# ESSAYS ON HIGHER EDUCATION, PARENTAL ALTRUISM, AND GRANGER CAUSALITY

by

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This dissertation is dedicated to my father, A. T. M. Abdullah; my mother, Nasima Akther; my caring wife, Rakiba Nabi; and my two lovely daughters, Tasfia Tahsina Hasnat and Tanjia Tavinah Hasnat, who sacrificed a lot of things in their life to bring me at this stage.

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# ABSTRACT

This dissertation consists of three distinct, publishable 'papers' included as separate chapters. Although all of the three chapters can be read and understood independently, the first two chapters are on related themes. The essence of the first two chapters are, therefore, to analyze the individuals' higher education expenditures over their life-cycle. It considers the standard life-cycle human capital accumulation theory as a benchmark model and tests the rationale and validity of it by applying empirical methodologies and calibration. The third topic employs the Granger causality test to find out what causes the stock price volatility to become more sensitive, whether it is for interest rate or exchange rate risk.

More specifically, the first chapter uses Consumer Expenditure Survey (CEX) data to estimate the effects of an individual's age on higher education expenditures. Using a synthetic panel created from the CEX data and controlling for fixed effects and different demographic level characteristics, the regression results are then used to construct the life-cycle profile for higher education expenditures based on age. The estimated coefficients of age and its polynomials are found to be statistically significant. Special emphasis is placed on finding the turning points by using the polynomial regression coefficients that indicate a change in the age pattern over the life-cycle. The turning point analysis indicates a "hump" shaped nature for education expenditures, counter to what theory suggests.

The second chapter considers the life-cycle higher education expenditures profile for individuals as a baseline model and investigates the possible impact of "parental altruism" and borrowing constraints on the human capital investment. The analysis is conducted by employing a standard overlapping-generations model in which parental altruism is emphasized over borrowing constrains for children. This quantitatively calibrated model predicts that the intertemporal substitution effect dominates the income effect for the individual of middle age groups, causing allocation of their income towards the higher education expenditures of their young-aged child. The latter effect generates the hump shaped human capital expenditures pattern for individual parents who decided to enroll for college education at a later period of their life.

The third and final chapter explores the Granger causality test and the bi-variate as well as the multivariate co-integration to determine the interactions between interest rates, exchange rates, and the composite stock volatilities of different traded contracts under the Chicago Board Options and Exchange. Using the daily sector data for all observed variables from the St. Louis Fed over the period of 2007-2017 and introducing the method of simple vector autoregression, this study examines various aspects of the correlation where the current and the previous values of volatility indices, interest rates, and exchange rates have shown significant Granger causality effects to the return behavior of those volatility indices, interest rates, and exchange rates. The estimated result indicates that, under the absence of any long-run relationships, interest rates have more unidirectional and bi-directional causal effects with the stock market volatility indices than in comparison with the exchange rates, although both of them are identified as significant determinants of stock price volatility.

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# CHAPTER I

# HUMPS IN HIGHER EDUCATION EXPENDITURES OVER THE LIFE-CYCLE: EVIDENCE FROM CONSUMER EXPENDITURE SURVEY DATA

## 1. Introduction

This paper constructs the empirical life-cycle profile for higher education expenditures. In general, life-cycle profile means how much an individual spends on average with respect to age. The purpose of constructing an empirical life-cycle higher education expenditures profile is so it may be compared to theoretical life-cycle profiles. Cahuc and Zylberberg (2004) present a theoretical life-cycle profile of education expenditures that is generated from the model of life-cycle human capital accumulation of Ben-Porath (1967), and Heckman (1978). In this case, the individual's schooling and hence their expenditures on education is decreasing everywhere based on age. Thus, construction of an empirical profile in this area would allow one to see the validity of this standard theory.

The economic reasons behind a decreasing life-cycle higher education profile are related to opportunity costs. If wages increase with an individual's age (e.g., experience, specialization, tenure, seniority) then the opportunity cost of attending college should increase with age. Alternatively, empirical research on consumption expenditures (as opposed to higher education) for both durable and nondurable goods has shown a hump shaped nature of consumption function; expenditures are low early in life, then rise considerably until about the age of 50 and fall again. These examples, which include Gourinchas and Parker (2002), Fernández-Villaverde and Krueger

(2007), Fernández-Villaverde and Krueger (2011), would suggest that higher education expenditures may be humped shaped as well, because they are a component of total consumption expenditures. These alternative views, strictly decreasing or humped shaped profiles, form the null and alternative hypotheses of this paper.

For the purpose of constructing the life-cycle profile of education expenditures, this paper employs Consumer Expenditure Survey (CEX) data. The CEX is a rotating panel of about 5,000 households, where each household is interviewed every three months over the five calendar quarters. Every quarter, 20% of the sample is replaced by the new households. The CEX data set provides details about individual characteristics, including expenditures, and has the advantage of covering the ages needed to estimate the life-cycle profiles. For the same reason, this paper did not use the potential data from the NLSY-79 or from the NLSY-97 cohorts, as the respondents are too young in their life-cycle.[1] For example, the maximum age in NLSY would be between 51 to 58 years (considering the NLSY-79 cohort) in respect to the observed time period.

The empirical strategy that this paper uses is the fixed effects panel regression. Due to the difficulties of forming the regular panel associated with the CEX data, this study constructs a pseudopanel or synthetic panel and uses the cohorts as the fixed effects. More specifically, this panel is constructed in line with Deaton (1985) who averages by year of birth (an obvious common characteristic). Then, these averages are further grouped into 15 cohorts with a range of five years. The total effect is a panel that represents 15 average individuals who are in different stages of their life-cycle. The shape of the profile is found by including a sequence of polynomials with respect to age. If the profile changes slope, the higher order polynomials would

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[1] NLSY means National Longitudinal Survey of Youth, that are comprised of the set of surveys conducted by Bureau of Labor Statistics and are designed to gather data on labor market activities as well as other significant life events for several groups of young men and women at several points in time.

capture these deviations. The ages at which the polynomials change slope are called turning points, and a purpose of the estimation strategy is to quantify the importance (if any) of these.

The main result of this paper is that it finds a clear hump in the higher education life-cycle profile. The turning points are statistically significant and occur firstly at age 49 followed by another at age 74. A second finding of this paper is that higher education spending on individuals who live “outside” the consumer unit peak when the head of the household is around 50. Presumably, the spending is for the children of the family who are attending college but no longer live at home. Taken together, the findings suggests that, with respect to higher education, children cause individuals to under-consume around the age of 50 and to over-consume before and after this period thus leading to a hump in the educational spending profile. A resulting puzzling question arises: if consumers desire a smooth and decreasing higher education consumption profile (as theory predicts), why don’t they use precautionary savings decisions to achieve this? For sure the studies of Guo (2014) and Bagchi (2011), which demonstrate that borrowing costs and overconfidence, respectively, can explain the consumption expenditures hump, are likely a starting point in the search for new theories.

The rest of this paper is structured as follows. Section 2 presents the related literature and the CEX data with a description of pseudopanel construction. Section 3 outlines empirical methodology with a linear fixed effects modeling of higher education expenditures and polynomials of age. Section 4 discusses the empirical findings. Section 5 offers summary and concluding remarks.

## 2. Related Literature and the CEX Data

### 2.1 Related Literature

Higher education consumption is an important component of the U.S. economy (4% of the total consumption),[2] and it is also well known that higher education determines generational income mobility.[3] Additionally, higher educational choices are not solely for the young. The Current Population Survey reports that adult students – ages greater than 25 – make up about 38 percent of all college students.[4] The survey also shows that about half of adult students are 35 years and older. Fowler and Young (2004) report that the choice to enroll in college is heavily influenced by current business conditions. Thus, understanding spending patterns in higher education is an important endeavor as it gives insight into a significant non-trivial choice individuals make over their lifetime and over the business cycle.

This paper builds on the sizable literature documenting empirical life-cycle consumption expenditure profiles, examples of which include: Attanasio and Weber (1995), Deaton (1997), Attanasio, Banks, Meghir, and Weber (1999), Gourinchas and Parker (2002), Fernández-Villaverde and Krueger (2011), and Kraft, Munk, Seifried, and Steensend (2014). Moreover, these studies pertain to the total consumption expenditures or the major categories of consumption (nondurable, durable, services) over the life-cycle. This study is therefore unique in that no previous research has constructed higher education expenditures profiles.

A re-occurring result of the literature is that consumption expenditures over the life-cycle is hump shaped. Studies show that individuals' delay consumption for a certain period of their life-cycle and accumulate precautionary savings when income

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[2] National center for Education Statistics;
available at http://nces.ed.gov/programs/coe/indicator_cma.asp

[3] Thirteen Economic Facts about Social Mobility and the Role of Education, Brookings; https://www.brookings.edu/research/thirteen-economic-facts-about-social-mobility-and-the-role-of-education/

[4] Author's calculations.

uncertainty is revealed to a large extent. Thus, different demographics and persistent income shocks generate hump shaped consumption expenditures profiles for both durable and non-durable goods. This inverted U-shaped pattern is robust to different data sources as documented in the extensive review of Attanasio and Weber (2010). They mention that, on average, the hump peaks at age 50. With regards to higher education expenditures, there are reasons to believe that the hump does not exist. The standard human capital model of Ben-Porath (1967) predicts a strictly downward sloping higher education profile. The main reason is opportunity costs: expending on college when the individual is young at the expense of foregoing current expenditures. Observed earnings are relatively low when young, and thus attending college has a low opportunity cost when young. In related literature, a hump in wages (with very different peak ages than consumption) has been documented in Murphy and Welch (1990, 1992), Johnson and Neumark (1996), and Casanova (2013). This would reinforce the no-hump view as opportunity costs are magnified in the middle ages.

As an alternative hypothesis, it is conceivable that the profile is humped shaped. As higher education is a component of total expenditures, it may very well be triggering the pattern. Studies such as Caliendo and Huang (2007), Feigenbaum (2007), Bagchi (2011), and Guo (2014) have developed theories to explain either the consumption or income humps. Example theories include mortality risk, overconfidence, borrowing costs, and income uncertainty. As these reasons may also pertain to higher education, one might expect a hump in higher education consumption.

## 2.2 The CEX Data

The CEX data is based on a comprehensive survey run by the Bureau of Labor Statistics (BLS), where each household is interviewed every three months over five calendar quarters, and at most four times over a period of a year. It is a rotating panel that interviews almost 5,000 households; 80% of them are then re-interviewed the following quarter, and the remaining 20% are replaced by new, random households. This study takes the data that was collected by conducting an interview survey from the BLS and that are recorded in their website under the name "interview." The interview portfolio contains three sets of files: (i) family, (ii) member, and (iii) expenditure. The family file contains family demographics such as family size, region, reference person's age, population size, state, and urban or rural. The member file contains the information on each member of the family such as their age, race, gender, marital status, status of being in armed forces, and educational background. The expenditure file contains detailed records of expenditures on education by the family as well by each individual of the family.

The other survey component of CEX data, namely the diary survey, is designed to collect data on smaller purchased items made by the households, including food and beverages, both at home and food establishments, housekeeping supplies, tobacco, non prescribed drugs, and personal care products and services. The expenditures by each consumer unit (CU) has been recorded in a diary for two consecutive 1-week periods. Alternatively, the interview survey is designed to obtain data on relatively large type of expenditures, such as those for property, automobiles, education, major durable goods, and those that occur on a regular time intervals, such as rent or utilities. As the interview survey is more reliable (since an interviewer is required) to collect and record large data sets like higher education expenditures, this study rely solely on interview survey.

The variables taken from the expenditure files are the net amount[5] paid for educational expenses (JEDUCNET), the member number (EDUCGFTC), and the family number (NEWID). Because JEDUCNET contains expenditures on all forms of edu-

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[5] Net amounts are total less reimbursements. Reimbursement payments come either from employers or from people outside the CU.

cation, higher education consumption is identified by using the CEX coded variables EDUC_AY (types of educational expenses) and EDSCHL_A (types of school) that define the expenditure into 1 of 16 educational types (tuition, housing, etc.) and the type of education (college, elementary, etc.), respectively. Out of the 16 categories, only 6 types of expenditures would constitute consumption on higher education. For example, nursery school expenses are not relevant. The names of the variables extracted are presented in Table 1.1 and are self explanatory. Housing while attending school includes any expenditures on room and board such as student apartment sharing.

Unfortunately, after the second quarter of the year 2009, EDUCGFTC was recorded in a different order where the specific family member for which the expenses was made cannot be identified. Thus, even though the CEX database continues to collect the data up to 2014, only the years from 1996 to 2009 are used in this paper. These exclusion restrictions leave a sample of 687,282 observations. Demographic variables such as age, year of birth, race, sex, educational background and regions are extracted from the family and member files and merged with higher education expenditures.

The member number is used as a track for the CU person whom the expenditures was for. From time to time, the CU will spend on someone outside the family unit. Presumably these are for family members not living in the CU. In these cases, there is no distinct way to determine the identity of those individuals and what their relationship with the CU reference person is as the variable EDUCGFTC is set to missing. For this reason, the education expenditures for individuals outside the CU are dropped from the main analysis. In the worst-case scenario, this produces a data set where not all the expenditures are accounted for. However, a subsequent section will examine the patterns of gross expenditures as they presumably include reimbursements from family members who are considered outside the CU.

### 2.3 Constructing the Pseudopanel

The empirical approach closely follows Deaton (1985) and Fernández-Villaverde and Krueger (2007). The psuedopanel approach is applied to construct representative groups or cohorts from the samples. This study considers a range of 15 cohorts considering the member's year of birth starting after 1921 and ending in the year 1987. The main outcome variable $reduexp_{i,t}$ represents the average of net real amount of spending for higher education at time $t$ by the $i$'th cohort. Formally, the cohort expenditures are computed as:

$$reduexp_{i,t} = N_{i,t}^{-1} \sum r_{g,t,i} eduexp_{g,t} / cpi_t$$

where $N_{i,t}$ is the number of members in cohort $i$ at time $t$, $eduexp_{g,t}$ is the nominal higher education expenditures of the $g$'th person at time $t$, $cpi_t$ is the time $t$ consumer price index (CPI) (all consumers), and $r_{g,t,i}$ is an indicator variable that is 1 for the $g$'th person if lies inside the $i$'th cohort. In this context, based on all individuals belonging to a given cohort, an average value of their net amount paid for higher education consumption has been recorded as the number of observations for the dependent variable. In this paper, cohorts are defined by a 5-year interval, and the sample covers 14 years, so each cohort overlaps with an adjacent cohort at the sixth year.

Table 1.2 produces summary statistics for a selection of variables. The mean value of the real net amount paid for higher education is around \$69 with a standard deviation of 133.52. The maximum amount spent on higher education (i.e., $reduexp_{i,t}$) is just above \$835. Though these amounts seem low, the actual mean expenditures by individuals is larger. Recall that $reduexp_{i,t}$ is the time $t$ cohort average, and given that the average cohort age is 51 years old, the opportunity cost theory suggests that this should be a low number. The typical view of a college age student, age 18-22, makes up a small fraction of the cohorts.

Other covariates that are included in the study, so as to control for changes in cohort demographics that might cause a pattern shift in expenditures, are presented in Table 1.2. The variables $white_{i,t}$, $male_{i,t}$, and $urban_{i,t}$ are dichotomous variables that take the value one when race is white, gender is male, and the household is in an urban area, respectively. These variables are constructed in the same way as the expenditure series. Thus, the cohort demographic variables represent the percentage rates of urban dwelling white males in each cohort. This study finds that a large proportion of the sample lives in urban areas and are white. Almost half of the samples are males, and the other half are females. Other binary variables are used in the estimation process; of these, three quarter dummies are created by normalizing the first quarter as there are four quarters in each year from 1996-2009. The quarter dummies are defined as: $quarter2, \ldots, quarter4$. Additionally, time dummies that represent the year are also constructed. The time dummies are denoted as: $year2$, $year3, \ldots, year15$.

The cross-sectional variations in $reduexp_{i,t}$, $white_{i,t}$, $urban_{i,t}$, and $male_{i,t}$ are presented in Table 1.3. This result provides useful information about the model covariates and summarizes the variation within individuals as well as across individuals in a panel. The variable $reduexp_{i,t}$ has higher variation between cohorts than within. This is logical and expected as there should be considerable gap between cohort variation; earlier cohorts (older) are further along in their educational life-cycle. The variables $white_{i,t}$ and $male_{i,t}$ also have higher variation between cohorts. Alternatively, the variation over time of individuals (within) is greater than the between variation for the variable $urban_{i,t}$. Thus, having a variable such as $urban_{i,t}$ may serve to capture the within variation of variable $reduexp_{i,t}$.

### 2.4 Spending on Outside of the Consumer Unit

The interview survey has a record on higher education expenditures over those individuals who are non-CU members (not living with the survey households anymore) that are not included in $reduexp_{i,t}$. Figure 1.1 illustrates the average of higher education spending on family members living outside the CU with respect to the age of individual reference person. The figure exhibits a peak starting around the age of 43 to the age 63 for the reference person (head of CU). Outside of this age range, expenditures are significantly smaller. The highest average expenditures by the household are approximately \$275 when the head's age is 53.

The characteristics of Figure 1.1 mimic the real life fact that heads of families who are ages 40-60 would most likely have children starting their journey as a college student. As stated before, because there is no distinct way to determine the identity of the individuals and their relationship within the CU (if at all), this study can not say with certainty the spending is for children who live outside the CU. However, economic reasoning suggests that the peak of Figure 1.1 is most likely due to spending on children of the family who are currently not living in the consumer unit.

## 3. Empirical Specification

### 3.1 Specification of the Estimation Models

To capture both cases of the null (downward sloping everywhere) and the alternative hypotheses (hump shaped) regarding the higher education expenditures profile, the regression model includes a third order polynomial. The polynomial which is nonlinear with respect to age takes the following form:

$$\mathbf{x}_{i,t}\boldsymbol{\beta} = age_{i,t}\beta_1 + age_{i,t}^2\beta_2 + age_{i,t}^3\beta_3$$

where $\mathbf{x}_{i,t}$ is the vector of polynomials of the main control variable age. Given this specification, this polynomial will possibly have two turning points (discussed in the following section) that are determined by the significance of $\beta_2$ and $\beta_3$.

Besides including $\mathbf{x}_{i,t}\beta$, the overall estimation model controls for cohort and time effects as well as general cohort level demographic characteristics, such as gender and race. In particular, this study specifies the following equation as the Fixed Effects (FE) model:

$$reduexp_{i,t} = \alpha + c_i + \boldsymbol{\lambda}_t\boldsymbol{\gamma} + \mathbf{x}_{i,t}\boldsymbol{\beta} + \mathbf{z}_{i,t}\boldsymbol{\delta} + \varepsilon_{i,t} \tag{1}$$

Though not reported in the estimation results, the parameter $c_i$ is included to capture unobserved variables that are heterogeneous with respect to cohorts. For example, members of older generations in greater numbers may have lower high school graduation rates than younger generations. Thus, these generations are "less able" to demand higher education. The parameters $\boldsymbol{\gamma}$ control for time effect: both time and quarterly seasonality on educational expenditures. Because the enrollment payment system of the U.S. higher education occurs in the first weeks of a semester, one would expect differences in the quarters when payments usually do not occur.

The previous definitions for the time variables, given by $\boldsymbol{\lambda}_t = \{quarter2,\ldots,quarter4\}$ and $\{year2,\ldots,year15\}$ show that this study is controlling for year-to-year changes but not changes from quarters of different years. As in Deaton (1997) and Fernández-Villaverde and Krueger (2011) the coefficients for the fixed effects, quarterly dummies, and time dummies are restricted to sum to zero. Additionally, the coefficients for the time dummies are restricted to be orthogonal to a year time trend (demeaned). The variable $\varepsilon_{i,t}$ is assumed to be an independent, zero mean, random error. This random term captures all the measurement error in higher education expenditures, as well as any unobserved cross-sectional heterogeneity.

The vector of covariates $\mathbf{z}_{i,t}\delta$ is intended to capture variation of $reduexp_{i,t}$ from changes in cohort demographics that have occurred over time. The specific form used is given as:

$$\mathbf{z}_{i,t}\boldsymbol{\delta} = white_{i,t}\delta_1 + male_{i,t}\delta_2 + urban_{i,t}\delta_3$$

The race parameter $\delta_1$ measures the impact of an individual who decides to attend college and therefore incurs different costs, depending on the choice of an educational institution, family background, and race or ethnicity. Hispanics and Blacks generally have lower college attendance rates than Whites as shown by previous studies.[6] Therefore, Hispanics and Blacks should have a lower average education expenditures than White households, suggesting $\delta_1$ is positive. The parameter $\delta_2$ captures the gender specific impact on $reduexp_{it}$. Though it is conceivable that there are differences in college enrollment by gender and retention rates, a-priori, it is difficult to sign $\delta_2$. The parameter $\delta_3$ is designed to capture the effect of urban or rural differences: assign all urban residents a value of 1 and all rural residents a value of 0.

The second model is the application of Correlated Random Effects (CRE) model of Mundlak (1978), Chamberlain (1982), and Wooldridge (2011). In this case the parameter $c_i$ is modeled by:

$$c_i = \psi + \bar{\mathbf{x}}_i\boldsymbol{\xi} + a_i$$

allowing equation (1) to be rewritten as:

$$reduexp_{i,t} = \alpha + \boldsymbol{\lambda}_t\boldsymbol{\gamma} + \mathbf{x}_{i,t}\boldsymbol{\beta} + \bar{\mathbf{x}}_{\mathbf{i}}\boldsymbol{\xi} + \mathbf{z}_{i,t}\boldsymbol{\delta} + \varepsilon_{i,t} \tag{2}$$

where $\alpha$ and $\varepsilon_{i,t}$ have absorbed the $\psi$ and $a_i$ variables respectively. Equation (2)

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[6] Investment in higher education by race and ethnicity.
http://www.bls.gov/opub/mlr/2014/article/investment-in-higher-education-by-race-and-ethnicity.htm

allows the benefit of the CRE specification is that it allows us to unify fixed and random effects estimation as an efficient estimator. With the CRE approach, one includes time-invariant variables and the parameter $\xi$ is the effect of the time averaged covariates (averaged across the unbalanced panel) on the response variables. Besides being the efficient estimator, this approach leads to a simple robust tests of the correlation between heterogeneity and covariates. This differentiated test, namely the Hausman test, compares random effects and fixed effects. If $\xi = 0$ the model will converge to the traditional random effects model and fixed effects is rejected. A downside to the CRE approach is that it complicates the estimation and testing of the turning points (if any) of the age polynomial. For this reason, if it is the case that $\xi \neq 0$, the FE is preferred to the CRE model.

The final model considered is the partially linear model (PLM) for panel data of Baltagi and Li (2002). The general setup is:

$$reduexp_{i,t} = c_i + \boldsymbol{\lambda}_t \boldsymbol{\gamma} + f(x_{i,t}) + \mathbf{z}_{i,t}\boldsymbol{\delta} + \varepsilon_{i,t}$$

The function $f(x_{i,t})$ is approximated by a $p^k$ series spline, which is a fractional polynomial with pieces defined by a sequence of knots which are smoothly joined. During the estimation, the data is differenced thus removing the fixed effects terms. A benefit of this approach is the lack of restrictive assumptions such as the degree of the polynomial and parameter restrictions of the type used by Deaton (1997). For example, parameter restrictions on the fixed effects are not necessary as the estimator differences the data. Additionally, the non-parametrics free-up identification for the time effects. A deficiency of the method arises in the PLM from the fact that there are no parameter estimates to test for turning points. For this reason, the PLM will only be used for a robustness check of whether the polynomials in the FE and CRE models have embedded too many restrictions to capture the true life-cycle profile.

### 3.2 Estimation of Turning Points

A prediction of the null hypothesis is that higher education expenditures are strictly decreasing with respect to age. In this case, the coefficient $\beta_1$ is expected to be negative, and the coefficients $\beta_2$ and $\beta_3$ are expected to be zero. Alternatively, under the research hypothesis of a hump, $\beta_2$ and $\beta_3$ do not necessarily have to be zero. In this case, $\beta_2$, and $\beta_3$ are expected to be positive and negative, respectively. This can be seen by taking the derivative of $\mathbf{x}_{i,t}\beta$ with respect to age and then solving for the roots of age. The resulting roots are denoted by the vector $\tau$ and given by:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} \frac{-\beta_2-\sqrt{\beta_2^2-3\beta_1\beta_3}}{3\beta_3} \\ \frac{-\beta_2+\sqrt{\beta_2^2-3\beta_1\beta_3}}{3\beta_3} \end{bmatrix} \tag{3}$$

where $\beta_1$, $\beta_2$ and $\beta_3$ are the unbiased estimators.

To determine the distributions of the turning points of $\tau_1$ and $\tau_2$ around the estimates $\beta_1$, $\beta_2$ and $\beta_3$, a first order Taylor series approximation is applied to the vector of equation (3) to give an approximation of the first order expansion and their variance of the form:

$$\tau_{1,2} = e - b\beta_1 + c\beta_2 - d\beta_3 \tag{4}$$

and

$$\begin{aligned} var\left(\tau_{1,2}\right) &= b^2 var\left(\beta_1\right) + c^2 var\left(\beta_2\right) + d^2 var\left(\beta_3\right) \\ &\quad + 2bcCov\left(\beta_1, \beta_2\right) + 2bdCov\left(\beta_1, \beta_3\right) + 2cdCov\left(\beta_2, \beta_3\right) \end{aligned}$$

Plassmann and Khanna (2007) show that the coefficients of the above two equations can be estimated by the following equation set:

$$e = a + b\beta_1 - c\beta_2 + d\beta_3$$

where

$$\begin{bmatrix} a = \frac{-\beta_2 \pm \sqrt{\beta_2^2 - 3\beta_1\beta_3}}{3\beta_3} & b = -\frac{\pm\sqrt{\beta_2^2 - 3\beta_1\beta_3}}{2} \\ c = \frac{\pm\sqrt{\left(\beta_2^2 - 3\beta_1\beta_3\right)}\beta_2}{3\beta_3} - \frac{1}{3\beta_3} & d = \frac{-\beta_2 \pm \sqrt{\beta_2^2 - 3\beta_1\beta_3}}{3\beta_3^2} + \frac{\pm\sqrt{\left(\beta_2^2 - 3\beta_1\beta_3\right)}\beta_1}{2\beta_3} \end{bmatrix}$$

For the CRE model, the turning points and their distributions are defined similarly, albeit slightly more complicated.

## 4. Results and Findings

### 4.1 Results from the Estimation Models

Table 1.4 presents the estimation results for net real education expenditures from the FE model illustrated in equation (1). With 742 observations, the $F$-value for testing of the joint significance of the coefficients rejects the null of zero as indicated by the low $p$-value (essentially 0). The coefficient of $age_{i,t}$, which is -85.72, has a negative sign, implying that higher education expenditures decreases as an individual's age increases. The coefficient of $age_{i,t}^2$ can be treated as an interaction effect of age multiplied by itself. Thus, the quadratic component $age_{i.t}^2$ indicates the direction and the steepness of the curvature when the rate of change of the linear age component is set equal to zero. The regression results find a positive estimate for $age_{i,t}^2$ of 1.46 which stipulates the slope of the higher education does, at some point, becomes less negative and may even become positive (upwardly sloped). Alternatively, the positive $age_{i,t}^2$ means when individual gets older the effect of age on the outcome variable is stronger. For the third polynomial, $age^3_{i,t}$ the coefficient estimate is -0.0079. All three age coefficients are highly significant at the lowest conventional level of significance.

For the set of demographic variables, the coefficient estimates of $white_{i,t}$ and $urban_{i,t}$ show a large positive difference in the mean of the outcome variable for the white individuals residing in urban areas as compared to the non-white and rural resident, holding all other predictors constant. However, only the coefficient on $white_{i,t}$ is statistically significant. The $male_{i,t}$ coefficient has a negative value which can be interpreted as the expected difference in the mean of the outcome variable decreases for that of the male individual as compared to the female, holding all other explanatory variables constant. Nevertheless, this coefficient is insignificant.

The effects of quarter dummies are large enough to affect net real education expenditures. The second and third quarter coefficients provide significant negative values, while the fourth quarter estimates offer significant positive values. This implies that the outcome variable decreases in the second and third quarter as compared with the first quarter, but increases in the fourth quarter compared to the first quarter. This pattern is plausible as it agrees with what one thinks is a standard college payment period, i.e. during the two vital periods of the U.S. education year, Fall and Spring semester.

Using the results from FE model, Figure 1.2 plots the estimated graph of the life-cycle profiles of real higher education expenditures over the age controlling for demographics, quarter and time effects. More specifically, the life-cycle profile is constructed assuming the economy is in the 4th quarter and last time period of the sample for a white male and urban dweller. The graph shows that the education expenditures have a hump. Instead of decreasing continuously (as theory suggests), at the age of approximately 50 expenditures begin to increase until a local peak is reached at around the age of 75. Then, the profile returns to its downward path. On one hand, one can view the pattern as deviating from theory in the range of ages 50-75; higher education expenditures are too high in this range. Another view is that education expenditures are too low prior to age 50. In any event, a statistical test for determination of turning point significance is needed. This is to be done in a subsequent section.

Table 1.5 provides the estimated results from the CRE approach as shown in equation (2). The coefficients of $age_{i,t}$, $age^2_{i,t}$, and $age^3_{i,t}$ have the same signs as compared to the FE model and thus include characteristics of the competing hypotheses. The corresponding magnitude of these parameters are almost identical to that of the FE model. For example, the $age_{i,t}$ coefficient for the CRE model is -88.96 whereas this was -85.72 in the previous FE model. The negative age coefficient for this model indicates that the real average education expenditures should decrease by almost 88.96 real dollars if the individual's age increases by one year holding all else constant. The estimated $age^2_{i,t}$ and $age^3_{i,t}$ coefficients are 1.48 and -0.008 respectively which echoes previous results of the FE model. All the age coefficients are highly significant in this case. Under the null hypothesis of a "no hump" situation, the estimates indicate that the curvature of the life-cycle profiles is interrupted at some age levels. Thus the null hypothesis is also likely to be rejected in the CRE approach.

The CRE model also incorporates the time in-variant variables $\overline{age}_i$, $\overline{age}^2_i$, and $\overline{age}^3_i$ and are in the spirit of imposing extra restrictions to estimate the parameters of the age variables. Notice that the impact of the average cohort age does not undo the negative slope coefficient of $age_{i,t}$; the sum of $age_{i,t}$ and $\overline{age}_i$ is still, as expected, negative. Likewise, the sum of the effects of remaining polynomials of age coefficients are consistent with the theory.

The impact of other demographic regressors on higher education expenditures are, for the most part, similar to those of the FE model. As before, the $white_{i,t}$

coefficient is found to be significant. The size of the coefficient indicates that the average difference in net spending on higher education is 443.15 real dollars higher for the white individual in comparison to the non-white individual. Additionally, the $male_{i,t}$ coefficient is negative -58.56 but insignificant. The last similarity is the positive $urban_{i,t}$ coefficient which shows that individuals living in an urban region spend 297.47 dollars more on net real education expenditures as compared to those in rural areas. However, unlike the FE method, the urban coefficient is significant under the CRE estimation technique.

Recall, when $\xi = 0$, the fixed effects estimation are not appropriate. Implementation of this Hausman test is given by the joint test of $\xi_1 = 0$, $\xi_2 = 0$, and $\xi_3 = 0$. The reported test statistic is the $\chi^2$ (chi-squared) with the value of 18.20 and 3 degrees of freedom has an extremely low $p$-value ($< 0.0004$). Thus, both the FE and CRE models are appropriate as they embed the fixed effects assumption. Because the FE is slightly easier to use (more parsimonious) than the CRE, we favor the FE over the CRE and thus base the remaining analysis on the results from the FE model.

Now consider the results from the partially linear model (PLM). Figure 1.3 plots the estimated non-parametric function of the PLM whose values have been normalized to sum to zero. Notice that the non-parametric function has the same shape as the parametric function estimated by the FE model in Figure 1.2. That is, the effect of age on net expenditures declines up to some point. Around the age of 50, expenditures begin to increase. At another age – around the mid 70's – the effect of age of is negative. Given the similarity across results, this study conclude that the functional form chosen to represent age's effect in the life-cycle is appropriate and not driving the apparent curvature in the life-cycle profile. The opportunity cost and the parental transfers are found to be the two important factors that plays the vital role in observing the hump shaped profile.

### 4.2 Results from the Polynomial Function Specification

To calculate the turning points, their corresponding standard deviations, $t$-values, and $p$-values, the coefficients for $age_{i,t}$, (all parts of the polynomial) are plugged into the equations (3) and (4) that were defined in the section 3.2. Panel A of Table 1.6 summarizes the estimation results for the two turning points. The findings are that there are two turning points, $\tau_1 = 48.58$ and $\tau_2 = 74.85$, with large $t$-values of 18.36 and 9.66, respectively. And, as a result, both turning points are highly significant as indicated by their low $p$-values. It can therefore be concluded that the expenditures on higher education have a statistically important hump starting roughly at age 49. The null hypothesis of this paper is rejected thus establishing the alternative hypothesis in favor of this point. Recall, the null hypothesis of a strictly downward sloping profile is driven by the opportunity costs. Clearly, the hump pattern suggests a wedge between the opportunity cost calculation at around the ages of 49 to 74.

### 4.3 Net versus Gross Expenditures

The CEX data set also has the information on how much of college expenditure was or will be reimbursed (EDREIMBX). By adding the values of JEDUCNET and EDREIMBX a gross higher education expenditures is constructed. The reimbursement payments can come either from the employers or from the people outside the CU. The gross data series can account for any missing expenditures that were discussed in the previous section 2.4.

Using the FE model and taking the gross real education expenditures as a dependent variable, the resulting sets of regression coefficients are then used to redraw the expenditures profile on age. Figure 1.4 plots the estimated graph of the life-cycle profiles of real gross higher education expenditures based on the individual age. As compared to Figure 1.2 of life-cycle profile, we see that the hump still occurring in the same range as with the net data. The only difference is that the hump is slightly bigger in size for the gross sample than for the net sample. These results suggest that applying the gross higher education expenditures did not make any distortion in shaping the life-cycle higher education expenditures profile.

To investigate whether the turning points are statistically significant, the same procedure has been applied to calculate the $t$-values and the standard errors. The summary of the estimation results of the two turning points are presented in Panel B of Table 1.6. The estimation results indicate that there are two turning points, $\tau_1 = 48.03$ and $\tau_2 = 74.85$, at the two age levels which are almost the same as previously found. The $t$-values are found to be 21.25 and 9.35, respectively. The extremely low $p$-values thus reject the null of the turning points being equal to zero.

### 4.4 Outside of CU Comparison

Consider the life-cycle profile together with spending on individuals outside the CU; Figure 1.5 displays both profiles. The peak of the outside spending occurs relatively close to the first turning point (around age 50). The figure suggests that the turning points are related to the family situation with regards to children. That is, children cause individuals to under-consume around the age of 50 and, quite possibly, to over-consume after this period thus leading to a hump in the educational spending profile.

## 5. Conclusion

In summary, this paper documents the life-cycle profiles of higher education expenses on age with special emphasis on the precision of the turning point estimates. Exploiting the CEX data from the first quarter of the year 1996 to the first quarter of the year 2009, the study estimates the higher education life-cycle expenditure profile. The preceding half (before the age of 49) of the profile can be explained by the theory of standard complete life-cycle model of education, one of the main workhorses of modern macroeconomics. However, this research finds that real higher education expenditures have a sizable hump, roughly starting at the age of 49 and peaking at the age of 74. The failure of this textbook theory calls into question whether opportunity costs are the sole driver of human capital accumulation as assumed in the models of Ben-Porath (1967) and Heckman (1978).

A second finding is that, when viewed together, the higher educational life-cycle profile and the profile that relates age of the head of the household and spending on individuals who live outside the family are related. The peak of the outside spending occurs relatively close to the first turning point (age 49). The conclusion is that individuals are making their own higher educational attainment un-smooth for someone else – their children. A puzzling question arises: if consumers desire a smooth and decreasing higher education consumption profile (as theory predicts), why don't they use precautionary savings decisions to achieve this? For sure, the education of children is costly but why incur more welfare loss from a destabilized life-cycle profile?

This study focused mainly on the empirics of the higher life-cycle profile. Finding a suitable theory that generates expenditure humps is a subject matter for future research. For example, non-labor income may be large at higher age levels and therefore inducing a dominant income effect. This will establish a way of forgoing work towards the other substitutes such as schooling and/or leisure. Other propositions may be embedded in the more advanced theories proposed by Guo (2014), Bagchi (2011), Feigenbaum (2007), and Caliendo and Huang (2007).

# REFERENCES

Attanasio, O. P., Banks, J., Meghir, C., and G. Weber. 1999. Humps and Bumps in Lifetime Consumption. *Journal of Business and Economic Statistics* 17(1): 22–35.

Attanasio, O. P., and G. Weber. 1995. Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey. *Journal of Political Economy* 103(6): 1121–1157.

Attanasio, O. P., and G. Weber. 2010. Consumption and Saving: Models of Intertemporal Allocation and their Implications for Public Policy. *Journal of Economic Literature* 48(3): 693-751.

Bagchi, S. 2011. Can Overconfidence Explain the Consumption Hump? A General Equilibrium Inquiry. *Journal of Economics and Finance* 35(1): 41–70.

Baltagi, B., and D. Li. 2002. Series Estimation of Partially Linear Panel Data Models with Fixed Effects. *Annals of Economics and Finance* 3(1): 103-116.

Ben-Porath, Y. 1967. The Production of Human Capital and the Life-Cycle of Earnings. *The Journal of Political Economy* 75(4): 352-365.

Cahuc, P., and A. Zylberberg. 2004. *Labor Economics.* vol. 1, 1 ed. Cambridge: The MIT Press.

Caliendo, F., and K. X. D. Huang. 2007. Overconfidence and Consumption Over the Life-Cycle. *Journal of Macroeconomics* 30(8): 1347–1369.

Casanova, M. 2013. Revisiting the Hump-Shaped Wage Profile. Manuscript, UCLA, Department of Economics.

Chamberlain, G. 1982. Multivariate Regression Models for Panel Data. *Journal of Econometrics* 18(1), 5-46.

Deaton, A. S. 1985. Panel Data from Time Series of Cross-Sections. *Journal of Econometrics* 30(1-2): 109–126.

Deaton, A. S. 1997. The Analysis of Household Surveys: A Microeconometric Approach to Development Policy. Washington D.C.: The World Bank.

Feigenbaum, J. 2007. Can Mortality Risk Explain the Consumption Hump? *Journal of Macroeconomics* 30(8): 844–872.

Fernández-Villaverde, J., and D. Krueger. 2007. Consumption Over the Life-Cycle: Facts from Consumer Expenditure Survey Data. *The Review of Economics and Statistics* 89(3): 552–565.

Fernández-Villaverde, J., and D. Krueger. 2011. Consumption and Saving over the Life-Cycle: How Important are Consumer Durables? *Macroeconomic Dynamics* 15(5): 725–770.

Fowler, S., and E. Young. 2004. The Acquisition of Skills Over the Life-Cycle. Manuscript, MTSU, Department of Economics and Finance.

Gourinchas, P.-O., and J. A. Parker. 2002. Consumption Over the Life-Cycle. *Econometrica* 70(1): 47–89.

Guo, N. L. 2014. Can Borrowing Costs Explain the Consumption Hump? Manuscript, University of Wisconsin-Whitewater, Department of Economics.

Heckman, J. 1978. A Life-Cycle Model of Earnings, Learning, and Consumption. *The Journal of Political Economy* 84(4): S11-44.

Johnson, R. W., and D. Neumark. 1996. Wage Declines Among Older Men. *Review of Economics and Statistics* 78(4): 740–748.

Kraft, H., Munk, C., Seifried, F. T., and M. Steensend. 2014. Consumption and Wage Humps in a Life-Cycle Model with Education. Manuscript, Goethe University Frankfurt, Sustainable architecture for finance in Europe (SAFE).

Mundlak, Y. 1978. On the Pooling of Time Series and Cross Section Data. *Econometrica* 46(1): 69-85.

Murphy, K. M., and F. Welch. 1990. Empirical Age-Earnings Profiles. *Journal of Labor Economics* 8(2): 202–229.

Murphy, K. M., and F. Welch. 1992. The Structure of Wages. *Quarterly Journal of Economics* 107(1): 285–326.

Plassmann, F., and N. Khanna. 2007. Assessing the Precision of Turning Point Estimates in Polynomial Regression Functions. *Econometric Reviews* 26(5): 503–528.

Wooldridge, J. M. 2011. Fractional Response Models with Endogeneous Explanatory Variables and Heterogeneity. CHI11 Stata Conference, Stata Users Group.

# APPENDICES CHAPTER I

# APPENDIX A: TABLES

**Table 1.1: Educational expenses type included in higher education.**

| Item Code | Type of EDUC_AY |
|---|---|
| 300 | Tuition |
| 310 | Housing while attending school |
| 320 | Food or board |
| 345 | Test Preparation |
| 350 | Books purchase, supplies or equipment |
| 360 | Other school related expenses |

**Table 1.2: Descriptive statistics.**

| Variable | Mean | Std. Dev. | Minimum | Maximum |
|---|---|---|---|---|
| $reduexp_{i,t}*$ | 68.54 | 133.52 | 0 | 839.43 |
| $age_{i,t}$ | 50.72 | 20.32 | 18 | 87 |
| $age^2_{i,t}$ | 2,985.11 | 2,114.676 | 324 | 7,569 |
| $age^3_{i,t}$ | 193,653.1 | 184,553.7 | 5,832 | 658,503 |
| $white_{i,t}$ | 0.8337 | 0.0310 | 0.7614 | 0.9308 |
| $male_{i,t}$ | 0.4660 | 0.0366 | 0.3312 | 0.5652 |
| $urban_{i,t}$ | 0.9113 | 0.0280 | 0.8325 | 1 |
| *No. of Obs.* | 742 | 742 | 742 | 742 |

Note: * means real education expenditures after adjusted for inflation.

Table 1.3: **Decomposition of standard deviations (between and within).**

| Variable | | Mean | Std. Dev. |
|---|---|---|---|
| $reduexp_{i,t}$ | Overall | 68.5383 | 133.52 |
| | Between | | 140.11 |
| | Within | | 72.14 |
| $white_{i,t}$ | Overall | 0.8336 | 0.0310 |
| | Between | | 0.0297 |
| | Within | | 0.0148 |
| $male_{i,t}$ | Overall | 0.4659 | 0.0366 |
| | Between | | 0.0352 |
| | Within | | 0.0163 |
| $urban_{i,t}$ | Overall | 0.9113 | 0.0280 |
| | Between | | 0.0171 |
| | Within | | 0.0234 |

**Table 1.4: Regression results from the FE model. Dependent variable: $\mathbf{reduexp_{i,t}}$**

| Variable | Coefficient | Std. Dev. | $t$-value | $p$-value |
|---|---|---|---|---|
| $age_{i,t}$ | -85.7297*** | 6.3979 | -13.40 | 0.000 |
| $age^2_{i,t}$ | 1.4603*** | 0.1236 | 11.81 | 0.000 |
| $age^3_{i,t}$ | -0.0079*** | 0.00072 | -11.04 | 0.000 |
| $white_{i,t}$ | 271.4435* | 150.4577 | 1.80 | 0.072 |
| $male_{i,t}$ | -114.9745 | 154.5859 | -0.74 | 0.457 |
| $urban_{i,t}$ | 82.4613 | 164.4187 | 0.50 | 0.616 |
| $quarter2$ | -16.2859*** | 3.3776 | -4.82 | 0.000 |
| $quarter3$ | -9.6839*** | 2.8955 | -3.34 | 0.000 |
| $quarter4$ | 25.9698*** | 3.8751 | 6.70 | 0.000 |
| $constant$ | 1346.084*** | 240.201 | 5.60 | 0.000 |
| $\sqrt{MSE}$ | 52.467 | | | |
| $N$ | 742 | | | |
| $Cohorts$ | 15 | | | |
| $F_{32,709}$ | 49.52<br>($p$=.0000) | | | |

Note: robust standard errors are recorded, *significant at 10% level, **significant at 5% level, ***significant at 1% level. Time dummies used in estimation are not reported.

Table 1.5: **Regression results from the CRE model. Dependent variable: $\mathbf{reduexp_{i,t}}$**

| Variable | Coefficient | Std. Dev. | $t$-value | $p$-value |
|---|---|---|---|---|
| $age_{i,t}$ | -88.9615*** | 6.5648 | -13.55 | 0.000 |
| $age^2_{i,t}$ | 1.4813*** | 0.1249 | 11.86 | 0.000 |
| $age^3_{i,t}$ | -0.0080*** | 0.0007 | -10.94 | 0.000 |
| $\overline{age}_i$ | 19.2352*** | 6.7570 | 2.85 | 0.005 |
| $\overline{age}^2_i$ | -0.2982** | 0.1337 | -2.23 | 0.026 |
| $\overline{age}^3_i$ | 0.0015* | 0.0008 | 1.86 | 0.063 |
| $white_{i.t}$ | 443.1503** | 154.5844 | 2.87 | 0.004 |
| $male_{i,t}$ | -58.5589 | 157.8138 | -0.37 | 0.711 |
| $urban_{i,t}$ | 297.4755** | 131.1777 | 2.27 | 0.024 |
| $quarter2$ | -16.3060*** | 3.3679 | -4.84 | 0.000 |
| $quarter3$ | -9.5024*** | 3.0911 | -3.07 | 0.000 |
| $quarter4$ | 25.8085*** | 4.1244 | 6.26 | 0.000 |
| $constant$ | 723.7424*** | 178.5764 | 4.05 | 0.000 |
| $\sqrt{MSE}$ | 54.6887 | | | |
| $N$ | 742 | | | |
| $Cohorts$ | 15 | | | |
| $F_{22,719}$ | 61.20<br>($p$=.0000) | | | |

Note: robust standard errors are recorded, *significant at 10% level, **significant at 5% level, ***significant at 1% level. Time dummies used in estimation are not reported.

**Table 1.6: Turning point analysis.**

| Turning points | Value | Std. Dev. | $t$-value | $p$-value |
|---|---|---|---|---|
| (A) Net Expenditures | | | | |
| $\tau_1$ | 48.5830*** | 2.6451 | 18.3673 | 4.9723e-62 |
| $\tau_2$ | 74.1601*** | 7.6744 | 9.6633 | 7.5669e-21 |
| (B) Gross Expenditures | | | | |
| $\tau_1$ | 48.0339*** | 2.2600 | 21.2542 | 4.2482e-78 |
| $\tau_2$ | 74.8594*** | 8.0049 | 9.3516 | 1.0817e-19 |

Note: *significant at 10% level, **significant at 5% level, ***significant at 1% level. Standard deviations are computed by the delta method.

# APPENDIX B: FIGURES

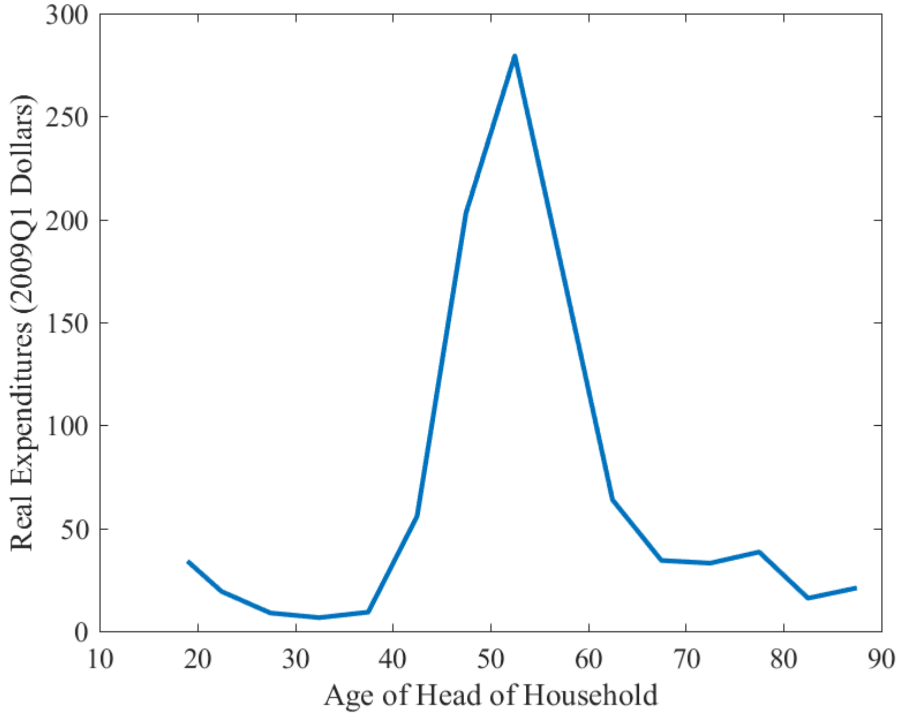

**Figure 1.1: Higher education spending on individuals outside the CU.**

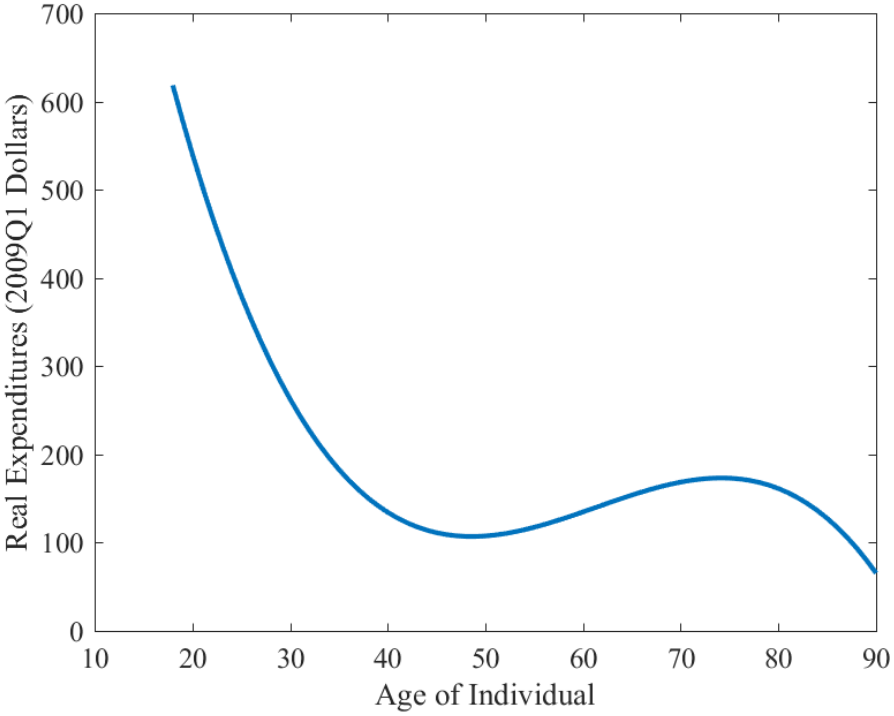

**Figure 1.2: The empirical life-cycle profile for net higher education expenditures.**

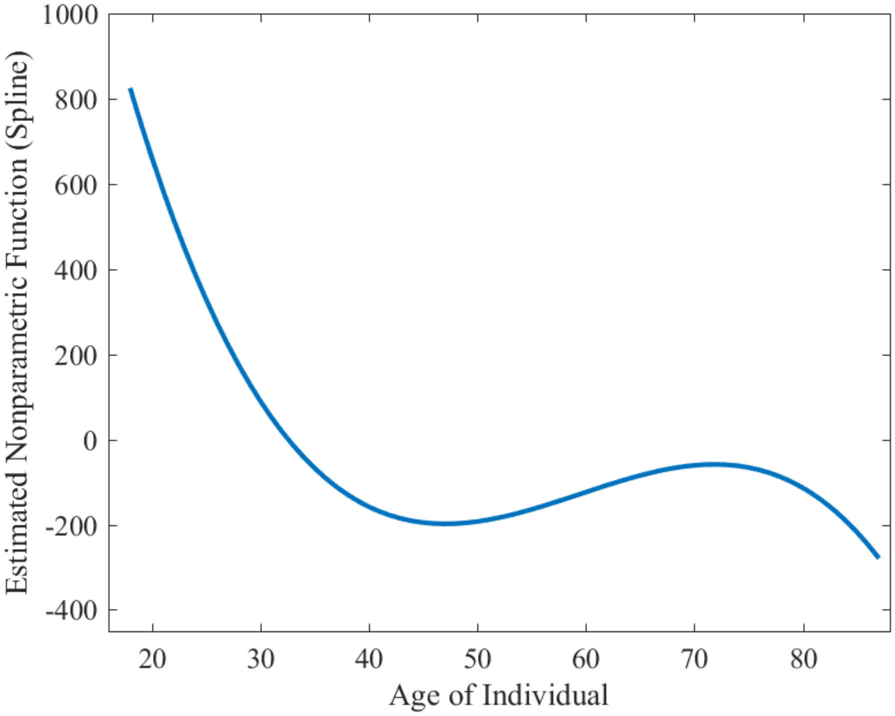

**Figure 1.3: Semi-nonparametric estimation of the life-cycle profile by the PLM (net expenditures).**

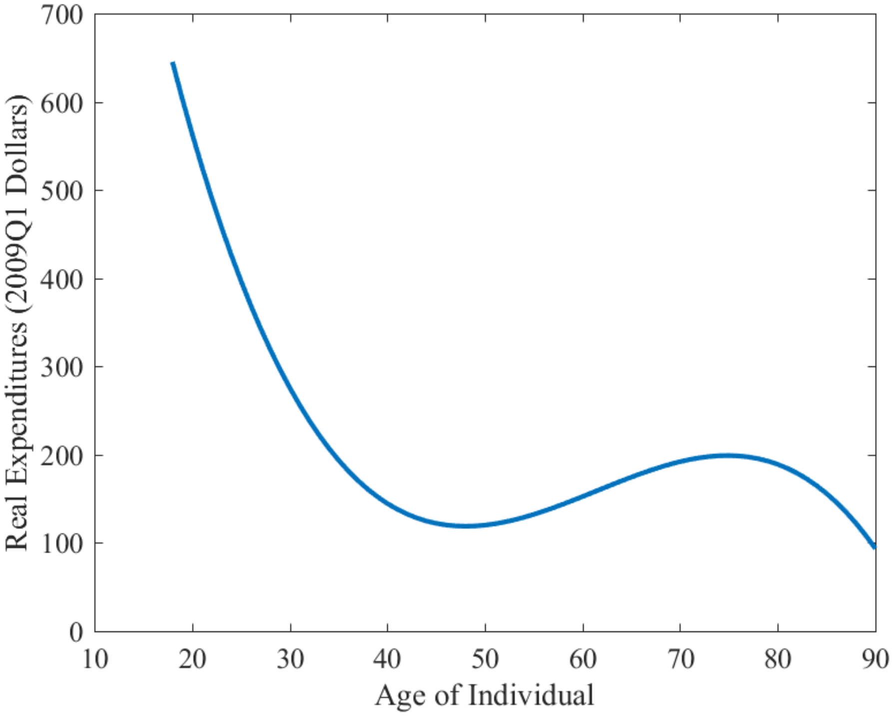

**Figure 1.4: The empirical life-cycle profile for gross higher education expenditures.**

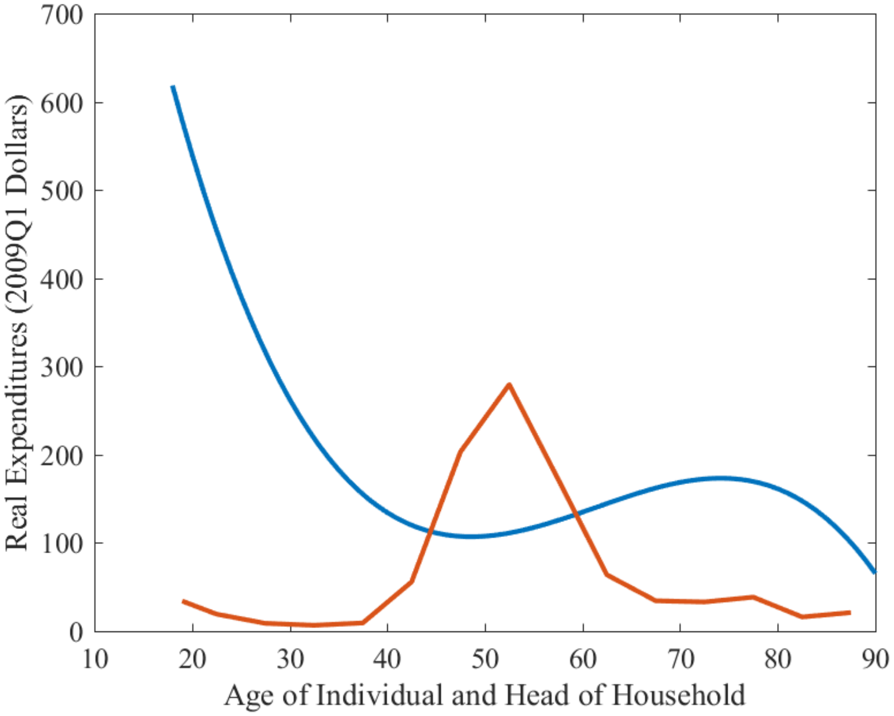

**Figure 1.5: Comparison of the life-cycle profile and spending outside the CU (net expenditures).**

# CHAPTER II

# DELAYS IN HIGHER EDUCATION CONSUMPTION: THE ROLE OF PARENTAL ALTRUISM AND BORROWING CONSTRAINTS

## 1. Introduction

The purpose of this paper is to quantify the impact of "parental altruism" on the individual life-cycle human capital expenditures profile. The recent empirical paper by Salimullah (2017) shows that individuals in their late fifties provide significant financial help to their family members (probably to be their children) when they are going to college, whether or not they live in the same household. Apparently, an individual parent makes financial transfer to their children to help their offspring financially when they are schooling. Each individual age-group allocates their time to perform their activities that produce earnings, physical capital and additions to the stock of human capital. However, this is not the case for all individuals who stay at different stages of their life-cycle.

For the case of altruistic parents—who care more about their children's welfare—individuals spend more for their offspring's higher education when they are at the middle age point of their life-cycle because for them the substitution effect dominates over the income effect. They supposed to substitute leisure with the work hours by spending more for their kids. This, in turn, compels them to work more labor hours for spending a significant amount for their own higher education consumption later after that age limit. This is because they can accumulate less physical capital due to transfers. This pattern of individuals' behavior causes a "hump" shaped life-cycle profile of the higher education expenditures instead of a continuously decreasing profile with respect to age. Starting from the past decade, human capital accumulation theory draws important attention to the field of economic literature when the impact of borrowing constraints and parental altruism is taken into consideration. Typically, borrowing constraints mean that children cannot borrow against their future income to finance their education when they are young, and the arguments in favor of this borrowing constraint leads to an inefficient accumulation of human capital, which causes a decrease in childrens' welfare.

Previous studies exhibit that the existence of borrowing constraints could be a better welfare mechanism for young aged people. This is because the majority of the parents care about the welfare of their descendants and it is true in most of the cases that they transfer or leave bequests accordingly for their successor. In an standard overlapping generations (OLG) model framework, Soares (2015) shows that if a borrowing constraint does not allow children to borrow against future income, the average level of individual as well as social welfare can be improved by raising aggregate savings that cause physical capital to rise. The imposition of borrowing constraints therefore, lead to a higher parental transfer, which in return raises children's welfare. On the other hand, if there are no borrowing hurdles, children could be able to fund their own higher levels of education, resulting in an increase in the aggregate level of human capital with a lower level of physical capital and parental bequests.[1]

The OLG model framework of this study has been constructed with the presumption that children are economic agents with the same preferences as adults, allocating resources to the acquisition of human capital and enjoying consumption, work, or leisure. Furthermore, it is assumed that parents care about their children's welfare

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[1] For instance, Ayiagari, Greenwood, and Seshadri (2002) make a representative statement in a model with parental altruism where they suggest that the average agent can not be better off in the incomplete market economy as opposed to the efficient one by comparing the equilibrium under borrowing constraints to that of unconstrained one.

in the sense that parents maximize their utility by taking into consideration their children's lifetime utility function. Therefore, being rational and forward-looking economic agents, parents make their decisions by taking into account the full impact of their children's future income levels.[2] In other research, Fowler and Young (2004) focus on the role of human capital acquisition and its possible impact on productivity boom and bust conditions. Young age individuals reduce their labor hours to forgo certain income due to the higher opportunity cost of obtaining skills when there is a productivity boom situation. During the productivity bust, young age people turn out to be both human capital and income poor agents. This will produce less consumption and savings over their lifetime because they will accumulate replacement human capital in their middle and older ages.

This paper calibrated the results between two parts. The first part of the results show the structure of the benchmark model with an OLG framework, which accounts for borrowing constraints that have effects on welfare through the aggregate levels of physical and human capital.[3] The regular hump shape in the physical capital profile intensifies the welfare gains of introducing borrowing constraints. As individuals grow older, they could earn and save more and the accumulation of capital also increases. The young faces the trade-off between low opportunity cost to work and relatively low opportunity cost to invest in human capital. Hence, the stock of human capital will be low with younger individuals but produce an upward trend with respect to age. Wages increase with tenure, seniority, experience and specialization, so labor hours will be a downward sloping curve based on age when an individual substitutes work by enjoying

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[2] In separate studies, Rangel (2003) and Boldrin and Montes (2005) convey that government support policies for education and social security can be a good option to imitate the possibility of unconstrained equilibrium. They argue that governmental support policies could help to optimize the allocations related to the unconstrained equilibrium and have predicted the idea in which liquidity constraints could be the inefficient competitive equilibrium and may be even worse than the unconstrained equilibrium.

[3] Earlier research reveals that an economy with borrowing constraints can increase aggregate savings and, hence, the aggregate level of physical capital and human capital.

more leisure. The benchmark model has produced a monotonically decreasing human capital expenditures profile because individuals' investment on human capital will decrease as they age more. For all age groups, the consumption profile generates the usual upward sloping shape which indicates that individuals desire to smooth out their consumption expenditures over their life-cycle.

The second part of the calibration results incorporate the parental transfers for the children and show the quantitative simulation over the baseline model. In this study, the explicit values for parental altruism have been computed by taking the weighted ratio of the marginal utilities of younger agents of different age groups who enter into the maximization problem of the older agents. The results show that when parental altruism is introduced then currently living younger agents are better off because they have more income to spend and face higher opportunity cost of attending school. This causes the young agents to accept more labor hours as compared to the agents in the middle age group. Nevertheless, the stock of human capital and the human capital expenditures profile for the young agents fall below that of the benchmark profile. On the contrary, the older agents (also to be the individual parents) who will bear less human capital expenditures in their middle age, will now accept a clear hump shaped profile for educational expenditures at the later part of their life due to the transfers made for their offspring at their mid 50's. Furthermore, the increase in human capital expenditures for higher age group individuals' amplifies the welfare loss of the parents as opposed to the welfare gains for their children.

The rest of the paper is organized as follows. Section 2 presents related literature and the empirical hump. Section 3 lays out the economic model of borrowing constraints and parental altruism. Section 4 presents the calibrations and discuss the solution method. Section 5 presents the modeling results, findings and comparisons. Section 6 states the concluding remarks.

## 2. Related Literature

This paper reviews a number of theoretical literature and empirical evidence that shows the relationship between the existence of borrowing constraints and human capital accumulation in a general equilibrium framework based on the overlapping generations model with parental altruism. The contribution of older generations of households of financing the education for their younger generations could be modeled in several ways depending on the socio-economic conditions under which the education regime is entertained. In earlier research, Kotlikoff and Summers (1981) estimate that intergenerational transfers accredit to 70-80% of private financial wealth in the U.S. and only 20-30% attribute to own life-cycle savings.[4]

This paper modeled the generous "parental altruism" and investigates the impact of financial transfers on individuals life-cycle profile for higher education expenditures in a standard OLG model. As noted earlier, one of the major researches in this area and more or less related to this present study has been conducted by Soares (2006, 2015). Indeed, Fowler and Young (2004) also study the cyclical behavior of the acquisition of skills for the individuals over their life-cycle. The calibrated model predicts that the substitution effect dominates the wealth effect for all age groups which implies the opportunity-cost considerations tend to make schooling countercyclical. However, their prediction is found incompatible with the data, which shows schooling by the young is procyclical. In the end, they replicate the procyclicality of schooling for the young and countercyclicality of schooling for the old only when human capital acquisition shocks positively correlated with the productivity shock. Their paper is closely related to this current research to the extent of its evolution about the human capital model.

In an overlapping generations model with altruism, Soares (2015) explains that

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[4] For more information see Christian Groth (2015) Macroeconomic Lectures Series.

borrowing constraints against the future income for children increases welfare in the long-run. His paper also explores the impact of parental altruism on welfare through their effect on the aggregate levels of physical and human capital. The author presumes that the inability to borrow against future income creates impediment for children's investment in education, decreasing the aggregate level of human capital. On the other side, the existence of borrowing constraints could allow aggregate savings to increase and, hence, the aggregate level of physical capital. His calibrated general equilibrium version of the model shows how parental altruism results in financial transfers that children allocate to consumption and education and raises children's welfare. In another earlier research, Altig and Davis (1989) employ the two-period model with altruism and find that borrowing constraints for children cause a substantial amount of parental transfers and therefore, increase the welfare of the agents in the long-run. However, in their long-run stationary equilibrium, the welfare gains for the younger agent is attributed to pecuniary effects of borrowing constraints where the increase in wages come from the rise in capital that results from the forced increase in savings.

Aiyagari, Greenwood, and Seshadri (2002) argue that if the currently living agents face the borrowing constraint then the average level of welfare for future agents will decrease. This is because parental altruism for children causes the older generations to make dissaving for the later part of their life. But, in their model, the authors only take into account the welfare for the adults to compute their average welfare measurement, and they did not consider the children as an active agents. Most of their papers are done by using OLG model over time instead of using life-cycle profile i.e. individuals age. In a separate paper, Accolley (2015) develops a dynamic deterministic general equilibrium model to investigate the contribution of human capital to economic growth characterized through the households caring about their children's welfare. His simulated model assumes that there is a permanent rise in the tuition rate for all the private educational institutions and in turn the ability of learning for households also rise. He found that each of these two shocks reveals a positive correlation between education, human capital, and output. The predictions of the model are used latterly to make statements on the student crisis due to the following decision to increase tuition fees that Quebec witnessed in 2012. His paper predicts that raising tuition will neither harm education nor negatively impact students' ability to pay.

Recently, Salimullah (2017) conducts empirical research that estimates the effect of an individual's age on higher education expenditures. The findings of this research come up with a clear hump in the higher education expenditures life-cycle profile for the later part of individual's age. The two turning points that occur at two different age periods namely, 49 and 74, are found statistically significant. A second finding of that paper is that higher education spending on individuals who live "outside" the consumer unit peak when the head of the household is around 50. Presumably, this spending was for the children of the family who are attending college but no longer live at home.

These two findings suggests that, with respect to higher education, children cause individuals to under-consume at the age limit of 50 to 53 and to over-consume before and after this age period. It therefore, leads to a hump in the educational spending profile. The paper empirically proves the theoretical life-cycle higher education expenditures profile by exploiting the consumer expenditure survey data and establishes the relationship between an individual's age and higher education expenditures with a missing hump over the later part (say, between the age limit of 49 and 75) of the individual's life. Figure 2.1 describes the intuitive idea of the education expenditures hump that are related with the turning points and the family situation with regards to children.

## 3. The Economic Model

Let us consider an economy where a large number of agents are born with identical preferences and identical initial capital stocks in each period. Agents from generation $t$ live for $\mathcal{I}$ periods and then disappear. At any point in time, there is a set of agents indexed by their life-cycle period number $\tau \in \mathcal{I} = \{0, 1, 2, \cdots, \mathcal{I}—1\}$. For example, at time $t$, the newly born agents are in life-cycle period $\tau = 0$ while the oldest agents are in life-cycle period $\tau = \mathcal{I} - 1$. Besides, the model has three other sectors, namely the household sector, the production sector, and the human capital acquisition sector. Individuals allocate their time for each period for work, education, and leisure.

### 3.1 The Benchmark Model

### 3.1.1 The Households

Let us assume the economy with a typical OLG model by taking into account the important assumption that the economic agent makes a choice of work, education, and leisure over his/her life. The theoretical formation of the utility function for a household born can be defined by the agent's utility function:

$$u(c^t, n_1^t, n_2^t) = u(c_{t+\tau}^t, n_{1,t+\tau}^t, n_{2,t+\tau}^t)$$

where c is the consumption, $n_1$ is time allocated as labor, and $n_2$ is time devoted to human capital accumulation.Therefore, the time $t$ dynamic expected utility maximization problem of an agent born in life-cycle period $\tau = 0$ is specified by the following discounted sum of lifetime utility:

$$\underset{\left\{c_{t+\tau}^t, n_{1,t+\tau}^t, n_{2,t+\tau}^t\right\}_{\tau=0}^{\mathcal{I}-1}}{max} E\left\{\sum_{\tau=0}^{\mathcal{I}-1} \beta^\tau \Psi_\tau u(c_{t+\tau}^t, n_{1,t+\tau}^t, n_{2,t+\tau}^t)\right\}$$

The term $\Psi_\tau = \prod_{i=0}^{\tau} \psi_i$ denotes the unconditional probability of surviving up to age $\tau$ with each $\psi_\tau$ representing the conditional probability of surviving from age $\tau - 1$ to $\tau$, $\beta$ is the subjective discount factor, and $u(.)$ is a momentary utility function. The control variables $c_{t+\tau}^t$, $n_{1,t+\tau}^t$ and $n_{2,t+\tau}^t$ denote time $t+\tau$ consumption, labor hours, and human capital acquisition hours for an agent born at time $t$, respectively. Finally, $k_{t+\tau+1}^t$ and $h_{t+\tau+1}^t$ are time $t+\tau+1$ physical capital and human capital stocks, respectively. The functional form of the utility function is assumed to take the following index:

$$u(c, n_1, n_2) = \frac{c^{1-\gamma}}{1-\gamma} + \rho \frac{(1 - n_1 - n_2)^{1-\mu}}{1-\mu} \tag{1}$$

The parameters $\gamma$ is the Arrow-Pratt coefficient of relative risk aversion; $\rho$ indicates the weight parameter on leisure; and $\mu$ determines the labor supply elasticity which is selected to match the calibration by Heathcote, Storesletten, and Violante (2004). This utility function is consistent with balanced growth only if one of the following two conditions hold: (i) $\gamma = 1$ and (ii) $\rho$ grows at the rate of technological progress. The marginal utilities with respect to $c$, $n_1$, $n_2$ can be formulated from equation (1) as:

$$u_c(c, n_1, n_2) = c^{-\gamma}$$

$$u_{n_1}(c, n_1, n_2) = -\rho (1 - n_1 - n_2)^{-\mu}$$

$$u_{n_2}(c, n_1, n_2) = -\rho (1 - n_1 - n_2)^{-\mu}$$

The budget constraints that a typical consumer born at time $t$ and any time $t+\tau$ where $\tau \in \mathcal{I}$ will face:

$$c_{t+\tau}^t + k_{t+\tau+1}^t \leq (1 + r_{t+\tau} - \delta_k) k_{t+\tau}^t + w_{t+\tau} h_{t+\tau}^t n_{1,t+\tau}^t \tag{2}$$

$$h_{t+\tau+1}^{t} \leq q_{t+\tau}^{t} + (1-\delta_h)\, h_{t+\tau}^{t} \tag{3}$$

where $w$ is the labor wage rate, $h$ is the stock of human capital accumulation, $r$ is the rate of return to physical capital, $\delta$'s are the depreciation rates, and $q$ is a human capital production function operated by each agent. Equation (2) denotes the standard expenditure identity; expenditures on consumption and investment must be less than or equal to income. Incomes are from previous capital holdings (net of depreciation $\delta_k$) and effective labor. Equation (3) relates human capital production to investment in human capital thus following Heckman (1978) and Fowler and Young (2004).

Human capital is produced by the following production function where previously accumulated human capital aids in the acquisition of new human capital when $\theta_1 > 0$:

$$q_{t+\tau}^{t} = \left(h_{t+\tau}^{t}\right)^{\theta_1} \left(n_{2,t+\tau}^{t}\right)^{\theta_2}$$

for all $t$ and $\tau \in \mathcal{I}$. The marginal products with respect to human capital and hours are denoted, respectively, as:

$$q_{h,t+\tau}^{t} = \theta_1 \left(h_{t+\tau}^{t}\right)^{\theta_1 - 1} \left(n_{2,t+\tau}^{t}\right)^{\theta_2}$$

$$q_{n_2,t+\tau}^{t} = \theta_2 \left(h_{t+\tau}^{t}\right)^{\theta_1} \left(n_{2,t+\tau}^{t}\right)^{\theta_2 - 1}$$

The input of goods into the production of human capital and the exogenous shock that may shift the efficiency of human capital technology are not considered in this case. This assumption facilitates equilibrium computations and is not restrictive as these goods could be concentrated out, re-interpreting $n_{2,t}$ as a goods-time investment composite (Heckman, Lochner, and Taber 1998).[5]

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[5] One restrictive assumption has been taken that is, the measure of physical capital is not included

### 3.1.2 The Firms

Firms combine the capital stock with labor services to produce goods according to a constant returns to scale production functions. Specifically, let us assume that the aggregate output from a firm is produced by following a Cobb-Douglas set up and technology:

$$Y_t = F\left(K_t, EN_t\right)$$

$$Y_t = K_t^{\alpha_k}\left(EN_t\right)^{1-\alpha_k}$$

where $K_t = \sum_{\tau=0}^{\mathcal{I}-1} k_t^{t-\tau}$ represents the aggregate capital stock and the inner product of $EN_t = \sum_{\tau=0}^{\mathcal{I}-1} h_t^{t-\tau} n_{1,t}^{t-\tau}$ represents the total effective labor input. The parameter of the production function $\alpha_k$ is assumed to satisfy the condition $0 < \alpha_k < 1$.

According to the competitive nature of the pricing policy, it can be ensured that all factors are paid with respect to their marginal products. The marginal productivity of an effective labor hour from the $\tau$th person will be equal to its real price:

$$w_t = MP_{EN_t} = \left(1-\alpha_k\right) K_t^{\alpha_k}\left(EN_t\right)^{-\alpha_k}$$

The marginal productivity of a unit of capital from the $\tau$th person will also be equal to its real price. Symbolically,

$$r_t = MP_{K_t} = \alpha_k K_t^{\alpha_k - 1}\left(EN_t\right)^{1-\alpha_k}$$

where $w$ is the labor wage rate, $r$ is the return to physical capital. All households receive the same return on capital, but there is a nontrivial distribution of wages over the life-cycle.

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in the production function of human capital. This can be excluded because human capital production is likely to be relatively labor-intensive.

## 3.2 OLG Model with Parental Altruism

### 3.2.1 A Simple Model with Altruism

Let us begin with a simple three period economy, where two types of agents live in the first two periods, first as a child then as an adult. The young agent (child) is born in each of the first two periods. The young agent becomes a parent in their second period and would be considered as an older agent (adult) and derives utility from their own consumption by incorporating the children's lifetime utility into their utility function. The older agent will not be available at the last period of the economy.

The young agent born in period $t \in \tau$ where $\tau = 1, 2, 3$ maximizes his/her discounted lifetime utility that can be expressed by the following dynamic equation:

$$\mathcal{V}_{t+\tau} = \underset{\{c_{t+\tau}\}}{max} \left\{ u\left(c^t_{t+\tau}\right) + \beta u\left(c^t_{t+\tau+1}\right) + \beta\beta_p \mathcal{V}_{t+\tau+1} \right\} \tag{4}$$

where $\mathcal{V}_{t+\tau}$ is the discounted lifetime utility of a child in period $\tau$ and $\beta > 0$ is the intertemporal discount factor and $\beta_p > 0$ is the altruism discount factor at which the child's lifetime utility is discounted by their parent. $u\left(c^t_{t+\tau}\right)$ is the utility function that is assumed to be strictly increasing and strictly concave. It is also assumed to be twice continuously differentiable and the Inada condition is assumed to be satisfied by this utility function.[6] $c^t_{t+\tau}$ denotes the consumption of the young individual in period $\tau$ and $c^t_{t+\tau+1}$ is the consumption to the next period. At the last period (i.e. at period 2) for this economy, the adult has no children, so $\mathcal{V}_{t+3} = 0$.

The adult that maximizes their discounted lifetime utility in the first period is given by:

$$\mathcal{V}_{t+\tau+1} = \underset{\{c_{t+\tau+1}\}}{max} \left\{ u\left(c^t_{t+\tau+1}\right) + \beta_p \mathcal{V}_{t+\tau} \right\}$$

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[6] Inada conditions, named after Japanese economist Ken-Ichi Inada, make assumptions about the shape of a production function and it is used to guarantee the stability of an economic growth path specifically for the neoclassical growth model in the field of macroeconomics.

Individuals earn labor income $w_{t+\tau}$ at the period $\tau$ and also accumulate assets. The budget constraints facing individuals at time $\tau$ can be written as:

$$c^t_{t+\tau} = g^t_{t+\tau} + w_{t+\tau} - k^t_{t+\tau}, \forall\tau = 1, 2$$

$$c^t_{t+\tau+1} = (1 + r_{t+\tau} - \delta_k)\, k^t_{t+\tau+1} - g^t_{t+\tau} + w_{t+\tau+1} n_{1,t+\tau+1}, \forall\tau = 1, 2, 3$$

where $g^t_{t+\tau}$ represents the resource transfer to their children by a parent (altruistic behavior of parents), $k_{t+\tau}$ denotes the physical capital holdings of an older agent at time $\tau$, and $r_{t+\tau}$ denotes the exogenous rate of return on these capital and $\delta_k$ denotes the rate of depreciation.

The first-order conditions obtained from equation (4) with respect to consumption $c^t_{t+\tau}$, asset holdings $k^t_{t+1+\tau}$, and parental transfers $g^t_{t+\tau}$, subject to the above two budget constraints will end up in the following two intra-temporal Euler equations:

$$u_1\left(c^t_{t+\tau}\right) = \beta\,(1 + r_{t+\tau} - \delta_k)\, u_1\left(c^t_{t+\tau+1}\right), \forall\tau = 1, 2$$

$$u_1\left(c^t_{t+\tau+1}\right) = \beta_p u_1\left(c^t_{t+\tau}\right), \forall\tau = 1, 2 \tag{5}$$

where $u_1(c^t_{t+\tau})$ and $u_1\left(c^t_{t+\tau+1}\right)$ denotes the marginal utilities of consumption by the young and older agents in period $\tau$ respectively. For any observed level of children's preference, the equation (5) can be used to determine the level of parental transfers and the two budget constraints can be used as the reflection of differential relation between the level of parental transfers, parents' accquisiton of wealth, and children's preference.

Soares (2015) evaluates that the marginal utility of transfers for parents can be computed by the difference between the marginal utility of their own consumption and the marginal utility of their children's consumption. The two intra-temporal Euler equations capture that marginal utility of consumption increases for children when there is a decrease in consumption without parental transfers, and this, in turn, increases parents' marginal utility of asset's transfer for their children. Hence, other things being constant, parental transfers are an increasing function of the parent's assets, $k^t_{t+\tau}$ and the children's savings, $k^t_{t+1+\tau}$.

### 3.2.2 An Extended Model with Altruism

Now, let us recall the economy where a large number of identical agents are born in each period and live at first as a children and then as adults for their timeline $\mathcal{I}$ periods, at most. Individuals for each generation maximize their discounted lifetime utility by including their own consumption and the lifetime utility of their children. Therefore, the maximum value function for someone born in period $\tau = 0$ is given by:

$$\mathcal{V}_{t+\tau} = \underset{\{c_{t+\tau}, n_{1,t+\tau}, n_{2,t+\tau}\}}{max} \left\{ u\left(c^t_{t+\tau}, n^t_{1,t+\tau}, n^t_{2,t+\tau}\right) + \beta \mathcal{V}_{t+\tau+1} \right\}, \tag{6}$$

while the value function for older agents will be:

$$\mathcal{V}_{t+\tau+1} = \underset{\{c_{t+\tau+1}, n_{1,t+\tau+1}, n_{2,t+\tau+1}\}}{max} \left\{ u\left(c^t_{t+\tau+1}, n^t_{1,t+\tau+1}, n^t_{2,t+\tau+1}\right) + \beta_p \mathcal{V}_{t+\tau} \right\},$$

subject to the budget constraint facing an individual of age $i$ at time $t$ will be:

$$c^t_{t+\tau} = g^t_{t+\tau} + w_{t+\tau} h^t_{t+\tau} n^t_{1,t+\tau} - k^t_{t+\tau};$$

$$c^t_{t+\tau+1} = \left(1 + r_{t+\tau} - \delta_k\right) k^t_{t+\tau+1} - g^t_{t+\tau+1} + w_{t+\tau+1} h^t_{t+\tau+1} n^t_{1,t+\tau+1}, \; \forall \tau = 0, 1, 2, \ldots, \mathcal{I} - 1$$

Recall, $\beta$ is the intertemporal discount factor, and $\beta_p$ is the discount factor for parental altruism. Older agent is assumed to have children in the second period of his/her lives. A parent values his/her children's consumption, human capital accumulation, and the hours devoted for labor because he/she cares for their well-being. Furthermore, children's preferences are considered to be the same as adults over their own consumption, labor hours, and human capital hours. As before, $g_{t+\tau}^{t}$ represents the resources (in terms of the consumption goods) given by the parents to their children. In another sense, these are the resources received by young agents from their older age parents.

Individuals have the available maximum amount of time in each period and allocate it for consumption, work, and education. In the first period of their lives, agents can choose how much time they allocate to these three categories. Individuals can work for $(\mathcal{I}-1)$ periods supplying $n_{1,t+\tau}^{t}$ hours of labor and earning $w_t h_{t+\tau}^{t} n_{1,t+\tau}^{t}$; where $w_t n_{1,t+\tau}^{t}$ is the real hourly wage rates per unit of labor hour and $h_{t+\tau}^{t}$ is the older agent's stock of human capital in period $t$, respectively. In the last period of their lives, they retire or may die in the next period with their assets taken as bequests.

If the value function denoted by equation (6) is solved for the first order conditions, subject to the budget constraints, then the following two intra-temporal Euler equations will be generated:

$$u_1\left(c_{t+\tau}^{t}, n_{1,t+\tau}^{t}, n_{2,t+\tau}^{t}\right) = \beta\left(1+r_{t+\tau}-\delta_k\right) u_1\left(c_{t+\tau+1}^{t}, n_{1,t+\tau+1}^{t}, n_{2,t+\tau+1}^{t}\right);$$

$$u_1\left(c_{t+\tau+1}^{t}, n_{1,t+\tau+1}^{t}, n_{2,t+\tau+1}^{t}\right) = \beta_p u_1\left(c_{t+\tau}^{t}, n_{1,t+\tau}^{t}, n_{2,t+\tau}^{t}\right);$$

where $u_1(.)$ denotes the marginal utilities of consumption for all agents in period $\tau$ where $\tau = 0, 1, 2, \ldots, \mathcal{I}-1$.

### 3.3 Characterization of the Equilibrium

The environment of the benchmark model and the model with parental altruism assumes that a typical agent makes a choice of consumption, savings, and leisure over his/her life. The formal dynamic programming problem of a household born agent at time $t$ and live for time $t+\tau$ is formulated by the following value function:

$$\mathcal{V}_{\tau}\left(s_{t+\tau}^{t}, S_{t+\tau}\right) = max\left\{u(c_{t+\tau}^{t}, n_{1,t+\tau}^{t}, n_{2,t+\tau}^{t}) + \beta\frac{\Psi_{\tau+1}}{\Psi_{\tau}}\left[\mathcal{V}_{\tau+1}\left(s_{t+\tau+1}^{t}, S_{t+\tau+1}\right)\right]\right\},$$

where $s_{t+\tau+1}^{t} = \left\{k_{t+\tau+1}^{t}, h_{t+\tau+1}^{t}\right\}$ and $S_{t+\tau+1} = \{K_{t+\tau+1}, H_{t+\tau+1}\}$;
subject to:

$$c_{t+\tau}^{t} + k_{t+\tau+1}^{t} \leq \left(1 + r_{t+\tau} - \delta_k\right) k_{t+\tau}^{t} + w_{t+\tau} h_{t+\tau}^{t} n_{1,t+\tau}^{t}\,;$$

$$h_{t+\tau+1}^{t} \leq q_{t+\tau}^{t} + \left(1 - \delta_h\right) h_{t+\tau}^{t}\,;$$

The above equation includes the additional term $\Psi$ for exogenous survival rate of the population and the aggregate state variables contain only the mean values for $K$ and $H$ but not the entire distribution. The next two sections of this paper will explain the calibrations in detail and also the life-cycle profiles for human capital expenditures by using the results from Krusell and Smith (1998), Fowler and Young (2004), and mostly revisiting the ideas from Soares (2015). Indeed, it turns out that the computed policy function ends up with a significant hump shaped human capital expenditures profile over the individuals' life-cycle, further reducing the computational burden of this model. Given the behavior of the higher education expenditure profiles presented by Salimullah (2017), this study is now in a position to explore the argument developed in the previous empirical life-cycle hypothesis.

Let us assume that the law of motion for the capital stocks, consumption, human capital, labor demand functions, initial conditions, and the transitions for the states variables are given. So, the solution of the following problem can be characterized as:

$$\mathcal{V}_{\tau}\left(t+\tau\right)=max\left\{u(c^{t}_{t+\tau},n^{t}_{1,t+\tau},n^{t}_{2,t+\tau})+\beta\frac{\Psi_{\tau+1}}{\Psi_{\tau}}\left[\mathcal{V}_{\tau+1}\left(t+\tau+1\right)\right]\right\},$$

subject to: (i) the terminal condition $\mathcal{V}_{\mathcal{I}}(t)=0\,\forall t$; (ii) non-negativity conditions $c^{t}_{t}, k^{t}_{t+\tau}, h^{t}_{t+\tau}\geq 0$, $\forall t,\tau$; (iii) $k^{t}_{t}=0$, $and\, k^{t}_{t+\mathcal{I}}=0$, $\forall t$; and (iv) the budget constraints;

Assume the budget constraints hold at each time $t+\tau$ for each agent $t$ then the optimal behavior of the households can be characterized by the following sets of the intra-temporal and intertemporal Euler equations:

$$u_{1}\left(t+\tau\right)=\beta\frac{\Psi_{\tau+1}}{\Psi_{\tau}}\left\{u_{1}\left(t+\tau+1\right)\left(1+r_{t+\tau+1}-\delta_{k}\right)\right\}; \tag{7}$$

$$-u_{2}\left(t+\tau\right)=u_{1}\left(t+\tau\right)h^{t}_{t+\tau}w_{t+\tau}; \tag{8}$$

$$-\frac{u_{3}\left(t+\tau\right)}{q^{t}_{n_{2},t+\tau}}=\beta\frac{\Psi_{\tau+1}}{\Psi_{\tau}}\left\{u_{1}\left(t+\tau+1\right)w_{t+\tau+1}n^{t}_{1,t+\tau+1}-\frac{u_{3(t+\tau+1)}}{q_{n_{2},t+\tau+1}}\left[q^{t}_{h,t+\tau+1}+1-\delta_{h}\right]\right\}; \tag{9}$$

where $u_{1}\left(t\right)=\dfrac{\delta u(.)}{\delta c_{t}}$, $u_{2}\left(t\right)=\dfrac{\delta u(.)}{\delta n_{1,t}}$ and $u_{3}\left(t\right)=\dfrac{\delta u\left(.\right)}{\delta n_{2,t}}$. The above equations (7)-(9) must hold at any time $t$ for each consumer born at time $t-\tau$ where for $\tau\in\mathcal{I}$.

## 4. Calibration and Solution Method

### 4.1 Calibration Benchmark Model

This section deals with a set of previously computed values of parameters that can be estimated independently by the model or are based on estimates provided by other relevant literature and data. Usually, these parameters are such that the predictions generated by the model can match with the given set of targets. This paper has taken the estimates that Fowler and Young (2004) also used to calibrate their model. Table

2.2 lists the group of calibrated parameters chosen for estimating the benchmark life-cycle profiles for all the choice variables of this research.

The relative risk aversion parameter (also the Arrow-Pratt coefficient of relative risk aversion) is found $\gamma = 1.44$, which falls in the range commonly used in the macroeconomics literature. The persistence parameter on leisure is set to $\rho = 0.75$, so the average fraction of time devoted to market activities is 0.33. The parameter $\mu$ determines the labor supply elasticity which is selected to match the calibration of Heathcote, Storesletten, and Violante (2004); the value is $\mu = 2.36$. The length of the life-cycle, $\mathcal{I}$, must be set a priori. In this OLG model, it is assumed that agents make economic decisions over, roughly, a 72 year period. If economic life is to start after their first year of college that is, at 18 years of age, then the terminal age is 89 for the old agents with the assumption that there is no change in the labor force supply schedule. To keep the computation compliant for the symmetry of the OLG model, however, the life-cycle is condensed so that $\mathcal{I} = 24$. In this case, each period represents 3 years with the same labor force participation occurring at the end of period 23.

The values for survival probabilities are presented in Table 2.1. These probabilities are found by converting the annual mortality probabilities from the U.S. Life Tables of the National Center for Health Statistics (1992). The probabilities for this paper are selected for $\mathcal{I} = 23$ life-cycle. To calibrate the parameters of the human capital function this paper employs the estimates from Heckman, Lochner, and Taber (1998) which is $\theta_1 = \theta_2 = 0.52$. This parametric prediction implies that there are increasing returns to scale in the human capital technology but conditional on current human capital hours. The technology shock is set to be constant which implies a mean value of $\overline{A} = 0.4049$ and the initial human capital level is set to $\overline{h}_t^t = 9.530$ and the human capital depreciation rate is $\delta_h = 0.0$.

The rest of the parameters that also need to be calibrated are $\{\beta, \phi, \sigma\}$ and $\{\alpha_k, \delta_k, k_t^t\}$. At first, $\alpha_k = 1/3 = 0.33$ is calibrated and defined as a share of income attributed to physical capital in the aggregate output for the United States. The average discount factor $\beta$ is calibrated as a fixed value compatible with a yearly psychological rate of 3% and gives the ultimate level of $\beta = (1.03)^{-3} = 0.9154$. The initial capital stock is set to $k_t^t = 0$. The rate of depreciation of capital $\delta_k$ is set at 6% per year; thus the three year depreciation rate is $\delta_k = 1-(1-0.06)^3 = 0.1694$. Finally, the parameters relative to the technology process is set to produce an autocorrelation of 50% for output and these targets imply $\sigma = 0.01$ and $\phi = (0.50)^3 = 0.125$.

## 4.2 Calibration Parental Altruism

The calibration of the altruistic discount factor $\beta_p$ is much more straightforward. Recall, the intra-temporal Euler equations from the OLG model with parental altruism and the assumption that households live for 24 life-cycle periods and each period is composed of three years. Clearly, it matches the ratio of the weighted average of the marginal utilities of consumption for 15 distinct groups of agents from the households of that 24 life-cycle periods. An agent in their first life-cycle period is assumed to be the age range of $\{18, 19, 20\}$ which is also equivalent to $\tau = 0$ and an agent in their last life-cycle period has the age range of $\{87, 88, 89\}$ which is equivalent to $\tau = 23$. Hence, the computation for parental altruism has been done by having the ratio of the marginal utilities of the older agents to the marginal utilities of the younger agents. i.e.

$$\beta_{p,\tau} = \frac{u_1(c_t^{t-\tau}, n_{1,t}^{t-\tau}, n_{2,t}^{t-\tau})}{u_1(c_t^t, n_{1,t}^t, n_{2,t}^t)}, \forall \tau = 9, 10, \cdots, 23$$

The 9 economic agents of that 72 year period, starting from age range of $\{18, 19, 20\}$ up to the age range of $\{42, 43, 44\}$ is represented by a three year interval and are considered to be the receivers of the parental transfers. The next 6 economic agents starting from the age range of $\{45, 46, 47\}$ up to the life-cycle period of $\{60, 61, 62\}$ are considered to be the receivers as well as the lenders of the parental transfers. This is shown in Table 2.3 by the values of $\beta_p^{(1,1)}$ to $\beta_p^{(1,9)}$. The rest of the 9 economic agents, starting from the life-cycle period of $\{63, 64, 65\}$ up to the age range of $\{87, 88, 89\}$ are considered to be the lenders of the parental bequest. Table 2.3 captures those calibration as the altruistic discount factors $\beta_p^{(1,10)}$ to $\beta_p^{(1,15)}$.

Figure 2.3 provides the plot of the values of altruistic parameters over the individual agents who are in the life-cycle period $\tau = 9$ i.e. age $\in \{45, 46, 47\}$ to the life-cycle period of $\tau = 15$ i.e. age $\in \{87, 88, 89\}$. It generates a faster increasing trend from the group of agent's age $\{45, 46, 47\}$ and reveals a decrease in the marginal utility of children's consumption by raising the parental transfers. The model in this study essentially captures a life-cycle framework and the interactions between "parental altruism" and children's welfare takes an important role in this case. Thus, Figure 3 highlights the joint interactions of different age group of agents by filling the values of parental altruism and the children in their different life-cycle stages.

## 5. Quantitative Results

### 5.1 Benchmark Life-Cycle Profile

This section presents the calibrated benchmark model and compares the results with the facet about the historical life-cycle human capital accumulation theory of Ben-Porath (1967) and Heckman (1978). All five panels of Figure 2.2 display the typical pattern behavior of the various macroeconomic sector's series over the individual life-cycle. The first panel (a) of Figure 2.2 plots the consumption for each age group for the simulated benchmark economy. The figure captures the continuously increasing and smooth profiles for consumption with a slight decreasing trend at the later period of the individual's age. This can be comparable with the standard model of consumption that simply suggests that households smooth their consumption across periods of high and low income to keep expected marginal utility constant.

Panel (b) engenders the predictable decreasing shape of the labor hour with respect to age. This means that individuals' when young work more and gradually cut off their labor hours when they get older. This is because wages increase with respect to experience, seniority, tenure and specialization and leisure becomes more superior goods for them. Thus, the willingness to substitute current leisure with earnings at the middle part of their life has a high opportunity cost. Panel (c) shows the correlation between human capital expenditures and the individual's age. As predicted, human capital expenditures profile generates a monotonically decreasing pattern based on all age groups. Individuals spend more on higher education when they are young at the cost of sacrificing earnings due to lower opportunit cost. This in turn, supports the previous idea about the theoretical life-cycle profiles.

Panel (d) observes the correlation between physical capital and age. The hump pattern of this graph is found consistent with the previous empirical results. The pattern of behavior is that individuals accumulate an increasing amount of capital with their age because typically individuals earn more income or assets when they grow old. The last panel (e) postulates that the stock of human capital acquisition is an increasing function of age. This fact is straightforward and complies with the prediction of the model. Individuals' acquire more knowledge along with their education and experience over their age and as a result, household's willingness to supply additional units of time for human capital acquisition causes the individual life-cycle human capital profiles to monotonically increase.

In total, these benchmark results are generally consistent with the analysis of previous findings by Campbell and Deaton (1989), Attanasio and Weber (1995), and

Gourinchas and Parker (2002). Additionally, the life-cycle model with human capital acquisition is generally consistent with the infinitely lived agent model studied by DeJong and Ingram (2001) where human capital acquisition is found to counteract with the fluctuations in an economic cycle. Though the baseline model can explain aggregate behavior and retains the behavior found in the infinitely lived framework, the model is unable to account for the parental caring about their children's welfare. Nevertheless, the "parental altruism" by the middle aged group who help their child financially with schooling and for attaining skills by forgoing their own higher education consumption is found indeterminate in many previous studies and literature. The next section will provide all of these human capital profiles by incorporating the parental altruism.

### 5.2 Life-Cycle Profile with Altruism

The central role played into the shape of human capital expenditures profile by the altruism mechanism is described in Figure 2.4. To make it comparable and easily understandable, the benchmark figures for each profile are also plotted simultaneously with the figures that included parental altruism. Panel (a) generates hump shaped consumption expenditures profile over the individuals' life-cycle as opposed to the baseline function. This is interpreted as the evidence that during the middle age of the individuals' life-cycle, the agents have more consumption expenditures in the form of durable goods such as educational expenditures and parental transfers. Panel (b) plots the new profile for labor hours with that of the baseline labor hour graphs. The profile with parental altruism shows the young employ for more hours to work as they become rich because they get transfers. The opportunity cost of investment for human capital is very high for young because they can earn more income and hence, the physical capital or savings for them also rise (panel d). However, their labor hours gradually decrease when they grow old and somewhat poorer when they reach middle age. The profile for labor hours eventually falls below the benchmark profile because middle aged individuals get transfers from the very old. At this age level those individuals spend more on their higher education to accumulate more human capital by sacrificing more hours to work.

The figure in Panel (c) represents the correlation between human capital expenditures and age. It shows that older agents come up with a hump shaped human capital expenditures profile at the later part of their age as compared with the baseline continuously decreasing profile. The profile falls below the baseline model because young have less college attendance and lower level of human capital expenditures and shows typical downward shape until the age, say 49. However, the human capital expenditures profile turns upward at the latter part of the middle aged individual and represents an increase of human capital expenditures by about 50%. This means, due to the parental transfers of resources by the middle-aged people for their children's education and having a lower level of physical capital, made the middle aged agents more poorer, and hence they have to spend more for their own higher education consumption. The profile has a downward shape again after the individual age of 70. Thus, the reader could observe a "hump" shaped pattern of expenditures profile instead of monotonically decreasing everywhere based on age.

The findings of this graph mimic the empirical results and data plot of the paper conducted by Salimullah (2017). On that study, it has been shown that the head of the household of a same consumer unit who is between the ages 40 to 60 would most likely have children starting their journey as a college student. This in turn, causes the transfers from parents to be significantly higher and leads to a hump in their higher education spending plot for individuals living outside of their families. The increase in parental transfers takes place in order to eliminate, mostly, the burden of schooling expenses of the younger generations and the parents use their transfers from older generations to finance their own current consumption and leisure.

Panel (d) captures the difference between the profiles of physical capital for both the benchmark model and the model with parental altruism. The shape of the profile with that of the parental altruism shows young have more savings because they get transfers from their parents and due to the high opportunity cost of attending schools they have a lower human capital stock. Since physical capital profiles are a reflection of an individual's consumption, work, and leisure hour, so the lower life-cycle profile of physical capital around the middle-age indicates that individuals cut back their labor hours and substitute it with more leisure and parental transfers. As the individuals within the age limit of 30 to 50 have low income due to lower labor hour and lower stock of human capital, their physical capital profiles fall below the benchmark model during that period. Panel (e) shows the profiles of human capital stock over the individual's age. Although it concurs to be an increasing shape over the individuals' age, it always falls below the shape of the benchmark model because young have the lower investment in human capital accumulation due to the parental transfers.

Panel (f) of Figure 2.4, the altruistic transfers profile, shows that when the resources transferred by the parents to their offspring at the middle age of an individual causes the middle age group to become more poorer. The intertemporal substitution effect dominates the income effect from a parental point of view because they must pay to the young generations by forgoing their own physical capital or savings. Although they receive transfers from the very old age generations, the transfers are not enough to outweigh the substantial amount of transfers from current parents to their kids. Therefore, this factor of altruism is one of the crucial ones and has been overlooked in many previous studies.

Finally, the analysis of those figures shows that emphasizing "parental altruism" over borrowing constraints can make individual's offspring better off by placing children's utility function into their parents' life-time utility function. This in turn raises the human capital expenditures for individual parents at the later part of their life. Additionally, the increase in parental transfers is reflected by the higher amount of consumption and education expenditures, and leisure activities of the parents at the beginning and the latter part of their life-cycles. This calibration technique provides the set of values for parental altruism parameters that has increased, somewhat, for the oldest generation in the short-run.

## 6. Conclusion

This paper mainly investigates the impact of "parental altruism" on individual's life-cycle human capital accumulation by setting up a quantitative OLG model through calibration. The findings of this human capital accumulation model shows that if there is a borrowing constraint for children while attending college then the altruistic transfers from parents cause young agents to spend less on their higher education due to the higher opportunity cost. This transfer causes the young to become rich and they work more hours for earning more physical capital and make less investment on the stock of human capital.

On the other hand, parental transfers cause the individual middle age parents to become more poor and face less amount of human capital stock as well as stock of physical capital. Although they get transfers from the very old agents, the intertemporal substitution effect dominates the middle age group's income effect in this case because they prefer more leisure than work and they make transfers for their offspring. Eventually, the middle age group faces a substantial plunge into their consumption and education expenditures profile with a lower amount of physical capital accumulation. That is, this type of agents (say, around the age of 49 to 70) needs to incur more human capital expenditures when they decide to go back to college for their own higher education consumption.

The paper uses the standard calibration technique and the parameter values from the previous literature to produce these quantitative results. The comparison of the result reveals that the benchmark OLG model produces the conventional shape of the consumption expenditures, the labor hours, stock of human capital and the physical capital accumulation life-cycle profile. The human capital expenditures profile for the benchmark model shows the monotonically decreasing shape with respect to individual's age while the expenditures profile with parental altruism falls below it and shows turning points for the middle age parents over their life-cycle, and thus individual parents ended up with a hump shaped human capital profile at the later part of their life.

Additionally, the study reveals that the increase in human capital expenditures for higher age group individuals' decrease the average welfare of currently living agents but increase the welfare for the young. In a calibrated model where children invest in human capital, the welfare impact of a borrowing constraint can be quite large. So, agents can be better off in an economy with incomplete credit markets, and setting public policy instruments in order to replicate the complete markets equilibrium that might not necessarily improve the welfare of the older generation.

# REFERENCES

Accolley, D. 2015. Altruistic Overlapping Generations of Households and the Contribution of Human Capital to Economic Growth. MPRA Paper (No. 69972). Munich Personal RePEc Archive. Munich, Germany.

Aiyagari, S. R., Greenwood, J., and A. Seshadri. 2002. Efficient Investment in Children. *Journal of Economic Theory* 102: 290-321.

Altig, D., and S. Davis. 1989. Government Debt, Redistributive Fiscal Policies, and the Interaction Between Borrowing Constraints and Intergenerational Altruism. *Journal of Monetary Economics* 24: 3-29.

Attanasio, O. P., and G. Weber. 1995. Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey. *Journal of Political Economy* 103(6): 1121–1157.

Ben-Porath, Y. 1967. The Production of Human Capital and the Life-Cycle of Earnings. *The Journal of Political Economy* 75(4): 352-365.

Boldrin, M., and A. Montes. 2005. The Intergenerational State Education and Pensions. *Review of Economic Studies* 72(3): 651-664.

Campbell, J. Y., and A. Deaton. 1989. Why is Consumption so Smooth? *Review of Economic Studies* 56: 357–374.

DeJong, D. N., and B. F. Ingram. 2001. The Cyclical Behavior of Skill Acquisition. *Review of Economic Dynamics* 4: 536-561.

Fowler, S. J., and E. R.Young. 2004. The Acquisition of Skills Over the Life-Cycle. Working Paper Series (No. 200402). Middle Tennessee State University, Department of Economics and Finance. Murfreesboro, TN.

Gourinchas, P.-O., and J. A. Parker. 2002. Consumption Over the Life-Cycle. *Econometrica* 70(1): 47–89.

Heathcote, J., Storesletten, K. and G. L. Violante. 2004. The Cross-Sectional Implications of Rising Wage Inequality in the United States. Working Paper, Georgetown University, Washington, DC.

Heckman, J. 1978. A Life-Cycle Model of Earnings, Learning, and Consumption. *The Journal of Political Economy* 84(4): S11-44.

Heckman, J., Lochner, L., and C. Taber. 1998. Explaining Rising Wage Inequality: Explorations with a Dynamic General Equilibrium Model of Labor Earnings with Heterogeneous Agents. *Review of Economic Dynamics* 1: 1-58.

Kotlikoff, L. J., and L. H. Summers. 1981. The Role of Intergenerational Transfers in Aggregate Capital Accumulation. *Journal of political economy* 89(4): 706-732.

Krusell, P., and A. A. Smith Jr. 1998. Income and Wealth Heterogeneity in the Macroeconomy. *Journal of Political Economy* 106(6): 867-896.

National Center for Health Statistics. 1992. U.S. Decennial Life Tables 1989-1991, Centers for Disease Control, U.S. Government, Washington D.C.

Rangel, A. 2003. Forward and Backward Intergenerational Goods: Why is Social Security Good for the Environment? *American Economic Review* 93(3): 813-34.

Salimullah, A. H. M. 2017. Humps in Higher Education Expenditure Over the Life-Cycle: Evidence from Consumer Expenditure Survey Data. Unpublished PhD. Dissertation Chapter. Middle Tennessee State University, Department of Economics and Finance. Murfreesboro, TN.

Soares, J. 2006. Borrowing Constraints, Parental Altruism and Human Capital Accumulation. Meeting Papers (No. 516). Society for Economic Dynamics.

Soares, J. 2015. Borrowing Constraints, Parental Altruism and Welfare. *Journal of Macroeconomics* 45: 1-20.

# APPENDICES CHAPTER II

# APPENDIX A: TABLES

Table 2.1: Calibrations for survival probabilities.

| | | |
|---|---|---|
| $\psi_0 = 1.0000$ | $\psi_1 = 0.9990$ | $\psi_2 = 0.9993$ |
| $\psi_3 = 0.9992$ | $\psi_4 = 0.9993$ | $\psi_5 = 0.9994$ |
| $\psi_6 = 0.9993$ | $\psi_7 = 0.9989$ | $\psi_8 = 0.9984$ |
| $\psi_9 = 0.9975$ | $\psi_{10} = 0.9963$ | $\psi_{11} = 0.9945$ |
| $\psi_{12} = 0.9925$ | $\psi_{13} = 0.9904$ | $\psi_{14} = 0.9872$ |
| $\psi_{15} = 0.9827$ | $\psi_{16} = 0.9759$ | $\psi_{17} = 0.9681$ |
| $\psi_{18} = 0.9600$ | $\psi_{19} = 0.9496$ | $\psi_{20} = 0.9364$ |
| $\psi_{21} = 0.9210$ | $\psi_{22} = 0.8999$ | $\psi_{23} = 0.8775$ |

**Table 2.2: Calibration parameters for benchmark model economy.**

| | | | | |
|---|---|---|---|---|
| Preferences | $\beta = 0.9151$ | $\gamma = 1.44$ | $\mu = 2.36$ | $\rho = 0.75$ |
| Human (Capital) Technology | $\theta_1 = \theta_2 = 0.52$ | $\overline{A} = 0.4049$ | $\delta_h = 0.0$ | |
| Physical Capital | $\alpha_k = 0.333$ | $\delta_k = 0.1694$ | | |
| Initial Stock | $k_t^t = 0.0$ | $\overline{h}_t^t = 9.530$ | | |
| Technology | $\phi = 0.125$ | $\sigma = 0.01$ | | |

**Table 2.3: Estimates of Parental Altruism Parameter.**

| Altruistic Discount Factor | Values | Altruistic Discount Factor | Values |
|---|---|---|---|
| $\beta_p^{(1,1)}$ | 0.2747 | $\beta_p^{(1,9)}$ | 0.6461 |
| $\beta_p^{(1,2)}$ | 0.2493 | $\beta_p^{(1,10)}$ | 0.7899 |
| $\beta_p^{(1,3)}$ | 0.2437 | $\beta_p^{(1,11)}$ | 0.9827 |
| $\beta_p^{(1,4)}$ | 0.2647 | $\beta_p^{(1,12)}$ | 1.1318 |
| $\beta_p^{(1,5)}$ | 0.3058 | $\beta_p^{(1,13)}$ | 1.1700 |
| $\beta_p^{(1,6)}$ | 0.3706 | $\beta_p^{(1,14)}$ | 1.1409 |
| $\beta_p^{(1,7)}$ | 0.4452 | $\beta_p^{(1,15)}$ | 1.0595 |
| $\beta_p^{(1,8)}$ | 0.5347 | | |

# APPENDIX B: FIGURES

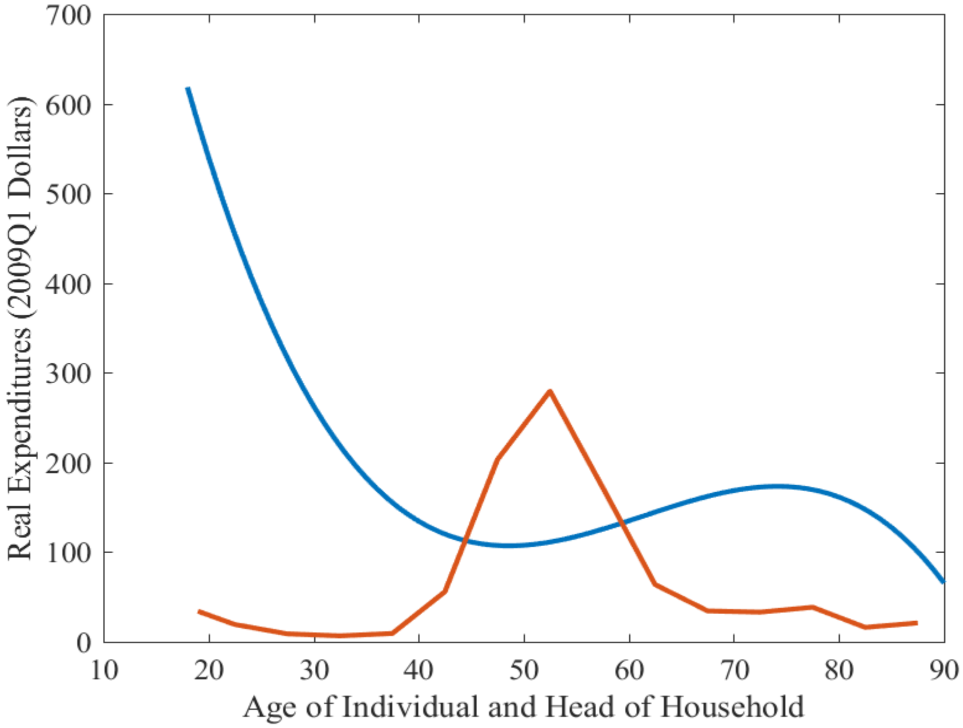

**Figure 2.1: Comparison of the life-cycle profile for higher education expenditures and parental transfers to children.**

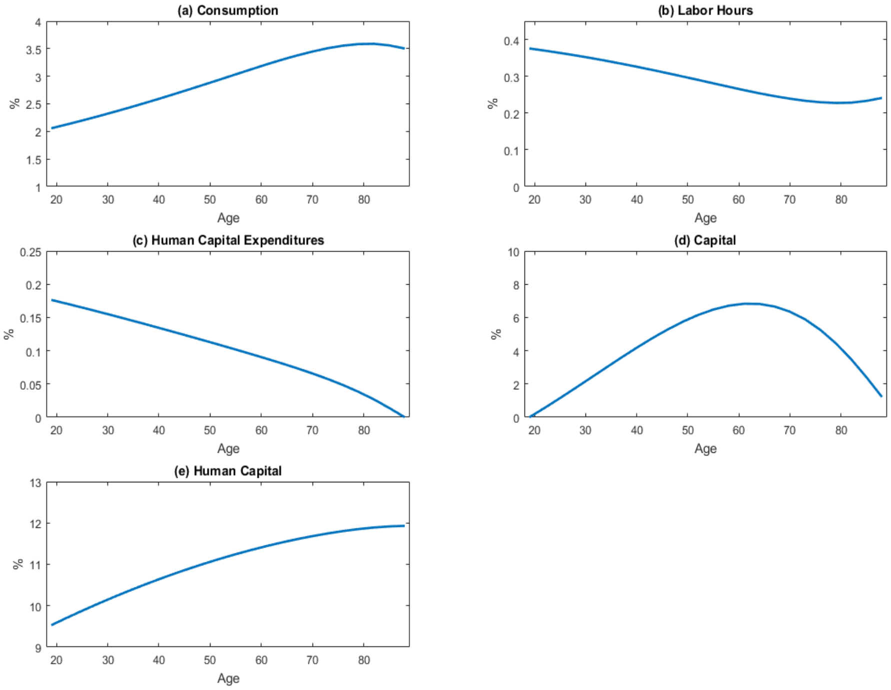

**Figure 2.2: Benchmark Life-cycle Profiles for Consumption, Labor Hours, Physical Capital and Human Capital.**

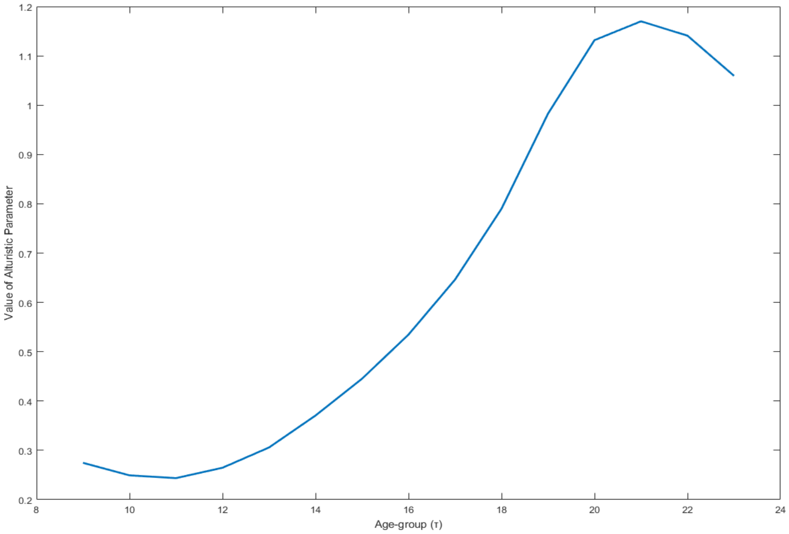

**Figure 2.3: Graphical Plot of Altruistic Parameter.**

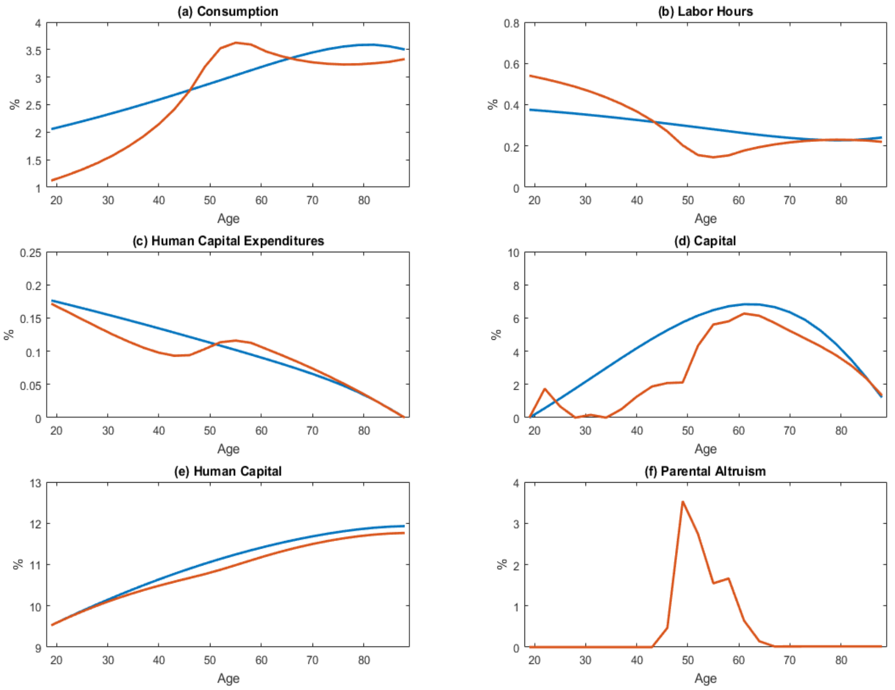

**Figure 2.4: Life-cycle Profiles with Parental Altruism Comparison.**

# CHAPTER III

# GRANGER CAUSALITY TESTS AND CO-INTEGRATION OF INTEREST RATE, EXCHANGE RATE, AND STOCK VOLATILITY AT THE CHICAGO OPTIONS MARKET

## 1. Introduction

Stock volatility is the relative rate at which the price of a security moves up and down. Volatility is found by calculating the annualized standard deviation of a daily change in price. If the price of a stock moves up and down rapidly over a short period of time, we say it has high volatility. On the other hand, if the price almost never changes, we say it has low volatility. In finance, volatility is a measure of the price variation of a financial instrument over time. Several indicators have been developed over the years, such as the S&P 500 Volatility Index, the NASDAQ Volatility Index, the Russell 2000 Volatility Index, etc. to track the status of broad market volatility and help investors decide when to buy or sell stocks. Stock prices rarely move in a straight line. Most of the time they move up and down, even sometimes trending higher and lower. Interest rate and foreign exchange rate risks are two significant economic and financial factors that affect the common stock value. The interest rates indirectly affect the valuation of the stock prices, and stock volatility directly creates a shift between the money market and capital market instruments.[1] The performance of the stock market can reflect the overall performance of a country's economy. When

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[1] The higher interest rates cause businesses and consumers to spend more on servicing debt. This makes borrowings from money and capital market more expensive and the availability of capital to investment goes down. As a result, the stock prices also move downward and thus, tends to create a change betweeen borrowing and lending of short-term assets and long-term assets.

the stock market is doing well, it may imply that the economy is experiencing high growth.

Gul and Ekinci (2006) state that advances in the stock market affect the exchange rate through liquidity and wealth effects; a rise in the interest rate increases the opportunity cost of holding cash balances and, therefore, creates a negative impact on money demand. This reduction in money demand creates an excess supply of credit and stimulates a decrease in stock prices. A decrease in stock prices reduces the wealth of domestic investors, which lowers their demand for money further. Banks then react by lowering interest rates, which dampens capital inflows, lessening the demand for domestic currency, and therefore depreciating domestic currency. Hence, this will cause an impact on the country's exchange rate situation. In a separate study, Md-Yusuf and Rahman (2012) argue that foreign exchange rates are a major source of macroeconomic uncertainty that affects stock volatility. Foreign exchange rates have become highly volatile since the abandonment of the fixed exchange rate system in 1973. In earlier research, Ibrahim (2000) lays out the interactions between Malaysian stock prices and the exchange rates using multivariate co-integration and the Granger causality test. He found that exchange rates and the stock prices have no long-run relationship by themselves, but in the presence of money supply and the foreign reserves these two variables showed co-integration; hence, Granger causes Malaysian stock volatility quite significantly. The general framework of this research is closely connected to his idea and therefore, extended by including several short-term rates of interest and foreign exchange for the United States.

This paper deals with the variables from the Chicago Board Options Exchange (CBOE) Global Markets—the largest U.S. options exchange with respect to annual trading volume—that calculates and updates several volatility indices implied by option prices, the exchange rates, and the interest rates. The three measures of market expectations of volatility indices are the CBOE Volatility Index: VIX, the NASDAQ 100 Volatility Index, and the S&P 500 3-Month Volatility Index. In the case of exchange rates, some influential rates of controlling currencies which are most commonly connected with U.S. business and exchange have been chosen. These are Japan/U.S. Foreign Exchange Rate, Mexico/U.S. Foreign Exchange Rate, and Canada/U.S. Foreign Exchange Rate. For the case of interest rates, the daily rates of Three-Month Treasury Bill: Secondary Market Rate, Three-Month Commercial Paper minus Federal Funds Rate, and One-Year Treasury Constant Maturity Rate have been taken as measures of effective rates. All of these data sets were collected as a daily form of data from the St. Louis Fed over the period between 2007 and 2017. There are other effective comparative measurements of interest rates that can be found at the St. Louis Fed website, but they cannot be used accurately in this study because they are recorded as quarterly or monthly.

This study explores the relationship between stock market volatility, exchange rate risk, and the interest rate uncertainty (risk). The motivation of this study is to find out whether it is the exchange rate or the interest rate that causes the stock price volatility to become more sensitive. Stock volatility has been considered a risk in the exchange rate and that risk has certain implications on the economic growth of a country, as the company's competitiveness within the country is generally affected by the exchange rate changes through their impact on input and output price. The findings of this paper show that under the co-integrating relationship Mexico/U.S. currency exchange rates share a common trend and form a stationary relationship in the long-run with the CBOE VIX volatility and NASDAQ 100 stock price volatilities. However, in the case of multivariate co-integration, the stock volatilities exhibit long-run relationship with the series of the exchange rates but their relationships are exogenously weak and do not move together to restore the equilibrium. On the other hand, having no long-run relationship between the stock volatilities and interest rates, more unidirectional and bi-directional causal relationships have been generated between them as compared to that of the exchange rates. This has been supported by the idea in which more positive changes in interest rates, sales and profits of the investor will decline, and stock prices will drop due to the fact that firms or investors will lose their international competitiveness. Obviously, interest rate volatility influences the value of the stock, since the future cash flows of the firm will change and affect their investment plan on stock or bond.

The rest of this paper is organized as follows. Section 2 deals with the related literature and the ex-ante discussion. Section 3 presents the data analysis and outlines the empirical methodology. Section 4 presents the results and findings of the paper. Finally, section 5 contains some concluding remarks.

## 2. Related Literature and *Ex-Ante* Discussion

### 2.1 Related Literature

A significant number of previous studies have been conducted in this area using a diverse amount of financial tools for various countries. Internationalization of stock markets, liberalized capital flows, and huge foreign investment in the U.S. equity markets have led stock and foreign exchange markets to become increasingly interdependent and caused several financial and currency crises across the world, especially in the U.S. during the fiscal year 2008-2009. Mishra et al. (2007) point out that different emerging markets around the globe have led academicians and practitioners to re-examine the nature of volatility spillovers between stock and foreign exchange markets that have seen largely correlated movements resulting in market contamination. One related study conducted by Kutty (2010) examines the relationship between stock prices and exchange rates in Mexico. The paper uses the Granger causality test and exhibits how stock prices lead exchange rates in the short-run, and there is no long-run relationship between these two variables. This finding substantiates the results of Bahmani-Oskooee and Sohrabian's (1992) conclusion but contradicts the findings of other studies, which reported a long-term relationship between exchange rates and stock prices (Kutty, 2010; Bahmani-Oskooee and Sohrabian, 1992).

Dimitrova (2005) shows in a separate paper whether there was a link between the stock market and exchange rates that might explain fluctuations in either market. The author uses a multivariate short-run model under the open economy and tests the assertions in which simultaneous equilibrium of goods, money, foreign exchange, and stock markets in two countries are allowed. Specifically, the paper focused on the U.S. and the U.K. over the period from January 1990 through August 2004. The findings of the paper state that firms' stock prices and the stock market condition may react to changes in the exchange rates situation. On the contrary, changes in stock prices may influence the movements in the exchange rates via firms' portfolio adjustments or outflow of capital. Earlier research by Neih and Lee (2001) examine the dynamic relationship between stock prices and exchange rates for the G7 countries using basic co-integration tests and vector error correction models (VECM) from 1993 to 1996. This research did not account for dual causality between the variables and their findings suggesting that there is no long-run relationship between the stock prices and the exchange rates in G7 counties. Yet, it has been observed that exchange rates have been used to explain the behavior of stock prices on the assumption of corporate earnings tending to respond to fluctuations in exchange rates.

Muller and Verschoor (2006) examine how multinational firms in the U.S. are affected by exchange rate fluctuations. They believe that currency movements are a major source of macroeconomic instability, which affects a firm's value, a situation they refer to as exchange rate exposure. They outline several theoretical reasons why the exchange rate and stock price interaction might be asymmetric. These include the asymmetric impact of hedging on cash flow, firms pricing to market-strategies, investors over-reaction, and nonlinear currency risk exposure, etc. In a separate research study, the relationship between Nifty returns and Indian Rupee/U.S. Dollar Exchange Rates has been widely examined by Agarwal et al. (2010). They find that the correlation between Nifty returns and exchange rates were negative. They employ the Granger causality test to carry out further investigation for the causal relationship between the two variables and their findings highlighted a kind of unidirectional relationship between Nifty returns and exchange rates, running from the former toward the latter.

Muradoglu et al. (2000) investigate the causal relationship between market returns, exchange rates, interest rates and inflation rates for nineteen emerging markets from 1976 to 1997. They explain how the scope of the importers competitiveness in domestic markets will increase, which would lead to the growth in profit and stock prices. Their findings supported that the interactions between implied stock volatilities and macroeconomic variables was mainly linked to the size of the stock market.[2] Another paper written by Zafar, Urooj, and Durrani (2008) exhibited that relating short term interest rates with stock returns and market volatility established that nominal one month T-bill yield had a significantly positive relation with market variance, but negatively correlated with future stock returns. In a different study, Çifter and Ozuna (2007) examine the impact of changes in the interest rates on the stock returns by applying the Granger causality test to the daily rates of the Istanbul Stock Exchange 100 index and the compounded rate of interest. Their findings come up with a proof of interest rate Granger causes ISE 100 index starting with

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[2] The superiority of the information content of implied volatility over historical volatility measure in various markets has been extensively documented by other researchers. (see among others, Blair, Poon and Taylor, 2001; Poon and Granger, 2003; Christensen and Prabbala, 1998; Jorion, 1995).

nine days' time-scale effect and specifies that the effects of interest rates on stock return increase with a higher time scale. Their research augmented the rationale of this current study. The study conducted by Ibrahim (2000) has formulated an idea that is closely related to this present study. He applies standard co-integration and Granger tests to investigate the interactions between exchange rates and the stock market index for Malaysia. The results established two opposite scenarios: There is no long-run relationship between the stock market indices and the exchange rates and there is evidence for co-integration when the model extends to include broader money supply and foreign reserves.

Another similar extensive study is conducted by Muktadir-al-Mukit (2012) where he took the volatility of the market index at the Dhaka Stock Exchange (DSE) and finds that, both in the long-run and in the short-run, interest rates are ranked first in terms of the Granger causes of market volatility index. Using the Johansen co-integration test, he showed that in the long-run, exchange rates have a positive impact and interest rates have a negative causal impact on stock price where all the coefficients are found statistically highly significant.The interest rate and the exchange rate have negative impacts on the stock market index in the long-run as well as the short-run, providing some useful insights into the effects on the stock market index of Malaysia, as shown in the research conducted by Thang (2009). The findings of his paper reveals that exchange rates and stock prices demonstrate a high relationship when returns in asset markets are lower and volatility is higher. In order to search for both the long-run and short-run impacts, Thang uses the standard econometrics time series model as the Johansen Juselius (JJ) co-integration test, the VECM, and the Granger causality test. The author divulges that interest rates have a negative impact on the stock market index. When the interest rate is high, investors will move their money from the equity market to savings, fixed deposits, and bond markets. On the other hand, when the interest rate is low, investors will shift their money into the stock market in order to gain higher profits.

## 2.2 ***Ex-Ante*** Discussion

The Granger causality testing procedure requires one set up while testing two equations. In each equation, the current value of one observed variable ($x_t$ or $y_t$) is formulated as an equation of the other variable(s) with all the different time lag and the previous lagged values of its own level one. Granger (1988) develops a relatively simple test that defined causality as follows: a variable $y_t$ is said to Granger causes $x_t$, if $x_t$ can be predicted with greater accuracy by using the past values of the variable $y_t$, all other terms remain unchanged. The thought process behind the Granger test is that- if previous values of variable $y_t$ significantly influence current values of variable $x_t$, then it can be said that $x$ causes $y$ and vice-versa. For instance, this study begins with the model by taking the three volatility indices of different measurements as dependent variables and then running the simple vector autoregression model to see whether the interest rates and the exchange rates have any Granger causality effects on the volatility indices.

Consider two time-series variables, $y_t$ and $x_t$ that are interrelated. Their combinations yield a sequence of equations that describe a system in which each variable is a function of its own lag, and the lag of the other variable. Together the equations constitute a system known as a vector autoregression (VAR). The following example includes the maximum lag of order two so we have a VAR(2):

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \varepsilon_t \tag{1}$$

$$x_t = \lambda_0 + \lambda_1 y_{t-1} + \lambda_2 y_{t-2} + \delta_1 x_{t-1} + \delta_2 x_{t-2} + \mu_t \tag{2}$$

If $y$ and $x$ are stationary, then equations (1) and (2) can be estimated by using the least squares method. On the other hand, if $y$ and $x$ are non-stationary in their levels but stationary in their first difference, then the difference operators can be used and the following system of equations can then be estimated:

$$\Delta y_t = \alpha_1 \Delta y_{t-1} + \alpha_2 \Delta y_{t-2} + \beta_1 \Delta x_{t-1} + \beta_2 \Delta x_{t-2} + \Delta \varepsilon_t \tag{3}$$

$$\Delta x_t = \lambda_1 \Delta y_{t-1} + \lambda_2 \Delta y_{t-2} + \delta_1 \Delta x_{t-1} + \delta_2 \Delta x_{t-2} + \Delta \mu_t \tag{4}$$

To make things simple, equation (3) and (4) are considered to test the following null hypotheses. The null hypothesis in this case is H$_0$: $x_t$ does not cause $y_t$; i.e.($x_t \nRightarrow y_t$).

$$\overbrace{\Delta y_t = \alpha_1 \Delta y_{t-1} + \alpha_2 \Delta y_{t-2} + \beta_1 \Delta x_{t-1} + \beta_2 \Delta x_{t-2} + \Delta \varepsilon_t}^{Unrestricted\, regression}$$
$$\overbrace{\Delta y_t = \alpha_1 \Delta y_{t-1} + \alpha_2 \Delta y_{t-2} + \Delta \varepsilon_t}^{Restricted\, regression}$$

From these two regression equations, the joint F-statistic will be calculated. If the calculated F-statistic is high enough at a lowest level of significance, then we can reject H$_0$ and conclude that $x_t\, causes\, y_t\, (x_t \Rightarrow y_t)$. The second form of equation is used to test another null hypothesis. H$_0$: $y_t$ does not cause $x_t$; i.e. ($y_t \nRightarrow x_t$).

$$\overbrace{\Delta x_t = \lambda_1 \Delta y_{t-1} + \lambda_2 \Delta y_{t-2} + \delta_1 \Delta x_{t-1} + \delta_2 \Delta x_{t-2} + \Delta \mu_t}^{Unrestricted\, regression}$$
$$\overbrace{\Delta x_t = \delta_1 \Delta x_{t-1} + \delta_2 \Delta x_{t-2} + \Delta \mu_t}^{Restricted\, regression}$$

A second F-statistic is calculated from the above two regressions. If the F-statistic is found higher than the critical F-value, then we can reject H$_0$ and conclude that $y_t\, causes\, x_t\, (y_t \Rightarrow x_t)$.

In each equation, the current value of one variable ($x_t$ or $y_t$) is a function of the previous time lag of other variables, and also its own values in previous time periods ('lagged' values). Recall that the purpose of this study is to find out whether there is any long run relationship between interest rates, exchange rates, and volatility indices and how these variables Granger causes each other. The regression procedure is designed for the possible causal interactions between nine variables: three are volatility indices of different measurement, three for representative exchange rates, and three for interest rates.

To capture these interactions, this paper employs co-integration viz-a-viz Granger causality approaches. The co-integration approach will capture the long-run co-movements or equilibrium relationship between those three different sets of variables. The Granger causality tests, on the other hand, shed light on the short-run dynamics of the variables concerned. Enders (1995) argues that co-integration between a set of variables does not necessarily imply the presence of the equilibrium relationship generated by market forces. The relationship of the variables can be causal or behavioral, or it can be characterized by a simple reduced-form relationship among other similar variables with common trends. The evidence for the relationship between the two variables may be persuasively obtained, the bi-variate framework usually employed in many studies may or may not be sufficient.

The bi-variate and multivariate framework for co-integration and causality testing, therefore, extend the analyses of this paper by investigating their potential relationship and creates the motivation towards the fact findings about what indices may receive considerable policy attention. The basic idea of co-integration relates closely to the concept of unit roots. For the set of above macroeconomic variables of interest, it may be the case that the variables in question are non-stationary, i.e. I(1) when taken individually, or there exists a linear combination of the variables that is stationary without taking the difference, i.e. the system of equations will be I(0).[3] This paper uses the Engle-Granger (EG) co-integration test to see whether two or more series are co-integrated i.e. two or more series are themselves non-stationary up-to certain order but tend to move together through time. Let us begin with the simple bi-variate VAR(p) model:

$$y_t = \alpha_1 + \beta_1 x_t + \sum_{i=1}^{k} c_i y_{t-i} + \sum_{i=1}^{k} d_i x_{t-i} + \varepsilon_{1t} \tag{5}$$

$$x_t = \lambda_1 + \delta_1 y_t + \sum_{i=1}^{k} e_i y_{t-i} + \sum_{i=1}^{k} h_i x_{t-i} + \varepsilon_{2t} \tag{6}$$

assuming that $\varepsilon_{1t}$ and $\varepsilon_{2t}$ are uncorrelated white-noise error terms. For testing the presence of a unit root and to see the case of any autocorrelation in the observed series, Dickey and Fuller (1979) proposed alternative regression equations known as Augmented Dickey-Fuller (ADF) Test. The ADF test is performed by estimating the following model:

$$\Delta y_t = \alpha + \delta y_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta y_{t-i} + u_t$$

The EG two-step co-integration test for bi-variate and multivariate case is represented by applying the ordinary least squares (OLS) technique into the equations (5) and (6) and then to include simply the ADF unit root tests on the residuals estimated from the step-one co-integrating regression.[4] If the variables are found co-integrated, the residuals from the equilibrium regression can be used to estimate the error correction model (ECM) model and used to analyze the long-run and short-run effects of the variables as well as to see the adjustment coefficient. For the multivariate case, this

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[3] A subset of the variables are individually integrated of order 1. In another sense the variables are non-stationary in their levels but their first differences are stationary. An I(0) variable indicates the series is stationary.

[4] The null hypothesis for the ADF test will be H0: delta=0.

study also performs the EG test for co-integration and estimating the co-integrating vectors which are also more straightforward.

## 3. Empirical Framework and Specification

### 3.1 Data Description

As previously stated, this paper uses the common data series from the Federal Reserve Bank of St. Louis website, namely St. Louis Fred economic data. Daily time point data ranging from December 2007 to December 2017 has been collected. The selected volatility indices are based on the common stock prices of the top publicly traded American company's equities or stocks. Three CBOE's volatility indices are: (*i*) VIXCLS, widely known as the first benchmark volatility index of Chicago Board Options Exchange, measures the market's expectation of future volatility. It is considered as the world's premier barometer of equity market volatility; (*ii*) VXVCLS, the implied volatility index on stocks constructed using Standard and Poor's 500 index options, widely known as S&P 500 3-Month Volatility Index; (*iii*) VXNCLS, the implied volatility index on domestic and international non-financial securities based on their market capitalization, with certain rules capping the influence of the largest components, popularly known as NASDAQ 100 Volatility Index. Under the CBOE Global Markets, these three vital indices mainly created as a suite of market expectations of volatility conveyed by the options prices based on all of the major U.S. broad-based stock indices.

Three predominant currency exchange rates that control the world's most trading volumes to the United States, have been chosen. These are Mexico/U.S. Foreign Exchange Rate (ticker symbol-DEXMXUS), Japan/U.S. Foreign Exchange Rate (ticker symbol-DEXJPUS), and Canada/U.S. Foreign Exchange Rate (ticker symbol-DEXCAUS). China/U.S. foreign exchange rate has been opted out because China has a strictly controlled currency policy which regulates their trading activity with the U.S. by controlling the daily movements of the yuan on the forex market. Unlike the others they follow an exchange rate adjustment system which can be identified as ‘managed floating’ or ‘crawling peg.’

The interest rate variables that have been used in this research are the three widely utilized short-term interest rates. These are the daily rates of 3-Month Treasury Bill: Secondary Market Rate (ticker symbol-DTB3), 3-Month Commercial Paper Minus Federal Funds Rate (ticker symbol- CPFF), and 1-Year Treasury Constant Maturity Rate (ticker symbol-DGS1). DTB3 is the interest rate on a three-month U.S. Treasury bill that is often used as one of the risk-free rates for U.S. based investors. CPFF is the spread of interest rate calculated by taking the difference between 3-Month AA rated Financial Commercial Paper Rate and Effective Federal Funds Rate and DGS1 is the index based on the average yield of various Treasury securities maturing at one year period. This study considers two government regulated interest rates as the useful proxies because the capital market usually operates on the belief that there is virtually no chance of the U.S. government defaulting on it’s obligations. Only the daily data for the interest rate has been downloaded from the FRED website. Therefore, other potential interest rates cannot be chosen because some of their values have been recorded either as a monthly, quarterly, or yearly basis. This restriction technique ends up with a total of 2,416 observations.

Table 3.1 summarizes and provides different statistical information about the data series used in this sample study. The preliminary analysis for all data series reveals that interest rate volatilities are highly persistent (according to their standard deviation) indicating that the values cluster closely to their average value. The spread between 3-month commercial paper and the federal funds rate (CPFF) has the lowest mean value ($\mu$ =0.17) as well as a lower standard deviation ($\sigma$ =0.283) as compared to the other two interest rates. The average mean value for all of the three interest rates is $\mu = 0.38$ and the standard deviation is $\sigma = 0.512$ (almost 51%). The one-year constant maturity rate is one of the most widely used indexes and is often used by lenders as a reference point for adjustable rate for mortgages. The mean and standard deviation for one-year constant maturity rate is found 0.60 and 0.663 respectively from this sample study.

All of the three option indices VIXCLS, VXNCLS and VXVCLS have much higher levels of implied volatility with the average mean of $\mu = 20.59$ and the standard deviation $\sigma = 8.405$ (almost 840%). The higher standard deviation means stock returns for the companies vary substantially from their average stock returns which can be considered as the reflection of revealing high levels of investor anxiety. The CBOE volatility index VIXCLS itself has the shortest level of maturity with $\mu = 19.50$ but S&P 500 3-Month Volatility Index has the lowest level of implied volatility with $\sigma = 7.705$. A sharp shift took place recently in the S&P 500 Index options as the credit crisis unfolded. The mean and standard deviation for the exchange rate DEXCAUS is found $\mu = 1.13$ and $\sigma = 0.132$ or 13% respectively. However, the overall measure of the market expectations by the two near-term volatility indices CBOE NASDAQ 100 Volatility and CBOE S&P 500 3-Month Volatility are quite different than expected, since they are based on some normalization scheme under CBOE. DEXJPUS has the highest mean and standard deviation with $\mu = 99.40$ and $\sigma = 13.619$ or 136%, over the class of three selected exchange rates. Nevertheless, the trend of exchange rate for Japan and Canada indicates an exchange rate appreciation against the U.S. dollar in this contemplated time period. The mean and standard deviation for the exchange rate between Mexico and U.S. DEXMXUS is found $\mu = 14.35$ and $\sigma = 2.727$ shows a moderate depreciation in the past few years as compared to the other two exchange rates. Other key statistics such as the lag length, minimum or maximum values, skewness, and kurtosis of the data sets are also displayed in Table 3.1 and are self explanatory.

### 3.2 Model Identification

As noted earlier, the EG test is a two-step procedure that involves an OLS estimation for a prespecified co-integrating regression and a unit root test of the residuals saved from this step. The null hypothesis of no co-integration is rejected if it is found that the residuals are non stationary. The EG test procedure for bi-variate and multivariate co-integration is shown by estimating the so called co-integrating regression of the following form. Equation (7) exhibits the first step of this test:

$$spv_{1,t} = \beta_1 + \beta_2 xrt_{2,t} + \beta_3 xrt_{3,t} + \beta_4 xrt_{4,t} + u_t \tag{7}$$

where $spv_t$ is the measurement for the stock price volatilities, $xrt_t$ is the exchange rate measurement. In the above regression it is assumed that all the variables are $I(1)$ and might be co-integrated to form a stationary relationship as well as a stationary residual term. This equation represents the assumed economically meaningful steady state or equilibrium relationship among the variables. If the variables are found co-integrating, they will share a common trend and form a stationary relationship in the long run. The second step in this procedure is to test for a unit root in the residual process to obtain the co-integrating regression. For this purpose, equation (8) for the estimated residual sequence is constructed.

$$\Delta\hat{u}_t = \alpha + \pi\hat{u}_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta\hat{u}_{t-i} + \upsilon_t \tag{8}$$

Under the null of no co-integration, the estimated residual will be $I(1)$ because $spv_t$

is $I(1)$, and all the parameters will be zero in the long run. Finding the optimal lag length is extremely important for the residual process to become the white noise.[5] If the unit root null hypothesis for the residuals is rejected then a significant $\pi$ implies $spv_t$ and $xrt_t$ are co-integrated. This means that the integrated variable $spv_t$ co-integrates at least with one of the variables on the right hand side. The estimated results might end up with a few co-integrating relationships between stock volatilities and the exchange rates. Once a co-integrating relationship is established then the next step is to see whether or not the stock price volatilities are weakly exogenous or hang together to restore the equilibrium with the exchange rates. The following equation (9) will estimate the ECM:

$$\Delta spv_{1,t} = \sum_{i=1}^{k} \delta_i \Delta spv_{1,t-i} + \sum_{i=1}^{k} \gamma_i \Delta xrt_{1,t-i} - \pi(spv_{1,t-1} - \beta_1 - \beta_2 xrt_{1,t-1}) + \Delta\varepsilon_{1,t} \quad (9)$$

where $\pi$ is the adjustment co-efficient that needs to be tested. If $\pi$ is found significant then there would be a long-run equilibrium relationship that exists between stock volatility and the exchange rate with a short-run dynamic adjustment mechanism. This implies that having some co-integration relationship, stock price volatility will not appear to be weakly exogenous and thus moves together to restore the equilibrium with exchange rates.

On the other hand, the time series of the interest rates are found stationary i.e.$irt_t \sim I(0)$ and the series for the stock volatilities are found to be non-stationary i.e. $spv_t \sim I(1)$. Therefore, there would be no genuine long run relationship to exist between them and hence, no co-integration will exist between interest rate and stock price volatility. As $irt_t$ will be more or less constant over time, $spv_t$ will increase over time.

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[5] In fact, the R programming software and it's applicable packages usually do the maximum work to generate the optimal lag length.

One of the important characteristics of VAR models is that they allow us to test the direction of causality. Causality in econometrics refers to the ability of one variable to predict (and therefore cause) the other. The standard methodology this paper uses is the two-way Granger causality test, which is an indistinct but familiar application of the $F$-test. As previously stated, an ADF test for unit root will identify whether a series is non-stationary or a random walk process. If a time series is stationary, the causality test is performed by using their level values, but in contrast if the time series are found non-stationary, then the transformation of the series is done by taking the first differences.

The number of lags are usually chosen by using some information criterion such as Akaike Information Criterion (AIC). Any particular 'lagged' value of a variable will be retained in the regression if it is significant according to the specific $t$-test and hence, the other 'lagged' values of the variable will jointly add an explanatory power to the model according to an $F$-test. For the bi-variate case, the directions of causation will be examined based on the following empirical models:

$$\Delta spv_{1,t} = \alpha_1 + \sum_{i=1}^{k_1} \delta_{1,i} \Delta spv_{1,t-i} + \sum_{i=1}^{k_2} \gamma_{1,i} \Delta xrt_{1,t-i} + \varepsilon_{1,t} \tag{10}$$

$$\Delta spv_{2,t} = \alpha_2 + \sum_{i=1}^{k_1} \delta_{2,i} \Delta spv_{2,t-i} + \sum_{i=1}^{k_2} \gamma_{2,i} irt_{2,t-i} + \varepsilon_{2,t} \tag{11}$$

where $irt_t$ is the measurement of the interest rate. The stock price volatilities and the exchange rate variables are expressed in their first differences. $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are both uncorrelated white-noise error terms. The null hypothesis of no Granger causation from $xrt$ to $spv$ and also from $irt$ to $spv$ that is $\sum \gamma_{1,i} = 0$ or $\sum \gamma_{2,i} = 0$, will be tested by using the lower level of significance for the $F$-statistic. The reverse causation from $spv$ to $xrt$ and $spv$ to $irt$ will be evaluated by reversing the role of $spv$, $xrt$, and $irt$ in equations (10) and (11). In this case, four alternative patterns of causality can be possible: $(i)$ $spv_t$ causes $xrt_t$ (unidirectional Granger causality from $spv$ to $xrt$), $(ii)$ $xrt_t$ causes $spv_t$ (unidirectional Granger causality from $xrt$ to $spv$), $(iii)$ there would be a bi-directional causality (causality among the variables), and finally $(iv)$ the two variables will be independent (no causality).

The appropriate causality model that allows us to test and statistically detect the cause and effect relationship among multivariate case, is expressed by the following VAR(p) models:

$$\Delta spv_{1,t} = \alpha_1 + \sum_{i=1}^{k_1} \delta_{1,i} \Delta spv_{1,t-i} + \sum_{i=1}^{k_2} \gamma_{1,i} \Delta xrt_{1,t-i} + \sum_{i=1}^{k_3} \lambda_{1,i} irt_{1,t-i} + \varepsilon_{1,t} \tag{12}$$

$$\Delta xrt_{2,t} = \alpha_2 + \sum_{i=1}^{k_1} \delta_{2,i} \Delta spv_{2,t-i} + \sum_{i=1}^{k_2} \gamma_{2,i} \Delta xrt_{2,t-i} + \sum_{i=1}^{k_3} \lambda_{2,i} irt_{2,t-i} + \varepsilon_{2,t} \tag{13}$$

$$\Delta irt_{3,t} = \alpha_3 + \sum_{i=1}^{k_1} \delta_{3,i} \Delta spv_{3,t-i} + \sum_{i=1}^{k_2} \gamma_{3,i} \Delta xrt_{3,t-i} + \sum_{i=1}^{k_3} \lambda_{3,i} irt_{3,t-i} + \varepsilon_{3,t} \tag{14}$$

where $\varepsilon_{1t}$, $\varepsilon_{2t}$, and $\varepsilon_{3t}$ are again uncorrelated white-noise error terms and $spv_t$ and $xrt_t$ are integrated of order one while $irt_t$ is integrated of order zero. From the above sets of equations the joint significance of the coefficients of 'lagged' independent variables will be evaluated to uncover the directions of causation between any pairs of the variables. If the coefficients of 'lagged' $xrt_t$ terms in equation (12) are statistically different from zero as a group and the coefficients of 'lagged' $spv_t$ terms in equation (13) are also statistically different from zero, but at the same time if the coefficients of 'lagged' $irt_t$ are not statistically different from zero both in equation (12) and (13) then two-way Granger causality will be seen between $xrt_t$ and $spv_t$. That means $xrt_t$ and $spv_t$ will have a bi-directional Granger causality. However, if the the coefficients of 'lagged' $irt_t$ are statistically different from zero in equation (12) but are not statistically different from zero in equation (13) then $irt_t$ will cause $spv_t$ in one direction and the reverse direction will be absent (unidirectional causality).

By the same token, if the ‘lagged’ terms of $irt_t$ in equation (13) are statistically different from zero as a group, and the ‘lagged’ $xrt_t$ terms in equation (14) are also statistically different from zero with a significant ’F-statistic’ then it can be concluded that $irt_t$ and $xrt_t$ will resulting a bi-directional Granger causality. Again, if the sets of ‘lagged’ $spv_t$ terms in equation (13) are not statistically different from zero but the sets of ‘lagged’ terms of $spv_t$ in equation (14) are statistically different from zero as a group then for a significant F-value, a unidirectional causality will be exhibited between $irt_t$ and $spv_t$. Finally, if both sets of ‘lagged’ $irt_t$ terms and ‘lagged’ $spv_t$ terms are not statistically different from zero for an insignificant F-value in either equations (13) and (14) then it can be concluded that $xrt_t$ is independent of $irt_t$ and $spv_t$ (no causality).

## 4. Empirical Results and Findings

### 4.1 Co-integration Analysis

The results for the bi-variate and multivariate co-integration tests are reported in Tables 3.3 and 3.4. The co-integration test depends on the ADF unit root test of the selected variables. The unit root test results for all the respective variables are shown in Table 3.2. The three interest rates are found to be stationary in their levels. Table 3.3 presents the results of co-integration for bi-variate case. In this study, the bi-variate EG test of co-integration between the stock volatility indices and the exchange rates are performed because they have a unit-root non-stationary process. The null hypothesis of no co-integration between the stock volatilities and the Mexico-U.S.$ exchange rates cannot be rejected in some cases. Table 3.3 reports the estimated coefficient from the co-integrating regression between different stock price volatilities measurement and the three exchange rates. Almost all of the coefficients are found to be statistically significant. It also shows the unit root test results for the estimated residuals from those co-integrating regression. It reports the error-correction estimation results through the adjustment coefficient only when the two series are found co-integrated. The null hypothesis of unit root in the estimated residuals of a co-integrating regression between the Mexico/U.S. exchange rate and the CBOE VIX volatility index is rejected at better than 1% level. It means that these two series are co-integrated or move together in the long run.

However, the null hypothesis of no co-integration cannot be rejected for S&P 500 3-month volatility index with the Mexico/U.S.$ exchange rate but is rejected for NASDAQ 100 volatility index. The unit-root hypothesis cannot be rejected for the estimated residuals from those two co-integrating regressions. Besides, all other exchange rates and the stock price volatilities illustrate zero co-integration or having no long-run relationship between them. The error-correction estimation for the co-integrating regression of CBOE VIX volatility index and CBOE NASDAQ 100 volatility index with the Mexico/U.S.$ rate are shown as an adjustment coefficient value of the residuals. In both cases the coefficients are found statistically significant; that means the CBOE VIX volatility variable is not weakly exogenous and moves together to restore the equilibrium with Mexico/U.S.$ rate. In summary, the results of this paper suggest that stock price volatilities exhibit zero co-integration with several other exchange rates but provides the evidence of co-integration with the Mexico/U.S.$ rate when investigated in a bi-variate context.

Table 3.4 presents the co-integration result for the multivariate case. In the multivariate framework there seems to be a long-run relationship between the stock price volatility indices and the exchange rate measures. But the statistically insignificant error correction estimation shows that S&P 500 3-month volatility index and NASDAQ 100 volatility index are weakly exogenous to the exchange rates and will not move together to restore the equilibrium in the long-run. Nevertheless, the finding of multivariate co-integration rules out the possibility of non-causality between the variables, since the presence of co-integration suggests that there must be at least one direction of causation in the Granger sense. The following two sections examine the issue in detail.

### 4.2 Results from Bi-variate Causality Test

This section investigates the empirical results produced by the bi-variate Granger causality tests. Table 3.5 generates some stimulating results of the short-run interactions between stock price volatility and exchange rate. The null hypothesis for this test is that the stock volatility index does not cause exchange rate and vice-versa . The null hypothesis of no causation from the CBOE VIX stock price volatilities (VIXCLS) to the Mexico/U.S. exchange rates is rejected at less than 1% level of significance. In contrast, the null hypothesis of no causation from the Mexico/U.S. exchange rates to the S&P 500 3-month volatility (VXVCLS) and NASDAQ 100 (VXNCLS) stock volatility cannot be rejected. However, for the cases of Canada/U.S. and Japan/U.S. exchange rates the changes in the stock prices have predictive power for those exchange rate changes. This paper documents unidirectional causality from the Japan/U.S. exchange rate and bi-directional causality from Canada/U.S. exchange rate to those of the stock price volatilities; most of them are found to be highly statistically significant at 1% to 5% level of significance. These findings, however, may not be convincing as the regressions may suffer from omitted variable bias or fail to account for the possibility of another variable driving both the exchange rates and the stock price volatilities.

Table 3.5 shows that CBOE VIX volatility index and Japan/U.S. foreign exchange rate have a unidirectional causal relationship. However, CBOE S&P 500 3-month volatility index establishes a bi-directional causality with Japan/U.S.$ exchange rate. Both of these relationships are found to be significant at a large value of F-statistic. Nevertheless, the positive and negative sign of causality are also determined by the significant value of the long-run multiplier (LRM). In the case of interest rates, the stock volatilities produce significant amount of unidirectional and bi-directional Granger causality with those preferred rates. Table 3.6 shows that almost every selected interest rate exhibits at least one unidirectional causality over stock prices although many of their relationships postulate a negative causality sign. This means that the stock price volatilities Granger cause interest rates more or less negatively.

Table 3.6 reports that the daily rates of 3-Month Treasury Bill (DTB3) shows a bi-directional Granger causality with all of the three CBOE volatility index, but it shows a negative causal relationship with the VIXCLS and VXVCLS. The spread between 3-month commercial paper and federal funds rate (CPFF) produce a feedback relationship for causality with VXVCLS. But VIXCLS, VXVCLS and VXNCLS indices show zero unidirectional causality for 1-year treasury constant maturity rate (DGS1); an index that is mostly used to set the cost of variable-rate loans such as adjustable rate mortgages. All the other test results of statistical significance or zero Granger causality have been ascertained from Table 3.6 by their respective lowest reported F-statistic and the corresponding probability values. Accordingly, it can also be predicted, at the same time, the causality sign of the test results by their LRM multiplier values and it's significance from the corresponding probability values.

The empirical results from those two tables produce two different bi-variate flow charts. Figure 3.1 incorporates the stock volatility and the exchange rate variables with a node; each node exhibits a causal relationship and the arrow sign represents the direction of the causality. Canada/U.S. exchange rate shows bi-directional Granger causality and Japan/U.S. exchange rate also displays two-way Granger causality with VXVCLS volatility index. Both of these are connected by two oppositely directed nodes. The interesting thing is that VXVCLS volatility index exhibits bi-directional Granger causality with the other two volatility rates. Finally, Japan/U.S. dollar rate shows a two-way Granger causality with Canada/U.S. dollar rate.

Figure 3.2 captures a number of bi-directional Granger causality and more unidirectional causality between interest rate and the stock price volatility. DTB3 rate establishes bi-directional Granger causality with all three stock volatilities VIXCLS, VXNCLS and VXVCLS. CPFF has two-way Granger causality with VIXCLS and VXVCLS volatility index. Two-way Granger causality is evident from VXVCLS to volatility indices VIXCLS and VXNCLS. DGS1 shows unidirectional causality to all three volatility indices. The feedback causality from those three indices towards DGS1 are absent in this case.

### 4.3 Results from Multivariate Causality Test

Table 3.7 illustrates causality results for the multivariate case. In the Granger sense, the results confirm the previous bi-variate findings that the stock price volatilities are causally linked to the effective interest rates with feedback effects. In the case of the DTB3 interest rate, there is bi-directional Granger causality with VIXCLS, VXVCLS and VXNCLS. The F-values for the joint significance test are found to be statistically significant and the sign of the causal relationship is determined by the significant values of LRM. Additionally, both the CPFF spread rate and the DGS1 turn out to be Granger caused by VIXCLS, VXVCLS and VXNCLS volatility index. But in return, those two interest rates do not Granger cause the three volatility indices of the interest. However, there is no evidence for feedback relationship from VIXCLS to CPFF and the DGS1 to VIXCLS.

The results appear to indicate that VIXCLS Granger causes the Mexico/U.S dollar exchange rate. But this causality is found to be statistically insignificant in the opposite direction. Additionally, the null hypothesis that there is no causation from VIXCLS and VXVCLS to that of Japan/U.S. and Canada/U.S. exchange rates respectively, is not rejected in both cases. This means the Japan/U.S.$ rate and Canada/U.S.$ rate is an insignificant indicator for VIXCLS and VXVCLS volatility indices. Similarly, there is no causality from the VXNCLS to Mexico/U.S., Japan/U.S., Canada/U.S.$ rate. Only a feedback relationship for causality is established for VXVCLS and Mexico/U.S.$ rate. The F-statistic is high enough to reject the null at 1% level of significance in both cases. These results indicate that the Chicago options market is informatively efficient. The findings of this study are also more or less consistent with some findings of Kim (2003) and Soenen and Hennigar (1988). Moreover, their results indicate that the importance of interest rate regulation and monetary policies could be more effective in explaining the movements of the U.S. equity market.

For the multivariate case, Figure 3.3 produces a total of six bi-variate Granger causalities. DTB3 shows three bi-directional Granger causality with VIXCLS, VXVCLS and VXNCLS. Mexico/U.S.$ rate produce the bi-variate causality with VXVCLS. DGS1 has created a feedback causal relationship with Mexico/U.S.$ rate. Japan/U.S. foreign exchange rate unidirectionally causes VXVCLS and VXNCLS. Mexico/U.S.$ rate creates a bi-directional causality with DTB3. However, the flow chart reveals that Canada/U.S.$ rate has a unidirectional Granger causality to VXNCLS and VXVCLS. On the other hand, CPFF has a unidirectional Granger causality to Mexico/U.S.$ rate and Japan/U.S.$ rate. VXVCLS has a unidirectional causal relationship with

DGS1 interest rate but there is a one-way Granger cause from VIXCLS to DGS1. However, DGS1 shows a two-way Granger causality effect with Japan/U.S.$ rate. In summary, it is apparent that three interest rates produce more bi-directional Granger causality in terms of the numbers with stock volatilities than in comparison with exchange rates. The findings of this study, therefore, substantiate that interest rate has discernible causal impact on stock price volatilities but exchange rates, although created few causal relationship, cannot be identified as noticeable factors.

## 5. Conclusion

This research empirically examines the dynamics between the volatility of stock returns, the movement of dollar exchange rates, and interest rates in terms of the extent of interdependence and causality. The standard co-integration test and Granger procedure has been applied to investigate the interactions between three major exchange rates (in terms of the U.S. trade-volumes), three important representative interest rates and three stock market indices for the Chicago options market. Daily data sets from December 2007 to December 2017 have been used. The results from the bivariate models indicate no long-run relationship between the stock volatility indices and Japan/U.S. and Canada/U.S. exchange rates but there is evidence of long-run relationships between VIXCLS, VXNCLS and the Mexico/U.S. currency exchange rate. However, there is some indication for co-integration when the models are extended to include all the exchange rates and the stock volatilities. On the other hand, co-integrating relationships between the stock volatilities and the interest rate cannot be possible because the interest rate variables are found stationary in nature.

However, the absence of co-integration in bi-variate cases subsequently exhibits a few Granger causalities in between exchange rates and the stock market volatilities. The empirical findings from the causality tests reveal strong statistical evidence by establishing a number of unidirectional and bi-directional causal relationships between the stock price volatility and the interest rate. First, there is a unidirectional causality from all the three stock price volatilities to the 3-month treasury bill. Additionally, a feedback effect from the 3-month treasury bill to the stock volatilities is also observed. Second, three stock volatility indices and the exchange rates are Granger caused by the one year treasury constant maturity rate and the three-month commercial paper rate minus federal funds rate. There is also evidence of bi-directional causality between the CBOE S&P 500 3-month volatility index and the Mexico/U.S.$ exchange rate. The extent of unidirectional causal relationships with stock volatility indices with the other exchange rates is insufficient when compared to the interest rates.

Finally, this study presents more significant unidirectional as well as bi-directional causalities from interest rates to stock returns and other significant unidirectional causalities from exchange rates to stock returns. However, in the opposite cases the feedback effect of exchange rates mostly did not exist. The findings of this paper suggests that the Mexico/U.S.$ rate might play an important role in the U.S. stock market in the short-run. The changes in the exchange rate reflects the international competitiveness of United States' exports, one of the factors in U.S. growth rate over the past two decades. Additionally, the results focus on the fact that the stock price volatility at the Chicago Options Market is driven considerably by government based interest rates regulations. These changes in interest rates mainly reflect the presence of any short-run bi-directional causal link from DTB3 and CPFF in the Chicago Options and Equity market.

# REFERENCES

Agarwal, G., Srivastav, A. K., and A. Srivastava. 2010. A Study of Exchange Rates Movement and Stock Market Volatility. *International Journal of Business and Management* 5(12): 62-73.

Bahmani-Oskooee, M., and A. Sohrabian. 1992. Stock Prices and the Effective Exchange Rate of the Dollar. *Applied Economics* 24(4): 459-464.

Blair, B. J., Poon, S., and S. J. Taylor. 2001. Forecasting S&P 100 Volatility: The Incremental Information Content of Implied Volatilities and High-Frequency Index Returns. *Journal of Econometrics* 105(1): 5-26.

Canty, A., and B. Ripley. 2016. boot: Bootstrap R (S-Plus) Functions (R package version 1.3-18) [Computer Software]. Cambridge: Cambridge University Press.

Christensen, B. J., and N. R. Prabhala. 1998. The Relation Between implied and realized volatility. *Journal of Financial Economics* 50(2): 125-150.

Çifter, A., and A. Ozuna. 2007. Estimating the Effects of Interest Rates on Share Prices Using Multi-scale Causality Test in Emerging Markets: Evidence from Turkey. MPRA Paper No. 2485.

Csardi, G., and T. Nepusz. 2006. The igraph Software Package for Complex Network Research: InterJournal, Complex Systems 1695 [Software]. Available from http://igraph.org

Davison, A. C., and D. V. Hinkley. 1997. Bootstrap Methods and Their Applications [Computer Software]. Cambridge: Cambridge University Press.

Dickey, D.A., and W.A. Fuller. 1979. Distribution of the Estimators for Autoregressive Time Series with a Unit Root. *Journal of the American Statistical Association* 74(366): 427- 431.

Dimitrova, D. 2005. The Relationship Between Exchange Rates and Stock Prices: Studied in a Multivariate Model. *Issues in Political Economy* 14: 1-25.

Enders, W. 1995. *Applied Econometric Time Series.* New York: John Wiley & Sons. Inc.

Engle, R.F., and C.W.J. Granger. 1987. Co-integration and Error Correction Estimates: Representation, Estimation, and Testing. *Econometrica* 55: 251–276.

Granger, C.W.J. 1988. Some Recent Developments in the Concept of Causality. *Journal of Econometrics 39*: 199-211.

Gul, E., and A. Ekinci. 2006. The Causal Relationship Between the Nominal Interest Rates and Inflation: The Case of Turkey. *Scientific Journal of Administrative Development* 4: 54-69.

Ibrahim, M. H. 2000. Co-integration and Granger Causality Tests of Stock Price and Exchange Rate Interactions in Malaysia. *ASEAN Economic Bulletin* 17(1): 36-47.

Jorion, P. 1995. Predicting Volatility in the Foreign Exchange Market. *The Journal of Finance* 50(2): 507-528.

Kim, K. 2003. Dollar Exchange Rate and Stock Price: Evidence from Multivariate Co-integration and Error Correction Model. *Review of Financial Economics* 12(3): 301-313.

Kleiber, C., and A. Zeileis. 2008. Applied Econometrics with R. New York: Springer-Verlag [Software]. Available from http://CRAN.R-project.org/package=AER

Kutty, G. 2010. The Relationship Between Exchange Rates and Stock Prices: The Case of Mexico. *North American Journal of Finance and Banking Research* 4(4): 1-12.

Md-Yusuf, M., and H. A. Rahman. 2012. Causality Effect Between Equity Market and Exchange Rate Volatility in Malaysia. *IPEDR 55*(22): 109-114.

Mishra, A. K., Swain, N., and D. K. Malhotra. 2007. Volatility Spillover Between Stock and Foreign Exchange Markets: Indian Evidence. *International Journal of Business* 12(3): 343-359.

Muktadir-al-Mukit, D. 2012. Effects of Interest Rate and the Exchange Rate on Volatility of Market Index at Dhaka Stock Exchange. *Journal of Business and Technology* 7(2): 1-18.

Muller, A., and W. F. C. Verschoor. 2006. Asymmetric Foreign Exchange Risk Exposure: Evidence from U.S. Multinational Firms. *Journal of Empirical Finance* 13: 495-518.

Muradoglu, G., Taskin, F., and I. Bigan. 2000. Causality Between Stock Returns and Macroeconomic Variables in Emerging Markets. *Russian and East European Finance and Trade* 36(6): 33-53.

Neih, C.-C., and C.-F. Lee. 2001. Dynamic Relationship Between Stock Prices and Exchange Rates for G-7 Countries. *The Quarterly Review of Economics and Finance* 41(4): 477-490.

Pfaff, B. 2008. VAR, SVAR and SVEC models: Implementation within R Package vars. [Software]. Available from http://www.jstatsoft.org/v27/i04/

Pfaff, B. 2008. Analysis of Integrated and Cointegrated Time Series with R. Second Edition [Computer Software]. New York: Springer.

Poon, S.-H., and C.W.J. Granger. 2003. Forecasting Volatility in Financial Markets: A Review. *Journal of Economic Literature* 41(2): 478-539.

R Core Team. 2015. foreign: Read data stored by Minitab, S, SAS, SPSS, Stata, Systat, Weka, dBase (R package version 0.8-66) [Software].

Ryan, J. A., and J. M. Ulrich. 2014. xts: eXtensible Time Series (R package version 0.9-7) [Software]. Available from https://CRAN.R-project.org/package=xts

Soenen, L., and E. Hennigar. 1988. An Analysis of Exchange Rates and Stock Prices—The U.S. Experience Between 1980 and 1986. *Akron Business and Economic Review* 19: 7-16.

Thang , F. Z. 2009. Impact of Interest Rate and Exchange Rate on the Stock Market Index in Malaysia: A Co-integration Analysis. Unpublished master's thesis, Universiti Sains, Penang, Malaysia.

Trapletti, A., and K. Hornik. 2016. tseries: Time Series Analysis and Computational Finance (R package version 0.10-35) [Computer Software]. New York: Springer.

Wuertz, D. 2013. fImport: Economic and Financial Data Import (R package version 3000.82) [Software]. Available from https://CRAN.Rproject.org/fImport

Zafar, N., Urooj, S. F., and T. K. Durrani. 2008. Interest Rate Volatility and Stock Return Volatility. *European Journal of Economics, Finance, and Administrative Sciences* 14: 135-140.

# APPENDICES CHAPTER III

# APPENDIX A: TABLES

## Table 3.1: Summary Statistics

| Variables[†] | Mean | Std. Dev. | Min. | Max. | Skewness | Kurtosis | Lag Length |
|---|---|---|---|---|---|---|---|
| VIXCLS | 19.497 | 8.8746 | 9.14 | 80.06 | 2.29 | 7.73 | 18 |
| VXNCLS | 21.168 | 8.6342 | 10.31 | 79.16 | 2.35 | 8.39 | 18 |
| VXVCLS | 21.118 | 7.7056 | 11.85 | 64.35 | 1.82 | 7.45 | 18 |
| DTB3 | 0.3734 | 0.5867 | -0.02 | 3.29 | 2.28 | 5.20 | 20 |
| CPFF | 0.1690 | 0.2831 | -0.53 | 2.91 | 4.79 | 29.55 | 20 |
| DGS1 | 0.5988 | 0.6625 | 0.08 | 3.49 | 1.79 | 2.62 | 17 |
| DEXMXUS | 14.3543 | 2.7271 | 9.92 | 21.89 | 1.05 | 12.56 | 20 |
| DEXJPUS | 99.4039 | 13.6198 | 75.72 | 125.58 | -0.41 | 4.69 | 19 |
| DEXCAUS | 1.1279 | 0.1318 | 0.94 | 1.46 | -0.25 | 8.38 | 10 |

Note: Number of Observations are 2,416; [†]all of the variables are measured in their levels; VIXCLS means CBOE Volatility Index: VIX; VXNCLS means CBOE NASDAQ 100 Volatility Index; VXVCLS means CBOE S&P 500 3-Month Volatility Index; DEXMXUS stands for Mexico/U.S. Foreign Exchange Rate; DEXJPUS means Japan/U.S. Foreign Exchange Rate; DEXCAUS means Canada/U.S. Foreign Exchange Rate; DTB3 stands for 3-Month Treasury Bill: Secondary Market Rate; CPFF stands for 3-Month Commercial Paper Minus Federal Funds Rate; DGS1 stands for 1-Year Treasury Constant Maturity Rate.

**Table 3.2: Unit Root Test**

| ADF Test | | |
|---|---|---|
| Variables[†] | *Tau*-Statistics in level | *Tau*-Statistics after difference |
| VIXCLS | -2.21881 | -23.95152 |
| VXNCLS | -2.10924 | -21.30651 |
| VXVCLS | -2.51276 | -19.03378 |
| DEXMXUS | 1.24498 | -30.6357 |
| DEXJPUS | -0.133204 | -13.4615 |
| DEXCAUS | 0.691829 | -24.1448 |
| DTB3 | -4.50269 | - |
| CPFF | -3.73479 | - |
| DGS1 | -3.94405 | - |

Note: [†]three stock price volatilities and the three exchange rate variables are found to be first difference stationary, i.e. they are I(1); the three interest rate variables are found I(0) at their levels.

**Table 3.3: Co-integration Tests (Bi-variate Case).**

| Engle-Granger Test | | | |
|---|---|---|---|
| Variables | Coefficient | Test-statistic for Residuals | Adjutment Coefficient |
| (a) VIXCLS, DEXMXUS | -1.1005*** | -3.9841* | -0.0234*** |
| | (-15.77)$^\dagger$ | (0.052)$^\ddagger$ | (-2.786)$^\dagger$ |
| VIXCLS, DEXJPUS | -0.1996*** | -2.5947 | |
| | (-14.80) | (0.196) | |
| VIXCLS, DEXCAUS | -6.4481** | -2.6815 | |
| | (-2.41) | (0.179) | |
| (b) VXVCLS, DEXMXUS | -0.9134*** | -2.8892 | |
| | (-15.27) | (0.139) | |
| VXVCLS, DEXJPUS | -0.1999*** | -2.8277 | |
| | (-17.65) | (0.157) | |
| VXVCLS, DEXCAUS | -5.9372** | -2.6134 | |
| | (-3.74) | (0.232) | |
| (c) VXNCLS, DEXMXUS | -1.0037*** | -3.4231** | -0.0213** |
| | (-14.83) | (0.039) | (-2.556) |
| VXNCLS, DEXJPUS | -0.1578*** | -2.6836 | |
| | (-11.97) | (0.169) | |
| VXNCLS, DEXCAUS | -4.1666** | -2.6627 | |
| | (-2.94) | (0.156) | |

Note: $^\dagger$figures inside the parentheses indicate t-ratio for the entire column; $^\ddagger$figures inside the parentheses indicate the asymptotic p-value for the entire column; the null hypothesis for testing residuals is that estimated residuals has a unit root; *significant at 10% level, **significant at 5% level, ***significant at 1% level.

**Table 3.4: Co-integration Tests (Multivariate Case).**

| Engle-Granger Test | | | |
|---|---|---|---|
| variables | Coefficient | Test Statistics for Residuals | Adjutment Coefficient |
| (a) VIXCLS | | | |
| DEXMXUS | -5.6129*** | -5.62805 | -0.02867*** |
| | (-49.53)[†] | (0.001)[‡] | (-2.58)[†] |
| DEXJPUS | -0.6441*** | | |
| | (-42.20) | | |
| DEXCAUS | 143.071*** | | |
| | (49.66) | | |
| (b) VXVCLS | | | |
| DEXMXUS | -4.7561*** | -4.91534 | -0.01463 |
| | (-51.57) | (0.003) | (-1.95) |
| DEXJPUS | -0.6068*** | | |
| | (-48.87) | | |
| DEXCAUS | 125.502*** | | |
| | (53.53) | | |
| (c) VXNCLS | | | |
| DEXMXUS | -5.4311*** | -5.37601 | -0.02534 |
| | (-48.20) | (0.006) | (-1.29) |
| DEXJPUS | -0.5773*** | | |
| | (-38.05) | | |
| DEXCAUS | 136.955*** | | |
| | (47.81) | | |

Note: [†]figures inside the parentheses indicate t-ratio for the entire column; [‡]figures inside the parentheses indicate the asymptotic p-value for the entire column; the null hypothesis for testing residuals is that estimated residuals has a unit root;*significant at 10% level, **significant at 5% level, ***significant at 1% level.

### Table 3.5: Bi-variate Causality Results (Stock Volatility and Exchange Rate).

| Null Hypothesis☆ | F-Value | pr (F≥f) | Long Run Multiplier (LRM)† | pr(≥LRM)‡ |
|---|---|---|---|---|
| VIXCLS dnc DEXMXUS | 3.6035 | 0.000*** | 1.00E-04 | 0.332 |
| DEXMXUS dnc VIXCLS | 2.4197 | 0.058* | 139.577 | 0.407 |
| VXVCLS dnc DEXMXUS | 0.8489 | 0.279 | 1.00E-03 | 0.049 |
| DEXMXUS dnc VXVCLS | 0.1577 | 0.138 | 135.504 | 0.417 |
| VXNCLS dnc DEXMXUS | 3.4884 | 0.006*** | 1.00E-04 | 0.024 |
| DEXMXUS dnc VXNCLS | 1.2649 | 0.159 | 93.7300 | 0.388 |
| VIXCLS dnc DEXCAUS | 5.7695 | 0.000*** | 0.84240 | 0.493 |
| DEXCAUS dnc VIXCLS | 6.5271 | 0.040** | 0.03127 | 0.017 |
| VXVCLS dnc DEXCAUS | 6.0528 | 0.000*** | 0.00312 | 0.041 |
| DEXCAUS dnc VXVCLS | 5.8241 | 0.001*** | 3144.694 | 0.309 |
| VXNCLS dnc DEXCAUS | 0.1056 | 0.579 | 0.003 | 0.493 |
| DEXCAUS dnc VXNCLS | 4.3491 | 0.000*** | 0.0958 | 0.039 |
| VIXCLS dnc DEXJPUS | 2.9811 | 0.000*** | -0.002 | 0.000 |
| DEXJPUS dnc VIXCLS | 0.294 | 0.587 | -2.377 | 0.449 |
| VXVCLS dnc DEXJPUS | 3.0822 | 0.000*** | -0.003 | 0.057 |
| DEXJPUS dnc VXVCLS | 4.0521 | 0.044** | 0.0006 | 0.037 |
| VXNCLS dnc DEXJPUS | 3.6385 | 0.000*** | -0.003 | 0.032 |
| DEXJPUS dnc VXNCLS | 0.0755 | 0.784 | -1.074 | 0.485 |

Note: dnc means does not cause; †LRM is used to find out the specific sign of the causality; ‡pr(LRM) is the probability value when the null of LRM is not different from zero; ☆all the variables are measured in their first differences; *significant at 10% level, **significant at 5% level, ***significant at 1% level.

**Table 3.6: Bi-variate Causality Results (Stock Volatility and Interest Rate).**

| Null Hypothesis$^{\star}$ | F-Value | pr (F≥f) | Long Run Multiplier (LRM)$^{\dagger}$ | pr(≥LRM)$^{\ddagger}$ |
|---|---|---|---|---|
| VIXCLS dnc DTB3 | 3.2528 | 0.002*** | -0.0046 | 0.027 |
| DTB3 dnc VIXCLS | 9.3967 | 0.003*** | 0.0043 | 0.078 |
| VIXCLS dnc CPFF | 5.8332 | 0.001*** | -0.0024 | 0.053 |
| CPFF dnc VIXCLS | 8.7911 | 0.000*** | 0.0061 | 0.021 |
| VIXCLS dnc DGS1 | 0.5595 | 0.583 | -0.0713 | 0.437 |
| DGS1 dnc VIXCLS | 3.9987 | 0.001*** | 0.0039 | 0.037 |
| VXVCLS dnc DTB3 | 2.0408 | 0.061** | -0.0478 | 0.041 |
| DTB3 dnc VXVCLS | 7.0199 | 0.000*** | 0.0003 | 0.048 |
| VXVCLS dnc CPFF | 5.5638 | 0.000*** | -0.0048 | 0.052 |
| CPFF dnc VXVCLS | 9.3164 | 0.003*** | 0.0080 | 0.027 |
| VXVCLS dnc DGS1 | 0.4336 | 0.283 | -0.0743 | 0.957 |
| DGS1 dnc VXVCLS | 2.3444 | 0.002** | 6.7477 | 0.426 |
| VXNCLS dnc DTB3 | 1.8918 | 0.025** | -0.000 | 0.043 |
| DTB3 dnc VXNCLS | 8.1392 | 0.000*** | 5.6498 | 0.387 |
| VXNCLS dnc CPFF | 0.7555 | 0.361 | 0.0011 | 0.483 |
| CPFF dnc VXNCLS | 8.2831 | 0.000*** | 24.8066 | 0.255 |
| VXNCLS dnc DGS1 | 0.0482 | 0.286 | -0.0818 | 0.389 |
| DGS1 dnc VXNCLS | 3.8616 | 0.000*** | 0.0009 | 0.032 |

Note: dnc means does not cause; $^{\dagger}$LRM is used to find out the specific sign of the causality; $^{\ddagger}$pr(LRM) is the probability value when the null of LRM is not different from zero; $^{\star}$only the stock volatilities variables are measured in their first differences; *significant at 10% level, **significant at 5% level, ***significant at 1% level.

**Table 3.7: Multivariate Causality Results.**

| Null Hypothesis[☆] | F-Value | pr (F≥f) | Long Run Multiplier (LRM)[†] | pr(≥LRM)[‡] |
|---|---|---|---|---|
| VIXCLS dnc DEXMXUS | 2.3219 | 0.023** | 2.00E-04 | 0.065 |
| VIXCLS dnc DEXJPUS | 0.0432 | 0.194 | -0.0031 | 0.796 |
| VIXCLS dnc DEXCAUS | 0.7995 | 0.287 | 0.1097 | 0.429 |
| VIXCLS dnc DTB3 | 3.2589 | 0.001*** | -0.0046 | 0.044 |
| VIXCLS dnc CPFF | 0.8426 | 0.581 | -0.0019 | 0.481 |
| VIXCLS dnc DGS1 | 3.5509 | 0.000*** | 0.0079 | 0.035 |
| VXVCLS dnc DEXMXUS | 2.6705 | 0.000*** | 1.00E-04 | 0.026 |
| VXVCLS dnc DEXJPUS | 0.6351 | 0.548 | -0.5231 | 0.459 |
| VXVCLS dnc DEXCAUS | 0.0812 | 0.694 | 0.000 | 0.254 |
| VXVCLS dnc DTB3 | 2.0481 | 0.061** | 0.0047 | 0.036 |
| VXVCLS dnc CPFF | 5.5677 | 0.000*** | -0.0043 | 0.044 |
| VXVCLS dnc DGS1 | 2.4259 | 0.024** | -0.0174 | 0.451 |
| VXNCLS dnc DEXMXUS | 0.4319 | 0.192 | 3.00E-04 | 0.257 |
| VXNCLS dnc DEXJPUS | 0.4541 | 0.385 | -0.0037 | 0.225 |
| VXNCLS dnc DEXCAUS | 0.7557 | 0.617 | 0.0062 | 0.474 |
| VXNCLS dnc DTB3 | 2.8774 | 0.014** | 0.0049 | 0.037 |
| VXNCLS dnc CPFF | 5.7779 | 0.000*** | 0.0014 | 0.049 |
| VXNCLS dnc DGS1 | 2.0465 | 0.062** | -0.0083 | 0.041 |
| DEXMXUS dnc VIXCLS | 0.6441 | 0.258 | 104.975 | 0.395 |
| DEXMXUS dnc VXVCLS | 3.5928 | 0.000*** | 198.248 | 0.361 |
| DEXMXUS dnc VXNCLS | 0.6302 | 0.383 | 72.0766 | 0.346 |
| DEXJPUS dnc VIXCLS | 1.0515 | 0.396 | 1.7526 | 0.493 |
| DEXJPUS dnc VXVCLS | 1.5075 | 0.173 | -6.5915 | 0.453 |
| DEXJPUS dnc VXNCLS | 2.9465 | 0.000*** | 0.00128 | 0.049 |
| DEXCAUS dnc VIXCLS | 0.2568 | 0.231 | 701.655 | 0.465 |
| DEXCAUS dnc VXVCLS | 2.6824 | 0.029** | 1655.61 | 0.414 |
| DEXCAUS dnc VXNCLS | 2.8983 | 0.015** | 0.00239 | 0.049 |
| DTB3 dnc VIXCLS | 9.3403 | 0.000*** | 0.00046 | 0.048 |
| DTB3 dnc VXVCLS | 6.9793 | 0.000*** | 0.00013 | 0.002 |
| DTB3 dnc VXNCLS | 8.1121 | 0.001*** | 0.00079 | 0.008 |
| CPFF dnc VIXCLS | 8.7873 | 0.000*** | 26.7801 | 0.114 |
| CPFF dnc VXVCLS | 9.3098 | 0.000*** | 29.8672 | 0.225 |
| CPFF dnc VXNCLS | 3.2889 | 0.025** | 24.9768 | 0.154 |
| DGS1 dnc VIXCLS | 0.9588 | 0.452 | 6.4349 | 0.702 |
| DGS1 dnc VXVCLS | 2.3437 | 0.001*** | 6.8468 | 0.4324 |
| DGS1 dnc VXNCLS | 3.8416 | 0.000*** | 6.7591 | 0.3361 |

Note: dnc means does not cause; [†]LRM is used to find out the specific sign of the causality; [‡]pr(LRM) is the probability value when the null of LRM is not different from zero; [☆]stock price volatilities and the exchange rates variables are measured in their first differences; *significant at 10% level, **significant at 5% level, ***significant at 1% level.

# APPENDIX B: FIGURES

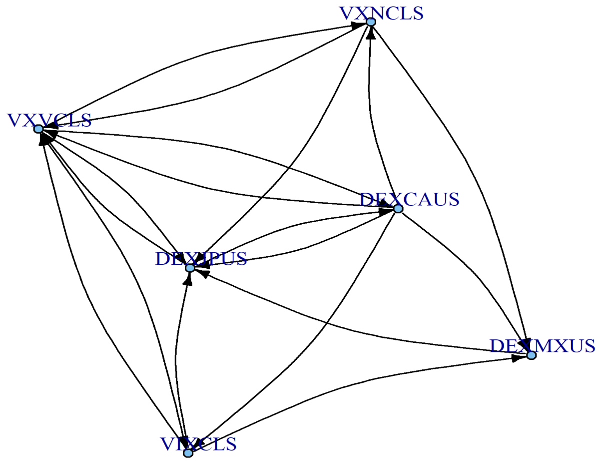

**Figure 3.1: Flow Chart of Bi-variate Granger Causality.**

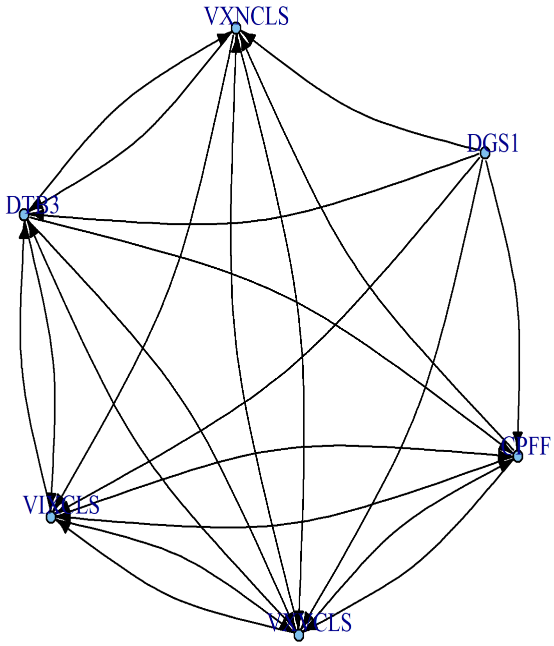

**Figure 3.2: Flow Chart of Bi-variate Granger Causality.**

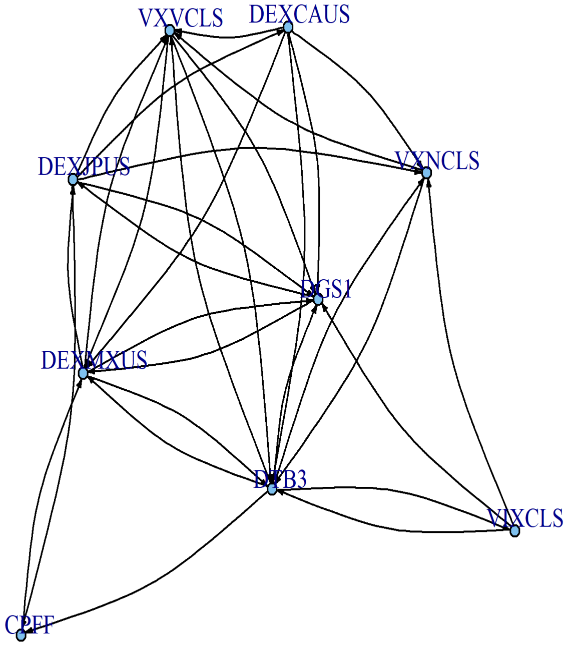

**Figure 3.3: Flow Chart of Multivariate Granger Causality.**