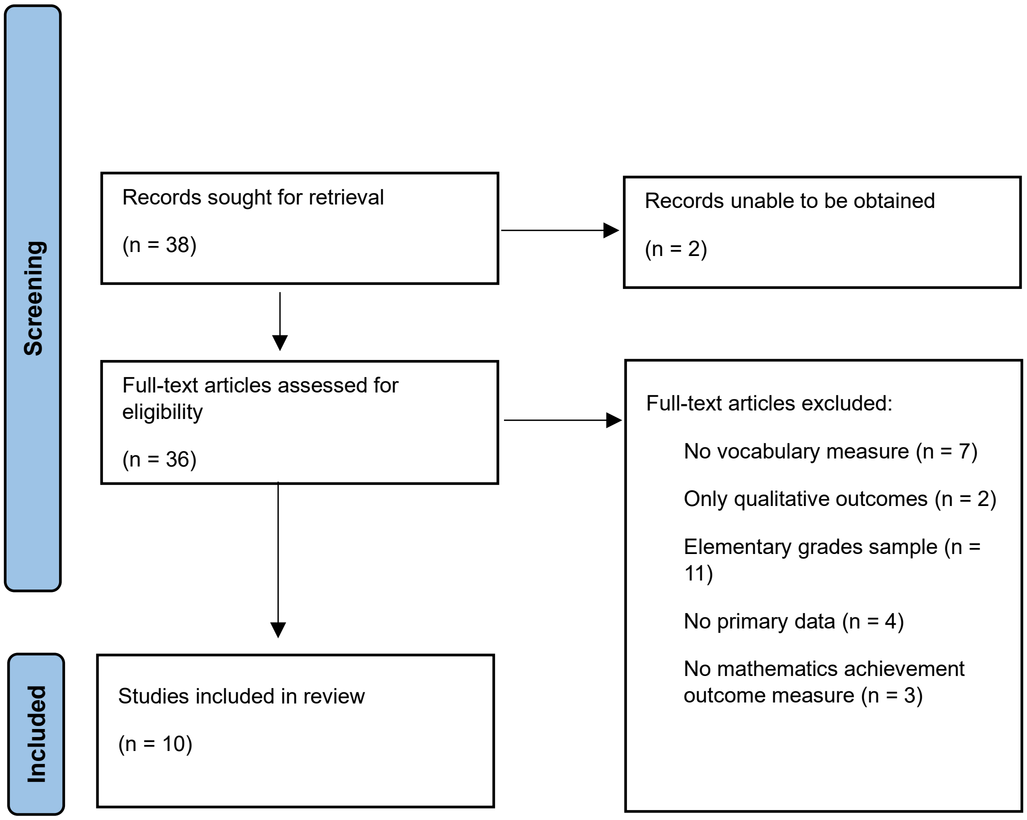


Figure 1. PRISMA flow diagram of search process.

Studies were identified for inclusion using the following criteria. First, they must investigate relationships between vocabulary (general or mathematics-specific) and mathematics performance or examine comprehension, cognitive skills (e.g., working memory, fluid intelligence), or task type (e.g., computation versus word problems) as predictors, moderators, or mediators. Second, the study population should focus on secondary or post-secondary students, particularly those aged 13 years or older, and involve mathematics or related academic domains (e.g., STEM fields). Third, eligible studies should use quantitative or mixed methods approaches, including randomized control trials, quasi-experimental, longitudinal, or cross-sectional designs. Finally, studies published within the last 20 years will be prioritized to ensure relevance to current educational practices and theories.

Studies included for synthesis used regression, path analysis, structural equation modeling, or other hierarchical linear modeling to establish a relationship between mathematics vocabulary and mathematics achievement. A measure of mathematics vocabulary and mathematics achievement such as a standardized assessment, professional educator created terminal assessment (e.g. end of unit exam), or researcher developed measure must be reported for full inclusion. Studies implementing an experimental comparison with a mathematics vocabulary treatment and mathematics achievement outcomes would be eligible for inclusion if enough data is provided for an effect size calculation. Randomized control trials are preferred, but studies can be included that implement quasi-experimental design or pre-post comparisons. Included measurements of working memory, traditional vocabulary, and reading comprehension will be evaluated if reported, but not mandatory for inclusion. Since this review will focus on older individuals in secondary and post-secondary education, studies involving sixth grade and below will be excluded.

Coding

Coding procedures were developed to guide the systematic review of literature examining the relationships between vocabulary knowledge and mathematics performance, with a focus on comprehension skills, cognitive abilities, task type, and vocabulary type as moderators. This review process systematically identified, extracted, and synthesized relevant findings from each study to facilitate analysis and comparisons. Essential information includes citation details (e.g., author, publication year). Key study elements such as research questions, theoretical frameworks, and population characteristics (e.g., age, grade level, demographics) will also be recorded. For methodology, the type of study design (e.g., RTC, Quasi, Pre-Post), sample size, and measures used to assess vocabulary, mathematics performance, comprehension, and cognitive abilities will be documented.

Findings were summarized in a manner that addresses and shapes research questions regarding the effects of general and mathematics-specific vocabulary on mathematics performance and the role of moderators such as comprehension, task type, vocabulary type, and cognitive skills. Key variables categorized for systematic analysis include unique identifiers for each article, year of publication, author(s), and type of publication. Variables related to study focus will detail the domain of focus (e.g., mathematics, language), moderators explored (e.g., task type), type of vocabulary assessed (e.g., mathematics-specific), and the nature of mathematical tasks (e.g., computation, word problems). Findings captured the strength and direction of the relationship between vocabulary and mathematics performance, the significance of moderation effects, and task-specific differences.

The extracted data was analyzed quantitatively and qualitatively. Quantitative summaries include descriptive statistics to highlight patterns across studies, such as the frequency of moderators examined, and effect sizes where available. Qualitative syntheses identify recurring themes, particularly regarding the role of vocabulary knowledge and cognitive skills in mathematics achievement. Findings will be mapped to the study's research questions, offering insights into both theoretical and practical implications.

Results

Ten studies were reviewed and synthesized highlighting the importance of both general and mathematics-specific vocabulary in predicting mathematics performance. For example, Cui et al. (2024) and Ünal et al. (2021) found that mathematics-specific vocabulary independently predicts achievement beyond general vocabulary. Studies like Townsend et al. (2020) further elaborated on how different dimensions of vocabulary (semantic, morphological, spelling) uniquely influence mathematical reasoning. Many studies explored how comprehension, cognitive skills, and task types moderate the vocabulary-math relationship. Neri et al. (2021) found that sentence processing and passage comprehension strongly predict math outcomes, especially when tasks involve linguistic complexity. Similarly, Prediger et al. (2022) showed that academic language proficiency moderates learning gains, particularly for multilingual students. The specifics of each study are laid out in this section along with a summary to be found in table 1.

Table 1

Summary of Literature Review Studies

Study	Main Research Questions	Theoretical Framework(s)	Methodology	Sample Characteristics	Author's Conclusions	Key Statistics	Implications
Koichu et al. (2007)	Does heuristic vocabulary improve problem-solving skills?	Heuristic strategies in problem-solving; Mathematical discourse theory	Classroom intervention with pretest-posttest and interviews	37 eighth-grade students in Israel	Heuristic vocabulary improves problem-solving and supports weaker students.	*g* = 0.88	Advocates for heuristic literacy integration in classrooms.
Neri et al. (2021)	What makes mathematics difficult for adults (role of vocabulary and comprehension)?	Construction-Integration Model; Mathematical Register Theory	Secondary data analysis with multilevel regression	368 German adults; diverse education and SES	Reading components predict math performance through comprehension complexity.	General Vocabulary: $\beta = 0.09$ Sentence Processing: $\beta = 0.27$ Passage Comprehension: $\beta = 0.24$	Differentiates vocabulary demands in adult math tasks.
Oyinloye & Popoola (2013)	Does activating prior knowledge improve vocabulary and math skills?	Schema Theory; Semantic Mapping	Quasi-experimental pretest-posttest design	260 secondary students; rural/urban Nigerian schools	Prior knowledge activation enhances vocabulary and math problem-solving.	*g* = 0.59*	Stresses importance of activating knowledge for ESL learners.
Prediger et al. (2022)	Can language-responsive instruction enhance math learning for multilingual students?	Academic Language Proficiency; Language-Responsive Instruction Principles	Cluster-randomized controlled trial with pretest-posttest	589 Grade 6-7 students; multilingual, diverse SES	Language-responsive instruction promotes learning for multilingual students.	*g* = 0.96*	Promotes inclusive, language-responsive math instruction.
Rolf (2022)	What is the impact of explicit vocabulary instruction for students with	Explicit, systematic instruction principles; Direct Instruction	Randomized controlled trial (RCT) with pretest-posttest	30 students; ages 11-14, students with disabilities	Explicit instruction significantly improves math-specific	*g = 1.99**	Validates systematic instruction for diverse learners.

	learning difficulties?				vocabulary outcomes.		
Rust (2011)	How does integrating content-area reading strategies improve math achievement?	Transactional Reading Theory; Construction-Integration Model	Control-treatment design with observational and interview-based analysis	179 community college students; developmental math	Reading strategies bridge gaps in comprehension and math problem-solving.	Reading Placement Score: β = 0.21 Prealgebra Placement Score: β = 0.25	Demonstrates the value of reading-math instructional alignment.
Townsend et al. (2020)	What is the role of linguistic resources in disciplinary achievement?	Academic Language Framework; Depth of Vocabulary Knowledge	Pretest-posttest design with regression analyses	306 11th-12th graders; 64% multilingual, diverse SES	Academic vocabulary explains unique variance in mathematics scores.	Spelling: *g* = 0.12 Semantic: *g = 0*.19 Morphological: *g = 0*.14	Suggests integrating multidimensional word knowledge into curricula.
Unal et al. (2021)	Does mathematics-specific vocabulary predict achievement across contexts?	Domain-general vs. domain-specific knowledge; Developmental Function Hypothesis	Cross-sectional regression with mediation models	277 eighth graders; U.S. and Turkey, split by SES	Math vocabulary mediates relationships between general language and performance.	US students: β = 0.27** Turkish High: β = 0.40** Turkish Low: β = -0.12	Encourages cultural and contextual adaptation of vocabulary measures.
Waite (2017)	Does directed vocabulary study improve standardized math test scores?	Constructivism; Marzano's Six-Step Vocabulary Instruction Model	Quasi-experimental pretest-posttest design	89 eighth graders; rural middle schools in Indiana	Directed vocabulary study significantly improves standardized test scores.	*g* = 1.19**	Reinforces benefits of guided, active vocabulary study.
Wanjiru & Miheso O-Connor (2015)	What are the effects of Frayer Model with ICT on vocabulary and math performance?	The Frayer Model; Constructivist learning principles	Non-equivalent control group pretest-posttest design	216 Form Two students; Kenyan secondary schools	Frayer Model with ICT integration boosts math performance significantly.	Boys: *g* = 0.61** Girls: *g* = 0.71**	Proves efficacy of ICT-supported vocabulary teaching.

*Significant at the .05 level. **Significant at the .01 level.

Koichu et al. (2007) introduced the concept of "heuristic vocabulary," defined as the language and concepts used to articulate and reflect on problem-solving strategies, positioning it as a mediating factor in mathematics performance. Their findings revealed that students who developed heuristic vocabulary demonstrated improved performance on both routine and non-routine tasks. This was particularly evident among lower-achieving students, for whom heuristic vocabulary served as a compensatory mechanism, enabling them to better reflect on and refine their approaches. Similarly, Neri et al. (2021) examined the relationship between vocabulary, text comprehension, and mathematical performance among adults and found that general vocabulary supported higher levels of comprehension required for interpreting mathematically complex texts. Their findings further demonstrated that vocabulary mitigated the adverse effects of lexical density, underscoring the critical interplay between linguistic and mathematical skills in problem-solving.

Prediger et al. (2022) emphasized the importance of academic language proficiency in fostering conceptual understanding of fractions, particularly among multilingual learners. Integrated approaches to teaching vocabulary, coupled with discourse-based scaffolding, facilitated equity in learning outcomes across language groups. Similarly, Oyinloye and Popoola (2013) highlighted the role of activating prior knowledge in enhancing vocabulary and conceptual understanding in geometry and algebra. Their intervention enabled students to draw connections between existing knowledge and new mathematical concepts, leading to significant gains in comprehension and problem-solving abilities. Rust (2011) extended this understanding by demonstrating that integrating reading strategies, such as guided text analysis, into mathematics instruction improved both vocabulary and problem-solving outcomes, especially for students in developmental mathematics courses.

Explicit vocabulary instruction has been shown to significantly enhance mathematics achievement across diverse populations. Rolf (2022) reported a large effect size for systematic vocabulary interventions, highlighting their capacity to improve access to mathematical content for students with learning disabilities. Similarly, Waite (2017) observed that scaffolded vocabulary study significantly enhanced eighth-grade students’ standardized mathematics assessment scores, emphasizing the value of teacher-directed feedback and self-assessments. Technology-enhanced approaches also showed promise. Wanjiru and Miheso O-Connor (2015) demonstrated that the Frayer Model, when integrated with ICT tools, was more effective than traditional definition-only methods in improving vocabulary and conceptual understanding. Their findings highlighted the importance of active engagement with mathematical vocabulary and the potential of multimodal strategies to support learning across genders.

Ünal et al. (2021) explored the differential roles of general and mathematics-specific vocabulary in predicting mathematics performance in diverse cultural contexts. They found that mathematics-specific vocabulary was a stronger predictor of achievement for high-achieving students, while general vocabulary played a more significant role for lower-achieving students, particularly in early learning stages. This suggests that general linguistic skills are foundational, but that domain-specific vocabulary becomes increasingly important as students advance. Townsend et al. (2020) similarly highlighted the role of academic vocabulary, finding that semantic and spelling knowledge uniquely predicted mathematics performance. However, morphological knowledge was more strongly associated with writing than mathematics, pointing to the need for further research into the nuances of discipline-specific vocabulary in mathematics education.

Across the ten studies, three central themes emerge: the distinct contributions of general and domain-specific vocabulary, the role of comprehension and cognitive moderators, and the effectiveness of instructional strategies in promoting vocabulary development and mathematical achievement. Multiple studies observed the roles of general and mathematics-specific vocabulary in mathematics performance. Ünal et al. (2021) and Townsend et al. (2020) demonstrated that mathematics-specific vocabulary uniquely predicts performance beyond general vocabulary, particularly for higher-achieving students. Conversely, general vocabulary playing a foundational role, especially for lower-achieving students or those in earlier developmental stages (e.g., Ünal et al., 2021; Neri et al., 2021). The findings suggest that general vocabulary serves as a gateway to more complex domain-specific knowledge, emphasizing the importance of sequential and integrated vocabulary instruction.

Several studies explored the moderating effects of cognitive and comprehension skills on the vocabulary-mathematics relationship. Neri et al. (2021) and Rust (2011) found that reading comprehension and linguistic complexity strongly influence mathematics outcomes, highlighting the intertwined nature of linguistic and cognitive processes. Similarly, Prediger et al. (2022) emphasized the role of academic language proficiency in promoting conceptual understanding, particularly for multilingual students. Other cognitive moderators, such as reasoning ability (Koichu et al., 2007) and general academic risk factors (Townsend et al., 2020), further showing the complexity of the vocabulary-mathematics relationship.

Effective instructional strategies were observed across studies, providing varied approaches to fostering vocabulary and mathematics achievement. Explicit and systematic instruction (Rolf, 2022), context-rich methods like the Frayer Model with ICT integration (Wanjiru & Miheso O-Connor, 2015), and knowledge activation strategies (Oyinloye & Popoola, 2013) all proved effective in enhancing both vocabulary knowledge and problem-solving abilities. Waite (2017) demonstrated that teacher-guided feedback and active learning strategies such as journaling significantly improved standardized test outcomes. These findings collectively highlight the need for structured, multimodal, and context-sensitive instructional approaches tailored to diverse learner profiles.

Despite these contributions, several gaps remain that this study seeks to address. Much of the existing research on mathematical vocabulary has been conducted with children or adolescents, leaving limited evidence on how vocabulary supports mathematics learning in postsecondary contexts. Furthermore, academically at-risk students enrolled in remedial algebra courses are underrepresented. Few studies have directly examined how mathematics-specific vocabulary predicts performance when controlling for general reading comprehension, despite evidence that comprehension and vocabulary are closely intertwined (Neri et al., 2021; Rust, 2011). In addition, little research has provided quantitative evidence for whether vocabulary knowledge operates differently across task types, such as computation and word problems, or whether its effects are stronger for items that explicitly contain aligned vocabulary. Finally, while many studies have measured general academic or expressive vocabulary, very few have isolated receptive mathematics vocabulary, the form most critical for accessing and interpreting algebraic content in assessment settings. Addressing these gaps can clarify the unique role of mathematics vocabulary as a cognitive and linguistic resource for algebra learning and inform instructional practices to support students' success in postsecondary mathematics.

Study Purpose

Building on prior research demonstrating the importance of linguistic skills in mathematics achievement, this study examined how vocabulary knowledge influences performance in college algebra. Prior studies have shown that mathematics-specific vocabulary is moderately associated with mathematics performance, with stronger effects for higher-order tasks and its influence persisting even after controlling for comprehension and cognitive skills (Lin et al., 2021). Related work has found that linguistic processing skills such as phonological awareness and rapid naming uniquely contribute to mathematics fluency and accuracy, suggesting a foundational role for language in numerical reasoning (Yang et al., 2022). Similarly, reading comprehension, general vocabulary, and working memory have been identified as significant predictors of mathematics achievement, with their effects intensifying as students progress to more advanced content (Lin & Powell, 2022). Collectively, this work positions vocabulary as both a gateway for accessing conceptual knowledge and a potential mediator of cognitive processes in mathematics learning.

Despite these findings, critical gaps remain within postsecondary mathematics education, particularly among academically at-risk students enrolled in remedial algebra courses. Most existing studies have focused on younger learners, leaving little evidence on how mathematics vocabulary supports college-level learning. Moreover, while prior research has highlighted the roles of comprehension and cognitive skills, few studies have tested these factors as moderators of the vocabulary and performance relationship. It also remains unclear whether the predictive strength of vocabulary varies by task type (computation vs. word problem) or by the alignment of known terms to those appearing on assessments. These gaps limit our understanding of how mathematics vocabulary functions as a cognitive and linguistic resource in college settings and how practitioners should approach mathematics vocabulary in the classroom.

This study addresses these gaps by examining how mathematics-specific vocabulary predicts mathematics performance in a college algebra course while accounting for reading comprehension. It further investigates whether reading comprehension, task type, and vocabulary alignment moderate this relationship. By clarifying how linguistic and cognitive factors jointly influence algebra performance, this study aims to inform the development of targeted instructional strategies that integrate both vocabulary and conceptual supports to enhance students' long-term success in college mathematics.

Research Questions

1. What is the relationship between mathematics vocabulary knowledge and mathematics performance in college algebra when controlling for reading comprehension skills?
2. Is the relationship between mathematics vocabulary knowledge and mathematics performance in college algebra moderated by reading comprehension skills?
3. Does task type (i.e., computation vs. word problem) moderate the relationship between mathematics vocabulary knowledge and mathematics performance in college algebra?

4. Does vocabulary that appears on the performance measure (aligned) predict college algebra performance differently than vocabulary that does not appear on the measure but is found in general algebra texts (general)?

Chapter 3

Methodology

This study employed a quantitative, cross-sectional design to examine the relationships between mathematics vocabulary knowledge, reading comprehension, and mathematics performance in college algebra. Specifically, the study investigated the extent to which mathematics vocabulary knowledge predicted algebra performance, and whether these relationships were moderated by reading comprehension, task type (computation vs. word problem), and vocabulary proximity (general vs. aligned). All assessments were administered in a single session to groups of students during their normally scheduled class time.

Participants

Participants were 107 undergraduate students enrolled in prescribed general education College Algebra courses (MATH 1710K) at Middle Tennessee State University. Students in these courses are placed based on ACT scores and/or placement exams. These students are considered at-risk and required to take “K” sections which, while covering the same material and standards as traditional sections, are given access to an exclusive tutoring lab and specialized instructors who are prepared to teach lower-achieving students. These students are all expected to learn the same material and take the departmental college algebra final exam as non-K students, fulfilling their general education course requirements and prerequisites for other courses (i.e. Precalculus). Students were recruited through convenience sampling with the assistance of course instructors, who shared an approved recruitment announcement during class sessions, email, and on the university’s learning management system (D2L). Participation was voluntary, with informed consent obtained prior to data collection. Incentives such as candy or extra credit were offered, with equivalent alternatives provided in accordance with Institutional Review Board (IRB) guidelines.

Measures and Procedures

Mathematics Performance

Mathematics performance was assessed using a researcher-selected set of items drawn from multiple subtests of the *KeyMath-3 Diagnostic Assessment* (Connolly, 2007). Only items in multiple-choice format were drawn from the *Applications* and *Basic Concepts* subtests to align with College Algebra standards, covering content such as solving linear and quadratic equations, interpreting functions, factoring, and modeling. No open ended items were selected to maintain a common response structure and allow efficient group administration. The measure included 26 items in total (see Appendix A for examples), divided into computation problems (items 1–16) and word problems (items 17–26). Computation items required students to apply symbolic procedures without interpreting verbal context, whereas word problems required students to interpret verbal information and translate it into a mathematical representation before solving Subscores for computation and word problems were calculated, along with a combined total score. Internal consistency reliability was evaluated using Cronbach's alpha.

Mathematics Vocabulary Knowledge

Mathematics vocabulary knowledge was measured using a researcher-developed multiple-choice assessment consisting of 30 multiple choice items (see Appendix B). Fifteen items targeted aligned vocabulary directly tied to the KeyMath-3 performance problems, while 15 items assessed general vocabulary commonly encountered in college algebra. Scores were computed for aligned, general, and a combined total. Content validity was supported through expert review by mathematics instructors also teaching the course.

Reading Comprehension

Reading comprehension was assessed using the Comprehension subtest of the *Gates-MacGinitie Reading Test, Fourth Edition* (GMRT-4; MacGinitie et al., 2000). The GMRT is a norm-referenced, group-administered reading assessment widely used with secondary and adult learners. Level 10/12, Form S was selected because the maturity and linguistic complexity of the passages are designed for students in late high school and beyond, aligning with the reading demands typical of college coursework. The Comprehension subtest includes short narrative and expository passages followed by multiple-choice questions assessing literal understanding, inferencing, and integration of ideas across text.

The GMRT-4 demonstrates strong psychometric properties at Level 10/12, with internal consistency reliability coefficients for the Comprehension subtest typically ranging from .88 to .94, and alternate-form reliability estimates from .80 to .87 (MacGinitie et al., 2007). In the present study, raw scores from the Comprehension subtest were used as an index of overall reading comprehension ability, and administration followed standardized GMRT procedures.

Measurement Plan

This study employed a battery of standardized and researcher-developed assessments to evaluate mathematics vocabulary knowledge, reading comprehension, and mathematics performance. All assessments were group-administered in a single classroom session lasting approximately 75 minutes. The administration order was standardized in order of mathematics performance, mathematics vocabulary, and reading comprehension. The principal investigator and course instructors monitored sessions using a standardized script and timing procedures in a quiet, distraction-free classroom. Answer sheets were checked for completeness before data entry.

Data Analysis

All analyses were conducted in R (R Core Team, 2023), with replication performed in SPSS (IBM Corp., 2023) to ensure comparability. Data were imported from Excel using the *readxl* package, and variables were cleaned, scored, and reshaped with functions from the *tidyverse* (e.g., *dplyr*, *tidyr*). Internal consistency reliability for mathematics performance, mathematics vocabulary, and reading comprehension measures were assessed using Cronbach's alpha in the *psych* package.

For inferential analyses, multiple linear regression models were estimated using the base R *lm()* function. Output summaries were extracted with *broom* to report regression coefficients, standardized beta weights, confidence intervals, and model fit statistics (R^2 and ΔR^2 where applicable). Interaction effects were probed and visualized using the *ggeffects* package to generate simple slopes and interaction plots. All continuous predictors were grand mean centered using constants obtained from SPSS Descriptives. Because the dataset contained no missing responses, the full sample ($N = 107$) was retained for all analyses.

Model assumptions were evaluated through residual plots (for linearity and homoscedasticity), Q–Q plots (for normality of residuals), and histograms of standardized residuals. Variance inflation factors (VIFs) were also examined to rule out problematic multicollinearity (all VIF < 5). To increase confidence in the findings, all primary analyses were replicated in SPSS using *Linear Regression* or *General Linear Model Repeated Measures*, and substantive conclusions converged across platforms.

Research Question 1: Mathematics Vocabulary Knowledge as a Predictor

A hierarchical linear regression was conducted to test whether mathematics vocabulary knowledge uniquely predicted mathematics performance beyond general reading ability. In the first step of the model, GMRT Reading Comprehension scores were entered. In the second step, mathematics vocabulary knowledge was added. The change in explained variance (ΔR^2) from Step 1 to Step 2 was used to evaluate the additional contribution of mathematics vocabulary knowledge. In addition to variance explained, unstandardized (B) and standardized beta weights (β) from the final model were interpreted to assess the relative strength of mathematics vocabulary knowledge and reading comprehension as predictors of mathematics performance.

Research Questions 2: Moderation by Reading Comprehension

To examine whether reading comprehension moderated the relationship between mathematics vocabulary knowledge and mathematics performance, a regression model was estimated that included mathematics vocabulary (grand-mean centered), reading comprehension (grand-mean centered), and their interaction term. This model tested whether the strength of the mathematics vocabulary and mathematics performance relationship differed across levels of reading comprehension. Unstandardized and standardized beta weights were interpreted for all predictors and the interaction term, with particular attention to the significance and direction of the interaction effect. Interaction effects were probed with simple slopes at −1 SD, mean, and +1 SD values of reading comprehension and visualized using the *ggeffects* package in R.

Research Question 3: Moderation by Task Type (Computation vs. Word Problem)

To examine whether the relationship between mathematics vocabulary knowledge and performance differed across computation and word problem tasks, a two-step analytic approach was used. First, a traditional repeated-measures analysis was conducted to verify the presence of within-subjects variation by task type. Next, mathematics performance scores were reshaped into long format to define task type (computation vs. word problem) as a within-subjects factor for use in a repeated-measures regression framework. A mixed-effects regression model was then estimated with mathematics performance as the dependent variable, mathematics vocabulary knowledge (centered) as the predictor, and task type as the moderator, while controlling for reading comprehension (centered). The mathematics vocabulary × task type interaction was tested to determine whether the strength of the mathematics vocabulary and performance relationship varied by task type. Beta weights from the model were examined to assess the relative contributions of mathematics vocabulary knowledge, task type, and reading comprehension, and predicted values were computed to evaluate the magnitude of mathematics vocabulary effects across task types.

Research Question 4: Moderation by Vocabulary Alignment Ratio

To test whether the predictive strength of mathematics vocabulary knowledge varied depending on whether vocabulary terms appeared on the performance measure, a hierarchical multiple regression analysis was conducted. Mathematics performance was entered as the dependent variable. Aligned and general mathematics vocabulary scores were simultaneously entered as predictors to estimate their unique contributions to performance. Beta weights were interpreted alongside model significance tests and changes in explained variance.

All models were estimated with $\alpha = .05$ as the threshold for statistical significance, and results were reported in terms of standardized beta weights, confidence intervals, and model fit statistics (R^2 and ΔR^2 where applicable). For any statistically significant interactions, simple slopes analyses were conducted by hand using the regression equation, with moderator values set at one standard deviation below the mean, the mean, and one standard deviation above the mean to estimate conditional effects. Diagnostic checks were conducted for linearity, normality of residuals, homoscedasticity, and multicollinearity. To increase confidence in the findings, each model was replicated in SPSS using Linear Regression or General Linear Model Repeated Measures, with results converging substantively across platforms. Detailed R scripts and SPSS syntax used for all analyses are provided in Appendix C to ensure transparency and enable reproducibility of the findings. Full diagnostic plots and statistics are provided in Appendix D.

In addition to the primary regression models, supplementary analyses were conducted to further examine the unique contributions of mathematics vocabulary knowledge and reading comprehension. For Research Question 1, an additional hierarchical regression was run with the order of predictors reversed, entering mathematics vocabulary knowledge first and reading comprehension second, to evaluate the unique variance explained by reading comprehension when controlling for mathematics vocabulary. For Research Question 3, an exploratory model was estimated testing the interaction between reading comprehension and task type (computation vs. word problems) while controlling for mathematics vocabulary. These supplementary models were analyzed using the same procedures as the primary models, and regression assumptions were evaluated through residual plots, normal probability plots, and variance inflation factors. No violations were detected.

Validity and Reliability

Content validity of the mathematics vocabulary measure was established through expert review and alignment with College Algebra learning objectives and standards to ensure that items reflected the terminology and concepts emphasized in the course curriculum. The measure included a mix of items directly aligned with terms present in the mathematics performance assessment and more general algebraic terminology to capture a broader range of course-relevant vocabulary. The mathematics performance items were drawn from standardized KeyMath-3 subtests, which have documented validity evidence for assessing mathematical problem-solving skills in older students.

Internal consistency reliability was evaluated for the mathematics performance and mathematics vocabulary measures using Cronbach's alpha. Reliability estimates were reported for the total scales as well as computation, word problem, aligned vocabulary, and general mathematics vocabulary subscores. Although alpha coefficients for some subscales were modest, they were consistent with values reported in prior research on heterogeneous mathematics constructs and were deemed sufficient for exploratory predictive modeling. The GMRT Reading Comprehension subtest was administered and scored according to standardized procedures outlined in the test administration manual and scoring key booklet, providing strong evidence of measurement validity for that construct.

Fidelity of Implementation

Fidelity of implementation was supported through both standardized administration and systematic data entry procedures. All sessions followed a scripted protocol that included uniform instructions, consistent order of assessments, and monitored timing to ensure comparability across groups. The principal investigator and classroom instructors supervised testing, provided clarification of procedures when needed, and confirmed that all participants adhered to task requirements.

Following data collection, fidelity was further supported through systematic data entry and scoring procedures. Explicit student responses were entered into a pre-formatted Excel workbook that incorporated embedded answer keys and automated grading functions. This system reduced human error by automatically coding responses as correct or incorrect, generating total and subscale scores for mathematics performance and mathematics vocabulary. After data entry, spot checks were performed against original response sheets to verify accuracy. These procedures ensured consistency, minimized scoring bias, and maintained a reliable dataset for subsequent analyses.

Ethical Considerations

This study adhered to all ethical guidelines approved by Middle Tennessee State University's Institutional Review Board. Informed consent was obtained prior to participation, and students were clearly informed of the voluntary nature of the study, their right to withdraw at any time without penalty, and the availability of equivalent alternatives to extra credit incentives. Consent forms emphasized that participation would not affect course grades or standing in the class.

To ensure confidentiality, all assessments were de-identified when transcribing data into the Excel spreadsheet through randomly assigned participant numbers. Reflection responses were anonymous and not linked to quantitative data, ensuring that students could share candid perspectives without risk of identification.

After collection, all response sheets were stored in a locked cabinet accessible only to the researcher. Data entry files were stored on password-protected computers, and datasets used for analysis were fully de-identified. Automated scoring in Excel reduced the risk of subjective bias in coding, while spot checks further protected against transcription errors. No individually identifiable results were shared with instructors outside of participation status.

Summary

This chapter outlined the study's research design, participant recruitment, measures, procedures, and analytic strategy. A cross-sectional, quantitative approach was used to examine the relationships between mathematics vocabulary knowledge, reading comprehension, and mathematics performance in college algebra. Analyses included multiple regression and repeated-measures regression models, structured around the four research questions. The following chapter presents the results of these analyses.

Chapter 4

Results

The purpose of this study was to examine the relationships among mathematics vocabulary knowledge, reading comprehension, and mathematics performance in college algebra with four research questions guiding analysis. All primary analyses were conducted in R. To further authenticate findings, results were cross validated in SPSS using equivalent Linear Regression and General Linear Model Repeated Measures procedures. Substantive conclusions were consistent across platforms, with minor differences attributable to variable centering and repeated-measures specification.

Preliminary Analyses

Data from 107 participants were included in the final analyses. All cases were retained after data screening; no participants were removed due to incomplete assessments. Any items with no response were coded as incorrect in analysis. Answer sheets were collected and reviewed for completeness prior to scoring. Responses were entered into a pre-formatted Excel workbook containing embedded answer keys and automated scoring functions. This auto grading process reduced transcription error by automatically coding responses as correct (1) or incorrect (0) for ease of verification and export. Following data entry, random spot checks were conducted against the original response sheets to verify accuracy. No discrepancies were identified. The coded responses were exported to a separate, clean Excel workbook to use as final input for R and SPSS.

Internal consistency reliability was estimated using Cronbach's alpha for each multi-item measure. The mathematics performance scale showed modest reliability overall ($\alpha = .58$). No comparison was made to published KeyMath-3 reliability expectations since items were hand selected from subtests rather than whole subtests being used. The mathematics vocabulary scale demonstrated similar internal consistency (*α = .64*). The GMRT Reading Comprehension subtest was administered and scored according to standardized procedures aligning with published reliability for the measure (*α = .89*).

Evidence for the validity of the measures was supported in multiple ways. Correlational analyses indicated positive associations among mathematics vocabulary, mathematics performance, and reading comprehension, consistent with theoretical expectations and prior research. Content-related validity was supported using items adapted from the KeyMath-3 Diagnostic Assessment to ensure coverage of core algebra concepts and terminology. In addition, all items were reviewed by experienced mathematics educators and content specialists to verify alignment with course objectives and appropriate difficulty for college algebra students. Collectively, these findings provide preliminary support for the measures' appropriateness for evaluating the hypothesized relationships in this study.

Descriptive Statistics

Table 2 presents descriptive statistics for all study variables, including the number of items contributing to each scale or subscale. Participants scored higher on computation items (*M* = 7.79, *SD* = 2.52, range = 2–14) than on word problem items (*M* = 4.17, *SD* = 1.80, range = 0–8), with a total mathematics performance score having a mean of 11.96 (*SD* = 3.56, range = 2–19). For mathematics vocabulary, participants answered more aligned items correctly (*M* = 11.18, *SD* = 1.85, range = 5–14) than general items (*M* = 10.45, *SD* = 2.39, range = 2–15), resulting in total mathematics vocabulary score mean of 21.63 (*SD* = 3.63, range = 10–28).

Reading comprehension scores on the GMRT were moderate overall (*M* = 29.95, *SD* = 8.93, range = 8–47). It is important to note that the measures were on different scales, and direct mean comparisons are not comparable as performance was relative to the measure scale.

Table 2

Descriptive Statistics for Study Variables

Variable	Number of Items	M	SD	Range
Computation	16	7.79	2.52	2–14
Word Problems	10	4.17	1.80	0–8
Mathematics Performance Total	26	11.96	3.56	2–19
Aligned Mathematics Vocabulary	15	11.18	1.85	5–14
General Mathematics Vocabulary	15	10.45	2.39	2–15
Mathematics Vocabulary Total	30	21.63	3.63	10–28
Reading Comprehension (GMRT)	48	29.95	8.93	8–47

Note. M = mean; SD = standard deviation.

Correlations

Bivariate correlations among all study variables are presented in Table 3. Computation and word problem scores were positively correlated (*r = .34, p < .001*). Both subscores were strongly, positively associated with total mathematics performance (computation: *r = .88, p < .001*; word problems: *r = .75, p < .001*). Aligned vocabulary was observed to have a statistically significant positive correlations to computation (*r = .35, p < .001*) and total mathematics performance (*r = .29, p = .002*), while its correlation with word problems was weak and not statistically significant ($r = .08$, $p = .387$). General mathematics vocabulary showed statistically significant correlations with computation ($r = .32$, $p < .001$) and mathematics performance total ($r = .32$, $p < .001$), and a marginal correlation with word problems ($r = .19$, $p = .054$). Total mathematics vocabulary was statistically significantly correlated with computation ($r = .39$, $p < .001$) and total mathematics performance ($r = .36$, $p < .001$), but not word problems ($r = .17$, $p = .088$).

Reading comprehension (GMRT) was statistically significantly positively correlated with computation ($r = .22$, $p = .025$), word problems ($r = .34$, $p < .001$), and total mathematics performance ($r = .32$, $p < .001$). GMRT also showed statistically significant correlations with aligned vocabulary ($r = .42$, $p < .001$), general mathematics vocabulary ($r = .42$, $p < .001$), and total mathematics vocabulary ($r = .49$, $p < .001$).

Overall, the magnitudes of the observed correlations were generally small-to-moderate, aligning with prior research examining the relation between mathematics vocabulary and mathematics performance. Lin et al.'s (2021) meta-analysis reported an average correlation of $r = .49$, characterized as a moderate association. In that work, the association tended to be stronger for higher-order tasks, such as word problems, than for foundational computation tasks. In the present study, correlations between vocabulary measures and computation and total mathematics performance were in the small-to-moderate range, consistent with these broader patterns, but correlations with word problems were smaller than typically reported.

Table 3

Pearson Correlations Among Study Variables (N = 107)

Variable	1	2	3	4	5	6	7
1. Computation	—						
2. Word Problems	.34**	—					
3. Performance Total	.88**	.75**	—				
4. Vocab Aligned	.35**	.08	.29**	—			
5. Vocab General	.32**	.19	.32**	.45**	—		
6. Vocab Total	.39**	.17	.36**	.81**	.89**	—	
7. GMRT Total	.22*	.34**	.32**	.42**	.42**	.49**	—

*p < .05. ***p < .01.*

Research Question 1

The first research question examined whether mathematics vocabulary knowledge uniquely predicted mathematics performance beyond general reading comprehension. A hierarchical regression was conducted in two steps, with GMRT Reading Comprehension scores entered in Step 1 and mathematics vocabulary knowledge added in Step 2.

In Step 1, reading comprehension statistically significantly predicted mathematics performance and accounted for 10.4% of the variance ($R^2 = .10, F(1, 105) = 12.23, p < .001$). Adding mathematics vocabulary knowledge in Step 2 statistically significantly improved the model (*ΔR^2 = .05, F-change(1, 104) = 6.47, p = .012*) increasing the explained variance to 15.7% (*R^2 = .16, F(2, 104) = 9.67, p < .001).*

In the final model (Table 4), mathematics vocabulary knowledge was a statistically significant positive predictor (β *= .26, t = 2.54, p = .012*), where reading comprehension did not retain statistical significance at the .05 level *(*β *= .19, t = 1.88, p = .063*). This indicates that for every one-point increase in mathematics vocabulary knowledge, mathematics performance increased by an estimated .26 standard deviations, holding reading comprehension constant. In contrast, each one standard deviation increase in reading comprehension was associated with an estimated .19 standard deviation increase in performance, and this effect did not reach statistical significance. In addition, mathematics vocabulary knowledge explained unique variance in mathematics performance beyond general reading ability.

Table 4

Hierarchical Regression Predicting Mathematics Performance From Vocabulary Knowledge and Reading Comprehension

Predictor	B	β	p
Step 1			
Reading Comprehension (GMRT)	.13	.32	< .001
Step 2			
Reading Comprehension (GMRT)	.08	.19	.063
Mathematics Vocabulary Knowledge	.26	.26	.012

Note. B = unstandardized coefficient; β = standardized beta

Research Question 2

The second research question examined whether reading comprehension moderated the relationship between mathematics vocabulary knowledge and mathematics performance. A multiple regression model was estimated that included centered mathematics vocabulary scores, centered reading comprehension scores, and their interaction term which can be found in table 5.

The overall model was statistically significant (*F(3, 103) = 6.42, p < .001*) explaining 13% of the variance in mathematics performance (Adj. $R^2 = .13$). Mathematics vocabulary knowledge was a statistically significant positive predictor (β *= .28, t = 2.45, p = .016*), whereas reading comprehension showed only a marginal, not statistically significant positive effect (β *= .19, t = 1.85, p = .067*). The interaction between mathematics vocabulary and reading comprehension was not statistically significant (β *= .03, t = .31, p = .756*), indicating that reading comprehension did not moderate the relationship between mathematics vocabulary knowledge and mathematics performance. The strength of the relationship between mathematics vocabulary knowledge and mathematics performance was observed to be consistent across levels of reading comprehension.

Table 5

Regression Predicting Mathematics Performance With Reading Comprehension as a Moderator

Predictor	B	β	p
Mathematics Vocabulary Knowledge	.27	.28	.016
Reading Comprehension (GMRT)	.08	.19	.067

Mathematics Vocabulary × Reading Comprehension	.003	.03	.756

Note. B = unstandardized coefficient; β = standardized beta

Research Questions 3

The third research question examined whether task type moderated the relationship between mathematics vocabulary knowledge and mathematics performance. A traditional repeated-measures analysis was first conducted to test for differences in performance across computation and word problem tasks and to evaluate potential moderation by task type. After evidence of moderation was observed, performance scores were reshaped into long format to define task type (computation vs. word problem) as a within-subjects factor, and a repeated-measures regression model was estimated with mathematics vocabulary knowledge, task type, reading comprehension, and their interactions. This restructuring allowed the inclusion of task type as a within-subjects factor in a regression framework, enabling the estimation of interaction effects with continuous predictors. Results from this model are presented in Table 6.

The overall model was statistically significant ($F(4, 209) = 49.18, p < .001$) accounting for 48% of the variance in performance scores (Adj. $R^2 = .48$). Mathematics vocabulary knowledge was a statistically significant positive predictor ($\beta = .28, t = 3.75, p < .001$), as was reading comprehension ($\beta = .12, t = 2.13, p = .034$). Task type was also statistically significant ($\beta = -.64, t = -12.89, p < .001$), indicating lower performance on word problems compared to computation items. Importantly, the interaction between mathematics vocabulary knowledge and task type was statistically significant ($\beta = -.17, t = -2.39, p = .018$), suggesting that the predictive strength of mathematics vocabulary knowledge differed by task type. The interaction indicated that the contribution of mathematics vocabulary knowledge was relatively stronger for computation items than for word problems.

Table 6

Repeated-Measures Regression Predicting Performance From Vocabulary Knowledge, Reading Comprehension, and Task Type

Predictor	B	β	p
Mathematics Vocabulary Knowledge	.22	.28	< .001
Reading Comprehension (GMRT)	.04	.12	.034
Task Type (0 = computation, 1 = word)	−3.63	−.64	< .001
Mathematics Vocabulary × Task Type	−.19	−.17	.018

Note. B = unstandardized coefficient; β = standardized beta

To examine whether the relation between mathematics vocabulary and mathematics performance differed by task type, a regression model including mathematics vocabulary, reading comprehension, and task type (coded 0 = computation, 1 = word problems), along with the interaction term vocabulary × task type, was estimated. From this model, the unstandardized simple slope equations for predicting performance (Y) at each level of task type were computed as:

$$Y_{Computation} = 7.79 + 0.222X + 0.039GMRT$$

$$Y_{Word\ Problems} = 4.16 + 0.035X + 0.039GMRT$$

These equations show the expected mathematics performance scores for each task type as vocabulary and reading comprehension vary. When task type is coded as 0 (computation), each one-point increase in mathematics vocabulary score is associated with a 0.222-point increase in computation performance, holding reading comprehension constant. When task type is coded as 1 (word problems), the corresponding increase is 0.035 points, indicating a substantially weaker relation between vocabulary and performance on word problems compared to computation.

A supplementary repeated measures regression (Table 7) was conducted to examine whether reading comprehension predicted mathematics performance differently for computation versus word problem items when controlling for mathematics vocabulary knowledge. Performance scores were again analyzed in long format with task type (0 = computation, 1 = word problems) as a within-subjects factor and mathematics vocabulary and GMRT scores as covariates.

The overall model was statistically significant ($F(4, 209) = 46.49, p < .001$) explaining approximately 47.1% of the variance in performance scores ($Adj.\ R^2 = .46$). Mathematics vocabulary knowledge was a statistically significant positive predictor of performance ($\beta = .16, t = 2.84, p = .005$), while reading comprehension showed a non-statistically significant positive association ($\beta = .11, t = 1.45, p = .148$). Task type was a statistically significant negative predictor ($\beta = -.64, t = -12.72, p < .001$), reflecting again lower scores on word problems than computation. Different from the primary model, the interaction was not statistically significant ($\beta = .01, t = 0.20, p = .843$), indicating that reading comprehension did not differentially predict performance on computation versus word problem tasks when controlling for mathematics vocabulary knowledge.

Table 7

Repeated-Measures Regression Predicting Mathematics Performance From Reading Comprehension and Task Type Controlling for Vocabulary

Predictor	B	β	p
Mathematics Vocabulary Knowledge	.13	.16	.005
Reading Comprehension (GMRT)	.04	.11	.148
Task Type	-3.63	-.64	< .001
Reading Comprehension × TaskType	.01	.01	.843

Note. B = unstandardized coefficient; β = standardized beta

Research Question 4

A multiple regression analysis was conducted to examine whether mathematics vocabulary that appeared on the performance measure (aligned) predicted college algebra performance differently than mathematics vocabulary that did not appear on the measure but is found in general algebra texts (general) controlling for reading comprehension. Both vocabulary scores were entered simultaneously as predictors. The model was statistically significant *($R^2 = .13, F(2, 104) = 7.65, p < .001$)* indicating that the two vocabulary measures together accounted for approximately 13% of the variance in mathematics performance.

General mathematics vocabulary emerged as a statistically significant predictor of performance ($\beta = .24, t = 2.29, p = .024$) whereas aligned vocabulary did not reach statistical significance ($\beta = .18$, $t = 1.79$, $p = .077$). This indicates that after accounting for aligned vocabulary, each one standard deviation increase in general mathematics vocabulary was associated with an estimated .24 standard deviation increase in mathematics performance. In contrast, a one standard deviation increase in aligned vocabulary was non-statistically significantly associated with an estimated .18 standard deviation increase in performance. The regression results are presented in Table 8.

Table 8

Hierarchical Regression Predicting College Algebra Performance from Aligned Vocabulary, General Vocabulary, and Reading Comprehension (GMRT)

Predictor	B	β	p
Aligned Vocabulary	.35	.18	.077
General Mathematics Vocabulary	.35	.24	.024

Note. B = unstandardized coefficient; β = standardized beta

Summary

The purpose of this study was to examine the relationships among mathematics vocabulary knowledge, reading comprehension, and mathematics performance in college algebra. Four research questions guided the analyses with all analysis estimated in R and cross-validated in SPSS with conclusions converging across platforms. Preliminary analyses confirmed that all participants (N = 107) completed the assessments with no missing data. Reliability estimates for mathematics performance and mathematics vocabulary measures were modest, and descriptive statistics indicated variability across computation, word problem, and vocabulary sub scores. Correlations revealed statistically significant associations among mathematics vocabulary, reading comprehension, and mathematics performance, supporting their inclusion in regression analyses.

Hierarchical regression revealed that mathematics vocabulary knowledge accounted for additional unique variance in mathematics performance beyond reading comprehension, with standardized beta weights indicating that mathematics vocabulary was the stronger predictor. Moderation analysis indicated that reading comprehension did not statistically significantly moderate this relationship.

Task type was examined as a potential moderator of the relationship between mathematics vocabulary knowledge and performance. Following initial repeated measures analyses, performance data were reshaped into long format and analyzed in a repeated measures regression model with task type (computation = 0, word problem = 1) as a within-subjects factor. Results revealed a statistically significant negative interaction, indicating that mathematics vocabulary knowledge predicted computation performance more strongly than word problem performance. Possible explanations for this pattern are addressed in the Discussion.

Regression results indicated that when aligned and general vocabulary were entered simultaneously, general vocabulary emerged as a statistically significant predictor of mathematics performance, whereas aligned vocabulary did not. This pattern suggests that students' performance was supported more by broad mathematics vocabulary knowledge drawn from general algebra than by familiarity with the specific terms embedded in the performance measure. In other words, the predictive strength of mathematics vocabulary appears to generalize beyond test-specific terminology, accentuating the role of transferable vocabulary knowledge in supporting algebra performance.

Together, these findings indicate that mathematics vocabulary knowledge is an important predictor of performance in college algebra, even when accounting for reading comprehension. Moderation effects were mixed, with task type emerging as a statistically significant moderator while reading comprehension ability and alignment were not observed to moderate the relationship.

Chapter 5

Discussion

The purpose of this study was to examine how mathematics vocabulary knowledge relates to mathematics performance in college algebra, while accounting for the potential roles of reading comprehension, task type, and vocabulary alignment. Building on prior research that has emphasized the foundational role of language and cognitive skills in mathematics learning (Lin et al., 2021; Lin & Powell, 2022; Cirino et al., 2013), this study sought to address critical gaps in understanding how these factors interact in postsecondary contexts. Using a cross-sectional design with 107 undergraduate students enrolled in prescribed College Algebra courses, the study administered assessments of mathematics performance, mathematics vocabulary, and reading comprehension, and employed multiple and repeated-measures regression analyses to address four research questions.

Several key findings emerged. First, mathematics vocabulary knowledge was consistently observed to statistically significantly predict and explain unique variance in mathematics performance even after controlling for reading comprehension. Reading comprehension contributed only marginal unique variance explanation and served as a non-statistically significant predictor. This suggests that mathematics-specific vocabulary may play a more central role than general literacy skills in supporting college students' access to algebraic content as well as a potential indicator of a deeper understanding of mathematical content. Reading comprehension was not observed to moderate the relationship between mathematics vocabulary knowledge and performance, indicating that mathematics vocabulary supported performance similarly across all levels of reading comprehension. Task type was observed to moderate the relationship with mathematics vocabulary knowledge a stronger predictor of computation than of word problem performance, which is notable given that word problems are often assumed to be more language intensive. Finally, when mathematics vocabulary was disaggregated, general but not aligned vocabulary predicted mathematics performance, suggesting that students' mathematics vocabulary knowledge supported performance even when the specific terms they knew were not directly represented on the assessment.

Collectively, these findings highlight the central role of mathematics vocabulary in college algebra performance, while suggesting that its effects may operate somewhat independently of general reading comprehension. The results also reveal that mathematics vocabulary knowledge may be especially critical not just for conceptually complex word problems but also for supporting procedural accuracy on computation tasks. These patterns underline the need to reconsider how linguistic factors are conceptualized within algebra learning, particularly in postsecondary at-risk student populations as a facilitator of procedural fluency and an indicator of understanding.

Research Question 1: Vocabulary and Performance

The first research question examined whether mathematics vocabulary knowledge predicted mathematics performance in college algebra after accounting for reading comprehension. Results showed that mathematics vocabulary was a statistically significant positive predictor of performance, whereas reading comprehension demonstrated only a marginal, not statistically significant relationship. This finding underscores the central role of mathematics-specific vocabulary in supporting students' ability to access and solve algebra problems, suggesting that vocabulary knowledge may be more directly tied to success in this domain than broader reading comprehension skills.

This result aligns with research emphasizing the importance of discipline-specific language for engaging with complex academic content (Fang, 2012; Gee, 1996, 1999; Kleemans et al., 2018; Moschkovich, 2015; Shanahan & Shanahan, 2008, 2020; Townsend et al., 2012), and with perspectives in discourse theory that position mathematics learning as mastery of its specialized representational and linguistic systems (Radford, 2008; Sfard, 2008). While reading comprehension supports general meaning-making across texts, mathematics vocabulary provides the conceptual and linguistic keys necessary to interpret algebraic representations, follow symbolic reasoning, and apply appropriate procedures. Prior studies have similarly found that mathematics vocabulary contributes unique variance to mathematics performance even when controlling for comprehension and other cognitive factors (Cirino et al., 2013; Lin et al., 2021). The current findings extend this work to a college algebra context, demonstrating that vocabulary remains a meaningful predictor beyond the secondary level.

The marginal contribution of reading comprehension suggests that general literacy skills, while foundational, may be less directly engaged during algebraic problem solving than vocabulary-specific knowledge. As students progress into higher-level mathematics, comprehension demands may become increasingly embedded within domain-specific language and symbolic reasoning (Fuchs et al., 2018; Peng & Lin, 2019). From this perspective, reading comprehension may function as an indirect prerequisite rather than a concurrent driver of performance in algebra, with vocabulary knowledge mediating access to the mathematical content itself. This aligns with the disciplinary literacy framework, which emphasizes that advanced learning depends not just on general comprehension but on mastering the specialized ways of using language unique to a discipline (Shanahan & Shanahan, 2008; Sfard, 2008).

Overall, these findings highlight that mathematics vocabulary knowledge plays a unique role in predicting college algebra performance, even beyond students' general comprehension ability. This has important implications for instructional design, suggesting that interventions aimed at improving algebra achievement should include explicit, sustained attention to mathematics vocabulary development alongside general literacy support. As Moschkovich (2015) emphasizes, mathematics vocabulary is not an isolated set of definitions but part of a broader disciplinary discourse through which students construct meaning, communicate reasoning, and engage in mathematical practices. From this perspective, supporting students in using mathematical language authentically, such as in discussions, written explanations, and problem solving, may be essential for fostering deeper conceptual understanding and long-term success in algebra.

Research Question 2: Reading Comprehension as Moderator

The second research question examined whether reading comprehension moderated the relationship between mathematics vocabulary knowledge and mathematics performance. Results indicated that reading comprehension did statistically significantly moderate this relationship. In other words, the positive association between mathematics vocabulary and performance was consistent across students with lower, average, and higher levels of reading comprehension. This finding suggests that mathematics vocabulary contributes to performance in a relatively uniform way, regardless of students' general comprehension ability.

This outcome contrasts with some theoretical models that position reading comprehension as a critical scaffolding skill for learning new vocabulary (e.g., Perfetti & Stafura, 2014; Duke & Cartwright, 2021). The Active View of Reading model (Duke & Cartwright, 2021) emphasizes the reciprocal interplay between linguistic knowledge and cognitive processes such as self-regulation and comprehension monitoring. From this perspective, stronger reading comprehension might have been expected to amplify the benefits of mathematics vocabulary by facilitating deeper processing of problem statements and supporting integration of mathematical terms into broader conceptual frameworks. However, the absence of an interaction effect suggests that once students have acquired mathematics-specific vocabulary knowledge, they may be able to leverage that knowledge to solve algebra problems independently of their general comprehension skills.

This pattern is consistent with research suggesting that vocabulary may act as a domain-specific gateway skill, enabling access to mathematical content even when broader comprehension skills are less developed (Lin et al., 2021; Cirino et al., 2013). It is also possible that the specific comprehension measure used (the GMRT) captured general reading skills that are less directly engaged during algebra problem solving, which often relies more on parsing symbolic language and numerical structures than on processing extended text. As algebraic tasks grow more symbolically dense, the relative contribution of general reading comprehension may diminish, while the contribution of domain-specific vocabulary knowledge persists (Fuchs et al., 2018; Peng & Lin, 2019).

These findings suggest the importance of treating vocabulary as a distinct construct within mathematical learning, rather than assuming it operates primarily through its relationship with comprehension. Instructionally, this highlights the potential value of targeted mathematics vocabulary instruction for supporting students at all reading levels, including those who may struggle with general comprehension but can still benefit from strengthening their mathematical lexicon.

Research Question 3: Task Type as Moderator

The third research question explored whether task type (computation versus word problems) moderated the relationship between mathematics vocabulary knowledge and mathematics performance. Results revealed a statistically significant interaction where vocabulary knowledge was unexpectedly observed as a stronger predictor of computation performance than of word problem performance. This finding is notable because it contrasts with the common assumption that word problems are more linguistically demanding and therefore more dependent on vocabulary knowledge (Pierce & Fontaine, 2009; Rubenstein & Thompson, 2002). Instead, students' mathematics vocabulary knowledge appeared to support their performance on symbolic computation tasks more strongly than on linguistically embedded word problems.

Bivariate correlations (Table 3) offered initial evidence of task-type differences in the role of mathematics vocabulary. Across vocabulary measures, correlations with computation were in the small-to-moderate range, whereas correlations with word problem performance were near zero and not statistically significant. This pattern indicates that students with stronger mathematics vocabulary tended to perform better on computation tasks, but this relationship did not extend to word problem performance at the bivariate level. The moderation analysis confirmed this pattern, showing that the slope relating mathematics vocabulary to performance was significantly steeper for computation than for word problems. Together, these results demonstrate that the contribution of mathematics vocabulary to performance is task-specific, exerting a stronger influence on computation than on word problem solving.

One possible explanation for this pattern is that computation tasks may rely more heavily on students' ability to rapidly and accurately map mathematical terms to specific procedures, a process directly supported by vocabulary knowledge. As Lin et al. (2021) argue, mathematics vocabulary contributes to automaticity, enabling students to allocate cognitive resources to executing procedures rather than deciphering terminology during problem solving. Students with stronger mathematics vocabulary knowledge may recognize terms like *factor, simplify,* or *distribute* as immediate cues for specific actions, facilitating efficient and accurate computation. In contrast, word problems require additional cognitive processes such as reading comprehension, inference making, and situation modeling (Kintsch, 1988), which may diffuse the direct impact of vocabulary and increase the influence of other skills not measured in this study, such as working memory or reasoning ability (Peng et al., 2020).

Another factor that may have influenced this pattern is the distribution of aligned vocabulary terms across task types. Most of the aligned terms on the performance assessment were directly tied to computation procedures, whereas relatively few were specific to word problem contexts. This imbalance may have allowed students with stronger mathematics vocabulary knowledge to capitalize on familiar terms as cues during computation tasks, while word problems required applying mathematical reasoning without explicit lexical prompts. As a result, the mathematics vocabulary assessment may have been more closely aligned with the language demands of computation (Pierce & Fontaine, 2009), which could help explain why mathematics vocabulary knowledge showed a stronger association with computation than with word problem performance.

This finding challenges the assumption that vocabulary primarily supports performance on linguistically complex problems and expresses the need to reconceptualize how vocabulary functions in mathematics learning. Rather than operating only as a language comprehension scaffold, mathematics vocabulary may also serve as a procedural cueing system, particularly in symbol-dense algebraic contexts. This interpretation aligns with disciplinary literacy perspectives that emphasize how specialized terminology can facilitate entry into discipline-specific routines and practices. Instructionally, this suggests that explicit vocabulary instruction could be especially valuable for strengthening procedural accuracy and fluency in computation, potentially serving as a foundation upon which deeper conceptual understanding and word problem-solving skills can be built.

Research Question 4: Vocabulary Proximity as Moderator

The fourth research question examined whether vocabulary that appeared on the performance measure (aligned) predicted college algebra performance differently than vocabulary that did not appear on the measure but is found in general algebra texts (general). This analysis was motivated by the findings from Research Question 3, which showed that mathematics vocabulary knowledge was a stronger predictor of computation performance than of word problem performance. Because most of the aligned vocabulary terms on the performance assessment were embedded within computation tasks, it was possible that the stronger vocabulary computation link reflected students' familiarity with the specific lexical items present on those tasks. This analysis tested whether students' performance was driven primarily by knowledge of those aligned terms or by broader mathematics vocabulary knowledge.

General mathematics vocabulary emerged as the only statistically significant predictor suggesting that the stronger vocabulary computation relationship observed was not simply a function of term overlap between the vocabulary and performance measures. Instead, students with stronger general mathematics vocabulary performed better overall, even when tasks did not contain those specific terms. This pattern indicates that mathematics vocabulary knowledge may support computation performance through transferable conceptual knowledge and procedural cueing, rather than through rote familiarity with the exact terms present on the test.

This interpretation falls within disciplinary literacy frameworks, which emphasize that mastering a discipline's specialized language enables students to engage in its characteristic ways of thinking, reasoning, and problem solving (Shanahan & Shanahan, 2008). From this perspective, knowing key algebraic terms may provide students with conceptual schemas that extend beyond isolated word–definition pairings to support flexible reasoning across diverse problem contexts. Koichu et al. (2007) described this as the development of heuristic vocabulary, language that not only labels procedures but also helps students plan and monitor their problem-solving strategies. Students with stronger mathematics vocabulary may therefore be able to approach unfamiliar problems by drawing on these broader conceptual frameworks, reducing their reliance on exact term overlap between mathematics vocabulary and performance measures.

The lack of a unique contribution from aligned vocabulary also suggests that performance may reflect deeper structural understanding rather than surface-level term recognition. This aligns with research showing that advanced mathematical proficiency requires moving beyond memorizing individual words to integrating them within conceptual networks and symbolic structures (Fang, 2012; Lin et al., 2021). Students who can flexibly apply vocabulary knowledge in novel contexts may demonstrate more adaptive reasoning, which has been positioned as a core strand of mathematical proficiency (Kilpatrick et al., 2001), explaining why their performance was not limited to tasks containing the specific vocabulary they had studied.

Findings also suggest that mathematics vocabulary knowledge functions as more than a test-specific skill. Rather than exerting its strongest influence only when familiar lexical items are present, vocabulary knowledge appears to serve as a generalizable foundation for algebraic thinking, supporting students' ability to interpret and solve problems even when the exact lexical items vary. Instructionally, this underscores the importance of teaching mathematics vocabulary not simply as isolated definitions but as interconnected conceptual tools, helping students build transferable frameworks they can apply across diverse algebraic tasks.

The findings from Research Question 4 help clarify the pattern observed in Research Question 1, which showed that overall mathematics vocabulary knowledge statistically significantly predicted college algebra performance, whereas reading comprehension contributed only marginally. By disaggregating the vocabulary measure into aligned and general components, results from research question 4 analyses demonstrated that this predictive effect was driven primarily by widely used algebra terms rather than by aligned vocabulary terms that appeared on the assessment itself.

The vocabulary and performance relationship observed in research question 1 reflects students' broader conceptual and linguistic knowledge of algebraic terms, rather than simple familiarity with the specific lexical items appearing on the assessment. Put differently, students with stronger general mathematics vocabulary knowledge likely brought transferable conceptual schemas and procedural language that supported their problem solving across tasks, even when the exact terms they knew were not present. This reinforces the interpretation that mathematics vocabulary knowledge functions as a generalizable foundation for algebraic reasoning, rather than as a test-specific cueing system, but also might indicate an overlap in mathematics vocabulary and mathematics knowledge.

Addressing the Gaps in the Literature

This study was designed to address several gaps identified in the literature on mathematics vocabulary and mathematics performance. Much of the existing research linking mathematics vocabulary to mathematics performance has been conducted with children and adolescents, particularly in elementary and middle school contexts (e.g., Fuchs et al., 2018; Hughes et al., 2018; Lin et al., 2021). As a result, there has been limited evidence about how mathematics vocabulary supports learning among postsecondary students, especially those who are academically at risk. These findings extend prior work by showing that vocabulary remains a critical cognitive linguistic resource beyond K–12 contexts, suggesting its role is not limited to early skill-building but continues to support access to algebraic content in college-level coursework.

Prior research has often linked vocabulary to broader language and cognitive skills such as reading comprehension, working memory, and reasoning (e.g., Lin & Powell, 2022; Neri et al., 2021; Rust, 2011). However, few studies have directly tested these factors as moderators of the vocabulary and performance relationship. In this study, reading comprehension did not moderate the association between mathematics vocabulary knowledge and mathematics performance, indicating that mathematics vocabulary contributed similarly to performance regardless of students' reading levels. This finding suggests that mathematics vocabulary played a unique role independent of general literacy skills, challenging the assumption that vocabulary primarily reflects broader comprehension ability.

Previous studies have typically examined mathematics performance as a single construct without differentiating between task types, leaving it unclear whether vocabulary contributes differently to computation tasks versus word problems. This study found that task types statistically significantly moderate the vocabulary and performance relationship with mathematics vocabulary predicting computation performance more strongly than word problem performance. This pattern contrasts with findings from Lin et al. (2021), who reported stronger vocabulary effects on higher-order, linguistically demanding tasks. One possible explanation is that for these academically at-risk college students, mathematics vocabulary knowledge may primarily support accessing and applying procedural content, whereas performance on word problems may depend more heavily on reading comprehension or other cognitive skills not measured in this study. This divergence highlights the importance of disaggregating task types when examining the role of mathematics vocabulary, as mathematics vocabulary may not operate uniformly across different kinds of mathematical demands in postsecondary contexts.

Another unexplored question in the literature has been whether the predictive strength of mathematics vocabulary depends on the presence of known terms on performance assessments. This study addressed that gap by disaggregating vocabulary into terms directly aligned with the performance measure and more general algebra vocabulary drawn from college-level texts. Results indicated that general mathematics vocabulary but not aligned vocabulary statistically significantly predicted mathematics performance. This suggests that mathematics vocabulary knowledge supported problem solving even when known terms were not directly present on the assessment, implying that mathematics vocabulary may operate as a broader conceptual access tool rather than as a cue for item-level recall. This finding challenges the intuitive expectation that vocabulary predicts performance mainly by helping students recognize terms they have seen before and instead points to the transferability of vocabulary knowledge to unfamiliar contexts.

Collectively, these findings address critical gaps in the literature by extending vocabulary research to a postsecondary population, testing comprehension as a moderator, differentiating between task types, and examining vocabulary alignment. They show that mathematics vocabulary functions as a robust predictor of algebra performance for academically at-risk college students, operating independently of reading comprehension in a way that generalizes beyond directly encountered terms. These contributions build on prior work linking vocabulary to mathematical reasoning (e.g., Lin et al., 2021; Cirino et al., 2013) and clarify the role of mathematics vocabulary as a cognitive linguistic resource for success in college algebra.

Integrative Interpretation

These findings extend prior work highlighting the importance of vocabulary in mathematical learning (Cirino et al., 2013; Lin et al., 2021; Moschkovich, 2015) by demonstrating that its influence persists at the postsecondary level and within an at-risk population. They also contribute to disciplinary literacy theory (Shanahan & Shanahan, 2008, 2020; Sfard, 2008), which theorizes that advanced academic success depends on mastering the specialized language and representational systems of a discipline. Within this framework, vocabulary is not merely a list of terms to be memorized but a gateway into the conceptual structures and symbolic routines that define mathematical thinking. Students who internalize this disciplinary language may be better able to recognize patterns, map terms to operations, and navigate algebraic representations, enabling them to solve problems even when tasks require transfer of concept knowledge or are linguistically sparse.

At the same time, these results complicate the common assumption that vocabulary's primary function is to support comprehension of linguistically dense word problems. Instead, vocabulary appeared to facilitate accuracy on symbolic computation tasks more than on word problems, suggesting that its role may be as much about procedural cueing and automaticity as about decoding linguistic information. This interpretation aligns with Lin et al.'s (2021) proposal that vocabulary contributes to efficiency by freeing cognitive resources for higher-order reasoning. It also aligns with Koichu et al.'s (2007) notion of heuristic vocabulary as a tool for planning and regulating problem-solving strategies. In this sense, mathematics vocabulary may serve as a bridge between linguistic knowledge and cognitive processes, enabling students to access algebraic structures quickly and reliably.

This study contributes to a growing body of evidence positioning mathematics vocabulary as a central and potentially malleable factor in algebra learning. By highlighting its robust association with performance across diverse task types and comprehension levels, these findings support viewing vocabulary not as a peripheral literacy skill but as a core disciplinary tool essential for mathematical proficiency. This perspective has important implications for both instruction and future research.

Implications for Practice

The findings of this study provide evidence of the critical role mathematics vocabulary plays as a foundational tool for supporting college algebra performance, especially for students considered academically at-risk. Because mathematics vocabulary predicted performance independently of reading comprehension, and even more strongly for computation tasks, instruction should prioritize explicit development of mathematics-specific vocabulary as an integral part of algebra curricula rather than treating it as a supplementary literacy skill. This aligns with research advocating systematic vocabulary instruction to enhance students' access to mathematical content (Rolf, 2022; Waite, 2017).

In practice, this does not require a complete change in pedagogy but may involve embedding direct instruction on key algebraic terms into daily lessons, providing opportunities for repeated exposure and use across contexts. Other approaches such as the Frayer Model, semantic mapping, and morphology-based instruction could help students link terms to conceptual meaning and procedural application (Wanjiru & Miheso O-Connor, 2015; Townsend et al., 2020). Because mathematics vocabulary contributed to computation more than word problem performance, instruction should especially emphasize how specific terms cue procedural steps (e.g., “factor,” “distribute,” “simplify”), building automaticity that frees cognitive resources for higher-level reasoning (Lin et al., 2021). Teachers might also incorporate retrieval-based practice, guided journaling, and self-explanation activities to strengthen both receptive and expressive vocabulary while simultaneously supporting self-regulation (Schoenfeld, 1992; Zimmerman, 2002).

Importantly, the null moderation by reading comprehension suggests that targeted vocabulary instruction may benefit students across a range of reading levels, including those who may struggle with general literacy skills. This reinforces the value of providing discipline-specific language supports directly within mathematics classrooms rather than relying on students to acquire the necessary vocabulary incidentally through general reading. Framing vocabulary as a core mathematical skill may also promote students’ sense of ownership and disciplinary identity (Herbel-Eisenmann et al., 2013; Horn, 2012), particularly for language learners and students from linguistically diverse or historically marginalized groups.

Implications for Research

The present study also highlights several key directions for future research. First, future studies should incorporate direct measures of cognitive skills, especially working memory and nonverbal reasoning, to clarify how these abilities interact with vocabulary knowledge in shaping algebra performance (Peng et al., 2020; Lin & Powell, 2022). Including such measures would allow researchers to test more comprehensive models of the cognitive-linguistic mechanisms underlying mathematics achievement.

Second, further refinement of the mathematics vocabulary measure is needed to more clearly distinguish vocabulary knowledge from broader mathematical knowledge. As noted in

Research Question 4, general mathematics vocabulary demonstrated a stronger association with college algebra performance than vocabulary aligned directly to the performance measure. This overlap makes it difficult to determine whether the observed effects reflect linguistic knowledge, conceptual knowledge, or a combination of both. Future research should expand the vocabulary item pool, vary the linguistic and conceptual demands of items, and use factor analytic approaches to investigate whether vocabulary and conceptual knowledge emerge as distinct dimensions. Clarifying this structure would strengthen construct validity and allow more precise interpretation of the role that mathematics vocabulary plays in supporting college algebra performance.

Third, longitudinal designs are needed to examine how the relationships among vocabulary, comprehension, and mathematics performance evolve over time as students advance through postsecondary mathematics coursework. A cross-sectional snapshot at the beginning of the semester cannot capture whether vocabulary effects remain stable, diminish, or intensify as students develop more sophisticated algebraic reasoning skills and encounter increasingly abstract content. Tracking students across a semester of study or multiple courses could clarify whether vocabulary primarily serves as an early gateway skill that enables access to foundational algebraic content or whether it continues to provide unique support for higher-level problem solving. Longitudinal data could also illuminate the durability of mathematics vocabulary knowledge (i.e. whether students retain and apply terms across semesters) and how growth in vocabulary interacts with improvements in comprehension and cognitive skills to shape long-term mathematics achievement.

Fourth, future research should investigate the distinct roles of receptive and expressive vocabulary in postsecondary mathematics learning. While receptive vocabulary may enable students to decode and interpret problems, expressive vocabulary could support their ability to justify reasoning, articulate procedures, and self-monitor problem solving (Powell, 2022; Lin et al., 2021). Parsing these components may clarify how vocabulary contributes to both procedural fluency and conceptual understanding in algebra. Building on the findings of this study, it will also be important to identify which specific terms are most essential for student success. Although the present results showed that the predictive value of vocabulary knowledge was not limited to terms directly appearing on the performance assessment, understanding which words serve as high-leverage entry points into core algebraic concepts could inform the design of targeted instructional supports. Mapping the vocabulary most strongly associated with computation and word problem performance may help educators prioritize which terms to teach explicitly and how to sequence them to support students' progression from foundational procedures toward conceptual problem solving.

Finally, future research should investigate the causal impact of mathematics vocabulary instruction on algebra performance through experimental or quasi-experimental designs. While correlational findings from the present study demonstrate a robust association between mathematics vocabulary knowledge and performance, experimental evidence is needed to establish whether strengthening students' vocabulary can directly improve achievement outcomes. Randomized controlled trials could compare traditional algebra instruction to versions that embed systematic, explicit mathematics vocabulary instruction to evaluate effects on both computation and word problem performance. Such studies could also examine whether vocabulary-focused interventions are particularly effective for students with lower general reading comprehension, given that vocabulary predicted performance independently of comprehension in the present study. This line of research would provide important causal evidence to guide the design of targeted instructional supports aimed at reducing algebra failure rates among at-risk college students.

Limitations

Although the findings of this study offer important insights into the role of mathematics vocabulary in college algebra performance, several limitations should be considered when interpreting the results. The study was intentionally designed to focus on a specific set of linguistic variables while excluding other cognitive factors such as working memory and nonverbal reasoning. This focused approach allowed for a clearer examination of the vocabulary and performance relationship, but it also means that the models did not capture the full range of cognitive processes known to support mathematical problem solving (Peng et al., 2020; Lin & Powell, 2022). Including such measures could have clarified whether vocabulary effects operate independently or interactively with cognitive abilities.

Limitations related to measurement should also be acknowledged. Although the mathematics performance items were selected from a standardized instrument (KeyMath-3), the internal consistency reliability for the adapted performance measure was lower than ideal which may have attenuated observed effect sizes. The researcher-developed vocabulary measure, although reviewed for content alignment, was not piloted prior to full administration, leaving its psychometric properties largely untested. As reflected in the findings for Research Question 4, this likely introduced construct overlap, making it difficult to determine whether aligned vocabulary items were measuring linguistic knowledge, conceptual understanding, or a combination of both.

Finally, the study's design and sample characteristics limit the generalizability of its findings. Data were collected from a single institution and included only academically at-risk students enrolled in prescribed College Algebra sections. This focus provided a valuable opportunity to examine an often-overlooked population, but the results may not generalize to students in other institutional contexts or with different achievement profiles. The study also relied on raw GMRT reading comprehension scores rather than standardized scores, which limits comparability to other studies, and the qualitative reflection data collected from participants were not integrated into the statistical models. Incorporating standardized reading scores and mixed methods approaches in future studies could provide a more nuanced understanding of how students perceive and apply mathematical vocabulary during problem solving.

Delimitations

Several delimitations shaped the design of this study and should be considered when interpreting its findings. The study focused specifically on college algebra rather than mathematics more broadly. This narrow scope allowed for close alignment between the vocabulary and performance measures and provided a clear context for investigating the role of discipline-specific language. However, this focus also means the findings may not generalize to other mathematics domains that place different linguistic and conceptual demands on students. Similarly, only receptive, multiple-choice measures of vocabulary were administered to ensure feasibility for group testing. This choice supported efficient data collection from a large sample but did not capture students' expressive vocabulary (i.e. their ability to produce and apply mathematical language) which may play a distinct role in supporting problem solving.

Other design decisions also shaped the scope of the study. All assessments were administered in a single session during class time, which helped control for testing conditions and minimize attrition but restricted the number and type of measures that could be included, particularly cognitively demanding or time-intensive assessments. In addition, the study employed a cross-sectional design. While appropriate for examining concurrent associations among variables, this approach cannot speak to developmental or causal relationships over time. As noted in the implications for research, future longitudinal designs could help clarify how the contributions of vocabulary and comprehension to mathematics performance may shift as students progress through advanced coursework.

These delimitations were intentional and necessary to maintain the feasibility and focus of the study, enabling a targeted examination of the vocabulary and performance relationship in an at-risk college algebra population. At the same time, they provide important context for interpreting the scope of the findings and for guiding the design of future studies that expand to other populations, settings, and methods.

Conclusion

This study examined how mathematics vocabulary knowledge relates to college algebra performance while accounting for reading comprehension, task type, and vocabulary alignment. Across analyses, mathematics vocabulary consistently emerged as a statistically significant predictor of performance, even when controlling for reading comprehension. Reading comprehension was not observed to moderate this relationship, and vocabulary exerted stronger predictive power on computation than on word problems. Furthermore, mathematics vocabulary's contribution to performance did not depend on knowing the exact terms present on the assessment, suggesting that students' vocabulary knowledge generalized beyond familiar terminology to support broader algebraic reasoning.

These findings contribute to the growing body of literature emphasizing the importance of discipline-specific language in mathematics learning. By demonstrating that mathematics vocabulary predicts performance in a postsecondary setting and among academically at-risk students, this study extends previous work largely conducted with younger populations. The results suggest that vocabulary may function not only as a support for comprehension but also as a cognitive tool for accessing algebraic structures, building procedural fluency, and navigating symbol-dense tasks. In doing so, the study advances disciplinary literacy theory by showing that mastery of mathematical language can directly support students' engagement with algebraic content, even when general comprehension skills or specific term familiarity are limited.

At the same time, these findings highlight important considerations for both instruction and future research. Instructionally, they point to the value of integrating explicit mathematics vocabulary instruction into college algebra curricula to support students across a range of reading abilities. Strategically embedding vocabulary development may help students access algebraic concepts more efficiently, freeing cognitive resources for higher-order reasoning and problem solving. Future research should build on this work by examining how cognitive skills such as working memory interact with vocabulary knowledge, by strengthening the validity and reliability of mathematics assessments, and by investigating the distinct roles of specific receptive and expressive vocabulary in supporting both procedural fluency and conceptual understanding.

Ultimately, this study provides evidence of the central role of mathematics vocabulary in college algebra performance and contributes to a more nuanced understanding of the linguistic foundations of mathematical proficiency. By clarifying how vocabulary supports students' engagement with algebraic content independently of general comprehension skills or specific term familiarity, the study provides a foundation for targeted instructional interventions to strengthen students' success in algebra, a critical gateway for continued achievement in mathematics and STEM fields. These insights can help inform efforts to reduce failure rates, close opportunity gaps, and promote equitable access to advanced mathematics pathways in higher education.

REFERENCES

Aiken, L. R., Jr. (1972, October). *Language factors in learning mathematics* (ERIC Report No. ED068340). ERIC Information Analysis Center for Science, Mathematics, and Environmental Education. https://eric.ed.gov/?id=ED068340

Archer, A., & Hughes, C. (2011). *Explicit instruction: Effective and efficient teaching.* Guilford Press.

Bae, Y. S. (2013). *Word problem solving of students with autistic spectrum disorders and typically developing students: The role of mathematics vocabulary and cognitive flexibility* (Doctoral dissertation, University of Iowa). ProQuest Dissertations Publishing.

Baddeley, A. (2000). The episodic buffer: A new component of working memory. *Trends in Cognitive Sciences, 4*(11), 417–423.

Bailey, D. H., Watts, T. W., Littlefield, A. K., & Geary, D. C. (2020). State and trait effects on individual differences in children's mathematical development. *Journal of Educational Psychology, 112*(2), 336–350. https://doi.org/10.1037/edu0000380

Baker, W. (2016). Validating mathematics vocabulary measures for diverse student populations. *International Journal of Educational Research, 80*, 135–147.

Barwell, R. (2016). Investigating stratification, language diversity and mathematics classroom interaction. *PNA: Revista de Investigación en Didáctica de la Matemática, 11*(1), 34–52. https://doi.org/10.30827/pna.v11i1.6080

Barwell, R. (2018). From language as a resource to sources of meaning in multilingual mathematics classrooms. *The Journal of Mathematical Behavior, 50*, 155–168. https://doi.org/10.1016/j.jmathb.2018.02.007

Bentler, P. M. (1990). Comparative fit indexes in structural models. *Psychological Bulletin, 107*(2), 238–246. https://doi.org/10.1037/0033-2909.107.2.238

Berch, D. B., & Mazzocco, M. M. M. (2007). *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities.* Paul H. Brookes.

Bloom, B. S. (Ed.). (1956). *Taxonomy of educational objectives: Handbook I: Cognitive domain* (2nd ed.). Longmans, Green.

Bradley, C. A. (1987). *The relationship between mathematics language facility and mathematics achievement among junior high school students* (Doctoral dissertation, University of California, Los Angeles). ProQuest Dissertations Publishing.

Brenner, M. E. (1981). *The role of language in mathematical problem solving: A study of bilingual students* (Doctoral dissertation, Stanford University). ProQuest Dissertations Publishing.

Briggs, S. R., & Cheek, J. M. (1986). The role of factor analysis in the development and evaluation of personality scales. *Journal of Personality, 54*(1), 106–148.

Brown, S. I., & Quinn, R. J. (2007). Fraction proficiency and success in algebra: What does research say? *Australian Mathematics Teacher, 63*(3), 23–30.

Brown, V. L., Cronin, M. E., & Bryant, D. P. (2012). *Test of Mathematical Abilities–Third Edition (TOMA-3).* PRO-ED.

Bruner, J. S., Olver, R. R., & Greenfield, P. M. (1966). *Studies in cognitive growth.* Wiley.

Bryant, B. R., Brown, L., & Rivera, D. P. (2012). *Test of Mathematical Abilities–Third Edition (TOMA-3).* PRO-ED.

Carpenter, T. P., Fennema, E., Peterson, P. L., & Carey, D. A. (1989). A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal, 90*(1), 3–20. https://doi.org/10.1086/461846

Cartwright, K. B., Taboada Barber, A., & Archer, C. J. (2022). What's the difference? Contributions of lexical ambiguity, reading comprehension, and executive functions to math word problem solving in linguistically diverse 3rd to 5th graders. *Scientific Studies of Reading*, 1–20. https://doi.org/10.1080/10888438.2022.2084399

Cattell, R. B. (1963). Theory of fluid and crystallized intelligence: A critical experiment. *Journal of Educational Psychology, 54*(1), 1–22. https://doi.org/10.1037/h0046743

Cheung, G. W., & Chan, D. (2005). Testing and evaluating latent variable interactions and quadratics. *Organizational Research Methods, 8*(1), 1–25. https://doi.org/10.1177/1094428104269369

Cirino, P. T., Tolar, T. D., & Fuchs, L. S. (2013). Arithmetic and cognitive contributions to algebra. *Society for Research on Educational Effectiveness*. https://eric.ed.gov/?id=ED563057

Cohen, J. (1962). The statistical power of abnormal–social psychological research: A review. *Journal of Abnormal and Social Psychology, 65*(3), 145–153.

Connolly, A. J. (2007). *KeyMath-3 Diagnostic Assessment.* Pearson Assessments.

Cortina, J. M. (1993). What is coefficient alpha? An examination of theory and applications. *Journal of Applied Psychology, 78*(1), 98–104.

Cosso, J., Purpura, D. J., Maeda, Y., & Bofferding, L. (2022). The home mathematics environment of dual-language learning children and their early mathematics skills.

Journal of Applied Developmental Psychology, 81, 101434. https://doi.org/10.1016/j.appdev.2022.101434

Cowan, R., Donlan, C., Shepherd, D. L., Cole-Fletcher, R., Saxton, M., & Hurry, J. (2011). Basic calculation proficiency and mathematics achievement in elementary school children. *Journal of Educational Psychology, 103*(4), 786–803. https://doi.org/10.1037/a0024556

Cromley, J. G., & Azevedo, R. (2007). Testing and refining the direct and inferential mediation model of reading comprehension. *Journal of Educational Psychology, 99*(2), 311–325. https://doi.org/10.1037/0022-0663.99.2.311

CTB/McGraw-Hill. (2006). *Acuity Assessment.* CTB/McGraw-Hill.

Cummins, J. (2000). Academic language learning, transformative pedagogy, and information technology: Towards a critical balance. *TESOL Quarterly, 34*(3), 537–548. https://doi.org/10.2307/3587742

Duke, N. K., & Cartwright, K. B. (2021). The science of reading progresses: Communicating advances beyond the simple view of reading. *Reading Research Quarterly, 56*(S1). https://doi.org/10.1002/rrq.411

Dunn, D. M., & Dunn, L. M. (2019). *Peabody Picture Vocabulary Test* (5th ed.). NCS Pearson.

Durkin, K., & Shire, B. (1991). Lexical ambiguity in mathematical contexts. *Journal of Child Language, 18*(2), 519–536.

Elleman, A. M., Lindo, E. J., Morphy, P., & Compton, D. L. (2009). The impact of vocabulary instruction on passage-level comprehension of school-age children: A meta-analysis. *Journal of Research on Educational Effectiveness, 2*(1), 1–44. https://doi.org/10.1080/19345740802539200

Engelmann, S., & Carnine, D. (1982). *Theory of instruction: Principles and applications.* Irvington Publishers.

Fang, Z. (2012). Language correlates of disciplinary literacy. *Topics in Language Disorders, 32*(1), 19–34. https://doi.org/10.1097/TLD.0b013e31824501de

Fox, J., & Weisberg, S. (2019). *An R companion to applied regression* (3rd ed.). Sage.

Fuchs, L. S., Fuchs, D., Compton, D. L., Hamlett, C. L., & Wang, A. Y. (2015). Is word-problem solving a form of text comprehension? *Scientific Studies of Reading, 19*(3), 204–223. https://doi.org/10.1080/10888438.2015.1005745

Fuchs, L. S., Fuchs, D., Compton, D. L., Hamlett, C. L., & Wang, A. Y. (2018). Is word-problem solving a form of text comprehension? *Scientific Studies of Reading, 22*(2), 152–166. https://doi.org/10.1080/10888438.2017.1398259

Fuchs, L. S., Gilbert, J. K., Fuchs, D., Seethaler, P. M., & Martin, B. N. (2018). Text comprehension and oral language as predictors of word-problem solving: Insights into word-problem solving as a form of text comprehension. *Scientific Studies of Reading, 22*(2), 152–166. https://doi.org/10.1080/10888438.2017.1398259

Fulda, J. S. (2009). Rendering conditionals in mathematical discourse with conditional elements. *Journal of Pragmatics, 41*(7), 1435–1439. https://doi.org/10.1016/j.pragma.2009.03.006

Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: A 5-year longitudinal study. *Developmental Psychology, 47*(6), 1539–1552. https://doi.org/10.1037/a0025510

Gee, J. (1996). *Social linguistics and literacies: Ideology in discourses.* The Falmer Press.

Gee, J. (1999). *An introduction to discourse analysis: Theory and method.* Routledge.

Gernsbacher, M. A. (1990). *Language comprehension as structure building.* Lawrence Erlbaum Associates.

Gershon, R. C., Wagster, M. V., Hendrie, H. C., Fox, N. A., Cook, K. F., & Nowinski, C. J. (2013). NIH Toolbox for Assessment of Neurological and Behavioral Function. *Neurology, 80*(11 Suppl 3), S2–S6. https://doi.org/10.1212/WNL.0b013e3182872e5f

Gibbons, P. (2003). Mediating language learning: Teacher interactions with ESL students in a content-based classroom. *TESOL Quarterly, 37*(2), 247–273.

Gibbons, P. (2006). *Bridging discourses in the ESL classroom: Students, teachers and researchers.* Continuum.

Güler, H., & Gelbal, S. (2022). Assessing open-ended mathematics items: Reliability and validity using Generalizability Theory. *Educational Sciences: Theory and Practice, 12*(3), 55–65.

Han, X. (2020). Improving mathematics vocabulary learning in the foundation phase. In *The Southeast Asian Conference on Education 2020: Official conference proceedings* (pp. 15–24). https://doi.org/10.22492/issn.2435-5240.2020.3

Harmon, J. M., Hedrick, W. B., & Wood, K. D. (2005). Research on vocabulary instruction in the content areas: Implications for struggling readers. *Reading & Writing Quarterly, 21*(3), 261–280. https://doi.org/10.1080/10573560590949377

Hedges, L. V. (1981). Distribution theory for Glass's estimator of effect size and related estimators. *Journal of Educational Statistics, 6*(2), 107–128. https://doi.org/10.3102/10769986006002107

Heimlich, J. E., & Pittelman, S. D. (1986). *Semantic mapping: Classroom applications.* International Reading Association.

Herbel-Eisenmann, B. A., Choppin, J., Wagner, D., & Pimm, D. (2013). *Equity in discourse for mathematics education: Theories, practices, and policies.* Springer.

Horn, I. S. (2012). *Strength in numbers: Collaborative learning in secondary mathematics.* National Council of Teachers of Mathematics.

Hornburg, C. B., Schmitt, S. A., & Purpura, D. J. (2018). Relations between preschoolers' mathematical language understanding and specific numeracy skills. *Journal of Experimental Child Psychology, 176*, 84–100. https://doi.org/10.1016/j.jecp.2018.07.005

Hu, L. T., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling, 6*(1), 1–55. https://doi.org/10.1080/10705519909540118

Hughes, E. M., Powell, S. R., & Lee, J.-Y. (2018). Development and psychometric report of a middle school mathematics vocabulary measure. *Assessment for Effective Intervention, 45*(3), 226–234. https://doi.org/10.1177/1534508418820116

IBM Corp. (2023). *IBM SPSS Statistics for Windows* (Version 29.0.1.0) [Computer software]. IBM Corp.

Ismay, C., & Kim, A. Y. (2022). *ModernDive: Statistical inference via data science.* CRC Press.

Jamison, R. E. (2000). Learning the language of mathematics. *Language and Learning across the Disciplines, 4*(1), 45–54. https://doi.org/10.37514/lld-j.2000.4.1.06

Johnson, K. E., & Alibali, M. W. (1999). Conceptual change in mathematics: Understanding children's learning. In E. Fennema & T. A. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 223–246). Lawrence Erlbaum Associates.

Kendeou, P., Savage, R., & van den Broek, P. (2009). Revisiting the simple view of reading in a transparent orthography: The role of listening comprehension and reading fluency. *Reading and Writing, 22*(1), 1–25. https://doi.org/10.1007/s11145-007-9103-1

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics.* National Academy Press.

Kintsch, W. (1988). The role of knowledge in discourse comprehension: A construction-integration model. *Psychological Review, 95*(2), 163–182. https://doi.org/10.1037/0033-295X.95.2.163

Kleemans, T., Segers, E., & Verhoeven, L. (2018). Role of linguistic skills in fifth-grade mathematics. *Journal of Experimental Child Psychology, 167*, 404–413. https://doi.org/10.1016/j.jecp.2017.11.012

Kline, R. B. (2015). *Principles and practice of structural equation modeling* (4th ed.). Guilford Press.

Koichu, B., Berman, A., & Moore, M. (2007). Heuristic literacy development and its relation to mathematical achievements of middle school students. *Instructional Science, 35*(2), 99–139. https://doi.org/10.1007/s11251-006-9004-3

Kotsopoulos, D. (2007). Mathematics discourse: "It's like hearing a foreign language." *Mathematics Teacher, 101*(4), 301–305. https://www.jstor.org/stable/27971502

Lai, E. R. (2011). *Critical thinking: A literature review.* Pearson Research Reports. https://images.pearsonassessments.com/images/tmrs/CriticalThinkingReviewFINAL.pdf

Lee, K., Ng, E. L., & Ng, S. F. (2004). Working memory and literacy as predictors of performance on algebraic word problems. *Journal of Experimental Child Psychology, 89*(2), 140–158.

Li, H., & Gan, Z. (2022). Reading motivation, self-regulated reading strategies and English vocabulary knowledge: Which most predicted students' English reading comprehension? *Frontiers in Psychology, 13*. https://doi.org/10.3389/fpsyg.2022.1041870

Lin, X. (2020). Investigating the unique predictors of word-problem solving using meta-analytic structural equation modeling. *Educational Psychology Review*. Advance online publication. https://doi.org/10.1007/s10648-020-09554-w

Lin, X., & Powell, S. R. (2022). The roles of initial mathematics, reading, and cognitive skills in subsequent mathematics performance: A meta-analytic structural equation modeling approach. *Review of Educational Research, 92*(2), 288–325. https://doi.org/10.3102/00346543211054576

Lin, X., Peng, P., & Luo, H. (2021). The deficit profile of elementary students with computational difficulties versus word problem-solving difficulties. *Learning Disability Quarterly, 44*(2), 110–122. https://doi.org/10.1177/0731948719865499

Lin, X., Peng, P., & Zeng, J. (2021). Understanding the relation between mathematics vocabulary and mathematics performance: A meta-analysis. *The Elementary School Journal, 121*(3), 503–532. https://doi.org/10.1086/712059

Long, J. A. (2019). *interactions: Comprehensive, user-friendly moderation and interaction analysis* [R package]. CRAN.

Loewenthal, K. M. (2001). *An introduction to psychological tests and scales* (2nd ed.). Psychology Press.

MacGinitie, W. H., MacGinitie, R. K., Maria, K., & Dreyer, L. G. (2000). *Gates-MacGinitie Reading Tests* (4th ed.). Riverside Publishing.

Martiniello, M. (2008). Language and the performance of English-language learners in math word problems. *Harvard Educational Review, 78*(2), 333–368. https://doi.org/10.17763/haer.78.2.70783570r1111t32

Marzano, R. J. (2004). *Building background knowledge for academic achievement: Research on what works in schools.* ASCD.

McEntire, N. (1981). *The relationship of mathematics vocabulary and reading comprehension to solving word problems in mathematics* (Doctoral dissertation, University of Maryland). ProQuest Dissertations Publishing.

Moher, D., Liberati, A., Tetzlaff, J., & Altman, D. G. (2009). Preferred reporting items for systematic reviews and meta-analyses: The PRISMA statement. *PLoS Medicine, 6*(6), e1000097. https://doi.org/10.1371/journal.pmed1000097

Monroe, E. E., & Orme, M. P. (2002). Developing mathematical vocabulary. *Preventing School Failure, 46*(3), 139–142. https://doi.org/10.1080/10459880209603359

Montague, M. (1992). The effects of cognitive and metacognitive strategy instruction on the mathematical problem solving of middle school students with learning disabilities. *Journal of Learning Disabilities, 25*(4), 230–248.

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning, 4*(2–3), 189–212. https://doi.org/10.1207/S15327833MTL04023_5

Moschkovich, J. N. (2015). Academic literacy in mathematics for English learners. *The Journal of Mathematical Behavior, 40*, 43–62. https://doi.org/10.1016/j.jmathb.2015.01.005

Moschkovich, J. N. (2015). Scaffolding student participation in mathematical practices. *ZDM—Mathematics Education, 47*(7), 1067–1078. https://doi.org/10.1007/s11858-015-0730-3

Moses, R. P., & Cobb, C. E. (2001). *Radical equations: Civil rights from Mississippi to the Algebra Project.* Beacon Press.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* National Council of Teachers of Mathematics.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics.* Authors.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.* U.S. Department of Education.

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* National Academy Press.

National Research Council. (2013). *The mathematical sciences in 2025.* National Academies Press.

Neri, N. C., Wagner, J., & Retelsdorf, J. (2021). What makes mathematics difficult for adults? The role of reading components in solving mathematics items. *Educational Psychology, 41*(9), 1199–1219. https://doi.org/10.1080/01443410.2021.1964438

Organization for Economic Co-operation and Development. (2023). *Description of the PISA mathematics literacy proficiency levels: 2022.*

https://nces.ed.gov/surveys/pisa/pisa2022/docs/DescriptionsOf2022ProficiencyLevels-TableI31.pdf

Partnership for Assessment of Readiness for College and Careers. (2015). *Math spring operational 2015: Grade 5 end-of-year released items.* https://previprc.org/system/files/5th%20grade%20Math%20-%20EOY%20-%20Item%20Set_Sept.%202016.pdf

Peng, P., & Lin, X. (2019). The relation between mathematics vocabulary and mathematics performance among fourth graders. *Learning and Individual Differences, 69*, 112–118. https://doi.org/10.1016/j.lindif.2018.11.006

Peng, P., Lin, X., Ünal, Z. E., Lee, K., Namkung, J., Chow, J., & Sales, A. (2020). Examining the mutual relations between language and mathematics: A meta-analysis. *Psychological Bulletin, 146*(7), 595–634. https://doi.org/10.1037/bul0000231

Peng, P., Namkung, J., Barnes, M., & Sun, C. (2019). A meta-analysis of mathematics and working memory: Moderating effects of working memory domain, type of mathematics skill, and age. *Psychological Bulletin, 145*(9), 921–953. https://doi.org/10.1037/bul0000200

Perfetti, C. A. (2007). Reading ability: Lexical quality to comprehension. *Scientific Studies of Reading, 11*(4), 357–383. https://doi.org/10.1080/10888430701530730

Perfetti, C. A., & Stafura, J. (2014). Word knowledge in a theory of reading comprehension. *Scientific Studies of Reading, 18*(1), 22–37. https://doi.org/10.1080/10888438.2013.827687

Piaget, J. (1952). *The origins of intelligence in children* (M. Cook, Trans.). International Universities Press. (Original work published 1936)

Pierce, M. E., & Fontaine, L. M. (2009). Designing vocabulary instruction in mathematics. *The Reading Teacher, 63*(3), 239–243. https://doi.org/10.1598/RT.63.3.7

Pina, V., Fuentes, L. J., Castillo, A., & Diamantopoulou, S. (2015). Disentangling the effects of working memory, language, and early quantity knowledge on mathematics achievement. *Learning and Individual Differences, 37*, 180–187.

Pólya, G. (1945). *How to solve it: A new aspect of mathematical method.* Princeton University Press.

Powell, S. R. (2017). Mathematics vocabulary instruction: A guide for educators. *Journal of Educational Psychology, 109*(4), 532–550.

Powell, S. R. (2022). The roles of initial mathematics, reading, and cognitive skills in subsequent mathematics performance: A meta-analytic structural equation modeling approach. *Review of Educational Research, 92*(2), 288–325. https://doi.org/10.3102/00346543211054576

Powell, S. R., & Field, S. (2015). Reliability and validity of mathematics vocabulary assessments. *Journal of Educational Measurement, 52*(2), 198–215.

Powell, S. R., Fuchs, L. S., & Fuchs, D. (2013). Reaching the mountaintop: Addressing the Common Core Standards in mathematics for students with mathematics difficulties. *Learning Disabilities Research & Practice, 28*(1), 38–48.

Powell, S. R., Fuchs, L. S., & Fuchs, D. (2016). The role of mathematics vocabulary in academic achievement. *Learning Disabilities Research & Practice, 31*(3), 123–135.

Prediger, S., Erath, K., Weinert, H., & Quabeck, K. (2022). Only for multilingual students at risk? Cluster-randomized trial on language-responsive mathematics instruction. *Journal for Research in Mathematics Education, 53*(4), 255–276. https://doi.org/10.5951/jresematheduc-2020-0193

Proctor, C. P., August, D., Carlo, M., & Snow, C. E. (2006). The intriguing role of Spanish language vocabulary knowledge in predicting English reading comprehension. *Journal of Educational Psychology, 98*(1), 159–169. https://doi.org/10.1037/0022-0663.98.1.159

Purpura, D. J., Logan, J. A. R., Hassinger-Das, B., & Napoli, A. R. (2017). Why do early mathematics skills predict later reading? The role of mathematical language. *Developmental Psychology, 53*(9), 1633–1642. https://doi.org/10.1037/dev0000375

Purpura, D. J., Napoli, A. R., Wehrspann, E. A., & Gold, Z. S. (2017). Causal connections between mathematical language and mathematical knowledge: A dialogic reading intervention. *Journal of Research on Educational Effectiveness, 10*(1), 116–137. https://doi.org/10.1080/19345747.2016.1204639

R Core Team. (2023). *R: A language and environment for statistical computing* [Computer software]. R Foundation for Statistical Computing.

Radford, L. (2008). The semiotics of culture and the cultural dimension of mathematics education. In *Proceedings of the 5th International Conference on Mathematics Education and Society* (pp. 169–178). Centro de Investigação em Educação.

Raven, J. C. (1938). *Progressive Matrices: A perceptual test of intelligence.* H. K. Lewis.

Revelle, W. (2023). *psych: Procedures for personality and psychological research* [R package]. Northwestern University.

Riccomini, P. J., Smith, G. W., Hughes, E. M., & Fries, K. M. (2015). The language of mathematics: The importance of teaching and learning mathematical vocabulary. *Reading & Writing Quarterly, 31*(3), 235–252.

Rittle-Johnson, B. (2017). Developing mathematics knowledge. *Child Development Perspectives, 11*(3), 184–190. https://doi.org/10.1111/cdep.12229

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93*(2), 346–362. https://doi.org/10.1037/0022-0663.93.2.346

Riverside Insights. (2024). *KeyMath-3 Diagnostic Assessment.* https://www.riversideinsights.com

Roberts, G. J., Solis, M., & Chance, B. (2019). Embedding self-regulation into reading interventions to support reading and behavior outcomes. *Teaching Exceptional Children, 52*(2), 78–86. https://doi.org/10.1177/0040059919874306

Rockhold, J. L. (2013). *The effects of English vocabulary mastery on geometry achievement* (Doctoral dissertation, Florida Gulf Coast University). ProQuest Dissertations Publishing. https://eric.ed.gov/?id=ED561692

Rolf, K. R. (2022). *Examining the effectiveness of explicit, systematic mathematics vocabulary instruction for students with learning difficulties and disabilities in a specialized setting* [Doctoral dissertation, Utah State University]. USU Digital Commons. https://digitalcommons.usu.edu/etd/8550

Rosenblatt, L. M. (1978). *The reader, the text, the poem: The transactional theory of the literary work.* Southern Illinois University Press.

Rosseel, Y. (2012). lavaan: An R package for structural equation modeling. *Journal of Statistical Software, 48*(2), 1–36. https://doi.org/10.18637/jss.v048.i02

Rubenstein, R. N., & Thompson, D. R. (2002). Understanding and supporting children's mathematical vocabulary development. *Teaching Children Mathematics, 9*(2), 107–113.

Rumelhart, D. E. (1980). Schemata: The building blocks of cognition. In R. J. Spiro, B. C. Bruce, & W. F. Brewer (Eds.), *Theoretical issues in reading comprehension* (pp. 33–58). Lawrence Erlbaum Associates.

Rust, A. H. (2011). *The impact of instruction incorporating content area reading strategies on student mathematical achievement in a community college developmental mathematics course* (Doctoral dissertation, University of Maryland, College Park).

Sayeski, K. L., & Paulsen, K. J. (2010). Mathematics reform curricula and special education: Identifying intersections and implications for practice. *Intervention in School and Clinic, 46*(1), 13–21.

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading and Writing Quarterly, 23*(2), 139–159. https://doi.org/10.1080/10573560601158461

Schmitt, N. (1996). Uses and abuses of coefficient alpha. *Psychological Assessment, 8*(4), 350–353.

Schoenfeld, A. H. (1987). What's all the fuss about metacognition? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 189–215). Lawrence Erlbaum Associates.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. *Journal of Education, 196*(2), 1–38.

Sfard, A. (2007). When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint. *The Journal of the Learning Sciences, 16*(4), 565–613.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing.* Cambridge University Press.

Shanahan, C., & Shanahan, T. (2020). Disciplinary literacy. In J. Patterson (Ed.), *The SAT® suite and classroom practice: English language arts/literacy* (pp. 91–125). College Board.

Shanahan, C., Shanahan, T., & Misischia, C. (2011). Analysis of expert readers in three disciplines: History, mathematics, and chemistry. *Journal of Literacy Research, 43*(4), 393–429.

Shanahan, T., & Shanahan, C. (2008). Teaching disciplinary literacy to adolescents: Rethinking content-area literacy. *Harvard Educational Review, 78*(1), 40–59. https://doi.org/10.17763/haer.78.1.v62444321p602101

Shanahan, T., & Shanahan, C. (2012). What is disciplinary literacy and why does it matter? *Topics in Language Disorders, 32*(1), 7–18. https://doi.org/10.1097/TLD.0b013e318244557a

Sharp, J. M. (1998). *Results of using writing tasks to enhance fourth-grade children’s acquisition of fraction knowledge* [Microform]. ERIC. https://eric.ed.gov/?id=ED449984

Sharma, M. C. (2013). Levels of knowing mathematics. In L. Cermak & A. Vickers (Eds.), *Levels of knowing mathematics and depth of knowledge.* ATLAS ABE. https://atlasabe.org/wp-content/uploads/2022/01/Levels-of-Knowing-Math_Cermak-Vickers.pdf

Sidney, P. G., Thompson, C. A., & Rivera, F. D. (2015). Making meaning of mathematics vocabulary: Using latent semantic analysis to measure the semantic content of mathematics instruction. *Journal of Educational Psychology, 107*(2), 380–390. https://doi.org/10.1037/a0037820

Siegler, R. S., Thompson, C. A., & Schneider, M. (2012). An integrated theory of whole number and fractions development. *Cognitive Psychology, 62*(4), 273–296. https://doi.org/10.1016/j.cogpsych.2011.03.001

Simpson, A., & Cole, M. W. (2015). More than words: A literature review of language of mathematics research. *Educational Review, 67*(3), 369–384. https://doi.org/10.1080/00131911.2014.971714

Smarter Balanced Consortium. (2023). *Smarter Balanced mathematics construct-relevant vocabulary terms.* https://portal.smarterbalanced.org

Snow, C. E. (2010). Academic language and the challenge of reading for learning about science. *Science, 328*(5977), 450–452. https://doi.org/10.1126/science.1182597

Snow, C. E., & Uccelli, P. (2009). The challenge of academic language. In D. R. Olson & N. Torrance (Eds.), *The Cambridge handbook of literacy* (pp. 112–133). Cambridge University Press.

Stahl, S. A., & Nagy, W. E. (2005). *Teaching word meanings.* Lawrence Erlbaum Associates.

Stanovich, K. E. (1986). Matthew effects in reading: Some consequences of individual differences in the acquisition of literacy. *Reading Research Quarterly, 21*(4), 360–407. https://doi.org/10.1598/RRQ.21.4.1

Star, J. R., & Rittle-Johnson, B. (2008). Flexibility in problem solving: The case of equation solving. *Learning and Instruction, 18*(6), 565–579. https://doi.org/10.1016/j.learninstruc.2007.09.018

Suggate, S. P., Schaughency, E. A., & Reese, E. (2014). Vocabulary reflects the breadth but not the depth of preschoolers' word knowledge. *Reading Research Quarterly, 49*(4), 443–460. https://doi.org/10.1002/rrq.79

Tannenbaum, K. R., Torgesen, J. K., & Wagner, R. K. (2009). Relationships between word knowledge and reading comprehension in third-grade children. *Scientific Studies of Reading, 10*(4), 381–398. https://doi.org/10.1207/s1532799xssr1004_3

Tolar, T. D., Lederberg, A. R., & Fletcher, J. M. (2009). Factors influencing success in solving word problems: A meta-analysis. *Review of Educational Research, 79*(4), 889–921.

Torgesen, J. K., Wagner, R. K., & Rashotte, C. A. (2001). Prevention and remediation of severe reading disabilities: Keeping the end in mind. *Scientific Studies of Reading, 5*(3), 257–291. https://doi.org/10.1207/S1532799XSSR0503_5

Townsend, D., & Collins, P. (2009). Academic vocabulary and middle school English learners: An intervention study. *Reading and Writing, 22*(9), 993–1019. https://doi.org/10.1007/s11145-008-9141-y

Townsend, D., Filippini, A., Collins, P., & Biancarosa, G. (2012). Evidence for the importance of academic word knowledge for the academic achievement of diverse middle school students. *The Elementary School Journal, 112*(3), 497–518. https://doi.org/10.1086/663301

Townsend, D., Taboada Barber, A., Carter, H., & Salas, R. (2020). More than words: Older adolescents' linguistic resources in the context of disciplinary achievement and academic risk. *Reading Psychology, 41*(8), 778–802. https://doi.org/10.1080/02702711.2020.1782291

Tunmer, W. E., & Chapman, J. W. (2012). Does set for variability mediate the influence of vocabulary knowledge on the development of word recognition skills? *Scientific Studies of Reading, 16*(2), 122–140. https://doi.org/10.1080/10888438.2010.542527

Ünal, Z. E., Powell, S. R., Özel, S., Scofield, J. E., & Geary, D. C. (2021). Mathematics vocabulary differentially predicts mathematics achievement in eighth-grade higher- versus lower-achieving students: Comparisons across two countries. *Learning and Individual Differences, 92*, 102061. https://doi.org/10.1016/j.lindif.2021.102061

Unsworth, N., Heitz, R. P., Schrock, J. C., & Engle, R. W. (2005). An automated version of the operation span task. *Behavior Research Methods, 37*(3), 498–505. https://doi.org/10.3758/BF03192720

van Dijk, T. A., & Kintsch, W. (1983). *Strategies of discourse comprehension.* Academic Press.

Vukovic, R. K. (2006). *Mathematics difficulties in young children: Contributions of language, visual-spatial skills, and working memory* (Doctoral dissertation, University of Toronto). ProQuest Dissertations Publishing.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes* (M. Cole, V. John-Steiner, S. Scribner, & E. Souberman, Eds.). Harvard University Press.

Waite, A. M. (2017). *Effect of directed study of mathematics vocabulary on standardized mathematics assessment questions* (Doctoral dissertation, Indiana State University).

Waller, P., & Flood, C. T. (2016). Mathematics as a universal language: Transcending cultural lines. *Journal for Multicultural Education, 10*(3), 294–306. https://doi.org/10.1108/JME-01-2016-0004

Walshaw, M., & Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. *Review of Educational Research, 78*(3), 516–551. https://doi.org/10.3102/0034654308320292

Wanjiru, B., & Miheso O-Connor, M. (2015). Effects of mathematical vocabulary instruction on students' achievement in mathematics in secondary schools of Murang'a County, Kenya. *Journal of Education and Practice, 6*(18), 201–207.

Watts, T. W., Duncan, G. J., Siegler, R. S., & Davis-Kean, P. E. (2014). What's past is prologue: Relations between early mathematics knowledge and high school achievement. *Educational Researcher, 43*(7), 352–360. https://doi.org/10.3102/0013189X14553660

Webb, S., Sasao, Y., & Ballance, O. (2017). The updated Vocabulary Levels Test: Developing and validating two new forms of the VLT. *ITL—International Journal of Applied Linguistics, 168*(1), 34–70. https://doi.org/10.1075/itl.168.1.02web

Wickham, H. (2016). *ggplot2: Elegant graphics for data analysis.* Springer.

Wickham, H., Averick, M., Bryan, J., Chang, W., McGowan, L. D., François, R., Grolemund, G., Hayes, A., Henry, L., Hester, J., Kuhn, M., Pedersen, T. L., Miller, E., Bache, S. M., Müller, K., Ooms, J., Robinson, D., Seidel, D. P., Spinu, V., Takahashi, K., Vaughan, D., Wilke, C., &

Yutani, H. (2019). Welcome to the tidyverse. *Journal of Open Source Software, 4*(43), 1686. https://doi.org/10.21105/joss.01686

Wilkinson, G. S., & Robertson, G. J. (2006). *Wide Range Achievement Test* (4th ed.). Psychological Assessment Resources.

Williams, K. T. (2019). *Expressive Vocabulary Test* (3rd ed.). NCS Pearson.

Woodcock, R. W., McGrew, K. S., & Mather, N. (2001). *Woodcock–Johnson III Tests of Achievement.* Riverside Publishing.

Yang, X., Yan, M., Ruan, Y., Ku, S. Y. Y., Lo, J. C. M., Peng, P., & McBride, C. (2022). Relations among phonological processing skills and mathematics in children: A meta-analysis. *Journal of Educational Psychology, 114*(2), 289–307. https://doi.org/10.1037/edu0000710

Zeileis, A., & Hothorn, T. (2002). Diagnostic checking in regression relationships. *R News, 2*(3), 7–10.

Zhang, D., Zhao, Y., & Wang, L. (2021). The emergence of mathematical understanding: Connecting to the cognitive structure. *Frontiers in Psychology, 12*, 764567. https://doi.org/10.3389/fpsyg.2021.764567

Zimmerman, B. J. (2002). Becoming a self-regulated learner: An overview. *Theory Into Practice, 41*(2), 64–70.

APPENDICES

Appendix A

Mathematics Vocabulary Assessment

This custom 30-item multiple-choice assessment measured students' knowledge of mathematics vocabulary relevant to college algebra. Fifteen items were designed to align directly with terminology used in the mathematics performance assessment (e.g., *factor, slope, reciprocal*), while fifteen items assessed general mathematics vocabulary not directly represented on the performance test (e.g., *variable, coefficient, inequality*). Each item consisted of a single term and four response options, with one correct answer scored as 1 point and incorrect answers scored as 0. Total scores ranged from 0 to 30, with subscores computed separately for aligned and general vocabulary items.

Mathematics Vocabulary Measure

Name: ____________________________

Directions: Read each question carefully and circle the best answer.

Section 1: Aligned Mathematics Vocabulary

1. Least Common Denominator
   What does least common denominator mean in mathematics?
      A. The smallest number that is a multiple of the denominators
      B. The number with the fewest digits
      C. The number that comes first in a list
      D. The largest number in the problem

2. Factor
   What is a factor in mathematics?
      A. A number you multiply with another to get a product
      B. A number you divide by to get a remainder
      C. A number that cannot be simplified
      D. A number written as a decimal

3. Simplify
   What does it mean to simplify an expression in mathematics?
      A. Rewrite the expression to make it more complicated
      B. Change all numbers to decimals
      C. Rewrite the expression using only addition

D. Rewrite the expression in a shorter or reduced form

4. Expression
What is a mathematical expression?
A. A sentence that has an equals sign
B. A group of numbers, symbols, and operations without an equals sign
C. A number that repeats
D. A number written as a fraction

5. Slope
What does slope mean in mathematics?
A. The number you multiply to find a product
B. The distance around a shape
C. The steepness or slant of a line on a graph
D. The space between two numbers

6. Reciprocal
What is the reciprocal of a number in math?
A. The number with the same digits reversed
B. A number that is squared
C. A number that has a percent sign
D. A flipped fraction or the inverse of a number

7. Product
What does the word product mean in mathematics?
A. The answer to an addition problem
B. The total number of digits in a number
C. The answer to a multiplication problem
D. The number that comes after the equal sign

8. Mean
What does mean refer to in mathematics?
A. The middle number in a data set
B. The number that occurs most often
C. The smallest number in the data set
D. The average found by adding and dividing

9. Right Angle Triangle
What is a right angle triangle?
A. A triangle with no equal sides
B. A triangle with one 90-degree angle
C. A triangle with three equal sides
D. A triangle with all angles less than 90 degrees

10. Linear
   What does the word linear mean in mathematics?
      A. Having a shape like a square
      B. Related to curves or circles
      C. Forming a straight line
      D. Changing direction at each point

11. Function
   What is a function in mathematics?
      A. A graph that forms a closed shape
      B. A relationship where each input has one output
      C. A list of random numbers
      D. A formula with only subtraction

12. Relation
   What does the term relation mean in mathematics?
      A. A connection between numbers and operations
      B. A set of ordered pairs or input-output values
      C. A type of polygon
      D. A repeating decimal

13. Exponent
   What does an exponent tell you in a number like 3^2?
      A. How many times to add the base
      B. The number to multiply by
      C. How many times to divide the number
      D. How many times to multiply the base by itself

14. Square Root
   What does square root mean in mathematics?
      A. A number that is multiplied by 4
      B. The opposite of an exponent
      C. A number that multiplies by itself to make a given number
      D. A number divided by 2

15. Percentage
   What does the word percentage mean in math?
      A. A number divided by 10
      B. A number compared to 100
      C. A whole number
      D. A number written as a fraction

Section 2: General Mathematics Vocabulary

16. Evaluate

What does it mean to evaluate an expression in mathematics?
A. To explain how to solve the problem
B. To check if the expression is correct
C. To find the value of the expression
D. To rewrite the expression using different numbers

17. Solve
What does it mean to solve an equation in mathematics?
A. To rewrite the equation as a fraction
B. To find the value of the variable that makes the equation true
C. To make both sides of the equation equal
D. To graph the equation on a number line

18. Variable
What is a variable in mathematics?
A. A number that always stays the same
B. A letter or symbol that stands for a number
C. A rule for multiplying numbers
D. A type of equation with two operations

19. Constant
What is a constant in mathematics?
A. A number that changes in every problem
B. A number added to a variable
C. A value that does not change
D. A number that is part of an exponent

20. Term
What is a term in mathematics?
A. A number, variable, or combination of both in an expression
B. A number that comes after a decimal
C. A value used only in inequalities
D. The result of a subtraction problem

21. Coefficient
In the expression 7x, what is the coefficient?
A. The exponent
B. The number that multiplies the variable
C. The variable
D. The sum of the numbers

22. Logarithm
What is a logarithm in mathematics?
A. A rule for converting decimals to fractions

B. A method for comparing equations
C. A quantity representing the power to which a base must be raised to produce a given number
D. A symbol used to group terms in parentheses

23. Substitute
What does it mean to substitute in mathematics?
A. To rewrite an equation backwards
B. To change all variables into fractions
C. To replace a variable with a specific number
D. To add parentheses to an expression

24. Integer
What is an integer in mathematics?
A. A number that includes fractions and decimals
B. A number that is always positive
C. A whole number that can be positive, negative, or zero
D. A number that can only be divided by one

25. Radical
What is a radical in mathematics?
A. A symbol used to show a square root or other root
B. A number that cannot be divided
C. A number greater than 100
D. A decimal rounded to the nearest tenth

26. Absolute Value
What does absolute value mean in mathematics?
A. The opposite of a number
B. The number divided by two
C. The distance a number is from zero on a number line
D. A number with a negative sign

27. Equation
What is an equation in mathematics?
A. A sentence that shows two expressions are equal
B. A list of numbers in order
C. A graph with two lines
D. A number written as a power

28. Quadratic
What is a quadratic expression in mathematics?
A. An expression that has a variable with an exponent of 2
B. An equation with three variables

C. A graph that forms a straight line
D. A number multiplied by a radical

29. Inverse
What is the inverse of a number in mathematics?
A. The number multiplied by zero
B. The opposite operation or reciprocal
C. The square of the number
D. The number rounded to the nearest whole

30. Distribute
What does it mean to distribute in mathematics?
A. To separate terms into smaller pieces
B. To solve an equation using subtraction
C. To multiply a number by each term inside parentheses
D. To divide both sides of an equation

Appendix B

Mathematics Performance Assessment

This26-item multiple-choice mathematics performance assessment was developed using the KeyMath-3 to measure students' problem-solving skills in college algebra. Sixteen items assessed computation tasks requiring application of core algebraic procedures (e.g., simplifying expressions, solving for a variable), and ten items assessed word problem tasks requiring interpretation and application of algebraic concepts in context. Each item had four response options, with one correct answer scored as 1 point and incorrect answers scored as 0. Total scores ranged from 0 to 26, with subscores computed separately for computation and word problem items. Examples of each question type are provided below:

Computation

Solve for x:

$4(2x-3)-(x+5)=3x+1$

A. 2

B. $\frac{9}{2}$

C. 5

D. $\frac{11}{2}$

Word Problem

A tutor charges a fixed booking fee and an hourly rate. A 2-hour session costs $110 and a 5-hour session costs $200. What is the tutor's hourly rate?

A. $25

B. $30

C. $35

D. $40

Appendix C

Data Analysis Workflow and Syntax

This appendix contains the full data analysis workflow, including all statistical syntax and scripts used to conduct the analyses reported in this study. Analyses were performed using both R (R Core Team, 2023) and SPSS (Version 29). The R scripts include code for data cleaning, scoring, reliability estimation, descriptive analyses, correlation analyses, multiple regression models, and moderation models, as well as diagnostic checks and visualization code. The SPSS syntax steps document the equivalent linear regression and General Linear Model (GLM) repeated-measures procedures used to cross-validate results. These materials are provided to support transparency and replicability of findings. All scripts are organized by research question (RQ1–RQ4) and include comments describing the sequence of analyses, variable naming conventions, and model specifications.

R Workbook

```
# ================================
# Setup & Data Import
# ================================
if (!requireNamespace("pacman", quietly = TRUE)) install.packages("pacman")
pacman::p_load(readxl, dplyr, tidyr, psych, car, broom, parameters, writexl, readr)

# Create output folders
dir.create("outputs", showWarnings = FALSE)
dir.create("outputs/reliability", showWarnings = FALSE, recursive = TRUE)
dir.create("outputs/descriptives", showWarnings = FALSE, recursive = TRUE)
dir.create("outputs/diagnostics", showWarnings = FALSE, recursive = TRUE)
dir.create("outputs/models", showWarnings = FALSE, recursive = TRUE)

# --- DATA PATH ---
data_path <- "C:/Users/wpuckett/Documents/Dissertation/Data Analysis Third Run/Raw
Data.xlsx"
raw <- readxl::read_excel(data_path)
```

```
# Keep a row id for safe joins
raw <- raw %>% dplyr::mutate(.row_id = dplyr::row_number())

# ================================
# Scoring (Totals) — item names fixed to your spec
# ================================
df <- raw %>%
  dplyr::mutate(
    CompTotal  = rowSums(dplyr::select(., PerfScore_1:PerfScore_16),  na.rm = TRUE),
    WPTotal    = rowSums(dplyr::select(., PerfScore_17:PerfScore_26), na.rm = TRUE),
    PerfTotal  = CompTotal + WPTotal,
    VocabAlign = rowSums(dplyr::select(., VocabScore_1:VocabScore_15),   na.rm = TRUE),
    VocabGen   = rowSums(dplyr::select(., VocabScore_16:VocabScore_30),  na.rm = TRUE),
    VocabTotal = VocabAlign + VocabGen,
    GMRTTotal  = rowSums(dplyr::select(., GMRTScore_1:GMRTScore_48),     na.rm = TRUE)
  )

# Long frame for RQ3 (task-type); join totals back by .row_id
df_long <- df %>%
  dplyr::select(.row_id, CompTotal, WPTotal) %>%
  tidyr::pivot_longer(
    cols = c(CompTotal, WPTotal),
    names_to = "TaskType",
    values_to = "PerfScore"
  ) %>%
  dplyr::mutate(
    TaskType = factor(
      TaskType,
      levels = c("CompTotal", "WPTotal"),
      labels = c("Computation", "Word problems")
    )
  ) %>%
  dplyr::left_join(
    df %>% dplyr::select(.row_id, VocabTotal, VocabAlign, VocabGen, GMRTTotal),
    by = ".row_id"
  )

# ================================
# Reliability
# ================================
```

```
alpha_perf  <- psych::alpha(dplyr::select(df, dplyr::starts_with("PerfScore_")))
alpha_vocab <- psych::alpha(dplyr::select(df, dplyr::starts_with("VocabScore_")))
alpha_gmrt  <- psych::alpha(dplyr::select(df, dplyr::starts_with("GMRTScore_")), check.keys =
TRUE)

capture.output(alpha_perf,  file = "outputs/reliability/alpha_performance.txt")
capture.output(alpha_vocab, file = "outputs/reliability/alpha_vocabulary.txt")
capture.output(alpha_gmrt,  file = "outputs/reliability/alpha_gmrt.txt")

# ================================
# Descriptives & Correlations (Totals)
# ================================
desc_tbl <- psych::describe(dplyr::select(
  df, PerfTotal, CompTotal, WPTotal, VocabTotal, VocabAlign, VocabGen, GMRTTotal
))
readr::write_csv(as.data.frame(desc_tbl), "outputs/descriptives/descriptives_totals.csv")

cor_mat <- stats::cor(
  dplyr::select(df, PerfTotal, CompTotal, WPTotal, VocabTotal, VocabAlign, VocabGen,
GMRTTotal),
  use = "pairwise.complete.obs"
)
utils::write.csv(cor_mat, "outputs/descriptives/correlations_totals.csv", row.names = TRUE)

# ================================
# Z-scores (uniform across all RQs)
# ================================
df <- df %>%
  dplyr::mutate(
    zPerf   = as.numeric(scale(PerfTotal)),
    zComp   = as.numeric(scale(CompTotal)),
    zWP     = as.numeric(scale(WPTotal)),
    zAlign  = as.numeric(scale(VocabAlign)),
    zGen    = as.numeric(scale(VocabGen)),
    zVocab  = as.numeric(scale(VocabTotal)),
    zGMRT   = as.numeric(scale(GMRTTotal))
  )

df_long <- df_long %>%
  dplyr::mutate(
    zPerfLong = as.numeric(scale(PerfScore)),
```

```
    zVocab    = as.numeric(scale(VocabTotal)),
    zGMRT     = as.numeric(scale(GMRTTotal)),
    zAlign    = as.numeric(scale(VocabAlign)),
    zGen      = as.numeric(scale(VocabGen))
  )

# ===============================
# Regression Models (All standardized)
# ===============================

# RQ1: Main effects of Vocabulary (TOTAL) + GMRT predicting Total Performance
m1 <- stats::lm(zPerf ~ zVocab + zGMRT, data = df)

# RQ2: Vocabulary (TOTAL) × GMRT interaction predicting Total Performance
m2 <- stats::lm(zPerf ~ zVocab * zGMRT, data = df)

# RQ3: Task-type model (Computation vs Word) with Vocab (TOTAL) and GMRT
# Note: If you add a subject ID, consider lmer(zPerfLong ~ zVocab*TaskType + zGMRT + (1|ID))
m3 <- stats::lm(zPerfLong ~ zVocab * TaskType + zGMRT, data = df_long)

# RQ4 (final): Blocked model with disaggregated vocabulary
# Block 1 = Aligned + General; Block 2 = + GMRT
m4_b1 <- stats::lm(zPerf ~ zAlign + zGen, data = df)
m4_b2 <- stats::lm(zPerf ~ zAlign + zGen + zGMRT, data = df)
anova_m4 <- stats::anova(m4_b1, m4_b2)  # ΔR^2 test

# ===============================
# Save Model Summaries (plain text)
# ===============================
sink("outputs/models/RQ1_summary.txt");    print(summary(m1));    sink()
sink("outputs/models/RQ2_summary.txt");    print(summary(m2));    sink()
sink("outputs/models/RQ3_summary.txt");    print(summary(m3));    sink()
sink("outputs/models/RQ4_Block1_summary.txt"); print(summary(m4_b1)); sink()
sink("outputs/models/RQ4_Block2_summary.txt"); print(summary(m4_b2)); sink()
sink("outputs/models/RQ4_anova_block_compare.txt"); print(anova_m4); sink()

# Also write tidy coefficient tables
utils::write.csv(broom::tidy(m1, conf.int = TRUE),    "outputs/models/RQ1_coefficients.csv",
row.names = FALSE)
utils::write.csv(broom::tidy(m2, conf.int = TRUE),    "outputs/models/RQ2_coefficients.csv",
row.names = FALSE)
```

```
utils::write.csv(broom::tidy(m3, conf.int = TRUE),    "outputs/models/RQ3_coefficients.csv",
row.names = FALSE)
utils::write.csv(broom::tidy(m4_b1, conf.int = TRUE),
"outputs/models/RQ4_Block1_coefficients.csv", row.names = FALSE)
utils::write.csv(broom::tidy(m4_b2, conf.int = TRUE),
"outputs/models/RQ4_Block2_coefficients.csv", row.names = FALSE)

# Pretty parameter table for RQ4 Block 2
params_m4b2 <- parameters::model_parameters(m4_b2, ci = 0.95)
utils::write.csv(params_m4b2, "outputs/models/RQ4_Block2_parameters.csv", row.names =
FALSE)

# ================================
# Diagnostics (VIFs, residuals, plots)
# ================================

# Robust VIF writer: works for vector/matrix/list; recovers names when missing; manual fallback
vif_write <- function(model, label){
  out_path <- file.path("outputs/diagnostics", paste0(label, "_VIFs.csv"))

  compute_vif_manual <- function(model){
    mm <- stats::model.matrix(model)
    if (is.null(mm) || ncol(mm) < 2) return(NA)
    keep <- colnames(mm) != "(Intercept)"
    mm <- mm[, keep, drop = FALSE]
    if (ncol(mm) == 0) return(NA)
    p <- ncol(mm)
    vifs <- numeric(p); names(vifs) <- colnames(mm)
    for (j in seq_len(p)){
      y  <- mm[, j]
      Xj <- mm[, -j, drop = FALSE]
      r2 <- tryCatch({
        fit <- stats::lm.fit(x = Xj, y = y)
        1 - sum(fit$residuals^2) / sum((y - mean(y))^2)
      }, error = function(e) NA_real_)
      vifs[j] <- ifelse(is.na(r2) || r2 >= 0.999999, Inf, 1/(1 - r2))
    }
    vifs
  }

  v <- tryCatch(
```

```
   car::vif(model, type = "predictor"),
   error   = function(e) e,
   warning = function(w) suppressWarnings(car::vif(model, type = "predictor"))
  )

  write_out <- function(df_out){
   try(utils::write.csv(df_out, out_path, row.names = FALSE), silent = TRUE)
  }

  mm_terms <- colnames(stats::model.matrix(model))
  mm_terms <- mm_terms[mm_terms != "(Intercept)"]

  if (!inherits(v, "error") && is.matrix(v)) {
   out <- data.frame(
    Predictor = rownames(v),
    GVIF     = v[, "GVIF"],
    Df       = v[, "Df"],
    GVIF_adj  = v[, "GVIF^(1/(2*Df))"],
    row.names = NULL
   )
   write_out(out); return(invisible(NULL))
  }

  if (!inherits(v, "error") && is.numeric(v)) {
   terms <- names(v)
   if (is.null(terms) || length(terms) != length(v)) {
    terms <- if (length(mm_terms) == length(v)) mm_terms else paste0("Predictor_",
seq_along(v))
   }
   out <- data.frame(Predictor = terms, VIF = as.numeric(v), row.names = NULL)
   write_out(out); return(invisible(NULL))
  }

  if (!inherits(v, "error") && is.list(v) && all(c("GVIF","Df") %in% names(v))) {
   gv  <- as.numeric(v$GVIF)
   dfv <- as.numeric(v$Df)
   terms <- names(v$GVIF)
   if (is.null(terms) || length(terms) != length(gv)) {
    terms <- if (length(mm_terms) == length(gv)) mm_terms else paste0("Predictor_",
seq_along(gv))
   }
```

```
    adj <- if ("GVIF^(1/(2*Df))" %in% names(v)) as.numeric(v[["GVIF^(1/(2*Df))"]]) else
gv^(1/(2*dfv))
    L <- min(length(terms), length(gv), length(dfv), length(adj))
    out <- data.frame(
      Predictor = terms[seq_len(L)],
      GVIF      = gv[seq_len(L)],
      Df        = dfv[seq_len(L)],
      GVIF_adj  = adj[seq_len(L)],
      row.names = NULL
    )
    write_out(out); return(invisible(NULL))
  }

  v2 <- compute_vif_manual(model)
  if (all(is.na(v2))) {
    note <- data.frame(Note = "VIFs unavailable (model too simple or aliased).")
    write_out(note)
  } else {
    out <- data.frame(Predictor = names(v2), VIF = as.numeric(v2), row.names = NULL)
    write_out(out)
  }
  invisible(NULL)
}

resid_write <- function(model, label){
  resids <- data.frame(
    Fitted = fitted(model),
    Residuals = resid(model),
    StdResiduals = rstandard(model)
  )
  utils::write.csv(resids, file.path("outputs/diagnostics", paste0(label, "_Residuals.csv")),
row.names = FALSE)
}

save_png <- function(path, expr, width = 900, height = 700, res = 120) {
  png(path, width = width, height = height, res = res)
  on.exit(dev.off(), add = TRUE)
  force(expr)
}

diag_plots <- function(model, label){
```

```
  base <- file.path("outputs/diagnostics", label)
  vif_write(model, label)
  resid_write(model, label)
  save_png(paste0(base, "_Residuals_vs_Fitted.png"), plot(model, which = 1))
  save_png(paste0(base, "_QQPlot.png"),             plot(model, which = 2))
  save_png(paste0(base, "_ScaleLocation.png"),      plot(model, which = 3))
  save_png(paste0(base, "_Residuals_Hist.png"),
         hist(rstandard(model),
              main = paste0(label, ": Histogram of Standardized Residuals"),
              xlab = "Standardized Residuals"))
}

invisible(lapply(
  c("RQ1","RQ2","RQ3","RQ4_Block1","RQ4_Block2"),
  function(nm){
    message("Running diagnostics for: ", nm)
    model <- switch(nm, RQ1 = m1, RQ2 = m2, RQ3 = m3, RQ4_Block1 = m4_b1, RQ4_Block2 =
m4_b2)
    diag_plots(model, nm)
  }
))

# ===============================
# APA Snippets (RQ4 emphasis) — write to text file
# ===============================
apa_num <- function(x) ifelse(is.na(x), "—", format(round(x, 3), nsmall = 3))
pull_row <- function(tab, term_label) tab[match(term_label, tab$Parameter), , drop = FALSE]

pm <- as.data.frame(params_m4b2)
row_align <- pull_row(pm, "zAlign")
row_gen   <- pull_row(pm, "zGen")
row_gmrt  <- pull_row(pm, "zGMRT")

apa_lines <- c(
  "APA-style snippets (RQ4 Block 2)",
  sprintf("Aligned vocabulary (β = %s, SE = %s, 95%% CI [%s, %s], p = %s)",
        apa_num(row_align$Coefficient), apa_num(row_align$SE),
        apa_num(row_align$CI_low), apa_num(row_align$CI_high), apa_num(row_align$p)),
  sprintf("General vocabulary (β = %s, SE = %s, 95%% CI [%s, %s], p = %s)",
        apa_num(row_gen$Coefficient), apa_num(row_gen$SE),
        apa_num(row_gen$CI_low), apa_num(row_gen$CI_high), apa_num(row_gen$p)),
```

```
  sprintf("GMRT (β = %s, SE = %s, 95%% CI [%s, %s], p = %s)",
        apa_num(row_gmrt$Coefficient), apa_num(row_gmrt$SE),
        apa_num(row_gmrt$CI_low), apa_num(row_gmrt$CI_high), apa_num(row_gmrt$p)),
  sprintf("Adding GMRT explained ΔR² = %s beyond vocabulary; F-change = %s; p = %s",
        apa_num(summary(m4_b2)$r.squared - summary(m4_b1)$r.squared),
        apa_num(stats::anova(m4_b1, m4_b2)$`F`[2]),
        apa_num(stats::anova(m4_b1, m4_b2)$`Pr(>F)`[2]))
)
writeLines(apa_lines, "outputs/models/RQ4_APA_snippets.txt")

# ================================
# SPSS-ready export (4 key totals)
# ================================
writexl::write_xlsx(
  df[, c("PerfTotal", "VocabAlign", "VocabGen", "GMRTTotal")],
  "totals_for_spss.xlsx"
)

cat("\nDone. Outputs written to:\n- outputs/reliability\n- outputs/descriptives\n-
outputs/diagnostics\n- outputs/models\nand 'totals_for_spss.xlsx'\n")
```

SPSS Analysis Steps

Data Preparation

- Imported Excel file into SPSS
- Created total and subscale scores using Transform → Compute Variable
- Created centered versions using the same menu

Reliability

- Analyze → Scale → Reliability Analysis
- Model = Alpha for each set of items

Descriptives and Correlations

- Analyze → Descriptive Statistics → Descriptives
- Analyze → Correlate → Bivariate (Pearson)

Regression Models

- RQ1: Analyze → Regression → Linear (hierarchical blocks: GMRT then Vocabulary)
- RQ2: Analyze → Regression → Linear (Vocab, GMRT, and Vocab×GMRT interaction)
- RQ3: Analyze → General Linear Model → Repeated Measures (Task Type within-subject; Vocabulary and GMRT as covariates)
- RQ4: Regression → Linear (hierarchical blocks: VocabAlign & VocabGen then GMRTTotal)

Diagnostics
- Collinearity diagnostics (VIF): Analyze → Regression → Linear → Statistics → Collinearity diagnostics
- Residuals: Plots → ZPRED vs ZRESID; Save → Standardized residuals; Graphs → Legacy Dialogs → Q–Q

Notes on Replication
- Predictors were grand mean centered using constants derived from SPSS Descriptives.
- Results converged across SPSS and R, with only minor p-value differences attributable to software-specific estimation routines.
- Interaction effects were probed with simple slopes and plots using the interactions and ggeffects packages in R.

Appendix D

Model Diagnostics and Assumption Checks

This appendix presents diagnostic plots and statistics used to evaluate model assumptions for each of the regression analyses (RQ1–RQ4). The diagnostic checks included visual inspection of residuals versus fitted plots, Q–Q plots, and histograms of standardized residuals to assess linearity, normality, and homoscedasticity, as well as variance inflation factor (VIF) values to assess multicollinearity. These diagnostics are provided to support the validity and reproducibility of the reported analyses.

RQ1: Vocabulary and Reading Comprehension as Predictors

Figure D1. Residuals versus fitted values plot

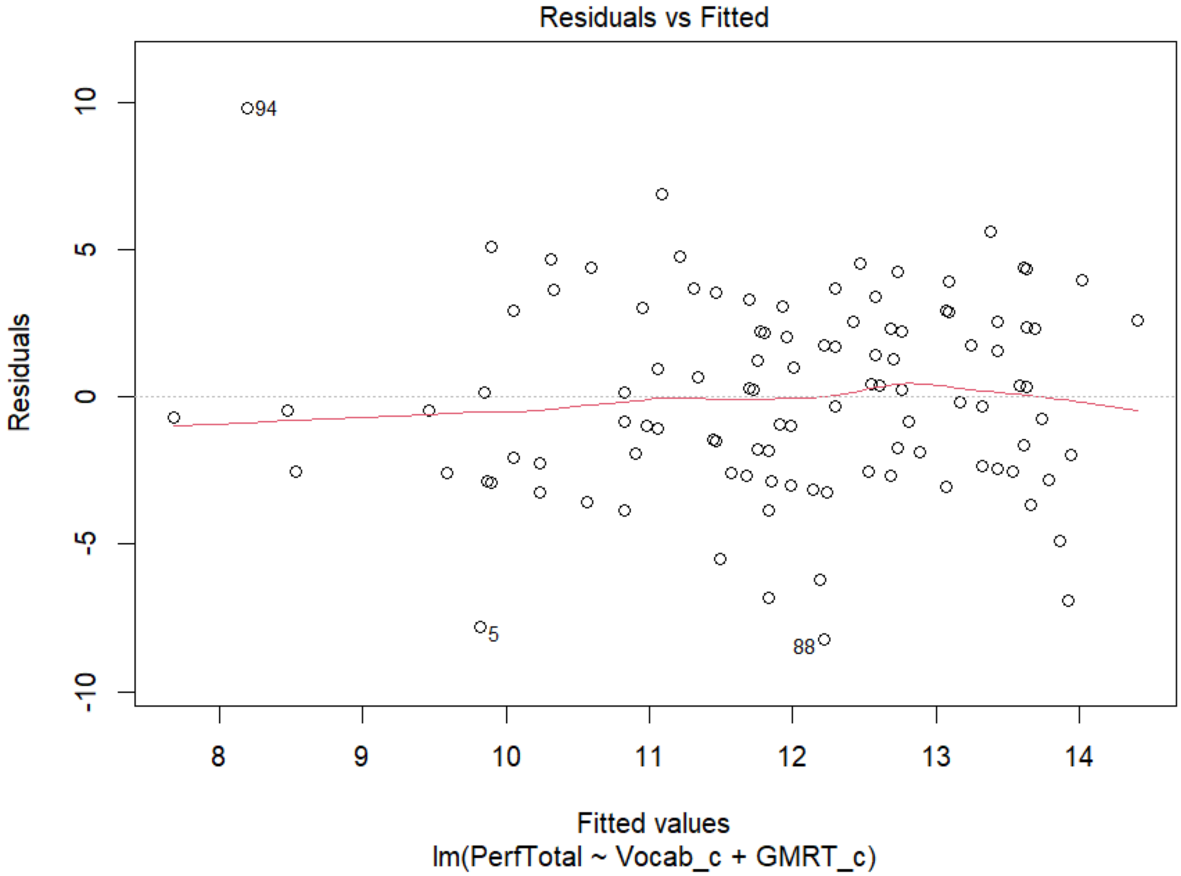


Figure D2. Q–Q plot of standardized residuals

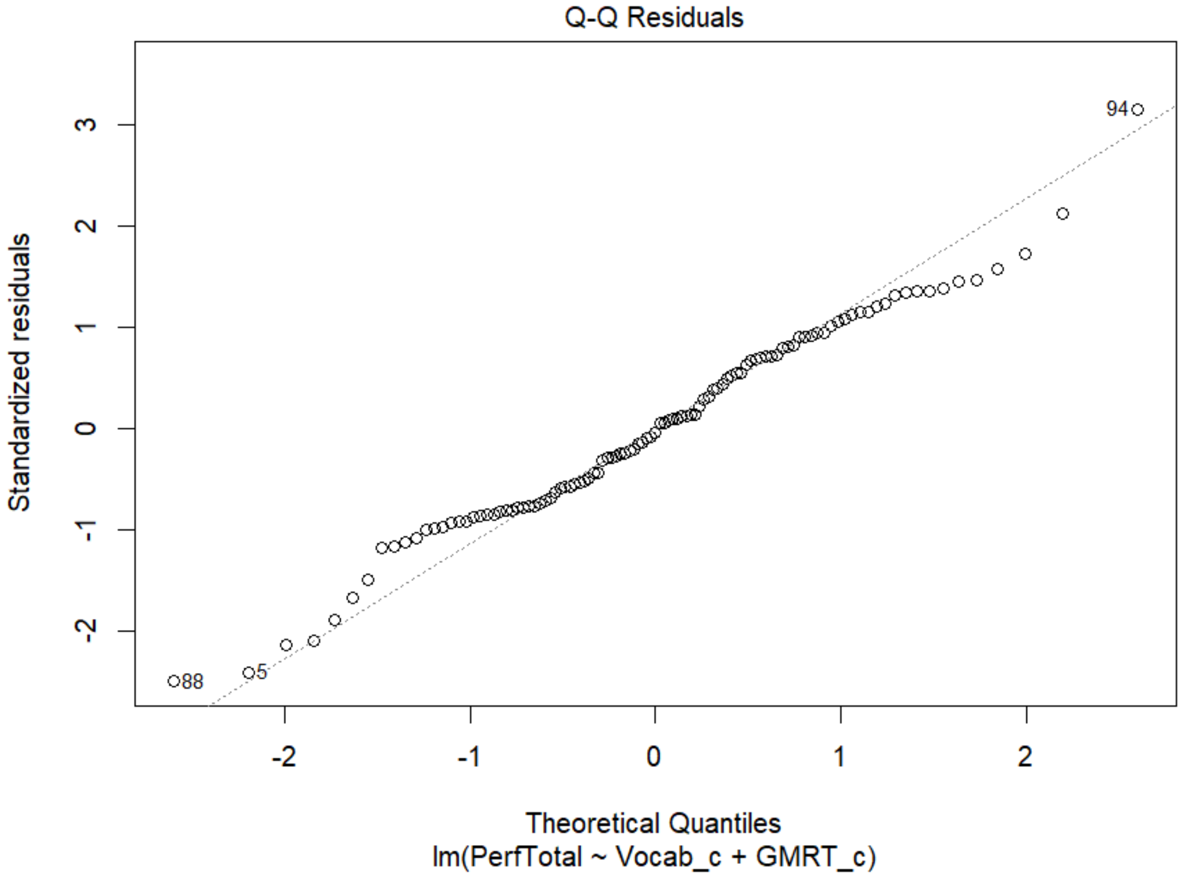


Figure D3. Histogram of standardized residuals

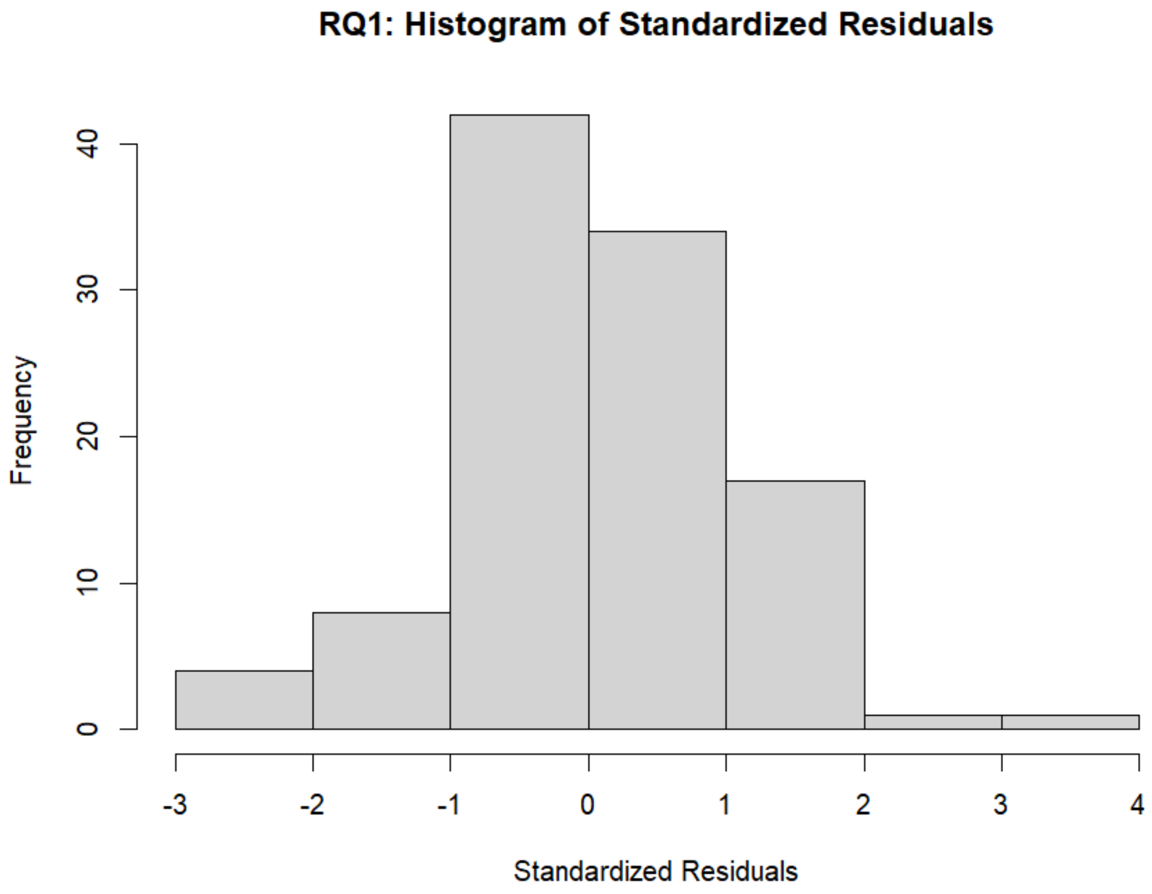


Table D1. Variance inflation factors (VIFs) for predictors

Predictor	VIF
Vocabulary (centered)	1.32
GMRT Reading Comprehension (centered)	1.32

Note. VIF = Variance Inflation Factor. Values below 5 indicate no problematic multicollinearity.

RQ2: Moderation by Reading Comprehension

Figure D4. Residuals versus fitted values plot

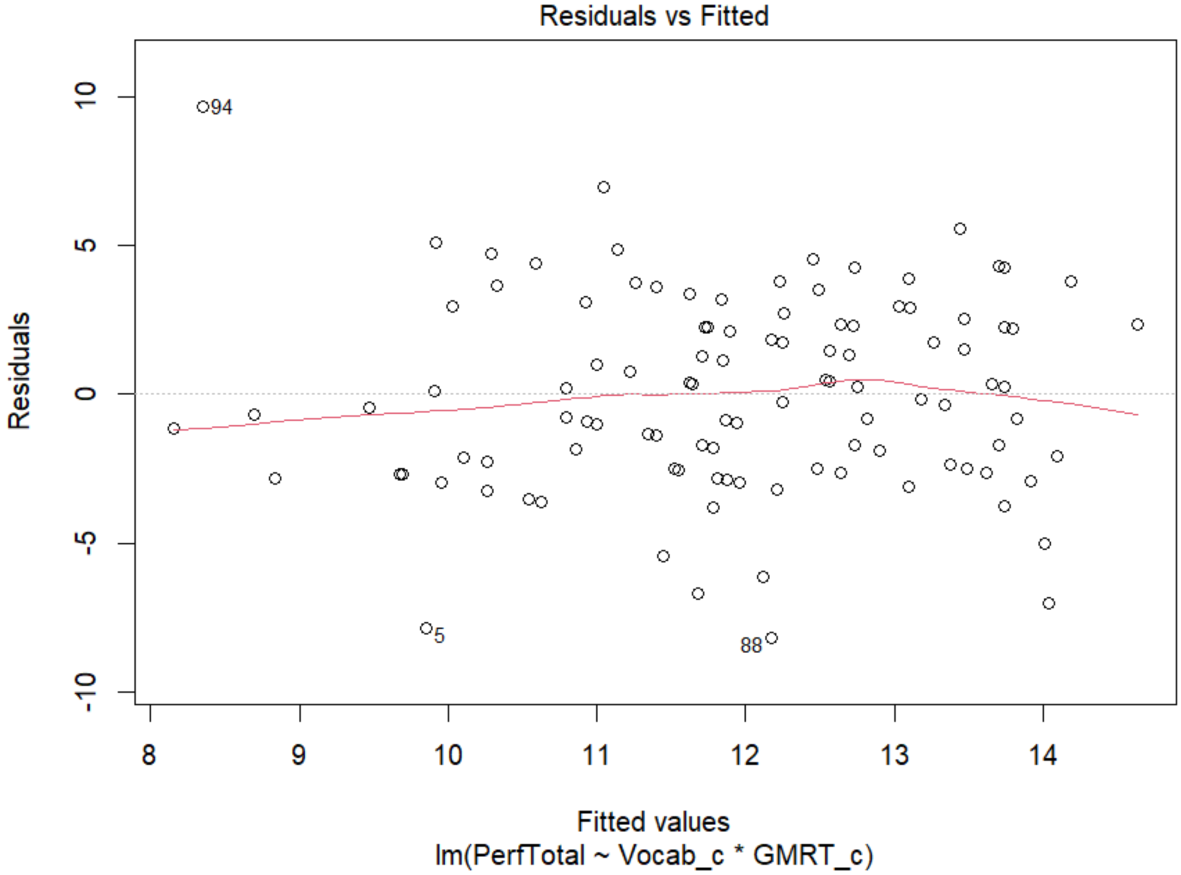


Figure D5. Q–Q plot of standardized residuals

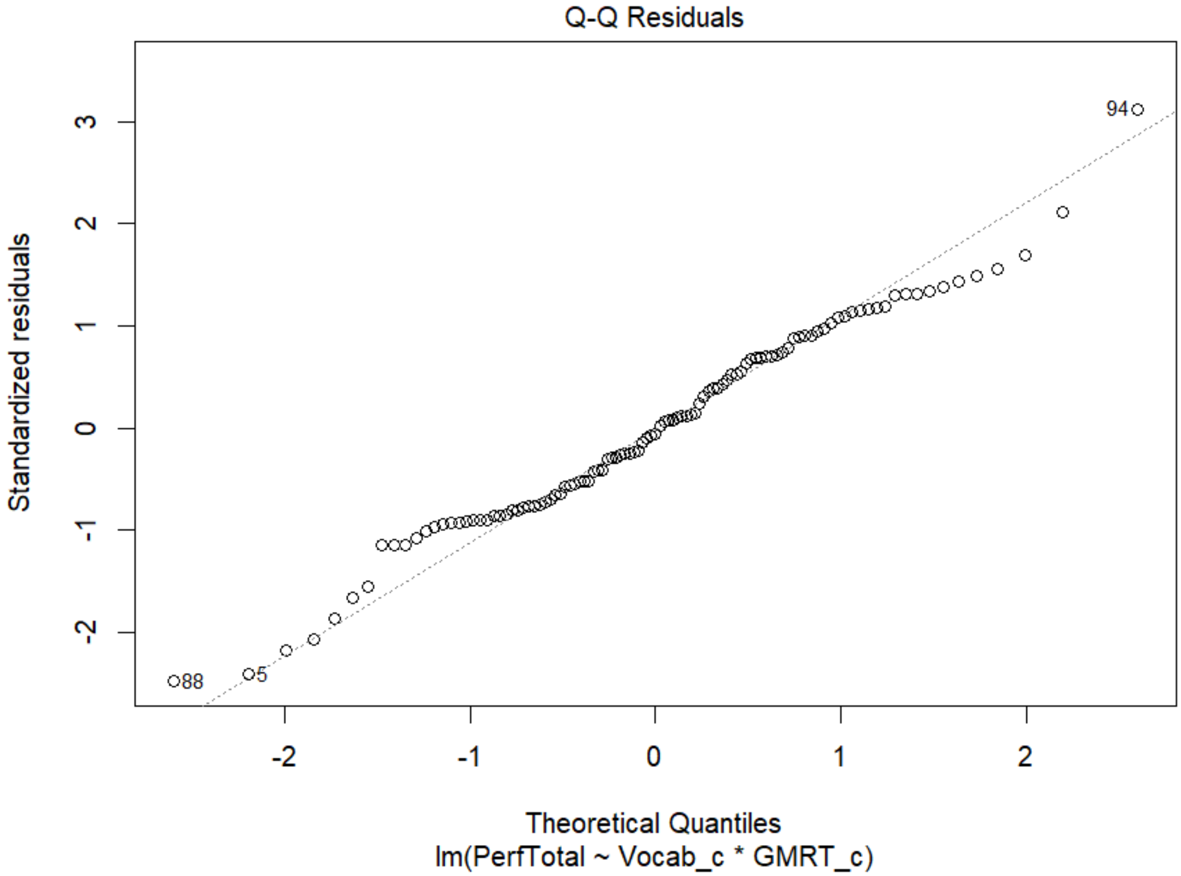


Figure D6. Histogram of standardized residuals

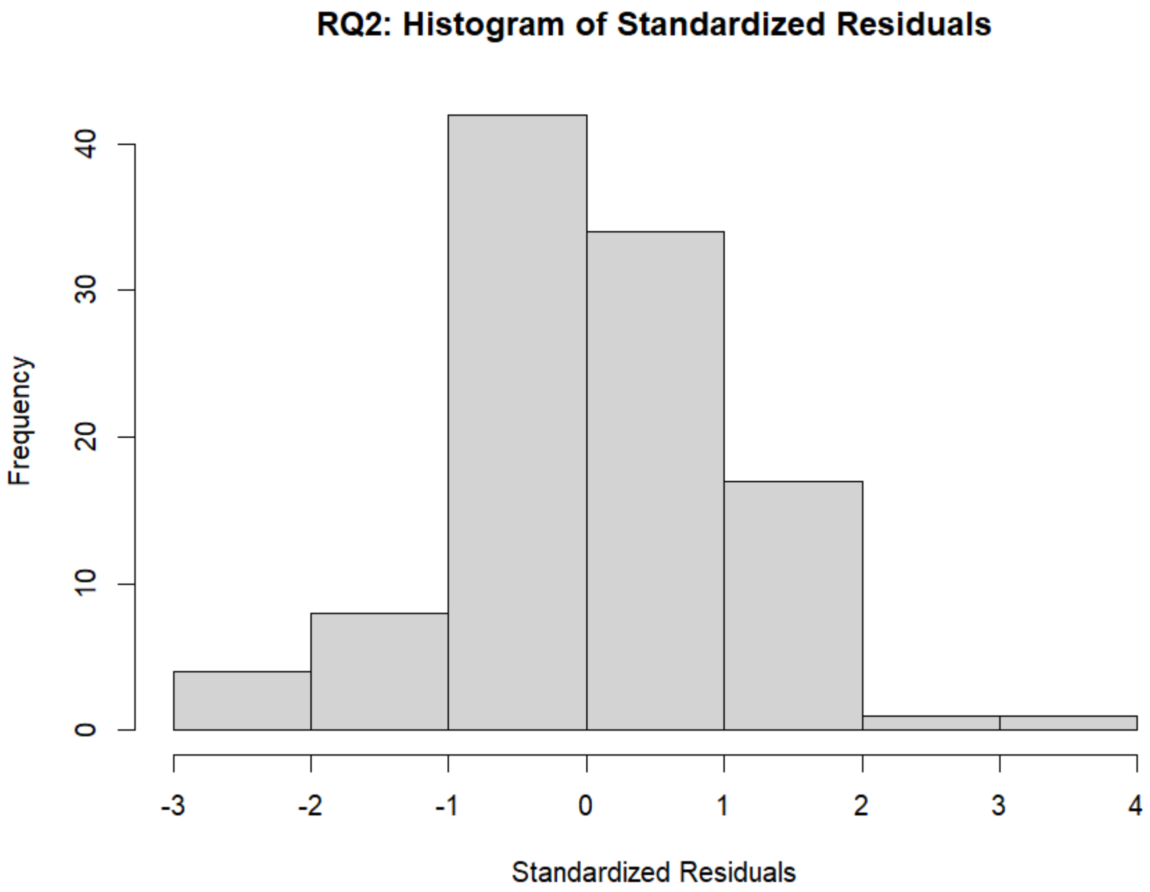


Table D2. Variance inflation factors (VIFs) for predictors

Predictor	VIF
Vocabulary (centered)	1.56
GMRT Reading Comprehension (centered)	1.32
Vocabulary × GMRT	1.22

Note. VIF = Variance Inflation Factor. Values below 5 indicate no problematic multicollinearity.

RQ3: Moderation by Task Type (Repeated Measures)

Figure D7. Residuals versus fitted values plot

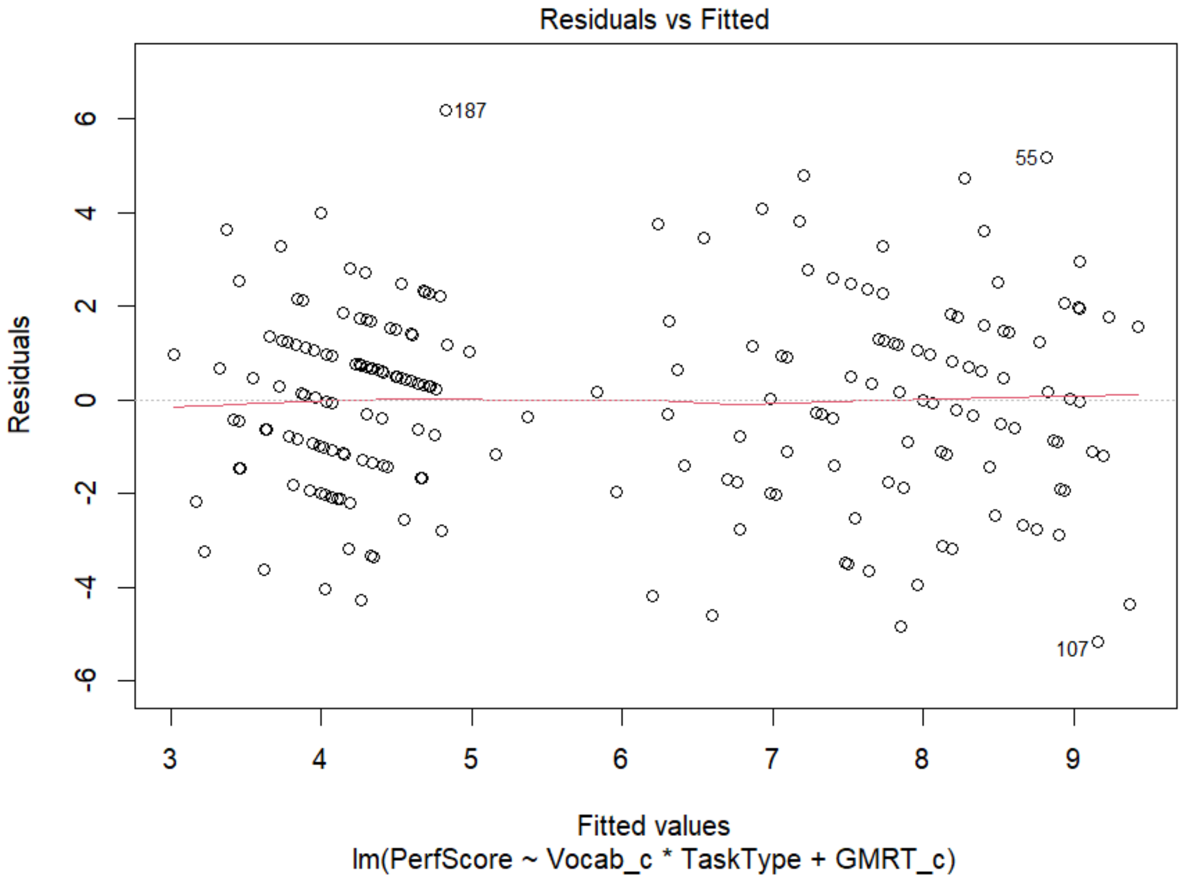


Figure D8. Q–Q plot of standardized residuals

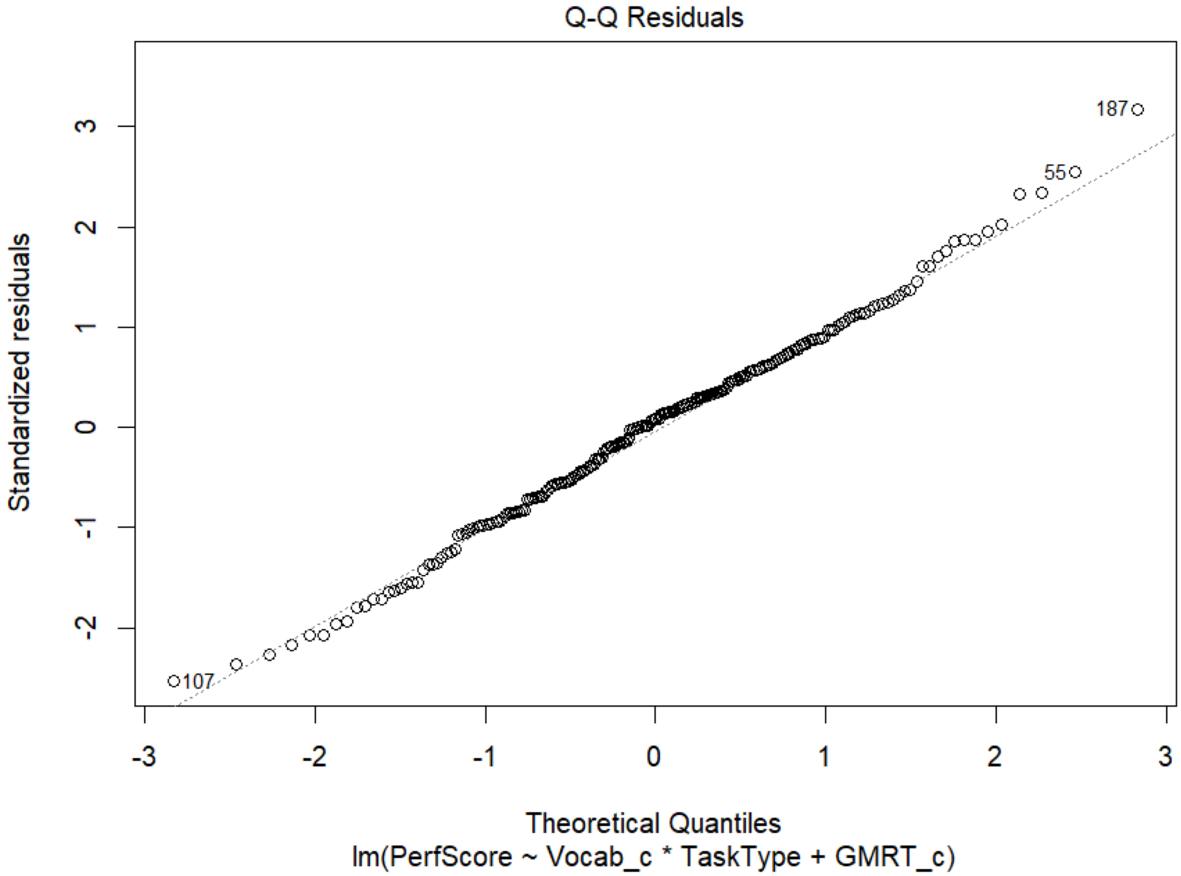


Figure D9. Histogram of standardized residuals

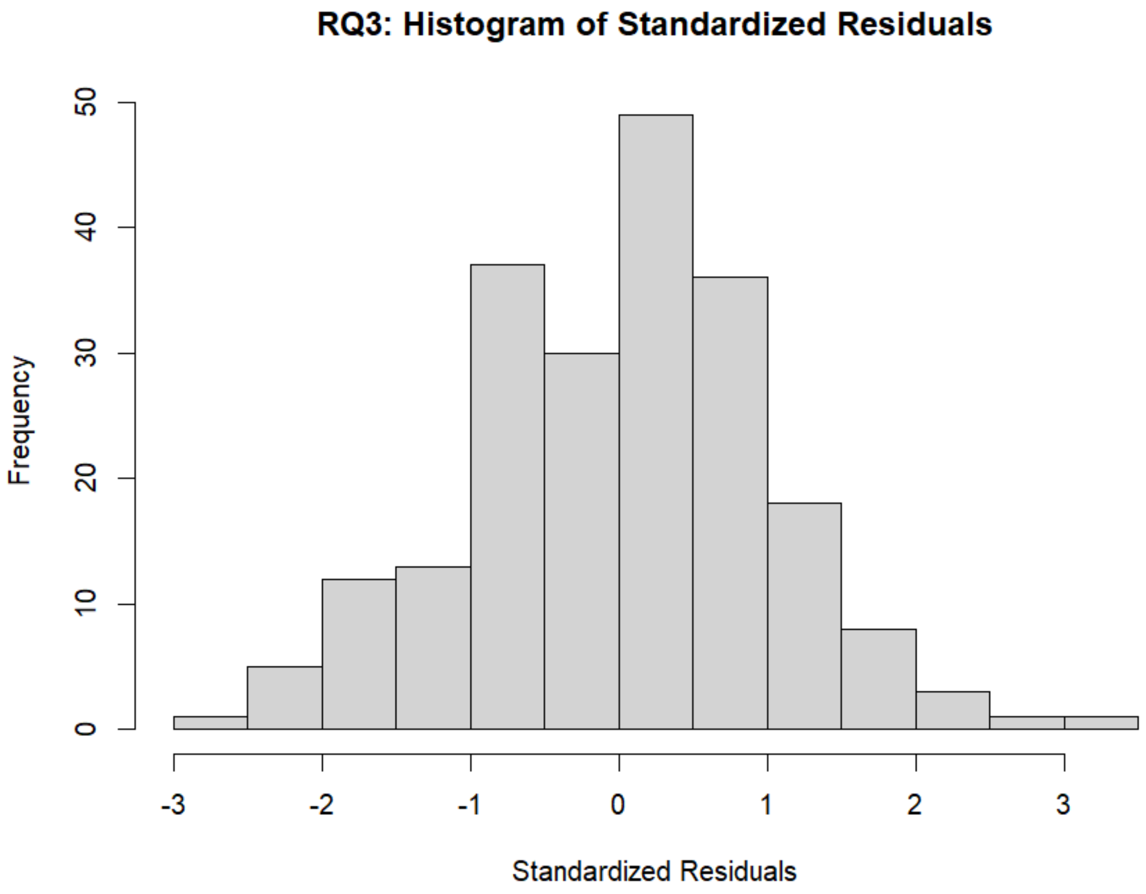


Table D3. Variance inflation factors (VIFs) for predictors

Predictor	VIF
Vocabulary (centered)	2.32
GMRT Reading Comprehension (centered)	1.32
Task Type	1.00
Vocabulary × Task Type	2.00

Note. VIF = Variance Inflation Factor. Values below 5 indicate no problematic multicollinearity.

RQ4: Moderation by Vocabulary Alignment Ratio

Figure D10. Residuals versus fitted values plot

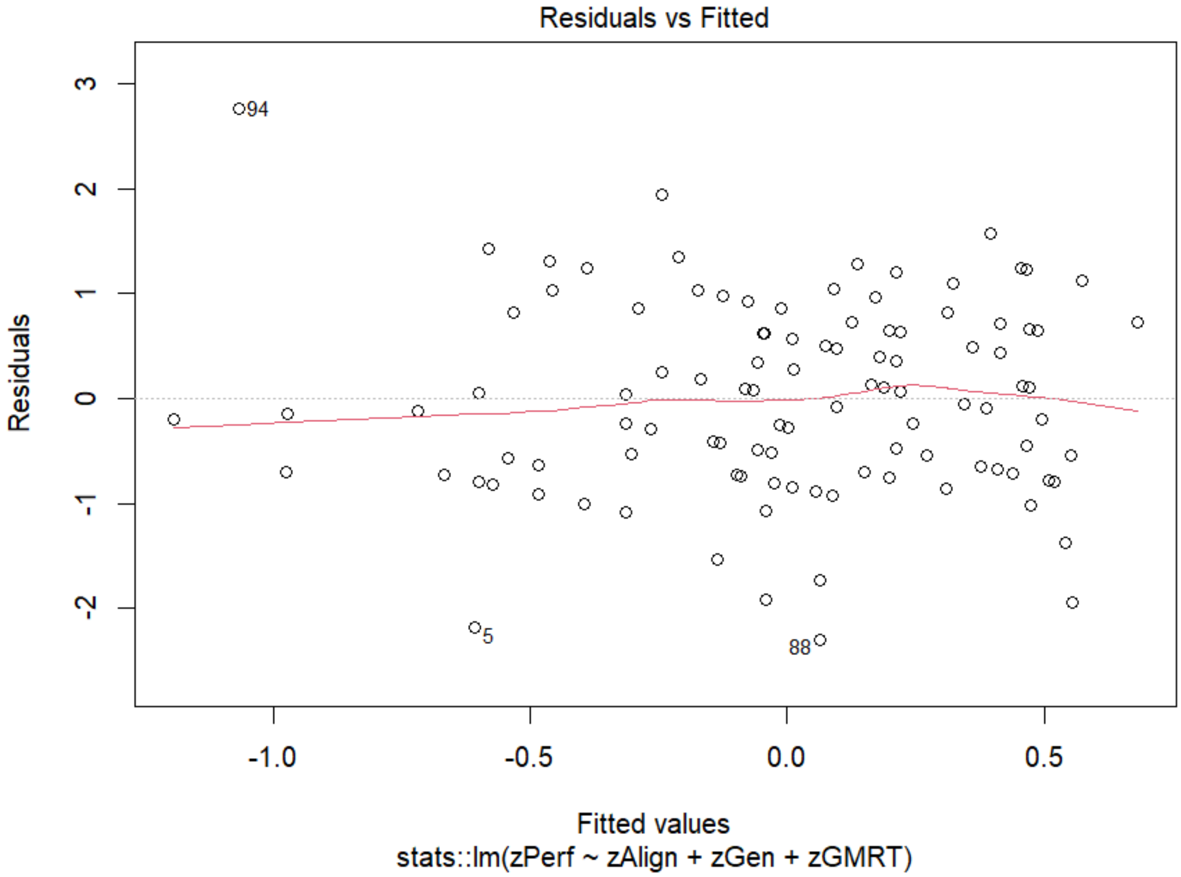


Figure D11. Q–Q plot of standardized residuals

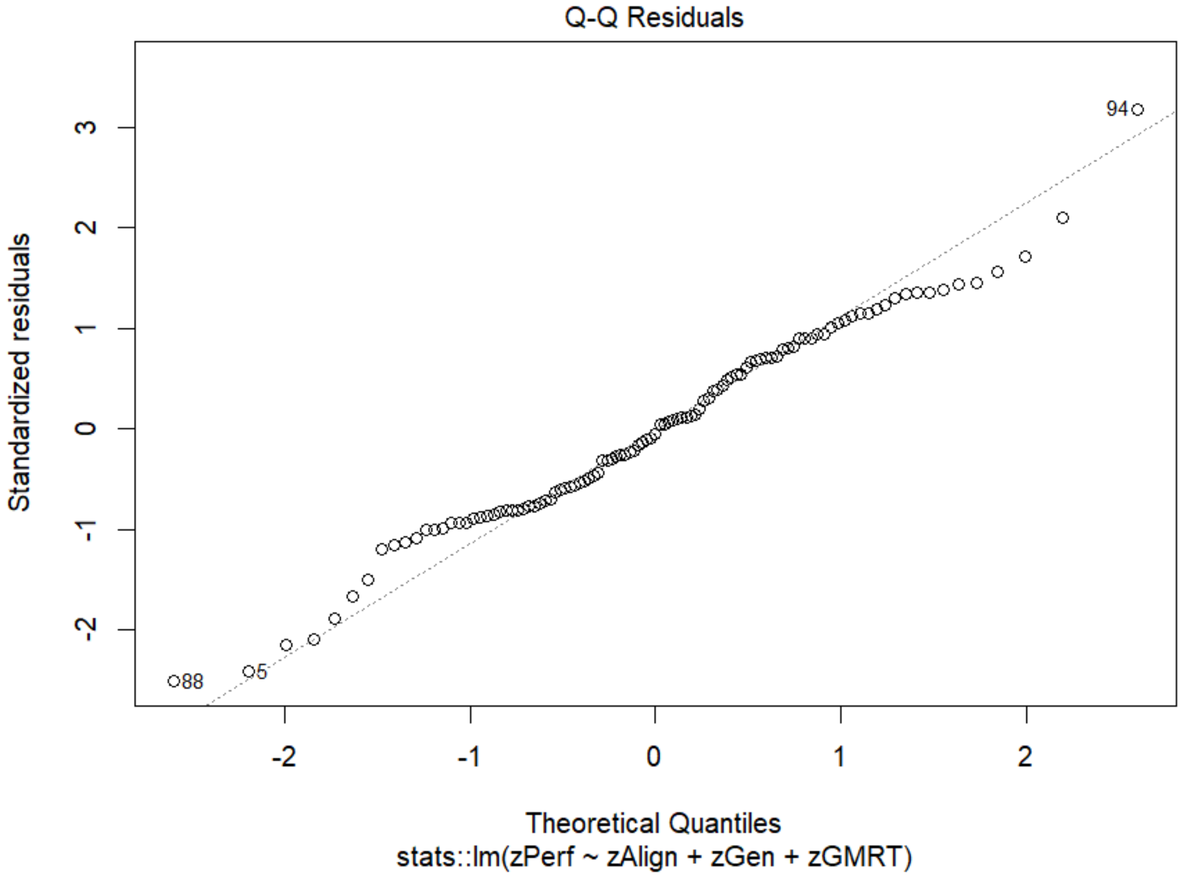


Figure D12. Histogram of standardized residuals

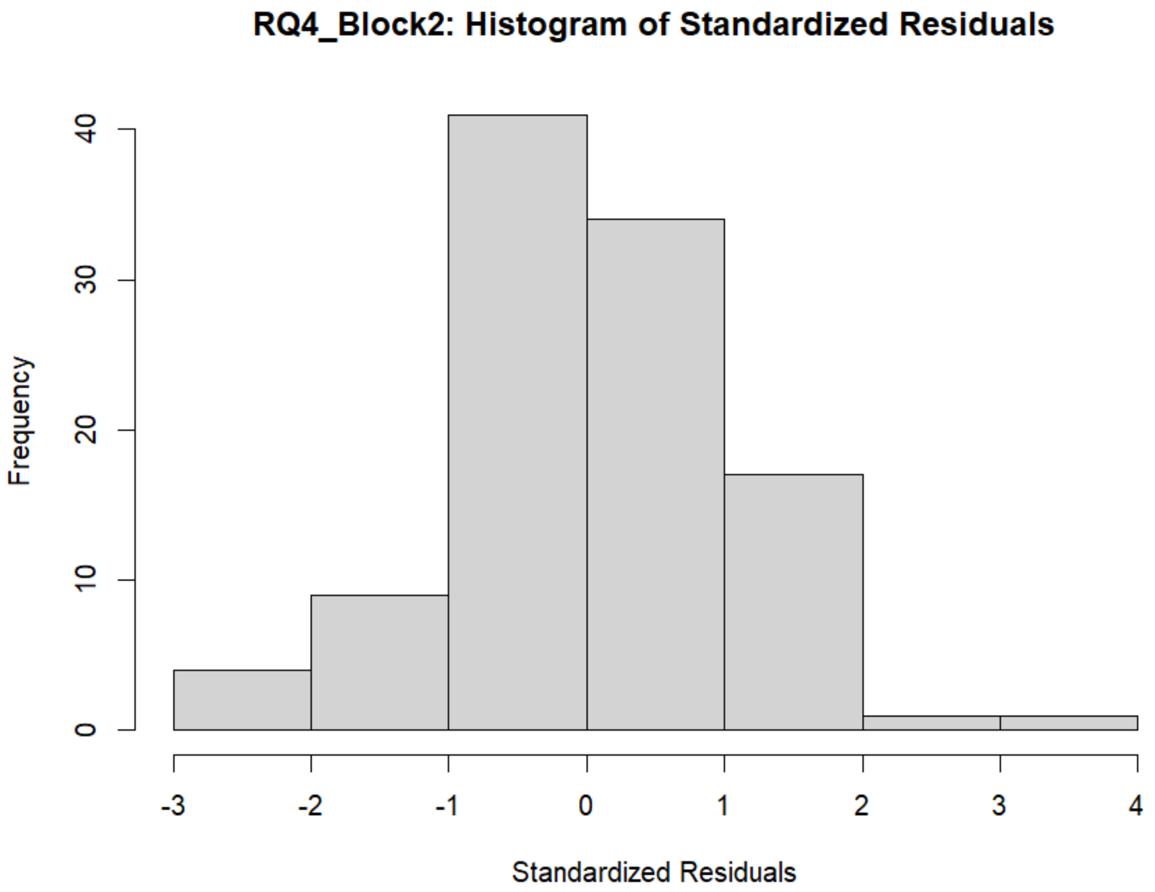


Table D4. Variance inflation factors (VIFs) for predictors

Predictor	VIF
Aligned Vocabulary	1.36
General Mathematics Vocabulary	1.37
GMRT Reading Comprehension	1.32

Note. VIF = Variance Inflation Factor. Values below 5 indicate no problematic multicollinearity.