Fostering Equity in the Secondary Mathematics Classroom

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ABSTRACT

This thesis highlights the importance of fostering equitable instruction in secondary mathematics classrooms. It focuses on key instructional practices designed to foster equity in the mathematics classroom. Research on the impact of de-tracking, implementation of rich math tasks, and orchestrating productive classroom discourse is reviewed and synthesized to create a professional development workshop unit plan to create a shared vision of equity among secondary mathematics teachers. The unit plan is presented in full and justified using research-based implications to foster equity in secondary mathematics classrooms.

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Chapter 1 – Introduction

According to the National Council of Teachers of Mathematics' (NCTM) *Catalyzing Change in High School Mathematics: Initiating Critical Conversations,* "Students should have an appreciation for the beauty and usefulness of mathematics and statistics. And students should see themselves as capable lifelong learners and confidents doers of mathematics and statistics." (NCTM, 2018, p. xi). Though this is not the current reality for most mathematics students, this is a goal for all mathematics teachers to strive to achieve: help all students see themselves as capable learners and confident doers of mathematics. As NCTM has noted, standards reform by itself will not achieve the goal of all students having high levels of mathematics understanding (NCTM, 2014). More is needed for students to develop a high level of understanding of mathematics and become confident doers of mathematics than simply rewriting the standards. To be true doers of mathematics, students should be positioned as "explorers" first (McCulloch & Lovett, 2024; Su, 2020).

All students, regardless of gender, race, or background, deserve access to the highest levels of mathematics and the opportunity to achieve excellence. "Excellence in mathematics education requires equity—high expectations and strong support for all students" (NCTM, 2000, p. 12). Thus, mathematics teachers should ensure they are engaging in equitable teaching practices. Though equity can be defined in multiple ways in multiple perspectives (Claiborne, n.d.; Hanna, 2003; Lubienski, 2002; Tan & Thorius, 2019), I will adopt the understanding of equity coined by Driscoll et al. (2016) that equity is about fairness is terms of access and potential. That is, teachers should be "providing each learner with alternative ways to achieve, no matter the obstacles they face" and allowing the "students to do challenging mathematical reasoning and problem solving" (Driscoll et al., 2016, p. ix-x).

Why should mathematics teachers ensure they are incorporating research-based, equity-oriented teaching practices? "It is through equitable instruction that students develop positive mathematics identities" (NCTM, 2018, p. 40). According to a study done by Good et al. on the impact of a student's sense of belonging in math, a student's feelings about their membership and sense of acceptance in math predicted whether or not they planned to pursue mathematics in the future (2012). Similarly, students with strong mathematical backgrounds have increased professional opportunities (D'Ambrosio, 2012; NCTM, 2018). Thus, teachers should ensure that they are developing their students' feelings of membership and mathematical identity. Anthony and Walshaw (2009) argue that teachers are the most important resource for students developing mathematical identities, which leads to greater confidence in their ability to learn mathematics.

True equity in mathematics classrooms will not be achieved until a student's mathematical achievement cannot be determined by his or her inherent characteristics such as race or gender (Gutierrez, 2002). NCTM identifies specific areas of current challenges facing mathematics education in the United States, including creating equitable system structures and supporting equitable instructional practices (NCTM, 2018). Implementing equitable instructional practices is key to a student's success because it is through equitable instruction that students develop positive mathematical identities, which then further supports the aims of NCTM for students to see themselves as lifelong learners and confident doers of mathematics (NCTM, 2018). Examples of equitable instructional practices include establishing high expectations, engaging students in tasks that are culturally relevant and draw upon their understandings, posing purposeful questions, engaging students in meaningful discourse, connecting procedural fluency and conceptual understanding, making student thinking public and providing justification, encouraging multiple representations or solution pathways, fostering a multidimensional classroom, using open materials, teaching students to be responsible for their own learning and valuing each other other's opinions (Boaler, 2006; Boaler & Staples, 2008; NCTM, 2014, 2017, 2018).

Furthermore, teachers engaged in the equitable practice of setting high expectations and communicating clear mathematical goals are equipping their students to be mathematical doers. If a teacher's expectations are not high, the students will not develop a deep understanding of the material (Ellis, 2008; Ellis & Berry, 2005). It is vital for mathematics teachers to communicate clear mathematical goals and set high expectations because this allows teachers to engage in other key equitable practices such as implementing tasks that promote reasoning and problem solving, posing purposeful questions, and facilitating meaningful mathematical discussions, which lead to deeper student understanding of mathematics (NCTM, 2017).

Current obstacles students face to access to equitable instruction include limited opportunities to learn mathematics at a high level and a lack of growth mindset due to placement in tracks (NCTM, 2018). One solution to providing alternative ways for students to have equal access and do challenging mathematical reasoning and problem solving is through rich mathematical tasks and classroom discourse. This thesis will focus on these three areas of concern noted as current struggles by NCTM: de-tracking, implementing rich math tasks, and orchestrating mathematical discourse (2014, p. 3). Thus, the purpose of this thesis is to provide secondary mathematics teachers with the information and tools needed to implement these equitable instructional practices in their classrooms to position students as capable doers and explorers of mathematics.

Consequently, this thesis will provide a brief summary and analysis of the research that has been done on de-tracking, implementing math tasks and orchestrating mathematical discourse in order to provide secondary mathematics teachers with the information necessary to build equity-based classrooms and position and support our students as doers and explorers of mathematics. The next chapter will focus on a review of relevant research of de-tracking, implementing rich math tasks, and facilitating mathematical discourse.

Overview of Thesis

This thesis focuses on enacting three instructional practices in secondary mathematics classes to promote equity through equal access and potential for all students. Chapter 2 will consist of the literature review which will have three main focuses: de-tracking, math tasks, and classroom discourse. The literature review will guide the creation and decisions made during the professional development lesson plan. Chapter 3 will contain the lesson plan for the five-session professional development workshop. Finally, Chapter 4 will provide an overall justification of the professional development workshop plan grounded in the findings of the research literature.

Definitions

Classroom discourse- conversation in the classroom with the purpose of exchanging mathematical ideas (NCTM, 2014)

Equity- fairness is defined in terms of access and potential (Driscoll et al., 2016)

Equitable instructional practices- instructional practices that include “high expectations, access to high-quality mathematics curriculum and instruction, adequate time for students to learn, appropriate emphasis on differentiated processes that broaden student’ productive engagement with mathematics, and human and material resources” (NCTM, 2014, p. 60)

Math task- a segment of classroom activity devoted to developing mathematical with the purpose of encouraging student thinking (NCTM, 2017; Smith et al., 2009)

Tracking- sorting of students into levels such as standard, honors, and advanced honors

Chapter 2 – Literature Review

This chapter will focus on the research behind de-tracking, math tasks, and classroom discourse in the secondary mathematics classroom in relation to equitable instruction. In each section, I will define the mathematical topic at hand, give the context of the topic in the mathematics classroom, and discuss the relevant research. I will begin by discussing findings and implications from relevant research about de-tracking then discuss two equitable instructional practices to implement once students are de-tracked.

De-tracking

Tracking in the mathematics classroom has been a hot topic as researchers and policymakers try to determine the best way to foster more effective mathematics instruction. This topic is a common theme in mathematics education today. According to the National Council of Teachers of Mathematics (NCTM) publication *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (2018), tracking is defined as placing students in a set sequence of classes, usually labelled as standard, honors, and advanced honors. The general purpose of tracking is to better differentiate instruction (Rubin, 2006). However, as found in numerous studies, tracking limits students access to quality mathematics instruction (Boaler, 1997; Yonezawa & Jones, 2006). The purpose of this section is to discuss major findings of researchers regarding tracking, specifically the cons of tracking, benefits of de-tracking, and student perspectives of tracking. I will discuss the first theme by defining tracking, summarizing the landmark study of Boaler and Staples (2008) on Railside, and synthesizing how tracking is detrimental to students.

Why do we track students?

According to NCTM's *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (2018), tracking is a standard practice instituted across the United States. The purpose of tracking in mathematics education is to better support instruction (Rubin, 2006), the success of which is measured through student achievement. However, research has found that there is no evidence that tracking raised achievement (Boaler, 1997; Hanushek & Wößmann, 2006). In a longitudinal study of ninth and eleventh grades from two schools, Boaler (1997) found no evidence that a student's placement in tracks raised achievement, but it did lower achievement for some students. Furthermore, Boaler found clear evidence that the more progressive models of teaching employed at Phoenix Park and the increased opportunity to learn enabled by the mixed ability classes led to the enhanced success of the students. Similarly, Hanushek and Wößmann (2006) found in a cross-country comparison that early tracking actually increases educational inequality in achievement.

Specifically, research has shown that there is a positive effect on achievement by rigor, not tracking (Tyson & Roksa, 2017). While exploring the association between grades and course rigor, Tyson and Roksa (2017) found that students that took a remedial Algebra I their freshman year did not reach the same math level as their peers with similar eighth grade standardized scores. Thus, students that took a less rigorous course and still earned an A, were as likely to reach Algebra II as students that earned an F in standard Algebra I. This highlights the impact of course placement on math attainment and the opportunities that should be open to all students.

Moreover, most secondary schools use tracking as a means of enrichment and remediation. For instance, high-achieving eighth graders are sometimes placed in an Algebra I class and are then able to progress further in the typical math sequence (Burris et al., 2006;

Dougherty, 2015). Gross (2006) strenuously advocates for early identification and tracking for gifted students. Similarly, students with lower achievement levels for eighth grade math have been placed in double-dose algebra, in which they take Algebra I in two years, essentially giving them twice as much exposure to the same content as their peers in one year (Cortes & Goodman, 2014). Also, tracking has been shown to negatively impact students of various ethnicities (Boaler & Staples, 2008; Faulkner et al., 2014). Now, I will summarize the cons research has found with respect to tracking in the last decades.

Cons for tracking

Research has found that tracking in mathematics classrooms does not have the desired impact on student achievement. Early research found no evidence that tracking raised achievement for any student, but did in fact, lower achievement for some students (Boaler, 1997). Specifically, when lower-tracked students were placed in a double-dose program with targeted instruction focused on building math skills and using improved pedagogy, the lower skilled students benefited less from the intervention than somewhat higher skilled students. Thus, tracking designed to help those students did not (Cortes & Goodman, 2014). This is supported by another study of twenty-three middle schools in California in which researchers found that students experience lower levels of achievement in similar-skills ability classes, and this effect is even more pronounced for lower-achieving students (Domina et al., 2019). Furthermore, this highlights the influence of track stability, or student movement between tracks, on student achievement (Domina et al., 2019).

In fact, research has shown tracking increases performance inequalities between tracks and demographics (Boaler, 1997; Fu & Mehta, 2018; Hanushek & Wößmann, 2006; Huang, 2009; Linchevski & Kutscher, 1998). This achievement gap increase is the result of the higher-ability students gaining at the expense of the lower-ability students (Fu & Mehta, 2018; Huang, 2009). In a quantitative study of the effectives of classroom homogeneity on math performance, Huang (2009) found that same ability classrooms are associated with lower math scores for low-achieving students, though this finding was insignificant at the 25th percentile. Therefore, the higher ability students are gaining from same ability classrooms, while the lower ability students are losing. Furthermore, this study found that homogenous ability relative to heterogenous ability grouping has no impact on the mean student performance (Huang, 2009). Thus, tracking does not serve its purpose of increasing achievement.

In contrast, a global study conducted by Hanushek and Wößmann (2006) found that all students lost with regard to achievement from the effects of tracking. While studying the impact of tracking using differences-in-differences approach to analyze 18 to 26 countries, Hanushek and Wößmann (2006) found that early tracking increases inequality and reduces performance, especially for lower achieving students. Moreover, the most interesting finding was that in no case did high achieving students gain at the expense of low achieving students. All students lost from the effects of tracking.

Additionally, tracking has been shown to have a negative impact on various minority groups (Faulkner et al., 2014; Oakes, 1990). Calling upon the data from the 1985-1986 National Survey of Science and Mathematics Education (NSSME), Oakes (1990) found that low-income, minority and low-ability students have less opportunity and access to mathematical knowledge. Compounding the issue is that many of these students are in schools and classrooms with fewer resources and less access to mathematical content. Furthermore, the National Science Board (NSB) asserts that the opportunity to learn is not fairly or evenly provided to all students and is further hindered by the mindset of the teachers that many students lack the ability to learn math and science (NSB, 1983). This is similar to the findings of Faulkner and colleagues, who found that teacher evaluations of a student’s mathematical ability were a greater predictor for placement in Algebra in the eighth grade for black students than it is for white students (Faulkner et al., 2014). While conducting a longitudinal study looking at the different math placements of black and white students up to eighth grade, Faulkner et al. found that the black students had lowered odds of being placed in eighth grade algebra even when controlling for performance (2014). Though tracking is meant to group students based on ability or performance, these students are at a disadvantage due to the impact of their teacher evaluations.

Furthermore, research in the past 20 years has analyzed how best to promote equitable practices in the classroom (Boaler, 2008; Rubin, 2006; Tabron, et al., 2021). Boaler has introduced the term “relational equity” to describe the equitable relationships occurring specifically in the classroom, such as students treating one another with respect (Boaler, 2008). Although it was not the original purpose of studying Railside High, it became evident that three practices Boaler and Staples witnessed played an important role in promoting relational equity in the classroom: commitment to the learning of others, respect for each other’s ideas, and learning ways of communicating (Boaler, 2008). These practices came into play as a result of Railside de-tracking its mathematics program. While the students were engaged in a reform-based curriculum, they worked in groups to solve longer, more conceptual problems. As a result of this, the students were engaging in practices that promoted relational equity within the classroom (Boaler, 2008). This culture would not have been created without de-tracking their mathematics program.

Similarly, Rubin (2006) found that successful de-tracked mathematics programs engage in instructional and institutional best practices. These include academic support for de-tracking for the students and professional and curriculum development support for the teachers. Moreover, Tabron et al. (2021) found an equity-oriented growth mindset is important to further support de-tracking reform. One teacher mentioned the importance of being willing to change, even after teaching for 20 years, because they are eager to push themselves and their students to the next level (Tabron et al., 2021). The emphasis cannot be on equity and growth mindset if the students are stuck in tracks. Now, I will present the research on the benefits of de-tracking mathematics classrooms.

Benefits of de-tracking

It is important to mention the ground-breaking findings of Boaler and Staples (2008) with regard to Railside High. While conducting a longitudinal study of roughly 700 students across four years, focusing on equitable practices, Boaler and Staples found remarkable results comparing three urban high schools in California: Railside, Hilltop, and Greendale. Although Greendale and Hilltop were more traditional in their approach to teaching mathematics, Railside was not. Greendale and Hilltop offered the traditional sequence of high school mathematics and the Interactive Mathematics Program (IMP), a more integrated and application curriculum. Railside only offered their own reform-oriented approach in which they used longer conceptual problems and collaborative and reflective practices without tracking the students. The students at Railside demonstrated higher overall achievement, and within two years, achievement gaps first evident between various ethnic groups disappeared, though Railside had been referred to before as “on the wrong side of the tracks” (Boaler & Staples, 2008, p. 609). These findings have had a huge impact on future research of the effects of tracking in mathematics classrooms (Horn, 2006; Tabron et al., 2021).

As seen from the Railside study, the benefits of de-tracking include incorporating more equitable practices (Boaler, 2008; Horn, 2006; Rubin, 2006). As Boaler (2008) mentions, teaching mixed ability students is challenging. Although all students in a track should be of similar ability, “all classrooms are heterogenous” (Rubin, 2006, p. 10). This means teachers need to focus more on equitable practices that engage and enrich all students (Boaler, 2008; Horn, 2006). This is supported by Horn (2006), who found four key characteristics dealing with equitable practices that supported the successful implementation of de-tracking. In a study of Franklin High, teachers focused on preparing all students to enter the International Baccalaureate (IB) pathway, most generally held as a pathway only for high-achieving students. In this study, the researchers found that de-tracking was impactful, but it was the equitable practices engaged in while de-tracking, specifically the growth mindset of the teachers and students, that truly made de-tracking successful (Tabron, et al., 2021). Similarly, Boaler (2008) found that the high-achieving students that had first complained about being forced to work with lower-achieving students found their depth of understanding and ability to explain and justify their answers was increased and subsequently helped their own achievement increase as well.

Another benefit of de-tracking is the shrinking achievement gap (Boaler & Staples, 2008; Fu & Mehta, 2018; Gamoran, 1992; Hanushek & Wößmann, 2006; Huang, 2009). In the case of Railside, the initial achievement gap between tracks and ethnic groups was almost completely eliminated at the end of two years (Boaler & Staples, 2008). This is also supported in Linchevski and Kutscher (1998). They found the added achievement from tracking was gone by the end of two years (1998). Additionally, two cross-country analyses of tracking found that early tracking increases educational inequality (Hanushek & Wößmann, 2006; Huang, 2009).

Regarding concerns about high-achievers being adversely affected by de-tracking, researchers used a longitudinal study to confirm earlier findings that initially high-achieving student's performance is not hurt if the curriculum remains the same and the mixed-ability level of the class increases (Burris et al., 2006). Moreover, research has found that an all-encompassing, accelerated curriculum is more beneficial to at-risk and low-achieving students because the high-achieving students are enriched by the curriculum and expectations, not their track placement (Burris et al., 2006). That is, it does not matter in which track the high-achieving students are placed because they are enriched by the curriculum and expectations. Thus, all students should be exposed to an all-encompassing curriculum and high expectations because all students, high-achievers and low-achievers, benefit from the curriculum and expectations.

Student perspectives

In researching tracking in middle and high school mathematics classrooms, a common theme of student perspectives emerged. As the ones primarily impacted by tracking, it is vital to know what students think (Boaler, 1997; Tabron et al., 2021; Yonezawa & Jones, 2006). In student interviews conducted through the course of a longitudinal study and student focus groups, students shared their opinions on the topic of tracking. Students in all studies expressed their feelings of stress and inadequacy from being placed in tracks (Boaler, 1997; Boaler et al., 2000; Yonezawa & Jones, 2006). Using student questionnaires and individual and group interviews, the studies found students thought tracking was not equitable. That is, it was not fair in how students were placed and the instruction they received (Boaler et al., 2000; Yonezawa & Jones, 2006). For instance, tracking creates a system in which the students must conform to an academic system in which they are either "that bad" or "that good" (Boaler et al., 2000, p. 645). In contrast, some students mentioned they thought tracking was a good idea as it established a sense of meritocracy and was difficult to learn in heterogenous ability classes (Yonezawa & Jones, 2006).

Furthermore, student participants in multiple studies remarked on the teachers' beliefs and attitudes (Boaler, 1997; Boaler et al., 2000; Tabron et al., 2021; Yonezawa & Jones, 2006). Students mentioned that it seemed some classes weren't as important as others when it came to tracked classes because it was obvious the teachers were investing more time and energy into the higher tracked classes. They were frustrated at the pedagogical gap apparent between the higher tracked and lower tracked teachers (Yonezawa & Jones, 2006). As one student mentioned, "Just because you're taking the regular class, [you] shouldn't get less of an education" (Yonezawa & Jones, 2006, p. 19). Similarly, in the case of Franklin High, research also emphasized the importance of teacher belief's (Tabron et al., 2021). This study found a link between de-tracking and the importance of growth mindset for the students and teachers. Specifically, the importance of de-tracking all aspects of the organization, including teacher assignments (Tabron et al., 2021). This research supports the student's perspectives of teacher beliefs and the role the teachers play in tracking.

Furthermore, studies found the students felt that the struggling students received subpar instruction, or their teachers mostly focused on procedures and the students frequently gave up (Boaler, 2011; Boaler et al., 2000; Yonezawa & Jones, 2006). They argued that de-tracking meant teachers should be teaching more equitably but were obviously not prepared to teach in such a way (Yonezawa & Jones, 2006). Horn (2006) found it important that teachers be able to distinguish between students doing math and students doing school. This requires administration and teachers to invest time in learning how to teach mixed ability classes and incorporate relational equity (Boaler, 2008). This is supported by earlier research, which found professional development workshops positively influenced teacher attitudes (Linchevski & Kutscher, 1998).

Mathematical Tasks

What is a mathematical task? A mathematical task is loosely defined as a segment of classroom activity devoted to developing mathematical ideas (Smith et al., 2009). The purpose of a mathematical task is to encourage student thinking (NCTM, 2017). Then, why should teachers go against the traditional instructional method of lecturing and implement mathematical tasks instead? Research has shown severe limitations to conventional mathematics instruction (Silver & Stein, 1996). Mathematics teachers need to expose and provide access to rigorous tasks to all students to achieve the aims of the NCTM (NCTM, 2014).

The benefits of implementing math tasks include a focus on big ideas and not rote memorization. This is a key characteristic of nations who are more high scoring on the Program for International Student Assessment Program (PISA) (Boaler, 2016; NCTM, 2014). Several studies have found evidence that increased exposure to and prolonged engagement with high-level cognitively demanding tasks increases students' learning (Hiebert & Wearne, 1993; NCES, 2003; Stein & Lane, 1996). Silver and Stein (1996) found clear evidence that students showed gains in mathematical thinking, problem solving and communication under the influence of the instructional changes during the QUASAR Project. Furthermore, national and international research has shown that student achievement is positively associated with the implementation of mathematics curriculum that contains a prevalence of cognitively challenging tasks (Boaler & Staples, 2008; Cai et al., 2011; Grouws et al., 2013; Schoen et al., 1999; Stigler & Hiebert, 2004).

In a study analyzing the effects on student learning using a reform-based curriculum, Connected Mathematics Program (CMP), and a traditional curriculum, Cai et al., (2011) found that teachers using CMP were more than three times as likely to implement high-level instructional tasks and that those tasks were more than three times as likely to be solved using multiple solution strategies. Also, the cognitive demand for the high-level instructional tasks that were implemented in the CMP classrooms was a significant predictor of the student's achievement gains. Similarly, a study of the implementation of another reform-based curriculum across 36 states found that students who experienced the reform-based curriculum significantly outperformed the other students in the ability to do quantitative thinking (ATDQT) category and was consistent across subgroups based on gender, race, and whether the students were tracked or not (Schoen et al., 1999). These findings were also consistent with students with an aptitude for mathematics placed at a magnet school, with demonstrated gains that were approximately double those of comparable students not participating in the reform-based curriculum.

Research has shown that reform-based curriculum is effective in increasing student achievement. The specific instructional changes that are being implemented that are causing such gains are the implementation of math tasks. Equitable teaching means all students should have equal access and opportunity to learn mathematics at a high level. Implementing high-level tasks is one way to accomplish this goal.

The next sections will address selecting high-level math tasks and identifying the cognitive demand of a task. This is important to note because research has shown the level of the task is important to allow students the opportunity to go deep with the mathematics. Thus, it is also important to discuss how to successfully implement a high-level math task without lowering its cognitive demand.

Selecting High-Level Math Tasks

Doyle joins other researchers in recognizing that mathematics "tasks serve as the proximal cause of student learning from teaching" (Stein et al., 1996, p. 459). Thus, it is important to select a task that accomplishes the goals of the lesson and is a tool to increase student learning. A framework suggested by Doyle (1988) is similar to a framework suggested by Stein et al. (1996). Doyle's framework focuses on four elements: the product, the operations to produce the product, resources and the weight or importance of the task in the accountability system. Stein's framework sees implementing tasks as a process that begins with how it is represented in the instructional materials or curriculum, then is interpreted by the teacher in the classroom, then is interpreted and implemented by the students, which leads to the students' learning. These frameworks are similar in that both pay attention to what the students should be creating, with what they will create it with and the expectations for how they will create it (Stein et al., 1996).

However, before a task can be implemented, it must be selected. What should a rich, high quality math task look like? Characteristics of a rich, or high quality, math task should include

1. Encourage thinking, reasoning and problem-solving
2. Allow for multiple entry points, solution pathways and representations
3. Have a low floor and high ceiling
4. Allow students to draw on previous knowledge
5. Encourage collaboration
6. Opportunities for extension (Liljedahl, 2021; Meyer, 2011; NCTM, 2017, 2018; Papert, 1993; Piggott, 2018; Wolf, 2015)

It is important to note that the content of the task does not dictate the quality of the task. The most important consideration while selecting a task is the effect it should have on the students' ability to be explorers of mathematics and the opportunity for them to develop a deep understanding of the mathematics at hand (NCTM, 2014). Examples of rich math tasks include Dan Meyer's 3-Act Math Tasks and Robert Kaplinsky's Open Middle problems. Both types of tasks focus on the student's mathematical thinking, problem-solving and conceptual understanding of the concept at hand through the use of storytelling and curiosity, which is an important factor to maintain student engagement—and thinking—at a high level.

Cognitive Demand

Additionally, a task analysis guide was also developed by Smith & Stein (1998) to classify mathematical types by the level and type of thinking required by the students engaged in the task: memorization, procedures without connections, procedures with connections, and doing mathematics. This is important because a successful implementation of a high-quality math task should foster student thinking and problem-solving, which leads to a deeper understanding and reasoning about the mathematics behind the task (Stein & Lane, 1996). Also, important to note is the prior knowledge and mathematical experience of the students engaged in the math task. As students become more mathematically mature, high cognitive demand tasks may dissolve into more routine exercises (NCTM, 2014).

Implementation

Task set up is defined by how the teacher intends and launches the task, while the task implementation is defined by how the students actually work on the task (Stein et al., 1996). Although a selected task may be of high cognitive demand, the way it is implemented can reduce the cognitive demand (NCTM, 2014). Using six teachers participating in the QUASAR project, Henningsen and Stein (1997) found some factors that affect the task implementation include classroom norms, task conditions, the teachers' instructional disposition, and the students' learning dispositions. In one classroom, the inappropriateness of the task with respect to clarity and task expectations led to student disengagement. This is one way in which the task conditions led to the decline of the cognitive demand of the task. Additionally, the students were given an entire class period to complete the task. However, this may have only exacerbated the decline in cognitive demand since the clarity of the task was in question.

As the mathematical authority in the classroom, teachers play a vital role in the decline or maintenance of the cognitive demand of a task (Johansson, 2007). As such, research has found several key factors that teachers can mitigate in their classrooms to maintain the cognitive demand of their selected tasks during the task launch and implementation. A key factor present in tasks that maintained their level of cognitive demand was a competent performance modelled by the teacher or a capable student, while classroom management problems were only judged to be a key factor in the cognitive demand decline in 18% of the observed tasks in a study conducted by Stein et al. (1996). Similar studies found that tasks that built upon the students' prior knowledge and were allotted sufficient time to complete the tasks maintained their cognitive demand, while the inappropriateness of the task and the removal of more challenging features of the task were associated with the cognitive decline of the tasks (Bennett & Desforges, 1988; Doyle, 1983, 1986; Henningsen & Stein, 1997; Sullivan et al., 2015). Teachers should also ensure they select an appropriate task for the students and continually monitor the student's progress through scaffolding and encouragement to provide meaningful explanations or connections (Anderson, 1989; Doyle, 1988; Henningsen & Stein, 1997). Additionally, teachers should guarantee they have prepared themselves and their classrooms to successfully implement high-level math tasks by carefully selecting tasks and creating a classroom environment that is conducive to successful task implementation. Some classroom factors that led to the decline of the cognitive demand and task implementation include a lack of time, focus on correct answers, superficial analysis of the problem (Henningsen & Stein, 1997).

Professional Development

In order to select rich math tasks and implement them in such a way that maintains their cognitive demand, mathematics teachers need further guidance through professional development (Arbaugh & Brown, 2005; Boston & Smith, 2009; Sullivan et al., 2015). Research has shown that teachers that engage in professional development focused on learning about the levels of cognitive demand had an impact on how they thought about math tasks and how they chose math tasks (Arbaugh & Brown, 2005). Another study focused on utilizing professional development through the Enhancing Secondary Mathematics Teacher Preparation (ESP) project workshop funded by the National Science Foundation, found similar evidence that teachers that participated in the professional development increased the level of cognitive demand of their instructional tasks and their ability to maintain the cognitive demand as evidenced by their students' work (Boston & Smith, 2009). Similarly, a study conducted by Sullivan et al. (2015) found that incorporating teacher moves that encourage persistence in lessons recommended through a professional development series allowed teachers to maintain the cognitive demand of the task.

Classroom Discourse

The third equitable instructional practice of focus for this literature review is classroom discourse. According to NCTM (2014), "Effective mathematics teaching engages students in discourse to advance the mathematical learning of the whole class", where discourse is defined as the "purposeful exchange of ideas" (p. 29). Through the purposeful orchestration of the exchange of ideas, math teachers are promoting access to high-level mathematics to all students. Thus, it is important for math teachers to have a shared understanding of how to identify and foster productive mathematical discourse as an equitable practice.

According to NCTM, students need to make connections to better understand the mathematical content and one way they can do this is through classroom discourse (2018). To be an equitable practice, the focus should be on how mathematical discourse allows access and opportunity for all students. As such, student discourse can lead to increased mathematical knowledge and understanding by allowing the teacher and other students the opportunity to monitor other students' thinking. Furthermore, the act of talking can help build understanding (Franke et al., 2009). According to Kazemi and Hintz (2014), classroom conversations are key to student's mathematics learning. From their analysis of mathematics discussions in elementary classrooms, four guiding principles emerged:

- Discussions should achieve a mathematical goal
- Students need to know what and how to share
- Teachers need to orient students to one another and the mathematical ideas so that every member of the class is involved in achieving the mathematical goal
- Teachers must communicate that all children are sense makers and that their ideas are valued (Kazemi & Hintz, 2014, p. 2)

These four guiding principles further illustrate how equitable instruction can be achieved through classroom discourse. Similarly, Smith and Stein (2011) developed a framework with five steps to successfully orchestrate mathematics discussions.

Five Practices

The five practices were developed from the 2001 ASTEROID project focused on what pre-service teachers learned, where Smith was the primary instructor who used these discourse practices with regularity. The researchers observing Smith and her students saw the effect these practices had on the quality of the discussions. Upon further analysis and tweaking, the five practices were created. The five practices of classroom discourse are anticipating possible student responses, monitoring students as they work, selecting students to present their work, sequencing the student responses and connecting student responses to each other and to the key mathematical ideas of the lesson (Smith & Stein, 2011). According to *The 5 Practices in Practice: Successfully Orchestrating Mathematics Discussions in Your Middle School Classroom*, "Discussion gives the students the opportunity to share ideas and clarify understandings, develop convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other people's perspective" (Smith & Sherin, 2019, p. xxv). The students are being given equal access and opportunity to learn through carefully orchestrated mathematical discussions that allow them to share ideas and develop a language for expressing important mathematical ideas while also developing arguments for how and why things work.

It is important to note that in order for the students to engage in high-level discourse and have the ability to make connections across their multiple representations, an appropriate task must be selected and the goals for the lesson must be clear. The mathematical discussion should help the students achieve the mathematical goal of the lesson (Kazemi & Hintz, 2014; Smith & Stein, 2011) and move beyond a "show and tell" (Ball, 2001). As mentioned by Smith and Stein (2011), the purpose of the five practices is to give the teacher doable moves to help the students achieve the mathematical goals of the lesson and move the discussion beyond a show and tell. Thus, it is important to have a carefully selected rich math task to support the learning goals of the lesson before planning a mathematical discussion around the topic.

Although there is some research that supports the relationship between discourse and students' mathematical understanding (Yimam & Dagnew Kelkay, 2022), there is little to no empirical research to clearly define the relationship between instruction incorporating discourse practices and its effect on student understanding (Pirie & Scharwzenberger, 1988). A year-long case study of an urban elementary school classroom led to the development of a discussion framework of four learning components: questioning, explaining mathematical thinking, source of mathematical ideas, and responsibility for learning (Hufferd-Ackles et al., 2004). Following one teacher's progress throughout the year, her class moved through each of the levels as evidenced by classroom observations and surveys. Another year-long study of an urban classroom found similar growth in students to engage in productive mathematical discourse (Staples & Truxaw, 2010). However, neither of these studies empirically categorized the relationship between the practice of classroom discourse and its effect on student understanding or achievement.

However, there are specific teacher discourse moves that have been shown to develop confident math learners. These will be discussed in the next section.

Teacher Moves

Teachers have the greatest impact on the success of math discourse in the classroom. As such, the discourse moves a teacher makes have the potential to develop classroom discourse that is productive and powerful (Curtis et al., 2021; Herbel-Eisenmann et al., 2013). The North

Carolina Collaborative for Mathematics Learning (NC^2ML) has gathered nine important discourse moves for fostering confident math learners:

1. Inviting student participation
2. Assessing/probing a student's thinking
3. Advancing/pressing a student's thinking
4. Orienting students to another's reasoning
5. Attributing student's mathematical ideas
6. Encouraging in-progress thinking
7. Assigning competence
8. Revoicing/asking students to revoice
9. Waiting (adapted from Curtis et al., 2021; Herbel-Eisenmann et al., 2013; Horn, 2012).

To facilitate meaningful mathematics discourse, teachers use questioning as a tool. Questioning can be used in a variety of ways in the classroom (NC^2ML, 2023; NCTM, 2014). Questions can be used to assess a student's content knowledge, help students develop thorough explanations, discover and correct misconceptions, make the mathematics visible and maintain classroom management (Kersaint, n.d.; McCarthy et al., 2016; NCTM, 2014). According to NCTM, there are four types of questions that are important to teaching mathematics: gathering information, probing thinking, making the mathematics visible and encouraging reflection and justification (2014). The type of question the teacher asks depends on its purpose.

Two question types are suggested by Herbel-Eisenmann and Breyfogle (2005). Funneling questions are questions meant to guide the student to a specific strategy or solution path, while focusing questions are meant to guide the students based on what they are thinking. Herbel-

Eisenmann and Breyfogle suggest teachers should pay attention to their patterns of questioning to incorporate more focusing questions, which supports student thinking and encourages the student to be clear and articulate. This suggestion is supported by one study of an elementary school teachers' use of questioning to elicit student thinking. The use of the teacher's general and specific questions provides evidence to show how teachers can provide support for students to be more detailed and thorough in their explanations and develop justifications (Frank et al., 2009; Kazemi & Stipek, 2001; Silver & Stein, 1996; Stein et al., 1996).

Furthermore, Smith and Stein suggest planning assessing and advancing questions before the discussion, which are two of the nine discourse moves for teachers (2011). NCTM (2014) also supports these teacher moves and further iterates that a teacher should hear the answer to an assessing question but should walk away to let the students ponder the advancing question. This allows all students the opportunity to explore the lesson's mathematical goal and go deep with it. Similarly, Liljedahl (2021) suggests a similar approach to students asking proximity and stop-thinking questions. Based on his research on building thinking classrooms, Liljedahl claims teachers should only answer keep-thinking questions and that answering a question with a question is only effective if the teacher immediately walks away.

Another teacher discourse move encourages teachers to orient students to one another's thinking, which promotes equitable instruction since "justification at any level promotes access and agency" (Bieda & Staples, 2020, p. 103). Furthermore, "Students who learn to articulate and justify their own mathematical ideas, reason through their own and others' mathematical explanations, and provide a rationale for their answers develop a deep understanding that is critical to their future success in mathematics and related fields" (Carpenter et al., 2003, p. 6). Thus, teachers pushing students for justification and explanation during whole group discussion encourages them to do this to their peers during small group collaboration and helps them orient to one another's thinking (Kazemi & Stipek, 2001; Smagorinsky & Fly, 1993). In a large racially, socially and ethnically diverse suburban public high school, three sophomore classes were selected to study their small group discussions. Smagorinsky and Fly (1993) found that teachers who participate with the students in the inquiry and do not take the role of modeler have a greater chance of encouraging students to internalize these skills and perform independently.

Other teacher discourse moves that contribute to knowledge mobility and positioning students as doers and explorers of mathematics are encouraging in-progress thinking and assigning competence (NC^{2}ML, 2023). In classroom observations done by Liljedahl, "knowledge mobility [was] accompanied by a decrease in groups' reliance on the teacher and an increase in reliance on themselves and other groups" (2021, p. 48). This is similar to the development of the framework created by Hufferd-Ackles et al. in which the last level is responsibility for learning (2004). Both qualitative studies found the importance of the shift of responsibility from the teacher to the students. Similarly, in a longitudinal study of a specific teacher's classroom practices designed to foster collaborative inquiry, Staples (2007) found that the teacher's role and instructional practices fell into three categories: supporting students in making contributions, establishing and monitoring a common ground and guiding the mathematics. Specifically, in attending to how the teacher guided the mathematics, Staples noted the importance of how the teacher kept the students positioned as the thinkers and decision-makers. Consequently, a trait of math classrooms rich with sense-making is the ability of the students to reach and justify conclusions based on their own knowledge without relying on the authority of the teacher (Kersaint, n.d.).

In the next chapter, a six-session professional development workshop lesson plan will be presented. This unit plan was created under the consideration and implications of the research provided in this literature review for the purpose of creating a shared vision of equity among secondary mathematics teachers and to provide them with actionable tools to implement in their classrooms.

Chapter 3 – Professional Development Workshop Plans

This chapter contains a 6-session plan on the three instructional practices discussed in the literature review: de-tracking, math tasks, and classroom discourse. The aims of this professional development are to equip secondary mathematics teachers with the knowledge and tools necessary to promote these equitable practices in their classrooms to allow all students equal access and promote their potential and create a shared vision of equitable instruction.

By eliminating tracking, we are giving all students the opportunity to go deep with mathematics through challenging mathematical reasoning and problem solving. This gives them the opportunity to see themselves as lifelong learners and confident doers of mathematics. Then, a key equitable instructional practice recommended by NCTM is implementing mathematical tasks that promote reasoning and problem solving. As the students are engaged in math tasks, they are also engaged in productive struggle and meaningful mathematical discourse. This builds shared understanding of mathematical ideas and develops deeper conceptual understanding.

The first day of the workshop introduces the idea of equity through de-tracking. The teachers will discuss and define equity then complete a carousel about de-tracking. The goal of the first workshop is to encourage teachers to consider the implications of tracking and the perspective of the students. The second workshop will focus on how to select appropriate math tasks to implement in the classroom and will provide examples of rich math tasks already available. Then, the third workshop will focus on identifying the level of cognitive demand of a task, while workshop four will focus on how to maintain the level of cognitive demand of a math task. Finally, the fifth and sixth workshops will focus on organizing productive classroom discourse to promote student understanding and building connections. The overarching goal of the professional development workshop is for teachers to take the tasks and strategies presented back to their classrooms to use with their students to engage in more equitable instruction.

The intended audience for the professional development workshop is high school mathematics teachers ranging in experience from 0 – 20 years. All teachers have a basic understanding of math tasks and orchestrating classroom discourse from prior professional development, but each teacher has a different perspective of the importance of math tasks and classroom discourse.

Each workshop has its own lesson plan that includes the learning objective, instruction and pacing, and reflection. All materials used for the professional development workshop are in the Appendices.

Professional Development- Day 1: Equity and De-tracking (140 minutes)

Learning objectives

1.) Teachers will be able to define equity in terms of access and potential.
2.) Teachers will give examples of the cons of tracking.
3.) Teachers will reflect on equitable practices in the classroom.

Materials

Powerpoint to display instructions (See Appendix A), Padlet link, chart paper, chart markers, notecard.

Instruction

- Activation (60 minutes)-

To establish the norms of the workshop and ensure everyone has the opportunity to participate, the facilitator will build trust throughout the workshop by recognizing that the workshop is a safe space and modelling appropriate listening skills. The norms of the workshop are

1. Be engaged at all times.
2. Do what’s best for everyone (including yourself).
3. Maintain a positive and willing attitude.

The workshop will begin with an introduction assignment in which the teachers will define equity using a Padlet. Then, the facilitator will lead a short discussion on what equity is based on the research, leading the teachers to think of equity in terms of fairness of access and potential. The facilitator will ask the teachers to share their personal experiences as students and as teachers with fairness in terms of access and potential. Expected responses include positive memories where a trusted adult encouraged them to reach their potential and more discouraging incidents in which they felt they had been slighted or overlooked as a student or have treated a former or current student in such a manner. Then, the teachers will engage in a short activity to emphasize the importance of perspective. Each table group will be given a set of four cards labelled North, South, East and West and a set of six colored blocks. The goal of the task is for each member to read their direction card and place the blocks in such a way that matches their perspective described on the card. Then, the person to the right will read their card and arrange the blocks to fit their perspective. Each member will take a turn doing this, going clockwise around the table until the six blocks are arranged in such a way that each person’s perspective matches that described on their card. The facilitator will be monitoring this activity for teacher participation and those groups that need assistance. Once each group is done, the facilitator will ask the teachers to think of three reasons why this activity was chosen to start the workshop. Anticipated responses include collaboration, group-work, building community and relationships, defining equity in terms of perspective, and engaging the teachers in a thinking task. The facilitator will focus the discussion on the importance of collaboration and perspectives. Though every teacher and student have a different background and perspective, all are valuable and should be considered. Discussion questions will include

- How does collaboration relate to equitable instruction?
- Whose perspective should we consider in the classroom? Why?
- How does considering other's perspective encourage equitable instruction?

- Exploration (60 minutes)-

The facilitator will instruct each group to create a poster for the pros and cons of tracking and a poster for the pros and cons of de-tracking. Since overlap is expected, teachers will be instructed to include overlapping ideas on their posters. As the teachers are working with their groups, the facilitator will be monitoring the group discussions for topics to mention whole group, including equity, student perspectives, personal experiences, and quantitative data to support either side. Once each group has completed both posters, the teachers will engage in a carousel to comment and question each other's ideas. Each teacher will be given a stack of post-it notes to leave questions or thought-provoking comments on each poster. Once all teachers have visited each group's posters, the facilitator will ask each group to share their comments and answer any questions left for them. Discussion questions include

- Why do we as mathematics teachers care about tracking?
- What are some effects of tracking?
- Is tracking equitable?
- If we can't do anything about it, what is the point?

Then, the facilitator will share about the research done at Railside High by Boaler and Staples (2008). The focus of the discussion will be on the instructional practices used by teachers at Railside and how they increased the equitable nature of the classroom. Then, the facilitator will ask the teachers to revisit their earlier thoughts about equity and tracking and ask for actionable measures that can be taken to implement more equitable practices in their own classrooms. Anticipated responses include

- More opportunities for enrichment, not just for the advanced students
- Allowing all students to participate in inquiry tasks, not just well-behaved classes
- Implement inquiry-based activities and use manipulatives
- Foster a sense of community among the students by having them use each other's names and implement accountable talk
- Not limiting the curriculum or what is taught based on the level of the class

- Closure (20 minutes)-

Then, the facilitator will encourage the teachers to reflect on the importance of equitable instruction. Emphasis will be placed on what the teachers can do to foster equity in their classrooms to combat the inequity perpetuated by systemic practices such as tracking.

- What are two specific ideas that you can implement with your students to promote fairness of access and potential?
- How can tracking negatively impact our students?

To prepare for the next session, the teachers should be prepared to share a math lesson that went well and why in the next session.

Assessment/Reflection

The teachers will complete a 3-2-1 exit ticket include three things they learned or thought differently about, two things they still wonder about and one question they still have about equity or tracking.

Professional Development- Day 2: Math Tasks part 1 (135 minutes)

Learning objectives

1.) Teachers will define a math task and give key characteristics of rich math tasks.

2.) Teachers will select a math task that is appropriate for their specific content area.

Materials

Powerpoint to display paper folding task, instructions and research on characteristics of rich math tasks (See Appendix B). Chart paper and markers. Notecard.

Instruction

- Activation (20 minutes)-

The workshop will begin with the teachers giving an example of a lesson/activity that they think went well in their classroom and why. The facilitator will encourage the teachers to think of the specific materials or actions that made the lesson or activity a success and how they know it was a success. Some expected teacher responses include student engagement, level of fun, hands on materials, level of rigor and openness, student data, etc. The purpose of this activity is to encourage teachers to think about aspects of their lesson that they think made the lesson successful. The facilitator is looking for the teachers to mention access and potential to link to equitable instruction. The facilitator will use guiding questions to prompt these responses:

- How do you know the lesson was successful?
- Were your students engaged in the lesson or activity? Why were they engaged?
- How did you incorporate student accountability?
- Did all students have access to the content presented in the lesson or activity? How do you know?
- What did you do to incorporate your students' personal preferences or backgrounds into your lesson or activity?

- Exploration (70 minutes)

The facilitator will engage the teachers in a rich math task, the paper folding task. Each teacher will be given a square piece of paper and then be asked to fold the paper in such a way to create different polygons with a specific area compared to the original square and must then convince his/her group that the shape meets the specified criteria:

1. Construct a square with exactly ¼ the area of the original square. Convince yourself and then your partner that it is a square and has ¼ of the area.

2. Construct a triangle with exactly ¼ the area of the original square. Convince yourself and then your partner that it has ¼ of the area.

3. Construct another triangle, also with ¼ the area, that is not congruent to the first one you constructed. Convince yourself and then your partner that it has ¼ of the area.

4. Construct a square with exactly ½ the area of the original square. Convince yourself and then your partner that it is a square and has ½ of the area.

5. Construct another square, also with ½ the area, that is oriented differently from the one you constructed in #4. Convince yourself and then your partner that it has ½ of the area.

As the teachers are working, the facilitator will be monitoring and asking guiding and advancing questions:

- How do you know that your square has ¼ of the original area?
- What strategies are you using to fold your shape?
- Why does this shape have the specified area?
- How can you convince your group that your shape meets the given criteria?

The facilitator will orchestrate a discussion focusing on key strategies and connections made while completing the task. The presenters will be selected and sequenced while the teachers are working. The anticipated sequence of presentations will be

- Guess and check
- Folding in half vertically/horizontally
- Folding along the main diagonals
- Folding along diagonals in rectangles that are created (not main diagonals)
- Folding the paper into 16ths and refolding/building up to make the specified shapes

At the end of the discussion, the facilitator will ask the teachers to think of characteristics of the task that make it a rich math task and create a poster for their table group. Anticipated responses include high floor or low ceiling, opportunities for enrichment, multiple solution paths, story-telling aspect, real-world application, draw on personal or cultural relevance to the student, etc. Then, the teachers will participate in a carousel to evaluate and critique each other's ideas. Once each group has visited the poster of all the groups and left feedback, the teachers will discuss whole group the characteristics of rich math tasks that they think a rich task must have. Then, the facilitator will share the characteristics of rich math tasks supported by research: encourage thinking, reasoning and problem-solving, low floor and high ceiling, allow for multiple entry points, solution pathways and representations, allow students to draw on previous knowledge, encourage collaboration, and include opportunities for extension. The facilitator will give the teachers time for a think-pair-share to discuss how the characteristics they listed are similar or dissimilar to the characteristics supported by research. Then, the facilitator will encourage the teachers to connect rich math tasks with equitable instruction using guiding questions:

  - Why does it matter that I implement math tasks in my classroom?
  - Why does it matter that this is a rich math task?

The facilitator will reiterate how equitable instruction begins with the teacher's lesson choices. Math tasks allow students the opportunity to go deep with the mathematics and develop conceptual understanding of the content at hand.

- Closure (45 minutes)

Then, the facilitator will provide time for the teachers to find or adapt a rich math task to use in their classrooms. Teachers that are struggling with where to start will be directed to explore tasks available at Three-Act Math Tasks, Open Middle, and Illustrative Mathematics. Then, the teachers will be given two short tasks that are very similar and asked to determine which one they think is the best example of a rich math task and why. Teachers will also share with the facilitator their chosen task and provide justification for why they believe it is a rich math task. Then, the facilitator will ask the teachers to reflect on how rich math tasks encourage equity in the classroom and ask for two actionable ideas the teachers can implement in their classrooms to encourage equity in the context of math tasks. Anticipated responses include

- Using short open-ended tasks for bellwork or early finishers to encourage problem-solving
- Using Open Middle problems instead of fluency worksheets

Assessment/Reflection

The teachers will complete a 3-2-1 exit ticket include three things they learned or thought differently about, two things they still wonder about and one question they still have about rich math tasks.

Professional Development- Day 3: Math Tasks part 2 (120 minutes)

Learning objectives

1.) Teachers will define the four levels of cognitive demand.
2.) Teachers will identify and provide justification for the level of cognitive demand of a task.

Materials

Powerpoint to display task and cognitive demand levels (See Appendix C). Copies of task handouts. Chart paper and markers. Notecard.

Instruction

- Activation (40 minutes)

  The workshop will begin with the teachers reflecting on the qualities of rich math tasks. Then, the teachers will engage in a mini task to create the equations of three lines that will form a right triangle by only using the digits -9 to 9. For time constraints, the facilitator will only debrief each group individually.

- Exploration (60 minutes)

  The facilitator will give the teachers the four example tasks and ask them to determine their level of cognitive demand. This is open for interpretation by each teacher and will be defined later in the workshop by the facilitator. Some expected responses are to evaluate the depth of knowledge of the question types, use the instructional focus documents from the TN Department of Education, or compare them to past assessment questions. Then, the teachers will create a poster ranking the four tasks from lowest to highest cognitive demand and provide justification. As the teachers are working, the facilitator will be monitoring their work and asking guiding and advancing questions:

  - Why did you decide to rank this task as lower than this task?
  - What aspects of this task make it more cognitively demanding than the others?
  - How could you adapt this task to increase its cognitive demand?

  Then, the teachers will participate in a carousel to evaluate and critique each group’s ideas. Each group will be required to use a sticky note to leave a question or feedback at each group's poster. The facilitator is looking for the teachers to describe the tasks in terms of memorization or application and will point those out during the carousel. Then, the facilitator will define and share with the teachers the 4 levels of cognitive demand as described by Smith and Stein (2011). Then, the teachers will revisit their posters to label the tasks with the appropriate level of cognitive demand while again justifying their choice. As a reflection moment for this workshop session, the teachers will be asked why they should consider the level of cognitive demand of a task. Expected responses include level of rigor, pushing all students to think mathematically, opportunities for enrichment, opportunity to be doers of mathematics, etc. Then, the teachers will be asked to evaluate the level of cognitive demand of the opening task and justify their answers. Anticipated responses include procedures with connections and doing mathematics.

- Closure (20 minutes)

  The teachers will be asked to evaluate the level of cognitive demand of their chosen task from the second workshop session and provide a justification. If the task is low-level, the teachers will be asked how they could adapt it to make it more high-level. Then, the facilitator will ask the teachers to reflect on how considering the cognitive demand of a task encourages equity in the classroom and ask for two actionable ideas the teachers can implement in their classrooms to encourage equity in the context of cognitive demand. Anticipated responses include

  - If we never expose our students to high-level tasks, they will never have the opportunity to access high-level content or go deep with the mathematics.
  - Use one level 3 (procedures with connections) task per chapter

Assessment/Reflection

The teachers will complete a 3-2-1 exit ticket including three things they learned or thought differently about, two things they still wonder about and one question they still have about cognitive demand.

Professional Development- Day 4: Math tasks part 3 (120 minutes)

Learning objectives

1.) Teachers will identify classroom factors that affect task cognitive demand.

2.) Teachers will identify teacher factors that affect task cognitive demand.

Materials

Powerpoint to display task and instructions (See Appendix D). Chart paper and markers. Notecard.

Instruction

- Activation (40 minutes)

  The workshop will begin with the teachers working on the frog puzzle with their table groups. Each pair will be given a copy of the frog puzzle in which three green frogs and three brown frogs are placed in a row with one space separating the two colors. The teachers need to determine the least number of moves to have the green and brown frogs switch sides. As the teachers are working, the facilitator will monitor their progress and ask guiding and advancing questions:

  - What is your strategy for moving the frogs?
  - Does it matter which frog you start with?
  - Does the order in which you move the frogs matter?
  - What is the minimum number of moves if there are 4 frogs on each side? 5 frogs?

- What patterns are you noticing?

The facilitator will lead a short discussion on determining the cognitive level of the frog puzzle to assess the teacher's current understanding of cognitive demand. The purpose is not to necessarily correctly identify the cognitive demand of the frog puzzle but to make sure the teachers are distinguishing between the four levels correctly and justifying their decision.

- Exploration (60 minutes)

  The facilitator will give the teachers the greatest/smallest solution task from Open Middle and have them create the equation with the smallest solution and the equation with the greatest solution. As the teachers are working, the facilitator will be monitoring their work and asking advancing and assessing questions:

  - How did you get that answer? Did you check your answer to make sure it is correct?
  - What does it mean by smallest solution?
  - What patterns are you noticing?
  - What strategies have you used so far? What about guess and check?
  - What is another way you could approach this task?
  - What would happen if you switched these values?
  - How does the constant affect the solution? The variable term?
  - Which part of the equation most affects the solution?
  - Could you create a simpler equation template to get you started?

Once all the groups are done with the task, the facilitator will lead a whole-group discussion to connect their solution strategies. Anticipated strategies include guess and check, working backwards, using more sophisticated means such as technology/systems of equations, and solving the equation in terms of constraints.

Then, the facilitator will ask the teachers to identify the level of cognitive demand of the task and justify their response. Then, the facilitator will direct the teachers to think of the perspective of the facilitator and determine what teacher moves maintained or decreased the cognitive demand of the task. Each table group will create a poster with the list of teacher moves that maintained the cognitive demand and a list of teacher moves that lowered the cognitive demand from what they observed, what they have experienced as students, or what has happened in their classrooms. Once each group is done, the teachers will complete a gallery walk to read each group's ideas. Anticipated responses include questioning, prompting to stay on task/persevere, task selection, etc. Once everyone has had time to think about each group's ideas, each group will be asked to determine what teacher moves would have specifically lowered the cognitive demand of the greatest/smallest solution task. Anticipated responses include giving the solution, changing the equation template, not holding students accountable for work/participation, classroom environment not conducive to inquiry, teacher disposition, student disposition, task set up, time, etc.

- Closure (20 minutes)

  The teachers will think of a time in their classrooms in which they feel that they lowered the cognitive demand of a task or assignment and how they would change what they did in order to maintain the cognitive demand of the activity. The teachers will also record three specific teacher moves they can do to not lower the cognitive demand of their chosen math task. Then, the facilitator will ask the teachers to reflect on how maintaining the level of cognitive demand encourages equity in the classroom and ask for two actionable ideas the teachers can implement in their classrooms to encourage equity in the context of implementing rich math tasks. Anticipated responses include

- Developing a sense of community within the classroom
- Start with less time for a task and add more if necessary
- Implement accountability procedures for students during tasks and group work

Reflection

The teachers will complete a 3-2-1 exit ticket include three things they learned or thought differently about, two things they still wonder about and one question they still have about classroom factors that affect cognitive demand.

Professional Development- Day 5: Classroom Discourse (120 minutes)

Learning objectives

1.) Teachers will identify the five practices for orchestrating mathematical discourse.

2.) Teachers will anticipate student responses to a selected task.

3.) Teachers will select and sequence student responses to a selected task.

Materials

Powerpoint to display sequence task and instructions (See Appendix E). Chart paper and markers. Notecard.

Instruction

- Activation (50 minutes)

  The teachers will engage in the sequence task where they are asked to draw the next two pictures in the sequence, identify the sequence as geometric, arithmetic or neither, and write a formula to represent the *n*th term of the sequence. As the teachers are working, the facilitator will monitor their progress and ask assessing and advancing questions as needed.

- "How many circles are in the next term? How do you know?"
- "Why did you say this is a geometric/arithmetic sequence? What is a geometric/arithmetic sequence?"
- "I see you separated your picture into parts. Why did you do that? Did it help?"
- "What does this variable represent in your formula? What does this number represent?"
- "Where is the number 4 from your formula visible in the picture?"

Once all the table groups have completed the task and put up their poster with their solution and work, the facilitator will lead a group discussion using the five practices on how the teachers determined the next picture in the sequence and how they created their formula to represent the *n*th term. The anticipated sequence of presentations is

- Incomplete solution- teachers only determined the number of circles in the next picture, not a general formula for the *n*th term
    - Facilitator will follow up with how they got their answer, leaving the work on the board to draw connections to the next group
- Table and using technology
    - Less common strategy but facilitator will follow up by comparing it to the previous group's answer and ask them to leave their work to draw connections to the next group
- Breaking into squares/building up

  - Minus the missing pieces/subtracting/breaking down

- Exploration (45 minutes)

  The facilitator will share the five practices for orchestrating mathematical discourse and ask the teachers to identify when each practice was used during the previous discussion. Then, the facilitator will give the teachers the pool problem and have them anticipate student responses. Once they have several strategies listed, they will swap papers and have another teacher work out the problem and share their anticipated strategies. Then, everyone will share their anticipated strategies as a whole group to ensure everyone has an understanding of the multiple ways the problem can be approached. Then, each group will be asked to select and sequence their anticipated solution strategies and create a poster to share with the group. Included on the poster should be the selected strategies and the order in which to present them. Once every group has made their poster, the teachers will do a carousel to view each group's ideas. Then, the facilitator will lead a discussion about why each group selected their strategies and order. Then, the facilitator will share her anticipated strategies chart for the activity and use it to debrief the teachers on how the previous discussion was orchestrated to focus on key aspects of the task and the progression of concrete to abstract in the solution strategies. Then, the facilitator will share her monitoring chart for the justification discussion focusing on how anticipating the reasons why to select and sequence certain strategies helped guide the discussion.

- Closure (25 minutes)

  The facilitator will share specific teacher moves from Smith and Stein (2011) to illustrate what the teacher is doing during the discussion and ask the teachers to identify and describe when the facilitator used each practice. Then, the facilitator will ask the teachers to reflect on how classroom discourse encourages equity in the classroom and ask for two actionable ideas the teachers can implement in their classrooms to encourage equity in the context of classroom discourse. Anticipated responses include

- Discourse allows the students the opportunity to justify their thinking in another way, which gives them the opportunity to go deep with the mathematics
- Discourse allows the students access to the mathematics in another way
- Keep track of who shares each time and make sure that every student has the opportunity to share their ideas

Reflection

The teachers will complete a 3-2-1 exit ticket include three things they learned or thought differently about, two things they still wonder about and one question they still have about the five practices or anticipating student responses.

Professional Development- Day 6: Teacher Discourse Moves (150 minutes)

Learning objectives

1.) Teachers will identify the nine teacher discourse moves.

Materials

Powerpoint to display instructions, prompts, and classroom videos (See Appendix F). Padlet link. Chart paper and markers. Notecard.

Instruction

- Activation (20 minutes)

The teachers will create a Padlet to share strategies and teacher moves they use to foster classroom discourse.

- Exploration (70 minutes)

The teachers will watch a 15-minute video of a teacher orchestrating a mathematical discussion around a technology task. As the teachers are watching, they will list teacher discourse moves they see being implemented. Then, each teacher will be given time to share their list with their group before each group shares with everyone. Anticipated responses include redirecting, questioning, push for justifications, revoicing or rephrasing, wait time, student accountability, push for connections, etc.

Then, the facilitator will share the nine discourse moves compiled by NC^2ML. The teachers will compare this list to their original observations and determine if there were any other discourse moves in the video from the nine discourse moves that they had missed the first time. Then, the facilitator will direct each group to write an edit to the video that would include the missing discourse moves. The edit should be no less than 20 lines (10 teacher and 10 student lines) and include realistic student responses based on what the student's understanding in the video. Then, each group will act out their script to share with everyone. The facilitator will call on teachers to identify the teacher discourse moves that the presenting group included.

- Closure (60 minutes)

The teachers will write a script, including identifying their discourse moves, of their anticipated classroom discussion for their selected task. The script should be clear and follow a logical sequence of events. Advancing and assessing questions should be included. Student understandings and misconceptions should be evident from the discussion. Multiple solution strategies should be incorporated. Teacher moves should be clearly identified and support the flow of the discussion and focus the students on connecting the mathematics. Then, the facilitator will ask the teachers to reflect on how these teacher moves encourage equity in the classroom. Anticipated responses include

- Assessing and advancing questions gives the students access to the mathematics
- Inviting student participation gives the students the opportunity to share their ideas and gives the students another perspective to consider
- Orienting students to another's reasoning gives them the opportunity to make sense of the problem in another way and allows them access to the mathematics at hand
- Assigning competence is a great way to invite student participation, encourage them to want to be capable learners of mathematics, and foster a sense of ownership that encourages them to want to go deep with the mathematics

Assessment/Reflection

The teachers will complete a 3-2-1 exit ticket include three things they learned or thought differently about, two things they still wonder about and one question they still have about teacher moves for engaging in math discourse.

The teachers will turn in their completed lesson protocol for their selected tasks. This should include the selected task, monitoring chart for the task, and script for orchestrating the discussion. The facilitator will visit each class to see how each teacher implements what they learned from the workshop in their classrooms.

Chapter 4 – Professional Development Workshop Justification

This chapter will provide justification for each workshop session's lesson plan decisions using research cited in the literature review chapter. It will begin with an overall discussion of the unit plan sequence and structure. Then, each lesson will be discussed further to explain the instructional methods.

The Professional Development Workshop Plan

The professional development workshop is sequenced to guide teachers through developing a shared vision of equitable instruction. According to NCTM, "An excellent mathematics program requires that all students have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential" (2014, p. 59) In order for all students to have access to high-quality mathematics teaching, teachers must ensure they are using equitable teaching practices that provide the opportunity for all students to maximize their learning potential. This is supported by NCTM's five equity-based mathematics teaching practices: go deep with mathematics, leverage multiple mathematical competencies, affirm mathematics learners' identities, challenge spaces of marginality, and draw on multiple resources of knowledge (2017). This professional development focuses on three instructional practices that will increase the equitable instruction occurring in the secondary mathematics classroom as supported by the five equity-based mathematics teaching practices.

Throughout the workshop, teachers are engaged in doing mathematics through participating in rich math tasks (Liljedahl, 2021) and participating in mathematical discourse orchestrated using the five practice (Smith & Stein, 2011). Also, the teachers will be engaged in multiple opportunities for discourse throughout each session because the act of talking can help build understanding (Frank et al., 2009). The facilitator uses teacher discourse moves throughout the workshop sessions to model how to further develop confident math learners (NC^2ML, 2023). The assessment each day will be a 3-2-1 exit ticket, which emphasizes reflection and inquiry as supported by NCTM (2014).

Workshop Session 1 – Equity and De-tracking

The first workshop introduces the idea of equity and the pros and cons of de-tracking. Tabron et al. (2021) states an equity-oriented growth mindset is vital to support de-tracking reform. Although tracking is a systemic practice that cannot be changed only at the school level, NCTM encourages heterogeneity (2014; 2017). Thus, the table groups created during the workshop will be heterogeneous by years of experience. Furthermore, Rubin (2006) found that successful de-tracked math programs engage in instructional best practices, so this workshop is designed to incorporate those practices. Also, the warm-up North-South-East-West activity considers the student perspectives as suggested by Yonezawa and Jones (2006). As the teachers are working, the facilitator will engage in discourse moves to promote collaboration and learning (NC^2ML, 2023; NCTM, 2014, 2017).

During the exploration, the teachers will create a list of pros and cons for tracking and leave feedback for each other. This is supported by the work of NCTM (2014) and Rubin (2006). The closure of the workshop asks the teachers to determine specific ideas they can implement to promote fairness of access and potential (NCTM, 2014).

Workshop Session 2 – Math Tasks Part 1: Selecting a Task

The second workshop focuses on identifying and selecting a rich math task. As stated by NCTM, “Tasks that promote reasoning and problem solving can provide access to all students and promote equity by providing students with multiple entry points and ways to demonstrate competence” (2017, p. 47). Therefore, three sessions of the professional development workshop will specifically address math tasks. Also, Hiebert and Wearne (1993) noted that prolonged exposure and engagement with high-level tasks increases student learning, so each day of the workshop will include a rich math task. Furthermore, engaging in math tasks encourages student inquiry (NCTM, 2014) and positions students as math explorers (McCulloch & Lovett, 2024).

The workshop begins with the teacher reflecting on a previous lesson that went well and why. This allows the teachers time to reflect on their teaching practices and gives them the opportunity to begin thinking about what makes a good activity or lesson. Then, the teachers will engage with the paper folding task because it allows them to explore the mathematics at hand at a deep level (NCTM, 2018). Also, Stein and Lane (1996) stated gains in mathematical thinking, problem solving, and communication are a result of instructional changes. Moreover, the implementation of cognitively challenging tasks is positively associated with student achievement (Cai et al., 2011; Grouws et al., 2013; Schoen et al., 1999). The task was selected based on the levels of cognitive demand iterated by Smith and Stein (1998) and on the qualities of rich math tasks (Liljedahl, 2021; Meyer, 2011; NCTM, 2017, 2018; Piggott, 2018; Wolf, 2015). As the teachers work, the facilitator will use teacher discourse moves such as revoicing, encouraging in-progress thinking, assessing and advancing questions and assigning competence to promote confident math learners (NC^{2}ML, 2023).

Once all the teachers have completed or mostly completed the paper folding task, the facilitator will lead the teachers through a whole group discussion following the five practices established by Smith and Stein (2011). The facilitator will expect the presenters to justify their solutions as this has been shown to encourage students to have the same expectations during small group collaboration (Smagorinsky & Fly, 1993) and promotes access and agency (Bieda & Staples, 2020). Then, the facilitator will share the qualities of a rich math task and have the teachers use these characteristics to select a rich math task to implement in their own classrooms.

Workshop Session 3 – Math Tasks Part 2: Cognitive Demand

As supported by Liljedahl (2021), students need to be repeatedly engaged in high-level thinking tasks. One aspect that is important to selecting a high-level thinking task is the task's level of cognitive demand. This workshop session of the professional development follows from a professional development focused on learning about the levels of cognitive demand and how this had an impact on how teachers thought about and chose tasks to implement in their classrooms (Arbaugh & Brown, 2005).

The workshop will begin with the teachers reflecting on the qualities of a rich math task from the previous session (Liljedahl, 2021; Meyer, 2011; NCTM, 2017, 2018; Piggott, 2018; Wolf, 2015). Then, the teachers will engage in a short mini task to create the equations of three lines that form a right triangle. This task was selected because it meets the qualifications of a rich math task. As the teachers are working, the facilitator will monitor their work and use teacher discourse moves to promote them to be confident math learners (NC^2ML, 2023). The facilitator will also be using this time to engage each group in a math discussion based on the five practices (Smith & Stein, 2011).

During the exploration, the teachers will evaluate the level of cognitive demand of four tasks used as examples by Smith and Stein (1998). To promote inquiry, the facilitator will not define the levels of cognitive demand before the activity begins (NCTM, 2014). As the teachers are working with their groups, the facilitator will be pushing for justification and asking advancing and assessing questions. Then, they will participate in a carousel activity to reflect on and consider the justifications for each task in the other groups. Once everyone has come to a consensus about the ranking of the four tasks, the facilitator will share the four levels of cognitive demand as determined by Smith and Stein (2011). Hiebert and Wearne (1993) concluded that increased engagement and prolonged exposure to high-level cognitively demanding tasks increases student learning. Therefore, the facilitator will include a time of reflection to consider why the level of cognitive demand of the task matters.

During the closing, the teachers will evaluate the level of cognitive demand of the opening task and of their chosen task from the previous session. This is supported by Boston and Smith's (2009) findings that teachers that participated in a professional development focused on levels of cognitive demand increased the level of cognitive demand of their instructional tasks.

Workshop Session 4 – Math Tasks Part 3: Implementation

According to Stein and Lane, the successful implementation of a rich math task should foster student thinking and problem solving, which leads to deeper understanding and reasoning about the math behind the task (1996). Therefore, it is important to be able to successfully implement a rich math task. This is the focus of the fourth workshop.

To begin, the teachers will engage with another rich math task, the frog puzzle. This task was selected because of its low floor, high ceiling characteristics (Papert, 1993). Also, it is a non-curricular task, which Liljedahl argues is important to implement to build thinking classrooms (2021). To use the five practices for orchestrating productive mathematics discussions, the facilitator will lead a group discussion on the cognitive demand of the frog puzzle (Smith & Stein, 2011). Also, since justification at any level promotes equity according to Bieda and Staples (2004), the facilitator will focus on justification during the group discussion.

During the exploration, the teachers will engage in another rich math task, the greatest and smallest solution task. To promote student thinking, the facilitator will use advancing and assessing questions while monitoring the teachers as they work (NCTM, 2014). Then, the teachers will participate in a group discussion focused on their solutions to promote equity and help them make connections (NCTM, 2018). To highlight teacher moves that maintained or lowered the cognitive demand of the task, the teachers will create a poster and share their ideas in a carousel. This is important because teachers play a vital role in the decline or maintenance of the cognitive demand of a task (Johansson, 2007). Stein et al. (1996) found that competent performance modelled by the teacher or capable student was a key factor in classrooms that maintained the cognitive demand of a task. During the group discussion, modelling is a key practice to call attention to.

During the closing, the teachers will determine three teacher moves that they can do to not lower the cognitive demand of a task. Sullivan et al. (2015) determined that teacher moves that encourage persistence allowed teachers to maintain the cognitive demand of the task, so the facilitator will encourage the inclusion of such teacher moves.

Workshop Session 5 – Classroom Discourse Part 1: The 5 Practices

The purposeful exchange of ideas allows math teachers to promote equity through equal access and opportunity for all students. According to NCTM, students need to make connections to better understand the mathematical content and discourse provides them this opportunity (2018). Similarly, Kazemi and Hintz (2014) have found that classroom conversations are critical to a student's mathematical learning. Therefore, the fifth workshop session is focused on defining and orchestrating mathematical discussion.

During the activation, the teachers will engage in a rich math task, the sequence task. This task was selected because it is a high-level math task and has multiple solution strategies to anticipate (Smith & Stein, 2011). Also, prolonged exposure to high-level math tasks increases student learning (Heibert & Wearne, 1993). As the teachers are working, the facilitator will monitor the teachers working according to the five practices (Smith & Stein, 2011) in order to select and sequence their work to best make connections across the solution strategies and go deep with the mathematics (NCTM, 2014). Then, the facilitator will have the teachers identify when she used the five practices during that session to organize the discussion.

Since the purpose of the five practices is to give teachers doable moves to help the students achieve the mathematical goals of the lesson, the teachers will practice anticipating using the pool task (Smith & Stein, 2011). Following another recommendation from Smith and Stein (2011), the teachers will also trade monitoring charts with their table groups to help devise as many solution strategies as possible. Then, they will use their anticipated strategies to select and sequence the presenters.

Workshop Session 6 – Classroom Discourse Part 2: Teacher moves

The focus of the final workshop session is teacher moves and questioning. It is important to consider what actions a teacher is making during a lesson because teachers can gain insights into what students know and the approaches they use in order to adjust their instruction accordingly (Kersaint, n.d.).

To begin, the teachers will engage in a Padlet to share teacher moves they have used to foster classroom discourse. Then, the teachers will analyze a short video of a classroom discussion about the mean as a balance point (McCulloch & Lovett, 2024). This gives the teachers the opportunity to look for the nine discourse moves gathered by NC^2ML to promote confident math learners (2023). Once the teachers have seen the video, they will compare their lists to the nine prescribed moves and determine which, if any, are missing. This allows them to become more familiar with these moves to better identify and implement them in their own classrooms. Also, this allows the teachers the opportunity to work on questioning (Kersaint, n.d.)

To end the final workshop session, the teachers will write a script including teacher discourse moves for their selected task. This follows the recommendation of Spangler and Hallman-Thrasher (2014) that teachers have a more difficult time determining student responses to tasks or topics that are not as familiar to the teacher.

Conclusion

The purpose of this thesis was to give secondary mathematics teachers a shared vision of equitable instruction and key equitable instructional practices to implement in their classrooms. The literature emphasized how to foster equity in the mathematics classroom through de-tracking, implementing high-level math tasks and orchestrating productive classroom discourse. Further, the unit plan in Chapter 3 was created to provide teachers with research-based practices to create more equitable outcomes for all students.

Throughout this thesis, it has been made clear that creating equitable outcomes in our mathematics classrooms has been a concern for several decades. However, it is through equitable instructional practices that we will finally achieve the aims of NCTM that all students should appreciate the beauty and usefulness of mathematics and see themselves as capable lifelong learners and confident doers of mathematics.

Through this process, I have gained a deeper understanding of the importance of fostering equity in the secondary mathematics classroom. This thesis has underscored the significance of addressing inequity and has equipped me with valuable insights into effective teaching strategies that incorporate equitable teaching practices. One of the key lessons I learned is the recognition that equitable instruction goes beyond simply diversity; it requires a deliberate and proactive approach to dismantling obstacles to learning such as tracking. Furthermore, this thesis has highlighted the importance of continual professional development and collaboration. Continuous learning about diverse pedagogies, staying informed about relevant research in equitable education, and engaging in meaningful discussion with fellow mathematics educators are essential for maintaining and advancing equitable instruction in the secondary mathematics classroom.

My own teaching practice has shifted to incorporate a mindfulness of how to foster equity in my classroom, including creating an environment in which all students feel valued and capable in their mathematical endeavors. My goal is to always create opportunities for students to go deep with the mathematics and see themselves as capable lifelong learners and doers of mathematics.

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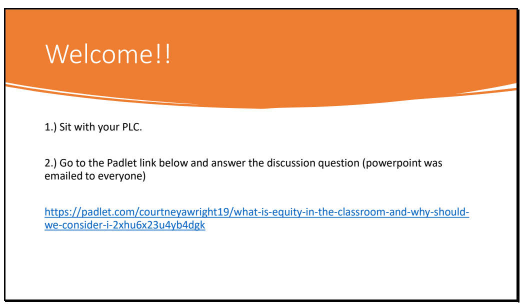
APPENDICES

APPENDIX A

Workshop Unit Plan PowerPoint Slides Day 1

Slide 1

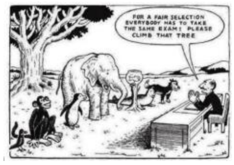
Slide 2



Slide 3

Debrief

- What were some popular responses to defining equity?
- What is the first word that comes to mind when "equity" is mentioned?



Slide 4

Research says…

We will adopt the understanding of equity coined by Driscoll et al. (2016) that equity is about fairness is terms of access and potential. That is, teachers should be "providing each learner with alternative ways to achieve, no matter the obstacles they face" and allowing the "students to do challenging mathematical reasoning and problem solving" (p. ix-x, 2016).

Slide 5

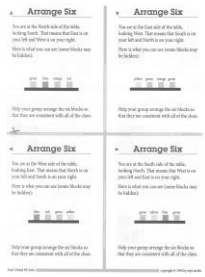
Personal experiences

- What has been your experience with equity (or denial of equity) as a teacher or student?

Slide 6

Arrange Six activity

- With your table group, each person will receive a directional card.
- Read your card aloud and arrange the six blocks to match the description on your card.
- Rotate clockwise, with each person reading their card and arranging the blocks.
- Continue until the blocks are arranged in such a way that each description is accurate from each direction.



Slide 7

What's the point?

- Why did we engage in this activity?
- Collaboration
  - How does collaboration relate to equitable instruction?
- Perspective
  - Whose perspective should we consider in the classroom? Why?

Slide 8

Research says…

- Why?
  - "Excellence in mathematics education requires equity—high expectations and strong support for all students" (NCTM, 2000, p. 12).
- Who?
  - Anthony and Walshaw (2009) argue that teachers are the most important resource for students developing mathematical identities, which leads to greater confidence in their ability to learn mathematics.
- Why do we care?
  - Students with strong mathematical backgrounds have increased professional opportunities (D'Ambrosio, 2012; NCTM, 2018).

Slide 9

Objectives

1.) Teachers will be able to define equity in terms of access and potential.

2.) Teachers will give examples of the cons of tracking.

3.) Teachers will reflect on equitable practices in the classroom.

Slide 10

Tracking

- What is tracking? How are students and teachers tracked?
- What are your personal experiences with tracking (as a teacher or student)?

Slide 11

Carousel Activity

Pros & Cons of Tracking	Pros & Cons of De-tracking
•	•
•	•
•	•
•	•

On your chart paper, create a list of pros and cons for Tracking. On your other piece of chart paper, create a list of pros and cons for De-tracking.

Slide 12

Carousel Activity

- Now that everyone is done, use your post-its to leave a comment or question at each poster.

Slide 13

Debrief

- What are some common themes you noticed?
- What ideas surprised you?
- What are some effects of tracking?

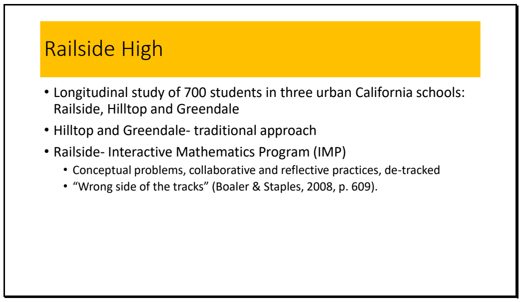
Slide 14

What's the point?

Why do we care about tracking or de-tracking?

Is tracking equitable?

Slide 15



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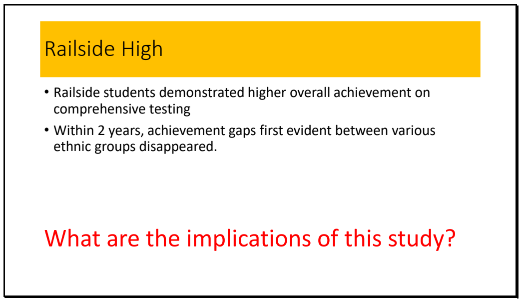

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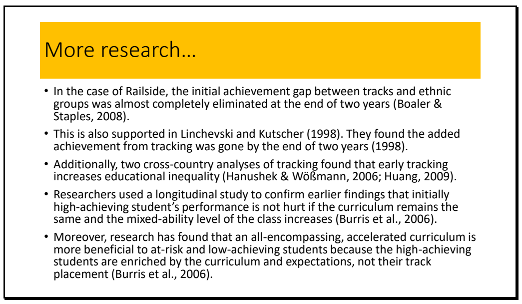


Slide 18

Again, why do I care?

While we know that tracking is a systemic problem, it is our job as teachers to make sure our students have access to high quality mathematics curriculum and instruction and the opportunity to go deep with the mathematics.

Personally, I know I can't change who is in which class, but I can make sure that all my students have access and opportunity to explore math at a high level and become life-long learners and confident doers of mathematics.

Slide 19

Closure

1.) What are two specific ideas that you can implement with your students to promote fairness of access and potential?

2.) How can tracking negatively impact our students?

3.) For the next session, think of a previous lesson that went well. Be prepared to share the highlights of the lesson and why you think it went well.

Slide 20

Exit ticket

On your notecard, write

3 things you learned or thought differently about

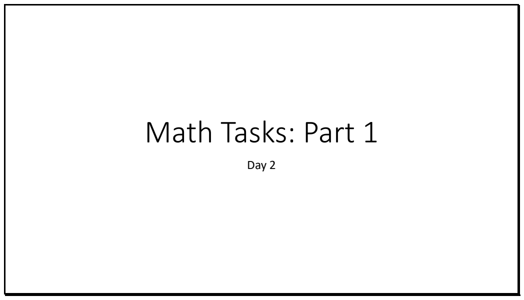
2 things you wonder or want to know more about

1 question you still have about equity or tracking

APPENDIX B

Workshop Unit Plan PowerPoint Slides Day 2

Slide 1



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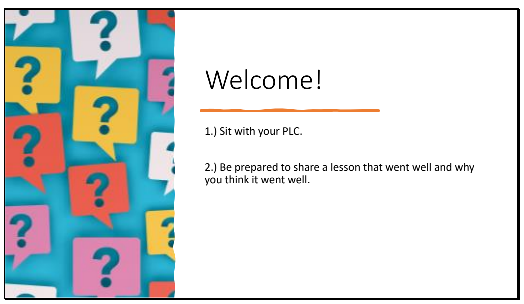

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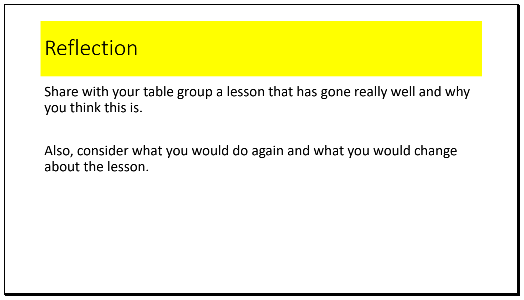

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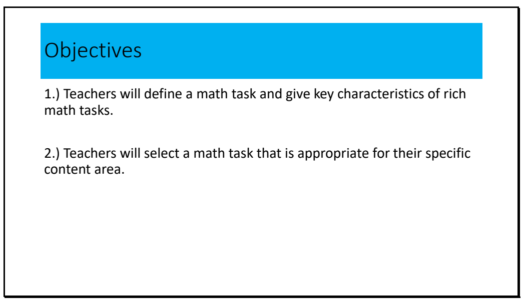

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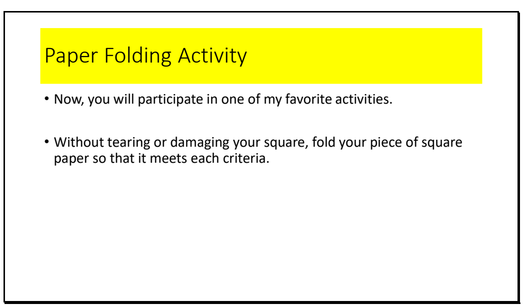


Slide 6


youcubed

Paper Folding

Work with a partner. Take turns being the skeptic or the convincer. When you are the convincer your job is to be convincing! Give reasons for all of your statements. Skeptics must be skeptical! Don't be easily convinced. Require reasons and justifications that make sense to you.

For each of the problems below one person should make the shape and then be convincing. Your partner is the skeptic. When you move to the next question switch roles.

Start with a square sheet of paper and make folds to construct a new shape. Explain how you know the shape you constructed has the specified area.

- 1. Construct a square with exactly ¼ the area of the original square. Convince yourself and then your partner that it is a square and has ¼ of the area.
- 2. Construct a triangle with exactly ¼ the area of the original square. Convince yourself and then your partner that it has ¼ of the area.
- 3. Construct another triangle, also with ¼ the area, that is not congruent to the first one you constructed. Convince yourself and then your partner that it has ¼ of the area.
- 4. Construct a square with exactly ½ the area of the original square. Convince yourself and then your partner that it is a square and has ½ of the area.
- 5. Construct another square, also with ½ the area, that is oriented differently from the one you constructed in #4. Convince yourself and then your partner that it has ½ of the area.

Slide 7

Debrief

- As you were working, I spoke to those I want to present their solutions and solution strategies.
- Make sure you are ready to share your strategy and are listening attentively to your peers while they are presenting.

Slide 8

Rich math task

- This is an example of a rich math task. What is a math task? What makes it a rich math task? What characteristics make this a quality task?
- With your table group, create a poster of the characteristics that you think makes this a rich math task. Then, we will do a carousel to share our thoughts and ask questions of each group.

Slide 9

Carousel

- What common characteristics did you notice?
- What common themes among the characteristics did you notice? (student engagement, accountability, teacher moves, etc.)

Slide 10

Research

Characteristics of a rich, or high quality, math task should include

1. Encourage thinking, reasoning and problem-solving
2. Allow for multiple entry points, solution pathways and representations
3. Allow students to draw on previous knowledge
4. Encourage collaboration
5. Opportunities for extension (Liljedahl, 2021; Meyer, 2011; NCTM, 2017; NCTM, 2018; Piggott, 2018; Wolf, 2015)

Slide 11

Why do I care?

- Why does it matter that I implement math tasks?
- Why does it matter that this is a rich math task?
- Equitable instruction
  - It is our job as teachers to provide opportunities for students to be "explorers" of mathematics and "providing each learning with alternative ways to achieve" and allowing the "students to do challenging mathematical reasoning and problem solving" (Driscoll et al., 2016, p. ix – x)

Slide 12

Research

- Several studies have found evidence that increased exposure to and prolonged engagement with high-level cognitively demanding tasks increases students' learning (Hiebert & Wearne, 1993; NCES, 2003; Stein & Lane, 1996).
- Silver and Lane (1996) found clear evidence that students showed gains in mathematical thinking, problem solving and communication under the influence of the instructional changes during the QUASAR Project.
- Furthermore, national and international research has shown that student achievement is positively associated with the implementation of mathematics curriculum that contains a prevalence of cognitively challenging tasks (Boaler & Staples, 2008; Cai et al., 2011; Grouws et al., 2013; Schoen et al., 1999; Stigler & Hiebert, 2004).

Slide 13

Again, why do I care?

- We want students to learn mathematics, not just memorize procedures, so we need them thinking and problem-solving. Math tasks give them the opportunity to develop these skills and go deep with the mathematics.

Slide 14

Examples of Rich Math Tasks

- Dan Meyer's 3-Act Math Tasks
- Robert Kaplinsky's Open Middle problems

Slide 15

Closure

- Find or adapt (look in the textbook, Dan Meyer, Open Middle) a task that you can implement with your classes this year.
- Make sure your task has the characteristics of a rich math task:
  - Encourage thinking, reasoning and problem-solving
  - Allow for multiple entry points, solution pathways and representations
  - Allow students to draw on previous knowledge
  - Encourage collaboration
  - Opportunities for extension

Slide 16

Exit ticket

On your notecard, write

3 things you learned or thought differently about

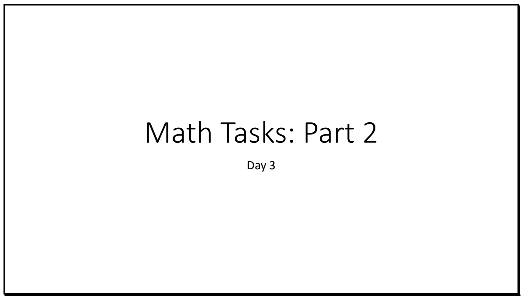
2 things you wonder or want to know more about

1 question you still have about rich math tasks

APPENDIX C

Workshop Unit Plan PowerPoint Slides Day 3

Slide 1



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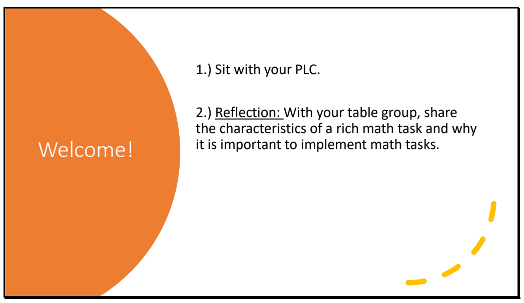

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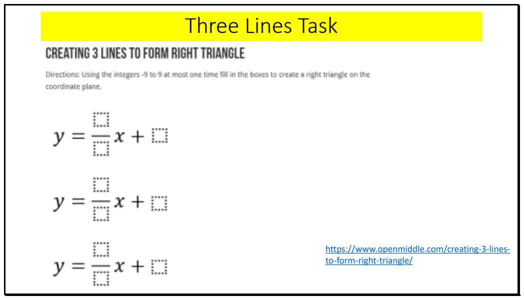

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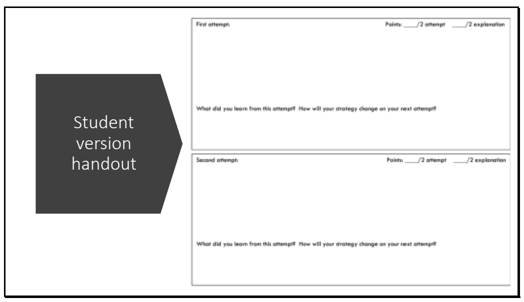

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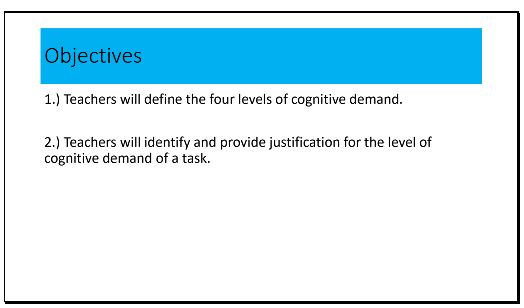


Slide 6

Cognitive Demand

- With your table group, determine the level of cognitive demand of each of the tasks (Smith & Stein, 1998).

Slide 7

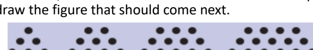
Why am I doing this?

- Why should we consider the cognitive demand of a task? How does this affect our students?
- How does this convey our expectations of our students?
- How does considering the cognitive demand of a task allow us to be more equitable in our instruction?

Slide 8

Task 1

For homework, Mark's teacher asked him to look at the pattern below and draw the figure that should come next.

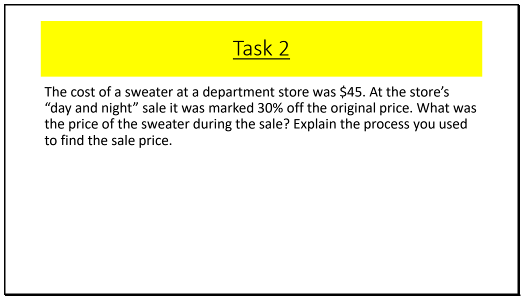


Mark does not know how to find the next figure.

A. Draw the next figure for Mark

B. Write a description for Mark telling him how you knew which figure comes next.

Slide 9



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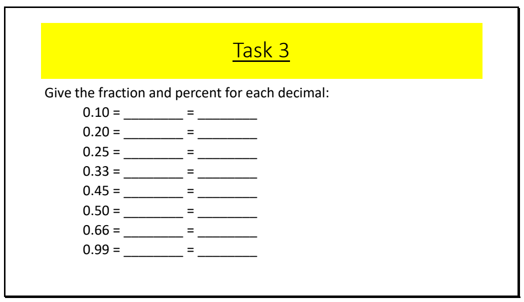

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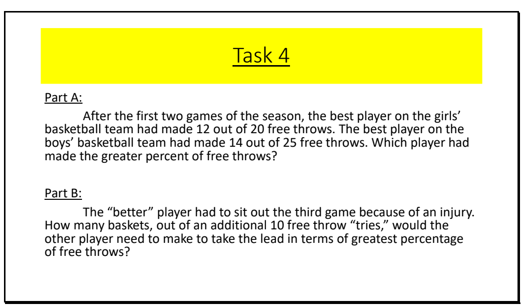


Slide 12

Carousel Activity

- When your table group is done, create a poster with your most cognitively demanding task at the top and the least cognitively demanding at the bottom including a brief justification.

Ranking by Cognitive Demand
Task 1- procedural
Task 2- application/enrichment
Task 3- (include your justification)
Task 4- (include your justification)

- When everyone is done, we will use the post-its to leave feedback and ask questions of each group.

Slide 13

Debrief

- What did you use to evaluate each task?
- What characteristics made you say one task was more rigorous or cognitively demanding that another?
- Should you consider how the students will approach the task? Why or why not?
- What factors outside of the task do you think you should consider that might affect the cognitive demand of the task?

Slide 14

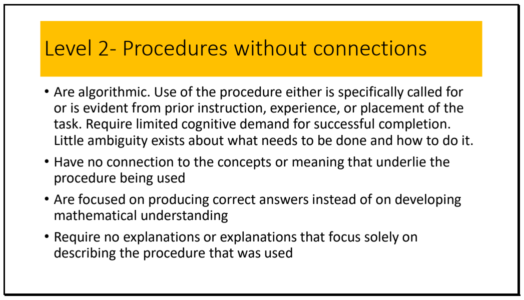
Level 1- Memorization

- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced

Smith, M. & Stein, M. (1998). Selecting and Creating Mathematical Tasks: From Research to Practice. *Mathematics Teaching in the Middle School*, 3, 344 – 50.

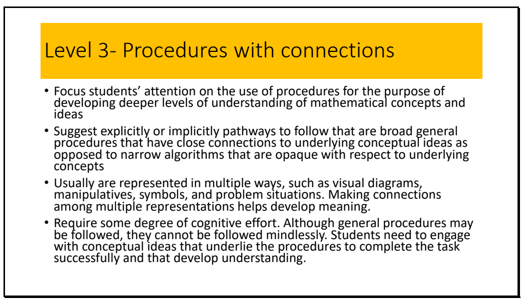
What are some key words or phrases that stand out to you? How do you expect the students to respond to such tasks?

Slide 15



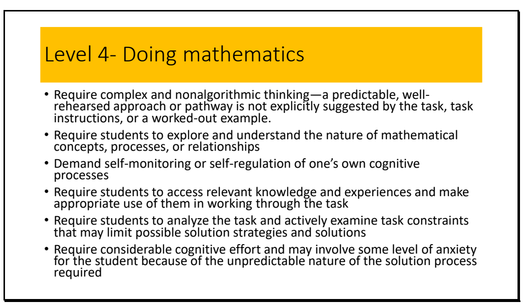
What are some key words or phrases that stand out to you? How do you expect the students to respond to such tasks?

Slide 16



What are some key words or phrases that stand out to you? How do you expect the students to respond to such tasks?

Slide 17



What are some key words or phrases that stand out to you? How do you expect the students to respond to such tasks?

Slide 18

Four tasks

- With your table group, categorize each of the four tasks with their level of cognitive demand.

Slide 19

Why does this matter?

- Cognitively demanding tasks challenge our students and give them access to high quality mathematics instruction (equity!)
- If we want our students to be explorers of mathematics and confident doers of mathematics, we need to give them the opportunity.
- **It is all about equity. We have to give students access to quality curriculum and the opportunity to go deep with the mathematics and be confident doers of mathematics.**

Slide 20

Research

- The cognitive demand for the high-level instructional tasks that were implemented in the Connected Mathematics Program (CMP) classrooms was a significant predictor of the student's achievement gains (Cai et al., 2011).
- Successful implementation of a high quality math task should foster student thinking and problem-solving, which leads to a deeper understanding and reasoning about the mathematics behind the task (Stein & Lane, 1996).

Slide 21

Examples- lower level tasks

Memorization

What is the rule for multiplying fractions?

Expected student response:

You multiply the numerator times the numerator and the denominator times the denominator.

or

You multiply the two top numbers and then the two bottom numbers.

Procedures without Connections

Multiply:

$$\frac{2}{3} \times \frac{3}{4}$$

$$\frac{5}{6} \times \frac{7}{8}$$

$$\frac{4}{9} \times \frac{3}{5}$$

Expected student response:

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12}$$

$$\frac{5}{6} \times \frac{7}{8} = \frac{5 \times 7}{6 \times 8} = \frac{35}{48}$$

$$\frac{4}{9} \times \frac{3}{5} = \frac{4 \times 3}{9 \times 5} = \frac{12}{45}$$

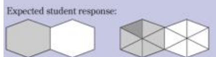
Slide 22

Examples- higher level tasks

Procedures with Connections

Find 1/6 of 1/2. Use pattern blocks. Draw your answer and explain your solution.

Expected student response:



First you take half of the whole, which would be one hexagon. Then you take one-sixth of that half. So I divided the hexagon into six pieces, which would be six triangles. I only needed one-sixth, so that would be one triangle. Then I needed to figure out what part of the two hexagons one triangle was, and it was 1 out of 12. So 1/6 of 1/2 is 1/12.

Doing Mathematics

Create a real-world situation for the following problem:

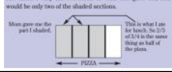
$$\frac{2}{3} \times \frac{3}{4}.$$

Solve the problem you have created without using the rule, and explain your solution.

One possible student response:

For lunch Mom gave me three-fourths of a pizza that we ordered. I could only finish two-thirds of what she gave me. How much of the whole pizza did I eat?

I drew a rectangle to show the whole pizza. Then I cut it into fourths and shaded three of them to show the part Mom gave me. Since I only ate two-thirds of what she gave me, that would be only two of the shaded sections.

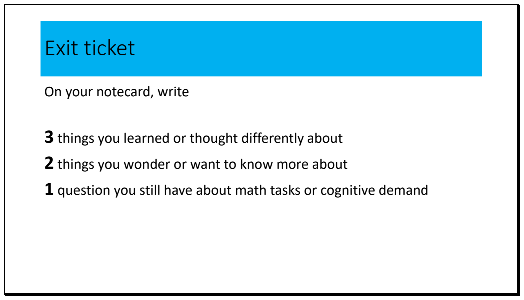


Slide 23

Closure

- What is the cognitive demand of the Three Lines Task? Why?
- Looking at your selected task to implement (from workshop 2), what is the task's level of cognitive demand? If the task is a lower level task, how can you adapt it to make it more cognitively demanding?

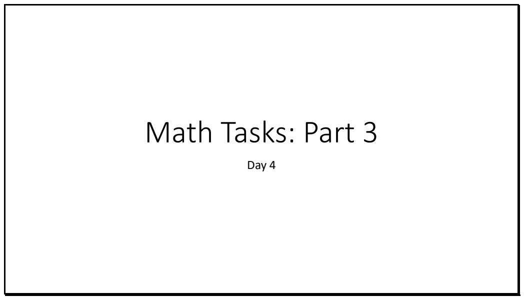
Slide 24


Exit ticket
On your notecard, write
3 things you learned or thought differently about
2 things you wonder or want to know more about
1 question you still have about math tasks or cognitive demand

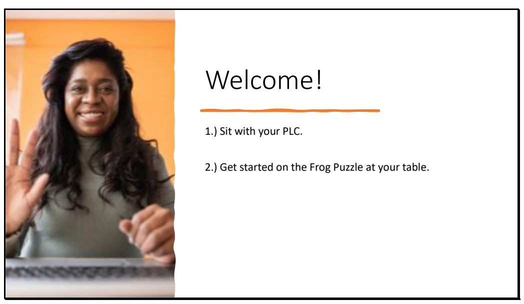
APPENDIX D

Workshop Unit Plan PowerPoint Slides Day 4

Slide 1



Slide 2



Slide 3

Frog Puzzle

Three green frogs (on the left) are trying to change position with 3 brown frogs (on the right). Green frogs and brown frogs can only move forward onto an empty lily pad or leap a single frog onto an empty lily pad.

How many moves are required to have all the brown frogs on the left and all the green frogs on the right with an empty lily pad in the middle?

Slide 4

Cognitive Demand

- What is the cognitive demand of the frog puzzle? Justify your answer.

Slide 5

Research

- Level 1- memorization
- Level 2- procedures without connections
- Level 3- procedures with connections
- Level 4- doing mathematics (Smith & Stein, 1998)

Slide 6

Research

- Peter Liljedahl argues for the use of non-curricular math tasks to spark engagement and teach students our expectations about thinking (2021).
- What does this imply for our instruction?
- How does this support equitable instruction in our classrooms?

Slide 7

Objectives

1.) Teachers will identify classroom factors that affect task cognitive demand.

2.) Teachers will identify teacher factors that affect task cognitive demand.

Slide 8

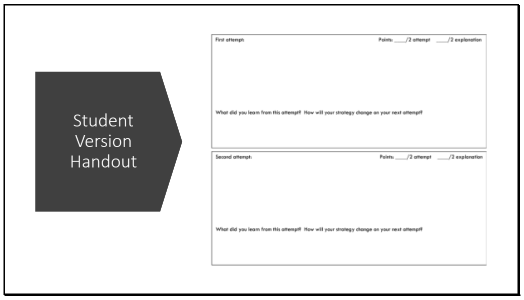
Greatest/Smallest Solution

MULTI-STEP EQUATIONS – SMALLEST (OR LARGEST) SOLUTION

Directions: Using the digits 1 to 9, at most one time each, create an equation where x has the smallest (or greatest) possible value.

$$\frac{\square}{\square}\left(\square x + \square\right) + \square x = \square x + \square$$

Slide 9



Slide 10

Debrief

- What was the greatest solution?
- What was the smallest solution? How did you define smallest solution?
- What were some popular solution strategies?
- How would you categorize the level of cognitive demand of this task? Why?
- How do you think your students would approach this problem?

Slide 11

So, who cares?

- Why should we consider the level of cognitive demand?
- Equitable instruction means access to quality instruction and curriculum for all students. Rich math tasks with high cognitive demand allow all students access to the content and the opportunity to go deep with the mathematics.
- They won't achieve those goals if you don't give them the opportunity.

Slide 12

Carousel Activity

Now, let's switch to our teacher perspective: How do we successfully implement these higher level tasks?

What factors (teacher and environment/classroom) affect the level of cognitive demand of a task?

Teacher moves that lower cognitive demand	Teacher moves that maintain cognitive demand
1.	1.
2.	2.
3.	3.
4.	4.

When everyone is done, we will use the post-its to leave feedback and ask questions of each group.

Slide 13

Debrief

- Did you think of more teacher or classroom factors? Why?
- What common themes/responses did you notice among the responses?
- What do you think is the most influential factor that lowers cognitive demand?
- What do you think is the most influential factor that maintains cognitive demand?

Slide 14

Reflect

- What teacher moves did I do that lowered or maintained the level of cognitive demand of the Greatest/Smallest Solution task?
- How might I have raised the cognitive demand of the task?
- What other classroom/environmental factors affected the level of cognitive demand?

Slide 15

Why does this affect me?

- Why should we consider how to effectively implement a high-level math task?
- The goal is to engage our students in equitable instruction!

Slide 16

Research

- Using six teachers participating in the QUASAR project, Henningsen and Stein (1997) found some factors that affect the task implementation include classroom norms, task conditions, the teachers' instructional disposition, and the students' learning dispositions. Some classroom factors that led to the decline of the cognitive demand and task implementation include a lack of time, focus on correct answers, superficial analysis of the problem.
- A key factor present in tasks that maintained their level of cognitive demand was a competent performance modelled by the teacher or a capable student, while classroom management problems were only judged to be a key factor in the cognitive demand decline in 18% of the observed tasks in a study conducted by Stein et al. (1996).
- Teachers should also ensure they select an appropriate task for the students and continually monitor the student's progress through scaffolding and encouragement to provide meaningful explanations or connections (Anderson, 1989; Doyle 1988; Henningsen & Stein, 1997).

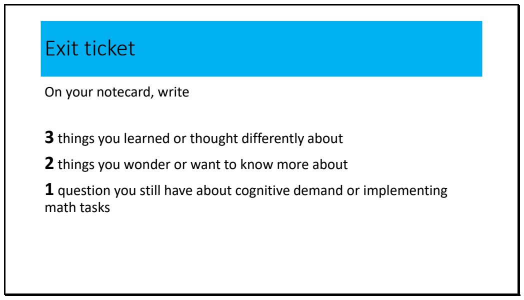
Which factors did you already consider? Do any of them surprise you? What do you think about the influence of classroom management?

Slide 17

Closure

- Think about a lesson or activity in which you lowered the cognitive demand. How would you do it differently now? What exactly would you change?
- Consider your chosen task to implement this year. What are three moves you can do to not lower the cognitive demand of your task?

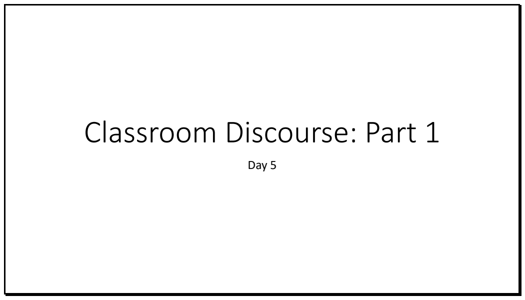
Slide 18


Exit ticket
On your notecard, write
3 things you learned or thought differently about
2 things you wonder or want to know more about
1 question you still have about cognitive demand or implementing math tasks

APPENDIX E

Workshop Unit Plan PowerPoint Slides Day 5

Slide 1



Slide 2

Slide 3

Objectives

1.) Teachers will identify the five practices for orchestrating mathematical discourse.

2.) Teachers will anticipate student responses to a selected task.

3.) Teachers will select and sequence student responses to a selected task.

Slide 4

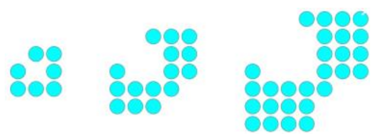
What's Next?

Given the sequence illustrated below,

a.) draw the next two pictures in the sequence

b.) identify the sequence as geometric, arithmetic or neither

c.) and write a formula to represent the nth term of the sequence.



Slide 5

Discussion

- Each selected group will share their solution and strategy to creating their formula to represent the nth term.
- Make sure you are listening and asking the presenters questions to clarify your understanding.

Slide 6

What just happened?

- Why did I select and sequence the presentations in that order?
- How did that make a difference than if I had called on groups at random or just gone around the room clockwise?

Slide 7

Why?

- So again, the question is "Why was I so careful in the presentations?"
- We want our students to have access to the content we are presenting. Allowing students to share and justify their thinking and carefully sequencing the presentations to scaffold the solution strategies gives students access to deep content and the opportunity to understand it themselves.

Slide 8

Research

- 5 Practices for Mathematical Discourse
  - Anticipating
  - Monitoring
  - Selecting
  - Sequencing
  - Connecting (Smith & Stein, 2011)

Slide 9

Anticipating

- Practice 0-setting goals/selecting a task
- Anticipating is about preparing yourself to engage with your students during the task.

Slide 10

Anticipating- action steps

1. Specify a clear goal for student learning and select a high-level task that is aligned to your learning goal.
2. Anticipate- get inside the problem, plan to respond, and plan to notice. If possible, get input from colleagues either virtually or face-to-face to gain additional insights into the task, its solutions and its challenges.
3. Create assessing and advancing questions that are driven by your goals for the lesson that build on where the students are in their thinking.
4. Produce a monitoring chart you can use to record data during the lesson that includes possible solutions, assessing and advancing questions that are driven by your goals, and questions you can ask students who cannot get started.

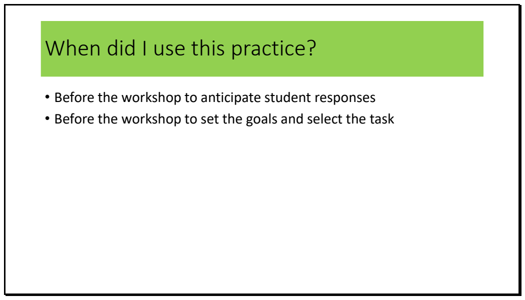
Smith & Stein, 2019, p. 64
The 5 Practices in Practice

What actions stand out to you? Why?
How else can you anticipate student responses?
What is the difference between advancing and assessing questions?
Why should we note which ones to ask students?
What is a monitoring chart?

Slide 11

Solution strategy	Assessing questions	Advancing questions	Who/what	Order
Building up- break into pieces and add	What does this variable/number represent in your formula? Why did you separate your picture into parts? Did it help?	Where is the number 4 from your formula visible in the picture?		
Breaking down- big square minus parts	What does this variable/number represent in your formula? Why did you separate your picture into parts? Did it help?	Where is the number 4 from your formula visible in the picture?		
Table	What patterns are you noticing? Where did this number come from in your table? What are your input and output?	How can you represent this pattern algebraically? How can you see your input and output in your picture?		
Technology approach (similar to table but added use of technology)	What patterns are you noticing? What does this number/variable mean? How has the technology helped you?	Where can you see this number/variable in your picture?		
Incomplete solution- only next term or not nth term	What type of sequence is this picture representing? How do you know?	How many circles are in the next term? How do you know? How can you represent an arithmetic sequence algebraically? A geometric sequence?		

Slide 12



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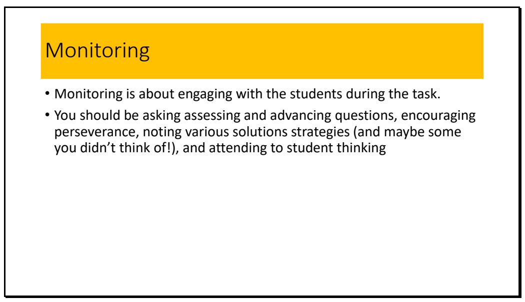

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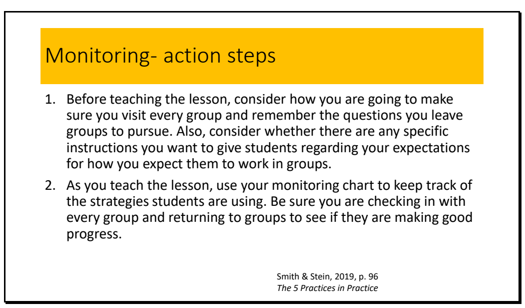
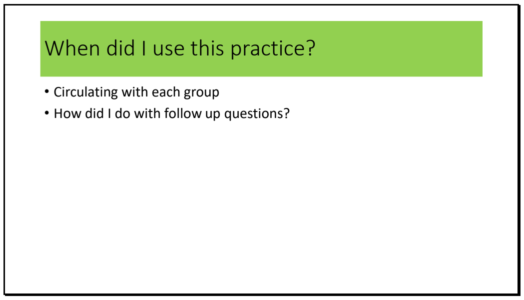


How do you establish group norms/classroom expectations for group work?
How do you ensure you visit and follow up with every group?
Peter Liljedahl encourages knowledge mobility by having groups post their work with extensions so everyone can see.

Slide 15



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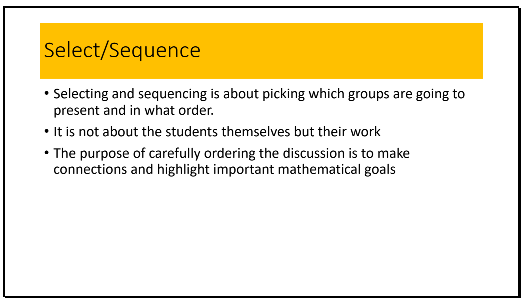

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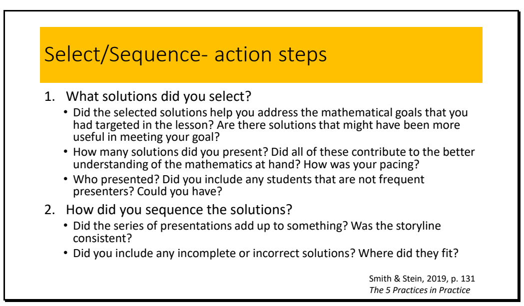


Slide 18

When did I use this practice?

- Solution presentations order
- Selecting presenters

Slide 19

Connecting

- Connecting is about helping your students see how different approaches and solutions strategies are connected and build a conceptual understanding of the mathematics at hand. You are scaffolding the presentations to highlight key concepts.

Slide 20

Connecting- action steps

1. What connections were made between student responses and the lesson goals?
2. What connections were made between different student responses?
3. To what extent were the mathematical ideas that were targeted in the lesson made public?
4. What did you do to ensure that all students in the class were participating in and accountable for making sense of the ideas being presented?
5. Would you do anything differently if you were going to teach this lesson again?

Smith & Stein, 2019, p. 162
The 5 Practices in Practice

Slide 21

When did I use this practice?

- During presentations- questioning, scaffolding, order of presentations
- During presentations- modelling, revoicing, rephrasing strategies

Slide 22

Why do I care?

- We just spent all this time identifying the 5 practices, but why should I care to implement this in my classroom?
- All students deserve the opportunity to learn. By engaging students in productive classroom discourse, you are giving them another opportunity to be explorers of the mathematics and building them up to be capable and confident doers of mathematics.

Slide 23

Research

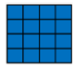
- "Discussion gives the students the opportunity to share ideas and clarify understandings, develop convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other people's perspective" (Smith & Sherin, 2019, p. xxv).
- "Knowledge mobility [was] accompanied by a decrease in groups' reliance on the teacher and an increase in reliance on themselves and other groups" (Liljedahl, 2021, p. 48).
- Questions can be used to assess a student's content knowledge, help students develop thorough explanations, discover and correct misconceptions, make the mathematics visible and maintain classroom management (Kersaint, n.d.; McCarthy et al., 2016; NCTM, 2014).

What do you notice?
Why should we engage our students in discourse?
Why do we care that our students are less reliant on us as teachers?
What role does questioning play in classroom discourse?

Slide 24

Pool Border Tile Problem

1. The given illustration is a model of a 4 x 4 pool. What is the number of tiles needed to border the pool?



2. Write an expression to represent the number of tiles needed to border a n x n pool.

Slide 25

Anticipate

- Working with your table group, create your own monitoring chart and fill in as many solution strategies as you can. Remember to consider different types of responses (visual, algebraic, verbal, etc.)

Strategy	Assessing Questions	Advancing Questions	Who/What	Order

Slide 26

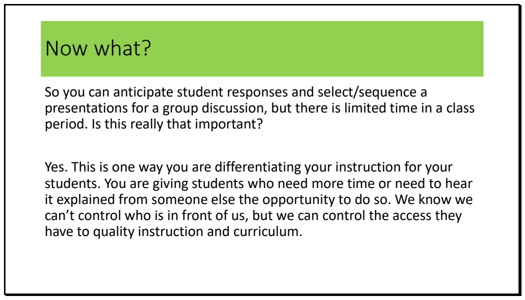
Select and Sequence

- Now that you have several solution strategies, select and sequence the strategies you would use to guide a class discussion.
- Create your poster, putting each strategy you selected in order and provide a brief justification of why you placed it in that order.

- We will do a carousel to view each group's ideas and use post-its to leave feedback and ask questions.

Strategy	Justification
1- visual	Most conceptual, open to all students
2- table	
3-	
4-	

Slide 27



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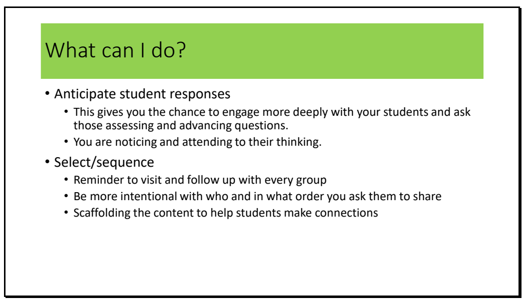

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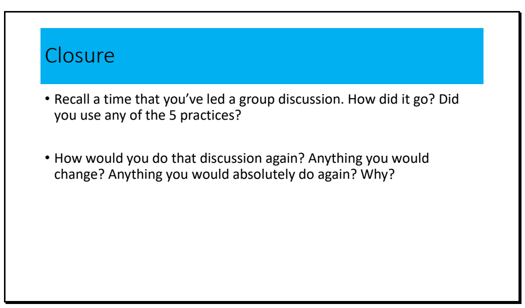

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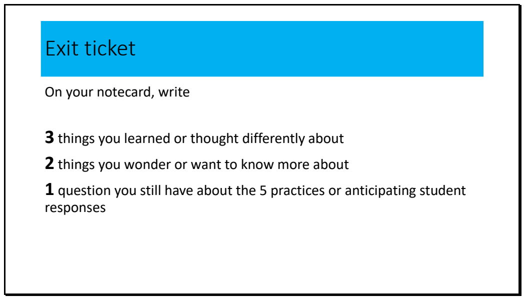

Exit ticket
On your notecard, write
3 things you learned or thought differently about
2 things you wonder or want to know more about
1 question you still have about the 5 practices or anticipating student responses

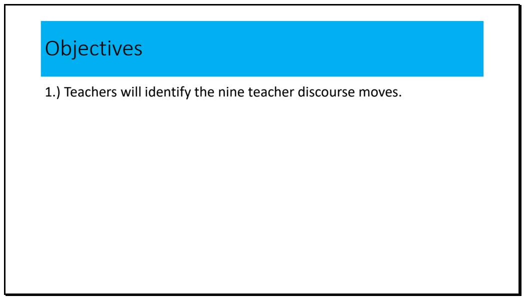
APPENDIX F

Workshop Unit Plan PowerPoint Slides Day 6

Slide 1

Slide 2

Slide 3



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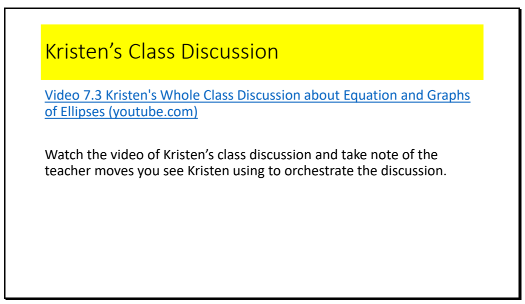

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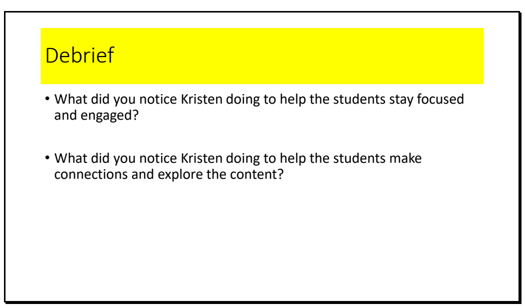

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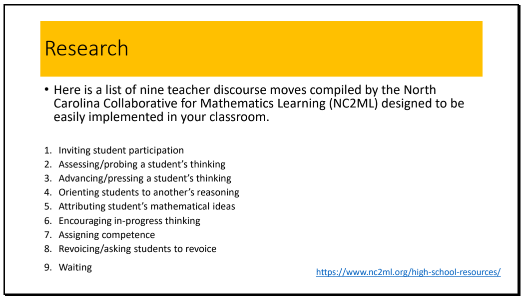

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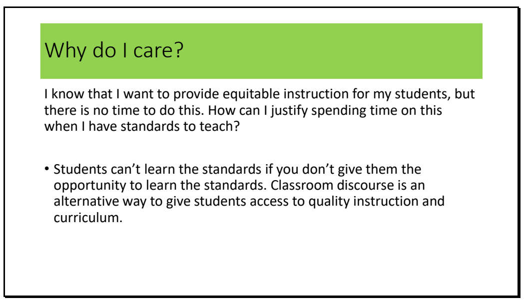

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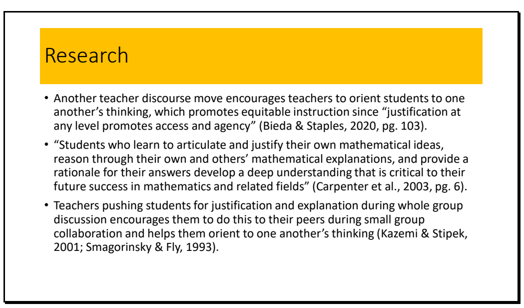


Why do we care about access and agency?
Why is it important to encourage self-reliance during small group collaboration?

Slide 9

What can I do?

1. Keep this handout where you can see it every day.
2. Then, just focus on using one discourse move at a time.

I would begin with inviting student participation and encouraging in-progress thinking.

Why do you think I started with these two practices?

Slide 10

Handout

Here is the link to NC2ML's website where you can access their handouts for free: https://www.nc2ml.org/high-school-resources/

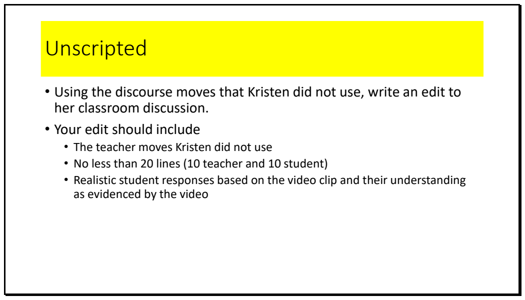
Slide 11

What did you notice?

Compare the list of teacher discourse moves to the list you created.

- Are there any moves that Kristen used that you think should be added to the list?
- Are there any moves that Kristen did not use but could have?

Slide 12



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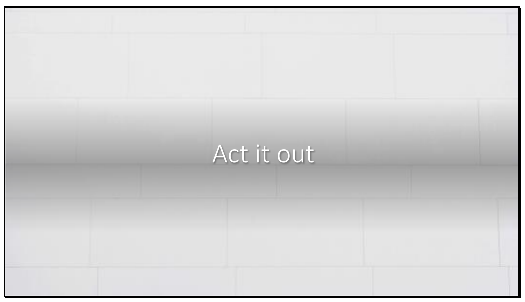

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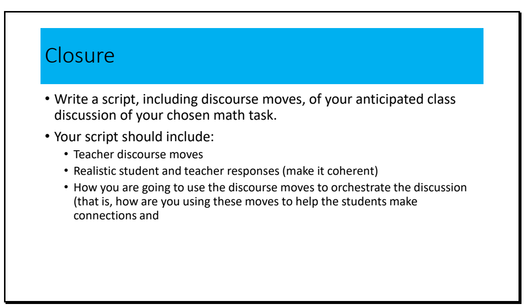

Slide 15

Exit ticket

On your notecard, write

3 things you learned or thought differently about
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