Abstract

This study focuses on how participants understood specific Common Core State Standards of Mathematics over the course of roughly two months when they engaged in concert modern dance and creative movement used to teach these standards. Weekly instructional rehearsals yielded copious notes that show the participants' growth in understanding, and pre and posttests in the form of concept maps demonstrate the participants' understanding before and after they learned the standards through dance. In all cases, the participants seemed to have a deeper understanding of the standards at the end of the project, and they often had a more precise understanding of related mathematical concepts as well. Many of the participants showed a more positive attitude for mathematics by the end of the study.

Introduction

Background

In the fall of 2013, I took a choreography class taught by Marsha Barsky for my dance minor. I used an Honors Contract in order to get honors credit for the class, and in doing so, I had to find connections between math and dance, find a mathematical formula for creating dance, or otherwise use math as a structure for my choreographies. By the end of the semester (in which I was also taking Calculus II), I had created a two minute and sixteen second dance that demonstrated the convergence of a specific geometric series to the integer two and a five minute piece built around a thirty second representation of the Fibonacci sequence.

My exploration of math and dance began in that class and has piqued my interest ever since. I considered doing my Honors Thesis on math and dance in some way since then, but as I continued to study secondary education, I settled on using math to structure dance as I had in the past and using those structures to teach math. As the project developed, the focus shifted from using structures of dances to teach mathematics to using creative movement to teach and structuring dances using that creative movement.

This research thesis and creative project combines three very different fields of study: math, dance, and education. In describing the combination of these three fields, the use of academic language from all three fields is necessary. A review of the literature dealing with education, math education, dance, dance education, the combination of math and dance, and teaching math through dance are all very important. Below you will find a literature review and a table of definitions that may be helpful in the discussion of the combination of these three fields.

Literature Review – Similarities in the Disciplines and Educational Benefits

Math and dance have several connections and similarities that allow the two disciplines to pair nicely and be used to support one another. The introduction to the book, *Math Dance,* states that both math and dance involve "creatively exploring patterns in time and space with an eye toward aesthetic potential" (Schaffer, Stern, & Kim, 2001, p.5). The same introduction states that internal consistency and harmony between intuition and analysis or perception and reasoning are important to the aesthetics of both fields, which can both be seen as abstract or practical (Schaffer, Stern, & Kim, 2001). Dance and math both require imagination, creativity, analytical thinking, form, and stylistic rules (Schaffer, 2011), and both can be seen as art and a study of patterns ("Math in your feet," 2013; Schaffer et al., 2001). Both disciplines impact society and daily life (Papacosta & Hanson, 1998), and both are present in all that we do (Schaffer et al., 2001).

There are several educational benefits of using dance to teach math as well. Dance inherently requires spatial awareness ("Math in your feet," 2013), and people tend to better understand "spatial imagery on a larger scale" (Thurston, 2006, p. 41). Dance also allows students to use knowledge about their bodies and movements to develop deeper understandings and discover new insights about math ("Math in your feet," 2013). Dance allows students to use movement and the way they experience the world around them to explore aspects of mathematics like shape, number, or graphs (Watson, 2005). It provides a means for a mathematical concept to be "understood mentally, physically, and emotionally" (Schaffer et al., 2001, p.5), and in so doing, it helps students to form connections between mathematics and the real world. For example, students can

recognize rotation better when they are physically facing different directions rather than when they are manipulating a geometric shape on a graph (Rosenfeld, 2011). Students of various learning styles, especially kinesthetic learners, can be reached through dance as a form of teaching mathematics (Schaffer, Stern, & Kim, 2001; Watson, 2005), and in some cases, dance makes math visible (Rosenfeld, 2011). Finally, dance "reache[s] students' multiple intelligences," teaches problem solving skills, and allows them to form "complex connections" (Werner, 2011, p. 1).

Literature Review – Mathematics Taught Through Dance

Much of the literature on the combination of math and dance or teaching math through dance reflects work being done at an elementary school level. A physical education teacher at Momauguin Elementary School in Connecticut aimed to teach third, fourth, and fifth grade Common Core State Standards in mathematics through choreographing a dance performance to popular music that students would showcase to parents and teachers. The students worked on units, distances, multiplication, division, fractions, and patterns all while learning choreography and histories of popular dances (Lips, 2014). A study of second, third, fourth, and fifth graders in Minneapolis showed that students who worked with a professional dancer to learn math concepts had a more positive or unchanged attitude toward math over the course of the school year according to a survey that students took once in the fall and again in the spring. The students who did not work with a dancer to learn the concepts had a more negative or unchanged attitude over the course of the school year according to the survey (Werner, 2011). Malke Rosenfeld, a professional percussive dancer, offers teacher workshops and school residencies in using percussive dance to teach elementary school students "grade-

appropriate" (Rosenfeld, 2011, p. 78) mathematics. She has also developed multiple

programs to this end and connects her work and teaching with the Common Core State

Standards for mathematics ("Math in your feet," 2013).

       In terms of secondary or post-secondary mathematics education through dance,

Chicago's Columbia College has a history of requiring their students, many of whom are

art or communication majors, to participate in a semester-long project to "build an

intellectual bridge" between the arts (including dance) and math and science (Papacosta

& Hanson, 1998, p. 250). The American Association for the Advancement of Science

(AAAS) and *Science Magazine* hold The "Dance Your Ph. D" Contest annually, where

science-related Ph. D theses (past or in progress) can be interpreted through dance for a

chance to win monetary prizes (Bohannon, 2010). Working with a variety of ages, Dr.

Karl Schaffer and Mr. Erik Stern of the Dr. Schaffer and Mr. Stern Dance Ensemble

teach math through dance to audiences and workshop participants. The mathematical

content that they teach ranges from kindergarten to college level mathematics, but they

consider their extension to secondary and post-secondary mathematics a hallmark of their

work (Schaffer, Stern, & Kim, 2001).

       For elementary school students, the math content learned through dance includes

units, distance, multiplication, division, fractions, and several other concepts that are

further discussed in Appendix A, Table A1. Similarly, the mathematical content observed

in dance for secondary and post secondary students includes circular motion, problem

solving, n-body problems, tessellations, and much more, all of which is discussed in

Appendix A, Table A2.

In terms of dance, the movement typically used for elementary school students is simple and involves jumping or making noise to recognize patterns. The work is done in a classroom or a workshop (Lips, 2014; "Math in your feet," 2013; Rosenfeld, 2011). The movement used for secondary school or college students ranges from pedestrian or everyday movements (Schaffer, 2010; Schaffer, 2011) to complex, highly developed concert dance (Papacosta & Hanson, 1998; Schaffer, 2010; Schaffer, 2011; Schaffer et al., 2001).

Literature Review – Educational Theory

Constructivist educational theory suggests that, "people construct new knowledge and understandings based on what they already know and believe" (Bransford, Brown, & Cocking, 2000). Ken Appleton (1993) expands on this idea and states that learners bring all of their previous knowledge, understandings, and experiences with them as they enter any given learning situation. As new information is presented to them, they use their existing knowledge to interpret and organize this new data. From there, they enter the process of assimilation, where they sort through their existing knowledge to see if any of it provides a good "explanation" for this new data. If it does, their existing knowledge, be it right or wrong, is reinforced with the new data. If it does not, the learners experience disequilibrium, which leads them to want to "resolv[e] the conflict" between the new data and the existing knowledge (Appleton, 1993, p. 269).

The learners then test the new data against other schemata (mental organizations of knowledge, understandings, and experiences) until they find one that they can restructure to incorporate the new data, experience, or idea (Appleton, 1993). This process is called accommodation. False accommodation happens when other students,

teachers, textbooks, or other sources of information give the learners the right answer

rather than the learners experiencing disequilibrium and working to explain new data or

ideas on their own (Appleton, 1993). After false accommodation occurs, the learners'

existing sets of knowledge are not changed, but they have new knowledge sets for school

situations (Appleton, 1993). Finally, if learners do not think that the new knowledge is

worthwhile or if they have experienced failure in a particular area before and do not want

to experience that failure again, they may opt out of a learning situation (Appleton,

1993).

 As it says in *How People Learn*, teachers need to pay attention to the existing

knowledge and preconceptions of students and build on these to help them gain a better

understanding through accommodation. If the students have misconceptions, these should

be addressed so as not to further this misunderstanding of the concept (Bransford et al.,

2000). *How People Learn* also says, "there are times, usually after people have first

grappled with issues on their own, that 'teaching by telling' can work extremely well"

(Bransford et al., 2000, p. 11), but the book first says that lecturing usually does not work

and thus admonishes against false accommodation, as does Appleton (1993).

 One method of allowing students to learn through accommodation rather than

from false accommodation through rote memory or lecture is through inquiry. Two

elementary school principals describe inquiry-based learning in math as partner work and

group work that allows students to create "mathematical explanations that make sense to

them" (Chapko & Buchko, 2004). They continue saying, "The dialog at the heart of the

inquiry math process allows students to solidify their thinking and allows the teacher to

discover and correct mistakes in their strategies" (Chapko & Buchko, 2004, p. 32), which

would prevent the students from assimilating incorrect information. In an article on

inquiry math lessons, two professors of math education say that in an inquiry math

lesson, the task, analysis of problems, revision of solutions, and presentation of findings

and information should all be student-centered and student-driven (Harper & Edwards,

2011).

   The same article claims that research suggests that students benefit from making

sense of the mathematics they encounter through discourse with other learners and

inquiry (Harper & Edwards, 2011). Student discourse includes conjecturing, questioning,

and discussing problems to "discover important mathematical concepts" (Stein, 2007, p.

285) and consists of sharing discussions and collaborative discussions (Staples and

Colonis, 2007). Sharing discussions allows students to listen to and understand their

classmates' reasoning while maintaining a focus on their own ideas and reasoning

(Staples & Colonis, 2007). Collaborative discussions allow students to hear and

understand their classmates' ideas and reasoning as well as respond and connect to

others' ideas" (Staples & Colonis, 2007), which encourages learners "to develop new

understandings of mathematics" (Staples & Colonis, 2007, p. 258).

   Teacher discourse with students includes addressing incorrect ideas so that

students do not hold on to their misconceptions (Bransford et al., 2000; Staples &

Colonis, 2007). It also includes what the teacher says to "promote conceptual

understanding of the mathematics itself" (Stein, 2007, p. 286), or cognitive discourse, and

what the teacher says to encourage or discourage participation in class discussions, or

motivational discourse (Stein, 2007). Questioning is important for teachers to discover

the students' reasoning processes and understanding. Closed questions lead students

toward a specific answer, while open questions do not and thus allow room for students to explain their thought processes and understandings of a particular subject. Thinking about questions to ask students ahead of time is important for teachers to make sure they are asking purposeful, open questions (Manouchehri & Lapp, 2003).

Summary and Application

The disciplines of math and dance are more similar and are more interconnected than they are typically thought to be. These interconnections give rise to multiple benefits of using dance and movement to teach mathematics. While there is work being done in combining math and dance at all levels of education, most of it seems to be focused on elementary school students and the most basic concepts of mathematics (Schaffer et al. 2001). It also seems that connections between dance and Common Core State Standards in mathematics are only being drawn at the elementary level (Lips, 2014; "Math in your feet," 2013). In my study, I extend this work by using dance to teach high school mathematics standards from the Common Core.

Constructivism refers to the idea that people learn and form new ideas based on their existing knowledge. For this reason, teachers need to pay attention to their students' prior knowledge and address any misconceptions they may have. Learners assimilate new information into their existing schemas, modify their existing knowledge to accommodate new information, obtain a specific answer to be used for school purposes in false accommodation, or opt out of learning situations. Using inquiry-based learning to teach students allows the students to form explanations and definitions that make sense to them, which may allow them to correctly accommodate new information better. Inquiry involves student discourse, which allows students to understand their own reasoning

processes and those of their classmates, and teacher discourse includes questioning,

addressing misconceptions, pushing students toward a conceptual understanding of a

concept, and encouraging (or sometimes discouraging) classroom discussion. The theory

of constructivism and literature related to inquiry-based learning informs this study in

that classroom discourse and inquiry-based learning were used to guide participants to

accommodate new mathematical knowledge into their existing schemas.

Definition of Terms

Since this thesis project is a combination of multiple fields of study, vocabulary

from these fields will be important in discussing the project. Therefore, I have included a

Table of Definitions (Table 1) including a section of dance terms and their definitions and

a section of math terms and their definitions to aid in the discussion of these fields.

Table 1

Table of Definitions

Dance Term	Definition
Inversion	One of Blom's sixteen ways to manipulate a motif, a motif performed upside-down. (Blom, 1982, pp.102-103)
Level	A spatial relationship being determined by closeness to the ground. Levels include low, middle, and high and can include artificially high (which involves partners, cables, or gimmicks). (Blom, 1982, pp. 31-32)
Motif	A single movement or short movement phrase… that is used as a source or a spark for development into an integrated [whole]. (Blom, 1982, p. 102)
Phrase	The smallest and simplest unit of form. It is short but complete and contains a beginning, middle, and end. As an analogy, a phrase is to dance as a sentence is to a book. (Blom, 1982, p. 23)
Plane	The result of joining any two of the three dimensions. This includes the vertical (height and width), horizontal (width and depth), and sagittal (depth and height) planes. (Blom, 1982, p. 35)
Retrograde	One of Blom's sixteen ways to manipulate a motif, a motif performed backwards. Start at the end, and follow it back through space. (Blom, 1982, p. 102)

Math Term	Definition
Complex Number	A complex number can be written in standard form as $a + bi$, where a and b are real numbers and i is the imaginary unit. (Rockswold, 2002, p. G-1)
Directed Line Segment	A line segment which has an initial point, a terminal point, magnitude (or length), and direction. (Larson & Hostetler, 2001, p. 511)
Domain	The set of all x-values of the ordered pairs in a relation. (Rockswold, 2002, p. G-4) The set of inputs of a function. (Larson & Hostetler, 2001, p. 125)
Function	A relation where each element in the domain corresponds to exactly one element in the range. (Rockswold, 2002, p. G-2)
Imaginary Number (Imaginary Unit)	A number denoted i whose properties are $i = \sqrt{(-1)}$ and $i^2 = -1$. A complex number $a + bi$ with b $\neq$ 0. (Rockswold, 2002, p. G-3)
Range	The set of all y-values of the ordered pairs in a relation. (Rockswold, 2002, p. G-4) The set of outputs of a function. (Larson & Hostetler, 2001, p. 125)
Real Number	All rational and irrational numbers; any number that can be written as a decimal. (Rockswold, 2002, p. G-4)
Relation	A set of ordered pairs. (Rockswold, 2002, p. G-4)
Vector	The set of all directed line segments that are in equivalent to a given directed line segment, which has an initial point, a terminal point, magnitude (or length), and direction. (Larson & Hostetler, 2001, p. 511)

Note. Almost every sentence in this table is a direct quote or a slightly adapted quote from the noted source in parentheses.

Thesis Question and Expected Outcomes

This thesis was designed to address the following question: What knowledge of

the content within Common Core State Standards for Mathematics do dancers

demonstrate before and after engaging in concert modern dance and creative movement

used to teach these standards? As I progressed through the project, I realized that I was

also gaining information on how participants understood Common Core State Standards

for Mathematics while engaging in creative movement used to teach these standards. I

then decided that it would be beneficial to assess their understandings during the process

of my study in addition to assessing their understandings before and after.

When planning for this thesis began, the question being addressed was more quantitative. For example, is it possible to effectively teach Common Core State Standards for Mathematics through concert modern dance? As planning continued, the thesis question diverged from this type of thought because defining "effectively learn" became difficult, especially considering that after some sort of weekly math intervention, the dancers were sure to learn something or have some sort of altered understanding. For this reason, the question became more qualitative, focusing more on how the dancers' understanding changed rather than whether or not it changed at all.

The dancers' understanding of their math concepts is expected to change and expand as the rehearsals for their dances continued throughout the semester. I expect that dancers' posttest concept maps will differ greatly from their pretest concept maps, and the dancers will be able to include definitions, terms, and examples they would not have been able to include in their concept maps before their rehearsals began.

Creative Project Description

Procedure and Timeline.

On February 16, 2015, I submitted the proposal for this thesis for the first time. I then met with the members of my thesis committee to review my work in late February and submitted a revised version of my proposal on March 30. Shortly thereafter, I reserved the dance studios in Murphy Center, room G040, belonging to MTSU's Department of Theatre and Dance for four rehearsals a week and my final concert. I arranged to use the sound system and performance lighting in the dance studios for my concert as well, and I began recruiting dancers for my project.

I began the process of obtaining approval from the International Review Board to do research on human subjects in April, and I submitted my first application on May 4. I spent late May and early June reading to become IRB certified, and was granted approval from the IRB on August 26 to begin my project.

I held an information meeting in Murphy Center on August 31 where I briefly explained to potential participants what I planned to do in my project, and I allowed those who no longer wanted to participate to leave the meeting. Those who did want to participate filled out a scheduling sheet so that I could begin scheduling rehearsals, and they signed an Informed Consent form that we carefully read and discussed as a group. Rehearsals for my thesis began on September 14, and with the exception of October 6-13, occurred weekly until the dress rehearsal and concert on November 6 and 7 respectively. The writing of this thesis occurred over this time as well as over winter break, and I submitted this thesis with a post-concert discussion on March 31, 2016, as was arranged previously to allow more time for rehearsals with the dancers and for my own reflection of the process. This thesis will be defended in Spring 2016.

Standards Taught Through Dance.

Because I wanted to use mathematics standards that could be applicable nationally rather than to just one state, I chose to draw the four standards that I would teach through dance from the Common Core State Standards for Mathematics (CCSSM, 2010). When I read through the high school mathematics standards, the four standards below stood out to me because I quickly thought of at least one way to address them through dance.

- **Number and Quantity – The Complex Number System (N-CN):** (1.) Know

  that there is a complex number *i* such that $i^2 = -1$, and every complex number has

  the form *a* + *bi* with *a* and *b* real (CCSSM, 2010, p. 60).

- **Number and Quantity – Vector and Matrix Quantities (N-VM): (**1.)

  Recognize vector quantities as having both magnitude and direction. Represent

  vector quantities by directed line segments and use appropriate symbols for

  vectors and their magnitudes. (4.) Add and subtract vectors. (5.) Multiply a vector

  by a scalar (CCSSM, 2010, p. 61).

- **Functions – Building Functions (F-BF):** (1.c.) Compose functions. For example,

  if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the

  height of a weather balloon as a function of time, then T(h(t)) is the temperature at

  the location of the weather balloon as a function of time (CCSSM, 2010, p. 70).

- **Geometry – Congruence (G-CO):** (4.) Develop definitions of rotations,

  reflections, and translations in terms of angles, circles, perpendicular lines,

  parallel lines, and line segments (CCSSM, 2010, p. 76).

General Methods

The Participants

Many of the participants of this study were in some way involved with the MTSU

Dance Program. This could mean that they were taking dance classes, were in MTSU's

pre-professional dance company The MTSU Dance Theatre, or were dance minors. Only

one of the participants did not fit into any of these categories at the time of the study, but

she had taken dance classes for many years prior to the study. For a full list of the

dancers' pseudonyms, majors, and minors, see Table 2. Table 2 also includes the math

topics that the dancers were engaging with and learning about in their instructional

rehearsals.

Table 2

Dancers and Their Majors and Minors

Pseudonym	Major(s)	Minor (if applicable)	Dance(s)
Amy	Psychology	Dance	Vectors
Brook	Integrated Studies	Dance, Entrepreneurship	Vectors
Cate	Liberal Studies	Dance	Vectors
Dana	Anthropology	Dance	Complex Numbers
Ella	Integrated Studies	Dance, Business Administration	Complex Numbers, Building Functions
Fay	Finance	Dance, Business Administration	Building Functions
Grace	Math	Dance	Building Functions, Congruence
Hope	Chemistry	Biology	Congruence
Ivy	Flute Performance, Music Industry	N/A	Congruence

PreTests

During the first week of rehearsals, I opened rehearsal by welcoming the dancers

and asking about their day or mood to make them feel more comfortable. I then reminded

the dancers that they were going to be pre and posttested over their math concept with a

concept map and that I would be recording them, which they already knew since they

read and signed an Informed Consent document. I instructed the students to brainstorm as

a group on the white board in the dance studio, and we discussed the importance of

talking through any disagreements or uncertainties even though after ten minutes they

would do their own individual concept maps on paper and could then write down only the

ideas from the brainstorm that they agreed with.

   I showed them two examples of what a concept maps looks like (see Appendix G)

from "Concept Map Assessment of Classroom Learning: Reliability, Validity, and

Logistical Practicality" (McClure, Sonak, & Suen, 1999, p. 479) and asked the dancers to

write on the line connecting two bubbles why those two bubbles are connected so that I

could see what they were thinking and how they were forming connections. The concept

map worksheet (see Appendices H-K) that the dancers were going to use contained an

overarching topic on the top of the paper as well as the Common Core State Standard in

the center bubble. I pointed these both out to the dancers so that if they felt they were

getting bogged down in the wording of the standard, they could easily refer to the

overarching topic. After this, I set up my laptop, began recording, passed out the concept

map assessments and dry-erase markers, and sat nearby taking notes while the dancers

brainstormed as a group.

   The group of dancers that learned about vectors brainstormed for about ten and a

half minutes, and had five minutes to work on their individual concept maps. The dancers

who learned about complex numbers asked for extra time on their group brainstorm and

were given fifteen minutes to brainstorm and ten minutes to work on their individual

concept maps. The girls who learned about building functions were given about eleven

minutes to brainstorm and six and a half minutes to complete their individual concept

maps, and the group that learned about congruence brainstormed for about fifteen

minutes and had about ten minutes to work on their individual concept maps. Interesting

information and events occurring in individual pretest rehearsals will be discussed below

in the "methods" section for each particular topic.

General rehearsal set up

Table 3			
Rehearsal Schedule			
Topic	Name of Dance	Day of Rehearsal	Time of Rehearsal
Vectors	Magnitude and Direction	Mondays	8:00 – 9:30 p.m.
Complex numbers	Sets of Numbers	Thursday	7:00 – 8:30 p.m.
Building functions	Inputs and Outputs	Friday	5:00 – 6:00 p.m.
Congruence transformations	Transformations	Friday	6:00 – 7:30 p.m.

       Table 3 provides a summary of the weekly rehearsal dates, times, and topic of

study for each of the four groups. Each rehearsal after the first one began with the

dancers and I sitting in a small circle. I would ask the dancers "What do you remember?"

and they would respond with a list of everything they remembered including dance

phrases they learned or came up with, definitions they derived in a previous rehearsal,

comments a dancer may have made that were for some reason or another memorable, and

processes or mathematical procedures the dancers may have learned. After these review

sessions, we would focus on learning the mathematical content or creating movement that

could be used to demonstrate the math content, and the rehearsal would end with another

discussion circle to review what we had done that day.

Evaluation of Concept Maps

In order to try to analyze the dancers' concept maps quantitatively, I decided to use what McClure, Sonak, and Suen call the "relational scoring method" in "Concept Map Assessment of Classroom Learning: Reliability, Validity, and Logistical Practicality" (McClure, Sonak, & Suen, 1999, pp. 482-483, 485, 488-491). This method originally came from "Effects of an Environmental Education-Related STS Approach
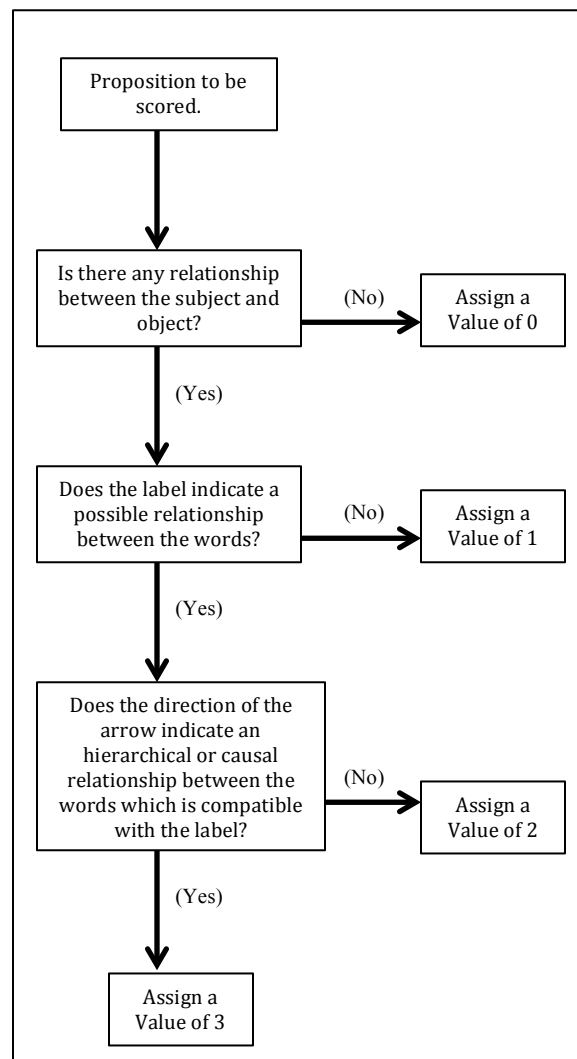
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│              │      scored.        │                        │
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│              │ relationship        │─────▶│  Value of 0 │   │
│              │ between the subject │      └─────────────┘   │
│              │ and object?         │                        │
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│                        │ (Yes)                              │
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│              │ between the words?  │                        │
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│              │ causal relationship │                        │
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│              │ which is compatible │                        │
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Figure 1: A reconstructed flowchart for scoring propositions (McClure & Bell, 1990, p. 10).

Instruction on Cognitive Structures of Preservice Science Teachers" by McClure and Bell. In this method, two topics or bubbles of the concept map connected with a line are called propositions, and propositions are scored on a scale of 0 to 3 with the help of a flowchart (see figure 1) (McClure & Bell, 1990).

I chose to use the relational method after reading McClure, Sonak, and Suen's article (1999) where they evaluated the reliability, validity, and practicality of six different methods of scoring concept maps used as a means of classroom assessment. The scoring methods they used they called the holistic, relational, and structural methods, and they evaluated each of these methods both with and without a "master map," which was a concept map completed by a professor to be compared with the concept maps completed by students. The results showed the relational method with a master map to be most reliable and valid (McClure, Sonak, & Suen, 1999), but I knew I did not want to use a master map in the evaluation of my dancers because I was concerned about evaluating them based on what my personal map looked like, and I knew that as I taught the material, I would be learning with the dancers, and my map was likely to develop along with our understanding. Still, the study recommended using one of the relational scoring methods because "the method is simple and [easily mastered] with little or no training" (McClure, Sonak, & Suen, 1999, p. 490). The study also says that the relational method makes scores easier to defend than in the holistic or structural methods because it does not require "broad subjective assessment of complex structures," (McClure, Sonak, & Suen, 1999, p. 491) but rather consideration of individual propositions (McClure, Sonak, & Suen, 1999).

Basically, in the relational scoring method, if there is no relationship between the two topics in a proposition, the proposition gets a score of 0. If there is a relationship but no label to indicate the relationship between two topics, the proposition gets a score of 1. If there is a label but the relationship is neither hierarchical (a superordinate/subordinate relationship) nor causal (one concept influences or causes the other), then the proposition is scored a 2 and deemed attributional, where one concept is an attribute or descriptor of the other. In my evaluation, if the relationship is labeled and is hierarchical or causal, then it is given a score of 3, and each invalid proposition was given a score of 0. For a table of the types of relationships the propositions might have, see Table 4 (McClure & Bell, 1990).

Table 4	
Table of relationship types for concept maps	
Relationship Type	Description
Hierarchy	Relationships expressing a superordinate/subordinate relationship between two expressions.
Attributional	Relationships describing one expression as an attribute or amplifying descriptor of the other expression.
Causal	Relationships which indicate that one expression influences or causes another, or occurs in a temporal sequence.
Note. This table is a modified version of Table 2 in McClure & Bell, 1990, p.3.	

In every rehearsal involving pretests or posttests, I told the dancers that I did not require them to write why they connected topics or bubbles on the line that connected them but it would benefit me greatly if they did. Because in several cases, the dancers wrote more labels (or more relevant ones) on the posttest than they did on the pretest, I decided that the dancers' labels or lack thereof were often indicative of their understandings of the mathematical topic and standard, so I decided to maintain the

original grading scale where a lack of label indicating the relationship between two concepts was assigned a score of 1. I also decided to count the number of hierarchical, causal, and attributional relationships since the scoring flowchart awards more points to hierarchical and causal relationships than attributional ones. If the dancers have an increased understanding of their topic, a greater percentage of their posttest propositions might be hierarchical or causal connections than attributional. It is also possible that they might have had more hierarchical and causal connections in the posttest than they did in the pretest.

In the sample maps from McClure, Sonak, and Suen's article (1999, p. 479) that I showed the dancers as a model for both the pre and posttests, the lines connecting the topics, concepts, or bubbles were directional arrows showing a subject and an object for each relationship (Appendix G). However, I did not explicitly tell the dancers to use directed segments to connect their concept bubbles, so none of them did, and I had to infer the direction of the connection when evaluating the concept maps. Each map began with an already filled out center bubble containing the mathematical content standard, which they learned about over the course of the semester (Appendices H-K), and most of the time, the dancers' thoughts sprouted from this first bubble. For this reason, I inferred that the line from the center bubble to a connecting bubble was directed toward the connecting bubble so that the original center bubble is the subject, and the connecting bubble is the object. Likewise, I assumed that any connection made from this connecting bubble was its object and so on. On rare occasions, a dancer would connect two bubbles sprouting from two different parts of the concept map (or two different sprouts of the

original bubble), and in these cases, comparing the two bubbles helped me to infer the

direction of the connecting line.

Methods for Specific Rehearsals

      Because each dance was used to teach a different concept and mathematical

standard, the process and discoveries made in rehearsals were very different for each

dance. Below is a detailed description of each rehearsal for each dance, which will

illustrate the discourse and tasks used in rehearsals. Beneath the description of the

instructional rehearsals for each dance are the qualitative and quantitative results about

the dancers' learning for each mathematical concept.

Complex Numbers

Number and Quantity – The Complex Number System (N-CN): (1.) Know that there

is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$

with a and b real (CCSSM, 2010, p. 60).

Instructional Rehearsals

Like every group of dancers in this project, we started the first day of rehearsals

with a pretest concept map. These girls, Dana and Ella, are both good at and excited by

problem solving, so when they had not come up with a rough draft of a concept map that

pleased them, they asked for more time to brainstorm. I allowed them an extra five

minutes, and when they wrote their individual concept maps, they took about ten minutes.

After the dancers had finished their concept maps, I began addressing

misconceptions and having the dancers define sets of numbers. We started with natural

numbers, so I drew a circle on the board, wrote "natural" inside of it, and asked the

dancers what they thought that might mean. After a few guesses and some discussion, I

told the dancers we tend to think of natural numbers as whole counting numbers or the

numbers used to count a certain whole number of objects. I asked for examples of

counting numbers, and they told me 1, 2, and 3.

At this point, the lesson took on a definite inquiry approach. I asked the dancers

how they would move natural numbers, and they had the task – which I was not even sure

was possible – of coming up with a way to represent natural numbers through dance. I

was pleasantly surprised that they found a way to do so that made sense to them when

they started discussing that natural numbers should be legato, continuous, and full

movements. They clarified by saying if they were to do an arm motion where they were

extending their right arm completely to the right, they would need to first extend it to the front and let it press out to the right because if they began the movement by simply trying to extend it to the right, they would have to pass through a position where their elbow would be pointing to the right without their forearm extended. They would then have to extend their forearm from their elbow to complete the movement, and this two-part motion getting the elbow to the right and then the rest of the arm would resemble a half of a movement and then the rest of the movement instead of one full movement.

Once the dancers had come up with a definition of natural numbers in terms of dance, I had them create six examples for me, using the numbers one through six. Some of their movements for these numbers were inspired by the numbers themselves, which made them easier for the dancers to remember down the road. For example, 1 was a step to the right along with a sweeping motion of the right arm from down across the body up and around to point straight up to make a straight line from the top of the arm to the feet. 4 included holding both the shoulder and elbow joints at right angles so that the hand is pointing to the ceiling and holding the right elbow with the left hand so that the arms make the shape of an upside down 4.

After they created six movements for the numbers one though six, I asked them to remind me what a natural number was. Once they said counting numbers, I drew another circle on the board around the first and added the number 0 to it. I asked the dancers what they thought this set of numbers including both 0 and natural numbers would be called. They guessed integers, so we discussed the term "whole numbers," and I asked the dancers to come up with a movement for whole numbers. The dancers and I decided that

since they already had movements for natural numbers, they just needed to make a movement for zero.

The dancers had a discussion about the definition of zero. Ella mentioned that it could be nothing or it could be space, but then Dana added that 1 divided by 0 is undefined, and anything multiplied by 0 is 0, so 0 is "powerful." With division by 0 in mind, Dana and Ella decided that 0 should be undefined movement that ends by going straight into 1. We modified the movement associated with 0 the following week, but for this first week, I wanted to let the dancers find definitions that made sense to them.

I added another circle to our diagram around the one containing whole numbers and wrote negative numbers in the circle (see figure 2 on page 28). I asked the dancers what this set was called, and they guessed integers correctly. Again, they noticed that the set of integers included whole numbers, which included natural numbers, so the negative numbers were the only new numbers for which to create movement. The dancers then decided that for negative numbers, they would simply move in the negative space of the shapes created by doing the movements for positive numbers. Negative space is "the unoccupied space surrounding a body, in the opening created by body shapes, or between bodies" (Ontario Ministry of Education, 2009). So for example, Dana might make a 2 shape, and Ella would find a way to move in the space between Dana's arms. This representation of negative numbers was also edited in the second rehearsal to better represent the concept of a negative number.

After discussing integers, I drew a circle around integers and wrote "rational" in it. I asked the dancers what they thought a rational number was. They did not have an answer, so I began writing some examples like $\frac{1}{2}$ and $\frac{3}{4}$, and Ella said, "so those are

rational fractions?" I asked Dana what she thought, and I underlined the root word "ratio"

in "rational." The girls realized that rational numbers are ratios or fractions. When I asked

them if integers were rational numbers, they told me they were because they are inside

the rational bubble. I asked them why, and they told me that any integer could be written

as a fraction over 1. For example, 3 is equal to $\frac{3}{1}$. At this point, we had not yet discussed

that the ratio must be of two integers.

With this discussion in mind, the dancers set to work on coming up with how to

represent fractions with movement. They decided that ratios and fractions show

relationships between two numbers, so they, as dancers, needed to have some sort of

relationship with one another, which tends to imply some sort of contact. Thinking

conceptually, Dana told Ella, "You be 1, and I will divide you." She then found a way to

divide the shape for 1 with the movement for 2 to create $\frac{1}{2}$. Following this, Ella divided

Dana's 2 with the movement for 3 and created $\frac{2}{3}$, and they did a few more examples.

After rational, I added a circle to the right and outside of all of the other sets of

numbers and wrote "irrational." When I asked the dancers what they thought this meant,

they said "not rational," so I asked them what it meant to not be rational. They decided

that irrational numbers could not be written as a ratio, and when asked for examples, they

gave π and $\sqrt{2}$. I wrote a few more examples in the circle on the board, and I asked the

dancers to find a way to represent irrational numbers through movement. They decided

that since the numbers after the decimal of an irrational number never end, that they

should use continuous (or "never-ending"), suspended movement. They also thought that

since rational numbers showed a relationship between two numbers, that irrational

numbers should not show a relationship, so they went to opposite sides of the room, faced

away from each other and improvised continuously until I asked them to come back together.

At this point, I added a giant oval containing every set of numbers the dancers had worked with so far, and wrote "real." The dancers were excited because they recognized the word "real" from the Common Core State Standard that they had made a concept map about that day, and I explained to them that we would work with the rest of the standard the next week. We then gathered around the white board, and I asked the dancers what they had learned that day.

They recounted that the set of real numbers included subsets of rational and irrational numbers, and within rationals are other subsets nestled into one another including integers, whole numbers, and natural numbers. Dana added that the set of integers can range from negative infinity to positive infinity, which is also something we discussed briefly when defining integers. This stood out because natural numbers started with 1, and whole numbers started with 0, so for a set of numbers to begin with negative infinity was quite a change. What stuck out for Ella was that rational numbers could be written as fractions, which she could remember by the root word "ratio." Dana then conjectured that imaginary numbers exist outside of real numbers, but real and imaginary numbers make complex.
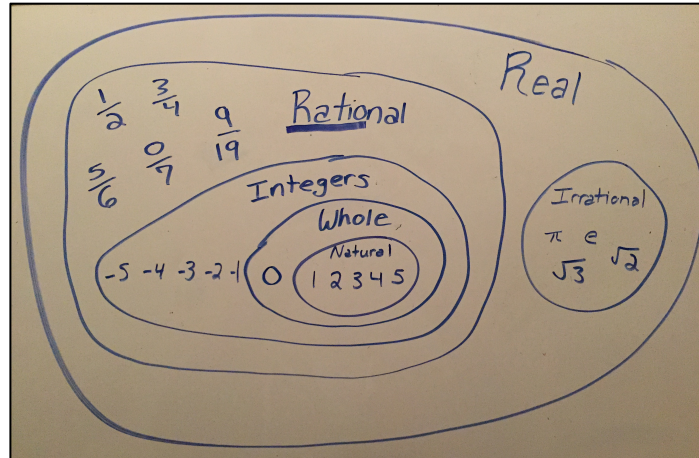
Figure 2: a recreation of my diagram from week 1

The next week when asked to define these sets of numbers, the dancers first drew the diagram that I drew the week before on the board (see figure 2). Ella said that natural numbers were counting numbers, and Dana added that natural numbers are all positive integers. Dana then defined whole numbers as "counting numbers plus zero," and Ella noted that whole numbers are all integers from zero to infinity, but since zero is not positive or negative, whole numbers would be all "integers greater than or equal to zero."

Ella defined integers as "all numbers with nothing to the right of the decimal," which I thought was excellent since we had not discussed the decimals of integers during the first week. Ella also said that integers "can be written as a ratio over 1," meaning that integers could be written as a fraction, where the numerator is the particular integer in question, and the denominator is 1. She said that rational numbers can be written as a ratio, and she and Dana both said that irrational numbers are numbers that cannot be written as a ratio.

After defining these terms, I drew a small circle to the right of the diagram they drew and wrote "Imaginary" inside it. I wrote that $i = \sqrt{-1}$, and I asked the dancers to

find a way to move *i*, as they had done for real numbers during the previous week. The

dancers struggled for a few minutes before I realized that I needed to clarify their

conceptual understanding of zero and negative numbers for this task to even be possible.

At this point, I asked the dancers to define negative numbers for me. The dancers

indicated that they understood that negative numbers are just like positive numbers with a

minus sign in front. They are to the left of 0 on a number line, and they run backwards.

For example, a number line would show "-3, -2, -1, 0, 1, 2, 3," from left to right.

        With this, the dancers decided to represent the opposite of their positive integers

by inverting the movement they were using to represent a particular positive integer. For

negative one, they started the movement with their right arm across their bodies like they

would for positive one, but then they swept their extended arms down in front of them, to

the right side of their bodies, and then straight up – ending similarly to how they ended

the movement for positive one. In other words, they did the movement "upside-down," as

inversion is described in *The Intimate Act of Choreography* (Blom & Chaplin, 1982, pp.

102-103). At this point, we had a discussion about the math terms "opposite" and

"inverse." Because the dancers were inverting their movement (or doing it upside-down),

I did not want the dancers to think they were taking the mathematical inverse of a

positive number to get a negative one. I explained to them that inverses are often

discussed when talking about functions because an inverse function would undo what the

original function did, but the opposite of a number is just its negative.

        The dancers nodded and affirmed that though they were inverting their movement

they were finding the opposite of a positive integer, not an inverse. At this point, I also

asked for their dance representation of 0 to be stillness to distinguish it from their dance

representation of an irrational number and to highlight zero's notion of nothingness. The

dancers agreed that it seemed to be a better option to represent 0, and they reviewed their

new representation for negative numbers. The dancers then decided that they could "take

the root of" – or isolate the initiation point of their movement for – negative one. Because

this movement starts in the shoulder, the dancers determined that i would be represented

with a small counter-clockwise circle with the right shoulder (the "root" of the new

movement for negative one).

After they had made this discovery and decision, I asked the dancers if it were

possible to have multiples of i. At first, they said that they did not think so, so I asked

them to solve two equations. Ella solved $x^2 - 4 = 0$ and found the solution to be $x = \pm 2$,

and Dana started to solve $x^2 + 4 = 0$ but got stuck at $x = \sqrt{-4}$. Recognizing Dana had

come to a stand still, which was not bad in anyway, especially since she had not had a

math class in years, Ella swooped in to help. Ella was not sure if what she was doing was

correct, but she had an idea she wanted to try, and she found that $x = \sqrt{-1(4)}$, so $x =$

$\pm 2i$. Both dancers recognized Ella's method as being the correct method to solve the

equation and understood that her solution showed us that it its possible to multiply by i. I

asked the dancers how we could represent $2i$, and again demonstrating their conceptual

understanding, the dancers said that they would simply repeat their movement for i twice.

If they were given $10i$, they would have repeated i ten times.

I asked the dancers what type of number 2 was, and they told me it was a natural

number, whole number, integer, rational number, and a real number. I asked because I

wanted them to recognize that real numbers and imaginary numbers could be multiplied

together, especially since their Common Core State Standard explicitly mentioned that a

and *b* are real. After this, I asked the dancers what happens when we have $0i$. They

answered that anything multiplied by zero is zero, so in terms of dance, it would be a

moment of stillness.

Finally, I drew a large circle on the board surrounding all of the sets of real

numbers and the imaginary number, and I labeled the circle complex numbers. The

dancers excitedly said, "oh!" since they knew that we had reached the main focus of their

Common Core State Standard, and they got visibly more excited. I asked them if they

remembered the form of a complex number from their concept map the previous week,

and they said they did not, so I wrote on the board $a + bi$. I asked the dancers what they

recognized and they said that they saw the imaginary number *i*. I then asked what type of

numbers they thought *a* and *b* were, and they said real and expressed their excitement of

knowing why they had spent time learning about all of those subsets of real numbers. I

then told them that *a* is considered the real component of a complex number, and I asked

them what we might consider *bi*. They said that *bi* was the imaginary component, and

they labeled the components on the board. Then, I asked Dana and Ella what would

happen if *a* equaled zero, and *b* did not. They agreed that we would then have some

multiple of *i*, so it would be an imaginary number. When I asked what would happen if *b*

were zero, they told me that we would be left with only *a*, a real number.

To finish up rehearsal, we sat down next to the board, and I asked the dancers to

define new terms for me. Ella defined a real number as a number that is not imaginary

and as a complex number, where *b* is equal to zero. Ella defined an imaginary number as

a complex number, where *a* equals zero, and Dana added that *b* cannot equal zero or we

would be left with zero, which is a real number. Ella mentioned that *i* is the square root of

negative one. Dana then said that a complex number is "a number with a real component

and an imaginary component" that can be written in the form $a + bi$. When asked what

stuck out and what they had learned from that rehearsal, Ella mentioned the difference

between the mathematical terms "inverse" and "opposite," and Dana talked about

revising their concept of a negative number, understanding that there can be more than

one i, and the components of a complex number.

On the third week, when I asked what we had talked about during the previous

week, the dancers mentioned complex numbers, real and imaginary components, and

what happened if a or b equaled zero. Dana then asked if a number was still complex if b

was equal to zero, and a was all that was left. Referencing the diagram I had drawn in the

first week and that they had drawn in the second, Ella explained that $a + 0i$, or just a,

would still be a complex number because it is a real number, which falls within the

"bubble" for a complex number, just like integers had fallen in the "bubble" for rational

numbers. This cleared up Dana's confusion and helped her clarify her understanding.

Then I asked the dancers to label the parts of a complex number. They wrote $a +$

bi on the board, and they labeled a a real number and the real component. They labeled b

a real number, i an imaginary number, and bi the imaginary component. After this I asked

them to define their sets of numbers, which they did, and then they reviewed the

movements they had come up with for different sets of numbers. When they finished

reviewing, I asked the dancers to come to the white board and write examples of real

numbers, on the board. The dancers wrote 1, 5, 0, 235, -2, $\frac{1}{2}$, $\frac{4}{7}$, $\sqrt{3}$, and π. I then used

these real numbers to create various complex numbers, and the dancers demonstrated

how they would represent the given number through dance.

I started them simply by writing $\frac{1}{2} + 5i$ on the board, and the dancers did their

movement for $\frac{1}{2}$ and followed it with i five times. I then wrote $\pi + i$, and the dancers

decided that they would begin their continuous improvisation with their movement for 3

since the only digit of pi to the left of the decimal point is 3. From there, they

continuously improvised until I said to stop. Then they did their movement for i once.

Things got a little tougher when I asked the dancers to move $235 + 0i$. The dancers had

movements for natural numbers one through 6, but nothing nearly as high as 235. They

realized that dancing the numbers 2, 3, and 5 would not get them what they wanted

because there was no way of determining a place value system. They also realized that 2,

3, and 5 might represent $2 + 3 + 5 = 10$ better than it would represent $200 + 30 + 5 = 235$.

At this point, the dancers were stuck, so I suggested that they finish out finding

movements for numbers one through ten, and then they make a movement for one

hundred. The dancers also decided to make a movement for fifty so that they would not

have to do their movement for ten eight times if they came across a number like eighty-

three. The dancers then experimented and created their new numbers. Fifty and one

hundred were essentially the same movement on the floor, but fifty travelled about half as

far as one hundred. Once they had figured out their new movements, they returned to the

question of $235 + 0i$, and did their movement for one hundred twice, ten three times, and

five once. Then they stood still to represent that zero multiplied with anything equals

zero. By this point, Dana and Ella had demonstrated a conceptual understanding of

multiplication being adding groups of the same number and of the additive nature of our

number system, where 235 is two groups of one hundred, three groups of ten, and 5

groups of one, which in this case was not needed because the dancers had movements for

numbers in the ones place.

From here, we picked up with $\frac{4}{7} - 2i$. One dancer did the movement for four, and

the other one "divided" her shape with the movement for seven, but then the dancers

pointed out that they didn't know how to dance negative i's. Because when they

multiplied i by a positive integer, they did that number of i's since multiplication is

adding a certain number of groups of a given number. The dancers recognized that they

had no way of moving a negative or irrational number of i's with the way we were

thinking about these numbers. I told them that I would go home and think about how we

might solve this problem so as not to waste rehearsal time, but in the end, the dancers and

I decided to leave our method the way it was because the dancers felt like it demonstrated

their understanding, and I agreed. After changing $\frac{4}{7} - 2i$ to the easier to dance $\frac{4}{7} + 2i$, The

dancers demonstrated $\sqrt{3} + \frac{1}{2}i$, beginning their continuous improvisation with their

movement for one since $\sqrt{3} \approx 1.732\ldots$ and finishing it with half of their movement for i.

After this, I asked the dancers to create a phrase with contact (touching, lifting,

weight sharing, or any other form of contact) using the movements they had created for

this piece. Since the onset of this project, I knew that the dance dealing with complex

numbers was going to be a duet so that we could create a partner phrase, and I could

assign one person to be the a in $a + bi$ and do some phrase, and assign the other person to

be b and have her do the partner phrase with an imaginary partner i. This contact phrase

was to become the partner phrase for person b. I noticed as the phrase developed (and the

dancers were not having as much contact as I intended) that the easiest way to ensure that

the dancers maintained contact, which was important to create a visually interesting

phrase to do with an imaginary partner, was to tell the dancers to use fractions so that they continually had to divide one another. As our rehearsal time ran out, the dancers recalled that the numbers used so far in their partner phrase were 36, 29, 0, -15, 50, $\frac{5}{8}$, $\frac{7}{19}$, and $\frac{4}{11}$. When asked what stuck out or what they had learned in rehearsal, Ella shared that the fact that b was a real number in the imaginary component of a complex number was solidified when they were dancing the complex numbers I was writing on the board.

In the fourth week of rehearsals for "Sets of Numbers," the dancers began by reviewing everything they knew about complex numbers and other sets of numbers. In this discussion, they defined rational numbers as numbers that can be written as a ratio and whose decimals terminated. For example, $\frac{1}{2}$ can be written as 0.5, but $\pi \approx$ 3.14159265… and goes on forever. I then asked the dancers how to write $\frac{1}{3}$ as a decimal. They paused and looked confused. They told me that $\frac{1}{3}$ is a rational number but its decimal is infinite since it is 0.3333333…, and three keeps repeating. They then decided that rational numbers have decimals that either terminate or repeat, and irrationals have infinite non-repeating decimals.

After this discussion, I asked the dancers to review their integer phrase – the phrase that developed from constantly reviewing their movements for the integers one to ten and negative one to negative ten in the same order every week. This became the "a phrase" that the person designated as person a would do while person b did the partner phrase with the imaginary partner. The dancers then reviewed and continued creating their partner phrase, or "b phrase," and they added the movements for $\frac{2}{3}$, 16, 150, and $\frac{5}{7}$ to the end of the phrase. At the end of this rehearsal, the dancers asked that I record both

phrases and email them to them so they could practice. Then, in our quick review, they told me what they had learned about the decimals of rational numbers.

After the review of everything they knew to start off the fifth week, I asked the dancers why they thought I had named the phrases a and b. I had told them my basic idea for structuring the piece early on, and they answered that a would be used as a real component somehow, and b would be used in an imaginary component probably danced with an imaginary partner. They did not mention that the b phrase was also completely made of real numbers, but that was an important aspect to me as well. I asked the dancers what music they thought they might want to dance to, and they told me they wanted something with a beat, but nothing too fast because they did not want to have to complete the movement for six (a big, spiraling turn) in one count of fast music. This was later used to pick the music for their piece.

After this brief discussion, the dancers and I started creating their dance. They started by doing both the a and b phrase together to let the audience wrap their minds around the phrases the dancers would be working with before they split apart and did two separate phrases at the same time. Then, Dana became person a in the downstage right corner (front left from the audience's perspective), and Ella became person b and danced the contact partner phrase with an imaginary partner in the upstage left corner (back right from the audience's perspective). This way, as an audience member, viewing the dancers left to right would show $a + bi$. If one dancer finished before the other, they would do $0+10i$ while they waited for the other dancer to finish, and then the other dancer would join in with a couple of i's to get her on time with the first dancer before they moved on.

We decided to bring Dana and Ella back together center stage after being separated to do the first $a + bi$ portion of their dance, and I asked them to do some weight sharing with one another. They decided to represent an irrational number by using this weight sharing and modifications to movements they had created to move across the space demonstrating a continuing, non-repeating decimal. In reality, they only showed three or four decimals since the dance had to end at some point, but I appreciated their mathematical intention behind the movement.

As we ran out of time, I asked the dancers to pause in figuring out how they would create this weight sharing portion so that I could finish the structure of the dance, and I told them we would complete that portion the following week. We discussed then that somehow the dancers would wind up doing the second $a + bi$ portion of their dance, but Ella would be a, and Dana would be b. From there, they would represent irrational numbers with continuous improvisation with no relationship to one another (as they had been doing since the first week of rehearsal) until the music and lights faded out. This was later changed, but in the fifth week, we had completed most of the structure of the dance.

The sixth week began with review, but since Dana could not attend, I simply typed terms for Ella to define into my laptop, and handed it over to Ella to type up her definitions. She defined integers as numbers with nothing after the decimal point, and included the decimals of rational and irrational in their respective definitions, but she defined many sets of numbers by using other sets of numbers. For example she wrote, "whole numbers = natural numbers and 0." This demonstrated a conceptual understanding of subsets of numbers. She defined real and imaginary numbers as

complex numbers where $b = 0$ and $a = 0$, respectively, and she noted that a and b were real numbers.

For this rehearsal, I brought the music (two songs) that Dana and Ella would be dancing to, but when running through what she knew of the dance, Ella and I realized that the songs switched at an awkward time in the movement. For this reason, Ella and I added walks and created more partner work to the beginning of the dance so that the change in music would happen as they were walking to their places for the first $a + bi$ part. We then finished the weight-sharing portion of the dance so that we could email both new parts of the dance to Dana and fill her in when she returned for the next rehearsal.

When Dana returned the next week, we had a long review of all the math that the dancers knew, and then Ella and I quickly taught Dana the parts of the dance that she had missed the previous week. The dancers then practiced their dance a couple of times, and we realized that the dance was too short because they were improvising at the end for an uncomfortable amount of time. Dana and Ella then decided to add a representation of rational numbers before the improvisation by repeating a series of numbers like the decimal of a rational number would repeat. This extended and completed the dance, and "Sets of Numbers" was finished on the last rehearsal.

Results

For the pretest, only about half of the propositions in each dancer's concept map were valid (see Table 5). By the posttest, not only did both dancers have an increased total number of propositions, both dancers had an increased number of valid propositions, and for both dancers, 100% of the propositions on the posttest were valid. For both

dancers, the number of hierarchical and attributional relationships in their propositions

increased, while the number of causal ones remained the same. Because the scoring

method allots more points to a hierarchical or causal relationship and I allotted zero

points to invalid propositions, the increase of valid propositions (no matter the

relationship) and increase of hierarchical propositions are factors contributing to the

overall increase of the dancers' scores. The definite increase in number of valid

propositions and in score demonstrates to me that the dancers certainly learned about

 their topic through the math and dance intervention.

Table 5

Concept Map Analysis of Sets of Numbers

Data Type	Dana (Pre)	Dana (Post)	Ella (Pre)	Ella (Post)
Total Propositions	12	26	9	23
Valid Propositions	5	26	5	23
Hierarchical	0	10	3	9
Causal	2	2	1	1
Attributional	10	14	5	13
Score	8	59	14	56

In the cases of the six causal relationships in all four concept maps, I deemed

most of them causal based on the wording the dancers used to describe the relationship

such as "if" and "then", "because", "makes it", and "thus." In the other two cases, the

dancers had written something that Ella described with the if/then relationship in her

pretest, and I wanted to award the same amount of points for the same information.

Both dancers' pretests involved incorrect definitions of complex numbers and invalid uses of i in algebra. Both dancers' posttests included definitions of types of numbers, examples of numbers in specific sets of numbers, distinctions of the real and imaginary components of a complex number, and notes of when a complex number could be purely real or purely imaginary. The lack of causal relationships in the dancers' concept maps likely relies on the fact that I did not focus on teaching causal relationships in complex numbers. A large portion of rehearsal and instructional time was spent discussing sets and subsets of numbers and their definitions. Whenever, the dancers discussed a subset relationship in their concept maps, I considered it a hierarchical relationship, which partially explains why there was such a large increase in the number of propositions describing hierarchical relationships in the posttests.

Having the dancers come up with a way to move their preliminary definitions for sets of numbers often allowed them to solidify or refine their original definitions. It also allowed for a way to keep the dancers engaged and exploring, which is often hard to do with a lesson based on definitions, but it kept the dancers actively learning (Edwards, 2015). Reflections from the dancers during the question and answer session at the end of the Math Dance Concert suggested that the dancers learned most of their information on complex numbers through forming verbal and physical definitions of sets of numbers on their own.

The revision of the movements the dancers originally came up with to represent zero and negative numbers also seemed to play a large part in clarifying the dancers' learning. Originally, the dancers had decided that a number divided by zero is undefined, so they wanted their movement to seem undefined. While I understood where their

thoughts were coming from, a lot of indefinite movement not only looked a lot like the

movement for irrational numbers, it did not reflect the idea that zero has a nothingness to

it. For example, if you have zero apples, you have no apples. There are none. To reflect

this idea of zero's nothingness I suggested that we change the movement for zero, and the

dancers immediately agreed that stillness would better represent the concept zero.

   Similarly, I stepped in as the teacher to clarify the dancers' movement choice for

negative numbers. Originally, they had decided that negative two, for example, would be

represented by one dancer moving in the negative space of the other dancer making the

shape of the ending of their movement for the number two. The dancers were using the

word "negative" and its meaning to them as dancers to inform their movement choices,

but they were not considering how their movement related to the definition of a negative

number. By asking them to change their movement for negative numbers, the dancers had

to focus on the mathematical definition, and how they could represent it through

movement. When they came up with the idea of inverting the movement, we were able to

discuss the difference between opposites and inverses in math and the connection

between opposites and negative numbers. This allowed the dancers to delve deeper into

their mathematical understanding of negative numbers rather than basing the

development of their dance and knowledge on the surface-level term "negative." It also

highlights the importance of the teacher's role in encouraging conceptual understanding

and participation (Stein, 2007).

   On the other hand, there was one rehearsal where the dancers and I decided not to

change particular movements even though we had run into a roadblock. I used the

dancers' suggestions of real numbers to form complex numbers that the dancers could

represent through movement using the movements and definitions they had already

created. When I asked the dancers to dance $\frac{4}{7} - 2i$, we realized that they had no way of

dancing a negative or irrational number of i's because of how they decided they should

dance i. Using their conceptual understanding of multiplication being groups (so 3×5

might be 3 groups of 5 items, which would produce 15 total items), they decided that $8i$,

for example, should be represented as repeating the movement for i eight times. That

meant that they did not have a means of dancing the movement for i a negative number of

times. For that matter, they could not dance the movement for i any number of times that

was not a natural number.

 Still the dancers asked me not to change anything. They decided that this

roadblock and the fact that they knew they could not overcome it with our current method

demonstrated their conceptual understanding of multiplication as well as their

understanding of the definitions of negative and irrational numbers (these were the sets

we discussed in rehearsal because we thought it could be interesting, although not

entirely accurate, to do roughly half or a quarter of the movement for i as a representation

of $\frac{1}{2}i$ or $\frac{1}{4}i$) since negative numbers are below zero and irrational numbers have an infinite

and non-repeating sequence of numbers after the decimal place.

 In rehearsals, in the question and answer section of the Math Dance Concert, and

in the posttest, it was exciting to see Dana's and Ella's confidence in their understanding

of sets of numbers. Each week in rehearsals the dancers seemed more certain of what

they had learned so far and more knowledgeable about their topic, and their

understanding was demonstrated both qualitatively – in the Math Dance concert,

rehearsals, and writings in their concept maps – and quantitatively – in their increase in

scores and number of valid propositions on their concept maps.

Congruence

Geometry – Congruence (G-CO): (4.) Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments (CCSSM, 2010, p. 76).

Instructional Rehearsals

The first rehearsal began with a pretest concept map. The group brainstorm began slowly, and it seemed that right off the bat, one of the three dancers was opting out and choosing not to participate. I was nervous that this dancer's lack of participation was going to prevent me from getting a good representation of what she (and they as a group) already knew about the topic. While two of the dancers stood up, walked to the white board and began to work, this dancer, Ivy, remained seated and watched the other two. As Grace and Hope continued to brainstorm, Ivy eventually began talking, pointing, and giving some input, and after about three minutes (I had started probing her to stand and help around two and a half minutes), she stood up and joined the other two dancers at the board.

Grace, who had already done a group brainstorm and pretest that day with the "Building Functions" group, immediately went to the board to get started. She began drawing a rough draft concept map on the board, which is probably an idea that she got from the previous group in which she had participated. Hope quickly followed Grace once she realized that she could include drawings, and she began illustrating perpendicular lines, parallel lines, angles, and other mathematical objects discussed in the standard. After that, she began drawing reflections, rotations, and translations on the

board. She represented each with a square being modified or transformed in some way on

a coordinate plane.

Once this process got going, I realized that Hope, a chemistry major with a

biology minor, already had a solid foundation of knowledge of congruence

transformations, and her brainstorming work was jogging the memories of the other two

dancers. She mentioned that perpendicular angles are always right angles and that a

translation is similar to a "slide." She even gave examples of congruent angles. I was

impressed with Hope's understanding, but I was worried that I would not have much to

teach this group of dancers.

Before moving on to a discussion about what was written on the board during the

brainstorm, the dancers did their individual concept maps, and Grace asked for more

time, which I granted her. When they were all done, with their individual maps, I asked

them questions about what they wrote on the board, specifically about concepts I thought

the dancers might have a misconception about or places where I thought there might be

gaps in their knowledge or understanding. I thought it would be best to get these

questions I had (and misconceptions they had) taken care of so we could all move on and

continue to learn to our greatest ability. Unfortunately, that meant that the dancers did not

get to dance much on our first day because we spent a lot of time going over what they

had written on the board. In particular, we spent the first rehearsal discussing reflections.

The dancers wrote "flip mirror image across x- & y-axis" to describe reflections

on the board during their group brainstorm concept map, so I asked them what they

meant. Hope proceeded to label the vertices of the two squares she had drawn and

explained to me that the vertices of one square corresponded to the vertices of the other in

a particular way so that the top square, which was above the x-axis, represented the

square before being reflected, and the square below the x-axis represented the reflected

square. After this demonstration I went back to the "across x- & y-axis" part of their

brainstorm, and I asked if they could reflect in other ways. Grace mentioned being able to

reflect across "the diagonal," so I asked her to draw what she was thinking on the board.

Like Hope, she found it easiest to draw two squares and label her vertices, and what she

drew was a reflection over the line $x = y$. When she was done, I asked her to sit back

down, and I asked everyone if they thought that what Grace drew was a reflection and

why. They all agreed that it was a reflection, so I asked if there were other ways to reflect

something. They said that it is possible to reflect over any line, so at this point, I

introduced the term "line of reflection."

   I got out a scarf, which we used as a line of reflection, and the dancers reflected

one another for a short time. Then we talked about where one person needs to be in the

room in relation to the scarf and the other person for the people's relationship to be

considered a reflection over the scarf. Everyone agreed that the people should be on

opposite sides of the scarf, but they could not agree on how far away from the scarf the

dancers needed to be. The dancers were in agreement that the relationship showed a

reflection if the two dancers were equally far away from the line of reflection, but one of

them suggested that it was still a reflection if the dancers were at different distances from

the line of reflection. One of the dancers agreed with her, and the other disagreed.

   At this time, I asked all three dancers to sit down. I placed the scarf between Ivy

and me (both she and I being roughly the same distance from the scarf), copied Ivy's

sitting position, and I asked the dancers if I was reflecting Ivy. All three said that I was. I

then asked Ivy to remain exactly as she was, and I scooted farther away from the scarf.

This way I was clearly farther away from the scarf than Ivy was. I asked again if I was

reflecting Ivy, and again two of the girls said I was, and one said I was not.

I came to the realization that I might have started with the wrong congruence

transformation since the explanation I needed was based on translations, so I went ahead

and told the dancers that in this example, I was both a reflection and translation of Ivy,

but I was not just a reflection. Hope quickly and fundamentally described a translation as

a slide when I asked if anyone knew what a translation was. Then I asked what we could

say about the points of an object and its reflection, now that they knew this new

information. The dancers repeatedly told me that the points of an object had to be the

same distance without any sort of reference with regards to what they should be the same

distance from. With this in mind, I asked what the points of an object and of its reflection

should be the same distance from, and one of the dancers decided that they should be

"equidistant" from the line of reflection.

Ivy mentioned that she had never heard the word "equidistant" used in such a

way, and from that point forward, the dancers used the term "equidistant" to describe the

distance from the points of an object to the line of reflection and from the points of the

object's reflection to the line of reflection. It would also be used to describe points

involved in rotations, and the word seemed to greatly transform the understanding of the

dancers in the piece since they brought up that term in every reflection beginning every

rehearsal from then on.

When I asked the dancers what we had talked about that day and what stood out

to them, Grace responded that "true reflections are equidistant." Hope then added that if

the points of an object and its reflection are not equidistant from the line of reflection, it might be a reflection and then a "slide." Grace mentioned that reflections can occur over the *x*- or *y*-axis or over any diagonal, and Hope mentioned that they learned about lines of reflection. Ivy threw in that she remembered the words "congruence" and "rotation" from the concept map, so she knew those were concepts we would discuss over the course of our rehearsals.

Rehearsal for the second week began with me asking Grace and Ivy what they remembered from the first week because Hope could not attend. Grace said that they discussed reflections and rotations, and Ivy said they discussed mirroring, which relates to reflections. Ivy also said that they talked about the points of a shape and its reflection being equidistant from the line of reflection. Grace recalled that they reflected objects across diagonals and labeled points of shapes on the board to see distances from the line of reflection and how a shape reflects over the line.

In order to allow the dancers to dance more and learn from movement in the second rehearsal, I asked Grace and Ivy to mirror each other over a line of reflection (my extra pair of sweatpants), and I watched from the side, in line with the line of reflection so I could call out for them to pause when their distance from the line of reflection changed. The dancers alternated who was mirroring who at first, and then I asked them to sense who was leading and who was following without talking. They could switch roles at any point. When they got to be different distances from the line of reflection, I would pause them, ask if what they were doing was reflecting one another, ask them to make necessary corrections in order to show a reflection and let them continue.

After ten minutes of that, I asked the dancers to reflect themselves using the line through their nose and bellybuttons as their line of reflection. I performed this exercise with them because I thought it could be useful in creating movement for the dance that we would later form, and we all found it to be an interesting and fun experience. Over the course of this project, this particular exercise led to the most laughter. We quickly found that walking was impossible and transitioning to different levels or positions like standing, sitting, squatting, kneeling, or lying was much more challenging when one side of the body was not allowed to move independently from the other. When reflecting on the experience, Grace with a laugh said that we looked silly, and Ivy added that it was hard since we had to think about everything being equal. Personally, I found that it was hard to find new ways to move while trying to reflect over my centerline. It seemed like I kept repeating the same movements and could not think of a new way to move that would still represent a reflection.

After this, we used the wall with the mirror as our line of reflection, and took note of our literal reflection in the mirror. We looked at how far we could get from the mirror and how close we could get to it, and we discussed if our reflection looked roughly the same distance to our line of reflection. We also combined exercises and played with using our centerline as our own personal line of reflection while looking in the mirror.

After I thought the dancers had a good grasp on reflections and ways we could represent them through movement, I asked them to describe a rotation. Grace said that rotations happen about an axis or a point at the middle of a circle. I asked her what she thought we might call such a point, given the term "line of reflection," and she quickly

called it a "point of rotation." Ivy gave an example saying that a square with vertices A, B, C, and D (called square ABCD) might be BCDA once rotated around a point.

At this point, I asked the dancers to stand up and make a circle by holding hands and rounding their arms. I put a small, round mint container on the floor in the center of the circle they had created, called it their point of rotation, and asked them to rotate around it until they were facing the same direction that they were facing when they started. The dancers laughed, looked at the mint container, and rotated around it. After this, I moved the container to be about two inches from Ivy's toes, and I asked Grace and Ivy if moving the point of rotation to this new position that was not in the center was acceptable. They said that it was, so I asked them to rotate around their new point of rotation. Ivy looked directly at the container so that she could keep an eye on how far away her toes were from the container, and Grace looked at Ivy's feet to make sure that her own feet were directly across from Ivy's since that is how they started. Ivy began taking tiny, scooting steps around the container, and Grace's steps were comparatively much larger. The dancers had decided without prior questioning or discussion that in a rotation, all points of a rotating shape must maintain their distance from the point of rotation. I was impressed and excited.

Once they had gone all the way around the container and returned back to their starting positions, I moved the container to be a few inches behind Grace's heels and asked if that was a valid place for a point of rotation. They thought about how they would move around the point and decided a point of rotation could be outside of the shape. When I asked them to rotate, Grace bent slightly forward and looked behind her heels at the container to make sure she was keeping the same distance between herself (a point on

the shape) and the container or point of rotation. Ivy watched Grace and the container in order to help Grace as much as she could.

At some point during this process, Ivy compared the rotating that she and Grace were doing around the mint container to the rotation of the Moon around the Earth, which is rotating around the Sun, a connection she easily made since she was taking an astronomy class at the time. She said that she and Grace were in their own little orbits around the mint container, and I had trouble seeing what she was seeing at first since I was looking at the big picture of a shape rotating around something rather than its points.

Since I was not sure if I agreed with how Ivy thought about rotations, and I wanted to be able to correct any misconceptions she might have, I asked her to draw her thought process on the board. She drew a triangle and a point of rotation, and she drew little circles with the point of rotation in the center. Each circle had a vertex of the triangle on it, and those circles were the paths that the points of the triangle traced around the point of rotation. After the explanation of how I thought about it, Grace, Ivy, and I all agreed that both of our methods were valid because they both insured that the points of the shape stayed the same distance from the point of rotation as the shape rotated. This discussion made Ivy happy and proud, and for weeks she said that it was her favorite event from rehearsals.

Once they had finished the rotating tasks, I was impressed, and we were almost out of rehearsal time, so I had the dancers sit down with me, and I asked how our bodies rotate. Ivy said that anything with a socket rotates, like an arm or a shoulder, and Grace mentioned our wrists and ankles. Ivy said that the spine does not rotate (unless the whole body is turning, like in a pirouette), and Grace clarified that the head rotates a little on top

of the spine. Grace mentioned that like we can move our fingers in a circular motion, our toes rotate, but Ivy and I discovered that this is not the case for us. Ivy then lay down on the ground and rolled over a few times, and cited that as an example of a rotation. Grace and I disagreed, so I asked Ivy what her point of rotation was. She claimed it was the top of her head, so I asked what happened to her point of rotation after she rotated. She did not seem to understand the question, so I simply pointed out that she was moving across the floor as she rotated, so she was both rotating and translating (which we would talk about the next week) at the same time, rather than just rotating.

To bring rehearsal to a close, I asked the dancers what they remembered about reflections. Grace immediately said the word "equidistant," and she explained that she had to keep the same distance to the line of reflection. Ivy then clarified that the points of an original shape and each mirrored point of the reflected shape have to be equidistant to the line of reflection. When asked to define a rotation, Grace said "a rotation is a turn about a point of rotation where each point of the object maintains its distance from that point." She also pointed out that the point of rotation must stay in one spot and that it does not also slide.

When I opened the next rehearsal by asking what the dancers remembered, Hope mentioned reflections, rotations, and translations, but Grace and Ivy mainly discussed tasks that they had done the previous week like rotating around and reflecting across different items. Since I wanted to find out more about the math that they remembered, I asked them to define a reflection. Grace said it is a "mirrored image across an axis or line of reflection." Grace and Ivy simultaneously said that the points of the object being reflected and of its reflection should be equidistant to the line of reflection. Hope added

that the reflection is an "exact copy." What she meant by this is that a reflection is a

congruence transformation. The shape itself does not change when it is reflected. We had

not yet talked about this concept.

When asked to define a rotation, Hope said that "an object [is] being turned or

persuaded a certain angle or a certain number of degrees." Grace said that it is important

to look at the points closest to and furthest away from the point of rotation. I think she

might have been headed toward saying that all of the points of the original shape should

be the same distance from the point of rotation when it is rotated; however, I really

wanted to make sure that she knew that this applied to all of the shape's points, so I asked

the other dancers what they thought about Grace's statement. Ivy said that she disagreed

because it was necessary to look at all of the points on the shape.

After this, I asked them to hypothesize about what they thought a translation

might be. Ivy thought linguistically that a translation is something said in a different way,

so she thought in math it might be something somehow done in a different way. Grace

thought that a translation was a slide, which is conceptually representative of a

translation, but it lacks any use of academic language or precision, which are important in

the Common Core State Standards for Mathematics, particularly in the Standards for

Mathematical Practice (CCSSM, 2010, p. 7).

To bring the dancers back together to a term they all had a general idea about, I

asked them what a point of rotation was. Ivy said that it is a point around which an object

rotates, and Hope added that an object moving around a point of rotation "stays the same

shape and moves the same," which again referenced the idea of congruence. This

discussion also allowed for an opportunity to transition into movement and let Ivy and

Grace catch up Hope on what they had done and learned during the previous week.

Having Grace and Ivy teach Hope also allowed me, as a teacher, to assess how well

Grace and Ivy understood the information and tasks from the previous week.

 The dancers walked Hope through each task from the previous week, but they

moved through them much quicker because we all knew we had other concepts to cover

that day. Luckily, the dancers were efficient and asked for materials I brought to

represent the point of rotation or line of reflection before beginning certain exercises, and

Hope caught on quickly.

 After they had finished catching up Hope on the events of the previous week, I

asked the dancers to help me make a grid on the floor using yarn for x- and y-axes. We

created a Cartesian coordinate plane with four quadrants on the floor of the dance studio.

We laid paper strips approximately four steps apart on all four axes to denote units on our

axes. Paper strips were laid on top of the yarn for positive numbers and under the yarn for

negative numbers. Once the coordinate plane was set up, I asked the dancers to hold

hands and straighten their arms to make a triangle. I positioned them clearly on the grid,

and I asked them what they would to if I asked them to translate positive three units on

the x-axis.

 The dancers collectively decided they would count three units in a certain

direction from where they started and move there without changing their shape. Then

they did just that. Still holding hands, the dancers moved as a triangle three units in a

specific direction. They counted each unit they moved out loud as a group, and once they

had quickly and successfully completed that example, I gave them a few more to try.

They seemed to think that this exercise was simple, and they seemed to grasp the mathematical material, so after a few examples, I allowed them to stop, and I asked them to create dance phrases that we could manipulate and work with to create our final dance. Ivy and Hope were having trouble coming up with anything individually, so they created a duet where they would reflect around Grace while she was doing her phrase. Grace's phrase was later used as the beginning of the dance, where Grace performs her phrase while Ivy and Hope walk in circles around the space. Later in the piece, Hope and Grace reflect around Ivy who does a specific phrase, so the time spent creating the phrases in this rehearsal was actually quite beneficial to our finished product.

For me, the biggest take-away from that rehearsal were the phrases and ideas that the dancers came up with, but for Hope, she said that she was excited that they got to put movement to their definitions. This was exciting to hear since putting movement to math and vice versa is a lot of the goal of this project.

When the rehearsal for the fourth week started, I asked Grace and Ivy what they remembered (Hope could not make it because of a prior engagement), and they started listing terms that they remembered, so I started asking them to define the terms that they were listing. Ivy defined the line of reflection as the line in the middle of a shape and its reflection, which I thought was an interesting way to think about it, and Grace added that the points of the two shapes (original and reflection) are equidistant from the line. I asked the dancers what a reflection was, and Ivy said that is was the exact object or shape, and Grace continued that the object was just mirrored. I asked what this object would be mirrored over, and they both said the object is mirrored over a line of reflection.

Grace defined a point of rotation as a "point around which an object turns," and Ivy defined rotation as "moving an object around the point of rotation [such that] each point [of the object] moves exactly the same distance or angle." The dancers worked together to define a translation, which they said was a slide where the points of an object move the same amount of space in the same direction.

After the dancers had defined terms, I decided to teach them a dance phrase that I had choreographed since I was not sure if I was going to use any of their phrases yet, and I felt like the dancers needed more material to reflect, rotate, and translate in the final dance. Plus, my phrase already had a lot of reflections, rotations, and translations built in, so we spent the majority of rehearsal learning my phrase, and making sure it really got into the dancers' bodies. We discussed some of the transformations that were built into my phrase, and the dancers practiced it several times with and without my help.

Once I felt like they had a good grasp on the phrase, I checked the time, and decided to start piecing together the dance. In this rehearsal, I started the dance with Grace in the center doing her phrase while Ivy and Hope were either walking in circles or standing still. I decided that all three dancers should somehow meet in the upstage corner on the right from the audience's perspective, and then they should have some sort of contact and a transformation (a group reflection, rotation, or translation). After this, they would all do my phrase as a group, which means they would finish the phrase in the same corner where they started, so they could form a triangle or circle and rotate outward. By the end of the fourth rehearsal, this is as far as we had gotten on the dance. Ivy and Grace walked through it a couple of times to make sure they knew the sequence, and one time Grace did not get to the spot where she needed to be as quickly as Ivy got to her spot.

This meant that Grace was standing too close to Ivy, and Ivy was ready to go. Ivy turned

to Grace and said, "Translate yourself over there." We all laughed for a long time, and

Ivy was still so proud of her joke that she wrote it on her posttest concept map.

The fifth week began with my asking the dancers what they remembered, and all

three of them defining every related term they knew. Then, Grace and Ivy again caught

up Hope. Ivy had already worked with her before rehearsal and began teaching her the

phrase, but rehearsal was spent clarifying and cleaning the phrase and pushing for more

dynamic movement. After Hope learned the phrase, the dancers and I taught her what we

had come up with for the dance so far, and we added a little to the end. By time everyone

was caught up and on the same page with regards to the dance, rehearsal time was over,

and we were only about halfway done with the dance.

The next week Hope was sick and contagious, so she had to miss our last

rehearsal before dress rehearsal and the show. According to the informed consent form

(see Appendix D), I was now able to remove Hope from the dance since she had missed

more than two rehearsals, which is detrimental to the progress, spacing, and cohesion of

the dance; however I knew Hope caught on quickly and had a perfectly valid reason for

why she could not attend rehearsal, so I allowed her to stay in the piece. She was grateful,

and she promised to learn everything as well as she could if I would video-record phrases

and parts of rehearsal and email them to her. All of that was acceptable according to the

informed consent form, so I recorded individual phrases and the finished dance (with

only Ivy and Grace dancing), typed instructions for phrases and spacing and visual

relationships, and I emailed it all to Hope after the sixth rehearsal. I emailed the video of

the dance from the sixth rehearsal to the other two dancers as well because they asked me

to so that they could practice. Again, this is acceptable according to the informed consent

form and the IRB.

During the sixth rehearsal, I asked Grace and Ivy to define all of the terms that

they knew, and once they were finished with that, I addressed some last minute points

that I thought might be important for the dancers to know that we had not gotten a chance

to discuss. I gave them the definitions of the words "transformation" and "congruence." I

thought these words would be especially beneficial to know for the question and answer

portion of the thesis concert, where someone might refer to reflections, rotations, and

translations as transformations or more specifically, congruence transformations.

After this, we finished the dance. Grace begins doing her phrase in the center of

the performance space while Hope and Ivy walk in circles around the space. All three

then meet in the corner and do my phrase as low as they can and finish in the corner

where they started. They then form a circle and rotate clockwise two places. In other

words, the circle rotates about 240 degrees. Then, Grace and Hope mirror each other

while Ivy does my phrase again, this time lightly and gracefully, coming between them

and changing directions to move all around the space. The dancers then meet in their

same familiar corner and travel on the diagonal to the downstage corner on the left (from

the audience's perspective) by making a circle and rotating over and over again,

alternating which person is being used as the point of rotation. After that, the dancers

remain in the circle and travel to the center of the performance space by imagining a line

of reflection in front of one dancer's toes, breaking the circle in order for the points to

reflect appropriately across that line of reflection, and reforming the circle. This

happened with alternating dancers representing the line of reflection until they got to the

center of the space and all three of them sat down and faced outward in a circle. They then performed floor work, which contained rotations and translations, and they all stood up and traveled using a different means to move around the space. Ivy performed my phrase again any way she wanted to perform it. Grace did lunges, which were featured at the end of my phrase, and Hope walked and did hand gestures.

Grace then did a second phrase that I choreographed while Ivy and Hope went to the back of the performance space and improvised using the exercise where they reflected themselves using the line through their noses and bellybuttons as their lines of reflections. When Grace finished the phrase she was doing, she joined them in the back, and they eventually spiral into a tight circle. This circle splits apart into a horizontal line across the center of the performance space where all three dancers perform the second phrase I wrote with different transformations. One dancer performs the phrase on the opposite side (meaning they switch their lefts and rights), representing a reflection. One dancer performs the phrase facing different directions, representing a rotation. Finally, one dancer does the phrase exactly as I choreographed it, representing a transformation. All three dancers repeat their transformation phrase until the music and lights dim, and the piece ends.

The three dancers met me an hour early before dress rehearsal the following Friday so that they could do the piece with all three of them there together. They ran the dance a couple of times, and I gave them notes, and while I was setting up everything in the performance space for a full run of the show, the three dancers were going through the dance in the dressing room. All three dancers knew the dance and performed it well the following night at the dance concert.

Results

For Hope and Ivy, the posttests demonstrated a greater total number of

propositions, number of valid propositions, number of attributional propositions, and

score than the pretests (see Table 6). All three dancers omitted causal relationships from

their pre and post concept maps and omitted hierarchical relationships from the pretests.

They all received points for three hierarchical relationships on their posttests because I

counted their connections of rotations, reflections, and translations to the center bubble or

congruence as demonstrating types of congruence transformations. In this case, rotations,

reflections, and translations would fall under the category of congruence transformations

and would therefore be subordinate to that category.

Table 6

Concept Map Analysis of Transformations

Data Type	Grace (Pre)	Grace (Post)	Hope (Pre)	Hope (Post)	Ivy (Pre)	Ivy (Post)
Total Propositions	29	22	14	26	12	25
Valid Propositions	23	18	13	24	11	23
Hierarchical	0	3	0	3	0	3
Causal	0	0	0	0	0	0
Attributional	29	19	14	23	12	22
Score	39	33	24	38	12	27

Grace's pre and posttests demonstrate a decrease in total propositions, valid

propositions, attributional propositions, and score. The decrease in number of valid

propositions, attributional ones, and score are most likely due to the fact that the total

number in her posttest was smaller than the number of valid or attributional propositions

in her pretests. This can be at least partially attributed to how she included the geometrical objects referred to in the standard. In the pretest, she included parallel lines, perpendicular lines, and line segments in separate bubbles each labeled as examples of objects found in geometry. From each of these bubbles came three connections labeled "can be" to rotated, reflected, and translated. This meant that parallel lines, perpendicular lines, and line segments, could all be rotated, reflected, and translated, and these nine propositions (each given a score of 2) were missing from Grace's posttest. On her posttest, she simply drew examples of parallel lines, perpendicular lines, and line segments in one bubble that was connected to the word "rotation" by a line that was labeled "looks." Based on her use of the phrase "looks like" to label a line connecting a word and an illustration elsewhere on the concept map, I assume that is what her label is intended to say, but even so, a rotation does not necessarily look like her illustration of these types of lines, so the proposition was given a score of zero. The treatment and organization of the concepts of parallel lines, perpendicular lines, and line segments on her pretest led to a score of 18 on that section of the map, while on the posttest, the treatment of these concepts led to a score of 0.

In rehearsals for "Transformations," the dancers and I mostly worked on formulating definitions of reflections, rotations, and translations rather than the geometric objects listed in the standard. The dancers all demonstrated their understanding of these objects through their answers to questions I asked, and we even sometimes discussed how these objects might be reflected, rotated, or translated, but these geometric objects were not the focus of the rehearsals. This could be the cause of such a drastic difference in at least Grace's inclusion and score concerning these objects, which in part contributed to

her decrease in numbers of total, valid, and attributional propositions. However, the other two dancers did not experience these decreases, and none of them included circles on their posttests, which is often the formation the dancers were in as a group to form the definitions of rotations, reflections, and translations since it was easy to tell them to hold hands and form a circle, and then I could tell them where to move or what type of transformation to perform with respect to a line of reflection, point of rotation, or graph on the floor.

In rehearsals, the dancers seemed to learn the terms "line of reflection" and "point of rotations," which are very important in defining rotations and reflections. However, Hope assumed that there was also some sort of "line of translation" and suggested this to the other dancers who seemed to think of her as the "smart one." Both Grace and Ivy included this in their concept maps, and were given a score of 0 for that proposition since I had even told them in a rehearsal or two that I did not think that "line of translation" was a precise term. Hope managed to write that an object could be translated over an "axis" in her map, which is true, and she may have made the suggestion of a "line of translation" and included it on the rough draft concept map during the group brainstorm because she might not have attended the rehearsal where this was discussed. In fact, Hope only attended one half of the rehearsals. She has a score higher than the other two dancers largely because she included more labels on her map than Ivy and more propositions (and valid ones) than Grace. Overall, all three dancers have very similar maps since they are all very similar to the rough draft map they created in their group brainstorm. This is common among the groups that chose to use their brainstorming time making a rough draft concept map, but Grace, Hope, and Ivy had particularly similar maps.

The dancers defined rotations, reflections, and translations in rehearsals, which showed growth since their pretests and accompanying group brainstorm only included examples of these concepts. The dancers also expressed their understanding of the fact that in reflections all the points of a shape (or line or other geometrical object) and the respective points of its image are the same distance from the line of reflection. However, all three of the dancers mentioned the word "equidistant" and related it to equal distance between the line of reflection and an object, which seems to imply that the points of the original shape are all the same distance from the line, so the terminology is not very precise.

The dancers said nothing but positive remarks in the question and answer portion of the Math Dance Concert. In fact, Grace and Hope talked about how they had applied what they had learned in rehearsals. Grace said, "I didn't even have to second guess myself because I was able to put the movement and the [math] together, I had ten times better of a background, and I knew exactly what I was talking about, and I didn't second guess myself once, which is very cool."

Shortly after, Hope talked about applying concepts learned through Math Dance to her physics test. She said, "When we were learning about [rotations and reflections], it was cool because on our physics exam, he had a lot of questions about it, and I knew it because of Morgan's [dance]."

Overall, the dancers' understanding of the topic did seem to change over the course of this project. However, their learning seems to be more emphasized through what they say about the topic and demonstrate through conversation than what they wrote on their concept maps. While in rehearsals, we focused on defining terms, the dancers

were able to apply what they had learned in their math and physics classes outside of

rehearsals. Two of the three dancers experienced an increase in total number of

propositions, number of valid propositions, and score from their pretests to posttests, and

all three dancers included a lot of the same information on their posttest concept maps

due to their referencing the rough draft map from their group brainstorm.

Vectors

Number and Quantity – Vector and Matrix Quantities (N-VM): (1.) Recognize

vector quantities as having both magnitude and direction. Represent vector quantities by

directed line segments and use appropriate symbols for vectors and their magnitudes. (4.)

Add and subtract vectors. (5.) Multiply a vector by a scalar (CCSSM, 2010, p. 61).

Instructional Rehearsals

On the first day of rehearsal, when we did concept maps, four dancers were

present. After this rehearsal one dancer would remove herself from the study due to more

pressing obligations, but she was present for the first rehearsal along with the three

dancers that remained in the vector dance and performed in the concert: Amy, Brook, and

Cate.

After concept mapping on this first day of rehearsal, I asked all four dancers to

create a dance phrase using only straight lines. Cate finished before the others, and I had

her perform her phrase for me. I stopped the other dancers early, before they could finish

their phrases, and the other three dancers and I learned Cate's phrase. For each movement

we discussed either our initial point in space and our terminal point, the initiation and

terminal points in our body, or the initial points and terminal points of our energy. When

we talked about our initial and terminal points in space, we talked about ideas like where

in the room we begin movement and that we finish it about two feet forward from where

we started. When we talked about initiation and terminal points in the body, we

discussed, for example, if a particular arm movement starts in the shoulder joint, the

finger tips, or the scapula and how it flows through the arm and finally finishes. When

discussing initial and terminal points of energy, we talked about an arm movement

starting in the shoulder but energy flows out of our fingertips and to the walls or the ceiling instead of coming to a halt at our fingers.

After talking through these ideas, we sat in a circle and derived our own definitions of initial and terminal points. Cate decided that the initial point would be like a "point of origin," and Amy said that an initial point is a "starting point." Brook then defined the terminal point as the destination, final point of movement, or where a movement will resolve. Cate added that the terminal point is probably the end of the vector.

For week two, we began by discussing what the dancers remembered from the first rehearsal. They told me that line segments have initial and terminal points, which can be used to describe where movements start and stop. Cate added that she thought vectors might be used to describe "movement pathways." After we had reviewed what we remembered from the first rehearsal, the dancers and I reviewed Cate's phrase. I gave Brook and Amy time to finish their phrases with straight lines from the previous week, and I let Cate play with extending her phrase. Brook finished before Amy, so we used Brook's phrase to learn about magnitude and direction in our movements. The dancers would, for example, lunge to the right and extend their arms wide to their right side, and I would simply ask how big that movement was, and with each movement after that, I was able to ask comparatively how big it was or what the length of the movement was. For each movement, I also asked which way we were facing or down which path our movement led us.

After we had discussed these ideas, I asked the dancers what else we might call the length of our movement or which way we were going. We discussed possible

terminology for a while, but as the dancers struggled and ran out of ideas, I decided to give them the terms "magnitude" and "direction." Then we got to discuss why those terms made sense. We used Brook's phrase to again go over magnitude and direction but also to review initial and terminal points from the first week. After this, we reviewed Cate's phrase, and we sat down to discuss what we had learned that day. Brook defined magnitude as the length of movement and lines, and Amy recognized that these could be lines in the body (a movement from your shoulder to your fingertips) or lines in space (walking from the back of the room four steps forward). Cate shared that there can be several initial and terminal points for one particular movement depending on whether the line we are discussing is in movement, in space, or in the lines we're making. By this, she meant that there could be different initial and terminal points for a movement in the body (shoulder to fingertip), a line in the space (back to four steps forward), or lines in energy (shoulder to the walls of the room). Amy informed us that the magnitude is the length between the initial and terminal points, and Brook agreed and supported her with the statement, "It's the distance between two points." With this information, Cate concluded that the distance is the magnitude.

The dancers and I began the third week by defining the terms we had discussed the previous week. Brook explained that magnitude is the extent of something with a beginning and an end, and Cate added that in dance terms, magnitude could be a quality (like when we used magnitude to describe lines of energy) or a quantity (like when we used it to discuss how many steps forward we took in space). Amy defined "direction" as trajectory, and Cate called it a path and gave the cardinal directions as examples. The initial point was defined as the beginning, the point of initiation of a movement, and the

starting point or possibly the "point of stillness before movement" by Amy, Cate, and

Brook respectively. When discussing terminal points, Amy and Cate simultaneously said,

"the end," and Brook described it as "the destination."

After this, the dancers were asked to draw what they thought a vector might look

like on a graph that I had drawn on the board and to show the initial point, terminal point,

direction, and magnitude. I got three drastically different pictures (see Figures 3 and 4),

and we discussed what we agreed with and disagreed with in all three graphs. The

dancers and I all agreed that the magnitude is the length between the initial and terminal

points, and all of the dancers seemed to agree that the initial point would be on one end of

their picture, and the terminal point would be on the other. There would not be an initial

or terminal point in the middle of a line. The debate that arose was whether or not a

vector had to be straight. The dancers and I had a discussion, and we determined that

since vectors have one direction, it has to be a straight line. If a line goes up and back

down again, it has at least two directions and would not be a vector. At this point, the

term "directed line segment" was introduced as a way to represent vectors on a graph,

and the dancers agreed that Amy's graph was the only graph that actually showed a

vector.
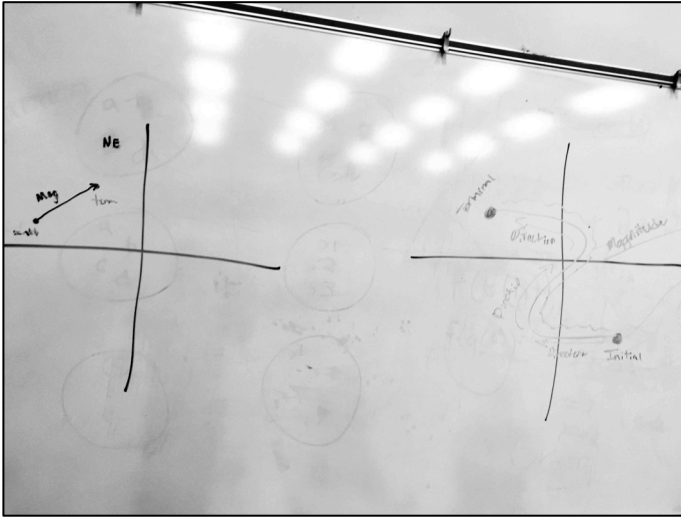
Figure 3: Amy's (left) and Brook's (right) drawing of what they think a vector looks like.
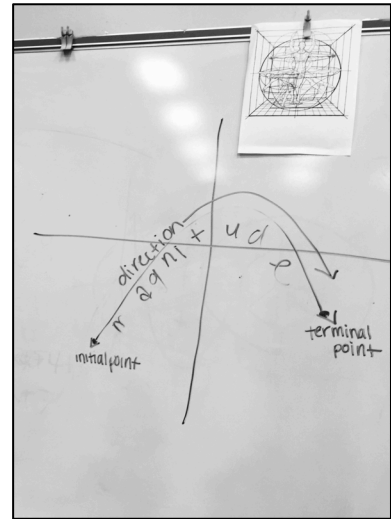
Figure 4: Cate's drawing of what she thinks a vector looks like.

After determining what a vector might look like, the dancers and I discussed the

component form of a vector. I drew a vector on the board, labeled the initial and terminal

points, and wrote the component form of the vector beside the graph. I then had the

dancers do some manipulations with the numbers on the board so that they could come

up with how to find the component form of a vector. The dancers decided that you

subtract the x-component of the initial point from the x-component of the terminal point,

and you go through the same process with the y-component. So in symbols, the initial

point might be (2, 1), and the terminal point might be (6, 7). Then the component form of

the vector v would be $< (6 - 2), (7 - 1) > = < 4, 6 >$. The dancers did another example, and

drew the vector on the graph for a visual representation.

Then I brought out two long pieces of yarn and purple paper strips. We created an

x-axis and a y-axis with this yarn and created a Cartesian coordinate plane with four

quadrants. We laid the paper strips approximately four steps apart on all four axes to

denote units on our axes. Paper strips were laid on top of the yarn for positive numbers

and under the yarn for negative numbers. After the graph was set up, I asked for a

volunteer to be the initial point. This person would walk to a point of their choice and tell me which point they were at. I would then ask for a second volunteer and have her go to a specific terminal point. The first two dancers would be asked to repeat the points at which they were standing and to face the direction of the vector. The initial point would then face the terminal point, and the terminal point would face away from the initial point. The third person would then be asked to tell me the component form of the vector that the other two girls were creating.

We did several more examples like this on the floor graph, and Brook often found that she needed to walk the *x*- and *y*-axes to determine how far the terminal dancer was from the initial one. Amy and Cate also took this approach at times. They also ran to the board at times to write out the coordinates and do the calculations to find the component form. The girls were having so much fun that they did not want rehearsal to end! When I asked them to gather together to discuss what we had done and learned that day, they whined and asked if they could do a few more.

I let them do one more example on the floor graph, and the dancers reluctantly gathered by the white board with me to discuss what they had done. Cate reminded us that they learned what a vector was. She said it was a directed line segment with magnitude, direction, an initial point, and a terminal point. Amy added that we can translate a vector anywhere on a graph as long as the magnitude and direction remain the same. Brook added that this works because vectors are equal (or their component forms are the same) if their magnitudes and directions are the same, even if their initial and terminal points differ.

After reviewing what they knew on the fourth week, we immediately got started multiplying vectors by scalars on the board. I asked the dancers what a scalar was, and we discussed the root word "scale" before we decided that a scalar is just a number. Brook used the word "integer" to describe a scalar several times, but every time I made sure to correct her that a scalar can be any real number and not just an integer. To learn to multiply vectors by scalars, I drew a graph and a vector and wrote its component form on the board, and then I asked the dancers what might happen if we multiplied that component form by, for example, three.

The dancers unanimously decided that both the x-component of the component form and the y-component would be multiplied by three. They used their knowledge of multiplying expressions using the distributive property to inform this decision. So, for example, $3 \times < 4, 6 > = < 12, 18 >$. I then had the dancers return to the floor graph, and we repeated the floor graph component form process, but after we created an original vector, I would tell the dancer at the initial point to stay in her spot and the dancer at the terminal point to move to where she thought the vector would be if we multiplied it by 2 or ½. I would then have the third person decide if the terminal point were correct or incorrect in their decision to move where they did and describe what happened to the vector. They decided that if a vector was multiplied by two, for example, the vector would grow to be twice as long, or the magnitude would also increase by two.

The dancers, particularly Amy, had quite a breakthrough when I asked them to multiply $< 2, 0 >$ by -3. For this particular problem, the dancer representing the initial point was standing on (0, 0), and the original terminal point was (2, 0). When I asked the dancers to multiply their vector by -3, the dancers knew to multiply both 2 and 0 by -3.

However, when they got -6, the terminal point dancer walked 6 places to the left (or -6 on

the *x*-axis), which made her final place at (-4, 0). At this point, all three dancers were

confused, and I was worried that I had lost them entirely. Then, the three dancers started

having their own group discussion, and I asked them how we find the new component

form if we multiply by a scalar. The girls answered my question, and I asked them what

our new component form would be. They answered < -6, 0 > and began to question their

current < -4, 0 >. They then had another group discussion where they determined that the

result of multiplying the original component form by a scalar does not tell you the

difference between the original and resulting vectors, but it tells you the resulting vector's

component form. After this discovery, Amy wrote the method of multiplying a vector by

a scalar on the board in general terms. She wrote " n < x, y > = <nx, ny >."

We spent time adding vectors and their component forms on the board and on the

graph as well that day, and the dancers learned the head-tail rule for adding vectors. That

is, they found out that if you add two vectors together, you can move the initial point of

the second vector to the terminal point of the first vector (maintaining the magnitude and

direction of both vectors), and the vector from the first's initial point to the second's

terminal point is the sum of the first two vectors. For example, if the component form of

the first vector is < 1, 4 >, and the component form of the second is < 3, 2 >, and both

vectors start at the origin, then we could translate the second vector to begin at the

terminal point of the first vector. This means the first vector would end and the second

would begin at the point (3, 2), and the second vector would end at the point (4, 6). The

component form of the resulting vector whose initial point is at the origin and terminal

point is at (4, 6), would be < 4, 6 >. This head-tail rule was used to begin and end the

"Magnitude and Direction" dance in the Math Dance Concert. The three dancers just took

turns being the first, second, and resulting vectors.

In the review of what we had learned that day, Brook brought up two other major

breakthroughs that had occurred in that rehearsal. She had discovered that when dealing

with multiplying vectors by scalars, that if the initial point is the origin (0, 0), then the

component form of a vector would have the same numeric components x and y as its

terminal point. She also pointed out that Cate had had a breakthrough in the definition of

component form. It was in this rehearsal that Cate actually recognized that the component

form of a vector actually only described the difference between the vector's terminal and

initial point, no matter what those points are. It is for this reason that if two vectors'

magnitude and direction are the same but their initial and terminal points are different,

then their component forms will still be the same.

When the fifth week opened with a review of what the dancers remembered,

Brook told me how we could move a vector "pretty much wherever on a graph," to which

Cate added that the vector could start anywhere because it still had the same distance or

magnitude. No one mentioned that for vectors to be the same anywhere on the graph,

they must also have the same direction, but I did not ask because I thought we would

come back to it. Before I began to ask the dancers to define terms, Cate also reminded us

that while we deal with distances and differences to find both the component form and

the magnitude of a vector, the component form and magnitude are two very different

things.

Before we started in on our moving and learning for the day, I asked the dancers

to define some terms they had been learning. Brook defined "initial point" as the starting

point of a vector, and she and Amy said that in dance, that could be the initiation of a

movement in the body, the initial point of a line of energy, or where you physically begin

in space. Brook defined "terminal point" as "where you end in space or where the

movement ends in the body."

When I asked them to define "directed line segment," Amy stated that it was a

line with a dot on one side and an arrow on the other, and Cate clarified that a directed

line segment is a way to represent a vector. Amy defined "direction" as the "facing or

movement pathway" from the initial point to the terminal point, and Brook said that

"magnitude" is the length or extent of a vector or movement. Amy shared that

"component form" is the "distance between the x- and y-coordinates of the initial and

terminal points."

Brook and Amy both excitedly defined "scalar" as simply "a number," and when

asked how to multiply a vector by a scalar, Amy wrote an example on the board. She

wrote: $4 < 4, 2 > = < 16, 8 >$. When I asked the dancers how to add two vectors, Amy

said that we "add the x-coordinates of both component forms and the y-coordinates of

both component forms, and that becomes the new component form."

I followed these questions by asking how they thought we would subtract one

vector from another. Brook came to the board and did an example of how she thought we

would subtract. She came up with an example using two vectors' component forms, and

she subtracted the second vector's x-component from the first's and the second's y-

component from the first's, which was correct so we did a few more examples. Then I

had the dancers draw vector pathways on pieces of paper (see Figures 5, 6, and 7). Some

of these pathways that they drew became pathways used in their dance. For example, in

the beginning of the dance, where the dancers demonstrate the head-tail rule for adding

vectors, some of Brook's and Amy's pathways are used. We ended this rehearsal by

reviewing the dance phrases that the dancers had created.
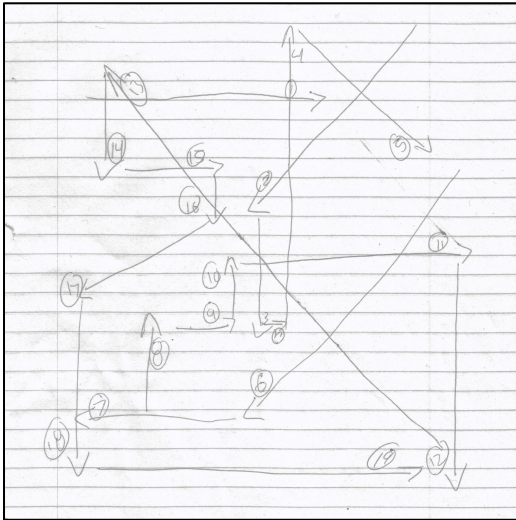
Figure 5: Vector pathways drawn by Brook
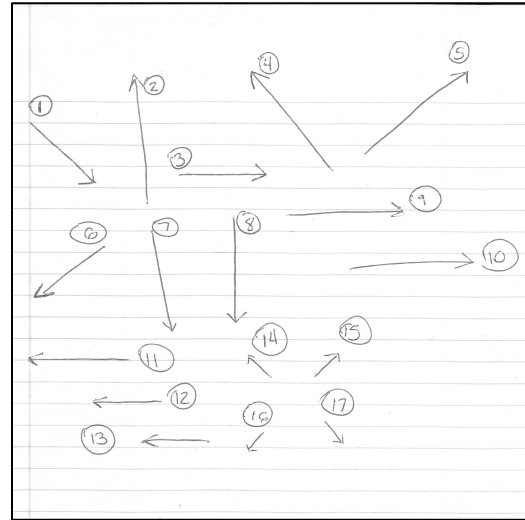
Figure 6: Vector pathways drawn by Amy
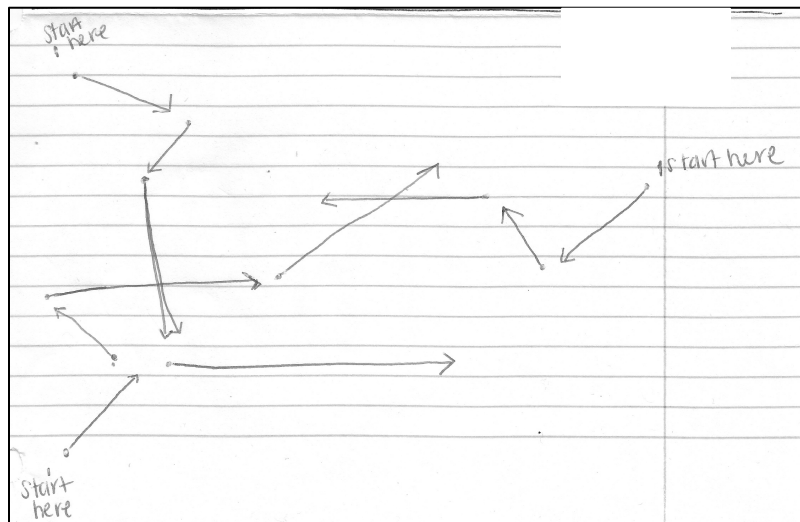
Figure 7: Vector pathways drawn by Cate

Amy was late to the next rehearsal so Brook and Cate were the only two

answering the reflective "what do you remember" and definition questions. In this

rehearsal, I asked the dancers to focus more on how to represent these terms through

dance. Brook explained how magnitude could be viewed as the energy used in

movement, and Cate called the magnitude the dynamics of movement. While neither of these connections focus on the magnitude being the length of something, the dancers focused on magnitude being how big something is. In this case, they connected magnitude with being the length of their line of energy, which often makes for a bigger, more dynamic movement.

After we finished reviewing for the week, Cate, Brook, and I began putting together "Magnitude and Direction," and when Amy walked in, she joined us in choreographing. In the first day of composing our dance, we used the head-tail rule for adding vectors to open the piece, and the dancers used their "arrow walks," which they came up with on the second day we used the floor graph, to complete their head-tail demonstration. After the head-tail rule, all three dancers' phrases were featured in someway or another in the dance, and Brook helped me come up with ways to get the dancers out of the lit space so that they could have breaks within the dance. All of the ways to get out of the space were linear to keep with the idea of a directed line segment and on a low level (usually sitting or kneeling) so that the dancers exiting would not distract the audience from what was going on in the lit space or the "stage."

The next (and final) rehearsal happened in much the same manner. When asked what they remember, the dancers pleasantly surprised me with all of the information they remembered having not learned anything new the week before and having missed a week of rehearsal. Amy said that all vectors have one direction, and Cate defined the component form as the difference between x_1 and x_2 and y_1 and y_2. In other words, Cate told us that in general, the component form of a vector with initial point of (x_1, y_1) and terminal point (x_2, y_2) is $<(x_2 - x_1), (y_2 - y_1)>$. With this in mind, Amy said that when we

add vectors, we should add x's with x's and y's with y's, and the same goes for subtracting the component forms of two vectors.

Once we had finished review, we finished the dance. The dancers created a literal arrow where all of the dancers had their torsos, arms, and right leg in a line parallel to the floor, and each of the back two dancers grabbed the ankle of the person in front of them to create a straight line. Brook was in front of the line with her arms slightly out to her side and behind her creating an arrow-like shape with her arms. If the dancer's arrow line was a vector, Amy's right foot would have been the initial point, Brook's head would have been the terminal point, and the height of all three dancers would have roughly been the magnitude.

We manipulated the dancers phrases throughout the piece, and the dancers repeated their original phrases several times. The piece both started and ended with "arrow walks" and the head-tail rule for adding two vectors. Overall, the dance that the dancers and I created seemed to be representative of the work the dancers did in rehearsal and the knowledge that they gained over the course of the semester.

Results

With the exception of the number of causal relationships in Cate's concept maps, each person saw an increase in every category from her pretest to posttest (see Table 7). For all three dancers, there were zero hierarchical relationships in their pretests and two in the posttests. Similarly, all three dancers have zero causal relationships in their pretests, and Amy and Brook have three and two, respectively, in their posttests. This may be because of how I chose to look at the concepts of initial points, terminal points, magnitude, and direction, as described next.

Table 7

Concept Map Analysis of Magnitude and Direction

Data Type	Amy (Pre)	Amy (Post)	Brook (Pre)	Brook (Post)	Cate (Pre)	Cate (Post)
Total Propositions	6	25	9	17	5	12
Valid Propositions	1	25	1	12	1	12
Hierarchical	0	2	0	2	0	2
Causal	0	3	0	2	0	0
Attributional	6	20	9	11	4	10
Score	1	53	1	13	1	12

Originally, I considered these concepts the fundamental parts of a vector, and I

considered the associated propositions as having hierarchical relationships because I felt

that these fundamental parts of a vector fell under the superordinate category of a vector.

However, as I continued to analyze the concept maps, I realized that both Amy and

Brook associated causal relationships with these concepts. Brook claimed that the initial

and terminal points formed a directed line segment, and this directed line segment is what

has magnitude and direction. This demonstrated a causal relationship in that if there were

no initial and terminal points, there would be no line segment to have direction and

magnitude. Similarly, Amy claimed that both the initial and terminal points "determine"

the magnitude (since the distance between these points would be the magnitude), but only

the terminal point "determines" the direction (since changing the terminal point of the

vector would change the direction). This implies that the initial point is chosen and

stationary while the terminal point can change, but eventually Brook agreed with this

argument and added it to her posttest concept map as well. Cate included it in a bubble

with a basic definition of direction, but she added, "according to [Amy]," so I treated the

bubble as if it had just contained the definition of direction since she did not seem to

believe Amy's argument. The use of the word "determines" in Amy's and Brook's maps

caused me to label the proposition as causal.

All three dancers had one valid proposition on their pretests, and that proposition

had to do with vectors or matrices being added and subtracted and, in some cases,

multiplied and divided. Brook is the only one of the three dancers whose propositions

were not all valid on her posttest. This seems to be due to a misunderstanding of what the

component form is and inaccurate phrasing of certain concepts. For example, Brook

wrote, "The component form is basically the difference between any two vectors, or the

difference between the two." This is not correct. The dancers all demonstrated

understanding of how to find a vector's component form in rehearsals, so when Cate

wrote that the component form is the "difference between [the] initial and terminal

points," I accepted that, despite its imprecision. Cate recognized that the initial and

terminal points of one vector are used to find the component form of a vector, not

subtracting two entire vectors. Amy was even more precise and wrote, "$<x_2 - x_1, y_2 -$

$y_1>$." These x's and y's represented the x- and y-components of the initial point (labeled

with a subscript 1) and terminal point (labeled with a subscript 2).

Amy's overall score was much higher than the other two dancers' partially

because she included more propositions, but largely because she labeled nearly all of the

lines in her propositions. On the other hand, Brook labeled only a few of her propositions,

and Cate did not label any. This greatly affected the dancers' scores since a lack of label

yields a score of 1, a labeled attributional relationship yields a score of 2, and a labeled

hierarchical or causal relationship yields a score of 3.

In rehearsals, the dancers demonstrated knowledge of finding component forms,

and adding two vectors, subtracting two vectors, and multiplying a vector by a scalar

using the vectors' component forms. Amy is the only one who demonstrated her ability to

perform these computations on her posttest, and she did so by doing examples like "$< 1,$

$3 > + < 2, 4 > = < 3, 7 >$."

All three dancers included definitions and terminology that they learned in

rehearsals, which is good because we focused on definitions and terminology in

discussions at the beginning and end of each rehearsal and when talking about the dance.

The dancers also discussed terminology a lot in the question and answer portion of the

Math Dance Concert, along with our process of creating the dance and how much they

felt like they benefitted from the use of the floor graph.

When talking about the floor graph, Brook mentioned how she felt she could

transfer the information she was learning back onto paper, which is often how we work

with mathematics. She said, "and then I could go back on the board and feel it in my

body to then write it out. And that's where it was a different experience: to be able to

translate it back onto that written paper."

When discussing how I got my participants, Brook wanted to add her reason for

wanting to participate in this project, and she said, "It was really because I wanted to

learn, um, something more about math, and it worked…" She delivered the quote of the

night, according to my friends, family, and professors who attended the Math Dance

Concert, when she said, "I kind of don't want to say doing the math because eventually I felt like I *was* the math…"

The dancers overall seemed to take away a lot from this project. They claim to have learned more about math, and they all demonstrated new knowledge on their posttests, which seem to be much more informed than their pretests that each had only one valid proposition. The dancers focused a lot on terminology and concepts like initial and terminal points, magnitude, and direction on their concept maps, which is a lot of what we focused on in discussion and movement in our rehearsals. They also demonstrated understanding of how to perform certain mathematical operations (like adding and subtracting) with vectors in rehearsals.

Building Functions

Functions – Building Functions (F-BF): (1.c.) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time (CCSSM, 2010, p. 70).

Instructional Rehearsals

The first rehearsal started with a pretest concept map. Ella, Fay, and Grace brainstormed on the their topic and Common Core State Standard, and like the dancers in "Sets of Numbers" and "Transformations," created a rough draft concept map from which to work on their individual concept maps. Because they discussed a lot of mathematical terminology like equations and constants in their group brainstorm, those are the ideas I addressed first after they finished their concept maps.

The dancers often used the term "equation" incorrectly, using it to describe an expression. For this reason, we discussed the difference between an equation and an expression. I gave them a few examples of expressions like 3+2 and $3x$ and $\frac{4y}{3x}$, and then I asked them what they noticed about expressions and what could be included in them. They told me that numbers, variables (or the alphabet, as Fay liked to say), and symbols (like the plus sign, minus sign, and division bar) could be included in expressions. I then showed the dancers an example of an equation (for example, $x+2 = 6$), and I asked what they thought the difference might be between an expression and an equation. They told me that an equation has an equals sign (=), but an expression does not, and an equation has something on both sides of the equals sign.

From this discussion, we moved on to functions. I wrote a function on the board, though imprecisely I wrote it in the form of an equation ($y = 2x + 1$) instead of in function notation ($f(x) = 2x + 1$), and from this example, the dancers asked if equations are functions. Again, imprecisely, I responded that some equations are functions, but not all of them are. I should have informed them that functions are relations and can be described by equations, which I never said, but I did fix the function notation issue in the next rehearsal.

In this first rehearsal, we discussed that for functions, each input only has one output. A simple example of this would be a linear function, or as I wrote on the board, an equation of a line. For the equation $y = 2x+1$, no matter what number is used as the input (what you put in for x), there is only one possible number that can come out in place of y, the output. So if x is 3, then $y = 2(3)+1 = 6+1 = 7$. For the input 3, there can only be one output, which is 7.

I explained this to the dancers, and for the rest of the rehearsal, we used this idea to experiment with linear functions and their inputs and outputs through movement. I would write an equation of a line on the board, and the dancers would pick one movement (their input) to do across the floor until they reached the "function" in the middle of the room, and then they would finish moving across the floor with their output movement. Using $2x+1$, Fay picked an x-movement that resembled an x. She took a step to the right, plied (or bent her knees) with her feet far apart, and moved both her arms up to make a "V" taking a pathway similar to that taken in a jumping jack. This was her input movement that she repeated until she reached the center of the room. Once she got to the center of the room, the dancer who played the role of the function would physically

adjust the dancer in some way to "add one." Fay would then do her input movement twice to signify $2x$, and then she would do the "add one" movement to represent $2x+1$. She would then repeat this sequence (two inputs plus the "add one" movement) until she reached the end of the room.

This exercise allowed the dancers to look at the difference between the input x and the output $2x+1$ with movement. The dancer designated as the function would be required to add the same "add one" movement to each dancer and input movement so that it was clear that the inputs were going through and being edited by the same function. To make the exercise fair, and to allow the same learning opportunity to all three dancers, we worked with three or four equations of lines, and for each equation, all three girls were the function once and the input twice. For example, when we were working with $y=2x+1$, Grace would be the function while Fay and Ella were the inputs and outputs, Ella would be the function while Grace and Fay would be the inputs and outputs, and finally Fay would be the function while the other two dancers were the inputs and outputs. After we went through this full cycle, I would change the equation on the board.

When we had about eight minutes left of rehearsal, I stopped the dancers so that we could discuss what they had learned in that rehearsal. They told me about the difference between equations and expressions, and Fay mentioned that if you put something into a function, it has an "effect" or output. The one question they asked me during the rehearsal was whether "building" in "building functions" was meant as a verb or an adjective. I told them that it was meant as a verb, but it did not matter much because the functions would start building on one another as we progressed through rehearsals.

The next rehearsal opened as usual with the dancers telling me what they

remembered. Grace and Ella brought up inputs and outputs, and Ella mentioned that we

discussed equations and expressions, which she defined as numbers that you can "do stuff

to, like plus or minus," without an equals sign. Fay added that expressions can or cannot

include variables, and started the discussion about equations by saying they had to have

an equals sign. Ella and Grace told me that equations had to have something on the other

side of the equals sign, and Ella explained an equation has to balance.

Then I decided to clarify the concepts we had discussed in the first rehearsal. I

introduced the dancers to function notation where an equation could be written as, for

example f(x) = 2x +1, a function of x. We discussed what could be used as an input to a

function, and Ella and Grace told me that numbers, variables, and expressions could be

inputs. We had not yet discussed that functions could be inputs of other functions in

composition or building, so I was proud that the dancers understood a basic function. To

clarify that each input of a function could only have one output, I drew three pairs of
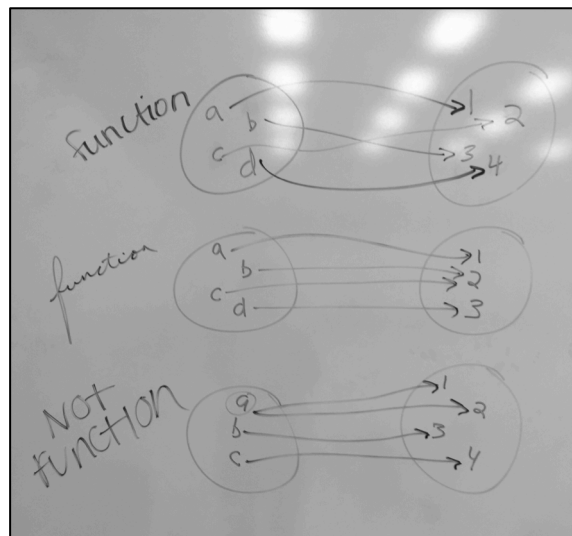
Figure 8: The dancers clarified understanding of
single output per input.

circles, wrote inputs in the circles on the left, wrote outputs in the circles on the right, and

drew arrows to connect inputs to outputs. I then asked for a volunteer to write if the top

relation drawn on the board was a function or not (see figure 8). After she wrote

"function," I asked the other dancers what they thought and whether or not they agreed,

and the dancers unanimously agreed since each input (in the left column) only had one

output (in the right column). We then repeated this process for the next two relations, and

after some discussion, the dancers all agreed to the correct answer each time. The dancers

said the second relation is a function since each input only has one output, despite the fact

that two inputs have the same output, and the last relation is not a function since the input

a has two outputs.

Once we had clarified all of these concepts, I asked the dancers how we could

dance functions because my thoughts on teaching them composition of functions through

movement were not solidified yet, and what they told me helped me form my ideas as

well as demonstrated what they had learned about the topic so far. Ella said, "We're each

a different input to this function, that would be the piece," and she insinuated that in

this case, each person would have a different output, so they would all end

differently. Grace thought about the dance like a linear function, specifically. She

said that like points that make a line, the output of each dancer's input would create

one thing, the dance. Fay added to this idea by saying that they could all start

differently and end the same, and she supported her logic with the exercise from

figure 8 by adding that it would be like two inputs having the same output, which is

still a function. This showed me that the exercise from figure 8 led Fay to a

breakthrough moment.

To make sure the dancers had something to work with and that they had the chance to dance in each rehearsal, I asked the dancers to create dance phrases that we would later manipulate to add to the piece. When time for rehearsal was almost over, I stopped the dancers and asked them to gather back around the board to discuss what they had learned and what we had done that day. Ella reminded everyone that a circle is not a function (I had used this example earlier as an equation where one input gives two outputs), and she told us that each input can only have one output. Fay said that two inputs can have the same output, and for a function, x would be the input while $f(x)$ would be the output.

When asked what they remembered at the beginning of the third rehearsal, Grace said that expressions and equations are not the same thing, but expressions can be plugged into certain equations as x, or the input. Fay then added that equations have to have an equals sign, and she said that in a function, one input cannot have more than one output.

By the third week, I had figured out how I could teach functions along with their inputs, outputs, and composition through movement, so after we had our review, we jumped right into reviewing the phrases they had previously started. Ella was the only one that remembered hers entirely, and her phrase looked good, so I had the three dancers work as a team to extend Ella's phrase, and this became known as the x-phrase. After the dancers had the x-phrase nailed down, we all came together and discussed ways to manipulate a motif. Specifically, we discussed three of Lynne Anne Blom and L. Tarin Chaplin's "Sixteen Ways to Manipulate a Motif," as printed in *The Intimate Act of Choreography* (1982, pp. 102-104). We discussed that

these manipulations would be the "functions" that we run the *x*-phrase through in

order to produce an output.

I chose the following manipulations:

"*Retrograde.* Perform it backward. Start at the end and follow it back through

space—like a movie run backward" (p. 102),

"*Inversion*: upside-down (⌈ becomes ⌊) or lateral (⌈ becomes ⌉). For upside-down

inversion, you may have to lie on the floor or stand on your head. (This can be tricky

and often impossible, but don't dismiss it on those grounds)" (pp. 102-103), and

"*Change of Planes/Levels.* Change the motif to a different plane: the horizontal, the

vertical, the sagittal plane, or any other slice of space. Do it on a different level. Trace

the path of the gesture and use it as a floor pattern. Move along that" (p. 104).

Once I was sure that Grace understood all three of these ways to manipulate

a motif (Ella and Fay had already taken Choreography I, a required course for dance

minors at MTSU, where you learn these manipulations), I let the dancers pick which

"function" they wanted to have. Fay wanted retrograde, and we thought change of

planes/levels would be the easiest for Grace to do, so Ella took inversion, which she

did not mind since she was used to working with inversions for the Complex

Numbers piece, where she and Dana had decided to invert the movements for

positive numbers to create the movements for negative numbers.

The dancers were then instructed to use their specific "function" to

manipulate the *x*-phrase. The new phrase Ella created from inversion became the

phrase *f(x)* (read "f of x"). The phrase Fay derived from retrograding phrase *x*

became *g(x)* ("g of x"), and the phrase Grace came up with after changing the planes

or levels of the *x*-phrase was the *h(x)* ("h of x") phrase.

At the end of rehearsal, I wanted to make sure the dancers understood what

they were doing mathematically when they were manipulating their phrases. I

asked them what their function was, and Ella and Fay responded that the functions

were represented by the ways they were manipulating their phrases. When asked

what the input of the function is, Grace said, "the original choreography," which in

this case, was the *x*-phrase. Ella then explained that the *f*, *g*, or *h(x)* phrases would be

the outputs of these functions. Grace described the process of manipulating the *x*

phrase with a "function" as "taking an original value and plugging it in… to create an

output."

In the opening review for the fourth week of rehearsals, I asked the dancers

what they remembered, and Ella started by saying that functions usually consist of

variables and constants, and *x* is the input that is put through the function, and *f(x)*

is the output. When she mentioned constants and variables, I assume she meant that

the variable is the input, since that is what varies, and the constants are the numbers

in the functions for example, in *f(x)* = 2*x*, 2 is a constant.

Ella followed her statements by saying that one input can only have one

output for it to be a function, and all three dancers agreed that two inputs could

have the same output. I asked the dancers what could be used as inputs, and Ella

said that numbers and expressions can be, but Grace added that functions could also

be inputs, which demonstrated Grace's prior knowledge on the subject of

composition of functions. I asked the dancers what the phrases they were making

represented, and Fay responded literally about retrograding and change of planes, and Ella said that the phrases they were creating represented functions.

After the review, I asked the dancers to review their *f(x)*, *g(x)*, and *h(x)* phrases. Ella was the first to finish her phrase (*f(x)*), so I had her perform it for me, and while she was demonstrating hers, Grace finished her phrase (*h(x)*). Ella and I both watched Grace's phrase, and when I decided that both phrases looked good, I had them teach each other their phrases. After Grace learned *f(x)*, I asked Grace to run it through her function (change the planes or levels of Ella's phrase) to create an *h(f(x))* phrase (read "h of f of x"). Similarly, once Ella learned *h(x)*, I asked her to use it as an input to her function (invert it) to create the output *f(h(x))* phrase (read "f of h of x").

By the time Ella and Grace had finished teaching each other their *f(x)* and *h(x)* phrases, Fay finished her *g(x)* phrase, so I watched and approved it so that she could also learn the *f(x)* phrase and retrograde it to create a *g(f(x))* phrase ("g of f of x").

Before rehearsal had ended, Ella had finished the f(h(x)) phrase, performed it for me and the other dancers, and gotten it approved. I also asked Fay and Grace to perform what they had so far on their new phrases before rehearsal ended, and we all discussed the phrases. After all three dancers had performed, I asked them what phrase Ella did, and Grace told me *f(h(x))*. Similarly, when I asked what phrase Grace was working on, she told me *h(f(x))*. Finally, I asked what phrase Fay was working on, and Fay told me that she was working on *g(f(x))*. Since the dancers were learning and manipulating so many different phrases with so many of the same movements

(these phrases were all, in some way, functions of *x*), I just wanted to finish up

rehearsal by making sure they knew which phrase they were creating given their

function and input.

As with several of the other dances, the fifth week of rehearsals is really when the

dance started to come together. As always, I asked the dancers what they remembered.

Grace told me that they were building functions. Ella told me that functions have inputs,

which can be "numbers, variables, or other functions." Grace added that expressions can

be inputs of functions, and Fay defined an expression as "a phrase that doesn't have an

equals sign," and she gave the example $6x+2$. I appreciated Fay's definition and the use

of the word "phrase" for two reasons. First, I like that Fay could have been comparing a

mathematical expression to a verbal phrase, whereas an equation would be a complete

sentence. I also like that Fay's use of the word "phrase" could have been referring to the

dance phrases that the dancers were using as inputs to their functions, and Grace had just

explained that expressions could be inputs to functions.

Ella said that the inputs of functions can only have one output, not two or more,

and when asked if two inputs could have the same output, all three dancers unanimously

agreed that they could again. I asked what the *x*-phrase represented, and Ella said that it

was an input. When I asked what the *f(x)* phrase represented, Grace said that it was an

output. I asked the dancers what the input of function *h* might be if the output was *h(f(x))*,

and one of them correctly responded that it was *f(x)*, and finally, Grace told me that *f, g*,

and *h* were the letters we were using to denote functions.

When we got started on the dance, the dancers had seven dance phrases to work

with: *x, f(x), g(x), h(x), f(h(x)), g(f(x))*, and *h(f(x))*. For example, *g(f(x))* would be Fay's

retrograde of Ella's inversion of the *x*-phrase. Ella's inversion of the *x*-phrase would be *f(x)*. I asked the dancers to find ways to make those phrases travel for a more visually interesting dance, and the dance opened with several of these traveling phrases. Once all three dancers were in the lit performance space, I asked them to do a dance improvisation exercise that we sometimes call "touch-response."

In this exercise, Ella may touch Fay's elbow with her hand. Then, Fay would have some sort of response to this touch, which could mean moving her arm away from Ella's touch, pushing into it, or any other response. This would cause Fay to eventually touch Ella, and the cycle would continue. With three people, there is much more opportunity for interesting contact, and the dancers were quick to recognize that I incorporated touch-response in the dance because the touches were bodily inputs, and the responses were outputs. In fact, at one point Grace missed Ella's arm in the touch-response section, and in a joking manner, Ella said, "I can't have an output without an input!" The dancers' understanding of how this exercise demonstrated inputs and outputs excited me because it made me realize that they had a firm grasp on how inputs and outputs related to each other, to functions, and to their dance.

The following week when I asked the dancers what they remembered, I got many of the same responses I had gotten in the fifth week, so to prepare them for the question and answer portion of the informal Math Dance Concert, I asked them how the math related to their dance. Grace told me that they had different expressions or phrases, and they manipulated them through functions. Ella took it a step further and said, "the functions are our brains [or their manipulations], and the phrases are our inputs and outputs."

I was happy with their answers, so we moved on and finished the dance that day. The portion of the project where the dancers learned the math through dance and movement was complete, and we were left to put what they had learned into a dance. The piece opened with Ella performing the $f(h(x))$ phrase (from here on, I will call the phrases by only their function names). Next Ella travelled with $f(x)$ to meet Fay, and together they performed and travelled x. The two then began the touch-response exercise, and Grace joined in. All three then performed $f(x)$.

Ella and Grace then continued to touch and respond on the floor or at a lower level while Fay performed $g(f(x))$. Ella and Grace then began to create a shape, and then the other person would find a way to create a new shape in contact with that person (again playing with inputs and outputs), and Fay joined in so that all three were taking turns. All three then performed $h(x)$, and then Grace and Fay did x facing different directions while Ella did a "non-distracting" motif, as I called it. Ella's non-distracting motif included crossing her forearms and using her left hand to push her right arm into initiating a circular movement with her crossed arms. She did this slowly and repeated this multiple times. Next Grace performed $h(f(x))$ while Fay and Ella did a "non-distracting" motif. Ella continued to repeat her motif, and Fay rubbed her left arm with her right hand repeatedly.

The three finally came back together in a diagonal downstage (the performance space closer to the audience) using a travelling phrase created with motifs from the other phrases. There they danced a combination of a few of their phrases as slowly as they could, and then they repeated the same combination of phrases as quickly as they could. They ended on the floor, so Grace stood up and used the new traveling phrase to exit the

performance space. Since Ella entered the space first, I wanted her to exit last, so she and

Fay used touch-response to help Fay exit the performance space. Finally, Ella repeated

her favorite phrase a few times until the lights and music faded, and the dance ended.

Results

All three dancers showed an increase in number of valid propositions, number of

hierarchical propositions, and score from their pretest to posttest (see Table 8). There

were only two concept maps that showed any causal relationships, and those were

deemed causal by the labels on the lines including "because", "makes it", and "after

composing."

Table 8

Concept Map Analysis of Inputs and Outputs

Data Type	Ella (Pre)	Ella (Post)	Fay (Pre)	Fay (Post)	Grace (Pre)	Grace (Post)
Total Propositions	11	14	13	18	14	13
Valid Propositions	9	14	8	18	11	13
Hierarchical	1	8	0	5	0	4
Causal	0	0	0	1	2	0
Attributional	10	6	12	12	12	9
Score	18	35	13	40	22	27

The increase in hierarchical relationships in the posttests is because two of the

dancers said that functions have inputs and outputs, which I understood as inputs and

outputs being included in functions, so I chose to label the dancers' inclusions of inputs

and outputs and what could be considered an input or output as demonstrating

hierarchical relationships. For all three dancers, the number of attributional relationships

did not increase from their pretests to posttests but either decreased or remained the same. It is possible that this is because of the focus on inputs and outputs as they relate to functions on the dancers' concept maps and in rehearsals. This emphasis on their maps would lead to a higher number of their propositions being labeled hierarchical, which could explain the decrease in attributional relationships.

The dancers' pretests included questions like whether or not the term "building" in "building functions" was meant as an adjective or a verb. They mentioned observing data and included equations that do not "build." On the other hand, their posttests included inputs and outputs, definition of a function, non-examples of functions, and basic representation of function notation for a composed function like "$f(g(x))$." Basically, the information on all three of the posttests demonstrated a deeper understanding of the concept than the questions on surface-level features of the standard included in the pretests.

In rehearsals, the dancers demonstrated understanding of inputs and outputs as they relate to functions and the basic definition of a function, and they used this knowledge to use functions as inputs of other functions in order to combine, build, and compose them. The dancers were able to list mathematical objects that can be used as inputs and outputs of functions both in rehearsals and on their concept maps, and they seemed to have a good grasp on the idea that each input can only have one output but two inputs can have the same output, as noted on all three posttest concept maps. This was also one of the discoveries I considered a "breakthrough moment" for at least one of the dancers in rehearsals.

In the question and answer portion of the Math Dance concert, Ella mentioned that the most memorable exercise for her was learning about inputs and outputs of functions on the first day. All three dancers were able to explain in detail how the math concept related to their dance and was visible in their dance, which demonstrated an understanding of the concept and of the ways movement from rehearsals or from the finished dance related to the math. Grace also explained in the Q&A that she was able to apply what she learned in rehearsals for this piece on a calculus test she took, and she did well on the test.

Ella, Fay, and Grace all seemed to learn most about inputs, outputs, functions, and composition of functions over the course of this project. Their posttests showed a higher number of valid propositions and a higher score than their pretests. Their posttests also included more hierarchical relationships and fewer attributional ones. Through discussion in rehearsals, writing on concept maps, and their answers in the question and answer portion of the Math Dance Concert, the dancers demonstrated a more conceptual understanding of their topic rather than the surface-level one their pretests demonstrated.

Summary of Process

Although each dance required a different process to address the mathematical concepts and standards on which the dance was based, the overall development of each dance was quite similar. The first rehearsal for each dance began with group brainstorming and the filling out of concept maps. After this, I tried to address any misconceptions the dancers seemed to have based on their group brainstorm, and then I gave the dancers movement tasks in which they could connect the mathematical ideas they would focus on over the course of the project to movement. After a few weeks of exploring mathematical concepts through creative movement and developing movement phrases that could be manipulated or altered in someway to address the mathematics, I began piecing together dances for the Math Dance Concert. Each rehearsal began with a review of what the dancers remembered from the previous week and closed with a review of what they had learned overall. As the concert approached, the dancers engaged more in review and rehearsal of the dances than they did in exploration of the concepts. After the concert, where the dancers performed the dances and explained what they had learned or how they had learned it in the question and answer session, the dancers completed their posttest concept maps.

Post-Concert Discussion

The Math Dance Concert, an exhibition of the work the dancers and I did in

mathematics through dance, was held on November 7, 2015 at 7:30 p.m. in Studio B in

Murphy Center G040. About sixty people were present. The doors of the dance studio

opened at 7:00 p.m., and slightly after 7:30, I stepped into the room, flipped the light

switch to turn off the lights in the room, and the tree lights surrounding the area in which

the dancers would performed were turned on so I could introduce the show as a whole,

point out the four Common Core State Standards printed in the program that the dancers

and I worked on, and remind the audience of the index card placed in the center of their

program for questions or comments.

I then introduced the first piece, "Sets of Numbers," by asking the audience a few

questions that I had the dancers think about while we were creating the dance. What is a

real number? What is an imaginary number? How do these relate to complex numbers,

and how can we represent this with dance? After these questions, the lights went down,

and the girls who were in the complex numbers piece entered the space for their dance.

The music began, the dancers started their dance, and the lights faded in on them.

When they were finished, the music and lights faded off on them, and the dancers

quickly moved to the center of the space to take a bow. They then walked off as I walked

on to introduce and ask some thought-provoking questions about the next piece,

"Transformations," and the process was repeated for each dance.

After the final dance, "Inputs and Outputs," the dancers exited the space, and I

explained to the audience what was to happen next. I announced that the dancers would

come out for a group bow all together, and when they did not because they could not hear

their cue, I went back to the door that they entered and exited through, and let them all in. The dancers came out, took a bow, and received a standing ovation for their performance. I let the dancers know that they could take a seat, which also signaled that the audience could take their seats, and the question and answer session began. I opened it by asking all of the dancers to describe our process of making these dances and the math that they learned through that process.

Starting with "Sets of Numbers" and then going in order of the show, each set of dancers told the audience what they knew and what we did for them to learn the math. After the concert I received several comments about how articulate and well-spoken the dancers were and about how it was apparent that the dancers really benefitted from this dance intervention. Several audience members also told me that they were struck by a comment that Brook made when she was describing the floor graph we used in learning about vectors about how she felt like she was not just learning the math but sometimes she *was* the math. Many audience members appreciated this comment and were excited by it and the work the dancers and I did.

At the end of the question and answer session, and thus, at the end of the show, I reminded the audience about the index cards in their programs so that they could write any other questions or comments they might have had but did not feel like sharing out loud and put them in a blue box outside the door. I received ten note cards with varying questions and comments.

Several comments said that the show was "amazing," and one said it "will be such an exciting idea to pass on to [my] students," which was great to read. One notecard reads, "I want to see the dances again after the explanation," which is also a comment a

got in person several times. While I agree this would have been ideal and definitely

beneficial to the audience as they tried to "understand" the dances, understanding the

dances is not a requirement for viewing a dance concert such as this one since the Math

Dance concert was meant as a way for the dancers to perform and demonstrate *their*

understanding of the math. The concert was not necessarily meant to teach the audience

members anything. Plus, viewing the concert again after the Q&A does not seem like it

would have been feasible in terms of time.

Along similar lines, one of the notecards commented that they did not understand

why I chose the standard I did for "Sets of Numbers." This particular audience member

noted that the standard made her feel like she was watching $a+bi$ the whole time, but she

does not think there was a better substandard that could have been chosen either. Reading

this, I realized that I should have emphasized more the purpose of the concert so that

audience members did not feel like they needed to recognize the math in the dances. In

fact, the dancers articulated their understanding of several sets of numbers, including

complex numbers of the form $a+bi$, but they had to learn those other sets of numbers to

understand what was meant when the Common Core State Standard said that "*a* and *b*

[are] real" (CCSSM, 2010, p. 60). The "Sets of Numbers" dance included the dancer's

knowledge of other sets of numbers, but was largely structured with the form $a+bi$,

where one dancer would be *a*, the real component, and do the integer phrase, and the

other partner would be *b* and do the partner phrase with an "imaginary" partner *i*. It

should also be noted that any real number the dancers demonstrated, for example 10,

could be written as $10+0i$. Likewise, the dancers had a movement for *i* and sometimes

repeated it multiple times, which would represent $0+10i$.

One notecard had several specific questions on it, many of which were also discussed in the question and answer session. For example, in the Q&A, I was asked how I made my music selection, and I explained that I tried to pick music drastically different for each piece to make the dances easily recognizable and easier to discuss; although some choices were made due to math being represented (the direction involved in vectors) or to the dancers' quality of movement (the graceful, light and prettiness of the dancers in "Transformations"). The card also asked about the process, which the dancers and I opened the Q&A with so they could describe some of our process and how they learned the math through movement.

Also on that notecard are questions for specific dances. The viewer wanted to know if the interaction between dancers meant anything in the first dance. It did. Most of the time the dancers were touching, they were creating a relationship between the two of them in which one movement for a specific integer divides another integer movement. A ratio of two integers, like the one they were representing, is a rational number, which is a type of real number, which is what *a* and *b* are in *a+bi*. The viewer was also curious about the slow walks around the edges of the performance space in "Magnitude and Direction" and the slow arm movements in "Inputs and Outputs." Typically, when dancers perform on a stage, there are curtains or wings on the side of the stage that the dancers are hidden behind when they exit the space. For these dances, since they were performed in a space without curtains to hide exited dancers, the dancers were asked to take a motif from the piece and perform it very slowly in order to be less distracting from the more energetic movement still happening toward the center of the space.

Another audience member asked what other topics could be taught through the floor graph. In this thesis, I taught vectors and congruence transformations (specifically translation) with it, but it could also be useful for learning functions and graphs thereof. Anything geometric might be easier to understand on the graph as long as it is two-dimensional, and the floor graph would be a great way to introduce students to the complex plane.

Finally, one concert viewer noted that "it would have been very interesting if there was a kind of short clip (explaining how the dance relates to the math standards) displayed before the dance [concert]." This is a great idea, and would in some ways fix the problem of wanting to see the dances again after the explanation, but I fear that if an explanation had been given before the dances, the audience would have felt the need to "understand" the dances or learn from them or would have been confused by an explanation of dances they had not yet seen. It would not have been possible in the space we were in, which I know because there was supposed to be a PowerPoint projection, but I could not find a projection screen large or smooth enough to project the presentation on legibly. The PowerPoint had to be removed from the concert less than twenty-four hours before the concert was presented, so a video would have probably had to be removed as well.

Aside from these note cards, I got varying verbal feedback. Several math professors told me that the concert was beautiful and that they were impressed, not only by the dances or how well the dancers spoke, but by how well they knew the math. One professor explained that it might have been expected, for example, for the dancers to have experienced this math dance intervention and left understanding kind of what a function

is and that they can be combined somehow. However, my dancers understood the difference between expressions, equations, and functions and knew what types of inputs and outputs a function might be able to have. They also knew exactly how they could represent those ideas through movement.

One person gave me some critical verbal feedback. This person wished I had explained to the audience during the question and answer session at the concert that dance is a field all its own, and (though it has been used for decades to teach various subject areas) is a subject area in and of itself full of well-educated professionals who are truly intelligent and should not be viewed as mere entertainment. In fact, dance is equally (if not more) as much a part of this project as math was. While I think this is a fact worth noting, and I hope it was evident to the audience without my having to mention it to them, I am not sure bringing all of this to the forefront of the conversation during the Q&A would have been beneficial to my project as it does not directly relate to my thesis question, which focuses on how dancers' understanding of math changes after using dance to teach the Common Core standards.

Conclusions

Over the course of this project, I found that by using creative movement and concert dance to teach mathematics standards, the students' role and the teachers' role became clearly defined. The students were to formulate mathematical definitions that made sense to them, use movement to further explore definitions or concepts, and make sense of concepts and examples of mathematical objects or procedures through dance. My role as the teacher was to facilitate discussion and exploration, help students find the mathematical meaning in their movements, and ensure that the students were accurately and precisely understanding the mathematics.

I found that in my experience, certain standards seem to be more feasible to use in an actual high school classroom. It seems logical to me to use movement to teach vectors, which are meant to describe an object's motion through space. Likewise, I think it makes sense to have students rotate, reflect, and translate themselves in order to teach transformations, in addition to having them draw these images on paper as would occur in a typical geometry classroom.

With that said, it seemed to me that finding a way to dance sets of numbers like Dana and Ella did would be difficult for students who did not have years of dance experience. If I were to try to teach sets of numbers through dance to high school students without dance experience, I would consider guiding the students more through the movement. For example, I might emphasize that whole numbers could be demonstrated with a whole movement, and I would ask students to create shapes for each number rather than a fluid motion. For rational numbers, I would allow students to find that a rational number is a ratio of two integers and that they could divide one person's shape for a

specific integer with another person's shape for a specific integer to create an example of a rational number through movement.

Composition of functions also seems like it would be more challenging to teach to students with less dance experience, so I would alter my methods to allow students to make shorter hand and arm gesture motifs – rather than full-body dance phrases – to "do in reverse" or "do above their heads" instead of using the choreographic manipulation tools. This way each student could be assigned a way to do the original gesture phrase, and then these methods of performing the original phrase could be combined to produce different outputs.

Finally, I have learned through this process that I will not always be able to anticipate students' responses and reactions, but anticipating as many as I can will help me to better understand the concept that I am trying to teach. It is best to come to each class or rehearsal prepared with a specific movement task for student exploration as well as questions to ask the students about their mathematical thinking. If I were to try to teach mathematics through dance again, I would be more prepared with specific movement tasks and an understanding of how those tasks relate to the mathematics in order to more efficiently reach my instructional goals.

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Appendix A

Math Content Taught Through Dance (from Literature)

Math content taught through dance is discussed in the tables below. Table A1 focuses on

concepts taught at an elementary school level, and Table A2 focuses on math content

taught at a secondary or post-secondary school level.

Table A1
Math Content Taught Through Dance at an Elementary School Level

Math Content	Application in Dance
Combinations	• One of the main themes established in teaching "Jump Patterns" is Combinations or finding "all the different ways something can be combined, such as toppings on a pizza or dance steps" (Rosenfeld, 2011, p. 82). For example, the girls that decided to perform their dance patterns in a B then A sequence, chose the BA combination over the AB combination (p. 78).
Communication	• One of the main themes established in teaching "Jump Patterns" is Communication or speaking "fluently about what is learned in both math and dance through written and spoken language, and to express ideas through kinesthetic, rhythmic, and symbolic language" (Rosenfeld, 2011, p. 82). Basically, students use math and dance terminology to communicate their ideas, give feedback to other teams, and reflect on what they are learning in journal prompts and workbooks.
Counting	• Schaffer and Stern's first math and dance performance included "an almost vaudevillian hand-shake routine, which…led to a discussion of estimation and number sense, as well as a workshop on counting handshakes" (Schaffer, Stern, & Kim, 2001, p. 7). • Students brainstorm all of the handshakes they know and come up with new ones. They then discuss hands involved in handshakes (for example, right to right, left to left, and right to left) and count the number of ways there are to shake hands (Schaffer, Stern, & Kim, 2011).
Fractions	• Third, fourth, and fifth graders use fractions to make patterns. Specific examples are not given (Lips, 2014).
Mapping on grid	• Students dance in a square taped off on the floor. They use this square "like a simple coordinate grid, consisting of four quadrants with [them] at the center of the x- and y-axis" (Rosenfeld, 2011, p. 85). The students then plot their positions in the square for a certain count on a grid worksheet (p. 81).

Measurement, units, and distance	• Students work on measurements in multiple units, usually related to distances. They are asked to stand certain distances away from each other. For example, the students had to estimate how far away from one another they needed to stand if they were a foot apart (Lips, 2014).
Multiplication and division	• Students form an even number of rows through multiplication and division. For example, one class had sixteen students, which they discovered was two rows of eight students. One fourth grader said the dancing made learning multiplication and division more fun. (Lips, 2014).
Patterns	• Third, fourth, and fifth graders use fractions to make patterns. Examples are not given (Lips, 2014). • Students create short dances or combinations using foot-based dance patterns that they can then transform and manipulate ("Math in your feet," 2013). • One of the main themes established in teaching "Jump Patterns" is Patterns or "anything with a unit—a shape, design, rhythm, or motif—that repeats in a predictable and organized manner" (Rosenfeld, 2011, p. 82). Students create four-count dance patterns or jump patterns and put them together to make a larger dance combination. • Schaffer and Stern's first math and dance performance included "a tap dance, which dealt with patterns and some of the arithmetic associated with those patterns, and led to an audience interaction with interwoven rhythms" (Schaffer, Stern, & Kim, 2001, p. 7). They also claim that choreography (like math) is basically "creatively exploring patterns" (p. 5). • Students follow the teacher or a partner in slapping their legs and clapping their hands to make rhythmic patterns (Schaffer, Stern, & Kim, 2011).
Problem solving	• One of the main themes established in teaching "Jump Patterns" is Problem Solving or "engaging in a task for which the solution or method is not known in advance" (Rosenfeld, 2011, p. 82).
Reflection	• Two fifth graders created a four-count dance pattern, Pattern A. For Pattern B, they reflect Pattern A and do their footwork backward. They start with count four and work to count one instead of moving from one to four (Rosenfeld, 2011, p. 78). • The students play "Congruent or Reflected," where the students watch one team dance their pattern, say whether they think the dancers were congruent or reflected, and explain why they think that. The students also play "Who's the Reflection," where the students watch one team dance their dance pattern congruently and then reflected, and then the students say who they think was the reflection (in other words, which dancer changed the original pattern or phrase) (p. 87).

Sequence	• Teams of students decide the sequence or order of the steps or counts within their own four-count dance patterns. They also decide the sequence of the four-count dance patterns in order to create a larger dance. For example, two fifth graders decided to perform patterns B then A rather than A then B (Rosenfeld, 2011).
Spatial reasoning	• A specific example of spatial reasoning is not given, though it is mentioned several times. The authors do note that using the whole body allows for new, different, and sometimes challenging ways to think spatially (Schaffer, Stern, & Kim, 2001). • Students use spatial reasoning to decide where to jump, slide, or otherwise move their feet (and in what direction) within their dance square ("Math in your feet," 2013).
Symmetry	• Students use symmetry in creating and transforming their four-count dance patterns. For example, one students may face the front and another may use turn symmetry to rotate 180° to face the back (Rosenfeld, 2011). • The first dance of Schaffer and Stern's first math and dance performance featured reflection and rotation, which are types of symmetry. They also used symmetry of scale by introducing fly swatters of various sizes (Schaffer, Stern, & Kim, 2001). • Students find a partner and mirror each other's movements. Then they practice rotational symmetry by a following one another and taking turns leading (similar to the mirroring), but instead of mirroring one another, if one moves his or her right arm, then the other should move his or her right arm (Schaffer, Stern, & Kim, 2011).
Transformations	• One of the main themes established in teaching "Jump Patterns" is Transformation or changing "the shape or position of an object" (Rosenfeld, 2011, p. 82). For example, one student may do a dance pattern (or even the entire combination) facing the front, and another may rotate 180° to face the back.
Variables	• Students are given movement variables from which to choose to create their four-count dance patterns (Rosenfeld, 2011). Movement variables seem to include five foot-positions, six movement types, and six directions (p. 88).

Table A2	
Math Content Taught Through Dance at a Secondary and Post-Secondary School Level	
Math Content	Application in Dance
Circular motion	• Participants hold a 8½ x 11 sheet of paper on a flat palm and move it in circular patterns, which keeps the hand pressed to the paper and the paper from falling (Schaffer, 2011).

Counting	• The same example of counting handshakes mentioned in the Elementary-level table can be used for higher levels. So students discuss hands involved in handshakes (for example, right to right, left to left, and right to left) and count the number of ways there are to shake hands (Schaffer, Stern, & Kim, 2011)
N-body problems	• In "the N-body equations[,] N equal masses chase each other around a fixed closed curve, equally spaced in phase along the curve." Simply put, a given number of dancers or participants ("N") walk in a particular pattern with equal distance between each dancer (Schaffer, 2011).
Patterns	• Schaffer and Stern's first math and dance performance included "a tap dance, which dealt with patterns and some of the arithmetic associated with those patterns, and led to an audience interaction with interwoven rhythms" (Schaffer, Stern, & Kim, 2001, p. 7). They also claim that choreography (like math) is basically "creatively exploring patterns" (p. 5).
Permutation	• Three people stand in a row. Two neighbors switch places without repeating, until they return to their original places. In the process of switching places, they stand in six different orders. They three people have accomplished all six permutations. The same can be done with all twenty-four permutations of four people (Schaffer, 2011).
Polyhedra	• Four people create a triangle, tetrahedron, octahedron, and cube with a piece of string (Schaffer, Stern, & Kim, 2011).
Problem solving	• Again, math and dance can both be thought of as creative problem solving, but as a specific example, in order to keep the same amount of distance between each participant in an N-body problem (see below), one would have to figure out when each dancer needed to being walking the pattern or where he or she would need to start walking the pattern (Schaffer, 2011).
Reflection	• In a dance by Schaffer and Stern called "Private Fly," they use multiple types of symmetry, including rotation, reflection, and scale (Schaffer, Stern, & Kim, 2001).
Rotation	• Rotation is mentioned and is even the title of a dance, but a specific example of rotation in that piece not provided. In a dance by Schaffer and Stern called "Private Fly," they use multiple types of symmetry, including rotation, reflection, and scale (Schaffer, Stern, & Kim, 2001). • The leader stands at a 180° rotation, but if the leader raises his or her right hand, then the followers must also raise his or her right hand (Schaffer, Stern, & Kim, 2011).
Scale	• Reflection, rotation, and symmetry of scale are all included in one of Schaffer and Stern's math dances called "Private Fly." The men introduce symmetry of scale by using fly swatters that get progressively larger (Schaffer, Stern, & Kim, 2001).

Sequences	• For example, participants can create a sequence of movements to perform in the formation (or spatial pattern) of the N-body problem (Schaffer, 2011).
Spatial awareness	• A specific example of spatial reasoning is not given, though it is mentioned several times. The authors do note that using the whole body allows for new, different, and sometimes challenging ways to think spatially (Schaffer, Stern, & Kim, 2001).
Symmetry	• Reflection, rotation, and symmetry of scale are all included in one of Schaffer and Stern's math dances called "Private Fly." In terms of symmetry of scale, the men use various sizes of fly swatters in the dance (Schaffer, Stern, & Kim, 2001).
	• There are four symmetries demonstrated. To demonstrate translation, the leader could face the same way as the followers and move. For reflection, the leader faces the followers, and the followers mirror the leader. For rotation, the leader faces the followers, and they all move the same side of their body (rather than opposite sides as in reflection). For glide symmetry, the leader faces the same way as the followers, ut they move from opposite sides of the body (Schaffer, Stern, & Kim, 2011).
Tessellations	• Video, rhythm, and movement tessellations are all mentioned. For video tessellation, participants will move using live projection of video tessellations of their movement to create multiple symmetries (Schaffer, 2011).

Appendix B

IRB Certificaiton

COLLABORATIVE INSTITUTIONAL TRAINING INITIATIVE (CITI PROGRAM)
COURSEWORK REQUIREMENTS REPORT*

* NOTE: Scores on this Requirements Report reflect quiz completions at the time all requirements for the course were met. See list below for details.
See separate Transcript Report for more recent quiz scores, including those on optional (supplemental) course elements.

- **Name:**   Catherine Davis (ID: 4723175)
- **Email:**   cmd5u@mtmail.mtsu.edu
- **Institution Affiliation:**   Middle Tennessee State University (ID: 714)
- **Phone:**   6158982616

- **Curriculum Group:**   Human Research
- **Course Learner Group:** Social & Behavioral Research
- **Stage:**   Stage 1 - Basic Course

- **Report ID:**   15474191
- **Completion Date:**   05/31/2015
- **Expiration Date:**   05/30/2019
- **Minimum Passing:**   80
- **Reported Score*:**   95

REQUIRED AND ELECTIVE MODULES ONLY	DATE COMPLETED	SCORE
Belmont Report and CITI Course Introduction (ID:1127)	05/08/15	3/3 (100%)
History and Ethical Principles - SBE (ID:490)	05/10/15	5/5 (100%)
Defining Research with Human Subjects - SBE (ID:491)	05/10/15	4/5 (80%)
The Federal Regulations - SBE (ID:502)	05/15/15	5/5 (100%)
Assessing Risk - SBE (ID:503)	05/22/15	5/5 (100%)
Informed Consent - SBE (ID:504)	05/24/15	5/5 (100%)
Privacy and Confidentiality - SBE (ID:505)	05/25/15	4/5 (80%)
Conflicts of Interest in Research Involving Human Subjects (ID:488)	05/31/15	5/5 (100%)
Middle Tennessee State University Module DEMO (ID:1073)	05/31/15	No Quiz

**For this Report to be valid, the learner identified above must have had a valid affiliation with the CITI Program subscribing institution
identified above or have been a paid Independent Learner.**

CITI Program
Email: citisupport@miami.edu
Phone: 305-243-7970
Web: https://www.citiprogram.org

COLLABORATIVE INSTITUTIONAL TRAINING INITIATIVE (CITI PROGRAM)
COURSEWORK TRANSCRIPT REPORT**

** NOTE: Scores on this Transcript Report reflect the most current quiz completions, including quizzes on optional (supplemental) elements of the course. See list below for details. See separate Requirements Report for the reported scores at the time all requirements for the course were met.

- **Name:**   Catherine Davis (ID: 4723175)
- **Email:**   cmd5u@mtmail.mtsu.edu
- **Institution Affiliation:**   Middle Tennessee State University (ID: 714)
- **Phone:**   6158982616

- **Curriculum Group:**   Human Research
- **Course Learner Group:** Social & Behavioral Research
- **Stage:**   Stage 1 - Basic Course

- **Report ID:**   15474191
- **Report Date:**   05/31/2015
- **Current Score**:**   95

REQUIRED, ELECTIVE, AND SUPPLEMENTAL MODULES	MOST RECENT	SCORE
History and Ethical Principles - SBE (ID:490)	05/10/15	5/5 (100%)
Defining Research with Human Subjects - SBE (ID:491)	05/10/15	4/5 (80%)
Belmont Report and CITI Course Introduction (ID:1127)	05/08/15	3/3 (100%)
The Federal Regulations - SBE (ID:502)	05/15/15	5/5 (100%)
Assessing Risk - SBE (ID:503)	05/22/15	5/5 (100%)
Informed Consent - SBE (ID:504)	05/24/15	5/5 (100%)
Privacy and Confidentiality - SBE (ID:505)	05/25/15	4/5 (80%)
Conflicts of Interest in Research Involving Human Subjects (ID:488)	05/31/15	5/5 (100%)
Middle Tennessee State University Module DEMO (ID:1073)	05/31/15	No Quiz

For this Report to be valid, the learner identified above must have had a valid affiliation with the CITI Program subscribing institution identified above or have been a paid Independent Learner.

CITI Program
Email: citisupport@miami.edu
Phone: 305-243-7970
Web: https://www.citiprogram.org

Appendix C

IRB Approval

IRB
INSTITUTIONAL REVIEW BOARD
Office of Research Compliance,
010A Sam Ingram Building,
2269 Middle Tennessee Blvd
Murfreesboro, TN 37129

MIDDLE TENNESSEE STATE UNIVERSITY

EXPEDITED PROTOCOL APPROVAL NOTICE

Wednesday, August 26, 2015

Investigator(s):   Catherine Morgan Davis (PI) and Sarah Bleiler
Investigator(s) Email: cmd5u@mtmail.mtsu.edu; sarah.bleiler@mtsu.edu
Department: Mathematics
Protocol Title: "Math dance: Teaching Common Core State Standards of Mathematics
                          through concert dance "
Protocol ID: 16-2007

Dear Investigator(s),

The MTSU Institutional Review Board (IRB), or its' representative, has reviewed the research proposal identified above. The MTSU IRB or its representative has determined that the study poses minimal risk to participants and qualifies for an **EXPEDITED** review under 45 CFR 46.110 and 21 CFR 56.110 within the category *(7) Research on individual or group characteristics or behavior* This approval is valid for one year from the date of this letter for **20 (TWENTY) participants** and it expires on **8/26/2016**.

Any unanticipated harms to participants or adverse events must be reported to the Office of Compliance at (615) 494-8918 within 48 hours of the incident. Any change(s) to this protocol must be approved by the IRB. The MTSU HRP defines a "researcher" as someone who works with data or has contact with participants. Anyone meeting this definition needs to be listed on the protocol and needs to complete the required training. New researchers can be amended to this protocol by submitting an Addendum request researchers to the Office of Compliance before they begin to work on the project.

Completion of this protocol MUST be notified to the Office of Compliance. A "completed research" refers to a protocol in which no further data collection or analysis is carried out. This protocol can be continued up to THREE years by submitting annual Progress Reports prior to expiration. Failure to request for continuation will automatically result in cancellation of this protocol and you will not be able to collect or use any new data.

All research materials must be retained by the PI or the faculty advisor (if the PI is a student) for at least three (3) years after study completion. Subsequently, the researcher may destroy the data in a manner that maintains confidentiality and anonymity. IRB reserves the right to modify, change or cancel the terms of this letter without prior notice. Be advised that IRB also reserves the right to inspect or audit your records if needed.

Sincerely,

Institutional Review Board
Middle Tennessee State University

IRBN001   Version 1.0   Revision Date 05.11.2015

Appendix D

Informed Consent Form

Ggg/

INSTRUCTIONS FOR INVESTIGATOR

The following is a template for a complete informed consent document. As a guide, it can be partially revised to fit your study. However, the first two (2) paragraphs and all questions need to be included, as required the by the Office of Human Research Protections.

If you choose to alter or waive consent for your study, you must provide justification to do so. Fill out the appropriate portion of the Request for Waiver or Alteration of Consent and attach it to your IRB application. The form can be accessed at http://www.mtsu.edu/irb/irbforms.shtml

If a question is not applicable to your study, simply insert n/a. You should also eliminate suggested language (in brackets and red type) if not pertinent to your study, to enhance participant comprehension. If used for a parent/legal guardian, alter language to refer to child.

Should you have any questions or need additional information, please do not hesitate to contact my office.

Compliance Officer

compliance@mtsu.edu

Box 134

Sam Ingram Building 011B

(615) 494-8918

Middle Tennessee State University Institutional Review Board
Informed Consent Document for Research

Principal Investigator: Catherine Morgan Davis
Study Title: Math Dance: Teaching Common Core State Standards of Mathematics Through Concert Dance
Institution: MTSU

Name of participant: _____ Age: _____

The following information is provided to inform you about the research project and your participation in it. Please read this form carefully and feel free to ask any questions you may have about this study and the information given below. You will be given an opportunity to ask questions, and your questions will be answered. Also, you will be given a copy of this consent form.

Your participation in this research study is voluntary. You are also free to withdraw from this study at any time. In the event new information becomes available that may affect the risks or benefits associated with this research study or your willingness to participate in it, you will be notified so that you can make an informed decision whether or not to continue your participation in this study.

For additional information about giving consent or your rights as a participant in this study, please feel free to contact the MTSU Office of Compliance at (615) 494-8918.

1. **Purpose of the study:**
   You are being asked to participate in a research study because I would like to know if it is possible to teach and learn Common Core State Standards for high school mathematics through dance. To this end, I am interested in how your understanding of a particular mathematical concept changes before and after you engage in modern dance and creative movement with the purpose of exploring a math concept. This project is an undergraduate thesis project to be presented to the Honors College of Middle Tennessee State University in partial fulfillment of the requirements for graduation from the University Honors College. The thesis focuses on teaching high school mathematics standards through dance, and will result in an informal concert consisting of four to five "Math Dance" pieces. Each piece will cover a specific mathematical standard. Topics will include complex numbers, vectors, transformations and congruence, and functions.

2. **Description of procedures to be followed and approximate duration of the study:**
   Before learning and creating these dances, all participants will be assessed for prior knowledge through concept maps. No prior math knowledge or interest is needed. This will just give us a good idea of where to start teaching and exploring the math concept in the dance. About a week before the concert, participants will be assessed in the same way again to determine learning or deeper understanding of the math concept. You will have a group brainstorming session before completing these concept maps, and the brainstorming and assessments will be video recorded so I can better understand your understanding of the concept.
   Rehearsals will consist of learning set dance phrases and structured improvisation in order to explore a mathematical concept, and all participants are expected to be present and dancing at every rehearsal. I will be taking notes during each rehearsal, so your questions, explorations, discoveries, and any mathematical connections you make will be noted and may be included in the written thesis. This project and these rehearsals will culminate in an informal dance concert in November (which is when research will end), and all participants are expected to be present for a mandatory dress rehearsal as well as the concert.
   The concert will be filmed and DVD copies of this film will be in the back cover of every printed copy of the thesis. Rehearsals may also be filmed for the purpose of remembering and practicing the dance if requested by a participant.

3. **Expected costs:**
   None

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Middle Tennessee State University Institutional Review Board
Informed Consent Document for Research

4. **Description of the discomforts, inconveniences, and/or risks that can be reasonably expected as a result of participation in this study:**
There are no risks associated with this project aside from the risks ordinarily associated with dance. Participants, please take care of your bodies and move intelligently.

5. **Compensation in case of study-related injury:**
Neither I nor MTSU will provide compensation in the case of study related injury.

6. **Anticipated benefits from this study:**
a) The potential benefits to science and humankind that may result from this study are:
If the dancers have a deeper understanding of the mathematical concept on which their dance is based at the completion of this project, dance and creative movement could be used as an effective teaching strategy to teach high school mathematics. This could benefit teachers and high school students, leading to a more mathematically literate society.
b) The potential benefits to you from this study are:
You will have the ability to put your work in my thesis project on your dance resume. You will also be listed as a dancer and given credit in the program of the informal concert.

7. **Alternative treatments available:**
Outside of this study you have probably learned mathematical concepts in a traditional math classroom, but within this study, there are no alternatives to learning math through dance.

8. **Compensation for participation:**
None.

9. **Circumstances under which the Principal Investigator may withdraw you from study participation:**
If you miss more than 2 rehearsals for the creative and teaching portion of this project, then you will be removed from the dance (for staging and spacing purposes for the concert). Thus, you will not take a second assessment, and there will be nothing to compare your first assessment to, so you will be removed from the research study. You may still be included in the printed discussion of the video of the brainstorming and assessments.

10. **What happens if you choose to withdraw from study participation:**
If you withdraw from only the research portion of this project, then any concept map assessments you would have completed would no longer be included in the data set or results of this project. If you withdraw from the project as a whole, your concept map assessments would no longer be included (as if you had only withdrawn from the research portion), but dances will also be adjusted to be performed without you, and your name will no longer appear in the concert program. If you withdraw in any manner, you may still be included in the printed discussion of the video of the brainstorming and assessments.

11. **Contact Information.**   If you should have any questions about this research study or possible injury, please feel free to contact **Morgan Davis** at **(423) 605-4251** or **cmd5u@mtmail.mtsu.edu** or my Faculty Advisor, **Dr. Sarah Bleiler** at **(615) 898-2616** or **sarah.bleiler@mtsu.edu**. You may also contact the **Research Compliance Office** at **(615) 494-8918** or **compliance@mtsu.edu**.

12. **Confidentiality.** All efforts, within reason, will be made to keep the personal information in your research record private but total privacy cannot be promised. Your information may be shared with MTSU or the government, such as the Middle Tennessee State University Institutional Review Board, Federal Government Office for Human Research Protections, if you or someone else is in danger or if we are required to do so by law. Dr. Bleiler of the mathematics department is my faculty advisor for this project, and all assessments will be kept in a file cabinet in her office. Any video recordings (those of the brainstorm and assessment and from rehearsals) will be kept on my password-protected laptop. Your

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Middle Tennessee State University Institutional Review Board
Informed Consent Document for Research

ideas and assessments may be shared in the written thesis along with an overall determination of growth or changed understanding of the mathematical concepts and your major(s) and/or minor(s), but your name will not appear with this information in the written thesis. Your names will be printed in the concert program so that you can include your participation in your dance resumes if you wish, but the programs (and your names) will not be included in the written thesis. There will be DVD copies of the final concert in the back cover of every printed copy of the final thesis, and if rehearsals are recorded by request, I cannot ensure the confidentiality of those videos as I cannot control if other members of the study show them to someone else. I do ask that all participants respect each other's privacy and keep the videos to themselves.

13. **STATEMENT BY PERSON AGREEING TO PARTICIPATE IN THIS STUDY**
   I have read this informed consent document and the material contained in it has been explained to me verbally. I understand each part of the document, all my questions have been answered, and I freely and voluntarily choose to participate in this study.

_____   _____
Date   Signature of patient/volunteer

Consent obtained by:

_____   _____
Date   Signature

                                        _____
                                        Printed Name and Title

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Appendix E

Recruitment Flier

(Photo from Boehm, 2014)

Appendix F

Information Meeting Flier

(Image by Quarles, 2011)

Morgan Davis's Honors Thesis
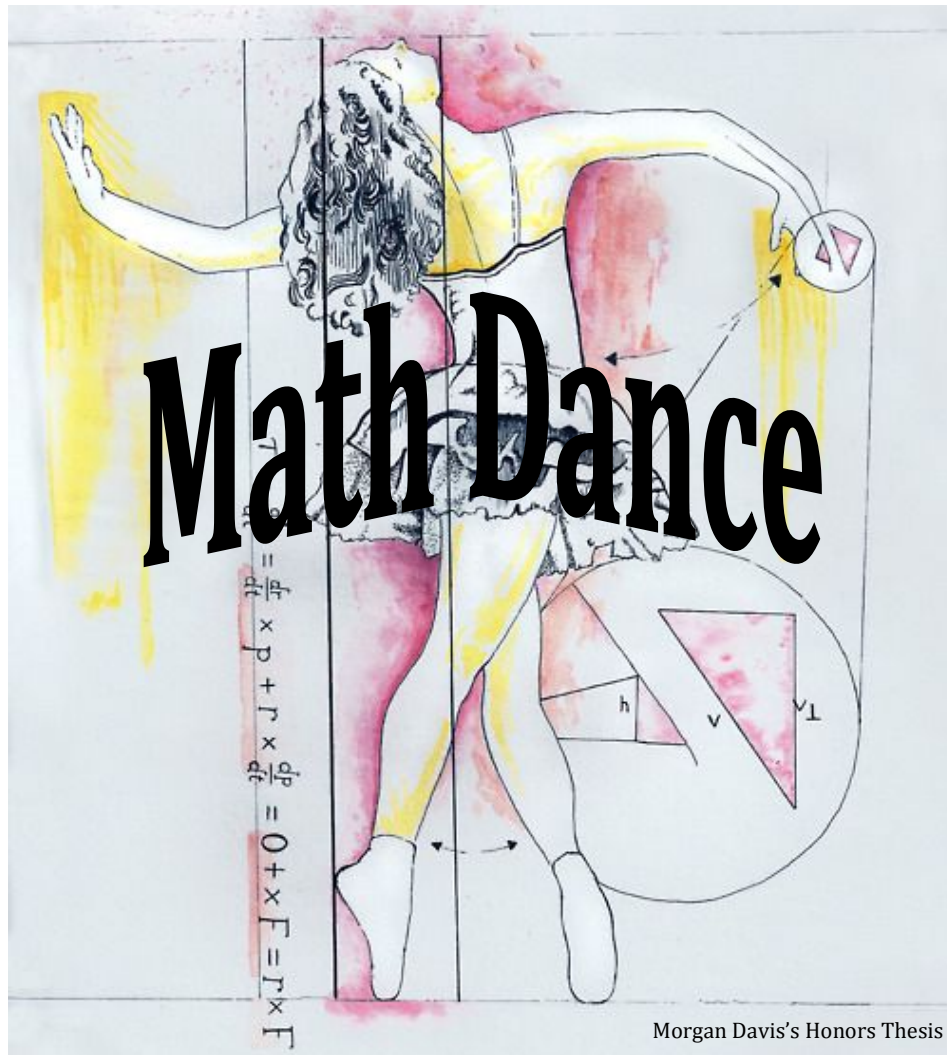
INFORMATION SESSION

Monday, August 31 at 7:30 in the studio (G040)

If you are interested in participating in my honors thesis, please come! Dance experience is required, but interest in mathematics is not! For those of you who definitely want to be in my thesis (or have already talked to me about it), we will schedule rehearsals, so please bring your schedule!

Appendix G

Example Concept Maps

(McClure, Sonak, & Suen, 1999, p. 479)
Sample A
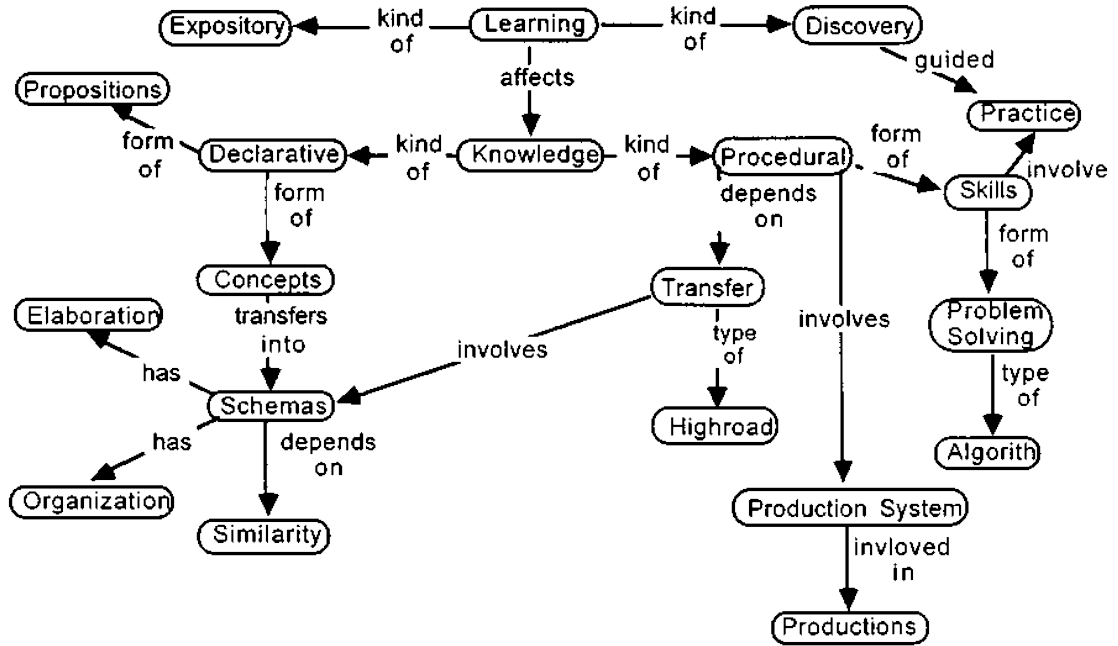
Sample B
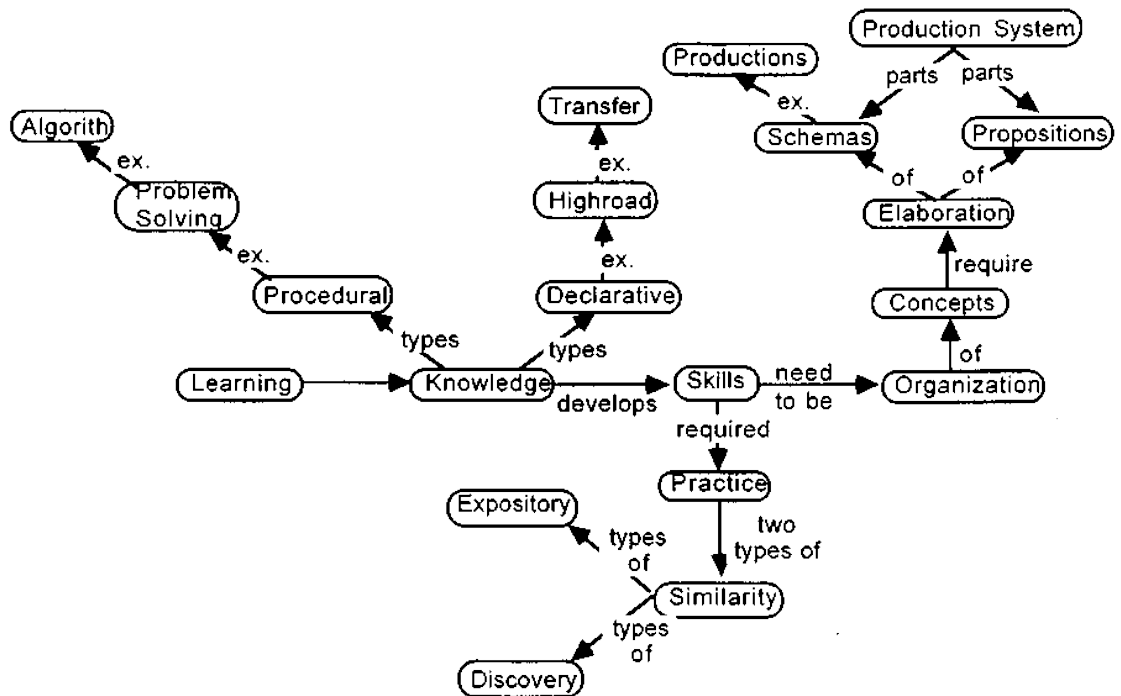
Appendix H

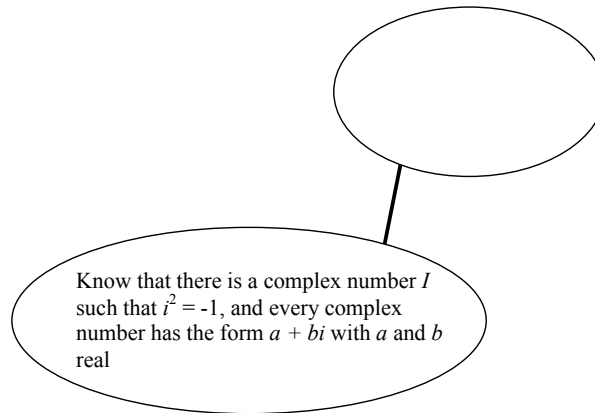## Blank Concept Map for Pre and Posttests (Complex Numbers)

Name: _____

Major/Minor: _____

Math Dance:
Teaching Common Core State Standards of Mathematics Through Concert Dance

The Complex Number System

Please construct a concept map with this math standard. You may include definitions, descriptions, characteristics, illustrations, examples, non-examples, any connections you make, and any previous knowledge. Please brainstorm as a group first, then include any information you wish to include in your concept map. I gave you one bubble to get you started, but please form as may connections as you can. You may connect bubbles in anyway you like or add any additional detail as you see fit.
(Source: CCSSM (2010). Common Core State Standards for Mathematics (p. 60). Washington, D.C.: Council of Chief State School Officers)

Know that there is a complex number I such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real

Additional comments:

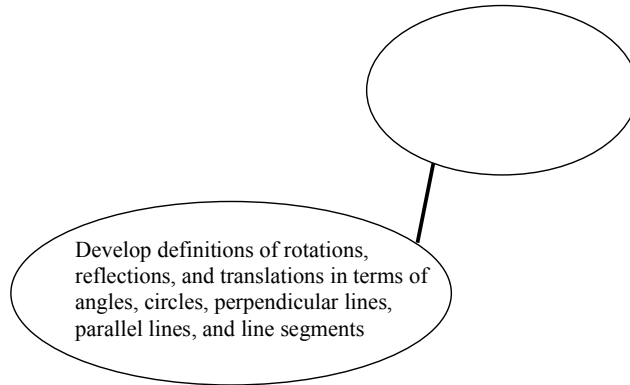Appendix I

Blank Concept Map for Pre and Posttests (Congruence)

Name: _____

Major/Minor: _____

Math Dance:
Teaching Common Core State Standards of Mathematics Through Concert Dance

Congruence

Please construct a concept map with this math standard. You may include definitions, descriptions, characteristics, illustrations, examples, non-examples, any connections you make, and any previous knowledge. I gave you one bubble to get you started, but please form as may connections as you can. You may connect bubbles in anyway you like or add any additional detail as you see fit.
(Source: CCSSM (2010). Common Core State Standards for Mathematics (p. 76). Washington, D.C.: Council of Chief State School Officers)

Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments

Additional comments:
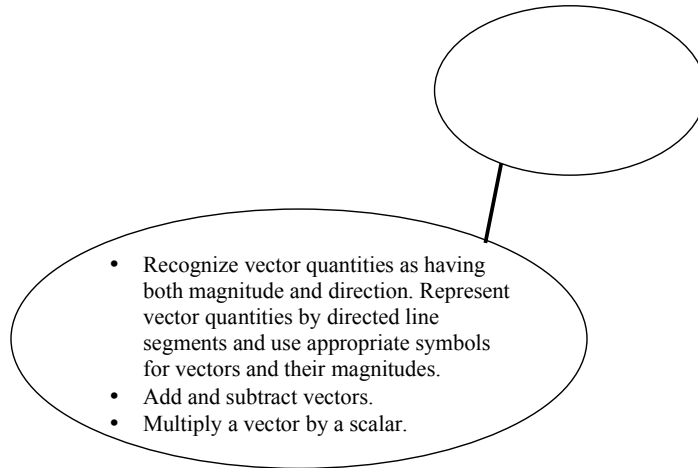
Appendix J

Blank Concept Map for Pre and Posttests (Vectors)

Name: _____

Major/Minor: _____

Math Dance:
Teaching Common Core State Standards of Mathematics Through Concert Dance

Vector and Matrix Quantities

Please construct a concept map with this math standard. You may include definitions, descriptions, characteristics, illustrations, examples, non-examples, any connections you make, and any previous knowledge. I gave you one bubble to get you started, but please form as may connections as you can. You may connect bubbles in anyway you like or add any additional detail as you see fit.
(Source: CCSSM (2010). Common Core State Standards for Mathematics (p. 61). Washington, D.C.: Council of Chief State School Officers)

- Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes.
- Add and subtract vectors.
- Multiply a vector by a scalar.

Additional comments:

Appendix K
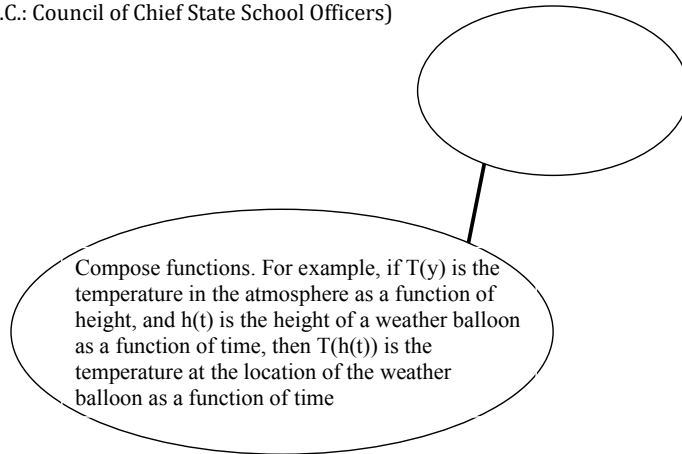
Blank Concept Map for Pre and Posttests (Building Functions)

Name: _____

Major/Minor: _____

Math Dance:
Teaching Common Core State Standards of Mathematics Through Concert Dance

Building Functions

Please construct a concept map with this math standard. You may include definitions, descriptions, characteristics, illustrations, examples, non-examples, any connections you make, and any previous knowledge. I gave you one bubble to get you started, but please form as may connections as you can. You may connect bubbles in anyway you like or add any additional detail as you see fit.
(Source: CCSSM (2010). Common Core State Standards for Mathematics (p. 70). Washington, D.C.: Council of Chief State School Officers)
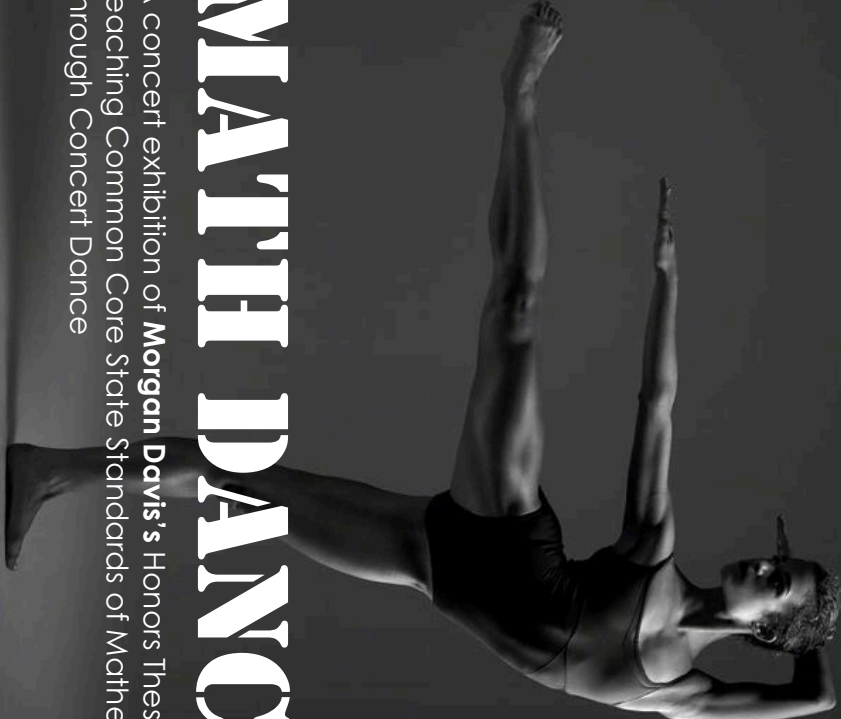
Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time

Additional comments:

Appendix L

Math Dance Concert Flier

(Photo by Boehm, 2014)

Appendix M

Math Dance Concert Program (Modified with Pseudonyms)

(Photo by Boehm, 2014)

Acknowledgements

First, I would like to acknowledge and thank God for the grace and blessings He has bestowed and the strength He has given when I needed it most.

I would also like to thank my family for their continued support and encouragement through everything I've done. Thank you for putting me in dance at a young age and always believing in me. I love you so much!

Special thanks go to the following:
Dr. Sarah Bleier-Baxter for advising me through this project and for her support,
**Jonathan Anderson** for talking through lesson plans with me and helping in any way he could,
Robbie Weaver and **Noa Medford** for filming this concert,
Drew Burlingham for editing music and operating the lights and projection,
Jessie Reeder for operating the sound system,
The MTSU Honors College and Faculty and **The MTSU Dance Program and Faculty** for the opportunity to create this thesis project,
And **the dancers** for their hard work and willingness to think and learn.

MATH DANCE

November 7, 2015 at 7:30 p.m.
MTSU's Murphy Center G040

Special thanks to the MTSU Dance Program and the MTSU Honors College
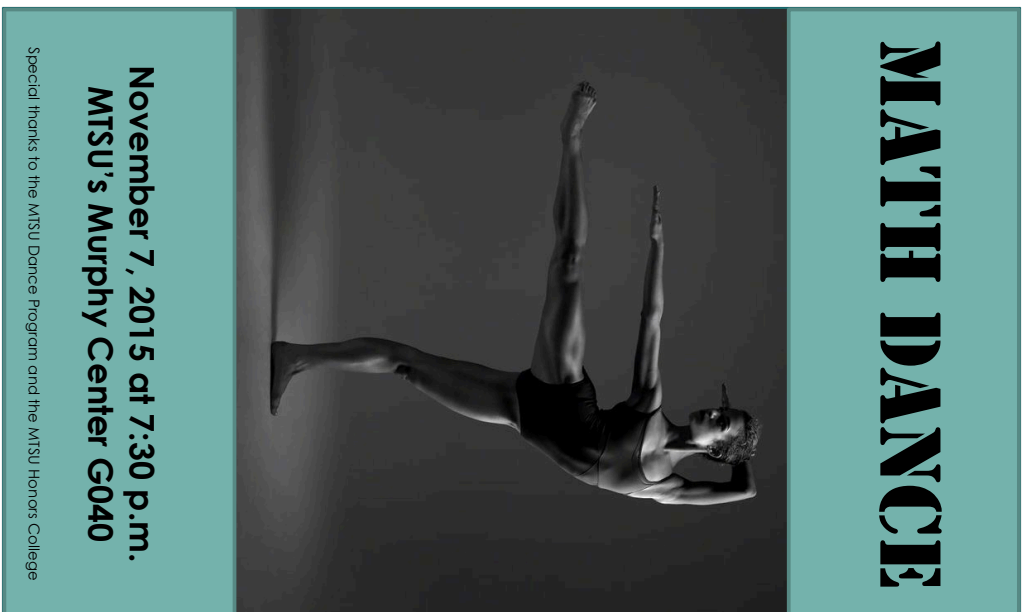
Thesis Summary

Over the past semester, Morgan Davis has been working with nine dancers to teach four high school standards from the Common Core State Standards for Mathematics (CCSSM) through concert modern dance and creative movement. The purpose of her undergraduate Honors thesis has been to answer the following question: *What knowledge of the content within Common Core State Standards for Mathematics do dancers demonstrate before and after engaging in concert modern dance and creative movement used to teach these standards?* Each week since September 14, Morgan and the dancers have worked toward understanding a mathematical standard through movement and discussed what they knew or remembered about their topic or standard. They also used their knowledge of the standards to create the dances in this concert.

About the Choreographer

Morgan Davis is a mathematics major, secondary education minor, and dance minor at Middle Tennessee State University. Morgan has been taking dance classes since she could walk and gained an interest in becoming a high school math teacher when she was in high school. Her first experience in connecting math and dance was in a Choreography I class when she experimented with structuring dance with sequences and series from her Calculus II class. After that, her interest in connecting math and dance grew, and the Honors thesis required by MTSU's Honors College provided a method for Morgan to continue exploring the connections between math and dance.

Math Dance: Teaching Common Core State Standards Through Concert Dance

Sets of Numbers

Number and Quantity – The Complex Number System (N-CN): (1.) Know that there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real (CCSSM, 2010, p. 60).
Dancers: Dana, Ella
Music: "Lakelight" and "Midnight" by Michael Price

Transformations

Geometry – Congruence (G-CO): (4.) Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments (CCSSM, 2010, p. 76).
Dancers: Grace, Hope, Ivy
Music: "I Will See You Again" and "Layover" by Andy McKee

Magnitude and Direction

Number and Quantity – Vector and Matrix Quantities (N-VM): (1.) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes. (4.) Add and subtract vectors. (5.) Multiply a vector by a scalar (CCSSM, 2010, p. 61).
Dancers: Amy, Brook, Cate
Music: "String Quartet No. 2, Op. 17, II. Allegro Molto Capriccioso" by Bela Bartok, performed by Tatrai Quartet

Inputs and Outputs

Functions – Building Functions (F-BF): (1.c.) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time (CCSSM, 2010, p. 70).
Dancers: Ella, Fay, Grace
Music: "Laundry" by edIT

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