Use of Dynamic Technology Tools in Statistics Education

By

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ABSTRACT

This paper examines the role of technology tools in statistics education over the past decades as well as provides recommendations for incorporating such tools into instruction. At this time, much software has been developed for the learning of statistics. However, students remain struggling to understand major statistical ideas such as center, shape, variation, sample size, sample, sampling distribution, and inference. Technology tools have numerous advantages over static demonstrations from textbooks or worksheets to assist with learning these major ideas.

Incorporating evidence from literature, this thesis demonstrates that technology tools serve as a crucial element for both students and teachers in teaching and learning statistical concepts. It argues that the use of dragging and measuring in dynamic statistical environments aid students in exploratory data analysis which increases their use of statistical reasoning. The same holds true for students when using dynamic statistical applets to view the processes of resampling from sample data to aid with statistical inference.

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Introduction

There has been a strong emphasis on science, technology, engineering, and mathematics (STEM) at all levels of education due to the growing sector of STEM jobs (Graduate STEM Education for the 21st Century, 2018). Since "nearly 8.6 million Americans were employed in STEM jobs in 2015, 93 percent of which paid better than the average national wage" (Graduate STEM Education for the 21st Century, 2018, p. 15) there is a great need to support the continued growth of this field. The growth of master's and doctoral degrees awarded in science and engineering grew 88% from 2000 to 2015 and, in particular, mathematics and statistics grew the most within this field at 151% (Graduate STEM Education for the 21st Century, 2018). Gibbs (2018) discussed the surge in the popularity of statistics jobs and how at the University of Toronto "undergraduate enrolment in statistics programs of study has outpaced growth, with an almost 7-fold increase" (p. 1). She goes on to state:

> The rise of data science has shown us that we need to be prepared to respond to the continuing evolution of applications, technology, and computational algorithms in a much more agile way than we have needed to before. Similarly, we must train our students to think beyond what they have already mastered, and to learn how to learn, so that they graduate from our programs with the agility to adjust to changing technologies and data, and the evolving nature of statistics and data science roles. (Gibbs, 2018, p. 1)

Carver et al. (2016) state nine major goals within the guidelines for assessment and instruction in statistics education report and, within goal number eight, state that "students should be given numerous opportunities to analyze data with the best available technology (preferably, statistical software)" (p. 11). It is also suggested that teachers "Use technology and show students how to use technology effectively to manage data, explore and visualize data, perform inference, and check conditions that underlie inference procedures" (Carver et al., 2016, p.16).

As an educator of both mathematics and statistics, I feel moved to provide and improve such opportunities for my current and future students. As such, my purpose in this thesis is to do a literature review not only on the impact of dynamic tools for statistics education but also on connections between the uses of dynamic technology within both mathematics and statistics education. Specifically, I will look at how dynamic technology has been used within a mathematics classroom, examine the differences between the content of mathematics and statistics, consider best teaching practices for statistics educators, synthesize the multiple frameworks provided to teach statistics to create a single list of recommendations, explore the variety of tools that have been used in the past to teach statistics, and then finally broadly explore the impact such tools have had on students learning statistics.

Use of Dynamic Technology in the Mathematics Classroom

The first exploration in this thesis is to uncover how dynamic technology has impacted the mathematics classroom since the invention and introduction of the internet, personal computing, and educational software in modern society. Kaput's (1992) article *Technology and Mathematics Education* has been pervasive throughout the work of mathematics education scholars and the article has an impressive 1429 citations at the time of this literature review. Although today's technology has eclipsed the capabilities of Kaput's era, his approach to using technology has stood the test of time (Roschelle et al., 2017). Roschelle et al. (2017) highlight some of the major calls to action by Kaput below:

1. Shifting from static media to dynamic media

2. Using “dynamic linking” to help students connect notation to models which also highlights mathematical structures and hides less important detail
3. Using calculators to offload routine calculations so that students can focus more on deeper learning goals
4. Using dynamic representations as a form of social participation, where students in a classroom could contribute to a dynamically evolving mathematical representation

Bearing these in mind, much of the research done on the impact of dynamic mathematics tools has shown a positive influence on the learning outcomes of students (Aventi, 2014; Bhatti et al., 2017; Hollebrands, 2007; Jaing et al., 2015; Roschelle et al., 2010; Sinclair, et al., 2009; Yeo & Webel, 2022). Specifically, technology’s allowances for teachers and researchers to shift from static to dynamic media has been studied and shown to provide specific benefits in student’s mathematical habits and understandings (Bhatti et al., 2017; Hollebrands, 2007; Jaing et al., 2015; Roschelle et al., 2010; Sinclair, et al., 2009). Sinclair et al. (2009) specifically noted how students were more capable of using “narrative thinking” while utilizing dynagraphs to describe types of functions and proposed “that the dynagraphs invited attention to the mathematical behavior of functions, in contrast to their properties or definitions” (p. 450). Narrative thinking emerges when students create accounts of the forms an object takes over time and the mathematical benefits of such thinking are that students may see the behaviors of such objects in contrast to the properties or definitions given by a book or instructor (Sinclair et al., 2009). “Dynamic linking” is a key aspect of dynamic technology tools that leads students towards thinking narratively. Hollebrands’ (2007) study involved students dragging and measuring geometric shapes within dynamic geometry software to discover properties and theorems. She discovered that students would behave either reactively or proactively with the tool based on how the teacher set up a mathematical task (Hollebrands, 2007). The tools' ability to quickly allow students to perform an action, view the results, and then perform another action aided in thinking narratively about the theorems and properties.

Researchers have studied the impact of tools such as SimCalc, Geogebra, and Geometer's Sketchpad (GSP) all of which have been developed to have characteristics that align with Kaput's calls to action (Aventi, 2014; Bhatti et al., 2017; Yeo & Webel, 2022). These studies found among students at various grade levels that using such tools creates an increase in participation, deepens learning, helps students grow in visualization techniques, improves their problem-solving and reasoning skills, and helps advance their mathematical language and responses (Aventi, 2014; Bhatti et al., 2017; Yeo and Webel, 2022). Despite these enhancements provided using tools, several studies emphasize the fact that the teacher's ability to provide quality tasks and instruction alongside the tool is what helps create the impact on student growth (Hollebrands, 2007; Jiang et al., 2015; Roschelle et al., 2010).

Specific Affordances of Dynamic Technology in the Geometry Classroom

The advent of dynamic geometry software is argued to have saved the geometry curriculum from being taken out of mathematics standards in many countries (de Villiers, 1996). For many years, geometry was taught in a static environment using compasses, rulers, and protractors which can be a difficult environment for students to think abstractly. Dynamic Geometry Environments (DGE) allow students to "move back and forth between the abstract theoretical field, and the perceptual graphical-spatial field" (Smith, 2010). The most welcome amenity of DGE is that students can engage in the art of problem posing and can take time to explore, conjecture, refute, reformulate, and explain models that they create or preconstructed sketches created by the instructor (de Villiers, 1996; Smith, 2010). Hadas et al. (2000) state in their study "the students ceased to be recipients of formal proofs, but were engaged in an activity of construction and evaluation of arguments" (p. 149). I would like to highlight the specific qualities that DGE affords to students when engaging in geometric discovery as outlined by Smith (2010):

1. Students can interact and use dragging and measuring tools to justify conjectures
2. Students can construct original sketches to solve a problem
3. Students can interact with preconstructed sketches to solve a problem

The following subsections explore the benefits of the listed features by reviewing the literature on how they enhance students' learning of geometry.

Dragging

Research on dragging items within a DGE has been linked to increased cognitive processes among students who use such tools in a geometry task (Arzarello et al., 2002; Baccaglini-Frank, 2010; Erbas & Yenmez, 2011, Yao, 2020). Smith (2010) defines seven types of dragging listed below:

1. Wandering dragging
2. Bound dragging
3. Guided dragging

4. Dummy locus dragging
5. Line dragging
6. Linked dragging
7. Dragging test (p.16)

When students use these different types of dragging, they can create a variety of mental models for thinking about geometrical figures and use them to achieve different goals (Erbas & Yenmez, 2011). The dragging actions allow students to view the geometry through a lens of variation and engages them in the act of seeking rather than stating (Leung, 2008). Students are able to learn geometric properties by experiencing what remains invariant when dragging a geometric construction and, therefore, dragging is a strong tool for inquiry (Saldano et al., 2019)

Measuring

The next affordance of DGEs is that it gives students the ability to measure constructed items such as angles, lengths, distances, perimeters, and areas. This is typically used in conjunction with the dragging tools to see changes in a geometric figure. Research has found that measurement within a dynamic geometry environment plays a key role in students conjecturing and constructing a formal proof (Erbas & Yenmez, 2011; Olivero & Robutti, 2007; Oner, 2013; Yao, 2020). Smith (2010) and Yao (2020) identify five classifications of measurement:

1. Wandering measuring
2. Guided measuring
3. Perceptual measuring
4. Validation measuring

5. Proof measuring (p. 19)

Students will use a combination of the measuring classifications while creating their own sketches or even interacting with preconstructed sketches. The exploratory tasks in a DGE will mostly involve the application of wandering, guided, and perception measuring, and the preconstructed tasks (made by the teacher) will mostly require students to apply validation and proof-measuring techniques. Students use these modalities to transfer between graphical and theoretical fields of thinking (Smith, 2010). Olivero and Robutti (2007) defined in their study how to notice students shifting between the two fields of thinking:

> If students are exploring the problem without having a clear idea in mind, their use of measurements is not directed towards a particular property; if at a certain point they see a regularity, or invariant, on the figure, it means that the measurements they are looking at have helped them to connect the graphical observation of the figure to a part of the theory: they may then be able to formulate a conjecture (p. 153).

It should be the ambition of geometry teachers to develop students into explorers of geometric objects that create conjectures, validate them, and construct formal proof (Leikin, 2004). Students' use of the variety of dragging and measurement techniques within a DGE serves them by helping them transfer between graphical and theoretical fields (Olivero & Robutti, 2007; Smith, 2010).

Creating Original Sketches and Interactions with Pre-Constructed Sketches

Exploring a geometric object is an important part of learning geometry. Whether on paper and pencil or a DGE, students must encounter specific scenarios when problem-solving. Part of the exploration process could involve students making their geometric sketches and discovering properties through dragging and measuring. Students in this type of setting have two trends of behavior: reactive or proactive; this all depends on how well the teacher sets up the task (Hollebrands, 2007). Too much freedom for students may lead them to "draw" figures instead of "construct" them properly (Smith, 2010). Thus, if teachers wish to give students that much freedom, students must be equipped with the necessary knowledge of the technology tool as well as the underlying geometric properties (Smith, 2010, Yao, 2020).

There seem to be two construction themes that are used in the literature as it relates to students creating their geometric objects: Open-ended (student-centered) and step-by-step (teacher-centered). First, Yao (2020) studied students' use of GSP to explore geometric ideas which led to significant evidence and states:

> I selected the tasks that demanded participants to construct geometric entities with GSP tools so they could move beyond using the dragging and measuring features…the tasks were open-ended to allow for multiple entry points and problem-solving approaches. These criteria for task selection were chosen to ensure that the participants engaged in the process of discovering new geometric knowledge and creating geometric objects by using digital tools (p. 4).

He found that designing tasks in this way helped students gain new math knowledge and new relevant ways of using the technology. Second, Baccaglini-Frank (2010) studied students working in steps to create a geometric construction and found that students in such an environment were able to be guided to reason about the relationships they perceived in their constructed object(s). Furthermore, she found that activities such as this could help prepare them for more open-ended explorations that require more flexibility using definitions and representations (Baccaglini-Frank, 2010). She had participants interact with the technology tools in a step-by-step process implying that the users were more likely to be geometry novices as opposed to Yao (2020) who had a more open-ended structure implying a more experienced geometry student. Both studies are different in this way, yet they yielded positive results when learning geometry concepts.

Much of the research, however, study the positive impact when students interact with preconstructed sketches (Erbas & Yenmez, 2011; Olivero & Robutti, 2007; Oner, 2013; Poon & Wong, 2017; Smith, 2010). Smith (2010) states two practical reasons for using preconstructed sketches:

1. All students are working and learning from the same diagram.
2. Teachers can limit the features of the software to keep students from going astray.

Poon and Wong (2017) developed, implemented, and studied student outcomes by using a set of pre-constructed dynamic geometry sketches to teach students geometry. They found that students were more motivated and stimulated to change their way of thinking as well as saw improvements in their abilities to visualize concepts (Poon & Wong, 2017). Oner (2013) studied the impact of students working collaboratively to learn geometric ideas and gave them a task in which they were investigating the properties of a given figure. The preconstructed graphs afforded students a common starting point for discussion and helped them begin their exploration and discussions immediately. Lastly, Erbas and Yenmez (2011) were comparing the use of GSP and traditional paper-and-pencil methods for learning geometry and found significant evidence that exploring preconstructed dynamic sketches provided superior learning over the traditional paper-and-pencil static worksheets.

Considering all the affordances of dynamic technology tools, they certainly have a place in mathematics and geometry classrooms by providing a means of interacting with geometric objects, exploring their properties, and gaining a deeper understanding. These themes will be referenced later as we explore dynamic statistical tools but for now, we will begin by comparing both mathematics and statistics.

Mathematics vs. Statistics

Having discussed the impact that dynamic technology has on the mathematics and geometry classroom, how does this translate to the statistics classroom? To understand these differences, I will now discuss how researchers define the key aspects of both mathematical and statistical reasoning and thinking, explain how context is the major aspect differentiating the two, and then list the main recommendations and considerations for teaching statistics. Finally, we will look at frameworks for teaching statistics and how the use of technology stands out within the frameworks for teaching statistics.

Two Types of Thinking

Statistics is a field of applied mathematics, and the underlying mathematics of fractions, decimals, and algebraic formulas are foundational (Ben-Zvi & Garfield 2004). Many fields rely on these ideas such as physics, economics, etc. However, model abstraction is a key connection between the disciplines of mathematics and statistics (delMas, 2004). Moore and Cobb (2000) explain that “statistics is distinguished from ‘mere computing’ by its extensive use of mathematical models” (p. 623). The mathematical thinking behind probability theory is where the marriage between mathematics and statistics happens. Regarding introducing formal inference in statistics, delMas (2004) explains that “Mathematics provides knowledge about the expected probability distribution of observed sample effects when there is no effect in the population. Statistics adds a probabilistic determination for the cutoff point that establishes when a probability is sufficiently low” (p. 88). In addition to the algebra and arithmetic learned from a math classroom, statistics takes the mathematical properties of probability and applies them to inference models. This connection, as well as the concept of statistical inference, is challenging for students to grasp and is generally flawed (Harradine et al., 2011). Mathematics and statistics, although related through these concepts, have more differences than similarities when it comes to applying statistical knowledge. delMas (2004) offers this succinct comment:

> A practicing statistician may use mathematics to assist in solving a statistical problem, but only after a considerable amount of work has been done to identify the question under investigation, explore data for both patterns and exceptions, produce a suitable design for data collection, and select an appropriate model for data analysis. (p. 84-85)

Perhaps one of the strongest summations defining the types of thinking required for both mathematics and statistics is that mathematics more regularly uses deterministic thinking whereas statistics typically employs probabilistic thinking (Scheaffer, 2006). Scheaffer (2006) defines both types of thinking explaining that deterministic thinking is described as “every result must have an explainable cause” (p. 310) and probabilistic is described as ”a result may be due to many unexplainable conjoined factors” (p. 310). Much of the mathematics taught in schools typically leaves students ingrained in deterministic ways of thinking about the world around them (Scheaffer, 2006). This idea of probabilistic thinking can be quite foreign and aggravating to students newly enrolled in a proper statistics course.

To complement the summarization just given, another way to phrase this key difference between mathematical and statistical thinking is that mathematics uses deductive reasoning and statistics uses inductive reasoning (Gattuso & Ottaviani, 2011; Rossman et al., 2013). Rossman et al. (2013) succinctly describe this difference:

> Mathematics involves rigorous deductive reasoning, proving results that follow logically from axioms and definitions. The quality of a solution is determined by its correctness and succinctness, and there is often an irrefutable correct answer. In contrast, statistics involves inductive reasoning and uncertain conclusions. Statisticians often come to different but reasonable conclusions when analyzing the same data. In fact, within these types of judgments lies the art of data analysis. All statistical inference requires one to use inductive reasoning, as informed inferences are made from observed results to defensible, but ultimately uncertain, conclusions. (p. 8-9)

With these perspectives now in our mind, I now discuss considerations that educators of statistics should be mindful of when instructing.

Recommendations to Statistics Educators

It is now constructive to delve into the common recommendations and advice for statistics educators, new and old, to consider and apply to their practice. Above all else, the teacher's knowledge of content and pedagogy as well as their disposition towards their subject will have the biggest impact on their classroom's ability to learn (Muijs & Reynolds, 2002). Regarding teaching statistics, instructors need to relay to their students the relevance of statistics, make clear the differences between statistics and mathematics, make statistical reasoning an explicit goal of instruction, present real data in all instruction and assignments, and continue to deepen their understanding of statistics and statistical pedagogy through professional development (Cobb & Moore, 1997; delMas, 2004; Hand, 2009; Lovett & Lee, 2017; Rossman et al., 2013; Scheaffer, 2006). These considerations aim to strengthen instruction, assessment, and the demeanor of educators as well as improve learner outcomes. In the sections to come, I will synthesize what the literature says about the recommendations above.

Relevance of Statistics

"When am I ever going to use this?" In my personal experience, this remark is commonly blurted out within the walls of a math classroom. Many mathematics teachers will boast about the practical application of mathematics standards, but as students progress through the curriculum this becomes less and less true for the average person. Fortunately for teachers of statistics, the application of statistical thinking and reasoning plays a central role in countless disciplines outside of physics, engineering, computer science, etc. Hand (2009) takes this idea of application and extends it by saying that "the statistics course is in fact probably the single course which will have the most influence on their careers" (p. 301). He continues by saying "as their careers progress, they will find themselves moving back from the front line into managerial roles … they will work their way up the chain, dealing with increasingly higher-level issues" (p. 301). As students eventually grow in their careers to take on administrative positions, data-driven decision-making is fundamental to being a modern leader. Driving home the relevance of statistics is a simple, yet powerful way to engage students with the content and view learning it as a worthwhile endeavor.

Besides the professional use of statistics, there is also a societal need. Utts (2003) states that "statistical studies are reported frequently in newspapers and magazines, so students are likely to encounter them on a routine basis" (p. 74). Utts (2003) warns that misunderstanding the core ideas of statistics "leads to cynicism among the population at best, and misuse of study results by policy-makers, physicians, and others at worst" (p. 74). Being competent and educated member of modern society requires them to have an understanding of how data is collected and analyzed (Gattuso & Ottaviani, 2011).

The Role of Context

Among researchers and educators, there is agreeance over a common theme when it comes to differentiating a novice student's first statistical experience from a traditional mathematical experience: context (Ben-Zvi & Garfield, 2004; Cobb & Moore, 1997; delMas, 2004; Hand, 2009; Rossman et al., 2013; Scheaffer, 2006). An enlightening perspective given by Hand (2009) is that ". . . statistical competence requires some maturity in understanding the world. Whereas we have mathematics prodigies, we do not have statistics prodigies" (p. 301). This is an important viewpoint to consider for educators of statistics. When introducing a statistical problem consider this: do the students know too little about the context or do they know too much? Are they mature enough to not let the context hinder their learning process? Ben-Zvi and Garfield (2004) reflect on the challenges new learners face and state that "The context in many statistical problems may mislead the students, causing them to rely on their experiences and often faulty intuitions to produce an answer, rather than select an appropriate statistical procedure" (p. 4).

While familiar contexts can provide engagement for many students, they may also distract them from thinking about the uncertainty that exists within the analysis of the data. For example, a student who is interested in sports may investigate relationships between two teams' points per game statistics and draw immediate conclusions based on their biases towards certain teams without thinking statistically. Conversely, a student forced to study the same relationship who has little interest in sports may not even understand the variables that go into scoring points in that sport. Thus, the difficulty of understanding the context hinders the learning of statistics. Educators should be aware of the knowledge their students have over a particular context and drive home the fact that all the conclusions drawn should be done so with uncertainty in mind.

Context, like the ideas discussed in the last section, is a key difference between mathematical and statistical reasoning. Like mathematics, statistics uses numbers, but numbers in context: data (Scheaffer, 2004). Data requires a different type of thinking. The variability within data sets statistics apart from mathematics and other mathematical sciences (Cobb & Moore, 1997).

Based on the evidence above, context must be on a focus of statistical instruction, lessons, and activities. Through context, students must connect the underlying mathematical ideas to reality to draw appropriate conclusions.

Make Clear Distinctions Between Mathematics and Statistics

As discussed earlier, mathematics and statistics are NOT the same. When instructing statistics within a classroom labeled as "math" the concepts should be identified as such so that students can see the separation between statistical and mathematical thinking (Scheaffer, 2006). Teachers that are stronger in mathematical thinking sometimes err in delivering content with an overemphasis on probability rules and the statistical process becomes "magic" (Cobb & Moore, 1997). If a teacher introduces statistical content with an automated and procedural approach, do the students know why the significance test yields the results? Do they understand how to interpret the results outside of memorization of a similar scenario? Cobb and Moore (1997) provide this remark:

> Students understand mathematics when they appreciate the power of abstraction, deduction, and symbolic representation, and can use mathematical tools and

> strategies flexibly in dealing with varied problems. Reasoning from uncertain empirical data is a similarly powerful and pervasive intellectual method. (p. 815)

Teachers must be mindful of their understanding of statistics and make the distinction between deterministic thinking used in mathematics and probabilistic thinking used in statistics clear to their pupils. Embracing uncertainty is key to progress in students' statistical reasoning (Scheaffer, 2006).

Statistical Reasoning is an Explicit Goal of Instruction

The term "cookbook math" is commonly used to refer to when a student follows a set of procedures to produce answers. They can complete the tasks, but do not know the underlying concepts. Similarly, a student could become a "cookbook" statistician allowing for the "magic" of the formulas and calculators to just produce answers. For both subjects, instruction should be grounded in concrete, physical activities to help students develop an understanding of abstract concepts and reasoning (Cobb & Moore, 1997; delMas, 2004). delMas gives the advice "to promote statistical reasoning, students must experience firsthand the process of data collection and explore the behavior of data … this should help students gain familiarity and understanding with concepts that are difficult to experience in everyday life" (p. 92). These experiences cannot be had by simply lecturing and practicing skills. Garfield (2002) states:

> It may be tempting to conclude that if students have been well taught and have performed well on exams, that they are able to reason correctly about statistical information. However, unless their reasoning is carefully examined, especially in applied contexts, these students may only be at the early stages of reasoning and not have an integrated understanding needed to make correct judgments and interpretations. (p. 10)

Teachers must make genuine efforts to provide experiences beyond rote memory of formulas and repetition of steps within a calculator. Engaging students in the investigative cycle (Carver et al., 2016) allows them to develop statistical reasoning. Such frameworks for engaging students in this cycle will be elaborated upon in a later section of this paper.

Present real data to students

No other approach gives the impression that "this is relevant" than using real data in meaningful contexts from genuine studies. As was discussed earlier, context is a crucial difference between mathematics and statistics. A student's statistical thinking grows when solving problems within context. Rossman et al. (2013) state that "instructors also need to help students learn to relate their comments to context and always consider data collection issues when stating their conclusion" (p. 11). To get students to understand the importance of context, measurement errors, and data collection strategies, it is crucial that real data are presented to students by their teacher (delMas, 2004; Rossman et al., 2013).

Statistics educators should seek to develop their skills

Professional development is an important part of a successful teaching career. There is little preparation for the instruction of statistics within programs that prepare upcoming math teachers and this is even more serious at the elementary level (Gattuso & Ottaviani, 2011; Lovett & Lee, 2017; Rossman et al., 2013). Gattuso and Ottaviani (2011) provide the following perspective:

> Teachers surely need to acquire statistical knowledge and develop their statistical thinking, but they also need training in the didactic of statistics to be able to follow their students' learning and reasoning and be able to spontaneously take advantage of classroom situations to promote student learning. The didactic of statistics will introduce teachers to misconceptions, difficulties, and common errors involved in learning statistics and will propose ways to handle them, thus

> allowing teachers to develop the self-assurance needed to teach adequately. (p. 129)

Lovett and Lee (2017) explain the difficulty that secondary teacher education preparation programs have met “the demands of state and national standards, accreditation, and licensure, as well as update programs based on new research in teacher education” (p. 299) and found that, overall, many teachers about to enter the mathematics education “did not demonstrate a strong understanding of high school statistics content” (p. 303). Their study was focused on pre-service teachers, but it seems reasonable to infer that this has been a steady trend for many years and that many current in-service teachers also would have a similar lack of understanding. With this in mind, teachers should seek opportunities to improve their understanding of statistics. Whether through professional learning communities, continuing education, or self-study, practicing the use of statistical reasoning to solve problems as well as extending the reach of one’s statistical content knowledge can only improve the quality of instruction that a student will receive. With these recommendations for statistics educators in mind, let's now turn our attention to summarizing various frameworks in the literature for teaching statistics.

Frameworks for Teaching Statistics

How should a statistics course be taught? Cobb and Moore (1997) as well as Gattuso and Ottaviani (2011) expressed a need for a framework for instructors when introducing the tools that statisticians use and apply to solve problems. Many such frameworks have been developed over the past decades (Ben-Zvi & Garfield, 2004; Lee & Tran, 2015; Carver et al., 2016; Scheaffer, 2006) and it is obvious that several of these frameworks have inspired each other. After considering the frameworks, common themes stand out. Below is a master list synthesizing the common principles and recommendations:

1. Use real data and emphasize the impact on the context and the purpose.
2. Conceptual understanding comes from active learning. Provide opportunities for students be active instead of listening to a lecture.
3. Teach students how to think statistically and emphasize concepts over procedures. Teach statistics as an investigative process of problem-solving and decision-making. The following are the habits to instill within students.
    a. Always consider the context of data
    b. Ensure the best measure of an attribute of interest
    c. Anticipate, look for, and describe variation
    d. Attend to sampling issues
    e. Embrace uncertainty, but build confidence in interpretations
    f. Use several visual and numerical representations to make sense of data
    g. Be a skeptic throughout an investigation
4. Use technology to emphasize concepts by automating computations and graphics to explore the concepts and analyze data in a deeper way
5. Use alternative assessment methods such as small group projects, peer review projects, and discussion sections for student presentations to better understand, improve, and evaluate student learning. (Bargagliotti et al., 2020; Ben-Zvi & Garfield, 2004; Lee & Tran, 2015; Carver et al., 2016; Scheaffer, 2006)

These are the main recommendations offered by the literature for teaching statistics. Note that item 4, the use of technology, was a common theme among the frameworks. To further emphasize its importance, Gattuso and Ottaviani (2011) state that “During their training in statistics, teachers should also be exposed to the use of technology tools. Technology can assist students in ‘doing’ and ‘seeing’ statistics and in reflecting on data” (p. 129).

Because the focus is on the use of technology in statistics class for this literature review, it is important to consider what the developers of each framework state about the use of technology. Specifically, technology can aid students in learning to think statistically by visualizing concepts or abstract ideas such as center, variation, sampling, and distribution (Bargagliotti et al., 2020; Ben-Zvi & Garfield, 2004; Carver et al., 2016). To accomplish this, students must “actively construct knowledge by ‘doing’ and ‘seeing’ statistics as well as reflecting on the observed phenomenon” (Ben-Zvi & Garfield, 2004, p.401). However, integrating technology can be challenging. There are some equity concerns for certain technologies being available in specific regions, but, overall, technology provides easy access to large, real data sets through dynamic statistical software and web-based applets (Bargagliotti et al., 2020; Ben-Zvi & Garfield, 2004; Carver et al., 2016).

The remainder of this thesis is dedicated to discussing a variety of statistical technology tools available and exploring the research of how these tools have impacted student learning for a variety of challenging standards.

Appropriate Technology Environments for Teaching Statistics

Before looking into the available technology tools and their impact on statistics education, let’s consider the early views of education researchers which focused on the creation of statistics tools for novices in a classroom.

The process of learning statistics through technology should go far beyond just having students allow a pre-generated program to carry out procedures but should instead focus on using tools that put the design and representation of data structures in the hands of the student (Shaughnessy, 2007). With this in mind, researchers in statistics education have been dedicated to analyzing the types of tools needed to produce effects as well as practical learning (Shaughnessy, 2007). Finzer (2002) advocates that software developers would, ideally, work with researchers to ensure the tool has effectiveness on student learning. However, this, for the most part, has not been the case. McNamara (2015) discusses a distinct gap between various technology tools for teaching and learning statistics, and those for legitimately doing statistics. Biehler et al. (2012) found that there are two main approaches taken regarding the gap unintentionally created by software developers: 1) a focus on learning statistical techniques for doing professional statistics and 2) using technology simply to illustrate statistical concepts. McNamara (2015) argues that "the gap between learning and doing statistics should be removed entirely by creating a new type of tool, bridging from a supportive tool for learning to an expressive tool for doing" (pg. 19). I will now discuss the various tools and review what the research has said about their impact on student learning in a statistics classroom.

Professional Tools

The most commonly used tools for professional statistics are SAS software, State Software, SPSS, Python, and JMP (McNamara, 2016). When considering their implementation into education, there are serious drawbacks like how prohibitively expensive they are and how unintuitive they are for novices (McNamara, 2016). However, several of these software programs have created education versions that are at a reduced cost and SAS OnDemand for Academics is free via the cloud for students and instructors (SAS Institute, 2022). Regardless of this, McNamara (2015) explains:

> Although Stata, SAS, and SPSS are commonly used in industry, none of them seem to be supportive of learners. They all provide specific types of graphics, and most work is done using menus and wizards, so they do not make clear what the tool is actually doing. Using these tools creates ‘users’ not ‘creators’ of statistics (p. 75).

Most of these tools seem to be intended for students who have some prior experience with statistics and/or computer programming.

A popular resource for academic statistics is R. It is a free, open-source software package for statistical computing and graphics (McNamara, 2015; Stemock and Kerns, 2019). In comparison to programs like SAS and SPSS, Stemock and Kerns (2019) found that “using the software package R in teaching introductory statistics is at least as effective as using SPSS; in fact, the students who used R earned higher grades on all tests compared to those taught using SPSS”(p. 63). McNamara (2015) praises R for being free, having a flexible framework, and using an interesting and unique language. Doi et al. (2016) provide examples of visualizations that can be coded in R and generated through the secondary app “Shiny” to present common statistical ideas to students, but this source only describes the teacher using the tool, not the students. In lies the problem, it is time-consuming and challenging for teachers to put these tools in students’ hands without major support. The tool requires a significant amount of computer programming knowledge for the teacher and student. As such, R does not completely bridge the gap between educational and professional statistical tools as it does not contain all the attributes defined by McNamara (2015, 2016, 2019).

The Available Tools for Learning

McNamara (2015) compiled a comprehensive list of tools available for learning statistics and provides commentary for each tool. Before going into the list, she states, “The tools currently available for learning and doing statistics generally break along that particular divide: those good for an introductory learner are generally not good for actually performing data analysis, and vice versa” (McNamara, 2015, p. 21). She recognized a need for a framework to critique statistical tools for both novices and expert users and provided the following list of attributes that should be incorporated into a statistical tool:

“1. Accessibility

2. Easy entry for novice users

3. Data as a first-order persistent object

4. Support for a cycle of exploratory and confirmatory analysis

5. Flexible plot creation

6. Support for randomization throughout

7. Interactivity at every level

8. Inherent documentation

9. Simple support for narrative, publishing, and reproducibility

10. Flexibility to build extensions” (McNamara, 2019, p. 376)

It would be ideal for one tool to have all of these attributes, but it is more reasonable for tools to take inspiration from one another’s strengths to help bridge the gap (McNamara, 2019).

How should students interact with these tools and what affordances should teachers look for when picking a specific tool for their students to use? Early in the boom of technology and its integration into statistics education, Joan Garfield and a team worked to outline the desired attributes of technological environments and how they could best aid teachers in illustrating statistical concepts. Shaughnessy (2007) explains and cites these attributes:

> A tool needs to allow students to view and explore data in different forms, allow students to experiment with and alter displays of data, change intervals on a graph, and explore different models that may fit the data, have access to the internet as to obtain software or data used in the study of other disciplines, and include representations (including dynamic ones) from which students may choose among different graphs in order to select the best way to interpret and display a data set (p. 992).

When considering a tool to use in instruction, the description above is an excellent guideline for teachers. The impact of such tools in statistics education is noted by Chance et al. (2007) stating that "It is hard to imagine teaching statistics today without using some form of technology…The technology revolution has had a great impact on the teaching of statistics" (p. 1). Next, consider the variety of tools available for learning statistics.

Graphing Calculators

In AP Statistics, students are expected to use technology tools throughout the coursework in preparation for the challenge exam, but most teachers limit technology use to graphing calculators such as TI-84 and TI-Nspire (McNamara, 2016). Research advocates for the effectiveness of handheld technology in the mathematics classroom to teach algebra, geometry, and statistics (Clark-Wilson, 2010; Graham, 2008; Wei & Johnson, 2018). Clark-Wilson (2010) specifies how TI-Nspire technology assists students with learning statistics stating:

> Two teachers provided examples of the use of TI-Nspire Navigator to enhance statistical work in mathematics by using the Screen Capture view to increase the visible sample size of the class data. This was used to support students' understanding of the relevance and importance of sample size when drawing

> assumptions from statistical data and also to appreciate how the TI-Nspire Random number generator and RandSeed setting influenced the resulting data when simulating dice experiments (p. 757).

Furthermore, Wei and Johnson (2018) found significant effects on students' ability to calculate normal probabilities and perform hypothesis testing.

Despite all the benefits cited above, there is also research that warns of the limitations of such tools. For example, McNamara (2016) states that "Calculators should not be considered appropriate tools for statistical computation… the analysis that is produced is not reproducible, and the 'computation' does not help students develop a deeper understanding of the underlying concepts" (p. 4). Graphing calculators are a very common tool in mathematics classrooms and many teachers might hope that they can meet all the needs of their learning objectives in statistics. However, students may only consider a statistical concept as a series of buttons instead of focusing on the statistical concept. The lack of easy, flexible dynamic representations has always been a shortcoming of graphing calculators. While they can serve students to a certain extent, they certainly do not "bridge the gap" into the professional sector of statistics and there are other tools that may help them gain a deeper understanding.

Spreadsheets

The next tool to be discussed is the spreadsheet. The first electronic spreadsheet, VisiCalc, was available in 1979 and since then, several other spreadsheet applications have been created such as SuperCalc, Multiplan, PlanPerfect, Quattro Pro, VP-PLANNER, AsEasyAs, and, likely the most well know these days, Microsoft Excel (Baker & Sugden, 2003). Baker and Sugden (2003) called for the need for more facilities to use spreadsheets in assessment contexts as well as the enabling of teachers to embrace spreadsheet technology and expose students to it. Much of the literature, however, highlights the deficiencies within a spreadsheet's ability to cater to exploratory data analysis, computational deficiencies, and overall lack of functionality to be a true tool for statistical programming (Baker & Sugden, 2003; John & Tony, 1996; McNamara, 2015). The underlying code for these spreadsheets is closed source which does not allow users to observe how methods are implemented (McNamara, 2015). However, McNamara (2015) praises Google Sheets and Open Office versions of excel for being accessible and equitable. Baker and Sugden (2003) remarked that "The invention of the spreadsheet made personal computers have real value in the marketplace and legitimated the personal computer industry" (p. 18). While it seems that there is a plateau when it comes to higher level statistical understanding with spreadsheets, the benefits and convenience in acquainting students to elementary statistics concepts make the spreadsheet a worthy tool for introducing traditional statistics concepts like basic hypothesis tests, one-way and two-way analysis of variance methods, simple and multiple regression analyses, a variety of probability and related functions, and the ability to generate random numbers to allow for simulation calculations (Baker and Sugden, 2003; John and Tony, 1996).

Dynamic Statistical Software

Dynamic statistical software allows for the "direct manipulation of mathematical objects and synchronous update of all dependent objects during the dragging operations" (Finzer, 2000, p.1). The early dynamic statistical tools are TinkerPlots and Fathom. "TinkerPlots and Fathom are essentially sibling software packages… are excellent tools for novices when learning statistics" (McNamara, 2015, p. 30). Finzer (2002) developed these tools for learning statistics and intended to allow students to be more creative when learning statistical concepts. Tinkerplots was designed for 4th grade up to secondary while Fathom is directed at secondary and college, but both focus on the way students think (McNamara, 2015). Research has shown that the use of both types of dynamic statistical software has improved learner outcomes (Ganesan & Eu, 2018; Watson, 2013). Ganesan and Eu (2018) found that “Fathom-based instructions do not isolate students from peers and teachers. Instead, it encourages communication between students and teacher when conducting the activities and the students will be confident to explore more during their learning.” (p. 21) Fitzallen (2007) as well as Watson and Donne (2009) analyze the software packages and classify them as being accessible, easy to use, assist in recalling data by displaying it in multiple forms, facilitate translating between mathematical expression and natural language, maintain extended memory when organizing and reorganizing data, provide multiple entry points for the abstraction of concepts, and produce visual representations that can be used for both interpretative and expressive learning activities. This aligns well with McNamara’s list ten of attributes. However, like spreadsheets, a limitation of these applets is that they eventually top out at the higher levels of statistics due to their inability to perform advanced statistical analyses, and McNamara (2015) states “Users may learn statistical concepts, but they are not developing any “computational thinking” or programming proficiency” (p. 35).

Another newer, dynamic statistical tool is the Common Online Data Analysis Platform (CODAP). CODAP, like TinkerPlots and Fathom, “were developed as tools for data analysis that offer learners an easy entry into data analysis by requiring no programming skills but of course, are, therefore, limited in their data exploration potential” (Frischemeier et al., 2021, p.183). However, CODAP is a free, web-based application that offers easier access for students and teachers. Frischemeier et al. (2021) report on the design and implementation of a unit in which secondary students used CODAP to explore real and meaningful data and concluded that "CODAP served as a valuable tool for initial data exploration… it also facilitated the data analysis process and decreased the cognitive load on the students, who could put their focus instead on data analysis and exploration rather than on the tool use" (p. 188).

Just like McNamara (2015) expressed concerns for Tinkerplots and Fathom, Frischemeier et al. (2021) acknowledge the limitations of CODAP and state that it "motivated our students to progress to a more complex tool" (p. 189). "The more freeform workspace can feel creative, but it makes it nearly impossible to reproduce analysis… There is no easy way to publish results from these programs" (McNamara, 2015, p. 51). CODAP checks many of the boxes on the list of attributes that should be incorporated into a statistical tool, but educators and professionals are still seeking a tool that can bridge the gap even more.

One other popular dynamic statistical software is TUVA which is similar to CODAP in its simplicity as well as being free but differs in some important ways. In 2016, Erickson (2016) makes some comparisons between the two software stating their differences with the following:

> "• Tuva is designed for easy access to curated data sets and "lessons"—series of screens with access to the live tool, where students can read instructions and answer questions. CODAP has no special data repository, though you can open files. CODAP is more geared towards getting data from "data interactives," which might generate data or get it from feeds.

• Tuva includes a wider variety of plot types including histograms, box plots, bar charts, pie charts, etc. CODAP, in contrast, currently makes all its plots with dots, though it allows adornments such as shading the IQR.

• Tuva has only one graph visible. CODAP allows any number of graphs, and supports synchronous selection among all views of the data.

• Tuva's data organization is flat, while CODAP's allows a hierarchical structure." (p. 4)

Since this workshop report, many updates have been made to both software and they have improved overall as great resources for novice learners of statistics.

Applets

The next tools up for discussion are applets. Web applets are direct competitors of CODAP, TUVA, TinkerPlots, and Fathom but have the advantage of being easy to access, mainly free, and simple to use by focusing on a single concept with some not requiring multiple steps to create a visualization (Tishkovskaya & Lancaster, 2012). Some researchers believe that a single tool cannot do it all and that many web apps can be combined to teach and learn statistics (Variyath & Nadarajah, 2022). The major web applets to note are Rossman and Chance Applet Collection and StatKey (Frischemeier et al. 2021; McNamara, 2015). Another notable web app for learning statistics created in 2016 is Stapplets which is described as "originally designed to be a graphing calculator replacement for students who already have internet-connected devices, such as laptops, iPads, or smartphones. Over the years, the collection has expanded to include activities and the ability to collaboratively collect data" (Tabor & Amar, 2022).

If a teacher wishes to show a visualization of a particular statistical concept, the Rossman and Chance applets are an excellent resource for teachers to share with a class or to allow students to play around with. These applets cover a wide variety of concepts but do not allow teacher or students to use their own data, thus is strictly used for learning (McNamara, 2015).

StatKey works with simulation-based methods to help students draw connections to the logic of statistical inference and improve understanding (Lock et al., 2018). Lock et al. (2018) concludes that the applet and methodology create a great starting point for inference and "paves the way for students to later more easily extend those important ideas to the more formula-driven, but still very common, traditional methods" (p. 6). StatKey will be explored further in the discussion of Sampling and Inference. McNamara (2015) remarks "StatKey applets do allow users to edit the example data sets or upload entirely new data, but they are necessarily limited to what they were programmed to do" (p. 38). While there are limitations to these applets, their accessibility and conciseness for teachers and students make them a desirable educational tool in a statistics classroom (Variyath & Nadarajah, 2022).

Regarding all the tools that have been discussed thus far, Konold (2007) emphasizes that statistics software must grow beyond its initial "static state" and explains that tools for learning statistics should not be reduced versions of professional tools for doing statistics. Instead, they should be developed with a bottom-up perspective, thinking about what features novices need to build their knowledge (Konold, 2007). Tools such as CODAP and StatCrunch have been developed with this bottom-up approach.
"StatCrunch" is companion software that is integrated within the Pearson publishing

company's "MyStatLab" (Mihai & Correa, 2019). Mihai & Correa (2019) found that students had favorable outcomes using the software and that they enjoyed features such as the ease of accessibility, immediate feedback, and hints to work problems. Such positive feedback and learning outcomes makes StatCrunch and MyStatLab a common choice for post-secondary educators.

Students Use of Dynamic Technology Tools to Learn Statistics

Burrill (2018) defines dynamic technology tools as "a tool that allows the user to link multiple representations- visual, symbolic, numeric, and verbal- and to connect these representations to support understanding" (p. 1). There is a significant amount of research describing the positive impact that the use of dynamic statistical tools has on students' learning of statistics (Bill, 2012; Dijke-Droogers et al., 2021; Fitzallen, 2012; Fitzallen & Watson, 2014; Frischemeier et al., 2021; McDaniel & Green, 2012; Mojica et al., 2019; Paparistodemou & Meletiou-Mavrotheris, 2008; Sánchez & Inzunza, 2006; Scranton, 2013; Wang et al. 2011; Watson, 2008; Watson & Donne, 2009). In this section, I will discuss specific research where students have benefited from the use of these tools. Among the various positive themes associated with students using these tools to learn statistics standards, two stand out. They are:

1) Use of the tool to create data representations and conduct exploratory analysis.
2) Interacting with a tool containing a preconstructed sketch to understand the ideas of sampling distributions and statistical testing as well as their prerequisite concepts such as center and variation.

These two themes are in alignment with the affordances discussed earlier about Dynamic Geometry Environments.

Student-Created Data Representations

Exploratory data analysis (EDA) is a major part of a student's statistics experience and involves them in generating or receiving a raw set of data and then creating multiple representations to answer statistical questions. Researchers have found evidence that the use of dynamic technology tools to engage students in EDA has improved their ability to create graphical representations, interpret the various created distributions, find and interpret measures of center and spread, see covariation in bivariate data, understand ideas of probability, and engage more easily with the complex idea of inference (Bill, 2012; Dijke-Droogers et al., 2021; Fitzallen, 2012; Fitzallen & Watson, 2014; Frischemeier et al., 2021; McDaniel & Green, 2012; Mojica et al., 2019; Paparistodemou & Meletiou-Mavrotheris, 2008; Scranton, 2013; Wang et al. 2011; Watson, 2008; Watson & Donne, 2009). Each of these studies had students take part in creating their data representations after giving them a set of raw data on a particular software such as Tinkerplots, CODAP, Fathom, etc. The common findings from the studies were that students could quickly create graphical representations. Following the construction of their graphs, students then either were given questions (Bill, 2012; Fitzallen, 2012; Fitzallen & Watson, 2014; Paparistodemou & Meletiou-Mavrotheris, 2008) or were generating their own questions (Frischemeier et al., 2021; Mojica et al., 2019) to then answer and provide graphical evidence. Examples of such questions are:

- Low-level questioning: Questions that address explicit material and can be answered by yes/no or by a single value. (Bill, 2012; Frischemeier et al., 2021)

  - “Do male or female students use Instagram more?” (Frischemeier et al., 2021, p. 185)
  - “Which grade shows the highest reading frequency?” (Frischemeier et al., 2021, p. 186)
  - “which runner had the most wins?”;” which country had the most wins?”; “did the times change over the years?” (Bill, 2012, p. 153)
  - “What’s the highest height?” (Mojica et al., 2019, p. 5)
- High-level questioning: Questions involving higher-order cognition of inference, synthesis, and evaluation that cannot be answered with the given data, or which concentrate on the distributions of one variable but do require an investigation of relationships between variables. (Bill, 2012; Frischemeier et al., 2021)
  - “In which ways do male and female students differ in their Instagram use?” (Frischemeier et al., 2021, p. 186)
  - “In which ways do the students differ in their reading habits across the grades?” (Frischemeier et al., 2021, p. 186)
  - “Do you think your die is fair? What in the data makes you think your die is fair or unfair? Or are you not convinced either way?” (Bill, 2012, p. 108)

Fitzallen & Watson (2014) discovered three strategies that students had when trying to answer such questions:

1) Snatch And Grab: The student clicks on buttons and moves things around anticipating that something helpful would appear on the screen. However, they often would not evaluate the differences made by their actions.
2) Proceed and Falter: Students use pre-established patterns of behavior to create graphs they are familiar with and then hesitate when they cannot use the graphs to answer the questions about the data. They appear not to be able to make the link between what is produced and what is needed to be produced to answer the questions.
3) Explore and Complete: Students are purposeful with their actions. They make correct decisions about which graph type would assist in answering the questions and add additional features to the graphs to determine if more information could be gleaned. (p. 3)

A very impressive result from several of the studies was the impact the tool TinkerPlots had on the ability of very young students (Year 3-6) to create graphs of real data and generate informal inferences (Fitzallen, 2012; Paparistodemou & Meletiou-Mavrotheris, 2008 Watson & Donne, 2009). Without the technology environments, the complexity of constructing the graphical representations by hand would have hindered the younger students from learning. Paparistodemou and Meletiou-Mavrotheris (2008) conclude that:

> Attributes of TinkerPlots like the ability to operate quickly and accurately, to dynamically link multiple representations, to provide immediate feedback, and to transform an entire representation into a manipulable object enhanced students' flexibility in using representations and provided the means for them to focus on statistical conceptual understanding. The visualization of the data helped children to express intuitive ideas about proportional reasoning, a fundamental topic in the school mathematics curriculum. The genuine endeavors of the young learners

> with multivariate data using TinkerPlots as an investigation tool helped them begin to develop their informal inferential reasoning. Furthermore, the software's design allows even young students to use what they already know to search for and detect group differences and trends (p. 101).

Fitzallen (2012) also found that Tinkerplots was able to support the students' abilities to describe covariation and generalize about a trend shown in the data and explains that it would be "pertinent to bring the initial introduction of scatterplots into the upper primary years of schooling" (p. 296).

In concluding these ideas, three major affordances of using dynamic statistical environments for EDA show up commonly across the literature (Bill, 2012; Dijke-Droogers et al., 2021; Fitzallen, 2012; Fitzallen & Watson, 2014; Frischemeier et al., 2021; McDaniel & Green, 2012; Mojica et al., 2019; Paparistodemou & Meletiou-Mavrotheris, 2008; Scranton, 2013; Watson, 2008) but Watson and Donne (2009) clearly define the tools as allowing students 1) flexibility of representation, 2) speed of analysis, and 3) exposure of levels of understanding. All these affordances provide rich experiences for students to learn and develop statistical reasoning skills. As such, having students create data representations within dynamic statistical environments should be a fundamental component of statistical learning environments. Exploratory data analysis gives students the opportunity to create these and help them to develop statistical reasoning.

Students' Interactions with "Preconstructed Sketches"

Some statistical standards (e.g. resampling, sampling distributions, formal inference procedures) are more complex and cannot be discovered by just exploring data and representations constructed by students. They require the teacher to pre-create a distribution of a set of data for students to interact with or present students with an online simulation of the event. Students struggle to learn about the concept of sampling distribution more than most other topics in statistics which is fundamental in the understanding of formal statistical inference (Shaughnessy, 2007). Researchers have found that when students interact with "preconstructed sketches" in a dynamic environment, they were able to overcome the difficulties of a sampling distribution (Bill, 2012; Chance et al., 2001, 2004; Dijke-Droogers et al., 2021; Fitzallen & Watson, 2014; McDaniel & Green, 2012; Sánchez & Inzunza, 2006). In several of the studies, the researchers concluded that software such as Fathom and TinkerPlots allows students to look for the structure and shape of distributions, better understand the Central Limit Theorem visually, understand sampling variability, see the effect of sample size, and distinguish the difference between "sample" and "sampling distribution" which helps students naturally transition into learning about hypothesis testing and inference (Bill, 2012; Chance et al., 2001, 2004; Dijke-Droogers et al., 2021; McDaniel & Green, 2012; Sánchez & Inzunza, 2006). Chance et al. (2004) state that "static demonstrations of sampling distributions are not sufficient to help students develop an integrated understanding of the processes involved, nor to correct the persistent misconceptions many students bring to or develop during a first statistics course" (p. 312). Furthermore, the concept of "resampling", which could be quickly and easily completed by students on the dynamic software using a "sampler" tool, helps students see the steps of creating a hypothetical sampling distribution on which to test hypotheses (Bill, 2012; Dijke-Droogers et al., 2021; Fitzallen & Watson, 2014; Wang et al. 2011). Dijke-Droogers et al. (2021) involved 9th grade students in using the Tinkerplots "resampler" tool and found that "In a short period of time, students—who were inexperienced in taking samples and working with digital tools—learned to carry out the modeling processes, including interpreting the simulated sampling distribution" (p. 257). Furthermore, they found that the preconstructed scenario within TinkerPlots enabled students to overcome the confusion and make the distinction between sample and sampling distributions (Dijke-Droogers et al., 2021)

In summary, students' interactions with preconstructed dynamic applets and software have a strong impact on their conceptual understanding of sampling as well as the center, spread, and shape of sampling distributions. This understanding leads to better sense-making within informal and formal statistical inference.

Conclusion

With the growth and demand for qualified statisticians and data analysts, it becomes the responsibility of statistics educators of all levels to use appropriate techniques and tools in their instruction (Gibbs, 2018). The findings from the literature suggest that mathematics education and statistics education are different and should be clearly defined as such. Statistics uses mathematics but that is not necessarily a reflexive relationship. Next, teachers should consider the frameworks available when beginning their instruction. The main emphases are to use real data, provide opportunities for active learning, stress concepts over procedures, make use of technology, and apply alternative assessment methods. Next, the research provided suggests the importance of using statistical tools to enhance students' experiences when learning statistics concepts. There are many tools available for learning as well as professional tools and a gap exists between them. McNamara (2015) discusses how software developers should work to find a way to bridge this gap. This would allow statistics students to be more prepared for the professional sector. Of the tools created over time, the literature suggests that dynamic technology tools greatly enhance student learning of statistical ideas such as center, shape, and variation. Also, the tools afford students the opportunity to explore distributions of data and their properties by dragging individual data points or whole variables to compare relationships and measuring specific statistics to observe changes.

Connections Between Mathematics and Statistics Dynamic Tools

A fascinating connection was drawn while comparing how dynamic tools are used in both mathematics and statistics classrooms. The three major themes we explored within dynamic geometry environments had parallels to dynamic statistics environments. Students can use dragging and measuring tools, create their own sketches, and interact with preconstructed sketches within DGE. The table below describes the connection these themes have to a dynamic statistics environment:

Measuring and Dragging	Constructing Original Sketches	Interactions with Pre-Constructed Sketches

Within dynamic statistical tools, the ability to easily find statistics such as measures of center (mean/median) and spread (Standard Deviation/IQR) coupled with dragging dynamically linked points to see changes in these measures helps students discover properties of distributions by engaging in narrative thinking (Sinclair et al., 2009).	Exploratory Data Analysis (EDA) involves students constructing various distributions and displays to make conjectures about a raw data set. In this environment, students can engage in various levels of questioning that lead to the beginnings of statistical inference starting at young ages (Bill, 2012; Dijke-Droogers et al., 2021; Fitzallen, 2012; Fitzallen & Watson, 2014; Frischemeier et al., 2021; McDaniel & Green, 2012; Mojica et al., 2019; Paparistodemou & Meletiou-Mavrotheris, 2008; Scranton, 2013; Watson, 2008).	Teacher-created scenarios or applets can assist students with learning complex topics such as the Central Limit Theorem, sampling variability, the effect of sample size, and distinguishing the difference between "sample" and "sampling distribution" which helps students transition into learning about hypothesis testing and inference (Bill, 2012; Chance et al., 2001, 2004; Dijke-Droogers et al., 2021; McDaniel & Green, 2012; Sánchez & Inzunza, 2006).

These connections suggest that teachers of mathematics could employ more familiar techniques of instruction to help their students learn statistics concepts as well.

In conclusion, the recommendations in this thesis were aimed at improving statistics education to better prepare students for their future education and careers. A critical finding from the literature is that dynamic technology tools serve as a crucial element for both students and teachers in learning and instruction. Furthermore, considering the suggested connection between dynamic geometry environments and dynamic statistical environments, it would be interesting to see future research search for evidence of improvement to the technological pedagogical content knowledge of pre-service and in-service teachers that use these three themes within geometry and statistics lessons.

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